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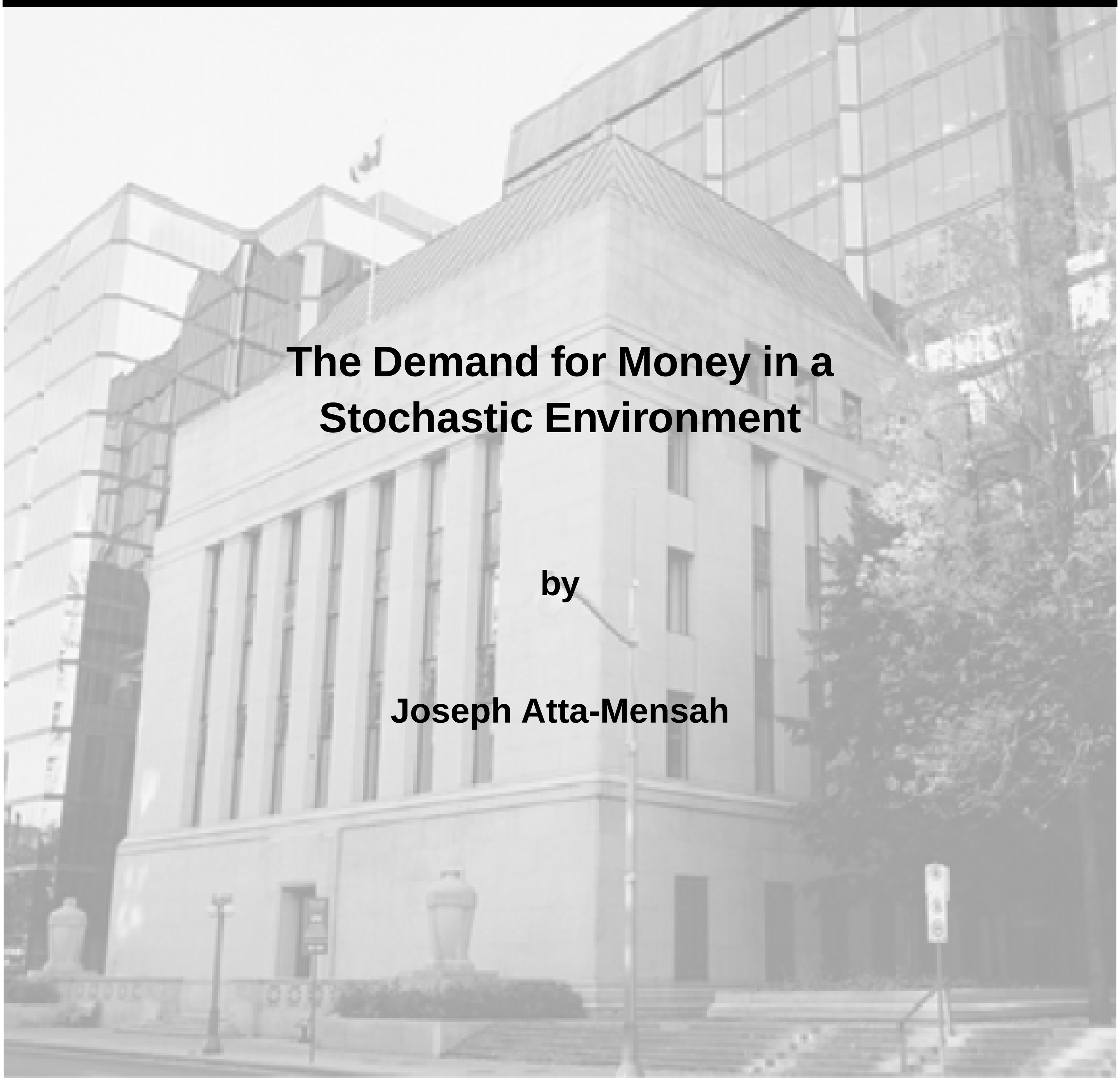

The Demand for Money in a Stochastic Environment

by

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Abstract

The author re-examines the demand-for-money theory in an intertemporal optimization model. The demand for real money balances is derived to be a function of real income and the rates of return of all financial assets traded in the economy. Unlike the traditional money-demand relation, however, where the elasticities are assumed to be constant, the coefficients of the explanatory variables are not constant and depend on the degree of an agent's risk aversion, the volatilities of the price level and income, and the correlation of asset returns. The author shows that the response of households to increased volatilities in the financial markets, economic activity, and prices cannot be predicted, because a rise in general uncertainties has an ambiguous impact on the demand for money. This suggests that increased uncertainty is not very helpful for the planning decisions of households, because the optimal level of money holdings in the period of uncertainty cannot be ascertained.

JEL classification: E41, E50, G11
Bank classification: Monetary aggregates

Résumé

L'auteur réexamine la théorie de la demande de monnaie à l'aide d'un modèle d'optimisation intertemporelle. La demande d'encaisses réelles est définie comme une fonction du revenu réel et des taux de rendement de l'ensemble des actifs financiers échangés au sein de l'économie. Toutefois, contrairement à ce qui est postulé dans la fonction traditionnelle de demande de monnaie, où les élasticités sont supposées fixes, les coefficients des variables explicatives ne sont pas constants et dépendent du degré d'aversion de l'agent pour le risque, de la volatilité du niveau des prix et du revenu et de la corrélation des rendements des actifs. L'auteur montre que la réaction des ménages à une hausse de la volatilité des marchés financiers, de l'activité économique et des prix est imprévisible, car l'accentuation de l'incertitude générale a une incidence ambiguë sur la demande de monnaie. Il semble donc qu'une incertitude accrue est loin de faciliter la planification des ménages étant donné que le niveau optimal des encaisses ne peut alors être établi.

Classification JEL : E41, E50, G11
Classification de la Banque : Agrégats monétaires

1. Introduction

The theory of demand-for-money balances constitutes an important part of monetary economics. Keynes (1936, chapter 13), who introduced the theory into economics, theorizes that economic agents hold money for precautionary, transactions, and speculative purposes. Both the precautionary and transactions demands are formulated as functions of income, whereas the speculative demand for money is influenced by the rate of return on traded securities. Baumol (1952) describes the microeconomic underpinnings of the Keynesian transactions demand for money. Using an inventory-control model, he derives the now-famous "square root rule" for calculating the optimum level of money that must be held by households for transactions purposes. Tobin (1958) describes the microeconomic foundations for the speculative demand for money. Applying the mean-variance analysis of the capital-asset-pricing model (CAPM), he shows that the demand for money depends on the expected return and riskiness of traded assets.[1]

Most of the theoretical derivations of the demand for money in the literature have been carried out in a static partial-equilibrium framework, in which economic agents choose the level of cash holdings that will minimize transactions costs. There are weaknesses to this framework. First, it assumes that the future rate of return of the financial assets is known with certainty. Second, economic agents do not undertake investment and consumption decisions simultaneously. Third, it is very difficult to understand the factors that make the traditional demand functions unstable. Fourth, the model is inadequate to analyze the impact of economic uncertainty on the demand for money. Fifth, the traditional models are static and do not allow for intertemporal substitution of financial assets. Sixth, empirical extensions assume that the parameters of the demand-for-money functions are constant and do not change over time.

This paper re-examines the theory of the demand for money by households, in a framework where an infinitely lived representative household simultaneously chooses an optimum level of consumption bundle and holdings of money, equities, and bonds. The source of income for the agent is the return on their financial assets and wage income. The prices of the consumption bundle, P, the wage income, and the return on the financial assets (equities and bonds) are assumed to change stochastically. The demand functions for money and the two assets are derived. Factors that influence the demand for money are then examined.

1. For other theoretical and empirical work on the demand for money, see Clower (1967), Akerloff and Milbourne (1980), and Frenkel and Jovanovic (1980). Also see Laidler (1993) for a survey on issues related to the demand for money.

Our results clearly show that, besides the traditional variables, the quantity of money held depends on an agent's aversion to risk, the rates of return of all assets in the economy, the riskiness of the assets, and the volatilities of the price level and income. Contrary to the traditional approach, which suggests that the demand function is linear, our framework indicates that the function is non-linear and that the parameters are not constant, which may explain the observed instability of estimated money-demand functions. Furthermore, our analysis demonstrates how changes in an agent's preferences have an impact on the quantity of money holdings, an important result that the traditional framework does not capture.

This paper is organized as follows. In section 2, we present a simple theoretical derivation of a money-demand function. In section 3, we analyze the factors that influence the demand for money. Section 4 offers some conclusions.

2. A Simple Derivation of a Demand-for-Money Function

In this section, we apply the framework of portfolio theory to derive a theoretical expression for the quantity of money that economic agents are willing to hold.[2] In this framework, households are assumed to choose simultaneously the optimum level of consumption bundle, money (currency, or transactions money), equities, and bonds.

2.1 The growth rates of financial assets and the price level

Let M, S, and B, respectively, represent the market value of the portfolio of money, equities, and bonds. The nominal rates of return of the financial assets and the price of the consumption good, P, are assumed to follow a stochastic process of the form:

$$\frac{dM}{M} = \alpha_m dt \, , \tag{1}$$

$$\frac{dS}{S} = \alpha_s dt + \sigma_s dz_s , \tag{2}$$

$$\frac{dB}{B} = \alpha_b dt + \sigma_b dz_b \, , \tag{3}$$

2. See Merton (1971, 1973) and Fischer (1975) on the methodology we follow.

$$\frac{dP}{P} = \alpha_p dt + \sigma_p dz_p, \tag{4}$$

where α_m is the expected instantaneous rate of return on money, α_s is the expected instantaneous rate of return on equities, σ_s is the instantaneous standard deviation of the return on equities, α_b is the expected instantaneous rate of return on bonds, σ_b is the instantaneous standard deviation of the return on bonds, α_p is the expected instantaneous rate of inflation, and σ_p is the instantaneous standard deviation of the inflation rate.[3] Also, dz_s, dz_b, and dz_p are standard Wiener processes with the following properties: $E(dz_s) = 0$; $E(dz_s)^2 = dt$; $E(dz_b) = 0$; $E(dz_b)^2 = dt$; $E(dz_p) = 0$; $E(dz_p)^2 = dt$; $E(dz_s dz_p) = \rho_{sp} dt$; $E(dz_b dz_s) = \rho_{bs} dt$; and $E(dz_b dz_p) = \rho_{bp} dt$; where dt is the change in time, ρ_{sp} is the instantaneous correlation between equity and the inflation rate, and ρ_{bp} is the instantaneous correlation between bonds and the inflation rate.[4]

The nominal rate of return on money, expressed by equation (1), has been modelled to be deterministic to reflect the liquidity and the predictable return of currency or transactions money in general. This implies that the definition of money in this paper excludes mutual funds, which are found in broad monetary aggregates. The return on bonds (equation (3)) is modelled to capture the stochastic behaviour of interest rates. The rate of inflation, in our framework, is also assumed to be stochastic. Equation (4) therefore captures the stochastic behaviour of the price level.

In an inflationary economy, economic agents are more concerned with the real return on an asset than the nominal return. Defining the real values of money, equities, and bonds, respectively, as $m = M/P$, $s = S/P$, and $b = B/P$, it is shown in the appendix that the real returns of the assets in the economy are:

$$\frac{dm}{m} = \beta_m dt - \sigma_p dz_p, \tag{5}$$

$$\frac{ds}{s} = \beta_s dt + \sigma_s dz_s - \sigma_p dz_p, \tag{6}$$

$$\frac{db}{b} = \beta_b dt + \sigma_b dz_b - \sigma_p dz_p, \tag{7}$$

where

3. Note that α_m is the average interest paid on the components of M1.
4. Malliaris and Brock (1982) describe the methodological and mathematical foundations of continuous-time stochastic modelling.

$$\beta_m = \alpha_m - \alpha_p + \sigma_p^2, \tag{8}$$

$$\beta_s = \alpha_s - \alpha_p - \sigma_{sp} + \sigma_p^2, \tag{9}$$

$$\beta_b = \alpha_b - \alpha_p - \sigma_{bp} + \sigma_p^2 . \tag{10}$$

σ_{sp} and σ_{bp}, which are, respectively, the covariances between the nominal rate of return on equities and the inflation rate, and the nominal rate of return on bonds and the inflation rate, are defined as:

$$\sigma_{sp} = \rho_{sp}\sigma_s\sigma_p, \tag{11}$$

$$\sigma_{bp} = \rho_{bp}\sigma_b\sigma_p. \tag{12}$$

Equations (8) to (10) generalize the Fisher equation and therefore give a more accurate estimation of real rates than the traditional estimation. Note that if inflation is deterministic, then the usual Fisher result—that the real return on an asset is equal to the difference between the nominal return and the inflation rate—will hold.

2.2 Budget constraint

The household is assumed to generate wealth from capital gains and wage income. Let ω_1, ω_2, and ω_3 be the proportions of the household's portfolio held in bonds, equities, and money. The budget constraint, as a flow, in real terms could then be expressed as:

$$dW = dY + \omega_1 W\frac{db}{b} + \omega_2 W\frac{ds}{s} + \omega_3 W\frac{dm}{m} - cdt , \tag{13}$$

where W is the instantaneous total wealth of the household, in real terms, c is the rate of consumption per unit time, and Y is real labour income, which is modelled to follow a stochastic process[5]:

$$\frac{dY}{Y} = \beta_y dt + \sigma_y dz_y, \tag{14}$$

5. In equation (13), consumption could be modelled to follow a stochastic process. Such an approach, however, would only complicate the model and not change the final outcome of the results of the paper.

where β_y is the expected instantaneous average real wage rate, and σ_y is the instantaneous standard deviation of the wage rate. Also, dz_y is a standard Wiener process with the following properties: $E(dz_y) = 0$; $E(dz_y)^2 = dt$; $E(dz_y dz_s) = \rho_{ys} dt$; $E(dz_y dz_p) = \rho_{yp} dt$; and $E(dz_y dz_b) = \rho_{yb} dt$. Moreover, dt is the change in time; ρ_{ys} is the instantaneous correlation between the wage rate and equity; ρ_{yp} is the instantaneous correlation between the wage rate and the inflation rate; and ρ_{yb} is the instantaneous correlation between the wage rate and the bond rate.

Also, the following condition must be met:

$$\omega_1 + \omega_2 + \omega_3 = 1. \tag{15}$$

Substituting equations (5), (6), (7), and (14) into equation (13), and using equation (15) to express $\omega_3 = 1 - \omega_1 - \omega_2$, the agent's intertemporal budget constraint takes the form:

$$\begin{aligned} dW &= \omega_1 W(\beta_b - \beta_m)dt + \omega_2 W(\beta_s - \beta_m)dt + (\beta_m W - c)dt \\ &+ \beta_y Y dt + \omega_1 W \sigma_b dz_b + \omega_2 W \sigma_s dz_s - W \sigma_p dz_p + Y \sigma_y dz_y . \end{aligned} \tag{16}$$

2.3 Household maximization problem

The representative agent is faced with the problem of choosing a portfolio of assets and a consumption rule that will maximize the expected value of a von Neumann-Morgenstern utility function. Thus, the agent's optimization problem can be summarized as:

$$\underset{c,\, \omega_1,\, \omega_2}{Max} E_0\left[\int_0^\infty e^{-\rho t} U(c(t), t) dt\right], \tag{17}$$

subject to equation (16), and

$$W(0) = W_0. \tag{18}$$

Also, the utility function $U(\cdot,\cdot)$ is restricted to be concave in c (i.e., $U_c > 0$ and $U_{cc} < 0$). E_0 is the conditional expectations operator conditional on $W(0) = W_0$ being known. A value function, J, is then defined as:

$$J(W, Y, t) \equiv \underset{c,\, \omega_1,\, \omega_2}{Max} E_0\left[\int_0^\infty e^{-\rho t} U(c(t), t)dt\right]. \tag{19}$$

Equation (19) is also constrained by equations (16) and (18). As shown in the appendix, the optimization problem facing the agent could be reduced to:

$$\underset{c,\, \omega_1,\, \omega_2}{Max} \Phi(c, \omega_1, \omega_2, W, Y, t) = e^{-\rho t} U(c(t), t) + L(J) , \tag{20}$$

where L, which is known as the Dynkin operator over W and Y, is defined in the appendix. The first-order conditions for the maximization problem are:

$$\Phi_c = e^{-\rho t} U_c - J_w = 0 , \tag{21}$$

$$\begin{aligned} \Phi_{\omega_1} &= WJ_w(\beta_b - \beta_m) + WY\sigma_{by}J_{wy} \\ &+ J_{ww}[W^2(\omega_1\sigma_b^2 + \omega_2\sigma_{bs} - \sigma_{bp}) + WY\sigma_{by}] = 0 , \end{aligned} \tag{22}$$

$$\begin{aligned} \Phi_{\omega_2} &= WJ_w(\beta_s - \beta_m) + WY\sigma_{sy}J_{wy} \\ &+ J_{ww}[W(\omega_2\sigma_s^2 + \omega_1\sigma_{bs} - \sigma_{sp}) + WY\sigma_{sy}] = 0 . \end{aligned} \tag{23}$$

Equation (21) restates the condition that, in equilibrium, the marginal utility of consumption can be equated to the marginal utility of wealth. Equations (22) and (23) are similar to the standard equations for deriving a generalized capital-asset-pricing model.

2.4 The demand for money

Given that ω_3 represents the proportion of real wealth held as money, the aggregate money held by the agent is $\omega_3 W$, which is equated to a familiar notation, M/P (m). Based on equation (A27), in the appendix, the relation for the demand for money can be expressed as:

$$m = A_o + A_1\beta_m + A_2\beta_s + A_3\beta_b + A_4 Y , \tag{24}$$

where

$$A_o = \frac{W[\sigma_s\sigma_y(1-\rho_{sb}^2) - \sigma_p\sigma_s(\rho_{bp} - \rho_{sb}\rho_{ps}) - \sigma_p\sigma_b(\rho_{ps} - \rho_{sb}\rho_{bp})]}{W\sigma_b\sigma_s(1-\rho_{sb}^2)} \tag{25}$$
$$- \frac{Y\sigma_y[\sigma_s(\rho_{sb}\rho_{sy} - \rho_{by}) + \sigma_b(\rho_{sb}\rho_{by} - \rho_{sy})]}{W\sigma_b\sigma_s(1-\rho_{sb}^2)},$$

$$A_1 = -\frac{(J_w/J_{ww})(\sigma_s^2 - 2\rho_{sb}\sigma_b\sigma_s + \sigma_b^2)}{\sigma_s^2\sigma_b^2(1-\rho_{sb}^2)}, \tag{26}$$

$$A_2 = \frac{(J_w/J_{ww})(\sigma_b^2 - \rho_{sb}\sigma_b\sigma_s)}{\sigma_s^2\sigma_b^2(1-\rho_{sb}^2)}, \tag{27}$$

$$A_3 = \frac{(J_w/J_{ww})(\sigma_s - \rho_{sb}\sigma_b)}{\sigma_b^2\sigma_s(1-\rho_{sb}^2)}, \tag{28}$$

$$A_4 = \frac{(J_{wy}/J_{ww})\sigma_y(\rho_{by}(\sigma_b\rho_{sb} - \sigma_s) - \rho_{sy}(\sigma_s\rho_{sb} - \sigma_b))}{\sigma_b\sigma_s(1-\rho_{sb}^2)}. \tag{29}$$

Before we examine the demand function for real money balances (equation (24)), it is important to note that ($-J_w/J_{ww}$) is the inverse of the household's degree of risk aversion. The degree of risk aversion is positive because of the concavity of the indirect utility function, which makes $J_{ww} < 0$.

We then interpret (J_{wy}/J_{ww}). From the first-order condition for consumption (equation (21)), we have:

$$e^{-\rho t}U_{cc}\frac{dc}{dy} = J_{wy}, \tag{30}$$

and

$$e^{-\rho t}U_{cc}\frac{dc}{dw} = J_{ww}. \tag{31}$$

Expressing equation (30) as a ratio of equation (31), we have:

$$\frac{J_{wy}}{J_{ww}} = \frac{(dc/dy)}{(dc/dw)}. \tag{32}$$

Equation (32) suggests that (J_{wy}/J_{ww}) is the ratio of the marginal propensity to consume out of income to the marginal propensity to consume out of wealth. The fact that these propensities are positive implies that (J_{wy}/J_{ww}) is also positive.

Equation (24) appears to be similar in spirit to the traditional demand for money. It also corroborates Friedman's (1956) view that the demand for money is a function of the rates of return of all financial assets traded in the economy. Unlike the traditional money-demand relation, however, where the elasticities are assumed to be constant, the coefficients of the explanatory variables are not constant and depend on the degree of an agent's risk aversion, the volatilities of the price level and income, and the correlation of asset returns. The functional form of the demand function implies that taste, risk appetite, and macroeconomic uncertainty determine the quantity of money holdings by households. Contrary to empirical results in the literature, the derived demand function clearly shows that the elasticities of money demand are not constant. This may explain why the money-demand functions have been observed empirically to be unstable. The properties of the money-demand function are examined in section 3.

3. Factors That Influence the Demand for Money

In section 2, we derived an expression for the demand for money. Although we have presented the demand for real money balances (equation (24)) in a linear form, we notice that it is a non-linear function of the rates of return and volatilities of the assets in the economy, the inverse of the degree of risk aversion and income. In this section, we examine the properties of this non-linear function.

Proposition 1: A rise in money's own rate of return leads to an increase in the real money holdings.

Proof: Differentiating the demand function (equation (24)) with respect to β_m yields:

$$\frac{\partial m}{\partial \beta_m} = -\frac{(J_w/J_{ww})(\sigma_s^2 - 2\rho_{sb}\sigma_b\sigma_s + \sigma_b^2)}{\sigma_s^2\sigma_b^2(1-\rho_{sb}^2)} > 0\,, \tag{33}$$

since $\rho_{sb} < 0$, because equity returns and bond yields are negatively related. Also, $\rho_{sb}^2 < 1$ and ($-J_w/J_{ww}$) > 0.

Remarks: Proposition 1 does not need any further elaboration, because it is very intuitive. It suggests that, all things being equal, economic agents' holdings of money rise with the rise in money's own rate of return.

Proposition 2: Money and equities are substitutes.

Proof: Differentiate the demand for money with respect to β_s:

$$\frac{\partial m}{\partial \beta_s} = \frac{(J_w / J_{ww})(\sigma_b^2 - \rho_{sb}\sigma_b\sigma_s)}{\sigma_s^2\sigma_b^2(1-\rho_{sb}^2)} < 0\,, \tag{34}$$

since $\rho_{sb} < 0$, $\rho_{sb}^2 < 1$, and $(J_w/J_{ww}) < 0$.

Remarks: The results confirm the traditional view that money and equities are substitutes, which implies that, as equity returns rise, economic agents hold more equity and less money.

Proposition 3: A rise in bond yields has a negative impact on the demand for money.

Proof: Differentiate the demand function with respect to β_b:

$$\frac{\partial m}{\partial \beta_b} = \frac{(J_w / J_{ww})(\sigma_s - \rho_{sb}\sigma_b)}{\sigma_b^2\sigma_s(1-\rho_{sb}^2)} < 0\,, \tag{35}$$

since $\rho_{sb} < 0$, $\rho_{sb}^2 < 1$, and $(J_w/J_{ww}) < 0$.

Remarks: The results demonstrate that bonds and money substitute. They also confirm empirical findings in the literature that the interest elasticity of money demand is negative.

Proposition 4: The demand for money rises with real income.

Proof: Differentiate with respect to real income:

$$\frac{\partial m}{\partial y} = \frac{(J_{wy} / J_{ww})\sigma_y(\rho_{by}(\sigma_b\rho_{sb} - \sigma_s) - \rho_{sy}(\sigma_s\rho_{sb} - \sigma_b))}{\sigma_b\sigma_s(1-\rho_{sb}^2)} > 0\,, \tag{36}$$

since $\rho_{by} < 0$, and $\rho_{sy} > 0$, because equity returns and economic growth are positively correlated: $\rho_{sb} < 0$, $\rho_{sb}^2 < 1$, and $(J_{wy}/J_{ww}) > 0$.

Remarks: The results confirm our intuition and validate empirical findings that the income elasticity of the money demand is positive.

Proposition 5: Changes in the volatility of the rate of return of equities has an indeterminate impact on the demand for money.

Proof: Differentiate with respect to σ_s:

$$\frac{\partial m}{\partial \sigma_s} = \frac{\partial A_o}{\partial \sigma_s} + \frac{\partial A_1}{\partial \sigma_s}\beta_m + \frac{\partial A_2}{\partial \sigma_s}\beta_s + A_2\frac{\partial \beta_s}{\partial \sigma_s} + \frac{\partial A_3}{\partial \sigma_s}\beta_b + \frac{\partial A_4}{\partial \sigma_s}Y, \tag{37}$$

but

$$\frac{\partial A_o}{\partial \sigma_s} = \frac{W\sigma_p\sigma_b(\rho_{ps} - \rho_{sb}\rho_{bp}) + Y\sigma_b\sigma_y(\rho_{sb}\rho_{by} - \rho_{sy})}{W\sigma_b\sigma_s^2(1 - \rho_{sb}^2)}, \tag{38}$$

which is clearly indeterminate;

$$\frac{\partial A_1}{\partial \sigma_s} = \frac{-(J_w/J_{ww})(2\rho_{sb}\sigma_b\sigma_s - \sigma_b^2)}{\sigma_s^3\sigma_b^2(1 - \rho_{sb}^2)} < 0, \tag{39}$$

since $\rho_{sb} < 0$, $\rho_{sb}^2 < 1$, and $(J_w/J_{ww}) < 0$;

$$\frac{\partial A_2}{\partial \sigma_s} = \frac{(J_w/J_{ww})(\rho_{sb}\sigma_b\sigma_s - 2\sigma_b^2)}{\sigma_s^3\sigma_b^2(1 - \rho_{sb}^2)} > 0, \tag{40}$$

since $\rho_{sb} < 0$, $\rho_{sb}^2 < 1$, and $(J_w/J_{ww}) < 0$;

$$A_2\frac{\partial \beta_s}{\partial \sigma_s} = -\rho_{sp}\sigma_p\left[\frac{(J_w/J_{ww})(\sigma_b^2 - \rho_{sb}\sigma_b\sigma_s)}{\sigma_s^2\sigma_b^2(1 - \rho_{sb}^2)}\right] < 0, \tag{41}$$

since $\rho_{sp} < 0$, $\rho_{sb} < 0$, $\rho_{sb}^2 < 1$, and $(J_w/J_{ww}) < 0$;

$$\frac{\partial A_3}{\partial \sigma_s} = \frac{(J_w/J_{ww})\rho_{sb}\sigma_b}{\sigma_s^2\sigma_b^2(1 - \rho_{sb}^2)} > 0, \tag{42}$$

since $\rho_{sb} < 0$, $\rho_{sb}^2 < 1$, and $(J_w/J_{ww}) < 0$;

$$\frac{\partial A_4}{\partial \sigma_s} = \frac{(J_{wy}/J_{ww})\sigma_y(\rho_{by}\sigma_b\rho_{sb} + \rho_{sy}\sigma_b)}{\sigma_b\sigma_s(1 - \rho_{sb}^2)} < 0, \tag{43}$$

since $\rho_{sb} < 0$, $\rho_{by} < 0$, $\rho_{sy} > 0$, $\rho_{sb}^2 < 1$, and $(J_{wy}/J_{ww}) > 0$.

Remarks: Equations (38) to (43) indicate that the sum effect of changes in the volatility of equities on the demand for money is very ambiguous. However, the impact on the coefficients of money demand is very interesting. Intuitively, one would expect that, in times of stock market volatility, money would be households' preferred store of value, because equities would be unattractive. This behaviour of households was observed in 2001 and 2002, when double-digit growth in the monetary aggregates coincided with heightened uncertainty in North American stock markets. Equations (38) and (43), however, show that the coefficients on the own-rate of interest and income fall with a rise in equity volatility, pulling down the demand for money. This result is a departure from the traditional view, in which the elasticities of the demand for money are held constant. We find that uncertainty in financial markets causes the parameters of the demand-for-money function to move around, making it difficult to predict the full impact of household holdings of money.

Proposition 6: A rise in the volatility of interest rates has an ambiguous impact on the demand for money.

Proof: Differentiate with respect to σ_b:

$$\frac{\partial m}{\partial \sigma_b} = \frac{\partial A_o}{\partial \sigma_b} + \frac{\partial A_1}{\partial \sigma_b}\beta_m + \frac{\partial A_2}{\partial \sigma_b}\beta_s + \frac{\partial A_3}{\partial \sigma_b}\beta_b + A_3\frac{\partial \beta_b}{\partial \sigma_b} + \frac{\partial A_4}{\partial \sigma_b}Y, \tag{44}$$

but

$$\frac{\partial A_o}{\partial \sigma_b} = \frac{W[\sigma_p\sigma_s(\rho_{bp} - \rho_{sb}\rho_{bp}) - \sigma_s\sigma_y(1-\rho_{sb}^2)] + Y\sigma_s\sigma_y(\rho_{sb}\rho_{sy} - \rho_{by})}{W\sigma_b^2\sigma_s(1-\rho_{sb}^2)}, \tag{45}$$

which is indeterminate;

$$\frac{\partial A_1}{\partial \sigma_b} = \frac{-(J_w/J_{ww})(2\rho_{sb}\sigma_b\sigma_s - \sigma_s^2)}{\sigma_s^2\sigma_b^3(1-\rho_{sb}^2)} < 0, \tag{46}$$

since $\rho_{sb} < 0$, $\rho_{sb}^2 < 1$, and $(J_w/J_{ww}) < 0$;

$$\frac{\partial A_2}{\partial \sigma_b} = \frac{2(J_w/J_{ww})\rho_{sb}\sigma_b\sigma_s}{\sigma_s^2\sigma_b^3(1-\rho_{sb}^2)} > 0, \tag{47}$$

since $\rho_{sb} < 0$, $\rho_{sb}^2 < 1$, and $(J_w/J_{ww}) < 0$;

$$\frac{\partial A_3}{\partial \sigma_b} = \frac{(J_w / J_{ww})(\rho_{sb}\sigma_b - 2\sigma_s)}{\sigma_s \sigma_b^3 (1 - \rho_{sb}^2)} > 0\,, \tag{48}$$

since $\rho_{sb} < 0$, $\rho_{sb}^2 < 1$, and $(J_w/J_{ww}) < 0$;

$$A_3 \frac{\partial \beta_b}{\partial \sigma_b} = -\rho_{bp}\sigma_p \left[\frac{(J_w / J_{ww})(\sigma_s - \rho_{sb}\sigma_b)}{\sigma_b^2 \sigma_s (1 - \rho_{sb}^2)}\right] < 0\,, \tag{49}$$

since $\rho_{bp} < 0$, $\rho_{sb} < 0$, $\rho_{sb}^2 < 1$, and $(J_w/J_{ww}) < 0$;

$$\frac{\partial A_4}{\partial \sigma_b} = \frac{(J_{wy} / J_{ww})[\sigma_s \sigma_y \rho_{by} + \sigma_b \rho_{sy} \rho_{sb}]}{\sigma_b^2 \sigma_s (1 - \rho_{sb}^2)} < 0\,, \tag{50}$$

since $\rho_{sb} < 0$, $\rho_{by} < 0$, $\rho_{sy} > 0$, $\rho_{sb}^2 < 1$, and $(J_{wy}/J_{ww}) > 0$.

Remarks: The sign of equation (44) is clearly indeterminate. Equations (45) to (50) demonstrate that changes in interest rate uncertainty cause the underlying parameters of the money demand to move in different directions, making it difficult to predict the full impact of the shock on the total quantity of money demanded by households. The results show that, when interest rates are volatile, the coefficients attached to the rates of return of alternative financial assets in the economy increase, and so push up the demand for money. On the other hand, the impact on the coefficients of the own-rate of return and income is negative, which suggests that households hold less money. The total impact depends on the net effect of the response of the changes in the parameters of the money-demand function.

Corollary: The results of propositions 5 and 6 suggest that the impact of the volatilities of monetary policy and financial markets on the demand for money produces both substitution and income effects. The substitution effect arises because, in times of uncertainty in financial markets, households prefer riskless assets, such as money, to their riskier counterparts. Economic agents demonstrate this substitution effect by raising the coefficients attached to the returns on the riskier assets. The income effect arises because, in times of financial uncertainty, agents could respond by moving away from nominal assets into real assets. As the results show, the income effect is registered through the negative relationship between the coefficient on income and the volatilities of the interest rate and the return on equity. The full impact of these uncertainties on the demand for money depends on the magnitude of the substitution and income effects.

Proposition 7: An increase in the volatility of income has an ambiguous impact on the demand for money.

Proof: Differentiate with respect to σ_y:

$$\frac{\partial m}{\partial \sigma_y} = \frac{\partial A_o}{\partial \sigma_y} + \frac{\partial A_4}{\partial \sigma_y} Y, \tag{51}$$

but

$$\frac{\partial A_o}{\partial \sigma_y} = \frac{W[\sigma_s(1-\rho_{sb}^2)] - Y[\sigma_s(\rho_{sb}\rho_{sy} - \rho_{by}) + \sigma_b(\rho_{sb}\rho_{by} - \rho_{sy})]}{W\sigma_b\sigma_s(1-\rho_{sb}^2)}, \tag{52}$$

$$\frac{\partial A_4}{\partial \sigma_y} = \frac{(J_{wy}/J_{ww})[\rho_{by}(\sigma_b\rho_{sb} - \sigma_s) - \rho_{sy}(\sigma_s\rho_{sb} - \sigma_b)]}{W\sigma_b\sigma_s(1-\rho_{sb}^2)} > 0, \tag{53}$$

since $\rho_{sb} < 0$, $\rho_{by} < 0$, $\rho_{sy} > 0$, $\rho_{sb}^2 < 1$, and $(J_{wy}/J_{ww}) > 0$.

Remarks: The results demonstrate that, in times of heightened economic uncertainty, households may or may not increase the quantity of their money holdings. An intuitive explanation for this result is that, in an uncertain economic environment, households, as a precaution, may hold excess money balances to meet unforeseen expenditures. On the other hand, economic agents may decide to hold less money and more real and financial assets. Hence, the total impact on the demand for money depends on which effect dominates.

Proposition 8: A rise in the volatility of the price level has an ambiguous impact on the demand for money.

Proof: Differentiate with respect to σ_p:

$$\frac{\partial m}{\partial \sigma_p} = \frac{\partial A_o}{\partial \sigma_p} + A_1\frac{\partial \beta_m}{\partial \sigma_p} + A_2\frac{\partial \beta_s}{\partial \sigma_p} + A_3\frac{\partial \beta_b}{\partial \sigma_p}, \tag{54}$$

but

$$\frac{\partial A_o}{\partial \sigma_p} = \frac{W[\sigma_s(\rho_{sb}\rho_{ps} - \rho_{bp}) - \sigma_b(\rho_{ps} - \rho_{sb}\rho_{bp})]}{W\sigma_b\sigma_s(1-\rho_{sb}^2)} > 0, \tag{55}$$

since $\rho_{sb} < 0$, $\rho_{bp} < 0$, $\rho_{ps} < 0$, and $\rho_{sb}^2 < 1$;

$$A_1 \frac{\partial \beta_m}{\partial \sigma_p} = 2\sigma_p \left[-\frac{(J_w / J_{ww})(\sigma_s^2 - 2\rho_{sb}\sigma_b\sigma_s + \sigma_b^2)}{\sigma_s^2 \sigma_b^2 (1 - \rho_{sb}^2)} \right] > 0\,, \tag{56}$$

since $\rho_{sb} < 0$, $\rho_{sb}^2 < 1$, and $(-J_w/J_{ww}) > 0$;

$$A_2 \frac{\partial \beta_s}{\partial \sigma_p} = (2\sigma_p - \rho_{sp}\sigma_p) \left[\frac{(J_w / J_{ww})(\sigma_b^2 - \rho_{sb}\sigma_b\sigma_s)}{\sigma_s^2 \sigma_b^2 (1 - \rho_{sb}^2)} \right] < 0\,, \tag{57}$$

since $\rho_{sp} < 0$, $\rho_{sb} < 0$, $\rho_{sb}^2 < 1$, and $(J_w/J_{ww}) < 0$;

$$A_3 \frac{\partial \beta_b}{\partial \sigma_p} = (2\sigma_p - \rho_{bp}\sigma_p) \left[\frac{(J_w / J_{ww})(\sigma_s - \rho_{sb}\sigma_b)}{\sigma_b^2 \sigma_s (1 - \rho_{sb}^2)} \right] < 0\,, \tag{58}$$

since $\rho_{bp} < 0$, $\rho_{sb} < 0$, $\rho_{sb}^2 < 1$, and $(J_w/J_{ww}) < 0$.

Remarks: Clearly, the sign of equation (54) is ambiguous. A plausible explanation for this result is that, in a volatile inflation environment, economic agents substitute out of nominal assets for real assets, causing the demand for money to fall. On the other hand, uncertain movements in the price level could increase the money held by agents for precautionary reasons to meet unplanned expenditures. Hence, the impact of the volatility of prices on the demand for money depends on which response is dominant.

4. Conclusion

This paper has re-examined the demand-for-money theory, because we believe that the traditional specification of money-demand functions as relationships between real money balances, a scale variable, and an opportunity cost of holding real money is very restrictive. We have argued that one of the weaknesses of the traditional demand function is the assumption that the coefficients of the explanatory variables are constant and not adequate to analyze the effects of macroeconomic uncertainty on household money holdings. Furthermore, if economic agents decide to hold money to find the proper mix for their investment portfolio, then the optimal level of money they hold will be influenced by both the level and the volatilities (variances) of the scale variable and the opportunity costs. Moreover, rational economic agents are generally risk-averse and require compensation for any additional risk they take. This suggests that the return on, and volatility of, financial assets play an important role in the quantity of money demanded by risk-averse economic agents.

Using portfolio theory, we have demonstrated theoretically that the demand for real money balances should be a function of real income and the rates of return of all financial assets traded in the economy. Unlike the traditional money-demand relation, however, where the elasticities are assumed to be constant, the coefficients of the explanatory variables are not constant and depend on the degree of an agent's risk aversion, the volatilities of the price level and income, and the correlation of asset returns. The nature of the underlying parameters may explain why the traditional demand function has been observed empirically to be unstable. Further results in the paper have shown that the response of households to heightened volatilities in the financial markets, economic activity, and prices cannot be predicted, because a rise in general uncertainties has an ambiguous impact on money demand. This suggests that increased uncertainty is not very helpful for the planning decisions of households, because the optimal level of money holdings in the period of uncertainty cannot be ascertained.

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Appendix

A1 Expressing returns in real terms

In an inflationary economy, economic agents are more concerned with the real return on an asset than the nominal return. Hence, we apply Itô's lemma to find the expressions for the real return of the assets in the economy. Define the real value of bonds as:

$$b = \frac{B}{P}, \tag{A1}$$

where B is the nominal value of the bonds and P is the price index. Since we have a one-good economy, however, the price index is the same as the price of the consumption good. Applying Itô's lemma, we get the following:

$$db = \frac{\partial b}{\partial t}dt + \frac{\partial b}{\partial B}dB + \frac{\partial b}{\partial P}dP + 0.5\frac{\partial^2 b}{\partial B^2}(dB)^2 + 0.5\frac{\partial^2 d}{\partial P^2}(dP)^2 + \frac{\partial^2 b}{\partial B \partial P}dBdP\ . \tag{A2}$$

Taking the appropriate partial differentials of b and substituting equations (1) and (4) from the text, equation (A2) becomes:

$$db = \frac{1}{P}[\alpha_b B dt + \sigma_b B dz_b] - \frac{B}{P^2}[\alpha_p P dt + \sigma_p P dz_p] + \frac{B}{P^3}[\sigma_p^2 P^2 dt] - \frac{1}{P^2}[\rho_{bp}\sigma_b\sigma_p PB]dt\ . \tag{A3}$$

Separating out the drift and the diffusion terms, equation (A3) becomes:

$$\frac{db}{b} = \beta_b dt + \sigma_b dz_b - \sigma_p dz_p, \tag{A4}$$

with

$$\beta_b = \alpha_b - \alpha_p - \sigma_{bp} + \sigma_p^2, \tag{A5}$$

and σ_{bp}, which is the covariance between the nominal rate of return on money and the inflation rate, is defined as:

$$\sigma_{bp} = \rho_{bp}\sigma_b\sigma_p. \tag{A6}$$

In a similar manner, we define the real values of the portfolios for equities and bonds as:

$$s = \frac{S}{P}, \tag{A7}$$

$$m = \frac{M}{P}. \tag{A8}$$

The application of Itô's lemma yields the expressions for real returns for the portfolios as:

$$\frac{ds}{s} = \beta_s dt + \sigma_s dz_s - \sigma_p dz_p, \tag{A9}$$

$$\frac{dm}{m} = \beta_m dt - \sigma_p dz_p, \tag{A10}$$

where

$$\beta_s = \alpha_s - \alpha_p - \sigma_{sp} + \sigma_p^2, \tag{A11}$$

$$\beta_m = \alpha_m - \alpha_p + \sigma_p^2, \tag{A12}$$

and σ_{sp}, which is the covariance between the nominal rate of return on equity and the inflation rate, is defined as:

$$\sigma_{sp} = \rho_{sp}\sigma_s\sigma_p. \tag{A13}$$

A2 The Dynkin operator

A representative household's optimization problem can be summarized as:

$$\underset{c,\,\omega_1,\,\omega_2}{Max} E_0\left[\int_0^{\infty} e^{-\rho t} U(c(t), t) dt\right], \tag{A14}$$

subject to the budget constraint defined in the text (equation (16)) and

$$W(0) = W_0. \tag{A15}$$

Also, the utility function, $U(\cdot)$, is restricted to be concave in c (i.e., $U_c > 0$ and $U_{cc} < 0$). E_0 is the conditional expectations operator conditional on $W(0) = W_0$ being known. Let $t = t_o + \Delta t$ and assume that the third partial derivatives of $J(\cdot)$ are bounded. Then, by applying Taylor's series theorem, the mean value theorem for integrals, and taking the limits as $\Delta t \to 0$, define a value function, J, as:

$$J(W(t_0), Y, t_0) \equiv \underset{c,\, \omega_1,\, \omega_2}{Max} [e^{-\rho t} U(c(t), t) + E(J(W(t_0), Y, t_0)) \tag{A16}$$

$$+ J_t dt + J_w E(dW) + J_y E(dY) + J_{wy} E(dWdY) + 1/2 J_{ww} E(dW)^2$$

$$+ 1/2 J_{yy} E(dY)^2] \ .$$

From the real income relation (equation (14)) and the budget constraint (equation (16)), we have:

$$E(dW) \ = \ \omega_1 W(\beta_b - \beta_m) + \omega_2 W(\beta_s - \beta_m) + (\beta_m W - c) + \beta_y Y\,, \tag{A17}$$

$$E(dW)^2 \ = \ [W^2(\omega_1^2 \sigma_b^2 + \omega_2^2 \sigma_s^2 + 2\omega_1\omega_2\sigma_{bs} - 2\omega_1\sigma_{bp} - 2\omega_2\sigma_{sp} + \sigma_p^2) \tag{A18}$$
$$+ WY(2\omega_1\sigma_{by} + 2\omega_2\sigma_{sy} - 2\sigma_{py} + Y\sigma_y^2)]\,,$$

$$E(dY) \ = \ \beta_y Y\,, \tag{A19}$$

$$E(dY)^2 \ = \ \sigma_y^2 Y^2\,, \tag{A20}$$

$$E(dYdW) \ = \ WY(\omega_1\sigma_{by} + \omega_2\sigma_{sy} - \sigma_{yp} + Y\sigma_y^2)\,. \tag{A21}$$

Also,

$$E(J(W(t_0), Y, t_0)) \equiv J(W(t_0), Y, t_0) \ . \tag{A22}$$

Substituting equations (A17) to (A22) into equation (A16), we obtain the continuous time version of the Bellman-Dreyfus fundamental optimality equation of the form:

$$\underset{c,\, \omega_1,\, \omega_2}{Max} \Phi(c, \omega_1, \omega_2, W, Y, t) \ = \ e^{-\rho t} U(c(t), t) + L(J)\,, \tag{A23}$$

where L, which is known as the Dynkin operator over W and Y, is defined as:

$$L(J) \ = \ J_t + J_w[\omega_1 W(\beta_b - \beta_m) + \omega_2 W(\beta_s - \beta_m) + (\beta_m W - c) + \beta_y Y] \tag{A24}$$
$$+ J_y \beta_y Y + J_{wy}[WY(\omega_1\sigma_{by} + \omega_2\sigma_{sy} - \sigma_{yp} + Y\sigma_y^2)]$$
$$+ 0.5 J_{ww}[W^2(\omega_1^2\sigma_b^2 + \omega_2^2\sigma_s^2 + 2\omega_1\omega_2\sigma_{bs} - 2\omega_1\sigma_{bp} - 2\omega_2\sigma_{sp} + \sigma_p^2)$$
$$+ WY(2\omega_1\sigma_{by} + 2\omega_2\sigma_{sy} - 2\sigma_{py} + Y\sigma_y^2)]$$
$$+ 0.5 J_{yy}\sigma_y^2 Y^2 \ .$$

Constrained by the budget equation, equation (A23) is the household optimization problem.

A3 Demand for financial assets

The demand for the three financial assets of the economy are derived by solving first-order conditions summarized by equations (22) and (23) for ω_1, ω_2, and ω_3. The expressions for the functions are:

$$\begin{aligned}\omega_1 = &-\frac{J_w}{WJ_{ww}}\left[\frac{(\beta_b-\beta_m)}{\sigma_b^2(1-\rho_{sb}^2)}-\frac{\rho_{sb}(\beta_s-\beta_m)}{\sigma_b\sigma_s(1-\rho_{sb}^2)}\right]\\ &+\frac{YJ_{wy}}{WJ_{ww}}\left(\frac{\sigma_y(\rho_{sb}\rho_{sy}-\rho_{by})}{\sigma_b(1-\rho_{sb}^2)}\right)\\ &+\frac{Y\sigma_y(\rho_{sb}\rho_{sy}-\rho_{by})+W\sigma_p(\rho_{bp}-\rho_{sb}\rho_{ps})}{W\sigma_b(1-\rho_{sb}^2)},\end{aligned} \tag{A25}$$

$$\begin{aligned}\omega_2 = &-\frac{J_w}{WJ_{ww}}\left[\frac{(\beta_s-\beta_m)}{\sigma_s^2(1-\rho_{sb}^2)}-\frac{\rho_{sb}(\beta_b-\beta_m)}{\sigma_b\sigma_s(1-\rho_{sb}^2)}\right]\\ &+\frac{YJ_{wy}}{WJ_{ww}}\left(\frac{\sigma_y(\rho_{sb}\rho_{by}-\rho_{sy})}{\sigma_s(1-\rho_{sb}^2)}\right)\\ &+\frac{Y\sigma_y(\rho_{sb}\rho_{by}-\rho_{sy})+W\sigma_p(\rho_{ps}-\rho_{sb}\rho_{bp})}{W\sigma_s(1-\rho_{sb}^2)},\end{aligned} \tag{A26}$$

$$\begin{aligned}\omega_3 = &-\frac{J_w}{WJ_{ww}}\left[\frac{(\sigma_b\rho_{sb}-\sigma_s)(\beta_b-\beta_m)}{\sigma_s\sigma_b^2(1-\rho_{sb}^2)}-\frac{(\sigma_s\rho_{sb}-\sigma_b)(\beta_s-\beta_m)}{\sigma_b\sigma_s^2(1-\rho_{sb}^2)}\right]\\ &+\left(-\frac{YJ_{wy}}{WJ_{ww}}\left[\frac{\sigma_s\sigma_y(\rho_{sb}\rho_{sy}-\rho_{by})-\sigma_b\sigma_y(\rho_{sb}\rho_{by}-\rho_{sy})}{\sigma_m\sigma_s(1-\rho_{sm}^2)}\right]\right)\\ &+\frac{W[\sigma_s\sigma_y(1-\rho_{sb}^2)-\sigma_p\sigma_s(\rho_{bp}-\rho_{sb}\rho_{ps})-\sigma_p\sigma_b(\rho_{ps}-\rho_{sb}\rho_{bp})]}{W\sigma_b\sigma_s(1-\rho_{sb}^2)}\\ &-\frac{Y\sigma_y[\sigma_s(\rho_{sb}\rho_{sy}-\rho_{by})+\sigma_b(\rho_{sb}\rho_{by}-\rho_{sy})]}{W\sigma_b\sigma_s(1-\rho_{sb}^2)}.\end{aligned} \tag{A27}$$

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