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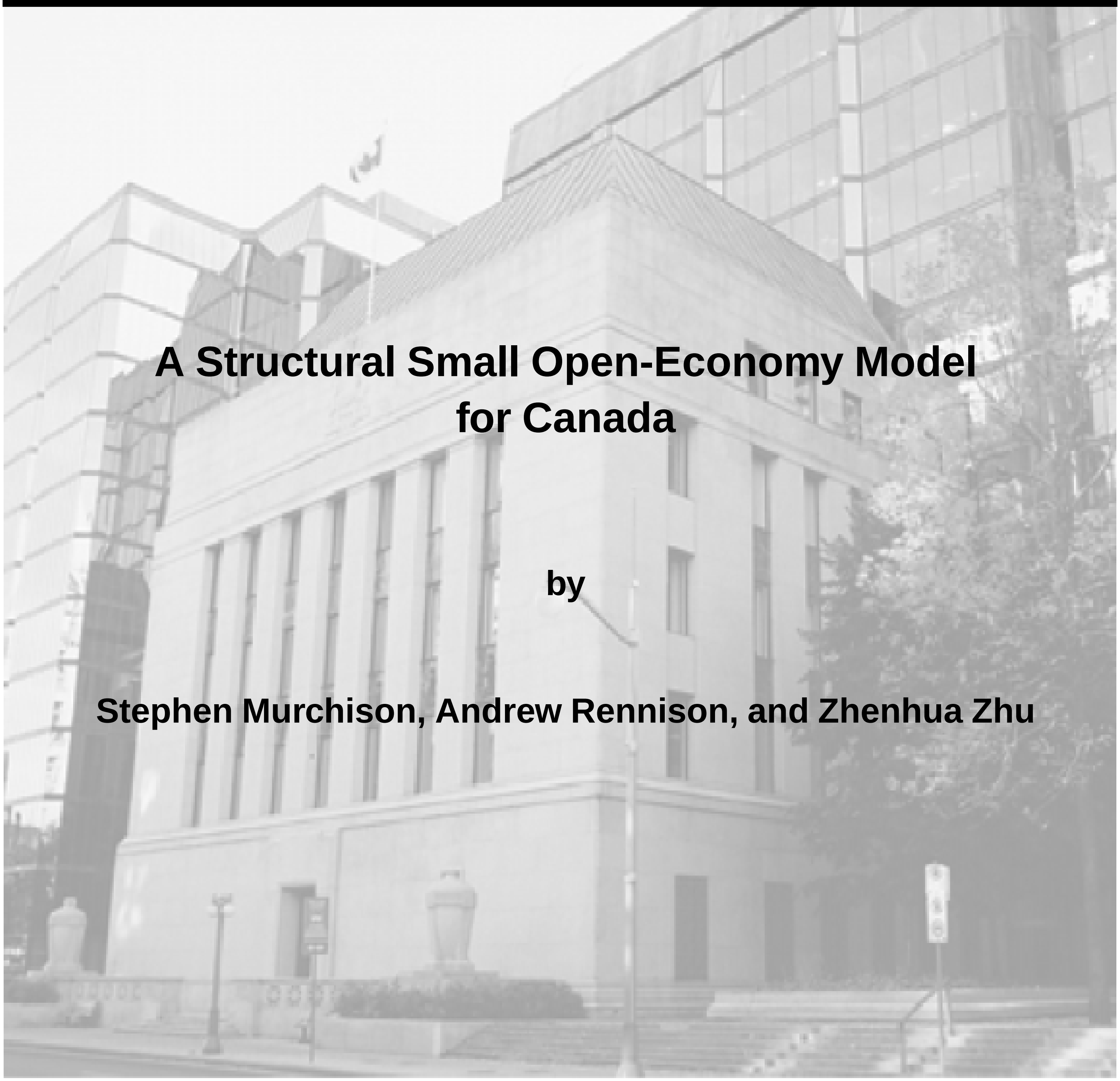

A Structural Small Open-Economy Model for Canada

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Contents

Acknowledgements iv
Abstract/Résumé v

1 Introduction 1

2 A Small Open Economy 3

2.1 Domestic production 3
2.2 Import sector 13
2.3 Consumers 15
2.4 Monetary policy 18
2.5 Foreign economy 19

3 Solution and Estimation 20

4 Model Dynamics 23

4.1 Inflation 24
4.2 Investment 29

5 Conclusion 32

Bibliography 34

Tables 38

Appendix A: The Steady State 40

Appendix B: The Stationary Dynamic Model 42

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Abstract

The authors develop a small open-economy dynamic stochastic general-equilibrium (DSGE) model in an attempt to understand the dynamic relationships in Canadian macroeconomic data. The model differs from most recent DSGE models in two key ways. First, for prices and wages, the authors use the time-dependent staggered contracting model of Dotsey, King, and Wolman (1999) and Wolman (1999), rather than the Calvo (1983) specification. Second, to model investment, the authors adopt Edge's (2000a, b) framework of time-to-build with ex-post inflexibilities. The model's parameters are chosen to minimize the distance between the structural model's impulse responses to interest rate, demand (consumption), and exchange rate shocks and those from an estimated vector autoregression (VAR). The majority of the model's theoretical impulse responses fall within the 5 and 95 per cent confidence intervals generated by the VAR.

JEL classification: E2, E3, E52
Bank classification: Business fluctuations and cycles; Economic models; Inflation and prices

Résumé

À l'aide d'un modèle d'équilibre général dynamique et stochastique qui décrit une petite économie ouverte, les auteurs cherchent à élucider les relations dynamiques existant entre les variables macroéconomiques canadiennes. Leur modèle diffère de deux façons fondamentales des récents modèles du genre. Premièrement, les auteurs n'ont pas recours au modèle à contrats échelonnés de Calvo (1983), mais bien à celui de Dotsey, King et Wolman (1999) et de Wolman (1999) où la probabilité d'ajustement des prix et des salaires au cours d'une période donnée est une fonction croissante du délai écoulé depuis le dernier ajustement. Deuxièmement, ils modélisent le comportement de l'investissement au moyen du schéma de Edge (2000a et b), qui comporte un délai de formation du capital et des rigidités *ex post*. Les paramètres du modèle sont établis de manière à minimiser l'écart entre les profils de réaction du modèle structurel aux variations des taux d'intérêt, de la demande (ou consommation) et du taux de change et ceux tirés de l'estimation d'un vecteur autorégressif (VAR). La majorité des profils de réaction théoriques issus du modèle se situent à l'intérieur des intervalles de confiance à 90 % générés par le VAR.

Classification JEL : E2, E3, E52
Classification de la Banque : Cycles et fluctuations économiques; Modèles économiques; Inflation et prix

1 Introduction

The goal of this paper is to better understand the observed dynamic behaviour of aggregate nominal and real variables in the Canadian economy. With this in mind, we develop a small open-economy dynamic stochastic general-equilibrium (DSGE) model for Canada. In the context of this model, we also ask whether, relative to the recent DSGE literature (for example, Christiano, Eichenbaum, and Evans 2001 and Smets and Wouters 2002), alternative explanations are possible for the observed dynamics of inflation and investment.

Our model is consistent with the New Open-Economy Model (NOEM) paradigm, in that it extends the basic closed-economy, optimizing-agent/sticky-price, or New Neoclassical Synthesis (NNS, see Goodfriend and King 1997), framework to allow for international trade in goods and credit. The model includes sticky nominal wages and sticky prices for domestically produced and imported goods (as in Smets and Wouters 2002), the latter implying incomplete exchange rate pass-through in the short run. Following Christiano, Eichenbaum, and Evans (2001) and Smets and Wouters (2002), our model includes habit formation in consumption and variable utilization for capital.

This model differs from most of the recent literature in two important ways. The first concerns wage- and price-setting, where we adopt the time-dependent staggered contracting model presented in Dotsey, King, and Wolman (1999) and Wolman (1999) as an alternative to the more conventional (and convenient) Calvo (1983) specification. We assume that the conditional probability of a price/wage change by a given firm/consumer, rather than stay constant, increases in the number of periods since the most recent price change. Furthermore, this probability goes to one after a fixed number of periods. These two differences allow our model to generate a more gradual, hump-shaped response of inflation without having to appeal to rule-of-thumb price-setters, as in Galí and Gertler (1999), or imperfect indexation, as in Christiano, Eichenbaum, and Evans (2001).

The second way in which we diverge from most of the recent DSGE literature is in our assumptions regarding investment. We assume that physical capital formation is constrained by time-to-build with ex-post investment inflexibilities, as in Edge (2000a, b). Relative to traditional time-to-build models (see, for example, Oliner, Rudebusch, and Sichel 1995), ex-post inflexibilities allow a firm to reoptimize after the decision to start an investment project. This feature is introduced by assuming that investment expenditures are complementary across time and implies that, unlike the traditional time-to-build model, optimal investment depends positively on its own lags. Relative to a model with only capital adjustment costs (for recent examples in a DSGE framework, see Kim 2000 and Bouakez, Cardia, and Ruge-Murcia 2002), these assumptions allow our model to better capture the observed dynamics of investment. At the same time, while recent DSGE models (for example, Christiano, Eichenbaum, and Evans 2001 and Smets and Wouters 2003) have incorporated costly investment adjustment to deliver hump-shaped responses of investment to various shocks, we view time-to-build as an alternative explanation for the behaviour of investment.

Overall, we view our results as promising in that our model explains reasonably well the observed dynamic behaviour of the Canadian macroeconomy. Most of the model's theoretical impulse responses considered in this paper are within the estimated 5 and 95 per cent confidence intervals generated by the vector autoregression (VAR). The majority of the large differences between the structural and VAR models' responses can be traced back to the excess sensitivity of the nominal (and real) exchange rate in the DSGE model, which is driven by an uncovered interest rate parity condition. Other interesting results concern the price and wage contract lengths implied by the model. The model predicts average contract lengths of 4 and 5 quarters, respectively, for domestic and import prices, and 6.5 quarters for nominal wages, slightly longer than those suggested by other similar studies (see, for example, Ambler, Dib, and Rebei 2003).

The paper is organized as follows. Section 2 describes the basic structure of the model, focusing on the pricing and investment problem faced by firms. Section 3 describes the solution and estimation strategy used in the paper and reports the values for each structural parameter used in the simulations. Section 4 compares the model's predicted behaviour of inflation and investment with a similarly specified model that uses Calvo-style pricing and a simple costly capital adjustment model for investment. Section 5 concludes.

2 A Small Open Economy

2.1 Domestic production

We begin by assuming a continuum of monopolistically competitive firms that each produce a differentiated domestic product and charge a price for their good that maximizes expected profits. Thus, the representative firm, i, $i \in [0,1]$, will produce Y_i and receive price $P_{d,i}$ in return. Aggregate domestic output, Y, and the domestic price level, P_d, are defined by

$$Y_t = \left[\int_0^1 Y_{it}^{\frac{\epsilon_p - 1}{\epsilon_p}} di \right]^{\frac{\epsilon_p}{\epsilon_p - 1}} \tag{1}$$

$$P_{d,t} = \left[\int_0^1 P_{d,it}^{1-\epsilon_p} di \right]^{\frac{1}{1-\epsilon_p}}. \tag{2}$$

Cost minimization in the production of a unit of Y implies that firm i faces the demand schedule

$$Y_{it} = \left(\frac{P_{d,it}}{P_{d,t}} \right)^{-\epsilon_p} \cdot Y_t \tag{3}$$

for its product. In addition, firms produce goods using a constant elasticity of substitution (CES) production technology that combines capital and labour:

$$\mathcal{F}(A_t L_{it}, u_{it} K_{it}) = \left[\delta^{\frac{1}{\sigma}} (A_t \cdot L_{it})^{\frac{\sigma-1}{\sigma}} + (1-\delta)^{\frac{1}{\sigma}} (u_{it} \cdot K_{it})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad \sigma \neq 1, \tag{4}$$

where A is labour-augmenting technology that is assumed to be common to every firm with

$$log(A_t) = log(A_{t-1}) + \varepsilon_t^A \qquad \varepsilon^A \sim N(0, \sigma_A^2). \tag{5}$$

σ is the elasticity of substitution between capital and labour and u_{it} is the utilization rate of capital, which we assume is variable.[1]

Capital accumulation is constrained by time-to-build with ex-post inflexibilities (Edge 2000a, b). We assume complementarity between investment expenditures in a given project across time, which discourages firms from diverging ex post from their original investment plan. We formally incorporate this interdependence by specifying what we call firm i's "effective investment," I_{it}^{E}, as a CES aggregator of past and current investment expenditures:

$$I_{it}^{E} = \left\{ \sum_{j=0}^{\tau} (\phi_j I_{i,t-j,t})^{\theta} \right\}^{1/\theta}. \tag{6}$$

The second subscript on the investment terms denotes the time of the investment expenditure; the third denotes the period in which the project is to be completed. θ controls the degree of intertemporal complementarity between investment expenditures: as $\theta \rightarrow -\infty$, investment expenditures become perfect complements, and the investment plan is completely inflexible ex post. The ϕ's can account for a planning phase at the start of a project in which expenditures are typically relatively small as, for example, building plans are drawn up (see Christiano and Todd 1996). We allow a 1-quarter planning period by allowing ϕ_τ to vary relative to $\phi_0 ... \phi_{\tau-1}$. For the project length, $\tau+1$, we assume 5 quarters.[2] In section 4.2, we further discuss the implications of our time-to-build assumptions for the dynamics of investment.

[1]The choice of CES production reflects several considerations (for a general discussion for Canada, see Perrier 2003). First, a lower elasticity of substitution makes real marginal cost behave more procyclically, as Murchison and Zhu (2003) discuss, which aids in explaining inflation movements. Second, fluctuations in labour that are driven by temporary structural shocks will have less effect on the marginal product of capital and therefore investment. Hence, we expect that investment will be less sensitive to temporary movements in the real interest rate. The exclusion of capital-augmenting technology growth ensures a stationary steady state for labour's share of income (a feature found in Canadian data).

[2]Previous studies have assumed that projects require 4 quarters from start to completion (see, for example, Oliner, Rudebusch, and Sichel 1995 and Fuhrer 1997). Based on survey evidence, Koeva (2000) finds that the typical structures investment project (which currently comprises roughly 43 per cent of Canadian business investment) requires 8 quarters from the decision to start to the completion of the project. Based on an assumption of 8 quarters for structures investment, 4 quarters for industrial equipment (17 per cent of Canadian investment), and 2 quarters for other equipment (40 per cent of Canadian

The firm's capital stock at the start of a period is the sum of the last quarter's depreciated capital stock plus the amount of effective investment, or new capital, installed at the end of the previous period:

$$K_{i,t+1} = (1-\omega)K_{it} + I_{it}^{E}. \tag{7}$$

Aggregate investment, I_{it}, is the sum of firm i's investment expenditures on projects currently underway:

$$I_{it} = I_{it,t} + I_{it,t+1} +, ..., I_{it,t+\tau} = \sum_{j=0}^{\tau} I_{it,t+j}. \tag{8}$$

Firms also incur a quadratic cost when they adjust the level of the capital stock, which takes the form of a deadweight loss of the produced good.

We also assume that firms can vary their rate of capital utilization at the cost of foregone output.[3] When we incorporate quadratic capital adjustment costs in addition to convex costs of capital utilization, output evolves according to

$$Y_{it} = \mathcal{F}(A_t L_{it}, u_{it} K_{it}) - \frac{\chi}{2K_{it}} I_{it}^{E^2} - \psi \left(1 - e^{\rho(u_{it}-1)}\right) K_{it}, \tag{9}$$

where χ determines the size of capital adjustment costs and ψ and ρ determine the costs of variable capital utilization.[4]

Turning to the firm's pricing decision, we follow the bulk of the literature in assuming the existence of multi-period price contracts. We follow Dotsey,

investment), we arrive at an average completion time for Canada of 5 quarters. Our time-to-completion assumptions for industrial and other equipment are consistent with those in Edge (2000b).

[3]This is consistent with the assumptions of Christiano, Eichenbaum, and Evans (2001) and Smets and Wouters (2003). As Christiano, Eichenbaum, and Evans (2001) point out, this assumption of costs in terms of the output good instead of in terms of capital depreciation helps to generate a rise in capital utilization after a positive monetary policy shock, which dampens the resulting rise in marginal cost.

[4]We impose a restriction on the parameter ψ such that, in steady state, the utilization rate is one and the cost of utilization is zero.

King, and Wolman (1999)[5] and Wolman (1999) and allow for the possibility that firms fix their prices for up to j ($j > 1$) periods (hereafter, this pricing model is referred to as the DKW model). This differs from the original Taylor (1980) specification in that Taylor assumed all firms set price contracts for exactly j periods. It also differs from the often-used Calvo (1983) specification in that j is finite. We first introduce the following notation. Let α be a j-dimensional vector in which the ith row, α_i, represents the probability that a firm adjusts its price, conditional on having last adjusted i periods ago. By assumption, $\alpha_j = 1$. The fraction of firms, ϖ_i, in a given period that charge prices that were set i periods ago is therefore given by

$$\varpi_i = (1 - \alpha_i) \cdot \varpi_{i-1} \quad i = 1, 2, ..., j-1, \tag{10}$$

or

$$\Lambda_i \equiv \frac{\varpi_i}{\varpi_0} = \prod_{q=0}^{i} (1 - \alpha_q) \qquad \alpha_0 = 0, \tag{11}$$

which states that the probability, Λ_i, of a contract price remaining in effect i periods in the future is equal to the product of the probabilities of not changing prices in each of the preceeding periods up to the ith. ϖ_0 represents the (constant) proportion of firms that adjust their price in any given period. Since each firm must fall into one of the categories (in terms of the number of periods since their last price change), we have

$$\varpi_0 = 1 - \sum_{i=1}^{j-1} \varpi_i. \tag{12}$$

The average contract length, $\mathcal{L}$, is given by

$$\mathcal{L} = \varpi_0^{-1} = \sum_{i=1}^{j} \Omega_i \cdot i \quad \text{with} \;\; \Omega_i \equiv \alpha_i \cdot \Lambda_{i-1} \;\; \text{and} \;\; \textstyle\sum_{i=1}^{j} \Omega_i = 1. \tag{13}$$

[5] For this version of the model, we exclude the state-dependent component discussed in Dotsey, King, and Wolman (1999). Thus, our price-change probabilities are invariant to the state of the economy.

Ω_i is very useful in that it captures the proportion of firms that will hold their prices fixed for exactly i periods. The distribution of Ω_i for $i = 1, 2, ..., j$ provides a very useful guide as to the plausibility of the pricing model considered.

In the Calvo model, $\alpha_i = \alpha \forall i$. In other words, the proportion of firms that change prices each period is simply α, so

$$\varpi_0 = \alpha \quad ; \ \varpi_i = \alpha \cdot (1-\alpha)^i \quad \text{and} \ \ \mathcal{L} = \alpha^{-1}. \tag{14}$$

Finally,

$$\Omega_i = \alpha \cdot (1-\alpha)^{i-1}, \tag{15}$$

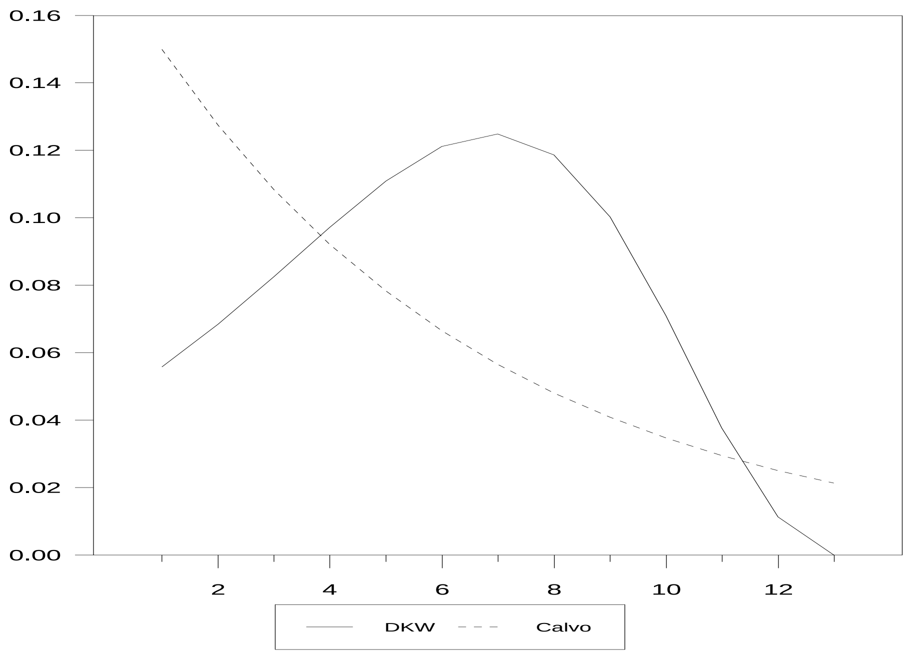
which is strictly decreasing in i. Therefore, it is impossible to obtain a bell-shaped distribution of price changers using the Calvo model. There will always be a higher proportion of price changers after one period than for any other length of time. This difference between the two models is illustrated in Figure 1. In the figure,"DKW" corresponds to the estimated distribution of wage contracts for Canada (see section 3), whereas "Calvo" corresponds to the Calvo model with $\alpha = 0.15$, the value necessary to produce the same average contract duration (slightly more than 6.5 quarters) as the DKW model. The figure clearly indicates that, whereas DKW provides a bell-shaped distribution of wage contracts, the Calvo model's distribution declines monotonically from the maximum at 1 quarter. This difference will turn out to have important implications for the dynamic response of inflation to certain shocks.

Of course, the Calvo specification is not intended to capture literally the pricing behaviour of firms, but rather to be a convenient approximation of their true behaviour. It is convenient in that (up to a log-linear approximation) one can combine the optimal pricing equation with the price aggregator to form a compact, Phillips-curve-style, inflation equation of the form

$$\pi_t = \beta \mathcal{E}_t \pi_{t+1} + b\lambda_t \qquad b > 0, \tag{16}$$

where λ_t is real marginal cost and b increases with the probability of price-change parameter α. Thus, the period t price level is a function of only one

Figure 1 - Distribution of Contract Lengths



predetermined state variable, the $t-1$ price level. The DKW specification, by contrast, requires $j-1$ state variables. Furthermore, the corresponding "Phillips curve" representation is far more complex, admitting multiple lags and leads of inflation as well as marginal cost (see Guerrieri 2001 and Bakhshi, Khan, and Rudolf 2003). Moreover, written in this form, the error terms will have a complicated moving-average structure that must be accounted for in empirical work.

In addition to having a parsimonious specification, the Calvo model also has the advantage of requiring that the econometrician identify only a single parameter (if β is taken as given). The average duration of price contracts can then be computed directly, without having to impose a maximum contract length on the structure of the model prior to estimation (as with the Taylor and DKW models).

These factors combined likely explain the popularity of the Calvo model in the literature. The vast majority of structural model builders employ this convenient approximation rather than use the DKW model. This would appear to be quite reasonable if the two models produced similar behaviour and one was much easier to use. It has been clearly demonstrated, however, that this is not the case.

Perhaps most notably, Wolman (1999) shows that the predictions of the two models following a representative shock to marginal cost[6] are significantly different along two important margins (see also Kiley 2002). First, the Calvo model predicts that the maximum response of inflation should arrive in the first period of the shock which, as will be discussed later, appears counterfactual for Canada. Second, it predicts that inflation should respond by considerably less in magnitude (from 20 to 50 per cent as much) with Calvo pricing than with the DKW set-up. Wolman concludes on this basis that, if the Calvo model is consistent with U.S. inflation data, as Sbordone (1998) and Galí and Gertler (1999) suggest, then the DKW specification could not be consistent with the data. The Calvo model has, however, not been found to be compatible with Canadian inflation (see Guay, Luger, and Zhu 2002), and we thus consider the DKW model as an alternative for explaining inflation in Canada.

Dotsey (2002) demonstrates that the significance of lagged inflation in estimated Calvo-based Phillips curves should be interpreted with caution if the true underlying model is of the DKW form. Specifically, a DKW model without indexation and full rationality assumed for all pricers will generate data that admits a lag of inflation into specifications of the form given by (16). In addition, Kozicki and Tinsley (2002) find that a version of the DKW model fits Canadian inflation dynamics better than hybrid models that assume that some fraction of agents form expectations non-rationally.

These findings are of considerable importance for those who view the assumptions of partial price indexation or "rule-of-thumb" behaviour not as a structural feature of the economy but rather as convenient modifications that are required for the Calvo model to replicate the persistence found in inflation. We will consider the implications of the Dotsey (2002) and Wolman (1999) findings in greater detail in section 4.

The representative firm's objective is to choose $P_{d,it}, Y_{it}, L_{it}, K_{i,t+1}, u_{it}$, and I_{it} subject to equations (3), (6), (7), and (9) to maximize the value of

[6]Wolman defines marginal cost to be labour's share of output or unit labour costs.

the firm:

$$\mathcal{V}_{it} = \mathcal{E}_t \sum_{s=t}^{\infty} \mathcal{R}_{t,s} \left[P_{d,is} \left(\frac{P_{d,is}}{P_d, s} \right)^{-\epsilon_p} Y_s - W_{is} L_{is} - P_s^I \sum_{k=0}^{\tau} I_{is,s+k} \right], \tag{17}$$

where P_s^I is the price of investment and the stochastic discount factor, $\mathcal{R}_{t,s}$, is defined as

$$\mathcal{R}_{t,s} \equiv \prod_{v=t}^{s} \left(\frac{1}{1+R_v} \right). \tag{18}$$

Abstracting for the moment from the optimal choice of price, the solution to (17) gives rise to the following optimality conditions (ignoring first-order conditions with respect to the Lagrangians):

$$W_t = \lambda_{it} \mathcal{F}_l(\cdot), \tag{19}$$

$$q_t = \mathcal{E}_t \mathcal{R}_{t,t+1} \left[\lambda_{i,t+1} \left(\mathcal{F}_k(\cdot) + \frac{\chi}{2} \left(\frac{I_{i,t+1}^E}{K_{i,t+1}} \right)^2 + \psi \left(1 - e^{\rho(u_{it}-1)} \right) \right) + (1-\omega) q_{t+1} \right], \tag{20}$$

$$I_{it,t+k} = \mathcal{E}_t \left[\frac{E I_{i,t+k}^{1-\theta} \phi^{\theta} \mathcal{R}_{t,t+k}}{P_t^I} \left(q_{t+k} - \chi \lambda_{i,t+k} \frac{I_{i,t+k}^E}{K_{i,t+k}} \right) \right]^{\frac{1}{1-\theta}}, \tag{21}$$

$$\mathcal{F}_u(\cdot) = -\psi \rho e^{\rho(u_{it}-1)} K_{it}, \tag{22}$$

where λ_{it} is the constraint that equates demand and supply, which may be interpreted as the marginal cost of production. The variable q_t is the shadow value of capital, or the discounted contribution of capital to future dividends. Given the production technology described by equation (4), we also have

$$\mathcal{F}_l(\cdot) = \left(\frac{\delta \mathcal{F}(\cdot)}{L_{it}}\right)^{\frac{1}{\sigma}} A_t^{\frac{\sigma-1}{\sigma}}, \tag{23}$$

$$\mathcal{F}_k(\cdot) = \left(\frac{(1-\delta)\mathcal{F}(\cdot)}{K_{it}}\right)^{\frac{1}{\sigma}} u_{it}^{\frac{\sigma-1}{\sigma}}, \tag{24}$$

$$\mathcal{F}_u(\cdot) = \left(\frac{(1-\delta)\mathcal{F}(\cdot)}{u_{it}}\right)^{\frac{1}{\sigma}} K_{it}^{\frac{\sigma-1}{\sigma}}. \tag{25}$$

Turning to the optimal pricing decision, we note that the assumption of random contract lengths induces uncertainty amongst firms as to how long their price will remain in effect. Using equation (11), we can recast $\mathcal{V}_{it}$ in terms of $P_{d,it}$ rather than $P_{d,is}$ weighted by the probability that that price will still be in effect in period s, which is Λ_{s-t}. Of course, the price chosen today will affect profits at most $j-1$ periods into the future, since at this point Λ_{s-t} goes to zero. The solution for the optimal price by firm i is then given as

$$P_{d,it} = \left(\frac{\epsilon_p}{\epsilon_p - 1}\right) \mathcal{E}_t \left(\frac{\sum_{s=t}^{t+j-1} \mathcal{R}_{t,s} \Lambda_{s-t} P_{d,s}^{\epsilon_p} \lambda_{is} Y_s}{\sum_{s=t}^{t+j-1} \mathcal{R}_{t,s} \Lambda_{s-t} P_{d,s}^{\epsilon_p} Y_s}\right). \tag{26}$$

Equation (26) states that firms, when resetting their price, will consider both the rate of inflation and the level of marginal cost over the expected life of the price contract. In steady state with zero inflation, this collapses to the standard static condition that price is a constant markup over marginal cost and the markup $\epsilon_p/(\epsilon_p - 1)$ is decreasing in the elasticity of demand for the firm's product. Perfect competition corresponds to the limit where ϵ_p approaches infinity.

Since all firms that reset their price at the same time choose the same price, we can replace $P_{d,it}$ in equation (26) with an aggregate contract price,[7]

[7]With firm-specific state variables such as capital this is not necessarily true. One solution to this problem is to assume complete contingent claims for firms.

$p_{d,t}$. Consequently, the aggregate price level can be expressed as a CES aggregate of the contract prices for the various cohorts:

$$P_{d,t} = \left(\sum_{k=0}^{j-1} \varpi_{d,k} (p_{d,t-k})^{1-\epsilon_p} \right)^{\frac{1}{1-\epsilon_p}}. \tag{27}$$

Equations (26) and (27) jointly describe the evolution of the domestic price level or GDP deflator.

2.1.1 Exports

Aggregate output is either consumed, invested, or exported to the foreign country,

$$Y_t = C_{dt} + I_{dt} + X_{dt}, \tag{28}$$

where C_d, I_d, and X_d are, respectively, domestic consumption, investment, and exports, which are distinct from their imported counterparts. Cost minimization on the part of foreigners implies an export demand function of the form

$$X_t = \gamma_{z^*} \cdot \left(\frac{P_t^{*x}}{P_t^*} \right)^{-\vartheta} \cdot Z_t^*, \tag{29}$$

where P_t^{*x} and P_t^* are, respectively, the foreign price of domestic output and the foreign general price level, and Z_t^* is foreign expenditures. Here, we assume that the foreign import price (P_t^{*x}) is determined in the same manner as the home import price, which is described in the next section; $-\vartheta$ is the elasticity of substitution between domestic exports and foreign-produced goods.

2.2 Import sector

In addition to domestic goods, we assume the existence of a continuum of intermediate imported goods, M_{jt}, $j \in [0,1]$, that are bundled into an aggregate import, M_t, by the aggregator and sold in the domestic market,

$$M_t = \left[\int_0^1 M_{jt}^{\frac{\epsilon_p - 1}{\epsilon_p}} dj \right]^{\frac{\epsilon_p}{\epsilon_p - 1}}, \tag{30}$$

with no value added.

Demand by the aggregator for the differentiated goods is given by the familiar cost-minimizing demand functions,

$$M_{jt} = \left(\frac{P_{m,jt}}{P_{m,t}} \right)^{-\epsilon_p} M_t, \tag{31}$$

where $P_{m,t}$ is the aggregate import-price deflator, given as[8]

$$P_{m,t} = \left[\int_0^1 P_{m,jt}^{1-\epsilon_p} dj \right]^{\frac{1}{1-\epsilon_p}}. \tag{32}$$

In addition, the imported good can either be consumed, invested, or re-exported. Using the example of consumption, we see that the total is a CES aggregate of its domestically produced and imported components,

$$C_t = \left[(1-\gamma_c)^{\frac{1}{\varphi}} C_{dt}^{\frac{\varphi-1}{\varphi}} + \gamma_c^{\frac{1}{\varphi}} C_{mt}^{\frac{\varphi-1}{\varphi}} \right]^{\frac{\varphi}{\varphi-1}}, \tag{33}$$

where C_{mt} is imported consumption. Similar expressions exist for investment and exports with weights γ_I and γ_x. Demand for C_{mt} is similarly given as

$$C_{d,t} = (1-\gamma_c) \left[\frac{P_{d,t}}{P_t^c} \right]^{-\varphi} C_t \qquad C_{m,t} = \gamma_c \left[\frac{P_{m,t}}{P_t^c} \right]^{-\varphi} C_t. \tag{34}$$

[8] We assume the same elasticity of substitution for imported intermediates as for domestic intermediates, $-\epsilon_p$.

The price of consumption (and of investment and exports) is a CES aggregate of the domestic and import prices where the relative weights are determined by the index of openness, γ_c:

$$P_t^c = \left[(1-\gamma_c)P_{d,t}^{1-\varphi} + \gamma_c P_{m,t}^{1-\varphi}\right]^{\frac{1}{1-\varphi}} . \tag{35}$$

Thus, for example, the price of investment will be more exposed to the exchange rate than will consumption, since the imported share of investment is considerably higher than the imported consumption share in Canada over history.

We follow Smets and Wouters (2002) in assuming that the price of the imported good is temporarily rigid in the currency of the importing country. Consequently, exchange rate pass-through to import prices is partial in the short run and complete in the long run. Exchange rate fluctuations are absorbed by the importers' profit margins in the short run, since they purchase goods according to the law of one price. Importers therefore take into consideration the future path of foreign prices and the nominal exchange rate over the expected duration of their contract when deciding on their time t price. Analogous to the domestic price-setter, the importers' price-setting rule is given as:

$$P_{m,it} = \left(\frac{\epsilon_p}{\epsilon_p - 1}\right)\mathcal{E}_t\left(\frac{\sum_{s=t}^{t+j-1}\mathcal{R}_{t,s}\zeta_{s-t}P_{m,s}^{\epsilon_p}(e_s P_s^*)M_s}{\sum_{s=t}^{t+j-1}\mathcal{R}_{t,s}\zeta_{s-t}P_{m,s}^{\epsilon_p}M_s}\right), \tag{36}$$

where we can again replace individual price-resetters with a cohort of firms, $p_{m,t}$, each of which resets at time t.[9] The aggregate import price level is then determined as a CES aggregate of past contract prices:

$$P_{m,t} = \left(\sum_{k=0}^{j-1}\varpi_{m,k}(p_{m,t-k})^{1-\epsilon_p}\right)^{\frac{1}{1-\epsilon_p}} . \tag{37}$$

[9] ζ is defined in a manner analogous to Λ for the domestic sector.

2.3 Consumers

A continuum of households indexed by h , $h \in [0,1]$, purchase domestically produced and imported goods and consume leisure to maximize their lifetime utility. Each household is assumed to supply differentiated labour services to the intermediate-goods sector. Furthermore, the labour market is assumed to be monopolistically competitive, which motivates the existence of wage contracts. Household labour services are purchased by an aggregator and bundled into composite labour according to the Dixit-Stiglitz aggregation function:

$$L_t \equiv \left[\int_0^1 L_{ht}^{\frac{\epsilon_w - 1}{\epsilon_w}} dh\right]^{\frac{\epsilon_w}{\epsilon_w - 1}}. \tag{38}$$

Similarly, the aggregate wage index is given as

$$W_t \equiv \left[\int_0^1 W_{ht}^{1-\epsilon_w} dh\right]^{\frac{1}{1-\epsilon_w}}. \tag{39}$$

The aggregator purchases differentiated labour services to minimize costs. Thus, the demand for labour services from individual h is given as

$$L_{ht} = \left(\frac{W_{ht}}{W_t}\right)^{-\epsilon_w} \cdot L_t. \tag{40}$$

Finally, we assume that wages are reset according to the same model presented for import and domestic prices in the previous section. Specifically, we allow for the possibility that households fix their wages for up to $q, (q > 1)$ periods. As with prices, the aggregate nominal wage, W_t, can be expressed as a CES aggregate of the individual "cohort" wage contracts signed up to $q - 1$ periods in the past:

$$W_t = \left(\sum_{k=0}^{q-1} \varpi_{w,k} (w_{t-k})^{1-\epsilon_w}\right)^{\frac{1}{1-\epsilon_w}}. \tag{41}$$

The instantaneous utility function for the h^{th} household is given as

$$\mathcal{U}_{ht} = \frac{\mu}{\mu - 1}(C_{ht} - H_t)^{\frac{\mu-1}{\mu}} \exp\left(\frac{\eta(1-\mu)}{\mu(1+\eta)} \cdot L_{ht}^{1+1/\eta}\right), \tag{42}$$

where H_t is the external habit, which is assumed to be proportional to lagged *aggregate* consumption:

$$H_t = \xi C_{t-1}. \tag{43}$$

Equation (42) is non-standard primarily in the sense that consumption and leisure are not additively separable (see King, Plosser, and Rebelo 1988 and Basu and Kimball 2000; for a model application, see Smets and Wouters 2003). Consequently, the marginal utility of consumption (leisure) will depend on labour (consumption). The exact nature of this relationship will, in turn, depend on the intertemporal elasticity of substitution of consumption, μ. For $\mu < 1$, consumption and labour will be complements, which implies that the marginal utility of consumption is increasing in hours worked. Additively separable utility functions, on the other hand, require the restriction that $\mu = 1$ to generate balanced long-run growth (i.e., constant per-capita hours worked) in the presence of trend productivity growth.[10]

In addition, household consumption will depend positively on lagged aggregate consumption according to the parameter ξ. Thus, we assume that individuals enjoy high consumption in and of itself (provided $\xi < 1$), but that they also derive utility from high consumption relative to that of their neighbours.

Households maximize lifetime utility according to

$$\mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \varepsilon_t^{\beta} \mathcal{U}_{ht}, \tag{44}$$

[10] One may also impose that $\xi = 1$ when habit formation takes the form $\frac{\mu}{\mu-1}\left(\frac{C_t}{C_{t-1}^{\xi}}\right)^{\frac{\mu-1}{\mu}}$, which implies that the level of consumption alone has no affect on utility. Setting $\mu = 1$ in this case has the undesirable effect of eliminating the effect of habit persistence in the consumption Euler equation.

where ε_t^β is a shock to the rate of time preference that is assumed to evolve according to an AR(1) process,

$$\varepsilon_t^\beta = a^\beta \varepsilon_{t-1}^\beta + \nu_t^\beta, \tag{45}$$

subject to the dynamic budget constraint,[11]

$$P_t^c C_{ht} + \frac{B_{ht}}{1+R_t} + \frac{e_t B_{ht}^*}{(1+R_t^*)(1+\kappa_t)} = B_{h,t-1} + e_t B_{h,t-1}^* + W_{ht} L_{ht} + \Pi_t, \tag{46}$$

where B_{ht}^* and B_{ht} are, respectively, the value of foreign (domestic) currency-denominated bonds held at time t and e_t is the nominal exchange rate. Π_t represents dividends paid by the firm. κ_t is interpreted as the country-specific risk premium and is assumed to have both a deterministic and stochastic component. More specifically, the risk premium will move with the home country's net foreign indebtedness as a share of nominal GDP, as in Schmitt-Grohé and Uribe (2003) (see also Ambler, Dib, and Rebei 2003 for an application), thereby ensuring a stationary dynamic path for the net-foreign-asset-to-GDP ratio about its steady-state value. In addition, we assume that the risk premium is subject to a shock process that represents unforecastable changes in investors' preferences:

$$\kappa_t = \varsigma \left[\exp\left(\frac{e_t B_t^*}{P_{dt} Y_t} \right) - 1 \right] + \varepsilon_t^\kappa, \tag{47}$$

where

$$\varepsilon_t^\kappa = a^\kappa \varepsilon_{t-1}^\kappa + \nu_t^\kappa. \tag{48}$$

Maximizing (44) with respect to $C_{ht}, W_{ht}, L_{ht}, B_{ht}^*$, and B_{ht} subject to (40) and (46) yields the following first-order conditions:

[11] In addition to the budget constraint, the no-Ponzi game condition is enforced for domestic and foreign bonds. Also, we assume that consumption is identical across households despite differences in wage income out of steady state.

$$P_t^c \Phi_t = (C_t - H_t)^{-1/\mu} \exp\left(\frac{\eta(1-\mu)}{\mu(1+\eta)} \cdot L_{ht}^{1+1/\eta}\right) \cdot \varepsilon_t^{\beta}, \quad (49)$$

$$\Phi_t = \beta \mathcal{E}_t \Phi_{t+1}(1+R_t), \quad (50)$$

$$e_t \Phi_t = \beta \mathcal{E}_t e_{t+1} \Phi_{t+1}(1+R_t^*)(1+\kappa_t). \quad (51)$$

Consumers, when given the chance to reset their wages, will do so according to the following dynamic rule:

$$W_{ht} = \left(\frac{\epsilon_w}{\epsilon_w - 1}\right) \mathcal{E}_t \left(\frac{\sum_{s=t}^{t+q-1} \mathcal{R}_{t,s} \Xi_{s-t} P_s^c \Phi_s W_s^{\epsilon_w(1+1/\eta)} L_s^{1+1/\eta}(C_s - H_s)}{\sum_{s=t}^{t+q-1} \mathcal{R}_{t,s} \Xi_{s-t} \Phi_s W_s^{\epsilon_w} L_s}\right)^{\frac{\eta}{\epsilon_w + \eta}} . \quad (52)$$

Moreover, since all consumers who choose to reset their wage at the same time will choose the same wage, we can replace W_{ht} with the cohort wage, w_t. Equation (52) can then be combined with (41) to solve for the behaviour of the aggregate nominal wage, W_t.

2.4 Monetary policy

For the purposes of this paper, we characterize monetary policy as behaving the same as in the VAR, which allows us to isolate differences in the behaviour of the variables of interest (inflation and investment) while holding the behaviour of policy constant across the two models. The nominal interest rate equation in the VAR is given as

$$R_t = 1.1\pi_t + (1-\varrho_1-\varrho_2)\overline{r} + \sum_{i=1}^{2} \varrho_i(R_{t-i} - \pi_{t-i}) + \sum_{i=1}^{2} \varrho_{i+2}(Y_{t-i}/\overline{Y} - 1) + \varepsilon_t^R, \quad (53)$$

where $\overline{r}$ is the steady-state real interest rate and $\overline{Y}$ is the steady-state level of output.[12] Thus, policy is seen as raising nominal interest rates in response to

[12] In the VAR, steady-state output corresponds to the HP-filter-based trend extracted from Y_t. For the purposes of the temporary shocks considered in this paper, one can reasonably think of $(Y_t/\overline{Y} - 1)$ as corresponding to an output gap.

higher inflation and above steady-state output. With respect to the former, determinacy in the structural model requires that the real interest rate rise in response to an inflationary shock; that is, the coefficient on inflation in our reaction function must be greater than one. However, initial estimates of this parameter were less than one, and we therefore impose a value of 1.1 for the subsequent estimation of the VAR. The term ε_t^R is interpreted as the monetary policy shock and is assumed to follow an AR(1) process[13]:

$$\varepsilon_t^R = a^R \varepsilon_{t-1}^R + \nu_t^R. \tag{54}$$

2.5 Foreign economy

To close our open-economy model, it is necessary to specify processes that generate foreign demand, Z_t^*, the general foreign price level, the foreign import price level, and the foreign nominal short-term interest rate. For the general price level and output, we estimate a bivariate-VAR model on U.S. data from 1980Q1 to 2002Q4, treating interest rates as exogenous. We then add to the model a Taylor-style rule for U.S. real interest rates of the specific form

$$R_t^* = c + 0.5 \cdot (Z_t^*/Z_t^{*p} - 1) + 0.5 \cdot \pi_t^* + \nu_t^{*R}, \tag{55}$$

where Z_t^{*p} is the HP-filter-based trend extracted from Z_t^*. For simplicity, we assume in deflating our model that Z_t^* and Y_t share a common stochastic trend generated by A_t. Also for simplicity, foreign import prices are assumed to be subject to the same contract-duration probabilities as domestic import prices:

$$p_{m,t}^* = \left(\frac{\epsilon_p}{\epsilon_p - 1}\right) \mathcal{E}_t \left(\frac{\sum_{s=t}^{t+j-1} \mathcal{R}_{t,s}^* \zeta_{s-t} P_{m,s}^{*\epsilon_p} (P_s^x/e_t) M_s^*}{\sum_{s=t}^{t+j-1} \mathcal{R}_{t,s}^* \zeta_{s-t} P_{m,s}^{*\epsilon_p} M_s^*}\right), \tag{56}$$

$$P_{m,t}^* = \left(\sum_{k=0}^{j-1} \varpi_{m,k} (p_{m,t-k}^*)^{1-\epsilon_p}\right)^{\frac{1}{1-\epsilon_p}}. \tag{57}$$

[13] An AR(1) process for the monetary policy shock in the structural model is necessary to ensure a similar interest rate dynamic to that of the VAR.

Consequently, exchange rate movements feed into foreign import prices only gradually, thereby slowing the response of domestic exports to shocks.

3 Solution and Estimation

The model presented here is non-linear and contains unobserved expectations of future state variables. Before solving the model we first log-linearize it numerically about its stationary steady state using a first-order Taylor-series expansion (implemented in Troll). Second, we solve the log-linear version of the model using Sparse AIM (see Anderson and Moore 1985 and Anderson 1997). The "solution" is essentially a structural VAR representation, the dynamic behaviour of which is identical to that of the linear forward-looking version of the model.

As is now the custom with DSGE models, we divide the unknown structural parameters into two sets. The first set is calibrated so that the model will generate steady-state ratios that conform to historical averages found in the data. The results are summarized in Table 1. The quarterly subjective discount rate is set to 0.99, the value typically chosen in the literature. The parameter ϵ_p is set to 5.0 based on the historical average markup of price to marginal cost of 20 per cent (from 1966 to 2003). The parameter δ was set to 0.60 to replicate the historical steady-state labour share of income of 0.56. The parameter ω, which is the quarterly depreciation rate of installed capital, is set to 0.05. In the absence of trend growth, it is necessary to calibrate this parameter at a level above the typical value of 0.025 to ensure a plausible steady-state investment-to-output ratio. ψ is simply a calibration parameter that is set so that the steady-state behaviour of the model is unaffected by the introduction of variable capacity utilization. θ is set to -20, implying an elasticity of substitution across investment expenditures of -0.05. This calibration ensures that investment plans are costly to revise ex post.[14] ϕ_4, which governs investment in the planning period, is estimated (see Table 2), while the remaining ϕ's (ϕ_0 through ϕ_3) are calibrated so that, in steady state, the capital stock generated by the model is equal to the traditional capital stock measure (i.e., that generated by replacing I_t^E in (7) with I_t). γ_c, γ_I, and γ_x

[14]This parameter was poorly identified by the data and therefore calibrated. Sensitivity analysis indicates that local alternatives to the chosen value have very little impact on the model's impulse responses.

were, respectively, chosen to replicate in steady state the historical average import shares for consumption, investment, and exports.

One of the drawbacks of the pricing model described in section 2.1 is that one is forced to (i) choose a maximum contract length, j, prior to estimation (in much the same fashion as one must choose the lag length for a VAR model), and (ii) estimate the entire j-dimensional *vector* of conditional price-change probabilities (the α_i's), rather than the single parameter, α, in the Calvo model.

To address the first issue, one can proceed as in the VAR literature by beginning with an arbitrarily high maximum contract length (within reason, and survey evidence can provide guidance) and then performing a set of sequential conditional likelihood tests to determine the smallest value of j not rejected by the data. In future work, we plan to proceed according to this strategy. For the purposes of this paper, we have chosen a maximum contract length of 3 years (12 quarters), in the expectation that this will capture the vast majority of price and wage contracts for Canada.

For the second issue, we have imposed a non-linear functional form on the α vector to reduce the free parameter set from $j-1$ to 1:

$$\alpha_k = \left(\frac{1}{1+\mathcal{S}}\right)^{j-k} \qquad \mathcal{S} > 0; \quad k = 1, 2, ..., j-1, \tag{58}$$

where $\mathcal{S}$ is a freely estimated parameter (subject to being positive). Furthermore, $\partial\alpha_k/\partial k > 0$ and $\partial^2\alpha_k/\partial k^2 > 0$, ensuring that the conditional probability of a price change is increasing (at an increasing rate) in the amount of time since the last price change. Table 2 provides the values for $\mathcal{S}_d$, $\mathcal{S}_m$, and $\mathcal{S}_w$, which correspond, respectively, to the domestic price, foreign price, and wage.[15]

Our estimation strategy is similar to that of Christiano, Eichenbaum, and Evans (2001), in that the parameters are chosen to minimize the distance between the model's impulse responses and those from an estimated model. Specifically, we attempt to match our model's impulse responses with those generated by an estimated "near" VAR. Our VAR is ordered as follows: core

[15] In addition, we impose that the probability of a wage change after one and two quarters is zero.

CPI inflation, HP-filtered log real consumption, investment, and output, the real Canada-U.S. exchange rate, the 90-day commercial paper rate deflated by the quarterly CPI inflation rate, and the quarter-over-quarter change in the GDP deflator. We include an external sector containing HP-filtered log real U.S. GDP, core inflation, the federal funds rate deflated by U.S. core inflation, and the Bank of Canada's commodity price index.[16] The model is estimated using data from 1980 through to the first quarter of 2003.

For this paper we consider three shocks, the first of which is a monetary policy shock, or an unanticipated change in the short-term interest rate. The second is an exchange rate shock, which in the structural model takes the form of a temporary unexplained movement in the country-specific risk premium. The third is a demand, or consumption, shock that in the structural model is the result of a temporary change in the households' rate of time preference. We attempt to match the responses of domestic variables for each shock at the 1- through 12-quarter horizons.

Formally, the estimation strategy can be represented as follows. Let $\tilde{\Phi}$ denote a vector of the estimated VAR's impulse responses, and let $\Phi(\kappa)$ denote the structural model's impulse responses as they map from the set of structural parameters, κ. Our estimator of κ is the solution to:

$$\Omega = \min(\tilde{\Phi} - \Phi(\kappa))' V (\tilde{\Phi} - \Phi(\kappa)), \tag{59}$$

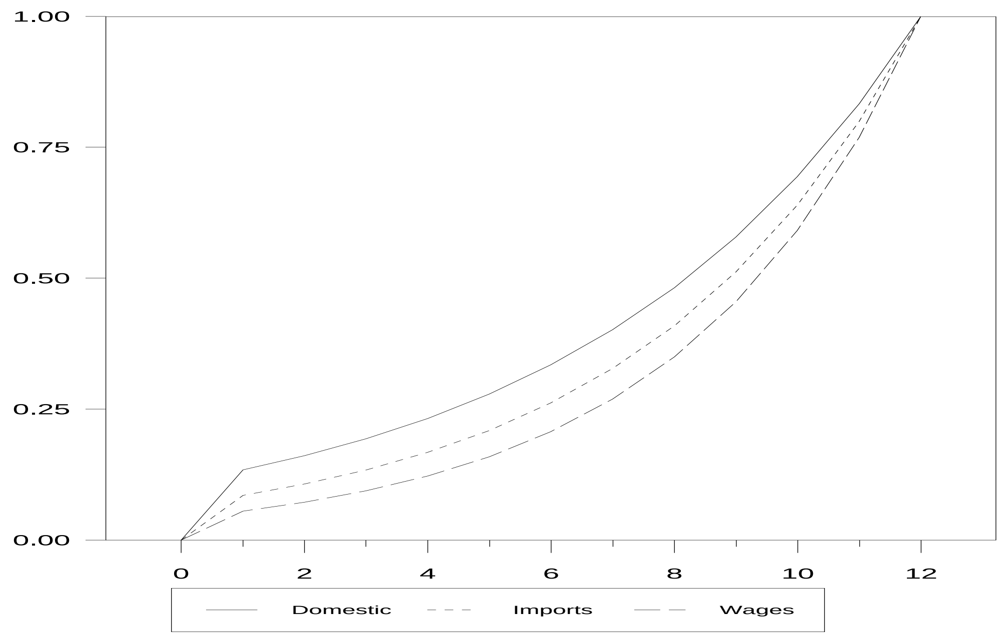
where V is the identity matrix.[17]

Table 2 gives the model's estimated parameters. In terms of the results for the pricing models, the estimated conditional probability and contract-duration distributions are given in Figures 2 and 3, respectively. In terms of average contract durations, wages are longest at just over 6.5 quarters, followed by import prices at about 5 quarters and domestic prices at just over 4 quarters. While this ordering accords with those of other recent studies, the estimates themselves are on the high side. Ambler, Dib, and Rebei (2003) estimate 4, 3, and 3 quarters for wages, import prices, and domestic prices,

[16]Block exogeneity is imposed to exclude feedback from domestic to foreign variables.

[17]We impose a value of 50 for the entries in V corresponding to the interest rate response to an interest rate shock, the consumption response to a consumption shock, and the exchange rate response to an exchange rate shock. This effectively ensures that the first-period impact on the variable being disturbed is equivalent across models.

Figure 2 - Conditional Price-Change Probabilities



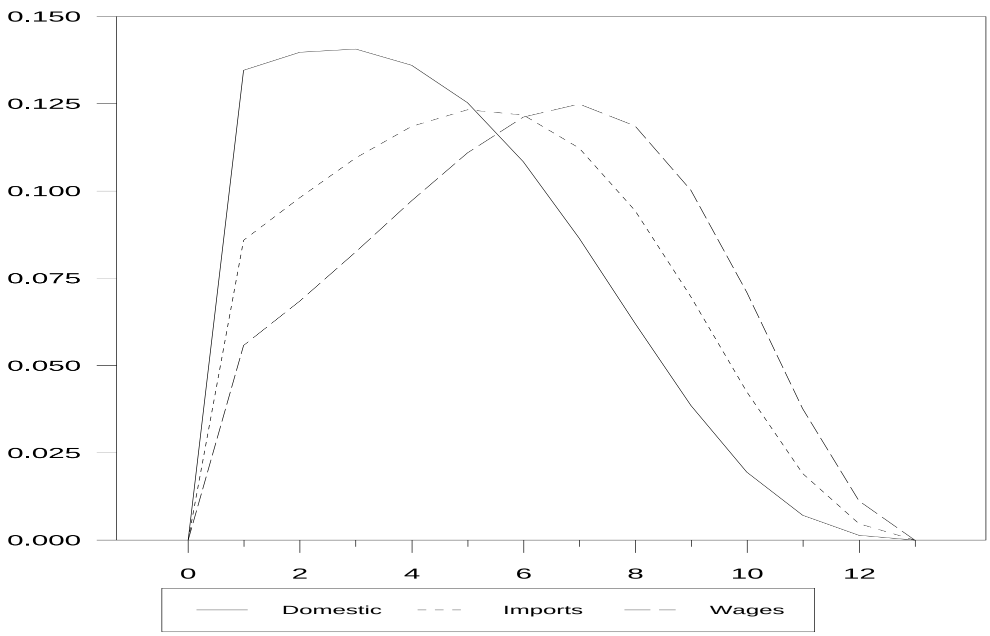
respectively, and Christiano, Eichenbaum, and Evans (2001) find 2 and 3 quarters for U.S. domestic prices and wages, respectively.

4 Model Dynamics

In this section we discuss the structural model's dynamics, focusing on the behaviour of inflation and investment. Figures 4 through 6 show the impulse responses from the model as well as the responses and associated 5 and 95 per cent confidence intervals from the VAR. Before we discuss inflation and investment dynamics in detail in sections 4.1 and 4.2, respectively, we briefly discuss some other interesting results.

The first concerns consumption, for which the model produces a hump-shaped response to both an interest rate and exchange rate shock (Figures 5 and 6), which is attributable to the assumption of habit formation in preferences for consumption (see Christiano, Eichenbaum, and Evans 2001). The model also matches well the magnitude and timing of the data-based response

Figure 3 - Distribution of Contract Lengths



to an interest rate shock. On the other hand, in an exchange rate shock the response predicted by the structural model is both larger and earlier than that suggested by the data.

Another key discrepancy is apparent between the structural and VAR models' exchange rate response to an interest rate shock; specifically, the initial response of the exchange rate is much larger in the structural model than in the VAR. This feature, which is related to the assumption of uncovered interest rate parity, has important implications for the dynamics of prices, which will be discussed below.

4.1 Inflation

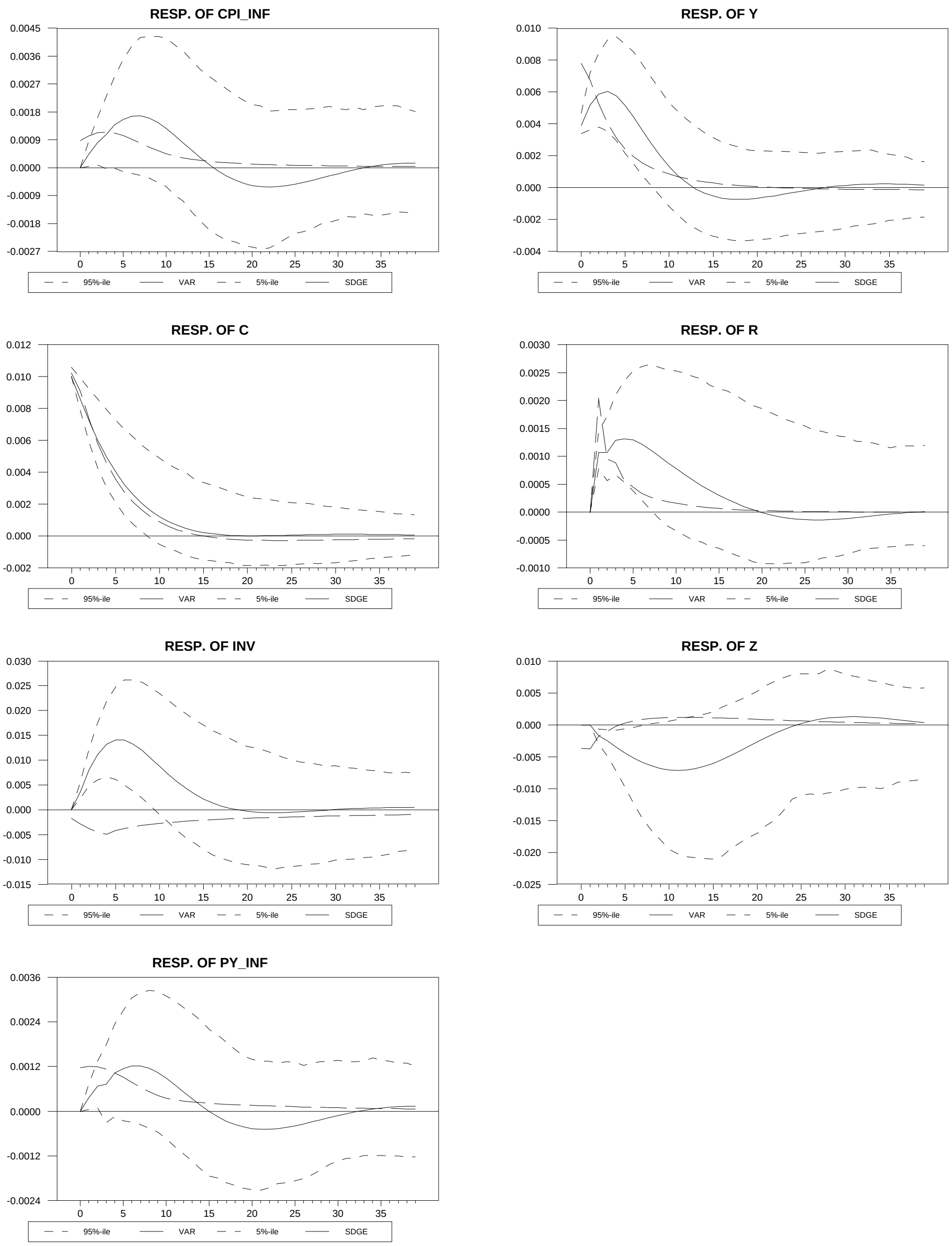
The issue of the inflation persistence of price/wage contracting models is discussed at length in Ball (1991), Fuhrer and Moore (1995), Galí and Gertler (1999), and Mankiw (2001). The issue stems from the fact that the Calvo and Taylor set-ups are fully forward looking when cast in inflation space. Consequently, inflation tends to be a "jump" variable, attaining its maximum response in the first period after a shock, which is counterfactual for Canada. Furthermore, inflation will tend to be only as persistent as its forcing variable(s). This has led authors such as Galí and Gertler (1999) and Christiano, Eichenbaum, and Evans (2001) to modify the basic Calvo framework in order to effectively introduce a lag of inflation into the Phillips-curve specification. Galí and Gertler assume that some fraction of price-setters do not form expectations about the future rationally but rather rely on a rule of thumb that relates current prices to lagged prices and lagged inflation. Christiano, Eichenbaum, and Evans (2001), on the other hand, assume that it is costly to reoptimize one's price and therefore this is done only periodically. Between optimizations, firms follow a simple updating rule that again relates current prices to lagged prices and lagged inflation. Both of these schemes lead to Phillips curves with a single lag and lead of inflation, thereby allowing them to generate "hump-shaped" impulse responses, provided the shock itself contains some intrinsic persistence.

These modifications would seem to suggest that overlapping contracts alone are not sufficient to generate sluggish adjustment of the inflation rate. However, as Figure 4 indicates, this is not necessarily the case. In the presence of a shock to consumers' rate of time preference, the initial response of CPI inflation is about 75 per cent of its peak response, which occurs in period 4 of the shock, despite an average price contract duration of only 4.4 quarters. While the peak response does occur sooner than in the VAR, the model response lies everywhere inside the VAR's confidence bands beyond the first 2 quarters. Thus, while it appears that we cannot explain the entire dynamic path of inflation for a demand shock, this model takes a step in that direction without having to appeal to ad hoc additions to the pricing structure. How is this possible? To isolate the exact difference in the DKW model that facilitates the delayed response of inflation, it is instructive to first return to the equation that relates current aggregate prices to current and past contract prices (equation (27)):

$$P_{d,t} = \left(\sum_{k=0}^{j-1} \varpi_{d,k} (p_{d,t-k})^{1-\epsilon_p} \right)^{\frac{1}{1-\epsilon_p}},$$

which, when log-linearized and differenced around a zero-inflation steady

Figure 4 - Demand Shock



Note: The solid line corresponds to the VAR response.

state, may be written as

$$\Delta \widehat{P}_{d,t} = \sum_{k=0}^{j-1} \varpi_{d,k} \Delta \widehat{p}_{d,t-k}.$$

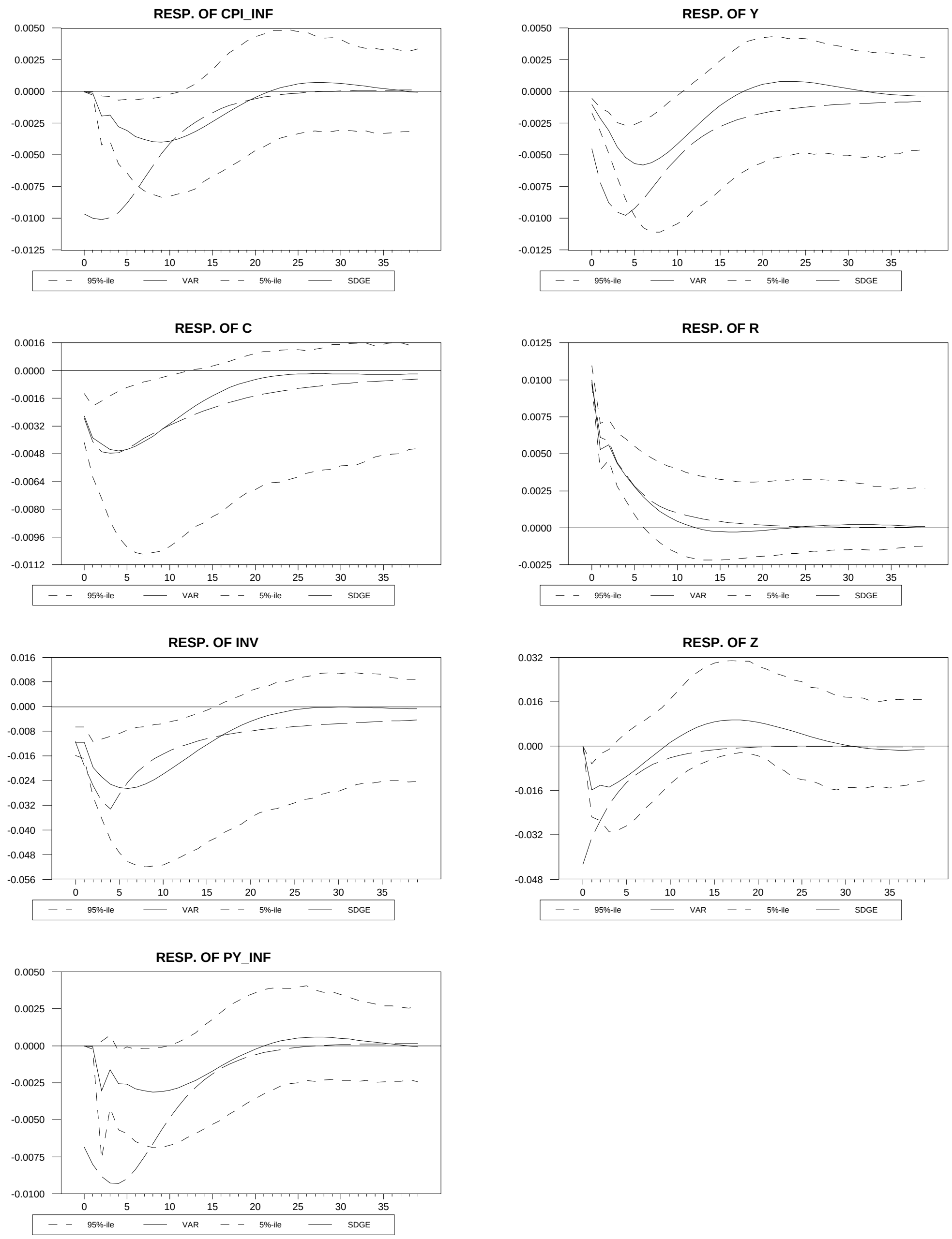
For simplicity, we consider two periods only and ask the question: under what circumstances will inflation (shock minus control) in period $t+1$ exceed that in period t, $\Delta \widehat{P}_{d,t+1} > \Delta \widehat{P}_{d,t}$? This will occur only if the following condition is satisfied:

$$\frac{\Delta \widehat{p}_{d,t+1}}{\Delta \widehat{p}_{d,t}} > \left(1 - \frac{\varpi_1}{\varpi_0}\right) = \alpha_1. \tag{60}$$

Thus, aggregate price inflation will rise by more in period $t+1$ than in t if and only if the ratio of contract price inflation rates across the two periods exceeds the conditional adjustment probability for contracts that were signed in period t. Of course, if $\Delta \widehat{p}_{d,t+1} < 0$, then this condition cannot be satisfied. But, provided that the contract price continues to rise in period $t+1$, the lower α_1 is, the more likely it will be to satisfy condition (60). Herein lies the key difference between the two pricing models. In order to have a sufficiently low value for α (recall that $\alpha_i = \alpha$ with Calvo pricing), such that (60) is fulfilled, the average contract length would be implausibly long. For instance, our estimated model suggests that, for import prices, the probability of a price change after 1 quarter is about 10 per cent (much lower than typically found). In the Calvo model, this would imply an average contract duration of 10 quarters, or 2.5 years, as opposed to 5 quarters in our model. In addition, such a long average contract length would have adverse consequences for the elasticity of inflation with respect to fluctuations in output, perhaps rendering the model incompatible with history.

For the other two shocks (Figures 5 and 6) we reach a similar conclusion, at least for GDP inflation. In the case of the exchange rate shock, GDP inflation attains its maximum deviation from steady state in period 8 and its initial response is about 50 per cent of its maximum. For the monetary policy shock, the response is slightly faster: 5 quarters and a first-period response that is 75 per cent of the maximum. CPI inflation, by contrast, peaks in the third (monetary policy shock) and first periods. These responses are

Figure 5 - Interest Rate Shock



Note: The solid line corresponds to the VAR response.

essentially due to two factors. The first, which is most evident in the interest rate shock, is the excess sensitivity of the exchange rate to interest rates relative to the VAR. Specifically, the real exchange rate appreciates by approximately three times as much as the VAR in the first quarter of the shock. The second factor relates to the speed with which the nominal exchange rate returns towards control (the monotonic depreciation in this case is consistent with uncovered interest parity). The VAR response, in contrast, is both more gradual and more persistent, helping to generate a hump-shaped response for both measures of inflation. How do these weaknesses manifest themselves into a jump-like response for CPI inflation? First, the speed with which the exchange rate returns to control means that import inflation behaves like a "jumper": those firms that sign price contracts in the period of the shock bear the brunt of the shock and therefore instigate the largest price change. Second, the size of the exchange rate movement means that import inflation dominates the CPI inflation response despite having a share of only about 20 per cent. This behaviour reveals a definite weakness of the model and is an issue that will be addressed in future versions.

4.2 Investment

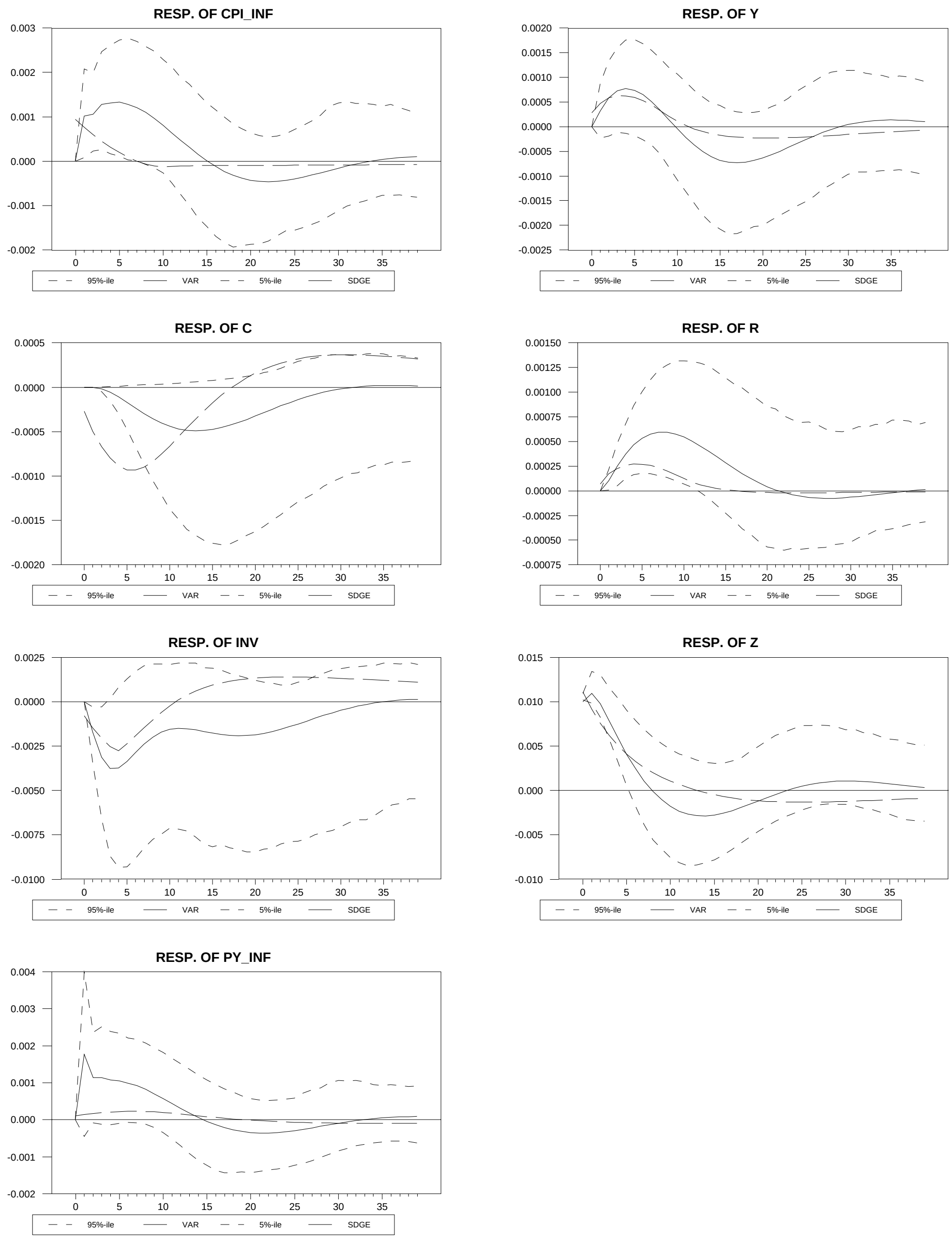
A common explanation for sluggish investment behaviour in optimizing models is the presence of capital adjustment costs. In models that incorporate costs of this type, typically referred to as q-models, firms incur a deadweight convex cost to adjusting the capital stock.[18] A key shortcoming of this assumption is that it is insufficient to explain the hump-shaped response of investment to shocks that is often found in the data.[19] Recent studies have pointed to higher-order adjustment costs (i.e., costs associated with changing the level of investment rather than the level of capital) to explain this phenomenon.[20] Christiano, Eichenbaum, and Evans (2001) show that this assumption is sufficient to deliver a response of investment to a monetary policy shock similar to that produced by an estimated VAR.

[18]For recent examples in a DSGE framework, see Bouakez, Cardia, and Ruge-Murcia (2002) and Kim (2000).

[19]Christiano, Eichenbaum, and Evans (2001) find that, with adjustment costs specified this way, the investment response to a monetary policy shock is greatest in the first period in which investment can respond.

[20]See, for example, Christiano, Eichenbaum, and Evans (2001) and Smets and Wouters (2003).

Figure 6 - Exchange Rate Shock



Note: The solid line corresponds to the VAR response.

Time-to-build provides an alternative explanation for the observed investment dynamics. Time-to-build implies that investment will affect the productive capital stock with a lag longer than the typical assumption of one quarter. In addition, to the extent that investment plans are inflexible ex post, current investment will be pinned down by past decisions. Investment will thus respond gradually to unforeseen changes in economic conditions, leading to behaviour similar to the hump shape seen in the data.

Indeed, Figures 5 and 6 indicate that our model is able to mimic well the investment impulse responses from the VAR for an interest rate and exchange rate shock. In particular, the structural model produces responses that are close to the VAR in shape, magnitude, and timing. However, Figure 5 shows that the structural model is unable to explain the VAR response to a consumption shock; the VAR produces a statistically significant increase in investment following the shock, whereas the structural model produces a slight decrease.[21]

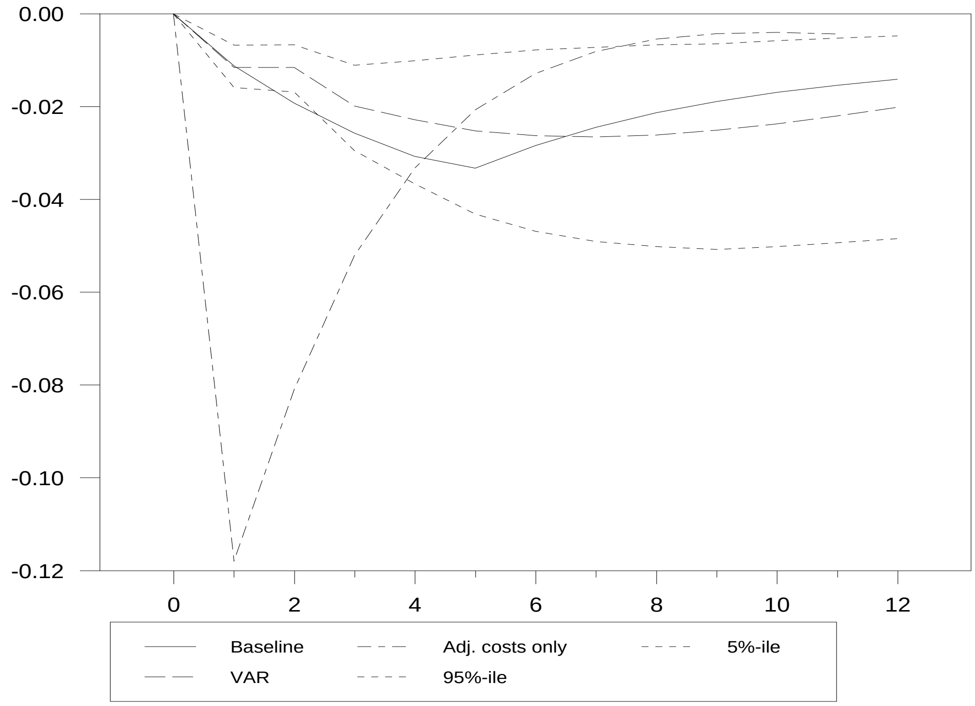
Figure 7 shows the investment response to an interest rate shock in the structural and VAR models as well as a version of the structural model in which we remove the time-to-build constraints.[22] Time-to-build enables the structural model to better explain the investment dynamics of the VAR relative to the model without time-to-build. Furthermore, the model without time-to-build requires a much higher burden of capital adjustment costs to match the magnitude of the VAR response; to produce the impulse response shown in Figure 7, the adjustment cost parameter, χ, was increased from 20 to 100 in the model without time-to-build. A χ of 100 implies that, in steady state, with a quarterly depreciation rate of 2.5 per cent and an investment-to-output ratio of 12 per cent, capital adjustment costs eat up 15 per cent of output, versus 3 per cent for a χ of 20.[23] Overall, the model with time-to-build is able to reproduce the magnitude of the VAR response under relatively small capital adjustment costs and the shape of the VAR response without the assumption of higher-order investment adjustment costs.

[21] This discrepancy is due to the correlation in the data between investment and output, the so-called "accelerator" effect, which our structural model does not capture. An alternative explanation is that the average correlation between output and investment is due to technology shocks that are not identified in the VAR.

[22] Formally, time-to-build is removed by setting $\phi_0 = 1$ and $\phi_1 ... \phi_4 = 0$ in (6).

[23] A χ of 20 is closer to, but still slightly higher than, previous estimates in the empirical q-theory literature. See Erceg and Levin (2003) for a discussion.

Figure 7 - Interest Rate Shock



5 Conclusion

In this paper, we have developed a structural small open-economy model in an attempt to understand the dynamic relationships in Canadian macroeconomic data. Particular attention has been paid to two key differences between the set-up of our model and that which is typical in the recent literature. First, for prices and wages, we used the time-dependent staggered contracting model described in Dotsey, King, and Wolman (1999) and Wolman (1999), rather than the Calvo (1983) specification. Second, we modelled investment in the framework of time-to-build with ex-post inflexibilities à la Edge (2000a, b), instead of assuming investment adjustment costs. In addition to sticky prices and time-to-build, the model contains many of the rigidities emphasized in the recent DSGE literature. We have shown that the model provides a reasonably good explanation for many of the dynamic properties of aggregate Canadian data, in that most of the model's impulse responses fall within the 5 and 95 per cent confidence intervals of the responses from an estimated VAR.

Although we view this study as a promising first step, our results reveal two key weaknesses. First, the model's initial exchange rate response to an interest rate shock is much stronger than is suggested by the data. On its own, this feature would result in too much inflation sensitivity, but it is compounded by the excessive speed with which import prices respond to exchange rate movements. Second, in the short run, inflation is generally more responsive to shocks in the structural model than in the data. One explanation for inflation's slower response in the data may be the indexation of wages to the inflation rate. If included in the structural model, wage indexation would likely slow the response of marginal cost, and thus inflation, to shocks.

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Table 1: Calibrated Parameters

Parameter	Value
Preferences	
β	0.99
Competition	
ϵ_p	5.0
ϵ_w	5.0
Production	
δ	0.60
ω	0.05
θ	-20
$\phi_0, ..., \phi_3$	4.85
Import shares	
γ_c	0.8
γ_I	0.5
γ_x	0.73

Table 2: Estimated Parameters

Parameter	Value
Contract duration	
$\mathcal{S}_d$	0.20
$\mathcal{S}_m$	0.25
$\mathcal{S}_w$	0.30
Production	
σ	0.53
χ	20
ρ	0.1
ϕ_4	10
Preferences	
μ	0.92
η	0.90
ξ	0.85
Trade	
φ	2.0
ϑ	0.5
Monetary policy	
ϱ_1	0.56
ϱ_2	0.31
ϱ_3	0.26
ϱ_4	-0.25
Risk	
ς	0.021
Shocks	
a^κ	0.88
a^β	0.00
a^R	0.29

Note: No standard errors are shown, due to convergence problems. To avoid convergence on a local minimum, it is necessary to provide starting values for the parameters that are thought to be quite close to the optimum values. Doing so, however, severely compromises the accuracy of the Hessian matrix, which is used to compute the standard errors. Future work will focus on this problem.

Appendix A: The Steady State

In this appendix, we describe the model's steady state under the assumption that, in the steady state, inflation and the growth rate of labour-augmenting technical progress are zero. Appendix B describes the stationary version of the dynamic model when these assumptions are relaxed.

We begin with the prices of domestically produced and imported goods. In steady state with zero inflation and no technological growth, the equation for the price of the domestically produced good (equations (26) and (27) in the dynamic model) breaks down to a relationship between the real marginal cost (λ/P_d, denoted $\tilde{\lambda}$) and the inverse of the markup:

$$\tilde{\lambda} = \left(\frac{\epsilon_p - 1}{\epsilon_p}\right), \tag{A1}$$

whereas the real price of the imported good (P_m/P_d, denoted $\tilde{P}_m$) is equal to the markup times the real exchange rate (eP^*/P_d, denoted $\tilde{e}$):

$$\tilde{P}_m = \left(\frac{\epsilon_p}{\epsilon_p - 1}\right)\tilde{e}. \tag{A2}$$

In the no-growth steady state, the nominal-wage equations (41) and (52) can be rearranged to yield an equation for the long-run labour supply in terms of the consumer wage and, owing to the additive non-separability of consumption and labour in the utility function, consumption:

$$L = \left(\frac{\left(\frac{\epsilon_w - 1}{\epsilon_w}\right)\frac{\tilde{W}}{\tilde{P}^c}}{\tilde{C}(1-\xi)}\right)^{\eta}, \tag{A3}$$

where $\tilde{P}^c = P^c/P_d$, $\tilde{W} = W/AP_d$, and $\tilde{C} = C/A$. This equation, together with the steady-state analogue to the dynamic consumption Euler equation (49),

$$\tilde{P}^c\tilde{\Phi} = (\tilde{C}(1-\xi))^{-1/\mu}\exp\left(\frac{\eta(1-\mu)}{\mu(1+\eta)}\cdot L^{1+1/\eta}\right), \tag{A4}$$

where $\tilde{\Phi} = A^{1/\mu} P_d \Phi$, jointly determine the household's long-run labour supply and consumption decisions. To ensure a stationary steady state, we require that $\tilde{\Phi}_t$ be stable, which, when combined with the Bond Euler equation (50), yields the stability condition

$$\beta = (1 + R)^{-1}, \tag{A5}$$

where R is the equilibrium real interest rate.

The domestic share of consumption depends on its relative price as

$$\tilde{C}_d = (1 - \gamma_c) \tilde{P}^{c\varphi}. \tag{A6}$$

The domestic share of investment and exports can be written in the same manner. Also, we can rewrite the equation for exports as:

$$\tilde{X}_t = \gamma_{z^*} \cdot (\tilde{P}_t^{*^x})^{-\vartheta} \tilde{Z}_t^*, \tag{A7}$$

where $\tilde{X} = X/A$, $\tilde{P}^{*^x} = P^{*^x}/P^*$, and $\tilde{Z}^* = Z^*/A$. Thus, the export share of the domestic economy is related negatively to the relative price of exports (in the foreign currency) and positively to the relative size of the foreign economy.

Appendix B: The Stationary Dynamic Model

To compute the stationary model, we deflate all real variables by the level of labour-augmenting technological progress and all nominal variables by the domestic price level.[1] The result is a model expressed only in terms of stationary ratios and growth rates.

We begin with some definitions that will simplify the exposition. First, we define $\tilde{\pi}_{d,s,t}$ as the cumulative change in the domestic price level between periods t and s and $\pi_{d,t}$ as the one-period change in the domestic price level:

$$\tilde{\pi}_{d,s,t} = \frac{P_{d,s}}{P_{d,t}}, \pi_{d,t} = \frac{P_{d,t}}{P_{d,t-1}}. \tag{B1}$$

Analogously, $\tilde{g}_{s,t}$ is the cumulative change in labour-augmenting technical progress and g_t is the one-period change:

$$\tilde{g}_{s,t} = \frac{A_s}{A_t}, g_t = \frac{A_t}{A_{t-1}}. \tag{B2}$$

We start with the stationary analog to equation (26), the equation for the aggregate domestic contract price. We re-express this equation as the contract price of domestic goods relative to the aggregate domestic price level. We denote this ratio $\tilde{p}_{d,t}$. The equation can be rewritten as:

$$\tilde{p}_{d,t} = \left(\frac{\epsilon_p}{\epsilon_p - 1}\right) \mathcal{E}_t \left(\frac{\sum_{s=t}^{t+j-1} \mathcal{R}_{t,s} \Lambda_{st} \tilde{\lambda}_{is} \tilde{\pi}_{d,s,t}^{1+\epsilon_p} \tilde{Y}_s \tilde{g}_{s,t}}{\sum_{s=t}^{t+j-1} \mathcal{R}_{t,s} \Lambda_{st} \tilde{\pi}_{d,s,t}^{\epsilon_p} \tilde{Y}_s \tilde{g}_{s,t}}\right), \tag{B3}$$

where $\tilde{Y}_s$ is output divided by labour-augmenting technical progress and $\tilde{\lambda}_s$ is real marginal cost $(\lambda_{is}/P_{d,s})$. Similarly, the aggregate contract price for importers can be rewritten as:

$$\tilde{p}_{m,t} = \left(\frac{\epsilon_p}{\epsilon_p - 1}\right) \mathcal{E}_t \left(\frac{\sum_{s=t}^{t+j-1} \mathcal{R}_{t,s} \zeta_{s-t} \tilde{P}_{m,s}^{\epsilon_p} \tilde{e}_s \tilde{\pi}_{d,s,t}^{1+\epsilon_p} \tilde{M}_s \tilde{g}_{s,t}}{\sum_{s=t}^{t+j-1} \mathcal{R}_{t,s} \zeta_{s-t} \tilde{P}_{m,s}^{\epsilon_p} \tilde{\pi}_{d,s,t}^{\epsilon_p} \tilde{M}_s \tilde{g}_{s,t}}\right), \tag{B4}$$

[1] For the most part, we present the stationary form of equations that contain lags and leads. The stationary transformations of those equations that contain only contemporaneous relationships are essentially unchanged from their non-stationary counterparts.

where $\tilde{e}_s$ is the real exchange rate, or $e_s P^*/P_{d,s}$, $\tilde{M}_s = M_s/A_s$, and $\tilde{P}_{m,s} = P_{m,s}/P_{d,s}$. The equation for the aggregate domestic price can be rewritten as:

$$1 = \left(\sum_{k=0}^{j-1} \varpi_{d,k} (\frac{\tilde{p}_{d,t-k}}{\tilde{\pi}_{d,t-k,t}})^{1-\epsilon_p} \right)^{\frac{1}{1-\epsilon_p}}, \tag{B5}$$

and the equation for the aggregate imported price can be rewritten as:

$$\tilde{P}_{m,t} = \left(\sum_{k=0}^{j-1} \varpi_{d,k} (\frac{\tilde{p}_{m,t-k}}{\tilde{\pi}_{d,t-k,t}})^{1-\epsilon_p} \right)^{\frac{1}{1-\epsilon_p}}. \tag{B6}$$

For the firm's labour demand and investment decisions, we begin by deflating output (dynamic equation (9)) by the level of labour-augmenting technical progress:

$$\tilde{Y}_{it} = \mathcal{F}(L_{it}, u_{it}\tilde{K}_{it}) - \frac{\chi}{2\tilde{K}_{it}} \tilde{I}_{it}^{E^2} - \psi \left(1 - e^{\rho(u_{it}-1)}\right) \tilde{K}_{it}, \tag{B7}$$

where $\tilde{I}_{it}^E = I_{it}^E/A_t$, $\tilde{K}_{it} = k_{it}/A_t$, and

$$\mathcal{F}(L_{it}, u_{it}\tilde{K}_{it}) = \mathcal{F}(A_t L_{it}, u_{it} K_{it})/A_t. \tag{B8}$$

In steady state, the nominal wage grows at the product of the domestic inflation rate and the growth rate of labour-augmenting technical progress. We therefore write the stationary form of the labour demand first-order condition (19) as:

$$\tilde{W}_t = \tilde{\lambda}_{it} \mathcal{F}_l(L_{it}, u_{it}\tilde{K}_{it}), \tag{B9}$$

where $\tilde{W}_t = W_t/A_t P_{d,t}$ and $\tilde{\lambda}_{it} = \lambda_{it}/P_{d,t}$ (real marginal cost). For investment, we begin by defining a stationary shadow value of capital, $\tilde{q}_t = q_t/P_{d,t}$, which evolves according to:

$$\tilde{q}_t = \mathcal{E}_t \pi_{d,t+1} \mathcal{R}_{t,t+1} \tag{B10}$$

$$\times \left[\tilde{\lambda}_{i,t+1} \left(\mathcal{F}_k(\cdot) + \frac{\chi}{2} \left(\frac{\tilde{I}^E_{i,t+1}}{\tilde{K}_{i,t+1}} \right)^2 + \psi \left(1 - e^{\rho(u_{it}-1)} \right) \right) + (1-\omega)\tilde{q}_{t+1} \right],$$

where $\tilde{I}^E_{i,t} = I^E_{i,t}/A_t$ and $\tilde{K}_{i,t} = k_{i,t}/A_t$. The stages of investment expenditure are also redefined as $\tilde{I}_{it,t+k} = I_{it,t+k}/A_t$, which evolves according to:

$$\tilde{I}_{it,t+k} = \mathcal{E}_t \left[\tilde{I}^E_{i,t+k} \tilde{g}_{t+k,t} \left[\frac{\phi^\theta \mathcal{R}_{t,t+k} \tilde{\pi}_{d,t+k,t}}{\tilde{P}^I_t} \left(\tilde{q}_{t+k} - \chi \tilde{\lambda}_{it+k} \frac{\tilde{I}^E_{i,t+k}}{\tilde{K}_{i,t+k}} \right) \right]^{\frac{1}{1-\theta}} \right], \text{(B11)}$$

where $\tilde{P}^I_t$ is the price of investment goods relative to the price of domestic goods. Capital evolves according to:

$$\tilde{K}_{it} = \frac{1}{g_t} \left((1-\omega)\tilde{K}_{i,t-1} + \tilde{I}^E_{i,t-1} \right). \tag{B12}$$

The firm's capital utilization decision is rewritten as:

$$\mathcal{F}_u(L_{it}, u_{it}\tilde{K}_{it}) = -\psi \rho e^{\rho(u_{it}-1)} \tilde{K}_{it}. \tag{B13}$$

We next turn to the consumer's problem, beginning with the consumption Euler equation (49). The stationary version of this equation is as follows:

$$\tilde{P}^c_t \tilde{\Phi}_t = (\tilde{C}_t - \tilde{H}_t)^{-1/\mu} \exp \left(\frac{\eta(1-\mu)}{\mu(1+\eta)} \cdot L_{ht}^{1+1/\eta} \right), \tag{B14}$$

where $\tilde{\Phi}_{ht} = \Phi_{ht} P_{d,t} A_t^{1/\mu}$, $\tilde{C}_{ht} = C_{ht}/A_t$, and $\tilde{H}_t = H_t/A_t$. To ensure a stable steady-state equilibrium, we require the term $\tilde{\Phi}_{ht}$ to be stationary in steady state, which in turn implies that, in steady state, the Lagrangian Φ_{ht} must grow by the rate $(g_t^{1/\mu}\pi_{d,t})^{-1}$. This can be combined with the first-order condition for bonds (50) to yield the stability condition, which states that, in equilibrium, the following relationship exists between the real interest rate, the growth rate of labour-augmenting technical progress, and the rate of time preference:

$$\beta(1+R_t) = g_t^{1/\mu} \pi_{d,t}. \tag{B15}$$

We can then write an equation for the deflated real wage ($\tilde{W}_{ht} = \frac{W_{ht}}{A_t P_{d,t}}$) as:

$$\tilde{W}_{ht} = \left(\frac{\epsilon_w}{\epsilon_w - 1}\right) \tag{B16}$$

$$\times \mathcal{E}_t \left(\frac{\sum_{s=t}^{t+q-1} \mathcal{R}_{t,s} \Xi_{s-t} \tilde{P}_s^c \tilde{\Phi}_{h,s} \tilde{W}_s^{\upsilon} L_s^{1+1/\eta} (\tilde{C}_{hs} - \tilde{H}_s) \tilde{\pi}_{d,s,t}^{\upsilon} \tilde{g}_{s,t}^{1-1/\mu+\upsilon}}{\sum_{s=t}^{t+q-1} \mathcal{R}_{t,s} \Xi_{s-t} \tilde{\Phi}_{h,s} \tilde{W}_s^{\epsilon_w} L_s \tilde{\pi}_{d,s,t}^{\epsilon_w - 1} \tilde{g}_{s,t}^{\epsilon_w - 1/\mu}} \right)^{\eta/(\epsilon_w+\eta)},$$

where $\tilde{P}_s^c$ is the consumption deflator divided by the domestic deflator, $\tilde{W}_s$ is the aggregate nominal wage divided by the product of labour-augmenting technology and the domestic deflator, and we define

$$\upsilon = \epsilon^w (1 + 1/\eta). \tag{B17}$$

Turning to the open-economy links, domestic consumption as a fraction of total consumption ($C_{d,t}/C_t$, denoted $\tilde{C}_{d,t}$) can be expressed as:

$$\tilde{C}_{d,t} = (1 - \gamma_c) \tilde{P}_t^{c\varphi}. \tag{B18}$$

The domestic share of investment can be written in the same manner. We can rewrite the equation for exports as:

$$\tilde{X}_t = \gamma_{z^*} \cdot (\tilde{P}_t^{*^x})^{-\vartheta} \tilde{Z}_t^*, \tag{B19}$$

where $\tilde{X}_t = X_t/A_t$, $\tilde{P}_t^{*^x} = P_t^{*^x}/P_t^*$, and $\tilde{Z}_t^* = Z_t^*/A_t$. The stationarity of this ratio is ensured by the assumption, as previously noted, that steady-state foreign expenditure and domestic labour-augmenting technological progress grow at the same rate.

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