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Abstract

This paper summarizes the results of recent research evaluating the Bank of Canada's Quarterly Projection Model (QPM). Because QPM consists of a steady-state model and a dynamic model, our evaluation work consists of two parts.

The first part assesses the calibration of QPM's core steady-state using a variant of Canova's (1994, 1995) Monte Carlo approach. Using parameter values drawn from prior distributions, we assess QPM's sensitivity to various plausible parameter values. Our approach differs somewhat from the recent literature in that it specifically takes into account the uncertainty that surrounds the estimates of the steady-state values we are trying to evaluate. Instead of attempting to match exactly the desired properties of the data, we calculate confidence intervals around the mean of the variable we wish to match, subsequently discarding parameterizations that result in simulated data falling outside this interval.

The second part of the evaluation uses artificial data, generated stochastically with QPM, to test the dynamic model's ability to replicate key historical moments. Autocorrelations, reduced-form regressions, and temporal bivariate correlations are used to compare historical data with data produced by QPM. We also assess the sensitivity of our results to the structure of the stochastic shocks and the specification of the monetary policy rule.

The results of the two evaluations reveal some strengths and weaknesses in the model. For example, while most of the parameter calibrations in the steady-state model appear reasonable, there are some parameters for which other values may be more appropriate. Similarly, while the dynamic model can replicate most of the key historical moments, some work is required to develop the linkages between foreign and domestic variables.

JEL classification: C52, E17, E30, E37
Bank classification: Economic models

Résumé

Les auteurs exposent les résultats des recherches qu'ils ont menées récemment sur le Modèle trimestriel de prévision (MTP) de la Banque du Canada. Comme le MTP se compose d'un modèle de régime permanent et d'un modèle dynamique, son évaluation se présente en deux volets.

Dans le premier, les auteurs évaluent l'étalonnage du régime permanent de base du MTP au moyen d'une variante de la méthode de Monte-Carlo utilisée par Canova (1994 et 1995). À l'aide de valeurs tirées de lois de probabilité a priori, ils estiment la sensibilité du MTP à diverses valeurs paramétriques plausibles. Leur méthode diffère quelque peu de celle employée dans les études récentes, en ce sens qu'elle tient compte spécifiquement de l'incertitude qui entoure les estimations des paramètres de régime permanent qu'ils tentent d'évaluer. Plutôt que d'essayer de reproduire exactement les propriétés souhaitées des variables, les auteurs calculent des intervalles de confiance autour de la moyenne de la variable qu'ils désirent reproduire, pour ensuite éliminer les valeurs des paramètres qui font déborder les données simulées de cet intervalle.

Dans le second volet de leur évaluation, les auteurs s'appuient sur des données artificielles, générées de manière stochastique à l'aide du MTP, pour tester la capacité du modèle dynamique à reproduire des moments historiques clés. Ils se servent d'autocorrélations, de régressions à forme réduite et de corrélations temporelles bivariées pour comparer les données historiques avec les données produites au moyen du MTP. Ils évaluent en outre la sensibilité de leurs résultats à la structure des chocs stochastiques et à la formulation de la règle de politique monétaire.

Les deux évaluations font ressortir certains points forts et certaines faiblesses du modèle. Par exemple, bien que l'étalonnage de la plupart des paramètres semble raisonnable dans le modèle de régime permanent, des valeurs plus adéquates pourraient être affectées aux paramètres dans certains cas. De même, si le modèle dynamique peut reproduire la plupart des moments historiques clés, les liens entre les variables intérieures et étrangères méritent d'être étoffés.

Classification JEL : C52, E17, E30, E37
Classification de la Banque : Modèles économiques

1. Introduction

Dynamic general-equilibrium models have become the standard method for analyzing many important economic questions, including those related to monetary policy issues. Following the influential work of Lucas (1976) and Kydland and Prescott (1982), many economists started moving away from reduced-form models towards dynamic general-equilibrium models to analyze different economic issues. The internal consistency of dynamic general-equilibrium models, which lends itself nicely to examining economic problems, often implies, however, a layer of complexity that makes it difficult to evaluate the performance of a model against a well-specified metric. The evaluation of such models concerns not only their internal coherence, which forces the researcher to work with the complete model, but also multiple sources of error when performing the following tasks: (i) selecting the specific functional forms linking the endogenous to the exogenous variables of the model, (ii) parameterizing the structure of the model, and (iii) specifying a distribution for the exogenous processes. Consequently, many of the reduced-form tests that have been used so effectively to evaluate 1970s-style models cannot be applied to dynamic general-equilibrium models. Other approaches must be used.

In this paper, we merge the results of two recent lines of research to report on our attempts to evaluate the Bank of Canada's Quarterly Projection Model (QPM). The bank developed QPM in 1993, and although it was evaluated before it went into production, the evaluation focused on the properties of the model in deterministic simulation. Important issues, such as confronting the stochastic properties of real-world data using formal statistical methods or evaluating the calibration of QPM, were not performed in a systematic manner. Improvements in computing speed and refinements in the model-evaluation literature allow us to assess the stochastic properties and calibration of QPM using formal metrics.

QPM is used by Bank staff to prepare economic projections and to conduct research on policy analysis. Its purpose is to bridge the gap between forecasting models and more structural models designed solely for policy analysis. In a strict sense, therefore, QPM is distinct from the dynamic general-equilibrium model typically observed in the economic literature. QPM is calibrated to match a wide variety of stylized facts of the Canadian economy. For example, estimated vector autoregressions (VARs) have been used to establish short-run responses that are consistent with the historical data. Empirical results from other research have also been used in selecting some key parameters.

QPM consists of a steady-state model (SSQPM) and a dynamic model. SSQPM, based on the Blanchard-Weil model of household behaviour, describes the determinants of long-term choices made by profit-maximizing firms and overlapping generations of consumers, given the policies of the fiscal and monetary authorities, all in the context of an open economy with important ties to the rest of the world. The economic behaviour of these agents, given their long-run budget constraints and the market-clearing conditions of an open economy, determine the long-run equilibrium or steady state to which the dynamic model converges. Black et al. (1994) provide a detailed description of SSQPM.

Dynamic QPM, in contrast, traces out the path of the economy from its initial conditions to the steady state determined by SSQPM. The dynamic model has two key features. First, agents are forward looking. In particular, they act based on intertemporal optimization, conditioned by expectations that are modelled as a mixture of forward-looking and backward-looking components. The evolution of these expectations plays an important role in the overall dynamic response to innovations. In addition, adjustment of both prices and quantities is assumed to be costly (more specifically, quadratic), so there is an intrinsic component to the model's dynamic properties.

The second key feature of the dynamic model is that it is stable and converges on the equilibrium defined by SSQPM. In other words, the model provides a complete and consistent solution for all stocks and flows. There are three key stocks in QPM: government bonds, private sector physical capital, and net foreign assets. The steady-state levels of these stocks are consistent with the economic theory in SSQPM, and the necessary flows are supported by relative price movements. In other words, if a shock affects a stock, then the flows will adjust such that the model moves to its steady state.

Our evaluation, like the QPM system, consists of two parts: (i) assessing the calibration of SSQPM, and (ii) evaluating the ability of dynamic QPM to replicate important statistical properties found in the historical data. The first part of our evaluation assesses the calibration of SSQPM using a variant of Canova's (1994, 1995) Monte Carlo approach. Our procedure formalizes the choice of parameters and the evaluation of the model, and provides a systematic way of conducting a sensitivity analysis of model parameters. The second part examines the dynamic properties of QPM by focusing on artificial data generated via stochastic simulations. In particular, we investigate the ability of QPM to generate artificial data that reproduces empirical correlations: autocorrelations, bivariate correlations, and partial correlations of key macroeconomic variables. From these exercises we hope to determine which areas of QPM we should focus on in our ongoing model development efforts. We emphasize that the purpose of this paper is to document the margins that the model performs well and the margins that it does not; we leave a full diagnosis of the unsatisfactory results that we uncover for future work.

This paper is organized as follows. Section 2 describes our methodology for evaluating SSQPM and reports the results of its application to the steady-state model. Section 3 details the construction of the stochastic environment in which we simulate QPM, and presents evidence highlighting the ability of dynamic QPM to match important statistical moments found in the historical data. Section 4 considers whether the evaluation of dynamic QPM is affected by altering certain aspects of the stochastic environment or the monetary policy reaction function embedded in QPM. Section 5 provides concluding comments and identifies areas for future work.

2. Evaluation of the Steady-State Model, SSQPM

SSQPM summarizes our beliefs about the steady-state structure of the Canadian economy. It is a set of theoretical relationships, involving different sectors of the economy, whose influences are quantified by the model's parameters. As such, the validity of SSQPM is a function both of its theoretical structure and of its parameterization. This paper deals primarily with the calibration of SSQPM, leaving an evaluation of its theoretical structure for future work.

When SSQPM was originally calibrated, the values that the model-builders gave to various parameters no doubt involved a trade-off among three objectives: (i) the parameters should correspond closely with what one expects from economic theory, (ii) the model properties that result from a given choice of parameters should be reasonable, and (iii) the simulated model should reasonably match some long-run properties of the data. The methodology that we use to evaluate the calibration of SSQPM should therefore reflect the same three objectives. Accordingly, we modify Canova's (1994, 1995) methodology, as it applies to the evaluation of steady-state models, to achieve this end.

Our methodology can best be described as an informal Bayesian approach to the evaluation of calibrated models. It is Bayesian because it takes into account two sources of information: (i) our prior beliefs, expressed through prior distributions of the parameters of SSQPM, and (ii) Canadian economic data. It is informal because the influence of the data does not enter the analysis via a formal likelihood function but, instead, is imposed through inequality constraints that the simulated data are required to satisfy.

Section 2.1 describes in greater detail our approach to the calibration evaluation, and section 2.2 discusses the results. Section 2.3 concludes the SSQPM evaluation and suggests some areas for future investigation.

2.1 The SSQPM evaluation methodology

2.1.1 Assigning prior distributions

Following Canova (1994, 1995), our methodology starts with the assignment of prior distributions to the model's parameters. By assigning prior distributions that, for the most part, reflect our interpretation of economic literature, we meet one of the objectives of SSQPM's model-builders: parameter values should correspond closely with economic theory. Furthermore, by allowing for a distribution rather than a single value for each parameter, we can evaluate the sensitivity of model properties to plausible alternative characterizations of SSQPM's calibration.

The objective of this exercise is to trace out the effects of varying the domestic non-policy parameters in SSQPM. As such, we do not allow all the parameters embodied in SSQPM to vary; parameters that apply to "rest-of-world" (ROW) variables, such as those that describe the foreign-production function, and domestic policy parameters, such as tax rates, are held constant.

Prior distributions are assigned on a parameter-by-parameter basis and, where possible, the literature is used to guide the choice of parameter values. For some parameters, economic theory or empirical evidence can help to pin down a range of possible values. In such cases, we can use a normal distribution with a mean and standard deviation reflecting the average and dispersion of estimates in the literature. The distribution can be truncated to reflect theoretical considerations that restrict the admissible parameter values. Alternatively, in cases where the theory and empirical evidence are less helpful, we adopt an uninformative prior, the uniform distribution, while still looking to the literature to provide some guidance for a feasible range of values. Table 1 shows the prior distributions chosen for the parameters of SSQPM.

2.1.2 Evaluating model properties

The end result of our evaluation is an assessment of the model properties of SSQPM. We typically analyze model properties in SSQPM by looking at the effects of various standard shocks. For example, based on the non-Ricardian structure of SSQPM as incorporated through the small-open-economy Blanchard-Yaari model, we would expect that an increase in the government debt-to-output-ratio would reduce consumption, output, and net foreign assets, and cause the exchange rate to depreciate. The government debt-to-output-ratio standard shock in SSQPM simulates the effect of such a shock on the model, and allows us to quantify its results on economic variables of interest.

We assess the sensitivity of model properties to various plausible choices for the parameters of SSQPM as follows: a set of parameters are drawn randomly from the prior distributions, and the effects of our standard shocks are simulated under each parameterization. By repeating this process (10,000 times), we can build up empirical distributions that describe the effects of the various shocks on key variables, and the corresponding likelihood of each outcome. In addition, we can assess the outcome of a shock under SSQPM's current parameterization relative to this distribution. For example, we can evaluate how close outcomes under the current SSQPM calibration are to the central tendency of the empirical distributions. Alternatively, we can gauge the uncertainty associated with the effects of any standard shock with, say, a 95 per cent confidence interval of the empirical distribution.

2.1.3 Matching the model to the data

The methodology we have described to this point follows closely Canova's methodology as he applies it to steady-state models. We introduce modifications that allow us to deal in a more satisfactory way with the question of matching the simulation outcomes to the data. As stated in the introduction to this section, the ability of the model to match the data[1] was undoubtedly an important influence on the original calibration of SSQPM. This follows directly from the fact that SSQPM, together with its dynamic counterpart, QPM, is used to produce an economic projection once each quarter. Nonetheless, the ability of the simulated model to match the data is not the only characteristic that is important. As noted above, the calibration of SSQPM no doubt resulted from trading off this objective against the other two objectives of achieving reasonable model properties and choosing parameters that accord with theory. Our evaluation procedure for the calibration of SSQPM must therefore balance these objectives.

Because of the role that QPM (and SSQPM) plays in producing projections, its data-matching requirements go beyond those of the more recent literature on dynamic general-equilibrium models. Davis and Espinoza (1998), for example, build empirical distributions of model properties by repeated sampling of parameters from their prior distributions, but they do not assess whether the parameterizations under consideration allow model simulations to reasonably match the data. Alternatively, studies where data-matching is a priority typically emphasize the model's ability to replicate a small set of features in the data; i.e., those that the researcher is most interested in for the purpose of their analysis. Canova (1995), for example, examines the sensitivity of the simulated effect of a change in taxation on consumer welfare. Obviously, this

1. In this section, "matching simulations to the data" means "matching a control simulation (e.g., with actual values of exogenous variables)," not the shock simulations (e.g., the effect of alternative policies) that we refer to in later sections.

approach is unsatisfactory for the purposes of this evaluation, because SSQPM should be able to replicate a broad range of features in the data.

Other studies match model outcomes to the data in different ways, one of which (e.g., Harrison and Vinod 1992, Abdelkhalek and Dufour 1998) is to divide the parameters of a model into two groups: calibration parameters and economic parameters. Calibration parameters are set to the values required (conditional on the choices for the economic parameters) to make the simulated data *exactly* match the actual data.[2] In these studies, the value of calibration parameters is of no economic interest; only the economic parameters are relevant and they are varied randomly according to their prior distributions. In SSQPM, however, almost all parameters embody well-defined theoretical concepts. To use this methodology, some of SSQPM's economic parameters would therefore have to be classified as calibration parameters; their values would be chosen solely by the requirement that simulated data match actual data, and all economic information about these parameters would be ignored. This seems inappropriate; in terms of the three objectives of SSQPM's model-builders, it overemphasizes the objective that the model be able to match the data at the cost of disregarding whether parameter values accord with economic theory.

In addition, an evaluation of a steady-state model's ability to match the data must deal with the fact that an economy's steady state is unobservable. We have only an actual time series, representing each variable's out-of-steady-state behaviour, and we therefore must construct estimates of the steady state. Of course, we can use the long-run average of a variable as an estimate of its steady state (as long as it is stationary), but this estimate has a degree of uncertainty that is, in practice, unknown. Our evaluation must acknowledge this uncertainty.

To deal with the issue of data-matching, we first choose five key variables to evaluate whether model simulations match the data: the consumption/income ratio, investment/income ratio, export/income ratio, import/income ratio, and financial assets/income ratio. For each variable we calculate a mean (our point-estimate of the steady state), standard deviation, and 95 per cent confidence interval over the period 1965–1999.[3] We then simulate the model repeatedly, drawing parameter values randomly from their prior distributions, and rejecting from our analysis those sets of parameters for which any of the five key variables fall outside their 95 per cent confidence interval.[4] We also impose an additional constraint on our simulations: parameters must be chosen

2. Typically, simulated data would be constrained to match actual data in a year that is chosen as a base year.
3. If a ratio shows a clear trend over the period, we detrend it (linearly) before calculating its standard deviation.
4. The choice of a 95 per cent confidence interval for our data constraints is quite generous, in that it amounts to agreeing to consider all but the most extreme values of the actual data as potential steady-state values. On the other hand, from a statistical viewpoint, the 95 per cent confidence interval is quite conventional. Were we to set up a hypothesis test, we would probably reject the hypothesis test that an actual value is a steady-state value only if it fell within the 5 per cent tail of the distribution.

in a way that meets the two stability conditions embodied in SSQPM.[5] Any potential parameterizations of the model that do not meet the stability conditions are rejected and omitted from further analysis.

We think that this approach provides the kind of balance, with respect to the objectives of calibration, that we outlined earlier. It gives the influence of prior beliefs a more prominent role in the evaluation than do some studies in the literature (e.g., Harrison and Vinod 1992, Abdelkhalek and Dufour 1998). It does not do so, however, at the expense of an assessment of the model's ability to fit the data, as in Davis and Espinoza (1998). At the same time, it concedes that there is considerable uncertainty surrounding the outcomes to which simulations should be benchmarked.

2.2 Results of the SSQPM evaluation

This section describes the results of applying our methodology to SSQPM. To start, we consider the impact of the data constraints, first by examining the results of SSQPM simulations under the prior parameter distributions, and then by examining, ex post, how the data constraints affect our view of plausible parameter values. We then analyze the model's response to various shocks, examining whether the model's properties accord with theory, and how dispersed the effects of those shocks are under various parameterizations.

2.2.1 The data constraints

Figures 1 to 5 illustrate the effects of our inequality data constraints on the model, showing the effects of alternative parameterizations of SSQPM on the control simulation when there are no data constraints. Superimposed on this graph is the SSQPM control value for the variable under consideration, as well as bands that show our data inequality constraints. This information therefore allows us to assess whether the current SSQPM model falls within our inequality constraints, and to evaluate roughly what proportion of the alternative parameterizations of SSQPM coming from our prior distributions would pass the data constraints.

Figure 1 shows the consumption/income ratio: the majority of the alternative parameterizations taken from the prior distributions generate consumption/income ratios that fall within a 95 per cent confidence interval of the long-run average ratio. The consumption/income ratio associated with the SSQPM parameterization is close to the midpoint of the 95 per cent confidence interval. We would therefore not expect the consumption/income ratio data restriction to have a large influence on the allowable parameter values. As Figures 2 to 5 show, the same is true for most of the other variables that we use in our data restrictions.

5. These stability conditions ensure that wealth will remain finite in the simulations.

The results of these simulations can provide additional information as to what, given the model's structure, are plausible parameterizations. To formally quantify this information, we construct posterior distributions for the model's parameters. Specifically, we discard those combinations of parameter values that result in simulations that do not meet our data constraints; those that remain are pooled to create a posterior distribution. The top panel in Figures 6 through 10 shows the initial random sample of parameter values taken from the prior distribution, while the bottom panel in those figures shows the corresponding posterior distribution. In addition, the thick solid vertical line shows how the SSQPM value for the parameter fits within that parameter's distribution.

Figure 6 shows the prior and posterior distributions for the parameter ASOELX_SS (the "almost-small-open-economy" parameter, which captures the effect of foreign economic activity on Canadian exports). In this case, the two distributions are very similar, indicating that the Canadian data impose very little constraint on the plausible parameter values. In fact, this result is typical of those obtained for many of the parameters in SSQPM; the data are simply not very helpful in pinning down plausible parameter values. In addition, the current SSQPM value for the parameter typically lies near the centre of both the prior and posterior distributions, reflecting that, for the most part, the current SSQPM parameter values are reasonable.

There are, however, some cases where the data are more informative. For example, Figure 7 shows the prior and posterior distributions for DEPRKBUS_SS, the steady-state rate of depreciation of capital. The difference between the top and bottom panels indicates that the data constraints allow us to pin down admissible values for this parameter much more precisely. The bottom panel also reveals that the current SSQPM value for this parameter is appropriate, in that it is close to the mean of the posterior distribution.

There are some cases where the data indicate that the current SSQPM parameterization may be less appropriate. Figure 8, for example, shows that the data are helpful in narrowing the feasible range of values for the parameter RKBUS_R, the risk premium on the cost of capital. It also shows that the SSQPM value for that parameter lies at the low range of admissible values. A similar observation can be made for the parameter SIGMA (Figure 9), the coefficient of relative risk aversion, for which the current SSQPM value also lies at the low end of the posterior distribution.

2.2.2 The model properties

We next analyze the effects of various standard shocks on key economic variables. While theory can often guide us on the direction of a shock's effect, it may have little to say about its magnitude.

We therefore consider qualitative aspects of these simulations, such as how frequently our Monte Carlo simulations generate, say, a negative response for a variable. The frequency with which a shock moves the artificial economy in the predicted direction provides some information on the robustness of the theory that underpins the model. We also examine how dispersed the effects of the shock are under different parameterizations. The distribution of a variable's response to a shock that is built up by the Monte Carlo simulations allows us to consider, for example, where the existing SSQPM estimate of the effect of a shock is situated within the empirical distribution. We do not use this to "test" whether the SSQPM value for the effect of a shock is accurate, because we would invariably expect the SSQPM value to fall within the 95 per cent confidence interval that we generate. This would always occur if the model-builders did their job well and met their objectives of building a model with parameters that seem reasonable from the perspective of economic theory (i.e., close to the central tendency of the prior distribution), that reasonably fit the data, and that produce reasonable model properties. Instead, we use the confidence interval to derive a measure of the uncertainty that remains regarding model properties even after we have given the parameters plausible values and ensured that the simulated model can reasonably fit the data.

The first shock we examine is a 10 per cent increase in the government debt-to-output ratio. The Blanchard-Yaari model for consumption, which is embodied in SSQPM, would have consumption decline (because of the non-Ricardian nature of the model) in response to this shock. The shock works its way through the system as follows: foreign indebtedness increases as agents shift towards holding more government debt and fewer net foreign assets. This higher foreign indebtedness results in higher interest payments on foreign debt, and the exchange rate must depreciate to allow net exports to rise and cover these higher interest payments. The exchange rate depreciation causes the cost of imported investment goods, and thus the cost of capital, to rise, and, as a result, lowers the levels of both the capital stock and output.

Figure 10 shows the effect of this shock on consumption and reveals that 100 per cent of the Monte Carlo simulations give the predicted negative response. The difference between the SSQPM response and the mean of the empirical distribution generated by the Monte Carlo simulations is minute (in fact, the two are indistinguishable in Figure 10). The empirical distribution is skewed, no doubt reflecting the fact that the theoretical economic structure of the model cuts off part of the distribution that would otherwise lead to positive values for consumption. The 95 per cent confidence interval taken from the empirical distribution gives a range of about -0.6 per cent to -0.2 per cent for the effects of the 10 per cent government debt shock. From an economic perspective, then, there seems to be considerable precision as to the effects of a government debt shock on consumption.

Figure 11 shows the effect that this shock has on output. Recall that the theoretical structure of SSQPM suggests that the capital stock and output should fall. Indeed, in our empirical distribution, output falls in 96 per cent of simulations of this shock, leaving, somewhat surprisingly, a few simulations with plausible parameterizations in which output rises. The SSQPM effect of the shock on output, -0.05 per cent, is very small. Using the effect of a government debt shock on output as a measure of the degree of Ricardian equivalence (though other definitions are possible), one would conclude that SSQPM is very close to being Ricardian equivalent, despite its Blanchard-Yaari structure. Furthermore, the 95 per cent confidence interval given by the empirical distribution indicates that this effect, again from an economic standpoint, is precisely estimated. In contrast, a recent U.S. survey paper on the effects of government debt indicated a consensus view that output would fall by about 0.4 per cent in response to a 10 per cent increase in debt (Elmendorf and Mankiw 1998). The SSQPM response is about 1/10th the size of the U.S. estimate.

For the other key economic variables, the impact of the government debt shock, in terms of the empirical distributions, is as expected. That is, the SSQPM results for the shock are consistent with the theory embodied in the model and generally sit near the central tendency of the empirical distributions.

Next, we examine the effect of a 1 per cent total factor productivity increase. In general, we expect this shock to have a fairly neutral effect on the model, with real variables rising by a total that reflects the labour share coefficient in the production function (which is random in our empirical distributions) combined with the increase in factor productivity. The almost-small-open-economy assumption dictates that, for exports to rise with domestic output, the exchange rate must depreciate slightly (i.e., the price of foreign exchange must rise).

The effects of this shock are shown in Figures 12 through 16. Figure 12 shows that the amount by which the exchange rate depreciates in SSQPM is slightly above the mean of the empirical distribution. This depreciation causes net exports to rise slightly, and the SSQPM response is about one standard deviation above the central tendency of the empirical distribution, although, economically, this difference is unimportant (Figure 13). The SSQPM response of the current account (Figure 14) is also well below the central tendency of the empirical distribution but, again, not by an economically significant amount. Similarly, while the relative price of imports (Figure 15) generated by SSQPM is above the central tendency of the empirical distribution, the difference is less than one standard deviation.

One anomaly in this simulation is the effect on financial assets (Figure 16); in SSQPM, financial assets (which include government debt, the capital stock, and net foreign assets) fall by about 1 per cent in response to the productivity shock, whereas the mean effect from the empirical distribution is close to 0. Though this difference is not "statistically significant" (i.e., outside the 95 per cent confidence interval of the empirical distribution), it is still economically important.

Next we examine the effect of a 10-basis-point decline in the rate at which consumers discount future utility (Figures 17 and 18). For almost all of the variables of interest, although the SSQPM values lie close to the central tendencies of the empirical distributions, there is a great deal of uncertainty, as evidenced by the wide confidence bands. Furthermore, differences between the SSQPM values and the central tendencies of the empirical distributions, which might not seem statistically significant, are sometimes fairly large from an economic viewpoint.

Figure 17 shows the effect of this shock on consumption. The entire distribution of consumption responses to this shock is positive, as one would expect, given that the lower discount rate encourages agents to accumulate more financial wealth, thereby supporting higher consumption in the new steady state. The SSQPM response is 0.30, compared with a mean response from the empirical distribution of 0.39. The 95 per cent confidence interval, which covers the range 0.1 to 1.3, is large, given the size of this shock. Likewise, the confidence interval for the effect of the shock on total wealth ranges from about 0.6 per cent to 4.5 per cent, with an SSQPM value of 1.2 (Figure 18).

2.3 Overall evaluation of SSQPM

Our evaluation methodology has pointed to a number of areas that require further investigation. For example, the posterior distributions for several parameters (e.g., SIGMA, the coefficient of relative risk aversion) indicate that the current SSQPM calibration is out of line with the central tendencies of these distributions. Our analysis also indicates which model properties we hold with a good deal of confidence, compared with those we are quite uncertain about, assuming that we are willing to agree that the basic structure of SSQPM is appropriate. For example, the empirical distributions of model properties for the government debt/output shock are fairly concentrated, indicating relatively little uncertainty about the quantitative effects of this shock. In contrast, the results of a discount rate shock are very uncertain, with wide confidence intervals for its effects on key macroeconomic variables.

There are, of course, some potential limitations to our analysis. One major concern is whether the number of simulations (about 10,000 successful simulations in each case) is sufficient to generate accurate estimates of the distributions we are seeking to characterize. While the number of simulations is similar to that used in other studies, there are more parameters in SSQPM than in most models in the literature. As a crude check on the accuracy of our empirical distribution, we resimulated 10,000 replications of the government debt shock using a different random draw of sets of parameter values. We found in this experiment that the shape of the empirical distributions was not greatly changed, leading us to conclude that 10,000 replications are sufficient to yield a reasonably accurate characterization of the distributions.

As we noted in the introduction to this section, our evaluation of SSQPM to date has focused exclusively on the calibration of SSQPM. Future work should evaluate various aspects of the structure of SSQPM against competing economic models in the literature. Perhaps such an evaluation could use a Bayesian-type approach similar to that in Schorfheide (1999).

3. Evaluation of Dynamic QPM

In this section, we set up a stochastic environment for dynamic QPM, conduct stochastic simulations of the model, and compare moments of simulated and actual data to evaluate how closely QPM can replicate key features of the historical data. The stochastic environment also allows us to evaluate certain overidentifying restrictions.

We view these evaluation procedures for dynamic QPM as very stringent tests of the model. We expect that any dynamic economic model, whether calibrated (as QPM is) or estimated, would “fail” many of the tests, both formal and informal, that we rely on here. One of the difficulties, then, in evaluating our results is that our tests provide an absolute standard of how well QPM fits certain important characteristics of the data, but no relative standard—that is, no measure of how QPM does relative to other plausible models of the Canadian economy. In future work, we plan to address this issue. For example, we could estimate a VAR of the Canadian economy that incorporates the variables that we consider in our present analysis. An estimated VAR should encompass many important dynamic features of the data. We would simulate the VAR stochastically, then conduct the same analysis of correlations between simulated and actual data that we consider here with QPM. A comparison of the correlation analysis based on QPM simulations with that generated by the VAR would then give us a measure of the relative fit of the model. For the present, however, we consider only the properties of QPM in isolation.

3.1 Stochastic specification

The first step in our evaluation of the dynamic model is to arrive at a base-case set of stochastic perturbations to simulate QPM. Since foreign variables in QPM are exogenous, we simulate statistically a four-variable VAR that loosely represents key ROW variables and use them as inputs into our QPM simulations. This represents an attempt to mimic, albeit crudely, the effect of foreign variables on the Canadian economy. For key domestic variables, we treat shock measurement as a method-of-moments problem and use an informal estimation-by-simulation approach to measure the magnitude and persistence of the shocks. The simulated ROW variables and final parameterized domestic shock terms constitute the stochastic environment.

3.1.1 VAR and ROW variables

A VAR is estimated using four key QPM ROW variables: world commodity prices, price level, output gap, and short-term interest rate. The estimated VAR is then simulated stochastically (using the estimated orthogonalized variance-covariance matrix) and the resulting dynamic paths are used to represent the exogenous ROW variables in QPM. This simple method allows us to include data-measured persistence and covariance in the ROW variables.

The variables in the VAR are the quarterly growth rate in world commodity prices (or $\Delta LPCOMROW$ in QPM parlance), the quarter-over-quarter G-6 GDP inflation rate ($\Delta LPROW$), the G-6 output gap ($LYROW$_GAP), and a measure of a G-6 short-term nominal interest rate ($R1ROWzz$). The VAR is estimated over the 1973Q1 to 1999Q4 sample period (to maintain consistency with our other empirical work) and is identified by a Wold causal ordering of commodity price inflation, GDP deflator inflation, output gap, and short-term interest rate.[6] To get a sense of the dynamic behaviour of the ROW variables within the VAR, we examine the resulting impulse-response functions (IRFs). Figure 19 displays the 20-period IRFs of each variable along with one standard error interval.[7] The main diagonal IRFs represent the variable that is being shocked, while each column traces out the responses to a particular innovation. Column 1, therefore, corresponds with the dynamic response of the variables to a one-standard-deviation world commodity price shock.

Overall, the IRFs conform broadly to what we would expect. Consider, for instance, column 3, which displays the IRFs to a one-standard-deviation (0.6 per cent) innovation in the G-6 output gap. The innovation induces world commodity prices to increase by 0.3 per cent and GDP inflation to rise gradually over six quarters. As a consequence of higher inflation and a positive output gap, the short-term interest rate rises by about 0.6 per cent within one year. Other impulse responses appear equally plausible, except for the one that corresponds with the G-6 interest rate

6. The optimal data-based lag choice for our VAR is 3. We use 2 lags, however, since this VAR offered us smoother IRFs without a loss in their general shape. The lag length does not affect our forthcoming conclusions.

7. The confidence intervals are generated using Monte Carlo integration. We sample antithetically, instead of randomly, from the posterior density. Geweke (1988) finds the antithetic approach to be more efficient than the random approach.

shock. In this case, a positive innovation to short-term interest rates leads to short-lived (2 to 3 periods) increases in prices and output, which suggests that we may be omitting an inflation indicator variable that is in the information set of the monetary authorities but not included in our VAR (Christiano and Eichenbaum 1995). While it would be useful to explore a larger-dimensional VAR, the limited number of foreign variables in QPM prevents us from doing so. An alternative and more viable approach to resolving the puzzle of the short-lived response of price and output to the interest rate shock is to relax our recursive-causal ordering. Along this margin, we examined many sets of IRFs based on different exclusion restrictions and were unable to find a set of restrictions that resolved the price and output puzzles without introducing other problems.[8] We also examine how well the simulated data approximates the actual by comparing some summary statistics (Table 2) across the two data sets. These statistics suggest that the simulated data appear to capture reasonably well both the volatility and persistence of the historical data.

To shed some light on the open-economy linkages within dynamic QPM, we simulated it with only ROW variables as stochastic disturbances.[9] One feature is especially notable: ROW variables account for only a small proportion of the variability of most real variables. For example, ROW shocks account for less than 25 per cent of the volatility in quarterly exports movements and less than 20 per cent of the quarter-over-quarter fluctuations in the real exchange rate. In other words, QPM displays very little propagation of external shocks. Later work suggests that the representation of international linkages in QPM is an area for future development.

3.1.2 Estimation by simulation and domestic shocks

We also need a set of stochastic shocks for domestic variables to simulate QPM. While there are several approaches to shock measurement, we choose a method that allows both QPM and the empirical data to influence the measurement of the innovations. More specifically, we start with a simple AR(1) representation of innovations and then reparameterize them until QPM produces standard deviations and autocorrelation coefficients of the economic variables that match approximately those in the data. Loosely speaking, we reparameterize the persistence (ρ) and variance (σ_u^2) of the following shock process,

$$\varepsilon_t = \rho\varepsilon_{t-1} + u_t;\ u_t \sim iid(0, \sigma_u^2),$$

8. The volatility and persistence of the artificial data are reasonably robust to alternative VAR models. For instance, we considered a VAR with exclusion restrictions that gave us an almost diagonal reduced-form variance-covariance matrix (the polar case to our base-case VAR), and did not find that this alternative VAR changed grossly the moments of the simulated data.
9. The following conclusion is based on 100, 108 period, simulations of QPM.

until we match approximately the population moments of the artificial (or model-generated) data with the empirical sample moments; that is, $\arg min\left\| \hat{W}_T - \tilde{W}_T(\tilde{\rho}, \tilde{\sigma}_u^2) \right\|$, where $\hat{W}_T$ are the moments under consideration calculated from empirical data and $\tilde{W}_T(\tilde{\rho}, \tilde{\sigma}_u^2)$ are QPM simulated moments based on a specific parameterization of ρ and σ_u^2. This approach can be considered an informal method-of-moment estimation, where we estimate the free parameters to set the distance between moments of the artificial and empirical data within some confidence interval (for a formal treatment of estimation by simulation, see McFadden 1989).[10] One potential problem with informal moment-matching is that parameters can be selected even if they are not identifiable. In other words, we are forcing innovations to be some AR(1) process even though their true representation may not be. Another potential concern is that the parameterization of the shocks may not be unique; another set of parameterizations could lead us to similar population moments.

The set of stochastic shocks that allow QPM to reproduce empirical standard deviations and autocorrelations will provide us with overidentifying restrictions to test the model properties of QPM. That is, since the set of shocks is designed only to match two sets of moments, other properties of the data provide testable overidentifying restrictions of QPM. For example, aggregate output and the terms of trade in QPM are built up from variables that have direct shock terms, so the ability of QPM to produce aggregate output and terms-of-trade data that match the empirical properties of their sample counterparts would constitute testable overidentifying restrictions.

In addition, the fact that the shocks designed in the first step remain fixed during the tests of restrictions strengthens the results from these tests, as it does not allow the shocks to be chosen to fit some restriction. In the current case, for instance, shocks are parameterized on their ability to match empirical standard deviations and autocorrelations but not, say, temporal correlations, so the ability of QPM to reproduce the latter moment is a strong test of the model, and one that will be explored later in this paper. One disadvantage to this approach is that both shocks and QPM constitute the model in stochastic simulation, so a rejection of an overidentifying restriction may be a rejection of the model, the shocks, or both. This disadvantage, however, arises regardless of the innovation measurement methodology.

Before we can proceed to the shock-estimation stage, we must decide on the number of shocks and their initial magnitude. For the former, we append shock terms to some important behavioural

10. Estimation by simulation is closely related to the approach of calibrating model parameters to match a statistic generated by the model with that in the data. Kydland and Prescott (1982), for instance, calibrate the coefficient of relative risk aversion in their real business-cycle model by matching the variance of detrended output.

variables: consumption, investment, exports, GDP price deflator, and real G-6 exchange rate.[11] For the initial magnitude of the shocks, we start with identically, independently distributed (iid) shocks parameterized to match the standard deviation of empirical residuals from an AR(k) model for the variables under consideration. The estimation period is 1973Q1 to 1999Q4. Moreover, we need to decide on a set of standard deviations and autocorrelations (of consumption, investment, exports, GDP price deflator, and real G-6 exchange rate) to match. Obviously, moments must exist for the comparison to be meaningful, so the data must often be transformed to induce stationarity. Singleton (1988) and Cogley and Nason (1995) show that the detrending method considered in calculating moments may itself have a large effect on conclusions, so in an effort to control for such problems we consider three methods for inducing stationarity: first difference, fourth difference, and Hodrick-Prescott (H-P) detrending.

For each simulation, QPM is run over 116 quarters, with the first 8 quarters omitted from analysis; this leaves 108 quarters, which corresponds with the length of the historical sample over which we calculate the moments to be matched. Each complete experiment is based on 100 successful (stable) replications. The distributions for variables of interest are built up by averaging across time and across replications. After each complete experiment, we compare the persistence and volatility of the artificial data with the historical data, and then reparameterize the shock terms accordingly, until we are able to match each moment within a 95 per cent confidence interval. Table 3 presents the final shock estimates and Table 4 reports moments of the artificial data (in bold type) relative to those from the historical data. The latter reports the standard deviations and autocorrelation coefficients surrounded by their upper and lower 95 per cent asymptotic confidence intervals.

The final calibration of the shocks allows us to examine our so-called overidentifying restrictions. As stated earlier, several important variables in QPM do not have shock terms, but are built up from those variables with direct shock terms, so the ability of QPM to produce artificial data for the former variables that match the empirical properties of their sample counterparts would constitute testable overidentifying restrictions. Table 5 reports the empirical standard deviations and first-order autocorrelation coefficients, along with their 95 per cent confidence intervals for detrended output, employment and imports, and the levels of the terms of trade and different interest rates. The values based on the artificial data are shown in bold type. In sum, except for

11. Admittedly, our choices are somewhat arbitrary, but previous work suggests that these five variables are sufficient to induce QPM to produce sufficiently variable volatility. An obvious omission is a productivity shock. In previous work, however, we found, given the current structure and calibration of the shocks, only a small effect arising from productivity shocks. In ongoing work we are attempting to find a more prominent role for productivity innovations.

those corresponding with employment fluctuations, all of the overidentifying variance restrictions are rejected. In particular, imports and the terms of trade display much less volatility than in the historical data, suggesting, perhaps, that QPM is relatively insensitive to ROW fluctuations. Also, there is too much volatility in output and interest rates. The excessive volatility in output suggests that there may be undue correlation among the components of aggregate output, as the variance of the components is either within or below its historical range. We explore this issue further in the next section.

The excessive movement in interest rates is likely caused by two factors: (i) the link between interest rates, output, and inflation embedded in QPM, and (ii) the monetary authority's goal of inflation targeting. The first factor means that excessive volatility in output will tend to be translated into excess volatility in inflation. The second factor means that, since the objective is to stabilize inflation, volatile output, which would otherwise make inflation volatile, will have to be offset by even more volatile interest rates. The variance of artificial CPI inflation (excluding food and energy) data is excessive when compared with the variance of historical CPI inflation calculated using its implicit target instead of its mean to centre the data.[12] This, presumably, better matches the definition of historical CPI inflation with that in QPM, which has an explicit inflation target of 2 per cent. However, the estimated price-shock term fell to zero when we attempted to match with these target-centred CPI inflation data. Since we believe that price shocks have been an integral feature of Canadian economic history, we choose to estimate our price-shock term by focusing on mean-centred GDP inflation data.

3.2 Dynamic model evaluation

3.2.1 Autocorrelations

In this section, we explore in more depth whether QPM is able to generate artificial data that has a similar degree of persistence to that found in the historical data. In particular, we consider the ability of QPM data to replicate empirical autocorrelation functions (ACFs). For many variables, the first-order autocorrelation is not particularly interesting, since the disturbance terms for these variables are parameterized such that the AR(1) coefficient of the simulated data matches its historical counterpart (see previous section). In other words, the results are biased towards finding a favourable result, at least for the first autocorrelation. There are, however, four key variables that do not have direct shock terms, but are built up from other aggregates: terms of trade; real

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12. As a measure of the implicit inflation target, we use the staff economic projection 2-year-ahead CPI inflation expectation over the 1974Q4 to 1992Q4 period. Thereafter, we use the Bank's stated inflation targets (see Amano 1997 for more details).

exchange rate; output; and consumer prices, excluding food and energy. These variables provide another set of overidentifying tests and should indicate the ability of QPM to generate persistence in some key data series. All data, except the terms of trade, are first differenced to induce stationarity.[13]

To test the match between empirical and artificial ACFs, we compute a series of the generalized Q-statistic, which is defined as:

$$Q = (\tilde{AC} - AC)^T V_{AC}^{-1} (\tilde{AC} - AC),$$

where the vector AC is the historical data ACF, and $\tilde{AC}$ is the QPM-generated ACF. The latter is estimated by averaging autocorrelations across our 100 artificial samples. The variance matrix V_{AC} is simply the autocorrelation variance estimated from the historical sample. The generalized Q-statistic has an approximate chi-squared distribution with degrees of freedom equal to the number of elements in AC.[14] As with a standard Q-statistic, the results are a function of the number of lags used in the analysis. In an effort to control for this problem, we present generalized Q-statistics over the 2-, 4-, 8-, and 20-period horizon (Table 6). A large value of Q indicates that the QPM ACF is a poor match for the historical ACF. Overall, the artificial data appear to match the historical data quite well, in that none of the calculated Q-statistics are significant at the 5 or 10 per cent level.

Figure 20 illustrates our previous conclusions by plotting the 20-period empirical ACF along with the ACF calculated using artificial QPM data. The dash-dot line represents the average autocorrelations based on artificial data, the thick solid line depicts empirical autocorrelations, and the dashed lines are the corresponding 95 per cent empirical confidence intervals. With the exception of the exchange rate at the three-quarter horizon, for which the QPM-generated autocorrelation coefficient falls just outside the historical confidence interval, the artificial data appear to capture historical persistence quite well.

3.2.2 Bivariate temporal correlations

In this section, we examine how well QPM is able to reproduce temporal bivariate correlations found in the historical data. Before we begin with this comparison, however, we must select correlations to consider. We use two criteria: (i) the correlations must be important from the perspective of a monetary authority, and (ii) the correlations must display a clear and consistent (or stable) pattern. We omit, for instance, the economically relevant correlation between the price

13. Unless otherwise noted, all level variables are in log form.
14. See Hogg and Craig (1978).

of commodities and exports, since the historical data do not provide a clear and consistent correlation pattern. Based on some preliminary work, the following key temporal correlations are found to be reasonably stable over time: output and consumption; output and investment; output and employment; output and interest rate; consumption and interest rate; output and exchange rate; price inflation and wage inflation; and output and inflation. In addition to the above moments, we examine other stable bivariate relationships and, although perhaps not of primary importance from a monetary policy perspective, they are still of interest, since they may help gauge QPM's ability to mimic properties of the empirical data. Again, since inferences drawn may depend on the moments used, we attempt to control for this problem by considering three different methods of inducing stationarity: first difference, fourth difference, and H-P detrending.[15]

In Figures 21 through 36, the upper, middle, and lower panels display dynamic correlations based on first-differenced, fourth-differenced, and H-P detrended data, respectively. The solid line in each figure represents correlations calculated using historical data over the 1973Q1 to 1999Q4 period, while the long dashed line represents the average temporal correlations based on the artificial data. The short dashed lines represent 95 per cent confidence intervals, based on the historical data and constructed using den Haan and Levin's (2000) data-dependent VARHAC estimator.

Each figure plots the correlation between the first variable identified in the figure title and eight leads and lags of the second variable identified. The vertical axis marks the degree of correlation and the horizontal axis represents the timing of the dynamic correlation. The number -4 along the horizontal axis, for example, represents a lead of four periods for the second variable. This exercise is not intended to examine QPM's ability to reproduce the historical data correlations exactly, but to determine its ability to replicate the broad correlation shapes found in the data.

The first set of figures we consider focus on the correlation between aggregate output and eight leads and lags of private consumption, investment, and employment (Figures 21 to 23). From Figure 21 it appears that QPM is able to generate correlations between output and private consumption that capture the broad shape found in the data. For both artificial and empirical data, the maximum positive correlation occurs at $t = 0$ and falls monotonically towards zero on either side. Unfortunately, the relationship between output and shorter lags of consumption generated by QPM is somewhat weaker than that found in the empirical data. QPM is less successful at capturing correlation between output and investment. In this case, both the broad shape and peak correlations are misaligned when compared with historical data correlations. The peak empirical

15. When calculating statistics with H-P-filtered data, we trim the sample on each side by eight periods to control for the start and end of the sample problem.

correlation, for instance, arises at frequency zero, whereas the maximum correlation generated by QPM occurs at investment leads between three and five. Output and employment correlations are presented in Figure 23. While QPM generates data that mimics the broad shape of the historical correlation, there is a two- to three-period phase shift. In particular, the historical data display a peak correlation at time $t = 0$, while the same correlation generated by QPM peaks at an employment lead of about two to three quarters; the difference is statistically significant. In a related fashion, the correlation based on artificial data is statistically lower over the $t = 0$ to $t = 3$ range.

Next, we consider QPM’s ability to match dynamic correlations of the yield spread (the interest rate measure that affects economic behaviour in QPM) and two key aggregates: private consumption and output. For these correlations, shown in Figures 24 and 25, it appears that QPM does a reasonably good job of capturing the negative correlation between output and lags of the yield spread. The relationship, however, is statistically different over shorter lags of the yield spread. Not surprisingly, a similar picture emerges when we consider the correlation between consumption and the yield spread. QPM produces data that capture the negative relationship between consumption and the yield spread, but fails to reproduce the magnitude found in the empirical data. We also consider the effect of another key relative price on Canadian output fluctuations: the real exchange rate. Figure 26 plots the temporal correlations between the real exchange rate and output fluctuations, and shows that QPM is able to generate a correlation structure that replicates the broad shape found in the data, but with a phase shift of about four periods, or one year. This suggests another timing problem in QPM.

Finally, we consider QPM’s ability to match dynamic correlations that are especially important to monetary policy-makers: wage and price inflation; and output fluctuations and inflation. Figure 27 shows the historical and artificial data correlations between wage and core CPI inflation. It is readily apparent that QPM generates excessive correlation between wages and inflation relative to that found in the historical data. The problem is especially noticeable for first-differenced data. Figure 28 shows the correlations between output fluctuations and CPI inflation. Unlike the previous figures, the lower panel of Figure 28 does not plot the correlations between two H-P filter detrended variables but, instead, the correlation between a quarterly growth rate (of consumer prices) and H-P-detrended output. This allows us to present correlations that are more economically relevant, in that we can interpret deviations from an H-P trend as deviations from some equilibrium level. Along this dimension, QPM does a remarkably good job of reproducing the correlation pattern found in the data except for, perhaps, the relationship between the output gap and inflation four to eight quarters into the future. The historical data indicate a consistent and reasonably strong positive correlation, whereas the artificial data admit a declining and weaker correlation between future inflation and the output gap. To the extent that the QPM monetary authority targets inflation, however, we should expect future inflation and the output gap to be close to orthogonal (see Rowe and Yetman 2000).

Overall, QPM appears to reproduce data that are reasonably consistent with history. There are, of course, areas where further development would be useful. Especially notable are the relationship between wage and price inflation; and the timing between employment and output fluctuations, and between output and real exchange rate movements.

As stated earlier, other statistical relationships are considered to obtain a clearer picture of the dynamic properties of QPM. Owing to the fact that Canada is a small open economy, the external sector plays a key role in the Canadian economy. We therefore analyze the ability of QPM to replicate some important open-economy correlations. Figures 29 and 30 show dynamic correlations between the exports and the real exchange rate, and imports and the real exchange rate. The historical data for the former show a clear pattern, indicating that depreciations are related to increased exports at all frequencies under consideration. The artificial data correlations suggest that real exchange rate depreciations lead to increased exports in the future and that declining past exports are related to an exchange rate depreciation. In this case, the artificial data correlations seem more sensible than the historical data correlations, but it is important to remember that these results are based on simple correlations and not careful empirical analysis. For the real exchange rate and imports relationship, the historical data are unable to provide a consistent pattern across the detrending methods, so we simply note that artificial data generate a mirror image of the correlation between exports and the real exchange rate.

Figures 31 through 34 show the relationship between imports and exports, ROW output and Canadian output, output and imports, and exports and ROW output. Figure 31 plots the import/export dynamic correlations, which attempt to capture, at least partially, the "import-for-export" phenomenon. The historical data display a triangular-shaped correlation structure that peaks on zero at about 60 per cent. In contrast, the artificial data generated by QPM display virtually no correlation at a frequency of zero between imports and exports, suggesting that the model fails to capture the import-for-export relationship found in the historical data. Similarly, the link between Canadian output and ROW output is statistically weaker in QPM than in the data. Figure 32 shows a historical correlation structure that peaks at $t = 0$ and declines monotonically towards zero over 4-period leads and lags. The artificial data, in contrast, show only a small positive correlation between ROW and Canadian output. The ability of QPM to match the correlation between output fluctuations and imports is somewhat better (Figure 33). While QPM is unable to reproduce the same timing as in the historical data, it appears to mimic the same broad correlation pattern.

Figure 34 displays the historical relationship between ROW output and exports, and indicates more success. Although the artificial data are unable to capture the maximum correlation found in the historical data, QPM is able to reproduce the broad correlation shapes found in the historical data.

Finally, as noted earlier, output fluctuations display an excessive degree of volatility in QPM, even though the volatility of the components of aggregate output is similar to those found in the historical data. Indeed, the standard deviation from the historical data is about 3.4 per cent, whereas that from QPM is about 6 per cent. We conjecture that the excessive volatility arises from undue correlation among some components of aggregate output in QPM, and we investigate here whether this conjecture is supported by the evidence. For the sake of brevity, we do not describe the results, but they indicate that QPM, in general, poorly reproduces the historical dynamic correlations between some of the components of real GDP. The contemporaneous correlation between government and private consumption, for instance, is statistically different and of a different sign. The other government consumption correlations admit similarly gross non-matches. Thus, the evidence suggests that the sources of excessive aggregate output variability are spurious relationships amongst some of the components of aggregate output.

3.2.3 Partial correlations

In this section, we investigate the ability of QPM data to reproduce estimation results of two well-researched reduced-form equations: inflation and output growth. We define inflation as the first difference of core CPI and specify it as a linear function of inflation at $t = -1$ to $t = -4$ and the following variables at $t = 0$ to $t = -4$: the output gap (calculated as deviations of log output from its H-P trend), the quarterly growth rate of the nominal exchange rate, and the quarterly change of log commodity prices. Using the same 4-lag structure, the quarterly growth rate of output is specified as a linear function of itself, the yield spread, the quarterly change in the log real exchange rate, the quarterly growth rate of commodity prices, and the quarterly growth rate of ROW output. Both equations are estimated using data from the 1973Q1 to 1998Q1 sample period. It is very likely that these equations are misspecified. To the extent that QPM is a good approximation of the Canadian economy, however, the equations based on historical and artificial data should be equally misspecified and thus produce similar results.

Table 7 shows the estimation results from the reduced-form inflation equation. To focus on the average effect of any particular variable on inflation, we report the summed parameter estimates, rather than any particular parameter estimate. In Table 7, the first column describes the regressors under consideration, the second column reports historical data (summed) point estimates along with their 95 per cent confidence intervals, and the third column presents the average (summed) parameter estimate from QPM artificial data.

Table 7 shows that QPM is able to produce inflation with approximately the same degree of persistence as the historical data (perhaps a little too much), but that it is unable to capture the magnitude of the effect of the other three regressors. In particular, the effect of the exchange rate and commodity prices lies within the 95 per cent confidence intervals, but their estimated average effect on inflation is substantially smaller than that estimated using historical data. Similarly, the effect of the artificial data output gap is about 20-fold smaller than that estimated in the historical data (0.005 versus 0.1). This is an especially odd result, since inflation, or more precisely the price level, is determined in QPM via a Phillips curve relationship, and the artificial data H-P-filter output gap and the QPM output gap are highly correlated (95 per cent) with a variance ratio of almost one. Future work is needed to explore this odd result.

Table 8 lists estimates from reduced-form output equations. The estimates based on the historical data indicate that output growth responds positively to movements in its regressors, although the effects of lagged output growth and the real exchange rate are not statistically significant. In contrast, QPM generates artificial output data, which has a negative partial correlation with simulated commodity price data. This result supports our previous conclusion regarding the weakness of the open-economy linkages in QPM.

4. Sensitivity Analysis

In this section, we report the results of two perturbations to the base-case simulation exercise. We consider, first, a modification to the structure of the stochastic shocks and, second, a change of the monetary policy rule. While the sensitivity analysis reported is by no means exhaustive, it does provide some useful information on directions for future research on QPM.

To assess the sensitivity of our results to the type of shocks, we replace the current base-case AR(1) calibration of the domestic shocks with iid disturbances.[16] We parameterize the latter such that their standard deviations are identical with those induced by the AR(1) innovations. In other words, we attempt to determine how much our conclusions are an artifact of including persistence into the estimation-by-simulation shock terms. Overall, the temporal correlations across the two disturbance structures are very similar, except for those corresponding with output and investment. Figure 35 plots correlations between investment and output in both historical data and data generated by QPM under iid shock parameterization. The solid and short-dash lines represent

16. The simulated ROW variables are not changed for these exercises.

dynamic correlations based on the historical data and their corresponding 95 per cent confidence interval, while the long-dash line corresponds with correlations based on iid innovations. From the figures, it is readily apparent that the iid innovations induce QPM to generate investment correlations that are positive contemporaneously and negative at the first lead and lag. This suggests that the investment aggregate in QPM needs some degree of external persistence, inter alia, to generate correlations closer to those in the historical data.

We also re-estimate the reduced-form output and inflation equations specified in the previous section. Table 9 reports the reduced-form inflation equation estimation results. The first three columns reproduce the information provided in Table 7, while the fourth column reports the summed average parameter estimate from the regressors under consideration. The parameter estimates are quite similar to the base-case ones. The reduced-form output equation results are given in Table 10. The most striking result is that output growth generated by the iid disturbances displays a negative autocorrelation, instead of the positive persistence estimated in the base-case. This result is not surprising, however, given the investment behaviour described in the previous paragraph.

We also consider the effect of a different monetary rule. In particular, we replace the base-case inflation-forecast-based rule with a simple Taylor rule estimated with Canadian data spanning roughly the same period as that used for our empirical work; that is:

$$rsl_t = rsl_t^* + \alpha(\pi_t - 2) + \beta\tilde{y}_t,$$

where rsl is the yield spread, rsl^* is the equilibrium yield spread, π is the annual consumer price (excluding food and energy) inflation, $\tilde{y}$ is the output gap, and α and β are parameters to be calibrated.[17] Following the empirical results in Amano (1997), we set both parameters to 0.5.[18]

Consistent with the conclusions from the iid disturbance exercise, the dynamic correlations are generally similar to the base-case. The correlation between the output gap and future inflation has deteriorated slightly relative to the base-case. An examination of these correlations (Figure 36) reveals coefficients at leads of inflation that are now closer to zero than those in the base-case (Figure 28). As stated earlier, however, we expect this result when a monetary authority targets inflation.

17. The current inflation-forecast-based rule has a parameter of 1.65 on deviation of future inflation from its target.

18. To allow comparison, we use the same set of shocks as in the base-case.

Again, we re-estimate our reduced-form output and inflation equations. Table 11 reports the estimation results for the inflation equation. The parameter estimates based on the Taylor-rule artificial data are remarkably similar to the base-case results. The estimation results from the reduced-form output equation (Table 12), however, are quite different from those estimated using the base-case artificial data (Table 9); the real exchange rate and ROW output growth now co-vary negatively with domestic output changes. Also, output growth generated using the Taylor rule exhibits more persistence and a smaller response to shifts in the other regressors.

Overall, our modest sensitivity analysis suggests that most of the moments considered are reasonably robust to perturbations in the innovation structure and monetary policy rule. The exceptions appear to be the investment aggregate, the behaviour of which seems very sensitive to the types of stochastic shocks, and the inflation and output relationship, which is, as expected, sensitive to the type and calibration of the monetary policy rule.

5. Conclusion

In this paper, we have presented the results of two recent lines of research aimed at evaluating QPM. The first line of evaluation focused on the steady-state calibration of QPM using a variant of Canova's (1994, 1995) Monte Carlo method. The approach allows us to formalize the choice of parameters and provides a systematic method to analyze the model's sensitivity to parameter perturbations. The second line of evaluation examined the ability of QPM to reproduce sample moments found in the historical data. In particular, we used stochastic simulations to investigate QPM's ability to generate artificial data that mimic empirical correlations: autocorrelations, bivariate correlations, and partial correlations of key macroeconomic variables.

The results of our two evaluation methods point to some strengths and weaknesses in the QPM system. The main result of our evaluation of the steady-state model is that most of the parameter calibrations appear reasonable, in that they lie within the empirical distribution and are close to the empirical mean. Nevertheless, there is still room for improvement. In cases where the SSQPM parameters are some distance away from the central tendencies of their respective posterior distributions, minor improvements in the parameterization of the model may be possible. Our analysis also highlights that the effects of some shocks (such as a government debt shock) seem to be known with a good deal of confidence, while for other shocks (such as a discount rate shock) there is more uncertainty regarding the magnitude of their effects.

The results corresponding with dynamic QPM indicate that the model is able to reproduce most of the temporal correlations that are important from a monetary policy-maker's perspective. One notable area that needs future development, however, is the wage/price relationship; another is the external sector. QPM appears to be unable to mimic many of the open-economy moments we see in the data. A modest sensitivity analysis suggests that these conclusions are robust both to the degree of persistence in the shock structure and to the monetary policy rule. These tests, however, are quite stringent, in that we have no other metric for comparison. We plan to compare the ability of QPM to reproduce the data's dynamic properties with an estimated model, perhaps a VAR. This approach should shed some light on how stringent our tests are, since most estimation approaches choose parameters by fitting the data. In other words, we would compare QPM's ability to reproduce data with a model that is designed (estimated) to match correlations in the data.

It would be useful to formally combine the results from the steady-state and dynamic QPM evaluations in some fashion. One area where we may be able to integrate the results is the external sector of QPM. In particular, the Monte Carlo evaluation of the steady-state model suggests that data constraints shed very little light on the ASOE assumption, while the dynamic analysis indicates that the open-economy linkages in QPM are somewhat weaker than those found in the empirical sample data. Perhaps combining these types of information would allow us to further refine our methodology and areas for future development.

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Table 1: Prior Distributions on Structural Parameters of SSQPM

Parameter name	SSQPM value	Range	Definition
asoelx	0.6000	U[0 1]	Coefficient on y/yrow in x_y:forex equation
asoepx	0.5/6	U[0 0.1]	Coefficient on yrow/y in px equation
crtshar	0.4000	U[0 1]	Per cent of rule-of-thumb consumers
delta	0.9646	U[0.9346 0.9946]	Consumer's discount factor
deprkbus	0.0803	U[0.05 0.11]	Rate of depreciation of capital stock
el	0.6551	N(0.66, 0.2^2) truncated [0,1]	Labour share in Cobb-Douglas production function
gamma	0.9600	U[0.90 0.99]	Probability of survival
ltfp_dq	0.0085	N(0.0085, 0.15^2)	Growth of total factor productivity
r1_r_ss	0.0040	U[0 0.01]	Real risk premium on 90-day rates
r40zz_t	0.0050	U[0 0.01]	Real risk premium on 10-year bonds
rcon_r	0.0400	U[0.025 0.055]	Real consumer discount rate
rgb_r	0.0020	U[0.0005 0.0035]	Risk premium on government debt
rkbus_r	0.1450	U[0.08 0.20]	Risk premium on the cost of capital
rnfa_r	0.0050	U[0.0013 0.0088]	Risk premium on net foreign assets
rslzz	0.0050	U[0, 0.01]	Slope of the yield curve
runemp	0.0715	N(0.07, 0.15^2)	NAIRU
sigma	0.6600	U[0.2 1.5]	Coefficient of relative risk aversion
xr40zz1	0.7500	U[0.4 1.0]	Weight on r40uszz in the r40zz equation
xr40zz2	0.7000	U[0.4 1.0]	Weight on r1zz in the r40zz equation
xr40zz3	1.0000	U[0.8 1.0]	Weight on the inflation differential in the r40zz equation
xrcc1	0.2500	U[0,1]	Weight on r1 in the cost of capital equation
xrgb1	0.3500	U[0,1]	Weight on r1 in rgb equation
xrnfa1	0.1600	U[0,1]	Weight on r1 in rnfa equation
xxlp2	0.9400	U[0.5 1.5]	Coefficient on yrow/y in px equation
xxlpcm2	0.7000	U[0.2 1]	Coefficient on real exchange rate in pcm equation
xxlpcx2	0.5300	U[0,1]	Weight given to commodity prices in price of exportables equation
xxlpinvm2	0.7000	U[0.3 1]	Weight on real exchange rate in pcinvm equation
xxlpm2	0.8250	U[0.5 1]	Weight on real exchange rate in pm_pfc equation

```
/* stab1_ss -- First stability condition */
E1:   (1+stab1_ss)**0.25   =   1+(1+rcon_ss)**0.25 - (gamma_ss*y_dq_ss)**0.25,
E2:   y_dq_ss              =   exp(ltfp_dq_ss/el_ss+lfullnemp_dq_ss)

/* stab2_ss -- Second stability condition */
E3:   (1+stab2_ss)**0.25   =   1+y_dq_ss**0.25 - (1+rcon_ss)**0.25*(1-((1+mpcw_ss)**0.25-1)),
```

Table 2: Summary Statistics from Actual and Simulated ROW Data

Variable	Actual variance (x1000)	Simulated variance (x1000)	Actual AR(1) coefficient	Simulated AR(1) coefficient
$\Delta LPCOMROW$	1.77	1.54	0.38	0.35
$\Delta LPROW$	0.04	0.03	0.94	0.91
$LYROW$_GAP	0.20	0.20	0.85	0.85
$R1ROWzz$	0.90	0.90	0.93	0.93

Table 3: Final Reparameterized AR(1) Shock Terms

Shock term	AR coefficient	Standard deviation of the residual term (in percentage form)
Consumption	0.45	1.00
Investment	0.90	2.61
Exports	0.40	2.40
GDP deflator	0.70	0.49
Nominal G-6 exchange rate	0.75	1.59

**Table 4:
Historical and QPM Moments
Standard Deviations and Autocorrelation Coefficient
With Their Corresponding 95 Per Cent Confidence Intervals for Selected Variables**

Variables[a]	Standard deviation	AR(1) coefficient
Consumption		
Quarterly	5.2 < **5.7** < 5.9 < 6.8	0.20 < 0.39 < **0.46** < 0.58
Annual	3.4 < 3.9 < **4.0** < 4.5	0.63 < 0.84 < **0.85** < 1.02
H-P filtered	2.2 < 2.5 < **2.6** < 2.9	0.66 < 0.85 < **0.86** < 1.04
Investment		
Quarterly	11.3 < 13.0 < **14.0** < 15.0	0.07 < **0.22** < 0.26 < 0.45
Annual	7.5 < 8.5 < **8.7** < 9.8	0.64 < **0.82** < 1.03
H-P filtered	5.0 < 5.7 < **5.9** < 6.6	0.68 < **0.83** < 0.87 < 1.06
Exports		
Quarterly	10.4 < **11.7** < 11.9 < 13.7	-0.22 < -0.03 < 0.16 < **0.22**
Annual	5.3 < 6.1 < **6.5** < 7.0	0.53 < 0.72 < **0.76** < 0.91
H-P filtered	3.4 < 3.9 < **4.1** < 4.4	0.54 < 0.72 < **0.76** < 0.91
GDP price deflator		
Quarterly	3.8 < **4.1** < 4.3 < 5.0	0.64 < **0.69** < 0.83 < 1.02
Annual	3.5 < **3.5** < 4.0 < 4.6	0.76 < **0.92** < 0.95 < 1.14
Real G-6 exchange rate		
Quarterly	7.2 < 8.2 < **8.6** < 9.5	0.21 < **0.38** < 0.40 < 0.59
Annual	4.9 < **5.5** < 5.6 < 6.5	0.66 < **0.82** < 0.85 < 1.04

a. Quarterly indicates quarterly growth at annual rates. Annual indicates year-over-year growth and H-P filter corresponds with H-P-detrended log variables.

Table 5:
Test of the Overidentifying Restrictions
Standard Deviations and Autocorrelation Coefficient
With Their Corresponding 95 Per Cent Confidence Intervals for Selected Variables

Variables	Standard deviation	AR(1) coefficient
Output		
Quarterly	2.9 < 3.3 < 3.8 < **5.9**	0.25 < **0.29** < 0.44 < 0.63
Annual	2.0 < 2.3 < 2.7 < **3.6**	0.67 < **0.80** < 0.86 < 1.06
H-P filtered	13 < 1.5 < 1.8 < **2.4**	0.69 < **0.80** < 0.88 < 1.07
Imports		
Quarterly	**4.2** < 11.0 < 12.6 < 14.5	0.04 < 0.23 < 0.42 < **0.84**
Annual	**3.6** < 6.7 < 7.7 < 8.9	0.60 < 0.79 < **0.91** < 0.98
H-P filtered	**2.5** < 4.0 < 4.5 < 5.2	0.61 < 0.80 < **0.92** < 0.99
Employment		
Quarterly	2.1 < **2.1** < 2.4 < 2.8	0.38 < 0.57 < 0.76 < **0.83**
Annual	1.6 < **1.8** < 1.9 < 2.1	0.70 < 0.89 < **0.91** < 1.08
H-P filtered	1.1 < **1.2** < 1.3 < 1.5	0.72 < 0.91 < **0.92** < 1.10
Terms of trade (level ratio)	**2.8** < 4.0 < 4.6 < 5.3	0.73 < 0.92 < **0.95** < 1.10
Yield spread	1.2 < 1.4 < 1.6 < **4.6**	0.55 < 0.74 < **0.91** < 0.92
10-year interest rate	2.1 < 2.4 < 2.7 < **4.3**	0.76 < **0.93** < 0.95 < 1.14
90-day interest rate	3.0 < 3.5 < 4.0 < **8.7**	0.74 < **0.92** < 1.11

Table 6: Autocorrelation Function Comparisons, Generalized Q-Statistics

	Horizon			
Variable	2	4	8	20
Output growth	3.51	7.66	8.81	16.96
Inflation	2.62	2.72	5.83	18.08
Exchange rate	0.35	6.27	8.01	16.85
Terms of trade	0.60	1.82	3.01	11.23

Table 7: Summed Partial Correlations with Inflation

$\sum_{i}^{k} \alpha_i$	Historical data $R^2 = 0.84$	Base-case artificial data $R^2 > 0.99$
Lagged inflation ($i = 1, k = 4$)	0.808 < 0.896 < 0.984	0.979
H-P filtered output gap ($i = 0, k = 4$)	0.044 < 0.100 < 0.155	0.005
Nominal exchange rate ($i = 0, k = 4$)	0.016 < 0.069 < 0.123	0.016
ROW commodity price ($i = 0, k = 4$)	-0.005 < 0.026 < 0.057	0.007

Table 8: Summed Partial Correlations with Output Growth

$\sum_{i}^{k} \alpha_i$	Historical data $R^2 = 0.60$	Base-case artificial data $R^2 = 0.45$
Lagged output growth ($i = 1, k = 4$)	-0.093 < 0.242 < 0.578	0.035
Yield spread ($i = 0, k = 4$)	-0.010 < 0.075 < 0.161	0.038
Real exchange rate ($i = 0, k = 4$)	-0.058 < 0.029 < 0.116	0.268
ROW commodity price ($i = 0, k = 4$)	0.016 < 0.055 < 0.094	-0.014
ROW output growth ($i = 0, k = 4$)	0.059 < 0.489 < 0.918	0.453

Table 9: Summed Partial Correlations with Inflation, iid Shock Perturbation

$\sum_{i}^{k} \alpha_i$	Historical data $R^2 = 0.84$	Base-case artificial data $R^2 > 0.99$	iid artificial data $R^2 = 0.96$
Lagged inflation ($i = 1, k = 4$)	0.808 < 0.896 < 0.984	0.979	0.939
H-P-filtered output Gap ($i = 0, k = 4$)	0.044 < 0.100 < 0.155	0.005	0.010
Nominal exchange rate ($i = 0, k = 4$)	0.016 < 0.069 < 0.123	0.016	0.040
ROW commodity price ($i = 0, k = 4$)	-0.005 < 0.026 < 0.057	0.007	0.016

Table 10: Summed Partial Correlations with Output Growth, iid Shock Perturbation

$\sum_{i}^{k} \alpha_i$	Historical data $R^2 = 0.60$	Base-case artificial data $R^2 = 0.45$	iid artificial data $R^2 = 0.46$
Lagged output growth ($i = 1, k = 4$)	-0.093 < 0.242 < 0.578	0.035	-0.874
Yield spread ($i = 0, k = 4$)	-0.010 < 0.075 < 0.161	0.038	0.102
Real exchange rate ($i = 0, k = 4$)	-0.058 < 0.029 < 0.116	0.268	0.329
ROW commodity price ($i = 0, k = 4$)	0.016 < 0.055 < 0.094	-0.014	-0.054
ROW output growth ($i = 0, k = 4$)	0.059 < 0.489 < 0.918	0.453	0.602

Table 11: Summed Partial Correlations with Inflation, Taylor Rule Perturbation

$\sum_{i}^{k} \alpha_i$	Historical data $R^2 = 0.84$	Base-case artificial data $R^2 > 0.99$	Taylor rule data $R^2 > 0.99$
Lagged inflation ($i = 1, k = 4$)	0.808 < 0.896 < 0.984	0.979	0.983
H-P-filtered output Gap ($i = 0, k = 4$)	0.044 < 0.100 < 0.155	0.005	0.008
Nominal exchange rate ($i = 0, k = 4$)	0.016 < 0.069 < 0.123	0.016	0.022
ROW commodity price ($i = 0, k = 4$)	-0.005 < 0.026 < 0.057	0.007	0.009

Table 12: Summed Partial Correlations with Output Growth, Taylor Rule Perturbation

$\sum_{i}^{k} \alpha_i$	Historical data $R^2 = 0.60$	Base-case artificial data $R^2 = 0.45$	Taylor rule data $R^2 > 0.99$
Lagged output growth ($i = 1, k = 4$)	-0.093 < 0.242 < 0.578	0.035	0.887
Yield spread ($i = 0, k = 4$)	-0.010 < 0.075 < 0.161	0.038	0.003
Real exchange rate ($i = 0, k = 4$)	-0.058 < 0.029 < 0.116	0.268	-0.007
ROW commodity price ($i = 0, k = 4$)	0.016 < 0.055 < 0.094	-0.014	-0.008
ROW output growth ($i = 0, k = 4$)	0.059 < 0.489 < 0.918	0.453	-0.006

Figure 1

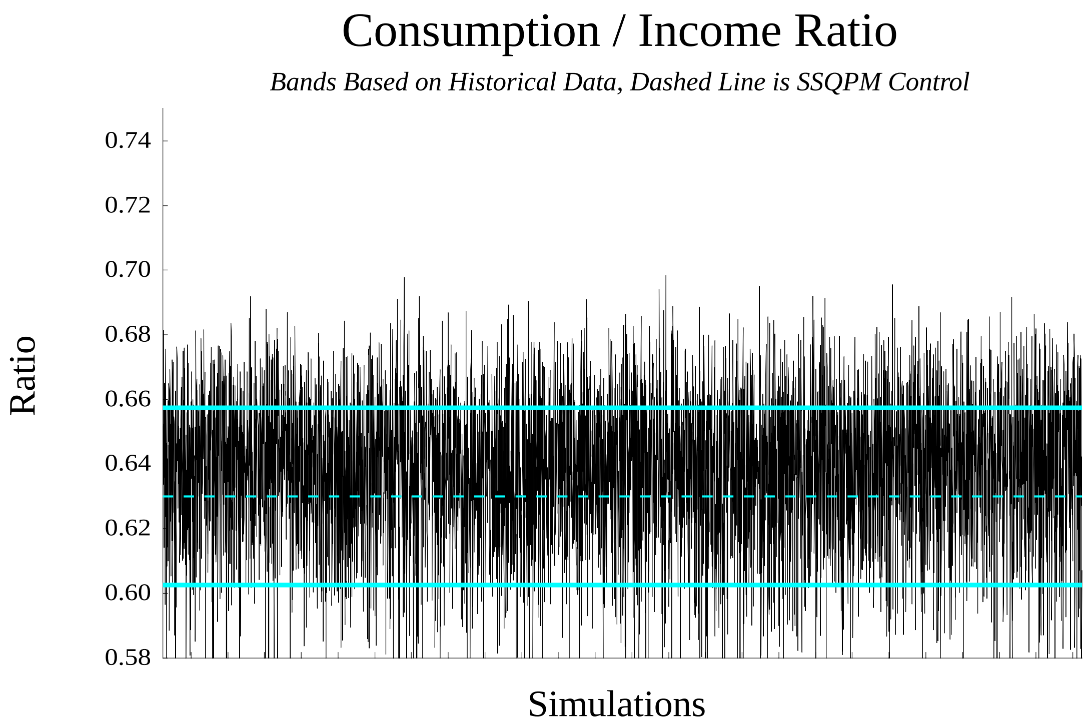


Figure 2

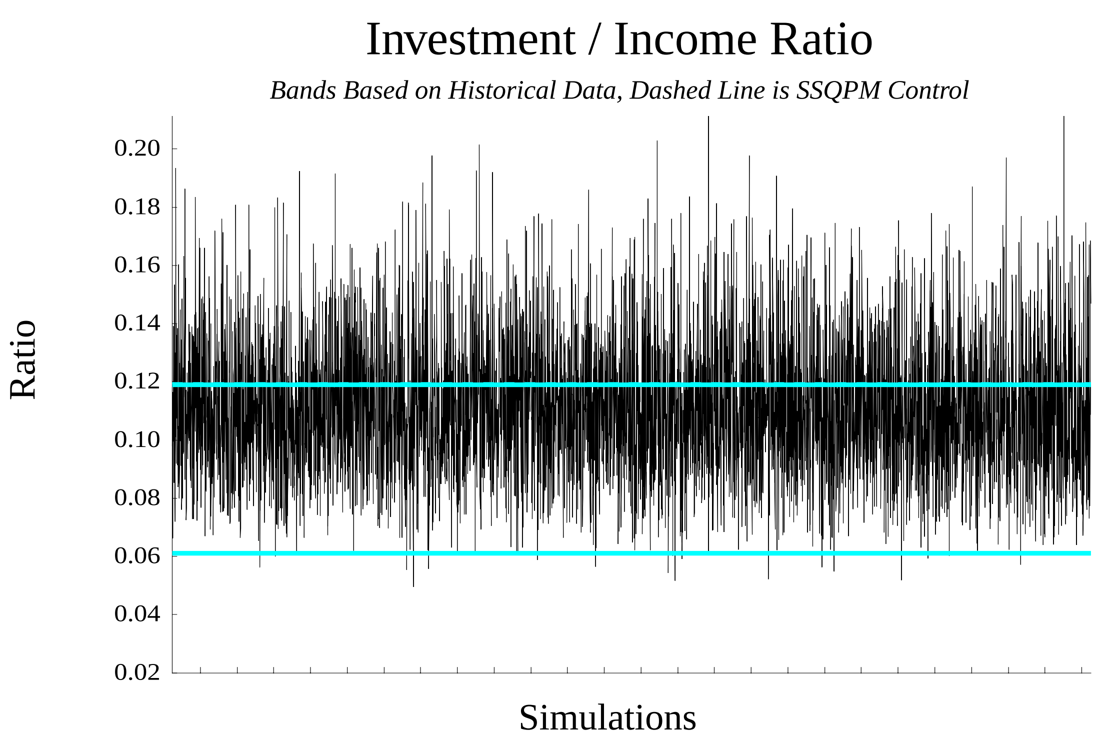


Figure 3

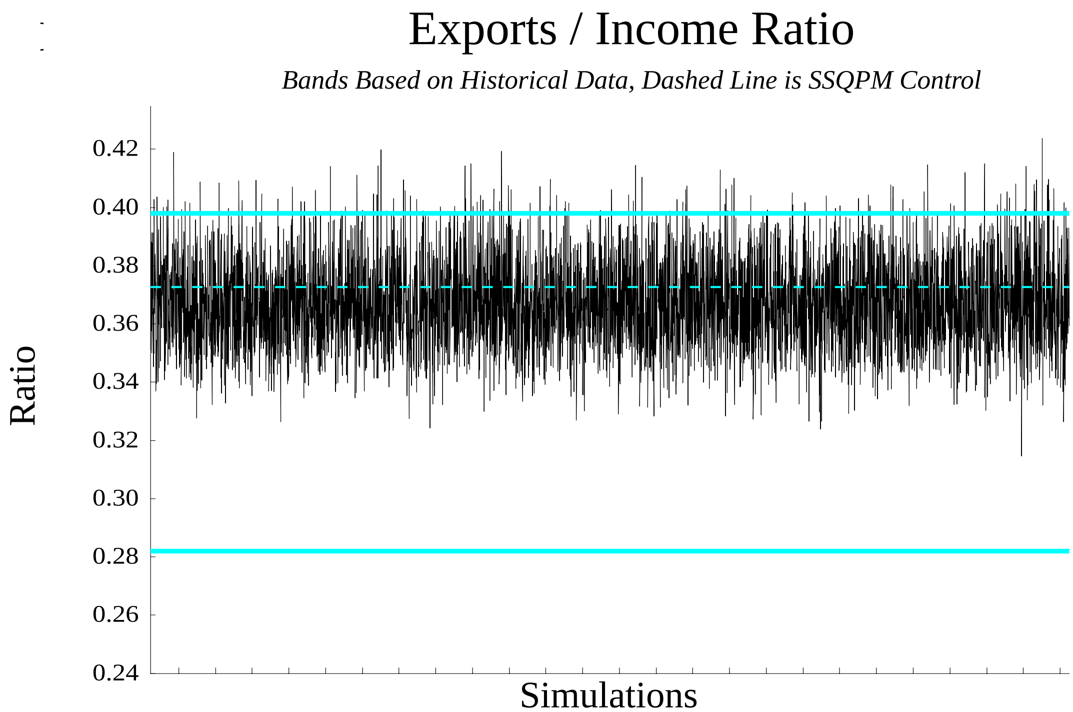


Figure 4

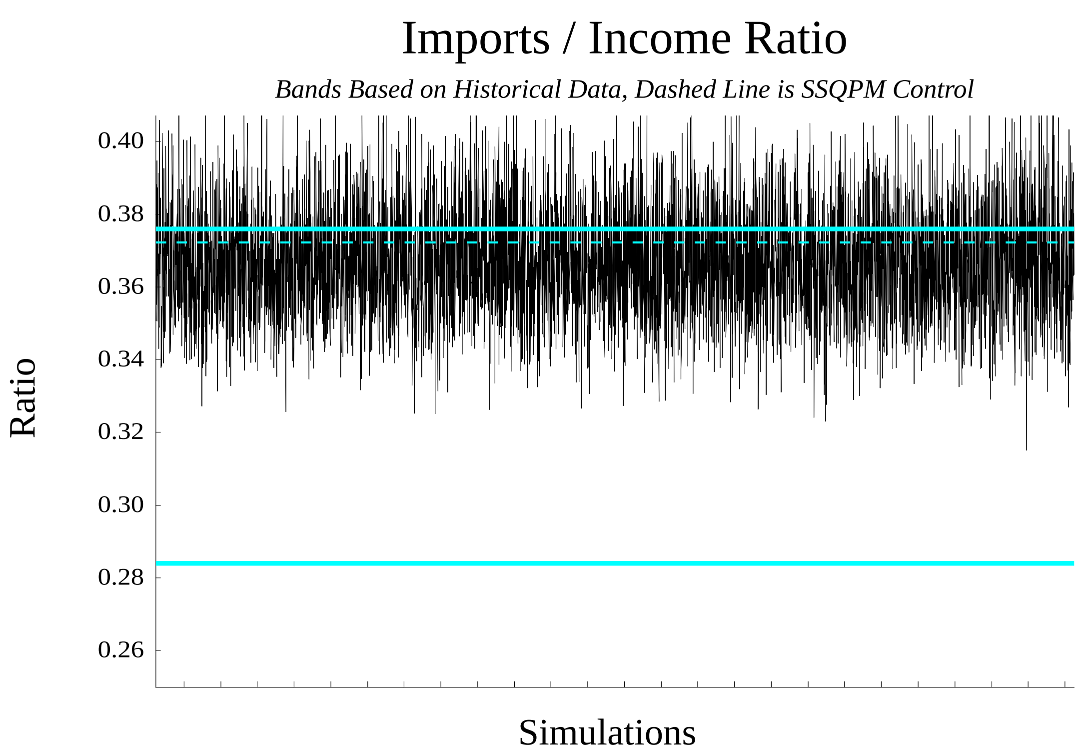


Figure 5

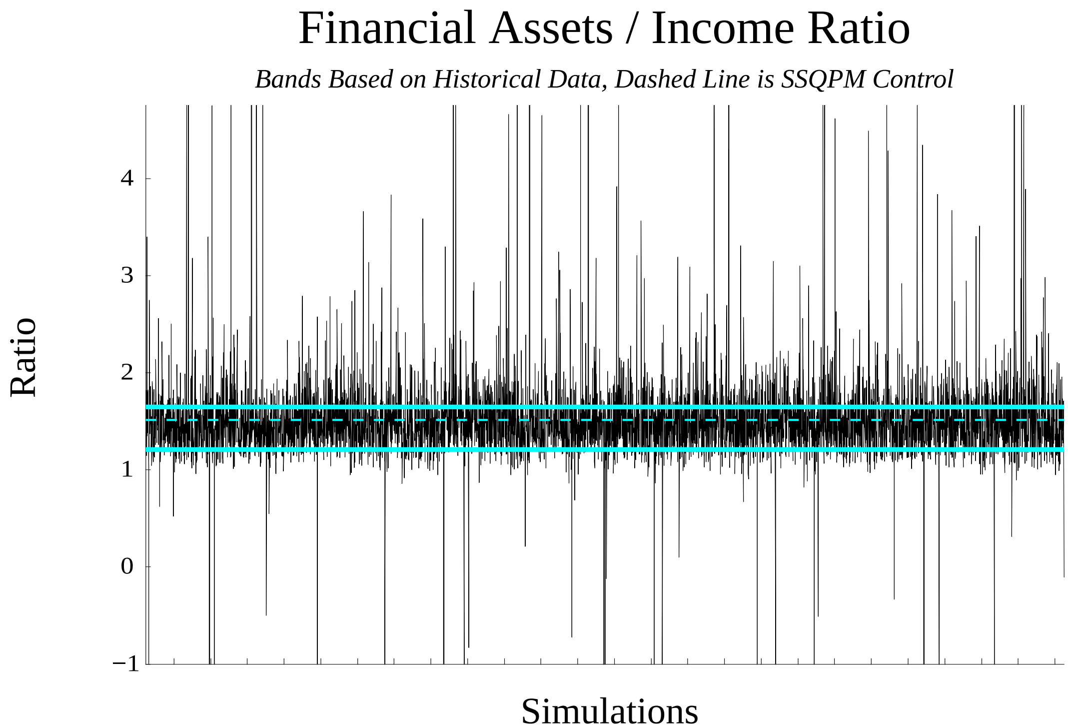
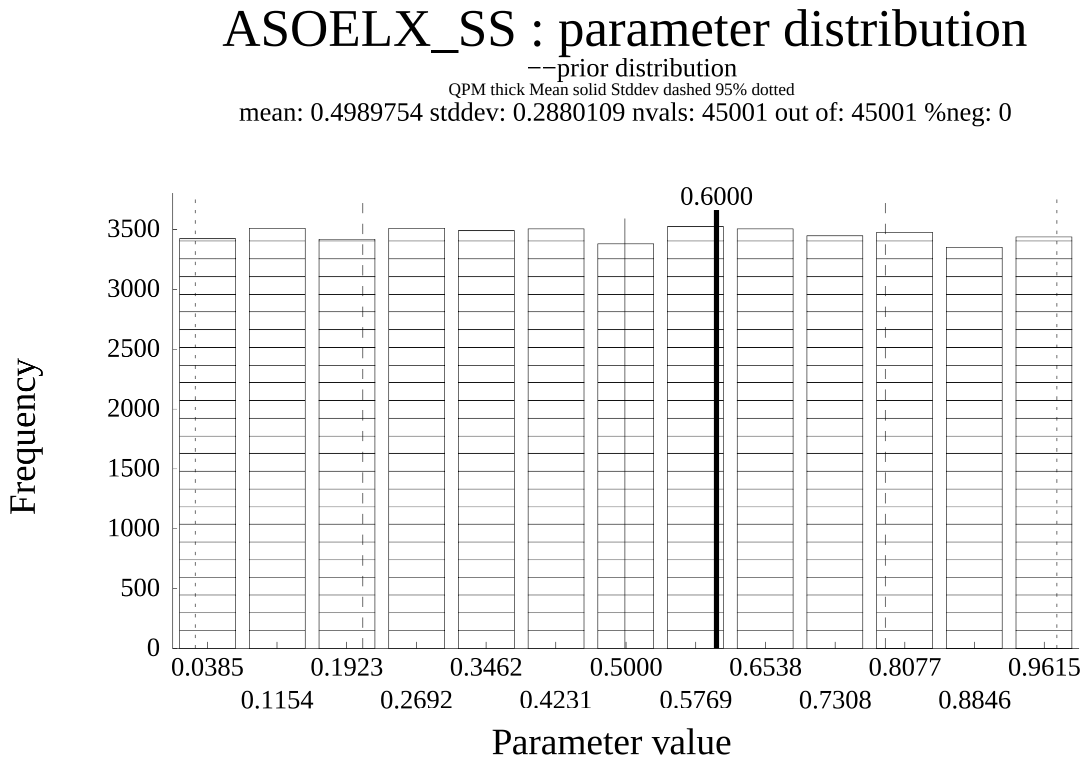

Financial Assets / Income Ratio
Bands Based on Historical Data, Dashed Line is SSQPM Control
Ratio
4
3
2
1
0
−1
Simulations

Figure 6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−−prior distribution
QPM thick Mean solid Stddev dashed 95% dotted
mean: 0.4989754 stddev: 0.2880109 nvals: 45001 out of: 45001 %neg: 0
0.6000
Frequency
3500
3000
2500
2000
1500
1000
500
0
0.0385
0.1154
0.1923
0.2692
0.3462
0.4231
0.5000
0.5769
0.6538
0.7308
0.8077
0.8846
0.9615
Parameter value

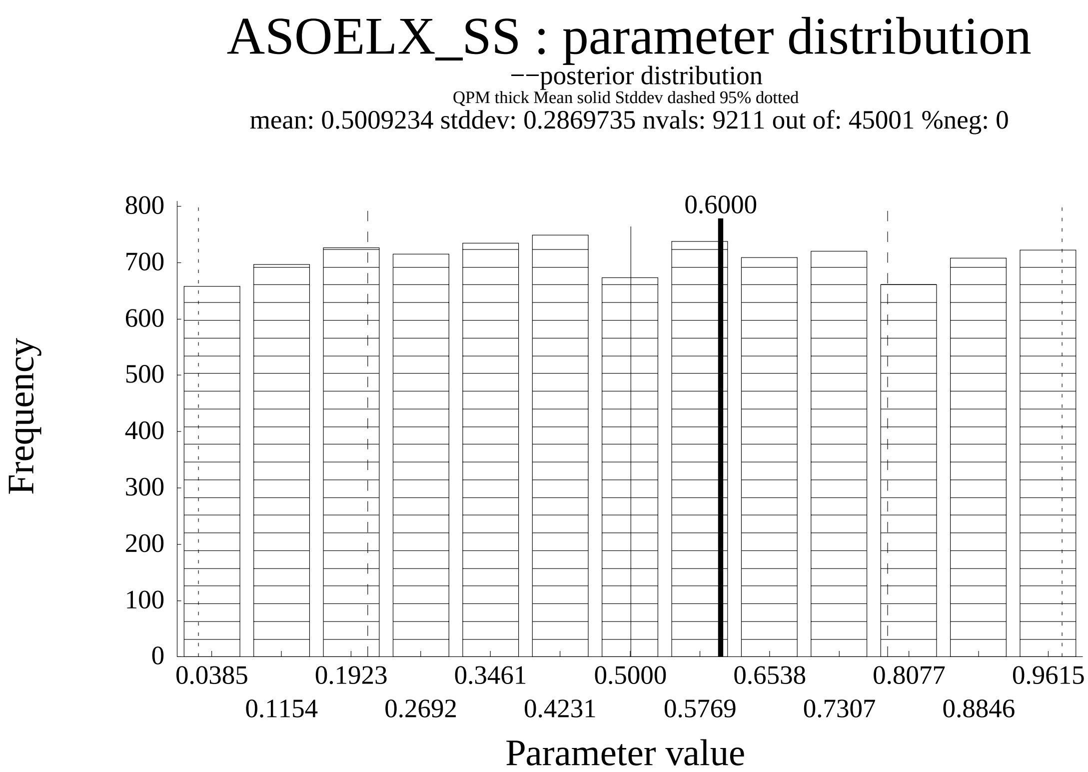
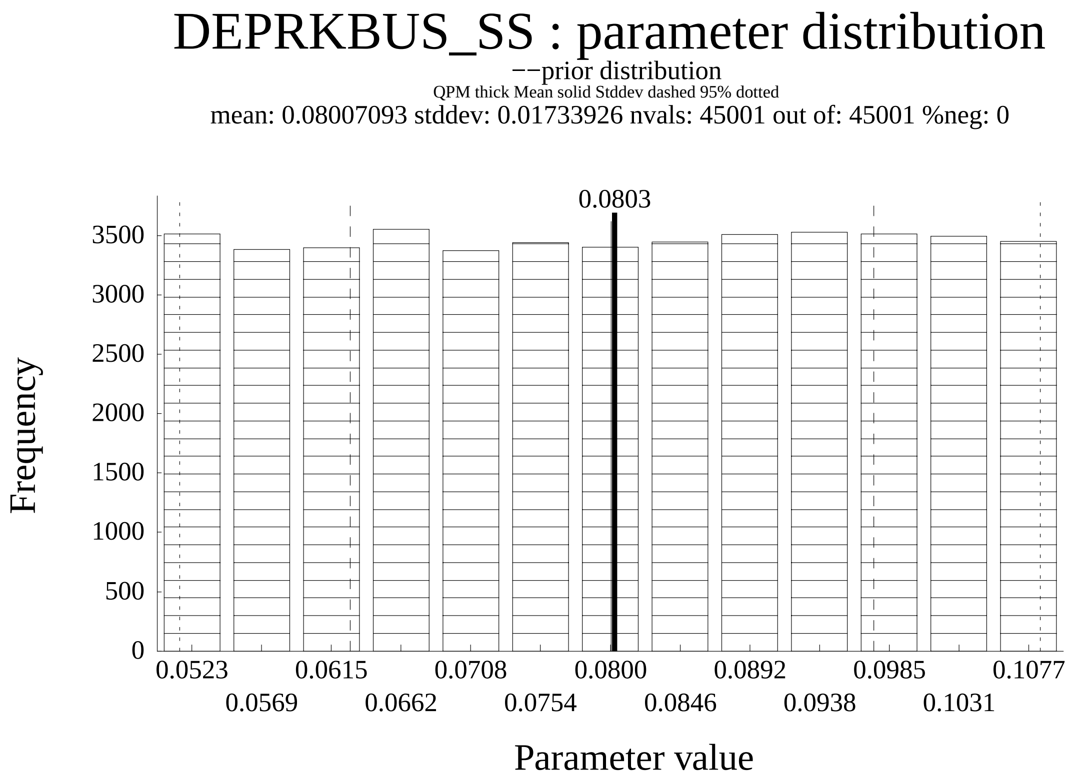

ASOELX_SS : parameter distribution
−−posterior distribution
QPM thick Mean solid Stddev dashed 95% dotted
mean: 0.5009234 stddev: 0.2869735 nvals: 9211 out of: 45001 %neg: 0
0.6000
Frequency
800
700
600
500
400
300
200
100
0
0.0385
0.1154
0.1923
0.2692
0.3461
0.4231
0.5000
0.5769
0.6538
0.7307
0.8077
0.8846
0.9615
Parameter value

Figure 7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−−prior distribution
QPM thick Mean solid Stddev dashed 95% dotted
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0.0803
Frequency
0
500
1000
1500
2000
2500
3000
3500
0.0523
0.0569
0.0615
0.0662
0.0708
0.0754
0.0800
0.0846
0.0892
0.0938
0.0985
0.1031
0.1077
Parameter value

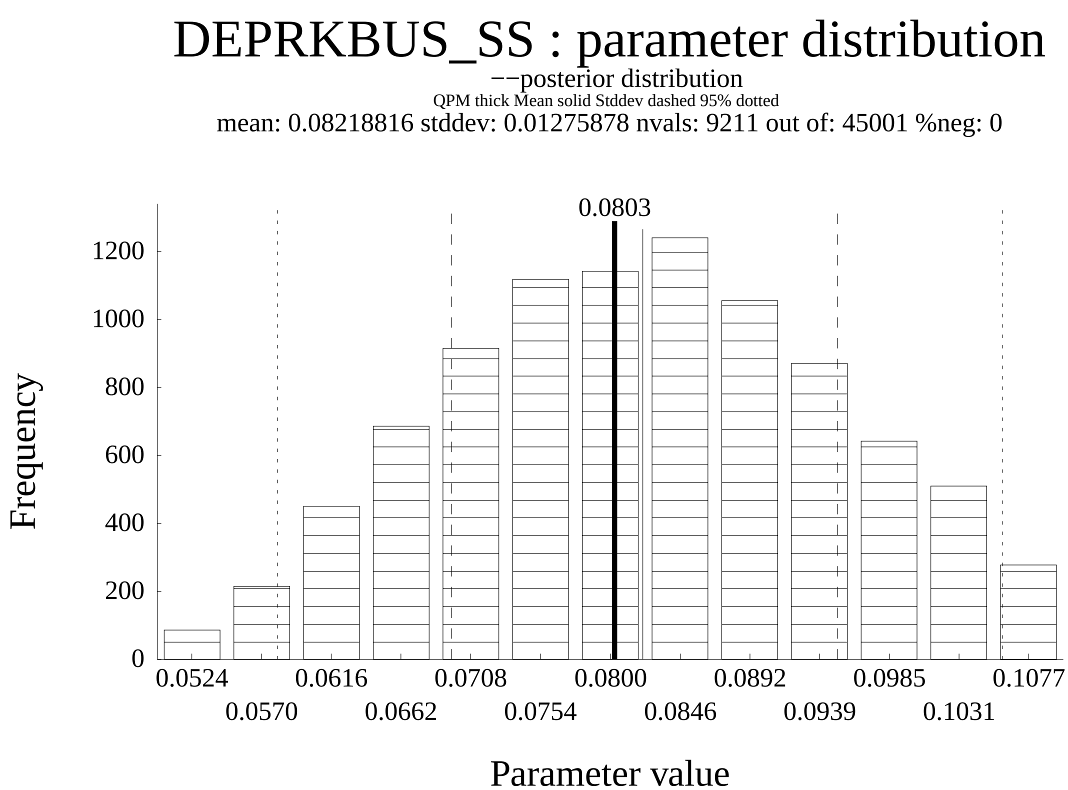
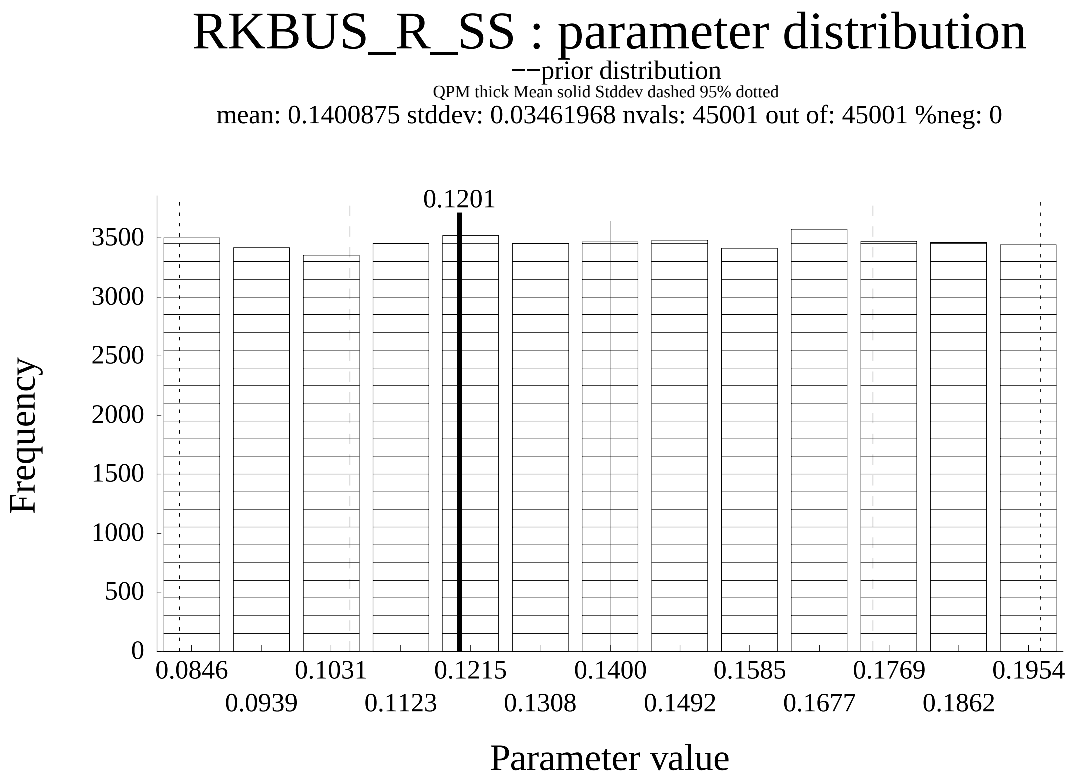

DEPRKBUS_SS : parameter distribution
−−posterior distribution
QPM thick Mean solid Stddev dashed 95% dotted
mean: 0.08218816 stddev: 0.01275878 nvals: 9211 out of: 45001 %neg: 0
0.0803
Frequency
0
200
400
600
800
1000
1200
0.0524
0.0570
0.0616
0.0662
0.0708
0.0754
0.0800
0.0846
0.0892
0.0939
0.0985
0.1031
0.1077
Parameter value

Figure 8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−−prior distribution
QPM thick Mean solid Stddev dashed 95% dotted
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0.1201
Frequency
0
500
1000
1500
2000
2500
3000
3500
0.0846
0.1031
0.1215
0.1400
0.1585
0.1769
0.1954
0.0939
0.1123
0.1308
0.1492
0.1677
0.1862
Parameter value

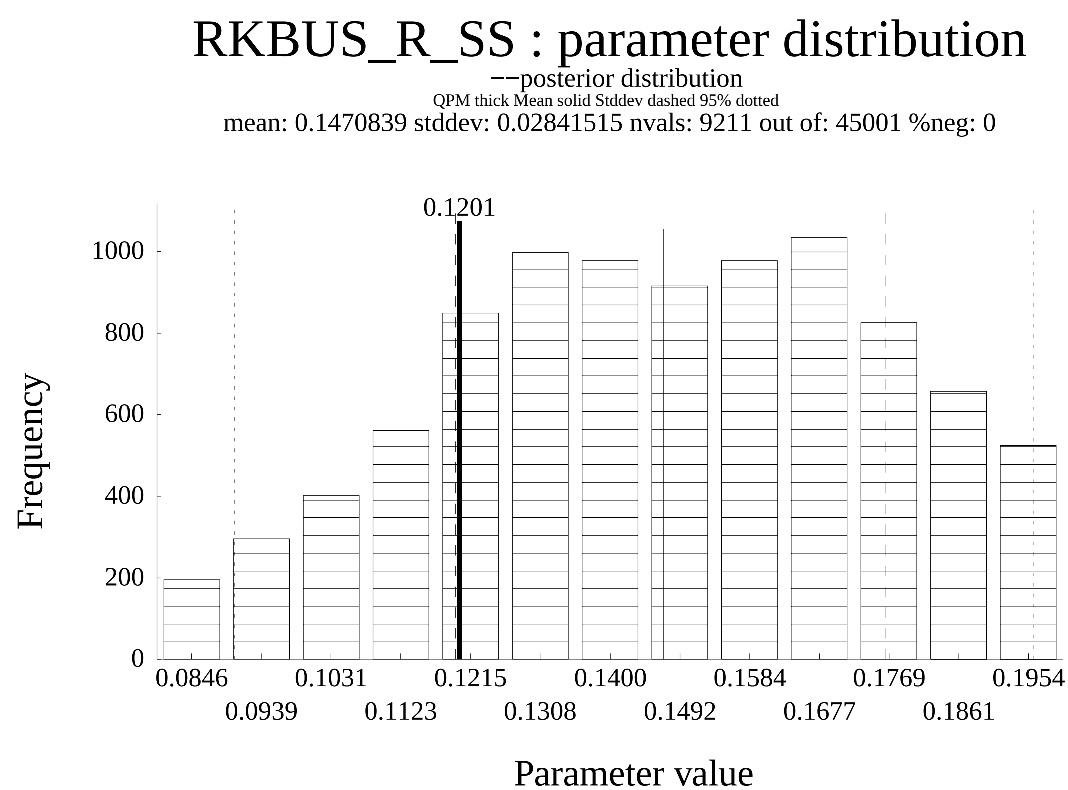
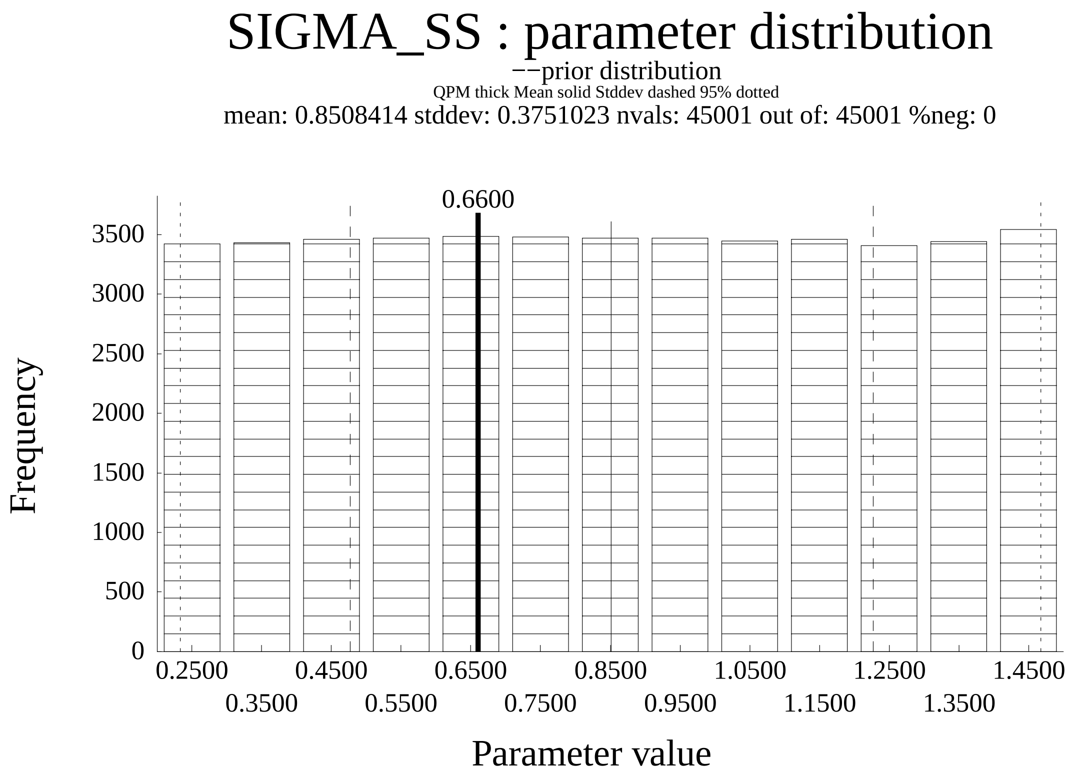

RKBUS_R_SS : parameter distribution
−−posterior distribution
QPM thick Mean solid Stddev dashed 95% dotted
mean: 0.1470839 stddev: 0.02841515 nvals: 9211 out of: 45001 %neg: 0
0.1201
Frequency
0
200
400
600
800
1000
0.0846
0.1031
0.1215
0.1400
0.1584
0.1769
0.1954
0.0939
0.1123
0.1308
0.1492
0.1677
0.1861
Parameter value

Figure 9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−−prior distribution
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0.6600
Frequency
0
500
1000
1500
2000
2500
3000
3500
0.2500
0.3500
0.4500
0.5500
0.6500
0.7500
0.8500
0.9500
1.0500
1.1500
1.2500
1.3500
1.4500
Parameter value

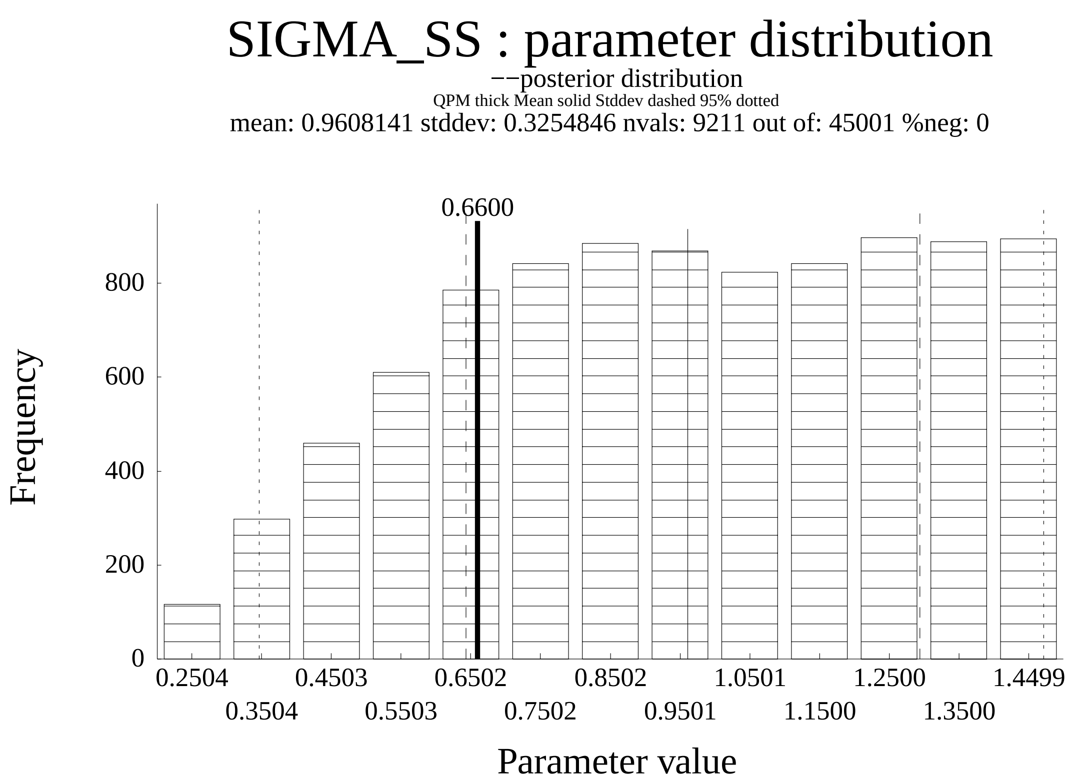

SIGMA_SS : parameter distribution
−−posterior distribution
QPM thick Mean solid Stddev dashed 95% dotted
mean: 0.9608141 stddev: 0.3254846 nvals: 9211 out of: 45001 %neg: 0
0.6600
Frequency
0
200
400
600
800
0.2504
0.3504
0.4503
0.5503
0.6502
0.7502
0.8502
0.9501
1.0501
1.1500
1.2500
1.3500
1.4499
Parameter value

Figure 10

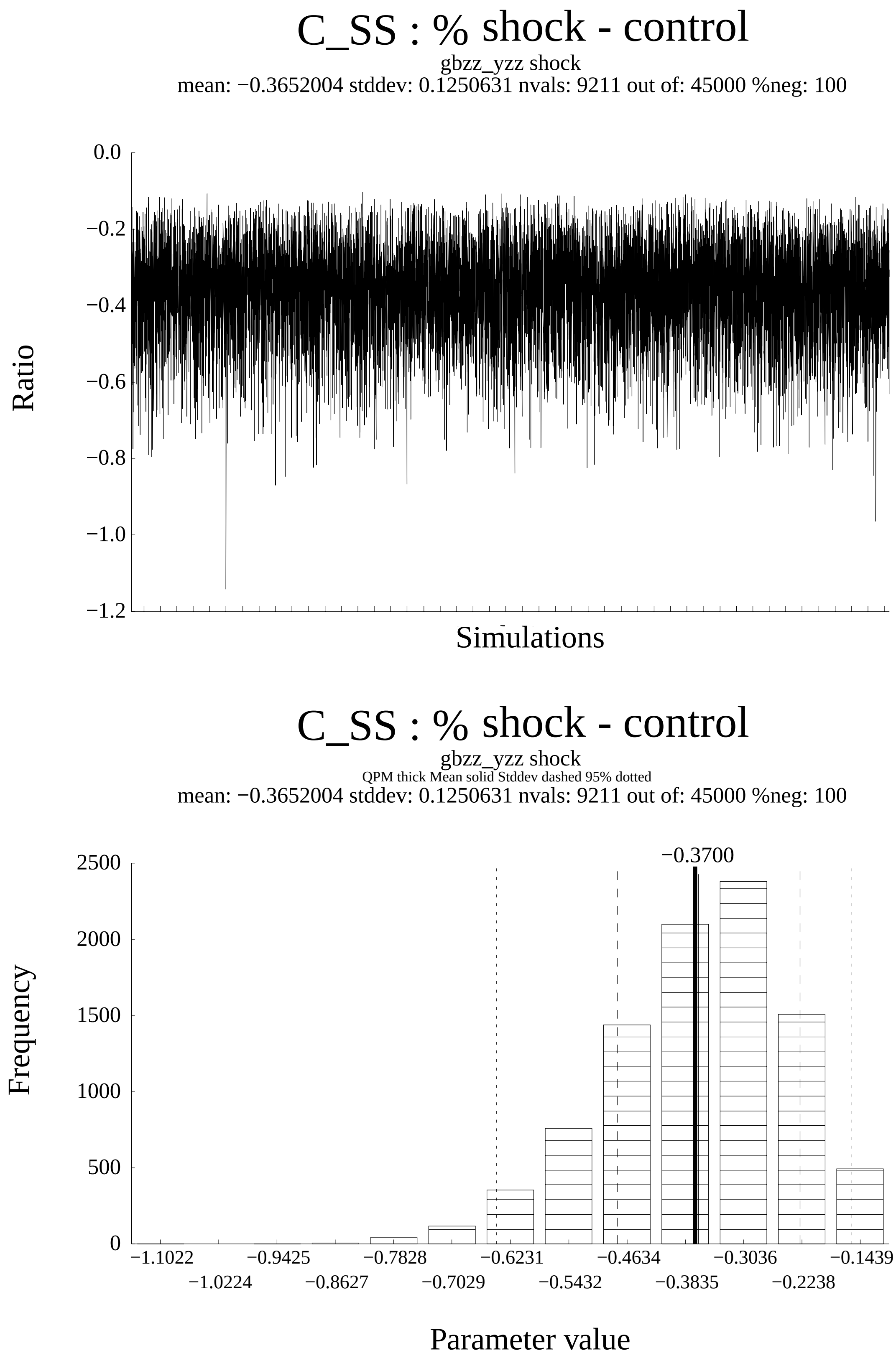
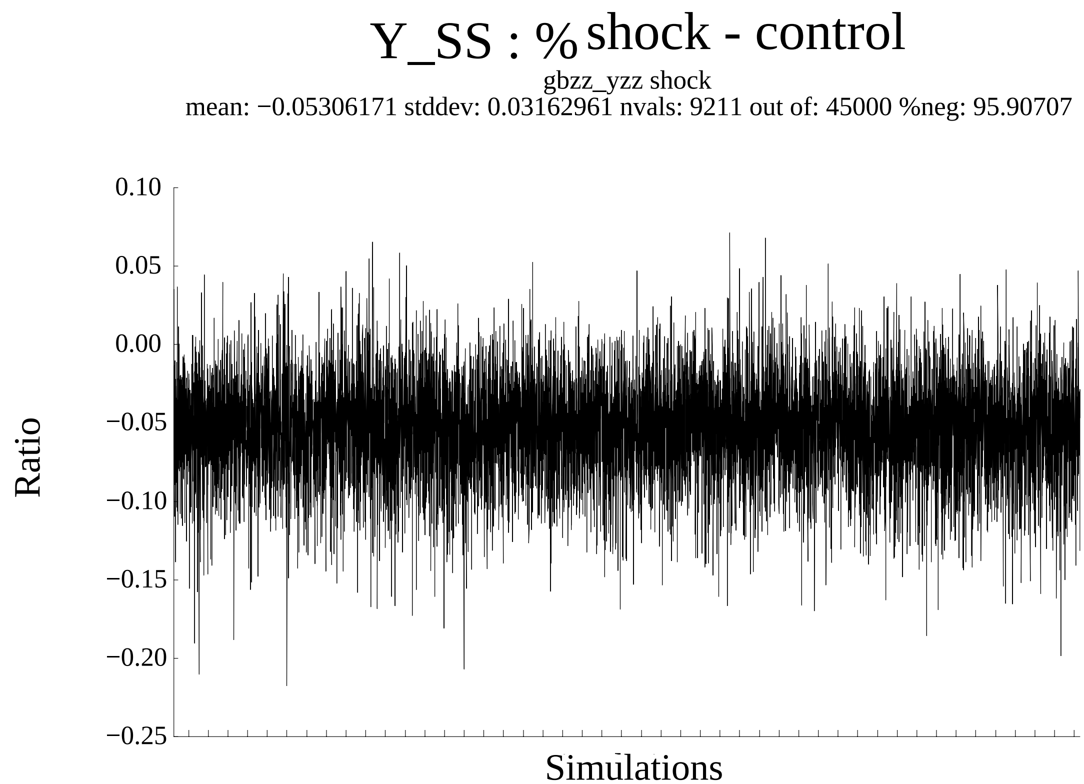

C_SS : % shock - control
gbzz_yzz shock
mean: −0.3652004 stddev: 0.1250631 nvals: 9211 out of: 45000 %neg: 100
Ratio
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.2
Simulations
C_SS : % shock - control
gbzz_yzz shock
QPM thick Mean solid Stddev dashed 95% dotted
mean: −0.3652004 stddev: 0.1250631 nvals: 9211 out of: 45000 %neg: 100
Frequency
2500
2000
1500
1000
500
0
−0.3700
−1.1022
−1.0224
−0.9425
−0.8627
−0.7828
−0.7029
−0.6231
−0.5432
−0.4634
−0.3835
−0.3036
−0.2238
−0.1439
Parameter value

Figure 11


Y_SS : % shock - control
gbzz_yzz shock
mean: −0.05306171 stddev: 0.03162961 nvals: 9211 out of: 45000 %neg: 95.90707
Ratio
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
−0.25
Simulations

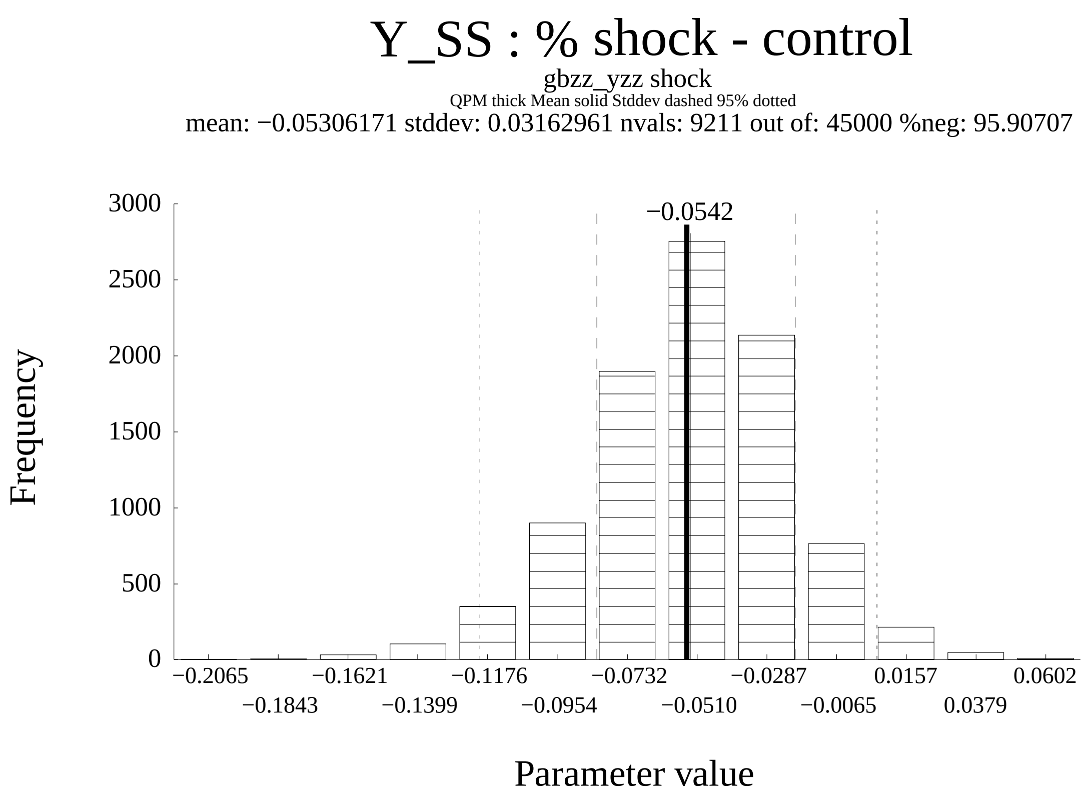
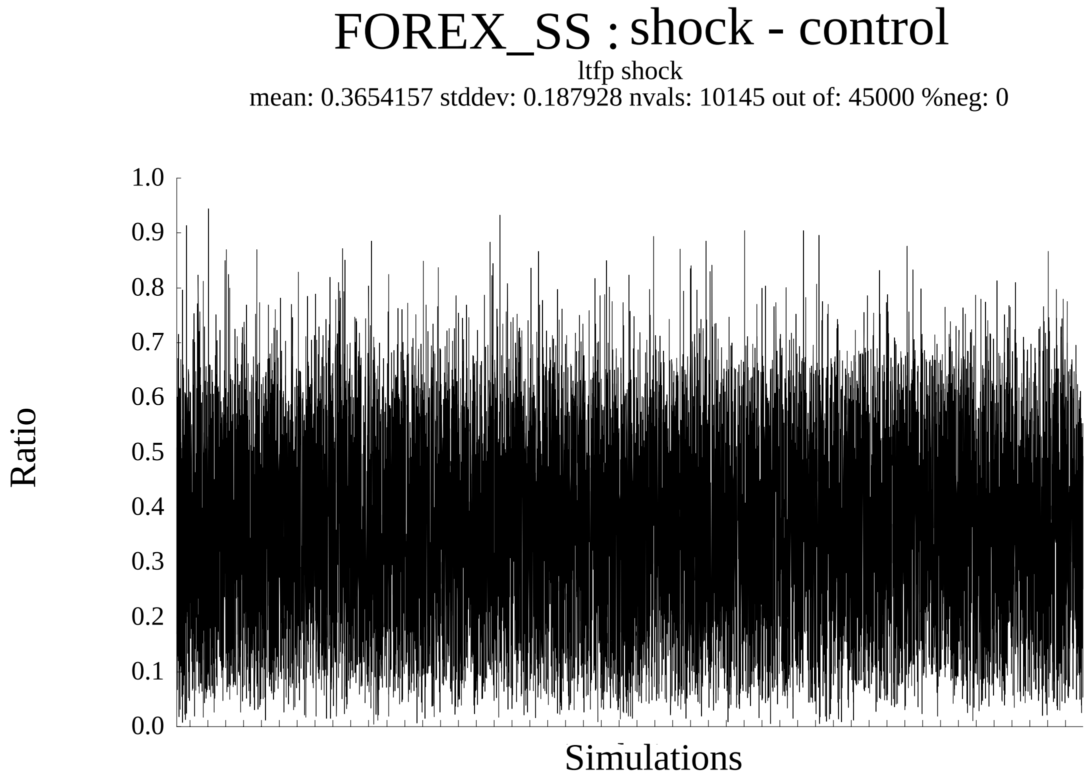

Y_SS : % shock - control
gbzz_yzz shock
QPM thick Mean solid Stddev dashed 95% dotted
mean: −0.05306171 stddev: 0.03162961 nvals: 9211 out of: 45000 %neg: 95.90707
Frequency
3000
2500
2000
1500
1000
500
0
−0.0542
−0.2065
−0.1843
−0.1621
−0.1399
−0.1176
−0.0954
−0.0732
−0.0510
−0.0287
−0.0065
0.0157
0.0379
0.0602
Parameter value

Figure 12


FOREX_SS : shock - control
ltfp shock
mean: 0.3654157 stddev: 0.187928 nvals: 10145 out of: 45000 %neg: 0
Ratio
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
Simulations

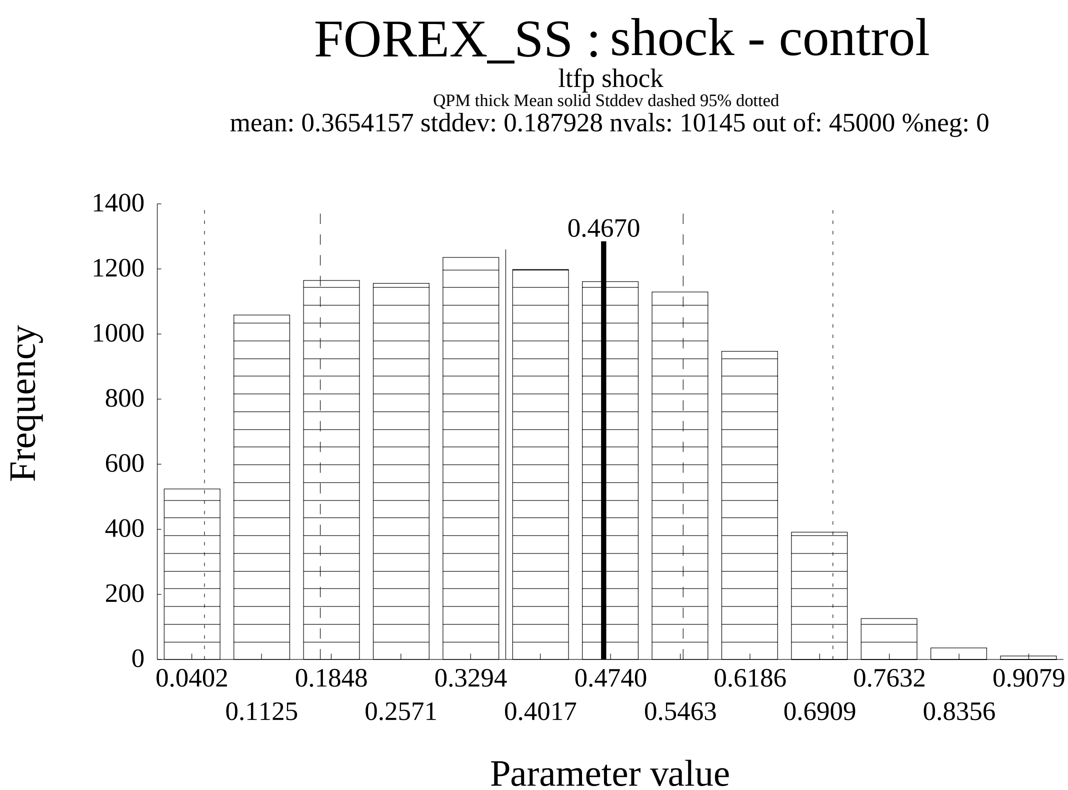
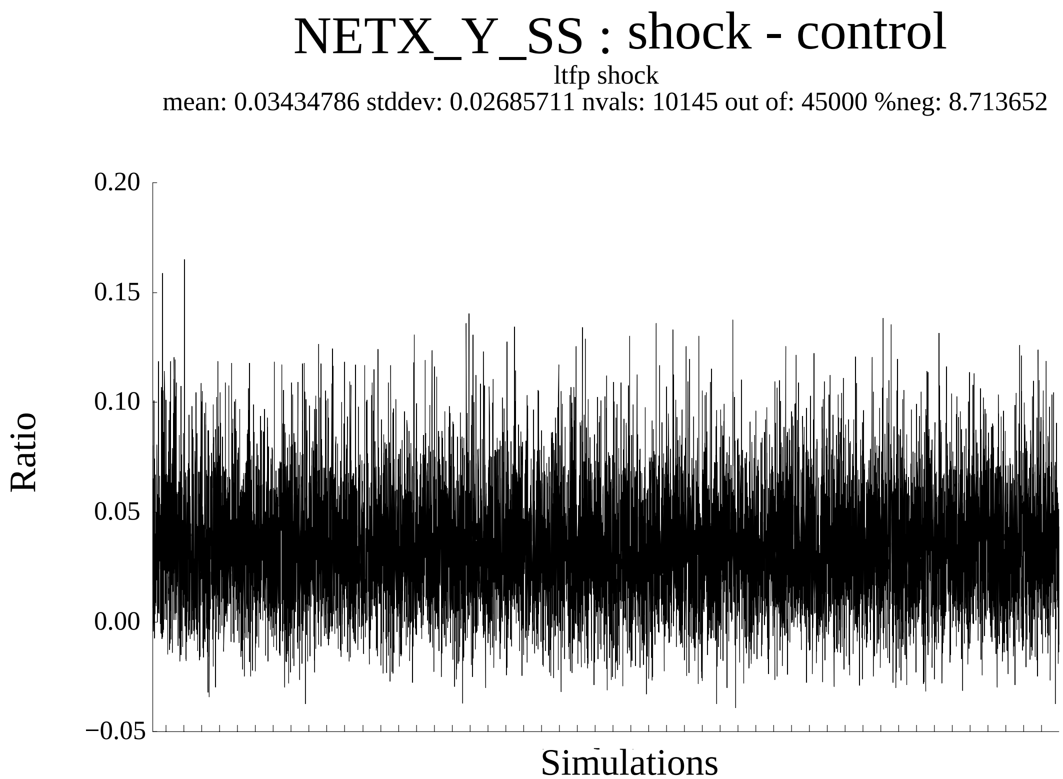

FOREX_SS : shock - control
ltfp shock
QPM thick Mean solid Stddev dashed 95% dotted
mean: 0.3654157 stddev: 0.187928 nvals: 10145 out of: 45000 %neg: 0
Frequency
1400
1200
1000
800
600
400
200
0
0.4670
0.0402
0.1125
0.1848
0.2571
0.3294
0.4017
0.4740
0.5463
0.6186
0.6909
0.7632
0.8356
0.9079
Parameter value

Figure 13


NETX_Y_SS : shock - control
ltfp shock
mean: 0.03434786 stddev: 0.02685711 nvals: 10145 out of: 45000 %neg: 8.713652
Ratio
0.20
0.15
0.10
0.05
0.00
−0.05
Simulations

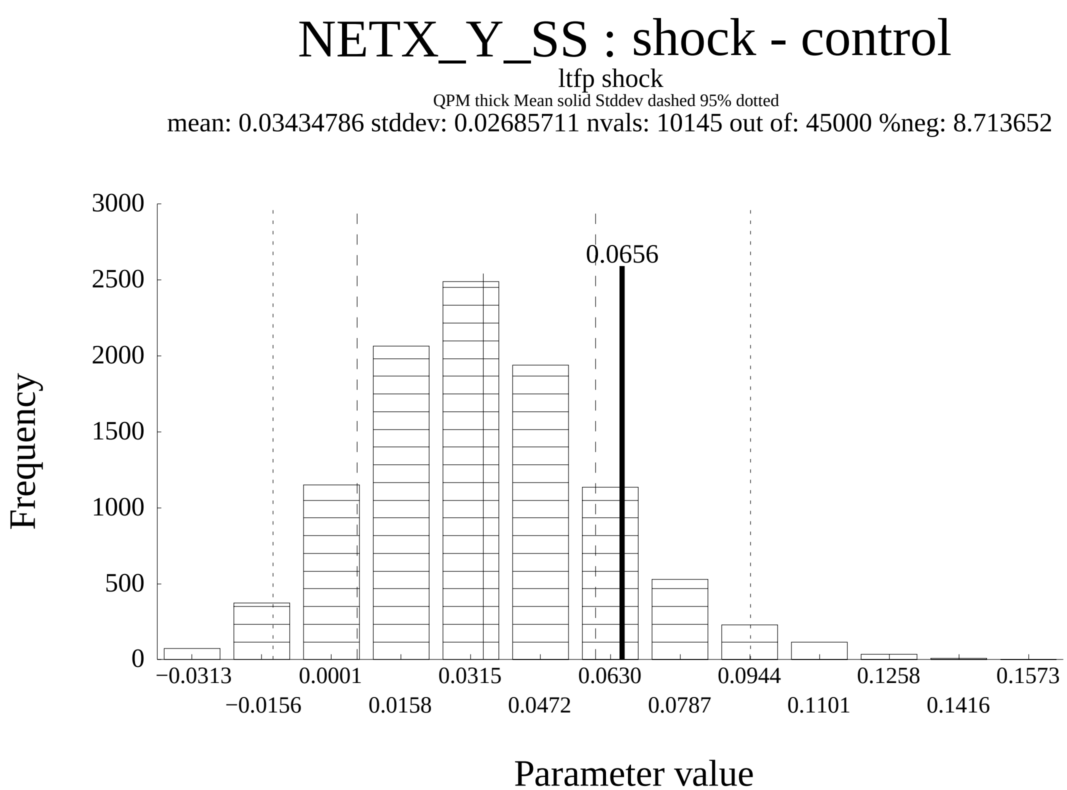
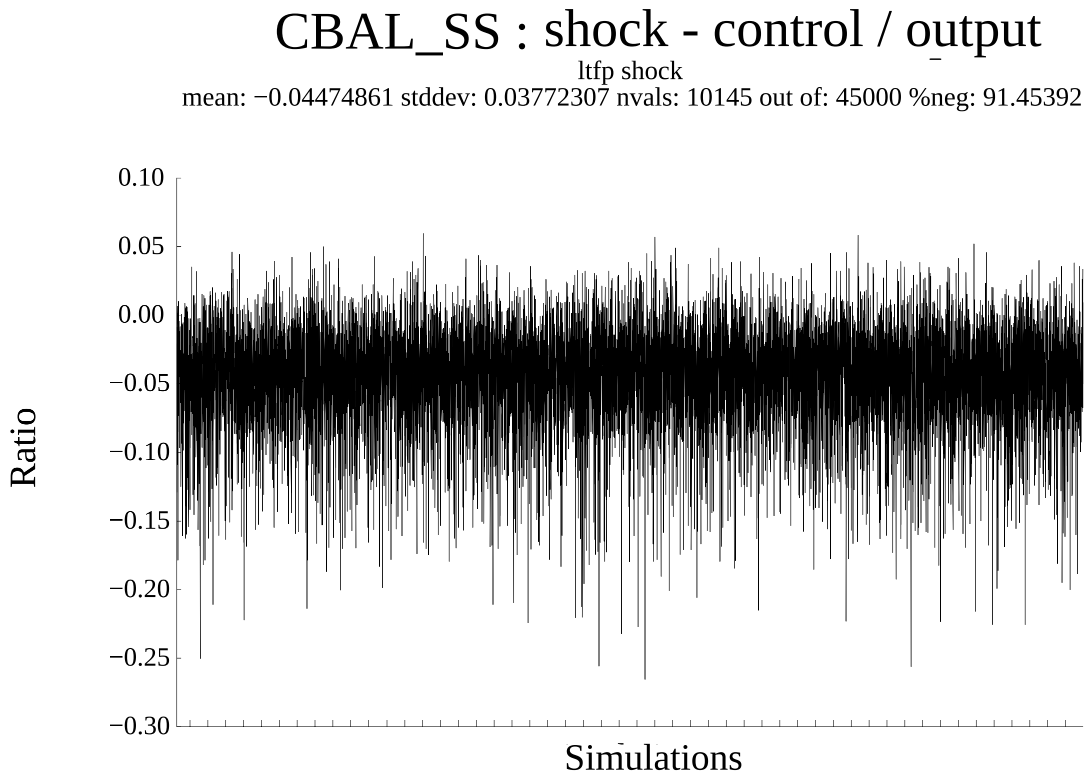

NETX_Y_SS : shock - control
ltfp shock
QPM thick Mean solid Stddev dashed 95% dotted
mean: 0.03434786 stddev: 0.02685711 nvals: 10145 out of: 45000 %neg: 8.713652
Frequency
3000
2500
2000
1500
1000
500
0
0.0656
−0.0313
−0.0156
0.0001
0.0158
0.0315
0.0472
0.0630
0.0787
0.0944
0.1101
0.1258
0.1416
0.1573
Parameter value

Figure 14


CBAL_SS : shock - control / output
ltfp shock
mean: −0.04474861 stddev: 0.03772307 nvals: 10145 out of: 45000 %neg: 91.45392
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
−0.25
−0.30
Ratio
Simulations

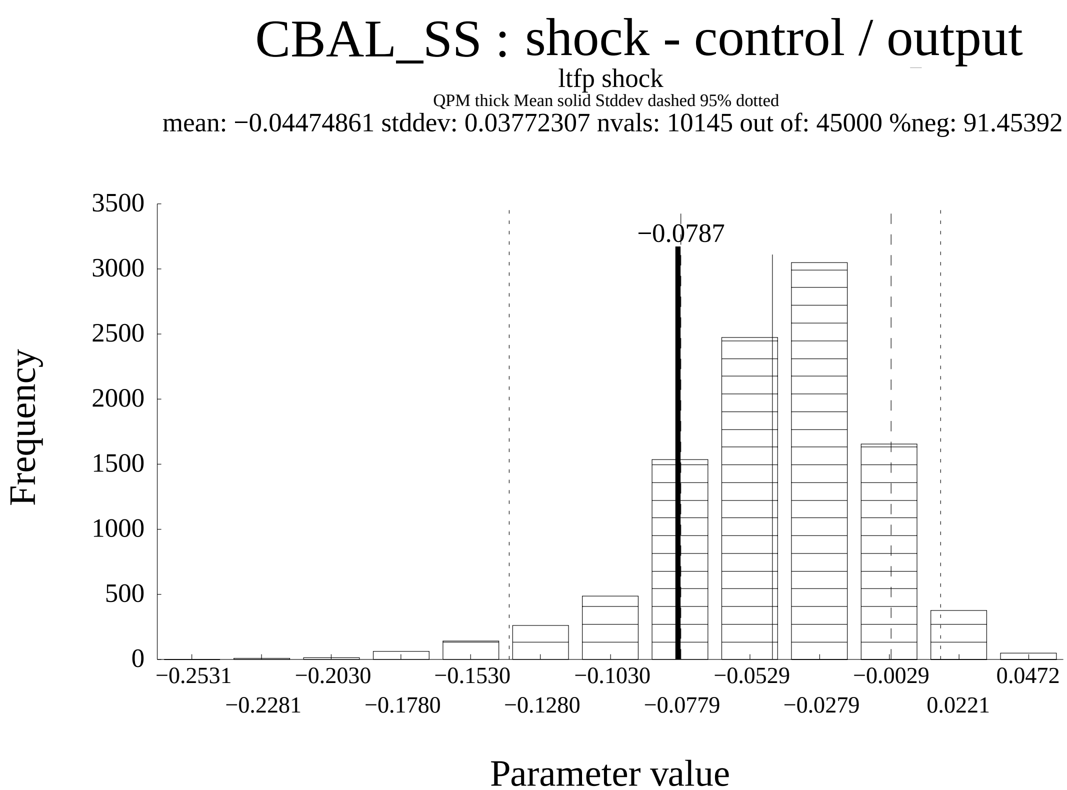
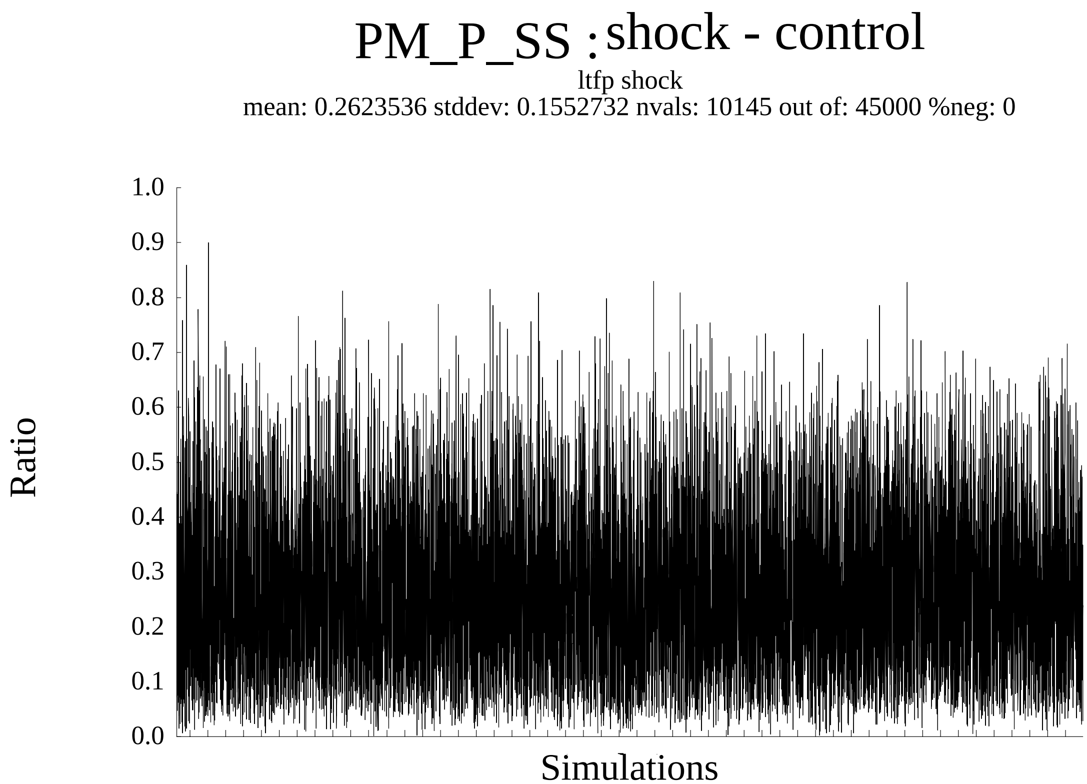

CBAL_SS : shock - control / output
ltfp shock
QPM thick Mean solid Stddev dashed 95% dotted
mean: −0.04474861 stddev: 0.03772307 nvals: 10145 out of: 45000 %neg: 91.45392
−0.0787
3500
3000
2500
2000
1500
1000
500
0
Frequency
−0.2531
−0.2281
−0.2030
−0.1780
−0.1530
−0.1280
−0.1030
−0.0779
−0.0529
−0.0279
−0.0029
0.0221
0.0472
Parameter value

Figure 15


PM_P_SS : shock - control
ltfp shock
mean: 0.2623536 stddev: 0.1552732 nvals: 10145 out of: 45000 %neg: 0
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
Ratio
Simulations

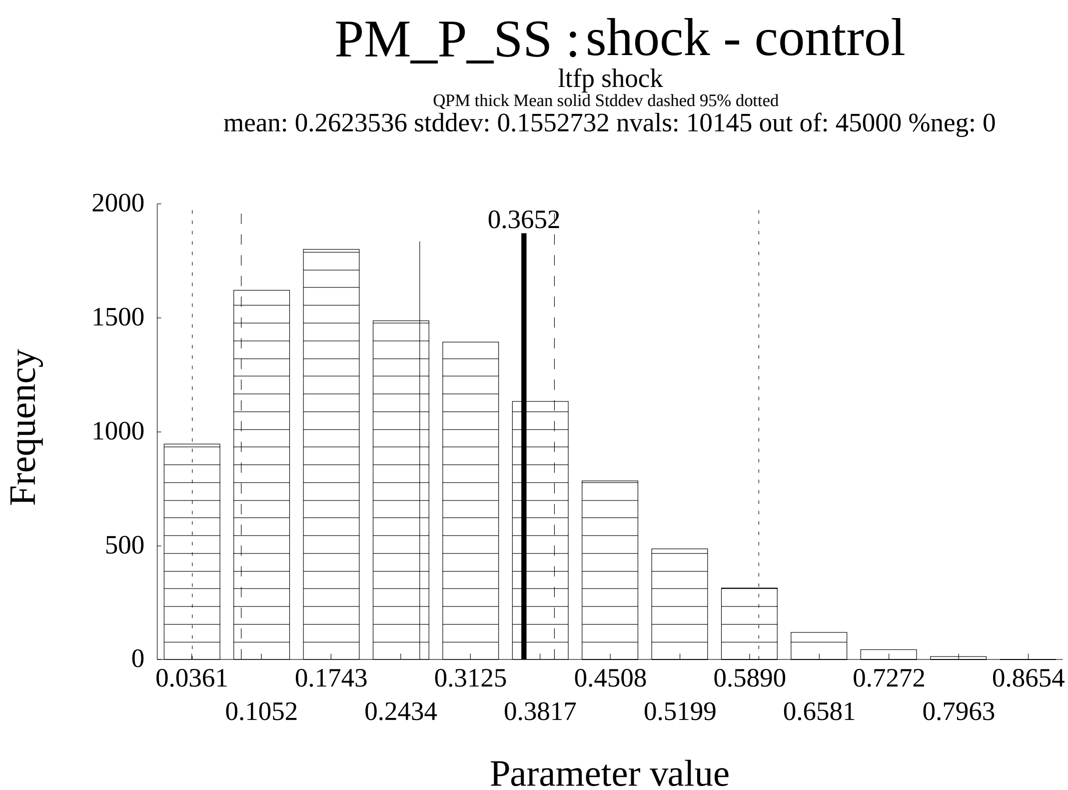
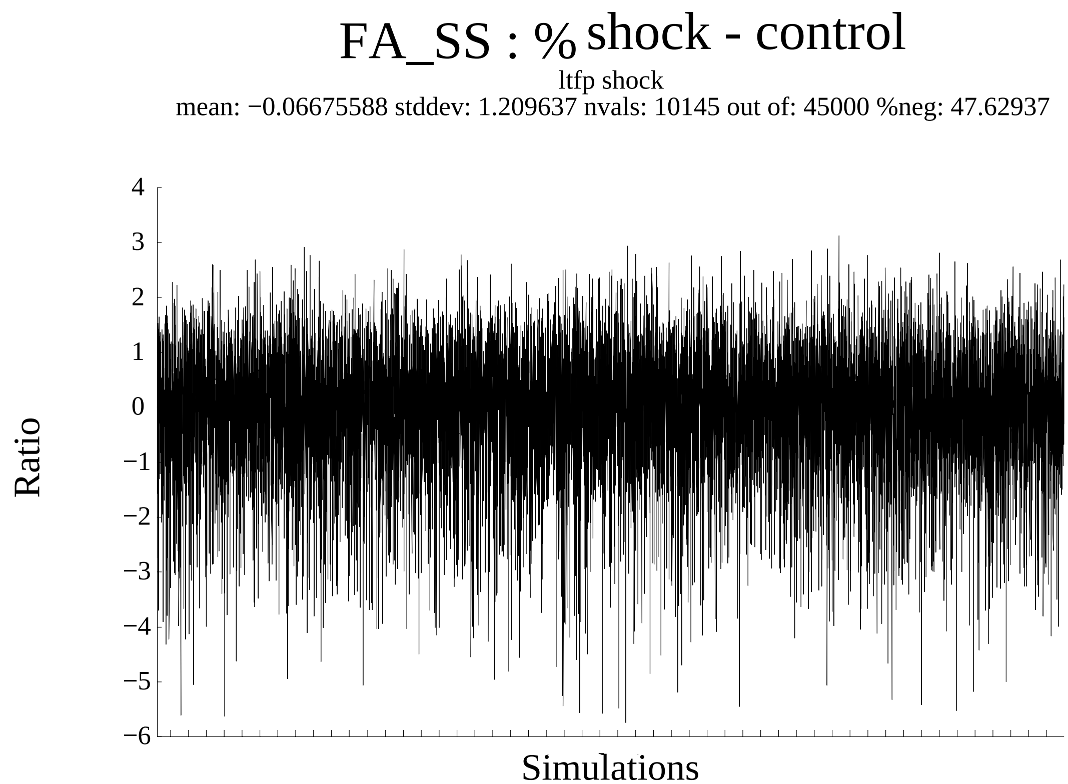

PM_P_SS : shock - control
ltfp shock
QPM thick Mean solid Stddev dashed 95% dotted
mean: 0.2623536 stddev: 0.1552732 nvals: 10145 out of: 45000 %neg: 0
2000
1500
1000
500
0
0.3652
Frequency
0.0361
0.1743
0.3125
0.4508
0.5890
0.7272
0.8654
0.1052
0.2434
0.3817
0.5199
0.6581
0.7963
Parameter value

Figure 16


FA_SS : % shock - control
ltfp shock
mean: −0.06675588 stddev: 1.209637 nvals: 10145 out of: 45000 %neg: 47.62937
4
3
2
1
0
−1
−2
−3
−4
−5
−6
Ratio
Simulations

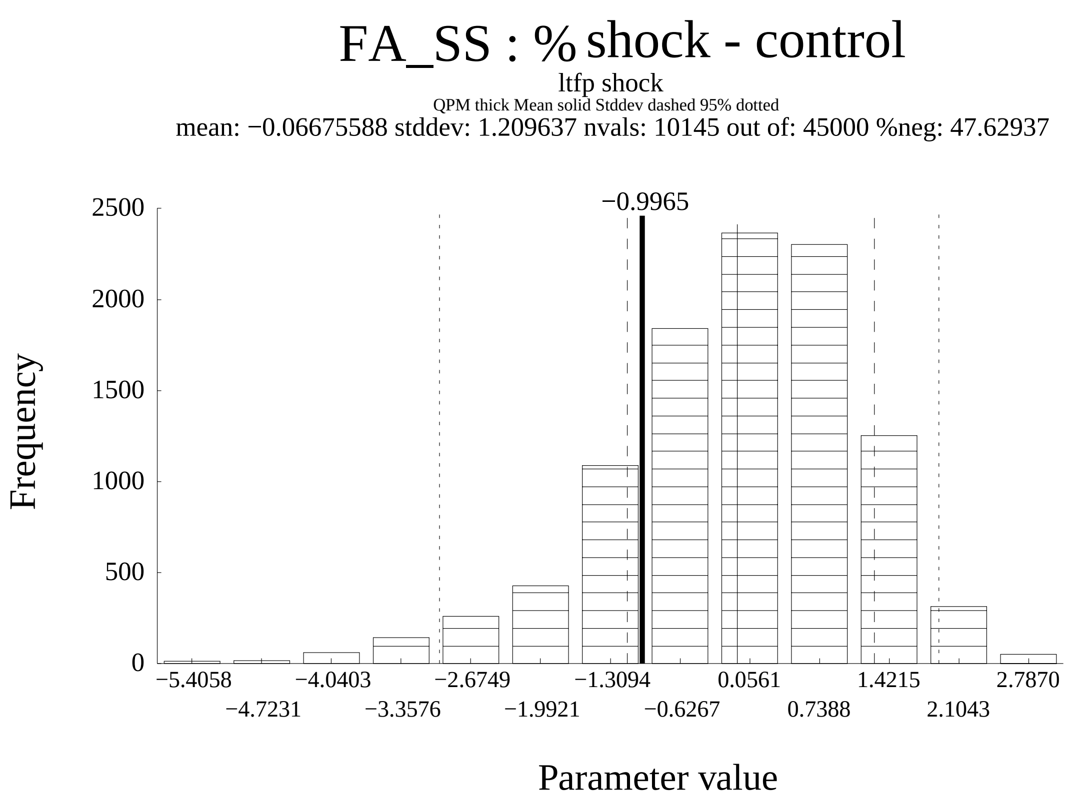
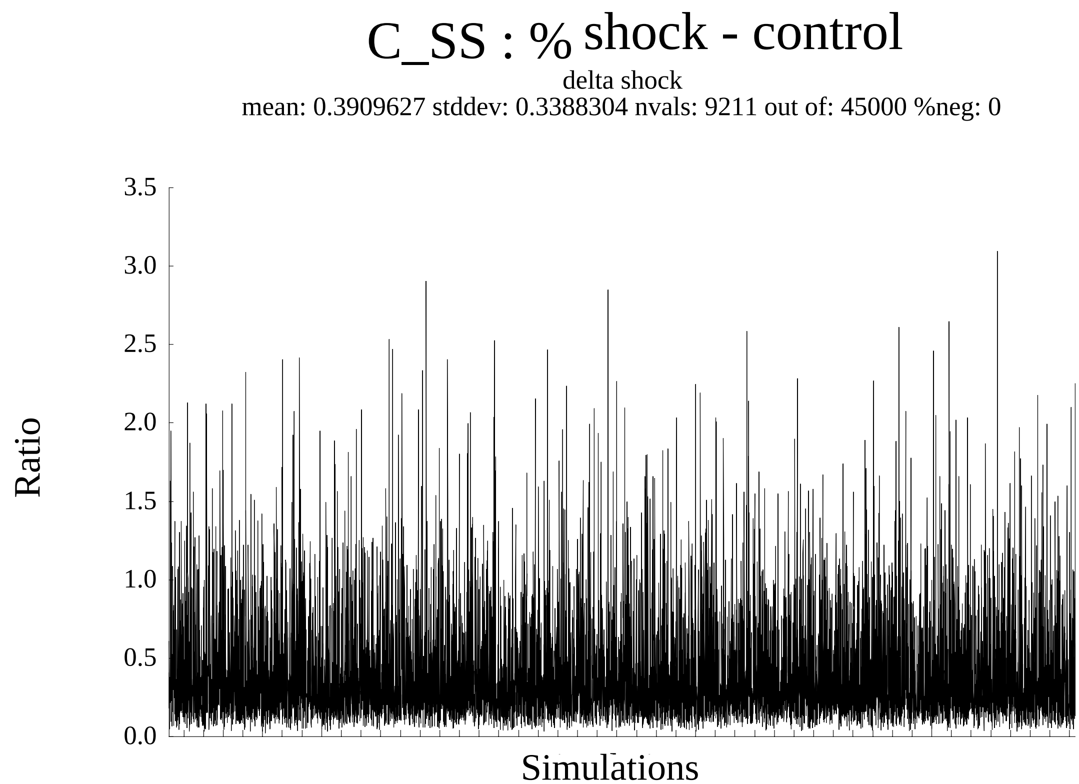

FA_SS : % shock - control
ltfp shock
QPM thick Mean solid Stddev dashed 95% dotted
mean: −0.06675588 stddev: 1.209637 nvals: 10145 out of: 45000 %neg: 47.62937
−0.9965
2500
2000
1500
1000
500
0
Frequency
−5.4058
−4.0403
−2.6749
−1.3094
0.0561
1.4215
2.7870
−4.7231
−3.3576
−1.9921
−0.6267
0.7388
2.1043
Parameter value

Figure 17


C_SS : % shock - control
delta shock
mean: 0.3909627 stddev: 0.3388304 nvals: 9211 out of: 45000 %neg: 0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Ratio
Simulations

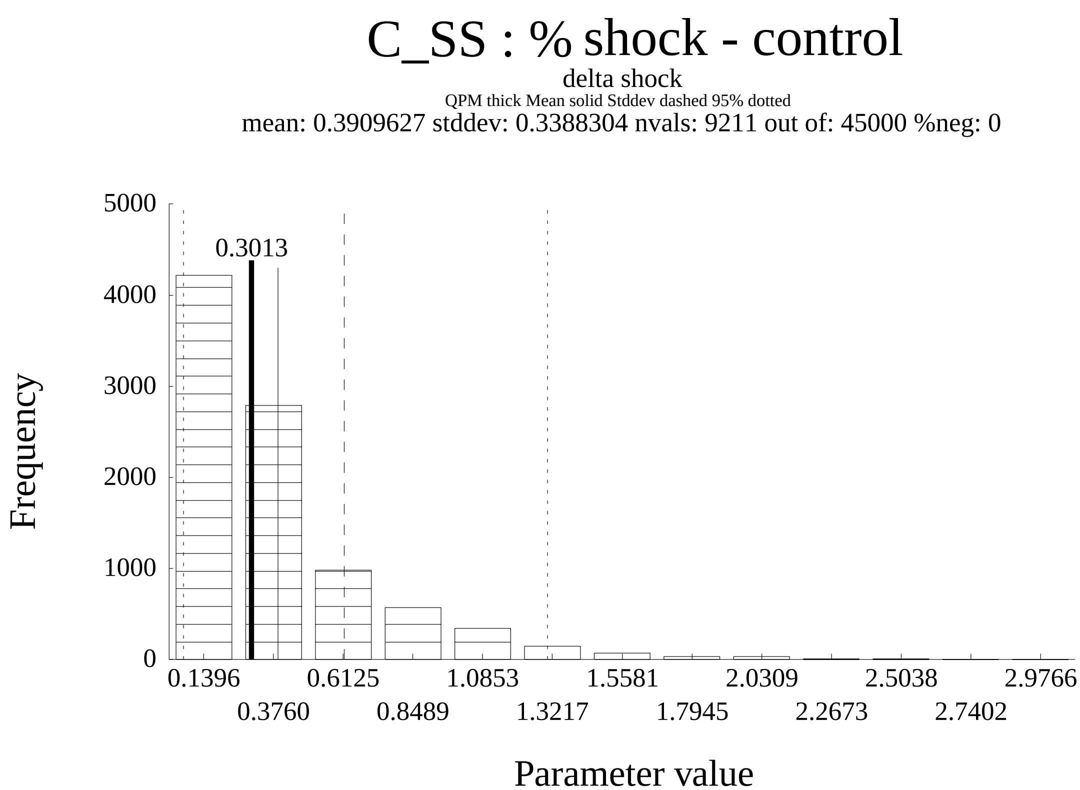
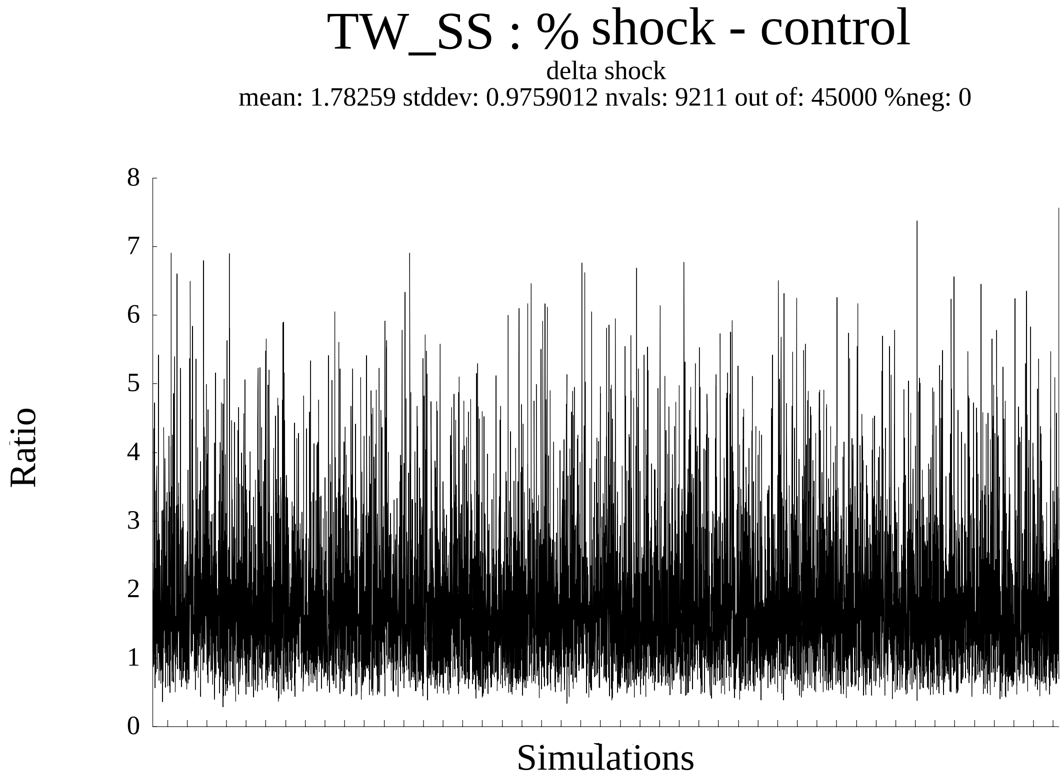

C_SS : % shock - control
delta shock
QPM thick Mean solid Stddev dashed 95% dotted
mean: 0.3909627 stddev: 0.3388304 nvals: 9211 out of: 45000 %neg: 0
5000
4000
3000
2000
1000
0
0.3013
Frequency
0.1396
0.6125
1.0853
1.5581
2.0309
2.5038
2.9766
0.3760
0.8489
1.3217
1.7945
2.2673
2.7402
Parameter value

Figure 18


TW_SS : % shock - control
delta shock
mean: 1.78259 stddev: 0.9759012 nvals: 9211 out of: 45000 %neg: 0
8
7
6
5
4
3
2
1
0
Ratio
Simulations

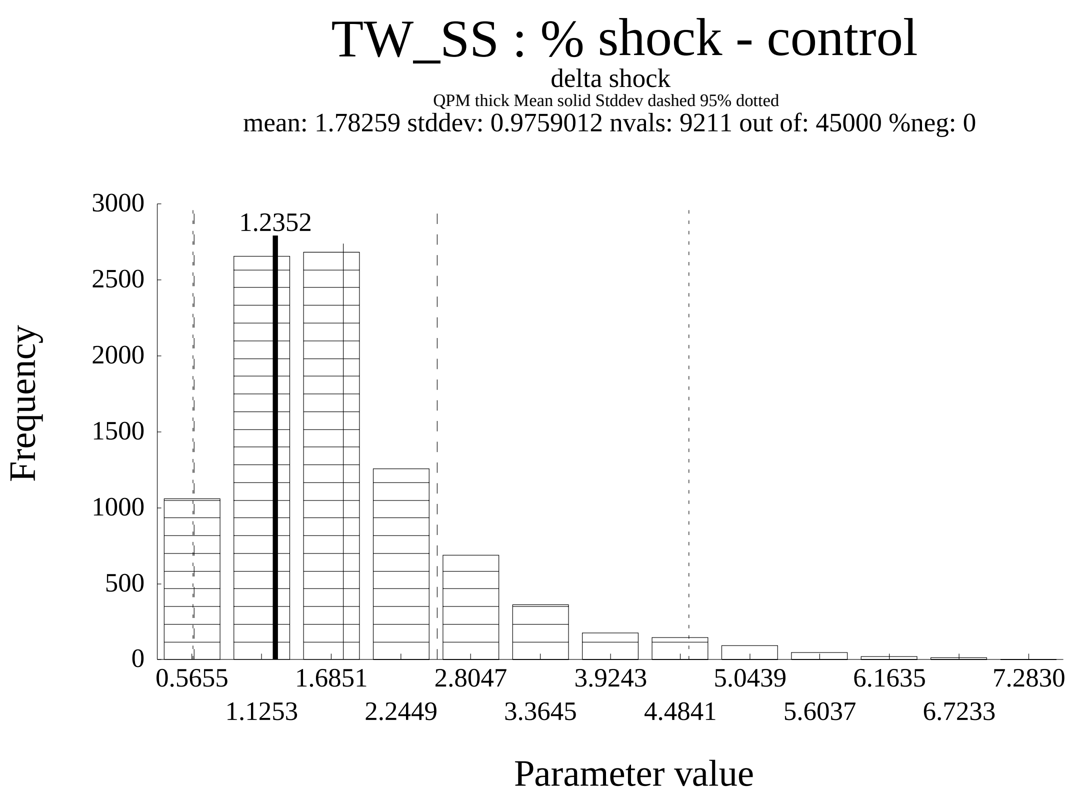
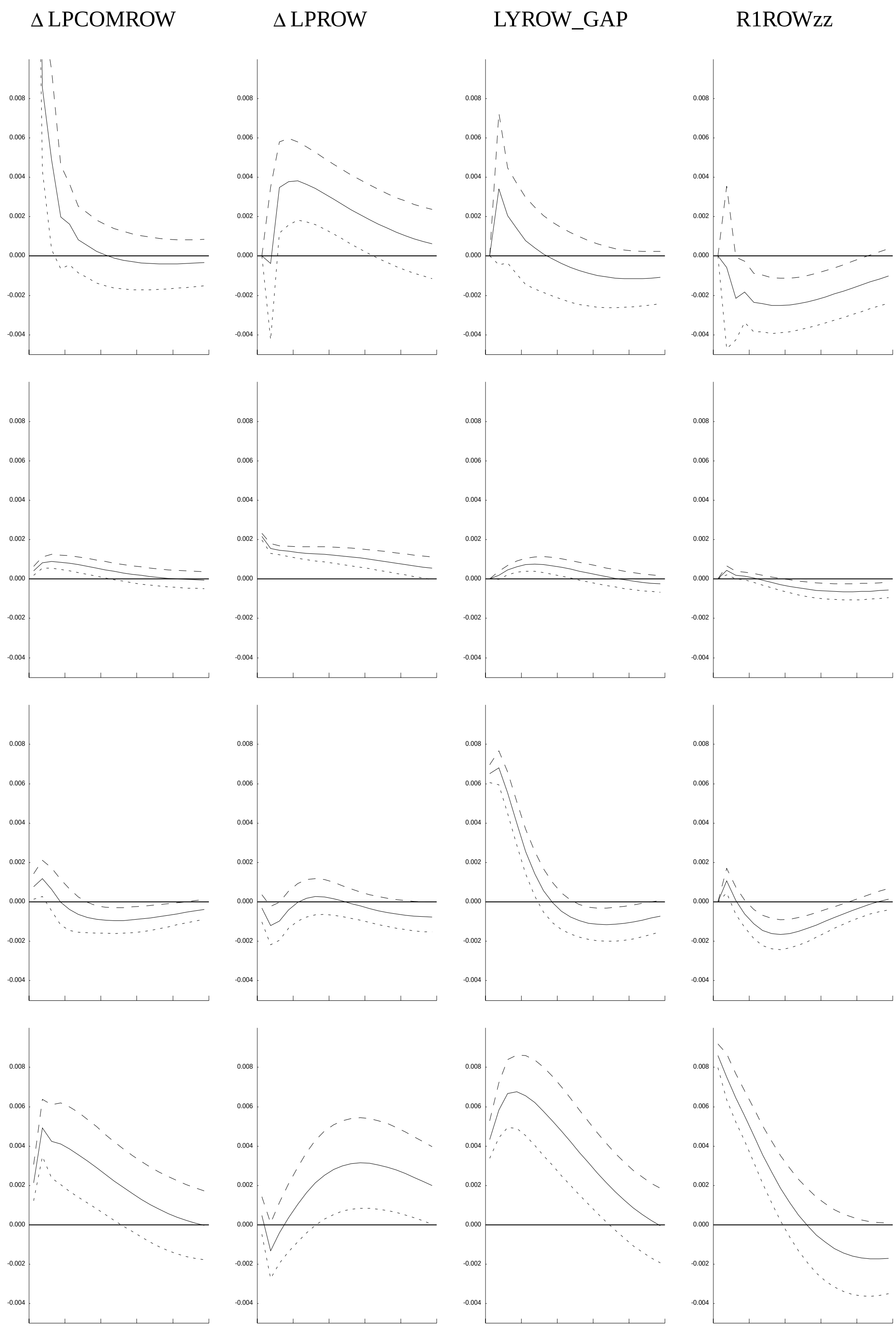

TW_SS : % shock - control
delta shock
QPM thick Mean solid Stddev dashed 95% dotted
mean: 1.78259 stddev: 0.9759012 nvals: 9211 out of: 45000 %neg: 0
3000
2500
2000
1500
1000
500
0
1.2352
0.5655
1.6851
2.8047
3.9243
5.0439
6.1635
7.2830
1.1253
2.2449
3.3645
4.4841
5.6037
6.7233
Frequency
Parameter value

Figure 19: ROW VAR Impulse-Response Functions



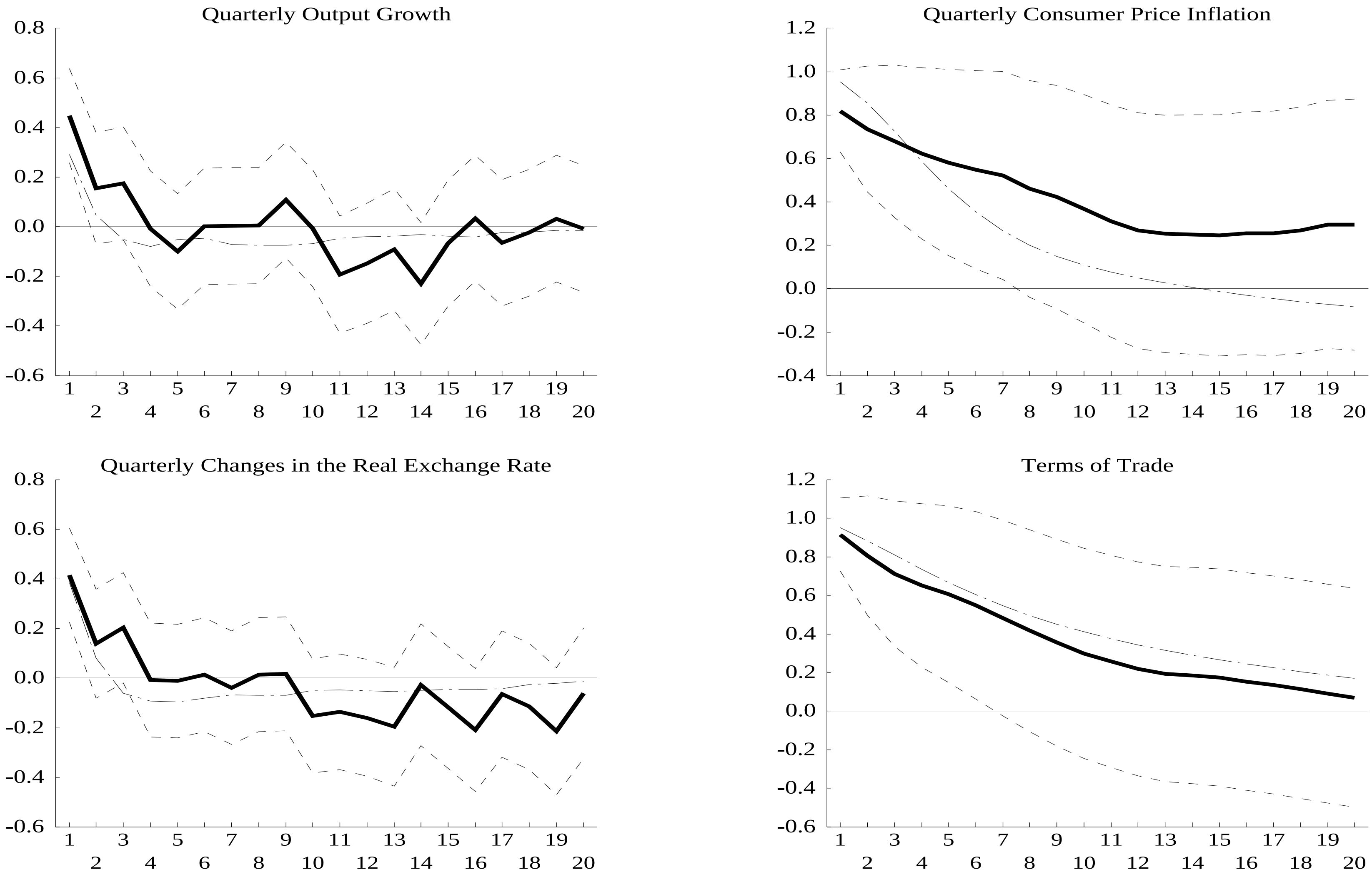
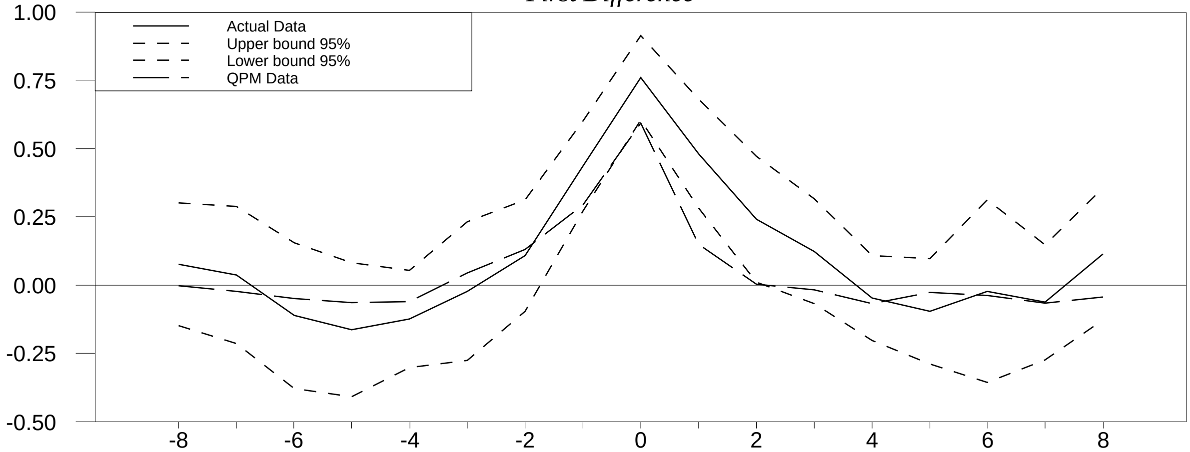


Figure 20: Empirical and QPM Autocorrelation Functions

Figure 21

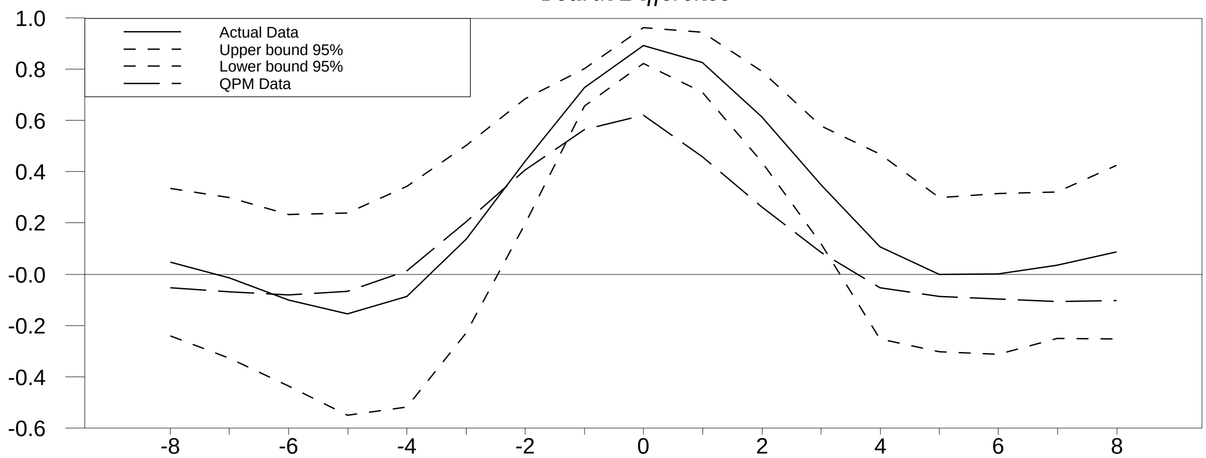
Output and Consumption

First Difference



Output and Consumption

Fourth Difference



Output and Consumption

H-P Filter

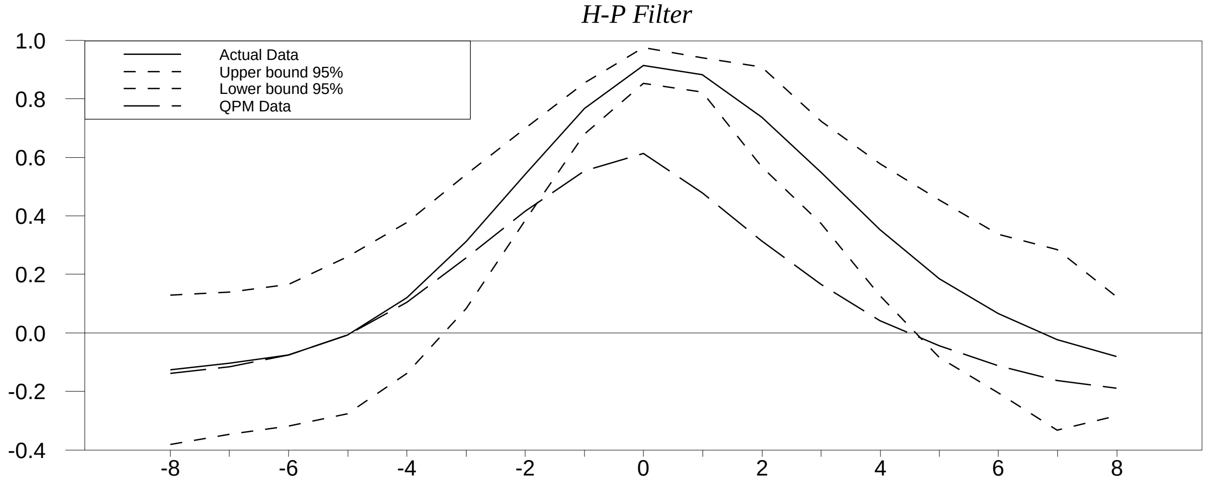
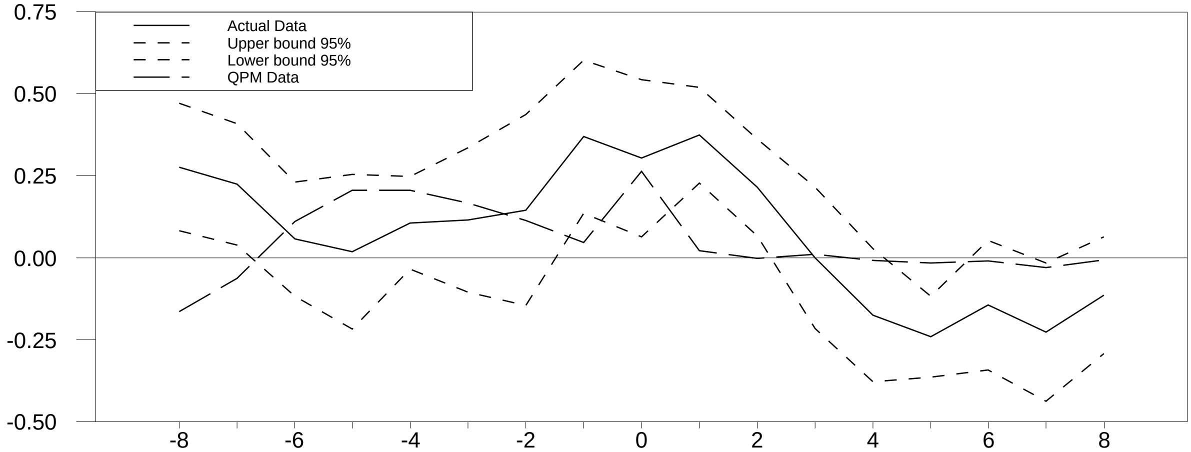


Figure 22

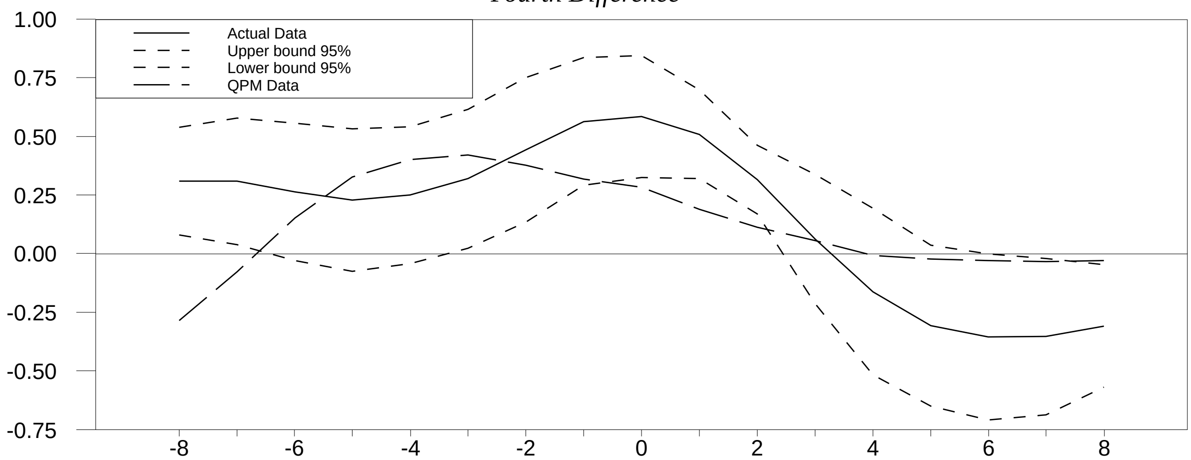
Output and Investment

First Difference



Output and Investment

Fourth Difference



Output and Investment

H-P Filter

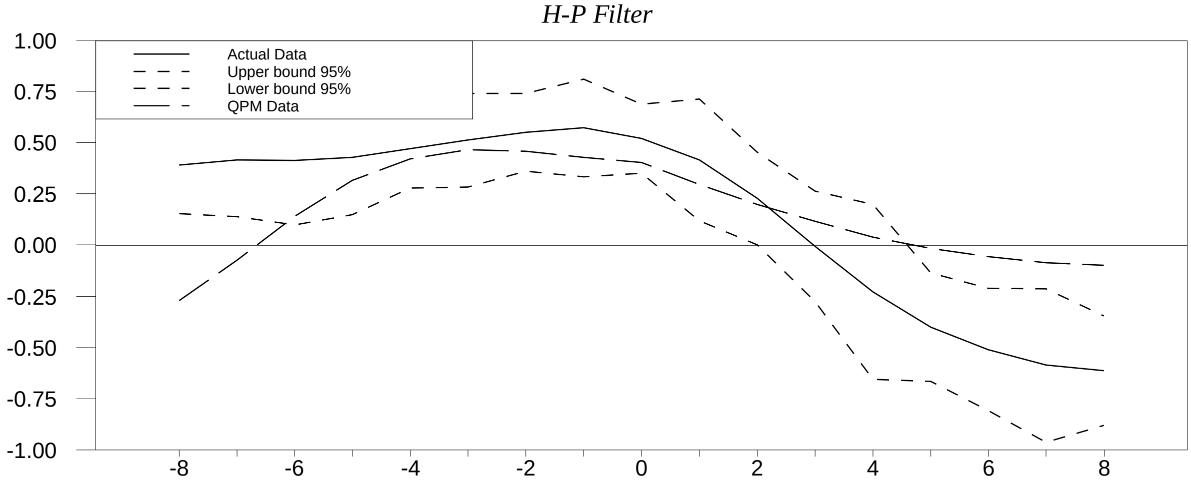
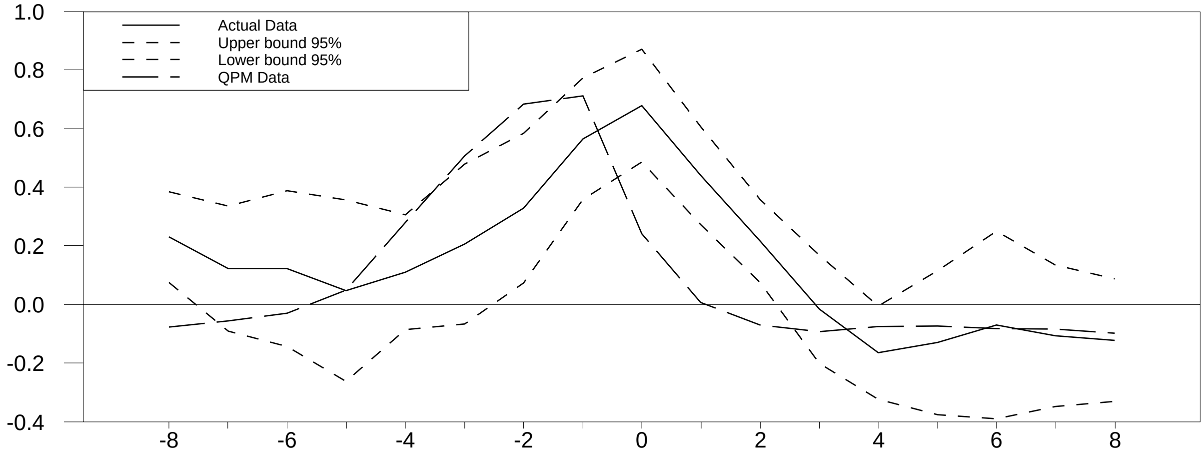


Figure 23

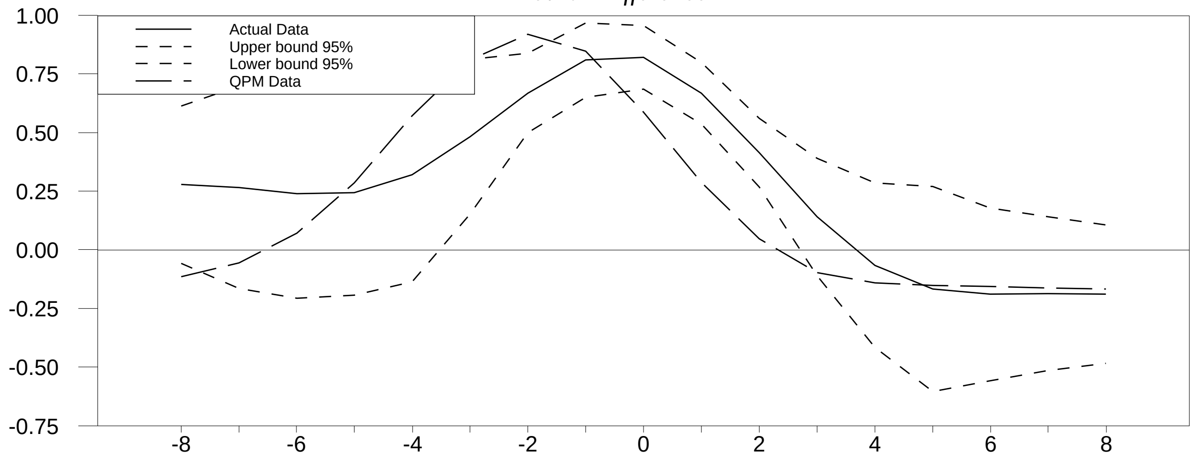
Output and Employment

First Difference



Output and Employment

Fourth Difference



Output and Employment

H-P Filter

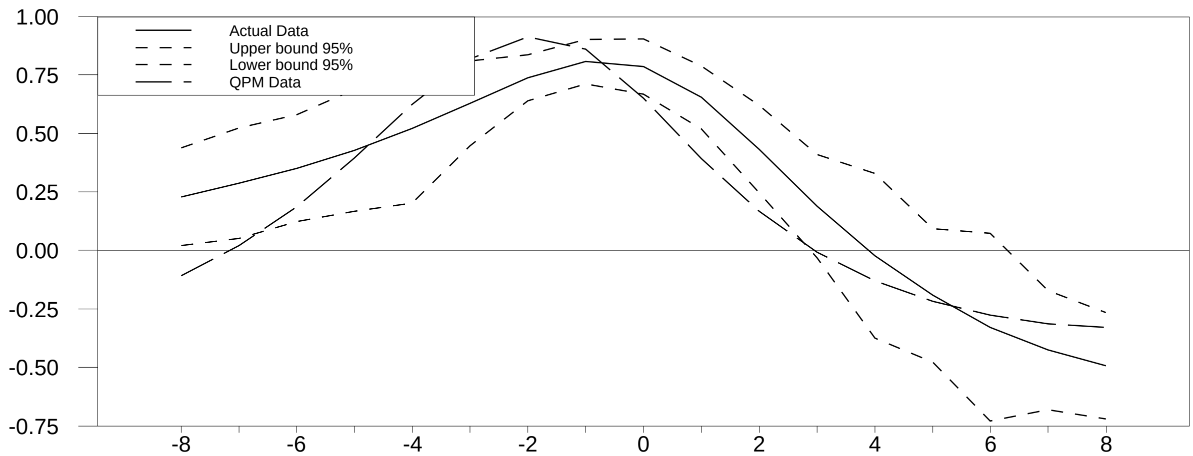
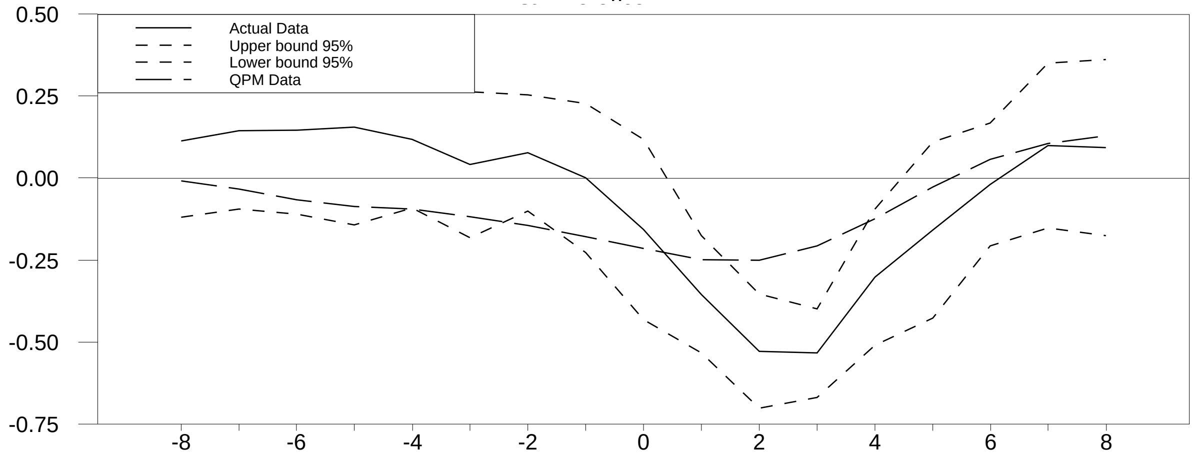


Figure 24

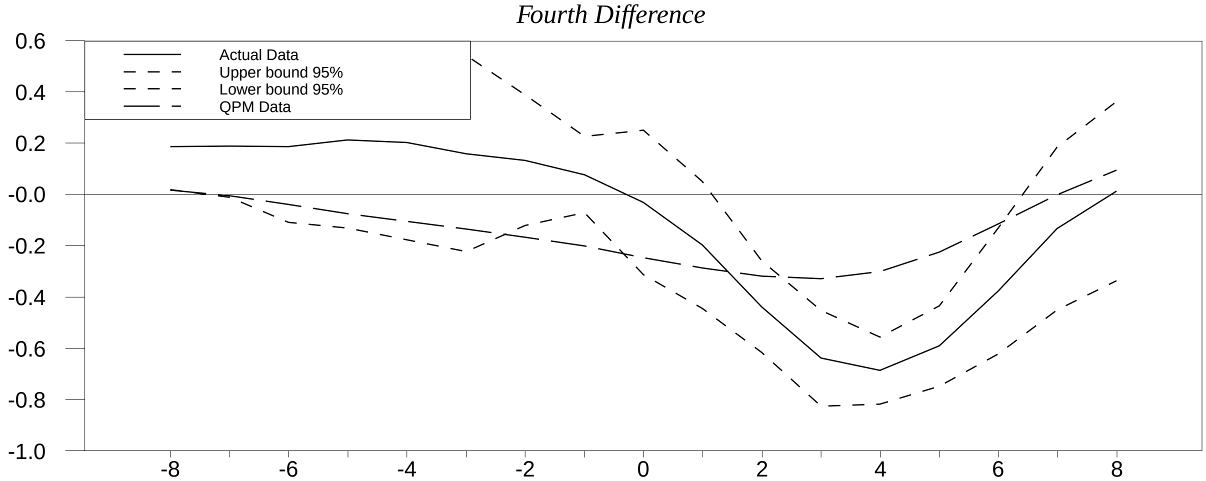
Consumption and Yield Curve Gap

First Difference



Consumption and Yield Curve Gap

Fourth Difference



Consumption and Yield Curve Gap

H-P Filter

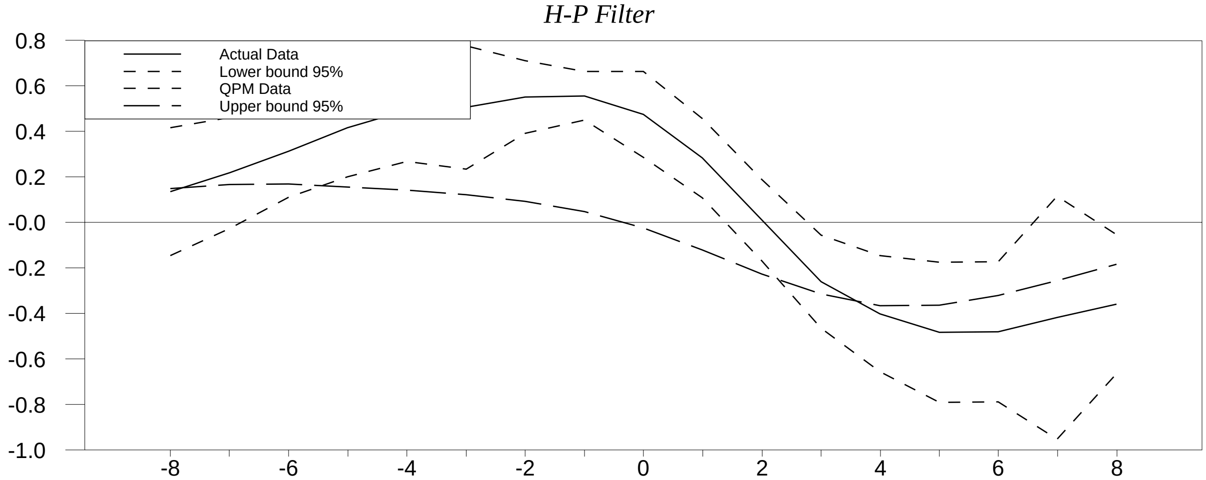
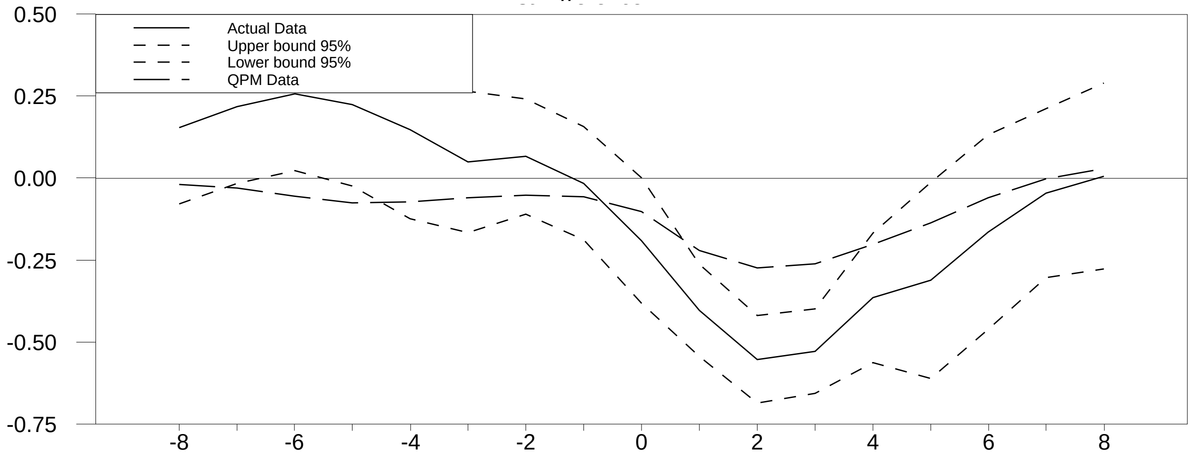


Figure 25

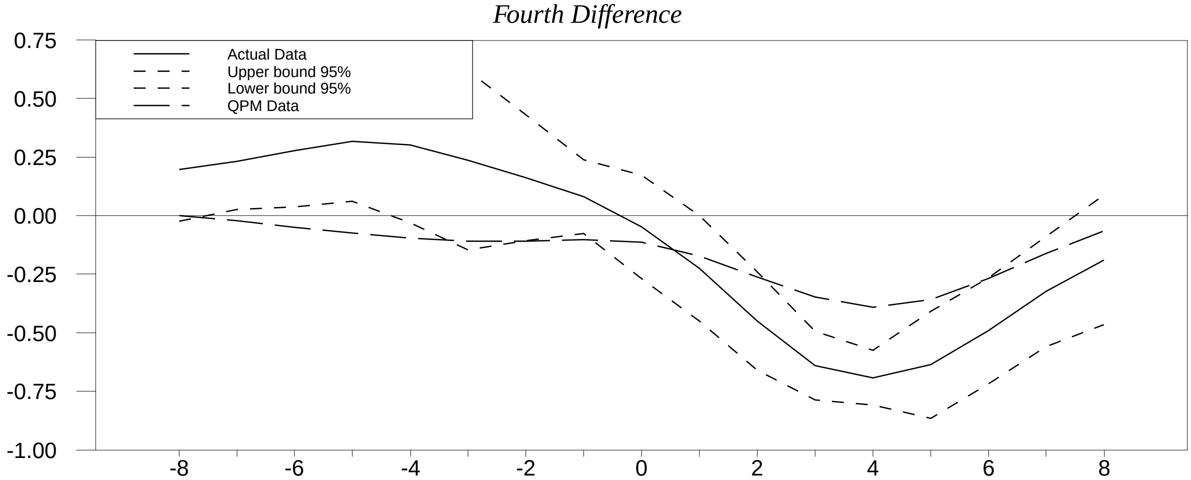
Output and Yield Curve Gap

First Difference



Output and Yield Curve Gap

Fourth Difference



Output and Yield Curve Gap

H-P Filter

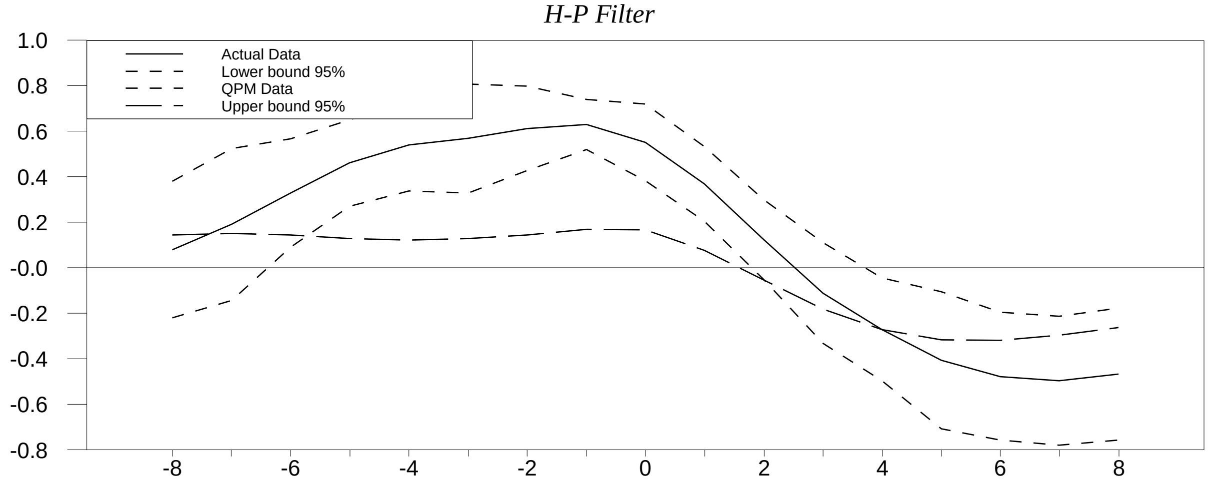
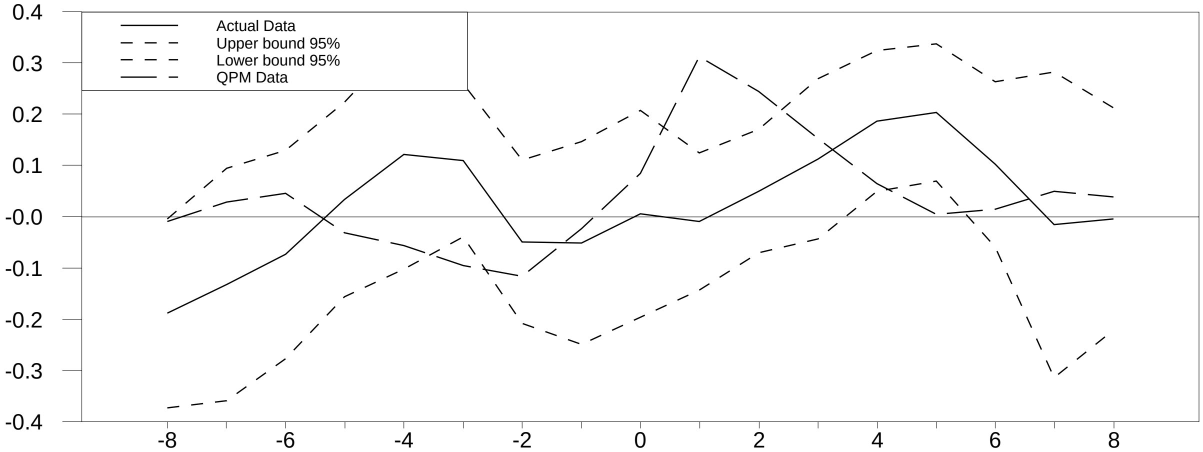


Figure 26

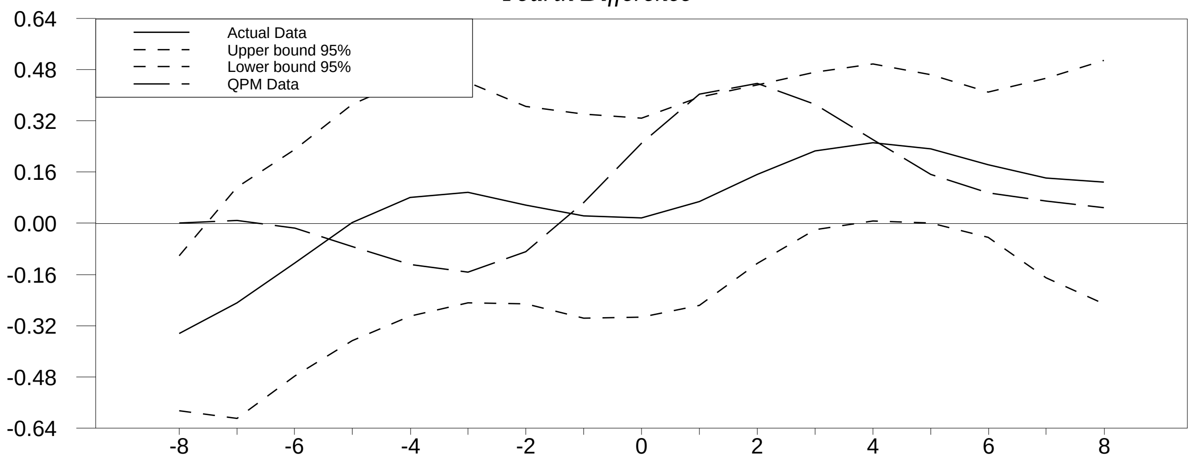
Output and Real Exchange Rate

First Difference



Output and Real Exchange Rate

Fourth Difference



Output and Real Exchange Rate

H-P Filter

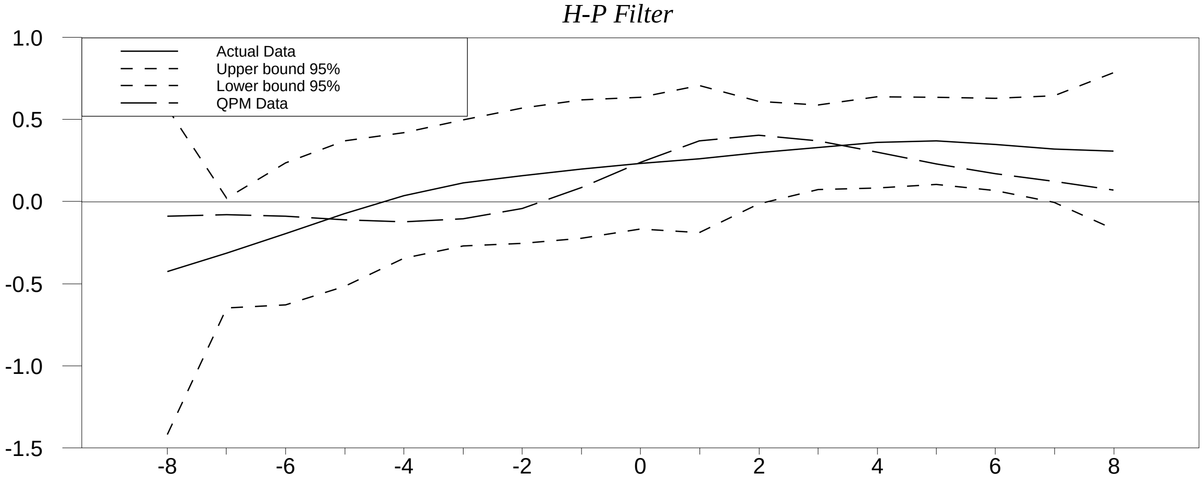
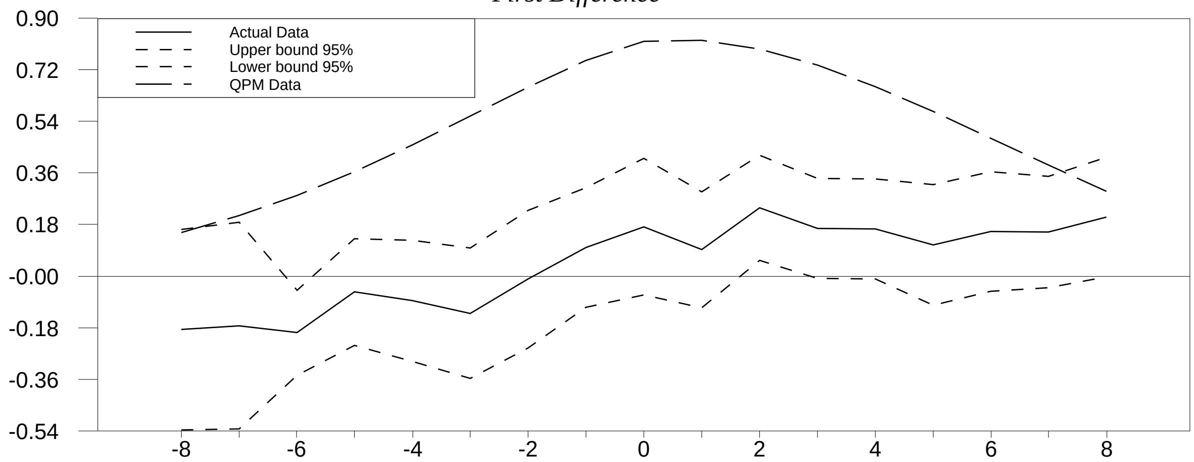


Figure 27

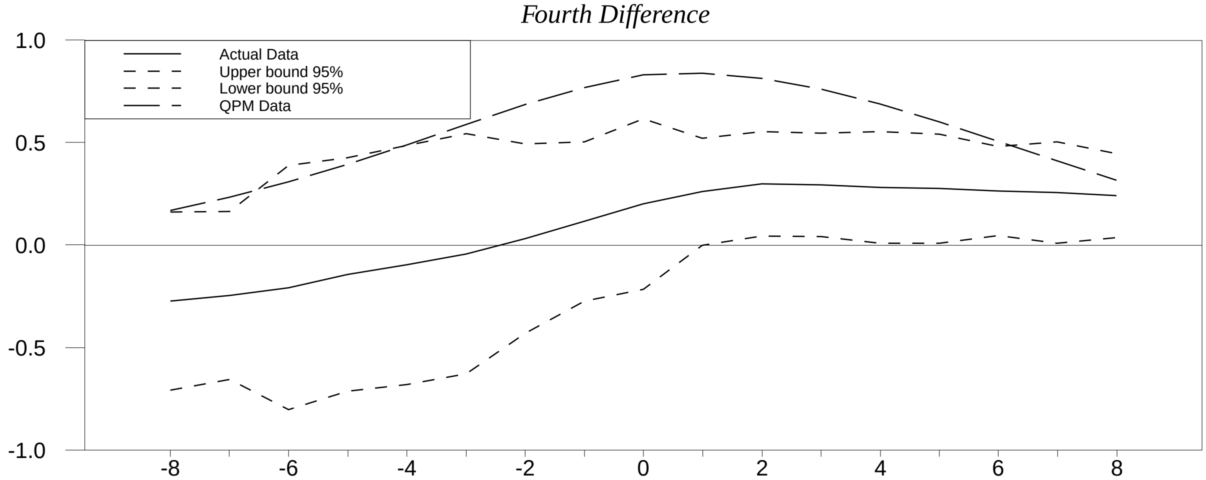
Wage and Price Inflation

First Difference



Wage and Price Inflation

Fourth Difference



Wage and Price Inflation

H-P Filter

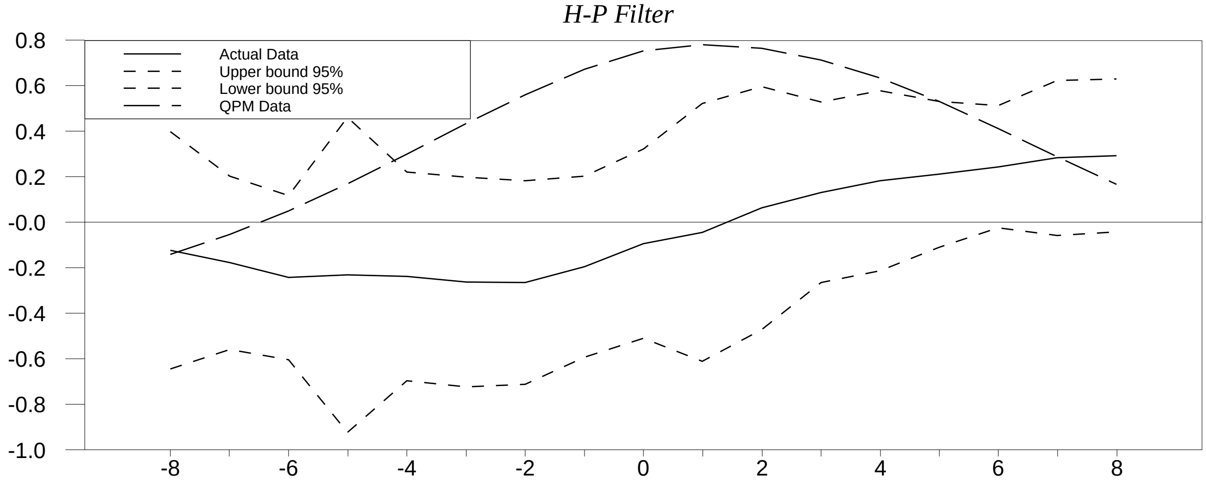
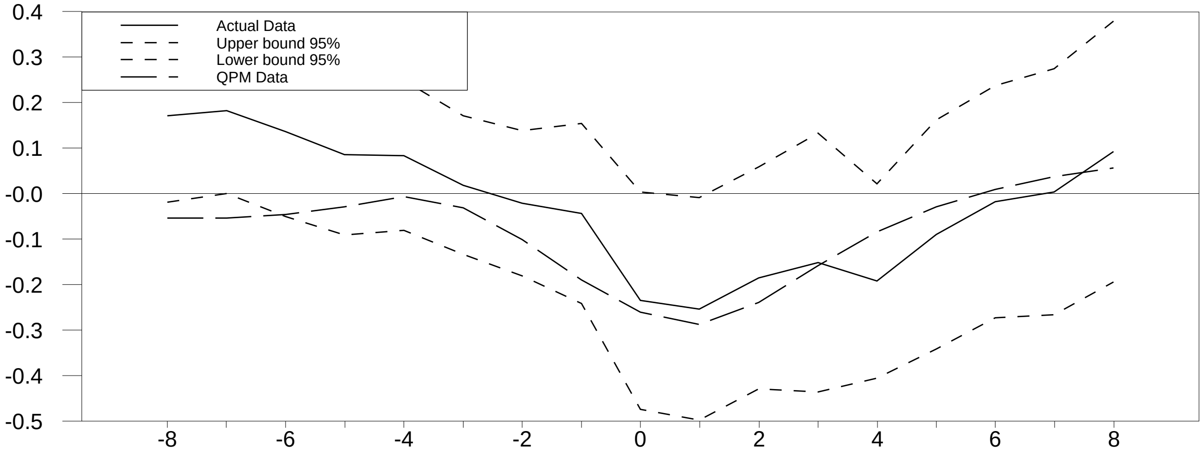


Figure 28

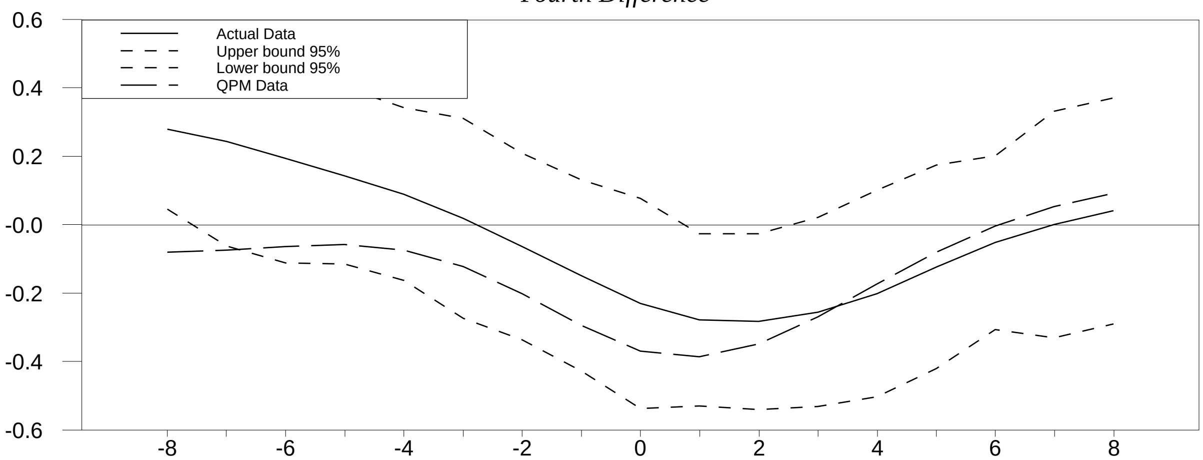
Output and Inflation

First Difference



Output and Inflation

Fourth Difference



Output and Inflation

Phillips Curve

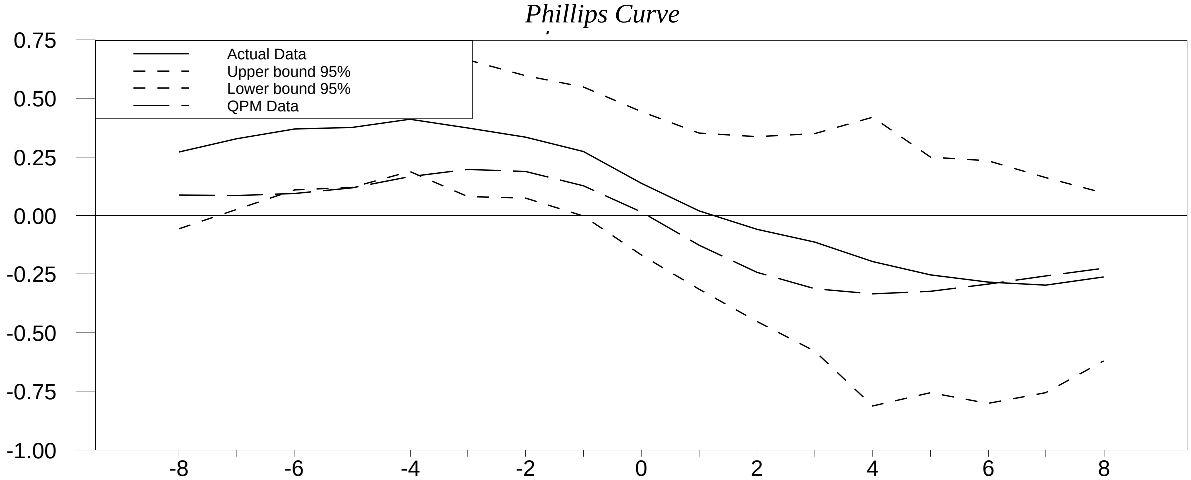
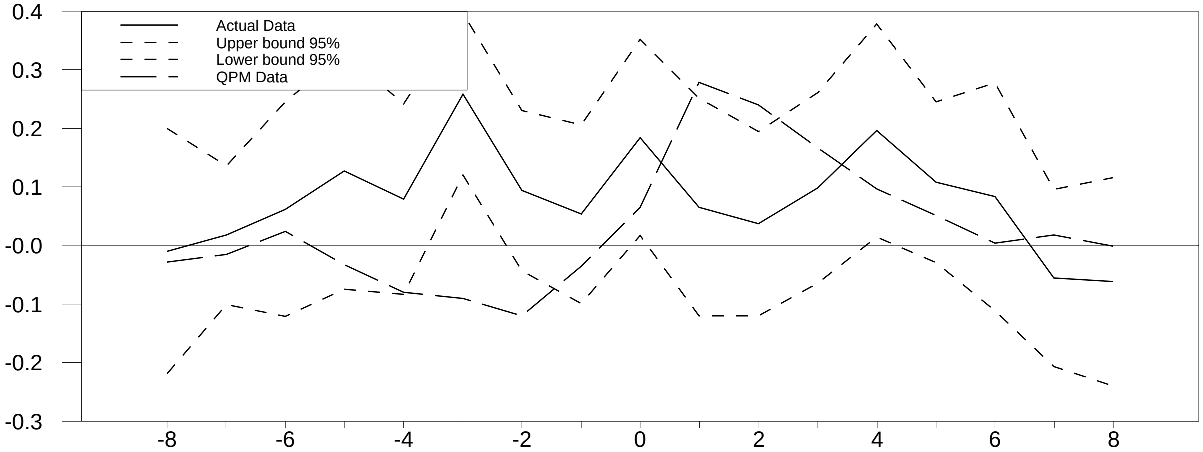


Figure 29

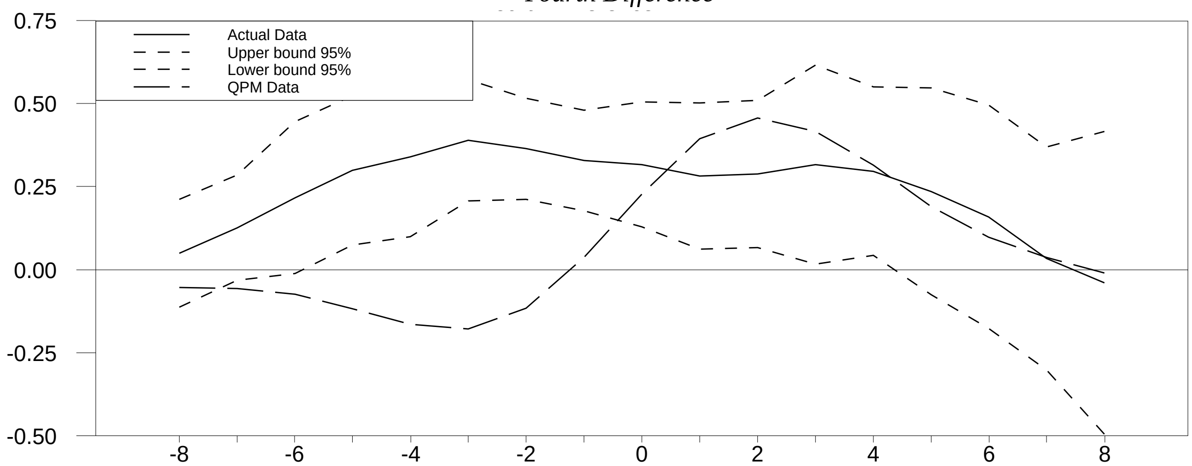
Exports and Real Exchange Rate

First Difference



Exports and Real Exchange Rate

Fourth Difference



Exports and Real Exchange Rate

H-P Filter

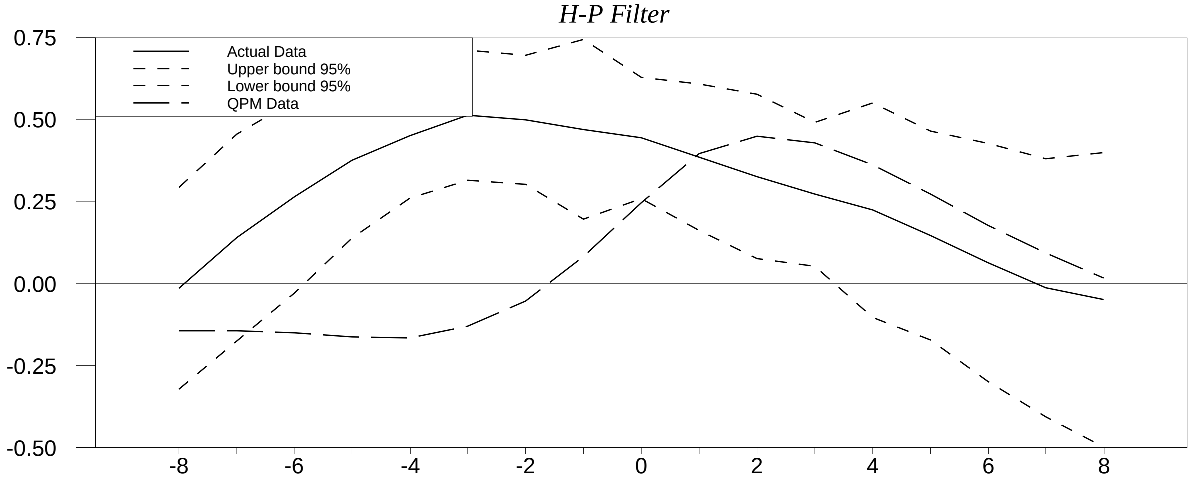
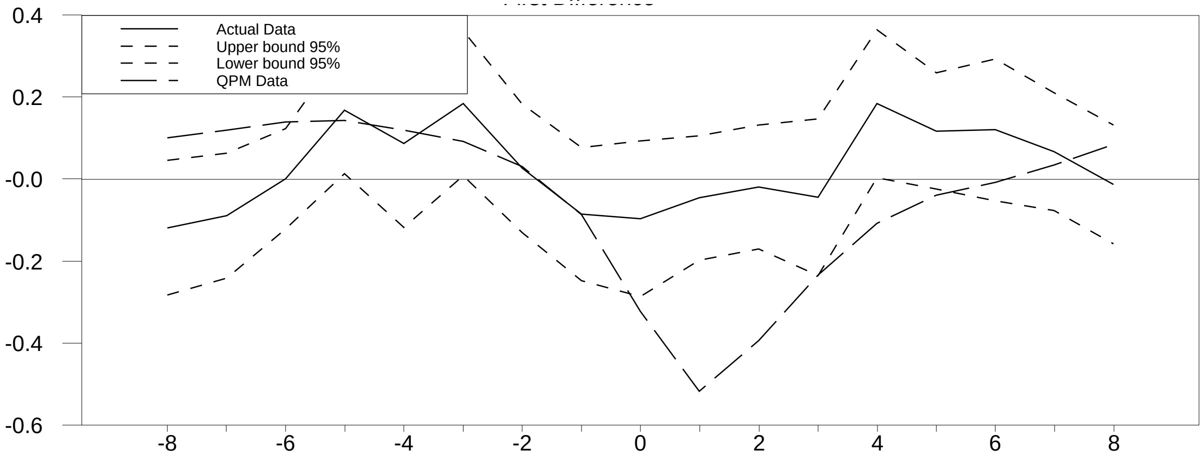


Figure 30

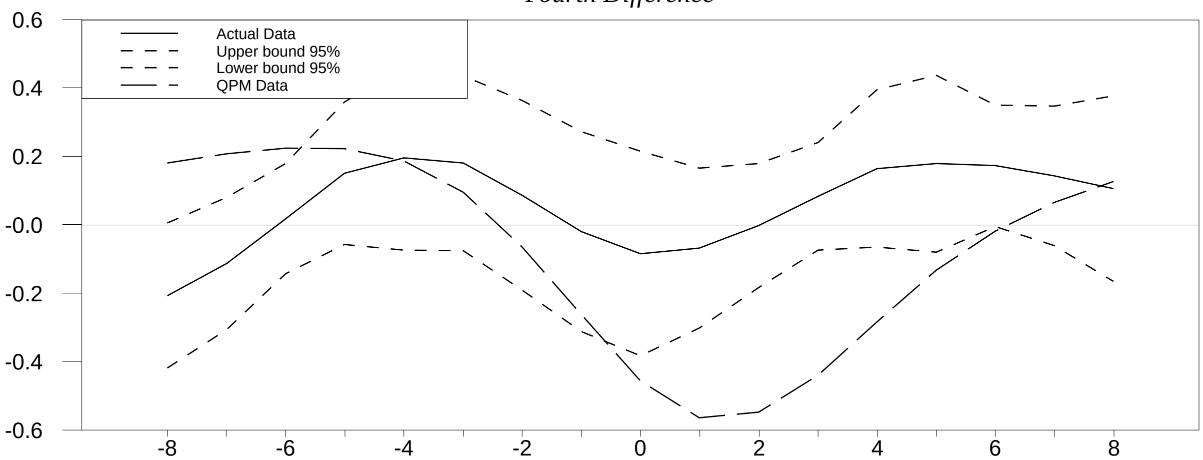
Imports and Real Exchange Rate

First Difference



Imports and Real Exchange Rate

Fourth Difference



Imports and Real Exchange Rate

H-P Filter

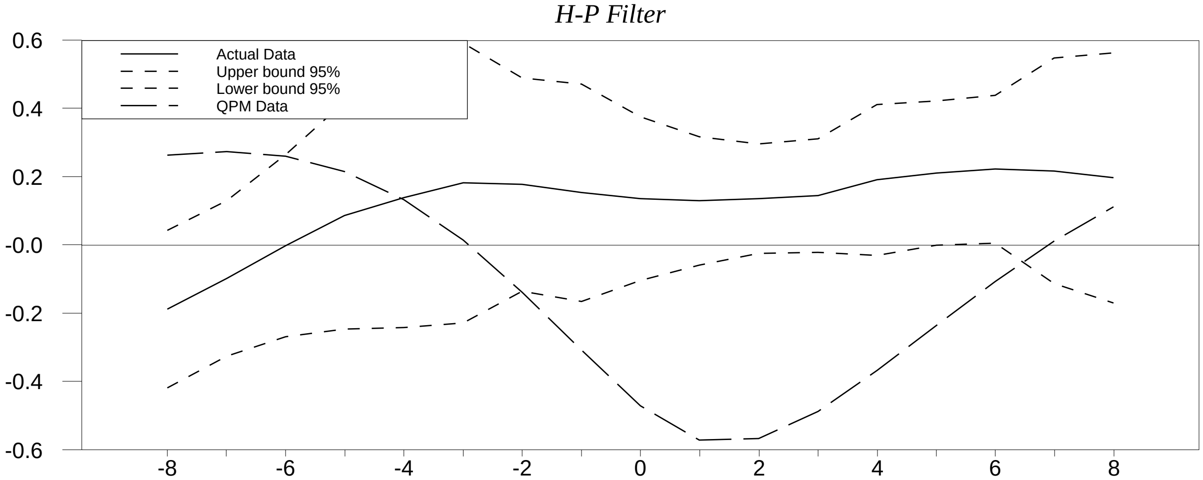
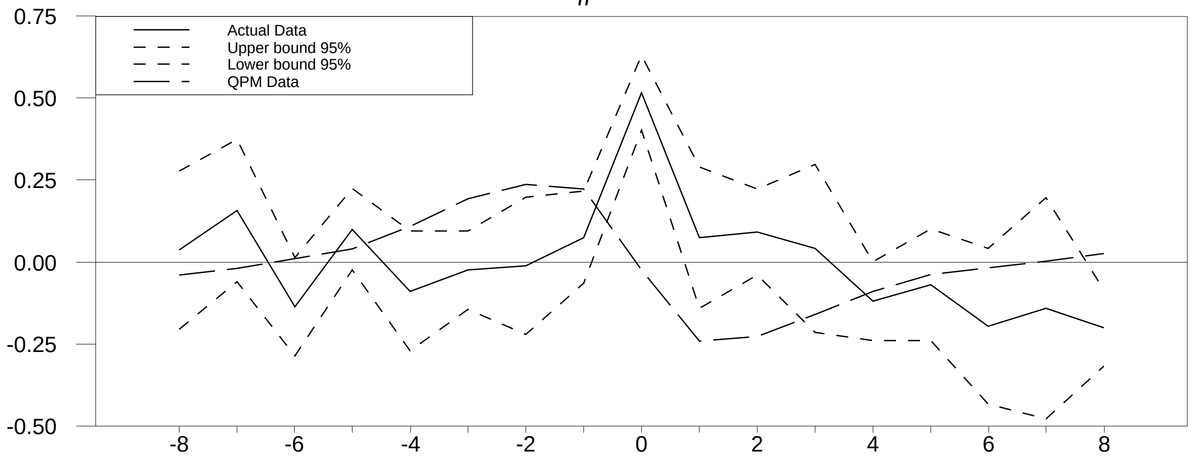


Figure 31

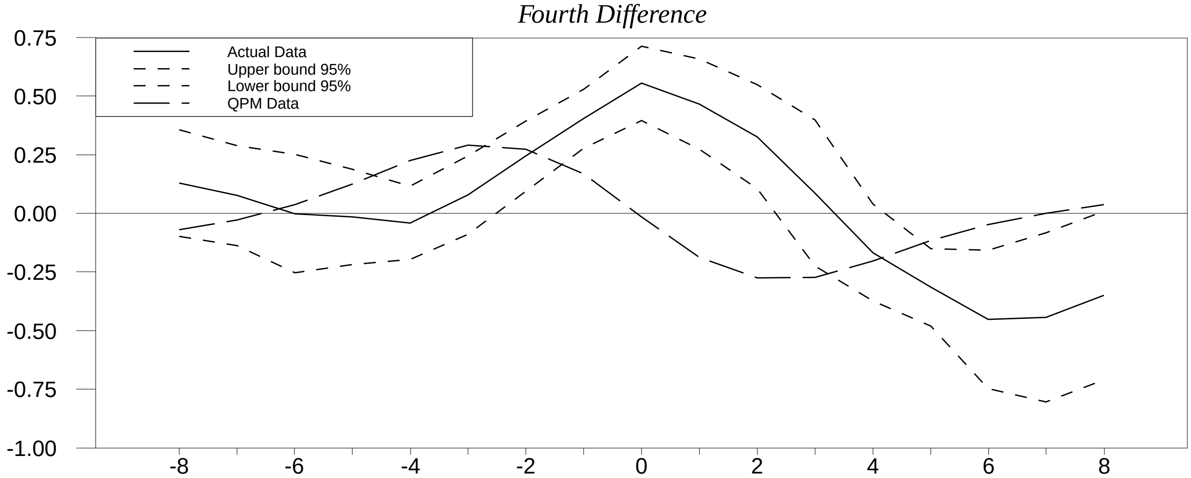
Exports and Imports

First Difference



Exports and Imports

Fourth Difference



Exports and Imports

H-P Filter

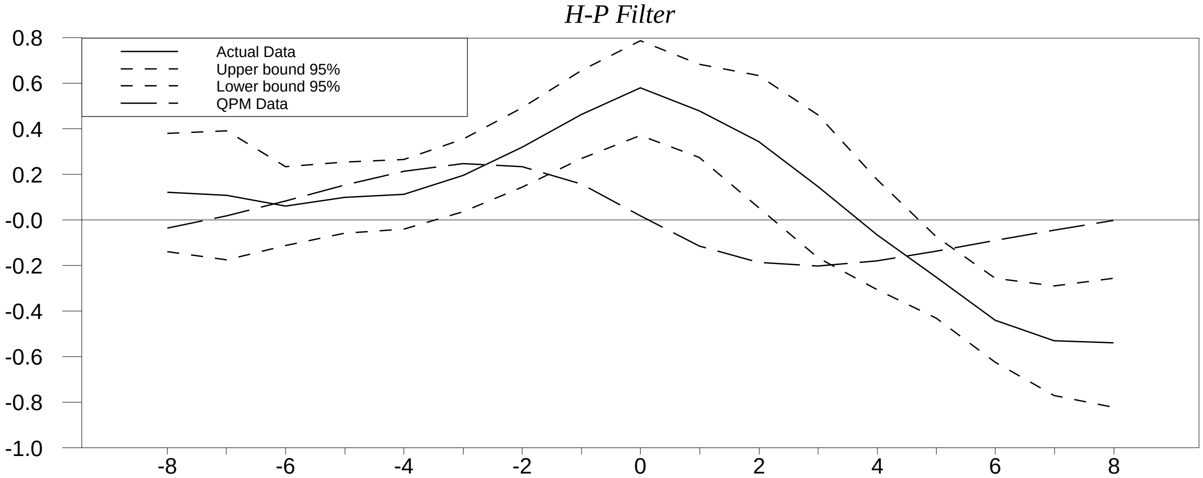
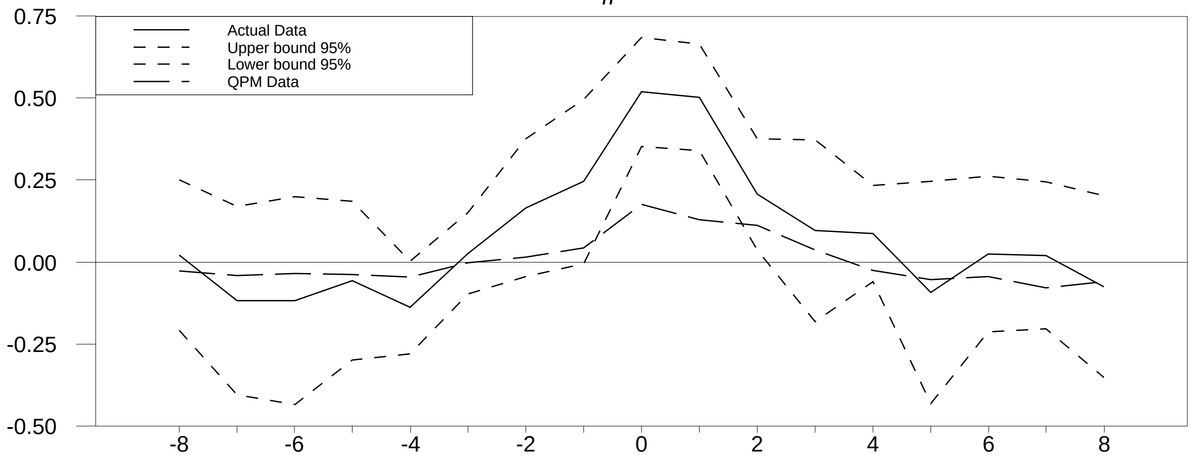


Figure 32

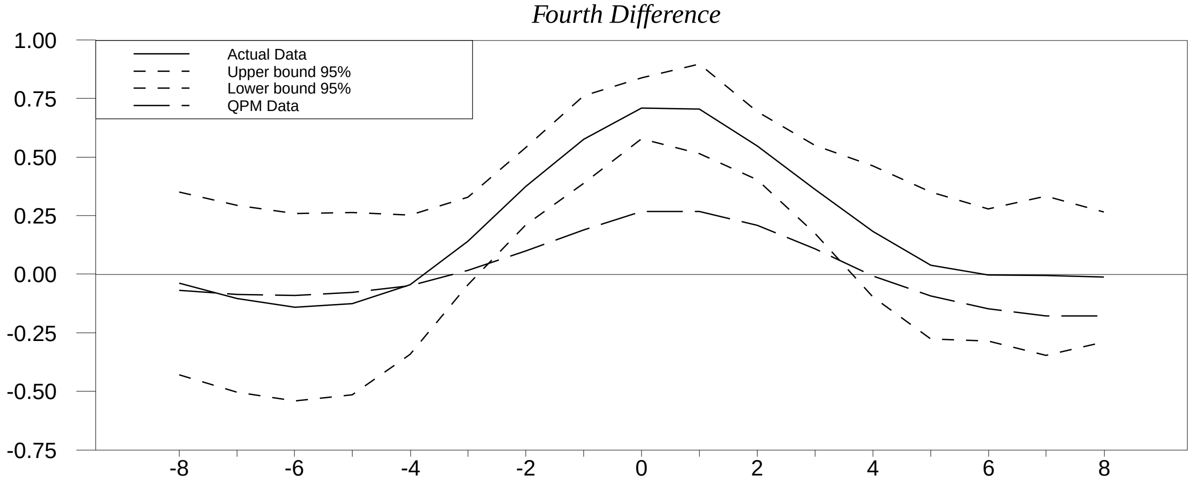
Output and ROW Output

First Difference



Output and ROW Output

Fourth Difference



Output and ROW Output

H-P Filter

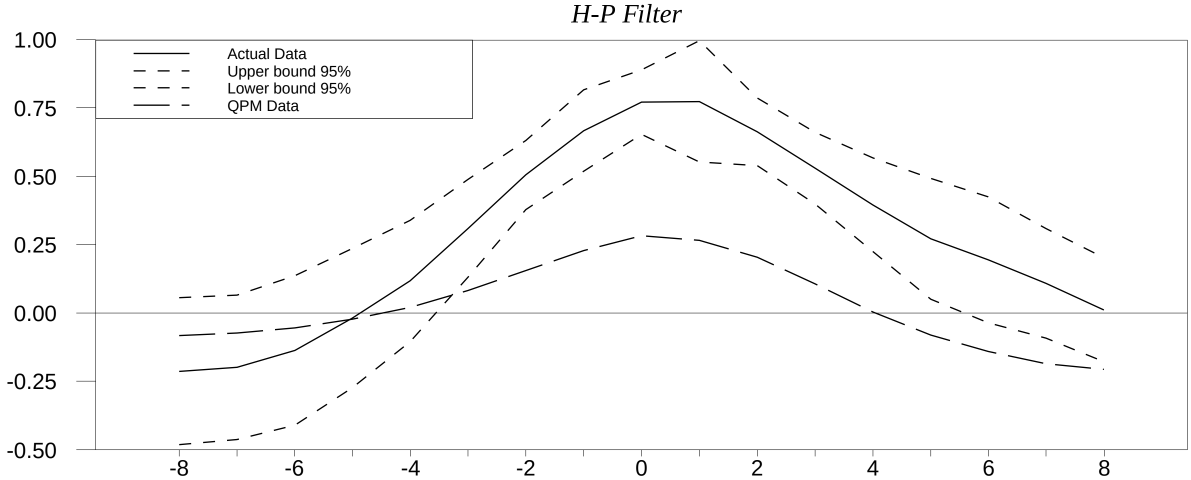
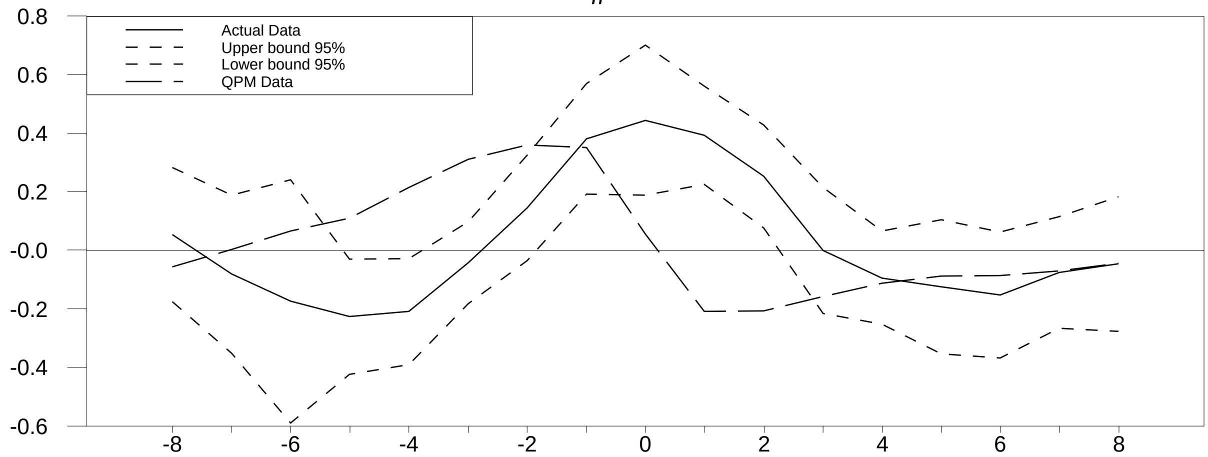


Figure 33

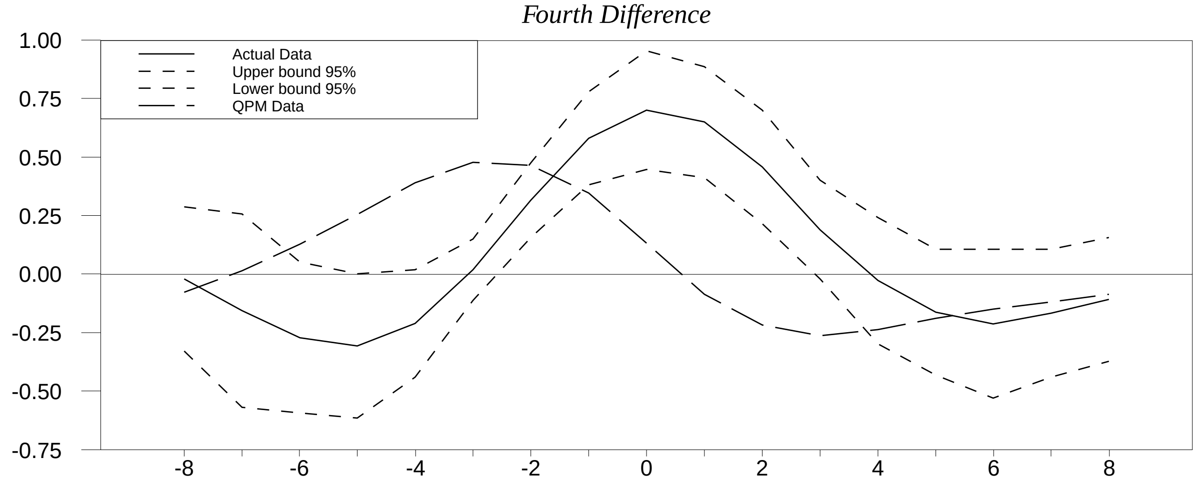
Output and Imports

First Difference



Output and Imports

Fourth Difference



Output and Imports

H-P Filter

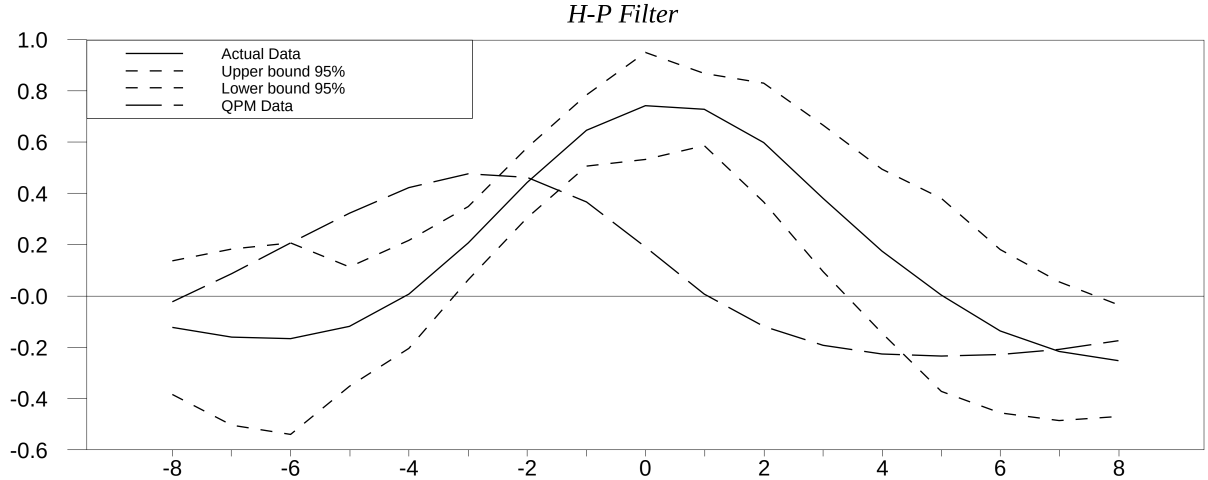
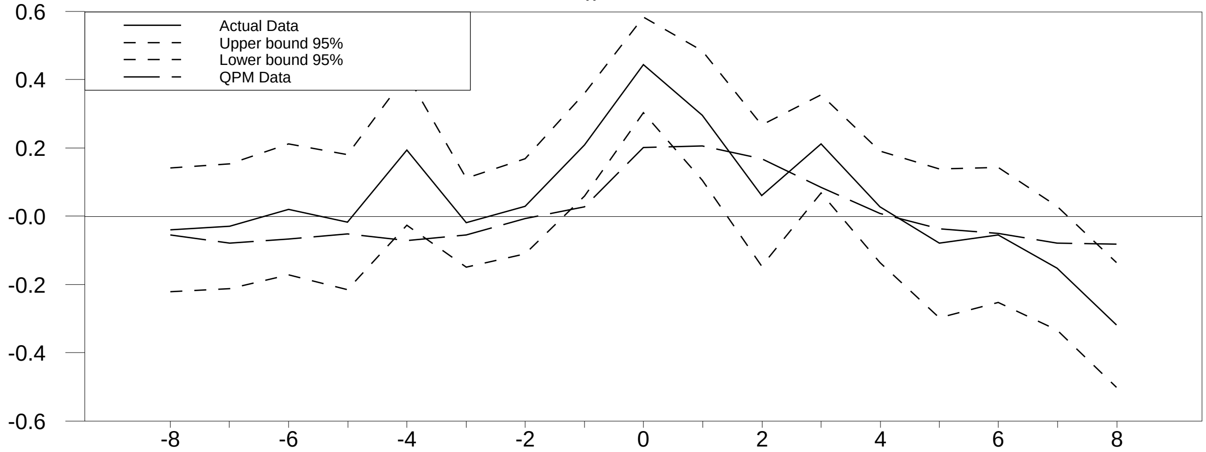


Figure 34

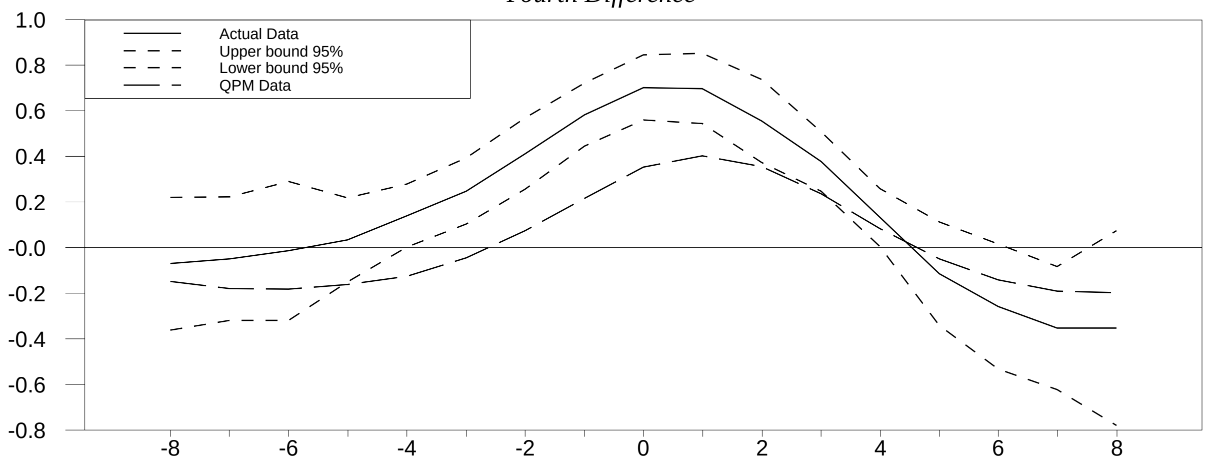
Exports and ROW Output

First Difference



Exports and ROW Output

Fourth Difference



Exports and ROW Output

H-P Filter

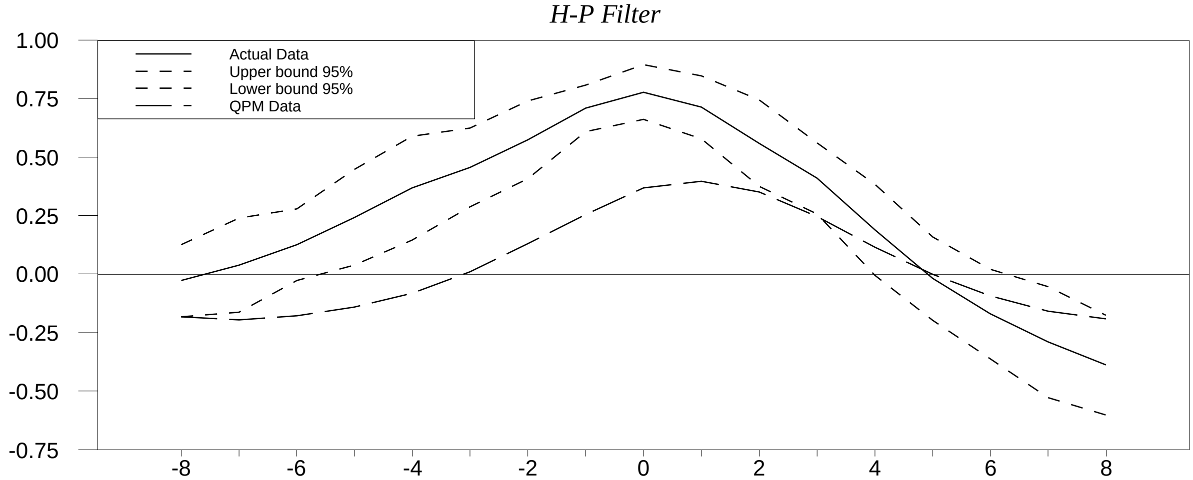
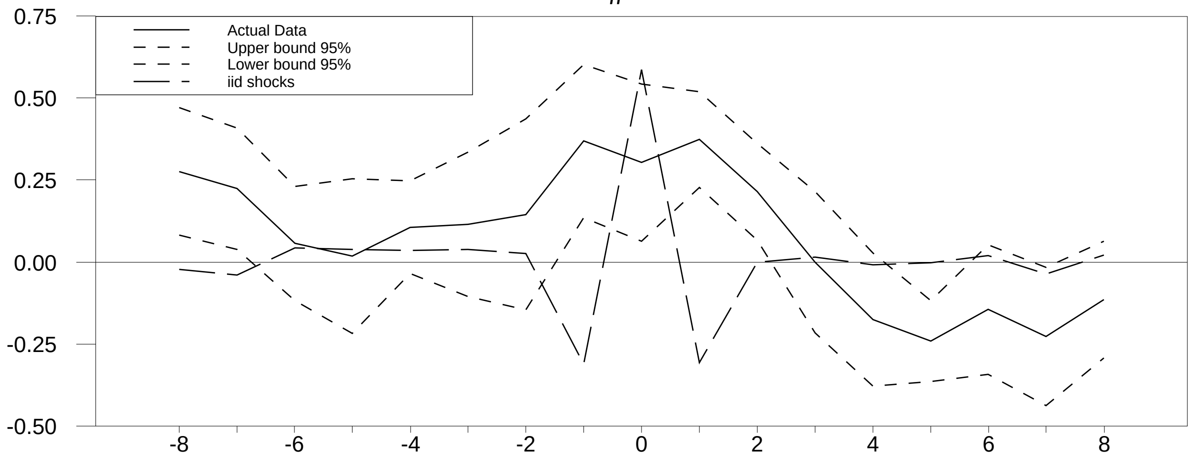


Figure 35

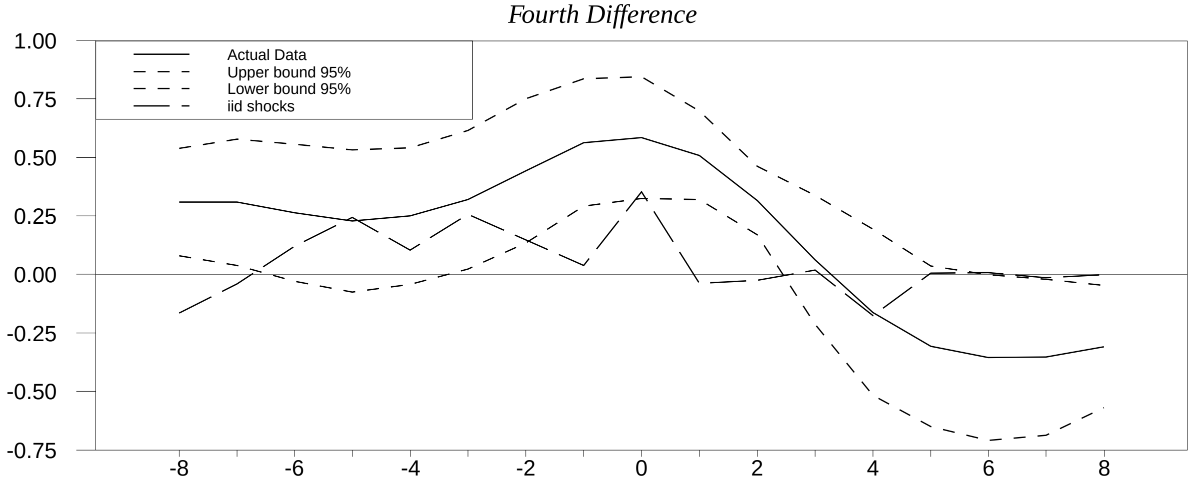
Output and Investment

First Difference



Output and Investment

Fourth Difference



Output and Investment

H-P Filter

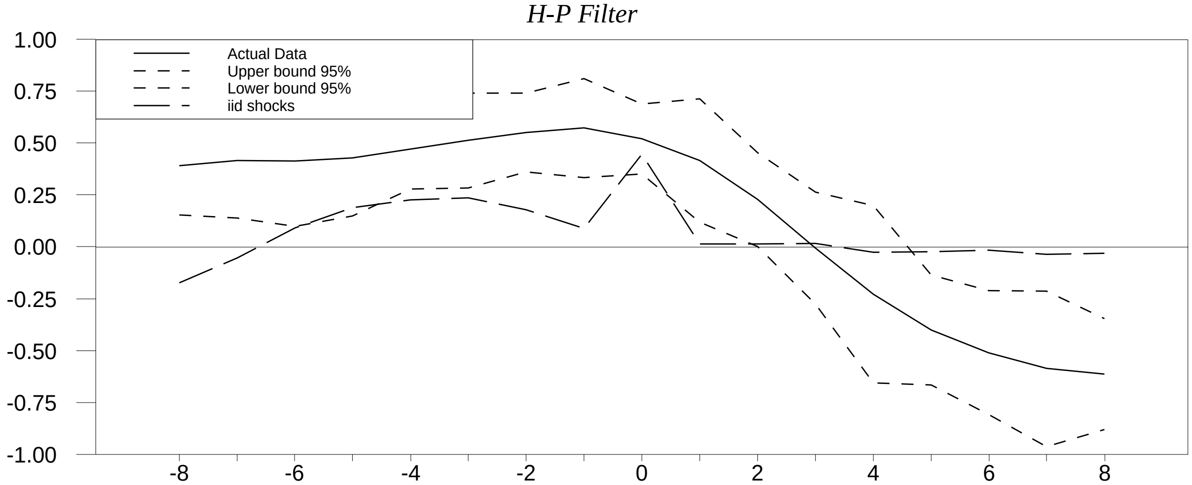
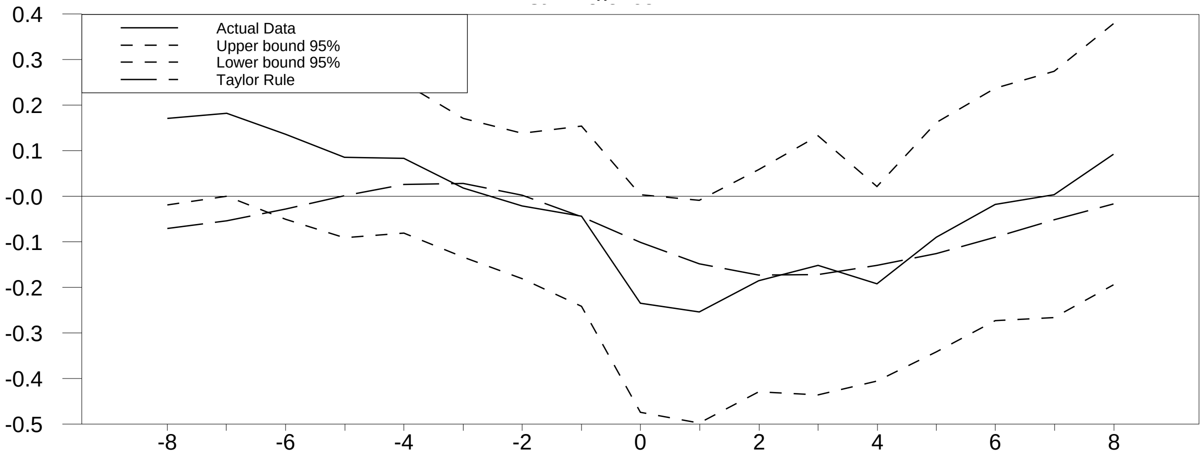


Figure 36

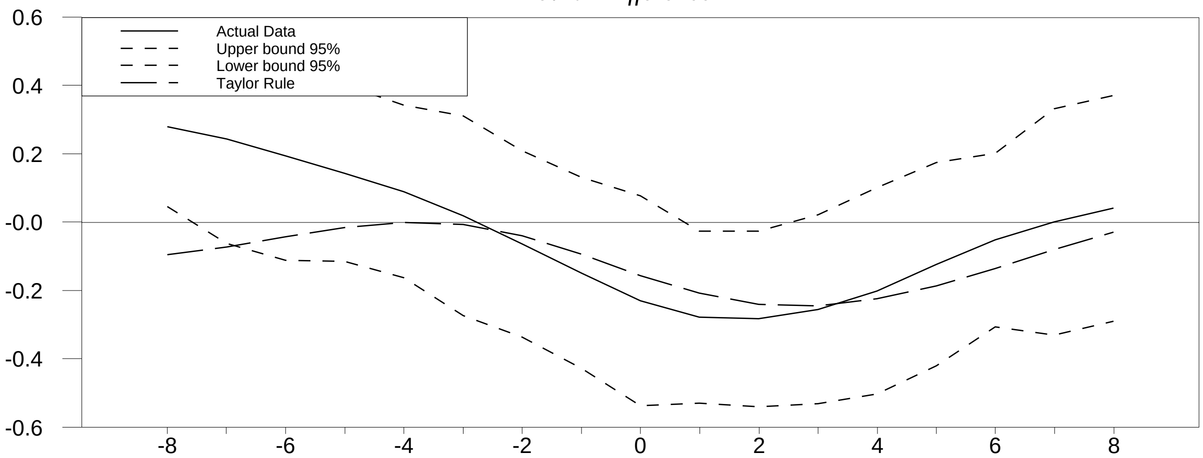
Output and Inflation

First Difference



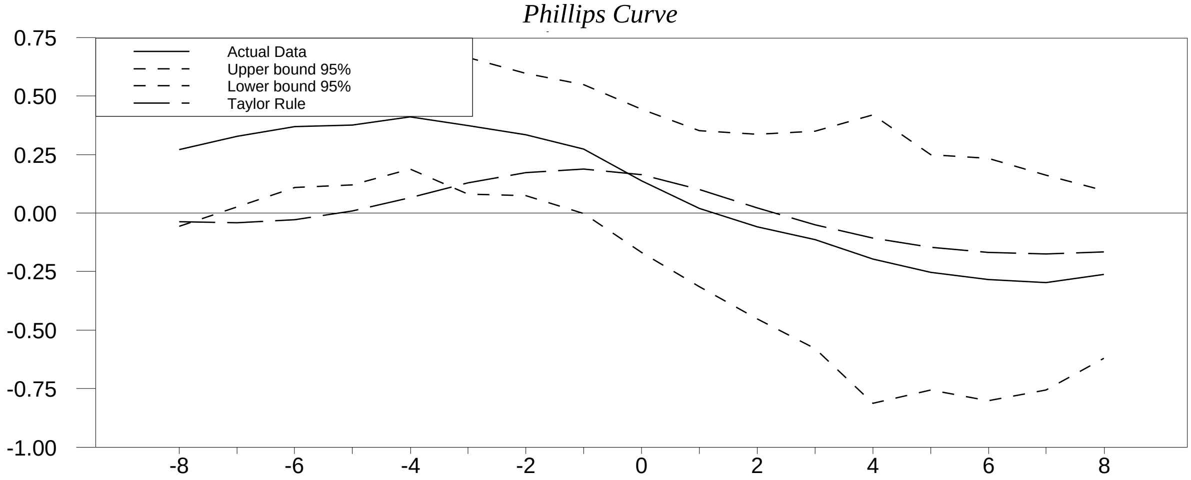
Output and Inflation

Fourth Difference



Output and Inflation

Phillips Curve



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