Bank of Canada Banque du Canada

Working Paper 2001-13 / Document de travail 2001-13

Predetermined Prices and the Persistent Effects of Money on Output

by

Michael B. Devereux and James Yetman

ISSN 1192-5434

Printed in Canada on recycled paper

Bank of Canada Working Paper 2001-13

August 2001

Predetermined Prices and the Persistent Effects of Money on Output

by

Michael B. Devereux[1] and James Yetman[2]

[1]Department of Economics
University of British Columbia
Vancouver, British Columbia, Canada V6T 1Z1
devm@interchange.ubc.ca

[2]Research Department
Bank of Canada
Ottawa, Ontario, Canada K1A 0G9

The views expressed in this paper are those of the authors.
No responsibility for them should be attributed to the Bank of Canada.

Contents

Acknowledgements

Michael Devereux thanks the Social Sciences and Humanities Research Council of Canada for financial support, and City University of Hong Kong for hospitality while this paper was being written.

Abstract

This paper illustrates a model of *predetermined pricing*, where firms set a fixed schedule of nominal prices at the time of price readjustment, based on the work of Fischer (1977). This type of price-setting specification cannot produce any excess persistence in a fixed-duration model of staggered prices, but we show that with a probabilistic model of price adjustment, as in Calvo (1983), a predetermined pricing specification can produce excess persistence. Moreover, in response to a money shock, the aggregate dynamics are very similar to those under a specification of *fixed prices*, the assumption underlying most recent dynamic sticky-price models.

JEL classification: E30
Bank classification: Transmission of monetary policy

Résumé

L'étude porte sur un modèle de *préétablissement des prix* qui s'inspire du travail de Fischer (1977) et dans le cadre duquel les entreprises, au moment de rajuster les prix, décident de leurs prix nominaux futurs pour différentes périodes. Un tel mode de détermination des prix ne peut produire une persistance excédentaire dans un modèle d'ajustement échelonné des prix sur une durée fixe, mais les auteurs démontrent qu'il peut le faire dans un modèle probabiliste d'ajustement des prix, comme celui de Calvo (1983). De plus, la dynamique globale consécutive à un choc monétaire est très semblable en régime de préétablissement des prix et en régime de *fixité des prix* (cette dernière hypothèse sous-tend les plus récents modèles dynamiques à prix rigides).

Classification JEL : E30
Classification de la Banque : Transmission de la politique monétaire

1. Introduction

In this paper, the authors develop a simple dynamic model of aggregate price and output adjustment under predetermined prices (PP), where firms can set different prices for each future period in which prices are pre-set.[1] We contrast this with the standard specification, in which a single price is set for all future periods (fixed prices, or FP). In our framework, both types of pricing arrangements involve price adjustment according to the probabilistic model of Calvo (1983), in which firms face an exogenous constant probability of readjusting their prices. Conventionally, it is argued that the PP model does not allow for excess persistence, in that the real effects of money shocks cannot persist at a higher rate than that implied by the exogenous frequency of price adjustment (e.g., Romer 2000). In contrast, it is well known (Taylor 1979, Ball and Romer 1990, Walsh 1998, and Romer 2000) that the FP model can allow for excess persistence in the presence of *real rigidities*.[2] Our results, however, show that the same property holds for the PP model. In the presence of real rigidities, the response to money shocks can display excess persistence, even though firms may set different prices for each future period during the life of the price contract. The critical difference between our results and previous versions of the PP model lies in the use of the Calvo (1983) specification for price adjustment.

Our results show that for a special case, in which the elasticity of real marginal cost to output is unity (and money follows a random walk), the two pricing specifications are exactly equivalent. More generally, in the response of the price level and output to money shocks, the two specifications are quantitatively very similar. When the degree of real rigidity is very high, the FP specification shows considerably more persistence in the real effects of money shocks, although the PP specification implies a greater impact effect on output. When the degree of real rigidity is low, the opposite conclusion holds.

The two price-adjustment specifications reflect different views of the underlying source of price stickiness. If menu costs were the most important cause of price stickiness, then firms would wish to set a single price pertaining to current and future periods. Alternatively, if contracting costs were more important (as in the original Fischer (1977) model), firms would be more willing to allow prices to be predetermined, but differently for future periods, reflecting their expected marginal costs in each period.

1. This is sometimes called the Fischer model, based on Fischer (1977).
2. Following Ball and Romer (1990), we define a *real rigidity* to be any mechanism that causes firms to be reluctant to adjust their price relative to the average prices of all other firms in the market.

Section 2 outlines the development of the model, section 3 illustrates our results, and section 4 concludes.

2. A Model of Predetermined Prices

The main elements of dynamic sticky-price economies are very familiar (see Walsh 1998 for many references). Here we set out the minimum structure that is necessary to compare the two different price-setting specifications discussed in the introduction. This class of models can be derived quite easily from an underlying dynamic general-equilibrium environment (Walsh 1998).

Under each pricing specification, firms set prices in advance based on desired or target prices. Desired prices depend on expected marginal cost, which itself depends on both current output (or the output gap) and prices of all other firms (or the price level). A simple quantity-theory equation relates output to the economy-wide price level.

The quantity-theory equation (or the aggregate-demand equation) is written in log terms as

$$y_t = m_t - p_t, \tag{1}$$

where y_t is aggregate output and $m_t - p_t$ represents real balances. The nominal marginal cost facing each firm is the same function of the aggregate price level and output. It can be written as

$$w_t = p_t + \upsilon y_t. \tag{2}$$

The parameter υ measures the elasticity of the real wage to output.

The desired price of any firm is just the marginal cost in any period. Using (1) and (2), we write the desired price level as

$$p_t^* = (1-\upsilon)p_t + \upsilon m_t. \tag{3}$$

Equation (3) says that the desired price level is equal to an average of the economy-wide price level and nominal aggregate demand. The parameter υ captures the extent to which the desired price level depends on aggregate demand, relative to the current economy-wide price level. The higher υ is, the more sensitive marginal cost is to movements in output (or the output gap), and the more willing individual firms are to adjust their desired price, relative to the aggregate price level (the average prices of all other firms). But when υ is very small, marginal cost is very insensitive to output and firms' desired prices are very close to the aggregate price level. In this case, firms are extremely reluctant to set prices that differ from the average prices of other firms in the economy. This occurs where there is significant *real rigidity*, in the terminology of Ball and Romer (1990) and Romer (2000).

We now focus on the pricing decision for the representative firm. Let firms face the constant discount factor, $\beta < 1$. Then a firm that must set its price in advance experiences a loss in expected profits, relative to a situation where price adjustment is instantaneous. Following Walsh (1998), it can be shown that the loss in profits is approximately given by the squared deviation of the log price from the desired log price.

Thus, any firm *i* faces an expected loss of

$$L_t(i) = E_t \sum_{j=t}^{\infty} \beta^j \Phi(p_{t+j}(i) - p^*_{t+j})^2, \tag{4}$$

where Φ is a constant. Loss function (4) must hold irrespective of the pricing regime in place.

2.1 Fixed prices

We now assume that nominal prices must be set in advance, as in Taylor (1979), Calvo (1983), Yun (1996), and many others. We term this specification FP. In addition, we follow Calvo and Yun in assuming that the time of price-setting is random for each firm. A firm can revise its price in each period with probability $(1 - \kappa)$, irrespective of how long its price has been fixed in the past. When adjusting its price at time *t*, the firm must set a fixed price $\hat{p}_t(i)$ that will hold for future periods until it faces an opportunity to revise its price again. The firm then faces an expected loss function given by

$$L_t(i) = E_t \sum_{j=t}^{\infty} (\beta\kappa)^j \Phi(\hat{p}_t(i) - p^*_{t+j})^2. \tag{5}$$

It is easy to establish that the optimal price for firm *i* is

$$\hat{p}_t(i) = (1 - \beta\kappa) E_t \sum_{j=0}^{\infty} (\beta\kappa)^j p^*_{t+j}. \tag{6}$$

At any time period, a fraction $(1 - \kappa)$ of firms will reset their price. Since all firms are identical, they set their prices equal to the right-hand side of (6). The aggregate price level for the economy is then given by

$$p_t = (1 - \kappa)\hat{p}_t + \kappa p_{t-1}. \tag{7}$$

From equation (6), the newly set price $\hat{p}_t$ satisfies

$$\hat{p}_t = (1 - \beta\kappa) p_t^* + E_t \beta\kappa \hat{p}_{t+1}. \tag{8}$$

We can combine (3), (7), and (8) to solve for the dynamics of p_t, $\hat{p}_t$, and p_t^* for an economy with fixed prices. The solution requires an assumption on the stochastic process determining nominal aggregate demand.

2.2 Predetermined prices

Now assume that each firm faces the same constant probability $1 - \kappa$ of revising its price, but when it does adjust its price, it may set a sequence of prices $\{\hat{p}_{t+j}\}_0^\infty$ for all periods in the future. Beginning with the next period, it will again face a constant probability of adjusting its prices. Thus, the key difference between this and fixed pricing is that the firm can set a different price pertaining to all future periods. We term this specification PP. The assumptions accord with the price-setting model of Fischer (1977) (see Romer 2000 for a discussion).

Under this price-setting arrangement, when setting a price sequence, the expected loss function of the firm is given by

$$L_t(i) = E_t \sum_{j=t}^{\infty} (\beta\kappa)^j \Phi(\hat{p}_{t+j,t}(i) - p_{t+j}^*)^2, \tag{9}$$

where $\hat{p}_{t+j,t}(i)$ is defined as the price set by firm *i*, at time *t*, pertaining to time period $t + j$ in the future. The optimal price sequence for firm *i* is

$$\hat{p}_{t+j,t}(i) = E_t p_{t+j}^*. \tag{10}$$

At time period *t*, $1 - \kappa$ firms will reset their prices. All firms set the same price sequence, given by the right-hand side of (10).

The aggregate price level for the economy with PP is given by

$$p_t = (1 - \kappa) \sum_{j=0}^{\infty} (\kappa)^j E_{t-j} p_t^*. \tag{11}$$

Equation (11) indicates that the price at any time *t* depends on a weighted sum of prices set during this period and in the past, where, in each case, the price is equal to the expected *desired* price, based on the information available at the time of price adjustment.

Using equations (3) and (11), we can obtain the solution for actual and desired aggregate prices for the economy with PP.

2.3 Monetary process

To compare the effects of the two pricing regimes, we must make an assumption about the stochastic process for the money stock. Assume that the money stock exhibits an AR(1) process in growth rates. Thus,

$$m_t - m_{t-1} = \rho(m_{t-1} - m_{t-2}) + u_t, \tag{12}$$

where $u_t \sim iid(0, \sigma^2)$. There is no drift in the money stock.[3]

3. A Comparison of the PP and FP Specifications

3.1 Solution: FP

Under the FP regime, we can solve equations (3), (7), (8), and (12) to obtain

$$p_t = \mu p_{t-1} + (1-\mu)m_t + \frac{\rho\beta\mu(1-\mu)}{(1-\rho\beta\mu)}(m_t - m_{t-1}), \tag{13}$$

where μ is the stable root of the dynamic system in $\tilde{p}_t$ and p_t implied by (3), (7), (8), and (12).[4] Then, using (1), we can write output as

$$y_t = \mu y_{t-1} + \mu u_t + \mu\rho\left(1 - \frac{\beta(1-\mu)}{(1-\rho\beta\mu)}\right)(m_t - m_{t-1}). \tag{14}$$

3. The introduction of drift in the monetary process would lead to a distinction between the FP and the PP, since PP can adjust prices for expected monetary growth without cost, while FP cannot. A more realistic approach to FP in the presence of expected inflation, utilized by Yun (1996), is to assume that firms can add a deterministic trend to their newly set price, based on expected trend inflation. If we use this interpretation of the FP model, then the two pricing regimes would treat trend inflation identically. Hence, there is no loss of generality in omitting a drift term in equation (12).

4. The expression for μ is $\mu = \frac{1}{2}\left(1 - \upsilon + \kappa\upsilon + \frac{\kappa(1-\upsilon)+\upsilon}{\beta\kappa} - \sqrt{\left(1 - \upsilon + \kappa\upsilon + \frac{\kappa(1-\upsilon)+\upsilon}{\beta\kappa}\right)^2 - \frac{4}{\beta}}\right)$.

3.2 Solution: PP

Under the PP regime, we can write the expression for the aggregate price level as

$$p_t = (1-\kappa)\sum_{j=0}^{\infty}\kappa^j E_{t-j}((1-\upsilon)p_t + \upsilon m_t) =$$
$$(1-\kappa)(1-\upsilon)\sum_{j=0}^{\infty}\kappa^j E_{t-j}p_t + (1-\kappa)\upsilon\sum_{j=0}^{\infty}\kappa^j m_{t-j}. \quad (15)$$

The general solution to equation (15), using (12), can be shown as

$$p_t = \sum_{j=0}^{\infty}\theta(j)u_{t-j}, \quad (16)$$

where

$$\theta(j) = \frac{(1-\rho^{j+1})\upsilon(1-\kappa^{j+1})}{(1-\rho)(1-(1-\kappa^{j+1})(1-\upsilon))}.$$

Then, the level of output can be obtained from equation (1) together with (16).

3.3 Equivalence of the two pricing schemes

The first result is that when $\upsilon = 1$ and $\rho = 0$, the solutions for (13) and (16) are equivalent. When $\upsilon = 1$ and $\rho = 0$, we obtain $\mu = \kappa$, so from (13) we have

$$p_t = \kappa p_{t-1} + (1-\kappa)m_t \quad (17)$$

and

$$y_t = \kappa y_{t-1} + \kappa u_t. \quad (18)$$

From (16), noting that when $\rho = 0$, we have $m_t = \sum_{j=0}^{\infty} u_{t-j}$, and solutions (17) and (18) also follow. Thus, when the elasticity of marginal cost to output is unity and the money stock follows a random walk, both pricing regimes have the same aggregate price dynamics and therefore the same behaviour for aggregate output. In this case, the dynamics of the price level and output are driven purely by the probability of price adjustment. A shock to the money stock at time *t*, u_t is absorbed into the aggregate price level up to the proportion $1-\kappa^{T}$ after T periods. Therefore, there is no persistence beyond that imparted by the probability distribution of price adjustment itself.

However, when $\upsilon \neq 1$, the two pricing regimes have different implications for the dynamics of prices and output. For simplicity, we continue to assume that $\rho = 0$. The dynamics of the price level and output under FP are well known (see Chari, Kehoe, and McGrattan 2000; Romer 2000; and Walsh 1998). In particular, it is easy to show that $\mu > \kappa (\mu < \kappa)$ as $\upsilon < 1 (\upsilon > 1)$.[5] In the first case, prices converge at a rate slower than that dictated by the exogenous frequency of price adjustment. As a consequence, there is more persistence in output than with the exogenous price-adjustment process. This excess persistence is driven by the presence of real rigidity. On the other hand, with $\upsilon > 1$, prices adjust more quickly and there is less persistence in output than that imparted by the exogenous price-adjustment process.

In the conventional version of the PP model, as presented by Romer (2000), there can be no excess persistence. For instance, when price contracts are adjusted every two periods and price-setters can set different prices for each period, an unanticipated money shock can have an impact on output that lasts at most for two periods. Once all contracts have been readjusted, the price level must fully adjust to a money shock.

The PP model does allow the possibility of excess persistence, in that the price level can adjust at a slower rate than with the exogenous price-adjustment probability. To see this, note that a permanent shock to the money stock at time t will increase the price level by $\frac{\upsilon(1-\kappa^{T})}{(1-(1-\kappa^{T})(1-\upsilon))}$ after T periods. This is less than (greater than) $(1-\kappa^{T})$ as $\upsilon < 1 (\upsilon > 1)$. Thus, the condition for excess persistence in response to money shocks in the PP model is equivalent to that in the FP model.

The difference between these results and previous versions of the PP model lies in the features of Calvo's (1983) probabilistic price-adjustment process. When all prices readjust after a known duration, then price-setters under a PP regime will take into account that the prices of all other firms will have adjusted to the information available at the outset of the oldest contract. But under the Calvo price-setting arrangement, all contracts are readjusted only asymptotically. Even though

5. To see this, let $a = 1-\upsilon+\kappa\upsilon+\frac{\kappa(1-\upsilon)+\upsilon}{\beta\kappa}$. From footnote 4, it follows that $\mu(\upsilon) = \frac{1}{2}\left(a-\sqrt{a^{2}-\frac{4}{\beta}}\right)$, where $\mu(\upsilon)$ reflects the dependence of the root on υ. Note that $\mu(1) = \kappa$ and $\mu'(\upsilon) < 0$.

only a small fraction of contracts are unadjusted after the average contract length $\left(\frac{1}{1-\kappa}\right)$ has elapsed, this can be an important determinant of the speed of aggregate price adjustment. This will be the case when the adjusting firms are unwilling to allow their prices to differ from those of all other firms (i.e., when υ is small). Thus, the presence of real rigidity can generate excess persistence in output, even in the PP model, when contracts are readjusted in the manner described.

3.4 Quantitative analysis

How do the two pricing regimes differ quantitatively? Figures 1 and 2 illustrate the impact of a permanent, unanticipated increase in the money supply on the price level and output under the two specifications. We use three different values for υ (Table 1). Setting $\upsilon = 3$ implies a high elasticity of marginal cost to output, and a low degree of real rigidity. $\upsilon = 1.2$ represents the parameterization used in Chari, Kehoe, and McGrattan (2000), based on a dynamic general-equilibrium version of Taylor's overlapping-contracts model. With $\upsilon = 0.1$, there is a much higher elasticity of marginal cost to output and a higher degree of real rigidity. These parameter assumptions are contained in the range used by Ball and Romer (1990).

Table 1: Calibrated parameter values

β	κ	υ	ρ
0.985	0.75	3, 1.2, 0.1	0.23

The rationale for the other parameter values shown in Table 1 is as follows. A value for β of 0.985 implies an annual real interest rate of 6 per cent, and κ equal to 0.75 implies an average length of price adjustment of four quarters. To choose a value for ρ, we directly estimated equation (12) on quarterly U.S. Federal Reserve non-borrowed reserves data over the 1959–2000 period. Non-borrowed reserves represent a widely used measure of an exogenous policy-determined monetary aggregate for the U.S. economy.

Figures 1 and 2 illustrate that, in general, the response of the price level and output is quite similar for the two different pricing schemes. With a very high value of υ, the immediate price impact is greater in the PP specification, and so the impact on output is smaller than in the FP specification. Of course, in this case, the overall persistence of output is very low for both specifications, as the above discussion implies. When $\upsilon = 1.2$, the two specifications exhibit almost identical price and output responses. However, as Figure 3 shows, when $\upsilon < 1$, the response of the price level and output is the reverse of that where $\upsilon = 3$: the immediate price impact of a money shock is less under PP than under FP. As a result, the immediate impact on output is larger under PP. But PP shows considerably less persistence. Both specifications display considerable *excess* persistence when $\upsilon = 0.1$. The output response is initially greater under PP. But after eight quarters, output under PP falls below that under FP, and subsequently adjusts towards its steady state at a much faster pace.

4. Conclusion

We have introduced a model of *predetermined pricing*, where firms set a fixed schedule of nominal prices at the time of price readjustment, based on the model of Fischer (1977). While this pricing specification cannot produce any excess persistence in a fixed-duration model of staggered prices (Romer 2000), we show that with a probabilistic model of price adjustment, as in Calvo (1983), the predetermined pricing specification can produce aggregate persistence. Moreover, the aggregate dynamics in response to a money shock are very similar to those under a specification of *fixed prices*, the assumption underlying most recent dynamic sticky-price models.

References

Ball, L. and D. Romer. 1990. “Real Rigidities and the Non-Neutrality of Money.” *Review of Economic Studies* 57 (2): 183–203.

Calvo, G.A. 1983. “Staggered Prices in a Utility Maximizing Framework.” *Journal of Monetary Economics* 12 (3): 383–98.

Chari, V.V., P.J. Kehoe, and E.R. McGrattan. 2000. “Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?” *Econometrica* 68 (5): 1151–79.

Fischer, S. 1977. “Long Term Contracts, Rational Expectations, and the Optimal Money Supply Rule.” *Journal of Political Economy* 85 (1): 191–205.

Romer, D. 2000. *Advanced Macroeconomics.* Montreal: McGraw Hill.

Taylor, J.B. 1979. “Staggered Wage Setting in a Macro Model.” *American Economic Review* 69 (2): 108–13.

Walsh, C. 1998. *Monetary Theory and Policy.* Cambridge: MIT Press.

Yun, T. 1996. “Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles.” *Journal of Monetary Economics* 37 (2): 345–70.

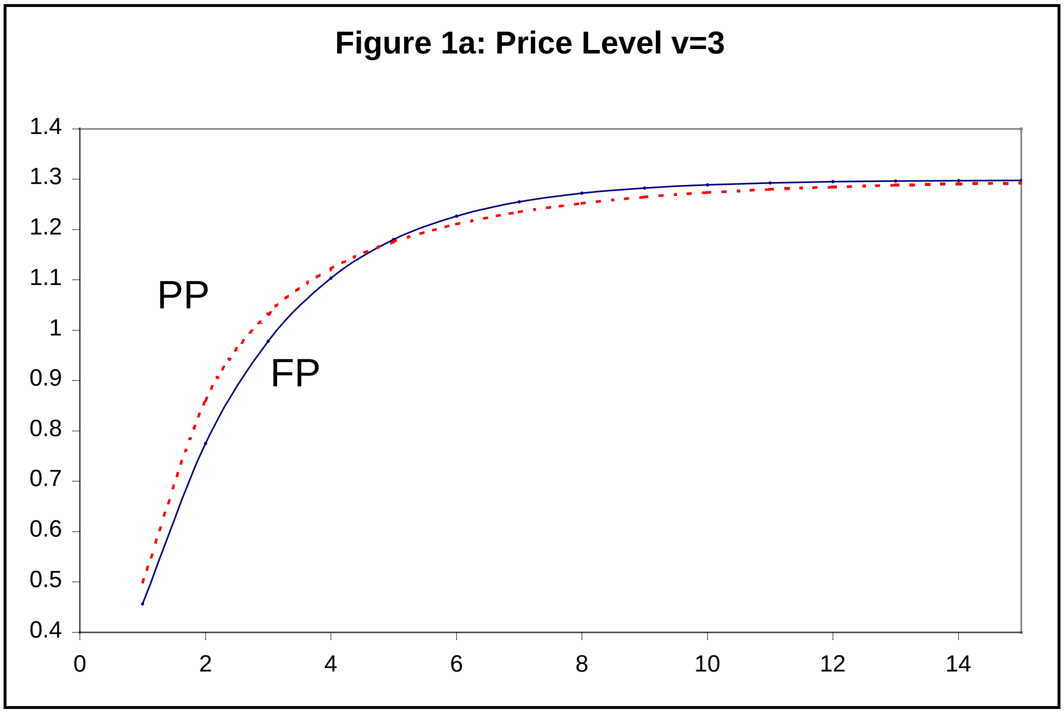


Figure 1a: Price Level v=3
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0
2
4
6
8
10
12
14
PP
FP

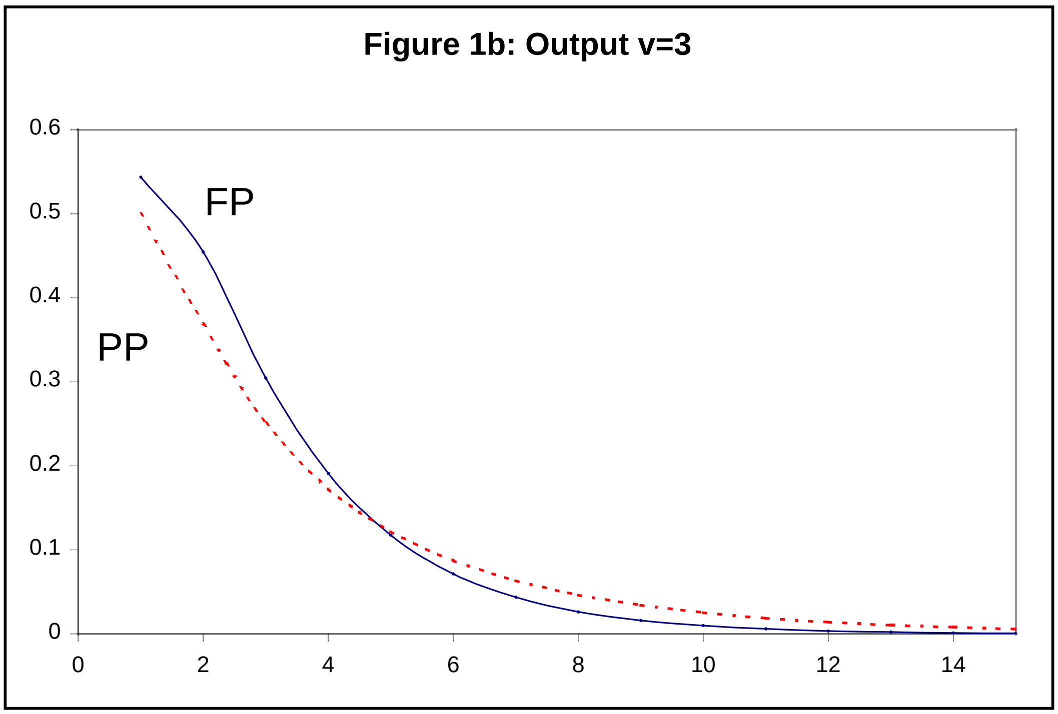


Figure 1b: Output v=3
0.6
0.5
0.4
0.3
0.2
0.1
0
0
2
4
6
8
10
12
14
FP
PP

Figure 2a: Price Level v=1.2

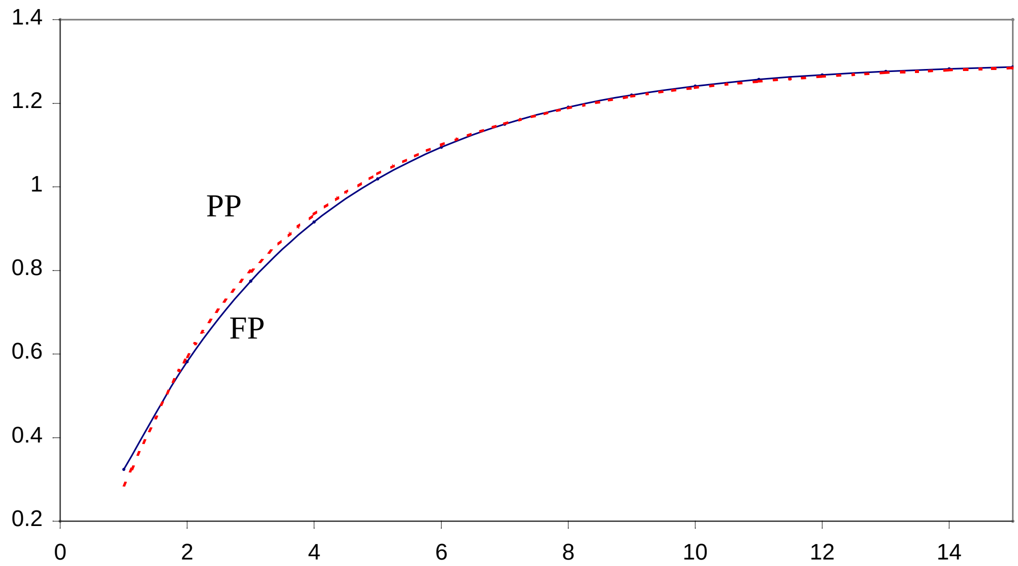
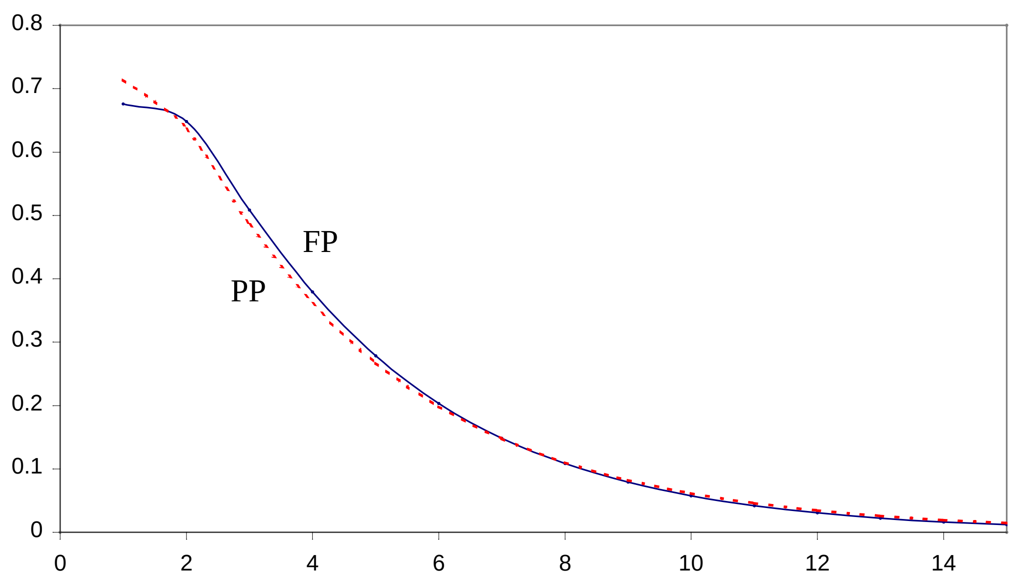


Figure 2b: Output v=1.2



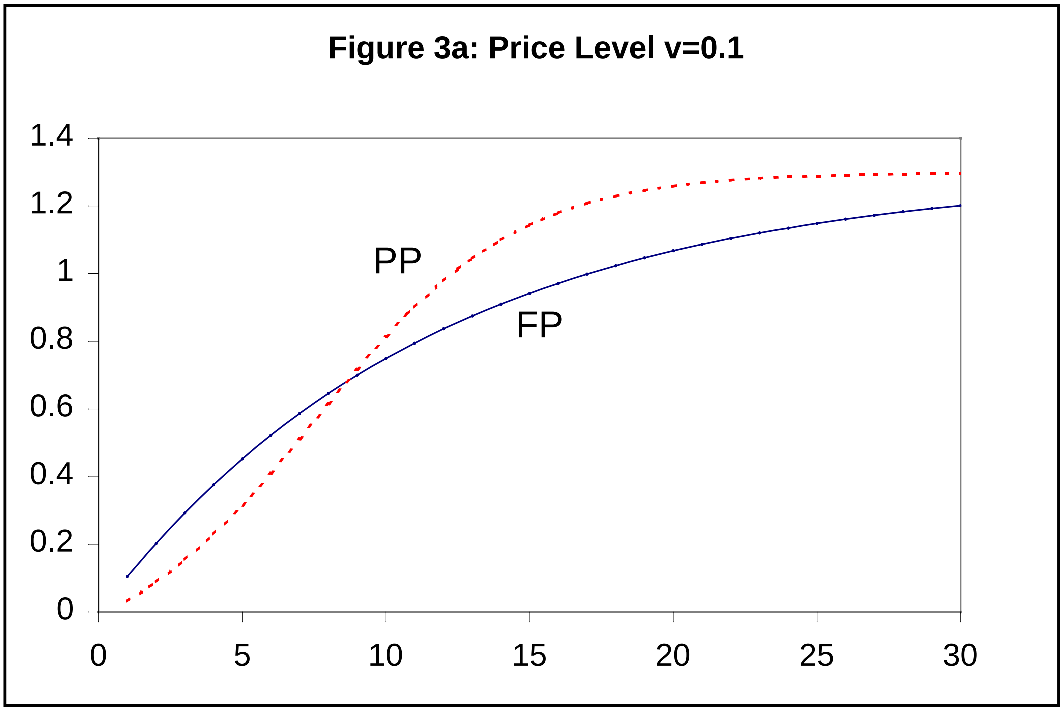


Figure 3a: Price Level v=0.1
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
30
PP
FP

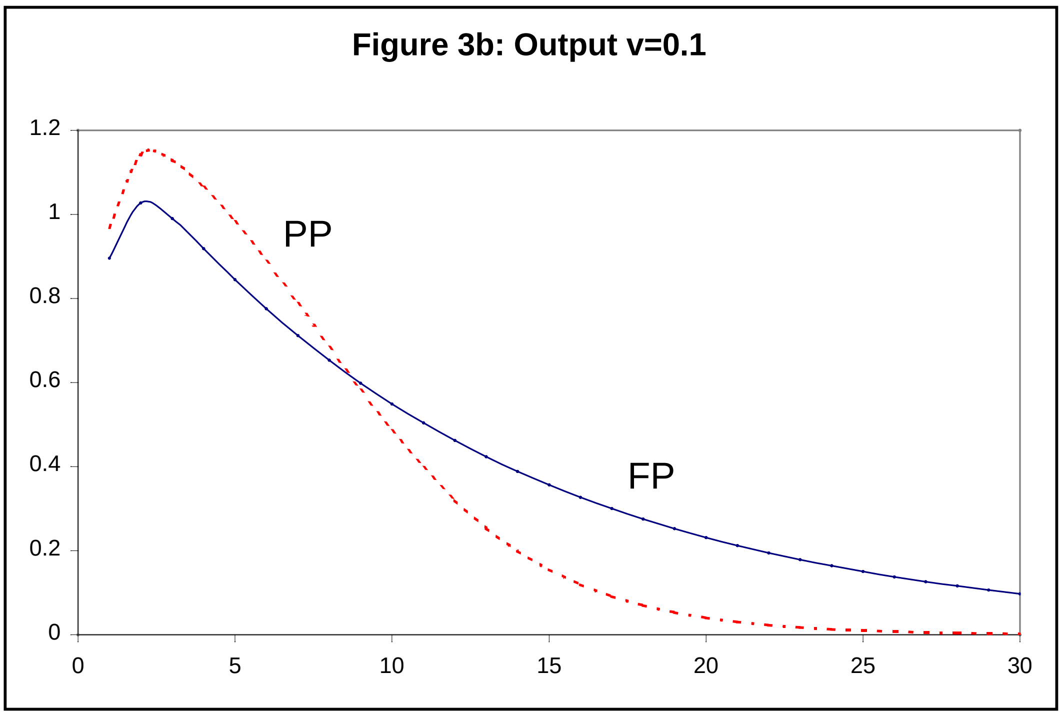


Figure 3b: Output v=0.1
1.2
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
30
PP
FP

Bank of Canada Working Papers
Documents de travail de la Banque du Canada

Working papers are generally published in the language of the author, with an abstract in both official languages. ***Les documents de travail sont publiés généralement dans la langue utilisée par les auteurs; ils sont cependant précédés d'un résumé bilingue*.**

2001

2001-12	Evaluating Linear and Non-Linear Time-Varying Forecast-Combination Methods	F. Li and G. Tkacz
2001-11	Gaining Credibility for Inflation Targets	J. Yetman
2001-10	The Future Prospects for National Financial Markets and Trading Centres	C. Gaa, S. Lumpkin, R. Ogrodnik, and P. Thurlow
2001-9	Testing for a Structural Break in the Volatility of Real GDP Growth in Canada	A. Debs
2001-8	How Rigid Are Nominal-Wage Rates?	A. Crawford
2001-7	Downward Nominal-Wage Rigidity: Micro Evidence from Tobit Models	A. Crawford and G. Wright
2001-6	The Zero Bound on Nominal Interest Rates: How Important Is It?	D. Amirault and B. O'Reilly
2001-5	Reactions of Canadian Interest Rates to Macroeconomic Announcements: Implications for Monetary Policy Transparency	T. Gravelle and R. Moessner
2001-4	On the Nature and the Stability of the Canadian Phillips Curve	M. Kichian
2001-3	On Commodity-Sensitive Currencies and Inflation Targeting	K. Clinton
2001-2	Exact Non-Parametric Tests for a Random Walk with Unknown Drift under Conditional Heteroscedasticity	R. Luger
2001-1	The Elements of the Global Network for Large-Value Funds Transfers	J.F. Dingle

2000

2000-23	The Application of Artificial Neural Networks to Exchange Rate Forecasting: The Role of Market Microstructure Variables	N. Gradojevic and J. Yang
2000-22	Une analyse empirique du lien entre la productivité et le taux de change réel Canada-É-U	D. Dupuis et D. Tessier
2000-21	Les effets réels du cours des actions sur la consommation	L. Pichette
2000-20	Steps in Applying Extreme Value Theory to Finance: A Review	Y. Bensalah

Copies and a complete list of working papers are available from:
Pour obtenir des exemplaires et une liste complète des documents de travail, prière de s'adresser à :

Publications Distribution, Bank of Canada
234 Wellington Street, Ottawa, Ontario K1A 0G9
E-mail: publications@bankofcanada.ca
Web site: http://www.bankofcanada.ca

Diffusion des publications, Banque du Canada
234, rue Wellington, Ottawa (Ontario) K1A 0G9
Adresse électronique : publications@banqueducanada.ca
Site Web : http://www.banqueducanada.ca