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Richard Luger

Research Department
Bank of Canada
Ottawa, Ontario, Canada K1A 0G9
luge@bankofcanada.ca

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Abstract

This paper proposes a class of linear signed rank statistics to test for a random walk with unknown drift in the presence of arbitrary forms of conditional heteroscedasticity. The class considered includes analogues of the well-known sign and Wilcoxon test statistics. The exactness of the proposed tests rests only on the assumption that the errors are symmetrically distributed. No other assumptions, such as normality or even the existence of moments, are required. Simulations confirm the reliability of the proposed tests, and their power is superior to that of the parametric variance-ratio test. The inference methods developed are illustrated by a test of the random walk hypothesis in exchange rates for five major currencies against the U.S. dollar.

JEL classification: C12, C22
Bank classification: Econometric and statistical methods

Résumé

L'auteur propose de se servir de tests des rangs de forme linéaire pour vérifier si une marche aléatoire avec dérive indéterminée est possible en présence de formes arbitraires d'hétéroscédasticité conditionnelle. La catégorie de tests qu'il envisage comprend des analogues de deux tests bien connus : le test des signes et le test de Wilcoxon. Une seule hypothèse, à savoir une distribution symétrique des erreurs, suffit pour assurer l'exactitude des tests proposés; aucune autre hypothèse, telle que la normalité ou même l'existence de moments, n'est requise. Les simulations confirment la fiabilité des tests, dont la puissance est supérieure à celle du test paramétrique du ratio des variances. À l'aide des méthodes d'inférence qu'il a mises au point, l'auteur teste l'hypothèse que l'évolution du cours de cinq grandes monnaies par rapport au dollar américain s'apparente à une marche aléatoire.

Classification JEL : C12, C22
Classification de la Banque : Méthodes économétriques et statistiques

1. Introduction

The random walk hypothesis is important in the analysis of economic and financial time series. Specifically, given a time series of random variables $y_1, ..., y_T$, the random walk hypothesis corresponds to $\phi = 1$ in the first-order autoregressive model

$$y_t = \mu + \phi y_{t-1} + \varepsilon_t, \tag{1}$$

where μ is an unknown drift parameter and the error terms ε_t will, in general, be neither independent nor identically distributed.

Among the many ways proposed to test the random walk hypothesis, Cochrane's (1988) variance-ratio methodology has become popular, especially in empirical finance (see, for example, Poterba and Summers 1988, Lo and MacKinlay 1988, Liu and He 1991, and Kim, Nelson, and Startz 1991). It exploits the fact that the variance of uncorrelated increments is linear in the sampling interval. The variance-ratio statistic at lag q is defined as the ratio of the variance of the q-period difference to the variance of the one-period difference divided by q. Under the null hypothesis of a random walk, this ratio is unity.

Lo and MacKinlay (1988) derive the asymptotic sampling theory for the variance-ratio statistic. In recognition of the time-varying volatilities that characterize financial time series, Lo and MacKinlay also derive a heteroscedastic-consistent estimator of the variance-ratio's asymptotic variance. The accuracy of this estimator depends on the relative homogeneity of the conditional variances. As Lo and MacKinlay show, the variance ratio is approximately a linear combination of autocorrelation coefficients similar to the Box-Pierce Portmanteau statistic. Hence, the variance-ratio statistic depends on the asymptotic properties of the empirical autocorrelations. Taylor (1984) argues that, to obtain accurate autocorrelation estimates, a series possessing reasonably homogeneous conditional variances should be used. Such a series results when the returns are rescaled by their conditional standard deviation. As the simulation results of Kim, Nelson, and Startz (1998) indicate, failure to properly rescale leads the variance-ratio test to have the wrong size, rejecting the null too often, depending on the degree of heterogeneity in the conditional variances. Moreover, it is well known that the kind of distributions encountered in financial applications typically exhibit fat tails and in some cases not even the second moment seems to exist. The simulation results described in this paper further reveal that when moments do not exist, such as with the Cauchy distribution, the variance ratio suffers severe size distortions, again rejecting much too often.

This paper proposes alternative methods of testing the random walk hypothesis that overcome these difficulties.[1] The approach exploits results from the theory of non-parametric statistics that show that the only tests about a median or mean that are valid under sufficiently general distributional assumptions, allowing for non-normal and possibly heteroscedastic observations, are based on sign statistics (see Pratt and Gibbons 1981, 218). Such results have been exploited by Dufour (1981) and Campbell and Dufour (1991, 1995, 1997), where several variants of signed rank tests are considered to test orthogonality restrictions, including the random walk hypothesis in time series.

Following Campbell and Dufour (1997), a test of the random walk hypothesis in the presence of an unknown drift parameter proceeds in two steps. First, one establishes an exact confidence interval for the drift parameter, μ, that is valid at least under the null hypothesis. Second, signed rank statistics based on the products $(y_t - y_{t-1} - b)[y_{t-1} - med_{t-1}]$, where med_t is the sample median up to

[1]Other parametric tests have been proposed that do yield exact inference methods for first-order autoregressive models, including the random walk hypothesis (see, for example, Dickey and Fuller 1979, Dufour and Kiviet 1998, and Dufour and Torrès 2000). However, the exactness of those methods rests on specific parametric assumptions, such as normality of the errors. Breitung and Gouriéroux (1997) obtain an exact rank test for a random walk with drift; however, the exactness rests on the assumption of independent and identically distributed errors.

time t, are computed for each value b element of the confidence interval. Campbell and Dufour show that the sign and Wilcoxon tests apply in this context such that, when combined using Bonferroni's inequality with the confidence interval for μ, a finite-sample bounds test can be performed.

As their simulation experiments reveal, the Campbell-Dufour approach can be conservative in finite samples such that power losses can result against certain alternatives. The signed rank statistics proposed in this paper have the virtues of those in Campbell and Dufour (1997): they have known finite-sample distributions, they are robust to departures from Gaussian conditions that underlie many parametric tests, and they are invariant to unknown forms of conditional heteroscedasticity. Additionally, Monte Carlo simulations show that the methods proposed here are more powerful than the Campbell-Dufour approach.

Section 2 defines the model and the assumptions under which the test statistics are developed. Section 3 derives the class of non-parametric test statistics along with their null distributions. Section 4 presents the results of a Monte Carlo study as evidence of the finite-sample performance of the proposed methods. Size-corrected power comparisons are made with Lo and MacKinlay's (1988) parametric variance-ratio statistics; the power of the non-parametric statistics proposed here is shown to be superior to that of the parametric variance ratio. Section 5 applies the methods to test the hypothesis of a random walk in exchange rates for five major currencies against the U.S. dollar. Section 6 concludes.

2. Definition of the Model

The observed process $\{y_t\}$ is assumed to be generated according to

$$y_t = \mu + \phi y_{t-1} + \varepsilon_t, \qquad \text{for } t = 1, ..., T \tag{2}$$

where $\mu = c(1 - \phi)$. This parameterization of the drift parameter μ follows from the framework in Andrews (1993) based on a latent AR(1) time series. Here it is assumed that $c \neq 0$ such that the autoregressive parameter may be expressed as $\phi = 1 - \mu/c$. To understand the reason for this, suppose that $y_t = \phi y_{t-1} + \varepsilon_t$ with $|\phi| < 1$, and that ε_t is symmetrically distributed about zero. In this case, the procedures developed below, designed specifically against non-zero median alternatives, lose their usefulness, since the marginal distribution of y_t is symmetric.[2] Excluding this case is not too restrictive, since, as Andrews (1993) argued, one rarely assumes that the drift parameter is known and equal to zero. In the framework established here, $\mu = 0$ implies $\phi = 1$. Note that the set of admissible values for the autoregressive parameter is unrestricted.

Let $f(\varepsilon_t, \varepsilon_{t-1}, ..., \varepsilon_0)$ denote the continuous multivariate density of the error terms, let ε_0^t denote the vector $(\varepsilon_t, ..., \varepsilon_0)$, and define $|\varepsilon_0^t|$ to be $(|\varepsilon_t|, ..., |\varepsilon_0|)$. As in Taylor (1984), it is assumed that the multivariate density of the error terms is symmetric, such that

$$f(\varepsilon_0^t) = f(|\varepsilon_0^t|), \tag{3}$$

for all ε_0^t, $t = 0, 1, ..., T$.

Several popular models of time-varying conditional variance satisfy the multivariate symmetry assumption. Suppose that the errors are governed by $\varepsilon_t = \sigma_t \cdot \eta_t$, where $\{\eta_t\}$ is an independent and identically distributed (i.i.d.) sequence drawn from a symmetric distribution such as a standard Gaussian or Student-t distribution. For example, suppose that $\eta_t \sim i.i.d.\ N(0,1)$ and

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2. \tag{4}$$

This is the Gaussian GARCH(p,q) model introduced by Bollerslev (1986), in which σ_t^2 depends linearly on past squared realizations of ε_t. In this case, (3) is satisfied, since

$$(\varepsilon_t \mid \varepsilon_0^{t-1}) \stackrel{d}{=} (|\varepsilon_t| \;\mid\; |\varepsilon_0^{t-1}|), \tag{5}$$

[2]Also presented is a modification of the procedures with discriminatory power against alternatives where $\mu = 0$. The non-parametric statistics based on this modification are, however, only exact in large samples.

where $\overset{d}{=}$ stands for the equality in distribution. More generally, any specification in which the conditional variance of ε_t is given by a function of its past realizations, such as

$$\sigma_t^2 = g(\varepsilon_{t-1}, \varepsilon_{t-2}, ..., \varepsilon_0),$$

where $g(\varepsilon_{t-1}, \varepsilon_{t-2}, ..., \varepsilon_0) = g(|\varepsilon_{t-1}|, |\varepsilon_{t-2}|, ..., |\varepsilon_0|)$, will satisfy (5) and thus (3). Of course, it is assumed that the function $g(\cdot)$ is strictly positive such that the conditional variance is well-defined. The conditional variance need not be finite nor even follow a stationary process. In fact, other than (5), and *a fortiori* (3), no restrictions are placed on the degree of heterogeneity and dependence of the conditional variance process. Notice that if the first moment exists, then the symmetry assumption in (5) implies the usual assumption of zero conditional expectation $E(\varepsilon_t|\varepsilon_{t-1}, \varepsilon_{t-2}, ..., \varepsilon_0) = 0$, which in turn implies $E(\varepsilon_t) = 0$.

Furthermore, the condition in (3) implies that $E(\varepsilon_t, \varepsilon_s) = 0$, for $t \neq s$ (see Randles and Wolfe 1979, Lemma 1.3.28). This zero covariance condition forms the basic building block of the variance-ratio statistic. However, the multivariate symmetry assumption is much more general, since finite covariances are not assumed. In fact, the existence of any moments need not be assumed for the validity of the procedures described next.

3. Description of the Test Statistics

The null hypothesis to be tested is $H_0 : \phi = 1$ in the context of model (2) defined above. To this end, consider the first-difference $\Delta y_t = y_t - y_{t-1}$ for $t = 1, 2, ..., T$. The basic building block of the inference method proposed here is the following quantity:

$$z_t = \Delta y_{t+m} - \Delta y_t, \tag{6}$$

defined for $t = 1, 2, ..., m$, where $m = T/2$. It will be assumed that T is even, so that the midpoint m is an integer. Define the sign function as $s[z] = 1$ if $z > 0$, and $s[z] = 0$ if $z \leq 0$, and consider the class of linear signed rank statistics defined by:

$$SR_m = \sum_{t=1}^{m} s[z_t] a_m(R_t^+), \tag{7}$$

where R_t^+ is the rank of $|z_t|$ when $|z_1|, |z_2|, ..., |z_m|$ are placed in ascending order and the corresponding set of scores $a_m(i)$, $i = 1, ..., m$ satisfy $0 \leq a_m(1) \leq ... \leq a_m(m)$ with $a_m(m) > 0$.

At first, it would seem to be difficult to establish the null distribution of the test statistics defined by (7), since the differences z_t are not necessarily independent. Despite this difficulty, the exact null distribution of any statistic defined by (7) is characterized next. Notice first that when $\phi = 1$, these statistics are a function only of $(\varepsilon_{t+m} - \varepsilon_t), t = 1, 2, ..., m$.

Lemma 0.1 *Let $\varepsilon_0, \varepsilon_1, ..., \varepsilon_T$ be a sequence of continuously distributed random variables that satisfy the symmetry condition (3). Then, $s[(\varepsilon_{t+m} - \varepsilon_t)]$ is distributed like a Bernoulli variable B_t, such that $\Pr[B_t = 1] = \Pr[B_t = 0] = 1/2$ for $t = 1, 2, ..., m$.*

Proof: Under the symmetry condition (3),

$$(\varepsilon_1, \varepsilon_2, ..., \varepsilon_T) \overset{d}{=} (-\varepsilon_1, -\varepsilon_2, ..., -\varepsilon_T). \tag{8}$$

Define $\delta(\varepsilon_1, \varepsilon_2, ..., \varepsilon_T) = ((\varepsilon_{m+1} - \varepsilon_1), (\varepsilon_{m+2} - \varepsilon_2), ..., (\varepsilon_T - \varepsilon_m))$. Then it follows that

$$\delta(\varepsilon_1, \varepsilon_2, ..., \varepsilon_T) \overset{d}{=} \delta(-\varepsilon_1, -\varepsilon_2, ..., -\varepsilon_T), \tag{9}$$

or

$$((\varepsilon_{m+1} - \varepsilon_1), ..., (\varepsilon_T - \varepsilon_m)) \overset{d}{=} (-(\varepsilon_{m+1} - \varepsilon_1), ..., -(\varepsilon_T - \varepsilon_m)), \tag{10}$$

since $\mathbf{X} \overset{d}{=} \mathbf{Y}$ implies $U(\mathbf{X}) \overset{d}{=} U(\mathbf{Y})$ for any measurable function $U(\cdot)$ defined on the common support of $\mathbf{X}$ and $\mathbf{Y}$ (see Randles and Wolfe 1979, Theorem 1.3.7). In turn,

$$E\left[s[(\varepsilon_{t+m} - \varepsilon_t)]\right] \overset{d}{=} E\left[s[-(\varepsilon_{t+m} - \varepsilon_t)]\right], \tag{11}$$

or

$$\Pr[\varepsilon_{t+m} - \varepsilon_t > 0] = \Pr[\varepsilon_{t+m} - \varepsilon_t < 0] = 1/2, \tag{12}$$

for $t = 1, 2, ..., m$, since $(\varepsilon_{t+m} - \varepsilon_t)$ is continuous, QED.

This result is a slight generalization of the lemma on medians found in Theil (1971, 618), in that here the marginal distributions are not assumed to be independent. The lemma establishes that, under the null, the elements of $(s[z_1], ..., s[z_m])$ are identically distributed. If they are also independent, then by exchangeability we have $(s[z_1], s[z_2], ..., s[z_m]) \stackrel{d}{=} (s[z_{d_1}], s[z_{d_2}]..., s[z_{d_m}])$ for all permutations $(d_1, d_2, ..., d_m)$ of the integers $(1, 2, ..., m)$. The next result establishes that this is indeed the case and that the null distribution, of any linear signed rank statistic defined by (7), is identical to the null distribution that would result if $z_1, z_2, ..., z_m$ were independent.

Theorem 0.1 *Let $\varepsilon_0, \varepsilon_1, ..., \varepsilon_T$ be a sequence of continuously distributed random variables that satisfy the symmetry condition (3). Then, the null distribution of any linear signed rank statistic defined by (7) has the property that*

$$SR_m = \sum_{t=1}^{m} s[z_t] a_m(R_t^+) \stackrel{d}{=} \sum_{i=1}^{m} B_i a_m(i), \tag{13}$$

where $B_1, ..., B_m$ are mutually independent uniform Bernoulli variables on $\{0, 1\}$.

Proof: Under the symmetry condition (3),

$$(\varepsilon_1, \varepsilon_2, ..., \varepsilon_T) \stackrel{d}{=} (-\varepsilon_1, \varepsilon_2, ..., \varepsilon_T) \stackrel{d}{=} \cdots \stackrel{d}{=} (-\varepsilon_1, -\varepsilon_2, ..., -\varepsilon_T),$$

where all 2^T such terms appear in this string of equalities in distribution. By applying the function $\delta(\cdot)$ defined in the lemma, it is seen that

$$\begin{aligned}
&((\varepsilon_{m+1} - \varepsilon_1), (\varepsilon_{m+2} - \varepsilon_2), ..., (\varepsilon_T - \varepsilon_m)) \stackrel{d}{=} \\
&(-(\varepsilon_{m+1} - \varepsilon_1), (\varepsilon_{m+2} - \varepsilon_2), ..., (\varepsilon_T - \varepsilon_m)) \stackrel{d}{=} \\
&((\varepsilon_{m+1} - \varepsilon_1), -(\varepsilon_{m+2} - \varepsilon_2), ..., (\varepsilon_T - \varepsilon_m)) \stackrel{d}{=} \\
&\cdots \stackrel{d}{=} \\
&(-(\varepsilon_{m+1} - \varepsilon_1), -(\varepsilon_{m+2} - \varepsilon_2), ..., -(\varepsilon_T - \varepsilon_m)),
\end{aligned}$$

where all 2^m such terms appear in this string of equalities in distribution. Let $\mathbf{E} = ((\varepsilon_{m+1} - \varepsilon_1), (\varepsilon_{m+2} - \varepsilon_2), ..., (\varepsilon_T - \varepsilon_m))$. It follows that the 2^m different values that the vector $s(\mathbf{E}) = (s[\varepsilon_{m+1} - \varepsilon_1], s[\varepsilon_{m+2} - \varepsilon_2], ..., s[\varepsilon_T - \varepsilon_m])$ may take in $\{0, 1\}^m$ have the same probability $(1/2)^m$. Therefore, the elements of $s(\mathbf{E})$ are mutually independent. Define d_i to be the position of the integer i in the realization of the vector $(R_1^+, R_2^+, ..., R_m^+)$, $i = 1, ..., m$. Thus

$$\sum_{t=1}^{m} s[z_t] a_m(R_t^+) = \sum_{i=1}^{m} s[z_{d_i}] a_m(i). \tag{14}$$

Now, conditionally on $|\mathbf{E}| = (|\varepsilon_{m+1} - \varepsilon_1|, |\varepsilon_{m+2} - \varepsilon_2|, ..., |\varepsilon_T - \varepsilon_m|)$, the vector of scores is a fixed permutation of $(a_m(1), a_m(2), ..., a_m(m))$. Under the null, conditionally on $|\mathbf{E}|$, it follows that

$$\sum_{i=1}^{m} s[z_{d_i}] a_m(i) \overset{d}{=} \sum_{i=1}^{m} B_i a_m(i), \tag{15}$$

since $(s[z_{d_1}], s[z_{d_2}], ..., s[z_{d_m}]) \overset{d}{=} (B_1, B_2, ..., B_m)$, where $B_1, ..., B_m$ are mutually independent uniform Bernoulli variables on $\{0, 1\}$. Moreover, given the symmetry established in the lemma, we have under the null that $s[z_t]$ is independent of R_t^+ and thus of $a_m(R_t^+)$ (see Randles and Wolfe 1979, Lemma 2.4.2). Therefore, under the null, it is the case also unconditionally that

$$SR_m = \sum_{t=1}^{m} s[z_t] a_m(R_t^+) \overset{d}{=} \sum_{i=1}^{m} B_i a_m(i), \tag{16}$$

since the distribution of $\sum_{i=1}^{m} B_i a_m(i)$ does not depend on $|\mathbf{E}|$, QED.

For similar extensions of the theory of linear signed rank tests, see Dufour (1981) and Campbell and Dufour (1991, 1995, 1997). Within the class of statistics defined by (7), consider the sign statistic, which is obtained from the constant score function $a_m(i) = 1$:

$$S_m = \sum_{t=1}^{m} s[z_t], \tag{17}$$

and the Wilcoxon signed rank statistic

$$W_m = \sum_{t=1}^{m} s[z_t] R_t^+, \tag{18}$$

obtained with $a_m(i) = i$. The following result, which is an immediate corollary to the theorem, establishes that the statistics defined in (17) and (18) have the usual distributions.

Corollary 0.1 *Let the model given by (2) hold with assumption (3). Then, when* $\phi = 1$, *we have:*

(i) The statistic S_m *defined by (17) is distributed according to* $B(m, 1/2)$ *a binomial distribution with number of trials* $m = T/2$ *and probability of success* $1/2$.

(ii) The statistic W_m *defined by (18) is distributed like* $W(m) = \sum_{i=1}^{m} iB_i$, *where* $B_1, ..., B_m$ *are mutually independent uniform Bernoulli variables on* $\{0, 1\}$.

The null distribution of the Wilcoxon variate has been tabulated for various values of m; see Table A.4 in Hollander and Wolfe (1973) for $m \leq 15$. For larger values, the standard normal distribution provides a very good approximation. In fact, following standard results found in Randles and Wolfe (1979, Section 10.2), it can be shown that under the null, the standardized statistic

$$SR_m^* = \left[SR_m - \frac{1}{2} \sum_{t=1}^{m} a_m(t) \right] \Bigg/ \sqrt{\frac{1}{4} \sum_{t=1}^{m} a_m^2(t)} \tag{19}$$

has a limiting standard normal distribution.

If the error distribution were known, it would be possible to select a set of scores to obtain a locally most powerful signed rank test (see Randles and Wolfe 1979, Theorem 10.1.19). Since in most applications the distribution of the errors is unknown, optimal scores are difficult to choose.

Besides the obvious dependence on sample size, the power of the proposed test statistics depends by construction on the value of the drift parameter. Power can be improved by modifying the basic building block to $\tilde{z}_t = z_t(y_{t+m-1} - y_{t-1})$. The modified building block,

$$\tilde{z}_t = (\phi - 1)(y_{t+m-1} - y_{t-1})^2 + (\varepsilon_{t+m} - \varepsilon_t)(y_{t+m-1} - y_{t-1}), \tag{20}$$

has a median shifted to the left or right, depending on the sign and magnitude of $(\phi - 1)$. Furthermore, the statistics based on $\tilde{z}_t$ will have power to detect $\phi = 0$, an alternative against which the statistics based on z_t only have trivial power. This modification follows one proposed in Campbell and Dufour (1997) to resolve similar problems. However, owing to the presence of the term $\varepsilon_t y_{t+m-1}$ in (20), the statistics based on this modification are only approximately exact. As the sample size increases, the effect of this term becomes negligible. In the next section, numerical evidence is presented that shows that the approximately exact versions reject at their nominal level, even with relatively small samples, and outperform in some cases the exact signed rank statistics.

4. Comparison with the Variance Ratio

There have been some Monte Carlo comparisons between parametric and non-parametric tests of the random walk hypothesis. For instance, Campbell and Dufour (1997) report evidence on the size and power of their non-parametric bounds procedures compared with the t-statistic based on the OLS estimate of β_1 in a regression of the form $y_t = \beta_0 + \beta_1 y_{t-1} + u_t$. Under various distributions of the error terms, they find that the non-parametric procedures based on signs and ranks outperform the parametric tests in terms of power, especially when the error distribution is fat-tailed.

Similar power gains from using non-parametric tests are reported in Wright (2000). He compares his non-parametric versions of the variance-ratio test statistic based on signs and ranks with the conventional variance-ratio test. The Monte Carlo simulations are performed in the context of a stochastic volatility model which differs from that used in Lo and MacKinlay (1989) by the alternative distributional assumptions for the error term. Wright finds that his non-parametric tests dominate even the heteroscedastic-robust version of the parametric variance-ratio test.

Similarly, the Monte Carlo simulations in this section compare the size and power of the signed rank test statistics introduced in the previous section with the parametric variance-ratio test. Specifically, the comparison is with Lo and MacKinlay's (1988) variance-ratio statistic, computed as:

$$VR(q) = \frac{\sigma^2(q)}{\sigma^2}, \tag{21}$$

where

$$\hat{\sigma}^2(q) = \frac{1}{M}\sum_{t=q}^{T}(y_t - y_{t-q} - q\hat{\mu})^2, \tag{22}$$

$$M = q(T-q+1)(1-q/T). \tag{23}$$

and

$$\hat{\sigma}^2 = \frac{1}{T-1}\sum_{t=1}^{T}(y_t - y_{t-1} - \hat{\mu})^2, \tag{24}$$

$$\hat{\mu} = \frac{1}{T}\sum_{t=1}^{T} y_t - y_{t-1}. \tag{25}$$

Lo and MacKinlay show that the following test statistic is asymptotically standard normal under the null hypothesis:

$$VR_1(q) = (VR(q) - 1)\left(\frac{2(2q-1)(q-1)}{3qT}\right)^{-1/2} \tag{26}$$

They further derive the following version of the variance-ratio statistic, which is consistent for certain forms of conditional heteroscedasticity:[3]

$$VR_2(q) = (VR(q) - 1)\left(\sum_{j=1}^{q-1}\left[\frac{2(q-j)}{q}\right]^2 \hat{\delta}(j)\right)^{-1/2} \tag{27}$$

where

$$\hat{\delta}(j) = \frac{\sum_{t=j+1}^{T}(y_t - y_{t-1} - \hat{\mu})^2(y_{t-j} - y_{t-j-1} - \hat{\mu})^2}{\left[\sum_{t=1}^{T}(y_t - y_{t-1} - \hat{\mu})^2\right]^2}. \tag{28}$$

[3]To ensure consistency, restrictions must be placed on the maximum degree of dependence and heterogeneity allowable while still permitting a law of large numbers and a central-limit theorem to hold. See Lo and MacKinlay (1988, 48-50) for more details.

The presence of the unknown drift parameter in the null hypothesis distinguishes this study from that of Wright (2000), which assumes for the validity of his sign-based statistics that the drift is known to be identically zero. In this respect, the results reported here are comparable with those of Campbell and Dufour (1997), who explicitly allow for an unknown drift parameter. Campbell and Dufour's approach ensures that the probability of a Type I error is bounded from above by the desired level α. As the simulations in Campbell and Dufour (1997) show, the bounds tests can be quite conservative in finite samples, such that power losses may result against certain alternatives.

The size and power comparisons made here are couched in terms of the following models:

Model 1: $y_t = \mu + \phi y_{t-1} + \varepsilon_t$.

Model 2: $y_t = \mu + \phi y_{t-1} + exp(h_t/2)\varepsilon_t$ with $h_t = \lambda h_{t-1} + \xi_t$.

As in Campbell and Dufour (1997), ε_t are i.i.d. according to either a $N(0,1)$, a $t(3)$, or a Cauchy distribution. The intercept is $\mu = 2$ such that the results pertaining to Model 1 are directly comparable with those in Campbell and Dufour (1997). The initial value is generated as $y_0 = \mu + \varepsilon_0$ under both the null and the alternative hypotheses. Model 2 is a stochastic volatility model similar to that in Lo and MacKinlay (1989) and Wright (2000). The persistence parameter is set as $\lambda = 0.99$ and the innovations to volatility ξ_t are i.i.d. $N(0,1)$ independent of ε_τ for all t and τ. Samples of size $T = 100, 200$ are simulated, and $\mu = 1$ is also considered, to contrast the power of the tests based on z_t with those based on $\tilde{z}_t$.

At the 5 per cent significance level, each test of the random walk null hypothesis $H_0 : \phi = 1$ is applied as two-sided. The empirical rejection probabilities reported in Tables 1 through 6 are based on 1000 replications of each data-generating process. In each case, the models were first simulated under the null hypothesis. The results of this first round are found in the rows where $\phi = 1$. Using the same sequences of error terms, data were generated under the alternative hypotheses considered such that the differences are entirely attributable to the change in ϕ. Each test was size-corrected using the empirical critical values obtained in the first round of simulations.

The results for model 1 are reported in Tables 1 through 3. The results in Table 1 are for the case where $\mu = 1$ and $T = 100$. Under Cauchy distributed error terms, the variance ratio behaves better than its heteroscedastic-robust version, which rejects in some cases at more than five times its nominal level. On the other hand, the non-parametric tests are seen to be robust, rejecting at the nominal level under the null. Holding the value of the drift fixed, Table 2 reports the results when the sample size is doubled to $T = 200$. In some cases, such as under normally distributed errors, power is nearly doubled. As ϕ tends to zero, the median of z_t also tends toward zero. For this reason, power is seen to decrease somewhat when $\phi = 0.98$ compared to $\phi = 0.99$.[4] As Table 7 shows, the modification to $\tilde{z}_t$ overcomes this problem.

Comparing the results of Table 3, where $\mu = 2$ and $T = 100$, with those of Campbell and Dufour (1997, Table 6), the non-parametric tests proposed here display superior power. Again, they also dominate the variance-ratio tests. It appears that, for thin-tailed error distributions, power increases more when the drift increases than when the sample size increases. When the errors are fat-tailed, power gains seem to be more appreciable when the sample size increases than when the drift increases.

The good performance of the non-parametric tests is repeated in Tables 4 through 6 for the stochastic volatility model. The results in these tables indicate that the conventional variance-ratio test is not robust to conditional heteroscedasticity. The robust version performs much better with empirical rejections closer to the nominal level. However, as in Tables 1 to 3, even the heteroscedastic-robust

[4]The proposed statistics have non-trivial power against non-stationary alternatives $|\phi| > 1$, regardless of the value of the drift parameter.

version of the variance-ratio test suffers from size distortions when the errors are Cauchy distributed. Although hardly surprising, considering that the central-limit theorem does not apply in this case, such results warn that the variance ratio should not be applied blindly without consideration for the conditions under which such parametric procedures are valid.

Table 7 reports the size and power of the approximately exact versions of the standardized sign and Wilcoxon statistics for various values of the drift parameter. These results are based on 10,000 replications. The tests behave well, rejecting at their nominal level in a manner similar to that of the exact signed rank statistics. The power results are remarkable. In particular, the performance of the Wilcoxon statistic is clearly superior to that reported in Tables 1 and 2 under the same generating configuration.

5. Application to Exchange Rates

Several authors, such as Meese and Singleton (1982) and Baillie and Bollerslev (1989), provide evidence to support the conjecture that the process generating the natural logarithm of nominal exchange rates is well approximated by random walks. This suggests that foreign exchange markets are informationally efficient[5] (see also Cornell and Dietrich 1978, Corbae and Ouliaris 1986, and Hsieh 1988). This view is challenged by Liu and He (1991), who found evidence to reject the random walk hypothesis when they applied the Lo-MacKinlay variance-ratio test to five major weekly nominal exchange rates: the Canadian dollar, French franc, German mark, Japanese yen, and British pound, all vis-à-vis the U.S. dollar. This challenge is further supported in the conclusions of Wright (2000). This section re-examines the random walk hypothesis for these nominal foreign exchange rates

[5]In the sense that if markets are informationally efficient then the best forecast of the future price is the current price. See Campbell, Lo, and MacKinlay (1997, Section 2.1) for more on market efficiency and the random walk hypothesis.

with the non-parametric tests proposed above.

The data consist of daily spot exchange rates for the five aforementioned currencies for the period covering 7 August 1974 to 29 May 1996. The foreign exchanges and the time period considered are the same as in Wright (2000), although the data source is different. These data were collected by the Federal Reserve Bank of New York and are the noon buying rates in New York.[6]

The exchange rate observed on Wednesday (or the next trading day if the market was closed on Wednesday) was used to construct the weekly returns. These were taken to be the first differences of the natural logarithm of the rates retained each week.

The values of the variance-ratio test statistic and its heteroscedastic consistent version, VR_1 and VR_2, for aggregation values $q = 2$, 5, 10, and 30, are reported in Table 8. Also reported is the value of the sign and Wilcoxon statistics based on $\tilde{z}_t$. This version is preferable given high-frequency returns where the drift is relatively small.

Except for the Canadian dollar, the random walk hypothesis cannot be rejected on the basis of the proposed non-parametric statistics. The results from the variance-ratio tests are in stark contrast with those based on the non-parametric statistics. On the basis of the parametric variance ratios,[7] the random walk hypothesis is rejected for some values of q. The case of the Japanese yen is particularly striking.

The results for the foreign exchange rates considered here provide evidence that is contrary to that obtained by Liu and He (1991) and Wright (2000). In

[6]The data are available on the Board of Governors of the Federal Reserve System Web site.

[7]The inference ignores the joint implications of the variance ratios for the various values of q. Chow and Denning (1993) and Cecchetti and Lam (1994) have proposed methods for testing multiple variance ratios. Fong, Koh, and Ouliaris (1997) found, contrary to Liu and He (1991), that once the joint implications of the variance-ratio statistics are taken into account, there is much weaker evidence against the random walk hypothesis.

general, it is found on the basis of non-parametric procedures, which are valid in finite-samples under sufficiently general assumptions, that the random walk hypothesis cannot be rejected.[8]

6. Concluding Remarks

This paper's main motive was to provide a generalization of the non-parametric bounds tests for a random walk with unknown drift proposed in Campbell and Dufour (1997). Their approach ensures that the probability of a Type I error is bounded from above by the desired level α. These bounds tests, however, can be quite conservative in finite samples such that power losses might occur. The distinguishing feature of the non-parametric tests proposed in this paper is that, while they retain the virtues of those proposed in Campbell and Dufour (1997), the probability of a Type I error is exactly α.

The robustness of the proposed tests to unknown forms of heterogeneity and dependence in the conditional variance has been contrasted against the parametric variance-ratio test. The results of simulation experiments further reveal that the power of the proposed tests is superior to that of the variance-ratio test for the alternatives considered here. A comparison of these results with those in Campbell and Dufour (1997) highlights the power gains of the non-parametric tests proposed in this paper.

Although the proposed methods have been illustrated with financial time series, they have general applicability. The methods will have high discriminatory power in the context of macroeconomic time series, for example, where the drift term is usually large.

[8] Implicitly, this is also a non-rejection of the hypothesis of symmetrically distributed innovations. This suggests that models of foreign exchange volatility that assume symmetric distributions, such as Gaussian or Student-t GARCH models, might not be misspecified at least in this respect.

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Table 1
Size and Power comparisons for Model 1 with $\mu = 1$, $T = 100$

ϕ	$q=2$ VR_1	$q=2$ VR_2	$q=5$ VR_1	$q=5$ VR_2	$q=10$ VR_1	$q=10$ VR_2	S_m^*	W_m^*
				$\varepsilon_t \sim N(0,1)$				
1.00	5.1	5.5	4.8	5.3	5.3	5.8	5.8	6.3
0.99	7.2	6.9	12.9	12.5	15.6	16.1	17.6	34.1
0.98	9.1	8.3	18.2	17.5	24.2	24.5	27.8	50.3
				$\varepsilon_t \sim t(3)$				
1.00	4.9	5.7	4.9	7.4	4.4	6.5	4.9	5.3
0.99	5.4	4.3	6.0	6.2	7.8	6.0	19.5	25.0
0.98	5.4	4.9	7.1	7.9	10.0	7.3	27.5	35.1
				$\varepsilon_t \sim Cauchy$				
1.00	3.8	11.4	4.7	16.7	3.7	22.4	5.5	4.2
0.99	5.5	4.3	5.3	4.1	5.2	4.5	25.1	30.8
0.98	5.6	3.2	5.4	3.6	5.5	3.6	34.1	38.0

Note: Entries are the empirical rejection probabilities (in percentage) of the random walk hypothesis $H_0 : \phi = 1$ in the model $y_t = \mu + \phi y_{t-1} + \varepsilon_t$ for $t = 1, ..., T$, where ε_t are independent and identically distributed.

Table 2
Size and Power comparisons for Model 1 with $\mu = 1$, $T = 200$

ϕ	$q=2$ VR_1	$q=2$ VR_2	$q=5$ VR_1	$q=5$ VR_2	$q=10$ VR_1	$q=10$ VR_2	S_m^*	W_m^*
				$\varepsilon_t \sim N(0,1)$				
1.00	5.7	6.2	5.8	6.1	4.7	5.1	3.8	4.0
0.99	11.8	10.6	27.2	27.1	44.8	44.6	62.6	86.4
0.98	10.7	9.4	25.1	24.4	40.9	40.8	53.6	83.6
				$\varepsilon_t \sim t(3)$				
1.00	5.0	4.8	4.2	4.3	4.7	5.8	4.7	5.0
0.99	6.0	6.5	9.4	10.0	13.0	13.8	44.2	57.9
0.98	5.3	5.6	7.8	7.7	10.1	10.8	39.0	48.9
				$\varepsilon_t \sim Cauchy$				
1.00	3.4	10.6	5.1	14.3	4.7	18.9	4.4	4.6
0.99	5.2	3.4	4.6	2.7	5.2	2.9	52.2	49.3
0.98	4.7	2.8	4.5	2.4	4.5	2.4	56.6	56.7

Note: See Table 1.

Table 3
Size and Power comparisons for Model 1 with $\mu = 2$, $T = 100$

	$q=2$		$q=5$		$q=10$			
ϕ	VR_1	VR_2	VR_1	VR_2	VR_1	VR_2	S_m^*	W_m^*
				$\varepsilon_t \sim N(0,1)$				
1.00	5.0	4.7	4.1	4.1	4.4	5.1	5.0	4.3
0.99	19.5	20.8	49.5	49.8	62.9	62.7	72.0	90.1
0.98	42.1	43.5	79.2	80.0	89.0	89.2	90.6	98.7
				$\varepsilon_t \sim t(3)$				
1.00	4.9	5.3	5.8	7.0	4.1	6.1	4.8	5.1
0.99	9.0	11.0	16.6	17.4	26.2	26.0	52.8	61.4
0.98	13.6	15.5	27.9	28.3	42.8	41.4	71.6	82.9
				$\varepsilon_t \sim Cauchy$				
1.00	3.3	10.0	3.7	15.4	2.9	21.2	6.1	4.6
0.99	5.2	3.6	5.3	4.3	6.1	4.8	32.2	38.6
0.98	5.4	3.3	5.4	3.2	6.1	3.8	41.7	45.8

Note: See Table 1.

Table 4
Size and Power comparisons for Model 2 with $\mu = 1$, $T = 100$

	$q=2$		$q=5$		$q=10$			
ϕ	VR_1	VR_2	VR_1	VR_2	VR_1	VR_2	S^*_m	W^*_m
				$\varepsilon_t \sim N(0,1)$				
1.00	39.0	5.2	35.6	5.5	26.3	5.3	6.1	5.1
0.99	19.4	21.6	23.3	24.6	24.1	24.5	41.3	40.0
0.98	21.0	23.7	24.9	25.5	26.0	26.2	46.3	43.6
				$\varepsilon_t \sim t(3)$				
1.00	33.4	6.3	31.8	7.9	24.5	6.7	5.9	5.6
0.99	14.3	18.2	18.5	18.6	20.8	20.8	38.9	36.8
0.98	16.5	20.2	20.1	20.8	21.2	21.9	44.0	39.5
				$\varepsilon_t \sim Cauchy$				
1.00	14.1	8.6	15.8	12.5	10.1	16.8	4.2	6.2
0.99	9.8	11.3	11.0	10.6	12.9	11.4	52.5	41.2
0.98	10.9	11.0	11.9	11.4	13.2	11.9	58.3	47.1

Note: Entries are the empirical rejection probabilities (in percentage) of the random walk hypothesis $H_0 : \phi = 1$ in the model $y_t = \mu + \phi y_{t-1} + exp(h_t/2)\varepsilon_t$ with $h_t = \lambda h_{t-1} + \xi_t$ for $t = 1, ..., T$, where $\lambda = 0.99$, and ξ_t is i.i.d. $N(0,1)$ independent of ε_t, which also are independent and identically distributed.

Table 5
Size and Power comparisons for Model 2 with $\mu = 1$, $T = 200$

	$q=2$		$q=5$		$q=10$			
ϕ	VR_1	VR_2	VR_1	VR_2	VR_1	VR_2	S_m^*	W_m^*
				$\varepsilon_t \sim N(0,1)$				
1.00	52.4	4.1	49.4	4.8	42.7	5.8	3.7	5.0
0.99	12.0	14.2	15.4	15.7	16.3	17.1	59.3	46.5
0.98	12.2	14.3	15.2	15.6	15.5	16.1	61.5	48.5
				$\varepsilon_t \sim t(3)$				
1.00	46.4	4.3	46.5	5.8	40.7	6.5	4.2	3.7
0.99	11.8	13.4	14.2	14.8	15.0	15.7	58.2	49.0
0.98	12.1	13.3	14.2	14.5	14.5	15.0	61.6	51.7
				$\varepsilon_t \sim Cauchy$				
1.00	17.3	6.8	18.7	9.6	18.2	11.6	4.0	5.5
0.99	6.7	7.1	7.2	7.7	7.1	7.4	72.0	61.1
0.98	6.8	6.9	7.1	6.8	6.9	6.9	73.1	65.8

Note: See Table 4.

Table 6
Size and Power comparisons for Model 2 with $\mu = 2$, $T = 100$

	$q=2$		$q=5$		$q=10$			
ϕ	VR_1	VR_2	VR_1	VR_2	VR_1	VR_2	S_m^*	W_m^*
				$\varepsilon_t \sim N(0,1)$				
1.00	39.2	5.5	34.3	5.7	27.2	5.5	5.3	5.0
0.99	24.0	26.6	28.5	30.0	32.1	31.2	50.0	49.5
0.98	27.0	28.9	31.1	32.5	34.6	34.3	54.2	51.6
				$\varepsilon_t \sim t(3)$				
1.00	32.0	4.4	28.0	5.1	20.3	5.5	3.4	4.6
0.99	20.5	23.4	23.3	25.1	26.3	25.8	49.7	46.1
0.98	22.0	24.0	24.9	27.2	28.5	28.2	54.4	49.2
				$\varepsilon_t \sim Cauchy$				
1.00	13.3	8.4	14.0	13.3	11.7	15.8	4.7	4.6
0.99	12.4	14.2	14.1	14.5	16.1	14.4	59.7	51.7
0.98	14.0	14.0	14.9	14.5	16.6	14.6	65.2	57.2

Note: See Table 4.

Table 7
Size and power of the approximately exact test statistics

ϕ	$\mu = 0.25$ $\widetilde{S}^*_m$	$\widetilde{W}^*_m$	$\mu = 0.5$ $\widetilde{S}^*_m$	$\widetilde{W}^*_m$	$\mu = 0.75$ $\widetilde{S}^*_m$	$\widetilde{W}^*_m$	$\mu = 1$ $\widetilde{S}^*_m$	$\widetilde{W}^*_m$
				$T = 100$				
1.00	5.46	7.10	5.01	5.52	5.02	5.32	5.03	5.19
0.99	8.08	13.05	11.15	18.11	17.21	27.92	26.16	41.47
0.98	9.34	15.20	15.33	27.53	24.78	43.19	38.03	61.43
				$T = 200$				
1.00	4.22	5.62	4.11	4.90	4.13	4.85	4.16	4.78
0.99	9.91	18.26	21.18	38.61	40.05	65.47	62.70	87.71
0.98	10.04	19.46	19.67	42.71	37.23	70.40	58.01	89.74

Note: Entries are the empirical rejection probabilities (in percentage) of the random walk hypothesis $H_0 : \phi = 1$ in the model $y_t = \mu + \phi y_{t-1} + \varepsilon_t$ for $t = 1, ..., T$, where $\varepsilon_t \sim i.i.d.\ N(0, 1)$.

Table 8
Tests for a random walk in weekly exchange rates

$q = 2$ VR_1	VR_2	$q = 5$ VR_1	VR_2	$q = 10$ VR_1	VR_2	$q = 30$ VR_1	VR_2	$\widetilde{S}^*_m$	$\widetilde{W}^*_m$
				Can$/US$					
2.69⋆	2.12⋆	1.74	1.43	0.60	0.51	0.47	0.43	-2.68⋆	-2.71⋆
				FF/US$					
1.69	1.54	2.21⋆	1.88	1.87	1.60	2.89⋆	2.59⋆	0.08	-0.24
				DM/US$					
1.53	1.40	2.04⋆	1.74	1.92	1.64	2.07⋆	1.86	1.59	1.00
				Yen/US$					
2.42⋆	1.92	4.87⋆	4.03⋆	4.83⋆	4.10⋆	4.10⋆	3.63⋆	0.75	0.94
				Pound/US$					
1.64	1.15	1.68	1.25	2.62⋆	2.01⋆	2.41⋆	1.97⋆	1.08	1.81

Note: Entries are the values of variance-ratio test statistics for various values of q and of the sign and Wilcoxon test statistics (see text for details). The star indicates a rejection at the 5 per cent level.

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