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Liquidity Effects and Market Frictions

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Abstract

The goal of this paper is to shed light on the nature of the monetary transmission mechanism. Specifically, we attempt to tackle two problems in standard limited-participation models: (1) the interest rate liquidity effect is not as persistent as in the data; and (2) some nominal variables are unrealistically volatile. To address these problems, we introduce nominal wage and price rigidities, as well as portfolio adjustment costs and monopolistically competitive firms, to better understand how each of these costs affects the size and length of the liquidity effect following a central-bank policy action.

Quantitative analysis shows that including these rigidities does improve the model, to some extent at least, in the expected manner. The main findings are: (1) wage and portfolio adjustment costs are able to deepen and lengthen the liquidity effect following a monetary policy action; (2) these two adjustment costs, especially wage adjustment costs, can reduce inflation volatility; (3) price adjustment costs, at least under money-growth policy rules, cause excessive interest-rate volatility and are unable to significantly reduce inflation volatility.

Résumé

L’étude cherche à clarifier la nature du mécanisme de transmission de la politique monétaire. Les auteurs s’attachent plus précisément à deux problèmes que posent les modèles traditionnels à « participation limitée » : 1) le fait que l’effet de liquidité sur les taux d’intérêt soit moins persistant dans ces modèles que selon les données; 2) le degré irréaliste de volatilité de certaines variables nominales. Afin de résoudre ces deux problèmes, les auteurs postulent la rigidité des salaires et des prix nominaux, ainsi que l’existence de coûts d’ajustement des portefeuilles et d’un cadre de concurrence monopolistique; leur objectif est de comprendre comment chacun de ces facteurs influe sur la taille et la durée de l’effet de liquidité produit par les mesures de politique monétaire de la banque centrale.

L’analyse quantitative montre que l’insertion de rigidités a pour effet d’améliorer le modèle de la façon prévue, du moins dans une certaine mesure. Voici les principaux résultats obtenus par les auteurs : l’incorporation dans le modèle de coûts d’ajustement des salaires et des portefeuilles permet d’accentuer et de prolonger l’effet de liquidité produit par une mesure de politique monétaire; 2) la prise en compte de ces deux types de coûts, en particulier ceux se rapportant aux salaires, peut réduire la volatilité de l’inflation; 3) l’addition de coûts d’ajustement des prix, à tout le moins dans le contexte de règles de politique avec croissance monétaire, entraîne une volatilité excessive des taux d’intérêt et ne parvient pas à atténuer de façon sensible la volatilité de l’inflation.

Contents

1. Introduction

Economists are only beginning to reach an understanding of the complexities of the monetary transmission mechanism. Achieving a thorough comprehension of the workings of monetary policies will require an exploration of the complex structure of the complete macroeconomy.[1] In this spirit, we use a general equilibrium model in which money plays an important role in each of the investment, production and consumption processes.

Economists have generally accepted the idea that money plays a role in the economy due to its asymmetric distribution to economic agents. That is, money is first distributed to financial intermediaries and then to firms before it finally reaches consumers' hands. This is the basic idea embedded in a standard limited-participation model. However, there are still some limitations with the basic version of this model. First, the liquidity effect is not as persistent as that observed in the data. For instance, most empirical estimates find that the interest rate should fall for several quarters following an expansionary monetary policy shock, specifically a money-growth shock or a series of unexpected money-level shocks. Second, stochastic simulations of limited-participation models generally find too much volatility of inflation and other nominal variables.

As we know, monetary policy shocks are transmitted through agents' decision-making processes via dynamic mechanisms, such as adjustment costs. If markets operated without any frictions, monetary policies would have no (persistent) effect on interest rates or any real variables. In addition, some economists have conjectured that price and wage rigidities may be a primary cause of the persistent liquidity effect of a monetary shock.[2]

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1. See the Presidential Address made by Michael Parkin at the 1998 CEA meeting.

2. As Williamson (1996) observes, "It is necessary to seriously confront the frictions which make monetary and financial factors matter."Similarly, Aiyagari (1997) points out that a modelling approach that considers frictions can be expected to have a significant impact on answers to questions of interest to macroeconomists and policymakers. Finn (1995) also argues that the combination of the assumptions of increasing return to scale and market frictions can lead to prolonged liquidity effects.

In this vein, Chari, Kehoe, and McGrattan (1996) introduce a staggered-price-setting mechanism into a money-in-the-utility-function model (Taylor, 1980). They show that such a model cannot generate persistent movements in output following monetary shocks if the model has any of the following features: zero-income effect preferences (Beaudry and Devereux, 1996); non-constant elasticity of demand for intermediate goods (Kimball, 1995); upward-sloping marginal cost curve for firms (Rotemberg, 1995); or an input-output structure (Basu, 1995).

Christiano, Eichenbaum, and Evans (1996) compare sticky-price models with limited-participation models. They conclude that any model equipped with only one type of friction cannot successfully account for the basic stylized facts unless unrealistic parameter values are assumed. Aiyagari and Braun (1997) speculate that a combination of the limited-participation model and a price-adjustment cost will lead to useful insights into business fluctuations. In this paper, we pursue our research along this avenue. More precisely, we attempt to determine the relative importance of three major frictions -- price, wage and portfolio adjustment costs -- in understanding the monetary transmission mechanism. Our model is developed from the basic limited-participation model originated by Lucas (1990) and Fuerst (1992).

This paper introduces different types of adjustment costs to investigate whether they improve the model's ability to replicate some of the major stylized facts of empirical impulse response functions and higher moments. First, portfolio-adjustment costs are introduced to prolong the interest-rate effects of a monetary policy shock. Second, nominal price and wage adjustment costs are also added to the model to dampen the volatility of the nominal side of the economy.[3]

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3. Dow (1995) has looked at the liquidity effects of monetary shocks by considering frictions in both commodity and credit markets. Unfortunately, his model does not lead to more persistent liquidity effects.

In general, the adjustment costs we introduce do improve the model, to some extent at least, in the expected manner. Wage and portfolio adjustment costs are able to lengthen and/or deepen the interest rate liquidity effect following a monetary policy action.

Wage adjustment costs, which can be wage negotiation costs or possibly information accumulation costs, are particularly effective in lowering the volatility of inflation and increasing the response of output to a monetary policy action. These costs reduce workers' power to increase wage rates following a positive money shock. With firms borrowing to pay wages, this implies that intermediaries must further cut interest rates in order to induce firms to borrow the new funds. Given lower wages but a fixed supply of funds, firms will increase hours worked and output compared to an economy with no wage adjustment costs.

Portfolio adjustment costs reduce the incentives for households to change the level of their cash holdings thereby limiting the adjustment of deposits and creating a persistent liquidity effect. The extended deviation of the interest rate from steady state induces persistent deviation of inflation and output as well.

In contrast, price adjustment costs are less effective, having basically no effect on inflation and output when wage and portfolio adjustment costs have already been introduced into the model. Price adjustment costs can be marketing costs, advertising costs, and information accumulation costs. In examples with only price adjustment costs, there were reductions of inflation volatility, but mostly in expected future volatility not the contemporary response. For small price adjustment costs there is a deepening of the interest-rate liquidity effect. However, as the costs are increased, the liquidity effect is reversed as firms try to avoid the costs by increasing hours and output and hence loan demand and the interest rate. In general, the firms attempt to reduce the price adjustment costs leads to excessive interest-rate volatility in the model.

In sum, this paper finds that real and nominal adjustment costs can greatly improve the characteristics of limited-participation models, but more work is necessary to properly calibrate these costs. Once this is accomplished, this model of the monetary transmission mechanism should be able to replicate more of the nominal and real characteristics of business cycles.

The rest of this paper is organized as follows. Section 2 provides a detailed description of the economic environment and the dynamic general equilibrium problem of money. Section 3 calibrates the model. The quantitative analysis is presented in Section 4. Finally, Section 5 summarizes the findings in this paper, and points to the direction of our future research.

2. The model

2.1 Economic Environment

2.1.1 Households

The preferences for a typical household, *i*, are given by:

$$E_0\left\{\sum_{t=0}^{\infty}\beta^t U(C_{it}, 1-L_{it}^s-AC_{it}^Q)\right\} \tag{1}$$

where $0<\beta<1$ and,

$$U(C_{it}, 1-L_{it}^s-AC_{it}^Q) = \begin{cases} \frac{1}{\phi}[C_{it}(1-L_{it}^s-AC_{it}^Q)^{\gamma}]^{\phi} & if \ \ \phi \neq 0 \\ \ln C_{it}+\gamma\ln(1-L_{it}^s-AC_{it}^Q) & if \ \ \phi = 0 \end{cases} \tag{2}$$

where C_{it}, $L^s{}_{it}$, and $AC^Q{}_{it}$ are the period *t* consumption, labour supply, and time cost of portfolio adjustment, respectively. We assume that households must take time to adjust their financial portfolios between periods. As with the standard limited-participation models, households cannot adjust their portfolios within a period. The portfolio adjustment cost is assumed to be a convex function of the following form:

$$AC_{it}^{Q} = \frac{\varphi_q}{2}\left(\frac{Q_{it+1}}{Q_{it}} - (1+x)\right)^2 \tag{3}$$

In deterministic steady state, the adjustment cost will be zero since all nominal variables will grow at the money growth rate, $1+x$.[4] As the growth rate of cash holdings deviates from its steady state level, the adjustment cost increases quadratically.

We assume that money is introduced in the economy by a modified cash-in-advance (CIA) constraint. That is, households make their consumption and investment purchases out of the sum of the nominal cash balance transferred from the last period and the labour income earned in the current period. This constraint is described in (4). The periodic budget constraint given by (5), which says that cash and deposits carried forward to period *t* is the sum of interest payments, dividends, capital income, and unspent cash from the goods market.[5]

$$P_t C_{it} + P_t I_{it} + P_t AC_{it}^{W} \le Q_{it} + W_{it} L_{it} \tag{4}$$

$$\begin{aligned} Q_{it+1} + N_{it+1} = {} & R_t^d N_{it} + D_{it} + F_{it} + R_{kt} K_{it} + \\ & [Q_{it} + W_{it} L_{it} - P_t C_{it} - P_t I_{it} - P_t AC_{it}^{W}] \end{aligned} \tag{5}$$

where I_{it} is period *t* investment; Q_{it} is the amount of cash that household *i* has at the beginning of period *t*; N_{it} is the household's level of demand deposits for period *t* determined in period *t-1*;[6] D_{it} and F_{it} are the dividends received from the firm and the financial

4. This function form is similar to that in Christiano and Eichenbaum (1992). Obviously, there are no adjustment costs in steady state. This applies to all the adjustment costs discussed in the model.

5. Our model treats money primarily as a medium of exchange, even though other factors, such as variation in velocity, might have important roles in the monetary transmission mechanism. In addition, a binding CIA constraint is not imposed in this model.

6. The sum of cash and deposit holdings is equal to the money supply so that $\sum(Q_{it}+N_{it})=M_t$.

intermediaries, respectively, at the end of period t; and $AC^{W}{}_{it}$ is the wage adjustment cost. The rental price of capital is given by R_{kt} so that households earn $R_{kt}K_t$ from their capital stock in period t.

Given a relative stable Canadian labour market, the employers do not often change their workers' wage rates. On the other hand, most workers do not frequently negotiate with their employers in order to avoid paying the costs involved in the negotiation process. We assume that a worker has to bear a real cost to propose and realize a change in their nominal wages; we can think of this as a negotiating cost. Even though the magnitude of the cost can be small, the impact of the cost on aggregate labour supply might be significant. This wage adjustment cost is very different from the nominal wage rigidities reflected in the long-term contracts between workers and firms. In this model, the wage can be changed at any time but only after a cost is paid.

During period t, households take as given from period t-1 their capital stock holdings (K_{it}) and their distribution of money holdings between deposits (N_{it}) and cash (Q_{it}). Assume that households must choose their period $t+1$ split of financial assets between cash (Q_{it+1}) and deposits (N_{it+1}) before the end of period t. This is the standard assumption of the limited-participation models.

The law of motion for the physical capital stock is given in equation (6).[7]

$$K_{it+1} = (1-\delta)K_{it} + I_{it} \tag{6}$$

7. We can easily introduce capital-adjustment costs in this model by assuming the following form,

$$AC^{k}_{jt} = I_{jt}\left\{-\frac{\varphi_k}{2}\left(\frac{K_{jt+1}}{K_{jt}} - \exp(\mu)\right)^2\right\}$$

Unlike in some other limited-participation models, these costs are found to be unnecessary for generating liquidity effects or reducing the volatility of investment.

Wage adjustment costs are assumed to have the following functional form:

$$AC_{it}^{w} = \frac{W_t}{P_t}\left\{-1 + \exp\left[\phi_{w1}\left(\frac{W_{it}}{W_{it-1}} - (1+x)\right)\right]\right. \qquad (7)$$
$$\left. - \varphi_{w1}\left(\frac{W_{it}}{W_{it-1}} - (1+x)\right) + \frac{\varphi_{w2}}{2}\left(\frac{W_{it}}{W_{it-1}} - (1+x)\right)^2\right\}$$

The adjustment costs are comprised of a symmetric component (when $\varphi_{w2}>0$) and an asymmetric component when $\varphi_{w1}\neq 0$. If $\varphi_{w1}<0$ then adjustment costs will be higher for wage decreases than wage increases. Since households have monopoly power over their differentiated labour, they recognize that their wage rate, W_{it}, is a function of their labour supply as shown in (11) and account for this when maximizing their utility. Consequently, the wage adjustment cost function can be written as a function of L_{it} by combining (7) and (11).

A household's decision problem, shown in the Appendix II, is to maximize utility given by (1) and (2) by selecting decision variables Q_{it+1}, N_{it+1}, C_{it}, K_{it+1}, and $L^s{}_{it}$ subject to the constraints in (11) and (3) to (7).

2.1.2 The Final Goods Firm

The final goods producer in this economy is simply an aggregator which buys inputs from the intermediate goods producers and combines them into the final good. Also, the aggregator hires labour from the differentiated households to generate a composite labour commodity which is used by intermediate goods firms in their production process. The final good can either be consumed or invested. The production technology for the final good is summarized by the following *constant return to scale* function

$$Y_t = J^{\frac{1}{1-\theta_y}} \left(\sum_{j=1}^{J} Y_{jt}^{\frac{(\theta_y - 1)}{\theta_y}} \right)^{\frac{\theta_y}{(\theta_y - 1)}} \quad (8)$$

where Y_t is the output of the final good, Y_{jt} is the amount of input acquired from intermediate producer j, and J is the number of intermediate firms. The parameter θ_y is the elasticity of substitution between any two different intermediate goods.

The final goods producer's profit maximization problem is given as

$$Max\ \Pi_t = P_t Y_t - \sum_{j=1}^{J} P_{jt} Y_{jt} \quad (9)$$

From its first order condition for Y_{jt}, we can easily derive the following simple relation between the price, P_t, for the final good and the price, P_{jt}, charged by the producer of the intermediate good j.

$$P_{jt} = P_t \left(\frac{J Y_{jt}}{Y_t} \right)^{-\frac{1}{\theta_y}} \quad (10)$$

Following the same logic, we can assume this final good producer also buys heterogeneous labour, L_{it}, from household i and sells a composite labour input to the intermediate goods producers.[8] Therefore, the relationship between the competitive wage rate, W_t, faced by the firms and the individual wage rate, W_{it}, set by household i is given by[9]

8. The composite labour is produced by the aggregator with constant return to scale technology,

$$L_t = J^{\frac{1}{1-\theta_l}} \left(\sum_{j=1}^{J} l_{jt}^{\frac{(\theta_l - 1)}{\theta_l}} \right)^{\frac{\theta_l}{(\theta_l - 1)}}$$

9. For simplicity, we assume there is a unit mass of both households and firms.

$$W_{it} = W_t \left(\frac{IL^s_{it}}{L^s_t} \right)^{-\frac{1}{\theta_L}} \tag{11}$$

where θ_L is the elasticity of substitution between any two types of labour.

2.1.3 The Intermediate Goods Firms

Each intermediate good producer j $(j=1,2,...,J)$ hires the aggregate labour commodity available from the aggregator, $L^d{}_{jt}$, in a perfectly competitive market at wage rate W_t. The firm also rents capital goods (K_{jt}) from the households, in a competitive market, at a rental price of R_{kt}. The firm must borrow cash from the financial intermediary each period, at interest rate R_t, in order to have the funds to hire labour and begin production. The total amount of borrowing in period t is the sum of the firm's nominal wage bills, W_tL_{jt}. The capital rental cost is assumed to be financed through internally generated revenue. Capital and labour inputs are combined in an increasing returns to scale production function to produce a differentiated product, Y_{jt}.

The firm sells its output in a monopolistically competitive market in which it has the power to set its own price, P_{jt}. However, it is costly for a firm to change the price of its product. This motivates us to assume that the intermediate firms face the price adjustment costs. This assumption might help to a certain extent generate the persistent effects of a policy shock.

Let $AC^p{}_{jt}$ represent a price adjustment cost which the firm must pay whenever it changes its price level at a rate different from the steady state growth rate. These are real costs measured in terms of the final good purchased by the firm and used up in the process of changing prices. These can be thought of as menu, advertising, or marketing costs for the setting of new prices. Firm j's objective is to maximize the present value of its lifetime stream of dividend payments to its shareholders, D_{jt}. These dividends are discounted by a factor, β, as well as by a term representing the marginal utility value of the dividends, λ_{2t}, for the households, which own the firms.

$$max\ E_0\left\{\sum_{t=0}^{\infty}\beta^t\cdot\lambda_{2t}\cdot D_{jt}\middle|\Omega_{1t}\right\} \tag{12}$$

where

$$D_{jt} = P_{jt}\cdot Y_{jt} - R_t\cdot W_t\cdot L^d_{jt} - R_{kt}\cdot K_{jt} - P_t\cdot AC^p_{jt}. \tag{13}$$

The price-adjustment cost function is described in equation (14) and has the same basic form as the other adjustment costs.[10]

$$AC^p_{jt} = Y_t\cdot\left\{\frac{\varphi_p}{2}\left(\frac{P_{jt}}{P_{jt-1}} - \frac{1+x}{\exp(\mu)}\right)^2\right\} \tag{14}$$

The production technology exhibits increasing return to scale with labour-augmented technological progress, that is,

$$Y_{jt} = F(K_{jt}, L^d_{jt}) = (K^{\alpha}_{jt}(z_t L^d_{jt})^{1-\alpha})^{\Phi} \tag{15}$$

where 0<a<1, $\Phi \geq 1$. The variable z_t represents labour-augmenting technological progress

$$z_t = \exp(\eta t + \theta_t). \tag{16}$$

The parameter η is the steady state growth rate of the technology level in the economy and the technology shock, θ_t, is assumed to follow the random process,

$$\theta_t = (1-\rho_\theta)\theta + \rho_\theta\theta_{t-1} + \varepsilon_{\theta_t}. \tag{17}$$

This is a simple AR(1) process for which $0<\rho_\theta<1$ and $\varepsilon_{\theta t}$ is i.i.d. with zero mean and standard deviation σ_θ.

10. Asymmetric price adjustment cost results have not been reported since the symmetric costs have only a small effect in the final model.

Firm j's dividend-maximization problem can be characterized by a set of marginal conditions for labour and capital as given in Appendix I.

2.1.4 Financial Intermediary

At the beginning of period t, the financial intermediary has demand deposits of N_t that it received from the households in the previous period. The financial intermediary uses these funds, along with any transfer from the government, $T^b{}_t$, to make loans to the intermediate firms, $B^f{}_t$.

$$B_t^f = N_t + T_t^b \tag{18}$$

At the end of each period, the financial intermediary pays back the household deposits, with interest, using the debt repayments collected from firms.[11] The objective of the financial intermediary is to choose the optimal amount of loans made to firms and the optimal level of demand deposits to maximize the expected present value of its dividend:

$$max \left\{ \mathrm{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \lambda_{2t} F_t \right] \right\} \tag{19}$$

where the dividend is given by,

$$F_t = (1 + R_t^l) B_t^f - (1 + R_t^d) N_t \tag{20}$$

Financial intermediation is assumed to be a costless activity. With no barriers to entry, competitive forces will ensure that the equilibrium interest rate on loans equals the rate paid on deposits, that is $R^l{}_t = R^d{}_t = R_t$. Consequently, in equilibrium, the financial intermediary will pay $F_t = (1+R_t)T^b{}_t$ in dividends to the households.

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11. We assume that there is no risk of default in this model.

2.1.5 The Central Bank and Government

Since the focus of this paper is on monetary policy, we assume that the government neither collects taxes nor issues debt.[12] However, the government makes transfer, $T^b{}_t$, to the financial intermediaries through the central bank. Specifically,

$$X_t = T_t^b. \tag{21}$$

In this case, new money is distributed directly by the central bank to only financial institutions.[13] This restriction on how money is distributed is the main cause of why money is non-neutral in the short run in this model. Again, money is not a source of revenue for the government; what matters in the model is the level of liquidity which has a real impact on economic activity.

Assume that the central bank follows a money growth rate rule as described in equation (22)

$$x_t = (1-\rho_x)x + \rho_x x_{t-1} + \varepsilon_{xt} \tag{22}$$

where x_t is the money growth rate, X_t/Mt.

An exogenous positive monetary shock, $\varepsilon_{xt}>0$, increases the funds available to the financial intermediary in period t. Consequently, the financial intermediary responds by lending more to firms for the employment of labour. To ensure that firms will borrow these excess fund, the banks lower the equilibrium interest rate thereby creating the liquidity effect.

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12. We can easily incorporate the fiscal side of the government into the current setup and conduct the relevant policy analysis.

13. In Canada, monetary expansions and contractions are carried out by the central bank's cash management: drawdowns and redeposits of federal government deposits with direct clearers.

2.1.6 Market Clearing

Assuming a unit mass of households and firms, the following equations describe the market-clearing conditions which must hold for an equilibrium to exist:

(1) Goods:

$$C_t + I_t + AC_t^w + AC_t^p = Y_t \tag{23}$$

(2) Loans:

$$W_t L_t = N_t + T_t^b \tag{24}$$

(3) Money:

$$M_{t+1} = M_t + X_t \tag{25}$$

(4) Labour:

$$L_t^s = L_t^d \tag{26}$$

2.2 A Notion of Competitive Equilibrium

A stationary competitive equilibrium is defined to be a sequence of allocations $\{C_{it}, L^s{}_{it}, N_{it+1}, Q_{it+1}, I_{it}\}$ for households, $\{K_{jt}, L^d{}_{it}\}$ for firms, a set of prices $\{P_{it}, P_t; W_{jt}, W_t; R_t; r_{kt}\}$, and a central bank reaction function $\{x_t\}$ such that, given the equilibrium prices and other parties' actions,

(1) households choose $\{C_{it}, L^s{}_{it}, N_{it+1}, Q_{it+1}, I_{it}, W_{it}\}$ to maximize (1) subject to constraints (2) to (7) and (11);

(2) firms choose $\{K_{jt}, L^d{}_{it}, P_{jt}\}$ to maximize (12) subject to constraints (13) to (17);

(3) financial intermediaries solve the maximization problem in (19);

(4) prices adjust to ensure that market-clearing conditions hold: P_t clears the final-goods market, W_t clears the composite labour market, R_t clears the loan (or credit) market.

The equilibrium allocations and prices are the solution to the system of 12 equations in the 12 unknowns $\{C_{it}, L^s{}_{it}, N_{it+1}, Q_{it+1}, K_{jt+1}, L^d{}_{it}, P_{it}, P_t, W_{jt}, W_t, R_t, \lambda_{1t}, \lambda_{2t}\}$. The equations are given by (4), (11), (12), (24), (25), (A1)-(A5), (A11), and (A12). Equation (28) can be used if the cash-in-advance constraint proves to be non-binding:[14]

$$\lambda_{1t}(Q_{it} + W_{it}L_{it} - P_tC_{it} - P_tI_{it} - P_tAC_{it}{}^w) = 0 \tag{27}$$

3. Calibration

The balanced growth path of the model is calibrated to quarterly Canadian data for the 1955 to 1996 period. The mean annual growth rate of per capita output during this period was 1.83%, so we set μ=0.004563 (1.83% = (exp(μ)-1)*4). The discount factor β was assumed to be 0.993, so the annual real rate of return on investment is about 2.8%. The annualized depreciation rate, δ, was set at 10% to approximately match the capital-output ratio observed in the Canadian data.

The parameters θ_y and θ_l represent the market power that a firm and a household have to adjust their price and wage, respectively. Smaller θ values represent stronger market power. A constant-return-to-scale (CRS) economy would set these θ values to infinity so that firms and households would be price and wage takers. However, with an increasing-return-to-scale (IRS) economy we choose values closer to those used in Kim (1996) and set θ_y=5.0 and θ_l=10.0.[15] The increasing return to scale parameter, Φ, is set at a moderate value of only 1.25. Finn (1995) has suggested that Φ

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14. According to the precautionary-saving argument, in a volatile market economy, households always want to hold a certain amount of cash in their saving and/or chequing accounts from time to time to accommodate the unexpected needs. That is why the cash-in-advance constraint is not necessarily binding.

should not be too large because the volatility of real variables is decreasing in the degree of IRS. The value of α=0.3369 is chosen such that the steady state labour share of income is 0.65, which is found in the data.

The preference parameters γ=3.8068 and ϕ=–0.4213 are calibrated such that the steady state employment and consumption-output ratio are equal to 0.171 and 0.745, respectively, as observed in the Canadian data.[16]

As described in equation (17), the technology shock is assumed to follow an AR(1) process. The autocorrelation parameter, ρ_θ, is set to be 0.95 based on the assumption that the technology shock is fairly persistent. The standard deviation of the technology shock is set so that the standard deviation of output from the model is close to that from the data (0.0166)

The monetary policy parameters describing the money growth rate process from equation (22) are calibrated to match the quarterly mean growth rate and autocorrelation coefficient of the Canadian money base. This implies x=0.011 and ρ_x=0.18. The standard deviation of the monetary policy shock is set such that the variance of money growth rate from the model is close to that from the data (0.0096).

For the adjustment-cost parameters, we use ϕ_p=1, ϕ_{w1}=0, ϕ_{w2}=10, and ϕ_q=1 during our computation but these coefficients do not affect the steady state. Future work will need to devote more attention to the calibration of these costs.

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15. To verify calibration which is used for remainder of the paper, we used the Generalized Method of Moments to estimate all of the parameters in the model. These estimates obtained are close to those we calibrate.

16. The consumption-output ratio may seem high since government and private consumption have been lumped together for simplicity. Future work will reintroduce government spending as a separate entity.

4. Results

4.1 Impacts of an expansionary policy shock: impulse responses

This section analyses the model through a discussion of the model's impulse responses to a one period monetary policy action.[17] The innovation, ε_{xt}, in equation (22) is set to 0.01, representing a 4 per cent annualized money growth rate shock, in period 5. The impulse response functions for the interest rate, inflation rate, and output are plotted in each of Figures 1 to 5 for different adjustment cost assumptions. Figure 1 shows the combined impact of price, wage, and portfolio adjustment costs while Figures 2 to 4 show the impact of these costs individually. The experiments consider price-adjustment costs of ϕ_p=1, wage costs of ϕ_{w1}=0, ϕ_{w2}=10, and portfolio costs of ϕ_q=1.

Figure 1 illustrates that the base case model with no adjustment costs does generate a liquidity effect after an increase in the money growth rate. However, the deviation of the interest rate and output from steady state lasts for only one period. The inflation rate is only slightly more persistent since it is above steady state for two periods. The addition of adjustment costs makes the deviation of each variable from steady state more persistent and also magnifies the initial responses of output and the interest rate.

With adjustment costs, the interest rate is at least one percentage point below steady state for about three quarters. The response of inflation is initially negligible but increases in subsequent periods. This result does not appear in an equilibrium model with only price rigidities, such as the model by Chari, Kehoe, and McGrattan (1996). It is obvious that the three costs help to reduce the upward pressure on inflation. Finally, a lower interest rate leads to a jump in output production. To understand how each

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17. Fung and Kasumovich (1998) provides the empirical evidence on the liquidity effects of money supply shocks in Canada.

adjustment cost works in the system, we need to examine individually its effects on the key macroeconomic variables.

Figure 2 shows the effect of adding price adjustment costs when wage and portfolio costs are already present. The magnitude of the interest-rate liquidity effect is increased by about one percentage point. In addition, there is only a very marginal dampening effect of the inflation response. The output response is also only marginally affected. This leads us to speculate that the price adjustment cost is not the crucial factor in generating persistent output fluctuations at least in this version of a monetary transmission mechanism model. To verify this, consider another set of experiments.

Figure 5 shows the effects of adding price adjustment costs when there are already wage costs but no portfolio adjustment costs. In this case, the price adjustment costs do have a minor impact on the inflation rate, output, and the interest rate. The interest rate liquidity effect is, once again, magnified by the adjustment costs but there is no change in the persistence of the liquidity effect. As well, including price adjustment costs has only a modest effect on the response of inflation, reducing it by about half a percentage point in the period after the shock. It seems that price adjustment costs are not a very effective way to reduce the volatility of inflation when portfolio adjustment costs have already been introduced into the model.[18] From Figures 2 and 5, we learn that a model built with a simple price adjustment cost does not help to generate persistence of the liquidity effect. Our finding is consistent with the findings in Chari, Kehoe, and McGrattan (1996).

Figure 3 plots the impulse responses with and without wage-adjustment costs. An expansionary policy shock increases equilibrium employment and wages. Because this cost is paid by households, slower wage adjustment implies there will be an

18. If all portfolio costs and restrictions are removed from the model so that households are free to adjust in response to a shock, then the price adjustment costs are effective at lowering the volatility of inflation. However, they still introduce relatively more volatility into the interest rate than is removed from the inflation rate.

increase in labour supply. Given that labour demand is relatively inelastic for the calibration used, the total wage bill will be decreasing in wage adjustment costs thereby requiring a larger drop in interest rates in order to maintain loan market equilibrium. Consequently, the output response is almost doubled over the entire period. That is why the wage adjustment cost is able to dampen the overall response of inflation. This cost is also largely responsible for postponing the peak inflation response from the first to the second period.

The effect of portfolio adjustment costs is shown in Figure 4. These costs reduce a household's time available for work. With a steep labour demand curve, there is an initial increase in the total wage bill, following a monetary injection, compared to the base case without portfolio adjustment costs. This implies an increase in the demand for loans at the initial lending rate. Because the supply of loans is fixed, the market will adjust the lending rate to a higher level to reach a new equilibrium. That is why the initial size of the liquidity effect is smaller with portfolio adjustment costs. However, in the presence of this cost, the liquidity effect becomes much more persistent, which is consistent with empirical studies on Canadian data. With the interest rate persistently below steady state, the economy is able to maintain output more significantly above steady state for a longer period as shown in Figure 4. These portfolio adjustment costs also dampen the initial response of inflation to a monetary policy action.

The formula in equation (7) for wage adjustment costs permits us to examine a model with asymmetric costs when $\phi_{w1} \neq 0$. Assuming $\phi_{w1} < 0$ yields a model with downward wage rigidity but only minimal costs if households wish to raise their wages. Figures 6 and 7 plot the impulse responses for a 1 per cent expansionary and contractionary shock, respectively, for both the symmetric and asymmetric wage cost cases. The impulse responses for an expansionary monetary policy action show that there are only minor costs imposed on the economy. The asymmetric case is closer to the no-wage-cost example although the cost does have some effect by reducing nominal volatility and increasing the output response slightly. In contrast, Figure 7 presents the effects of a contractionary policy action on the key macroeconomic variables. The asymmetric cost results are now almost the same as the symmetric cost results (by construction) with much less nominal response of inflation and wages than in the no-cost case. The deviation of output in response to the shock has almost doubled relative to the base case. In sum, these two graphs illustrate that the model is capable of replicating the real world observation of downward wage rigidity. The model economy would thus experience, on average, deeper recessions than booms.

4.2 Higher Moments

This section analyses the higher moments of a number of different versions of the model to illustrate some of the relative contributions of each of the adjustment costs. Table 1 contains some summary statistics for Canada from 1955 to 1996 for the primary real and nominal variables.

A 42-year sample was replicated 50 times assuming σ_θ=0.01 and σ_x=0.0096. The model's trend was reintroduced into the simulated data before it was logged and HP detrended. The summary statistics for certain variables of interest are given in Table 2. The first set of columns in Table 2 shows the results of the model when there are no adjustment costs included. The next two columns add price adjustment costs, followed by wage adjustment costs, and finally, in the last column, portfolio adjustment costs are added.

On the real side, comparing Tables1 and 2 shows that the standard deviations of output and consumption are quite close to that found in the Canadian data although the simulated investment series are not as volatile as they should be.[19] The data on hours worked is also not quite volatile enough although the addition of

19. Capital adjustment costs were not added in any of the higher moment exercises because investment never became volatile enough to warrant these costs. A preliminary examination of policy rules which smoothed interest rates revealed much more volatile investment data which may justify the reintroduction of capital adjustment costs.

wage and portfolio adjustment costs helps to substantially increase the standard deviation of this variable.

Investment and consumption were correlated much more closely with output than observed in the Canadian data. This is because investment and consumption are closely tied to the movement in output as described by the goods-market clearing condition, (25). The number of hours worked was correlated with output to the same degree as in the data. In general, our model, like the other dynamic general equilibrium models, can mimic well the cyclical behavior of the real economy.

On the nominal side, all nominal variables are more volatile compared with their counterparts in the data. This is caused partially by the monopolistic-competition structure, where workers and intermediate producers are not price takers. All three adjustment costs reduce the volatility of inflation but not by enough to replicate the variance observed in the Canadian data. The price adjustment costs greatly increase the volatility of the nominal interest rate. This implies that the real interest rate also becomes more volatile given that price adjustment costs reduce the inflation volatility. That is why the real variables also fluctuate more with the addition of price adjustment costs, as shown in the second column of Table 2. The last two columns of the table present the effect of the wage and portfolio adjustment costs. It is clear that the wage and portfolio adjustment costs offset some of the volatility of the interest rate, but not enough to allow the price adjustment costs to be increased until the model's inflation volatility matched the Canadian data.

All versions of the model exhibit negative correlations of the interest rate, price level, and inflation rate with output while the actual data has a negative correlation between only output and the price level. This is determined by the basic feature of a limited-participation model, that is, the strong negative correlation between interest rate and output. The inclusion of the adjustment costs strengthens this negative relation, as we can see in Table 2. As well, the wage adjustment costs are most useful for trying to reverse the negative correlations of the inflation rate and interest rate with output.[20] This finding indicates that wage adjustment costs play a more important role than portfolio and price adjustment costs in replicating the stylized facts.

The basic version of the model also has a negatively autocorrelated interest rate, contrary to the Canadian data, which the portfolio adjustment cost is capable of reversing. However, the actual interest rate was still much more persistent than could be generated by the model. The inflation rate in the model was also negatively autocorrelated until portfolio adjustment costs were added, but, again, none of the costs could really create the desired degree of inflation persistence. Our findings imply that the market adjustment costs can help generate the persistence in output fluctuation, but cannot successfully replicate the variation of the nominal side of the Canadian economy.

5. Concluding remarks

This paper examines how different types of market frictions can affect the transmission of monetary policy. It is shown that different types of real and nominal side adjustment costs can be used to improve the impulse response functions and higher moment characteristics of limited-participation models.

Perhaps surprisingly, a sticky price assumption is not crucial for the model to produce a persistent liquidity effect. In addition, the price adjustment cost could not effectively reduce inflation volatility without introducing excessive interest rate volatility. In contrast, wage and portfolio adjustment costs are relatively more effective

20. In a version of the model in which firms owned capital instead of households, the inclusion of the wage adjustment cost generates results with a positive correlation between inflation and output and a negative correlation between the price level and output, as in the data.

than the price rigidity in reducing the nominal volatility in the economy.

Future research will extend the current model by considering various specifications of the monetary policy reaction function, including money growth rules, interest rate rules, as well as inflation or price level targeting. In particular, the policy rules would be ranked to determine whether an inflation-targeting rule dominates the others according to a central banks objective function. Also it would be interesting to completely endogenize the money growth rule and determine the globally optimal policy reaction function when faced with a particular set of shocks.

Another related extension would be to incorporate a more finely articulated banking sector, which would inform our understanding of the role of financial intermediation, and the interaction of money and credit, in the transmission of monetary policy.

Table 1: Cyclical Behaviour of the Canadian Economy: 1955:01 to 1996:04

Variables	Standard Deviation (%)	Correlation with real GDP	Autocorrelation
Real:[a]			
GDP	1.66	1.00	0.82
Consumption	0.75	0.61	0.60
Investment	5.38	0.88	0.77
Hours	1.85	0.85	0.93
Nominal:			
Money base growth rate	0.96	0.07	0.18
Interest rate	0.38	0.32	0.75
Price	1.40	-0.45	0.94
Inflation	1.93	0.29	0.42

a. All the real variables and the price level are logged and then HP detrended. Money growth, the interest rate and the inflation rate are HP detrended. The real variables are in per capita terms. The data on hours worked spans the 1976 to 1996 period only. We also apply this same procedure to the time series generated from the model.

Table 2: Cyclical Behavior of the Simulated Monetary Economies

Variables	No adjustment cost (50 Replications)			Add price adjustment costs (ϕ_p=1;ϕ_{w2}=0;ϕ_q=0)			Add wage adjustment costs (ϕ_p=1;ϕ_{w2}=10;ϕ_q=0)			Add portfolio adjustment costs (ϕ_p=1;ϕ_{w2}=10;ϕ_q=1)		
	S.D. (%)	Corr. with Y	Auto corr.	S.D. (%)	Corr /Y	Auto corr.	S.D. (%)	Corr. with Y	Auto corr.	S.D. (%)	Corr. with Y	Auto corr.
Real:												
Output	1.71	1.0	0.54	1.73	1.0	0.58	1.63	1.0	0.61	1.77	1.0	0.69
Consumption	0.75	0.97	0.63	0.76	0.97	0.65	0.72	0.97	0.68	0.82	0.98	0.70
Investment	4.60	0.99	0.50	4.69	0.99	0.55	4.42	0.99	0.59	4.63	0.99	0.69
Hours	1.03	0.88	0.20	1.05	0.88	0.36	1.28	0.78	0.45	1.51	0.82	0.64
Nominal:												
Money growth rate	0.96	0.33	0.18	0.96	0.33	0.18	0.96	0.52	0.18	0.96	0.46	0.18
Interest rate	0.51	-0.23	-0.07	1.30	-0.46	-0.16	1.15	-0.45	-0.15	0.99	-0.61	0.20
Price	2.07	-0.72	0.71	2.03	-0.71	0.72	1.62	-0.60	0.77	1.55	-0.63	0.79
Inflation	6.16	-0.30	-0.07	5.92	-0.27	-0.05	4.22	-0.12	-0.002	3.86	-0.11	0.02

a. All the real variables and the price level are logged and then HP detrended. Money growth, the interest rate, and the inflation rate are HP detrended. The real variables are in per capita terms.

Figure 1: Positive Monetary Action: With and Without Adjustment Costs

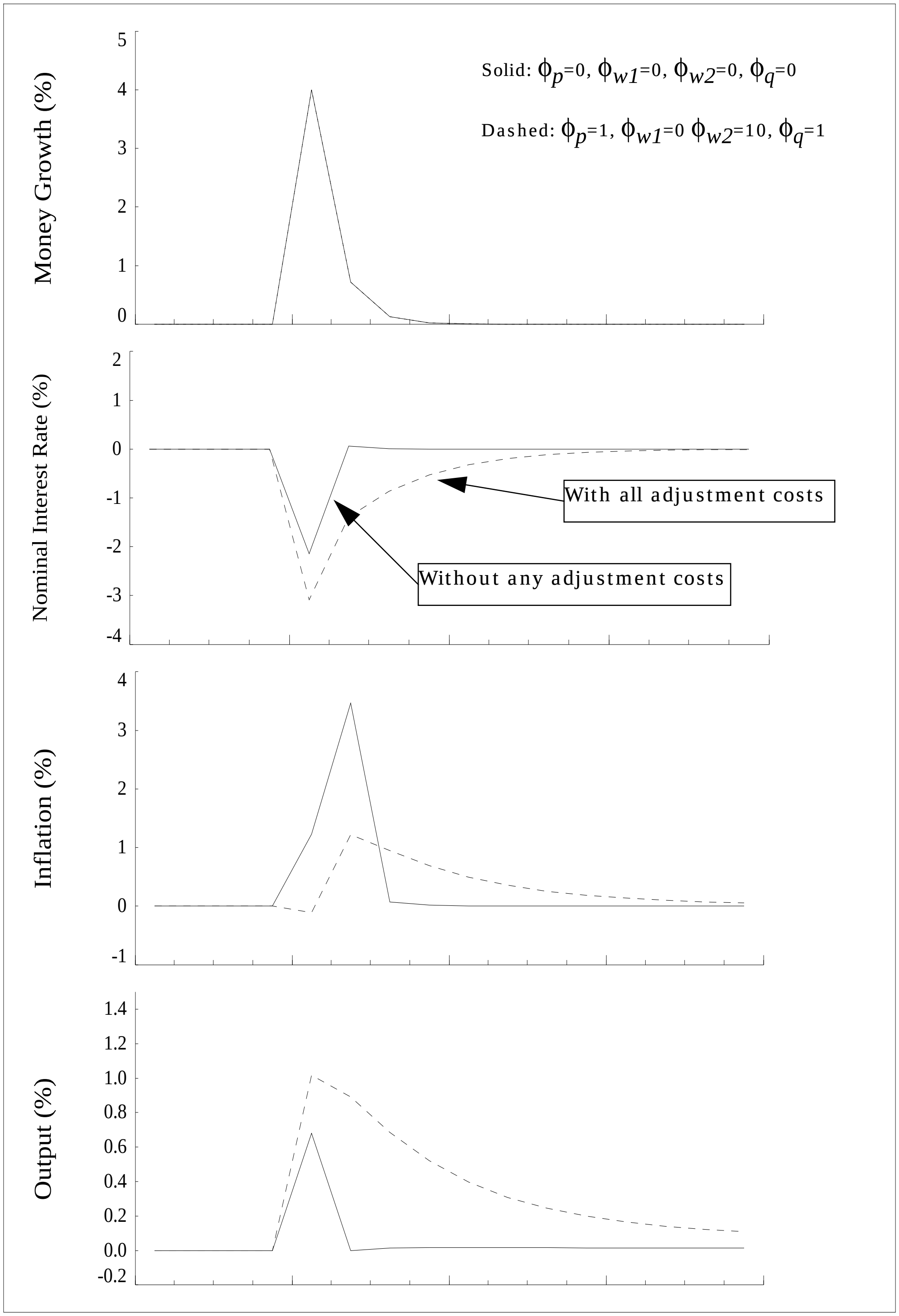


Figure 2: Effect of Price Adjustment Cost

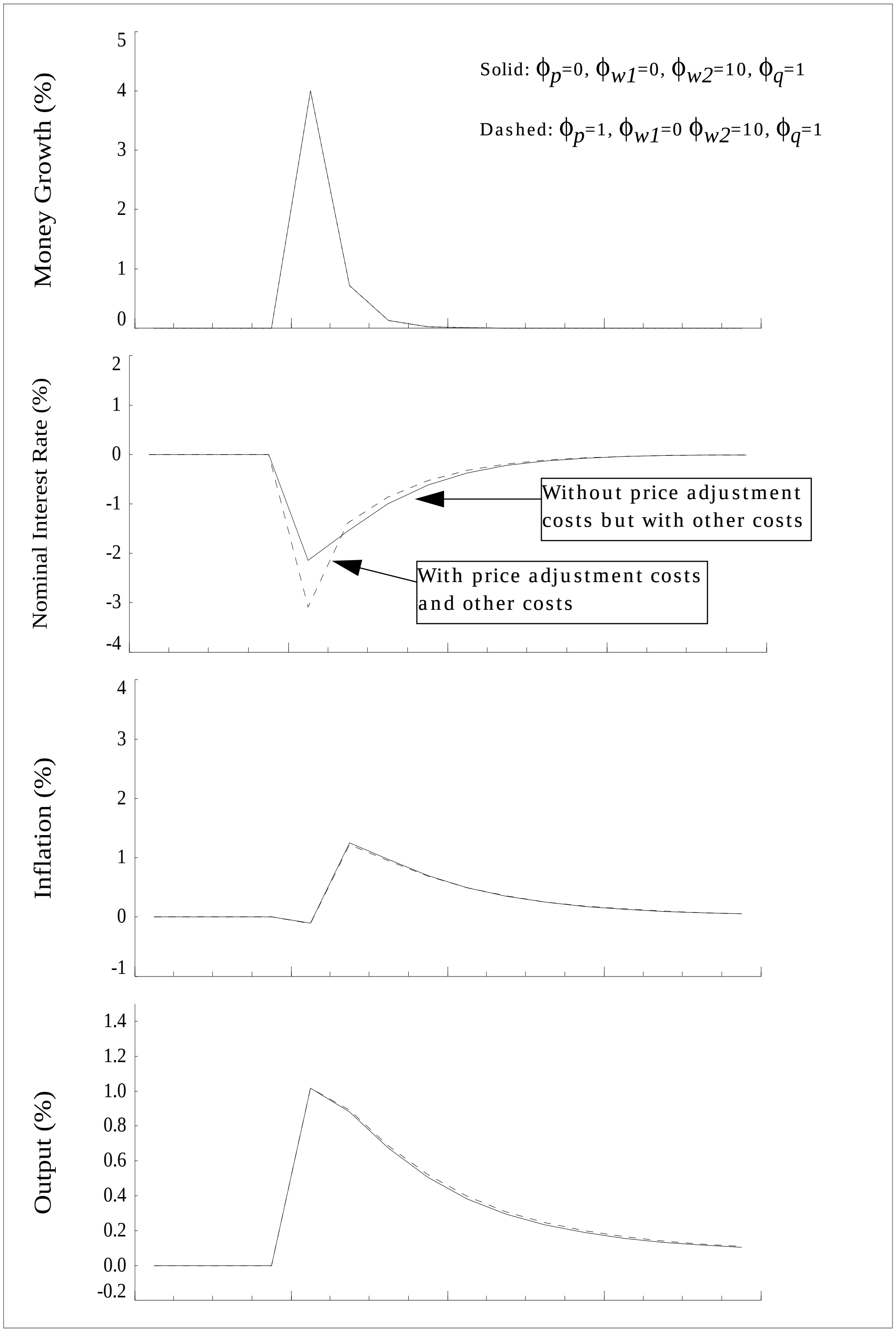


Figure 3: Effect of Wage Adjustment Cost

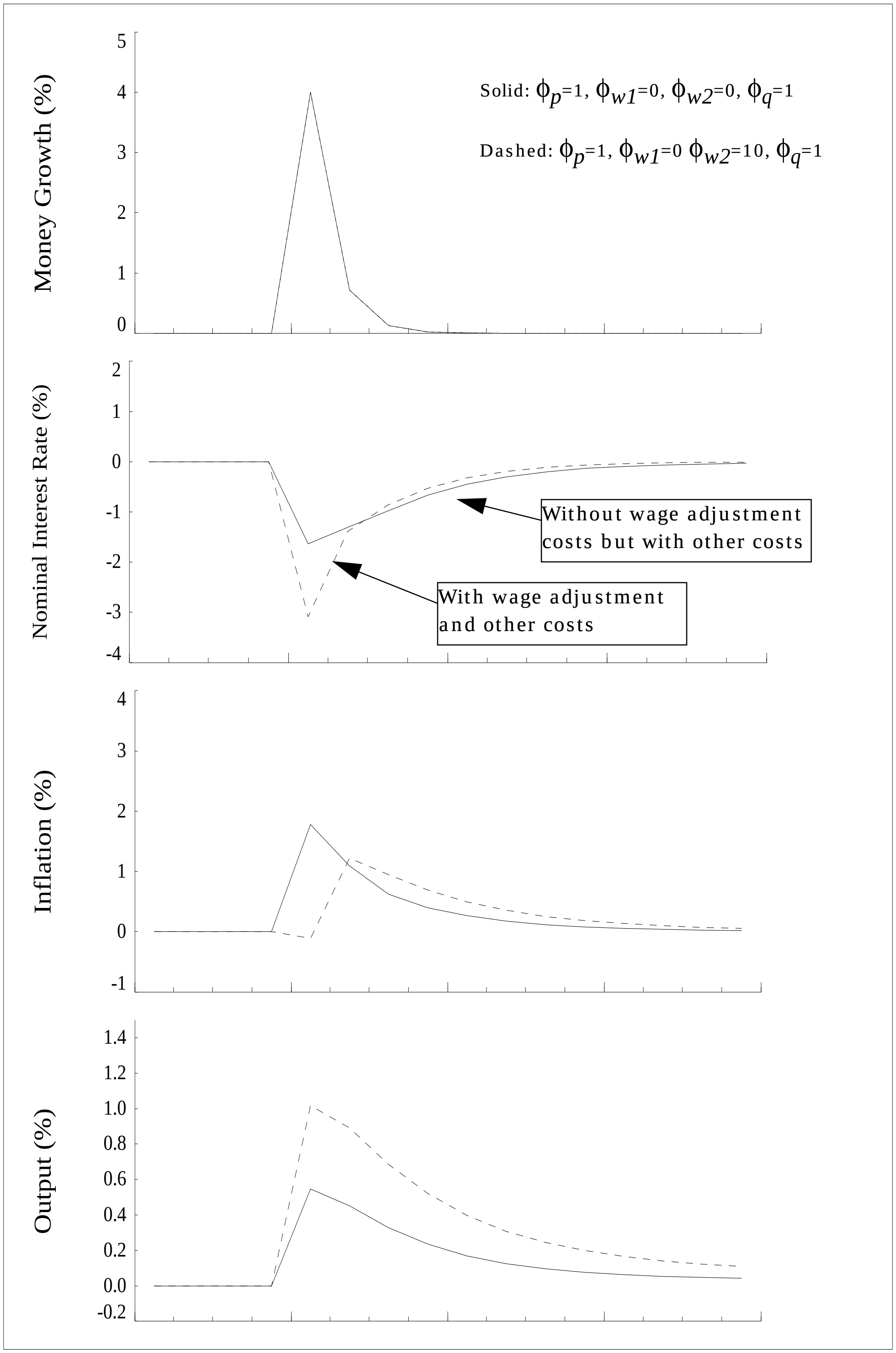


Figure 4: Effect of Portfolio Adjustment Cost

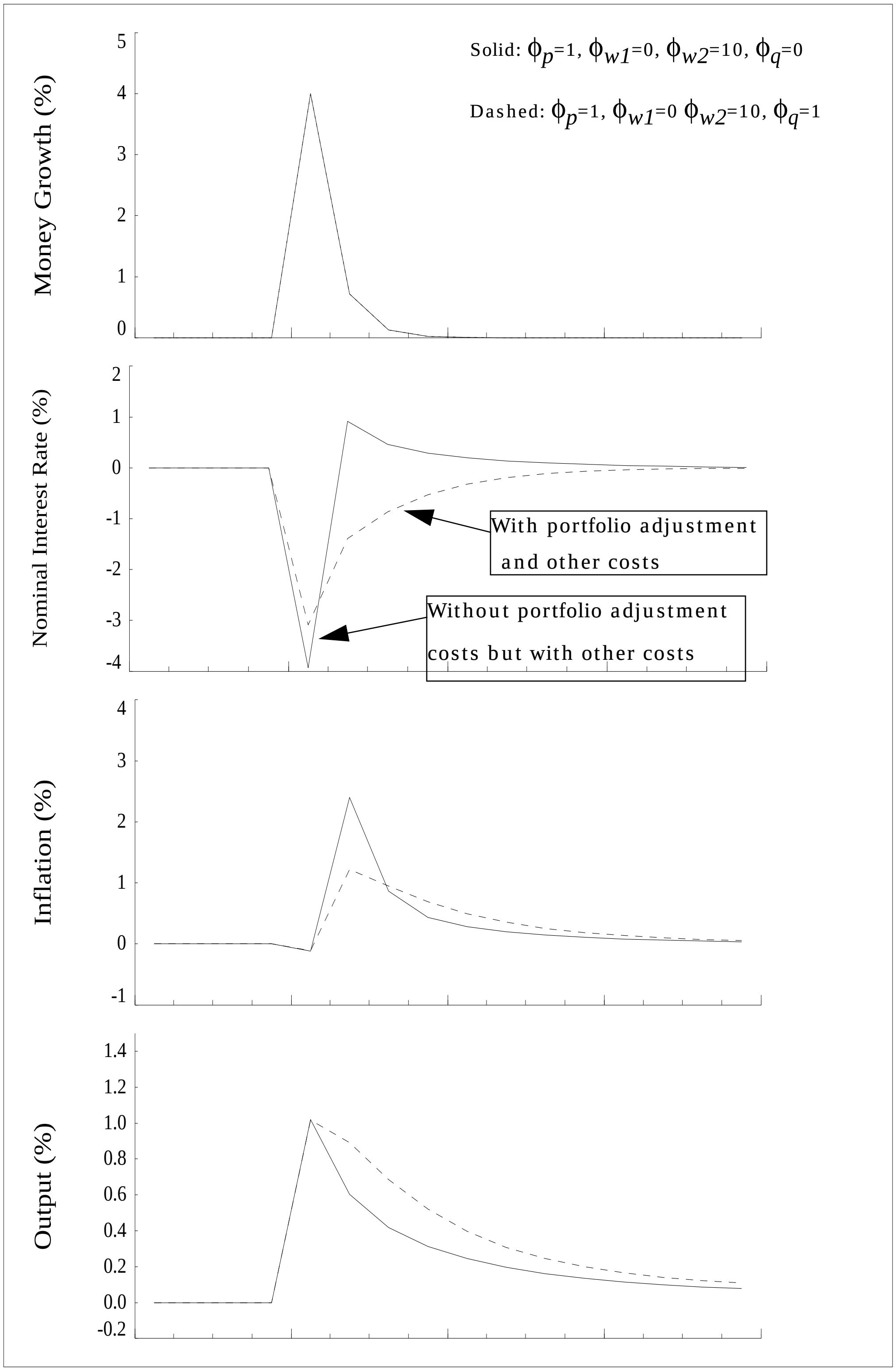


Figure 5: Effect of Price Adjustment Costs (No Portfolio Costs)

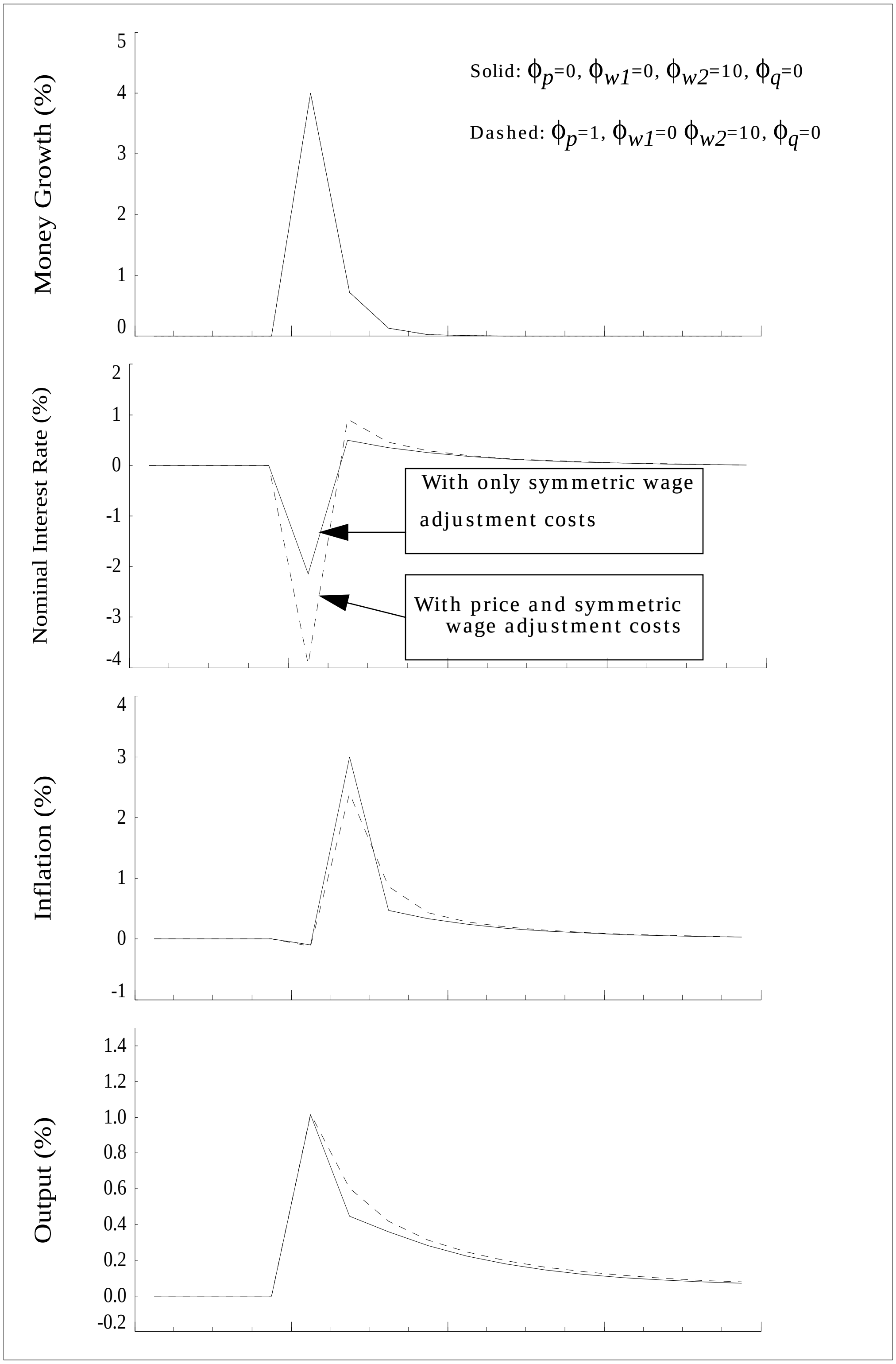


Figure 6: Effect of Asymmetric Wage Adjustment Costs (Positive)

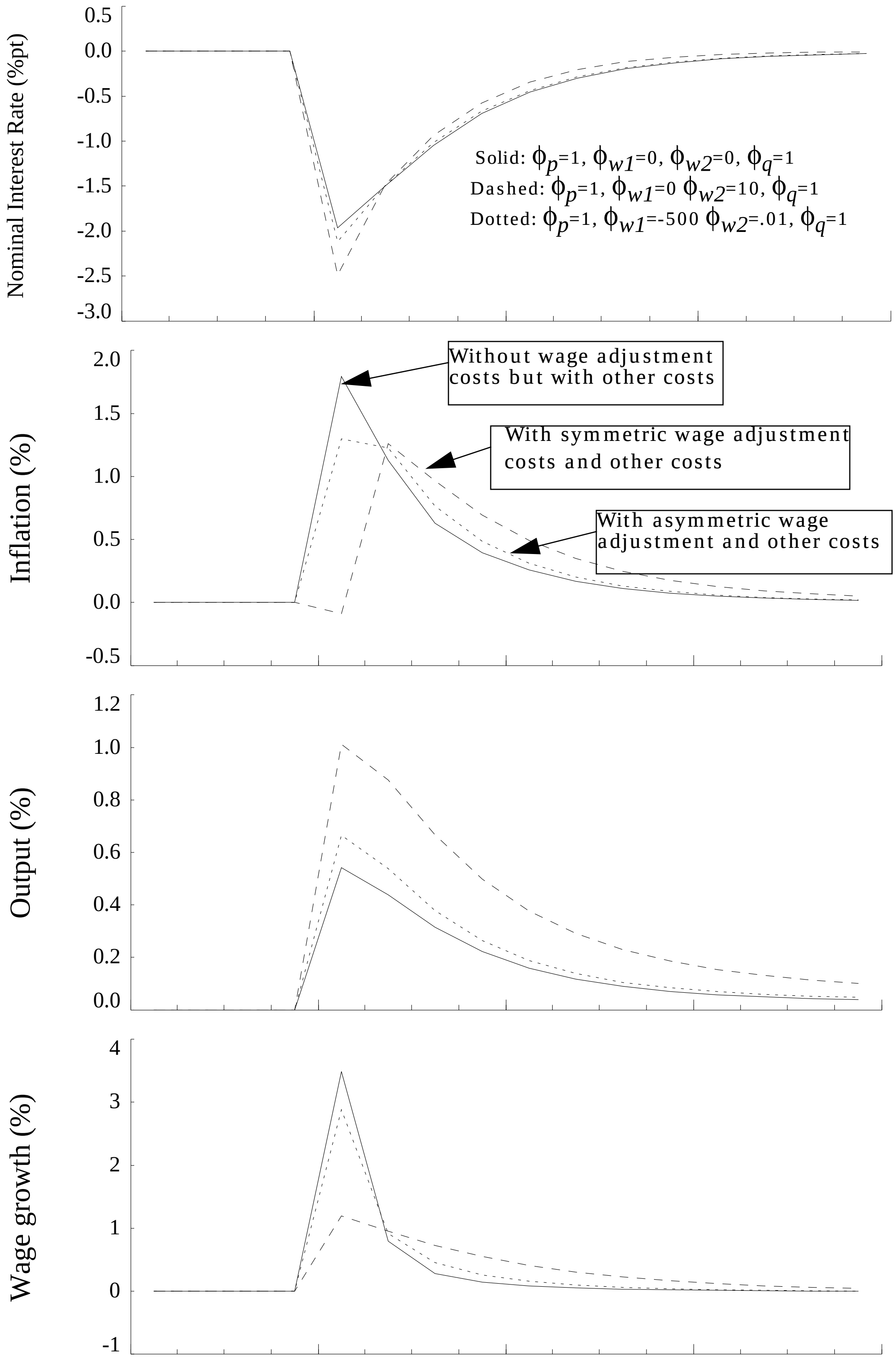
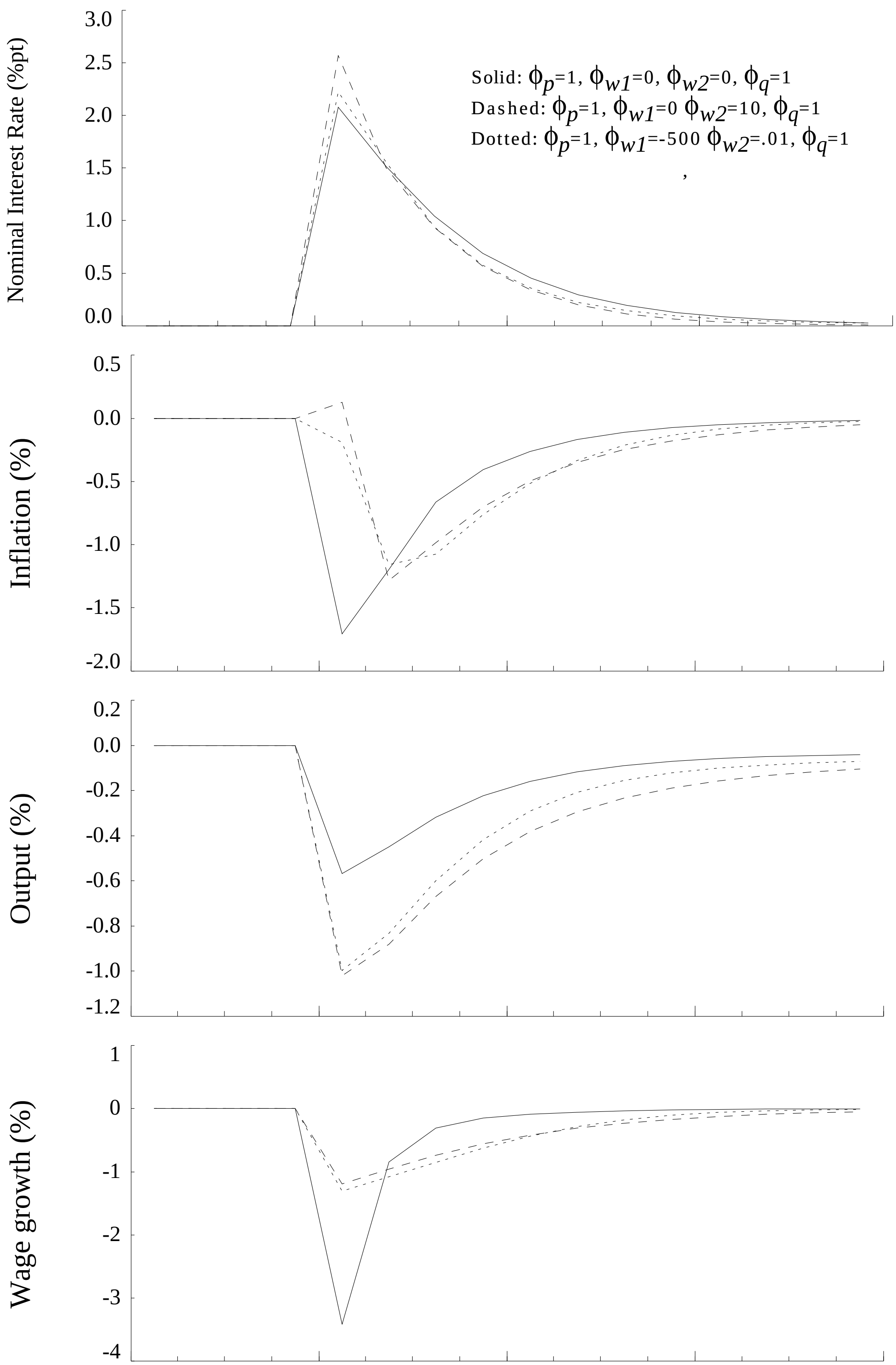


Figure 7: Effect of Asymmetric Wage Adjustment Costs (Negative)



Appendix: Model Details

Appendix I: Model Solution Technique

The model is solved using a technique which “stacks” the first order conditions and market clearing conditions, one for each endogenous variable at each period of a proposed horizon, and then solves the complete system simultaneously using a Newton procedure. A more complete description of this ‘stacked time’ methodology and a comparison to similar techniques can be found in Armstrong et al. (1995). Two of the primary benefits of this technique are that it does not involve a linear approximation so that the model’s important nonlinearities are not lost, and that it converges on a solution relatively easily and quickly.

Nonlinear solutions for basic versions of the model were also generated using Coleman’s algorithm (see examples of this in Coleman (1991), Gomme and Greenwood (1995) and Andolfatto and Gomme (1997)) which finds approximations for the policy rules of the economy across a grid of relevant state values. While having expressions for the policy rules would make doing impulse response functions and stochastic simulations much easier, it was found that only the very basic versions of the model without adjustment costs could be solved in realistic time frames with Coleman’s algorithm.

All of the results shown in Section 4 use the ‘stacked time’ technique described above.

Appendix II: Euler Equations

Household *i*’s decision problem is summarized by the following set of marginal conditions, where λ_{1t} and λ_{2t} represent the lagrangean multipliers associated with the CIA and individual budget constraint, respectively.

$$U_{1t} = P_t(\lambda_{1t} + \lambda_{2t}) \tag{A1}$$

$$\lambda_{2t} + U_{2t}\frac{\partial(AC_{it}^{Q})}{\partial Q_{it+1}} = \beta E_t\left[\lambda_{1t+1} + \lambda_{2t+1} - U_{2t+1}\frac{\partial(AC_{it+1}^{Q})}{\partial Q_{it+1}}\right] \tag{A2}$$

$$\beta E_t(\lambda_{2t+1}R_{t+1}^{d}) = \lambda_{2t} \tag{A3}$$

$$U_{2t} + P_t(\lambda_{1t} + \lambda_{2t}) \cdot \frac{\partial(AC_{it}^{w})}{\partial W_{it}}\frac{\partial W_{it}}{\partial L_{it}^{s}} = \tag{A4}$$

$$(\lambda_{1t} + \lambda_{2t}) \cdot \left(W_{it} + L_{it}\frac{\partial W_{it}}{\partial L_{it}^{s}}\right) -$$

$$\beta E_t\left[P_{t+1}(\lambda_{1t+1} + \lambda_{2t+1})\frac{\partial(AC_{it+1}^{w})}{\partial W_{it}}\frac{\partial W_{it}}{\partial L_{it}^{s}}\right]$$

$$P_t \cdot (\lambda_{1t} + \lambda_{2t}) = \beta E_t[P_{t+1}(\lambda_{1t+1} + \lambda_{2t+1})(1-\delta) + R_{kt+1}\lambda_{2t+1}] \tag{A5}$$

where

$$\frac{\partial(AC_{it}^{Q})}{\partial Q_{it+1}} = \frac{1}{Q_{it}} \cdot \left\{\varphi_q\left(\frac{Q_{it+1}}{Q_{it}} - (1+x)\right)\right\} \tag{A6}$$

$$\frac{\partial(AC_{it+1}^{Q})}{\partial Q_{it+1}} = -\frac{Q_{it+2}}{Q_{it+1}^{2}} \cdot \left\{\varphi_q\left(\frac{Q_{it+2}}{Q_{it+1}} - (1+x)\right)\right\} \tag{A7}$$

$$\frac{\partial(AC_{it}^{w})}{\partial W_{it}} = \frac{1}{W_{it-1}}\frac{W_t}{P_t} \cdot \left\{\varphi_{w1}\exp\left[\varphi_{w1}\left(\frac{W_{it}}{W_{it-1}} - (1+x)\right)\right]\right. \tag{A8}$$

$$\left. - \varphi_{w1} + \varphi_{w2} \cdot \left(\frac{W_{it}}{W_{it-1}} - (1+x)\right)\right\}$$

$$\frac{\partial(AC_{it+1}^{w})}{\partial W_{it}} = \frac{-W_{it+1}}{W_{it}^{2}}\frac{W_{t+1}}{P_{t+1}} \cdot \left\{\varphi_{w1}\exp\left[\varphi_{w1}\left(\frac{W_{it+1}}{W_{it}} - (1+x)\right)\right]\right. \tag{A9}$$

$$\left. - \varphi_{w1} + \varphi_{w2} \cdot \left(\frac{W_{it+1}}{W_{it}} - (1+x)\right)\right\}$$

$$\frac{\partial W_{it}}{\partial L_{it}^{s}} = -\frac{1}{\theta_L} \cdot \frac{W_{it}}{L_{it}^{s}} \tag{A10}$$

Firm j's dividend maximization problem can be characterized by the following set of marginal conditions for labour, $L^{d}{}_{jt}$, and capital, K_{jt}.

$$\lambda_{2t}\left(P_{jt}F_{L,jt} + Y_{jt}\frac{\partial P_{jt}}{\partial L_{jt}^{d}}\right) - \beta E_t\left[\lambda_{2t+1}P_{t+1}\frac{\partial AC_{jt+1}^{p}}{\partial L_{jt}^{d}}\right] = \tag{A11}$$

$$\lambda_{2t}\left\{R_t^{l}W_t + P_t \cdot \frac{\partial(AC_{jt}^{p})}{\partial L_{jt}^{d}}\right\}$$

and

$$\lambda_{2t}P_{jt}F_{K,jt} - \beta E_t\left[\lambda_{2t+1}P_{t+1}\frac{\partial(AC_{jt+1}^{p})}{\partial K_{jt}}\right] = \tag{A12}$$

$$\lambda_{2t}\left(R_{kt} + P_t\frac{\partial(AC_{jt}^{p})}{\partial K_{jt}} - Y_{jt} \cdot \frac{\partial P_{jt}}{\partial K_{jt}}\right)$$

where

$$\frac{\partial P_{jt}}{\partial L_{jt}^{d}} = -\frac{1}{\theta_y}\frac{P_{jt}}{Y_{jt}} \cdot \frac{\partial Y_{jt}}{\partial L_{jt}^{d}} \tag{A13}$$

$$\frac{\partial P_{jt}}{\partial K_{jt}} = -\frac{1}{\theta_y}\frac{P_{jt}}{Y_{jt}} \cdot \frac{\partial Y_{jt}}{\partial K_{jt}} \tag{A14}$$

$$\frac{\partial(AC_{jt}^{p})}{\partial L_{jt}^{d}} = \frac{Y_t}{P_{jt-1}} \cdot \frac{\partial P_{jt}}{\partial L_{jt}^{d}}\left\{\varphi_p\left(\frac{P_{jt}}{P_{jt-1}} - \frac{1+x}{\exp(\mu)}\right)\right\} \tag{A15}$$

$$\frac{\partial(AC_{jt+1}^{p})}{\partial L_{jt}^{d}} = -Y_t \cdot \frac{P_{jt+1}}{P_{jt}^{2}} \cdot \frac{\partial P_{jt}}{\partial L_{jt}^{d}}\left\{\varphi_p\left(\frac{P_{jt+1}}{P_{jt}} - \frac{1+x}{\exp(\mu)}\right)\right\} \tag{A16}$$

$$\frac{\partial(AC^p_{jt})}{\partial K_{jt}} = \frac{Y_t}{P_{jt-1}} \cdot \frac{\partial P_{jt}}{\partial K_{jt}}\left\{\varphi_p\left(\frac{P_{jt}}{P_{jt-1}} - \frac{1+x}{\exp(\mu)}\right)\right\} \tag{A17}$$

and

$$\frac{\partial(AC^p_{jt+1})}{\partial K_{jt}} = -Y_t \cdot \frac{P_{jt+1}}{P^2_{jt}} \cdot \frac{\partial P_{jt}}{\partial K_{jt}}\left\{\varphi_p\left(\frac{P_{jt+1}}{P_{jt}} - \frac{1+x}{\exp(\mu)}\right)\right\} \tag{A18}$$

Appendix III: The Stationary Representation of the Equilibrium

We assume that along the balanced growth path all the real variables grow at the rate of μ and nominal variables at the rate of x. To solve the model we need to find the stationary representation for the economy. All stationary variables, except L_t and R_t, along the balanced growth path are denoted by lower case letters. Notice that there is no population growth in the model economy.

$$c_{it} = \frac{C_{it}}{\exp(\mu t)}, \quad k_{jt+1} = \frac{K_{jt+1}}{\exp(\mu t)}, \quad i_{jt} = \frac{I_{jt}}{\exp(\mu t)}$$

$$n_{it} = \frac{N_{it}}{M_t}, \quad \mathrm{q}_{it} = \frac{Q_{it}}{M_t}, \quad \mathrm{p}_{jt} = \frac{\exp(\mu t)P_{jt}}{M_t}, \quad \mathrm{r}_{kt} = \frac{\exp(\mu t)R_{kt}}{M_t}, \quad \mathrm{w}_t = \frac{W_t}{M_t}$$

Define the stationary marginal utilities as follows for the case in which $\phi < 0$,

$$u_{1t} \equiv \frac{U_{1t}}{\exp(\mu(\phi-1)t)} \equiv c_{it}^{\phi-1}(1 - L^s_{it} - ac^q_{it})^{\gamma\cdot\phi} \tag{A19}$$

$$u_{2t} \equiv \frac{U_{2t}}{\exp(\mu\phi t)} \equiv \gamma c_{it}^{\phi}(1 - L_{it}^{s} - ac_{it}^{q})^{\gamma \cdot \phi - 1} \quad \text{(A20)}$$

Now define the following stationary variables

$$\hat{\lambda}_{2t} = \frac{M_{t+1}}{\exp(\mu\phi t)} \cdot \lambda_{2t} \quad \text{(A21)}$$

$$\beta^{*} = \beta \exp(\mu\phi) \quad \text{(A22)}$$

$$1 - \delta^{*} = \frac{1 - \delta}{\exp(\mu)} \quad \text{(A23)}$$

The stationary representation of the adjustment costs and their derivatives is given in the following equations.

$$ac_{it}^{q} \equiv AC_{it}^{Q} \equiv \frac{\varphi_q}{2}\left(\frac{q_{it+1}}{q_{it}}(1 + x_t) - (1 + x)\right)^2 \quad \text{(A24)}$$

$$ac_{it}^{w} \equiv \frac{AC_{it}^{W}}{\exp(\mu t)} \equiv \frac{w_t}{p_t}\left\{-1 + \exp\left[\varphi_{w1}\left(\frac{w_{it}}{w_{it-1}}(1 + x_{t-1}) - (1 + x)\right)\right]\right. \quad \text{(A25)}$$

$$\left. - \varphi_{w1}\left(\frac{w_{it}}{w_{it-1}}(1 + x_{t-1}) - (1 + x)\right) + \frac{\varphi_{w2}}{2}\left(\frac{w_{it}}{w_{it-1}}(1 + x_{t-1}) - (1 + x)\right)^2\right\}$$

$$\frac{\partial(ac_{it}^{q})}{\partial q_{it+1}} \equiv M_t \cdot \frac{\partial(AC_{it}^{q})}{\partial Q_{it+1}} \equiv \frac{1}{q_{it}}\left\{\varphi_q\left(\frac{q_{it+1}}{q_{it}}(1 + x_t) - (1 + x)\right)\right\} \quad \text{(A26)}$$

$$\frac{\partial(ac_{it+1}^{q})}{\partial q_{it+1}} \equiv M_{t+1}\frac{\partial(AC_{it+1}^{q})}{\partial Q_{it+1}} \equiv -\frac{(1 + x_{t+1})q_{it+2}}{q_{it+1}^{2}} \quad \text{(A27)}$$

$$\left\{\varphi_q\left(\frac{q_{it+2}}{q_{it+1}}(1 + x_{t+1}) - (1 + x)\right)\right\}$$

$$\frac{\partial(ac_{it}^{w})}{\partial w_{it}} \equiv \frac{M_{t-1}}{\exp(\mu t)}\frac{\partial(AC_{it}^{w})}{\partial W_{it}} \equiv \frac{w_t}{p_t w_{it-1}}$$

$$\left\{\varphi_{w1}\exp\left[\varphi_{w1}\left(\frac{w_{it}}{w_{it-1}}(1+x_{t-1})-(1+x)\right)\right]-\varphi_{w1}\right.$$

$$\left.+\varphi_{w2}\left(\frac{w_{it}}{w_{it-1}}(1+x_{t-1})-(1+x)\right)\right\} \quad \text{(A28)}$$

$$\frac{\partial(ac_{it+1}^{w})}{\partial w_{it}} \equiv \frac{M_t}{\exp(\mu(t+1))}\frac{\partial(AC_{it+1}^{w})}{\partial W_{it}} \equiv -(1+x_t)\frac{w_{it+1}}{w_{it}^2}\cdot\frac{w_{t+1}}{p_{t+1}} \quad \text{(A29)}$$

$$\left\{\varphi_{w1}\exp\left[\varphi_{w1}\left(\frac{w_{it+1}}{w_{it}}(1+x_t)-(1+x)\right)\right]-\varphi_{w1}+\varphi_{w2}\left(\frac{w_{it+1}}{w_{it}}(1+x_t)-(1+x)\right)\right\}$$

$$\frac{\partial w_{it}}{\partial L_{it}^{s}} \equiv \frac{1}{M_t}\cdot\frac{\partial W_{it}}{\partial L_{it}^{s}} \equiv -\frac{1}{\theta_l}\cdot\frac{w_{it}}{L_{it}^{s}} \quad \text{(A30)}$$

$$i_{jt} = k_{jt+1}-(1-\delta^{*})k_{jt} \quad \text{(A31)}$$

The household's first order conditions, equations (A1)-(A5), can be combined to obtain the following set of equations.

$$U_{2t}+U_{1t}\cdot\frac{\partial(AC_{it}^{w})}{\partial W_{it}}\frac{\partial W_{it}}{\partial L_{it}^{s}} = \frac{U_{1t}}{P_t}\cdot\left(W_{it}+L_{it}\frac{\partial W_{it}}{\partial L_{it}^{s}}\right) \quad \text{(A32)}$$

$$-\quad \beta E_t\left[U_{1t+1}\frac{\partial(AC_{it+1}^{w})}{\partial W_{it}}\frac{\partial W_{it}}{\partial L_{it}^{s}}\right]$$

$$\lambda_{2t} = -U_{2t}\frac{\partial AC_{it}^{Q}}{\partial Q_{it+1}}+\beta E_t\left[\frac{U_{1t+1}}{P_{t+1}}-U_{2t+1}\frac{\partial(AC_{it+1}^{Q})}{\partial Q_{it+1}}\right] \quad \text{(A33)}$$

$$\beta E_t(\lambda_{2t+1}R_{t+1}^{d}) = \lambda_{2t} \quad \text{(A34)}$$

$$U_{1t} = \beta E_t[U_{1t+1}(1-\delta) + R_{kt+1}\lambda_{2t+1}] \tag{A35}$$

The stationary representation of the household's first order conditions is given by the following set of three equations.

$$u_{2t} + u_{1t} \cdot (1 + x_{t-1}) \cdot \frac{\partial(AC_{it}^w)}{\partial w_{it}}\frac{\partial w_{it}}{\partial L_{it}^s} = \tag{A36}$$

$$\frac{u_{1t}}{p_t} \cdot \left(w_{it} + L_{it}\frac{\partial w_{it}}{\partial L_{it}^s}\right) - \beta^* E_t\left[u_{1t+1}\frac{\partial(AC_{it+1}^w)}{\partial w_{it}}\frac{\partial w_{it}}{\partial L_{it}^s}\right]$$

$$\hat{\lambda}_{2t} = -(1 + x_t)u_{2t}\frac{\partial ac_{it}^q}{\partial q_{it+1}} + \beta^* E_t\left[\frac{u_{1t+1}}{p_{t+1}} - u_{2t+1}\frac{\partial(ac_{it+1}^q)}{\partial q_{it+1}}\right] \tag{A37}$$

$$\beta^* E_t\left[\frac{\hat{\lambda}_{2t+1}R_{t+1}^d}{1 + x_{t+1}}\right] = \hat{\lambda}_{2t} \tag{A38}$$

$$u_{1t} = \beta^* E_t\left[u_{1t+1}(1-\delta^*) + \exp(-\mu)r_{kt+1}\frac{\hat{\lambda}_{2t+1}}{1 + x_{t+1}}\right] \tag{A39}$$

The binding CIA constraint becomes

$$p_t \cdot (c_{it} + i_{it} + ac_{it}^w) = q_{it} + w_{it}L_{it} \tag{A40}$$

For the firm's problem, let

$$ac_{jt}^p = \frac{AC_{jt}^P}{\exp(\mu t)} \equiv y_t \exp(-2\mu)\frac{\varphi_p}{2}\left(\frac{p_{jt}}{p_{jt-1}}(1 + x_{t-1}) - (1 + x)\right)^2 \tag{A41}$$

The stationary representation of the production function is given by the following:

$$y_{jt} \equiv \frac{Y_{jt}}{\exp[(\alpha\mu + \eta(1-\alpha))\Phi t]} \quad \text{(A42)}$$

where capital grows at rate m and productivity at rate h in steady state. Along a balanced growth path with all real variables growing at the same rate, the following restriction must be satisfied.[1]

$$\Phi = \frac{\mu}{\alpha\mu + \eta(1-\alpha)} \quad \text{(A43)}$$

Under this condition, the stationary representation of output can be written as

$$y_{jt} \equiv \frac{Y_{jt}}{\exp(\mu t)} \equiv \exp(-\alpha\mu\Phi)[k_{jt}^{\alpha}(\exp(\theta_t)L_{jt}^{d})^{1-\alpha}]^{\Phi} \quad \text{(A44)}$$

The resulting stationary representations for the marginal productivities of labour and capital are

$$f_{l,jt} = \frac{F_{L,jt}}{\exp(\mu t)} \equiv (1-\alpha)\Phi\frac{y_{jt}}{L_{jt}^{d}} \quad \text{(A45)}$$

and

$$f_{k,jt} \equiv \frac{F_{K,jt}}{\exp(\mu)} \equiv \alpha \cdot \Phi \cdot \frac{y_{jt}}{k_{jt}}. \quad \text{(A46)}$$

The stationary representation of the derivatives of firm prices are given by

$$\frac{\partial p_{jt}}{\partial L_{jt}^{d}} \equiv \frac{\exp(\mu t)}{M_t} \cdot \frac{\partial P_{jt}}{\partial L_{jt}^{d}} \equiv -\frac{1}{\theta_y} \cdot \frac{p_{jt}}{y_{jt}} \cdot f_{l,jt} \quad \text{(A47)}$$

$$\frac{\partial p_{jt}}{\partial k_{jt}} \equiv \frac{\exp(2\mu t)}{M_t \exp(\mu)} \cdot \frac{\partial P_{jt}}{\partial K_{jt}} \equiv -\frac{1}{\theta_y} \cdot \frac{p_{jt}}{y_{jt}} \cdot f_{k,jt} \quad \text{(A48)}$$

1. Given $\Phi \geq 1$, then $\mu \geq \eta$.

Similarly, we can show the representation for the marginal adjustment costs with the following equations.

$$\frac{\partial(ac^p_{jt})}{\partial L^d_{jt}} = \frac{1}{\exp(\mu t)} \cdot \frac{\partial(AC^p_{jt})}{\partial L^d_{jt}} \equiv \frac{1+x_{t-1}}{\exp(\mu)} \cdot \frac{y_t}{p_{jt-1}} \cdot \frac{\partial p_{jt}}{\partial L^d_{jt}} \cdot \left\{(\varphi_p \cdot \exp(-\mu))\left(\frac{p_{jt}}{p_{jt-1}}(1+x_{t-1})-(1+x)\right)\right\} \tag{A49}$$

$$\frac{\partial(ac^p_{jt+1})}{\partial L^d_{jt}} = \frac{1}{\exp(\mu(t+1))} \cdot \frac{\partial(AC^p_{jt+1})}{\partial L^d_{jt}} = -\frac{(1+x_t)}{\exp(\mu)} \cdot y_{t+1} \cdot \frac{p_{jt+1}}{p^2_{jt}} \cdot \frac{\partial p_{jt}}{\partial L^d_{jt}} \left\{\varphi_p \exp(-\mu)\left(\frac{p_{jt+1}}{p_{jt}}(1+x_t)-(1+x)\right)\right\} \tag{A50}$$

$$\frac{\partial(ac^p_{jt})}{\partial k_{jt}} \equiv \frac{1}{(1+x_{t-1})}\frac{\partial(AC^p_{jt})}{\partial K_{jt+1}} \equiv \frac{y_t}{p_{jt-1}} \cdot \frac{\partial p_{jt}}{\partial k_{jt}} \cdot \left\{\varphi_{p1}\exp\left[\exp(-\mu)\varphi_{p1}\left(\frac{p_{jt}}{p_{jt-1}}(1+x_{t-1})-(1+x)\right)\right] - \varphi_{p1} + \varphi_{p2}\exp(-\mu)\left(\frac{p_{jt}}{p_{jt-1}}(1+x_{t-1})-(1+x)\right)\right\} \tag{A51}$$

$$\frac{\partial(ac^p_{jt+1})}{\partial k_{jt}} \equiv \frac{1}{(1+x_t)\exp(\mu)} \cdot \frac{\partial(AC^p_{jt+1})}{\partial K_{jt}} \equiv -y_t \cdot \frac{p_{jt+1}}{p^2_{jt}} \cdot \frac{\partial p_{jt}}{\partial k_{jt}} \cdot \left\{\varphi_p \exp(-\mu)\left(\frac{p_{jt+1}}{p_{jt}}(1+x_t)-(1+x)\right)\right\} \tag{A52}$$

The firm's first order conditions from equations (A11) and (A12) can be written as,

$$\frac{\hat{\lambda}_{2t}}{1+x_t}\left[p_{jt}\cdot f_{l,jt}+y_{jt}\cdot\frac{\partial p_{jt}}{\partial L^d_{jt}}\right]-\beta^* E_t\left[\frac{\hat{\lambda}_{2t+1}}{1+x_{t+1}}\cdot p_{t+1}\cdot\frac{\partial(ac^p_{jt+1})}{\partial L^d_{jt}}\right] \quad \text{(A53)}$$

$$=\frac{\hat{\lambda}_{2t}}{1+x_t}\left\{R^l_t\cdot w_t+p_t\cdot\frac{\partial(ac^p_{jt})}{\partial L^d_{jt}}\right\}$$

(A54)

$$\frac{\hat{\lambda}_{2t}}{1+x_t}p_{jt}\cdot f_{k,jt}\cdot\exp(\mu)-\beta^* E_t\left\{\frac{\hat{\lambda}_{2t+1}}{1+x_{t+1}}\cdot(1+x_t)\cdot p_{t+1}\cdot\frac{\partial(ac^p_{jt+1})}{\partial k_{jt}}\right\}$$
$$=\frac{\hat{\lambda}_{2t}}{1+x_t}\left(r_{kt}+p_t\cdot(1+x_{t-1})\cdot\frac{\partial(ac^p_{jt})}{\partial k_{jt}}\quad-\exp(\mu)y_{jt}\cdot\frac{\partial p_{jt}}{\partial k_{jt}}\right)$$

(A55)

The stationary presentation of the market clearing conditions are:

$$c_{it}+i_{it}+ac^p_{jt}+ac^w_{it}=y_t \quad \text{(A56)}$$

$$w_tL_{jt}=1-q_{jt}+t^b_t \quad \text{(A57)}$$

$$p_tc_{it}+p_tac^w_{it}=q_{it} \quad \text{(A58)}$$

$$t^b_t=x_t \quad \text{(A59)}$$

For simplicity, we assume that the government expenditure is exactly financed by the tax revenue from the households and firms. This leads to x_t=$t^b{}_t$.

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