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Canadian Short-Term Interest Rates and the BAX Futures Market:

An Analysis of the Impact of Volatility on Hedging Activity and the Correlation of Returns between Markets.

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Analysis of the Impact of Volatility on Hedging Activity and the Correlation of Returns between Markets

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Abstract

This paper analyses how Canadian financial firms manage short-term interest rate risk through the use of BAX futures contracts. The results show that the most effective hedging strategy is, on average, a static strategy based on linear regression that assumes constant variances, even though dynamic models allowing for time-varying variances are found to have superior explanatory power. The results also show a rise in the correlation of the returns to three-month bankers' acceptances and three-month treasury bills with the returns to BAX futures contracts during periods of increased money market volatility, suggesting that hedging activity should increase during market volatility.

Résumé

La présente étude analyse la façon dont les entreprises financières canadiennes gèrent le risque de taux d'intérêt à court terme par l'entremise de contrats BAX. Les résultats de cet examen montrent que la stratégie de couverture la plus efficace, en moyenne, est une stratégie statique fondée sur une régression linéaire qui suppose des variances constantes, même si des modèles dynamiques autorisant des variances variables dans le temps ont un pouvoir explicatif supérieur. Ces résultats montrent également une augmentation de la corrélation entre les rendements des acceptations bancaires à trois mois et des bons du Trésor à trois mois et ceux des contrats BAX au cours de périodes de volatilité accrue sur le marché monétaire, ce qui donne à penser que les opérations de couverture devraient s'accroître pendant de telles périodes.

Contents

1: Introduction and Summary

The management of interest rate risk is an important task for financial managers as unanticipated increases in interest rates can lead to substantial capital losses on holdings of short-term fixed rate securites[1]. There have been several periods of volatility in short-term interest rates in Canada in the 1990s, due to domestic and international influences. Non-financial firms tend to protect themselves from this volatility by purchasing swap and forward rate agreements (FRAs) with financial firms, usually banks.[2] The study of interest rate risk management focuses on how financial institutions manage such risk.[3]

The purpose of this analysis is to provide information on the relative effectiveness of hedging strategies in responding to volatility in short-term interest rates and greater understanding of the movement and the dynamic co-movement of Canadian money market interest rates in response to volatility. This paper specifically examines how Canadian financial institutions manage interest rate risk through the use of 3-month bankers' acceptance (BAX) futures contracts that have been offered on the Montreal Exchange since April 1988.[4] This, in turn, should lead to better understanding of money market developments and more effective monetary policy operations. This is important to the Bank of Canada that, like central banks of most developed countries, relies on money market operations to implement and signal monetary policy.

A hedging strategy that uses futures first requires a decision on the required number of futures contracts. The number of contracts depends on the optimal hedge ratio, defined by Myers (1991) as the "proportion of the cash position that should be covered with an opposite position on a futures market." The optimal hedge ratio can be estimated by a linear regression of the cash return upon the futures return, and is referred to in this paper as the OLS Hedge model or the conventional model.[5]

Several authors have demonstrated problems in estimating the optimal hedge ratio by linear

1. See Hanweck and Shull (1996) and Zelmer (1996).
2. See Downie, McMillan and Nosal (1996).
3. Miville (1996) presents the results of a survey on the use of derivatives by Canadian financial firms for the month of April 1995, which indicates that futures represented the largest amount outstanding, followed by swaps and forward rate agreements, at horizons of less than one-year.
4. The BAX futures contract is an exchange-traded contract based on a C$1 million bankers' acceptance with a maturity of three months. Contracts mature two business days prior to the third Wednesday of the month for March, June, September and December over a two-year period. BAX futures are the most actively traded financial futures contract in Canada.
5. See Johnson (1960), Stein (1961), Edderington (1960) and Hill and Schneeweis (1981).

regression as this technique assumes that the variances of the returns are constant.[6] This leads to a constant hedge ratio and, consequently, a static hedging strategy. The assumption of constant variances has been empirically violated by many financial time series and indicaes the need for a dynamic hedging strategy and an optimal hedge ratio that responds to the volatility of returns. The models commonly used to estimate time-varying optimal hedge ratios are the multivariate GARCH (MGARCH) models of Bollerslev (1990), Engle and Kroner (1995) and Bollerslev, Engle and Wooldridge (1988).

Gagnon and Lypny (1995) applied MGARCH techniques to analyse one-week hedges of three-month Canadian bankers' acceptances using BAX futures. They found the dynamic hedging strategy to be more effective at reducing the interest-rate risk with both in- and out-of-sample measures than the conventional hedging strategy. This paper extends the analysis of Gagnon and Lypny by looking at one-day hedges (daily returns). This is useful for three reasons. First, a financial institution may on any given day enter into a swap agreement or an FRA as it responds to the needs of its clients. The optimal hedge ratio for managing the risk in that day's position can be calculated by analysing daily returns and may or may not indicate a need to transact in the BAX futures market. Second, daily returns provide information more quickly on the transmission of volatility between the futures market and the money market. Third, the impact of volatility on hedging activity will be better reflected in optimal hedge ratios estimated from daily returns as developments in the Canadian money market and the BAX futures market occur very quickly. For example, there is evidence that hedging activity on the BAX futures market tends to increase as a result of an increase in money market volatility[7], which can be better captured using daily data.

The results of this study show that the conventional model provides an effective hedging strategy on average except during times of increased volatility.

This study also shows a rise in the correlation between the cash and futures returns, and a rise in the estimated optimal hedge ratio during periods of increased volatility that corresponds to an increase in the effectiveness of hedging with BAX futures contracts and explains the increase in hedging activity noticed on the Montreal Exchange.

The paper is organized into five sections. Section 1 is the introduction and summary.

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6. For example, Cumby, Cecchetti, Cumby and Figlewski (1988), Kroner and Sultan (1993), Baillie and Myers (1991), Sephton (1991) and Gagnon and Lypny (1995).
7. see Harvey (1996).

Section 2 discusses hedging activity using futures as an optimization problem facing an investment manager. The solution is shown to be the time-varying optimal hedge ratio, which can be estimated by MGARCH models. Section 3 provides a preliminary univariate analysis of the daily returns, estimates the MGARCH models, and analyses the estimated time-varying optimal hedge ratios. Section 4 discusses hedging effectiveness analyses and the correlation of returns. Section 5 offers suggestions for further research.

2: Optimal Hedge Ratios and the MGARCH Model

In this section, the time-varying optimal hedge ratio is derived and shown to depend on (1)the covariance between the cash and futures returns, and (2)the variance of the futures return. MGARCH models are then used to estimate the time-varying optimal hedge ratios.

2.1: Derivation of the Time-Varying Optimal Hedge Ratio

Let S_t be the time *t* cash price of an asset and F_t the time *t* price of a futures contract to deliver the underlying security on the delivery date. The log differences of the cash and futures prices give the random rates of return to the respective positions from the end of period *t–1* to the end of period *t*.[8] The returns may be expressed as:

$$\begin{aligned} s_t &= ln(S_t) - ln(S_{t-1}) \\ f_t &= ln(F_t) - ln(F_{t-1}) \end{aligned} \tag{1}$$

The random rate of return to the investment manager’s hedged portfolio from the end of period t–1 to the end of period t will equal:

$$x_t = s_t - \gamma_{t-1} f_t \tag{2}$$

where γ_{t-1} is the proportion of the cash position covered by futures contracts, the hedge ratio, and indicates the number of futures contracts in which a short position should be taken. The hedge ratio is indexed by *t–1* since the hedge must be implemented given the information available at the beginning of each period.

An investment manager’s goal in implementing a hedging strategy is to maximize the expected utility from the portfolio.[9] Expected utility is modelled as a trade-off between the mean

8. Taking the logarithm removes the effect of the level of interest rates on the volatility.
9. According to Rothstein and Little (1984), there are two other hedging techniques: (i) the face value approach in which the number of futures purchased have nominal value equal to the nominal value of the cash investment, and (ii) the dollar-equivalency approach, in which the number of futures contracts are chosen such that the change in the dollar value of the futures position offsets the change in the dollar value of the cash position. These techniques are not considered in this paper.

and the variance of the random return to the portfolio, expressed as:

$$E(U(x_t)|I_{t-1}) = E(x_t|I_{t-1}) - \phi Var(x_t|I_{t-1}) \tag{3}$$

where ϕ is the degree of risk aversion ($\phi > 0$).[10] $E(x_t|I_{t-1})$ is the expected return to the hedged portfolio based on all the information available up to the end of period *t–1*, denoted I_{t-1}. The variance of the return to the hedged portfolio is given by:

$$Var(x_t|I_{t-1}) = Var(s_t|I_{t-1}) + \gamma_{t-1}^2 Var(f_t|I_{t-1}) - 2\gamma_{t-1} Cov(s_t, f_t|I_{t-1}). \tag{4}$$

The investment manager's problem is to choose γ_{t-1} so as to maximize the expected utility function, given by (3):

$$\max_{\gamma_{t-1}} \left[E(U(x_t)|I_{t-1}) \right] = \max_{\gamma_{t-1}} \left[E(x_t|I_{t-1}) - \phi Var(x_t|I_{t-1}) \right]. \tag{5}$$

The solution γ_{t-1}^* can be shown to equal:

$$\gamma_{t-1}^* = \frac{Cov(s_t, f_t|I_{t-1})}{Var(f_t|I_{t-1})} - \frac{E(f_t|I_{t-1})}{2\phi Var(f_t|I_{t-1})}. \tag{6}$$

The first term on the right side of (6) is the variance-minimizing hedge ratio, while the second term is the speculative demand for futures. The optimal hedge ratio thus depends on the degree of risk aversion of the investment manager, i.e., the more risk averse the investment manager, the lower the speculative demand for futures.

It will be assumed that the logarithm of the price of a BAX futures contract follows a martingale, $E(ln(F_t)|I_{t-1}) = ln(F_{t-1})$. This assumption implies that the speculative demand for BAX futures is zero as the conditional expectation of the return to holding a futures contract will be zero, $E(f_t|I_{t-1}) = 0$.[11] The time-varying optimal hedge ratio then becomes:

$$\gamma_{t-1}^* = \frac{Cov(s_t, f_t|I_{t-1})}{Var(f_t|I_{t-1})}. \tag{7}$$

Equation (7) demonstrates that the optimal hedge ratio is a function of the conditional covariance between the returns and the conditional variance of the futures return. It further demonstrates that the optimal hedge ratio will be time-varying as both the covariance term and the variance term

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10. Kroner and Sultan (1993) and Gagnon and Lypny (1995) define the optimization problem with regard to changes in the prices of the cash position and the futures contract, rather than with regard to the rate of return to the hedged portfolio.
11. McCurdy and Morgan (1988) show that the futures prices for foreign currency futures follows a martingale for weekly data but not for daily data. If the futures price follows a martingale, the logarithm of the futures price cannot strictly follow a martingale. The assumption of a martingale is made for expositional purposes in comparing the methods of estimating the variance-minimizing hedge ratio.

depend upon the information available to the investment manager.[12]

2.2: The Multivariate GARCH Model

To calculate the time-varying optimal hedge ratio, one must obtain estimates of the conditional variance of the futures return, $Var(f_t|I_{t-1})$ and the conditional covariance of the returns, $Cov(s_t, f_t|I_{t-1})$. A flexible class of models for jointly estimating these variables is the Multivariate GARCH (MGARCH) models of Bollerslev, Engle and Wooldridge (1988), Bollerslev (1990), and Engle and Kroner (1995).

To introduce the MGARCH model (following Gagnon and Lypny 1995), define y_t to be a 2x1 random vector of the returns to cash and futures positions. A simple model of the interaction between the returns is given by:

$$y_t = \mu + \varepsilon_t \qquad \varepsilon_t \sim N(0, H_t) \tag{8}$$

where μ is a 2x1 vector of constants, $\varepsilon_t = \begin{bmatrix}\varepsilon_{1,t} & \varepsilon_{2,t}\end{bmatrix}^T$ is a 2x1 vector of errors and H_t is a 2x2 symmetric matrix representing the conditional covariance matrix between the returns. H_t can be expressed as:

$$H_t = \begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{12,t} & h_{22,t} \end{bmatrix} = \begin{bmatrix} Var(s_t|I_{t-1}) & Cov(s_t, f_t|I_{t-1}) \\ Cov(s_t, f_t|I_{t-1}) & Var(f_t|I_{t-1}) \end{bmatrix}. \tag{9}$$

Equations (7) and (9) demonstrate that the conditional covariance matrix provides all the information necessary to estimate the time-varying optimal hedge ratio.

Two forms of MGARCH model will be employed for the empirical application: Engle and Kroner's (1995) BEKK model (MGARCH-BEKK), and Bollerslev's (1990) Constant Correlation model (MGARCH-CC).[13] These models differ in their specification of the conditional covariance matrix H_t, in that the BEKK model allows for dynamic correlation between the returns (see Appendix). As the structure of the MGARCH models is quite complicated, simple representations are presented here; more complicated representations are outlined in the Appendix.

For the BEKK model, the elements of the conditional covariance matrix are generated from the following matrix equation:

$$H_t = C^TC + A^T\varepsilon_{t-1}\varepsilon_{t-1}^TA + G^TH_{t-1}G \tag{10}$$

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12. If the variances of the cash and futures returns and the covariance of the returns are assumed constant, the OLS Hedge model may be used to calculate the hedge ratio as: $\gamma^* = Cov(s, f)/Var(f)$.
13. The MGARCH model of Bollerslev, Engle and Wooldridge (1988), the VECH model, is used less frequently as the estimated covariance matrix is not guaranteed to be positive definite.

where C is a 2x2 upper triangular matrix, while A and G are 2x2 matrices. The structure of equation (10) shows that the BEKK model allows for a wide range of dynamic interaction between the conditional variances of the returns and the conditional covariance of the returns and that eleven parameters characterize the covariance matrix. McCurdy and Morgan (1991, 1992) assume that A and G are symmetric matrices, reducing the number of parameters to nine. However, this assumption implies that the cross-equation effects from the past squared errors and the past conditional variances cannot be separately determined.

The elements of the conditional covariance matrix for the Constant Correlation model are generated by the following equations:

$$h_{11,t} = c_1 + a_{11}\varepsilon^2_{1,t-1} + g_{11}h_{11,t-1} \tag{11a}$$

$$h_{22,t} = c_2 + a_{22}\varepsilon^2_{2,t-1} + g_{22}h_{22,t-1} \tag{11b}$$

$$h_{12,t} = \rho\left(\sqrt{h_{11,t}}\sqrt{h_{22,t}}\right) \tag{11c}$$

where ρ represents the assumed constant correlation between the cash and futures returns. The Constant Correlation model imposes the restriction that only own past lags enter into the equations of the conditional covariance model, and from (11) it is evident that seven parameters are required to characterize the conditional covariance matrix. The Constant Correlation model contains features of both the OLS Hedge model (constant correlation) and features of the BEKK model (time-varying variances and optimal hedge ratios).

The optimal hedge ratio takes on a somewhat different form with the assumption of constant correlation between the cash and futures returns. Using equations (9) and (11c) the conditional covariance of the cash and futures returns can be expressed as:

$$Cov(s_t, f_t | I_{t-1}) = \rho\left(\sqrt{Var(s_t|I_{t-1})}\sqrt{Var(f_t|I_{t-1})}\right) \tag{12}$$

Substituting (12) into equation (7) leads to the following expression for the time-varying optimal hedge ratio for the Constant Correlation model:

$$\gamma^*_{t-1} = \rho\left(\frac{\sqrt{Var(s_t|I_{t-1})}}{\sqrt{Var(f_t|I_{t-1})}}\right) \tag{13}$$

Equation (13) shows that under the constant correlation assumption, the time-varying optimal hedge ratio will be a function of the correlation between the cash and futures returns, ρ, and the ratio of the conditional standard deviations of the returns.[14]

3: Estimating Optimal Hedge Ratios

To obtain a broader understanding of co-movements in the Canadian money market and the BAX futures markets, two short-term Canadian money market instruments are analysed: three-month bankers' acceptances and three-month treasury bills. Three-month bankers' acceptance rates are used in domestic interest rate swaps and three-month treasury bill rates are used as a reference for Canadian short-term interest rates. As the BAX futures contract is based on the three-month bankers' acceptance, managing bankers-acceptance risk is called direct hedging, while managing treasury-bill risk is called cross hedging.[15]

The preliminary analysis of the data indicates that daily returns to money market instruments and BAX futures cannot be adequately analysed assuming normally distributed errors. The Student's *t* distribution, which allows for fatter tails than the normal, proves to be of more use in modelling daily returns from both a univariate and a multivariate perspective. This follows from Terasvirta (1996) who showed that in modeling high frequency time-series, leptokurtic distributions should be used.

3.1: Data and Univariate Analysis

Since the BAX futures market was not very liquid prior to 1992, the data used are the Montreal Exchange's daily closing prices for the closest-to-maturity BAX futures contract for the period January 2, 1992 to December 31, 1995. [16] The daily closing interest rates, on an annualized basis, are employed for the three-month bankers' acceptances and three-month treasury bills. Figures 1, 2 and 3 plot the annualized interest rates and the daily holding period returns (first differences of the logarithms of the prices) for each of the time-series.[17]

Over the 1992-1993 period, short-term interests rates in Canada were generally declining with infrequent increases. Interest rates began to rise in early 1994, and again in late 1994, and

14. This equation can also be found in McNew and Fackler (1994).
15. Since three-month bankers' acceptances and three-month treasury bills are competing three month instruments, the results should be quite similar.
16. The data from January 2, 1992 to December 28, 1994 are employed in the estimation exercise while the data for 1995 are kept back for out-of-sample forecasting analysis.
17. The price of the BAX futures contract is quoted as 100 minus the annualized yield on a three-month bankers' acceptance. To create a time-series for the BAX futures returns, the current contract is followed up to one week prior to maturity when a switch to the next contract is made. In calculating the returns to the BAX futures position, care is taken to ensure that the correct contract is used in the return calculation on the dates when contracts are switched.

generally declined throughout 1995. The figures also indicate episodes of heightened volatility in September-November 1992, August 1993, March 1994, late 1994-early 1995 and October-November 1995 that reflect the influence of international disturbances, fiscal and constitutional concerns and are discussed in greater detail in Zelmer (1996) and various issues of the *Bank of Canada Annual Report* and the *Bank of Canada Review*.

Table 1 presents preliminary statistics on the daily returns to three-month bankers' acceptances, BAX futures and three-month treasury bills. The results indicate that each of the series exhibit serial correlation, significant departures from Normality, skewness and excess kurtosis, and autoregressive conditional heteroskedasticity (ARCH). The results also indicate, as is usually observed in financial time series, that the returns to the cash positions, the bankers' acceptances and three-month treasury bills, exhibit greater deviations from Normality than the BAX futures returns.

Table 2 presents the results of testing for unit roots in the log prices of each series[18] using the Phillips and Perron (PP) and the Augmented Dickey-Fuller (ADF) unit root tests. Given the evidence of ARCH in the time-series, Haldrup (1992) demonstrates that the distribution of unit root tests is shifted to the left for GARCH models as the roots of the GARCH process approach unity, but concludes that GARCH effects do not cause many difficulties in the application of unit root tests.[19]

For the PP tests, 22 lags were included in the correction for autocorrelation, which corresponds to roughly one month of data, and is the same as that used in Baillie and Bollerslev (1989). The number of lags in the ADF test regressions is chosen by a method suggested by Campbell and Perron (1991); it involves starting with a large number of lags of the dependent variable (30), and testing the statistical significance of the last lag. If the last lag is not significant, it is dropped, and the regression re-estimated. The procedure is continued until the last lag is statistically significant. The Campbell and Perron procedure led to 27 lags being included for the bankers' acceptances and the treasury bills and 30 lags for the BAX futures. The null hypothesis

18. Granger and Hallman (1991) demonstrate that the log transform of an integrated series may lead to the null of a unit root being rejected too often, but that the autocorrelation will still indicate nonstationarity. Thus, if the prices themselves contain a unit root, a finding of a unit root in the log of the prices would indicate nonstationarity.
19. Haldrup warns that there may be problems if the intercept term is very small, and also against using White's standard errors in the application of the ADF tests, as White's correction will move the distribution of the tests to the right. If there are asymmetries in the GARCH process, the correction will be too large and result in too few rejections of the null of a unit root.

of a unit root is not rejected for any of the series, supporting the use of daily holding period returns.[20]

From the preliminary analysis, AR(5)-GARCH(1,1) models with Student t-distributed errors are estimated for each series with the results presented in Tables 3, 4 and 5. The parameter υ is referred to as degrees of freedom, or the shape parameter.[21] The results indicate "explosive" GARCH processes for each of the returns series.[22] Table 6 presents Likelihood Ratio tests of the Student's *t* distribution against the Normal distribution, showing that the Student's *t* distribution is statistically superior.[23]

3.2: Multivariate Analysis

Both the Constant Correlation and the BEKK models are estimated using the conditional bivariate Student's *t* distribution of Press (1982, pg. 136). The contributions to the log-likelihood function for the bivariate Student's *t* are given by

$$lnL_t = C - \frac{1}{2}ln|H_t| - \left(\frac{\upsilon+2}{2}\right)ln\left(1 + \left(\frac{1}{\upsilon-2}\right)\varepsilon_t^T H_t^{-1}\varepsilon_t\right). \tag{14}$$

The constant C is a function of the shape parameter and is expressed as

$$C = ln\Gamma\left(\frac{\upsilon+2}{2}\right) - ln\Gamma\left(\frac{\upsilon}{2}\right) - ln(\pi(\upsilon-2)), \tag{15}$$

where Γ represents the gamma function.

The estimation results are presented in Tables 7, 8 and 9, where the Maximum Likelihood estimation was carried out using the BFGS algorithm.[24] Table 9 presents the results for the BEKK model in a more convenient form. The results from the Constant Correlation model and the BEKK model are quite similar, with both models indicating the presence of 'explosive' behaviour in the returns.

Table 9 shows that there is little evidence of dynamic interactions between the bankers' acceptance and BAX futures returns. The only evidence of dynamic interaction is the effect of the

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20. The issue of potential cointegration between the cash and futures log prices is not discussed.
21. The smaller the shape parameter, the 'fatter' the tails of the distribution. As each of the estimated models has the value of the shape parameter less than 3, the measure of kurtosis for the Student's *t* does not exist. The estimated kurtosis is $3 + (6/(\hat{\upsilon} - 4))$, which exists only if $\hat{\nu} > 4$ (see Theil (1971) p.82). This suggests that allowing for a 'fat-tailed' distribution is not sufficient to model the returns analysed.
22. An "explosive GARCH" process is one that has the sum of the GARCH parameters $(\alpha + \beta)$ exceeding 1.0.
23. The estimation of the models under the assumption of Normality is available from the author.
24. The estimated degrees of freedom, $\hat{\upsilon}$, reflects the kurtosis in the bivariate distribution, where the degrees of freedom for each of the marginal distributions equals $\hat{\upsilon}$.

past product of the errors on the BAX futures return. Table 10 presents Likelihood Ratio test results indicating that the Student's *t* distribution fits the bivariate distribution of the bankers' acceptances and BAX returns better than the Normal.

3.3: Estimated Variances and the Optimal Hedge Ratios

Figure 4 presents the estimated conditional variances from the BEKK model for the returns to bankers' acceptances and BAX futures.[25] It shows that increased volatility affected both the bankers' acceptance and the BAX futures market. The correlation between the estimated conditional variances is 0.94, indicating a very high degree of co-movement between the conditional variances of the returns to bankers' acceptances and BAX futures.[26]

The OLS Hedge model is estimated and the results are presented in the upper panel of Table 11, where the hedge ratio is shown to be 0.803, indicating that investment managers should have covered 80.3% of their bankers' acceptances position with BAX futures over the 1992-1994 period.

The optimal hedge ratios for the Constant Correlation and BEKK model are presented in Figures 5 and 6 respectively, where the dotted lines represent the OLS hedge ratio. The OLS hedge ratio typically exceeds the hedge ratios from the dynamic hedging models but falls below the dynamic hedge ratios during periods of heightened volatility. The OLS hedge model leads to more coverage of the cash position in general, however both of the dynamic hedging models indicate that an investment manager should increase the coverage of the cash position during periods of increased volatility.[27]

3.4: Cross-Hedging Treasury Bills

The OLS cross hedge model is estimated and the results are presented in the lower panel of Table 11. The estimated cross hedge ratio is 0.721, which indicates that investment managers should have covered 72.1% of their treasury bill positions with BAX futures over the 1992-1994 period.

The results of estimating the time-varying cross-hedge of three-month treasury bills are

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25. The conditional variances from the Constant Correlation model are qualitatively similar; the figures are available upon request.
26. The correlation between the conditional variances is greater than the correlation between the unconditional variances which is 0.75.
27. Harvey (1996) shows that the trading volume on the Montreal Exchange of BAX futures contracts for hedging purposes does rise during periods of heightened volatility, which is when an investment manager would want a more effective hedge.

presented in Tables 12 and 13, with the more convenient results of the BEKK model presented in Table 14. The results are qualitatively very similar to those estimated using bankers' acceptances, except with regard to some of the cross-equation influences that occur through the treasury bill returns and the covariance of the returns.

Figure 7 presents the estimated conditional variances of the returns to bankers' acceptances and the three-month treasury bills. The pattern of volatility for these two instruments is very similar, further indicating that volatility in one part of the money market affects other parts of the market. The results also indicate that the returns to bankers' acceptances are somewhat more susceptible to volatility than are treasury bill returns. The Bank of Canada through this period was attempting to target the three-month treasury bill rate (and hence the Bank Rate) through its control over settlement balances provided to direct-clearers[28] and, on occasion, the Bank of Canada would engage in outright sales or purchases of three-month treasury bills to moderate movements in the three-month treasury bill rate. Since the Bank of Canada does not operate in the market for bankers' acceptances, the variability of the rate of interest on bankers' acceptances is not directly affected by Bank of Canada activity in the treasury bill market, and bankers acceptance rates tended to be more variable.

Figure 8a shows the time-varying optimal hedge ratios for both bankers' acceptances and treasury bills from the BEKK model, and indicates a very similar pattern for both three-month instruments. Figure 8b shows that one would typically have a higher optimal hedge ratio for bankers' acceptances than one would for treasury bills. There are two reasons for this result. First, as noted, the rate of return on bankers' acceptances was more volatile and led to greater demand for hedging services of bankers' acceptance positions. Second, the BAX contract is priced relative to bankers' acceptances. To cross-hedge treasury bills, a hedger would have to deal with the basis, or the spread between the prices of bankers' acceptances and BAX futures contracts, and with the spread between the rates of interest on bankers' acceptances and treasury bills.

28. From March 1980 to February 1996, the Bank Rate was set at 25 basis points above the weekly tender for three-month treasury bills. Since the middle of 1994, the Bank has no longer targeted three-month interest rates; instead its operating procedures have been focused on keeping the overnight rate within a 50-basis-point band. Since February 1996, the Bank Rate has been tied to the top of the operating band for the overnight rate.

4: Hedging Effectiveness and Dynamic Correlation

This section analyzes hedging effectiveness and the correlation between the cash and futures returns. The effectiveness of a hedging strategy depends on its ability to create a portfolio whose return has a lower variance than the cash position. Out-of-sample hedging effectiveness is also analysed by using the one-step ahead variance forecasts produced by each of the models.

The one-day ahead variances, the one-day ahead optimal hedge ratio, and the return on the hedged portfolio over the next day were calculated. One day was then added to the sample and the models were reestimated and the one-day forecasts recalculated. This exercise was repeated for each day of 1995.

To estimate hedging effectiveness (following Kroner and Sultan 1993), the return to the hedged portfolio is calculated each day and the variance of the hedged portfolio is estimated over the full sample by

$$var(s_t - \hat{\gamma}_{t-1} f_t) \tag{16}$$

where $\hat{\gamma}_{t-1}$ is the estimated hedge ratio from one of the models. The following table presents the estimated in-sample and out-of-sample variances of the hedged portfolio, where the lower the variance the more effective the hedging strategy.

Variance of Bankers' Acceptances Hedged Portfolio		
	1992-1994	**1995**
Unhedged	0.0214	0.0217
OLS Hedge	0.0046	0.0064
BEKK	0.0056	0.0066
Constant Correlation	0.0060	0.0110

The results indicate that the OLS hedge model produces the hedged portfolio with the lowest variance, on average, both in-sample and out-of-sample, despite the fact that the time-varying variance models have more explanatory power for the movement of returns. Of the time-varying hedging strategies, the best hedging performance is provided by the model allowing for dynamic correlation.

To analyse the effectiveness of cross-hedging treasury bills, the variance of the cross-hedged portfolio is calculated using equation (16). The following table presents the estimated in-sample and out-of-sample variances of the cross-hedged portfolio.

Variance of Treasury Bills Cross-Hedged Portfolio		
	1992-1994	**1995**
Unhedged	0.0186	0.0163
OLS Hedge	0.0051	0.0051
BEKK	0.0063	0.0059
Constant Correlation	0.0062	0.0097

As with the direct hedge of the bankers' acceptances, the OLS Hedge model provides the best hedging performance, and unlike the results for bankers' acceptances the Constant Correlation model works marginally better than the BEKK model in-sample, though the BEKK performs better out-of-sample.

Edderington (1979) demonstrated that hedging effectiveness and the correlation between the returns are closely related, as the effectiveness of a hedge corresponds to the square of the correlation, ρ^2, between the cash and futures returns. For the OLS Hedge model, effectiveness can be estimated by the R^2 from the hedge regression:

$$R^2 = \rho^2 = \frac{Cov^2(s, f)}{Var(s)Var(f)}. \tag{17}$$

Equation (17) demonstrates, assuming constant variances, that the higher the correlation between the returns to the cash and futures positions, the more effective the hedge. Using equation (17) Senchak and Easterwood (1983) derive the following equation:

$$Var(x_t) = Var(s_t)\left(1 - \rho^2\right) \tag{18}$$

This equation illustrates that the variance of the hedged portfolio will be lower than the variance of the cash position, so long as the correlation between the cash and the futures returns is non-zero. Equation (18) further illustrates that the higher the correlation between the cash and futures returns the lower will be the variance of the hedged portfolio, or the more effective will be the hedging strategy.

The assumption of a constant correlation has been challenged by Hegde (1982), who hypothesizes that the correlation between the cash and the futures returns will rise during periods of increased volatility. To analyse the correlation during periods of increased volatility, the hedging model would have to allow for the correlation between the cash and futures returns to vary with time. This is not a feature of the OLS Hedge model nor of the Constant Correlation model, but is a feature of the BEKK model.

The dynamic correlation between the cash and futures returns model is given by

$$\rho_t = \frac{Cov(s_t, f_t | I_{t-1})}{\sqrt{Var(s_t | I_{t-1})}\sqrt{Var(f_t | I_{t-1})}}. \tag{19}$$

According to Hegde, ρ_t would be expected to rise during periods of heightened volatility, unless the link between the cash and the futures markets breaks down, which might happen if the bid-ask spreads in the futures market increased to such an extent that trading activity ceased.[29]

Figure 9a plots the dynamic correlation calculated by the BEKK model applied to bankers' acceptances, along with the estimated correlations from the OLS Hedge model (0.885) and from the Constant Correlation model (0.795). Note that the correlation estimated from the OLS Hedge model is higher than that from the Constant Correlation model at all points, and exceeds the dynamic correlation from the BEKK model most of the time. As suggested by Hegde, it is during the periods of increased volatility that the correlation from the BEKK model exceeds the correlation from the OLS Hedge model, supporting the fact that hedging activity on the Montreal Exchange increases during periods of increased volatility. That is, during periods of increased volatility, the BAX futures become more effective hedging instruments.

Figure 9b plots the dynamic correlation from the treasury bill model and is qualitatively similar to the Figure for bankers' acceptances. Also, the estimated correlation from both the OLS Hedge model (0.849) and the Constant Correlation model (0.740) are lower than the comparable values from the bankers' acceptance evaluation.

4.1: Interpretation and Implications for the Bank of Canada

The results indicate that models that assume constant variances and constant correlations provide adequate hedging performance on average, but fail to provide adequate hedging performance during periods of heightened volatility by failing to take into account the dynamic interaction between the cash and futures returns.

This has implications for the Bank of Canada's use of interest rate derivative securities prices. Prices of interest rate derivative securities (FRAs and BAX) are used to derive market expectations of future interest rates. These expectations are derived from models that assume

29. See Garber and Spencer (1995) and references therein for a discussion of dynamic hedging in periods of market volatility, such as the 1987 stock market crash.

constant variances and, by implication, constant dynamic interaction. There is some evidence that these models perform adequately on average but less well during periods of increased volatility. The results show that this is not surprising, as models which do not allow for dynamic correlation between short-term interest rates and interest rate derivative instruments apparently provide adequate information on average but will not be as useful during periods of heightened volatility. This indicates that care should be exercised in deriving information about interest rate movements from models that assume constant variances.

Further, periods of increased volatility of short-term interest rates will also be periods where the value of portfolios of short-term instruments will be volatile. Knowing how volatility behaves between markets and how markets intereact will improve the Bank’s tactical response to these episodes and help the market find viable trading ranges, easing the pressures arising from excessive volatility.

5: Further Research

This paper shows that a static hedging model based on linear regressions that assume the variances of returns are constant, produces, on average, a more effective one-day hedge than dynamic hedging models which allow for time-varying variances and leptokurtosis. This occurs even though the dynamic hedging models do a better job of explaining the relationship between the returns.

The paper also shows that during periods of increased volatility, dynamic hedging, based on the BEKK model, is more effective than static hedging, and, further, that the correlation between the cash and futures returns typically rises. These results suggest that the BAX futures become more effective hedging instruments during periods of increased volatility and corresponds to the evidence that hedging activity using BAX futures rises during these periods.

Two directions of research that might prove fruitful in extending the analysis in this paper are related to “spillovers” of volatility from the futures market to the money market,[30] and the interest rate differential and exchange rate effects on the volatility of Canadian money markets and hedging activity.

30. This follows from Harvey (1996) where it was shown that the BAX futures market responded to ‘news’ faster than the market for treasury bills.

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Appendix

These pages expand upon the presentation of the conditional covariance matrices given in the main text for the Constant Correlation and BEKK models.

The three equations for the MGARCH-CC model are given by:

$$\begin{aligned} h_{11t} &= c_1 + a_{11}\varepsilon^2_{1t-1} + g_{11}h_{11t-1} \\ h_{22t} &= c_2 + a_{22}\varepsilon^2_{2t-1} + g_{22}h_{22t-1} \\ h_{12,t} &= \rho\left(\sqrt{h_{11,t}}\sqrt{h_{22,t}}\right) \end{aligned} \quad (A1)$$

which can be re-written as a somewhat more general equation system, such as

$$\begin{aligned} \begin{bmatrix} h_{11t} \\ h_{22t} \end{bmatrix} &= \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \varepsilon^2_{1t-1} \\ \varepsilon^2_{2t-1} \end{bmatrix} + \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} h_{11t-1} \\ h_{22t-1} \end{bmatrix}. \\ h_{12,t} &= \rho\left(\sqrt{h_{11,t}}\sqrt{h_{22,t}}\right) \end{aligned} \quad (A2)$$

For the MGARCH-CC model, it is typically assumed that $a_{12}=a_{21}=g_{12}=g_{21}=0$, so that the conditional variances are functions of own lagged variances and own lagged squared errors.

The equations for the MGARCH-BEKK model are somewhat more complicated. To begin, we present Equation (13) once again

$$H_t = C^TC + A^T\varepsilon_{t-1}\varepsilon^T_{t-1}A + G^TH_{t-1}G. \quad (A3)$$

Expanding the matrices leads to the following representation:

$$\begin{aligned} \begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{12,t} & h_{22,t} \end{bmatrix} &= \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix}^T \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^T \begin{bmatrix} \varepsilon_{1,t-1} \\ \varepsilon_{2,t-1} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1} \\ \varepsilon_{2,t-1} \end{bmatrix}^T \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \\ &+ \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}^T \begin{bmatrix} h_{11,t-1} & h_{12,t-1} \\ h_{12,t-1} & h_{22,t-1} \end{bmatrix} \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \end{aligned} \quad (A4)$$

which upon calculating the matrix expressions, we obtain the equations for the conditional variances and the conditional covariance.

Conditional Variance Equation 1:

$$\begin{aligned} h_{11,t} = {} & c^2_{11} + a^2_{11}\varepsilon^2_{1t-1} + 2a_{11}a_{21}\varepsilon_{1t-1}\varepsilon_{2t-1} + a^2_{21}\varepsilon^2_{2t-1} \\ & + g^2_{11}h_{11t-1} + 2g_{11}g_{21}h_{12t-1} + g^2_{21}h_{22t-1} \end{aligned} \quad (A5)$$

Conditional Variance Equation 2:

$$h_{22,t} = (c_{12}^2 + c_{22}^2) + a_{12}^2\varepsilon_{1t-1}^2 + 2a_{12}a_{22}\varepsilon_{1t-1}\varepsilon_{2t-1} + a_{22}^2\varepsilon_{2t-1}^2 + g_{12}^2 h_{11t-1} + 2g_{12}g_{22}h_{12t-1} + g_{22}^2 h_{22t-1} \tag{A6}$$

Conditional Covariance Equation

$$h_{12,t} = (c_{11}c_{12}) + a_{11}a_{12}\varepsilon_{1t-1}^2 + (a_{12}a_{21} + a_{11}a_{22})\varepsilon_{1t-1}\varepsilon_{2t-1} + a_{21}a_{22}\varepsilon_{2t-1}^2 + g_{11}g_{12}h_{11t-1} + (g_{12}g_{21} + g_{11}g_{22})h_{12t-1} + g_{21}g_{22}h_{22t-1} \tag{A7}$$

(A5), (A6) and (A7) illustrate the complicated interaction between the conditional variances, the conditional covariances, and the cross-equation restrictions among the parameters.

The parameters that generate the most interest are those on the lags of a returns own variances and squared errors. Equation (A5) illustrates that the parameters of interest in discovering whether the estimated GARCH parameters of the first conditional variance equation sum to less than 1.0 are not a_{11} and g_{11}, but rather a_{11}^2 and g_{11}^2. The comparable parameters for the second conditional variance equation, from (A6), are a_{22}^2 and g_{22}^2, while the parameters for the conditional covariance equation, from (A7), are $(a_{12}a_{21} + a_{11}a_{22})$ and $(g_{12}g_{21} + g_{11}g_{22})$.

The BEKK model might best be referred to as a Dynamic Correlation model. This can be seen by writing the correlation as the conditional covariance over the product of the conditional standard deviations:

$$\rho_t = \frac{h_{12,t}}{\sqrt{h_{11,t}}\sqrt{h_{22,t}}} \tag{A8}$$

From equations (A5)-(A7), each of the elements on the right-hand side of equation (A8) have an autoregressive structure. Thus ρ_t can be referred to as the dynamic correlation, or as the conditional correlation, since it is a function of conditional variances and the conditional covariance.

Figure 1: 3-month Bankers' Acceptances

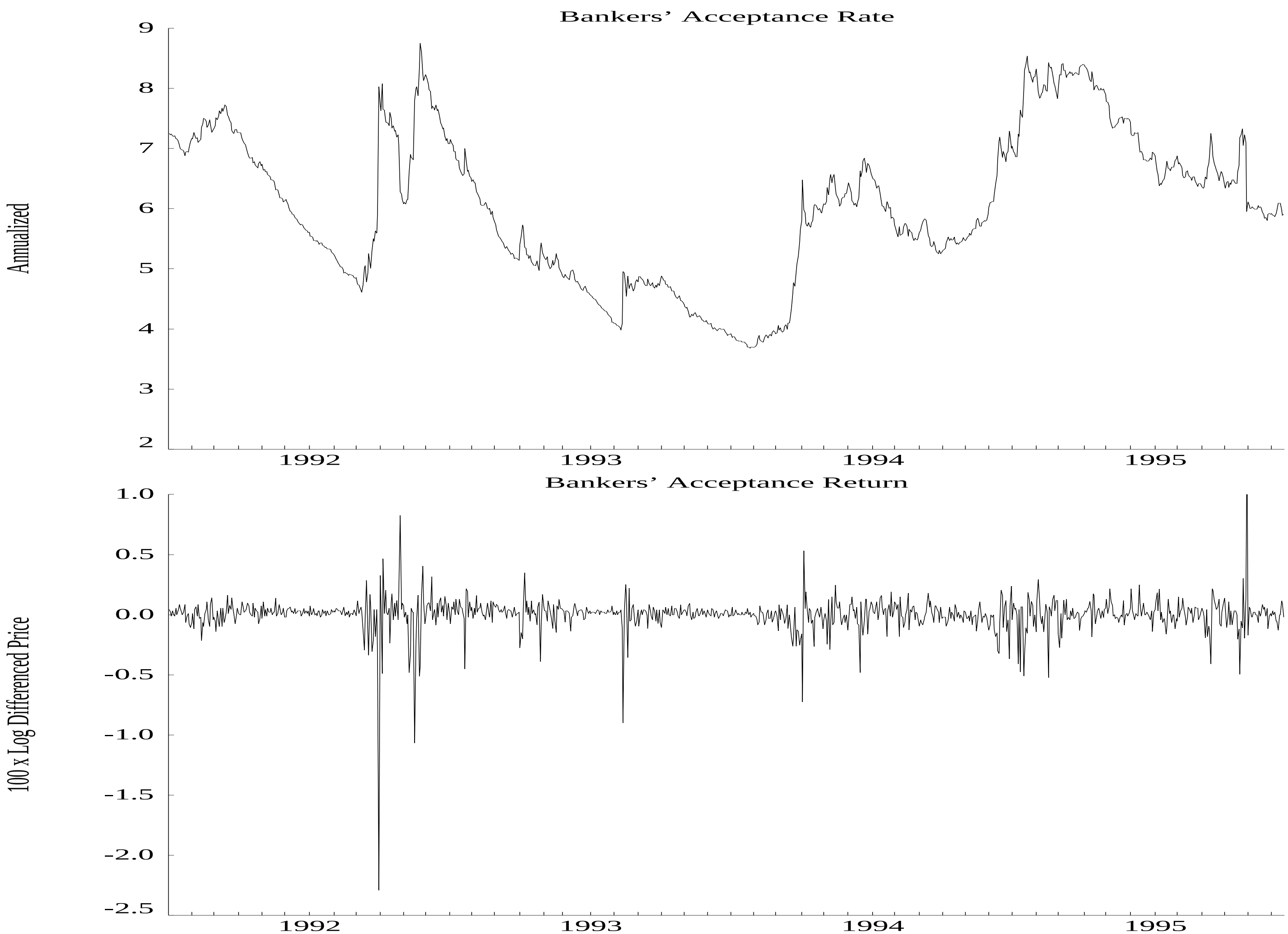

Figure 2: BAX Futures

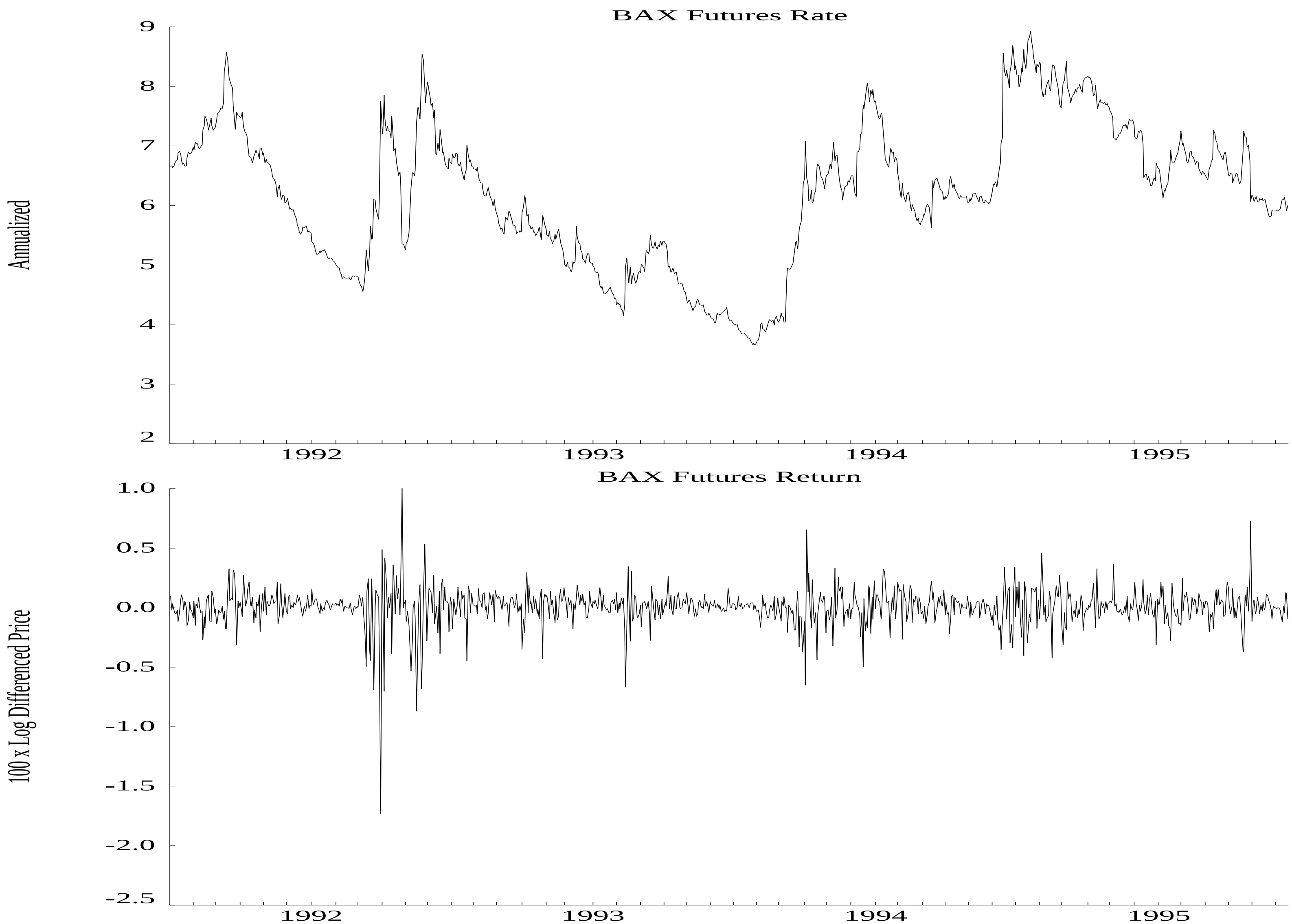

Figure 3: 3-month Treasury Bills

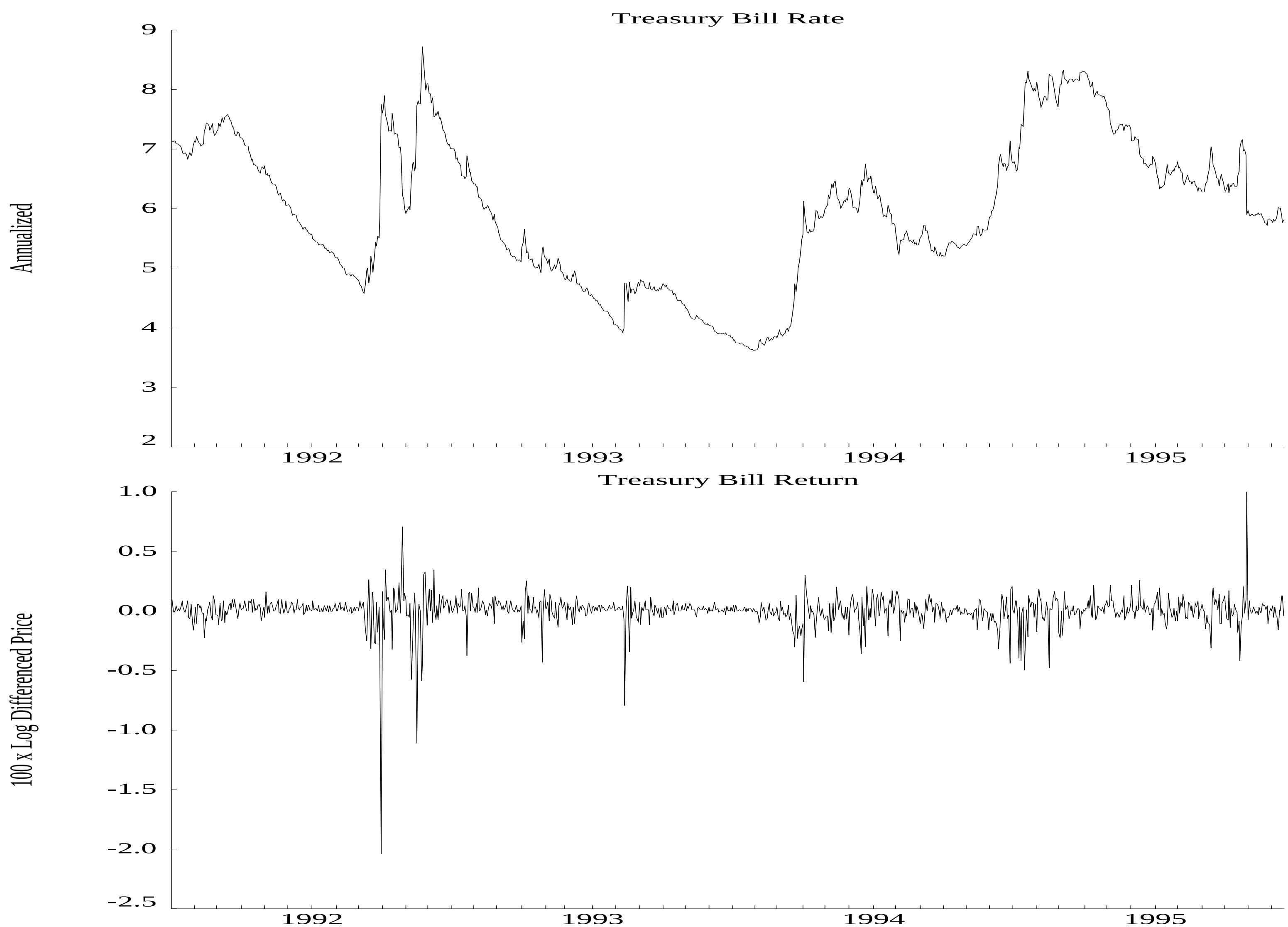

Figure 4: Estimated Conditional Variances
MGARCH-BEKK

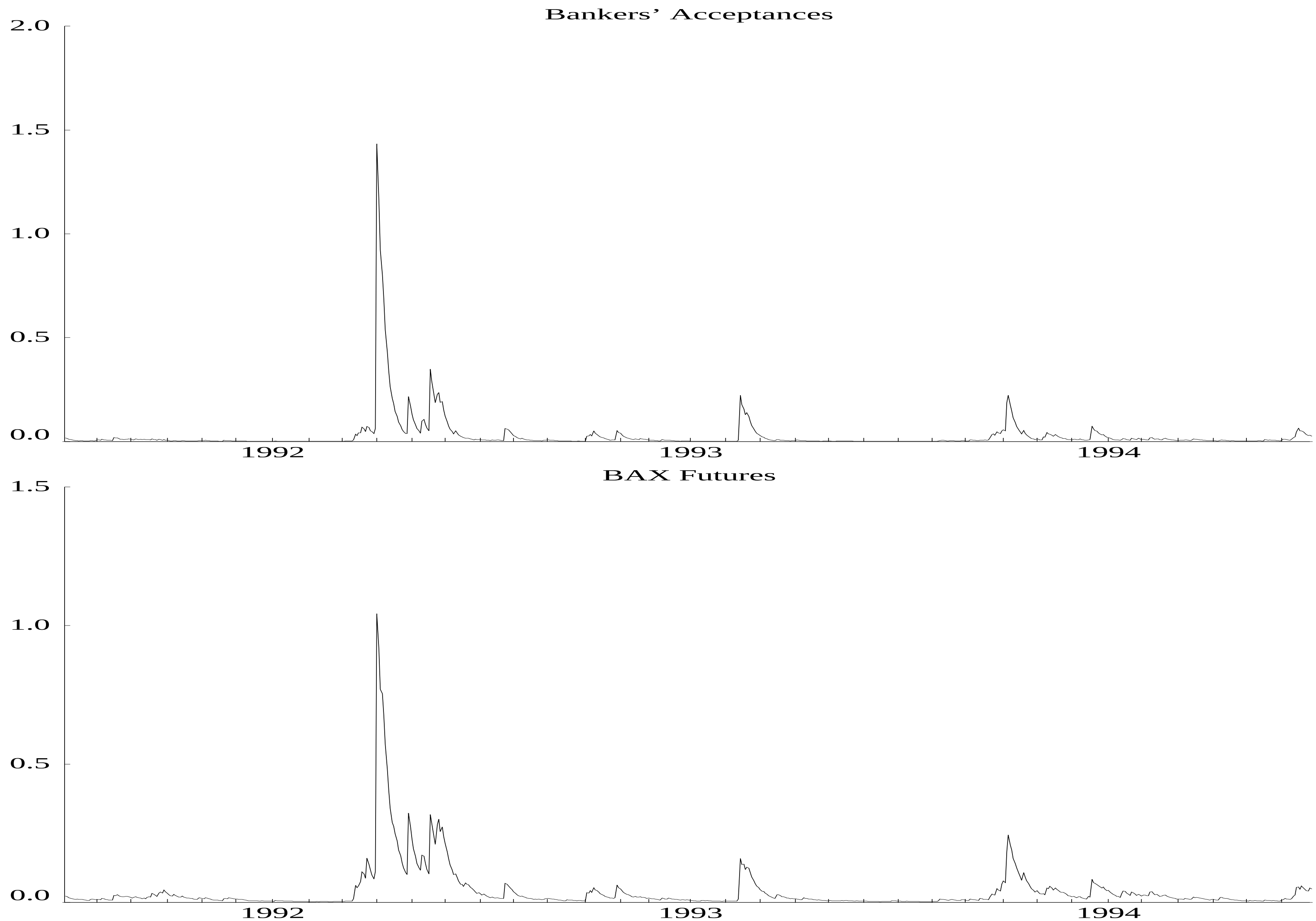

Figure 5: Optimal Hedge Ratio
Solid Line - Constant Correlation
Dashed Line - OLS Hedge Ratio

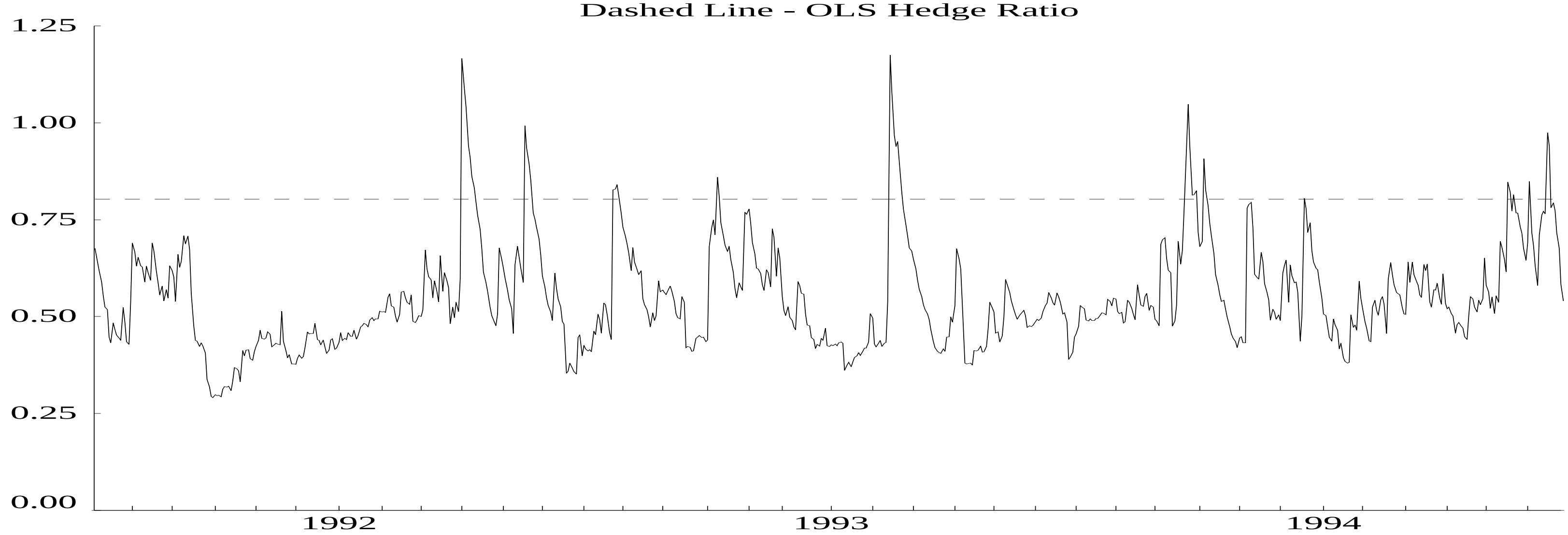

Figure 6: Optimal Hedge Ratio
Solid Line - BEKK
Dashed Line - OLS Hedge Ratio

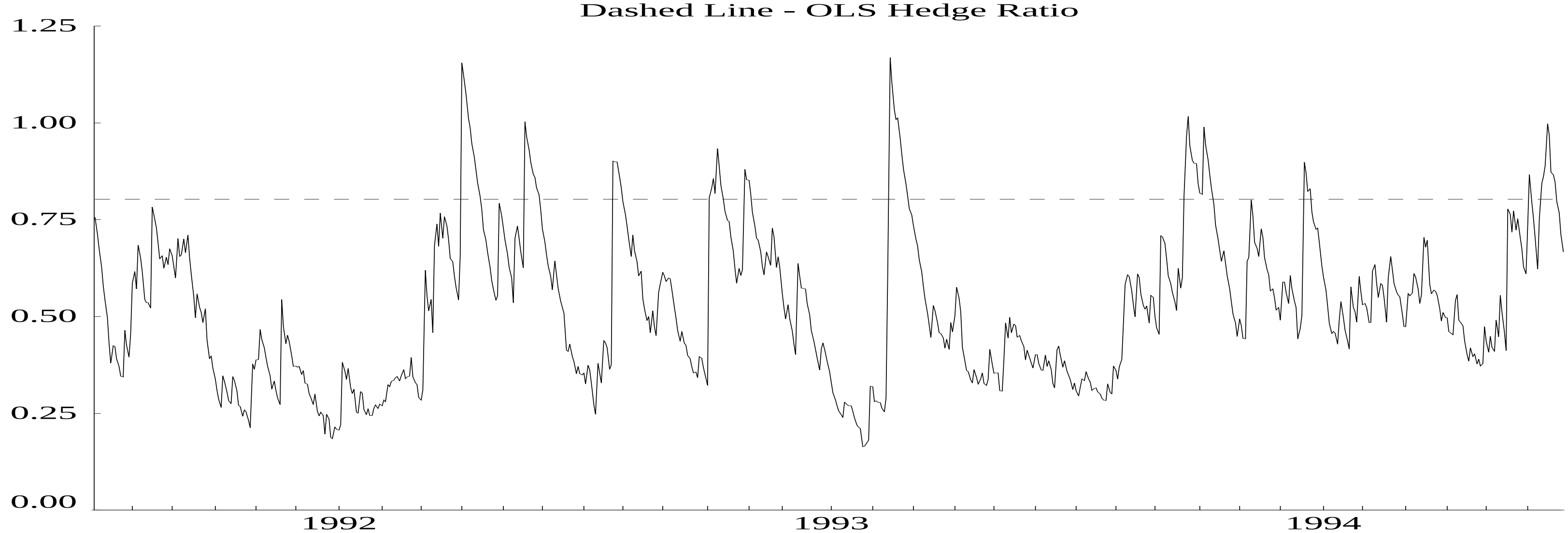

Figure 7: Estimated Conditional Variances

Solid Line - Treasury Bills
Dashed Line - Bankers' Acceptances

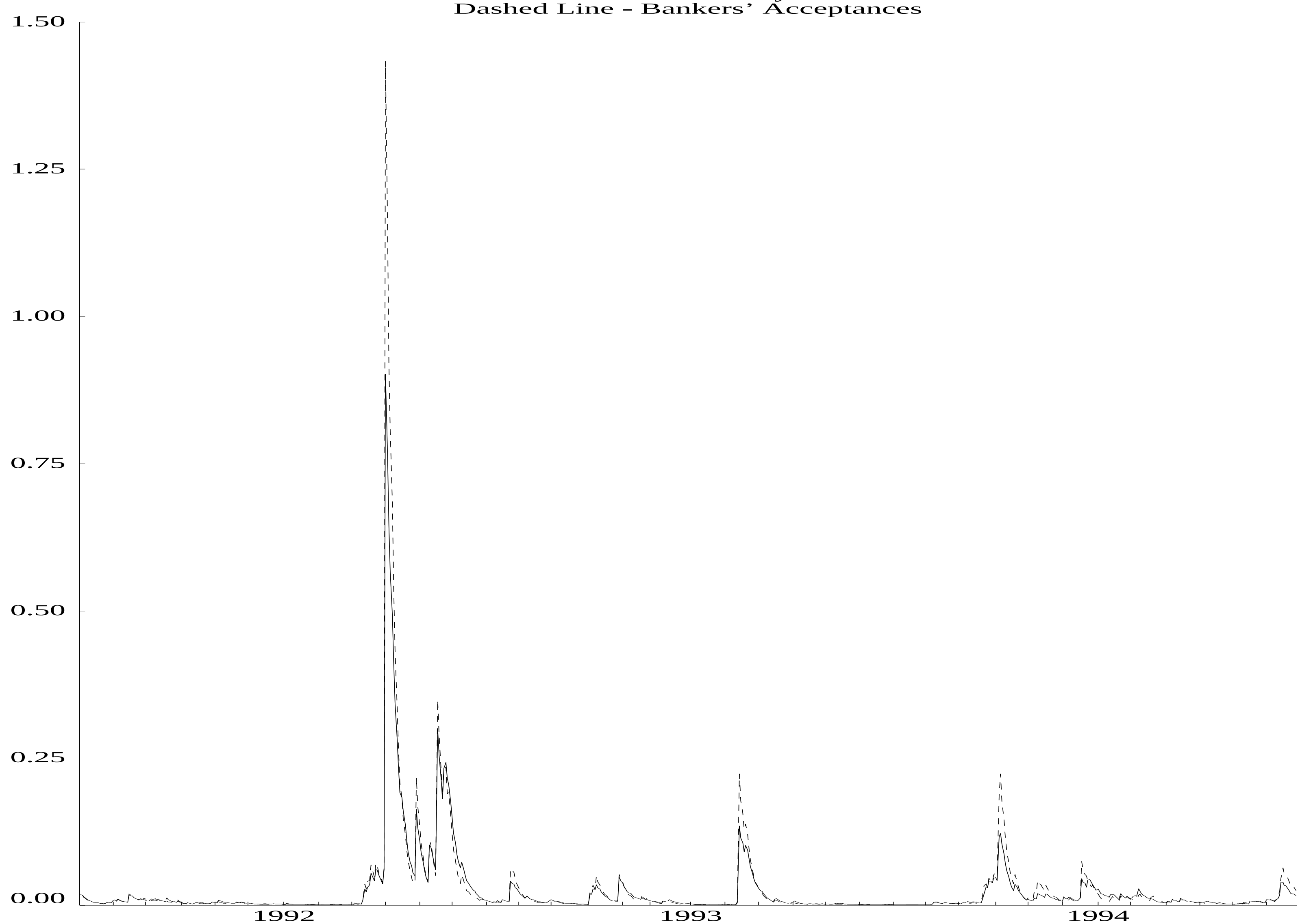

Figure 8a: Optimal Hedge Ratio
Solid Line - Bankers' Acceptances
Dashed Line - Treasury Bills

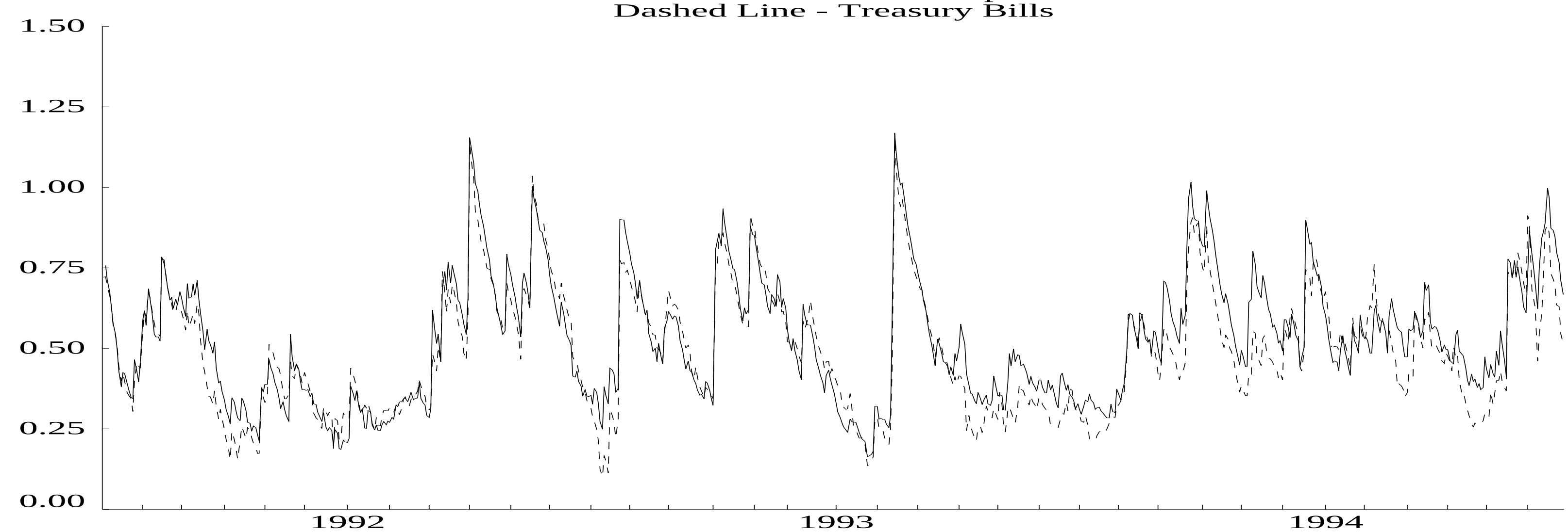

Figure 8b: Optimal Hedge Ratio Spread
Bankers' Acceptances minus Treasury Bills

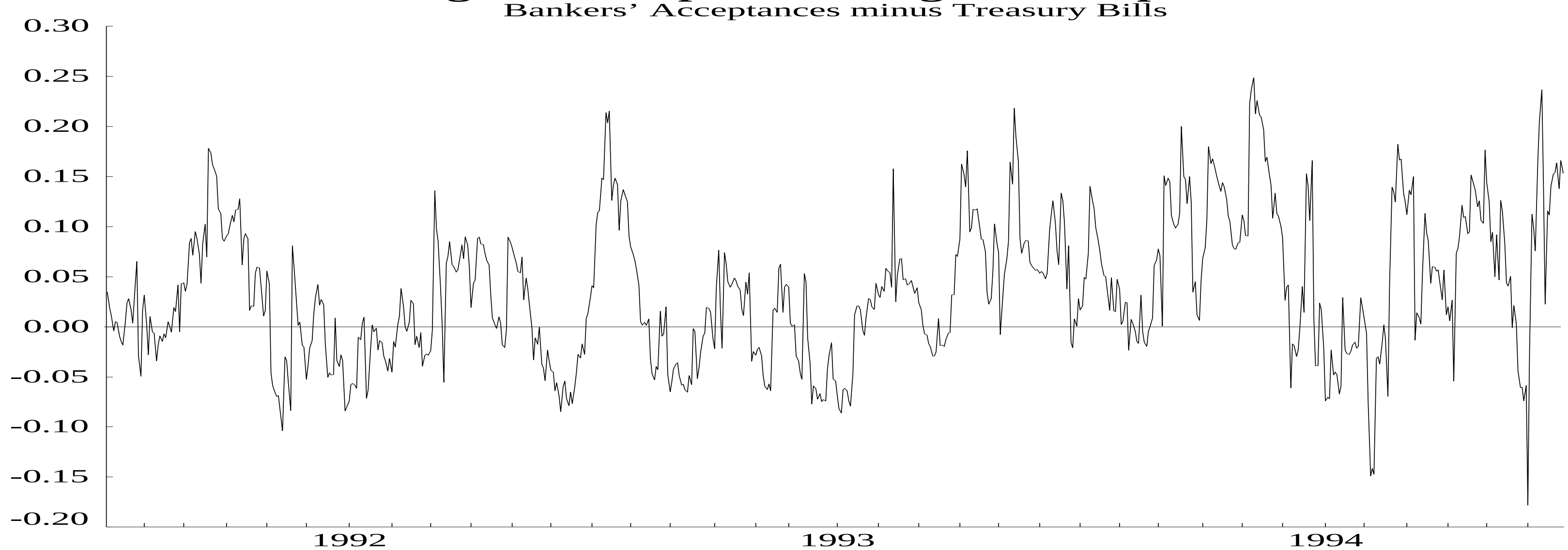

Figure 9a: Dynamic Correlation

Bankers' Acceptances and BAX Futures Returns

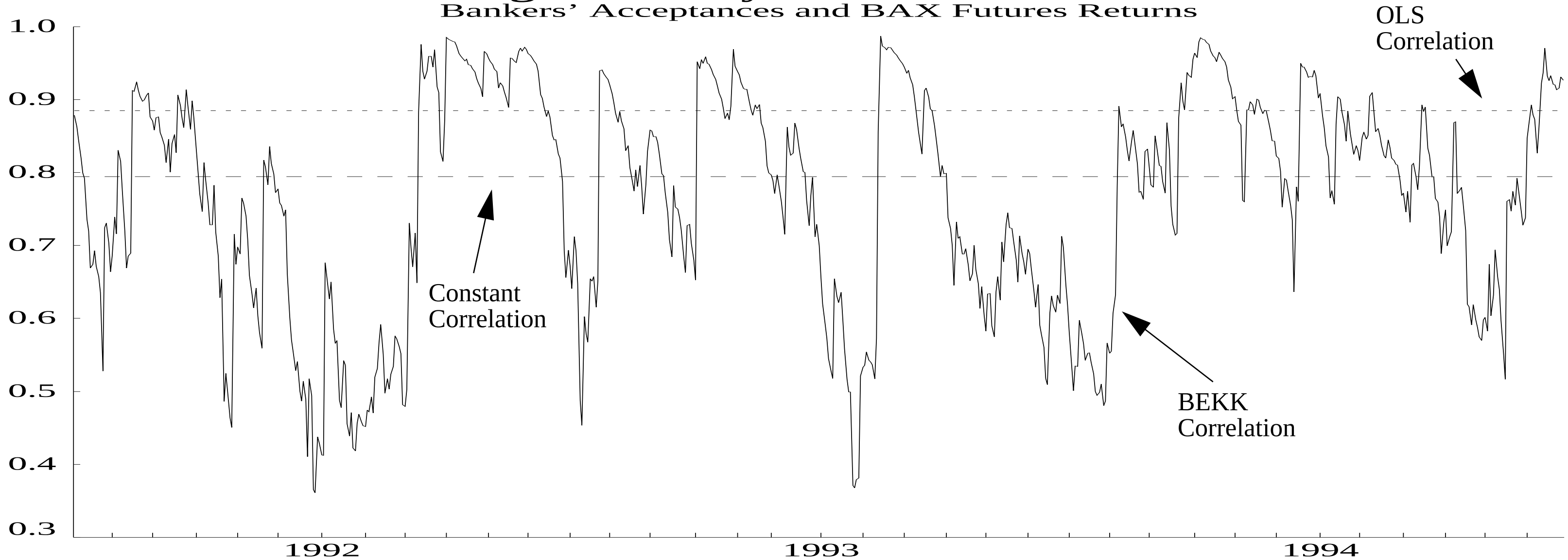

Figure 9b: Dynamic Correlation

Treasury Bills and BAX Futures

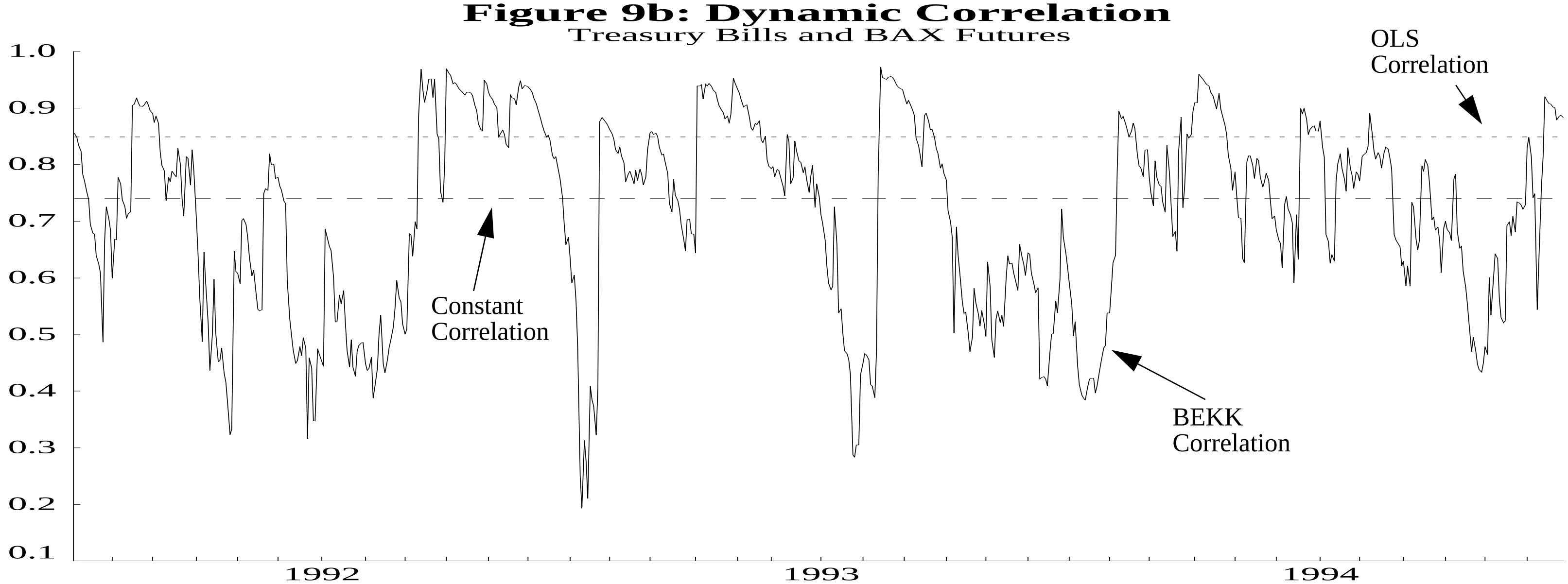

Table 1: Summary Statistics

	Q(36)	Q(10)	Skew	Kurt	$Q^2(10)$	ARCH
Bankers'Acceptances	84.79	6.32	-5.69	83.80	31.62	5.63
	(0.00)	(0.79)	(0.00)	(0.00)	(0.00)	(0.00)
BAX Futures	49.33	7.13	-2.03	20.06	25.31	4.13
	(0.07)	(0.71)	(0.00)	(0.00)	(0.00)	(0.00)
Treasury Bills	125.82	10.59	-5.22	69.80	51.92	7.71
	(0.00)	(0.39)	(0.00)	(0.00)	(0.00)	(0.00)

Notes: Values in parentheses are p-values. Q(36) is the Ljung-Box test for autocorrelation. Q(10) is the same test for the residuals from an AR(5) model applied to each time-series. Skew and Kurt are individual tests for Skewness and Kurtosis respectively, and are applied to the residuals from the preliminary autoregression. $Q^2(10)$ is a Ljung-Box test for autocorrelation applied to the squared residuals from the preliminary AR(5). ARCH is a artificial regression test for ARCH(5) from the residuals.

Table 2: Unit Root Tests

	Phillips/ Perron	Dickey/ Fuller	Augmented Dickey/Fuller
Bankers'Acceptances	-2.053	-1.698	-2.020
BAX Futures	-1.934	-1.500	-1.560
Treasury Bills	-2.051	-1.649	-2.050

Notes: The Phillips/Perron test included 22 lags in the autoregressive correction. The Augmented Dickey-Fuller test included 27 lags for bankers' acceptance and treasury bills, and 30 lags for the BAX. Lags chosen by testing down from 30 lags following Campbell and Perron (1991). The Asymptotic 1%, 5% and 10% critical values are -3.43, -2.86, -2.57 respectively. (See Davidson and Mackinnon (1993)).

Table 3: Bankers' Acceptances

Autoregressive Equation						Garch Equation			
μ	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	κ	β	α	υ
.0118	.0716	-.0023	.0719	.0799	.0521	.0002	.7625	.4204	2.677
(.002)	(.034)	(.034)	(.033)	(.034)	(.032)	(.000)	(.036)	(.141)	(0.285)

Notes: Standard errors in parentheses. The model estimated is an AR(5)-GARCH(1,1) assuming conditional Student's *t* errors:

$$r_t = \mu + \sum_{j=1}^{5} \rho_j r_{t-j} + \varepsilon_t \qquad \varepsilon_t \sim t(0, h_t, \upsilon)$$
$$h_t = \kappa + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}$$

Table 4: BAX Futures

Autoregressive Equation						Garch Equation			
μ	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	κ	β	α	υ
.0138	.1187	-.0243	.0117	.0316	-.0048	.0002	.8488	.1971	3.176
(.003)	(.035)	(.036)	(.034)	(.035)	(.024)	(.000)	(.029)	(.054)	(.406)

Notes: Standard errors in parentheses. The model estimated is an AR(5)-GARCH(1,1) assuming conditional Student's *t* errors:

$$r_t = \mu + \sum_{j=1}^{5} \rho_j r_{t-j} + \varepsilon_t \qquad \varepsilon_t \sim t(0, h_t, \upsilon)$$
$$h_t = \kappa + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}$$

Table 5: Treasury Bills

Autoregressive Equation						Garch Equation			
μ	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	κ	β	α	υ
.0101	.0416	-.0113	.0374	.0879	.1070	.0002	.7994	.3092	2.780
(.002)	(.036)	(.030)	(.033)	(.034)	(.035)	(.000)	(.032)	(.102)	(.328)

Notes: Standard errors in parentheses. The model estimated is an AR(5)-GARCH(1,1) assuming conditional Student's *t* errors:

$$r_t = \mu + \sum_{j=1}^{5} \rho_j r_{t-j} + \varepsilon_t \qquad \varepsilon_t \sim t(0, h_t, \upsilon)$$
$$h_t = \kappa + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}$$

Table 6: Student's *t* and Normal

Test for $\frac{1}{\hat{\nu}} = 0$	Bankers' Acceptances	BAX Futures	Treasury Bills	Critical Value
t-test	9.01	10.71	8.45	1.96
Likelihood Ratio	372.71	319.29	323.70	3.84

Notes: These tests are discussed in Bollerslev (1987) and Engle and Bollerslev (1986). The Likelihood Ratio test is a one-degree of freedom test. A test statistic above the appropriate critical value indicates a rejection of the null hypothesis of Normality.

Table 7: Constant Correlation

Student's *t* distribution				
	Estimate	Standard Error	T-Statistic	p-value
μ_1	.0161	.002	**9.77**	0.00
μ_2	.0203	.003	**7.73**	0.00
c_1	.0004	.000	**3.50**	0.00
c_2	.0007	.000	**3.04**	0.00
rho	.7947	.014	**55.78**	0.00
g_{11}	.7621	.034	**22.40**	0.00
g_{22}	.8319	.028	**29.30**	0.00
a_{11}	.2244	.048	**4.72**	0.00
a_{22}	.1622	.034	**4.73**	0.00
ν	3.275	.274	**11.94**	0.00
Maximized Loglikelihood = 1888.41				

The basic model estimated is given by:

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

The MGARCH-CC model was estimated using the following specifications for the covariance matrix:

$$\begin{bmatrix} h_{11t} \\ h_{22t} \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} g_{11} & 0 \\ 0 & g_{22} \end{bmatrix} \begin{bmatrix} h_{11t-1} \\ h_{22t-2} \end{bmatrix} + \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} \begin{bmatrix} \varepsilon^2_{1t-1} \\ \varepsilon^2_{2t-1} \end{bmatrix}$$

$$h_{12t} = \rho(\sqrt{h_{11t}}\sqrt{h_{22t}})$$

The contributions to the loglikelihood function for the Student's *t* distribution are of the form:

$$\ln L_t = C - \frac{1}{2}\ln|H_t| - \left(\frac{\upsilon+2}{2}\right)\ln\left[1 + \left(\frac{1}{\upsilon-2}\right)\varepsilon_t^T H_t^{-1}\varepsilon_t\right]$$

where the constant in the loglikelihood will be given by:

$$C = \ln\Gamma\left(\frac{\upsilon+2}{2}\right) - \ln(\pi(\upsilon-2)) - \ln\Gamma\left(\frac{\upsilon}{2}\right)$$

Table 8: BEKK

Student's *t* distribution				
	Estimate	Standard Error	T-Statistic	p-value
μ_1	.0157	.001	**10.59**	0.00
μ_2	.0199	.002	**8.29**	0.00
c_{11}	.0130	.003	**4.86**	0.00
c_{12}	.0138	.006	**2.32**	0.02
c_{21}	.0140	.003	**5.22**	0.00
g_{11}	.90.95	.039	**23.11**	0.00
g_{12}	-.0067	.046	-0.15	0.88
g_{21}	-.0233	.026	-0.88	0.37
g_{22}	.9227	.029	**32.37**	0.00
a_{11}	.4740	.095	**4.97**	0.00
a_{12}	.1659	.112	1.48	0.13
a_{21}	.0478	.067	0.71	0.47
a_{22}	.3378	.078	**4.31**	0.00
ν	3.348	.285	**11.52**	0.00
Maximized Loglikelihood=1891.81				

The basic model estimated is given by:

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

The MGARCH-BEKK model estimated was using the following specifications for the covariance matrix:

$$\begin{bmatrix} h_{11t} & h_{12t} \\ h_{12t} & h_{22t} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix}^T \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix} + \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}^T \begin{bmatrix} h_{11t-1} & h_{12t-1} \\ h_{12t-1} & h_{22t-1} \end{bmatrix} \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$$
$$+ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^T \begin{bmatrix} \varepsilon_{1t-1}^2 \\ \varepsilon_{2t-1}^2 \end{bmatrix} \begin{bmatrix} \varepsilon_{1t-1}^2 \\ \varepsilon_{2t-1}^2 \end{bmatrix}^T \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

The contributions to the loglikelihood function for the Student's *t* distribution are of the form:

$$lnL_t = C - \frac{1}{2} ln|H_t| - \left(\frac{\upsilon + 2}{2}\right) ln\left[1 + \left(\frac{1}{\upsilon - 2}\right) \varepsilon_t^T H_t^{-1} \varepsilon_t\right]$$

where the constant in the loglikelihood will be given by:

$$C = ln\Gamma\left(\frac{\upsilon + 2}{2}\right) - ln(\pi(\upsilon - 2)) - ln\Gamma\left(\frac{\upsilon}{2}\right)$$

Table 9: BEKK

	Variable	Estimate	S.E.	t-statistic
BA Equation	Constant	0.016	0.001	**10.60**
BAX Equation	Constant	0.020	0.002	**8.30**
Shape	ν	3.348	0.285	**11.74**
BA Conditional Variance Equation	Constant	1.7e-4	6.9e-5	**2.43**
	$\varepsilon^2_{1,t-1}$	0.225	0.093	**2.49**
	$\varepsilon_{1,t-1}\varepsilon_{2,t-1}$	0.045	0.056	0.81
	$\varepsilon^2_{2,t-1}$	0.002	6.4e-3	0.36
	$h_{11,t-1}$	0.827	0.072	**11.55**
	$h_{12,t-1}$	-0.042	0.050	-0.86
	$h_{22,t-1}$	5.4e-4	1.2e-3	0.44
BAX Conditional Variance Equation	Constant	3.9e-4	1.6e-4	**2.44**
	$\varepsilon^2_{1,t-1}$	0.028	0.037	0.74
	$\varepsilon_{1,t-1}\varepsilon_{2,t-1}$	0.112	0.052	**2.18**
	$\varepsilon^2_{2,t-1}$	0.114	0.053	**2.15**
	$h_{11,t-1}$	4.5e-5	6.1e-4	0.074
	$h_{12,t-1}$	-0.012	0.084	-0.14
	$h_{22,t-1}$	0.8514	0.053	**16.19**
Conditional Covariance of Returns Equation	Constant	1.8e-4	8.5e-5	**2.12**
	$\varepsilon^2_{1,t-1}$	0.079	0.067	1.17
	$\varepsilon_{1,t-1}\varepsilon_{2,t-1}$	0.168	0.032	**5.33**
	$\varepsilon^2_{2,t-1}$	0.016	0.026	0.62
	$h_{11,t-1}$	-6.1e-3	0.041	-0.149
	$h_{12,t-1}$	0.8394	0.020	**42.68**
	$h_{22,t-1}$	-0.022	0.024	-0.91

Notes: The parameters are from Equations (A5), (A6) and (A7) in the Appendix using the estimates from Table 7. Boldface indicates significance at the 5% level. The standard errors of the parameter estimates are calculated by the delta method of Greene (1993).

Table 10: Student's *t* vs. Normal

Test for $\frac{1}{\hat{v}} = 0$	BA BEKK	BA Constant Correlation	TB BEKK	TB Constant Correlation	Critical Value
t-test	13.67	13.30	13.59	13.22	1.96
Likelihood	919.82	976.16	778.44	909.42	3.84

Notes: These tests are discussed in Bollerslev (1987) and Engle and Bollerslev (1986). The Likelihood Ratio test is a one-degree of freedom test. A test statistic above the appropriate critical value indicates a rejection of the null hypothesis of Normality.

Table 11: OLS Hedge Models

	Constant	BAX	R^2	Q(22)	$Q^2(22)$	ARCH(5)
Bankers'Acceptances	-0.003	0.803	0.783	59.14	33.30	2.09
	(0.002)	(0.015)		[0.000]	[0.056]	[0.064]
Treasury Bills	-0.003	0.721	0.728	141.36	70.45	7.72
	(0.002)	(0.015)		[0.000]	[0.056]	[0.064]

Notes: Results of linear regressions. Q(22) is a test for residual serial correlation from the OLS Hedge model estimation. $Q^2(22)$ is a test for serial correlation in the squared residuals and ARCH(5) is a Lagrange Multiplier test for autoregressive conditional heteroskedasticity of order 5.

Table 12: Constant Correlation Treasury Bills

Student's *t* distribution				
	Estimate	Standard Error	T-Statistic	p-value
μ_1	.0133	.002	**7.68**	0.00
μ_2	.0177	.003	**6.44**	0.00
c_1	.0003	.000	**3.05**	0.00
c_2	.0005	.000	**2.68**	0.02
rho	.7400	.018	**43.00**	0.00
g_{11}	.8175	.028	**28.70**	0.00
g_{22}	.8613	.023	**36.91**	0.00
a_{11}	.1753	.042	**4.22**	0.00
a_{22}	.1437	.033	**4.32**	0.00
ν	3.200	.298	**10.66**	0.00
Maximized Loglikelihood = 1822.17				

The basic model estimated is given by:

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

The MGARCH-CC model estimated was using the following specifications for the covariance matrix:

$$\begin{bmatrix} h_{11t} \\ h_{22t} \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} g_{11} & 0 \\ 0 & g_{22} \end{bmatrix} \begin{bmatrix} h_{11t-1} \\ h_{22t-2} \end{bmatrix} + \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} \begin{bmatrix} \varepsilon^2_{1t-1} \\ \varepsilon^2_{2t-1} \end{bmatrix}$$

$$h_{12t} = \rho(\sqrt{h_{11t}}\sqrt{h_{22t}})$$

The contributions to the loglikelihood function for the Student's *t* distribution are of the form:

$$lnL_t = C - \frac{1}{2}ln|H_t| - \left(\frac{\upsilon+2}{2}\right)ln\left[1 + \left(\frac{1}{\upsilon-2}\right)\varepsilon_t^T H_t^{-1}\varepsilon_t\right]$$

where the constant in the loglikelihood will be given by:

$$C = ln\Gamma\left(\frac{\upsilon+2}{2}\right) - ln(\pi(\upsilon-2)) - ln\Gamma\left(\frac{\upsilon}{2}\right)$$

Table 13: BEKK Treasury Bills

Student's *t* distribution				
	Estimate	Standard Error	T-Statistic	p-value
μ_1	.0135	.002	**8.64**	0.00
μ_2	.0179	.003	**6.96**	0.00
c_{11}	.0113	.003	**3.83**	0.00
c_{12}	.0166	.006	**2.97**	0.01
c_{22}	-.0117	.004	**-3.35**	0.00
g_{11}	.9522	.028	**33.69**	0.00
g_{12}	.0355	.042	0.84	0.40
g_{21}	-.0421	.022	*-1.95*	0.05
g_{22}	.9061	.026	**34.94**	0.00
a_{11}	.3770	.088	**4.28**	0.00
a_{12}	.0140	.124	0.11	0.91
a_{21}	.0853	.065	1.31	0.19
a_{22}	.4130	.079	**5.20**	0.00
ν	3.241	.292	**11.10**	0.00
Maximized Loglikelihood=1822.38				

The basic model estimated is given by:

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

The MGARCH-BEKK model estimated was using the following specifications for the covariance matrix:

$$\begin{bmatrix} h_{11t} & h_{12t} \\ h_{12t} & h_{22t} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix}^T \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix} + \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}^T \begin{bmatrix} h_{11t-1} & h_{12t-1} \\ h_{12t-1} & h_{22t-1} \end{bmatrix} \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$$

$$+ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^T \begin{bmatrix} \varepsilon^2_{1t-1} \\ \varepsilon^2_{2t-1} \end{bmatrix} \begin{bmatrix} \varepsilon^2_{1t-1} \\ \varepsilon^2_{2t-1} \end{bmatrix}^T \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

The contributions to the loglikelihood function for the Student's *t* distribution are of the form:

$$lnL_t = C - \frac{1}{2} ln|H_t| - \left(\frac{\upsilon + 2}{2}\right) ln\left[1 + \left(\frac{1}{\upsilon - 2}\right) \varepsilon_t^T H_t^{-1} \varepsilon_t\right]$$

where the constant in the loglikelihood will be given by:

$$C = ln\Gamma\left(\frac{\upsilon + 2}{2}\right) - ln(\pi(\upsilon - 2)) - ln\Gamma\left(\frac{\upsilon}{2}\right)$$

Table 14: Treasury Bills

	Variable	Estimate	S.E.	t-statistic
TB Equation	Constant	0.135	0.002	**8.63**
BAX Equation	Constant	0.179	0.003	**6.98**
Shape	ν	3.240	0.292	**11.10**
TB Conditional Variance Equation	Constant	1.3e-4	6.7e-5	*1.91*
	$\varepsilon^2_{1,t-1}$	0.142	0.067	**2.13**
	$\varepsilon_{1,t-1}\varepsilon_{2,t-1}$	0.064	0.038	*1.71*
	$\varepsilon^2_{2,t-1}$	0.007	0.011	0.66
	$h_{11,t-1}$	0.907	0.054	**16.85**
	$h_{12,t-1}$	-0.080	0.043	*-1.86*
	$h_{22,t-1}$	1.8e-3	1.8e-3	0.97
BAX Conditional Variance Equation	Constant	4.1e-4	1.6e-4	**2.65**
	$\varepsilon^2_{1,t-1}$	1.9e-4	0.003	0.06
	$\varepsilon_{1,t-1}\varepsilon_{2,t-1}$	0.012	0.071	0.16
	$\varepsilon^2_{2,t-1}$	0.171	0.066	**2.60**
	$h_{11,t-1}$	0.001	0.003	0.42
	$h_{12,t-1}$	0.064	0.074	0.87
	$h_{22,t-1}$	0.821	0.047	**17.47**
Conditional Covariance of Returns Equation	Constant	1.9e-4	8.2e-5	**2.27**
	$\varepsilon^2_{1,t-1}$	0.005	0.048	0.11
	$\varepsilon_{1,t-1}\varepsilon_{2,t-1}$	0.157	0.032	**4.93**
	$\varepsilon^2_{2,t-1}$	0.033	0.033	1.08
	$h_{11,t-1}$	0.034	0.041	0.83
	$h_{12,t-1}$	0.861	0.015	**57.05**
	$h_{22,t-1}$	-0.038	0.019	**-2.06**

Notes: The parameters are from Equations (A5), (A6) and (A7) in the Appendix using the estimates from Table 7. Boldface indicates significance at the 5% level. Italicized indicates significance at 10% level. The standard errors of the parameter estimates are calculated by the delta method of Greene (1993).

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