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The Structure of Interest Rates in Canada: Information Content about Medium-Term Inflation

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Abstract

This paper examines the relationship between the term structure of interest rates and future changes in inflation for Canada using a newly constructed par-value yield series. The main conclusion of the empirical work is that the slope of the nominal term structure from 1- to 5-year maturities is a reasonably good predictor of future changes in inflation over these horizons. This result is similar to that obtained for the United States and other countries.

Results for models that also include competing indicators of inflation suggest that the medium-term structure of interest rates contains unique information about future inflation. Although there is additional information about future changes in inflation in M2+, commodity prices, and the output gap, this does not affect the predictive content of the medium-term structure.

Résumé

Les auteurs examinent la relation entre la structure des taux d' intérêt et l'évolution future de l'inflation au Canada en se servant d'une nouvelle série de taux de rendement construite au moyen des valeurs nominales. La principale conclusion qu'ils tirent de cet examen empirique est que la pente de la courbe des rendements (mesurés en termes nominaux) pour les échéances de 1 à 5 ans est un indicateur relativement fiable de l'évolution future de l'inflation à ces horizons. Différentes études réalisées au sujet des États-Unis et d'autres pays aboutissent à la même conclusion.

Selon les résultats obtenus à l'aide de modèles faisant intervenir d'autres indicateurs de l'inflation, la structure des taux d'intérêt à moyen terme recèle des renseignements distincts au sujet de l'évolution future de l'inflation. Bien que l'agrégat M2+, les prix des produits de base et l'écart de production aident aussi à prévoir l'inflation à venir, la valeur informative de la structure des taux d'intérêt à moyen terme est indépendante de celle de ces variables.

CONTENTS

1 Introduction

Recent empirical research for the United States and other countries has found a close relationship between the slope of the term structure of interest rates beyond one year and future changes in medium-term inflation. This is in strong contrast to the shorter maturities, where changes in short-term interest rates tend to reflect changes in real interest rates and the stance of monetary policy more than they reflect changes in expected inflation. In this paper, we examine whether the strong relationship between the medium-term structure of interest rates and future changes in inflation holds for Canada using a newly constructed par-value yield series. Since monetary policy actions affect economic activity and inflation with a significant lag, the changes in inflation beyond one year are the most relevant for policy.

The theoretical framework for this research is the well-known Fisher hypothesis, which states that nominal interest rates adjust one-for-one with expected inflation. The Fisher hypothesis, under rational expectations and the assumption of a constant slope of the real term structure over time, implies that the nominal term structure of interest rates should be associated with changes in future inflation. However, the slope of the real term structure is not constant but stationary, so that the relationship between the nominal term structure and changes in expected inflation will not necessarily hold in any particular short run; that is, real shocks can have temporary effects on the slope of the term structure of interest rates.

The main conclusion of our empirical work is that the slope of the nominal term structure from 1- to 5-year maturities contains considerable information about future changes in inflation over these horizons. For example, today’s 5-year rate minus today’s 1-year rate is a reasonably good predictor of the average inflation rate over the next five years minus the average inflation rate over the next year. Furthermore, this information about future changes in inflation is unique to the medium-term structure of interest rates. When the constraints of the rational expectations theory of the yield curve are relaxed to allow for information from other indicators of inflation (such as M2+ and the output gap), the slope of the medium-term structure of interest rates retains its predictive power for future changes in inflation.

The following section outlines the basic theory underlying the estimated equations. Section 3 briefly reviews the recent research and Section 4 describes the construction of the data set and the properties of the data. Section 5 discusses some econometric problems and presents the estimation results for the basic term-structure model. Section 6 presents the estimation results for term-structure models that also include competing indicators of inflation, and Section 7 analyses the forecasting performance of the models. The final section summarizes the estimation results and discusses how the term-structure models may be used to monitor inflation expectations and to forecast future changes in inflation.

2 The Basic Model of the Term Structure and Inflation

This first part of this section outlines the theoretical relationship between the term structure and future changes in inflation based on the Fisher equation and the rational expectations theory of the term structure of interest rates. This is the methodology used in the literature and the interpretation used in this paper. The second part briefly outlines an alternative interpretation, which is based on a simple macroeconomic framework.

2.1 The Expectational Interpretation

The Fisher equation relates nominal interest rates, real interest rates and inflation, so that expected inflation over *k* years is equal to the *k*-year nominal interest rate minus the *k*-year real interest rate

$$E_t(\pi_{k,t}) = i_{k,t} - r_{k,t},$$

where E_t is expectations at time *t*, $\pi_{k,t}$ is the inflation rate from time *t* to $t+k$, $i_{k,t}$ is the *k*-year nominal interest rate at time *t*, and $r_{k,t}$ is the *k*-year (ex ante) real interest rate. Assuming that expectations are rational, the realized inflation rate over the next *k* years is

$$\pi_{k,t} = E_t(\pi_{k,t}) + \varepsilon_{k,t},$$

where $\varepsilon_{k,t}$ is the forecast error over the *k* years with a mean value of zero. These errors will be autocorrelated with monthly data because of overlapping observations. Substituting into the Fisher equation above yields

$$\pi_{k,t} = i_{k,t} - r_{k,t} + \varepsilon_{k,t}.$$

In order to examine the information in the term structure of interest rates, the equation for the n-year inflation rate is subtracted from the equation for the k-year inflation rate to yield an equation for the future change in inflation

$$\pi_{k,t} - \pi_{n,t} = i_{k,t} - i_{n,t} - r_{k,t} + r_{n,t} + \varepsilon_{k,t} - \varepsilon_{n,t}.$$

Following Mishkin and others,[1] we assume that the slope of the real term structure is constant and estimate the following regression equation for the change in inflation

$$\pi_{k,t} - \pi_{n,t} = \alpha_{k,n} + \beta_{k,n}(i_{k,t} - i_{n,t}) + \mu_t.$$ [2]

The $\beta_{k,n}$ coefficient in this regression reveals how much information there is in the slope of the nominal term structure of interest rates about future changes in inflation. A value of $\beta_{k,n}$ statistically different from zero provides evidence that the term structure contains significant information about future changes in inflation and that slopes of nominal and real term structures do not move one for one with each other. A value of $\beta_{k,n}$ statistically different from one indicates that the slope of the real term structure of real interest rates is not constant over time and that the nominal term structure contains information about the real term structure.

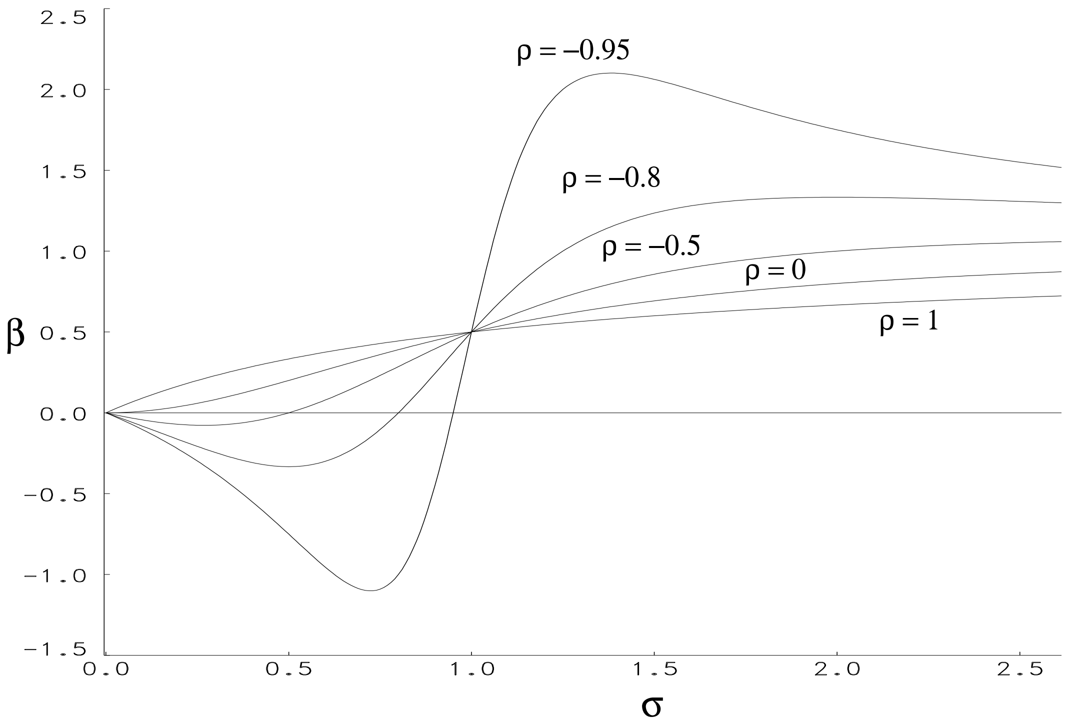
An interpretation of the size of the β-coefficients follows from the chart below. It plots the theoretical relationship between the β-coefficient on the term structure under the assumption of rational expectations, the ratio of the standard deviations of the change in expected inflation and of the slope of the real term structure σ, and the correlation between the change in inflation and the slope of the real term structure ρ.[3]

As the variability of inflation increases relative to that of the real term structure, the coefficient on the term structure increases. Consequently, since inflation rates are less variable than real interest rates in the short run,

1. See Gerlach (1995), Jorion and Mishkin (1991), and Mishkin (1990a; 1990b; 1991).
2. The error term for this equation contains a stochastic element if the slope of the real term structure is not constant over time.
3. This simple relationship falls out of the standard formula for the β-coefficient in a forecast equation under the assumption of rational expectations, which sets the covariance of the inflation forecast error with the slope of the real term structure equal to zero. See Gerlach (1995), Hardouvelis (1988) and Mishkin (1990a; 1990b; 1991).

coefficients on the term structure can be expected to be smaller for shorter-term maturities than for longer maturities. Furthermore, the coefficient is larger for higher correlations between the change in expected inflation and the slope of the real term structure.

$$\beta = (\sigma^2 + \rho\sigma)/(1 + \sigma^2 + 2\rho\sigma) \ .$$



For example, the coefficient on the term structure would be expected to be larger for the period of rising inflation during the 1970s than for the period since the early 1980s. The coefficient would also be larger during the earlier period because of the large oil-price shocks, which probably increased the negative correlation between the change in expected inflation and the slope of the real term structure. A value above 1 for the ratio of the standard deviations and a value close to -1 for the correlation coefficient would give slope coefficients of 1.5 to 2.0.

2.2 A Causal Interpretation

The theoretical relationship between the structure of interest rates and changes in medium-term inflation presented above is based on the assumption that the medium-term structure reflects agents' rational expectations of future changes in inflation. An alternative interpretation is that changes in the slope of the term structure cause future changes in inflation because the slope proxies the stance of monetary policy.

Recent empirical research by Cozier and Tkacz (1994) for Canada, Estrella and Hardouvelis (1991) for the United States, and Harvey (1991) and Hu (1993) for the G-7 countries finds that the slope of the term structure is a good predictor of future real economic activity and inflation. The common explanation for this strong link is that the spread between short- and long-term rates is a good measure of the real interest rate relative to its equilibrium. The term spread is viewed as an indicator of monetary stance, since monetary actions mainly affect the short-term rate.

The link with real economic activity exists because aggregate demand depends on changes in the slope of the real term structure through an IS-curve relationship with a lag of several quarters. The changes in the level of real output relative to the level of potential output in turn affect inflation through an augmented Phillips-curve relationship, which allows for a short-run trade-off between output and inflation.

3 Previous Research

Much of the empirical literature on the relationship between the term structure of interest rates and inflation expectations is associated with the work of Mishkin and Fama. Overall, this research finds that the term structure beyond one year contains considerable information about future inflation, while the term structure less than one year contains very little information.

Mishkin (1990a) finds for the United States that the difference between yields on maturities of Treasury bills from 6-12 months contains some information about the path of future inflation, while the term structure for maturities of 6 months or less contains almost no information. The fit of his equations is modest, predicting less than 10 per cent of the change in inflation. For Canada, Mishkin (1991) also finds that the maturities of less than 12 months provide virtually no information about future inflation.

Applying the same methodology to the term structure of maturities from 1-5 years, Mishkin (1990b) finds significant information content about future inflation, with the term spread predicting as much as 45 per cent of the change in inflation over the 3- and 4-year horizons in the 1980s. The coefficients on the term spreads are not significantly different from 1, suggesting that the nominal term structure over this maturity range contains very little information about the real term structure. Fama (1990) similarly finds that the spread between the 1- and 5-year yields predicts as much as 30 per cent of the 2- and 3-year changes in inflation.

Jorion and Mishkin (1991) extend the research on the medium-term structure to Germany, the United Kingdom, and Switzerland, and similarly find strong evidence that the medium-term structure has significant ability to forecast changes in inflation. More recently, Gerlach (1995) extends the work for Germany out to the 10-year maturity and finds considerable information about future changes in inflation, with spreads against 2-year rates being more informative than spreads against 1-year rates.

The results of this research on the slope of the term structure and medium-term inflation are consistent with the work by Fama and Bliss (1987) on the rational expectations theory of the medium-term structure of interest rates. They found that spreads for maturities longer than one or two years have some predictive content for movements in future interest rates.

4 Data Description and Analysis

4.1 Theoretical Par-Value Yields

Previous research for Canada on the term structure of interest rates used the readily available average maturity-class yield series of 1-3 years, 3-5 years, 5-10 years, and over 10 years.[4] Since the series is based on simple averages of the bond yields within a given maturity class, they potentially can suffer from large swings within-class average term to maturity.

One of the contributions of our empirical work is the estimation of a theoretical par-value or constant-maturity yield series for Canada going back to 1967.[5] The yield series were estimated on a data set made up of end-of-month (last Wednesday) observations for all domestic-pay Government of Canada bonds. The bond price data were originally collected from the

4. See Browne and Manasse (1989) and Hardouvelis (1994).
5. Currently, the Bank of Canada maintains a theoretical par-value series for all maturities out to 30 years on a daily frequency going back to July 1985.

Bank of Canada Review (Table G7) by Rose and Schworm (1980) for the period 1967-79, and were updated by Boothe (1987) for the period 1980-83 and by the Bank of Canada for the period thereafter.

Hypothetical par-value yield curves were estimated for each month with a spline-function model that was developed by Bell Canada (1969) and modified by the Bank of Canada to estimate a daily yield curve.[6] A sample of "close-to-par" bonds chosen by a number of filters is used for the estimations.

4.2 Inflation Data

The consumer price index (CPI) is the appropriate measure of inflation since it is the one that is most closely watched by the bond market. The term-structure model of inflation was estimated over the period January 1967 to February 1995. The CPI measure excludes food and energy and, for the period since 1984, also excludes the effects of indirect taxes.[7]

The estimations use the cumulative inflation rate, expressed at continuously compounded annual rates, for the forecast horizons out to 5 years

$$\pi_{k,t} = (100/k)[\log(P_{t+k}/P_t)],$$

where k= 1, 2, 3, 4, or 5 years and P is the level of the consumer price index.

Even though monthly inflation is only known for certain when the data is published in the following month, the inflation data for the month was aligned with the interest rate data for last Wednesday of the month. This alignment reflects the view that the market has developed a fairly good forecast of the current level of inflation by the end of the month.[8] Thus, the expected 1-year-ahead inflation rate for January 1982 is the change in the CPI from January 1982 to January 1983; the 2-year-ahead inflation rate is from January 1982 to January 1984; and so on. Since the last observation is February 1995, the last observation for the regression with a 2-year horizon is February 1993, and for the 5-year horizon it is February 1990.

6. A more parsimonious specification developed by Echols and Elliot (1979) was used for some months in 1980-81 when there were not many observations.
7. Inflation data excluding the effects of indirect taxes are only available from 1984.
8. Preliminary estimations with the end-of-the month interest rate data aligned with the previous month's inflation data were very similar to those presented in the paper.

4.3 Properties of the Data

The slope of the term structure of interest rates and changes in 2- to 5-year-ahead inflation relative to 1-year-ahead inflation are presented in Chart 1. The charts show rather large movements in the change in inflation for the different forecast horizons over the sample period. The seesaw-type movements reflect the effects of the oil-price shocks in 1973 and 1979 and the subsequent monetary policy accommodation. For example, in the chart for the 3-year horizon, the change in inflation rises as the 3-year-ahead inflation rate is affected by the shocks and then drops abruptly as the 1-year-ahead rate is affected. The inflation series were also affected by a smaller negative oil-price shock in early 1986.

The charts indicate that the slope of the term structure missed the big swings in inflation that were associated with the oil-price shocks, which could not have been foreseen by the market. The charts also suggest that the market may have underestimated the longer-run effects of the shocks or, possibly, the subsequent monetary accommodation, since the term structure also appears to miss the periods of persistent high inflation following the shocks. The relationship appears to be much closer during the recent period of more stable inflation.

Table 1 presents the augmented Dickey-Fuller t-tests for unit roots for the spreads against the 1-year interest rate out to 5 years and changes in inflation relative to the 1-year rate out to 5 years for the full sample from 1967.[9] The autoregressive order for each test statistic was based on a recursive t-statistic procedure with a 5 per cent critical level for choosing the last lag. The ADF statistics for the full sample suggest that the slope of the nominal term structure and changes in inflation follow a mean-stationary process at the 5 per cent level. This also suggests that the real term structure was mean-stationary over the sample period.

9. Monte Carlo evidence suggests that ADF t-tests perform quite well under the null even when the process generating the error includes a large (negative) moving-average component (Davidson and Mackinnon 1993).

5 Empirical Results for the Basic Term-Structure Model

5.1 Some Econometric Issues

This section discusses the correction for the serial correlation of the forecast errors mentioned earlier and the problem of small-sample bias due to the large data overlap.

The problem of serial correlation of the error terms in these regression equations arises because the monthly frequency of the data is finer than the forecast horizon for inflation and the forecast errors are realized only at the end of the forecast horizon. Although rational expectations rule out any correlation between the forecast error of inflation over the k years ($\varepsilon_{k,\,t}$) and the variables entering the conditioning information set at time t, they do not rule out autocorrelation in the forecast errors realized from t+1 to t+k since they do not enter the conditioning set. Consequently, these error terms are likely to follow a moving-average process of order (12k-1) with monthly data.[10] This problem is corrected in the estimations with the Newey and West (1987) adjustment methodology.

The corrected standard errors will in general lead to correct inference asymptotically. However, as the degree of overlapping increases with the longer forecast horizons, the number of fully independent observations diminishes. Consequently, differences between finite-sample distributions and asymptotic distributions may be large when the data have a large moving-average component,[11] ranging from 23 periods with monthly observations for a 2-year forecast horizon to 59 periods for a 5-year horizon.

Jorion and Mishkin (1991) identified the possible magnitude of the small-sample bias using Monte Carlo test simulations. They found that a standard 5 per cent critical value from the asymptotic distribution wrongly rejects the null hypothesis on β-coefficients about 20 per cent of the time for the 2-year forecasting horizon and about 50 per cent of the time for the 5-year horizon. They found that the critical values for t-tests from Monte Carlo simulations on the β-coefficient at the 5 per cent level range from 3.5

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10. The error term, μ_t, could be serially correlated for lags greater than (12k-1) if the slope of the real term structure is not constant.
11. See Richardson and Stock (1989), and Huizinga and Mishkin (1984).

for k=2 years to 5.0 for k=5 years. These critical values are based on estimates from four countries on the same model as above for a 15- year period. In the estimations presented below, the t-statistics on the estimated slope coefficients for the recent sample period are considerably larger than the critical values that are calculated by Jorion and Mishkin (1991).

5.2 Estimation Results

The estimates of the equations for the future change in inflation for horizons out to 5 years for the period 1967 to 1990-93 are presented in Table 2. The results indicate that there is information in the medium-term maturity structure about future inflation, with the $\bar{R}^2$ rising from 0.09 for the 2-year horizon to 0.33 for the 5-year horizon. This is in sharp contrast to the lack of predictive power in money market rates for Canada (Mishkin 1991).

The coefficients on the term spreads are all positive as expected and increase with the term to maturity, a result similar to that found for other countries. The coefficients rise from 0.87 for the 2-year horizon to 1.73 for the 5-year horizon. The t-statistics based on Newey-West standard errors suggest that β-coefficients are significantly greater than zero for all horizons. Furthermore, the β-coefficients are not significant different from one for all horizons, suggesting that the slope of the real term structure was constant for the sample period from 1967. The constant term is insignificant for all horizons, suggesting that real rates over the 1- to 5-year horizon were not statistically different over this period; that is, the real term structure was flat over this maturity range.

The rolling Chow tests on the forecast residuals presented in Chart 2 indicate parameter instability in the early 1970s and through most of the 1980s for all horizons. The instability appears to be associated mainly with oil-price shocks. These shocks were concentrated over a few quarters and triggered higher levels of inflation for a number of years. The price shocks were not forecastable and the failure of the term structure to predict them is expected. In order to exclude the errors associated with the unforecastable oil-price shocks, slope dummy variables were introduced into the forecast equations. The introduction of oil-price dummy variables follows in spirit Perron (1991), who argues that oil price shocks are exogenous and not a realization of the underlying data-generating mechanism of a series. The oil-price dummy variables take out the bias due to these the unforeseen shocks so that the term structure can more closely reflect inflation expectations.

Inspection of the price of oil suggested that the first shock covered the period from about 1973M06 to 1974M12 and the second from 1979M05 to 1980M03. The dummy variables equal one when inflation rates from 1- to 5-years ahead fall within these periods. The results presented in the final panel in Table 2 indicate that the β-coefficients are now noticeably lower for all horizons and much closer to one for the 3- to 5-year horizons.[12]

The forecast equations were re-estimated for a more recent period in order to ascertain the information content in the term structure of interest rates when inflation and/or real interest rates were not subjected to large and unforeseen shocks. The results presented in Table 3 are for the total CPI, excluding food, energy and the effects of indirect taxes, for the period 1984 to 1990-93.[13] The coefficients are highly significant and closer to one across all horizons.[14] The $\bar{R}^2$ s peak at 0.59 at the 3- and 4-year horizons, much higher than the peak of 0.33 for the full sample. The negative constant terms are now significant for the three longer horizons, suggesting that term premiums increased with the term to maturity over the recent period.

6 Augmented Term-Structure Models of Inflation

6.1 Inflation and Forecast Errors

Although we find evidence of a one-to-one relationship between the nominal term structure and changes in inflation for Canada, the relationship will not necessarily hold in any particular short run because the real term structure over this maturity range is not constant, but stationary; that is, stochastic disturbances to real rates can have temporary effects on the nominal term structure of interest rates. In addition, the one-to-one relationship may not hold if the market misses important information about inflation. An attempt to capture deviations in the one-to-one relationship between the term structure and changes in inflation is made by introducing

12. The first dummy variable was significant for all horizons and the second one for the two longer horizons.
13. The results for the total CPI and the CPI excluding food and energy are quite similar.
14. Although the slope of the term structure tested I(1) over this period, the slope and the change in inflation were cointegrated for all horizons, with slope coefficients not statistically different from one. These results are available upon request.

additional information and short-run dynamics into the basic equations.

As a preliminary test of the extent to which information other than the nominal term structure may be important (perhaps because of the stochastic portion of real rates and / or because the market does not incorporate all relevant information in predicting inflation), the current level of inflation and lagged forecast errors are introduced into the term-structure equations.

The term-structure equation with the current level of inflation is

$$\pi_{k,t} - \pi_{1,t} = \alpha_k + \beta_k(i_{k,t} - i_{1,t}) + \gamma_k \pi_t + \varepsilon_{k,t},$$

where π_t is the year-over-year inflation rate at time t. The current level of the inflation rate also proxies lagged inflation in this forecast equation. The estimates presented in Table 4 indicate that the current level of inflation captures important information about the change in medium-term inflation that is missed by the term structure spreads. The $\bar{R}^2$s are much higher for all forecast equations and the t-statistics on the level of inflation are highly significant. The size and significance of the β-coefficients for those horizons are not materially affected by the presence of the level of inflation. The negative sign suggests that the level of inflation has a tendency toward mean reversion over these horizons.

The basic equation with the forecast errors realized at time t is

$$\pi_{k,t} - \pi_{1,t} = \alpha_k + \beta_k(i_{k,t} - i_{1,t}) + \gamma_k e_{k,t-k} + \varepsilon_{k,t},$$

where $e_{k,t-k}$ is the inflation forecast error for the k-year inflation rate from time t-k that is realized at time t minus the 1-year inflation rate from time t-k. The results presented in Table 5 indicate that the information content in the forecast errors for the 3- and 4-year horizons increased the predictive content of the term-structure equations for those maturities by around 50 per cent. As with the level of inflation, the nominal term structure retains its predictive content for future changes in inflation.

6.2 Financial and Real Variables

In this section, competing indicators of inflation are introduced into the basic equations in an another attempt to identify relevant missing information.[15] These augmented regressions also allow for an assessment of the marginal predictive content of the medium-term structure of interest rates about inflation.

The competing financial variables include the M1 gap created from Hendry’s (1995) monthly long-run demand-for-money function for M1 (*m1gap*), M2+ (*m2+*) as in Clinton (1995), the Toronto Stock Exchange Index (*tse*), as a measure of asset prices, and the Bank of Canada Commodity Price Index (*bcpi*), as a measure of commodity prices.

The only real variable considered is the output gap (*ygap*), defined as the residual from a regression of monthly output against a linear and a quadratic time trend. GDP at factor cost was used since it is the only aggregate available on a monthly basis. The output gap is included in the basic model in order to assess the robustness of the expectational interpretation taken in the paper. The causal interpretation of the relationship between the term spread and the change in inflation stresses that the empirical link exists because the term spread affects future inflation via the short-run trade-off against the output gap. In this case, the presence of the output gap would be expected to affect the predictive content of the nominal term spreads.

The augmented equations add other variables as follows

$$\pi_{k,t} - \pi_{1,t} = \alpha_k + \beta_k(i_{k,t} - i_{1,t}) + \sum_j \gamma_{k,j}(Z_{j,t} - Z_{j,t-p}) + \varepsilon_{k,t},$$

where the Zs are financial and real variables. These variables correspond to the levels of the M1 gap and the output gap, and to the growth rates for M2+, asset and commodity prices. Using the levels of the gap variables is consistent with an accelerationist view of inflation. These competing indicator variables are expressed in difference form since the term-structure equations are used to predict the k-year-ahead inflation rate relative to the 1-year-ahead rate. The values for p in $Z_{j,t-p}$ are chosen on the basis of fit.

The difference between the 3- and 1-year growth of the indicator variables provided the best fit for the two shorter horizons and the 2-year

15. Specifications that included short-run dynamics, such as an error-correction mechanism, did not noticeably improve the in-sample fit of the models.

difference generally did better for the longer horizons.[16] The results for the current level of the gaps are slightly better than those that also included lagged gaps.The estimation results are quite similar across forecast horizons. The results for the augmented model of the change in 3-year-ahead inflation relative to the 1-year-ahead inflation rate are presented in Table 6.

The term-structure spreads maintain their predictive content in the presence of the competing indicators of inflation in virtually all models for all forecast horizons. In addition, the size and significance of the β-coefficients are not generally affected, with virtually all of them remaining not statistically different from one. A significant amount of additional information about the future change in inflation is contained in the difference in the growth of M2+ for all horizons and of commodity prices for the 2- to 4-year horizons. The output gap contains additional information for the 3- and 5-year horizons. The difference in the growth of asset prices and the M1 gap are insignificant across all horizons.

In general, the improvement in fit due to the inclusion of the competing indicators of inflation is similar to that resulting from adding the current level of inflation. The improvement suggests that the market at times misses information about future inflation that is contained in these indicators. The improvement may also reflect that these variables capture fluctuations in the real rates.

7 Forecasting Performance

The estimation results in the previous sections focussed on in-sample goodness-of-fit. In this section, rolling regression tests are used to assess the ability of the term structure of interest rates to forecast out-of-sample. The first set of regressions focusses on the stability of the forecast errors of the medium-term structure. The second set assesses the incremental forecasting ability of the term structure relative to competing inflation indicators.

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16. The fit of the 2- and 3-year growth specifications is noticeably better than the difference between the current 1-year rate and the 3-year lag of the 1-year rate. However, the additional information in the 2- and 3-year growth rates is relatively small.

The stability of the forecast errors is assessed by calculating rolling root-mean-squared errors (RMSE). The period covered by each RMSE corresponds to the length of the forecast horizon.[17] This rolling test procedure reflects the way that bond market participants with specific horizons would likely evaluate their forecast errors at each point in time.

Chart 3 presents the out-of-sample RMSEs for forecasts of 2- to 5-year-ahead inflation relative to the 1-year-ahead inflation rate. The dates on the horizontal axis correspond to the date of the last forecast error used in the calculation, and for a 12*k*-month horizon the RMSE rolls over *k* years. For example, the first RMSE reported in the second panel for the 3-year horizon is for 36 consecutive 3-year-ahead forecasts and is dated 1982M12.

The rolling RMSEs indicate that the forecasting performance of the term structure has been relatively more stable and accurate since the late 1980s, especially for the 2- and 3-year horizons.

The incremental forecasting ability of the longer-term structure spreads is assessed by calculating RMSEs from recursive regressions of the basic term-structure model and an augmented model that also includes either the current level of inflation or the difference in the growth of M2+. Both variables were statistically significant across all forecast horizons. The recursive method involves making a *k*-year-ahead forecast, where $k = 2, 3, 4$, and 5, and then sequentially updating the sample by one month, re-estimating and forecasting *k* years ahead. The out-of-sample RMSEs for the recursive regressions are presented in the table below.

The RMSEs indicate that the errors for both the basic and augmented indicator models increase with the forecast horizon, especially beyond the 3-year horizon. Despite the better in-sample fit, the augmented model with the current level of inflation has slightly poorer forecast power for the two

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17. The models for all forecast horizons were estimated for a 10-year period from 1967 to 1976. The estimated coefficients from these regressions were used to generate a *k*-year-ahead forecast with the *k*-maturity term spread starting in 1977M01. The sample was then updated by one month, the model re-estimated, and a second forecast was generated from 1977M02. The process of updating the estimation and forecast was continued until there were 12*k*-consecutive, *k*-year-ahead forecasts. A RMSE was then calculated for this set of 12*k*-consecutive forecasts. Subsequent sets of *k*-year-ahead forecasts are generated and RMSEs calculated by dropping the first forecast in the previous set and adding a forecast for another month.

shorter horizons.[18] The model with the growth of M2+ has slightly better out-of-sample fit for the 3-year horizon.

Root Mean Squared Errors for 1989M01 to 1995M02

Indicator Models	$\pi_{2,t} - \pi_{1,t}$	$\pi_{3,t} - \pi_{1,t}$	$\pi_{4,t} - \pi_{1,t}$	$\pi_{5,t} - \pi_{1,t}$
$i_{k,t} - i_{1,t}$	0.41	0.57	0.92	1.04
$i_{k,t} - i_{1,t}$: $\beta = 1$	0.42	0.39	0.55	0.63
$i_{k,t} - i_{1,t}$, π_t	0.57	0.65	0.91	1.06
$i_{k,t} - i_{1,t}$, π_t : $\beta = 1$	0.60	0.62	0.92	0.98
$i_{k,t} - i_{1,t}$, m2+	0.38	0.47	0.89	1.05
$i_{k,t} - i_{1,t}$, m2+: $\beta = 1$	0.38	0.39	0.54	0.72

Variable *m2+* is the difference between the 3- and 1-year growth rates for the 2- and 3-year horizons and the difference between the 2- and 1-year rates for the longer horizons.

Nevertheless, there is a marked improvement in the forecasting power of the basic term-structure models beyond the 2-year horizon when the β-coefficients on the term spreads are restricted to equal one. The incremental forecasting power of M2+ at the 3-year horizon disappears with the restriction. Since the coefficients are noticeably greater than one (but not statistically different) for the full sample period, this restriction reduces the volatility of the term structure forecasts for the recent period of more stable inflation.[19]

8 Concluding Remarks

8.1 Summary of the Results

The estimation results for the period from 1967 to 1995M02 suggest that a relatively close relationship between the slope of the term structure

18. The better in-sample fit and poorer out-of-sample performance may be a symptom of "overfitting."
19. The slope coefficients that are estimated on the sample period since 1984 are much smaller and quite close to one, especially for the 3- to 5-year horizons (see Table 3).

and future changes in inflation exists in the Canadian data. The term-structure equations predict up to one-third of the variation in the changes in inflation out to 5 years. The forecasts based on estimations for the period from 1984 predict as much as 59 per cent of the variation in the future changes in inflation. The coefficients are highly significant and are not significantly different from one for both periods as predicted by the rational expectations theory.

Results for models that also include competing indicators of inflation suggest that the medium-term structure of interest rates contains unique information about expected inflation. Although there is additional information about future changes in inflation in M2+, commodity prices and the output gap, this does not affect the predictive content of the medium-term structure.

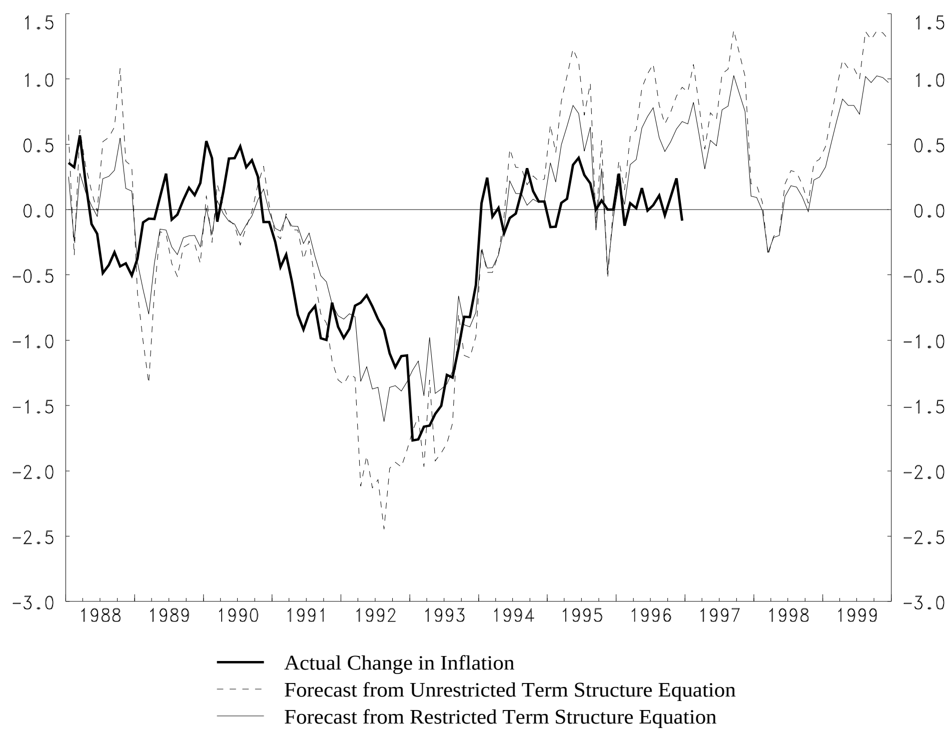
The out-of-sample performance of the term-structure models was assessed on a sample period that includes the effects of the large oil-price shocks in the 1970s. These shocks and the subsequent monetary policy accommodation were not anticipated by the market. Consequently, the forecasting performance is inferior to that of models estimated over a more recent period. Rolling out-of-sample forecasts indicate that the additional information in competing indicators does not noticeably improve the forecast performance. Rolling root-mean-squared errors that correspond to the length of each forecast horizon indicate that the forecast values have been generally more stable and accurate since the late 1980s.

8.2 Usefulness and Limitations as an Indicator

The overall conclusion from the estimation results is that a persistent change in the slope of the medium-term structure strongly indicates a change in expected inflation. Thus, if a steeper yield curve persists, inflation will likely rise several years in the future by the amount of the steepening. A negatively sloped term structure that persists indicates that inflation will decline.

The chart below plots the actual values of the change in inflation from 1 to 3 years from 1988 to late 1995 and the forecast values for both unrestricted and restricted versions of a term-structure model out to 1999. The models are based only on the spread between the 3- and 1-year yields. The out-of-sample values are generated from recursive regressions beginning in 1985 that are re-estimated after each monthly forecast. The intercept and slope coefficients in the unrestricted model are based on estimations on the sample from 1967. In the restricted model, the slope coefficient is constrained to equal one and the (negative) intercept term is re-estimated.

Actual and Forecast Change in Inflation
(3-year minus 1-year inflation rate)



The actual and forecast values from the models suggest that the term structure from 1 to 3 years tracks the change in inflation during periods when inflation is relatively stable as well as it does during periods when it is changing rapidly. The term structure in 1995 was predicting relatively stable inflation through 1998, while the term structure in 1996 was predicting inflation to accelerate in the coming years.

The restriction on the slope coefficient noticeably improves the out-of-sample performance of the term structure over the 1988-95 period, probably because of the absence of large supply-side shocks.

The medium-term structure, however, has some limitations as a predictor of inflation. Since the term spreads from 1- to 5-year maturities are not assumed to have a causal link with inflation, they cannot be expected to be useful when the economy faces large unforeseen shocks. In addition, when the term structure is responding to historically atypical information and to atypical expectations about changes in real returns or atypical term premiums, it might be less useful for predicting variations in inflation.

Chart 1

Change in Inflation and Slope of the Term Structure of Interest Rates

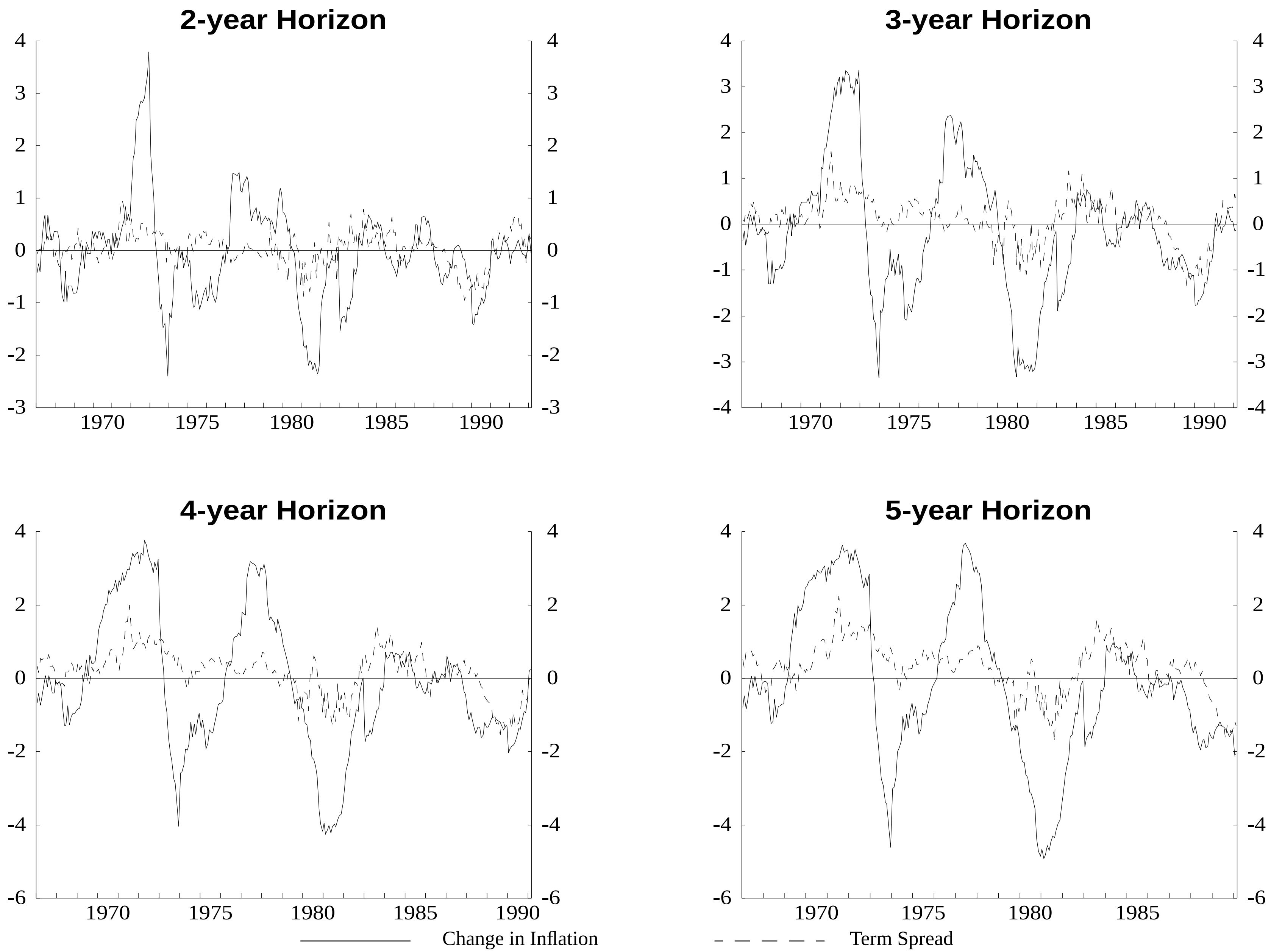
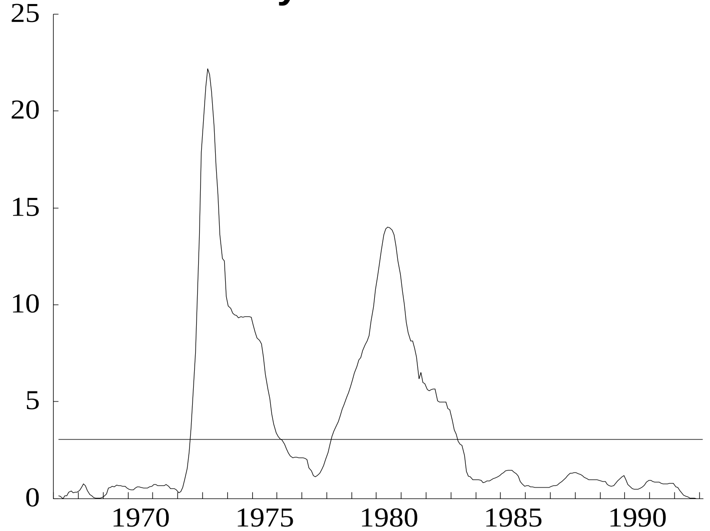
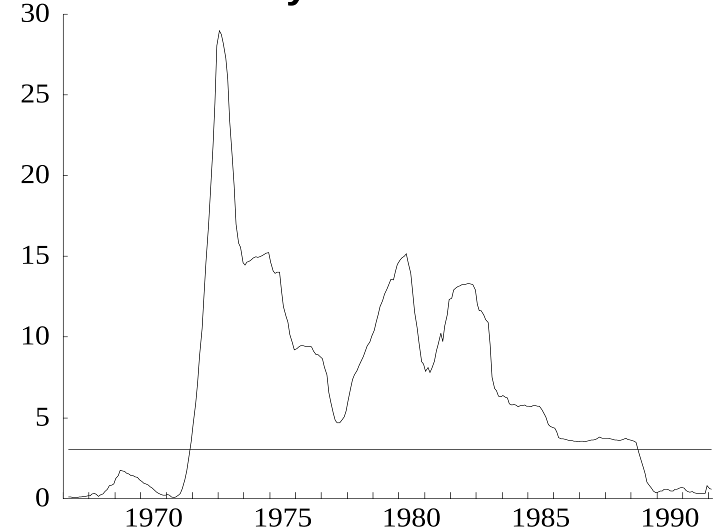


Chart 2
Rolling Chow Tests

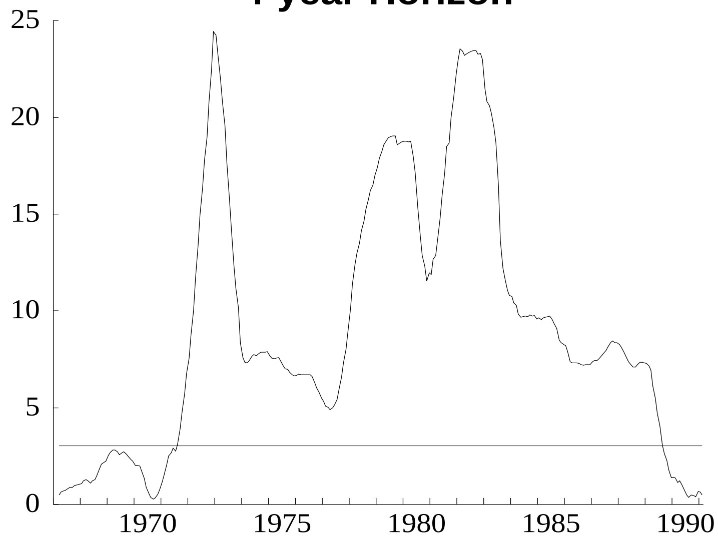
2-year Horizon



3-year Horizon



4-year Horizon



5-year Horizon

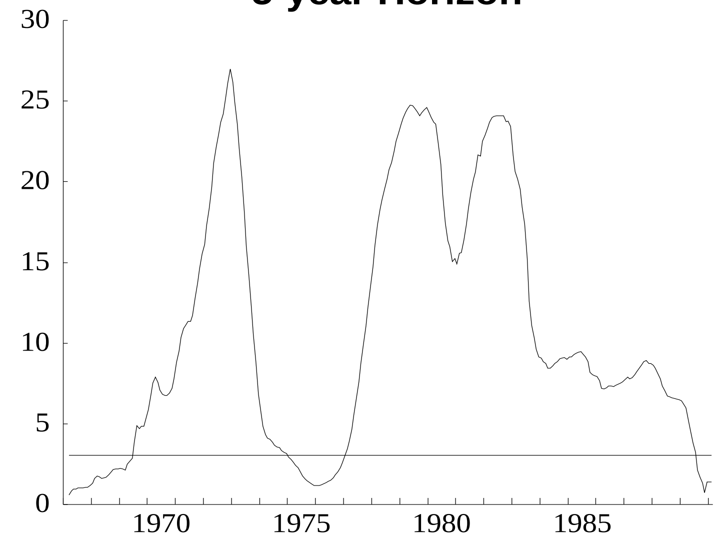


Chart 3
Rolling RMSEs

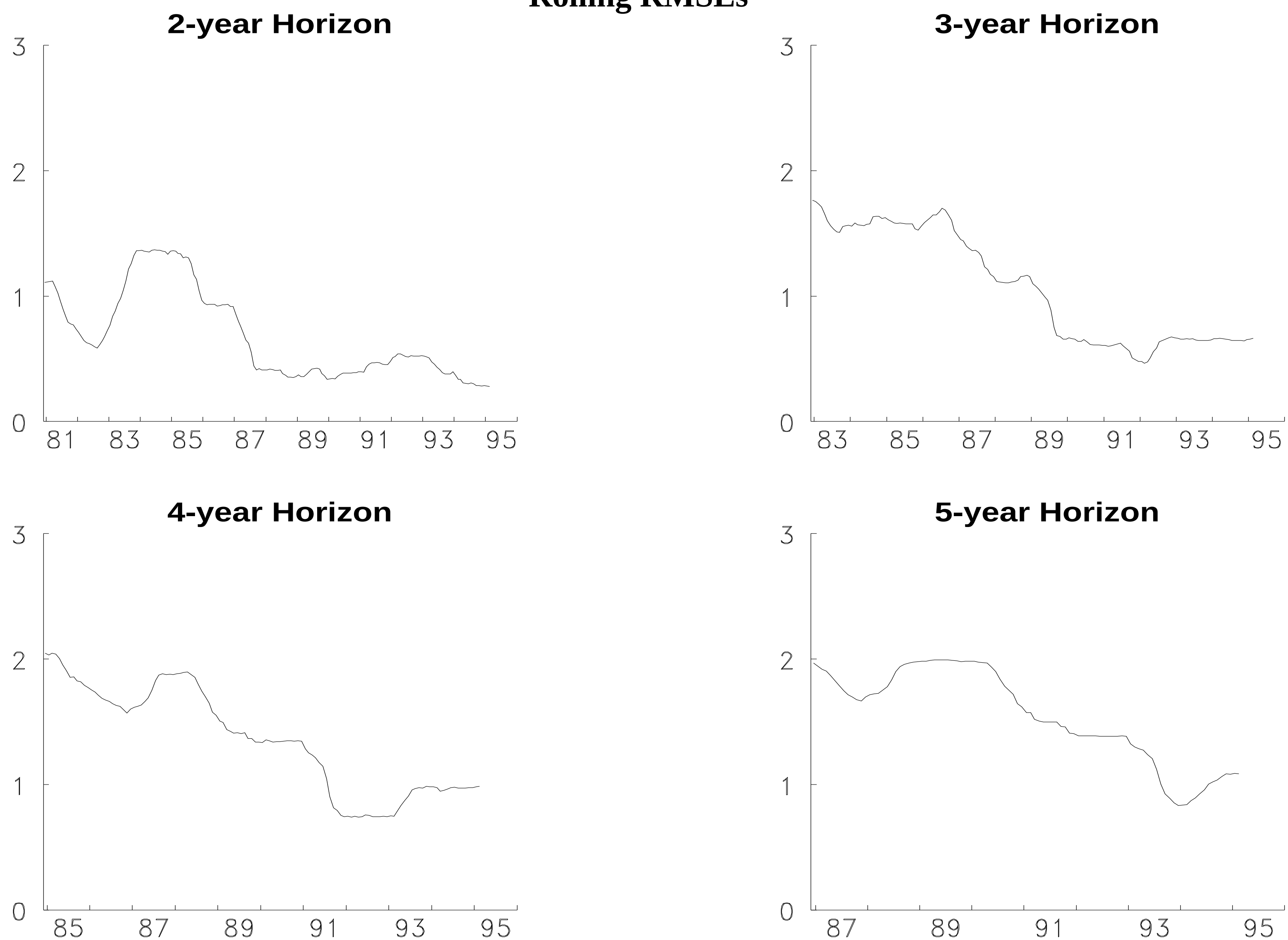


Table 1
Unit Root Tests for the Slope of the Term Structure of Interest Rates and the Change in Inflation

Series	W/O Trend	W/ Trend
1967-1995M02		
$i_{2,t}$ - $i_{1,t}$	-3.75*	-3.74*
$i_{3,t}$ - $i_{1,t}$	-3.14*	-3.14
$i_{4,t}$ - $i_{1,t}$	-3.24*	-3.28
$i_{5,t}$ - $i_{1,t}$	-3.28*	-3.27
$\pi_{2,t}$ - $\pi_{1,t}$	-3.33*	-3.57*
$\pi_{3,t}$ - $\pi_{1,t}$	-3.61*	-3.91*
$\pi_{4,t}$ - $\pi_{1,t}$	-3.63*	-3.94*
$\pi_{5,t}$ - $\pi_{1,t}$	-3.23*	-3.75*

* Rejection of the null hypothesis of non-stationarity at the 5 per cent level using the asymptotic critical values of 2.86 and 3.41 for the unit-root test without and with trend, respectively.

Table 2: 1967 to 1990-1993M02

$$\pi_{k,t} - \pi_{1,t} = \alpha_k + \beta_k (i_{k,t} - i_{1,t}) + \varepsilon_{k,t}$$

Horizon (k)	2-year	3-year	4-year	5-year
α	-0.08	-0.29	-0.43	-0.68
(t-stat)	(0.54)	(1.11)	(1.38)	(1.95)
β	0.87	1.41	1.66	1.73
(t-stat)	(2.27)	(3.31)	(4.13)	(4.60)
RBAR^2	0.09	0.22	0.31	0.33
SEE	0.89	1.25	1.51	1.69
β	1.0	1.0	1.0	1.0
(p-value)	(0.73)	(0.34)	(0.10)	(0.05)
$\left(\beta + \sum_i \beta_i D_i\right) : \beta$	0.54	0.86	1.41	1.29
(t-stat)	(1.88)	(2.85)	(4.29)	(3.14)
$\left(\beta + \sum_i \beta_i D_i\right) : \beta$	1.0	1.0	1.0	1.0
(p-value)	(0.10)	(0.64)	(0.21)	(0.51)

The D_i s are oil-price dummy variables that equal one when inflation rates out to 5-years ahead fall within the periods 1973M06-1974M12 and 1979M05-1980M03. T-statistics are based on Newey-West standard errors of the coefficients. P-values for the restriction $\beta = 1$ are distributed as $\chi^2(1)$.

Table 3: 1984 to 1990-1993M02

Horizon (k)	2-year	3-year	4-year	5-year
α	-0.08	-0.23	-0.36	-0.57
(t-stat)	(1.50)	(3.71)	(3.15)	(3.55)
β	0.66	0.87	0.90	0.85
(t-stat)	(3.94)	(10.66)	(16.06)	(7.44)
RBAR^2	0.32	0.59	0.59	0.50
SEE	0.38	0.41	0.78	0.63
β	1.0	1.0	1.0	1.0
(p-value)	(0.05)	(0.13)	(0.09)	(0.18)

Coefficient t-statistics are based on Newey-West standard errors of the coefficients. P-values for the restriction $\beta = 1$ are distributed as $\chi^2(1)$.

Table 4: 1967 to 1990-1993M02

$$\pi_{k,t}-\pi_{1,t} = \alpha_k + \beta_k(i_{k,t} - i_{1,t}) + \gamma_k \pi_t + \varepsilon_{k,t}$$

Horizon (k)	α (t-stat)	β (t-stat)	γ (t-stat)	$\bar{R}^2$
2 - year	-0.08	0.87		0.09
	(0.54)	(2.27)		
	0.51	0.70	-0.10	0.16
	(1.88)	(2.24)	(2.47)	
3 - year	-0.29	1.41		0.22
	(1.11)	(3.31)		
	0.97	1.12	-0.21	0.35
	(2.55)	(3.97)	(4.12)	
4 - year	-0.43	1.66		0.33
	(1.38)	(4.13)		
	1.19	1.35	-0.26	0.44
	(2.47)	(5.96)	(3.88)	
5 - year	-0.68	1.73		0.33
	(1.95)	(4.60)		
	1.03	1.40	-0.26	0.42
	(1.66)	(6.55)	(2.78)	

Table 5: 1969-72 to 1990-1993M02

$$\pi_{k,t}-\pi_{1,t} = \alpha_k + \beta_k(i_{k,t} - i_{1,t}) + \gamma_k e_{k,t-k} + \varepsilon_{k,t}$$

2 - year	-0.08	0.87		0.09
	(0.54)	(2.27)		
	-0.08	0.89	0.06	0.09
	(0.45)	(2.21)	(0.41)	
3 - year	-0.29	1.41		0.22
	(1.11)	(3.31)		
	-0.36	1.10	-0.21	0.41
	(1.69)	(3.19)	(4.12)	
4 - year	-0.43	1.66		0.31
	(1.38)	(4.13)		
	-0.67	1.43	-0.52	0.51
	(2.85)	(4.77)	(3.83)	
5 - year	-0.68	1.73		0.33
	(1.95)	(4.60)		
	-0.98	1.61	-0.07	0.31
	(2.43)	(3.79)	(0.68)	

Table 6
Indicator Model Regressions
Sample: 1975-1992M02

$$\pi_{3,t} - \pi_{1,t} = \alpha_3 + \beta_3(i_{3,t} - i_{1,t}) + \sum_j \gamma_{3j}(Z_{j,t} - Z_{j,t-3}) + \varepsilon_{3,t},$$

where Z_t is the current level of a gap or a 1-year growth rate and, for growth variables, Z_{t-3} is the 3-year growth rate at time t.

Model	α	β	m1-gap	ygap	m2+	tse	bcpi	π_τ	$\bar{R}^2$
1 (t-stat)	-0.38 (1.22)	0.94 (3.28)							0.14
2 (t-stat)	0.35 (1.19)	0.74 (3.57)	-3.86 (1.64)						0.21
3 (t-stat)	-0.37 (1.33)	1.13 (3.59)		7.69 (2.47)					0.17
4 (t-stat)	-0.27 (0.97)	1.11 (4.28)			0.19 (3.27)				0.22
5 (t-stat)	-0.40 (1.31)	0.85 (3.82)				0.01 (0.66)			0.14
6 (t-stat)	-0.28 (1.02)	0.87 (3.82)					0.05 (2.97)		0.28
7 (t-stat)	0.88 (1.79)	0.80 (3.56)						-0.20 (2.98)	0.30
8 (t-stat)	0.65 (1.55)	0.78 (3.06)	-1.15 (0.54)	-1.81 (0.44)	0.07 (0.97)	-0.00 (0.10)	0.04 (2.33)	0.11 (1.55)	0.36

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