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WEALTH, DISPOSABLE INCOME AND CONSUMPTION:

Some Evidence for Canada

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CONTENTS

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ABSTRACT

The author develops a measure of aggregate private sector wealth in Canada and examines its ability to explain aggregate consumption of non-durables and services. This wealth measure includes financial, physical and human wealth.

The author measures human wealth as the expected present value of aggregate labour income, net of government expenditures, based on a discrete state-space approximation of an estimated bivariate vector autoregression for the real interest rate and the growth rate of labour income, net of government expenditures. His general approach to measuring financial and physical (non-human) wealth is to consolidate the assets and liabilities of the various sectors of the economy so as to measure the net worth of the ultimate owners of private-sector wealth – households. To the extent possible, the author measures non-human wealth at market value.

The relationship between consumption and wealth is explored as a means both of gauging the usefulness of the wealth measures developed in this report and of improving on empirical consumption models for Canada. The principal empirical finding is that both wealth and disposable income are important determinants of consumption. Including wealth in the consumption function is particularly helpful in explaining the consumption boom of the late 1980s.

RÉSUMÉ

Dans le présent rapport, l'auteur élabore une mesure de la richesse globale du secteur privé au Canada et examine sa capacité d'expliquer la consommation globale de biens non durables et de services. Cette mesure englobe la richesse financière, la richesse physique et la richesse humaine.

L'auteur définit la richesse humaine comme la valeur actuelle anticipée du revenu global du travail, nette des dépenses publiques. Cette mesure est basée sur une approximation discrète d'espace-état d'une autorégression vectorielle à deux variables, estimée pour le taux d'intérêt réel et le taux de croissance du revenu du travail net des dépenses publiques. L'approche générale utilisée par l'auteur pour mesurer la richesse financière et la richesse physique (non humaine) consiste à regrouper l'actif et le passif des divers secteurs de l'économie de façon à obtenir une estimation de la situation nette des détenteurs ultimes de la richesse du secteur privé, à savoir les ménages. Dans la mesure du possible, l'auteur mesure la richesse non humaine à la valeur du marché.

L'auteur cherche à déterminer la relation entre la consommation et la richesse afin de pouvoir évaluer l'utilité des mesures de la richesse élaborées dans le présent rapport et d'améliorer les modèles empiriques de consommation au Canada. Le principal résultat empirique auquel il parvient est que tant la richesse que le revenu disponible constituent des déterminants importants de la consommation. L'inclusion de la richesse dans la fonction de consommation est particulièrement utile pour expliquer l'essor de la consommation à la fin des années 80.

1 INTRODUCTION

Empirical research on consumption behaviour over the past 15 or so years has been strongly influenced by two papers, both published in 1978. Hall (1978) examined the stochastic implications of the permanent income hypothesis, and Davidson et al. (1978) developed dynamic econometric models for aggregate consumption based on an error-correction (EC) specification.

Hall's innovation was to exploit the restrictions implied by the first-order conditions of a representative consumer's intertemporal optimization problem. The attractive feature of this approach is that the resulting Euler equations do not require the specification of all future variables relevant to the household's consumption decisions, so the difficulties associated with measuring wealth and obtaining closed-form relationships between consumption and wealth can be largely avoided.

These advantages are, however, achieved at some cost. While the estimation of Euler equations allows the empirical researcher to examine whether consumers adjust optimally between adjacent periods, it provides little direct evidence on the determinants of consumption or on how various random disturbances or policy changes are transmitted to consumption. Consequently, the Euler equation approach to modelling consumption is not particularly well-suited to policy analysis and forecasting. In addition, the stochastic permanent income model also suffers the practical problem that it is frequently rejected by the data (see, for example, Flavin 1981, Hayashi 1982, Nelson 1987, and Wirjanto 1991). These rejections are often interpreted as evidence that some consumers face binding liquidity constraints (see Hall and Mishkin 1982, Flavin 1985, Zeldes 1989, and Campbell and Mankiw 1990).

Partly as a result of these limitations, policy makers and practitioners in general have largely turned to the more eclectic EC consumption model proposed by Davidson et al. (1978). The EC model is not usually derived from the micro foundations of optimizing behaviour, but is proposed as a robust empirical relationship between macro aggregates that is formulated using both the cointegration analysis of Engle and Granger (1987) and the principles of dynamic econometric modelling as outlined in Hendry and Richard (1982). Error-correction consumption models have been estimated for a wide range of countries – see Davidson et al. (1978) and Davidson and Hendry (1981) for the United Kingdom, Harnett (1988) for the United States, Sawyer (1991) for Canada, Rossi and Schiantarelli (1982) for Italy, Steel (1987) for Belgium, and Thury (1989) for Austria.

The standard EC model follows a long tradition in macroeconomics of specifying a static or long-run relationship between consumption and disposable income, and then augments this long-run consumption function with additional variables aimed at explaining short-run deviations from the long-run trend. Some researchers have also included the real interest rate in the long-run consumption function (see Davidson and Hendry 1981 and Sawyer 1991), in part to capture wealth effects. Measures of liquid assets have also been included (Hendry and von Ungern-Sternberg 1981 and Patterson 1991) for the same reason. To a large degree, however, EC models have ignored the implication of the life cycle-permanent income hypothesis that consumption is determined by wealth or its flow equivalent, permanent income. This report attempts to redress this situation.

This report has two main goals: to develop a measure of aggregate private sector wealth in Canada that includes financial, physical and human wealth, and to examine the ability of this wealth measure to explain aggregate consumption. The relationship between consumption and wealth is explored both as a means of gauging the usefulness of the wealth measures developed in this report and to improve upon empirical consumption models for Canada. By augmenting the standard EC consumption model with a comprehensive measure of wealth, this study goes part way towards bridging the gap between life cycle–permanent income consumption equations, and the more empirically motivated EC consumption models based on disposable income.

Many important economic questions are connected with the behaviour of wealth. These include the effects of tax policies on savings, the impact of fluctuations in the prices of equity, bonds and housing on aggregate demand, and the consequences of changes in the level of foreign indebtedness. Comprehensive measures of wealth that include both human and non-human wealth have been previously constructed for Canada at an annual frequency by Irvine (1980) and Beach, Boadway and Bruce (1988). The more recent Beach, Boadway and Bruce study provides annual estimates of human and non-human wealth from 1964 to 1981 for each of 10 age groups. Human wealth is constructed by taking the present value of the projected after-tax earnings of each age group based on a time trend and their life-cycle earnings profile. Non-human wealth is constructed as the sum of the capitalized flow of capital income and the accumulated present value of tax-sheltered savings. Aggregate human wealth and non-human wealth are obtained by summing across age cohorts.

At a minimum, the current study provides alternative measures of human and non-human wealth that extend from 1963 through to 1993 at a quarterly frequency. Relative to the earlier work by Irvine and Beach, Boadway and Bruce, the construction of human wealth takes a more aggregated approach, which allows for greater recognition of the joint statistical properties of innovations in income and interest rates. Human wealth is computed as the expected present value of aggregate labour income net of government expenditures based on an estimated bivariate vector autoregression (VAR) for the real interest rate and the growth rate of labour income net of government expenditures. Owing to the non-linear nature of the present-value formula, the estimated VAR is approximated as a discrete-value finite-state Markov chain, thereby permitting expectations to be computed as a weighted sum over possible outcomes instead of as an intractable integral. Non-human wealth in the current study is based on actual stock data obtained largely from the national balance sheet accounts and adjusted to obtain market values, rather than on cumulated capital and savings flows, as in Beach, Boadway and Bruce.

The ability of the new wealth measures to explain consumption is explored in the context of a general consumption equation that includes both disposable income and wealth. Implicit in this approach is the presumption that the stock of wealth is exogenous to the portfolio-allocation problem so that the composition of wealth does not influence the consumption-savings decision.[1] The empirical analysis begins by examining the long-run relationship between consumption, wealth and disposable income using standard tests for cointegration, and provides asymptotically efficient estimates of the long-run propensities to consume out of wealth and disposable income using two alternative estimation procedures. This long-run analysis is complemented by estimating dynamic EC models for consumption that include wealth.

The principal finding is that wealth is a significant determinant of consumption, particularly in the long run, but disposable income continues to exert an important influence on consumption in both the short run and the long run. The influence of disposable income may be interpreted as evidence of liquidity constraints, but it could also be proxying in part for wealth itself. There is also some evidence that consumption responds differently to different components of wealth. This may reflect the different stochastic behaviour of alternative assets, the importance of consumer heterogeneity, or the absence of separability between the portfolio-allocation and consumption-savings decisions.

1. The alternative is to model the portfolio-allocation and consumption-savings decisions in an integrated framework as in Poloz (1986).

2 MEASURING WEALTH

In a competitive economy with perfect capital markets the representative consumer's wealth is the sum of his financial and physical assets net of liabilities (non-human wealth) plus the expected discounted value of his current and expected future after-tax labour income (human wealth). In an open economy, non-human wealth will generally include both domestic and foreign assets net of liabilities. Summing across consumers and dividing by the population yields aggregate per capita real wealth. If we define all aggregate variables in real per capita terms, real per capita wealth (*W)* can be written as:

$$W_t = A_t + D_t^d + (L_t - T_t) + E_t\left[\sum_{i=1}^{\infty}\prod_{j=1}^{i}\left(\frac{1}{1+r_{t+j}}\right)(L_{t+i} - T_{t+i})\right] \quad (2.1)$$

where A is net domestic and foreign physical and financial assets, D^d is domestic holdings of government debt, L is labour income, T is taxes net of transfers, r is the real interest (or discount) rate, and E_t is the expectations operator conditioned on information available at time t.

If we add the assumptions of intergenerational altruism and lump sum taxes, wealth is characterized by the well-known Ricardian equivalence proposition. For a given path for government expenditures and the foreign debt, wealth is invariant to the timing of taxes and the size of the government debt. Domestically held government debt nets out of wealth, since rational consumers realize that the value of the government debt they hold is offset by future tax liabilities. This can be demonstrated by combining (2.1) with the government's budget constraint. The government's lifetime budget constraint requires that the present value of taxes is sufficient to pay for the present value of government expenditures plus any outstanding government debt:

$$T_t + E_t\left[\sum_{i=1}^{\infty}\prod_{j=1}^{i}\left(\frac{1}{1+r_{t+j}}\right)T_{t+i}\right]$$
$$= D_t^d + D_t^f + G_t + E_t\left[\sum_{i=1}^{\infty}\prod_{i=1}^{i}\left(\frac{1}{1+r_{t+j}}\right)G_{t+i}\right] \tag{2.2}$$

where G is real government expenditures on goods and services and D^f is the government debt held by foreigners. Substituting (2.2) into (2.1) yields aggregate wealth as a function of government expenditures and foreign holdings of government debt:

$$V_t = A_t - D_t^f + (L_t - G_t)$$
$$+ E_t\left[\sum_{i=1}^{\infty}\prod_{i=1}^{i}\left(\frac{1}{1+r_{t+j}}\right)(L_{t+i} - G_{t+i})\right] \tag{2.3}$$

The measurement of wealth developed in this study adopts the definition of wealth given in (2.3). While (2.1) and (2.3) are equivalent, (2.3) provides a more convenient basis for measurement because it allows us to ignore difficult issues regarding the timing of future taxes and deficits. Wealth depends on real government expenditures and not on how they are financed.[2]

2.1 Human wealth

According to the definition of wealth given in (2.3), human wealth is the present discounted value of current and future labour income net of government expenditures. If we let $X = L - G$ and denote its growth rate by x, human wealth can be written as the product of current labour income net of government expenditures, and a term that captures the effects of expected future growth:

2. Admittedly, this representation of wealth does not address several real-world features. In particular, consumers and governments typically cannot borrow at the same interest rates, taxes are not lump sum, and some consumers may have finite horizons. The test will be whether (2.3) provides an empirically useful measure of wealth.

$$H_t = X_t \left[1 + E_t \left[\sum_{i=1}^{\infty} \prod_{j=1}^{i} \left(\frac{1 + x_{t+j}}{1 + r_{t+j}} \right) \right] \right] \equiv X_t \Gamma_t \qquad (2.4)$$

where H is human wealth and Γ represents the term in the outer square brackets. The problematic aspect of measuring human wealth is that the cumulative growth factor Γ is not directly observable, because it depends on expectations.

One approach to measuring Γ would be to develop a fully articulated macro model and then invoke the rational expectations hypothesis. This approach would be very ambitious. A more modest approach, and the one pursued in this study, is to model the behaviour of variables over which expectations must be taken using time-series techniques and then use the resulting time-series model to compute expectations of the future. This approach is implemented by estimating a bivariate VAR to characterize the joint distribution of the growth rate of labour income net of government expenditures (hereafter, net income) and the real interest rate. The estimated bivariate VAR is then approximated as a discrete-valued finite-state vector Markov chain and this approximated system is used to compute the expected discounted value of future net income growth.

2.1.1 A time series model for net income and the real interest rate

The first step is to define net income and the interest rate and to examine their time-series properties. All the variables used in this study together with the raw data sources are described in Appendix 1, but the most important details deserve some comment.

In an effort to get a more complete measure of aggregate labour income, the published labour income series is augmented to include labour income in the farming and unincorporated business sectors. It is assumed that the share of labour in these two categories is the same as for the overall economy.

Government expenditure on goods and services is measured as the reported quarterly government expenditure series less the fraction of the reported series which has been paid historically by corporations, non-residents and government interest income. The decision to subtract these components from the reported government expenditure series reflects an attempt to obtain a measure of government expenditures that accurately reflects the tax liabilities of households. The resulting government expenditure series was then smoothed using a four-quarter moving average because the raw quarterly data exhibited large high-frequency movements that have more to do with timing considerations than with the future tax liabilities of households. Labour income and government expenditures are both deflated by the GDP price deflator and the population to put them on real per capita terms. [3]

Measuring the real interest rate that discounts future net income growth is complicated by two factors. First, in practice we observe a wide array of interest rates on a broad menu of different assets, so it is not immediately clear what is *the* real interest rate. Second, the real interest rate depends on expected inflation and is not therefore directly observable.

With respect to the first issue, since the aim is to use wealth to explain consumption, an obvious route is to use a consumer-oriented rate such as a mortgage rate or a consumer loan rate. Unfortunately, even measuring *the* mortgage or consumer loan rate is not straightforward. Institutional changes in consumer loan markets and shifts in the maturity structure of mortgages make it difficult to define a consistent mortgage or consumer loan rate series.

As a second-best alternative, the interest rate used in this study is the 90-day prime corporate paper rate plus the average spread (equal to

3. The choice of the GDP deflator is motivated by the fact that in a diversified economy such as Canada's, the GDP deflator provides a broad-based measure of the aggregate price level. However, given that the ultimate objective in this study is to examine the ability of wealth to explain the behaviour of consumption of non-durables and services, an attractive alternative is to use the deflator for this component of consumption. In the empirical analysis of consumption and wealth to follow, the choice of deflator is treated largely as an empirical matter by including the ratio of these two deflators among the explanatory variables in the consumption function. The finding that this relative price has little explanatory power for consumption suggests that the GDP deflator is appropriate.

2.3 percentage points) between the mortgage rate and the prime corporate paper rate. This nominal interest rate is converted to a real rate by subtracting expected inflation. Consistent with the measurement of real income and the other components of wealth, inflation itself is measured using the GDP deflator. Expected inflation is proxied by a two-period moving average of past inflation. This proxy is motivated by the fact that, over the sample in question, GDP inflation is well-described by a second-order autoregressive, AR(2), process with approximately equal coefficients that sum to unity.[4]

Figures 1 and 2 plot the log of net income and the real interest rate over the period 1956:01 to 1993:03. Figure 1 suggests that shocks to net income permanently alter its trend so it would not be sensible to base expectations of future net income on a deterministic time trend. In contrast, the real interest rate depicted in Figure 2 appears to be stationary, which suggests that the real interest rate can be reasonably expected to return to its mean (on average) in the future.

4. Estimating an AR(2) model for GDP inflation (π) over the sample 1963:01 to 1993:03 yields

$$\pi_t = 0.52\pi_{t-1} + 0.43\pi_{t-2}$$

The sum of the AR coefficients is 0.95, which is insignificantly different from unity on the basis of an augmented Dickey-Fuller (ADF) test. Adding further lags of inflation to the AR model fails to add significant explanatory power for current inflation.

Figure 1
Real per capita labour income net of government expenditures
(in logs; quarterly rates)

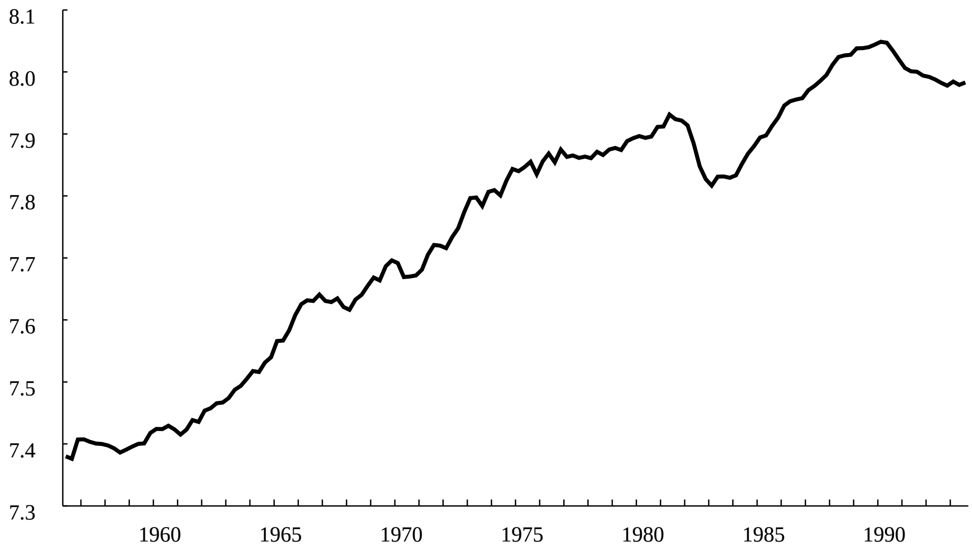
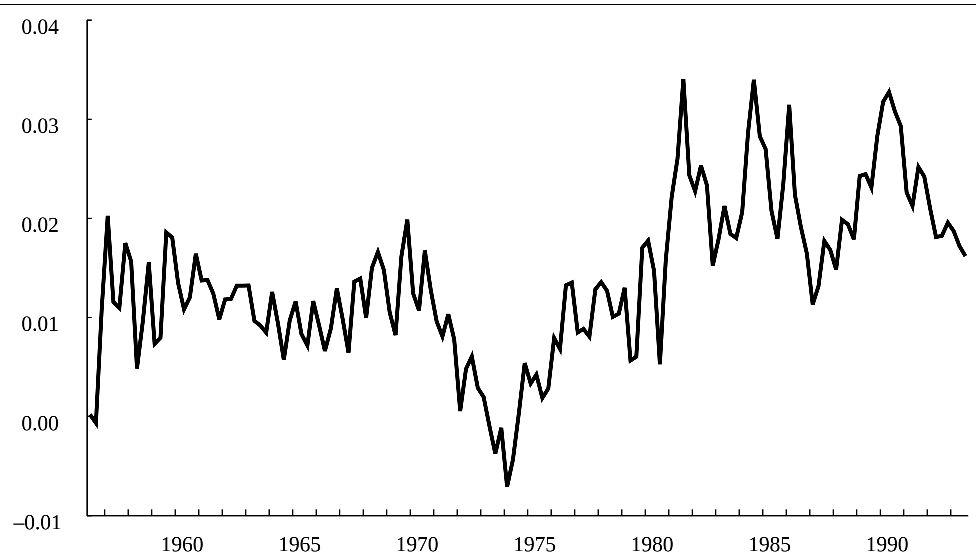


Figure 2
The real interest rate
(quarterly rates)



The time-series properties of net income and the real interest rate are investigated more formally using two popular tests for unit roots – the ADF test advocated by Dickey and Fuller (1979) and Said and Dickey (1984), and the non-parametric test proposed by Phillips and Perron (1988). The test results are reported in Table 1 and are largely consistent with the inferences made on the basis of Figures 1 and 2. The ADF test and the Phillips-Perron (PP) test both fail to reject the null of a unit root in the log of net income but convincingly reject this null in the case of its first difference. This suggests that the trend in net income is indeed stochastic and that the growth rate of net income is a stationary process.

Table 1
Tests for unit roots –
augmented Dickey-Fuller (ADF) and
Phillips-Perron (PP) tests

Sample: 1956:03–1993:03

Variables	ADF lags	ADF t–ratio	PP t–ratio	5% critical
log of net income	3	–1.66	–2.21	–3.43
Δ log of net income	1	–6.45	–11.22	–3.43
real interest rate	2	–2.52	–5.32	–3.43

Note: All tests are conducted with a time trend included in the unit-root regression. In the case of the ADF test, the number of lagged first differences of the dependent variable to include on the right-hand side was chosen following the selection procedure advocated by Hall (1989). This involves sequentially reducing the number of lags included until the t-statistic on the highest-order lag included is significantly different from zero. The lag selection began with four lags and used a 10 per cent level for the t-test. For the PP test, the number of lags used in the non-parametric correction for serial dependence is set to the square root of the number of observations used, following the suggestion of Andrews (1991). The critical values for the ADF and PP statistics are the same and are computed for the actual number of observations available, based on the response surface estimates given in MacKinnon (1991).

In the case of the real interest rate, the evidence is not as clear, but on balance it appears to favour stationarity. The more powerful PP test rejects the null of a unit root at the 5 per cent level, while the reported ADF t-ratio does not. However, the ADF test in this case is very sensitive to the number of lagged first differences that are included in the regression. Based on Hall's (1989) selection procedure, two lags were included yielding an ADF t-ratio of –2.54.[5] When only one lag is included, the ADF t-ratio falls to –4.12, which then rejects the null of a unit root at the 5 per cent level.

The joint distribution of net income growth and the real interest rate is estimated using a VAR. Owing to computational constraints associated with the finite-state Markov chain approximation to be discussed below, the VAR is restricted to be of first order. Fortunately, this constraint does not appear to be too serious, since the first-order model captures most of the predictive content of past income growth and real interest rates. The estimated VAR together with some basic diagnostics are reported in the top panel of Table 2. Both net income growth and the real interest rate exhibit positive persistence, with the interest rate being the more serially dependent of the two. The coefficient on the lagged real interest rate in the net income growth equation is significantly negative, consistent with the conventional view that raising real interest rates dampens economic activity.

2.2 Computing the cumulative growth factors

The VAR can be used to forecast net income growth and the real interest rate, thereby providing an estimate of their expected future path. This, however, is not sufficient for the measurement of human wealth, since the cumulative growth factor Γ is a non-linear function of net income growth and the real interest rate. The expected value of a non-linear function is not the function of the expectation, so we cannot simply replace x_{t+j} and r_{t+j} in (2.4) with their j-period ahead forecasts from the VAR and then drop the

5. See the notes accompanying Table 1 for a short description of Hall's (1989) lag-length selection procedure.

expectation operator. Instead, the expectation in (2.4) must be evaluated directly.

This is accomplished by approximating the continuous-valued VAR as a discrete-valued finite-state vector Markov chain. If net income growth, the real interest rate and their joint distribution are made to be discrete, expectations may be solved as a probability weighted sum rather than as an intractable integral. The approximation procedure is due to Tauchen (1986) and it is implemented by means of 25-point grids for net income growth and the real interest rate. The finite-state system therefore has $25^2 = 625$ states, and the transition matrix describing the dynamics of the system is 625×625. The statistical properties of the discrete-valued system are summarized in the bottom panel of Table 2. The circumflex on $\hat{x}$ and $\hat{r}$ distinguishes the series of discrete values from the true series of continuous values. From a comparison of the upper and lower panels of Table 2, it is apparent that the finite-state vector Markov chain closely mimics the statistical properties of the VAR.

With the approximated system, the cumulative growth factors Γ can be computed directly for every state of the system. The explicit solution for Γ is given in Appendix 2, and a sample of the results is provided in Table 3. The values for $\hat{x}$ and $\hat{r}$ in the second column and row respectively are the discrete values that net income growth and the real interest rate (both measured at quarterly rates) can take on. For example, suppose, $\hat{x}$ is in state 3 and $\hat{r}$ is in state 24. The net income growth is –2.19 per cent at quarterly rates, the discount rate is 3.41 per cent at quarterly rates, and the cumulative growth factor is 85.5. If the level of net income was, say, \$3,000 at quarterly rates, then real per capita human wealth would be \$3,000 x 85.5 = \$256,500.

Table 2
Estimated VAR for net income growth and the real interest rate with its vector Markov chain approximation

Estimated VAR 1956:03–1993:03*

$$r_t = 0.0024 + 0.839 r_{t-1} - 0.005 x_{t-1}$$
$$\quad (0.0007) \quad (0.04) \qquad (0.03)$$

$$x_t = 0.0079 - 0.338 r_{t-1} + 0.209 x_{t-1}$$
$$\quad (0.002) \quad (0.11) \qquad (0.08)$$

***r*-equation**	***x*-equation**
$\bar{R}^2 = 0.72$	$\bar{R}^2 = 0.12$
DW = 1.91	DW = 2.03
stand. dev. resid. = 0.0043	stand. dev. resid. = 0.0108

Statistical properties of the discrete-valued vector Markov chain

$$\hat{r}_t = 0.0025 + 0.833 \hat{r}_{t-1} - 0.006 \hat{x}_{t-1}$$

$$\hat{x}_t = 0.0079 - 0.335 \hat{r}_{t-1} + 0.208 \hat{x}_{t-1}$$

***** Bracketed terms below estimated coefficients are standard errors.

Table 3 Cumulative growth factors used to evaluate human wealth							
state $\hat{x}\backslash\hat{r}$		**1**	**2**	**3**	**...**	**24**	**25**
	value $\hat{x}\backslash\hat{r}$	–0.0067	–0.0049	–0.0031	...	0.0341	0.0359
1	–0.0270	114.1	112.9	111.7	...	85.4	84.5
2	–0.0244	114.2	113.0	111.7	...	85.5	84.6
3	–0.0219	114.3	113.1	111.8	...	85.5	84.7
...	...	...	...	...	...	...	...
24	0.0320	116.1	115.0	113.7	...	86.9	86.0
25	0.0346	116.2	115.0	113.8	...	87.0	86.1

Comparing the cumulative growth factors across states we see that the higher is the real interest rate in the current state, the lower is the cumulative growth factor and, thus, human wealth. This largely reflects the persistence of real interest rate movements as captured in the estimated VAR. An above average realization of r_t raises the probability of an above average realization of r_{t+1} and thus lowers the expected present value of future net income growth. Reinforcing this own effect is a cross-effect stemming from the negative relationship between the real interest rate and net income growth. An above average realization of r_t also lowers the expected value of future net income growth, thereby further reducing human wealth. The quantitative impact on human wealth of a rise in the interest rate depends on the initial state of the system, which reflects the non-linear nature of the problem. To be more concrete, the response of the cumulative growth factor to a positive 25-basis-point shock to the real interest rate (at quarterly rates) ranges between –1.4 and –1.8 per cent, with the larger responses coming when r is near its mean.

The effects of innovations in net income growth are much smaller, which reflects the fact that shocks to net income provide very little information about the future. Net income growth exhibits very little serial correlation, and there are no significant cross-effects in the estimated VAR of net income growth on the real interest rate. Quantitatively, a positive 25-basis-point shock to net income growth (at quarterly rates), raises the cumulative growth factor by about 0.07 per cent.

With the cumulative growth factors summarized in Table 3, a historical series for human wealth is constructed. First, it is determined what state of the system the economy was in at each point in time, and then the product $X_t\Gamma_t$ is formed. The state of the economy is determined by picking the $\hat{x}$ and the $\hat{r}$ that are closest to the observed x_t and r_t in each period. The resulting series for human wealth is depicted in Figure 3. Figure 4 plots the annual estimates of human wealth reported by Beach, Boadway and Bruce as a basis of comparison.

Figure 3
Estimated real per capita human wealth

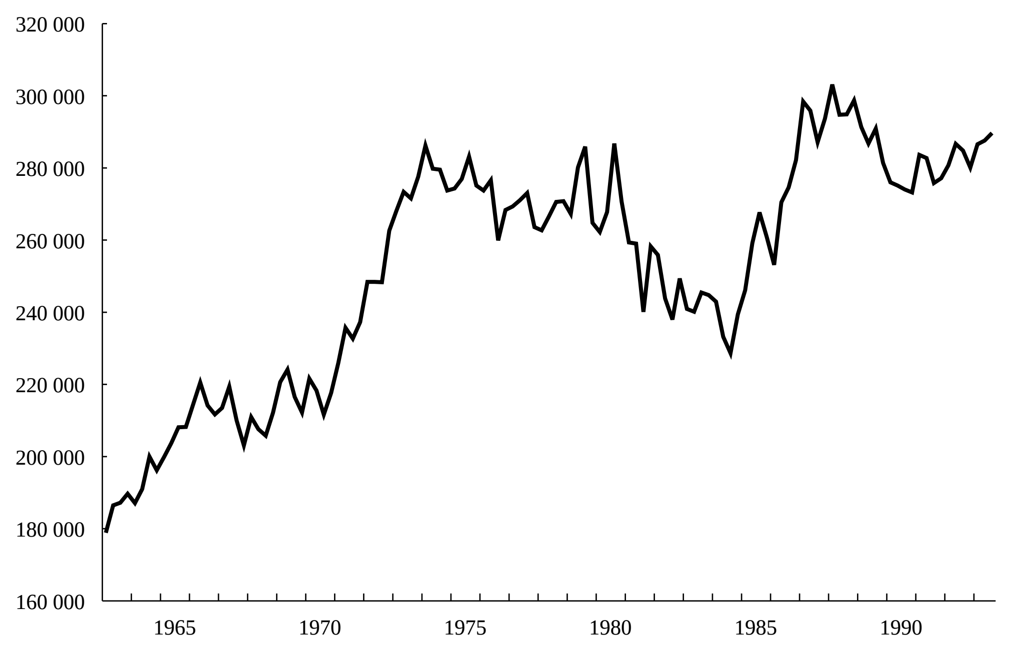
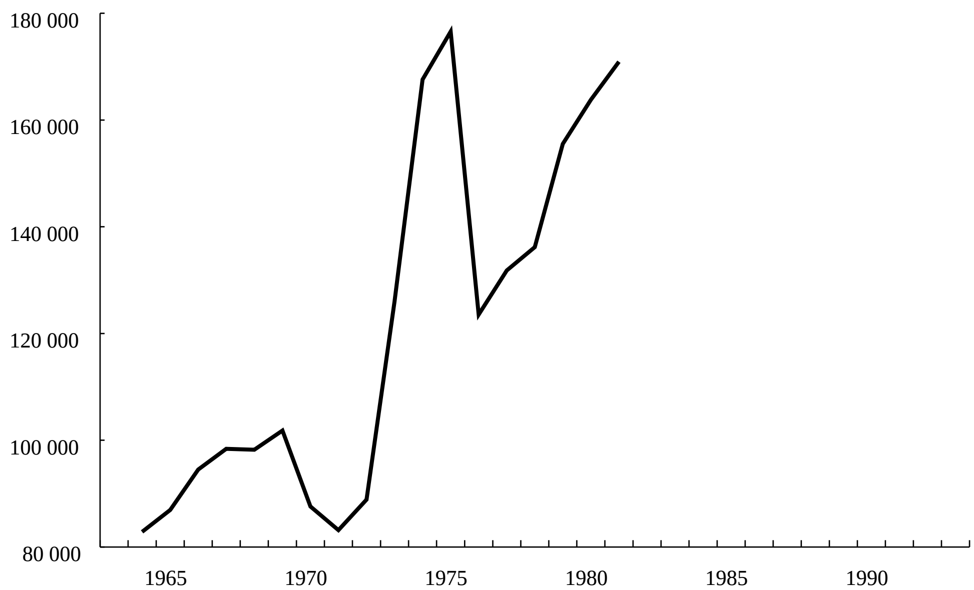


Figure 4
Beach, Boadway and Bruce's estimated real per capita human wealth



The most striking feature of the human wealth series is that its average level has changed very little since the mid-1970s. This largely reflects three facts. First, the real interest rate was very low in the mid-1970s and reasonably high in the 1980s (Figure 1). Second, the level of government expenditures grew rapidly in the early 1980s, which lowered both the level and growth rate of net income over this period. Third, labour income fell sharply in the early 1980s as a result of the 1981–82 recession.

Comparing Figures 3 and 4, we see that this measure of human wealth exhibits the same broad pattern as Beach, Boadway and Bruce's measure. Both rise sharply between 1971 and 1973, decline through the mid-1970s and then rise again at the end of the decade. The principal difference between the two is that the decline in human wealth in the mid-1970s is considerably more marked for the Beach, Boadway and Bruce series.

In the second half of the 1980s human wealth grew very rapidly owing to the combination of strong growth in net income (Figure 1) and falling real interest rates (Figure 2). This growth slowed as the decade ended and reversed somewhat in the 1990–91 recession.

2.3 Non-human wealth

In principle, measuring non-human wealth is relatively straightforward. The main task is to assemble the data and in some cases make adjustments to obtain market-value measures. The general approach to measuring non-human wealth is to consolidate carefully the assets and liabilities of the various sectors of the economy in an effort to "see through" the financial structure of the economy and to measure only the net worth of the ultimate owners of private-sector wealth – households. This "balance sheet" approach is implemented using quarterly national accounts data and by combining annual data from the national balance sheet accounts with quarterly data from the financial flow accounts. Unfortunately, the annual stock data and the quarterly flow data do not match up in the sense that year-to-year changes in the stock do not equal the sum of the flows over the year, so an adjustment was made to reconcile the stock-flow series.[6] From these consistent stock-flow data, the components of non-human wealth are constructed on a real per capita basis through division by the GDP price deflator and the population. The construction of these variables is described in detail in Appendix 1, but an overview is provided below.

Government debt held by foreigners (D_t^f) is the sum of treasury bills and federal, provincial and municipal government bonds held by non-residents. Net domestic and foreign assets (A) are defined to be the sum of non-financial and financial assets held by persons and unincorporated businesses, less the liabilities of this sector, plus the value of the Canada and Quebec Pension plans, and less the value of domestically held outstanding government debt. Non-financial assets include residential and

6. More specifically, the difference between the four-quarter cumulated flow and the year-end stock is allocated to each of the four quarters in proportion to the size of the flow in each quarter.

non-residential structures, machinery and equipment, consumer durables, inventories and land. Financial assets are defined as the sum of currency and deposits, corporate bonds, life insurance and pensions, foreign investments and equity. The principal liabilities of persons and unincorporated businesses are consumer and mortgage loans and other loans.

To understand the construction of net assets, it is important to distinguish between the components of A and their sum. This point is well illustrated by the treatment of deposits and government debt. Deposits are a component of A, which implies that this variable includes inside money. This, however, should not be the case, since inside money should be offset by consumer and business loans. Consumer and business loans are a liability to consumers, either directly or indirectly through their equity holdings in firms. In the case of government debt, A includes the government debt held directly by persons and unincorporated businesses, and then subtracts the total outstanding stock of domestically held government debt. As a result, both the government debt held directly by households and the government debt held by firms (and thus indirectly by households through their equity holdings) net out.

In the case of the three largest components of non-human wealth, adjustments are made to improve the quality of the market valuation of these assets. Equity in the national balance sheet accounts is measured at "current" value, which is defined as the sum of book value and cumulated retained earnings. To obtain a market-value measure, the current value of equity reported in the national balance sheet accounts is replaced with a measure of the book value of equity that is scaled by the growth rate of the TSE 300 composite stock price index. Bonds are reported in the national balance sheet accounts at book value. In the case of treasury bills, this is not a serious problem, since book and market values do not differ substantially for these short-term bonds. In the case of federal, provincial and municipal bonds with a longer term to maturity, the book value series reported in the national balance sheet accounts is replaced with a market value series which is constructed by multiplying the original book value series by a market price index that is constructed using Rose and Selody's (1985) present-value model. In the case of corporate bonds, no comparable market adjustment was made, since holdings of corporate bonds by persons and unincorporated businesses is relatively small. Housing is measured at market value by multiplying the constant dollar stock of housing by the multiple listings housing price index.

Figure 5 plots the constructed non-human wealth series. Non-human wealth has a more pronounced upward trend than does human wealth. Relative to Beach, Boadway and Bruce's measure of non-human wealth (Figure 6), which is defined as the sum of the sheltered and unsheltered capital stock series, the new measure shows more growth through the late 1960s, but the broad trends are similar. The new series reveal that fluctuations in non-human wealth have been considerably more pronounced since the end of the 1970s. The three main downturns in the new non-human wealth series coincide with the 1981–82 recession, the October 1987 stock market crash and the 1990–91 recession. Note also that each of these drops in non-human wealth follows a period of significantly above-average growth in non-human wealth. In terms of its relative size, non-human wealth is on average about one-sixth the size of human wealth. Thus, total wealth, which is depicted in Figure 7, looks more similar to human wealth than to non-human wealth. Figure 8 depicts Beach, Boadway and Bruce's total wealth series by way of comparison.

Figure 5
Estimated real per capita non-human wealth

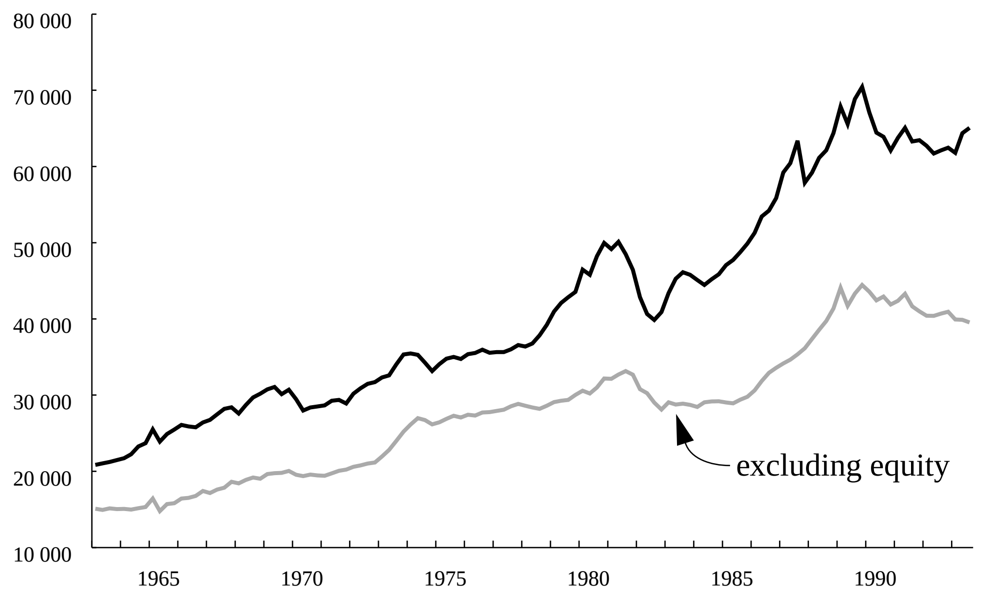


Figure 6
Beach, Boadway and Bruce's estimated real per capita non-human wealth

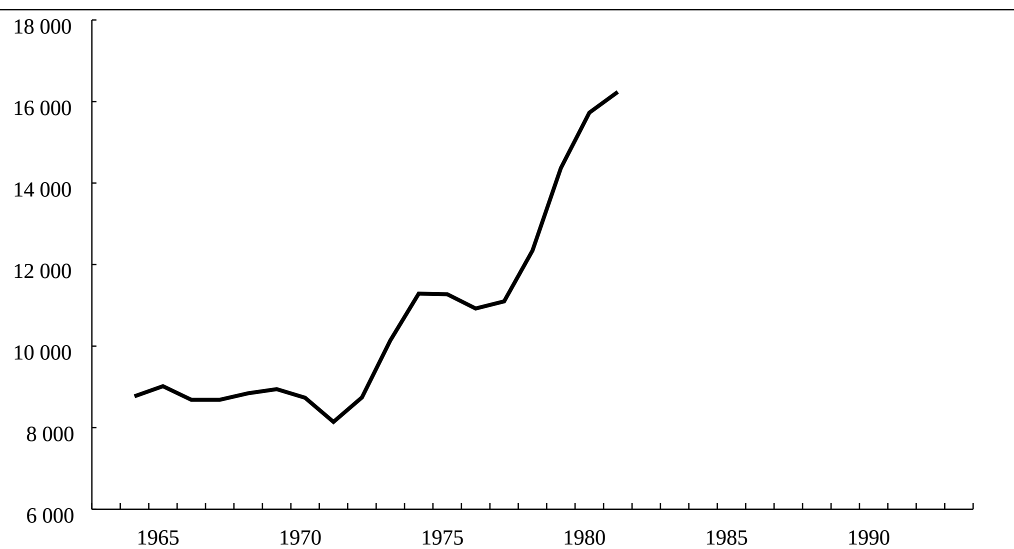


Figure 7
Estimated real per capita total wealth

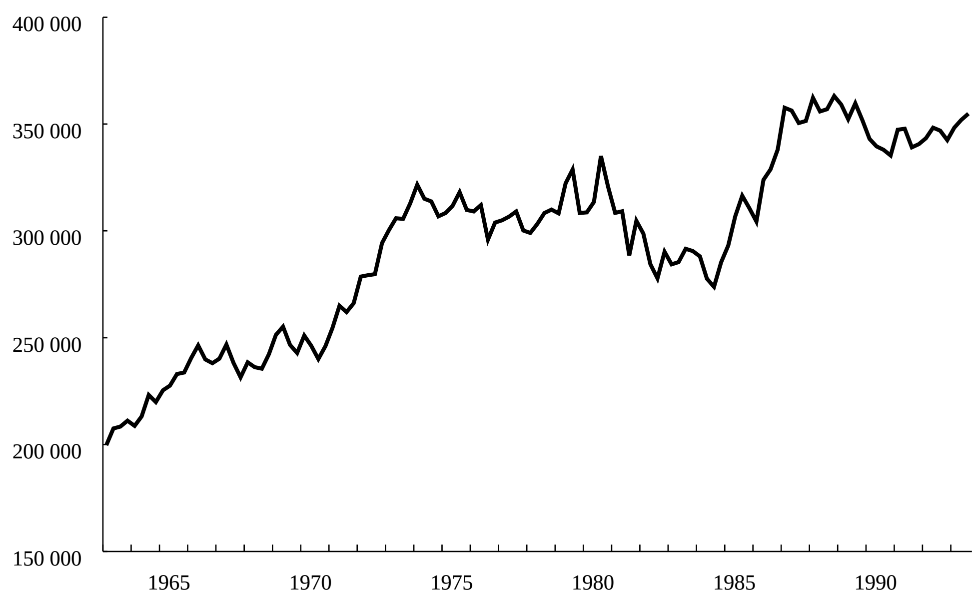


Figure 8
Beach, Boadway and Bruce's estimated real per capita total wealth

3 IS WEALTH USEFUL IN EXPLAINING CONSUMPTION?

Reliable measures of aggregate human and non-human wealth are potentially useful for addressing a broad range of macro issues, but perhaps their most obvious application is in determining consumption and savings. Specifically, one might ask, is measured wealth useful in explaining the time-series behaviour of aggregate consumption?

An alternative model with a long tradition in macroeconomics is the Keynesian consumption function that specifies consumption as a function of disposable income. The usefulness of wealth in explaining consumption is therefore evaluated relative to this alternative over both long and short horizons. The long-run analysis examines the relationship between consumption, wealth and disposable income using static linear regressions and standard tests for cointegration. This long-run analysis then serves as a precursor for the estimation of EC consumption equations designed to explain the short-run dynamics of consumption around its long-run trend.

At the outset it is worth stressing that this approach to assessing the usefulness of wealth is relatively stringent, since the disposable-income and wealth-based consumption functions may be (nearly) observationally equivalent. If innovations in disposable income are permanent or at least very persistent, then most of the information regarding their future path, and thus the information embodied in human wealth, is contained in the current observations of disposable income. Indeed, using Monte Carlo simulations, Davidson and Hendry (1981) have shown that Hall's (1978) random-walk model and the EC models developed by Davidson et al. (1978) and Hendry and von Ungern-Sternberg (1981) are nearly equivalent.

Moreover, at a theoretical level, both Campbell (1987) and Mitchener (1984) have demonstrated that a cointegrating relationship between consumption and disposable income is consistent with the life cycle–permanent income model. The results below cannot, therefore, be viewed as definitive tests of the permanent income model. Rather, they address the more modest question: Does measured wealth provide significant explanatory power for consumption beyond the information already contained in disposable income?

3.1 Long-run analysis

Unit root and cointegration tests are used to examine the long-run relationship between consumption, wealth and disposable income. Three basic cointegrating relationships are considered: a long-run Keynesian consumption function, a wealth-based consumption function, and a hybrid model that includes both disposable income and wealth. All three alternatives are specified in log-linear form:

$$c_t = \alpha_0 + \alpha_1 y_t + \alpha_2 p_t + \upsilon_t \tag{3.1}$$

$$c_t = \gamma_0 + \gamma_1 w_t + \gamma_2 p_t + \nu_t \tag{3.2}$$

$$c_t = \delta_0 + \delta_1 y_t + \delta_2 w_t + \delta_3 p_t + \vartheta_t \tag{3.3}$$

where c is the log of real per capita consumption on non-durables and services, y is real per capita disposable income, p is the price of consumption of non-durables and services relative to the price of GDP, and w is the log of real per capita total wealth. Disposable income, like wealth, is measured in real terms by deflating by means of the GDP deflator. In order to permit human and non-human wealth to have different effects on consumption and to minimize the impact of any level errors in the measurement of either component of wealth, versions of (3.2) and (3.3) are also considered, with human wealth (h) and non-human wealth (k) entered separately.

All the variables are measured at a quarterly frequency and are seasonally adjusted (see Appendix 1 for details). Table 4 reports the results of unit root tests. For c, y, w, h, and p the ADF and PP tests provide no evidence against the null of a unit root, but reject the null of a unit root in the first-differences of these variables. This suggests that these variables are all integrated of order one, that is, they are I(1), and it is appropriate to examine the possibility that they are cointegrated. Non-human wealth, in contrast, appears to be trend-stationary, which suggests it cannot account for the stochastic trend in consumption.

Table 4
Tests for unit roots – augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests

Sample: 1963:01–1992:04

Variables	ADF lags	ADF* t–ratio	PP t–ratio
c	3	–1.34	–2.05
y	1	–1.32	1.54
p	2	–0.70	–0.66
w	2	–2.06	–3.06
h	2	–1.95	–2.81
k	2	–3.50	–3.71
k'	3	–2.27	–3.10
u	1	–2.27	–4.15
Δc	2	–4.52	–13.85
Δy	0	–13.34	–12.15
Δp	2	–4.80	–9.63
Δw	1	–9.59	–11.19
Δh	1	–10.56	–11.03
Δk	0	–9.28	–8.59
$\Delta k'$	2	–4.49	–14.76
Δu	0	–5.94	–5.08

* The 5 per cent and 10 per cent critical values for the ADF and PP t-ratios are respectively –3.44 and –3.15.

The evidence of cointegration is evaluated on the basis of the ADF t-ratio test recommended by Engle and Granger (1987) and the normalized bias (or z_α) test recommended by Phillips and Ouliaris (1990). Gregory (1991) has recently evaluated the finite sample properties of a broad range of cointegration tests and concludes that these two tests are the most reliable in terms of their size and power. The latter test has a power advantage but is subject to size distortions in the presence of negative serial correlation, so there is somewhat of a trade-off between the two tests. The results are presented in Table 5. Several conclusions emerge.

Table 5
Test for the null of no cointegration – augmented Dickey-Fuller (ADF) and Phillips-Ouliaris (PO) tests

Sample 1963:01–1993:03

Variables	ADF lags	no trend				with trend			
		ADF t-ratio	ADF 5%	PO stat.	PO 5%	ADF t-ratio	ADF 5%	PO stat.	PO 5%
c,y,p	3	–2.53	–3.81	–34.71	–24.66	–2.51	–4.22	–34.67	–29.89
c,w,p	2	–1.97	–3.81	–18.07	–24.66	–1.97	–4.22	–20.31	–29.89
c,y,w,p	0	–5.26	–4.19	–51.99	–29.36	–5.25	–4.55	–52.02	–34.10
c,h,k,p	0	–3.02	–4.19	–20.42	–29.36	–3.07	–4.55	–20.70	–34.10
c,y,h,k,p	0	–5.07	–4.53	–51.32	–33.86	–5.05	–4.86	–51.33	–38.25
c,y,h, k',p	0	–5.40	–4.53	–51.65	–33.86	–5.37	–4.86	–51.53	–38.25
c,y	3	–2.46	–3.39	–26.07	–19.29	–2.50	–3.86	–25.33	–25.06
c,w	2	–1.88	–3.39	–15.85	–19.29	–2.30	–3.86	–19.97	–25.06
c,y,w	0	–5.15	–3.81	–49.86	–24.66	–5.12	–4.22	–49.81	–29.89
c,h,k	0	–2.84	–3.81	–18.72	–24.66	–2.84	–4.22	–19.00	–29.89
c,y,h,k	0	–5.06	–4.19	–49.53	–29.36	–5.03	–4.55	–49.50	–34.10
c,y,h, k'	0	–5.34	–4.19	–50.66	–29.36	–5.31	–4.55	–50.43	–34.10

Note: As with the unit root ADF tests considered above, the number of lagged differences included in the ADF regression are chosen following the lag-selection procedure advocated by Hall (1989), and critical values for the ADF t-ratios are calculated using the response-surface estimates of MacKinnon (1991) for the actual sample used. The Phillips and Ouliaris (PO) normalized bias test statistics are compared to detailed critical values for the PO test reported by Haug (1992).

First, there is little evidence that consumption is cointegrated with either total wealth or its human and non-human components – both the ADF test and the Phillips-Ouliaris (PO) test fail to reject the null of no cointegration at even a 10 per cent level.

Second, the evidence that consumption is cointegrated with disposable income is mixed. The ADF tests fails to reject the null of no cointegration at a 10 per cent level, while the PO test rejects the same null at the 5 per cent level.

Third, when disposable income and either total wealth or human and non-human wealth are included in the cointegrating vector, there is more convincing evidence of cointegration. Both the ADF and PO statistics reject the null of no cointegration at the 5 per cent level in all the long-run regressions that include wealth, and for the PO statistic this null is also rejected at the 1 per cent level. The one caveat is that the ADF statistics are somewhat sensitive to the number of lagged differences that are included in the ADF regression, but this problem should be minimized through use of the lag-length selection procedure advocated by Hall (1989).

A fourth finding is that the relative price of consumption *p* plays a minor role in the cointegration results. The lower panel of Table 5 reports the results when *p* is omitted from the cointegrating vector, and the test statistics change very little. Overall, the results suggest that both disposable income and wealth are important long-run determinants of consumption, while the relative price of consumption is not. An examination of the cointegrating vector itself bolsters this conclusion.

Table 6 reports the estimated long-run coefficients on disposable income, wealth and the relative price of consumption. The top panel of the table reports the Engle-Granger static ordinary least squares (OLS) estimates on which the Table 5 cointegration tests are based. These parameter estimates, while super-consistent (Engle and Granger 1987), are not efficient, and their distributions are unknown. To obtain more efficient estimates and perform valid inference on the long-run parameters, the cointegrating parameters are also estimated by means of the prewhitened fully modified estimator of Phillips and Hansen (1990) and the “leads-and-lags” procedure suggested by Stock and Watson (1993).

Table 6
Estimated long-run consumption functions

Dependent variable: *c*

Engle-Granger static OLS															
1.44	+	0.82*y*	–	0.24*p*					1.54	+	0.81*y*				
0.47	+	0.67*y*	+	0.19*w*	–	0.08*p*			0.37	+	0.65*y*	+	0.21*w*		
1.05	+	0.60*y*	+	0.13*h*	+	0.08*k*	–	0.17*p*	0.59	+	0.62*y*	+	0.18*h*	+	0.05*k*
1.46	+	0.55*y*	+	0.11*h*	+	0.12*k*’	–	0.06*p*	0.36	+	0.54*y*	+	0.13*h*	+	0.13*k*’
Phillips-Hansen fully modified estimator															
1.26	+	0.84*y*	–	0.23*p*					1.37	+	0.83*y*				
(0.22)		(0.02)		(0.12)					(0.27)		(0.03)				
0.25	+	0.63*y*	+	0.24*w*	–	0.01*p*			0.26	+	0.63*y*	+	0.24*w*		
(0.26)		(0.04)		(0.04)		(0.07)			(0.26)		(0.03)		(0.04)		
0.50	+	0.59*y*	+	0.21*h*	+	0.06*k*	–	0.06*p*	0.28	+	0.65*y*	+	0.21*h*	+	0.02*k*
(0.44)		(0.06)		(0.05)		(0.04)		(0.10)	(0.35)		(0.06)		(0.04)		(0.03)
1.76	+	0.52*y*	+	0.09*h*	+	0.15*k*’	–	0.06*p*	1.62	+	0.53*y*	+	0.09*h*	+	0.15*k*’
(0.37)		(0.05)		(0.04)		(0.03)		(0.06)	(0.39)		(0.05)		(0.03)		(0.04)
Stock-Watson leads-lags estimator															
1.36	+	0.83*y*	–	0.20*p*					1.53	+	0.81*y*				
(0.10)		(0.01)		(0.09)					(0.15)		(0.02)				
–0.11	+	0.55*y*	+	0.33*w*	–	0.04*p*			0.19	+	0.63*y*	+	0.24*w*		
(0.21)		(0.04)		(0.04)		(0.06)			(0.21)		(0.03)		(0.04)		
–0.30	+	0.57*y*	+	0.32*h*	+	0.02*k*	+	0.14*p*	0.21	+	0.62*y*	+	0.22*h*	+	0.03*k*
(0.43)		(0.05)		(0.03)		(0.04)		(0.08)	(0.32)		(0.05)		(0.03)		(0.03)
1.77	+	0.45*y*	+	0.12*h*	+	0.18*k*’	–	0.03*p*	1.49	+	0.53*y*	+	0.11*h*	+	0.14*k*’
(0.58)		(0.04)		(0.06)		(0.04)		(0.07)	(0.53)		(0.04)		(0.05)		(0.04)

Notes: Bracketed terms below estimated coefficients are standard errors.

The Engle-Granger and Phillips-Hansen results are based on the sample 1963:01–1993:03, while the Stock-Watson results use the shorter sample 1964:02–1992:02 to accommodate the leads and lags that are included in the regression. The Phillips-Hansen fully modified estimation uses a first-order VAR to prewhiten the residuals in all cases except for the regression of *c* on *y* and *p* alone. In this latter case, a second-order VAR was required to remove the serial correlation in the residuals. For the Stock-Watson estimates, the reported standard errors are computed by means of the Newey-West procedure, which uses a fourth-order process for the residuals.

Both estimators correct for the endogeneity bias that is likely to be present in this application, given that the right-hand-side variables are unlikely to be strictly exogenous, and both have the same asymptotic distributions. In all but one case, the Phillips-Hansen estimator uses residuals that are prewhitened with a VAR(1) to correct for serial correlation, since the first-order VAR is sufficient to capture most of the serial correlation in the residuals. The exception is the regression of *c* on *y* and *p*, which required a second-order VAR.

The Stock-Watson procedure is implemented using leads and lags of four quarters, and the reported standard errors are based on the Newey and West (1987) procedure, since there is evidence of serially correlated residuals. Four features of the results stand out.

First, the estimates for which valid standard errors are reported indicate that both disposable income and wealth are significant determinants of trend movements in consumption at any reasonable level of significance.[7]

Second, while including wealth among the right-hand-side variables reduces the coefficient on disposable income considerably, disposable income remains an important determinant of consumption. In the simple Keynesian model, the coefficient on disposable income is always slightly above 0.80; when wealth is added, this coefficient ranges from 0.45 to 0.65.

Third, almost all the explanatory power of wealth is coming from the human wealth component – the coefficient on non-human wealth (*k*) while positive, is small and within a standard error of zero. This finding is consistent with the evidence that non-human wealth is I(0), and cannot therefore explain the stochastic trend in consumption. Consumers, cognizant of the fact that fluctuations in non-human wealth are temporary

7. In contrast, Beach, Boadway and Bruce’s measures of total, human and non-human wealth were not found to be significant determinants of consumption of non-durables and services when they were added to a Keynesian consumption function estimated on annual data from 1964 to 1981 – the frequency and sample for which Beach, Boadway and Bruce’s wealth measures are available.

shocks, view fluctuations in non-human wealth as transitory income and therefore absorb them through savings.

This latter finding deserves more scrutiny. Disaggregating non-human wealth into its principal components reveals that fluctuations in non-human wealth are dominated by changes in equity prices. Equity comprises about one-quarter of non-human wealth on average and is by far its most volatile component. Note, in particular, the sharp drop in non-human wealth following the October 1987 stock market crash (Figure 5).

This importance of equity for the stochastic behaviour of total non-human wealth suggests at least two interpretations of the weak link from non-human wealth to consumption. Consumers may view much of the volatility in equity prices as short-run in nature, and therefore "see through" fluctuations in equity prices when making consumption decisions.

Alternatively, aggregate consumption may respond very little to changes in equity prices simply because a majority of people do not hold equity. Mankiw and Zeldes (1990) point out that only about one-fourth of American households own stocks. Moreover, they find that the consumption of stockholders in the United States is in fact more highly correlated with stock returns than is the consumption of non-stockholders. Assuming Canadian households are similar to their American counterparts, the low correlation between consumption and non-human wealth may reflect these distributional issues.

To test the possibility that the weak link between non-human wealth and consumption stems largely from equity, an alternative measure of non-human wealth (k') that does not include equity is constructed. In contrast to total non-human wealth, this alternative variable appears to be I(1), both on the basis of a simple inspection of the series (see Figure 5) and more formal tests. As reported in Table 4, ADF and PP tests on k' fail to reject the null of a unit root at the 10 per cent level, although the PP statistic is close. Cointegration tests reported in Table 5 suggest that consumption is cointegrated with y, h and k', but the more interesting results are the cointegrating parameters reported in Table 6. Note that when *k'* is substituted for *k*, non-human wealth has significant explanatory power for consumption based on the Phillips-Hansen and Stock-Watson estimates, and all three estimation techniques produce very similar coefficient estimates. Moreover, when the value of equity is entered separately with the consumption function including *y, h* and *k'*, it has a negative coefficient that is statistically indistinguishable from zero. The suggestion is that once we restrict attention to assets such as housing, currency and deposits, which have less variable returns and are more widely held, fluctuations in both human and non-human wealth have important effects on consumption.

A fourth noteworthy result in Table 6 is that while the coefficient on the relative price of consumption is typically negatively signed as expected, it is only significant in the Keynesian consumption function. Moreover, even in the Keynesian consumption function, the coefficient on the relative price variable is significantly below that on disposable income (which is nominal disposable income deflated by the GDP deflator). This suggests that the GDP deflator is preferred empirically in both the Keynesian and wealth-based consumption functions.

The above inferences all assume that the long-run parameters reported in Table 6 are constant, but if in fact they are changing through time, these inferences are invalid. In order to test for this type of misspecification, Hansen's (1992) tests for parameter non-constancy for I(1) processes are applied to the three most interesting long-run equations. Hansen proposes three tests – *SupF, MeanF* and *Lc* – which all share the null of parameter constancy but differ in their alternatives. *SupF* tests for a structural break of unknown timing and is therefore appropriate for determining if there has been a swift shift in regime, while *MeanF* and *Lc* model the parameters as a martingale under the alternative, so change is viewed as a gradual process. The test statistics together with their probability values are reported in Table 7 and refer to the Phillips-Hansen parameter estimates reported in Table 6.

Table 7
Hansen's test for parameter stability

Test	Cointegrating vector			
	c,y,p	*c,y,w*	*c,y,h,k*	*c,y,h, k'*
SupF	9.54 (>0.20)	5.18 (>0.20)	6.22 (>0.20)	8.41 (>0.20)
MeanF	4.82 (0.12)	1.76 (>0.20)	2.75 (>0.20)	3.53 (>0.20)
Lc	0.15 (>0.20)	0.13 (>0.20)	0.20 (>0.20)	0.26 (>0.20)

Note: Bracketed terms are the probability values for the associated test statistic.

The principal finding is that the null of stability cannot be rejected at a 10 per cent level for cointegrating vectors of *c* with *y* and *p*, *c* with *y* and *w*, and *c* with *y*, *h* and *k'*. There is some weak evidence of a gradual change in the parameters of the Keynesian consumption function – the probability value for the *MeanF* statistic is 12 per cent – and results for this consumption model did prove somewhat sensitive to the order of the VAR used to prewhiten the residuals. This may be suggestive of misspecification.

Once wealth is included in the cointegrating vector, the evidence of instability vanishes and the prewhitening procedure has less impact on the results. Since Hansen suggests that his tests for parameter constancy can also be interpreted as tests of the null of cointegration against the alternative of no cointegration, these results provide further support for the view that consumption and disposable income may be cointegrated and that adding wealth strengthens the evidence of cointegration.

3.2 Dynamic consumption functions

The evidence that wealth is helpful in explaining the stochastic trend in consumption suggests that empirical EC consumption models that embody a long-run relationship between consumption and disposable income may benefit from the inclusion of wealth. Two alternative EC models are considered: one that specifies a long-run relationship between consumption, disposable income and total wealth, and a second that enters human wealth and non-human wealth (excluding equity) separately. As a basis of comparison, a traditional EC model that excludes wealth is also estimated.

The dependent variable in all the dynamic equations is the first difference of consumption, and it depends on the first lag of the OLS estimates of the residual in the long-run consumption function – the EC term – and the current and lagged first differences of the long-run determinants of consumption. The coefficient on the EC term should be significantly less than zero if c is cointegrated with its long-run explanatory variables. Additional variables can also be included in the dynamic equation if these variables are thought to exert a short-run influence on consumption.

Empirical studies over the past decade and a half have furnished an expansive and colourful menu of would-be regressors that are likely to contribute to short-run fluctuations in consumption around its long-run trend. The present analysis, however, confines the specification search to two of the most popular explanatory variables: the real interest rate and the unemployment rate or its first difference.

The interest rate is typically included to capture both the wealth effect associated with revaluations of human and non-human wealth (since wealth is not usually included in the regression) and the intertemporal substitution effect associated with fluctuations in the relative price of current versus future consumption. In principle, the sign of the interest rate effect is ambiguous, but the usual presumption is that it is negative. The extended EC models considered here that include wealth provide the opportunity to separate the revaluation effect from the intertemporal substitution effect. The unemployment rate is generally thought to proxy either liquidity effects or uncertainty and its expected sign is therefore negative.

In order to ensure that the impact of these short-run variables is in fact confined to the short run, they should be I(0). The real interest rate and the unemployment rate are therefore pretested for stationarity. Evidence supporting the stationarity of the real interest rate was discussed above in the measurement of wealth and is presented in Table 1. The results from ADF and PP unit root tests on the unemployment rate and its first difference are reported in Table 4. The null of a unit root in the first difference of the unemployment rate is convincingly rejected, but the ADF and PP tests give conflicting results in the case of the level of the unemployment rate, so both u and Δu are considered.

The estimation of the dynamic consumption function began with fourth-order lags on all right-hand-side dynamic variables and was then sequentially simplified with the aim of producing a parsimonious equation with sensible economic properties, reasonable fit and well-behaved residuals. The estimated final-form dynamic equations are reported in Table 8.

The impact of including wealth in the EC model is best discussed with reference to the standard EC model in which the long-run behaviour of consumption is determined by disposable income alone. Results for this model are presented in the second column of Table 8; several features are noteworthy.

The EC term is negatively signed and significant at the 1 per cent level, which is consistent with evidence of cointegration. The estimated coefficients on the first difference of the relative price of consumption, the real interest rate and the change in the unemployment rate are all negatively signed as expected and significant at the 5 per cent level. The statistical significance of the coefficients on the third lag of the dependent variable and the fourth lag of the first difference of disposable income is difficult to interpret but is consistent with rejections of Hall's (1978) random walk model. Finally, the residual diagnostics reported at the bottom of Table 8 suggest that autoregressive conditional heteroscedasticity is a feature of the residuals. Accordingly, heteroscedastic-consistent standard errors are reported based on the Newey-West procedure.

Table 8
Error-correction models for consumption

Dependent variable: Δc – Sample: 1964:01–1993:03

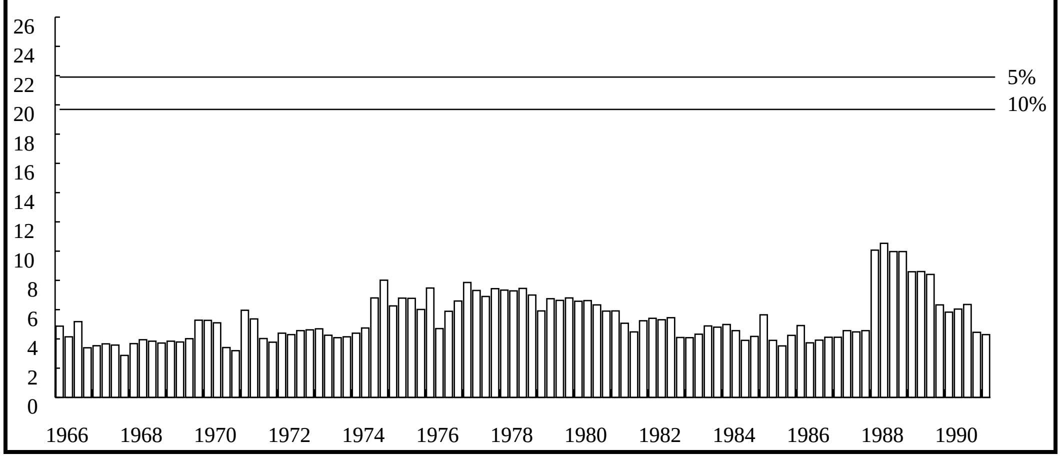
Short-run variables	**Long-run variables** *y,p*	*y,w*	*y,h, k'*
constant	0.005 (0.001)*	0.004 (0.001)	0.003 (0.001)
EC_{t-1}	–0.121 (0.033)	–0.164 (0.052)	–0.170 (0.063)
Δy_t	0.142 (0.038)	0.151 (0.038)	0.131 (0.042)
Δy_{t-4}	–0.131 (0.040)	–0.129 (0.041)	–0.122 (0.041)
Δp_t	–0.394 (0.116)	–0.373 (0.116)	–0.364 (0.115)
r_{t-1}	–0.133 (0.064)	–0.038 (0.069)	–0.022 (0.073)
Δu_t	–0.004 (0.001)	–0.004 (0.001)	–0.004 (0.001)
Δc_{t-3}	0.185 (0.083)	0.178 (0.079)	0.200 (0.085))
$\Delta k'_t$	–	–	0.042 (0.030)
$\bar{R}^2$	0.37	0.38	0.36
DW	2.10	2.03	2.02
Q(15)**	0.15	0.11	0.08
ARCH(1)	0.03	0.02	0.02
ARCH(4)	0.23	0.16	0.19
RESET	0.22	0.21	0.12
NORMAL	0.43	0.65	0.66

* Bracketed terms are Newey-West standard errors based on a first-order process for the residuals.

** The following four statistics are the probability values for the Box-Ljung Q-test (Q(15)) for autocorrelated residuals using 15 lagged autocorrelations, Engle's test for autoregressive conditional heteroscedasticity of orders one and four (ARCH), Ramsey's RESET specification test, and the Jarque-Bera test for normality (NORMAL). A value at or below 0.05 means that the null can be rejected at the 5 per cent level.

The stability of this traditional EC model is explored using rolling Chow tests and recursive estimation. Figure 9 depicts the χ^2 statistics associated with the rolling Chow test together with the 5 and 10 per cent critical values for the null of stability; the test statistics reveal no evidence of a shift in the coefficient estimates.[8] Recursive estimates obtained by reestimating the model sequentially, adding one observation each time, are depicted in Figure 10 for the most interesting coefficients. The dotted lines in the graphs mark the associated 95 per cent confidence interval. The graphs reveal that the inclusion of the 1981–82 recession in the sample is important for identifying significant evidence that the real interest rate and the change in the unemployment rate influence consumption. The other parameters in the model are more stable through time after some initial sample sensitivity, owing to the very low degrees of freedom near the start of the sample.

Figure 9
Rolling Chow tests: Keynesian ECM



8. The critical values depicted in Figure 9 are not the standard ones. As observed by Andrews (1990), if a break point is determined by a search over the sample, the standard critical values are too small and inference is biased against the null of stability. The reported critical values are taken from Andrews (1990) and control for this effect.

Figure 10
Recursive parameter estimates: Keynesian ECM

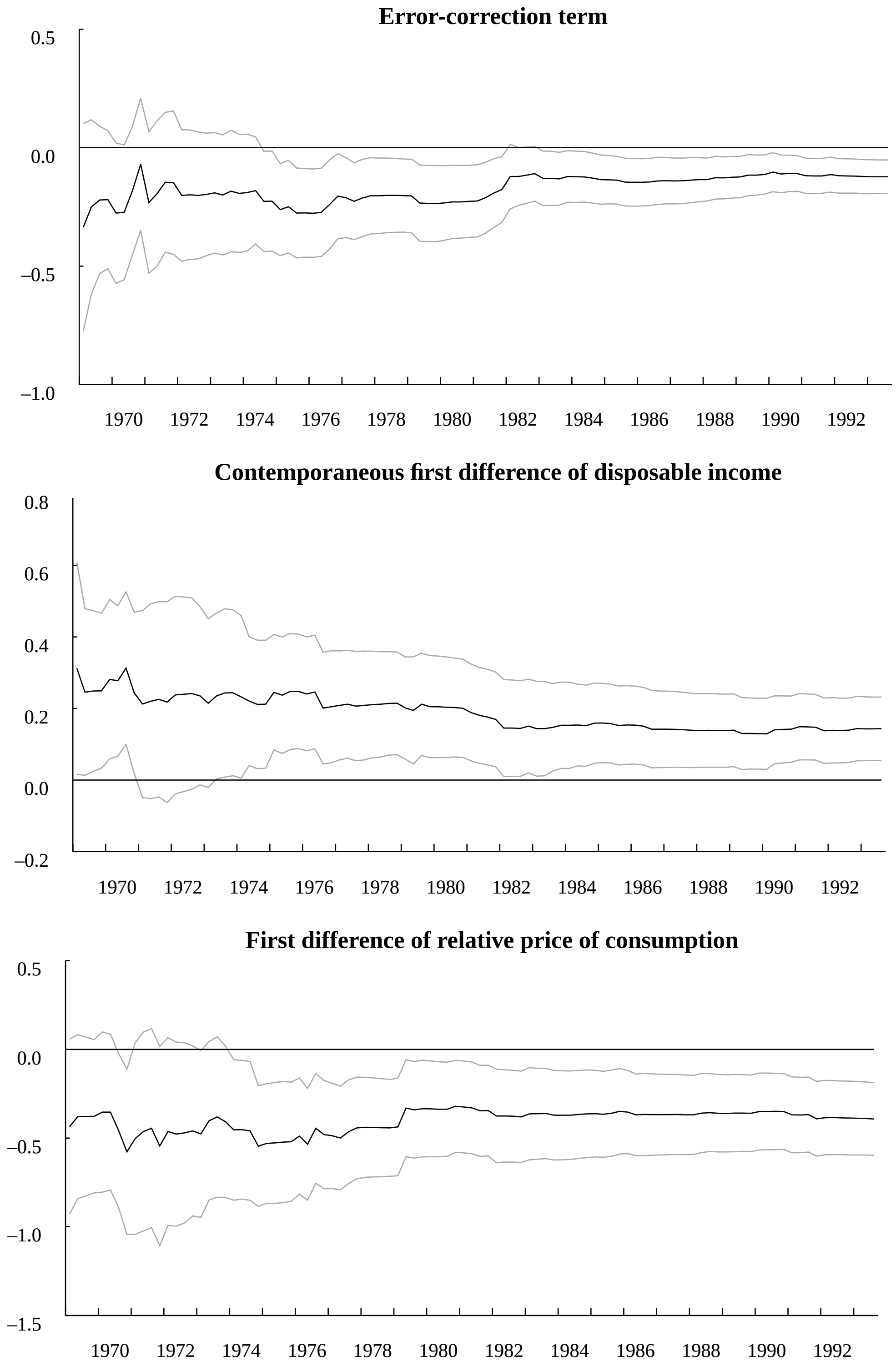
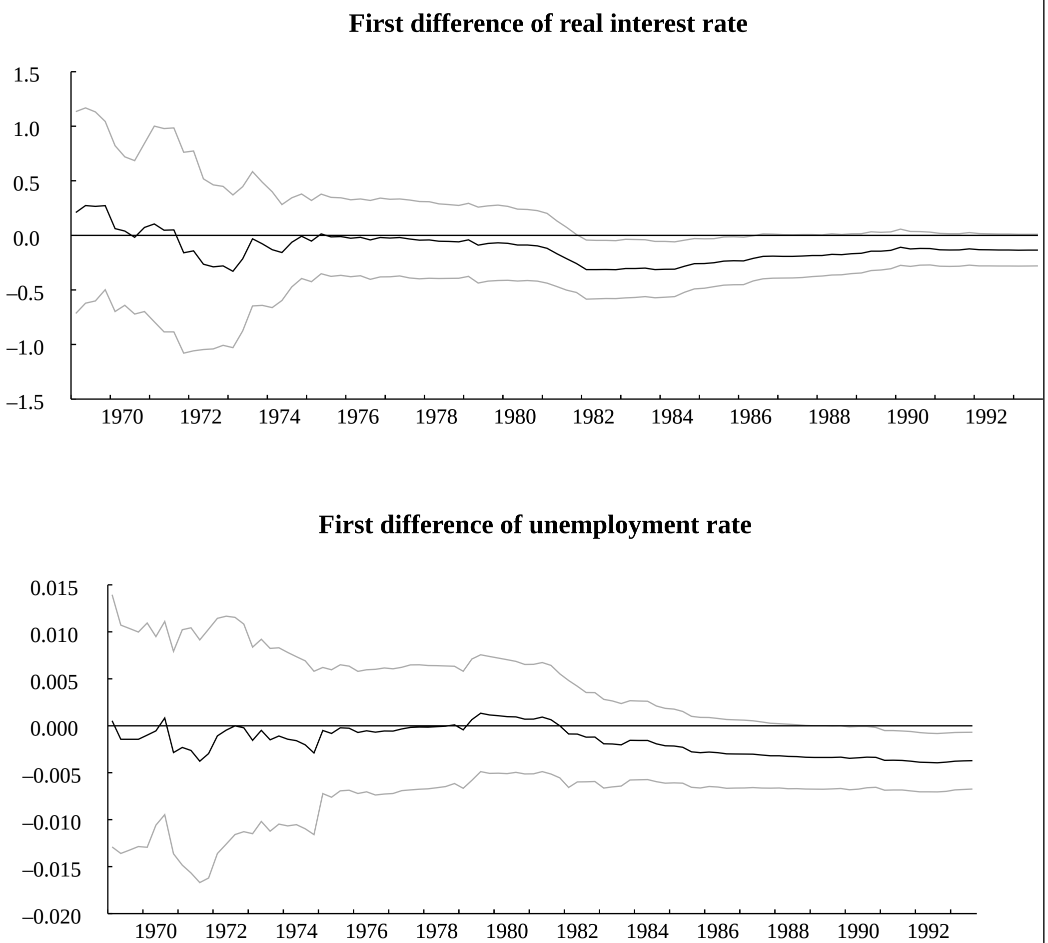


Figure 10 (*continued*)



The second-column results in Table 8 present an estimated EC model that is based on a long-run consumption function that includes both disposable income and total wealth. The most noticeable feature of this dynamic consumption function is that the EC term is larger in magnitude; this is consistent with the stronger evidence of cointegration reported in Table 5 when wealth is included.

A second feature of the EC model that includes wealth is that the coefficient on the real interest rate is about a third as large as in the Keynesian model, and is now no longer significantly different from zero at conventional levels. Presumably this reduced interest rate effect reflects the fact that the inclusion of wealth in the equation absorbs the wealth effect of interest rate changes, leaving only the pure substitution effect to be picked up by the coefficient on the real interest rate. The point estimates therefore suggest that most of the interest rate effect in the Keynesian equation is due to the wealth effect.

Third, note that the coefficient on the first difference of the relative price of consumption is virtually unaffected by the inclusion of wealth, so while relative price effects are captured by wealth in the long run, the relative price of consumption has significant dynamic effects on consumption. Finally, the third column of Table 8 considers the impact of substituting total wealth in the cointegrating vector for human wealth and non-human wealth (excluding equity) entered separately. The impact of this substitution is relatively minor; the coefficient on the real interest rate falls marginally and there is a small dynamic effect of changes in non-human wealth.

As for the Keynesian model, rolling Chow tests do not reject the null of stability for the EC equations that include wealth (see Figures 11 and 12). In addition, recursive estimates of the parameters reveal that the parameters are reasonably stable through time. Figure 13 reports recursive estimates for the model with human and non-human wealth (excluding equity) included separately, but very similar results are obtained when total wealth is used instead. Note in particular that the coefficient on the change in unemployment is now always negative, though only significantly so near the end of the sample. The residuals show little evidence of significant serial dependence, but autoregressive conditional heteroscedasticity continues to be present.

Figure 11
Rolling Chow tests: ECM including total wealth

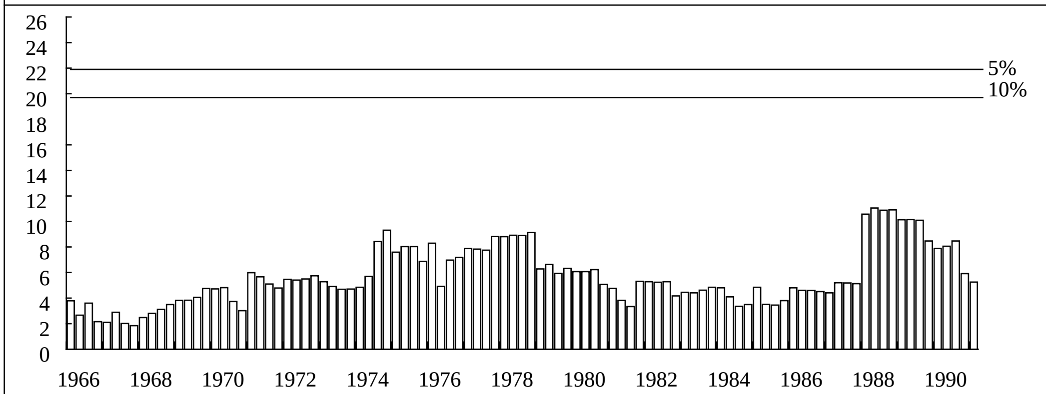


Figure 12
Rolling Chow tests: ECM including human wealth and non-human wealth excluding equity

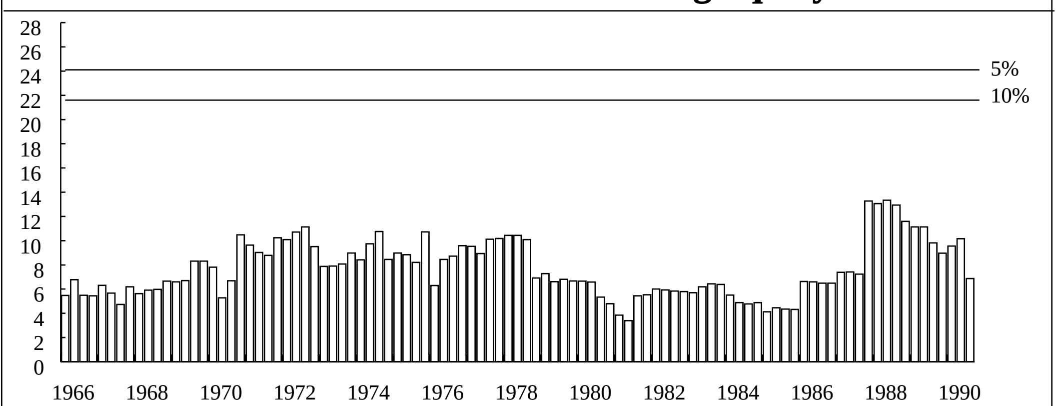


Figure 13
Recursive parameter estimates: human wealth and non-human wealth excluding equity

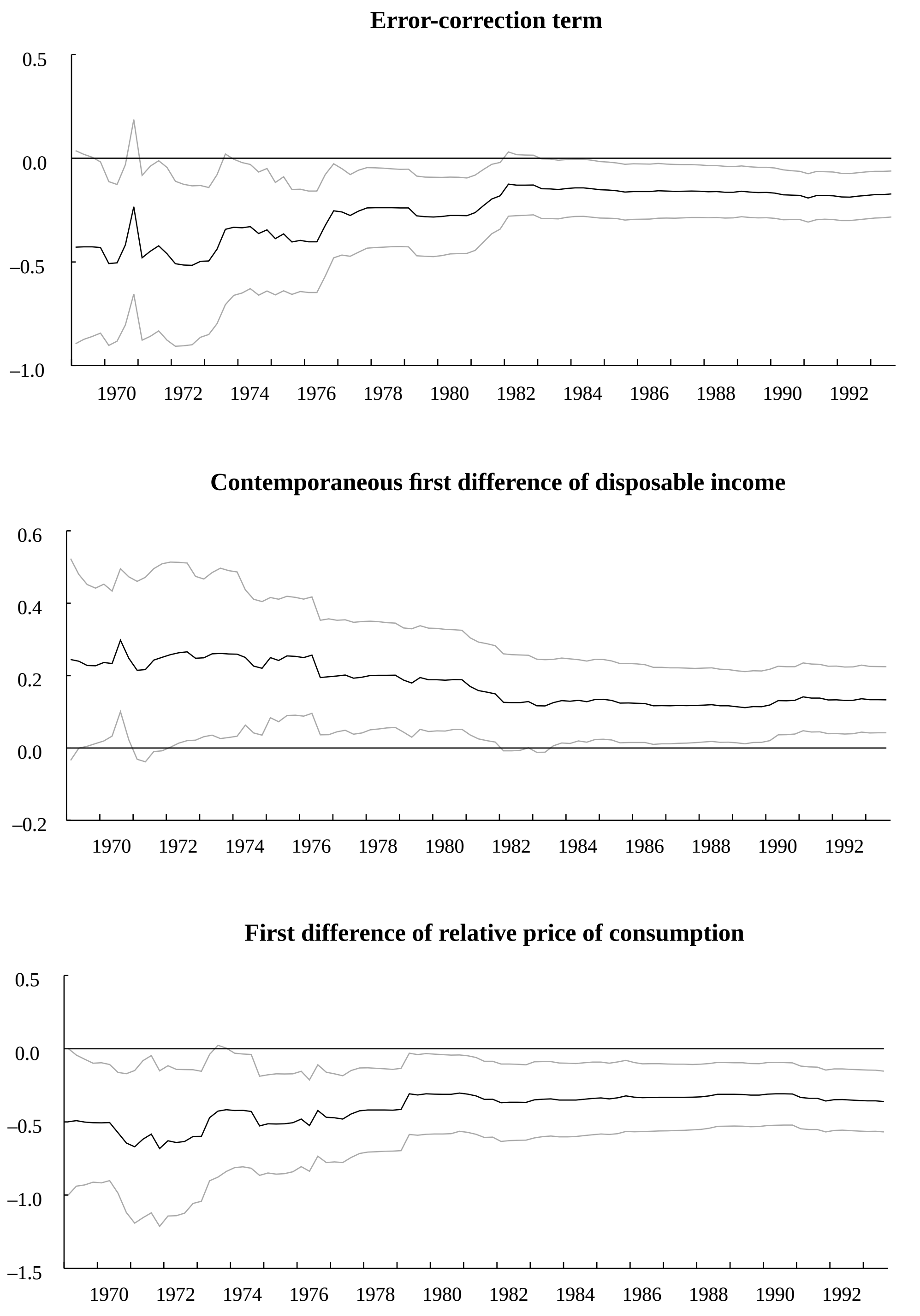
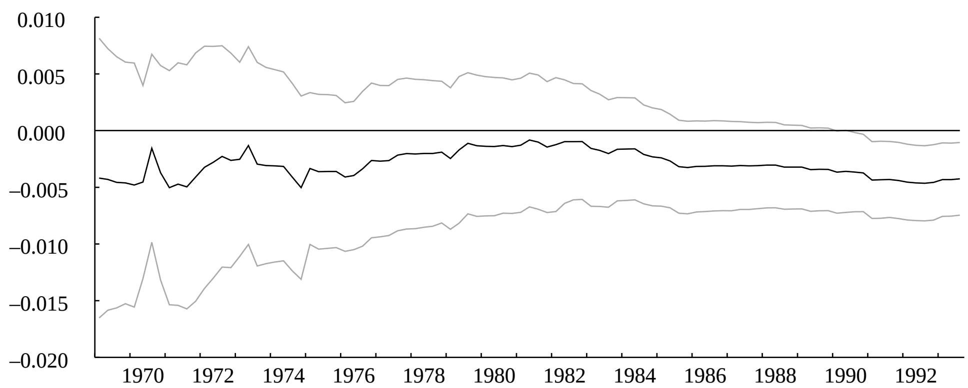
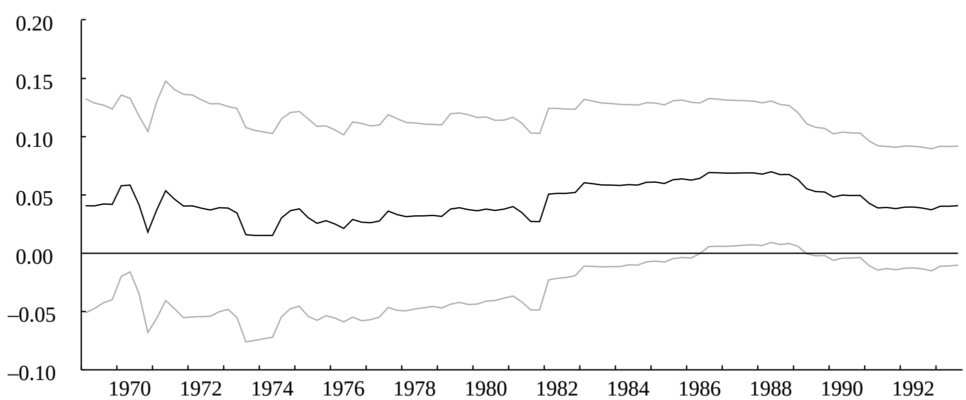


Figure 13 *(continued)*

First difference of unemployment rate



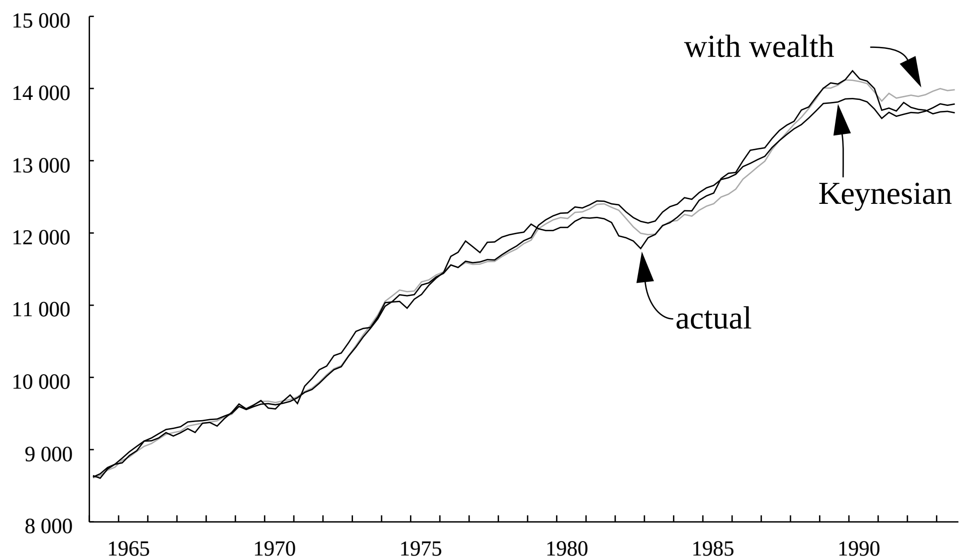
First difference of non-human wealth excluding equity



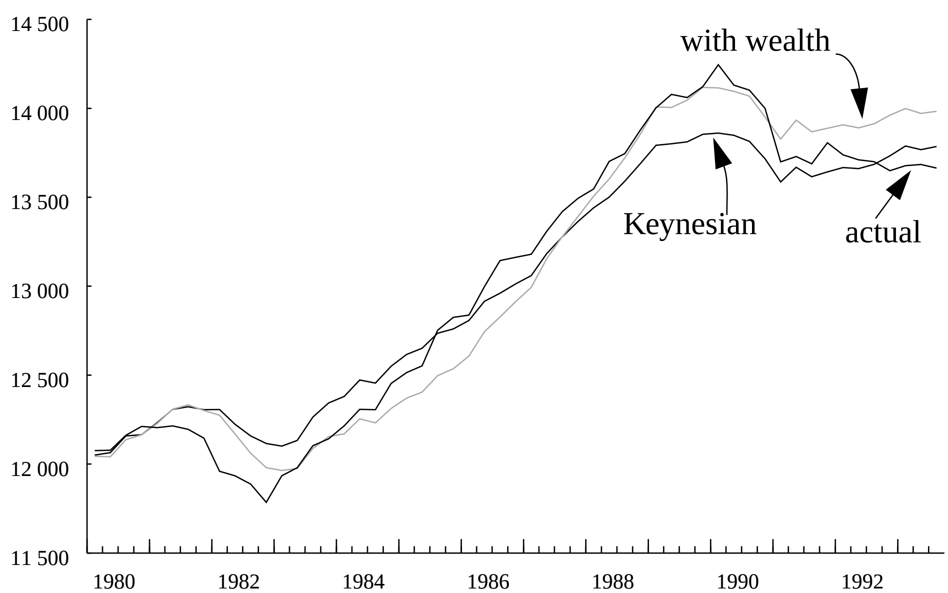
As a final exercise, the in-sample dynamic forecasting performance of the Keynesian and wealth-augmented consumption functions is examined. The top panel of Figure 14 plots actual real per capita consumption of non-durables and services over the full 1964–1993 sample, together with the predicted values of real per capita consumption from dynamic simulations of the Keynesian EC model and the augmented model, which also

Figure 14
Dynamic simulations of the Keynesian and wealth-augmented EC consumption equations

1964–1993



1980–1993



includes human wealth and non-human wealth (excluding equity).[9] The lower panel of Figure 14 focusses attention on the two most recent business cycles, reporting dynamic simulations starting in 1980.

The graphs reveal that both EC models track the broad movements in consumption reasonably well. Prior to the 1980s, the Keynesian EC equation and wealth-augmented model have very similar dynamic forecasts, but in the 1980s there are some more marked differences. Although both equations underpredict the decline in consumption experienced during the 1981–82 recession, the equation including wealth explains a considerably larger proportion of the observed peak-to-trough decline – 85 per cent as compared with only 51 per cent for the Keynesian equation. The equation including wealth also tracks observed consumption more closely in the latter half of the 1980s.

Strong growth in both human wealth and non-human wealth excluding equity through the 1987–88 period results in a considerably higher predicted level of consumption by the start of 1989, when wealth is included. As a result, the wealth-augmented model does a much better job of explaining the consumption boom in the late 1980s. This is consistent with the view that rising asset values, particularly housing prices, fuelled high levels of consumption over this period.

Looking at the most recent recession, we see that neither equation can account for the magnitude of the fall in consumption through 1990 – both equations predict about half the observed decline. For the period following the recession, both equations predict little consumption growth through 1991 and 1992, with a slight pickup in 1993. While actual consumption growth remained weak in 1993, the tempting conclusion is that stronger consumption growth is just around the corner.

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9. The dynamic simulations (not shown) for the model using total wealth are very similar though marginally inferior to those with human and non-human wealth (excluding equity) entered separately.

4 CONCLUSION

The wealth measure developed in this study has significant explanatory power for consumption over and above the information already contained in current disposable income. This suggests that expected future income is an important determinant of consumption for at least some households. At the same time, disposable income remains an important determinant of consumption in both the short and the long run. One interpretation of this finding is that a significant proportion of households are liquidity constrained and therefore consume out of disposable income. An alternative interpretation is that wealth is measured with error, and this error is correlated with disposable income. Unfortunately, the results presented in this study cannot be used to discriminate between these two alternatives.

A second finding is that fluctuations in equity prices have no significant impact on aggregate consumption of non-durables and services. A measure of non-human wealth that includes equity is found to have little explanatory power for consumption in either the long or the short run, while a measure that excludes equity is a significant long-run determinant of consumption. This finding may reflect the fact that most consumers simply do not hold equity or that a large proportion of the changes in equity prices are viewed as transitory.

Estimates of dynamic EC consumption functions suggest that the negative effect of interest rates on consumption in a traditional Keynesian consumption function is due principally to the effect of interest rates on human wealth. When wealth is included among the regressors, the coefficient on the real interest rate remains negative, owing presumably to an intertemporal substitution effect, but this effect is not significant.

For policy makers and practitioners, the most convincing evidence that wealth is an important determinant of consumption is probably that it is helpful in explaining both the severity of the 1981–82 recession and consumption boom of the late 1980s, both of which appear anomalous based on a traditional EC consumption equation that does not include wealth. While future work will no doubt improve empirical consumption functions further, by augmenting a standard EC consumption model with wealth, this analysis succeeds both in placing empirical consumption models for Canada on a firmer theoretical foundation and in improving our ability to explain observed consumption behaviour.

On a broader level, the usefulness of wealth explaining consumption suggests that the measures of wealth developed in this report may be useful in other applications. These include the impact of public policy on consumption and saving, and the effect of fluctuations in asset prices on aggregate demand and inflation.

The evidence that aggregate consumption and saving respond differently to different components of non-human wealth also suggests that distributional issues deserve more attention. In principle, heterogeneity in consumer portfolios could be addressed by disaggregating consumers into various groups, but this approach is hampered by the lack of adequate data. An alternative is to disaggregate non-human wealth into its principal components and examine the sensitivity of these individual components to aggregate consumption.

Evidence that consumption is influenced by the composition of wealth may also indicate that adjustment costs associated with portfolio allocation differ across assets. In the absence of separation between the portfolio-choice and consumption-savings problems, an integrated approach to modelling wealth allocation and consumption behaviour is appropriate. These and other extensions, however, are left for future work.

APPENDIX 1: The data

This appendix describes the construction of all the variables in this study. The raw data series are all drawn principally from CANSIM, but some series are from the RDXF and QPM data bases at the Bank of Canada. Reference numbers for CANSIM data are provided. To the extent possible, seasonally adjusted data were used, and in cases where the data were unadjusted, seasonal adjustment was performed using ARIMA X-11, if a stable seasonal pattern was found to exist (these variables are suffixed with *SA*).

L = real per capita labour income = *(LBINC/(PGDP*NPOP*4))*

LBINC = labour income in millions of current dollars
= *(YW+(YW/(YGDP–YENAR)))*(YFA+YNFNC)*

YW = labour income excluding military pay (wages, salaries and supplementary labour income in millions of current dollars). D20088–D20091

YGDP = gross domestic product in millions of current dollars. D20011

YENAR = residual error. D20029

YFA = farm income. D20005

YNFNC = unincorporated business income. D20006

PGDP = GDP price deflator. D20011 / D20463

NPOP = non-institutional population (15 years and over); quarterly average of the monthly series D767284 / 1000

r = real interest rate = *RR90*+2.3 / 400

RR90 = *R90* / 400–*EINF*

R90 = nominal interest rate on 90-day prime corporate paper. B14017

EINF = expected inflation = log$(PGDP(t)PGDP(t–2))/2$

G = real per capita government expenditures on goods and services to be paid for by households = $(GOVS/(PGDP*NPOP*4))$

GOVS = $(GOV(t)+GOV(t–1)+GOV(t–2)+GOV(t–3))/4$

GOV = government expenditures on goods and services less fraction not paid for by households historically
= $GOVEXP(1–\Omega)$

Ω = rolling historical average of the proportion of government expenditures paid by corporations, non-residents and government interest income
= $(\omega_t + \omega_{t-1} + \ldots + \omega_{t-7})/8$

ω = $((CPTAX–CPSUB–CPTRAN)+(NRSTAX–NRSTR)+GOVIN)/GOVEXP$

CPTAX = direct taxes from corporations and government enterprises. D20156

CPSUB = transfer payments to businesses (subsidies). D20164

CPTRAN = transfer payments to businesses (capital assistance). D20165

NRSTAX = direct taxes from non-residents. D20157

NRSTR = transfer payments to non-residents. D20166

GOVIN = government’s investment income. D20160

NETAX = $(INCTAX+SALTAX+OTHTAX–TRANF)$

TAX	=	taxes net of transfers
	=	*(NETAX/(PGDP*NPOP*4))*
INCTAX	=	direct taxes from persons. D20155
SALTAX	=	indirect taxes. D20158
OTHTAX	=	other current transfers from persons. D20159
TRANF	=	transfer payments to persons. D20163
GOVEXP	=	current expenditure on goods and services. D20162

A = net domestic foreign assets excluding government debt

= *(NETWORTHSA/PGDP*NPOP)*

NETWORTHSA	=	net worth at market value
	=	*HOUSESA+FINAQ1SA+EQUITQSA +HDEBTQSA–HLIABQSA+CQPPQSA –DDEBTQSA*
HOUSESA	=	non-financial assets of persons and unincorporated businesses. Includes total consumer durables and residential structures.
	=	*RSTRUCSA+KDUR*
RSTRUC	=	residential structures = *(PMLS/*100*)* * *KRC*
PMLS	=	multiple homes listing price index. RDXF data base = *PMLS*
KRC	=	stock of housing in constant 1986 dollars. QPM data base = *KRC*
RSTRUCSA	=	residential structures

KDUR	=	total stock of consumer durables (includes motor vehicles, household durables and miscellaneous durables)
	=	*(KMV*PCMV)+(KDHSHD*PHSHD) + (KDMISC*PCDMIS)*
KMV	=	stock of motor vehicles held by consumers *KMV* = 16 734.82 [1960:4] set *KMV* = 0.933**KMV* [*t*–1]+(D20490 / 4)
PCMV	=	price of consumer motor vehicles and parts
	=	(D20115 / D20490)
KDHSHD	=	stock of household durables held by consumers *KDHSHD* = 9 237.448 [1960:4] set *KDHSHD* = 0.944**KDHSHD* [*t*–1] +(D20491 / 4)
PHSHD	=	price of consumer household durables
	=	(D20116 / D20491)
KDMISC	=	stock of durables excluding motor vehicles and parts, and household durables held by consumers *KDMISC* = 3 539.846 [1960:4] set *KDMISC* = 0.944**KDMISC* [*t*–1] +(D20492 / 4)
PCDMIS	=	price of other consumer durables
	=	(D20117 / D20492)
FINAQ1SA	=	financial assets of persons and unincorporated businesses (seasonally adjusted). Includes currency and deposits, consumer credit, other Canadian bonds (short- and long-term), life insurance and pensions, foreign investments, mortgages and other financial assets.

	=	*FINAQ_CD+FINAQ_CC+FINAQ_OBDS + FINAQ_LP + FINAQ_FI+FINAQ_OF + FINAQ_M*
FINAQ_CD	=	currency and deposits. D160006+D160007+D160008, D150032+D150033+D150034
FINAQ_CC	=	consumer credit. D160010, D150048
FINAQ_OBDS	=	other bonds (short- and long-term). D160016+D160022, D150036+D150065
FINAQ_LP	=	life insurance and pensions. D160023, D150066
FINAQ_FI	=	foreign investments. D160028, D150068
FINAQ_OF	=	other financial assets. D160029, D150049
FINAQ_M	=	mortgages. D160017, D150128
EQUITYQ	=	market value of equity held by persons and unincorporated businesses
	=	*EQUITYQ(t*–1*)*TSE_Q(t)/TSE_Q(t*–1*) + (BEQUITY(t)–BEQUITY(t*–1*))*
TSE	=	TSE300 composite stock price index. B4237
BEQUITY	=	book value of equity held by persons and unincorporated businesses
	=	*EQUITQ–(YCR/*4*)*(EQUITQ/TEQUITQ)*
EQUITQ	=	“current” value of shares held by persons and unincorporated businesses. Current value is measured as the sum of book value and cumulated retained earnings. D160027, D150067

TEQUITQ	=	total outstanding stock of equity. D162906, D151988
YCR	=	retained earnings. D20068
HDEBT	=	book value of government debt held by persons and unincorporated businesses
	=	*HTBQ+HGBQ+HPBQ+HMBQ*
HDEBTQ	=	market value of government debt held by persons and unincorporated businesses
	=	*HTBQ+PBQ*(HDEBT–HTBQ)*
H_BQ	=	holdings of treasury bills (*T*), Government of Canada bonds (*G*), provincial bonds *(P*) and municipal bonds (*M*) by persons and unincorporated businesses. D160015, D150035; D160019, D150062; D160020, D150063; D160021, D150064
HLIABQ	=	total liabilities of persons and unincorporated businesses. Includes consumer credit, trade payables, bank loans, other loans, finance and other short-term paper, other Canadian bonds, and mortgages. D160031, D150050
NNWPFIQ	=	net worth of public financial institutions less their holdings of government debt
	=	*NWPFIQ–PFIGBQ–PFIPBQ–PFIMBQ* – *PFITBQ*
PFI_BQ	=	holdings of treasury bills (*T*), government of Canada bonds (*G*), provincial bonds (*P*) and municipal bonds (*M*) by public financial institutions. D161975, D151326; D161979, D151330; D161980, D151331; D161981, D151332

CPPQ	=	Canada Pension Plan. D162590, D151760
QPPQ	=	Quebec Pension Plan. D162660, D151784
CQPPQ	=	aggregated Canada and Quebec pension plans
	=	*(CPPQ+QPPQ)*
DDEBTQ	=	domestically held government debt at market value
	=	*TBILL–FORTBQ+PBG*(OTHDEBT –FORGBQ–FORPBQ–FORMBQ)*
TBILL	=	net teasury bills outstanding at book value
	=	*TTBQ–GTBQ–PGTBQ–PFITBQ–GETBQ –BCTBQ*
TTBQ	=	total outstanding stock of treasury bills. D162894, D151976
_TBQ	=	holdings of treasury bills by the federal government *(G*), provincial governments (*PG*), public financial institutions (*PFI*), government enterprises *(GE)* and the Bank of Canada (*BC*). D162185, D151482; D162255, D151544; D161975, D151326; D160155, D150147; D160435, D150350
OTHDEBT	=	net other debt outstanding at book value
	=	*TGBQ+TPBQ+TMBQ–GGBQ–GPBQ–GMBQ –PGBQ–PPBQ–PMBQ–PFIGBQ–PFIPBQ –PFIMBQ–GEGBQ–GEPBQ–BCGBQ*
T_BQ	=	total outstanding stock of federal government bonds (*G*), provincial government bonds (*P*) and municipal government bonds (*M*). D162898, D151980; D162899, D151981; D162900, D151982

G_BQ	=	federal government holdings of federal government bonds (*G*), provincial government bonds (*P*) and municipal government bonds (*M*). D162189, D151486; D162190, D151487; D162191, D151488
P_BQ	=	provincial government holdings of federal government bonds (*G*), provincial government bonds (*P*) and municipal government bonds (*M*). D162259, D151548; D162260, D151549; D162261, D151550
PFI_BQ	=	holdings by public financial institutions of federal government bonds (*G*), provincial government bonds (*P*) and municipal government bonds (*M*). D161979, D151330; D161980, D151331; D161981, D151332
GE_BQ	=	holdings by government enterprises of federal government bonds (*G*), provincial government bonds (*P*) and municipal government bonds (*M*). D160159, D150151; D160160, D150152; D160161, D150153
BCGBQ	=	Bank of Canada's holdings of federal government bonds. D160439, D150353
PBG	=	bond price index constructed following Rose and Selody (1985)
	=	*RAC/RG_Q+((*1–*RAC)/RG_Q)* exp[–*RG_Q* (*1–*RATAX)**19.6]
RG	=	average yield to maturity on 3–5 year Government of Canada bonds. B14010
RG_Q	=	quarterly average of the monthly series *RG*

RAC = average coupon rate
= *RAC(t*–1*)* if *(DIFDEBT*+1/19.6*)* < 0
= *RAC(t*–1*)*((DEBT(t*–1*)/DEBT(t))*–1 / 19.6*)*
+RG(((DEBT(t)–DEBT(t*–1*))/DEBT(t))*+1/19.6*)*
otherwise

RATAX = average tax rate = *INCTAX/LBINC*

DIFDEBT = *(DEBT(t)–DEBT(t*–1*))/(DEBT(t*–1*)*
+DEBT(t)) / 2+1 / 19.6

FDEBTQ = foreign debt at market value
= *(FORTBQ+PBG*(FORGBQ*
+FORPBQ+FORMBQ))

FOR_BQ = rest of the world – Government of Canada treasury bills (*T*), federal government bonds (*G*), provincial government bonds (*P*) and municipal government bonds (*M*).
D162745, D151811; D162749, D151814; D162750, D151815; D162751, D151816

c = log of real per capita consumption of non-durables and services
= log [*(CON–CDUR)/NPOP*]

CON = real total consumption. D20488

CDUR = real consumption of durables. D20489

y = log of real per capita disposable income = log [*YDP/PGDP/NPOP*]

YDP = current dollar disposable income. D20111

p = relative price of non-durables and services

= log [*((CON$ – CDUR$) / (CON – CDUR)) / PGDP*]

CON$ = total consumption in current dollars. D20113

CDUR$ = consumption of durables in current dollars. D20114

u = unemployment rate. Statistics Canada, Labour Force Division. *(LFSA601/LFSA101)**100

APPENDIX 2: Closed-form solution for the cumulative growth factors

This appendix outlines the closed-form solution for the cumulative growth factor Γ from (2.4). Expanding out the expression for Γ we have

$$\Gamma_t = E_t[q_{t+1} + q_{t+1}q_{t+2} + q_{t+1}q_{t+2}q_{t+3} + \ldots] \tag{A1}$$

where

$$q_{t+i} = \left(\frac{1 + x_{t+i}}{1 + r_{t+i}}\right) \tag{A2}$$

Using Markov approximation due to Tauchen (1986), the VAR for x and r is approximated as a finite-state discrete-valued system. Using grids of N_x and N_r to approximate the continuous-valued series x and r respectively, the state space of the dicrete system is $N_x \times N_r \equiv N$. The dynamics of the system are described by an $N \times N$ matrix of transition probabilities Φ with typical element

$$\phi_{k,l} = \text{prob}[state = k \mid state = l] \tag{A3}$$

where $\mid$ denotes “conditional on.” The discrete-valued system can be used to form the discrete variable $\hat{q}$ that approximates the continuous-valued variable q. Let Q be the $N_x \times N_r$ matrix of $\hat{q}$’s in the system and define $\vec{Q}$ as the $N \times 1$ vector obtained by stacking the columns of Q one on top of the other. If we index the elements of $\vec{Q}$ by $k = 1, \ldots, N$, the typical element of $\vec{Q}$ can be written as $\hat{q}(k)$. With this investment in notation, the expected geometric averages of $\hat{q}$ can be computed as follows:

$$E[\hat{q}_{t+1} \mid \hat{q}_t = \hat{q}(k)] = \sum_{l=1}^{N} \phi_{k,l}\hat{q}(l) \tag{A4}$$

$$E[\hat{q}_{t+1}(\hat{q}_{t+2}\,|\hat{q}_t) = \hat{q}(k)] = \sum_{l=1}^{N}\sum_{m=1}^{N} \phi_{k,l}\phi_{l,m}\hat{q}(l)\hat{q}(k) \qquad \text{(A5)}$$

Terms further into the future can be constructed in a similar fashion. The solutions (A4) and (A5) bring home the magnitude of the task of computing the cumulative growth factors. The terms (A4) and (A5) are only the first two terms of an infinite summation, and this infinite summation has to be computed in every state of the system.

Fortunately, there is a closed-form solution for Γ based on the approximated system, which makes the task of computing the cumulative growth factors much more manageable. Let $\vec{\Gamma}$ be an $N \times 1$ vector of all the cumulative growth factors in the discrete system. In addition, let Ω be an $N \times N$ matrix with all its rows being $\vec{Q}^{\tau}$, where τ is the transpose operator. The vector of cumulative growth factors then has the following representation:

$$\vec{\Gamma} = \sum_{\alpha=1}^{\infty} (\Phi \bullet \Omega)^{\alpha}\iota = ([I - \Phi \bullet \Omega]^{-1} - I)\iota \qquad \text{(A6)}$$

where $\bullet$ denotes element-by-element multiplication, ι is an $N \times 1$ vector of ones, and I is the identity matrix. Computing the vector of cumulative growth factors in every state of the system therefore amounts to inverting a large matrix. With grids of 25 points, the matrix to be inverted is 625×625.

APPENDIX 3: Wealth and its components (per capita in constant 1986 dollars)

Date	Total wealth	Human wealth	Non-human wealth	Non-human wealth excluding equity
63:1	199713.98	178880.23	20833.75	15090.25
63:2	207541.22	186487.74	21053.48	14923.26
63:3	208428.97	187203.91	21225.06	15137.56
63:4	211173.54	189694.84	21478.70	15048.76
64:1	208800.39	187084.90	21715.49	15068.68
64:2	213238.12	190997.70	22240.42	14980.54
64:3	223218.29	199965.46	23252.83	15156.46
64:4	219905.72	196221.88	23683.84	15311.88
65:1	225412.75	199904.33	25508.42	16426.69
65:2	227615.22	203713.34	23901.88	14782.94
65:3	232976.86	208095.57	24881.30	15693.68
65:4	233718.72	208239.08	25479.64	15821.16
66:1	240516.15	214424.28	26091.87	16417.15
66:2	246413.03	220525.00	25888.03	16510.28
66:3	239908.66	214143.18	25765.48	16760.85
66:4	238114.97	211701.46	26413.51	17414.80
67:1	240242.43	213486.80	26755.63	17147.33
67:2	246867.35	219391.21	27476.14	17611.66
67:3	238242.91	210038.17	28204.75	17854.39
67:4	231509.99	203105.68	28404.31	18630.22
68:1	238551.34	210972.48	27578.86	18426.17
68:2	236282.54	207589.08	28693.46	18865.70
68:3	235490.21	205796.90	29693.31	19185.44
68:4	242326.26	212130.56	30195.71	19035.31
69:1	251365.10	220614.90	30750.20	19637.60
69:2	255180.29	224102.27	31078.02	19763.47
69:3	246654.56	216538.64	30115.92	19793.65
69:4	242901.54	212182.26	30719.28	20042.29
70:1	251070.90	221603.55	29467.35	19560.01
70:2	246254.08	218283.04	27971.04	19376.86
70:3	240017.62	211637.02	28380.60	19568.82
70:4	246127.32	217608.52	28518.80	19462.32
71:1	254595.55	225955.36	28640.19	19405.45
71:2	264955.92	235702.81	29253.12	19720.54
71:3	262041.62	232671.96	29369.66	20071.68
71:4	266140.29	237243.88	28896.42	20235.65
72:1	278619.07	248425.40	30193.67	20600.78
72:2	279282.93	248399.27	30883.67	20771.96
72:3	279764.81	248293.98	31470.82	21023.59
72:4	294275.50	262577.65	31697.85	21151.73
73:1	300439.82	268113.84	32325.98	21957.40
73:2	305962.37	273370.45	32591.92	22816.64
73:3	305584.51	271549.85	34034.66	23981.93
73:4	312904.85	277572.30	35332.55	25200.14
74:1	321640.90	286172.89	35468.01	26141.46

74:2	315047.75	279761.08	35286.68	26967.40
74:3	313807.43	279537.24	34270.19	26722.16
74:4	306866.49	273723.28	33143.21	26168.82
75:1	308341.15	274276.51	34064.64	26441.25
75:2	311705.59	276926.70	34778.89	26874.54
75:3	318170.64	283155.46	35015.18	27298.02
75:4	309847.91	275116.42	34731.49	27068.78
76:1	309107.84	273724.93	35382.90	27420.00
76:2	312098.65	276561.92	35536.73	27312.23
76:3	295875.11	259903.44	35971.67	27718.01
76:4	303889.97	268334.99	35554.98	27760.00
77:1	304943.06	269300.86	35642.20	27925.98
77:2	306670.25	271023.92	35646.33	28089.86
77:3	309080.80	273066.02	36014.78	28524.96
77:4	300171.00	263595.09	36575.91	28845.98
78:1	299067.32	262709.63	36357.69	28597.69
78:2	303301.08	266518.14	36782.94	28376.72
78:3	308399.18	270567.75	37831.43	28206.42
78:4	309970.62	270757.97	39212.65	28595.03
79:1	308237.12	267267.53	40969.59	29083.68
79:2	322243.40	280155.07	42088.33	29253.62
79:3	328721.31	285872.05	42849.25	29376.25
79:4	308322.96	264786.03	43536.93	30025.43
80:1	308704.93	262239.91	46465.02	30581.92
80:2	313528.94	267742.52	45786.42	30213.95
80:3	335004.87	286781.38	48223.49	31014.03
80:4	320641.55	270664.18	49977.36	32176.22
81:1	308474.68	259318.59	49156.08	32128.56
81:2	309145.64	259029.55	50116.09	32694.27
81:3	288572.56	240072.29	48500.27	33151.53
81:4	304690.37	258238.69	46451.68	32670.88
82:1	298703.18	255851.14	42852.04	30770.50
82:2	284443.72	243791.24	40652.48	30247.37
82:3	277815.27	237967.64	39847.63	29004.26
82:4	290231.75	249326.37	40905.37	28097.29
83:1	284331.74	240935.78	43395.96	29059.28
83:2	285381.75	240116.77	45264.98	28761.24
83:3	291613.09	245480.81	46132.28	28881.16
83:4	290547.59	244758.95	45788.64	28709.15
84:1	288035.01	242932.51	45102.50	28449.09
84:2	277649.28	233209.41	44439.87	29064.71
84:3	273870.41	228666.52	45203.88	29161.83
84:4	285292.58	239430.32	45862.26	29188.33
85:1	293158.24	246095.88	47062.36	29035.56
85:2	306851.71	259123.47	47728.24	28927.78
85:3	316425.16	267677.14	48748.02	29387.03
85:4	310608.97	260720.10	49888.86	29783.46
86:1	304437.69	253145.50	51292.18	30619.53
86:2	323897.93	270467.02	53430.91	31835.94
86:3	328778.43	274581.25	54197.19	32933.66
86:4	338003.39	282153.19	55850.20	33592.52
87:1	357627.58	298408.21	59219.37	34155.63

87:2	356297.31	295864.21	60433.10	34653.21
87:3	350484.23	287102.30	63381.92	35340.39
87:4	351493.12	293623.24	57869.88	36140.68
88:1	362271.41	303096.20	59175.21	37361.89
88:2	355871.15	294741.20	61129.95	38568.60
88:3	356962.94	294831.17	62131.77	39711.03
88:4	363093.31	298708.11	64385.20	41362.44
89:1	359172.46	291311.83	67860.63	44095.75
89:2	352292.81	286731.84	65560.97	41708.22
89:3	359725.54	290878.09	68847.45	43323.68
89:4	351797.83	281352.02	70445.81	44452.88
90:1	343077.84	275980.88	67096.96	43556.16
90:2	339563.46	275123.85	64439.62	42423.73
90:3	337919.27	274018.82	63900.45	42930.06
90:4	335291.15	273184.94	62106.21	41894.65
91:1	347347.21	283589.50	63757.71	42359.89
91:2	347803.07	282722.90	65080.17	43293.55
91:3	339077.59	275786.95	63290.64	41652.16
91:4	340602.75	277153.79	63448.96	40976.95
92:1	343385.90	280678.60	62707.30	40417.54
92:2	348329.90	286640.59	61689.32	40394.84
92:3	346892.35	284791.78	62100.56	40694.73
92:4	342518.68	280058.66	62460.01	40948.46
93:1	348262.22	286535.98	61726.24	39918.64
93:2	351924.72	287575.41	64349.30	39866.69
93:3	354835.65	289673.15	65162.50	39539.28

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