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Tests of Market Efficiency in the One-Week When-issued Market for Government of Canada Treasury Bills

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ABSTRACT

This report presents different tests of market efficiency in the when-issued market for Government of Canada treasury bills and examines the effectiveness, in this market, of Bank operations over the 1986 to mid-1992 period. The when-issued market, which is a combination of a forward and futures market, enables market participants to buy or sell treasury bills up to one week in advance of the regular weekly auction. The theoretical pricing relations presented show that forward and futures yields, even in informationally efficient forward markets, may not be unbiased estimates of future spot yields. This occurs because a sweetener may be necessary to entice one side of the market into covering the other and because cash flows are uncertain at the time investment decisions are made.

In general, the ordinary least squares regressions reveal only marginal justification for rejecting the composite null hypotheses, which include market efficiency and rational expectations. However, the report shows that when efficiency gains are exploited, through modelling of the time-varying volatility, as generalized autoregressive conditional heteroscedasticity (GARCH), yield changes tend to be followed by yield changes of the same sign for the three when-issued bills. The empirical evidence also suggests that unexploited profitable hedging opportunities may have existed in the three- and six-month coincident-to-when-issued bills. Finally, the report shows that when the null hypothesis is rejected, when-issued yields understate treasury bill yields at auction and that open market operations influence when-issued yields and auction yields differently.

RÉSUMÉ

La présente étude expose différents tests portant sur l'efficience du marché des bons du Trésor négociés avant l'émission et examine l'efficacité des opérations effectuées par la Banque du Canada sur ce marché entre 1986 et le milieu de 1992. Le marché des bons du Trésor négociés avant l'émission, qui s'apparente à la fois au marché des opérations à terme et à celui des contrats à terme d'instruments financiers, donne aux participants la possibilité d'acheter ou de vendre des bons du Trésor pendant la semaine précédant l'adjudication hebdomadaire de ces titres. Les relations théoriques de détermination des prix montrent que, même dans un marché à terme efficient au point de vue de l'information, le rendement des opérations à terme ou des opérations sur contrats à terme peut ne pas constituer une estimation non biaisée du rendement des futures opérations au comptant. Cette situation résulte soit du fait qu'un incitatif peut être nécessaire pour amener une des parties au marché à prendre une position de couverture à l'égard de l'autre soit de l'incertitude entourant les flux financiers au moment où se prend une décision d'investissement.

En général, les résultats obtenus à l'aide des moindres carrés ordinaires permettent à peine de rejeter les hypothèses nulles conjointes d'efficience des marchés et de rationalité des attentes. L'auteur montre cependant que, lorsque, pour exploiter les gains d'efficience, on modélise, à l'aide de l'hétéroscédasticité conditionnelle autorégressive généralisée (GARCH en anglais), la volatilité, non constante dans le temps, qu'on observe si souvent dans les données financières produites à fréquence élevée, les variations des taux de rendement tendent à être suivies de variations de même signe du rendement des bons du Trésor négociés avant l'émission. Les résultats empiriques indiquent également qu'il existe des possibilités inexploitées d'opérations de couverture rentables sur le marché des bons du Trésor à trois et à six mois négociés avant l'émission. L'auteur montre enfin que, lorsque l'hypothèse nulle est rejetée, le rendement estimatif des bons du Trésor négociés avant l'émission est inférieur au rendement effectif établi à l'adjudication et que les opérations d'open market influencent différemment les taux de rendement du marché des titres négociés avant l'émission et ceux qui ressortent des séances d'adjudication.

EXECUTIVE SUMMARY

This report examines the when-issued market for Government of Canada treasury bills, describes its participants and explains its importance in money markets. It also examines efficient markets hypotheses of the when-issued yield. Because the when-issued market is a hybrid of a forward and futures market, efficiency tests for both forward and futures markets are examined, in addition to arbitrage restrictions.

The when-issued market facilitates the trade of the to-be-issued treasury bills up to one week in advance of the regular weekly auction. Hence, the when-issued market provides a link between the coincident-to-when-issued bill (the previously issued six- or twelve-month bill that has wound down) and the expected treasury bill yield at tender, and between current market yields and those that are expected to prevail at the upcoming auction. The Bank of Canada uses the yield on the when-issued bill as a gauge of the market's expectation of the upcoming treasury bill yield at tender (and subsequently of Bank Rate), while participants use the when-issued market to presell some expected auction winnings or to ensure that they have locked in an inventory of bills at a yield that would otherwise not be known until a later date.

Tests of forward market efficiency seem to indicate that the when-issued yield is an unbiased predictor of the upcoming treasury bill yield at tender. They appear to show as well that Bank operations affect the when-issued yield and the tender yield differently. Simple futures markets tests indicate that own-lagged when-issued yield changes help predict current six- and twelve-month when-issued yield changes, while more sophisticated futures markets tests indicate that the when-issued yield approaches the tender yield from below (it is less than the tender yield during the week leading up to the auction).

The existence of a whole array of treasury bill maturities implies that participants may choose to take a position in the coincident-to-when-issued market, rather than the when-issued market itself, for hedging purposes. Tests of arbitrage relationships in the when-issued and coincident-

to-when-issued markets show that there may have been (subject to liquidity and transaction costs) profitable hedging opportunities in both three- and six-month markets. Finally, it is shown that the when-issued–current treasury bill yield differential can be partially explained by a yield curve slope term and a net carry term (over the coincident-to-when-issued bill).

The findings of this note suggest *(i)* that market participants could improve their estimate of the treasury bill yield at tender by using the when-issued bill in conjunction with the current three- and six-month bills and the call loan rate and *(ii)* that the when-issued yield does not fully respond to open market operations in the same way that the treasury bill yield does at auction.

1 INTRODUCTION

Until late last year, the Bank of Canada auctioned on Thursdays a previously announced dollar amount of treasury bills having three-, six- and twelve-month maturity dates. Since 24 November 1992, however, treasury bills have been auctioned on Tuesdays for delivery on Thursdays.

For the Government of Canada, the weekly auction has become an important instrument in debt management, while for individual investors, large corporations and financial institutions, the treasury bill market provides a source of short-term and high-quality liquid assets. Participation in this market has widened considerably since 1980 with the increase in supply.

Used by the Bank of Canada for conducting open market operations and accepted by the Bank as collateral for lending purposes, treasury bills have become one of the most important securities in the implementation of monetary policy. The three-month treasury bill tender rate is particularly important, as the weekly Bank Rate[1] is set at 25 basis points (one quarter of one percentage point) above the average tender yield on it.

When the Bank of Canada makes the results of each auction public, it announces the quantity of treasury bills of each maturity to be auctioned the following week. For the next week, up to and including the morning of the auction, market participants are able to engage in forward trading of the treasury bills in what is called the when-issued market by writing when-issued contracts.[2] A when-issued treasury bill contract combines elements of both financial forward and futures contracts. Like a futures contract, it has a fixed *date* of maturity but like a forward contract it

1. The Bank Rate is the interest rate at which directly clearing members of the Canadian Payments Association and selected investment dealers are able to borrow collateralized funds from the Bank of Canada on a short-term basis.

2. There is some evidence suggesting that market participants may like to trade at horizons longer than one week. However, it is not clear how successful this would be as the dollar amounts of each tranche are known only one week prior to the auction.

is not marked-to-market[3] and its contracts are not standardized with respect to quantity. When-issued contracts are traded on an over-the-counter basis and not on a centralized exchange.

The when-issued market allows market participants to speculate on the future course of interest rates and to hedge both current and future treasury bill inventories; it also provides arbitrage opportunities against outstanding treasury bill yields. Forward (or futures) markets are efficient when all relevant information is contained in the forward price and forward and spot prices are tied by relationships that do not allow the systematic occurrence of abnormal profit. In the when-issued market, market efficiency requires that all available information be used in forming expectations of Thursday's treasury bill tender yield and that no potential positions yielding systematic and predictable abnormal profits exist. However, because future cash flows are uncertain at the time the investment decision is made and because when-issued contracts are net zero supply assets (every long position is matched by a short position), non-zero profits may arise at contract maturity.

While the when-issued yield can be used as a gauge of the market's expectation of the upcoming treasury bill yield at tender, expectations that are not sufficiently forward-looking may cause the when-issued yield to deviate from the future treasury bill yield. This can occur because forward and futures yields contain an expectation of the future spot yield as well as a component reflecting the riskiness of the derivative or the spot market. Hence, without knowledge of the extent of the bias in expectations or riskiness of the markets involved, the when-issued yield may be a biased barometer of the upcoming tender yield. In other words, because every short position is matched by a long position in the when-issued market and because future cash flows are uncertain, a risk premium may

3. Marking-to-market is the daily addition (or removal) of margin funds in accordance with futures price increases (or decreases). For example, in the event of a price increase, the clearing house adds the amount of the price change (times the number of contracts) to the margin deposit belonging to the buyer of the contract, with the funds coming from the margin deposit belonging to the seller of the contract.

be necessary to entice one side of the market into covering the other side if either the forward or spot market is significantly risky.

This potentially time-varying risk premium is one explanation of the deviation between forward (or futures) and spot yields. Another is the financing costs associated with long positions in the spot market for outstanding treasury bills. No such costs exist for similar positions in the when-issued market.

This report examines the when-issued market for Government of Canada treasury bills, describes its participants, and explains its importance in money markets. It also examines efficient markets hypotheses of the when-issued yield. In Section 2 a description of when-issued market participants and transactions is provided. Sections 3 and 4 look at forward market tests and futures market tests of efficiency. Section 5 considers arbitrage relations in the when-issued market and Section 6 examines the behaviour of when-issued–treasury bill spreads. In conclusion, Section 7 gives a summary and some final thoughts.

1.1 Monetary policy and the when-issued market

In conducting monetary policy operations, the Bank of Canada uses the yield on when-issued bills as one of the gauges of the market's expectation, at any given point in time, of the upcoming treasury bill yield at tender. This requires confidence that the when-issued yield accurately reflects expectations concerning the upcoming tender yield. If expectations were inefficiently formed and a biased when-issued yield resulted, the Bank's selection of appropriate day-to-day policy actions might be hampered. For example, if when-issued yields were not accurate predictors of treasury bill yields at the upcoming auction, the Bank might have difficulty in interpreting the market's perception of the upcoming tender yield. This, in turn, could hinder the Bank's implementation of appropriate policy operations.

While the Bank of Canada uses the when-issued yield as a barometer for its activities, it has not intervened in the when-issued market through forward purchases and sales. The Bank prefers instead to implement monetary policy through cash management and by direct intervention in the cash treasury bill market.[4] For policy purposes, open market operations typically involve current three-month treasury bills. On rare occasions and usually just before the new treasury bill tender, some transactions are in the coincident-to-when-issued treasury bill.[5] From January 1991 to January 1992 only 2 per cent of the number of policy transactions in the cash treasury bill market were in the coincident-to-when-issued bill. All such transactions occurred on tender mornings. Of course, to the extent the Bank's monetary policy operations provide the market with information on the likely yield of treasury bills at the upcoming auction, they will be reflected in the when-issued yield.

1.2 Empirical tests of efficiency in the when-issued market

Because of the unique characteristics of the when-issued market, three notions of market efficiency are considered: *(i)* speculative efficiency, *(ii)* hedging efficiency, and *(iii)* arbitrage efficiency. Speculative efficiency is examined in a futures market framework that implies that the best guess of future when-issued yields is the current when-issued yield pertaining to that contract.

The notion that the when-issued market can be used as an efficient hedge against future treasury bill price uncertainty is examined in two regression-based tests. The unbiased-expectations hypothesis suggests that the forward price (or yield) should systematically equal the future spot price (or yield), while tests of orthogonality restrictions examine the predictability of the forecast error that market participants make each week.

4. Monetary policy intervention is discussed in more detail in the following: Kevin Clinton and Kevin Fettig, "Buy-back Techniques in the Conduct of Monetary Policy," *Bank of Canada Review* (July 1989); "Bank of Canada Cash Management: The Main Technique for Implementing Monetary Policy," *Bank of Canada Review* (January 1991).

5. The previously issued six- and twelve-month treasury bills now having just over three months to maturity are called the coincident-to-three-month-when-issued bills. At the auction, these coincident-to-three-month-when-issued bills have three months remaining to maturity. An analogous situation exists for the six-month bill.

To test for arbitrage efficiency, a cash-and-carry arbitrage portfolio is created so as to render zero expected profits at the time the portfolio decision is made. In addition, a cost-of-carry and term-premium model is set forth as an explanation of the spread between the when-issued yield and the current treasury bill yield.

1.3 Summary of results

With respect to speculative efficiency, ordinary least squares (OLS) regressions revealed marginal evidence of when-issued yield changes that were predictable by a constant and by own-lagged yield changes. Much stronger evidence to this effect was found in generalized autoregressive conditional heteroscedasticity models – specifically, GARCH(1,1) models – where the variance-covariance matrix of when-issued yield changes is allowed to vary over time. Here, Bank of Canada market intervention was found to influence both the mean and variance of when-issued yield changes: three-month treasury bill sales tend to reduce volatility, while large foreign exchange intervention tends to be accompanied by increased volatility.[6]

Regarding hedging efficiency, the forward market tests of efficiency generally suggest that on average the when-issued yield is an unbiased predictor of the upcoming treasury bill yield at each weekly auction and that the forecast error that market participants make each week cannot be systematically predicted by the lagged forecast error, a yield-curve slope factor nor a constant. However, as in most tests of orthogonality restrictions, useful information may have been omitted from the tests. Along with the futures markets tests, it is found that open market operations affect the when-issued and treasury bill tender yields differently.

With respect to arbitrage efficiency, the results indicate that, depending on liquidity and transaction costs, profitable arbitrage

6. This does not mean that foreign exchange intervention causes the increased volatility. More likely, it is the volatility in the foreign exchange market that causes both intervention and volatility in the treasury bill market. No relationship was found between treasury bill purchases and the volatility of when-issued yields.

opportunities may have occurred in the coincident-to-when-issued market over the 1989 to mid-1992 test period. It may have been profitable to take a short hedge in the coincident-to-three-month-when-issued bill, coupled with the equivalent long when-issued position, or a long hedge in the coincident-to-six-month-when-issued bill. It was also found that the when-issued–current cash bill differential could be partially explained by a yield-curve slope factor and a net carry factor.

2 DESCRIPTION OF THE WHEN-ISSUED MARKET

Formal trading of when-issued treasury bills began in the fall of 1978 and, in conjunction with the supply of treasury bills, has grown considerably since.[7] Some when-issued trading takes place through broker services, which enter bids and offers directly onto computer screens indicating the best yield bid (lowest yield bid by a buyer) and the best yield offer (highest yield offered by a seller) as well as the dollar amounts to be traded at these yields. Other when-issued trading is done directly over the telephone. In this way, major dealers and large institutions engage in forward trading among themselves, contracting yields, quantities and maturities without broker intermediation.

The cost of participating in the when-issued market without a broker is equal to the bid-offer yield spread. For example, a participant wishing to buy a when-issued contract may bid 5.46 per cent against a counterparty's offer to sell at 5.44 per cent. An actual exchange will occur when the two yields coincide, which could come about, in this example, by the buyer's taking the seller's offer of 5.44 per cent. The difference of 0.02 per cent represents the cost to the buyer of completing the transaction. Conversely, a participant who enters the market wishing to sell may have to hit a buyer's bid. Hence, the smaller the bid-offer spread, the less yield one party must give up to attract a counterparty and the lower the cost of completing a transaction.

Trades that require broker assistance have the additional cost of broker commissions. Bid-offer yield spreads are typically one to two basis points, while broker commissions are about one-half of a basis point on each side of the transaction.

Participants in the when-issued market include investment dealers, chartered banks and major institutional investors such as non-bank deposit-taking institutions and pension and mutual funds. Generally speaking, the first two groups play a large role in the primary market (the

7. For an account of the treasury bill market in Canada, see "The Market for Government of Canada Treasury Bills," *Bank of Canada Review* (December 1987).

auction) for treasury bills. The others generally purchase treasury bills in the secondary market.

Dealers typically hold short positions in the when-issued market to hedge against anticipated winnings at the upcoming tender. Limited evidence concerning when-issued positions shows that as the conclusion of the when-issued trading period approaches, dealers are often short several hundred million dollars, of which approximately half is in the three-month bill. On occasion, however, a relatively large short position is held in the six- and twelve-month bills. In other words, the when-issued market allows dealers to presell some of their anticipated auction winnings, at a predetermined yield, even before they know the outcome of the auction. Those on the long side of the when-issued contract include institutional investors who want to lock in a treasury bill position each week but who do not have access to the weekly treasury bill auction.

2.1 When-issued market transactions

Market participants hedge positions in a market to lock in yields that otherwise would not be known until a future date. Hedgers typically have a position or a potential position in the underlying cash market (as a regular part of their business activity) and wish to minimize yield risks of an existing position by selling when-issued contracts or to lock in current yields for a future position by buying when-issued contracts.

A long hedge is accomplished through the purchase of a when-issued contract for a specified dollar amount and maturity structure of treasury bills to be delivered the following Friday. The motivation for doing so may be to lock in a known yield or to ensure that an adequate amount of treasury bills will be on hand for use, say, as collateral for borrowing purposes.

In a short hedge, a market participant, a dealer for example, presells some of the anticipated treasury bill winnings by arranging a when-issued contract requiring delivery, to a counterparty, of the appropriate dollar amount and maturity structure of treasury bills. Although no money changes hands at the time, the when-issued contract that is arranged is an obligation by the market participant to deliver the contracted bills. In order to meet the delivery requirements of the when-issued contract, the market participant will use either the auction winnings, if they materialize, an in-house inventory or treasury bills purchased in the secondary market. Hedging, by its nature, provides a link between the current when-issued yield and the expected treasury bill yield at tender.

In addition, hedgers must also consider the yield on the coincident-to-when-issued bill to ensure that they have made the most effective hedge. That is, when wishing to lock in yields on, or inventories of, treasury bills, market participants are faced with a choice: either purchase the seasoned six-month (or twelve-month) treasury bill to hedge against both inventory needs and yield movements in the three-month treasury bill market or purchase a when-issued contract for delivery next Friday. Hence, because a whole spectrum of treasury bill maturities exists, allowing arbitrage to take place, there is a link between the current when-issued yield and the current treasury bill yield.

The when-issued market can also be used to speculate on the future course of interest rates arising from the weekly auction or simply on the when-issued yield itself. Speculators do not generally have a position in the underlying cash market. Thus, while hedging is done to lock in a known quantity or yield, speculating is typically undertaken in an attempt to profit from yield movements in the derivative market itself. Suppose an investor expected when-issued yields to increase over the week leading up to the auction. The investor would initially take a short position in the when-issued market, on say Friday, and then wind down the position a few days later by taking a long position for an equal par value and same maturity structure of treasury bills as in the initial contract. If correct, the investor would profit from the increase in when-issued yields (and therefore fall in price) and use the treasury bills from the long position to cover the short position.

Finally, participants may wish to close out their positions so as to limit their losses, if yields move adversely with respect to their investment goals or if inventories are such that making or taking delivery becomes unappealing. To do so, an investor takes the opposite position (selling if initially holding a long position and buying if initially holding a short position) for the equal principal amount of treasury bills as in the original contract.

3 FORWARD MARKET THEORY AND TESTS

Alternative efficient markets hypotheses concerning the one-week when-issued market for Government of Canada treasury bills are examined through an analysis of the characteristics of both forward and futures markets. Efficiently determined forward and futures prices (or yields) can be biased predictors of future spot prices (or yields), if no assumptions about agents' behaviour toward risk are made. The bias is typically interpreted as a potentially time-varying risk premium and arises from essentially two sources: *(i)* the fact that cash flows are uncertain at the time the investment decision is made and *(ii)* the possibility that forward (and futures) contracts, being net zero in supply, may require a sweetener beyond the bid-offer spread. Even in informationally efficient markets, forward prices may not be unbiased predictors of future spot prices.

Similarly, the when-issued yield may persistently overstate or understate the upcoming treasury bill tender yield, even in an efficient market. The bias, if it exists, may arise simply because of the costs of completing transactions and can be known to all participants at the time the investment decision is made.

A discrete-time equilibrium model based on those from Richard and Sundaresan (1981), Cox, Ingersoll and Ross (1981), Dunn and Singleton (1986) and McCurdy and Morgan (1992) is employed[8] to show that equilibrium forward and futures yields are equal to the values of particular assets and that time-varying risk premiums are not inconsistent with market efficiency in the riskless arbitrage sense. This implies that investment strategies that have the same payoff at maturity must have the same current value.

As Dunn and Singleton note, quite general utility functions lead to the following stochastic Euler equation from the first-order conditions for maximum expected utility:

8. The model is adapted so that nominal assets may be priced; see Lucas (1982) and Hansen and Hodrick (1983), which show the link between pricing real assets and pricing nominal assets.

$$E_t(M_{t+1} \cdot R_{t+1}) = 1 = R_{t+1}E_t(M_{t+1}), \tag{3.1}$$

where M_{t+1} is the discounted (by an assumed constant-time preference discount factor), intertemporal (between t and $t+1$), marginal rate of substitution of consumption in nominal terms, R_{t+1} is one plus the (deterministic) riskless nominal rate of interest, and E_t is the mathematical expectations operator conditional on current and past information (E henceforth).[9] Equation (3.1) represents the present value of the cash flow generated by investing one dollar in a one-period bond having a risk-free return of R_{t+1}.

By denoting the time $t+1$ price of the riskless one-period bond as B_{t+1}, the right-hand side of expression (3.1) also simplifies to

$$E(M_{t+1}) = R_{t+1}^{-1} \equiv B_{t+1}, \tag{3.2}$$

which describes bond prices in terms of the intertemporal marginal rate of substitution of the consumption good.

The discrete time version of theorem 1 of the continuous-time equilibrium model of Richard and Sundaresan gives, using equation (3.1),

$$V_t = E(M_{t+1} \cdot S_{t+1} \cdot Q_{t+1}), \tag{3.3}$$

where V_t is the time t deterministic value of a random quantity, Q_{t+1}, having a spot price of S_{t+1} upon delivery at $t+1$. Here, M_{t+1} can be interpreted as a present value operator that discounts the $t+1$ (or future) value of a random flow back to a known time t value. Simply put, the time $t+1$ random cash flow of Q_{t+1} units of the spot commodity is converted to a time t random cash flow by multiplication by the marginal rate of substitution of consumption. This time t random payoff (or outlay) is then converted to a deterministic value through the expectations operator.

9. It is important to note that the left-hand equation (3.1) holds for all assets, while the right-hand equation (3.2) holds only for the riskless asset.

3.1 Forward market efficiency relations

As previously noted, an agent holding a long position in a forward or futures contract has the obligation to buy one unit of the underlying asset on a specified date for the previously arranged forward price (or yield). Further, forward contracts initiated on the maturity date for immediate delivery must have a price equal to the spot price of the underlying asset; otherwise, riskless and instantaneous arbitrage opportunities would exist. Hence, the equilibrium forward price (or yield) is the one that ensures newly created forward contracts will have zero value when initiated. In other words, forward contracts are fair bets, making market participants indifferent, at the margin, about entering forward markets or exposing themselves to future spot price or inventory uncertainty.

While no particular model of risk premiums has been accepted, models that are based on arbitrage arguments are quite general and show that the existence of a risk premium can be consistent with market efficiency. By simple arbitrage arguments, forward prices (or yields) can be expressed in terms of assets that render certain payments on the date of maturity. Thus, they contain expectations of future spot prices (or yields) and, possibly, an associated risk premium – a component that reflects the riskiness of the contract or of the market or the presence of a sweetener.

Consider the following strategy: *(i)* buy $1/B_t$ forward contracts for the delivery of a bond at T and *(ii)* invest the amount equivalent to F_t^T dollars in bonds that mature at T. The former requires no cash outlay while the latter requires an investment of F_t^T dollars, which is the forward price at time t for delivery of the underlying asset at T (maturity). As there are no interim payoffs of this hypothetical portfolio, the payoff at time T is

$$B_t^{-1} \cdot (F_T^T - F_t^T) + F_t^T \cdot B_t^{-1}. \tag{3.4}$$

The first term represents the amount gained or lost on the forward contract, and the second term is the return from investing the amount F_t^T with a return of R_t (which follows from equation 3.2). Hence, the time T payoff (equation 3.4) simplifies to $F_T^T.B_t^{-1}$, which must equal $S_T.B_t^{-1}$, where S_T is the spot price at T, since $S_T \equiv F_t^T$ by arbitrage.

Since the initial investment outlay is F_t^T, this strategy shows that the forward price, F_t^T, is the *present value* of the contract, which pays the amount $S_T.B_t^{-1}$ (or $S_T.R_t$) at T. Fortunately, the asset pricing paradigm above provides a present value operator that converts time T dollar payoffs into dollar values at t. As shown by equation (3.4), the forward price, F_t^T, is the present value of the payoff of $S_T.R_t$. Thus, the following equilibrium pricing relationship is found by application of the time t to T analogue of equation (3.3):

$$F_t^T = E(M_T(S_T \cdot R_t)) . \tag{3.5}$$

In words, the unknown payoff $S_T \cdot R_t$ is discounted back to time t and made deterministic through $E(M_T)$, that is, F_t^T is the present value of a portfolio that pays $S_T \cdot R_t$ at maturity.

Applying a covariance decomposition to equation (3.5) and using the Euler condition (3.1) yields

$$E(S_T) = F_t^T - Cov(M_T \cdot R_t \,, S_T) , \tag{3.6}$$

since $E(M_T \cdot R_t) = 1$, where R_t is deterministic. Equation (3.6) suggests that unless Cov(·) = 0 (which will always be the case under risk neutrality and a deterministic price level), forward prices are, in general, biased predictors of future spot prices. If the conditional covariance in equation (3.6) were constant, then forward markets would be characterized by a constant unconditional bias. However, the conditional distribution of the random variables S_T and M_T typically varies over time, so no constant bias need be present. In such cases and in an efficient market, a time-varying risk premium, Cov(·), will be present.

To illustrate this, let us suppose that the marginal utility of a dollar's worth of consumption and the evolution of the spot price (of the asset in question) were negatively correlated. This would imply that the covariance term in equation (3.6) is negative and therefore $F_t^T < E(S_T)$. In other words, if the gains from holding a long position in a forward contract are higher than average when the marginal utility of consumption is lower than average, then the forward contract is not a very good hedge against welfare losses. To see this, note that the forward price causes the value of holding a forward contract to be zero.[10] Hence, from equation (3.3):

$$0 = E(M_T \cdot (S_T - F_t^T)), \tag{3.7}$$

as $S_T - F_t^T$ is the profit (or loss) at T realized from holding a forward contract. Applying the same methodology as above, equation (3.7) can be simplified to yield

$$F_t^T = E(S_T) + R_T \cdot Cov(M_T, (S_T - F_t^T)), \tag{3.8}$$

by making use of the time T analogue of the Euler condition (3.1). Therefore, the forward price is biased downward as an estimate of the expected future spot price when the profits from a forward contract, $S_T - F_t^T$, are negatively correlated with the marginal utility of consumption. The holder of the long position is essentially providing insurance against a welfare loss to the agent holding the short position. Therefore F_t^T must be less than $E(S_T)$ in order to induce the long holder to issue the insurance. Keynes (1930) and Hicks (1946) suggested that in "normal" times, $F_t^T < E(S_T)$ would characterize most forward markets – a situation Keynes described as "normal backwardation." In short, normal backwardation will characterize a forward market when forward contracts are relatively poor consumption hedges.

Alternatively, when forward contracts are relatively good consumption hedges – profits from a forward contract are positively correlated with the marginal utility of consumption – then long investors will buy a hedge against welfare losses. Since the investor is buying insurance, F_t^T must be greater than $E(S_T)$ to induce the other side of the market into taking a position. This situation is known as contango.

Tests of forward market efficiency typically examine the ability of forward contracts to hedge against spot price uncertainty (see, for example, Bilson 1981; Fama 1984; Gregory and McCurdy 1984, 1986; and

10. Since neither the current forward price nor the risk-free rate affects the Cov(·) term, this can be seen as another way of deriving equation (3.6).

Boothe and Longworth 1986). A perfect hedge, where the forward price equals the future spot price, ensures zero expected profit from the hedging activity. After an examination of some descriptive statistics on when-issued yields, the results from testing forward market efficiency relations, as implied by equation (3.6), are presented.

3.2 Descriptive statistics

The ability of the when-issued market to predict the upcoming tender rate accurately depends on, among other things, the forecasting horizon, the degree of market homogeneity and market liquidity. For example, the longer the time is to maturity of the contract (one week versus one day), the greater the chance is of new information arising that could change investors' expectations concerning the upcoming tender yield.

Some basic statistics on the average mid-market[11] when-issued yield ($\overline{WI_t}^T$), the bid-offer yield spread (B-O), the forecast error (WI_t^T - TB_T), and the current when-issued–treasury bill yield spread (WI_t^T - TB_t) are given in Table 1. These data cover the period from January 1986 to June 1992, when auctions were held each Thursday. As explained below, liquidity and the forecasting horizon each play an important role in establishing the bid-offer spread and the forecast error. For example, the more liquid a market is, the smaller the B-O spreads and the forecast errors, because a trading partner can be easily found without having to build liquidity premiums into yields. Shorter forecasting horizons lead to smaller forecast errors, since there is less time for the arrival of new information to change expectations concerning the upcoming tender yield.

While only limited when-issued trading immediately follows the announcement of the issue amount for the upcoming auction, investors usually take positions later in the preauction period, when there is more information about likely yield movements and inventory requirements and less time for the arrival of additional information to materially

11. The mid-market yield is the average of the bid and offer yields.

Table 1
Some descriptive statistics

	3-month				6-month				12-month			
	Fri.	Mon.	Tue.	Wed.	Fri.	Mon.	Tue.	Wed.	Fri.	Mon.	Tue.	Wed.
Sample period: 1986 to 1988												
$\overline{WI}_t^T$:	8.85	8.84	8.86	8.87	9.14	9.11	9.14	9.14	9.43	9.39	9.44	9.45
B-O:	2.72	2.07	2.06	1.75	3.13	2.56	2.24	1.94	3.61	3.46	3.12	2.85
	(17.11)	(19.19)	(20.82)	(17.47)	(19.49)	(18.58)	(18.49)	(19.08)	(12.96)	(13.43)	(12.90)	(11.60)
WI_t^T-TB_T:	0.03	0.70	0.76	0.67	-2.63	-2.11	-2.37	-2.57	-4.49	-4.02	-4.53	-3.73
	(0.02)	(0.45)	(0.79)	(1.15)	(1.30)	(1.09)	(3.13)	(3.14)	(1.94)	(1.82)	(4.12)	(5.30)
WI_t^T-TB_t:	-0.01	-0.21	-0.23	0.07	-1.02	-0.48	-0.83	-1.30	-0.65	-1.53	-1.59	-1.81
	(0.01)	(0.38)	(0.47)	(0.15)	(2.87)	(0.58)	(2.79)	(3.10)	(1.48)	(3.77)	(4.48)	(5.47)
Sample period: 1989 to June 1992												
$\overline{WI}_t^T$:	10.59	10.56	10.59	10.65	10.52	10.50	10.52	10.58	10.44	10.44	10.45	10.50
B-O:	1.50	1.42	1.34	1.26	1.81	1.76	1.61	1.56	2.08	1.93	1.76	1.77
	(25.17)	(23.06)	(28.37)	(28.83)	(24.63)	(26.74)	(28.13)	(28.98)	(26.57)	(25.66)	(25.57)	(26.26)
WI_t^T-TB_T:	0.18	0.77	0.66	0.24	-2.26	-1.76	-2.19	-2.24	-2.86	-1.79	-2.87	-3.16
	(0.29)	(1.21)	(1.39)	(0.73)	(2.10)	(1.69)	(2.84)	(4.25)	(2.09)	(1.43)	(3.04)	(4.86)
WI_t^T-TB_t:	-2.79	-2.15	-2.19	-2.05	-3.34	-2.94	-2.94	-2.42	-2.91	-2.84	-3.24	-3.37
	(6.90)	(4.90)	(5.58)	(5.50)	(9.59)	(7.84)	(9.34)	(5.51)	(6.82)	(6.50)	(7.29)	(9.36)

Notes: T-statistics are in parentheses. The spread variables are scaled by multiplication by 100 and are therefore in basis points.

change these prospects. Consequently, the volume and frequency of trades generally pick up considerably between Tuesday and Thursday mornings. The increase in the number and diversity of when-issued market participants has improved liquidity over time and led to a narrowing of the closing bid-offer yield spread.

The tables indicate that the bid-offer yield spread for each maturity narrows during the week as the tender day approaches. These spreads have also been narrower in recent years than earlier in the period for every trading day. For example, for the 1989–92 period, the average bid-offer spread on three-month when-issued bills was 1.50 basis points on Fridays and only 1.26 basis points on Wednesdays – these spreads were substantially larger during the 1986–88 period.

The bid-ask spread can be taken as an indication of the relative liquidity of different when-issued bills (for example, the three-month versus the six-month bill. The liquidity of these bills in turn depends largely on the liquidity of the corresponding cash bills. Since approximately twice as many three-month treasury bills as six-month treasury bills and approximately four times as many three-month treasury bills as twelve-month treasury bills (by par value) are auctioned each week, the three-month when-issued bill is more liquid. As a result, narrower bid-offer spreads are found in the three-month when-issued bill: for example, on Mondays the average spread, during the more recent period, was 1.42 basis points on the three-month when-issued bill, 1.76 basis points on the six-month when-issued bill and 1.93 basis points on the twelve-month when-issued bill.

On the whole, the average forecast error for each maturity and day of the week is relatively small, although much larger (in absolute value) for the six- and twelve-month bills. While the average forecast errors for the three-month bill were never significantly different from zero, there are cases when the average forecast errors were significantly different from zero for the six- and twelve-month bills. The forecast errors for each of the bills have fallen over time. Finally, over the latter period, the when-issued–cash treasury bill spread was always significantly negative for each of the bills, while over the first sample, this was the case for the six- and twelve-month bills.

3.3 Forward market tests and results

The forward and spot price relationship derived from the asset pricing valuation method can be tested by means of two popular regression models. The first examines the ability of the when-issued yield to predict the upcoming treasury bill yield at tender – the unbiased expectations hypothesis (UEH). Subtracting the current treasury bill yield from both sides of equation (3.6) to ensure stationarity, replacing the expected treasury bill yield by its realized value less a rational expectations (RE) forecast error and setting the risk premium term to zero yields the standard regression equation (save scaling by multiplication by 100) for testing the UEH:

$$TB_T - TB_t = \alpha + \beta\,(WI_t^T - TB_t) + \varepsilon_t \tag{3.9}$$

with $H_0: \alpha = 0, \beta = 1, \varepsilon_t \sim iid\,(0, \sigma^2)$

where TB_T is the average treasury bill yield at the weekly tender, TB_t the current treasury bill yield, WI_t^T the current when-issued yield, and α and β are regression coefficients. This relationship is examined for each of the three maturities of bills and for t = Friday,..., Wednesday.

The OLS results of testing the UEH for each of the three when-issued bills are given in Table 2a, for the 1986 to 1988 period, and Table 2b, for the 1989 to June 1992 period.[12] In general, the joint F, F(2,.) and (robust) joint chi-square, $\chi^2(2)$, tests reveal that the null hypothesis concerning the coefficient estimates cannot be rejected at any reasonable level of significance for the three- and six-month when-issued bills during

12. The sample is split in order to test the stability of the regression equation (which is really an examination of how the market has changed over time).

Table 2a

Results of testing the unbiased expectations hypothesis

$$(TB_T - TB_t) \cdot 100 = \alpha + \beta(WI_t^T - TB_t) \cdot 100 + \varepsilon_t \text{ with } H_0\text{: } \alpha = 0, \beta = 1 \text{ and } \varepsilon_t \sim iid(0, \sigma^2)$$

Sample period: 1986 to 1988

	3-month				6-month				12-month			
	Fri.	Mon.	Tue.	Wed.	Fri.	Mon.	Tue.	Wed.	Fri.	Mon.	Tue.	Wed.
α:	-0.03	-0.71	-0.76	-0.65	2.11	2.03	1.89	2.02	3.47	3.16	4.08	3.17
	(0.02)	(0.46)	(0.80)	(1.12)	(0.99)	(1.06)	(1.82)	(2.79)	(1.45)	(1.40)	(3.47)	(4.31)
β:	0.83	0.93	0.99	0.79	0.50	0.84	0.43	0.56	-0.57	0.44	0.72	0.69
	(2.76)	(3.29)	(4.60)	(3.57)	(0.84)	(4.93)	(1.11)	(2.94)	(0.76)	(0.80)	(2.08)	(3.52)
$\bar{R}^2$:	0.06	0.09	0.19	0.28	0.00	0.11	0.01	0.07	0.00	0.00	0.04	0.09
$\chi^2(2)$:	0.86	0.86	0.72	0.23	0.26	0.38	0.07	0.00	0.00	0.13	0.00	0.00
F(2,.):	0.82	0.86	0.73	0.06	0.24	0.40	0.02	0.00	0.00	0.11	0.00	0.00
GNR:	0.02	0.46	0.18	0.91	0.67	0.48	0.69	0.10	0.37	0.93	0.54	0.96
Q(5):	0.01	0.98	0.67	0.09	0.97	0.39	0.75	0.00	0.91	1.00	0.84	1.00
$Q^2(5)$:	1.00	1.00	0.97	0.28	1.00	0.48	0.00	0.00	1.00	0.93	0.10	0.99

Notes: Numbers in parentheses are robust standard errors. Tests of the null hypothesis and the diagnostic tests are given as marginal significance levels (p-values). $\chi^2(2)$ is a robust chi-square test of the null hypothesis, F(2,.) is a standard F-test, GNR is a Gauss-Newton regression test for first order autocorrelation, Q(5) is the Ljung-Box test for fifth-order autocorrelation, and $Q^2(5)$ is the same for the squared residuals.

Table 2b

Results of testing the unbiased expectations hypothesis

Sample period: 1989 to June 1992

	3-month				6-month				12-month			
	Fri.	**Mon.**	**Tue.**	**Wed.**	**Fri.**	**Mon.**	**Tue.**	**Wed.**	**Fri.**	**Mon.**	**Tue.**	**Wed.**
α:	-0.12	-0.38	-0.33	-0.01	1.29	1.76	2.51	2.72	0.92	0.62	2.71	3.18
	(0.14)	(0.46)	(0.55)	0.02)	(1.03)	(1.38)	(2.83)	(5.31)	(0.60)	(0.41)	(2.62)	(4.44)
β:	1.02	1.18	1.15	1.12	0.71	1.00	1.11	1.20	0.34	0.59	0.95	1.01
	(7.13)	(8.25)	(11.47)	(15.29)	(2.79)	(4.34)	(6.69)	(21.36)	(1.29)	(2.31)	(6.54)	(8.74)
$\overline{R}^2$:	0.30	0.40	0.48	0.62	0.05	0.11	0.16	0.50	0.01	0.03	0.16	0.23
$\chi^2(2)$:	0.93	0.05	0.03	0.11	0.08	0.24	0.01	0.00	0.01	0.06	0.01	0.00
F(2,.):	0.94	0.14	0.10	0.17	0.05	0.25	0.02	0.00	0.00	0.07	0.01	0.00
GNR:	0.28	0.63	0.59	0.95	0.34	0.21	0.20	0.62	0.66	0.91	0.61	0.53
Q(5):	0.09	0.91	0.30	1.00	0.47	0.20	0.05	0.96	0.95	1.00	0.88	0.92
$Q^2(5)$:	0.53	0.63	0.00	0.00	0.92	0.96	0.00	0.80	0.64	0.42	0.90	1.00

Notes: Numbers in parentheses are robust standard errors. Tests of the null hypothesis and the diagnostic tests are given as marginal significance levels (p-values).

either period.[13] The only troublesome areas are for the twelve-month bills and six-month bills during the latter part of the week. Here, the UEH can be rejected at the 1 per cent level for the six-month when-issued bill on Wednesdays and for the twelve-month when-issued bill on Tuesdays and Wednesdays during both sample periods. Rejection of the null hypothesis in these cases can likely be attributed to the significant constant term and not to the deviation of β from its hypothesized value, 1. This possibly reflects a constant risk premium – which may be required to entice a counterparty into accepting one side of the contract – rather than systematic forecast errors.

In general, there is little evidence against model specification in the form of autocorrelation, as examined with a Gauss-Newton regression (GNR) for first-order serial correlation and the adjusted Ljung-Box test for autocorrelation in the first five lags, Q(5). The adjusted Ljung-Box test for autocorrelation in the first five lags of the squared residuals, $Q^2(5)$, indicates no evidence of heteroscedasticity.

The regressions typically explain less than 25 per cent of the variation in treasury bill yield changes, as shown by the $\bar{R}^2$ measures. The $\bar{R}^2$s are, however, rather variable, ranging from a high of 62 per cent to less than 1 per cent. As expected, the coefficients of determination from the regressions for the three-month bills are substantially higher than those for the other bills.

The second test examines orthogonality restrictions of the forecast errors $(TB_T - WI_t^T)$ to previously chosen information sets, Ω_t. The idea here is that the mistakes market participants make should not be predictable, if agents are making use of all available information. To see this, set the risk premium term to zero, subtract the forward yield from both sides and

13. Koenker (1982, 214) defines robustness as follows: "In statistics and more loosely in economics [robustness] has come to signify a certain resilience of conclusions to deviations from assumptions of hypothetical models...." It is important to realize that inferences concerning the null hypothesis are based on the implicit assumptions (or restrictions) of the test relation. Failure of the data to conform to OLS assumptions implies that inferences could be misleading.

replace the expected treasury bill yield by its realized value less a rational expectations forecast error in equation (3.6):

$$TB_T - WI_t^T = \xi_t \quad \text{where} \quad E(\xi_t | \Omega_{t-1}) = 0. \tag{3.10}$$

Essentially, the UEH is imposed on this structure. However, it is not clear what to include in Ω, because finding a stable relationship is usually difficult and because parsimony is forced upon the test relationship, given the finiteness of data. Tests that include only past histories of the dependent variable in Ω have become known as tests of weak-form market efficiency (Roberts 1959 and Fama 1970). If Ω contains contemporaneous public information, then orthogonality tests of the nature considered here are known as "semistrong-form tests," and enlarging Ω to include inside or restricted information can be thought of as a strong-form test.

Since few restrictions are placed on the selection of relevant information, it is not inconceivable to find, ex post, an information set Ω that is significantly correlated with the forecast error. In addition, however, it is necessary to find a systematic and stable relationship between the forecast error and the a priori chosen information set. Following Hansen and Hodrick (1983) and Gregory and McCurdy (1984, 1986), three instruments are taken from the agent's information set: a constant vector, the lagged forecast error,[14] $TB_{T-1}-WI_{t-1}^{T-1}$, and the known when-issued–treasury bill spread,[15] $WI_t^T-TB_t$. With the exception of scaling by multiplication by 100, the test regression can be written as:

$$TB_T - WI_t^T = \alpha + \gamma(TB_{T-1} - WI_{t-1}^{T-1}) + \phi(WI_t^T - TB_t) + \sum_i \varphi_i X_{i,t} + \varepsilon_t \tag{3.11}$$

14. Recall that these regressions are tested for t=Friday,...,Wednesday separately. Hence, the lagged forecast error is that corresponding to the previous week, that is, t-1 and T-1 correspond to data from a week earlier.

15. Note that this is not the forward premium, since TB_t is not the yield on the deliverable asset. However, it can be interpreted as a proxy for the slope of the yield curve at the very short end.

with H_0:$\alpha = \gamma = \phi = \varphi_i = 0$, $\varepsilon_t \sim iid(0, \sigma^2)$, and where X includes lagged time-dependent variables and current open-market-operations indicator variables that should influence both WI and E(TB) in the same manner.[16]

Note that α is the unconditional mean of the forecast error and can be interpreted, assuming an adequate test specification,[17] as the unconditional bias in when-issued yields. As indicated by the theoretical outline, if the forward market is a poor hedge against spot price uncertainty, then the forward price must be less than the expected spot price in order to induce an implicit insurance from the long side of the contract. Conversely, if when-issued contracts are good hedges, then long holders of the contract are purchasing a hedge against welfare losses. In such cases one would expect α to be significantly positive (or negative), as the forward yield (or price) would be less than the expected spot yield (or price).

The results of testing the orthogonality restrictions are presented in Tables 3a and 3b. In general, the results here are consistent with those from the test of the UEH: the null hypothesis that the previously chosen elements of the information set (corresponding to α, γ and ϕ) are irrelevant in predicting the forecast error cannot be rejected with reasonable confidence, as suggested by the F(3,.) and $\chi^2(3)$ statistics. The exceptions are mainly for the twelve-month when-issued bill and for the six-month bill on Wednesdays only. Evidence of heteroscedasticity in the earlier period disappeared in the later period with the exception of the three-month bill just prior to the auction day.

There is also some evidence that the current when-issued–treasury bill spread is useful for predicting the three-month forecast error. However, even in these cases, there is little explanatory power in the test regression, suggesting that even with the additional indicator variables, these variables can not likely be used in any economically significant way.

16. For example, treasury bill purchases by the Bank should cause both WI_t^T and $E(TB_T)$ to fall. In this case φ should be insignificantly different from zero.

17. OLS estimates are particularly sensitive to departures from normality and to the presence of outliers, which could come about as a result of trading on the basis of noisy information, such as the stock market crash or intervention by the Bank.

Table 3a

Tests of orthogonality restrictions

$$(TB_T - WI_t^T)\cdot 100 = \alpha + \gamma(TB_{T-1} - WI_{t-1}^{T-1})\cdot 100 + \phi(WI_t^T - TB_t)\cdot 100 + \sum_i \varphi_i X_{i,t} + \varepsilon_t \text{ with } \mathcal{H}_0\text{: } \alpha = \gamma = \phi = \varphi_i = 0 \text{ and } \varepsilon_t \sim iid(0,\sigma^2).$$

Sample period: 1986 to 1988

	3-month				6-month				12-month			
	Fri.	Mon.	Tue.	Wed.	Fri.	Mon.	Tue.	Wed.	Fri.	Mon.	Tue.	Wed.
α:	-1.39	-0.07	-0.31	-0.60	1.19	7.51	2.30	1.30	4.32	13.37	3.44	2.70
	(0.62)	(0.05)	(0.36)	(0.95)	(0.42)	(2.37)	(2.45)	(1.60)	(1.57)	(3.38)	(2.95)	(3.33)
γ:	0.18	0.07	0.12	-0.01	-0.11	-0.10	-0.03	-0.21	0.15	-0.02	0.14	-0.03
	(2.74)	(0.73)	(1.24)	(0.09)	(0.85)	(0.89)	(0.32)	(1.82)	(2.85)	(0.10)	(1.07)	(0.25)
ϕ:	-0.24	-0.87	-0.01	-0.22	-0.40	-0.14	-0.54	-0.43	3.04	0.07	-0.10	-0.37
	(0.76)	(0.30)	(0.03)	(0.92)	(0.71)	(0.86)	(1.44)	(2.71)	(1.69)	(0.10)	(0.38)	(1.49)
φ_i:	7.39_3		-65.73_1	-13.54_1	-43.50_1	-9.94_3	-46.59_1	-31.33_1		-18.10_4	-36.42_1	-21.00_1
	(2.24)		(79.58)	(10.10)	(2.09)	(2.90)	(19.02)	(35.47)		(3.46)	(15.60)	(24.26)
	-11.00_5			0.12_{10}	7.75_3	-6.17_9		4.94_2		-10.75_9		-4.04_5
	(3.28)			(2.26)	(2.12)	(1.66)		(2.43)		(2.47)		(2.22)
				-6.60_6								0.13_{10}
				(3.32)								(1.90)
$\bar{R}^2$:	0.03	0.00	0.21	0.06	0.01	0.01	0.09	0.19	0.10	0.04	0.09	0.14
$\chi^2(3)$:	0.05	0.86	0.41	0.42	0.67	0.03	0.05	0.02	0.00	0.01	0.01	0.00
F(3,.):	0.20	0.76	0.32	0.20	0.15	0.15	0.51	0.00	0.00	0.04	0.02	0.00
χ^2(all):	0.01	0.86	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.01	0.00	0.00
GNR:	0.97	0.67	0.87	0.77	0.56	0.81	0.82	0.54	0.39	0.03	0.65	0.82
Q(5):	0.62	0.95	0.12	0.01	0.72	0.95	1.00	0.60	0.37	0.00	0.86	0.99
$Q^2(5)$:	1.00	0.00	0.00	0.72	1.00	0.00	0.00	0.01	1.00	0.00	0.00	0.04

Notes: See notes at the bottom of Tables 2a and 3b. Numbers in parentheses are robust standard errors. Tests of the null hypotheses and the diagnostic tests are given as marginal significance levels (p-values). $\chi^2(3)$ is a test of $\alpha=\gamma=\phi=0$, while χ^2(all) is a test that all coefficients are zero.

Table 3b

Tests of orthogonality restrictions

Sample period: 1989 to June 1992

	3-month				6-month				12-month			
	Fri.	Mon.	Tue.	Wed.	Fri.	Mon.	Tue.	Wed.	Fri.	Mon.	Tue.	Wed.
α:	0.32	-0.09	-1.02	-0.17	0.84	1.64	1.02	2.73	3.32	1.21	2.29	2.53
	(0.35)	(0.10)	(1.29)	(0.50)	(0.63)	(1.15)	(1.01)	(5.50)	(1.33)	(0.71)	(1.99)	(3.42)
γ:	0.12	0.07	0.07	0.02	0.09	0.12	0.13	0.04	-0.03	0.01	0.04	0.05
	(1.26)	(0.73)	(0.53)	(0.21)	(1.07)	(1.40)	(1.27)	(0.61)	(0.38)	(0.13)	(0.60)	(0.64)
ϕ:	-0.03	0.19	0.27	0.11	-0.34	-0.04	0.18	0.17	-0.68	-0.33	-0.17	-0.08
	(0.18)	(1.19)	(2.82)	(1.74)	(1.29)	(0.14)	(0.92)	(3.01)	(2.62)	(1.20)	(1.01)	(0.74)
φ_i	-4.09_3	-17.53_6	2.26_2	5.03_4		-16.51_6	3.23_2		-6.46_5	-9.82_6	-4.82_6	5.44_4
	(2.75)	(2.12)	(2.44)	(1.93)		(2.23)	(1.88)		(2.18)	(1.90)	(2.24)	(1.95)
			-5.92_6	-3.78_6			-5.91_6					
			(2.26)	(3.94)			(2.01)					
$\bar{R}^2$:	0.03	0.08	0.03	0.08	0.01	0.02	0.02	0.02	0.04	0.00	0.00	0.01
$\chi^2(3)$:	0.66	0.12	0.00	0.14	0.13	0.25	0.26	0.00	0.02	0.18	0.01	0.00
$F(3,.)$:	0.52	0.20	0.01	0.20	0.07	0.19	0.01	0.00	0.01	0.26	0.01	0.00
χ^2(all):	0.06	0.06	0.00	0.00	0.13	0.08	0.01	0.00	0.01	0.05	0.01	0.00
GNR:	0.89	0.95	0.94	0.97	0.99	0.89	0.99	0.87	0.99	0.71	0.99	0.92
Q(5):	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98	1.00	1.00
$Q^2(5)$:	0.59	0.07	0.00	0.00	0.76	1.00	0.01	0.60	0.86	0.87	0.86	0.98

Notes: Subscripts on the indicator variables refer to the following dummy variables: 1, the market crash during the week of 17 October 1987; 2, PRA; 3, SPRA; 4, SRA; 5, three-month treasury bill sale; 6, purchase; 7, coincident-to-when-issued sale; 8, purchase; 9, foreign exchange intervention. The following subscripts refer to time-dependent variables: 10, the weekly rate of change of supply at the tender; 11, an increasing trend; 12, a decreasing trend; 13, three-month commercial paper less the three-month treasury bill (lagged) spread; 14, six-month less three-month treasury bill (lagged) spread; 15, the three-month treasury bill yield change. Numbers in parentheses are robust standard errors. Tests of the null hypotheses and the diagnostic tests are given as marginal significance levels (p-values). See other notes at the bottom of Table 3a.

Rejection of the null hypothesis was quite often accompanied by a significantly positive unconditional mean of the forecast error, α. In fact, almost one-third of the regressions contained a positive unconditional mean of the forecast error that was more than two standard deviations from zero. This could indicate that the when-issued yield (price) is less (greater) than the expected treasury bill yield (price) for the six- and twelve-month bills – a situation described earlier as contango. There were fewer occurrences of a significant constant term in the later sample.

Unfortunately, it is difficult to interpret the meaning of the significance of the additional indicator variables (corresponding to φ_i) and the corresponding "all coefficients zero" tests. The significant indicator variable for the October 1987 stock market crash shows that the treasury bill yield fell relative to the when-issued yield, perhaps indicating that agents thought this to be a purely transitory occurrence that would be reversed the following day. The significantly negative indicator variable corresponding to φ_6 for days when the Bank of Canada purchased three-month treasury bills for policy purposes – especially during the later sample – implies that WI_t^T did not capture the full effect of the operation that would later show up in TB_T. Hence, injection of liquidity, through this means, is thought to be temporary, as it is not fully reflected in WI_t^T.

In summary: Overall, tests of the UEH and of orthogonality restrictions in three- and six-month when-issued forecast errors suggest that agents do not generally make significant errors in predicting the upcoming treasury bill yield associated with these maturities at tender, and that the mistakes they do make are not systematic. Evidence against the null hypotheses was found for the twelve-month bill (and occasionally the six-month bill) in the form of an unconditional and constant bias.[18] Since the open-market-operations indicator variables should affect both $E(TB_T)$ and

18. I suspect that as the six- and twelve-month when-issued markets become more liquid, these constant forecast errors will disappear.

WI_t^T in the same manner, the significance of these variables suggests that the kinds of effects of intervention considered here are probably only temporary.

Earlier work by Poitras (1991, 621), concluded that for the three-month bill, the when-issued yield is an unbiased predictor of the tender outcome, but that the "same could not be said for the six-month bill." However, Poitras had not examined either market efficiency in the when-issued market for twelve-month treasury bills, orthogonality restrictions imposed by the UEH, or the effect of open market operations on the forecasting ability of market participants.

It should be noted that the statistical tests employed are joint tests of the RE hypothesis, the UEH and the absence of risk premiums. The theoretical framework makes it clear that rejection of the null hypothesis, if this is the case, might imply rejection of the UEH and/or of the RE hypothesis but may also suggest evaluation of the risk premium hypothesis. The risk premium hypothesis (or the alternative hypothesis in this case) simply states that market fundamentals (forward or spot) require a risk premium in order to entice one side of the market into covering the other side of the market. Evidence of a significant intercept could be an indication of this.

4 FUTURES MARKET THEORY AND TESTS

As in the case of forward contracts, futures contracts initiated on the maturity date must, in equilibrium, have a yield equal to the yield on the underlying asset; otherwise, riskless and instantaneous arbitrage opportunities exist. Since futures contracts have a fixed date of maturity, a portfolio consisting of a long position initiated at time t offset by a short position at t+1 must have *zero* expected cash flows at maturity to ensure that speculative profits do not persist. The position will be closed out when delivery at T is accepted, and delivery of the spot asset just received will close out the short position.

4.1 Futures market efficiency relations

Application of Theorem 1 of Richard and Sundaresan to this net zero investment portfolio gives

$$0 = E(M_{t+1}(G_{t+1}^T - G_t^T)), \tag{4.1}$$

where G_t^T is the futures price (yield) contracted now for payment at T. As before, $E(M_{t+1})$ can be interpreted as a present value operator that discounts the future value of a random flow back to a known value. The random time t+1 payoff (or outlay) is then converted to a deterministic value through the expectations operator.

From equation (4.1), using the left-hand side of equation (3.2) and the definition of covariance, we have

$$E(G_{t+1}^T) = G_t^T - R_{t+1} \cdot Cov(M_{t+1}\ ,\ G_{t+1}^T). \tag{4.2}$$

If the conditional covariance term in equation (4.2) is non-zero, then the one-period expected futures price change can be mainly attributed to a time-varying risk premium, which may account for the variability of expected returns. However, if this covariance term is zero, equation (4.2) states that the best guess of tomorrow's futures price (or yield) is today's futures price (or yield).

4.2 Futures market tests and results

Efficient futures markets, by definition, do not allow persistent profits to arise from speculating. The net zero supply characteristic implies that gains from speculation are exactly offset by losses, and the fixed date of maturity allows the futures position to be wound down very easily. A long position in a futures contract today offset by a short position tomorrow has expected cash flows of zero at maturity and must therefore have a present value of zero. The martingale hypothesis, or hypothesis of zero expected change (Samuelson 1965), formalizes this notion.

The martingale hypothesis for when-issued yield changes can be derived by setting the risk premium term to zero and subtracting the lagged when-issued yield from both sides of (a lagged) equation (4.2):

$$E_{t-1}WI_t^T - WI_{t-1}^T = \xi_t \quad \text{where} \quad E_{t-1}\xi_t \mid \Omega_{t-1} = 0 \text{ under } H_0. \tag{4.3}$$

Hence, when-issued yield changes are forecast errors that are orthogonal to information at t-1. In other words, the best guess of tomorrow's when-issued yield is today's when-issued yield (corresponding to the same contract). One obvious choice for inclusion in the information set is the lagged, when-issued yield change, since if when-issued yields are efficient, this variable will embed information concerning all other relevant variables.[19]

With the assumption of rational expectations, the unscaled OLS test regression, arising from (4.3), can be written as

$$WI_t^T - WI_{t-1}^T = \alpha + \gamma(WI_{t-1}^T - WI_{t-2}^T) + \phi Crash_t + \sum_i \varphi_i X_{i,t} + \varepsilon_t \tag{4.4}$$

and the null hypothesis, H_0, as: $\alpha = \gamma = \varphi_i = 0$ and $\varepsilon_t \sim iid(0, \sigma^2)$. The indicator variable *Crash* takes on a value of one during the week of the October 1987 stock market plunge and is included in the regression to remove obvious trending that occurred in the first and second moments of financial variables during that week. As in equation (3.11), X includes lagged, time-dependent variables and current open-market-operations indicator

19. See, for example, McCurdy and Morgan (1987, 1988).

variables that, if perceived to have permanent effects on the expected treasury bill yield, should cause the when-issued yield to adjust once and for all.

Tests of the martingale hypothesis in futures markets make the use of high frequency data desirable, as information in yield changes may otherwise be lost. However, futures prices (and therefore yields) sampled at high frequency may show significant time-varying volatility – see Figure 1 for three-month when-issued yield changes. The heteroscedasticity in when-issued yield changes can be modelled as generalized autoregressive conditional heteroscedasticity – see Figure 2 for the three-month when-issued GARCH(1,1) variance. This parsimonious specification (Bollerslev 1986) has been very successful in capturing time variation in financial data (Bollerslev, Chou and Kroner 1992). With this time-varying volatility assumption, the univariate GARCH(1,1) model for testing the martingale hypothesis can be written as:

$$(WI_t^T - WI_{t-1}^T) = \alpha + \gamma(WI_{t-1}^T - WI_{t-2}^T) + \varepsilon_t, \quad (4.5)$$

$$\varepsilon_t \sim iid(0, \sigma_t^2) \text{ where } \sigma_t^2 = \mu + \psi\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2. \quad (4.6)$$

except for additional explanatory variables in both the mean and variance and scaling. The conditional variance of when-issued yield changes, σ_t^2 in equation (4.6), is a function of a constant mean, lagged squared innovations and lagged conditional variances. The parameters from (4.5) and (4.6) are jointly estimated by maximum likelihood.

A natural extension to this is the GARCH-in-mean (GARCHM) model, proposed by Engle, Lillian and Robins (1987) in order to examine mean-variance trade-offs in financial cash flows. In this model, Engle, Lillian and Robins suggested that a function of the conditional variance be included in the mean equation:

$$WI_t^T - WI_{t-1}^T = \alpha + \gamma(WI_{t-1}^T - WI_{t-2}^T) + \delta(\sigma_t^2) + \varepsilon_t. \quad (4.7)$$

Figure 1
Three-month when-issued yield change in basis points
$(WI^T_t - WI^T_{t-1})*100$

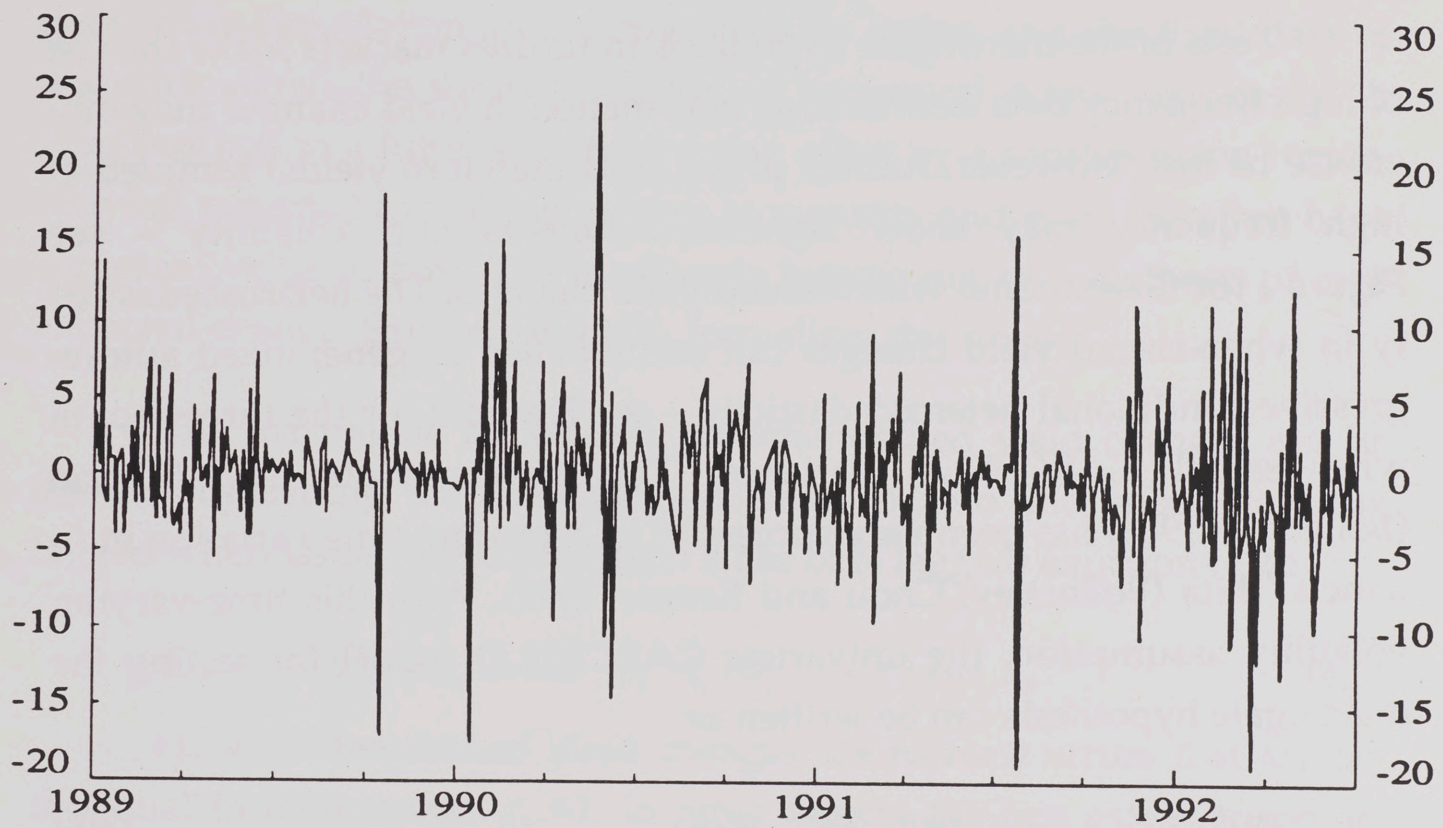
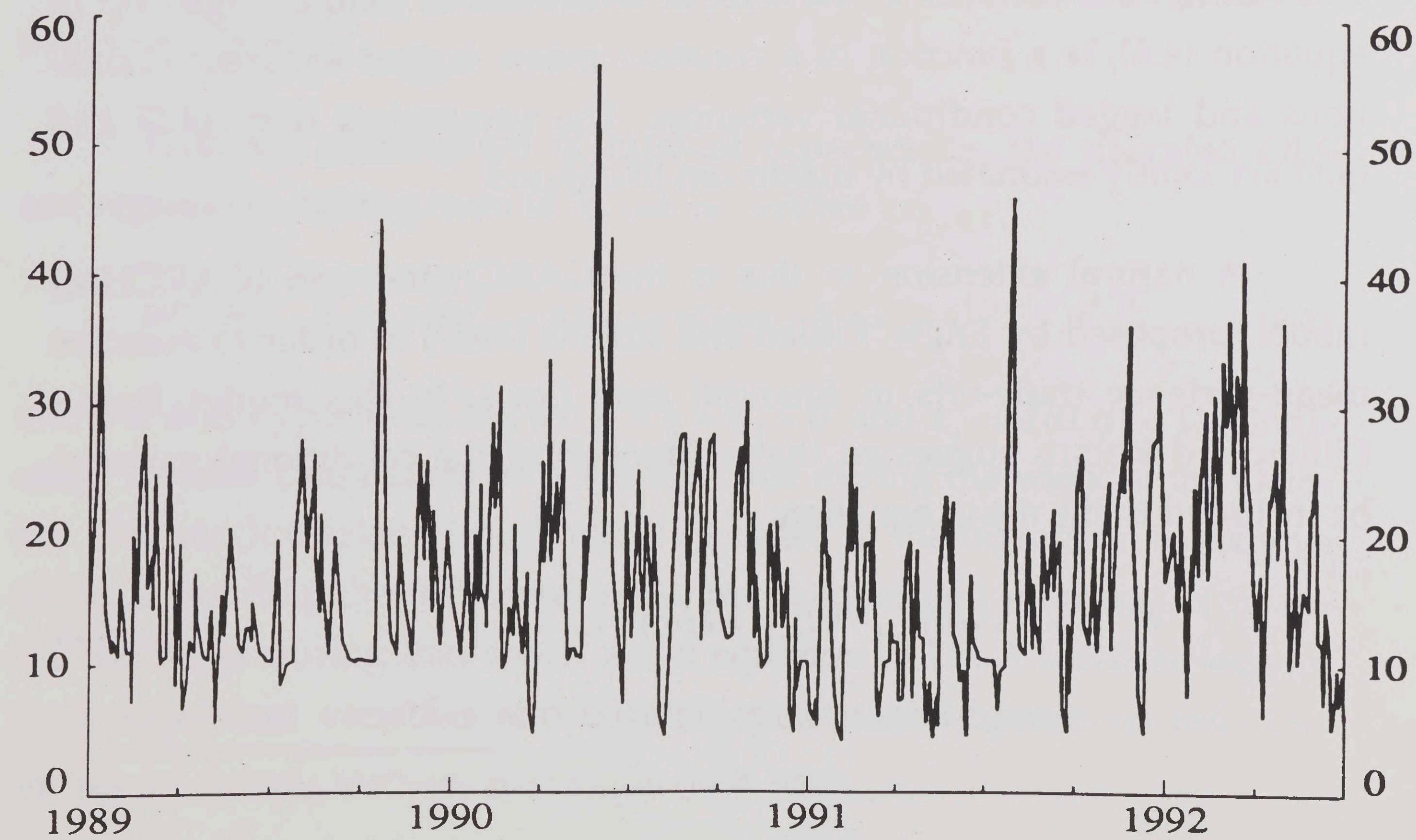


Figure 2
Conditional variance in three-month when-issued yield changes from the bivariate GARCH(1,1) model



This methodology can be used to see if the own conditional variance influences when-issued yield changes.[20] The GARCH model can be used to examine the effects of Bank of Canada intervention on the volatility of financial markets and then to examine these effects on when-issued yield changes using the GARCHM model.

By relaxing the assumption of zero covariance between the time series, the bivariate GARCH(1,1) model pairs the three- and six-month when-issued yield changes (ΔWI3 and ΔWI6 respectively) so as to allow the lagged yield changes of each series to influence each of the dependent variables. The bivariate test model can be written (with the exception of scaling and the additional explanatory variables) as:

$$\begin{bmatrix} \Delta WI3_t^T \\ \Delta WI6_t^T \end{bmatrix} = \begin{bmatrix} \iota & \Delta WI3_{t-1}^T & \Delta WI6_{t-1}^T \\ \iota & \Delta WI3_{t-1}^T & \Delta WI6_{t-1}^T \end{bmatrix} \Theta + \begin{bmatrix} \varepsilon_{3,t} \\ \varepsilon_{6,t} \end{bmatrix}, \ \varepsilon \sim iid \begin{bmatrix} 0, & \sigma_{3,t}^2, & \sigma_{3;6,t} \\ 0, & 0 & \sigma_{6,t}^2 \end{bmatrix}, \qquad (4.8)$$

where Θ is the parameter matrix and ι, a vector of ones. The conditional covariance between three- and six-month when-issued yield changes is GARCH(1,1):

$$\sigma_{3;6,t} = \mu + \psi \varepsilon_{3,t-1} \varepsilon_{6,t-1} + \beta \sigma_{3;6,t-1}. \qquad (4.9)$$

The OLS and univariate GARCH(1,1) coefficient estimates and tests of the martingale hypothesis are presented in Table 4a, for the 1986 to 1988 period, and Table 4b, for the 1989 to June 1992 period.[21] The basic OLS regressions, which do not include indicator variables for Bank operations, and tests show that there is, at best, weak evidence for rejecting the martingale hypothesis. Here the null hypothesis, $\alpha=0$ and $\gamma=0$, can be rejected for the three-month bill in the early sample period and can be rejected for the six- and twelve-month bills in the later sample period at about

20. This cannot be interpreted as a time-varying risk premium, as risk premiums must be able to take on both positive and negative values, while $\delta\sigma_t^2$ will take on the value given by δ for all time periods.

21. Yield changes must be made from the same contract. Hence, in this case Thursday's data is irrelevant, since yield changes are from two different contracts. Friday's data must be deleted because the lagged dependent variable would have been Thursday's yield change.

Table 4a
Tests of the martingale hypothesis

$$(WI_t^T - WI_{t-1}^T) \cdot 100 = \alpha + \gamma(WI_{t-1}^T - WI_{t-2}^T) \cdot 100 + \phi Crash_t + \sum_i \varphi_i X_{i,t} + \varepsilon_t \text{ with } H_0\text{: } \alpha = \gamma = \varphi_i = 0 \text{ and } \varepsilon_t \sim iid(0, \sigma_t^2).$$

$$\sigma_t^2 = \mu + \psi\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \kappa Crash_t + \sum_i \lambda_i X_{i,t}$$

Sample period: 1986 to 1988

	OLS basic			OLS augmented			GARCH basic			GARCH augmented		
	3-mo	6-mo	12-mo	3-mo	6-mo	12-mo	3-mo	6-mo	12-mo	3-mo	6-mo	12-mo
α:	0.63	0.49	0.88	0.32	0.85	1.53	0.44	0.39	0.66	0.12	0.11	1.08
	(1.62)	(0.95)	(1.49)	(0.73)	(1.60)	(2.32)	(1.76)	(1.21)	(1.88)	(1.10)	(0.30)	(2.71)
γ:	0.10	0.02	-0.08	-0.48	-0.03	-0.12	0.13	0.09	0.07	0.05	0.04	0.06
	(1.28)	(0.27)	(0.90)	(3.26)	(0.46)	(1.37)	(2.21)	(1.60)	(0.73)	(2.23)	(0.78)	(0.86)
ϕ:	-46.35	-46.60	-56.25	-64.24	-49.69	-56.93	-41.26	-45.35	-51.14	-45.89	-45.82	-49.24
	(3.40)	(1.96)	(2.12)	(5.56)	(1.99)	(2.08)	(4.70)	(5.38)	(4.13)	(6.57)	(3.39)	(3.36)
φ_i:				3.20_{3}	3.63_{3}	-5.95_{4}				0.58_{3}	2.25_{3}	4.39_{4}
				(2.46)	(2.33)	(4.03)				(2.85)	(3.02)	(4.02)
				0.54_{15}	-3.69_{4}	-5.96_{5}				1.02_{15}	-4.08_{5}	5.36_{5}
				(2.43)	(2.27)	(3.43)				(3.47)	(3.64)	(5.01)
				-2.92_{5}	-6.11_{5}	8.57_{6}				-2.04_{5}		
				(2.44)	(4.00)	(2.19)				(3.26)		
μ:							2.19	2.49	4.69	0.28	1.71	1.69
ψ:							0.27	0.19	0.23	0.35	0.13	0.17
β:							0.73	0.80	0.76	0.61	0.83	0.80
κ:							192.18	350.42	495.34	200.81	311.55	427.24
λ_i:										7.99_{5}	1.34_{5}	1.73_{5}

Notes: See notes at the bottom of Tables 3a and 3b. Some simple statistics on the explanatory variables are given at the bottom of Table 4a continued on the next page.

Table 4a (continued)

Hypothesis tests and diagnostic checks from the martingale models

Sample period: 1986 to 1988

	OLS basic			OLS augmented			GARCH basic			GARCH augmented		
	3-mo.	6-mo.	12-mo.	3-mo.	6-mo.	12-mo.	3-mo.	6-mo.	12-mo.	3-mo.	6-mo.	12-mo.
$\bar{R}^2$:	0.21	0.11	0.16	0.41	0.17	0.20	0.21	0.17	0.18	0.67	0.27	0.42
$\chi^2(2)$:	0.12	0.62	0.20	0.00	0.25	0.03	0.01	0.11	0.19	0.01	0.66	0.01
$F(2,.)$:	0.01	0.59	0.13	0.00	0.34	0.01						
$\chi^2(\text{all})$:	0.00	0.16	0.08	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00
GNR:	0.49	0.32	0.70	0.32	0.34	0.40	0.84	0.60	0.83	0.16	0.61	0.30
Q(5):	0.04	0.00	0.01	0.04	0.00	0.00	0.82	0.91	0.29	0.45	0.99	0.63
$Q^2(5)$:	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	0.79	0.99	1.00	0.95
ARCH:	0.00	0.07	0.11	0.00	0.04	0.03	0.82	0.88	0.81	0.88	0.93	0.80
$\chi^2(\text{GARCHM})$:							0.61	0.39	0.44	0.00 (δ=0.03)	0.67	0.60

Sample means of X_i (in basis points, t-statistics in parentheses)

$WI3^T_t - WI3^T_{t-1}$:	0.29 (0.66)	compaper - tb3:	20.71 (31.93)	WI3-tb3:	-0.10 (0.35)
$WI6^T_t - WI6^T_{t-1}$:	0.15 (0.27)	call - tb3:	2.73 (0.90)	$tb3_t - tb3_{t-1}$:	0.20 (0.48)
$WI12^T_t - WI12^T_{t-1}$:	0.36 (0.51)	tb6 - tb3:	28.26 (21.05)	compaper - $WI3^T_t$:	20.81 (31.37)
call - WI3:	2.83 (0.87)				

Table 4b

Tests of the martingale hypothesis (continued)

Sample period: 1989 to June 1992

	OLS basic			OLS augmented			GARCH basic			GARCH augmented		
	3-mo.	6-mo.	12-mo.	3-mo.	6-mo.	12-mo.	3-mo.	6-mo.	12-mo.	3-mo.	6-mo.	12-mo.
α:	-0.01	0.01	0.00	0.84	3.88	0.35	0.09	0.01	0.00	0.57	0.51	0.85
	(0.05)	(0.03)	(0.01)	(2.42)	(3.20)	(0.58)	(0.62)	(0.01)	(0.27)	(3.53)	(1.46)	(2.42)
γ:	0.34	0.12	0.13	-0.03	0.06	0.05	0.10	0.11	0.09	0.09	0.03	0.05
	(0.54)	(2.86)	(2.87)	(0.47)	(1.50)	(0.81)	(1.52)	(2.29)	(1.57)	(1.59)	(1.64)	(0.82)
φ_i:				1.73_{2}	-0.13_{13}	2.75_{3}				-1.55_{5}	3.11_{3}	1.60_{3}
				(3.38)	(2.34)	(2.47)				(5.10)	(3.97)	(1.79)
				-0.003_{12}	2.87_{3}	-5.17_{4}					-0.002_{12}	-2.84_{4}
				(2.01)	(3.34)	(2.27)					(2.61)	(2.02)
				-1.39_{5}	-0.01_{12}	-0.04_{14}					-2.32_{5}	-4.13_{5}
				(2.93)	(2.21)	(2.02)					(3.30)	(6.50)
					-2.81_{5}	-4.24_{5}						
					(3.73)	(4.56)						
μ:							6.59	5.73	2.93	7.58	1.00	0.24
ψ:							0.20	0.08	0.15	0.26	0.01	0.08
β:							0.45	0.81	0.83	0.17	0.98	0.90
λ_i:										12.09_{9}	4.21_{9}	7.53_{9}
										-0.46_{5}	-0.41_{5}	-0.42_{5}

Notes: See notes at the bottom of Tables 3a and 3b. Some simple statistics on the explanatory variables are given at the bottom of Table 4b continued on the next page.

Table 4b (continued)

Hypothesis tests and diagnostic checks from the martingale models

Sample period: 1989 to June 1992

	OLS basic			OLS augmented			GARCH basic			GARCH augmented		
	3-mo.	6-mo.	12-mo.	3-mo.	6-mo.	12-mo.	3-mo.	6-mo.	12-mo.	3-mo.	6-mo.	12-mo.
$\overline{R}^2$:	0.00	0.02	0.02	0.06	0.08	0.12	0.02	0.02	0.02	0.04	0.14	0.18
$\chi^2(2)$:	0.87	0.02	0.02	0.05	0.00	0.59	0.05	0.01	0.07	0.00	0.46	0.07
F(2,.):	0.68	0.01	0.00	0.08	0.01	0.53						
χ^2(all):			0.00	0.00	0.00				0.00	0.00	0.00	
GNR:	0.10	0.16	0.16	0.25	0.21	0.61	0.01	0.09	0.46	0.00	0.22	0.30
Q(5):	0.00	0.05	0.33	0.01	0.11	0.25	0.07	0.55	0.87	0.02	0.28	0.73
$Q^2(5)$:	0.00	0.13	0.38	0.00	0.06	0.00	0.96	0.61	0.88	0.95	0.79	0.94
ARCH:	0.94	0.01	0.02	0.02	0.08	0.08	0.01	0.96	0.64	0.89	0.95	0.87
χ^2(GARCHM):							0.42	0.35	0.40	0.64	0.67	0.38

Sample means of X_i (in basis points, t-statistics in parentheses)

$WI3^T_t - WI3^T_{t-1}$:	-0.01 (0.07)	compaper - tb3:	16.05 (58.51)	WI3-tb3:	-2.11 (8.93)
$WI6^T_t - WI6^T_{t-1}$:	-0.11 (0.36)	call - tb3:	13.42 (6.28)	$tb3_t - tb3_{t-1}$:	-0.24 (1.34)
$WI12^T_t - WI12^T_{t-1}$:	-0.18 (0.47)	tb6 - tb3:	-5.49 (5.34)	compaper - $WI3^T_t$:	18.16 (61.63)
call - WI3:	15.53 (6.66)				

the 2 per cent level, because the lagged yield change is significantly positive.

Results from the augmented OLS regressions reveal that on days when the Bank undertook SRAs or three-month treasury bill sales to counter large declines in yields, the net effect was to limit the decline and not to offset the decline entirely (as indicated by φ_4 and φ_5). SPRAs were undertaken on days when when-issued yields increased (as shown by the coefficient estimates φ_3) in order to limit the change. When these variables are included in tests of the martingale hypothesis, results based on the $\chi^2(2)$ tests are generally not altered. They do, however, add a great deal to the explanatory power of the regressions.

Evidence of misspecification in both the basic and augmented OLS regressions can be found from the tests for autocorrelation, Q(5), and for heteroscedasticity, $Q^2(5)$ and ARCH(5) in both periods. Autoregressive heteroscedasticity is prevalent in the regression residuals for all three maturities, which suggests it would be useful to examine a GARCH model. Evidence of autocorrelation is found in the regressions for the earlier period and for the three-month bill in the later period.

As a result of parameterization of the autoregressive heteroscedasticity in the OLS regressions, the GARCH(1,1) results show no evidence of autocorrelation nor of heteroscedasticity in the standardized residuals, ε_t/σ_t, as the marginal significance levels for these tests are well above 0.01. However, there is again marginal evidence against the null, (α=0 and γ=0), for the three-month bill in the first sample period and for the six-month bill in the later sample. In these cases, evidence of a positive and statistically significant coefficient on the lagged dependent variable implies that when-issued yield changes help predict future when-issued yield changes. There is no evidence, however, of a significant intercept, so that the unconditional mean of when-issued yield changes is zero. In addition, as shown by the χ^2(GARCHM) tests, there is little evidence of own volatility effects in the mean of when-issued yield changes as given in equation (4.7).

The conditional variance parameterization[22] shows the persistence in variance in yields for all three maturities, especially in the first sample period where the sum of the ψ and β coefficients is close to one.[23] The conditional variance of each of the yields was substantially larger during the week of the October 1987 stock market crash (as indicated by the large and significant coefficients on the dummy variable, $Crash_t$, in the variance equations), while when-issued yields were trending lower (as were most yields) during this period. The negative and significant coefficients on $Crash_t$ in the mean equations show this.

The results from the augmented GARCH models are slightly different: the null is rejected at the 1 per cent level for the three-month bill and for the twelve-month bill in the earlier sample. Hence, once when-issued yield changes have been purged of other effects, such as SPRAs, the $\chi^2(2)$ tests show that the constant and lagged yield changes do contribute significantly to the likelihood function, at least for the three-month bill. Finally, there is evidence of own variance effects in the three-month bill for the earlier period where δ is estimated to be 0.03.

It is likely that when-issued market participants make use of the information contained in the first and second moments of yield changes from other when-issued bills. However, the univariate models assume that these correlations between bills are zero. The bivariate model of the martingale hypothesis is able to incorporate the correlation of the means and variances (and therefore covariance) of when-issued bills written on treasury bills with two different terms to maturity. Results of estimation of the bivariate model over the 1989 to June 1992 period, which pairs the three- and six-month when-issued bills, are given in Table 5a, for the basic model, and Table 5b, for the augmented model.

22. All variables in the conditional variance parameterization are significant at the 1 per cent level, although t-statistics are not reported.

23. The integrated GARCH process constrains this sum to equal one and implies that shocks to volatility never die away completely but do become less important for future realizations of σ^2.

Table 5a
Bivariate test of the basic martingale hypothesis

Sample period: 1989 to June 1992

$$\begin{bmatrix} \Delta WI3^T_t \\ \Delta WI6^T_t \end{bmatrix} = \begin{matrix} \underset{(0.04)}{0.00} & + \underset{(2.08)}{0.06}\,\Delta WI3^T_{t-1} & - \underset{(1.56)}{0.03}\,\Delta WI6^T_{t-1} & + \varepsilon_{3,t} \\ \underset{(3.93)}{0.02} & - \underset{(2.35)}{0.12}\,\Delta WI3^T_{t-1} & + \underset{(5.32)}{0.16}\,\Delta WI6^T_{t-1} & + \varepsilon_{6,t} \end{matrix}$$

$$\begin{bmatrix} \sigma^2_{3,t} \\ \sigma_{3;6,t} \\ \sigma^2_{6,t} \end{bmatrix} = \begin{matrix} \underset{(3.34)}{6.13} & + \underset{(2.63)}{0.09}\,\varepsilon^2_{3,t-1} & + \underset{(9.91)}{0.55}\,\sigma^2_{3,t-1} \\ \underset{(4.00)}{5.25} & + \underset{(1.86)}{0.08}\,\varepsilon_{3,t-1}\varepsilon_{6,t-1} & + \underset{(8.95)}{0.69}\,\sigma_{3;6,t-1} \\ \underset{(3.21)}{7.68} & + \underset{(2.14)}{0.06}\,\varepsilon^2_{6,t-1} & + \underset{(10.07)}{0.75}\,\sigma^2_{6,t-1} \end{matrix}$$

Nobs	494		
$Q_3(5)$	0.08	$Q^2_3(5)$	0.71
$Q_6(5)$	0.64	$Q^2_6(5)$	0.57
$Q_{3,6}(5)$	0.42	$Q^2_{3,6}(5)$	0.51
χ^2 (*mean*)	0.01	χ^2 (*own-lagged*)	0.00
χ^2 (*constants and own*)	0.01	χ^2 (*cross-lagged*)	0.35

Notes: See the notes at the bottom of Tables 3a and 3b. $Q_{3,6}(5)$ is the Ljung-Box test for autocorrelation in the cross products of the residuals, $Q^2_{3,6}(5)$ is the same for the squared cross products, χ^2(mean) is a robust chi-square test for testing the null hypothesis that all coefficients in the mean equations are zero, χ^2(own-lagged) is the same for both of the own-lagged variables – analogously for χ^2(cross-lagged) – and χ^2(constants and own) is a simultaneous test that the coefficients on the intercepts and own-lagged variables are zero.

Table 5b
Bivariate test of the augmented martingale hypothesis

Sample period: 1989 to June 1992

$$\begin{bmatrix} \Delta WI3_t^T \\ = \\ \Delta WI6_t^T \end{bmatrix} \begin{matrix} 0.40 + 0.04\,\Delta WI3_{t-1}^T + 0.01\,\Delta WI6_{t-1}^T - 1.39\,TBSale_t + \varepsilon_{3,t} \\ (6.41)\ (2.92)\qquad (1.49)\qquad (8.92) \\ \\ 0.87 - 0.13\,\Delta WI3_{t-1}^T + 0.14\,\Delta WI6_{t-1}^T + 0.05\,SPRA_t - \\ (35.27)(2.51)\qquad (13.91)\qquad (2.45) \\ 3.08\,TBSale_t + \varepsilon_{6,t} \\ (9.32) \end{matrix}$$

$$\sigma_{3,t}^2 = 5.37 + 0.07\,\varepsilon_{3,t-1}^2 + 0.47\,\sigma_{3,t-1}^2 - 0.39\,TBSale_t + 0.95\,FXInt_t$$
$$(7.15)(3.38)\qquad (12.61)\qquad (8.65)\qquad (5.30)$$

$$\sigma_{3;6,t} = 4.34 + 0.09\,\varepsilon_{3,t-1}\varepsilon_{6,t-1} + 0.59\,\sigma_{3;6,t-1} - 0.31\,TBSale_t + 1.07\,FXInt_t$$
$$(3.65)\ (8.35)\qquad (11.33)\qquad (6.48)\qquad (7.36)$$

$$\sigma_{6,t}^2 = 7.21 + 0.11\,\varepsilon_{6,t-1}^2 + 0.64\,\sigma_{6,t-1}^2 - 0.39\,TBSale_t + 1.63\,FXInt_t$$
$$(2.86)(12.46)\qquad (10.04)\qquad (5.24)\qquad (6.32)$$

Nobs	494		
$Q_3(5)$	0.48	$Q_3^2(5)$	0.86
$Q_6(5)$	0.56	$Q_6^2(5)$	0.91
$Q_{3,6}(5)$	0.77	$Q_{3,6}^2(5)$	0.81
χ^2 (Mean)	0.00	χ^2 (own-lagged)	0.01
χ^2 (constants and own)	0.01	χ^2 (cross-lagged)	0.38
χ^2 (mean dummies)	0.00		

Note: See notes at the bottom of Table 5a.

The basic bivariate system captures the positive significance of the own-lagged dependent variable, as the robust t-statistics (2.08 and 5.32 respectively) are greater than their 1 per cent significance level counterpart. Their contribution to the likelihood function is confirmed, because the χ^2(own-lagged) test statistic for testing the null hypothesis that the coefficients on each of the own-lagged dependent variables are zero indicates that this hypothesis can be rejected, that is, the marginal significance level is less than 0.01.[24] The χ^2(constants and own) test indicates that the constant terms and own-lagged terms contribute jointly to the likelihood function as well, although for the three-month bill it is insignificantly different from zero. Hence, there is evidence against the martingale hypothesis, as one-period, lagged, when-issued yield changes help predict current when-issued yield changes.

When the effects of Bank of Canada open market operations on when-issued yield changes are considered, as in the augmented bivariate tests, the story remains unchanged: own-lagged, when-issued yield changes help predict current yield changes. In addition, there is now evidence of significantly positive constant terms, indicating that when the effects of Bank operations are removed from when-issued yield changes, the when-issued yield tends to approach the treasury bill yield at tender from below. Along with the results from the augmented univariate models, these results also show that three-month treasury bill sales by the Bank tend to reduce the volatility in when-issued yields, whereas intervention in the foreign exchange market may increase volatility.[25]

The diagnostic statistics from both models reveal no autocorrelation nor heteroscedasticity nor cross-correlation in the standardized residuals from the bivariate systems. That is, the GARCH(1,1) process has adequately captured the time variation in the conditional variances of

24. The null hypothesis that the coefficients on the "cross"-lagged dependent variables are zero cannot be rejected.

25. Foreign exchange intervention by the Bank of Canada on behalf of the government may come about as a result of relatively high volatility and may not be the cause of increasing volatility in when-issued yield changes.

when-issued yield changes and the covariance between yield changes. Evidence of strong persistence in the conditional variances and covariance between the two bills can be found in the coefficient estimates from the GARCH equations and Figures 3 and 4. These figures show that, aside from being highly time-varying, the conditional variance from the three-month bill (Figure 2) and the conditional covariance between the bills (Figure 4) were on occasion rather large: end of 1989, mid-1990, mid-1991 and early 1992, and that the conditional covariance between bills was always positive. Figure 3 shows that with the exception of mid-1991, the conditional variance in six-month bills was substantially higher than that in three-month when-issued bills. This is perhaps a reflection of lower liquidity.

In summary: Generally, the univariate models (both OLS and GARCH) revealed, at best, weak evidence either of a constant bias in when-issued yield changes or of dependence on own-lagged yield changes. However, there is strong evidence of dependence in yield changes from the basic bivariate GARCH model and, in addition, strong evidence of a constant bias in the augmented bivariate GARCH model. This suggests that the risk premium hypothesis could be evaluated as $WI_{t-1}^T - WI_{t-2}^T$ or that the constant term could be acting as a proxy for the risk premium.

Figure 3
Spread between the three- and six-month conditional variances from the bivariate GARCH(1,1) model

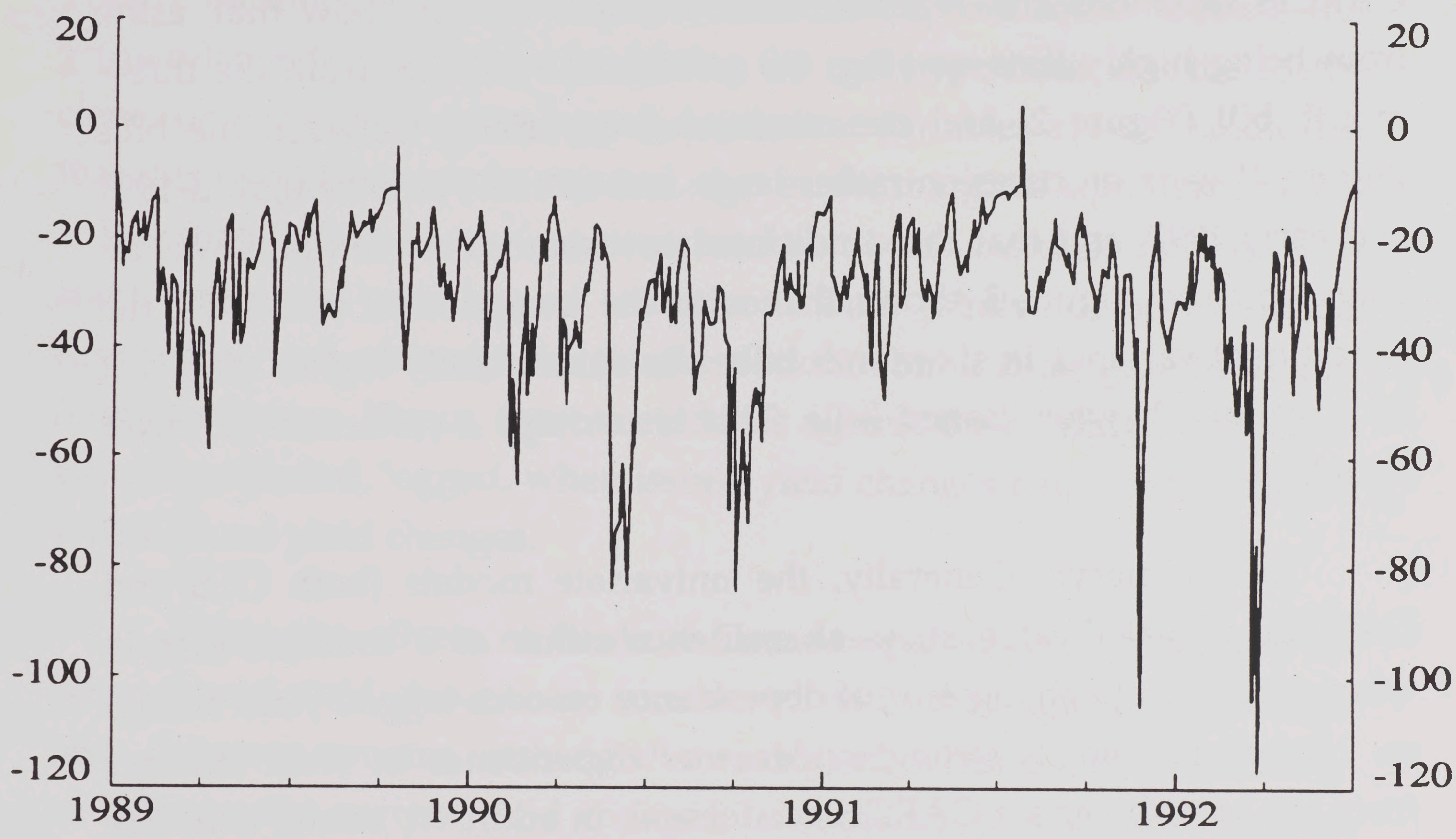
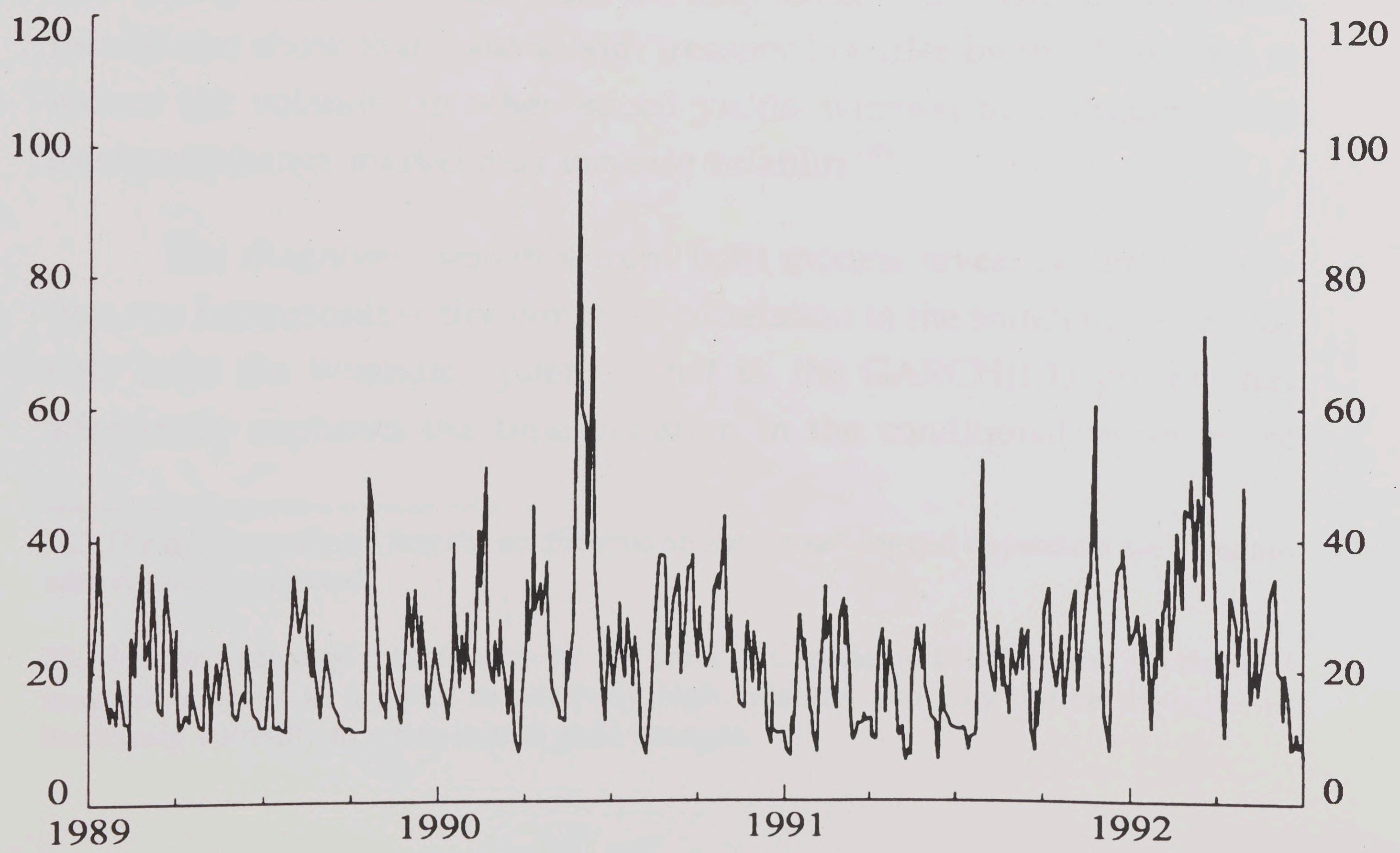


Figure 4
Conditional covariance between three- and six-month when-issued yield changes from the bivariate GARCH(1,1) model



5 ARBITRAGE RELATIONS AND TESTS

The link between forward or futures prices and spot prices can also be motivated by the existence of profitable arbitrage opportunities (see, for example, Harrison and Kreps 1979, Duffie 1986, and McCurdy and Morgan 1990). Market efficiency guarantees the absence of profitable arbitrage opportunities. In its simplest notion, arbitrage efficiency requires a hedged portfolio to have an initial value equal to the present discounted value of the expected cash flows. Similarly, portfolios requiring zero initial investment outlay must generate cash flows that, on average, have an expected present value of zero; otherwise arbitrage opportunities exist that yield non-zero profits.

5.1 Theoretical arbitrage relations

The most direct derivation of an expression for a riskless portfolio is through a hedged position in the spot market, with no initial investment outlay. Market efficiency ensures that the present value of the terminal cash flow is zero. Consider the following portfolio transactions: at time t, an investor "shorts" (or sells) one unit of the spot commodity and invests the proceeds, S_t, at the risk-free rate of R_t. At the same time the investor "longs" (or buys) one forward contract written on the same underlying spot asset. The time t investment outlay is zero. Next, at maturity, the spot asset having price S_T is delivered to the lender to close out the short position. The cash flows arising from this position at T must have an expected present value of zero to ensure market efficiency – the lack of profitable and riskless arbitrage opportunities.

This strategy can be applied to a hedged portfolio of a three-month treasury bill and a three-month when-issued contract (for example) to yield cash flows having an expected present value of zero, as follows: At time t, the investor sells a "three-month-plus-N-day" treasury bill, promising to repay in N days, and invests the proceeds, PTB_t^{3M+N}, at the overnight riskless interest rate R_t. The instrument being shorted is actually the seasoned six- or twelve-month treasury bill (the coincident-to-when-issued bill) that has three months and N days left to maturity and that will be re-opened in N days, that is, next Thursday at the auction.[26] At the same time, the investor buys one when-issued contract at price PWI_t^T for the acceptance of a three-month treasury bill on the following Thursday. No money changes hands on the when-issued contract until maturity, at which time the when-issued contract will be used to repay the short position. The cash flow from this portfolio at maturity (T, or Thursday) is

$$PTB_t^{3M+N} \prod_{t=1}^{N} R_t - PWI_t^T + (PTB_T^{3M} - PWI_t^T). \tag{5.1}$$

The first term in expression (5.1) is the cash flow arising from the use of the funds from the seasoned treasury bill for N days, the second is the cash disbursement from the long position in the when-issued contract, and the term in parentheses is the potential profit from taking delivery of the three-month treasury bill in exchange for the when-issued contract.[27] Now, after accepting delivery to close the when-issued contract, the market participant delivers the new three-month treasury bill to close out the short sale of the three-month-plus-N-day treasury bill, which now has a maturity date three months away.

Since the cash flows at t are zero, the cash flows of this portfolio at maturity must have a present value of zero to ensure arbitrage efficiency. If the cash flows generated at T were negative, then the opposite strategy (buying a three-month-plus-N-day treasury bill and simultaneously selling a three-month when-issued contract) would bring about riskless profits.

In expression (5.1), neither PTB_T^{3M} nor R_{t+i} for i >1 are known at the time the portfolio decision is made, but the present value operator (3.1) allows the direct comparison of the cash flows at initiation and maturity for a zero net investment portfolio. Since the cash flows from this

26. Recall that treasury bill auctions are currently held on Tuesdays. The empirical work for this paper was done when treasury bill auctions were held on Thursdays.

27. Note that a similar expression can be found for a hedge with a six-month treasury bill. However, this sort of portfolio cannot be constructed for twelve-month treasury bills, since treasury bills having "twelve months and N days" left to maturity do not exist.

strategy at t are zero, application of the present value operator to the time T cash flows yields

$$0 = E\{M_T\left(PTB_t^{3M+N}\prod_{i=1}^{N} R_{t+i} - 2PWI_t^T + PTB_T^{3M}\right)\}, \tag{5.2}$$

as the component in parentheses is the return on a net zero investment portfolio. Simplifying equation (5.2) by assuming that the risk-free overnight interest rate is non-stochastic, so that $E(R_t) = R_t$ and making use of the Euler condition (3.2) leads to

$$0 = R_T^{-1} \cdot PTB_t^{3M+N}\prod_{i=1}^{N} R_{t+i} - R_T^{-1} \cdot 2PWI_t^T + E(M_T) \cdot E(PTB_T^{3M}) + Cov(M_T, PTB_T^{3M}, \tag{5.3}$$

by a covariance decomposition. Further simplification finds

$$0 = PTB_t^{3M+N}\prod_{i=1}^{N} R_{t+1} - 2PWI_t^T + E(PTB_T^{3M}) + R_T \cdot Cov(M_T, PTB_T^{3M}). \tag{5.4}$$

Hence, a time-varying risk premium, Cov, is consistent with a short hedge in treasury bills and arises because the terminal payoff is unknown at the time the portfolio decision is made. This arbitrage valuation method does not require specific stochastic processes for either when-issued prices or risk-free bonds. However, the arbitrage relationship (5.4) assumes frictionless markets. In some cases, capital requirements, taxes, trading restrictions, transaction costs, bid/ask spreads and illiquidity[28] may significantly influence the arbitrage relation.

5.2 Test results of the arbitrage relations

The forward and spot price relationship derived from the arbitrage efficiency methodology (that is, discrete-time and risk-adverse agents) requires that the cash flows from a short hedge be zero. Hence, investigation of the time-series properties of the distribution of profits from this position is one way of examining how effective the hedge is. However, because equation (5.4) is stochastic, it is necessary to evaluate

28. An example is a situation where the bulk of the supply of coincident-to-when-issued bills is typically locked away in investment accounts and not available to trading accounts.

the expectation of the treasury bill price at the upcoming auction in order to examine the cash flows from this portfolio. Under rational expectations, the expectation of the future treasury bill price can be replaced with the actual treasury bill price less a rational expectations forecast error.[29] Under the null hypothesis of no risk premiums, the cash flows from this portfolio[30] should be white noise:

$$\varepsilon_t = PTB_t^{3M+N}\prod_{i=1}^{N} R_{t+i} - 2PWI_t^T + PTB_T^{3M} \sim iid(0, \sigma^2)\,. \tag{5.5}$$

An alternative way of examining the efficiency of a short cash-and-carry hedge is through a test regression:[31]

$$\frac{PTB_T^{3M} - PTB_t^{3M}}{PTB_t^{3M}} = \alpha + \beta\left(\frac{2PWI_t^T - PTB_t^{3M+N}\prod_{i=1}^{N} R_{t+i} - PTB_t^{3M}}{PTB_t^{3M}}\right) + \varepsilon_t, \tag{5.6}$$

where $\alpha=0$, $\beta=1$ and $\varepsilon_t \sim iid(0, \sigma^2)$ under the null hypothesis.

As noted, the three-month-plus-*N*-day treasury bill (the coincident-to-three-month when-issued bill) is not a regularly traded instrument and hence the Bank does not collect data for this bill. Thus, PTB_t^{3M+N} and therefore TB_t^{3M+N} (its corresponding yield) are not observable. The tests examined in this section assume a linear yield curve and set the price of the coincident-to-when-issued bill as a linear combination of the two closest treasury bills for which interest rate data are available.[32] The average

29. An ARMA(2,2) process, thought to capture adaptive expectations, gives similar results.

30. Poitras (1991) derives a similar expression for the when-issued–coincident-to-when-issued yield spread but in a risk neutral world. Unfortunately, he does not actually test the cash flow boundary conditions implied by the strategy, owing to lack of coincident-to-when-issued yield data.

31. Since the variables are in price format, each side of the equation must be scaled by PTB_t^{3M}, the current three- or six- month treasury bill, to ensure stationarity.

32. For example, in the case of the six-month coincident-to-when-issued bill, its price will be $(1-x)PTB_t^{6M} + xPTB_t^{12M}$, where x is approximately 25/26.

call loan rate is used as the overnight risk-free interest rate. Figures 5 and 6 plot the expected weekly profit from the theoretical hedged portfolio, under the assumption of rational expectations, as executed on Fridays and Wednesdays respectively, while the estimation results of equation (4.12) and primary statistics on the distribution of expected profits are given in Table 6 for the 1989 to June 1992 period.

In general, the figures indicate that the profits from a short three-month-plus-*N*-day cash-and-carry hedge are slightly positive, regardless of the day of the week that the portfolio is constructed. Those from a short six-month-plus-*N*-day hedge are negative. While the profits are on average significantly different from zero,[33] as indicated at the bottom of Table 6, they are likely to be economically insignificant and unpredictable – even for the six-month bill, expected profits are only in the twenty-cent-per-week-per-contract range for a long cash-and-carry hedge.

Even though the null hypothesis (α=0, β=1) is strongly rejected for both bills, as indicated by the $\chi^2(2)$ p-values), the test regression (5.6) has substantially better explanatory power for the expected treasury bill yield at the auction than the when-issued bill alone. None of the other regressions were able to produce $\overline{R}^2$s even close to those in Table 6. Hence, to get a better indication of the market's expectation of the treasury bill yield at the auction, it may be advantageous to use the when-issued yield in conjunction with a linear combination of current treasury bills.

Finally, it should be noted that it is not necessary for market participants to engage in constructing this portfolio to keep when-issued and treasury bill prices in line. They need only be aware that it is possible to form such a hedged portfolio to take advantage of informationally inefficient prices, should they come about. There may also be substantial difficulties in forming the portfolio due to thin markets in non-current bills. While physical ownership of the old treasury bill is not necessary for the creation of the portfolio suggested here, it may very well become

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33. As indicated by the relatively small p-values for the measure of kurtosis, the cash flow distribution has fatter tails than the normal distribution would suggest.

Figure 5
Expected profit from a short cash-and-carry Friday hedge

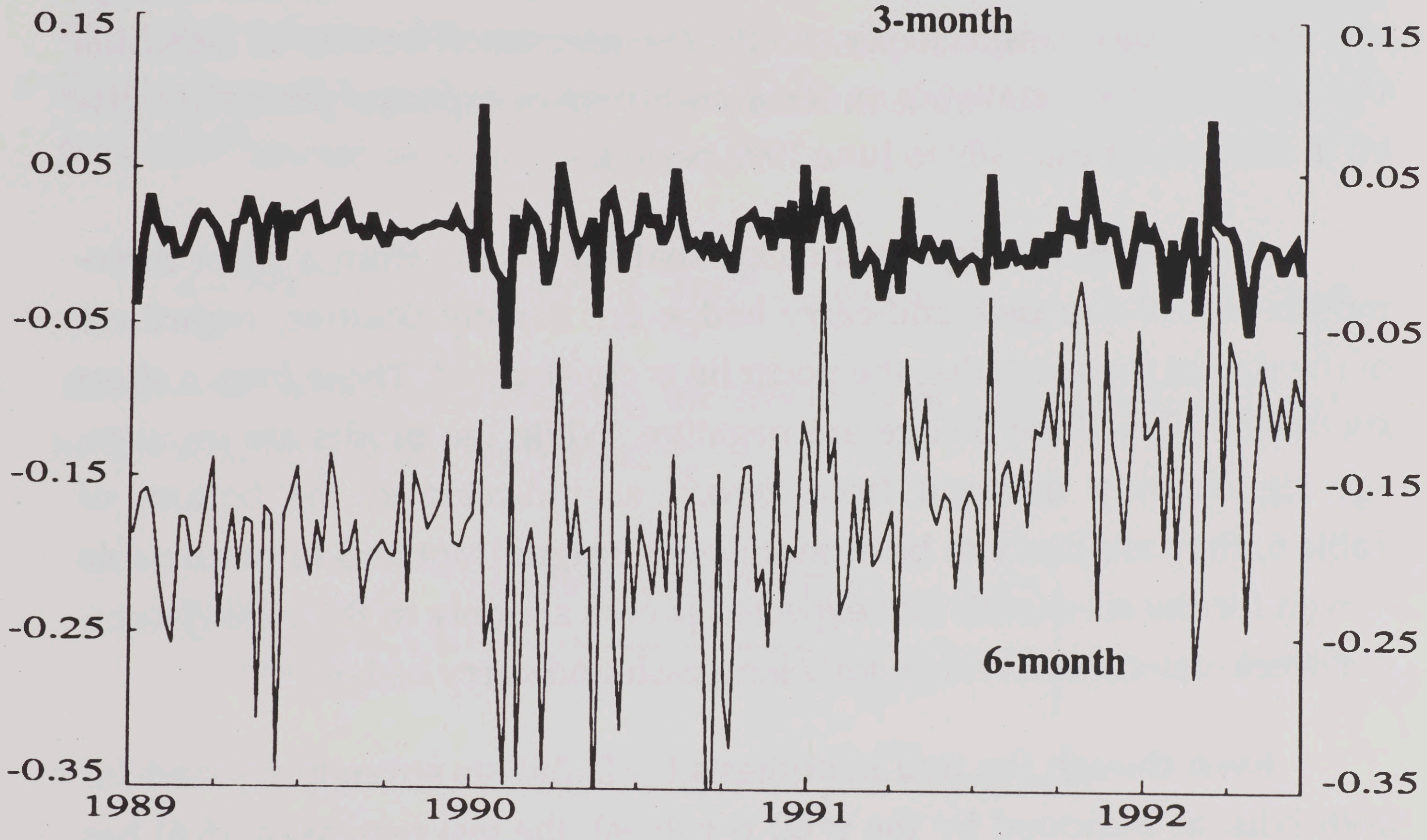


Figure 6
Expected profit from a short cash-and-carry Wednesday hedge

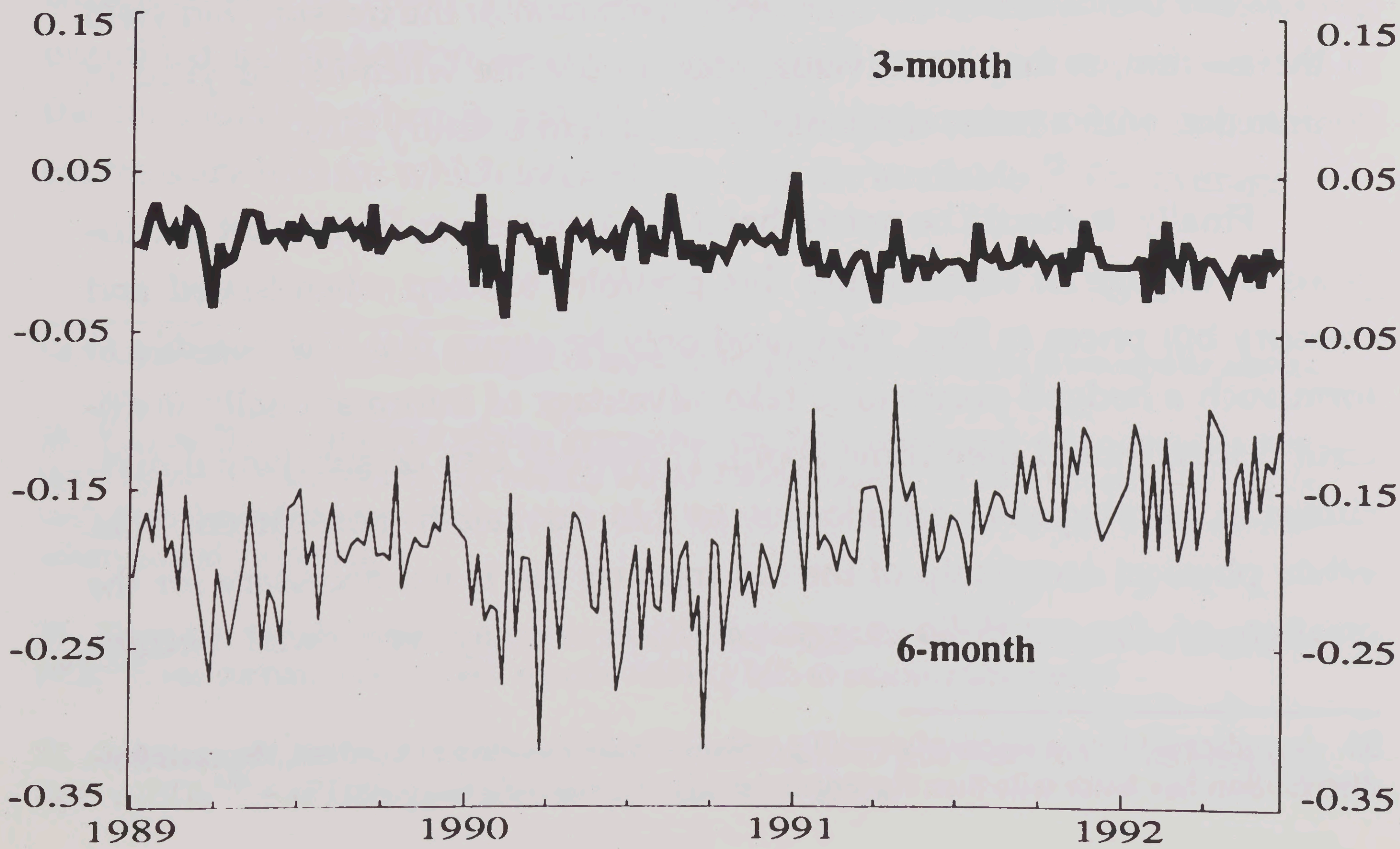


Table 6

Tests of a short cash-and-carry hedge

$$\frac{PTB_T^{3M} - PTB_t}{PTB_t} = \alpha + \beta \left(\frac{2 \cdot PWI_t^T - PTB_t^{3M+N} \prod_{i=1}^{N} R_{t+i}(-PTB_t)}{PTB} \right) + \varepsilon_t \text{ where } PTB_t^{3M+N} = xPTB_t^{3M} + (1-x)\,PTB_t^{6M} \text{ and } x \approx \frac{12}{13}$$

Sample period: 1989 to June 1992

	3-month				6-month			
	Fri.	Mon.	Tue.	Wed.	Fri.	Mon.	Tue.	Wed.
α:	-0.001	-0.01	-0.02	-0.02	-0.06	-0.06	-0.18	-0.19
	(0.49)	(2.85)	(4.24)	(7.63)	(2.53)	(12.07)	(23.36)	(50.66)
β:	0.69	0.84	0.85	0.83	0.26	0.73	0.85	0.92
	(9.18)	(21.41)	(31.07)	(42.52)	(1.62)	(4.99)	(7.05)	(11.43)
$\overline{R}^2$:	0.40	0.74	0.85	0.92	0.01	0.13	0.21	0.39
$\chi^2(2)$:	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
F(2,.):	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
GNR:	0.11	0.90	0.74	0.96	0.02	0.02	0.01	0.01
Q(5):	0.05	1.00	0.89	1.00	0.01	0.03	0.01	0.01
$Q^2(5)$:	1.00	0.96	0.01	0.06	1.00	0.99	0.03	0.99
PROFIT:								
Mean:	0.008	0.01	0.01	0.03	-0.17	-0.18	-0.18	-0.18
	(4.82)	(3.40)	(3.90)	(3.05)	(32.45)	(33.19)	(44.66)	(58.99)
Skew:	0.46	0.27	0.00	0.24	0.14	0.58	0.27	0.15
Kurt:	0.00	0.00	0.00	0.02	0.00	0.23	0.03	0.06
Min:	-0.09	-0.07	-0.08	-0.04	-0.42	-0.35	-0.35	-0.32
Max:	0.09	0.09	0.06	0.05	0.02	0.03	-0.03	-0.08
Sum:	0.04	0.001	0.003	0.005	-0.19	-0.21	-0.22	-0.21

Notes: Numbers in parentheses are robust t-statistics. See also the notes at the bottom of Table 3a.

necessary if a market participant wishes to take the opposite hedge, should market fundamentals suggest such a transaction. As well, closing prices are not always representative of the prices at which the majority of trades do take place. Even with these qualifications, it seems unlikely that the expected profits from a short cash-and-carry hedge portfolio deviated significantly from their theoretical values over the 1989 to June 1992 period.

6 WHEN-ISSUED–TREASURY BILL SPREADS

There are alternative ways to derive testable relationships arising from the cash flows from cash-and-carry style portfolios, as in equation (5.4). One of those is to impose efficiency conditions in advance and to postulate processes for the evolution of prices of financial assets.

6.1 When-issued–treasury bill spreads in a risk neutral world

Consider the case of a treasury bill maturing in three months and N days. In a frictionless and risk neutral world, its price per one dollar face value will be

$$PTB_t^{3M+N} = e^{-tb_t^{3M+N}(3M+N)}, \tag{6.1}$$

where tb_t^{3M+N} is its continuously compounded daily yield. For example, the holder of the coincident-to-three-month-when-issued bill expects to receive tb_t^{3M+N} per day over three months and N days. As this is the deliverable asset for a three-month when-issued contract, the price of the when-issued contract maturing at T can be calculated as

$$PWI_t^T = PTB_t^{3M+N} \cdot e^{r_t(N)} = e^{\{r_t(N) - tb_t^{3M+N}(3M+N)\}} \tag{6.2}$$

in a riskless environment and where r_t is the overnight borrowing rate. The latter part of expression (6.2) follows from substitution of (6.1). The unbiased expectations hypothesis allows PWI_t^T to be written as

$$PWI_t^T = e^{-E(tb_T^{3M}(3M))} = e^{-wi_t^T(3M)}. \tag{6.3}$$

Setting equations (6.3) and (6.2) equal, taking natural logarithms and rearranging the expression defines the following process for the when-issued yield:

$$wi_t^T = tb_t^{3M+N}\{\frac{3M+N}{3M}\} - r_t\{\frac{N}{3M}\}. \tag{6.4}$$

Adding to and subtracting $tb_t^{3M+N}(N/3M)$ from the right-hand side of (6.4) yields

$$wi_t^T = tb_t^{3M+N} + \{\frac{N}{3M}\} \cdot (tb_t^{3M+N} - r_t), \tag{6.5}$$

which shows that, in perfect markets, the when-issued yield must equal the yield on the coincident-to-when-issued bill (tb_t^{3M+N}) plus the net carry return on the coincident-to-when-issued bill over the life of the when-issued bill (N days) as a fraction of the term to maturity on the to-be-issued treasury bill (3M).[34] Subtracting the current treasury bill yield from both sides of (6.5) shows that the spread between the when-issued yield and the current treasury bill yield depends on the slope of the yield curve and the weighted net carry return on the coincident-to-when-issued bill:

$$wi_t^T - tb_t^{3M} = tb_t^{3M+N} - tb_t^{3M} + \{\frac{N}{3M}\} \cdot (tb_t^{3M+N} - r_t). \tag{6.6}$$

Note that while equation (5.4) also shows how the when-issued yield can be found from the coincident-to-when-issued yield adjusted for the net carry return, equation (6.5) is deterministic (everything is known at t) and holds only under the more restrictive assumptions of frictionless markets and risk neutrality.

Unfortunately, data for the coincident-to-when-issued bill are not readily available, as the bill is locked away and not typically traded. However, an expression can be found for the yield on the coincident-to-when-issued bill if the first part of equation (6.3) is made equal to equation (6.2), the natural logarithm is used and the expression is rearranged:

$$tb_t^{3M+N} = \{\frac{3M}{3M+N}\} \cdot E(tb_T^{3M}) + \{\frac{N}{3M+N}\} r_t. \tag{6.7}$$

Hence, the coincident-to-when-issued bill can be priced as a linear combination of the expected treasury bill tender yield and the overnight

34. This is derived from Hull (1989).

borrowing cost, the call loan rate.[35] Alternatively, assuming a linear yield curve, the three-month-plus-N-day bill could be priced as a linear combination of the three- and six-month bills:

$$tb_t^{3M+N} = x\,(tb_t^{3M}) + (1-x)\,(tb_t^{6M}) \quad \text{where } x \approx \frac{12}{13}, \tag{6.8}$$

since the coincident-to-when-issued bill has a term to maturity one week greater than the current three-month treasury bill.

While this seems unduly complicated, it simply shows that when participants wish to buy or sell the to-be-issued treasury bill during the week leading up to the regular weekly auction, they can either *(i)* take a position in the when-issued market at no current cost or *(ii)* take a position in the coincident-to-when-issued market, which may involve borrowing funds, thereby incurring current but no future costs. In a risk neutral world, yields will adjust, so that at the margin, individuals will be indifferent between these choices.

6.2 Regression results for when-issued–treasury bill spreads

The when-issued–cash treasury bill spread can be examined when equation (6.6) is written in regression format:[36]

$$wi_t^T - tb_t^{3M} = \alpha + \beta\,(tb_t^{3M+N} - tb_t^{3M}) + \gamma(\{\frac{N}{3M+N}\}\,(tb_t^{3M+N} - r_t)) + \varepsilon_t, \tag{6.9}$$

where lower-case letters refer to daily, continuously compounded yields and r_t the overnight risk-free rate (that is, the average call loan rate). In Table 7a this regression is examined. The daily rate on the coincident-to-when-issued bill is calculated as a linear function of the two nearest

35. There is anecdotal evidence suggesting that dealers do this. However, pricing the coincident-to-when-issued bill as a linear combination of the three- and six-month bills or extrapolating with the two- and three-month bills has a greater intuitive appeal. Empirically, these methods produce daily coincident-to-when-issued yields that differ by about one one-thousandth of a basis point.

36. Even though equation (6.6) is deterministic, it is not completely illogical to write it in regression format, since the theoretical daily yields are yields expected to prevail over the next N days, for example. Hence, these yields can be replaced by their realized values plus an innovation.

Table 7a

Determinants of the when-issued–treasury bill spread

$$wi_t^T - tb_t^{3M} = \alpha + \beta\,(tb_t^{3M+N} - tb_t^{3M}) + \gamma(\{\frac{N}{3M+N}\}\,(tb_t^{3M+N} - r_t)) + \varepsilon_t$$

where $tb_t^{3M+N} = x\,tb_t^{3M} + (1-x)\,tb_t^{6M}$ and $x \approx \frac{12}{13}$

Sample period: 1989 to June 1992

	3-month				6-month			
	Fri.	Mon.	Tue.	Wed.	Fri.	Mon.	Tue.	Wed.
$\alpha x 10^3$:	-0.09	-0.03	-0.07	-0.02	-0.20	-0.25	-0.34	-0.04
	(0.97)	(0.41)	(0.94)	(2.67)	(3.45)	(3.74)	(5.73)	(5.04)
β:	-0.01	0.002	-0.01	-0.01	-0.07	-0.004	0.01	0.02
	(0.14)	(0.03)	(0.19)	(0.24)	(1.01)	(0.05)	(0.25)	(0.28)
γ:	1.31	2.72	3.91	7.48	1.61	3.22	4.10	8.07
	(14.09)	(13.92)	(16.58)	(17.25)	(13.21)	(11.76)	(14.27)	(11.83)
$\overline{R}^2$:	0.60	0.62	0.66	0.66	0.60	0.60	0.60	0.51
$\chi^2(3)$:	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
GNR:	0.00	0.02	0.00	0.00	0.22	0.06	0.00	0.01
Q(5):	0.00	0.00	0.00	0.00	0.13	0.00	0.00	0.00
$Q^2(5)$:	0.93	1.00	0.97	1.00	0.99	0.02	0.00	1.00

Notes: See notes at the bottom of Table 3a.

actively traded bills. In Table 7b the results of estimation are presented, and it is assumed that the rate on the coincident-to-when-issued bill follows a process as in equation (6.7):

$$tb_t^{3M+N} = \{\frac{1}{3M+N}\} (3M \cdot tb_T^{3M} + N \cdot r_t), \tag{6.10}$$

with the additional assumption of rational expectations. Figures 7 and 8 plot some daily, continuously compounded Tuesday rates (N=2), including the rate on the coincident-to-when-issued bill as calculated by the two different methods, for selected periods. As shown, the coincident-to-when-issued yield calculated by equation (6.10) occasionally deviates substantially from the current treasury bill yield while, as expected, the other method produces a coincident-to-when-issued yield very similar to that on the three-month treasury bill. This difference can, in part, be attributed to the volatility of the average call loan rate, which is significantly more volatile than the yields on treasury bills.

The tables show that the null hypothesis (α=0, β=γ=1) can be soundly rejected. An interesting characteristic of the tables concerns the explanatory power of the coincident-to-when-issued–treasury bill spread: when the coincident-to-when-issued bill is calculated as a linear combination of the two nearest bills, the when-issued–treasury bill spread is explained mainly by the net carry term, and the alternative formulation for the coincident-to-when-issued yield produces a significant coefficient on the coincident-to-when-issued–treasury bill spread term, that is, β becomes significantly different from 0. A comparison of the $\overline{R}^2$s between tables shows that, indeed, this latter way of modelling the coincident-to-when-issued yield explains perhaps 10 per cent more of the when-issued–current treasury bill spread than the way of modelling it as a linear combination of the two bills.[37] Finally, as indicated by the GNR and Q(5) test

37. Furthermore, substituting equation (6.10) into (6.9) and rearranging the components gives an expression for the expected treasury bill yield at the upcoming auction, under the null hypothesis:

$$E(tb_T^{3M}) = \frac{(3M+N)^2}{3M(3M+2N)} \cdot wi_t^T + \frac{N^2}{3M(3M+2N)} \cdot r_t, \tag{6.9a}$$

which implies that $wi_t^T < E(tb_T^{3M})$ for days prior to Wednesdays (that is, where N=1).

Table 7b

Determinants of the when-issued–treasury bill spread (continued)

$$wi_t^T - tb_t^{3M} = \alpha + \beta\,(tb_t^{3M+N} - tb_t^{3M}) + \gamma(\{\frac{N}{3M+N}\}\,(tb_t^{3M+N} - r_t)) + \varepsilon_t$$

$$\text{where } tb_t^{3M+N} = \{\frac{1}{3M+N}\}\,(3M \cdot (tb_T^{3M}) + N \cdot r_t)$$

Sample period: 1989 to June 1992

	3-month				6-month			
	Fri.	Mon.	Tue.	Wed.	Fri.	Mon.	Tue.	Wed.
$\alpha x10^3$:	-0.06	-0.01	-0.02	-0.01	-0.19	-0.24	-0.03	-0.04
	(0.70)	(0.15)	(0.34)	(2.12)	(3.29)	(3.74)	(6.26)	(6.96)
β:	0.09	0.10	0.16	0.27	0.01	0.01	0.05	0.14
	(2.33)	(2.72)	(3.85)	(5.54)	(0.59)	(0.89)	(2.12)	(3.63)
γ:	1.31	2.43	3.29	5.32	1.65	3.19	3.96	6.74
	(13.52)	(11.37)	(14.32)	(12.02)	(14.98)	(12.64)	(15.64)	(12.39)
$\overline{R}^2$:	0.68	0.71	0.76	0.80	0.64	0.64	0.64	0.63
$\chi^2(3)$:	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
GNR:	0.00	0.46	0.01	0.10	0.21	0.26	0.01	0.05
Q(5):	0.00	0.55	0.00	0.01	0.10	0.06	0.00	0.00
$Q^2(5)$:	1.00	0.65	0.43	1.00	0.82	0.02	0.02	0.98

Notes: See notes at the bottom of Table 3a.

Figure 7
Some daily, continuously compounded Tuesday rates

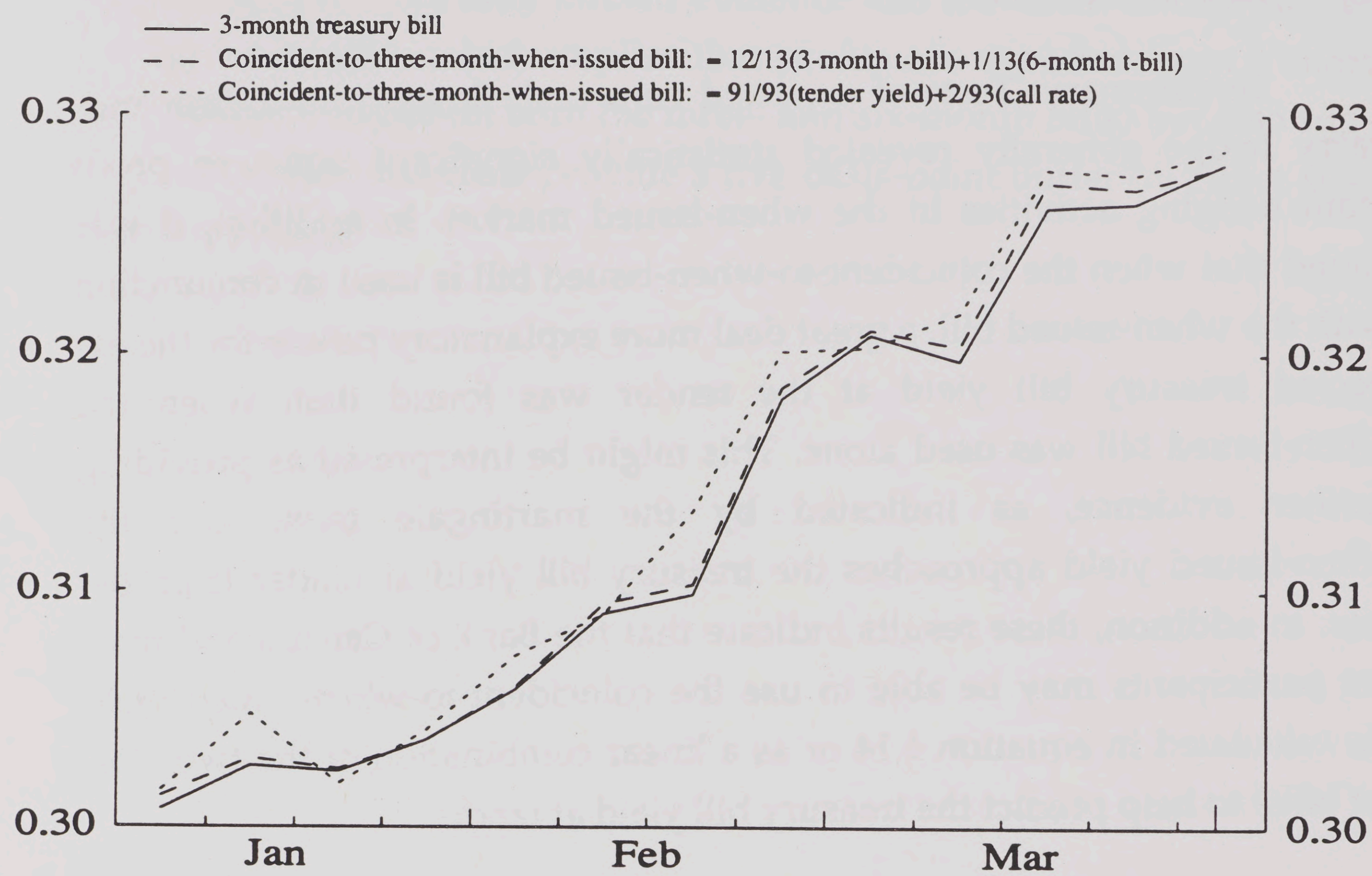
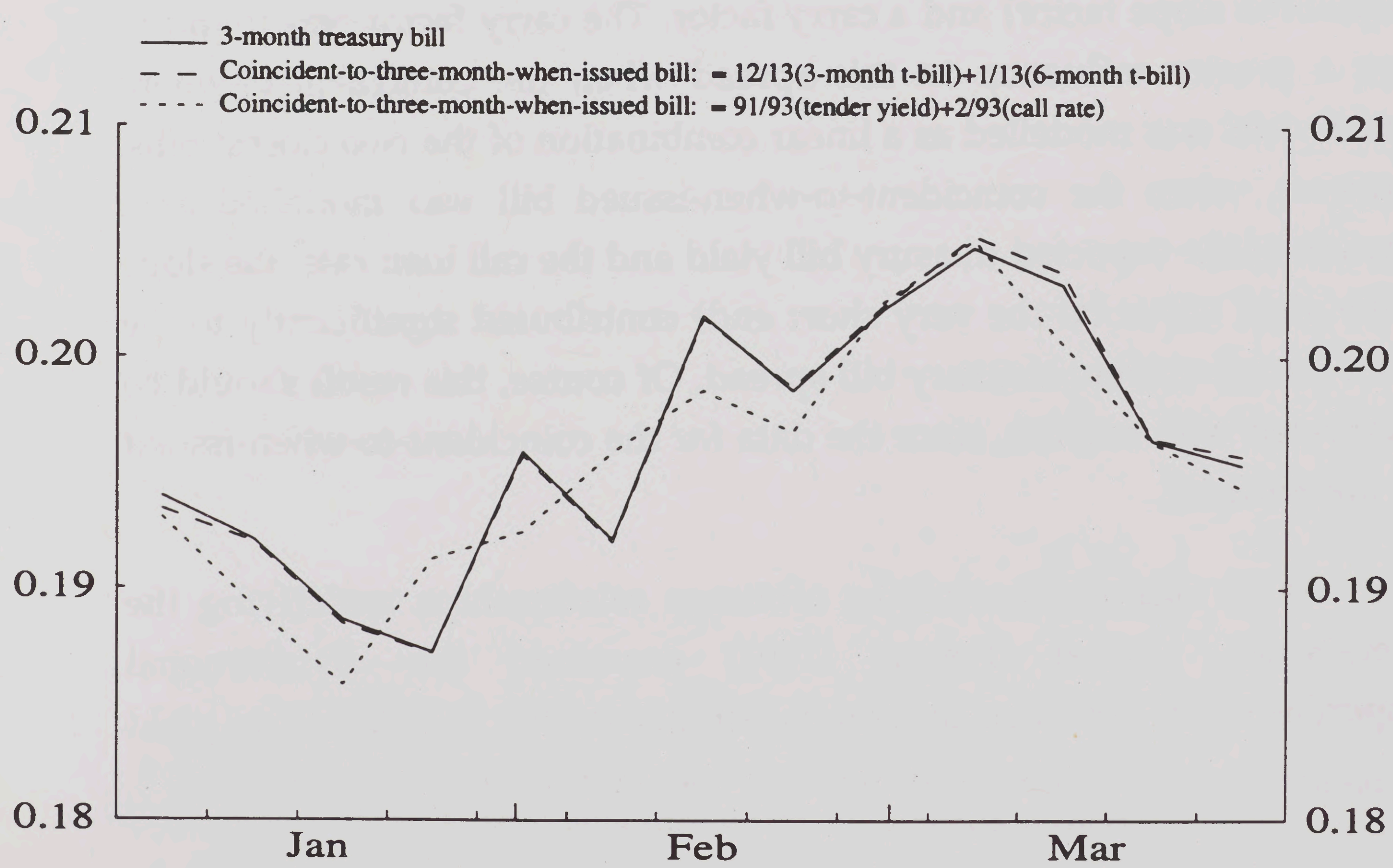


Figure 8
Some daily, continuously compounded Tuesday rates



statistics, autocorrelation is present in all but about one-quarter of the regressions – inferences are not affected by this, as the standard errors and the χ^2 tests are robust.

In summary: Tests of expected zero cash flows from a cash-and-carry hedge generally revealed statistically significant non-zero profit from hedging activities in the when-issued market. In addition, it was found that when the coincident-to-when-issued bill is used in conjunction with the when-issued bill, a great deal more explanatory power for the expected treasury bill yield at the tender was found than when the when-issued bill was used alone. This might be interpreted as providing further evidence, as indicated by the martingale tests, that the when-issued yield approaches the treasury bill yield at tender from below. In addition, these results indicate that the Bank of Canada and market participants may be able to use the coincident-to-when-issued yield (as calculated in equation 4.14 or as a linear combination of the two closest bills) to help predict the treasury bill yield at tender.

The theoretical outline showed that under the assumptions of perfect markets, risk neutral investors and continuous time trading, the when-issued–current treasury bill spread consists of a bill term factor (or yield-curve slope factor) and a carry factor. The carry factor was found to have a greater influence on this spread when the coincident-to-when-issued yield was modelled as a linear combination of the two closest bills. However, when the coincident-to-when-issued bill was modelled as a function of the expected treasury bill yield and the call loan rate, the slope of the yield curve (at the very short end) contributed significantly to the when-issued–current treasury bill spread. Of course, this result should be interpreted with caution, since the data for the coincident-to-when-issued bill are artificial.

As an examination of the arbitrage relationships underlying the when-issued market, Poitras (1991) examined the distributional properties of an arbitrage differential similar to that in (5.5) but in yield form.[38] He neglected the opportunity cost of tying funds up in the coincident-to-when-issued bill and assumed that all cash flows occur at portfolio initiation. He finds only limited evidence that the when-issued market is under-arbitraged, which implies that profits are expected from a short cash-and-carry hedge (in both the three- and six-month bills) but also suggests that transaction costs provide a five-basis-point buffer on either side of the cash flows.

38. Poitras uses the secondary market "cash t-bill rate" as the yield on the coincident-to-when-issued bill. Hence, he does not explicitly test the arbitrage relation.

7 CONCLUSION

This paper set forth and tested efficient markets hypotheses in the one-week when-issued market for Government of Canada treasury bills. In an efficient forward and futures market the forward/futures price should contain all relevant information so as to predict the future spot price accurately and systematically as well as to prohibit arbitrage profits. As shown in the theoretical outline, a systematic bias in forward or futures prices does not necessarily imply that the forward market is inefficient in the sense that systematic and unexploited profit opportunities exist. The risk premium, as this bias is typically called, may arise because of risk aversion, because cash flows are not known at the time the portfolio decision is made or because forward and futures contracts are net zero in supply. It may represent a "normal" market evaluation of risk compensation necessary to complete transactions.

In general, the forward market tests (namely, the unbiased expectations hypothesis and orthogonality restrictions) show that the three-month-when-issued rate is an unbiased predictor of the upcoming three-month tender rate, and that for the most part, except on the day before the auction, the six-month-when-issued rate is as well. These tests reveal that the twelve-month-when-issued bill is an inefficient predictor of its corresponding treasury bill yield – likely because of thin trading markets.[39]

The martingale hypothesis for when-issued yield changes was examined as a test of weak-form futures market efficiency. The basic univariate models, both OLS and GARCH(1,1), revealed only marginal evidence that a constant term and the own-lagged yield change could be used to predict current yield changes. However, the augmented univariate models showed that there was some evidence for rejection of the martingale hypothesis in the three- and six-month when-issued bills. This

39. On 1 June 1993 the government began a new two-week issuing cycle for one-year treasury bills, whereby each initial 52-week offering is reopened at the following week's regular treasury bill auction. This change was directed at improving the liquidity and efficiency of the one-year treasury bill market.

result suggests that the when-issued yield approaches the treasury bill yield at tender from below, that is, buyers are willing to give sellers a premium in order to be guaranteed an inventory of treasury bills at a prespecified yield. The bivariate GARCH model, which allowed the conditional covariance between the three- and six-month-when-issued yield changes to be time-varying and allowed a "cross" when-issued yield effect in each of the mean equations, generally confirmed this result. This model also showed that the conditional covariance between the three- and six-month yield changes was substantially time-varying, but always positive, and that the conditional volatility of six-month-when-issued yield changes was greater than that of the three-month bill.

The apparent contradiction of the forward and futures tests as well as the OLS technique versus the GARCH model can be attributed to the relative power of the tests and method. The forward market tests make use of data sampled only at weekly intervals and may therefore omit valuable information pertaining to daily when-issued yields. However, the information in daily when-issued yield changes is captured in the futures market tests, which use less restrictive modelling methods than the other tests.

Tests of zero expected profits in a short cash-and-carry hedge contained tentative evidence of profitable hedging opportunities, based on proxies rather than actual yields for the coincident-to-when-issued bill. These tests also showed that the expected treasury bill yield could be better predicted when the coincident-to-when-issued bill was combined with the when-issued bill rather than with the when-issued bill alone.

Finally, it was shown that the when-issued–current treasury bill spread could be decomposed into a yield-curve slope term (the yield on the coincident-to-when-issued bill over the yield on the treasury bill with the term to maturity closest to the coincident-to-when-issued bill) and a net carry term. With the assumption of a linear yield curve (at the short end), the net carry factor was the main contributor to the when-issued–treasury bill spread. Modelling the yield on the coincident-to-when-issued bill as the expected treasury bill yield at tender plus the cost of carry showed that both factors help explain the when-issued–treasury bill spread.

While the results of this report may suggest that the when-issued yield is a biased predictor of the upcoming treasury bill tender yield, it is unlikely that market participants can make use of other information in updating their expectations. Furthermore, transaction costs would diminish the economic value of any potential arbitrage gains.

REFERENCES

Backus, K., A. W. Gregory and C. I. Telmer. 1992. "Accounting for Forward Rates in Markets for Foreign Currency." Discussion paper, Department of Economics, Queen's University, Kingston, Ontario.

Bilson, J. F. O. 1981. "The 'Speculative Efficiency' Hypothesis." *Journal of Business* 54:435-51.

Black, F. 1976. "The Pricing of Commodity Contracts." *Journal of Financial Economics* 3:167-79.

Bollerslev, T. 1986. "Generalized Autoregressive Conditional Heteroskedasticity." *Journal of Econometrics* 31: 307-27.

Bollerslev, T., R. Y. Chou and K. F. Kroner. 1992. "ARCH Modelling in Finance: A Review of the Theory and Empirical Evidence." *Journal of Econometrics* 52:5-59.

Booth, P. and D. Longworth. 1986. "Foreign Exchange Market Efficiency Tests: Implications of Recent Empirical Findings." *Journal of International Money and Finance* 5:135-52.

Cox, J. C., J. E. Ingersoll and S. A. Ross. 1981. "The Relation Between Forward Prices and Futures Prices." *Journal of Financial Economics* 9:321-46.

Domowitz, I. and C. S. Hakkio. 1985. "Conditional Variance and the Risk Premium in the Foreign Exchange Market." *Journal of International Economics* 19:47-66.

Duffie, D. 1986. "Stochastic Equilibria: Existence, Spanning Number and the 'No Expected Financial Gains from Trade' Hypothesis." *Econometrica* 54:1161-83.

Dunn, K. B. and K. J. Singleton. 1986. "Modelling the Term Structure of Interest Rates under Non-separable Utility and Durability of Goods." *Journal of Financial Economics* 17:27-55.

Engle, R. F. 1982. "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation." *Econometrica* 50:987-1007.

Engle, R. F., D. M. Lillian and R. P. Robins. 1987. Estimating Time-Varying Risk Premia in the Term Structure: The ARCH-M Model, *Econometrica* 55:391-407.

Fama, E. F. 1970. "Efficient Capital Markets: A Review of Theory and Empirical Work." *Journal of Finance* 25:383-417.

Fama, E. F. 1984. "Forward and Spot Exchange Rates." *Journal of Monetary Economics* 14:319-38.

Gendreau, B. C. 1985. "Carrying Costs and Treasury Bill Futures." *Journal of Portfolio Management* 12:58-64.

Gregory, A. W. and T. H. McCurdy. 1984. "Testing the Unbiasedness Hypothesis in the Forward Foreign Exchange Market: A Specification Analysis." *Journal of International Money and Finance* 3:357-68.

Gregory, A. W. and T. H. McCurdy. 1986. "The Unbiasedness Hypothesis in the Forward Foreign Exchange Market: A Specification Analysis with Application to France, Italy, Japan, the United Kingdom and West Germany." *European Economic Review* 30:365-81.

Hansen, L. P. and R. J. Hodrick. 1983. "Risk Averse Speculation in the Forward Foreign Exchange Market: An Econometric Analysis of Linear Models." In *Exchange Rates and International Macroeconomics*, edited by J. A. Frenkel, 113-52. Chicago: University of Chicago Press.

Hansen, L. P. and S. F. Richard. 1987. "The Role of Conditioning Information in Deducing Testable Restrictions Implied by Dynamic Asset Pricing Models." *Econometrica* 55:587-613.

Harrison, J. M. and D. M. Kreps. 1979. "Martingales and Arbitrage in Multiperiod Securities Markets." *Journal of Economic Theory* 20:381-408.

Hicks, Sir J. 1946. *Value and Capital: An Inquiry into Some Fundamental Principles of Economic Theory*. 2nd. ed. Oxford: Clarendon Press.

Hull, J. 1989. *Options, Futures and Other Derivative Securities*. Englewood Cliffs, NJ: Prentice-Hall.

Jacobs, R. L. and R. A. Jones. 1980. "The Treasury Bill Futures Market." *Journal of Political Economy* 88:699-721.

Jones, F. J. 1981. "The Integration of the Cash and Futures Markets for Treasury Securities." *The Journal of Futures Markets* 1:33-57.

Keynes, J. M. 1930. *A Treatise on Money*. Vol. 2. New York: Harcourt, Brace.

Koenker, R. 1982. "Robust Methods in Econometrics." *Econometric Reviews* 1:213-55.

Lucas, R. E. 1982. "Interest Rates and Currency Prices in a Two-Country World." *Journal of Monetary Economics* 10:335-59.

McCurdy, T. H. and I. G. Morgan. 1987. "Tests of the Martingale Hypothesis for Foreign Currency Futures with Time-Varying Volatility." *International Journal of Forecasting* 3:131-48.

McCurdy, T. H. and I. G. Morgan. 1988. "Testing the Martingale Hypothesis in Deutsche Mark Futures with Models Specifying the Form of Heteroscedasticity." *Journal of Applied Econometrics* 3:187-202.

McCurdy, T. H. and I. G. Morgan. 1992. "Evidence of Risk Premiums in Foreign Currency Futures Markets." *Review of Financial Studies* 5:65-83.

Newey, W. K. 1985. "Maximum Likelihood Specification Testing and Conditional Moment Tests." *Econometrica* 53:1047-70.

Poitras, G. 1991. "The When-issued Market for Government of Canada Treasury Bills." *Canadian Journal of Economics* 24:604-23.

Pugh, D. G. 1991. "An Empirical Investigation of Risk Premia in Alternative Futures Markets' Positions: The Case of COMEX Gold and Silver, 1984 to 1988." PhD thesis, Department of Economics, Queen's University, Kingston, Ontario.

Pugh, D. G. 1992. "The When-issued Market for Government of Canada Treasury Bills: A Technical Note." *Bank of Canada Review* (November): 3-22.

Richard, S. F. and M. Sundaresan. 1981. "A Continuous-Time Equilibrium Model of Forward Prices and Futures Prices in a Multigood Economy." *Journal of Financial Economics* 9:347-72.

Roberts, H. V. 1959. "Stock Market 'Patterns' and Financial Analysis: Methodological Suggestions." *Journal of Finance* 14:1-10.

Samuelson, P. A. 1965. "Proof That Properly Anticipated Prices Fluctuate Randomly." *Industrial Management Review* 6:41-49.

Tauchen, G. 1985. "Diagnostic Testing and Evaluation of Maximum Likelihood Models." *Journal of Econometrics* 30:415-43.

White, H. 1982. "Maximum Likelihood Estimation of Misspecified Models." *Econometrica* 50:1-25.