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# *Government Debt in an Open Economy*

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## ABSTRACT

This paper introduces the CORE model, a prototype for a new quarterly model of the Canadian economy, designed for projections and policy analysis with focus beyond the very short run. The model has a clearly defined equilibrium and explicit adjustment mechanisms, primarily through relative prices, that are dynamically stable. Overlaid on a neo-classical growth model are shorter-term dynamics, roughly calibrated to reflect the Canadian data. Careful distinction is drawn between dynamics that arise from adjustment costs and other rigidities in the economy and dynamics that arise from the perceptions and expectations of economic agents. The model's properties are illustrated through simulations of the effects of two fiscal policy changes: an increase in the level of government spending, relative to total spending, with the debt-to-income ratio held fixed such that the extra spending must eventually be financed by taxation; and an increase in the debt-to-income ratio with the spending ratio held fixed such that there is a decline in taxation in the short term. The focus of these experiments is the path taken by private agents in their consumption and debt decisions and the macro adjustments that take place in national net foreign debt and the exchange rate.

# RÉSUMÉ

Voici le modèle CORE, le prototype d'un nouveau modèle trimestriel de l'économie canadienne, conçu à des fins de projection et d'analyse des politiques au-delà du très court terme. Il est doté de propriétés d'équilibre bien définies ainsi que de mécanismes d'ajustement explicites qui opèrent principalement par les prix relatifs et qui permettent d'atteindre une structure dynamique stable. Il s'agit d'un modèle néo-classique de croissance auquel se superpose une dynamique de court terme reflétant les caractéristiques principales des variables canadiennes. Une distinction y est soigneusement établie entre la dynamique liée aux coûts d'ajustement et aux autres rigidités de l'économie et celle qui découle des perceptions et des attentes des agents économiques. Pour illustrer les propriétés du modèle, deux modifications de la politique budgétaire sont simulées : une augmentation du niveau des dépenses publiques par rapport à la dépense globale, le ratio Dette/Revenu étant maintenu constant de sorte que les dépenses supplémentaires doivent tôt ou tard être financées au moyen de l'impôt; et une hausse du ratio Dette/Revenu, le rapport entre les dépenses publiques et la dépense globale demeurant fixe, ce qui donne lieu à une diminution des impôts dans le court terme. Ces simulations visent à mettre en évidence le comportement des agents économiques privés en matière de consommation et d'endettement et les ajustements macroéconomiques ayant trait à la dette extérieure nette et au taux de change.

# 1. INTRODUCTION

The experiences of the 1980s have provided fertile ground for macroeconomic research. Exchange rate volatility, productivity shocks, inflation, recession and disinflation have all wrought their effects on the economy. No economic development, however, has attracted more attention than the buildup of public debt in many countries. Notwithstanding this attention, much of the discussion has lacked the formal structure necessary to analyze the related policy issues. This paper provides an analysis of the impact of government debt and, in doing so, presents the properties of a prototypical model called CORE.[1] The CORE model is a relatively small macroeconomic model designed to assess the impact of policy decisions both in the short-run and beyond.

Since the 1960s, the workhorse of open economy models has been the IS-LM-BP model. It has proved very useful for examining the impact effects of small disturbances around a fixed steady state. Some strong assumptions, however, have limited its broader applicability. In particular, stock-flow relationships and intertemporal aspects of behaviour are ignored. As a consequence, the model's dynamics are very *ad hoc*, and these restrictions have undermined the credibility of its medium-term predictions. The model described in this paper reflects a great deal of attention in just these two areas through borrowing from dynamic optimization theory to establish meaningful steady-state conditions. Although blessed with a richness of theoretical rigour, dynamic optimization models often pay the price of having empirically implausible dynamics for a policy simulation model. Fortunately, the salient points of optimization models can be fused onto models that include the features that practitioners find necessary. This compromise is accomplished at a relatively low cost by building up a steady-state version of the complete model and then overlaying "Keynesian" dynamics. Along the way, we include two complications that are of growing empirical interest, but which do not detract from the central ideas contained in the model's

1. The CORE model is a prototype of PAQM (policy analysis quarterly model), which is being developed at the Bank of Canada. The CORE model contains all the essential structure of its larger offspring but with substantially less disaggregation. Some of the methodological seeds of these models can be found in Masson *et al.* (1980).

basic structure. The first of these is the *almost-small-open-economy* assumption. With the spread of multinational corporations, product differentiation and scale-augmenting technical change, the optimal industry configuration calls more and more for "world scale" firms producing specialized, differentiated products in various locations around the world. Thus, it is possible for even relatively small countries to have some influence on their export price.[2] The second is the *country-risk assumption*. Agency theory tells us that shareholders will appropriate value from bondholders by, among other things, selecting overly risky projects. Several direct analogies exist for nations whose claimants include governments, domestic residents and foreign investors. Governments can choose "inefficient" policies that transfer rents from international creditors to shareholder-voters. In recognition of this phenomenon, the willingness-to-pay of foreigners for domestic bonds may be written as a function of a country's relative international indebtedness.[3] Either or both of these features of the model can be turned off, but both provide interesting insights into such issues as the sustainability of fiscal regimes.

The second section of this paper begins with a description of the theory behind the model's steady state. The emphasis is on the consumption function and its role in linking consumption *flows* with asset *stocks*. There are important implications derivable from the imposition of a consistent accounting framework and the existence of a steady state. Section 3 begins the process of overlaying the dynamic structure of the model. Care is taken to draw a distinction between dynamics that are driven by intrinsic features of the economy -- such as costs of adjustment and other physical limitations -- and expectational dynamics. Throughout the paper, much of the detail of the model is eschewed in favour of more general characteristics and those features that are important for understanding the simulations that follow. The fourth section illustrates the model's usefulness as a tool for policy analysis by considering the short- and long-run implications of fiscal policy. Spe-

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2. This assumption has been adopted for a variety of different models. See, for example, Turnovsky (1977) chapter 10. Empirical evidence casting doubt upon the smallness assumption for Canada can be found in Applebaum and Kohli (1979) and Schembri (1989).

3. For arguments along these lines see Aizenman (1989) and Alesina and Tabellini (1989) and references therein.

cifically, two shocks are considered: a government expenditure shock with a maintained target ratio of government debt to income and a shock to the debt-to-income ratio holding government expenditures constant. Most other fiscal shocks are combinations of these two. Finally, some concluding thoughts are provided in the fifth section.

## 2. THE ANALYTICS OF THE STEADY STATE

The CORE model is a dynamic simulation model with well-identified equilibrium properties. Overlaid on a neo-classical supply side are Keynesian dynamics calibrated to reflect the stylized facts for Canada. A major objective in the model's construction is to identify the key relative prices that act as mechanisms to re-establish an equilibrium following a shock. This emphasis has made it convenient to ignore a number of complicating factors that would be more important for a larger model. Thus, the model uses mainly artificial data and omits some of the national accounts concepts. As well, it does not include inventories or housing and there is no distinction between durables and non-durables in consumption. Finally, the only tax allowed is a (non-distortionary) tax on income.

Understanding a macroeconomic model requires comprehension of its equilibrium properties and the mechanisms by which those properties are established following a shock. Even in those cases where one is primarily interested in a model's short-run predictions -- and therefore preoccupied by disequilibrium phenomena -- the unifying concept must be one of equilibrium, with relative prices determined to support that equilibrium. In referring to a model's equilibrium properties, it is the steady-state solution that one generally has in mind. It is important to note that the steady state of a model is not something written in stone. As with model dynamics, the steady state, too, will shift under the influence of shocks such as those to tastes, technology and exogenous foreign variables. But a full understanding of a model requires a distinction between disturbances that are fundamentally permanent in their effects and those that are temporary. To do otherwise is to abandon the paths of variables to predictions based merely on statistical correlation instead of models and the theories they embody.

We must emphasize that "steady state" should not be thought of as something relevant only to the far-distant future. We are not building a model to handle issues with very long dynamics, where things like demographics and multi-generation issues become paramount. Nor are we preparing to deal directly with issues like resource exhaustion or environmental change. We are dealing with standard macroeconomic dynamics, but with careful attention to the stocks of producing capital and of financial assets. The notion of steady state that applies has, in our view, much relevance in the discussion of macroeconomic adjustment over the horizons normally considered in policy analysis.

To outline the significance of these points, we begin the next subsection with a brief digression defining the concept of steady-state equilibrium and what restrictions its existence imposes on model structure. Of particular importance in this regard is the model's consumption function, which pins down the steady-state trade balance through the latter's relationship to the country's net foreign asset position. We show that it is consumer preferences, and not some *ad hoc* restrictions, that determine how non-Ricardian the model is.[4] With this background established, the steady-state version of the model is specified. We then describe its dynamic characteristics.

## 2.1 On Steady-State Equilibrium[5]

Policy analysis frequently involves interventions that have permanent effects on the long-run solution of a model. As we shall see, this is particularly true of fiscal policy interventions. Since changes in the long-run solution of a model have implications for what must hold over shorter horizons, an understanding of a policy model's steady state is often a prerequisite for coherent policy analysis. Our goal is to put some structure on the familiar income-expenditure identity in order to provide restrictions on model dynamics that have economic content.

$$Y_t^{SS} \equiv C_t^{SS} + I_t^{SS} + G_t^{SS} + X_t^{SS} - M_t^{SS} \tag{1}$$

The mnemonics of (1) are straightforward, except to note that an SS superscript indicates a steady-state value for the variable in question. In the con-

4. One could simply assume that the interest rate faced by consumers is higher than that faced by the government. This is factually true and would be sufficient to eliminate Ricardian equivalence. We chose not to do this in order to focus on the general underpinnings of Ricardianism and the *theoretical* reasons why it may not hold. In doing so we are able to identify sources of comparative statics coming from tastes and technology rather than from institutional sources. Finally, as King (1986) shows, modelling asymmetries in interest rates faced by different agents is quite difficult.

5. The discussion in this subsection centres around real variables. Price index number and currency denomination issues are not highlighted because they interfere with the central point of the exposition. At the level of abstraction of this subsection, it is sufficient to note that steady-state equilibrium implies many of the same things for nominal variables as for real variables: all price levels grow at the same rate (except for the nominal exchange rate) and aggregate inflation is super neutral.

struction of the CORE model we have assumed the existence of a neo-classical steady state. This assumption requires that (i) flow variables grow at a common rate so that their ratios are constant; (ii) the ratios of stock variables to income or output be constant; and (iii) rates of return be constant. From here we pin down, one by one, the values of the variables in equation (1), using our definition of the steady state and a minimal number of assumptions.

We take the left-hand side of equation (1) as being given by aggregate supply considerations, that is, by a production function, while the right-hand side is aggregate demand. This characterization is not perfect; it is not difficult to think of "supply" shocks that have important demand-side implications and vice versa. Nevertheless, the standard output-expenditure identity provides a useful framework for the discussion.

***Aggregate supply***

Let us begin with aggregate supply,[6] the determination of $Y_t^{SS}$. We have assumed that the economy's production technology can be represented by a simple Cobb-Douglas production function:[7]

$$Y_t^{SS} = \Theta_t K_t^{\alpha} L_t^{(1-\alpha)} \qquad (2)$$

where $K_t$ and $L_t$ are capital and labour, $\alpha$ is the elasticity of output with respect to capital, and $\Theta_t$ is the level of total factor productivity (TFP), which is taken as exogenous. The summation restriction on the output elasticities imposes constant returns to scale in production. Adding the assumption of free entry is sufficient to establish linear homogeneity and thus the equality of production income with factor payments in the steady state.

6. For the purposes of this section we assume a pure, small open economy with domestically produced capital. The impact of relaxing some of these restrictions is taken up later in the paper.

7. The assumption of the existence of a steady state together with Harrod-neutral technical change are those of the neo-classical growth model popularized by Solow (1957). The popularity of the Cobb-Douglas specification in growth models stems from the fact that technical change is both Hicks and Harrod neutral under the Cobb-Douglas specification and only under this specification. The broad implications of the neo-classical steady state appear to hold with some qualifications. One of these qualifications is that one must assume that the same proportion of income accrues to labour in the farm and unincorporated businesses sectors as in the rest of the economy. A second is that one must allow for an exogenous decrease in the price of capital investment due to the worldwide decrease in computer prices from 1975 to about 1985.

Pinning down aggregate supply requires us, therefore, to pin down factor inputs. We begin by assuming that long-run labour supply is independent of the real wage and wealth. The steady state requires that this labour supply be fully employed. Differentiating the firm's profit function evaluated at steady state, we obtain:

$$\frac{w_t^{SS}}{cc_t^{SS}} = \frac{(1-\alpha)}{\alpha}\left(\frac{L_t^{SS}}{K_t^{SS}}\right) \tag{3}$$

or $$K_t^{SS} = \bar{L}_t^{SS}\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{w_t^{SS}}{cc_t^{SS}}\right) = \Phi\left(cc_t^{SS}, \bar{L}_t^{SS}, \Theta_t\right) \tag{4}$$

where $w_t^{SS}$ is the steady-state real wage and $cc_t^{SS}$ is the steady-state cost of capital. Equation (3) gives the familiar correspondence between the capital-labour ratio and the wage-rental ratio, while (4) restates this relationship using the exogeneity of labour -- denoted by the overstrike -- to state that given the cost of capital and the level of productivity, the steady-state capital stock is determined by the available supply of labour. This relationship uses the fact that the steady-state real wage can also be written in terms of these variables.[8] The cost of capital is written as:

$$cc^{SS} = \frac{P_K^{SS}}{P^{SS}}\left(r^{SS} + \psi + \delta\right) \tag{5}$$

where $r$ is the required real return on financial capital, $\psi$ is a risk premium,[9] $\delta$ is the geometric depreciation rate of capital, and $(P_K/P)$ is the relative price of capital goods. In the case of a country that is a price taker in world financial markets, $r$ is exogenous; the assumption of a steady state fixes $(P_K/P)$ and $\psi$ as constants; thus the steady-state cost of capital is fixed anytime the steady-state world real interest rate is fixed. Taking this as so, $K_t^{SS}$ is pinned down, as is the real wage. With both capital and labour determined, output and income (the left-hand side of equation (1)) are pinned down by the production function. Linear homogeneity implies that in

8. The model here is a special case of the one analyzed in Rose and Selody (1985). The reader is referred to Chapter 4, section 4.4 of that study for the proofs of these assertions.

9. Fitting the stylized facts for the capital-output ratio requires that a steady-state risk premium, $\psi$, be incorporated as part of the cost of capital. Otherwise the implied capital-output ratio is much too large. This risk premium is a well-known feature of the data; Mehra and Prescott (1985) refer to it as the equity premium. Also note that although there is no allowance for taxation in (5) there is no loss of generality to the argument of this section from this omission.

steady state there are zero excess profits; both factors are rewarded according to their marginal products.

We have calibrated the model to the stylized facts for Canada, imposing exogenous growth in both total factor productivity and the labour force over time. Given relative prices, these in turn elicit steady-state growth in output, the capital stock and in all other real variables of the model.

***Aggregate demand***

We turn now to the expenditure side of the income-expenditure identity. We specify the law of motion for capital in the usual way:

$$K_t^{SS} = I_t^{SS} + (1 - \delta) K_{t-1}^{SS} \tag{6}$$

which implies that, given a constant rate of depreciation, $\delta$, once $K_t^{SS}$ is pinned down so is $I_t^{SS}$. Steady-state investment is simply that level of investment necessary to replace depreciating capital and to allow for growth of the net stock at the same rate that output grows.

Government spending, taxation and deficits are within the realm of policy variables. Therefore, only the minimum restrictions required to maintain stability are imposed on the model's government sector. It is the essential choice of government to decide exactly what ratio of government consumption to income is to be maintained in the steady state, but a constant finite ratio *is* required for a steady state to exist. Similarly, a constant steady-state ratio of government debt to income is also required although, once again, we permit broad discretion as to the exact ratio. Finally, these two choices imply, through the government budget constraint, a steady-state tax rate. We therefore require that the steady-state tax rate solve the government budget constraint to allow the policy-determined ratios of government spending and government debt to income to co-exist. That such a tax rate is feasible is our only constraint on the range of fiscal policy choices.[10]

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10. Nothing in our discussion of the restrictions on fiscal policy in the steady state should be regarded as constraining in any significant way the short-term dynamics of spending, debt and taxes. There remains a great deal of latitude in specifying the short-term behaviour of government to highlight, for example, the role of fiscal policy as an automatic stabilizer. What is required is that, at some point, government deficits be financed and that the implicit fiscal plan include this requirement.

Our assumption regarding the social role of government expenditures is a minimalist one: the government absorbs resources from the private sector to produce a perishable consumption good that is then transferred to the consumer. Arguments concerning the effects of government expenditures, positive and negative, on both private consumption and investment have been made in the past. Given the multiplicity of government objectives, a middle-ground path between the two seems prudent. Accordingly, we assume the government consumption good to be separable in the private sector's utility function.[11]

We are left with the determination of steady-state consumption and the trade balance. By definition, the steady-state trade balance reflects a gap between the output produced and that which is absorbed domestically; the difference is shipped abroad. These are not proportions that can be chosen arbitrarily; the economy's stock-flow identities impose restrictions on their values. In particular, whether a country can absorb more than it produces (or vice versa) depends on whether it is a net creditor (net debtor) with respect to the rest of the world. If the home country is a net debtor, payments necessary to maintain the steady-state ratio of net foreign assets (or liabilities) to income, $(NFA^{SS}/Y^{SS})$, must be made and this can come only at the expense of domestic absorption.[12] This shortfall of domestic absorption to income *is* the steady-state trade surplus, and its existence depends on no more than the existence of a steady state and the rules of accounting. It is this fundamental stock-flow relation that connects the trade balance to the consumption function as we shall now demonstrate.

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11. It is undoubtedly true that some of government expenditure involves creating infrastructure that increases the productivity of private capital and labour. Other expenditures sacrifice efficiency in the name of redistribution of income. Since the goals of government are various and often contradictory, we cannot carry over the notion that the government will invest in infrastructure until all net marginal benefits are exhausted. Also note that separability of the government's good in our representative agent's utility function allows us to omit it from the formal consumption theory.

12. This casts quite a different perspective on trade deficits from that of the layman. An uninitiated reader would assume -- because surpluses are commonly regarded as good things -- that thriftiness in the form of a positive net foreign asset position would be "rewarded" by a trade surplus. Just the opposite is the case. The simple accounting argument in the text, which is elaborated below, shows that the price of steady-state foreign indebtedness is, loosely speaking, that the home country must sell its exportable goods cheaply in order to finance its debts. This implication of foreign indebtedness is a more familiar phenomenon to residents of heavily-indebted third world countries but holds axiomatically for all countries.

Consolidating the household and business sectors, we define *financial* wealth of the private sector, $FW$, as follows:

$$FW_t^{SS} = B_t^{SS} + MN_t^{SS} + K_t^{SS} + NFA_t^{SS} \tag{7}$$

$MN_t$ is the stock of high-powered money, a fixed proportion of income (in a steady state) by assumption.[13] We have already fixed government debt, $B_t$, as a policy variable. Bonds are assumed to be issued in domestic currency units. The capital stock is pinned down, leaving $NFA_t$, the stock of net foreign assets, as the only undetermined category in financial wealth. The appearance of government bonds in the definition of financial wealth does not in any way prejudge the degree of Ricardian behaviour in the model. What counts is whether government bonds are considered *net* wealth; expected future taxes have yet to be considered. Denoting the real interest rate by $r_t$ and net tax payments by $\tau_t$, we write the private-sector budget constraint as:

$$Y_t^{SS} + r_t^{SS} NFA_{t-1}^{SS} + r_t^{SS} B_{t-1}^{SS} = C_t^{SS} + \tau_t^{SS} + S_t^{SS} \tag{8}$$

where: $S_t^{SS} = \Delta B_t^{SS} + \Delta MN_t^{SS} + \Delta NFA_t^{SS} - \delta K_{t-1}^{SS} + I_t^{SS} = \Delta FW_t^{SS}$. Steady-state savings is that level of foregone consumption necessary to ensure that the steady-state capital stock (net of depreciation) is financed and that all other asset stocks maintain their required ratios.

We also have the government's budget constraint:

$$G_t^{SS} + r_t^{SS} B_{t-1}^{SS} = \tau_t^{SS} + \Delta MN_t^{SS} + \Delta B_t^{SS}. \tag{9}$$

Letting $g$ represent the growth rate of all real, non-stationary variables in

13. There are differences among macro models with consistent government accounts with regard to whether the monetary authority can pursue an independent policy or whether it is compelled to finance part of the government deficit. Rose and Selody (1985), pp. 53-57, allow seigniorage to be created by an independent central bank, while Turnovsky (1977) and Scarth (1988) among others consider money-financing rules in conjunction with the issue of the sustainability of fiscal policy. Steady-state equilibrium requires that real money balances growth at the same rate as does the economy. As discussed below, we allow the monetary authority to directly manipulate the short-term interest rate as its policy instrument and so the technology of money markets is not a primary concern. However, linear homogeneity of money demand and exogenous velocity are sufficient conditions for a steady state with a constant inflation rate.

the steady state; that is: $Z_t^{SS} = (1+g)Z_{t-1}^{SS}$, equations (1) and (6)-(9) can be combined, with a small amount of manipulation, to form:

$$C_t^{SS} = Y_t^{SS} - I_t^{SS} - G_t^{SS} + (r_t^{SS} - g)NFA_{t-1}^{SS}. \tag{10}$$

By comparing equations (1) and (10) we see that the level of the steady-state trade balance is, as noted above, identified with the payments required to maintain the steady-state stock of net foreign assets, i.e., $X_t^{SS} - M_t^{SS} = NETX_t^{SS} = (r_r^{SS} - g)NFA_{t-1}^{SS}$. The ratio of net foreign assets to income must be constant in the steady state. Otherwise, the stock of wealth will drift over time towards positive or negative infinity. There is a link, given solely by accounting rules, between net foreign assets and the trade balance. Therefore, the steady-state trade balance is pinned down by the choice of the net foreign asset position. That being the case, the income-expenditure identity gives us steady-state consumption. A country can borrow to finance higher levels of domestic absorption today, but steady-state consumption is reduced by the resulting steady-state payments to foreigners.[14] It is clear therefore that, taking government as given, *steady-state consumption and NFA are jointly determined by the consumption-savings decisions of the private sector.* With savings being dual to consumption and every aspect of savings other than *NFA* being pinned down in the steady state, almost every intertemporal policy issue must be resolved through consumption decisions and their influence on net foreign assets.

We have shown that steady-state consumption can be thought of as the level of income left over after financing the steady-state growth of stock variables, including depreciation, and the operations of government. It serves our purposes to dub this income concept *consumable income*, $YC_t$. It should be understood that consumable income is related to, but different from, the more familiar notions of permanent income or disposable income. Permanent income is commonly defined as the expected present value of future per-

14. In accordance with the data which show Canada as having persistent trade surpluses and negative net foreign asset positions, we have assumed for the model that the domestic country is a net debtor in the steady state. This amounts to assuming that domestic residents have a higher rate of pure time preference than the rest of the world. In order to avoid confusion it should be understood for the main text that steady-state NFA is *negative*; calibration to the data for 1988 fixes a value of the ratio of nominal net foreign assets to nominal output of -0.35.

sonal income streams, while disposable income is usually thought of as after-tax income. With consumable income, we have in mind a general equilibrium concept that embraces the intertemporal nature of consumption-savings decisions. It shares with permanent income the idea that choosing consumption today is conditioned upon expected future income, and like disposable income, taxes matter. It differs from both in its attention to the requirements of equilibrium and the implication that a portion of personal income must be set aside to provide for asset growth.

The consumption function plays a particularly important role in the fiscal shocks discussed in this paper. When the fiscal authority decides to allow the ratio of government debt to income to rise, holding expenditures constant and thereby reducing taxes temporarily, consumer preferences determine both the proportion of public debt that is considered real wealth and how much of that debt will be held by domestic residents. The consumers' problem amounts to deciding the extent to which an increase in public debt will result in higher consumption today and a trade surplus tomorrow, or vice versa -- that is, how Ricardian is their behaviour.

If, in a closed economy, consumers are not Ricardian, the excess supply of bonds will result in lower bond prices (higher real interest rates), crowding out of investment, and hence a lower capital-output ratio. The analogue to this equilibrating mechanism in an open economy is a *higher* ratio of net foreign *liabilities* to income. The counterpart to the lower bond price in the closed economy is a lower steady-state real exchange rate necessary to generate a trade surplus just large enough to finance the higher payments on the foreign liabilities. The lower real exchange rate means that the home country sells its exportables for less than before, and that import and consumer prices rise. The open economy counterpart to long-run crowding out of investment is, therefore, crowding out of consumption.[15] We illustrate these issues below.

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15. It should be kept in mind that throughout this section we have assumed that all capital goods are domestically produced and that the economy is small and open in both financial and goods markets. Relaxing the former assumption to allow for imported capital introduces a channel whereby the real exchange rate affects the cost of capital. This means that a depreciation of the real exchange rate crowds out investment as well as consumption. The implications of relaxing these assumptions are described later in the paper.

***Consumption in the long run***

We now turn to our model of consumption and how it determines steady-state consumption and NFA simultaneously.[16] We have already identified the accounting link between the two, underscoring the fact that consumption today is foregone consumption in the future. Here, a theory is specified that identifies the relative price that consumers place on consumption today versus consumption in the future. This provides the structure necessary to pin down the ratio of net foreign assets to income, the trade balance and consumption, all as a function of expected future income, taxes, interest rates and the parameters of the representative agent's utility function.

We propose a discrete-time analogue to the Blanchard (1985) model. The representative agent is assumed to maximize utility over an infinite horizon subject to a lifetime budget constraint and a probability of death, $p$. This last feature of the model effectively reduces the model to a finite planning horizon and, in doing so, introduces elements of the overlapping generations model without the analytical difficulties with which traditional overlapping generations models are burdened. If the probability of death is 0.02 each year, the effective lifetime planning horizon is $(1+p)/p$ or 51 years.[17] Blanchard did not, however, provide a solution to the model for economies characterized by productivity and population growth or more general utility functions than the logarithmic form, assumptions that would be too restrictive to obtain realistic results for a model such as this one. Consequently, we have extended the results to relax these restrictions. The representative agent's problem is:

$$\underset{\langle c \rangle}{MAX}\ E_0 \sum_{t=0}^{\infty} \beta^t U(C_t) = E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} C_t^{1-\gamma} \qquad (11)$$

16. For present purposes our focus remains on the steady-state determinants of consumption. Our dynamic theory of consumption can be thought of as nested within the steady-state version; we consider the dynamics of consumption later in the paper.

17. Blanchard shows that if agents have access to life insurance markets then the effective rate that the consumer faces will equal the rate of interest on government bonds plus the probability of death. This assumption was maintained in the more recent variations on the Blanchard model presented by Weil (1989) and Buiter (1987, 1989). Refinements included the important observation that, contrary to the claim by Blanchard, the source of the breakdown in Ricardian equivalence in the model was not the possibility of death but rather the (implicit) assumption that those who die are replaced by new citizens, whose welfare current citizens do not value. This feature is attractive in that it formalizes the popular notion of public debt as a "burden to future generations."

subject to:

$$E_0 \sum_{t=0}^{\infty} \alpha^t C_t - E_0 \sum_{t=0}^{\infty} \alpha^t (Y_t - \tau_t - \Delta K_t) \leq FW_0\,, \tag{12}$$

where $$E_0 Z_t = (1+g) Z_{t-1} \qquad (Z_t \in Y, \tau, K)\,, \tag{13}$$

and where $\beta = 1/(1+\phi+p)$ is the agent's personal discount factor with $\phi$ being the subjective rate of time preference and $p$ being the constant, instantaneous probability of death; $\alpha = (1+g)/(1+r^{SS}+p)$ with $r$ representing the real interest rate applicable to consumers, set -- for convenience -- equal to the risk-free government rate. The utility function is taken to be of the constant relative-risk-aversion class, with $\gamma$ taken as the coefficient of relative risk aversion and its inverse as the elasticity of intertemporal substitution. As well, $FW_0$ represents the agent's initial endowment of financial wealth; $Y$ and $\tau$ are gross income and tax payments, respectively, just as before. The solution to this problem can be simplified to produce an Euler equation of the form:

$$C_{t+1}/C_t = (\alpha/\beta)^{1/\gamma} \tag{14}$$

which shows that the *consumption tilt* -- the time profile of consumption, given expected lifetime resources -- is determined by the real interest rate and such deep parameters as productivity growth, the probability of death and characteristics of the utility function. We solve equation (14) along with a terminal condition that disallows insolvency to arrive at an expression for consumption as a constant proportion of total wealth:

$$C_t = \kappa (FW_t + HW_t) \tag{15}$$

where $FW_t$ is, as before, financial wealth; $HW_t$ is human wealth defined, in aggregate, as the expected present value of future labour income net of taxes and provisions for capital.[18] Equation (15) along with (7) and a variant of (12) shows that, for example, an increase in government debt as described in the previous subsection affects financial wealth positively, because bonds are valued, and human wealth negatively, because expected future taxes are

18. Aggregation up from the representative agent to the aggregate economy does not change equation (15) but does change the calculation of human wealth. Human wealth for the individual agent is equivalent to the second term on the left-hand side of (12). Aggregate human wealth is the same except for the inclusion of an extra term in the denominator of $\alpha$ to account for population growth.

taken into consideration. The important difference is that a swap of bonds today for higher steady-state taxes over the whole course of the future increases total wealth from the perspective of the representative agent. This is so because new citizens from births or immigration are increasing the tax base, and because the possibility of death means the representative agent might not survive to pay off the debt. Thus, the agent discounts future taxes more heavily than the financial markets would. To put it another way, future taxes are not earmarked to be paid by those who benefited from the expenditures that the taxes finance. That being the case, there is a public-good nature to taxes while, in the present model, government expenditures are private goods. This is the reason why this economy does not exhibit Ricardian equivalence.

The average (and marginal) propensity to consume out of wealth is:

$$\kappa = \{ [ (\gamma - 1) / \gamma ] r + p + \phi / \gamma \} . \qquad (16)$$

The average propensity to consume out of wealth is determined by the coefficient of risk aversion, the real interest rate, the probability of death and the pure rate of discount. More importantly, equation (16) reveals $\kappa$ to be an important determinant of how much of an innovation in government debt, say, will be allowed by consumers to turn up in financial wealth (or $NFA^{SS}$) and how much will be consumed. If utility were logarithmic, so that $\gamma = 1$, then the average propensity to consume would differ from the pure rate of time preference, $\phi$, only by the probability of death. Thus, the same overdiscounting phenomenon described above influences the value of $\kappa$. The value of $\kappa$ is not, however, the only determinant of consumer response to an innovation. The exact proportion of the innovation that will be considered consumable income is determined in general equilibrium. For example, it will be influenced by what happens to relative prices.

The existence of a steady state imposes important restrictions on models and particularly on their dynamics. It is important to recognize, however, that the steady-state solution of the model is itself subject to disturbances. The steady-state consumption-income ratio is, for example, a function of the parameters in $\kappa$ plus the population growth rate and the rate of technical progress as well as exogenous foreign variables. Each steady-state solution is supported by a set of relative prices that establishes that particular solution as the correct one. Hence, equilibrium attains following a shock because relative prices move the economy towards the steady-state allocation of its resources.[19] As befits an open economy model, the key equilibrating price in the CORE model is, for most shocks, the real exchange rate. However, the mechanisms by which the exchange rate induces the components of the income-expenditure identity to settle down on steady-state values are various. Some of these mechanisms have been included as basic features of the model, while others come into play only as optional features at the user's discretion.

It is useful to identify five relative-price mechanisms in the model that act to support equilibria. These five are: (i) the direct effect of relative prices on imports and exports (*pure substitution effect*); (ii) the indirect effect of price changes on the real purchasing power of consumable income (*wealth effect*); (iii) the effect of relative prices on the cost of capital (*imported capital goods effect*); (iv) the *almost-small-open-economy assumption*; and (v) the *country-risk premium*. The first three points are included as features of the basic model and, in the context of the shocks considered in this paper, concern the effects of exchange rate changes on the supply and demand for goods. The fourth and fifth points refer to finite elasticities and non-linearities in the foreign country's offer curve, the first for offers in the goods market, the second for bond market offers. Both of these are optional features in the model.[20]

We shall illustrate these five steady-state price effects by way of example. Consider an increase in the steady-state ratio of government debt to income, holding government expenditures constant. The government budget constraint implies a higher level of steady-state taxation in order to sustain the

19. This contrasts sharply with the structure of many other models, such as most traditional Keynesian macroeconometric models, where the precise long-run solution of the model -- if one exists -- is an artifact of the dynamic structure, rather than given by theory. See Masson *et al.* (1980), pp. 7-16, for a discussion of this issue. Other models, based on the error-correction framework, include a mathematical requirement that flows differ from their long-run levels until "gaps" close. While models of this sort may appear to have a steady state, re-establishment of the steady state following a shock is often mechanical and not founded on the relative price movements that economists normally think of as fundamental to the adjustment.

20. This list of relative prices is not exhaustive. We could have included the relative price of government goods as affected by procurement policies as well as other steady-state shocks. The mechanics of the model are aptly demonstrated by the five points above.

higher debt load.[21] Assume initially that there is no country-risk effect, that no capital is imported and that the domestic country can sell any amount of its export good at the world price. (We shall relax these assumptions, one by one.) Table 2 summarizes the comparative statics of this shock under the assumption that the monetary authority targets nominal spending. Agents consider part of the debt to be wealth; this is the case not because of misperceptions but because agents understand that they will die, and others will be born, before all the debt needs to be repaid. (Misperceptions are introduced into the model in a later section of this paper.) This being the case, domestic consumers do not save enough income from the tax cut to finance the increase in public debt, leaving foreigners to purchase the remaining bonds at the world real interest rate. The higher steady-state level of net foreign *liabilities* requires a larger steady-state trade *surplus* in order to generate income to finance the debt. Such an increase can come about only through a relative price change: a depreciation of the steady-state exchange rate. The depreciation increases the price of imports and lowers the price of exports in domestic currency so that by the direct substitution effect, import consumption falls and exports increase.[22] This effect is buttressed by an income effect working through the feedback of import prices into consumption prices, thereby reducing the real purchasing power of consumable income, further affecting the consumption-savings decision. This is so because consumers choose the appropriate level of $NFA^{SS}$ (or, equivalently, $YC^{SS}$) based on deflation using the consumer price index.[23] Both of these channels work to crowd out domestic consumption (and crowd in exports) through the

21. This experiment can be executed by means of a temporary tax cut followed by a smaller, but permanent, increase in steady-state taxes. Since, in this section, we are interested solely in the comparative statics of this shock, the dynamics of the temporary tax cut are of no concern to us.

22. This is so, despite the small-open-economy assumption, because although the direct effect of demand shocks on export prices is zero -- just as in a pure flow model like the IS-LM-BP model -- stock equilibrium requires an exchange rate change, which produces an indirect (substitution) effect on prices measured in domestic currency units. This indirect effect, which is implied by the accounting and the requirements of steady state, is just one example of what is missed by a pure flow model.

23. An anchor of some sort is required to pin down the aggregate price level. Table 2 is based on the assumption that the monetary authorities target nominal spending. With this in mind, consider the nominal income-expenditure identity, evaluated under steady-state conditions: $P^{SS}Y^{ss} = \bar{k} = PC^{SS}C^{SS} + PK^{SS}I^{SS} + P^{SS}G^{SS} + PX^{SS}X^{SS} - PM^{SS}M^{SS}$. This equation, which is one of the equations of the steady-state version of the model in Table 1, shows that shocks that change potential output will show up in the aggregate price level, $P^{SS}$, with implications for relative prices via consumer prices. Changes in consumer prices then affect wealth valuation and so on.

exchange rate. None of the other effects is pertinent at this point.

As an aside, we should note that the model assumes that net foreign liabilities pay interest in foreign currency. This means that the depreciation *increases* the burden of initial stock of net foreign *liabilities* from the perspective of the domestic country. For this shock, therefore, the foreign currency denomination of *NFA* is a destabilizing influence in the comparative static sense, when the domestic country is a net debtor in equilibrium.[24]

If we allow specifically for imported capital goods, it will also be the case that the shock will change the cost of capital and hence the desired capital stock, thus providing a third channel for real exchange rate effects. The depreciation increases the price of imported capital and reduces potential output and export capacity. This makes the required increase in the trade balance more difficult to obtain and, in general equilibrium, consumption prices must rise even more than before to choke off domestic absorption.

Adding the *almost-small-open-economy (ASOE) assumption* relaxes the restriction that the domestic country's export price is exogenous. Although individual firms may not have market power, if the entire Canadian pulp and paper industry, for example, were to increase its production simultaneously, this would have the effect of decreasing export prices somewhat: $\{dX^{SS}/dY^{SS} > 0, dPX^{SS}/dY^{SS} < 0\}$, where $PX^{SS}$ is the steady-state price of domestically produced goods sold in the export market, holding the real exchange rate constant. This means that the domestic country's offer curve is no longer perfectly elastic and that its position -- being a function of potential output -- is important. The offer curve shifts as production possibilities change, resulting in changes in the export price. Thus, in the ASOE case, the exchange rate depreciation increases the cost of capital (the imported capital goods effect), decreases potential output, shifting the home country's offer curve inward, increasing the price of exportables relative to what it would

24. The destabilizing influence described in the text is an interesting topic particularly in the context of highly indebted economies. The CORE model, however, is parameterized for the Canadian economy, where the destabilizing effect is not currently large. We shall show below that fiscal expansion tends to bring about an exchange rate *appreciation* in the short run. This means that foreign currency denomination is also destabilizing in the dynamic sense, under the maintained assumption of static expectations. This concurs with Masson (1981), in that at least some small amount of forward orientation in expectations is required for dynamic stability. See also Masson (1983).

have been without this effect. As the third column of Table 2 indicates, the ASOE assumption has a direct effect on export prices that facilitates return to equilibrium by allowing prices -- as opposed to volumes -- to do more of the adjusting than in the pure SOE case.

Finally, let us consider the introduction of the country-risk effect by allowing foreign countries to base their willingness to pay for domestic bonds on the level and path for $NFA$ of the domestic country. Then both $r^{SS}$ and $NFA^{SS}$ are functions of $B^{SS}/Y^{SS}$ and both private consumption and government decisions will be affected by this recognition. In addition to the effects described above, the increased international indebtedness raises the real interest rate offered by the rest of the world, thereby raising the cost of capital and reducing potential output. This shrinkage of the production set makes generating the required trade surplus that much harder; much more of the adjustment must come about through relative price movements to induce further decreases in consumption. This is a case that has been studied extensively in the context of developing countries and is gaining increased attention for developed countries now that so many of them appear to be running unsustainable budget deficits.

We have shown, in an earlier subsection, the implications of the existence of steady-state equilibrium and the conventional accounting identities. They imply a relation between what we call consumable income and net exports or the steady-state stock of net foreign assets. In this subsection, we have outlined a theory of consumption that solves the net foreign assets-cum-consumption problem as one of intertemporal choice of consumption over uncertain lifetimes. Taken together, these concepts allow us to write down the steady state of the most general version of the model solely as a function of parameters and exogenous variables. To summarize our knowledge thus far we rewrite (10) showing the arguments of each variable:

$$C_t^{SS} = Y^{SS}(r^{SS}, \psi, \delta, \bar{L}, \Theta, f, \Gamma) - I_t^{SS}(Y^{SS}(\ldots)) - G_t^{SS}(Y^{SS}(\ldots), \Gamma)$$

$$+ [r(f, \Gamma) - g] NFA^{SS}(\gamma, r^{SS}, p, \phi, \Gamma, f), \tag{17}$$

where $\Gamma = \{B^{SS}/Y^{SS}, G^{SS}/Y^{SS}, \tau^{SS}\}$ is a vector of government control variables, $\psi$ is the risk premium demanded by investors to hold real capital, $f$ is a vector of foreign variables, and all other parameters are as defined previously. The arguments of consumption clearly depend on the arguments of everything else in the model. Still, there are some general statements that can be made. For example, we will show in the analysis of the fiscal policy shocks that the ratio of net foreign assets to income is strictly decreasing in the ratio of government debt to income.

When the purest, small open economy assumptions are retained, $\Gamma$ drops out as an argument to $r^{SS}$ and hence to $Y^{SS}$ as well; foreign offer curves are perfectly elastic and none of the domestic country's policies can directly affect either the terms of trade or the cost of capital.[25] Including imported capital is sufficient to result in $f$ and $\Gamma$ becoming arguments of $Y^{SS}$, since the cost of capital then depends on the real exchange rate.[26]

25. This is so not solely because of the absence of the imported capital effect but also because our tax is non-distorting. Also note that in this special case output and consumption are linear homogenous in productivity and the labour force.

26. Since it is assumed that government purchases are entirely domestically biased, they too can have an effect on output in the almost-small-open-economy case or if debt is used to finance the purchases.

# 3. MODEL DYNAMICS

From the outset our approach has been to begin with the broadest possible framework, derive some theoretical restrictions from that framework and then dig a little deeper. We began with only the accounting structure and added the minimal requirements of the neo-classical steady state. We then closed the steady-state model with a theory of consumption. We now consider the dynamic adjustment that takes place between the steady states, overlaying model dynamics on the neo-classical steady-state conditions described above. This method of model construction differs from that of traditional macroeconometric models and requires some motivation. Consequently, we begin this section with a digression on the role of the steady state in dynamic policy simulation models. This is followed by sections delineating the intrinsic and expectational dynamics of the model and why it is important to keep the two separate.

## 3.1 From Steady States to Dynamics

The steady state is important not only in its own right, but also because it conditions the dynamics of the model; steady-state solutions place restrictions on the dynamic paths of variables rather than having dynamic paths settle down on some long-run solution by happenstance. The traditional macroeconometric model is built around the latter method: one estimates a general dynamic form for each equation using regressors that are generally agreed upon. Statistical criteria are used to evaluate lag length and specification on an equation-by-equation basis and the long-run solution, if one exists, is determined principally by the estimated lag structure of the model. In focussing on statistical criteria for model evaluation with an emphasis on dynamics, this method assumes that estimated dynamics produce the best short-term forecasts. But for medium-term forecasts and policy analysis, it is necessary to know where the economy will settle down in order to interpret the path taken to get there. Moreover, the short- to medium-term dynamics in response to shocks where the steady state of the model is expected to change should be expected to be much different from those where it is not. Most traditional macroeconometric models do not impose sufficient restrictions to ensure that co-movements of variables make economic sense over longer horizons.

As an example of this, consider the hypothetical example of the expected impact on investment of a (temporary) demand shock and a (permanent) supply shock, both positive and beginning from steady state. It is well known that investment is considerably more volatile than is output. Investment booms are also associated with the late stages of business cycles. These observations probably reflect the fact that most capital investments are close to irreversible, necessitating efforts by entrepreneurs to discover how long-lived a shock is before changing an investment program. When the economy undergoes a temporary but long-lasting demand shock, there is usually a boom in investment, although the capital stock desired in the long term is unchanged. This means that the investment boom must be reversed. That is, abstracting from steady-state growth, every dollar of investment above the steady-state path during the boom must later be offset one-for-one by investment below the steady-state rate in order to return the capital stock to its desired level. If, however, there is a supply shock such that the desired capital stock increases, the investment boom need not be reversed. The two cases will obviously have very different medium-term dynamics. More importantly, models without steady-state restrictions would typically have investment dynamics driven by strong accelerator effects so that the capital stock would have no tendency to return to its original level following a demand shock; such models do not normally allow for a distinction between supply and demand shocks.[27]

In addition to the influence of the steady state, the CORE model contains several other mechanisms that allow the model's dynamics to differ under different kinds of shocks. The most important of these is the separation of dynamic adjustment mechanisms that are intrinsic to the economy from those that are driven by expectations. This allows the type of shock --

27. The omission of this distinction is likely the most important reason why the vast majority of large-scale macroeconometric models are unstable. In most instances, however, the instability arises out of a confluence of factors, and disentangling the sources of instability is difficult. This observation is the major reason for the strategy used in the construction of this model, of overlaying dynamics on top of a clear steady state.

whether it is permanent or temporary, anticipated or unanticipated -- to affect expectations and hence the overall model dynamics. Moreover, we provide mechanisms that allow a wide range of interpretations of the shock in question. This, in turn, allows substantial flexibility in the model's dynamic responses.

## 3.2 The Importance of Expectations

Owing to their short-run nature, most models make no distinction between intrinsic and expectational dynamics. Some models, such as the simplest forms of the IS-LM-BP model, are essentially devoid of dynamics and consequently assume static expectations. This may be sufficient for the analysis of pure impact effects of disturbances, but for longer-term forecasting and policy analysis such models are not satisfactory. Moreover, tagging on *ad hoc* dynamics is not likely to be successful, because the methodology behind the model's original construction is antithetical to forecasting and policy questions that involve any substantive changes in real or financial stocks. In the same way, omitting the distinction between real and expectational dynamics is methodologically inconsistent with the full range of policy questions, since expectations formation is not permitted to vary with policy. For example, in experiments seeking the monetary policy that minimizes the output cost of bringing down inflation, the influence of the candidate policies on agents' expectations is critical.[28] In his Hansen Memorial Lecture, Bank of Canada Governor John Crow (1988) alludes to the fact that expectations and economic dynamics are indeed conditional on monetary policy:

> In the light of the devastation brought about in the 1970s by severe inflation, world financial markets have tended to be extremely sensitive to any signs of an increase in price pressures. Any fears of a pickup in inflation have seemed to lead quickly to a rise in long-term bond rates ... Such fragility [of expectations] limits the short-term flexibility of monetary policy. This situation may be contrasted with one in which people have strong expectations of stable prices... With apparently persisting expectations of stable prices, the commodity price surge associated with the Korean War boom did not trigger expectations of a continuing wage-price spiral. As a result,

28. This issue is the focus of a series of articles by Buiter and Miller (1982,1983,1985). In addition, there is an extensive literature on the credibility of policy and its role in determining what has sometimes been called the sacrifice ratio. A survey is provided by Blackburn and Christensen (1989).

> the policy actions that were eventually taken were able to return the economy to price stability in a short period of time and without any pronounced economic slowing.
>
> pp. 11-12

The Governor describes markedly different statistical properties of inflation generated by a single economy. The phenomena described can only come about with different policy regimes supporting different expectations in each case. In the 1950s, following a period of relatively stable prices, agents expected monetary authorities to return inflation to its former rate, which made the disinflation less costly. In the 1980s, after an extended period of looser monetary policy, agents expected price shocks to propagate. The need to fight against those expectations meant there was a relatively high output cost of disinflation. It is impossible to model expectations with fixed autoregressive processes and still encompass these two episodes -- or any other substantial change in policy rules.

It what follows we begin, once again, with as broad a structure as is feasible by assuming initially that agents' expectations, however formed, are correct *ex post*. This allows us to focus on the *intrinsic dynamics* stemming from physical and psychic adjustment costs. Following our discussion of intrinsic dynamics, we go on to discuss the model's *expectational dynamics*.

## 3.3 Intrinsic Dynamics:

Intrinsic dynamics result from the costs borne by agents when they adjust variables, be they consumption levels, prices or something else. Included in this definition are mundane physical restrictions, many summarized by accounting rules and inequality restrictions. Since, with few exceptions, we model all intrinsic dynamics the same way, in this subsection we discuss consumption dynamics. Most of the discussion contained in this subsection carries over to other variables.

We think of the dynamic consumption problem as being nested within the steady-state model described above. The steady-state theory of consumption focussed on the implications of uncertain lifetimes on intertemporal allocation of consumption. Income uncertainty, interest rate movements and most aspects of intertemporal substitution were set aside in order to isolate the steady-state solution. Here we re-introduce those issues.

Consider a representative agent planning consumption expenditures over the infinite future in a world where adjustment is costly. Owing to past shocks, the agent will generally begin in a position away from steady state. Moreover, the agent faces an uncertain future income path. The agent must make a judgment about the current situation and form expectations about the future. To allow us to concentrate initially on intrinsic dynamics, let us assume that the expectations are correct, *ex post*. If the economy were undisturbed, the agent would maximize utility by attaining the steady-state consumption given by equation (15). Similarly, equation (14) gives the optimal consumption tilt over time, in the steady state. However, in the face of adjustment costs, the agent will not generally choose to jump immediately to the steady-state consumption path. A gradual approach will be less costly. Assuming adjustment costs to be quadratic in *logarithms*,[29] we model the agent's problem as follows:

$$\underset{\langle c \rangle}{MIN} \sum_{t=0}^{\infty} (\beta\rho)^t \{ (1-\lambda)(c_t - c_t^d)^2 - \lambda [c_t - c_t^{SS} - (c_{t-1} - c_{t-1}^{SS})]^2 \} \qquad (18)$$

The first term in (18) represents the cost to our agent of being away from the *desired* consumption level and carries a relative weight of $1-\lambda$. The desired level of consumption $t$ periods in the future, $c_t^d$, is based on the expectation today of the state of the economy over the future. The desired consumption level could be the steady-state value, but need not be. Ordinarily, desired consumption will differ from the steady-state level due to current and expected future dynamics in forcing variables. For example, interest rates that are temporarily below their equilibrium level may tempt agents to reduce savings and consume more goods than they would in the steady state, even though this action implies reduced consumption in later periods.

The term in square braces in (18) is the cost of changing consumption. It carries a relative weight of $\lambda$. In the limiting case where there are no adjustment

29. Henceforth, lowercase letters are used to designate the natural logarithms of variables. We must note that we make a small simplification in deriving equations in logarithmic form such as (19) below by allowing ourselves the approximation: $E(\log x) \approx \log(Ex)$ .

costs, i.e. $\lambda = 0$, the solution to (18) is to consume at the "desired" level and to jump immediately to the new desired level when circumstances change. The $c_{t-i}^{SS}$ are just the logarithms of the steady-state solutions from equation (15). Their presence here indicates that we are abstracting from the costs of adjusting consumption along the steady-state growth path.[30] Both terms are discounted over time: the parameter $\beta$ is a pure discount factor while $\rho$ is a forecast weighting factor. The weighting factor captures two important ideas. The first is that even though we have assumed, temporarily, that our agent makes no forecasting errors *ex post*, the confidence in those forecasts *ex ante* would decrease as the horizon was extended.[31] The second is that in some instances it is necessary to use forecasts long into the future even though they may be poor ones. For example, the acquisition of producing capital or consumer durables necessitates a lengthy planning horizon.

The first-order conditions for the solution to (18) form an Euler equation that can be solved along with a terminal condition to find:

$$c_t = \eta + \theta c_{t-1} + \theta (1-\lambda) \lambda^{-1} \sum_{i=0}^{\infty} (\beta\rho\theta)^i c_{t+i}^e \tag{19}$$

where $c_{t+i}^e$ are the future desired consumption levels conditional on information available at time zero: i.e., $c_{t+i}^e = E_0 c_{t+i}^d$, and where $\theta$ is a function of $\beta$, $\rho$ and $\lambda$. The parameter $\theta$ is strictly less than unity for $0 \le \lambda, \beta, \rho < 1$. The "constant term," $\eta$, is also a function of the same three parameters plus the steady-state growth rate, $g$.

According to the model, the decision rule for consumption is a function of lagged consumption and a geometrically declining distributed lead of expected future desired consumption. The extent to which lagged variables

30. To see this a little more clearly, note that we can rewrite the term in square braces in (18) as follows: $[c_t - c_t^{SS} - (c_{t-1} - c_{t-1}^{SS})] = [c_t - c_{t-1} - (c_t^{SS} - c_{t-1}^{SS})] = c_t - c_{t-1} - g$ where $g$ is again the steady-state growth rate. Note that our subtracting $g$ in (18) should not be interpreted as meaning that steady-state growth in real variables is costless. As with everything else in the steady state, the ratio of steady-state adjustment costs to output must be a constant. A more accurate description of where they are, therefore, would be to say that the costs of steady-state growth of all real variables are buried in the *ex post* observations of total factor productivity.

31. It is well known that the data for income and output for Canada are non-stationary in both aggregate and *per capita* terms. Since the forecast variance of a non-stationary process grows without bound as the forecast horizon increases, a weighting factor that, other things being equal, reduces the weight on future variables as the horizon is extended is justifiable.

determine current decisions depends on the relative cost of adjustment, $\lambda$. Increases in $\lambda$ affect (19) through both their direct effect on $(1-\lambda)/\lambda$, which is negative, and an indirect effect operating through $\theta$, which is also negative for credible values for $\beta$ and $\rho$. The presence of $\theta$ within the geometrically declining lead term as well as in front of the summation means that the indirect effect of increases in $\lambda$ will be stronger for the second term than the first. Thus the indirect effect complements the direct effect, and intrinsic consumption dynamics are increasingly dominated by past levels of consumption, rather than by expected future conditions, as $\lambda$ increases. In the limit, as $\lambda \rightarrow 1$, the forward terms of (19) drop out completely.[32] This influence of $\lambda$ on the determinants of adjustment dynamics is reminiscent of the standard partial adjustment model and, in this way, the *target-seeking* model presented here can be thought of as a generalization of the partial adjustment model.

Equation (19) is also affected by $\beta$ and $\rho$. The higher these parameters are, other things being equal, the larger the influence of lead variables on consumption. Being a pure discount factor, $\beta$ is taken as fixed in all sectors of the model; $\rho$ and $\lambda$ (and hence $\theta$ and $\eta$), however, differ across cases. Numerical simulation requires that in practice we truncate the infinite geometric lead structure and use some finite planning horizon, $N$.[33]

There are other aspects of intrinsic dynamics that are germane to particular sectors of the CORE model, but for our immediate purposes only the general flavour of the intrinsic dynamics is necessary. Thus, we leave these other factors until we come to the discussion of the shocks themselves. On top of the

32. Specifically, $\theta = \{(\beta\rho+\lambda)/\lambda - [((\lambda+\beta\rho)/\lambda)^2 - 4\beta\rho]^{0.5}\}/(2\beta\rho)$, with $0<\theta\leq 1$ being the smaller of two roots associated with the solution of the second-order difference equation derived from the first-order conditions to (18). See Sargent (1987), pp. 177-209. It can be shown that $\theta(\lambda, \beta, \rho)$ is monotonically decreasing in each of its arguments. There are at least two special cases of note. Suppose that $\lambda, \beta, \rho = 1$ implying no weight on the target-seeking objective of the agent and no discounting; i.e., the cost of adjusting (the second term in (18)) is all that matters. In terms of (19) this gives $\{\theta, \eta\} = \{1, 0\}$; with all of the lead terms dropping out, $c_t$ follows a pure random walk. If target seeking does not matter $(\lambda = 1)$ but there is discounting $(0<\beta, \rho<1)$, then the lead terms of (19) still drop out but agents' caring about the timing of adjustment costs implies $\eta > 0$, $0 < \theta < 1$. Equation (19) becomes an AR(1) with a constant, and consumption converges asymptotically on a steady-state value at a rate determined by discounting.

33. We choose the lead length for truncation so that beyond $N$ further leads carry coefficients small enough to be deemed inconsequential. Exactly which lead this is depends on $\rho$ and $\lambda$.

real adjustment model discussed in this section are fused expectational dynamics. These are obtained by positing methods by which agents formulate $c^e_{t+i}$ or their equivalents, a subject to which we now turn.

## 3.4 Expectational Dynamics

In the previous subsection we presented a consumption decision rule, equation (19), taking the future as given. Completing the model's dynamic structure requires specification of how expectations are formed. There are several methods that could be used. We could replace the $c^e_{t+i}$ with $c^{SS}_{t+i}$, that is use the equilibrium values as expected future values. This would leave the dynamic specification for consumption as a linear combination of lagged consumption and steady-state consumption. Since the former is pinned down by history and the latter is smooth by construction, however, there is not enough variation in either to produce the variety of dynamic responses called for in the full range of forecasting and policy simulation experiments. A dynamic economy with adjustment costs requires non-linear adjustment. A second method would be to express expectations as an autoregressive process using past values: $c^e_{t+i} = \phi(L)\, c_{t+i-1}$ thereby invoking autoregressive expectations everywhere in equation (19). This would effectively remove the steady-state condition from the equation. More importantly, autoregressive expectations have the disadvantage of being fixed; that is, they do not change to reflect the policy environment that is in place. They also do not distinguish whether a shock is expected or unexpected. A third approach is to assume that expectations are formed rationally, which amounts to substituting future solution values $c_{t+i}$ for $c^e_{t+i}$ in simulations of the model. For most economists, however, expectations that are fully rational in this sense are not viewed as compatible with the sort of dynamics that are in the macroeconomic data. Most shocks are unexpected and difficult to identify precisely. Therefore, especially in the short run, the restriction that agents correctly foresee the consequences of shocks is not seen as appropriate. We do not find any of these approaches adequate.

From a theoretical standpoint, perhaps the best way to handle the expectations problem would be to embed a formal theory of behavioural learning within a stochastic model. In a very small empirical model it might be possible to do justice to the theoretical developments in this area. In this model, however, to make such a concept operational would require simplifications that would make the chosen mechanism too particular to given shocks.[34] Instead, we offer a practical alternative in the form of a flexible mixture of adaptive and forward-looking expectations. This mixture allows for changes in expectations formation, while retaining the discipline of the steady state. At the same time, it provides the stickiness that is often necessary to capture the stylized facts of short-term movements in macroeconomic variables.

We model agents' expectations as a linear combination of three components: a backward-looking component, based on autoregression; the expected steady-state level of the variable in question; and the model's perfect foresight solution (i.e., a "rational" component). To this we add some terms specific to the particular variable. Again taking the consumption equation as our example, we have:

$$c^e_{t+i} = [(1-\mu_{c1}-\mu_{c2})A(L)yc_{t+i-1}+\mu_{c1}yc^{SS}_{t+i}+\mu_{c2}yc_{t+i}] + E_{t-1}\sum_{j=i}^{i+4}\vartheta_j(r_{t+j}-r^{SS}_{t+j}) + E_{t-1}\sum_{j=i}^{i+4}\zeta_j(NFA_{t+j}-NFA^{SS}_{t+j}) \quad (20)$$

where $yc$ is the log of consumable income, and A(L) is a polynomial in the lag operator.[35] In the above, $\mu_{c1}$ is the weight on the (log of the) steady-state level of consumable income; $\mu_{c2}$ is the weight on the perfect-foresight solution for the date in question, $t+i$; and $(1-\mu_{c1}-\mu_{c2})$ is the weight attached to a distributed lag of past (logs of) consumable income. The term in the square braces is expected consumable income. The lagged income term reflects the empirical observation that there is significant persistence in consumption. The three income terms and the constraints on their sum capture *consumption augmenting effects*, whereby permanent shocks to the level of production are translated, in part, into a parallel shift in the entire consump-

34. This conclusion is based on a variety of stochastic simulations we have done using very small models. The results, while promising, have also been marked by slow simulation turnaround and occasionally by model failure; i.e., non-convergence of the algorithm in simulation. These problems grow as the model becomes more complex and so it was decided that formal error-learning models would not be practical here.

35. That is for any variable $x$, $A(L)x = a_0 + (a_1L + a_2L^2 + a_3L^3 + \ldots)x = a_0 + a_1x_{t-1} + a_2x_{t-2} + \ldots$.

tion profile. Were $\mu_{c2} = 1$, expected consumable income would be identically equal to the perfect-foresight solution. The presence of terms other than the perfect-foresight term implies that agents usually make errors in their expectations. However, under some circumstances the $yc_{t+i}$ term can move $c^e_{t+i}$ around a great deal more than an equation without it, because it is the only term in the equation that is not pinned down by either steady-state conditions or by history.[36] Of course, with the correct restrictions on $A(L)$, all three terms converge eventually on the correct (steady-state) solution; the perfect foresight term only ensures that the dynamics are led well away from control when the forcing variables call for it, and that expectational errors do not result in dynamics long after the primal source of those errors has past.

The last two terms on the right-hand side of (20) capture expected deviations of real interest rates and expected deviations of the stock of net foreign assets from their steady-state values. By construction, they approach zero as a model solution approaches a steady state. The real interest rate gaps are included to pick up intertemporal *substitution effects* during periods of disequilibrium. This tends to offset the income effects of real interest rates captured in the definition of consumable income. The other gap terms, in $NFA$, represent *consumption tilting* effects, the shifting forward (or backward) of consumption in resolution of shocks to the stock of net foreign assets. A positive shock to $NFA$ will tilt the path of consumption upward until the gap disappears. The inclusion of these extra terms may be justified by appeal to recent developments in the theory of precautionary savings by risk-averse agents in the presence of uncertain income.[37] When agents are risk averse and cannot insure against income losses, transitory income is valuable for the information it conveys about the path of future income. This information is only correct on average, however. It follows that an economic agent facing shocks under incomplete information will respond initially using some gen-

36. This is the source of the non-linear dynamics mentioned in the first paragraph of this subsection.

37. The seminal reference on precautionary saving is Leland (1968) with elaborations on the theme applied to stochastic settings being provided by Barsky *et al.* (1986), Kimball and Mankiw (1989), Skinner (1988), and Zeldes (1989). Our characterization of this literature within the context of our model is a very loose one. The process described in the literature is one of Keynesian co-ordination failure with an error-learning process to provide dynamic structure. As noted in footnote 34, this was judged impractical for the CORE model.

eral rule and then gradually correct any errors.[38] As information regarding the nature of the shock unfolds, the representative agent comes to understand the steady-state levels of net foreign assets (or liabilities) and of real interest rates, and begins to look more toward the future, speeding the path of equilibration.

The model user has considerable control over the short-term dynamic responses of the model. This control comes from two related sources. First, users have the choice of varying $\mu_i$ in each expectational equation. In this way, users can tailor each sector's persistence to the desired assumption about how much information and foresight the representative agent has so that, for example, asset markets can be made to respond more quickly to shocks than goods markets. Second, users are also provided with a mechanism whereby they can alter these weights *over time as the shock plays out*. In doing so, users can characterize their beliefs as to the precise nature of the shock in the eyes of the representative agent. For example, if a shock is unanticipated but its effects are quickly learned once the shock is recognized, the user would initially place a large weight on the autoregressive expectations process and then allow that weight to fall quickly in favour of higher weights on the steady-state and perfect-foresight solutions. This would have the effect of producing a relatively large amount of initial stickiness but faster adjustment towards equilibrium in the medium term.[39] Substituting equation (20) into (19) yields an equation with a rich mixture of expectations and intrinsic dynamics, capable of encompassing a wide variety of dynamic responses to shocks. Moreover, in every case the model's construction ensures convergence without the long, deterministic cycles that plague purely adaptive models.[40]

38. The strength of the initial response would depend on the perceived costs of mistakes. It follows, therefore, that consumption responses are likely to be frequent, relatively small and not very autocorrelated, while investment responses are likely to be infrequent, relatively large and autocorrelated.

39. This is achieved very simply and automatically using a TROLL macro that we have written for the purpose. The macro prompts the user for the required information and provides suggestions on how to phase in the learning process. We should also note that one is not required to exercise the variable weights option to get convergence of model variables on their correct, steady-state levels.

40. For example, in the Longworth-Poloz model, a money growth shock produces real cycles that last for several decades. (See Longworth and Poloz (1986) especially pp. 49-52.) This is because the purely adaptive nature of expectations in LPMOD does not allow agents to learn from their mistakes. Properties of this sort will be observed in almost every model that is both linear and purely autoregressive in its expectations.

# 4. SOME FISCAL SHOCKS

In policy simulations it is important to understand the steady-state implications of a shock, especially when the shock has permanent effects. The adjustment, working through stock-flow dynamics, expectations and possibly other mechanisms, is strongly conditioned by the steady state. In understanding how an economy gets from "here" to "there," it is obviously very helpful to know where "there" is. One important example of the clarity such a perspective provides is that one can identify when dynamic forces unleashed by a shock require impact responses that are in the "wrong" direction from the perspective of the steady-state comparative statics. We describe one such case in this section. For these reasons, and in keeping with the way in which the description of the model has unfolded above, we analyze the effects of our shocks from back to front, looking first at the comparative statics results and then adding the dynamics.

The fiscal policy shocks described in this section were chosen specifically because fiscal policy very often has permanent effects on the economy, either through the share of expenditure commanded by the government or through the impact of financing deficits. Accordingly, we present two shocks. In one, government expenditures are changed permanently and the steady-state tax rate adjusts to maintain the original ratio of government debt to income. In the other, the ratio of government debt to income is increased, with government expenditures held at the original share of income. We begin, as before, with a discussion of the steady state, this time providing a broad outline of the whole steady-state model.

## 4.1 The Steady-State Model

Table 1A provides a list of model mnemonics and Table 1B, a list of the equations of the model's steady state.[41] In this subsection we augment our earlier discussion with a somewhat more complete description of the steady-state version of the model. Our discussion nonetheless will be brief. For a com-

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41. For the rest of the paper, we use the TROLL syntax of our simulation programs. We have omitted the more obvious identities from Table 1B, e.g., those that state that LJ=log(J) or that J=exp(LJ) for any variable J.

plete understanding of the model, interested readers are invited to study Table 1B.

Equation (1) states that the level of nominal net foreign assets (NFA) is proportional to nominal consumer income by the factor NFACNRAT; the optimal value of NFACNRAT is determined by consumer preferences and government debt. In Table 1B, the real side of this determination is given by equations (4) to (7) and (10), which pin down the wealth-to-income ratio of the economy. Equations (2) and (3), the income-expenditure identities, real and nominal, can be thought of as solving implicitly for $YC^{SS}$ and $PC^{SS}$, respectively. The fiscal policy assumptions of the model are contained within equations (8), (9) and (11), where it is assumed that government expenditures are a constant share of output, that the fiscal authority targets a constant debt-to-income ratio in the long run, and that taxes are adjusted to achieve that long-run target. The shares of imported investment goods (18) and consumption goods (19) are made functions of the exchange rate, with both absolute exchange-rate elasticities assumed to be unity. The pass-through of exchange rates to imported capital prices and to the cost of capital is contained in equations (24) and (29). The almost-small-open-economy (ASOE) assumption is captured by the parameters XPORT01 in equation (22) and $PX_1$ in equation (25), with XPORT01 < 1 implying that the domestic country's offer curve shifts with changes in potential output and $PX_1 > 0$ implying that a movement of the home country's offer curve along the foreign country's involves a change in price. Finally, in those cases where it is relevant, the adjustment of interest rates for country risk is shown by equation (30). Following a form suggested by Dornbusch (1987) and applied by Bhandari *et al.* (1989), foreign investors will demand a premium that is linear in the ratio of NFA to disposable income, NFACNRAT, below some critical value, NFATAR (i.e., when the debt ratio rises above some critical value). Portfolio or agency theory would probably suggest a more complex, non-linear structure, but the main point is ably captured by the structure portrayed in (31). The rest of the equations are quite standard.

## 4.2 Some Comparative Statics

***A public debt shock***

The top panel of Table 2 shows the comparative statics for a shock wherein the ratio of government debt to income rises from 0.8 to 1.0, with the share of government expenditures held at control by means of a temporary tax cut. The second and third columns of the table show results from the basic model -- that is, the model without a country risk premium -- under the small-open-economy (SOE) assumption and under the almost-small-open-economy (ASOE) assumption. The first column shows the effects of the shock in an interdependent world, with the potential for risk premia and with the ASOE assumption. Beginning with the simplest case shown in the second column, the first thing to note is that the level of foreign *indebtedness* increases in absolute terms by significantly more than the increase in government debt. This arises from two related influences.

First, the public perceives a large proportion of the government debt to be wealth and so chooses to consume from it in the short run. For the economy as a whole, this cannot be done without incurring further foreign indebtedness. To generate the revenue to service this higher level of foreign indebtedness, a larger trade surplus is required, which necessitates a depreciation of the real exchange rate (Z) or, equivalently, a deterioration in the terms of trade (TOFT). The deterioration is passed through into higher prices for imported consumption and capital goods. The higher consumer price level induces domestic residents to reduce consumption, including consumption of imports, freeing resources to increase exports. In the long run, therefore, the higher indebtedness crowds out domestic consumption, especially imports, through a terms-of-trade deterioration. This can be thought of as an inward shift of the consumption possibilities frontier combined with a rotation of the relative price line along that frontier.

Second, the higher prices for imported capital translate into a higher cost of capital, even at constant real interest rates. Thus the equilibrium capital stock (K) falls, as does potential output (Y). This represents an inward shift of the production possibilities frontier.

In a closed economy, the *steady-state* real interest rate could rise to choke off this change and reduce production possibilities.[42] For a small open economy without imported capital, there is a foreign sector that stands willing to lend at constant interest rates so that the attempt to increase consumption in the near term results instead in foreign indebtedness and diminished consumption possibilities in the long run. In summary, the long-run effect of the increase in government debt is reduced production possibilities (through the imported capital effect) and wealth, and higher exports of that production at lower prices than before. The decline in wealth and in consumption is, therefore, greater than the decline in production. Consumers attempt to deal with the shock to government debt by changing the time profile of their expenditures; they try to consume more of their lifetime resources in the immediate future and less later on.

The ASOE assumption changes matters slightly. The interdependency of the home country with the rest of the world means that a small amount of the welfare-reducing increase in consumer prices described for the SOE case is shifted onto foreign purchasers of the home country's exportable good. This is introduced by allowing the terms-of-trade deterioration to affect exports directly, thereby limiting the consumption price increase necessary to re-establish equilibrium.

Finally, we turn to the ASOE economy with an endogenous country-risk premium. Recall that under this scenario, foreign lenders will require a risk premium on debt when the (absolute) NFA-to-disposable-income ratio becomes large. Ideally, such an effect would be based not on the actual NFA-to-income ratio, but on the expected path of NFA and the time consistency, or lack thereof, of fiscal policy. The computational demands of such a model rule this out. The principal elements of the country-risk effect can be modelled by assuming that a premium is demanded that rises linearly once the ratio of net foreign debt to income exceeds some critical point. In this simulation, the critical value is -0.65 while the control value is -0.57. The shock does push the NFA ratio beyond its critical value, resulting in an increase in the real domestic interest rate and a larger increase in NFA relative to gov-

42. See Duguay and Rabeau (1987) for a closed-economy simulation model devoted to these issues.

ernment bonds (GBN) than in either of the two simpler cases. In the steady state, all public debt is rolled over at the new interest rates, and so the higher indebtedness implies an even higher financial burden in the long run. The public's tendency is again to shift the bulk of this obligation into foreign indebtedness. A correspondingly larger trade surplus is required, necessitating a larger deterioration in the terms of trade and a large increase in consumer prices. Partially offsetting this is the ASOE effect, which shifts the domestic country's offer curve vis-à-vis its foreign counterpart, promoting exports and mitigating the decline in domestic consumption.

In all instances there is a monotonic relationship between the ratio of government debt to nominal expenditures and the ratio of NFA to nominal spending. The exact curvature of the relation is a function of various parameters of the utility function and export market conditions as captured by PX1 and XPORT01. As well, the country-risk premium can significantly change the slope of the relation, resulting in a kink at the point where country risk begins to become a factor.

### *A government expenditure shock*

As noted above, the CORE includes only a rudimentary government sector. We assume that the government owns no capital and employs no labour directly. Rather, the government merely chooses a level of absorption in proportion to aggregate output. Because the government does not import, our assumption implies a strong domestic bias in resource utilization relative to the private sector. The assumption here is admittedly strong, but it does correctly capture an observed relative domestic-good concentration in government expenditures. The output of government is assumed to be additively separable in private agents' utility functions. Finally, the government imposes a lump-sum tax to finance the interest obligations on its debt.

The bottom panel of Table 2 shows the steady-state impact of an increase in government absorption from 20 per cent of real output to 21 per cent, holding constant the steady-state ratio of government debt to income by altering taxes as required.

The shock has the effect of driving out imports because of the higher domestic bias of government expenditures relative to private consumption. At the same time, the higher steady-state tax burden reduces consumable income. Thus, government expenditures crowd out private consumption. The agent's heavy discounting of the future leads to a choice of higher net foreign indebtedness. As before, this requires a depreciation of the real exchange rate in order to generate the trade surplus to service the foreign debt. This produces a relatively large reduction in steady-state consumption; an accentuation of the crowding out of consumption. The imported capital effect works to crowd out investment and reduce production possibilities, just as in the government debt shock. Thus both output and consumption are lower in the new steady state. The differences between the three cases shown in the table follow quite closely those described in the analysis of the government debt shock.

## TABLE 1A
## CORE MODEL MNEMONICS

*Preliminaries:*

LJ = the log of any variable J

$J^{SS}$ = the steady-state value of the variable J

JUS = the foreign equivalent of the variable J

XPORTj = constant in the export equation

IMPORTj = constant in the import equation

*Price Indices:*

P = GDP deflator

PC = price of consumption goods

PX = price of domestic goods in the export market (C$)

PM = price of imported goods in the domestic market (C$)

PINV = price of investment goods

PG = price of government goods (government expenditure deflator)

RPC = ratio of consumption price index to the GDP deflator

*Income and Wealth Concepts:*

Y = real GDP

YN = nominal expenditures

YC = consumable income

HW = human wealth (NPV of expected after-tax labour income)

FW = financial wealth

TW = total wealth (the sum of human and financial wealth)

MPC = the marginal propensity to consume out of wealth

*Supply-side Concepts:*

K = the stock of capital

INV = investment

CC = the cost of capital

MC = marginal cost

RRj = the annual real return on the jth asset

DELTA = the geometric rate of capital depreciation

CCRISK = risk premium on fixed capital

ET = trend total factor productivity

NF = full employment labour force

*Public and Private Finance:*

GBN = nominal value of government bonds outstanding

MBN = high powered money (exogenous)

M1 = private sector holdings of M1

TAXN = nominal total tax revenues

NFA = nominal value of the stock of net foreign assets

NFAI = nominal interest receipts on net foreign assets

NFAINRAT = ratio of interest payments on NFA to expenditures

GBRAT = ratio of nominal government bonds to expenditures

NFATAR = critical level of NFA relative to consumer income

RNWT = weighted average nominal interest rate on debt

*Foreign Trade and Payments:*

CBALN = nominal current account balance

TBALN = nominal trade balance

CBALNRAT = ratio of CBALN to nominal expenditures

TBALNRAT = ratio of TBALN to nominal expenditures

TBAL = real trade balance

X = exports

M = imports

INVM = imported investment goods

CM = imported consumption goods

IMSHAR = the share of investment goods that are imported

CMSHAR = the share of consumption goods that are imported

TOFT = the real terms of trade

S = the price of foreign exchange (C$/FC)

SE = expected nominal exchange rate

ZE = expected real exchange rate

DEPE = expected rate of exchange depreciation

RNFA = weighted average nominal return on net foreign assets

**TABLE 1B**
**THE STEADY-STATE VERSION OF THE CORE MODEL**

(1) $NFA = NFACNRAT \bullet YC^{SS} \bullet PC^{SS}$

(2) $Y^{SS} = YC^{SS} + INV^{SS} + G^{SS} + TBAL^{SS}$

(3) $P^{SS} \bullet Y^{SS} = PC^{SS} \bullet YC^{SS} + PG^{SS} \bullet G + PINV^{SS} \bullet INV^{SS} + (PX^{SS} \bullet X^{SS}) - (PM^{SS} \bullet (INVM^{SS} + CM^{SS})$

(4) $FW = K^{SS} \bullet PINV^{SS} / PC^{SS} + GBN / PC^{SS} + NFA / PC^{SS} + MBN / PC^{SS}$

(5) $HW = (ELEQ \bullet Y^{SS} \bullet P^{SS} / PC^{SS} - TAXN / PC^{SS}) \bullet (1 + RRCON + PROB) / (RRCON + PROB - GPERCAP)$

(6) $TW = FW + HW$

(7) $MPC = (1/PI) \bullet [ ( PI - 1) \bullet RRCON + PI \bullet PROB + TAU]$

(8) $TAXN^{SS} = G \bullet PG^{SS} - (GBN_t^{SS} - GBN^{SS}_{t-1}) + RNWT \bullet GBN^{SS}_{t-1} - (MBN_t^{SS} - MBN^{SS}_{t-1})$

(9) $GBN^{SS} = GTAR \bullet YN^{SS}$

(10) $YC^{SS} = MPC \bullet TW$

(11) $G^{SS} = GSHAR \bullet Y^{SS}$

(12) $PG^{SS} = P^{SS}$

(13) $LY^{SS} = (1-ELEQ) \bullet LK^{SS} + ELEQ \bullet LNF + LET$

(14) $LK^{SS} = 1 / ELEQ \bullet [ -LCC^{SS} + LET + LOG (1-ELEQ) ] + LNF$

(15) $INV^{SS} = 4 [K^{SS} - (1-DELTA) \bullet K^{SS}_{t-1}]$

(16) $CBALN^{SS} = NFACNRAT \bullet (YC_t^{SS} \bullet PC_t^{SS} - YC^{SS}_{t-1} \bullet PC^{SS}_{t-1})$

(17) $NFAI^{SS} = RNFA \bullet NFACNRAT \bullet YC^{SS}_{t-1} \bullet PC^{SS}_{t-1}$

(18) $LINVM^{SS} = LINV^{SS} - LZ^{SS} + IMPORT00$

(19) $LCM^{SS} = LYC^{SS} - LZ^{SS} + CMPORT00$

(20) $TBALN^{SS} = CBALN^{SS} - NFAI^{SS}$

(21) $TBALN^{SS} = PX^{SS} \bullet X^{SS} - PM^{SS} \bullet M^{SS}$

(22) $LX^{SS} = XPORT01 \bullet LYUS^{SS} + (1-XPORT01) \bullet LY^{SS} + LZ^{SS} + XPORT00$

(23) $LPM^{SS} = LPUS^{SS} + LS^{SS}$

(24) $PINV^{SS} = IMSHAR \bullet PM^{SS} + (1-IMSHAR) \bullet P^{SS}$

(25) $LPX^{SS} = PX1 \bullet (LYUS^{SS} - LY^{SS}) + LP^{SS}$

(26) $LZ^{SS} = LPUS^{SS} + LS^{SS} - LP^{SS}$

(27) $LTOFT = LPX^{SS} - LPM^{SS}$

(28) $M^{SS} = CM^{SS} + INVM^{SS}$

(29) $LCC^{SS} = LPINVM^{SS} - LP^{SS} + LOG(RR^{SS} + DELTA + CCRISK)$

(30) $RR_j^{SS} = 0.045 + Z_1 \bullet (NFACNRAT - NFATAR)$

(31) $Z_1 = -0.005$ IF NFACNRAT < NFATAR, ZERO OTHERWISE.

(32) $LM1^{SS} = LYN^{SS} + \chi\, RN^{SS}$

**TABLE 2**
**STEADY-STATE SHOCK MINUS CONTROL RESULTS**
**(in per cent, except as noted)**

| | Risk Premium | Without Risk | |
|---|---|---|---|
| | ASOE | SOE | ASOE |
| **Increase in GBN/YN from 0.80 to1.0** | | | |
| NFA/GBN | - 2.01 | - 1.28 | - 1.31 |
| PC | 5.86 | 3.08 | 2.71 |
| PX | 3.88 | 1.76 | 1.79 |
| PM | 12.90 | 8.66 | 7.81 |
| Y | - 3.45 | - 1.76 | - 1.59 |
| C | - 10.58 | - 6.96 | - 6.65 |
| K | - 8.62 | - 4.39 | - 3.97 |
| Z | 9.46 | 6.90 | 6.22 |
| X | 7.74 | 5.15 | 5.42 |
| M | - 19.77 | - 12.99 | - 11.96 |
| TW | - 14.13 | - 6.72 | - 6.44 |
| TOFT | - 9.02 | - 6.90 | - 6.02 |
| RR* | 0.33 | 0.00 | 0.00 |
| **Increase in G/Y from 0.20 to 0.21** | | | |
| NFA | - 19.59 | - 15.78 | - 15.92 |
| PC | 0.52 | 0.35 | 0.31 |
| PX | 0.38 | 0.20 | 0.20 |
| PM | 1.31 | 0.95 | 0.86 |
| Y | - 0.34 | - 0.20 | - 0.18 |
| C | - 3.14 | - 2.83 | - 2.79 |
| K | - 0.85 | - 0.50 | - 0.45 |
| Z | 0.97 | 0.76 | 0.68 |
| X | 0.79 | 0.56 | 0.59 |
| M | - 3.36 | - 2.80 | - 2.68 |
| TW | - 3.45 | - 2.79 | - 2.76 |
| TOFT | - 0.92 | - 0.76 | - 0.66 |
| RR* | 0.03 | 0.00 | 0.00 |

* absolute difference; see Table 1B for mnemonics.

## 4.3 Dynamic Simulation Results

To illustrate the CORE model's dynamics we consider an experiment that combines the two shocks described previously. Specifically, we consider the case where the authorities increase the ratio of government absorption from 0.20 to 0.21, while allowing the steady-state ratio of government debt to income to drift up from 0.80 to 0.90. For this exercise, we use a version of the model that assumes no country-risk premium.[43] There are still two important policy issues to address prior to conducting the simulation. The first concerns choosing among the infinite number of paths for taxes that will establish the new, steady-state ratio of government debt to income. We assume that taxes begin to adjust with a two-year delay and then change only slowly, reaching the level necessary for equilibrium after five years. Figure 1 shows the path for the ratio of government debt to income implied by our simulation rule. The shock occurs in the first quarter of year 11, with the tax rate response beginning in year 13.[44]

The second important policy issue concerns the handling of the monetary response to this fiscal intervention. The Keynesian macroeconometric models of the 1960s and 1970s usually specified interest rates as exogenous policy variables in spite of their key roles as equilibration mechanisms in dynamic economies. Monetary policy, however, is not usually considered as operating directly on these variables, or at least not without some notion of constraints operating on the feasible choices of these variables. Rather than

43. It is interesting and worthwhile to consider what mechanisms exist that prevent economies from maintaining policies that imply an exploding path for debt or net foreign liabilities. Calibrated correctly, a mechanism such as a country-risk premium can be used to unify the mathematical notion of instability with the economic idea of unsustainability. Under the assumption that governments respond in a reasonable fashion to increasing costs of government finance, the country-risk premium could be the device that forces governments to choose, in their own best interests, a sustainable policy regime. As Dornbusch (1987) has noted, issues in this general area have become prominent in the academic literature. See Bhandari *et al.* (1989) for an example in the context of an intertemporal growth model. For this paper, we have omitted the country-risk premium from the dynamic simulation. The feedback mechanism from policy to the interest rate and then to potential output is numerically difficult and it is not always easy to interpret in simulation results. Thus we leave this issue to a later paper.

44. The simulations were conducted using the mainframe computer software package TROLL on an IBM 3090 running under MVS/AX. TROLL's forward-looking simulator uses a slightly modified version of the Fair-Taylor (1983) algorithm. To speed the simulations we have written several TROLL macros that, among other things, provide starting values and terminal conditions for the forward-looking simulator.

associate monetary policy with the specific values of endogenous variables -- like interest rates and exchange rates -- we identify monetary policy with reaction functions. The monetary authority has targets, but does not have perfect foresight and operates under constraints. Accordingly, monetary policy actions are modelled by specifying: (i) a target level for a nominal variable (in this case nominal money growth); (ii) a short-term instrument through which policy is deemed to operate (the nominal interest rate, RN); and (iii) a weighting factor within a quadratic loss function that measures the monetary authority's willingness to tolerate interest rate volatility in order to attain the desired target sooner rather than later. The exchange rate is an endogenous variable and does not directly enter the monetary policy reaction function. Since the model has significant lags between monetary conditions and their effects on intermediate targets, the monetary authority must look ahead if it wishes to achieve its longer-term goals with minimum disruption to the economy. Given the lags, the monetary authority will generally not try to attain its underlying target at all times in the face of shocks, but will tend to choose relatively smooth, short-run paths for the nominal interest rate. Such paths minimize the real adjustment that the economy must undergo to re-attain the nominal target following a shock.

The foregoing illustrates that in all fiscal policy experiments there are in fact two shocks at work: the fiscal shock and the monetary response. The short-run "government expenditure multiplier" will always be a function of both. Consequently, the oft-heard claim that "monetary policy is held constant" for a given shock is open to interpretation. Does monetary policy being held constant during the course of a shock mean that model responses are constrained to the same *instrument* or *control variable* settings as in the base case, or does it mean that the same reaction function is used? In experiments with forecasting models, it has been quite common to assume that the monetary authorities target the nominal or real interest rate. In models with constraints imposed by theory, arbitrary restrictions on the values of endogenous variables must be confined to limited periods.[45] We therefore prefer

45. McCallum (1981) has made this point in the context of a discussion of the feasibility of nominal interest rate pegging. See also Coulombe and Fortin (1989).

to think of "unchanged monetary policy" as analogous to an unchanged reaction function. This approach specifies policy as a set of rules where policy instrument settings are contingent on the short-term dynamics of the economy but always subject to achieving a longer-term goal.

For the fiscal shocks in this paper, we assume that the monetary authorities allow some accommodation in the short run (Figure 3). Over the medium term, interest rates are set to return money growth to a rate consistent with zero inflation. The choice of money growth as the intermediate target and the provision for some short-run accommodation of the shocks facilitates comparisons with simulations from other models, where some form of short-run accommodation is usually allowed.[46] These policy assumptions produce a government expenditure multiplier that is positive for the first four years of the simulation (Figure 2). To restrain inflationary pressures, monetary conditions have to tighten at some point. This ends up putting a small cycle in the economy. The combined effects of higher interest rates and a stronger dollar work to reduce excess demand below control by the fifth year. The ratio of government debt to income, shown in Figure 1, displays a substantial amount of persistence, with the cycles in the debt ratio damping at a constant rate. This is a consequence of our backward-looking tax rule and is characteristic of autoregressive adjustment mechanisms. This mechanistic response contrasts sharply with the dynamics of much of the rest of the model, where the flexible expectations mechanism ensures that shocks do not persist unrealistically.

One objective of this section is to explain how a new steady state is achieved following a shock. Before doing so, however, we must describe briefly some of the dynamic features of the model. The expectations-generation features were described above and are an important source of price stickiness. There are other sources of "Keynesian" dynamics. These include wage and price equations written in log-levels that are consistent with models of costly

46. We have conducted empirical work using vector autoregressions on quarterly data for Canada that show that positive government expenditure impulses have been associated with *decreases* in interest rates and increases in money growth, Moreover, this result appears to be quite robust to the selection of candidate variables in the VAR. One might conjecture that this result is picking up the fact that monetary and fiscal policy in Canada have tended historically to work in the same direction. In any case, the initial decline in nominal interest rates in Figure 3 is very small and short lived.

price adjustment such as those of Rotemberg (1982) and Cozier (1989).[47] The final form is reminiscent of the Blanchard (1986) model except that there are leads in our equations similar to those in our dynamic consumption equation. There are also "demand gaps" in both the price and wage equations, with the relative influence of the gaps being a function of the nature of the shocks. In the long run, of course, the real wage equals the marginal product of labour. A special case of these equations is the traditional accelerationist Phillips curve.

Investment and employment decisions depend upon expected profitability. If firms expect high levels of demand in the future, they will increase the demand for labour and capital goods. As discussed in section 3, we assume that there are quadratic adjustment costs to changing both of these inputs, with capital being the more costly. Domestic firms are assumed to use both domestically produced and imported capital. The shares of these two investment goods in the aggregate capital bundle are assumed to be functions of the real exchange rate. Households are also assumed to consume a bundle of imported goods. This means that the share of imported goods in the aggregate consumption bundle is a function of the real exchange rate and that aggregate imports are a function of domestic demand and the expected real exchange rate. The level of exports is a function of domestic supply, foreign demand and the real exchange rate expected over the planning horizon.

As mentioned earlier, the simulations reported here assume that there is no endogenous country-risk premium. In this version of the model, the nominal exchange rate satisfies the interest parity condition. This theory of the exchange rate simply states that the expected percentage change in the exchange rate between two currencies will equal the interest rate differential thereby imposing the arbitrage condition that the net return on similar instruments be the same in the two currencies (up to a risk premium). The model uses a mixture of backward- and forward-looking processes for all

47. As an aside we might note that costly *price* adjustment is neither necessary nor sufficient to establish costly disinflation in the form of a price Phillips curve in the usual rate-of-change form. Without extra assumptions, one needs costly *inflation* adjustment. This point is cogently explained by Buiter and Miller (1985), pp. 15-21.

measures of expectations. If expectations were perfectly forward-looking, the exchange rate would behave in a fashion similar to the Dornbusch (1976) overshooting model: shocks would result in an immediate jump in the exchange rate so as to allow asset markets to be continuously in equilibrium; thereafter the exchange rate would glide smoothly toward its new long-run level in order to establish goods market equilibrium. The extent of foresight can be varied by adjusting the weight on the perfect-foresight solution, thereby determining the extent of jumps in the nominal exchange rate. However, there has to be some weight on the perfect-foresight solution for the exchange rate to find a path consistent with convergence to an equilibrium (i.e., for the model to be dynamically stable). In this sense it is the exchange rate that is the fundamental price that moves the economy back to equilibrium.

In the combined expenditure/debt-ratio shock, the exchange rate appreciates in the very short run and then depreciates in the long run (Figure 4). This path reflects the simple Keynesian notion that excess demand causes an appreciation as well as the idea that in the short run government expenditures have to crowd out trade with foreigners (Figure 6). Indeed, this short-run crowding out of real net exports is essential to re-establish a new stock-flow equilibrium with higher foreign debt: to reduce net foreign assets as desired, domestic absorption must exceed its equilibrium value in the short run. This means the net export position must deteriorate temporarily. This, in turn, requires a short-run appreciation of the exchange rate. In the long run, however, a trade balance improvement is required and with it a long-run depreciation of the real exchange rate. Thus the real exchange rate *must* initially move in the opposite direction from its long-run solution, before turning around.

The short-run expansionary impact of the shock is shown in Figure 2. Note that the sum of private and government goods consumption increases in the short run, although private goods consumption falls somewhat. The combined effects of aggregate demand, interest rates and exchange rates result in an increase in investment in the short run (Figure 8). The pass-through of exchange rates to the relative prices of imported consumption goods and imported capital goods is indicated by LRPC and LRPINV in Figure 9. In the long run, the terms-of-trade improvement results in a slightly lower capital stock and a slightly lower level of real output (Figure 2). The real wage displays virtually no cyclicality, a result that is consistent with empirical findings (Figure 10).[48]

These results show the contribution of models with well-specified steady-state conditions. The IS-LM-BP model would have treated expansionary fiscal policy as just another (pure flow) injection of expenditure and omitted the effects of stock equilibration. The medium-term depreciation of the exchange rate, the below-control swing in capital investment and the negative shock-minus-control effect on steady-state output would all have been missed by a model that omitted the stock-flow dimension of equilibrium in the economy.

48. See for example Bils (1985) and Keane *et al.* (1988)

Figure 1: Government Debt/Nominal GDP

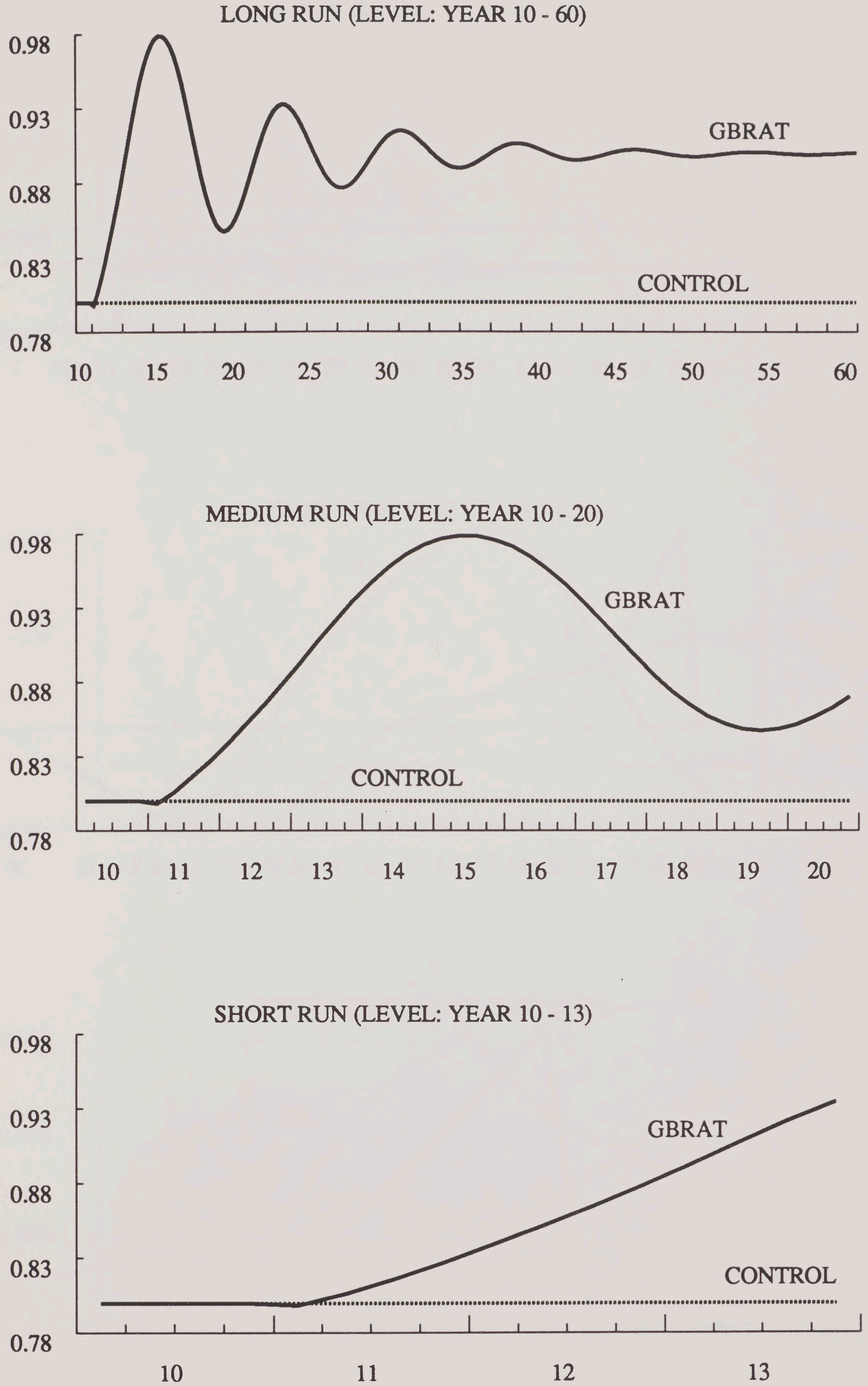


Figure 2: Shock minus control for real GDP

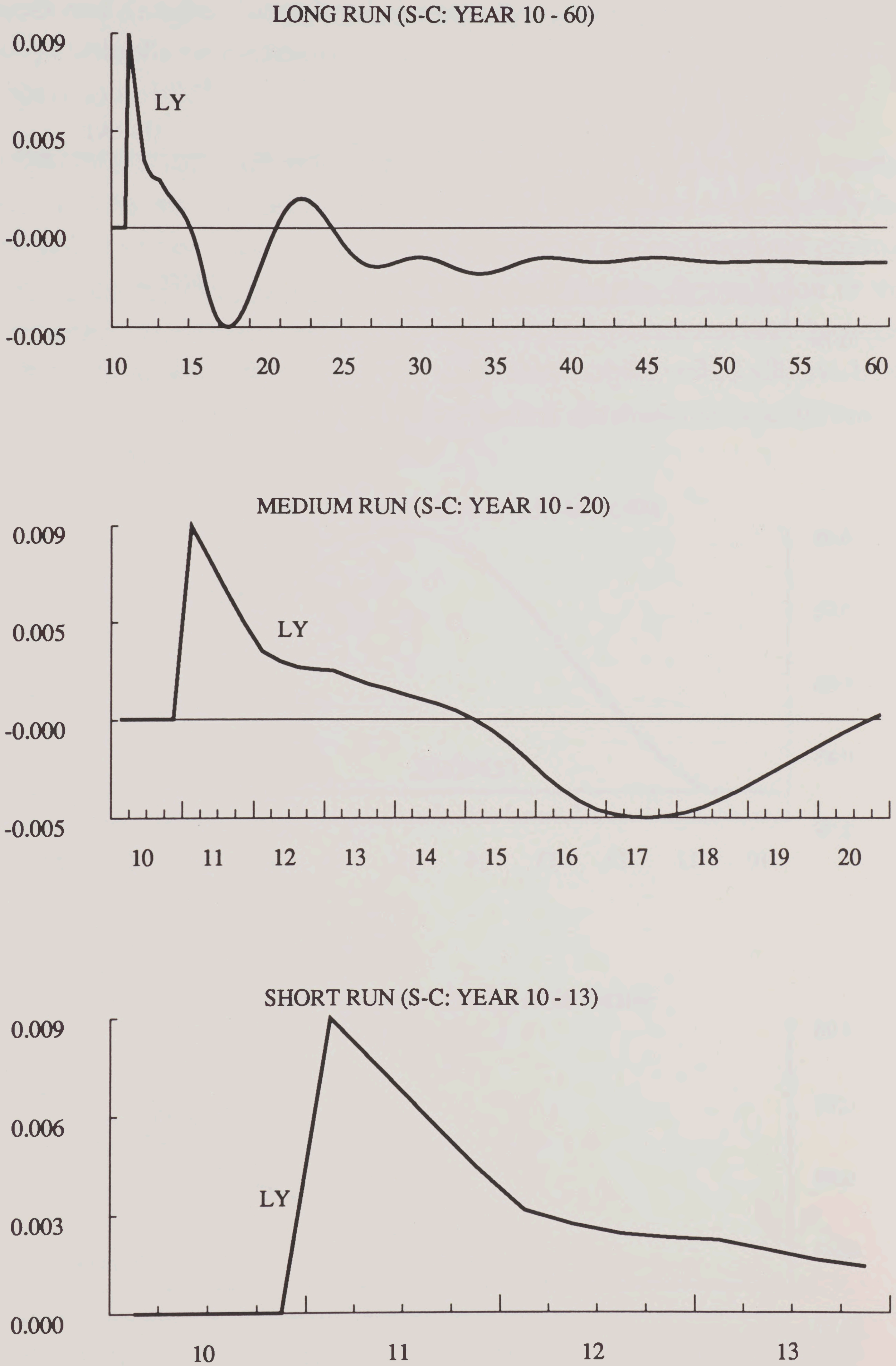


Figure 3: Shock - control for money growth and the nominal interest rate

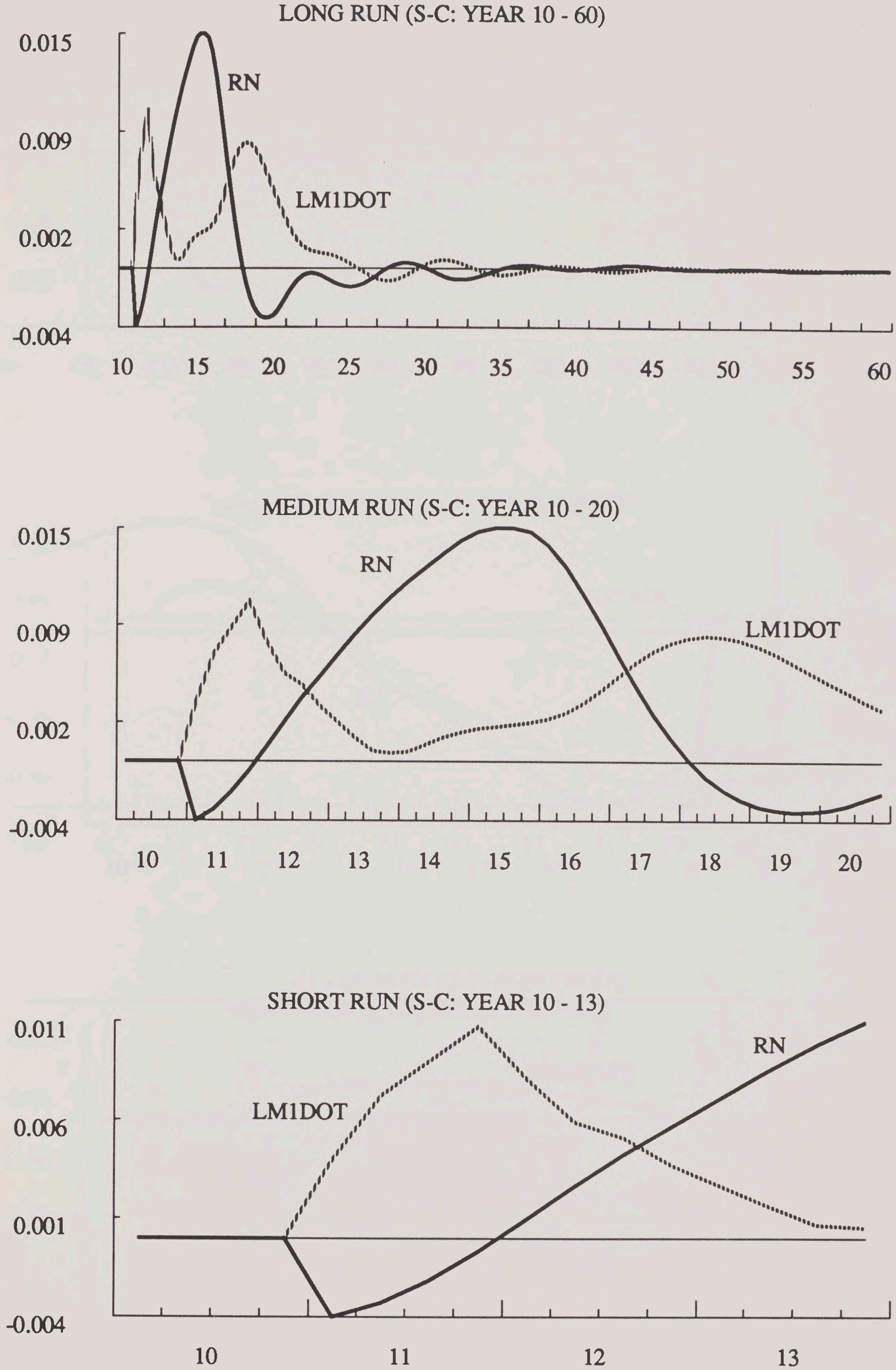


Figure 4: Shock minus control for the real exchange rate (+ depreciation)

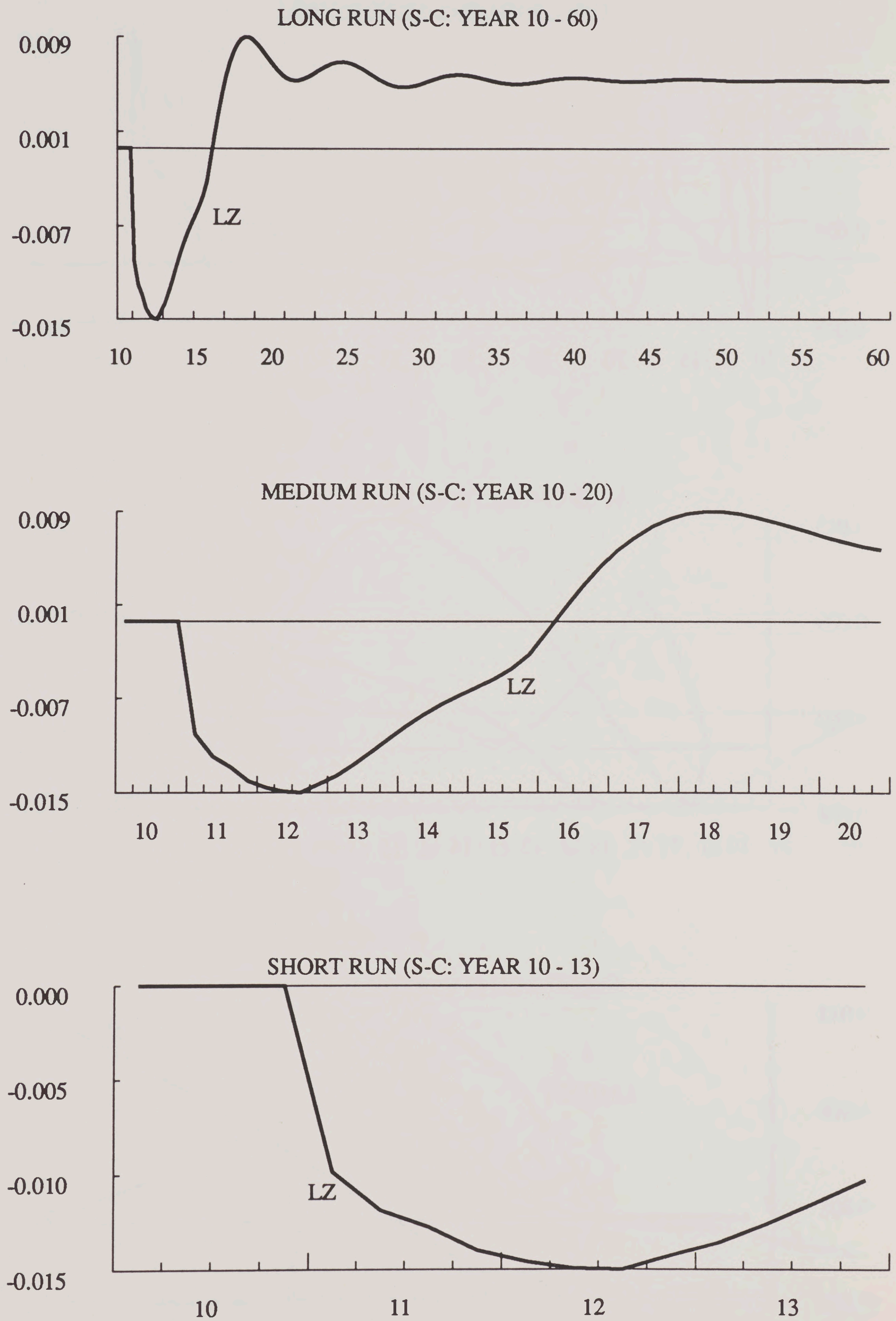


Figure 5: Net Foreign Assets/Nominal GDP

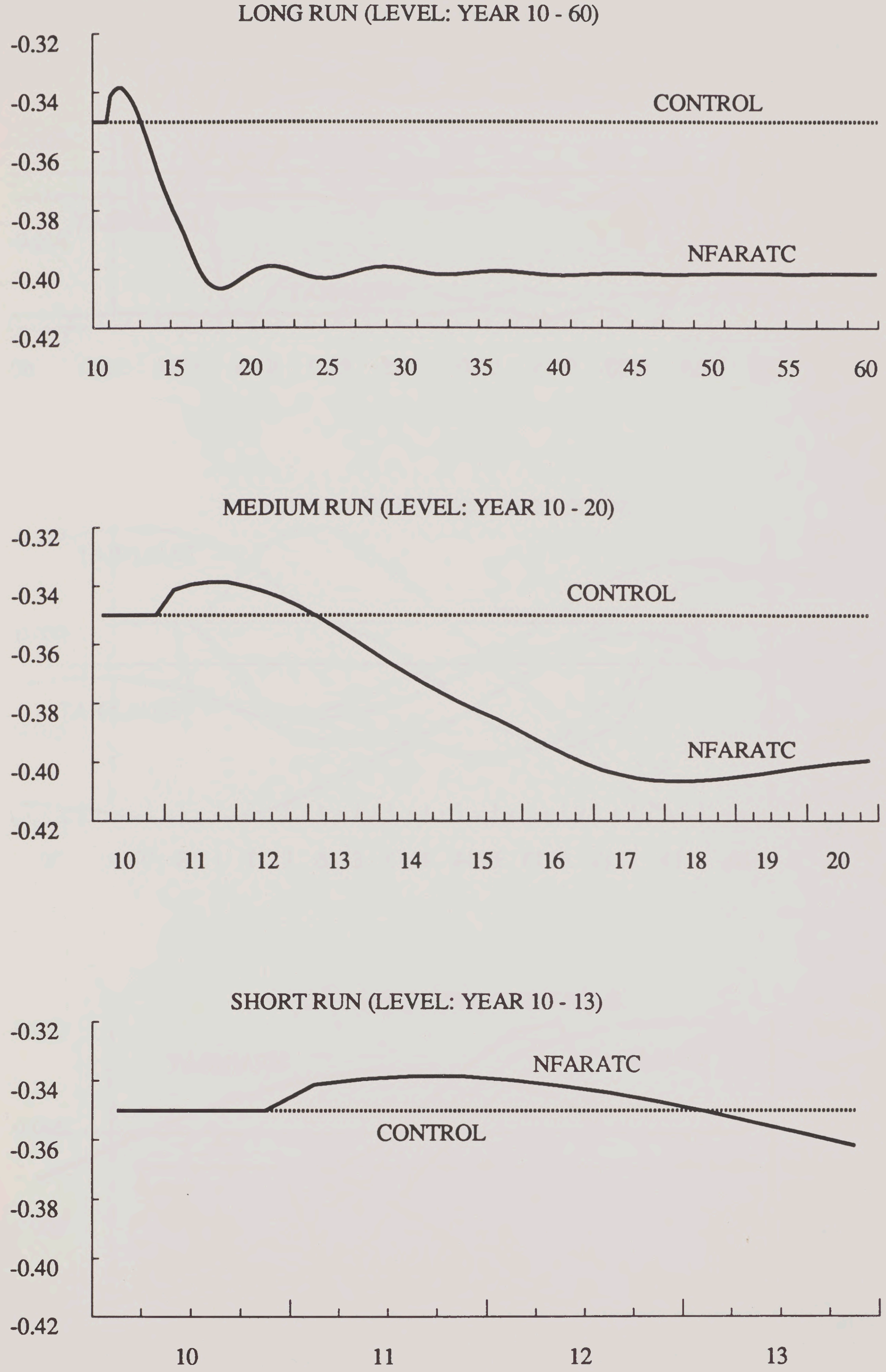


Figure 6: Shock minus control for CBALNRAT, NFAINRAT, TBALNRAT

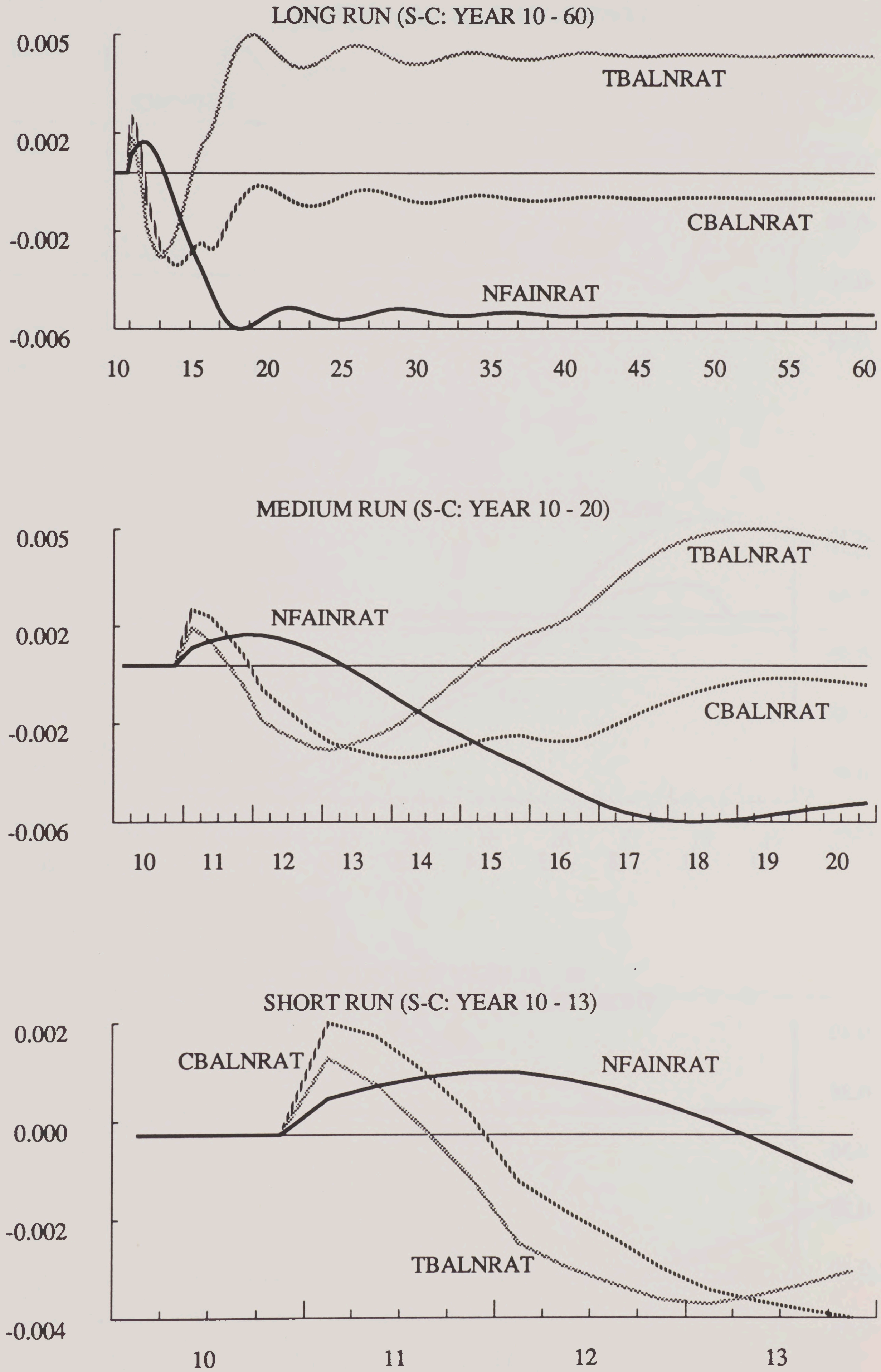


Figure 7: Shock minus control for LZ, LX, LM

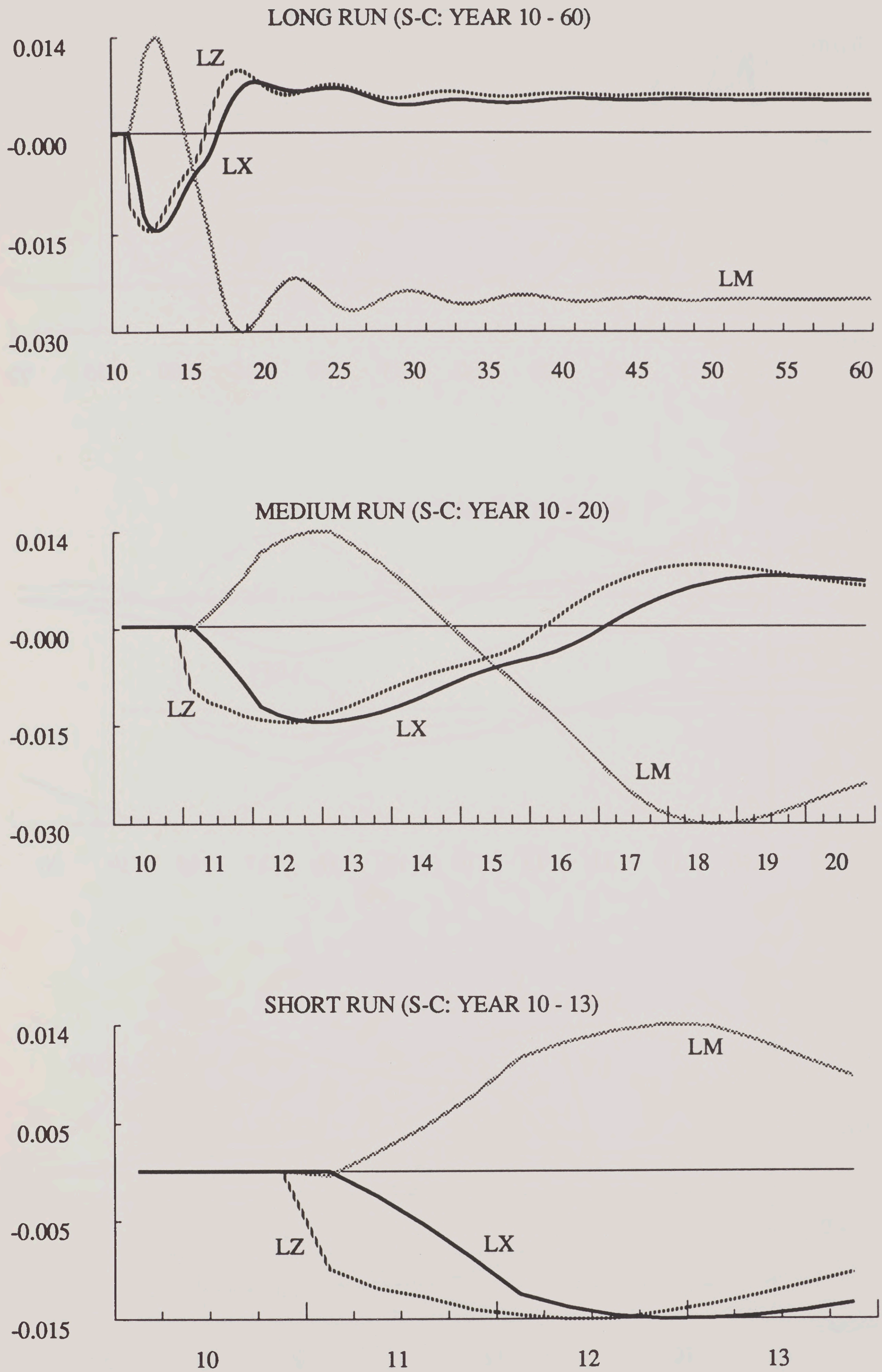


Figure 8: Shock minus control for LC, LK, LY, LINV

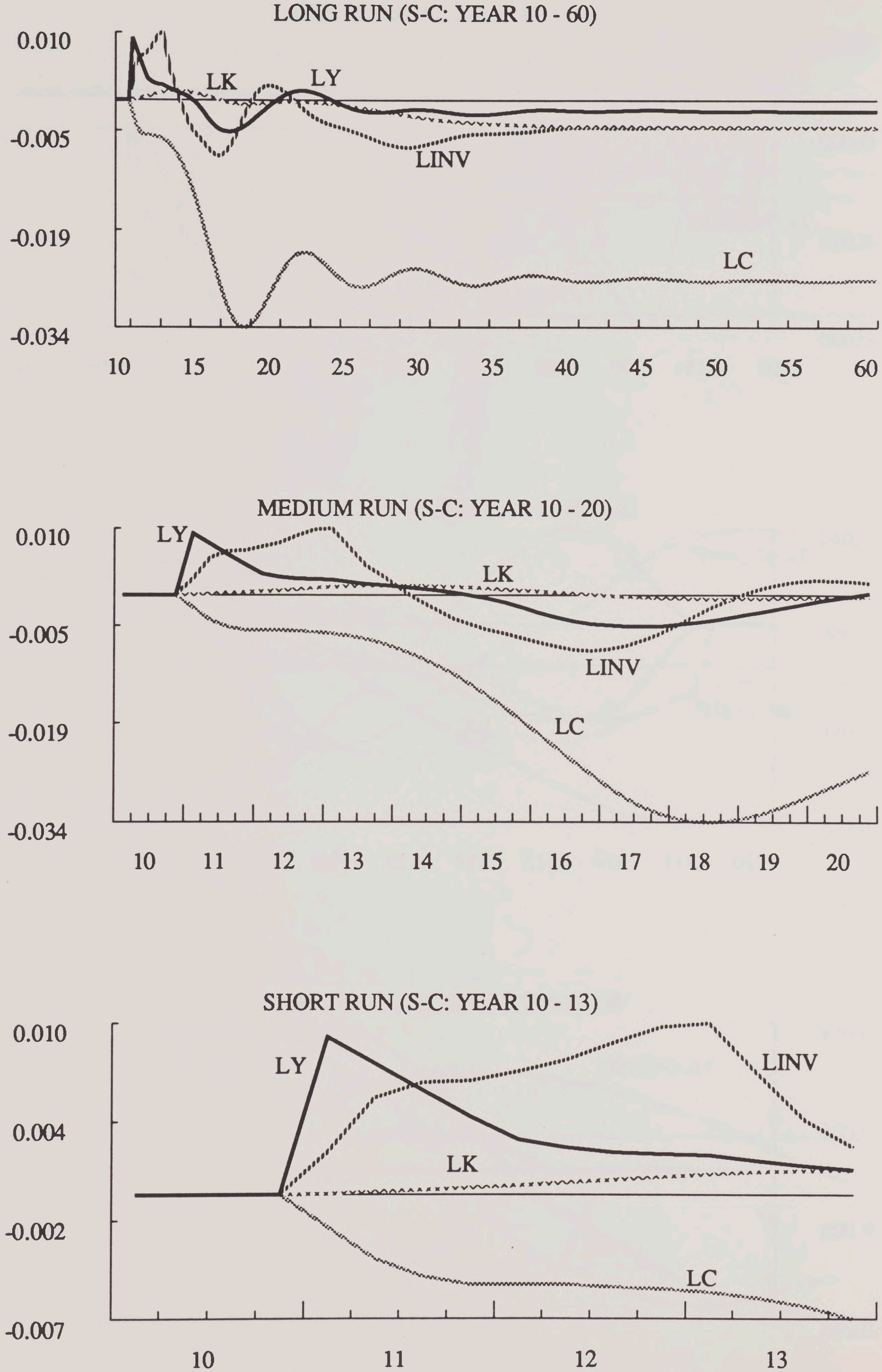


Figure 9: Shock minus control for LRPINV, LRPC, LTOFT

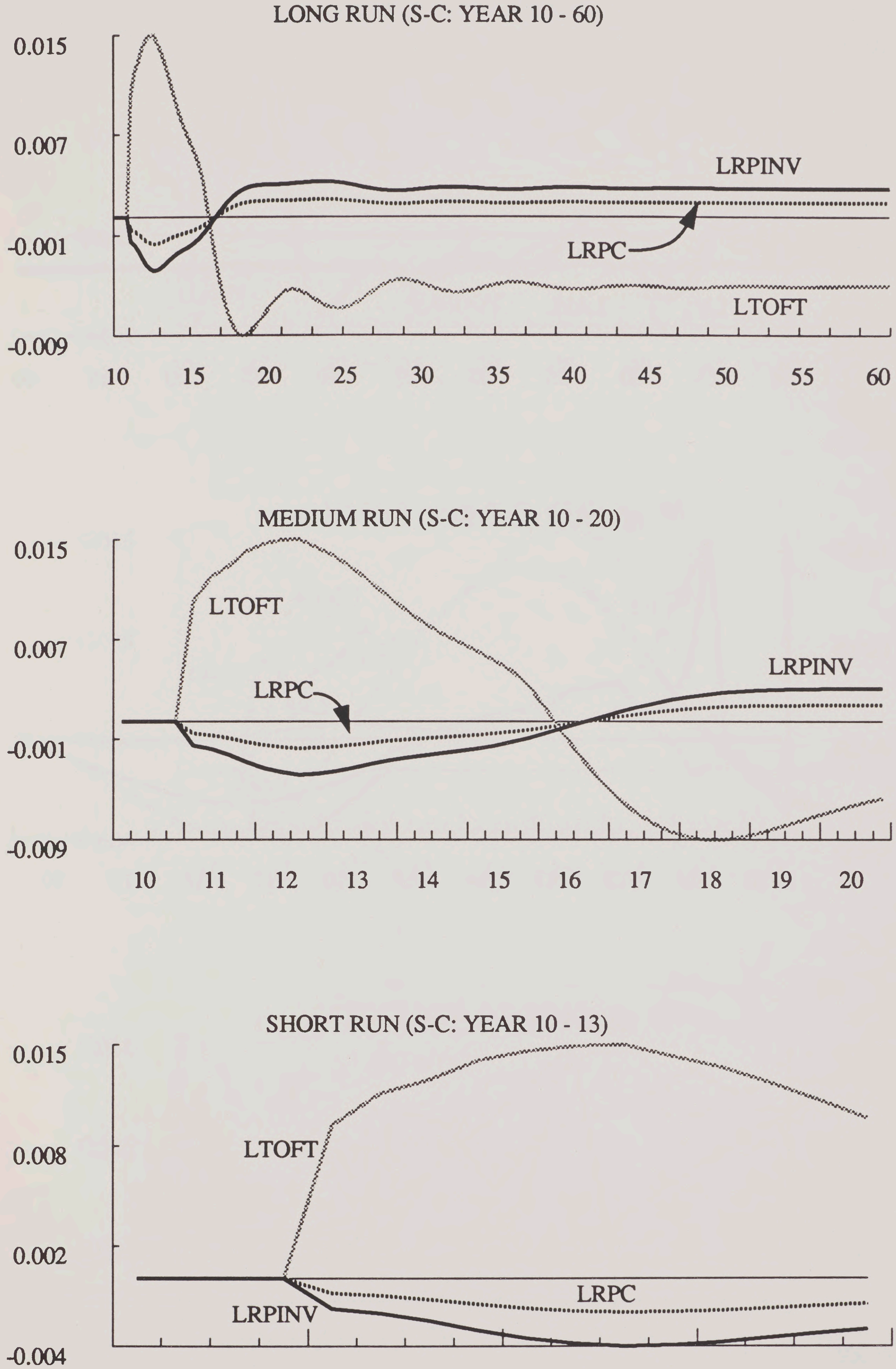
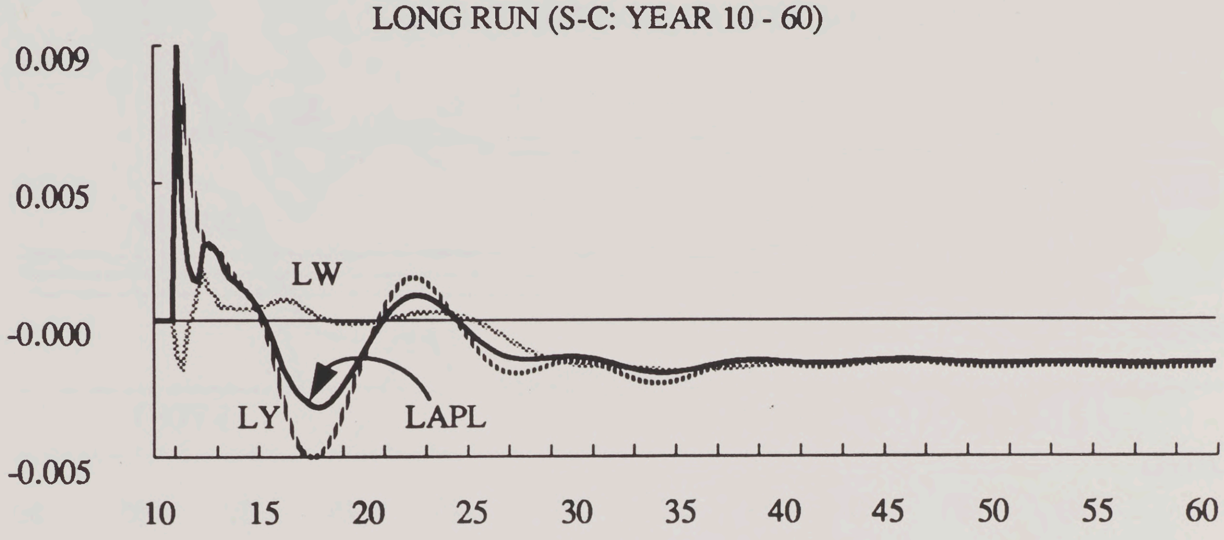
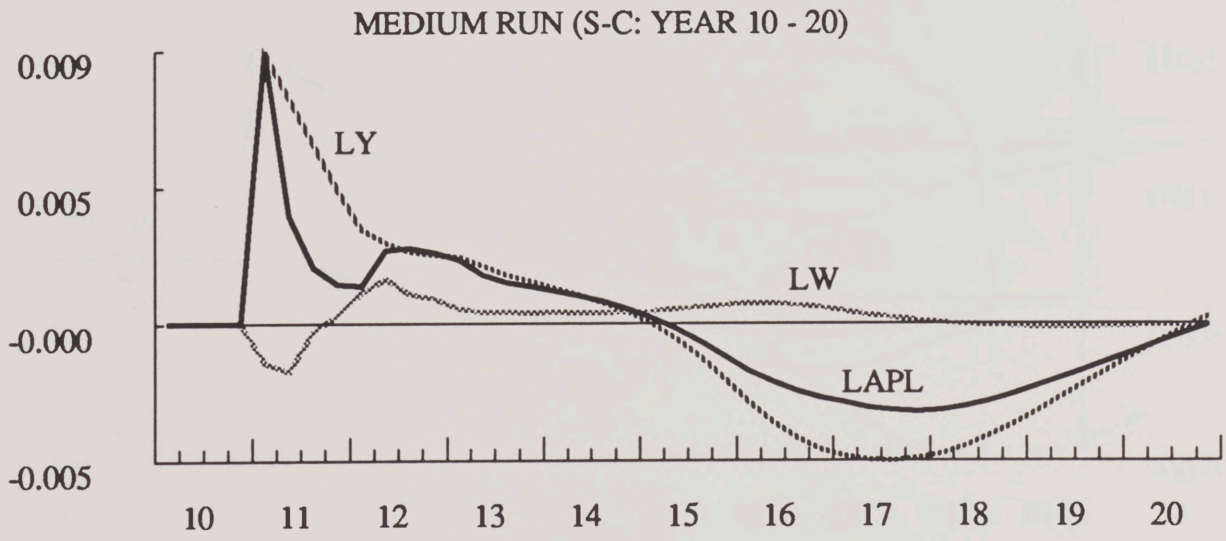


Figure 10: Shock minus control for LY, LW, LAPL



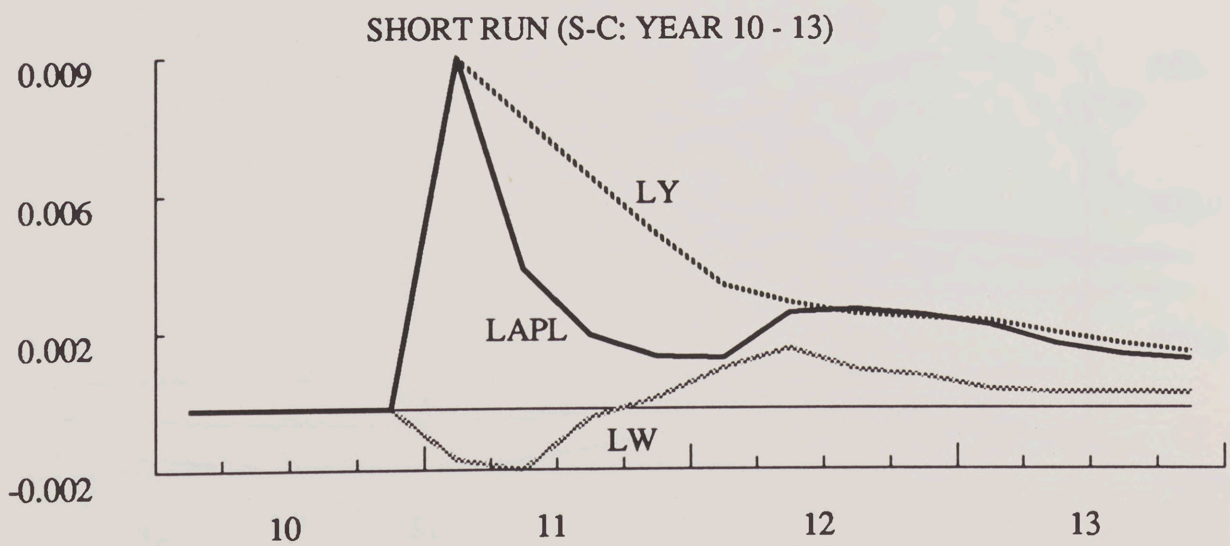
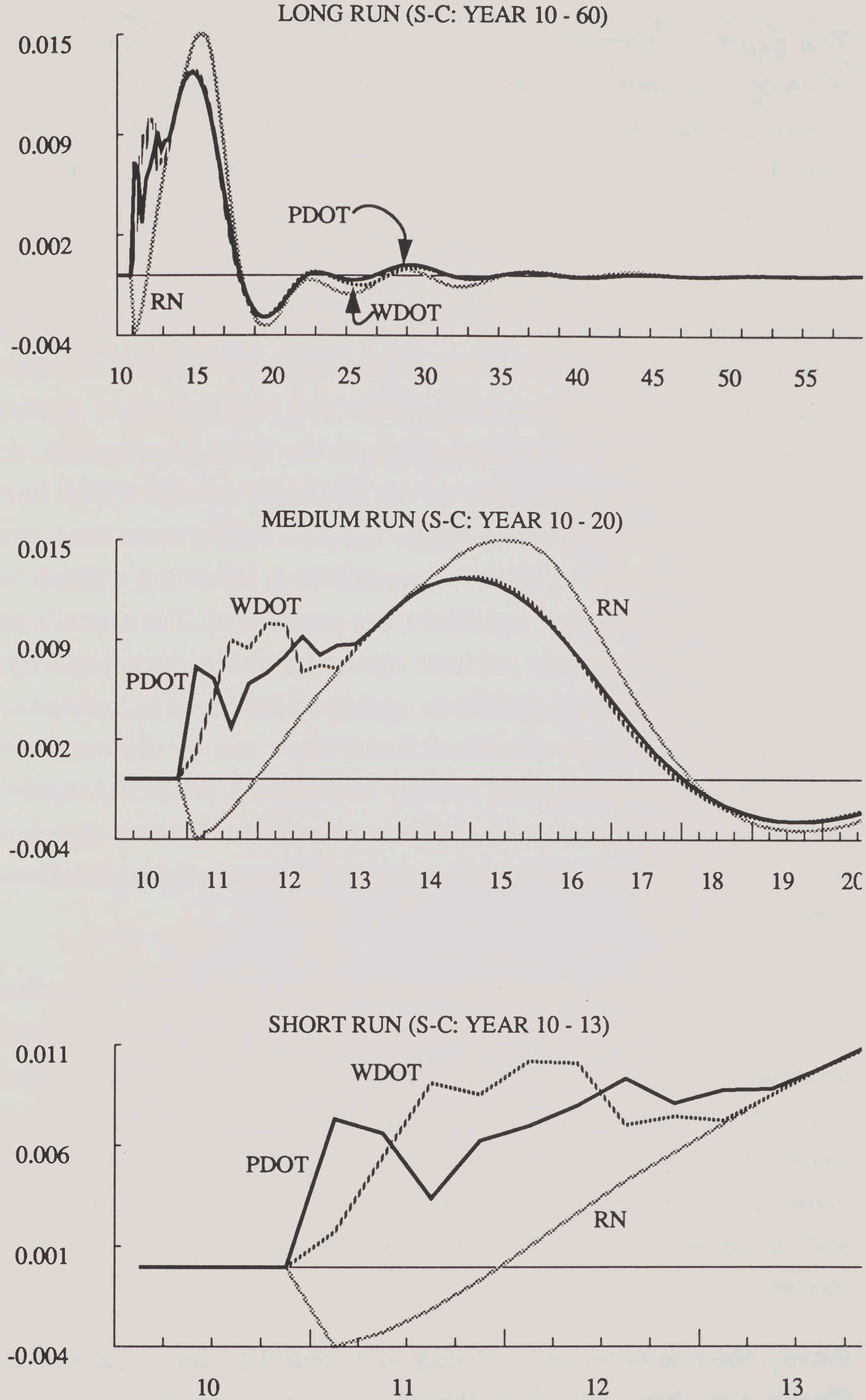



Figure 11: Shock minus control for PDOT, WDOT, RN



# 5. CONCLUDING REMARKS

This paper has presented some comparative static and dynamic responses of an open economy model to a range of fiscal interventions. The CORE model is a relatively small, dynamically consistent model of an open economy. It embraces the neo-classical growth model and buttresses it with meaningful asset accounts, stock-flow dynamics, adjustment costs and expectational dynamics.

It was demonstrated that it is important to understand the steady-state implications of shocks in order to trace out meaningful medium-term dynamics. The significance of this point was highlighted, in particular, by the response of the real exchange rate to the fiscal intervention, since the standard short-run prediction of the IS-LM-BP model would have been completely misleading over a longer horizon. At the same time, constraining model responses to obtain an equilibrium following a shock begs the question of how the new equilibrium is established. The model's attention to the difference between intrinsic dynamics and expectational dynamics addresses this issue. Shocks where agents are assumed to anticipate or correctly identify the hypothesized disturbance can be differentiated from those that are not anticipated or fully understood. While the capabilities of the model in this regard were not demonstrated in dynamic simulation, the flexibility of the model to handle a variety of assumptions was discussed at some length.

Care was taken to distinguish the methodology necessary for medium-term forecasting and policy simulation models from that of macroeconometric models used only for short-term forecasting. Most policy simulation experiments have significant medium-term consequences. (If this were not true, policy-making would be a good deal simpler.) And, most medium-term projections must deal with important issues of stock adjustment. Analyzing such issues requires attention to the intertemporal aspects of economies and models.

Finally, the role of monetary policy as a control problem was underscored. This, in turn, brought to light the role of policy rules, as opposed to simple instrument settings, and the fact that the short-run multipliers that are so often the focus of model experiments are conditional on the expectations of the economy's agents and the responses of policy authorities as well as on the shock itself. It was emphasized that if a policy authority has a target for policy outputs (e.g., inflation or the ratio of government debt to income) the sense in which holding instrument settings (e.g., short-term interest rates, or taxes) at their base-case levels is "holding policy constant" is obscured.

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