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The Canadian Experience with Weighted Monetary Aggregates

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ABSTRACT

This paper compares the empirical performance of Canadian weighted monetary aggregates (in particular, Fisher ideal aggregates) with the current summation aggregates, for their information content and forecasting performance in terms of prices, real output and nominal spending for the period 1971Q1 to 1989Q3. The properties of money-demand equations for these aggregates, particularly their temporal stability, are also examined. The major aggregates considered are M1, M2, M3, M2+, and their Fisher ideal counterparts. Also considered are M3+ (which adds near-bank deposits to M3) and two liquidity aggregates, as well as their Fisher ideal counterparts.

Over all, on the basis of the in-sample fit of indicator models, the out-of-sample forecasts by indicator models, the specification of money-demand functions, and the temporal stability of money-demand functions, Canadian simple-sum monetary aggregates appear to be empirically superior to Fisher ideal aggregates. Specifically, broad monetary aggregates are generally the best in predicting inflation, M1 works well in predicting nominal spending, and real M1 is the best predictor of real output. These conclusions generally agree with earlier studies, which have shown that weighted monetary aggregates rarely do better than simple-sum aggregates in predicting major Canadian macroeconomic variables.

RÉSUMÉ

Dans cette étude, les auteurs comparent les agrégats monétaires pondérés (en particulier, les agrégats idéaux de Fisher) aux agrégats traditionnels obtenus par sommation en termes de leur contenu informatif et leur capacité de prédire les prix, la production réelle et la dépense nominale sur la période allant du premier trimestre de 1971 au troisième trimestre de 1989. Les auteurs examinent aussi les propriétés – en particulier la stabilité temporelle – des équations de demande de monnaie ayant trait à ces agrégats. Les principaux agrégats pris en compte sont M1, M2, M3 et M2+, ainsi que leurs variantes établies selon la formule idéale de Fisher. Sont également étudiés l'agrégat M3+ (c'est-à-dire M3 plus les dépôts tenus dans les établissements parabancaires) et deux agrégats de liquidités, de même que leurs variantes établies selon la formule idéale de Fisher.

Si l'on tient compte de l'ajustement des modèles indicateurs sur la période d'estimation, des prévisions de ces modèles en dehors de la période d'estimation ainsi que de la spécification et de la stabilité temporelle des fonctions de demande de monnaie, il ressort que les agrégats monétaires établis par simple sommation sont dans l'ensemble supérieurs, du point de vue empirique, aux agrégats obtenus selon la formule idéale de Fisher. Il

ressort, plus précisément, que les agrégats monétaires au sens large sont généralement les meilleurs indicateurs de l'évolution de l'inflation, que M1 prévoit bien la dépense nominale et que M1 réel est l'agrégat qui prévoit le mieux la production réelle. Ces conclusions viennent en gros confirmer les résultats d'études antérieures montrant que le pouvoir prédictif des agrégats monétaires pondérés surpasse rarement celui des agrégats établis par simple sommation pour ce qui est de la prévision de l'évolution des principales variables macroéconomiques au Canada.

CONTENTS

1.0 Introduction

One of the alternative methods for constructing monetary aggregates that has received much attention in the literature is the method proposed by Barnett (1980), which uses statistical index number theory. His approach makes use of aggregation theory to compute indexes of financial assets that reflect the total utility, relative to some base period, attributable to the monetary services obtained from these assets.

Unlike the simple-summation monetary aggregates, these alternative “superlative” weighted monetary aggregates are derived from the optimization behaviour of economic agents and thus have stronger theoretical underpinnings. However, it is unclear whether their empirical performance is superior to the summation aggregates. Poorer empirical performance could arise, for example, because of difficulties in translating the theory into empirical counterparts.

The purpose of this paper is to compare the empirical performance of Canadian weighted monetary aggregates (in particular, Fisher ideal aggregates) and the current summation aggregates in terms of their information content and forecasting performance for prices, real output and nominal spending for the period 1971Q1 to 1989Q3. (The data on the Fisher ideal aggregates end at this point.) The major aggregates to be considered are M1, M2, M3 and M2+. As well, we consider M3+ (which adds near-bank deposits to M3) and two liquidity aggregates, which we call LL and LL+.

Work on earlier periods by Cockerline and Murray (1981, based on data from 1968Q2 to 1980Q4) and Hostland, Poloz and Storer (1988, based on data from 1969Q1 to 1986Q4) has shown that weighted monetary aggregates rarely do better (and never do much better) than simple-sum aggregates in predicting major Canadian macroeconomic variables. This paper examines whether the addition of data from the late 1980s changes these conclusions.

The Bank of Canada believes that it can best promote good overall economic performance by pursuing price stability. In this regard, in February 1991 the Bank and the Canadian government jointly set out a path for the reduction of inflation. The goal was to have inflation, as measured by the consumer price index (CPI), come down gradually to the midpoint of a 1 to 3 per cent band by the end of 1995. Furthermore, this target band was extended to the end of 1998 in a joint agreement with the government announced in December 1993. In the short run, the Bank concentrates on a core measure of inflation, which is defined using the CPI excluding food, energy and the effect of indirect taxes. Both before and after the adoption of the inflation-control targets, the Bank followed the growth of monetary aggregates very closely for the information they contain about future inflation, nominal spending growth and output growth. Although the Bank pays most attention to core inflation and overall CPI inflation, it is also concerned with the overall inflationary process. Thus it is interested in forecasts of inflation measured by the GDP deflator, as well as in what the forecasted growth of output would imply about the level of excess demand or excess supply in product markets. Given that the model used for producing the staff economic projection has no direct role for the monetary aggregates but has a direct role for the output gap in the inflationary process, it is important to have other forecasting models based on monetary aggregates that give information on both inflation and output growth. The types of indicator models discussed in this paper play this role.

The paper is organized as follows. Section 2 presents a brief literature review of the theory and empirical work on the weighted monetary aggregates, with emphasis on the Canadian evidence and the difficulties in translating theoretical concepts into empirical counterparts. Section 3 presents and discusses further empirical work on short-run indicator models. Section 4 discusses empirical results on the estimation of long-run demand-for-money equations. Section 5 presents tests of causality between the aggregates and selected variables in a vector error-correction model (VECM). Section 6 summarizes our conclusions.

2.0 Literature Review

The monetary aggregates currently used at the Bank of Canada are constructed by the simple summation of their various component assets. However, the components of the aggregates do not have the same degree of substitutability for one another, and some are clearly less liquid than currency and demand deposits. Hence, the simple summation of the various components of the aggregates does not accord with economic theory. This is because simple summation assumes that the assets in the aggregates are perfect substitutes (infinite elasticities of substitution). The alternative methods for constructing monetary aggregates that are examined below address this problem by using economic theory to derive, for components of the aggregates, weights that are consistent with the monetary services they provide.

2.1 Theoretical Construction of Superlative Monetary Aggregates

Consistent with Barnett's (1980) proposal, "superlative" monetary aggregates have been developed that are based on the index number theory.[1] This method defines money as a monetary quantity index. As noted by Barnett, under this approach aggregates are measured in terms of the flow of services that constitute the output of the economy's monetary transactions technology. The flow of monetary services is determined by weighting the quantity of each component asset with its unique rental cost. Second, superlative indexes are exact for flexible functional forms. Thus they avoid the restrictive assumptions required to justify the linear form of the summation aggregate. Third, superlative aggregates internalize the pure substitution effects of changes in user costs such that the index will not change unless an income effect is present. Income effects are reflected in the form of utility or monetary service changes. However, changes in user costs alone can cause simple-sum aggregates to change, even if the

1. Diewert (1976, 1978) introduced these indexes to the literature. He suggests that an index is superlative if it is exact for some aggregator function. In other words, there is a close correspondence between the aggregator function and the index number formula.

value of the underlying monetary services subutility function is unaffected (i.e. the income effect is zero). Thus, the potentially key empirical difference between the two will be changes in the simple-sum measure (akin to a shift variable in a regression) when no change in the economy's monetary service flow has occurred.

Two superlative indexes used in the literature are the (Tornquist-Theil) Divisia and the (chain-linked) Fisher ideal.[2] One advantage of the Fisher ideal index over the Divisia index is that, as an index measured in *levels*, it can handle the introduction of new assets and changes to the characteristics of the financial assets in the indexes. The change in the Divisia index, however, is based on the changes in the logarithms of its components, and because the logarithm of zero is minus infinity, the formula for computing the Divisia index implies that the growth rate of the Divisia aggregate equals infinity when a new asset is introduced. Thus, in a period when a new monetary asset is introduced, one can use the Fisher ideal index by setting the growth rate of the new asset to zero.

2.2 Translating Theory into Empirical Counterparts

In recent years, financial innovations have fundamentally altered the characteristics of many monetary assets. These innovations have increased the liquidity of most of the deposit liabilities of deposit-taking institutions. Such developments are a major reason why proponents of Divisia and other superlative indexes have called for new ways of defining and measuring the monetary aggregates.

Cockerline and Murray (1981) and Fisher, Hudson and Pradhan (1993) have argued that, despite their theoretical appeal, the superlative indexes have a number of drawbacks. First, Cockerline and Murray find that rates posted on savings deposits and other monetary components can exaggerate the effective rate economic agents expect on their investments. They argue that minimum balance requirements for certain accounts, early

2. See Barnett, Fisher and Serletis (1992) for other superlative indexes and the exact formula for constructing each one.

encashment penalties on some fixed-term assets, and other service charges all tend to reduce the measured own rates of return on monetary assets. These measurement problems are complicated further by the possibility that financial institutions cross-subsidize activities, such that service fees or interest rates may vary as a customer does other business with the institution.

A second measurement problem is that calculating user costs will be complicated by aggregating across assets with different maturity dates. Cockerline and Murray explain that if the yield curve is downward-sloping, say as a result of future inflation being expected to fall, then current short-term interest rates will be higher than long-term rates. Hence, the rental price of some of the monetary assets may be negative. However, since the rental price, $((R_{Bt} - R_{it})/(1 + R_{Bt}))$, is used in the superlative index as a measure of liquidity, it is meaningless when the rental price is negative. Cockerline and Murray address this problem by adjusting all the own interest rates with maturity greater than one year to an effective 91-day holding period return.[3]

Third, the method of constructing the superlative indexes assumes that economic agents hold the optimal values of assets in their portfolio and makes no allowance for portfolio adjustment costs. However, in practice, investors constantly readjust their portfolio holdings in response to changes in interest rates. Since the superlative method measures user cost of an asset by the difference between the rate on a benchmark asset and its own rate of interest, and the portfolio adjustment costs are not captured in the interest rates, the "true" user cost is underestimated. Spencer (1994) suggests that a theoretical and consistent way of addressing the portfolio-adjustment-cost problem is to assume that economic agents optimize the current distribution of monetary assets, not with respect to the actual returns observed, but with respect to permanent or trend returns on each asset. Spencer defines the trend return of an asset, which is more smooth than the

3. Barnett, Fisher and Serletis (1992), Thornton and Yue (1992) and Farr and Johnson (1985) suggest other ways to avoid this problem.

actual return, as a weighted average of the current and past returns of the asset. In order to allow for portfolio disequilibrium, Spencer recalculates the Divisia weights using the smoothed user costs derived from trend returns. When using U.K. data, Spencer finds the measure of Divisia M4-aggregate to perform better empirically than its simple-sum counterpart.

Fourth, the calculation of the user cost of any asset assumes that the benchmark asset is completely illiquid. This implies that an asset traded in secondary markets does not qualify as a benchmark, because a secondary market would enable that asset to be readily converted into more liquid assets that could be used for transactions. In practice, it is difficult to come by such an asset. Also, the benchmark asset must be chosen such that the user costs are non-negative.[4] Cockerline and Murray argue that a negative user cost would imply that economic agents would be prepared to sacrifice some of the returns on a purely non-monetary asset in order not to receive monetary services.

Fifth, Fisher, Hudson and Pradhan mention that superlative indexes have not been widely accepted, for one, because of their interpretation in the short run. The weights on the component assets (i.e. the expenditure shares, S_i) are very sensitive to changes in interest rates. A rise in interest rates will increase the user cost of currency and therefore lead instantaneously to a higher weight. However, as the higher interest rates cause investors to hold less cash in their portfolio, the weight for currency will fall over time. Owing to this lag, current weights are not optimal, unless investors adjust their portfolio instantaneously with changes in interest rates. Furthermore, in the short run, in a situation in which the amount of currency held by economic agents grows more rapidly than the amount of interest-bearing deposits, an increase in interest rates will instantaneously increase the weight for currency and reduce that on interest-bearing assets, thereby leading to an increase in the superlative index growth rate. As a result, Fisher, Hudson and Pradhan note that the superlative index could be a misleading indicator of the stance of monetary policy, in the short run.

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4. Barnett, Fisher and Serletis (1992, p. 2105, footnote 31) suggest a way to guarantee this.

2.3 Empirical Evidence for Canadian Monetary Aggregates

Cockerline and Murray (1981) were the first to apply Canadian data to evaluate the empirical properties of Divisia monetary aggregates. The authors compare the performance of the Divisia and the summation aggregates in terms of their information content, money-income causality and stability in money-demand equations. Their study finds that although the Divisia aggregates follow smoother time paths than the summation aggregates, their overall performance is unclear. For example, the Divisia aggregates contain less information on contemporaneous and future levels of income than the summation aggregates. In causality tests, the study also finds the Divisia aggregates to be inferior to their summation counterparts. However, the demand functions for the Divisia aggregates are found to be more stable than those of the summation aggregates.

After the study by Murray and Cockerline, the Bank of Canada switched to maintaining a data base for the Fisher ideal index rather than the Divisia index. The switch was due to the fact that Divisia indexes cannot handle the introduction of new assets.

Hostland, Poloz and Storer (1988) use Canadian data to conduct studies on the information content of the Fisher ideal monetary aggregates. Consistent with the earlier results of Cockerline and Murray, the authors' work finds that the Fisher ideal monetary aggregates generally contain less information than the summation aggregates. It compares the information content of alternative monetary aggregates with respect to nominal GDP. The study looks at 46 monetary aggregates, of which half are summation aggregates and the other half Fisher ideal indexes of monetary services, and finds M1 to be the most informative aggregate for both nominal and real GDP.[5] Fisher ideal M1ALD (M1 plus non-personal notice deposits at banks) is found to be the most informative for prices. Consistent with the results of Cockerline and Murray, the study finds that broad aggregates (M2+, M3BC and M3+) have the highest contemporaneous correlation with nominal GDP, while narrow aggregates (M1 and Fisher ideal M1ALD) have the most leading information about nominal GDP. Hostland, Poloz and Storer also find that the superlative monetary aggregates add little information to the summation aggregates. Hence there is no significant information loss in using summation aggregation.

Serletis and King (1993) examine the empirical relationships between monetary aggregates (summation or Divisia), income and prices in Canada.[6] The study finds none of the monetary aggregates to be cointegrated with the price level or nominal income. Based on the criterion of the smallest test tail area in Granger-causality tests, Serletis and King find the growth rate of simple-sum M2+ to be the best leading indicator of inflation; the growth rates of the Divisia aggregates and simple-sum M1 appear to be more useful than the growth rates of the other simple-sum aggregates for anticipating future movements in nominal income. Simple-sum M1 and Divisia M1 are the best leading indicators of real output.

Chrystal and McDonald (1994) compare the summation monetary aggregates to the Divisia aggregates for different countries, including

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5. The 46 aggregates are: (1) currency, (2) monetary base, (3) M1, (4) M1ALD (M1 plus non-personal notice deposits at banks), (5) M13 (M1ALD plus daily interest chequing and personal savings deposits at banks), (6) M2 (M13 plus personal fixed-term deposits at banks), (7) PHMS (M2 plus non-personal fixed-term deposits at banks), (8) PHMSB (PHMS plus bankers' acceptances), (9) PHMSBC (PHMSB plus commercial paper), (10) M3 (PHMS plus foreign currency deposits of residents booked in Canada at banks), (11) M3B (M3 plus bankers' acceptances), (12) M3BC (M3B plus commercial paper), (13) LL (M3BC plus Canada Savings Bonds, treasury bills held by the public, and 1- to 3-year government of Canada bonds), (14) - (24) are M1 through LL plus corresponding deposits held at trust and mortgage loan companies, credit unions, and caisses populaires (mnemonics add a "+") and (25) - (46) are Fisher ideal monetary indexes corresponding to the same level of aggregation for M1 through LL and M1+ through LL+.

6. What Serletis and King refer to as Divisia aggregates are actually Fisher ideal aggregates.

Canada. Their study involved the application of non-nested testing methods to the St. Louis equation to determine the relative information content of alternative monetary aggregates. Their test compares this equation, on the one hand, with simple-sum money and, on the other hand, with Divisia indexes or the currency equivalent index derived by Rotemberg, Driscoll and Poterba (1995). The study shows that summation M1 has the highest information content for nominal GDP, closely followed by Divisia M1.[7] Their results also show that, although summation M2, M3 and L do not have significant information content for nominal income, all their Divisia equivalents do. (Chrystal and McDonald do not examine the “plus” aggregates, which include deposits in non-bank financial institutions.)

In order to carry out causality tests, Chrystal and McDonald use the Johansen and Juselius (1990) methodology to estimate the number of cointegrating vectors in a vector autoregressive model (VAR) comprising the various measures of money, real GDP, the GDP deflator and the treasury bill rate. They also estimate a vector error-correction model (VECM) implied by the cointegration results. The VECM is then subjected to exclusion tests on the lags of each the differenced variables and on the lagged cointegrating terms. The exclusion tests are carried out using the linear Wald statistics, which have a central chi-squared distribution.

Among the simple-sum and “Divisia” aggregates they examined, constructed from data from Canadian chartered banks, Chrystal and McDonald (1994) found that (at a 5 per cent level of significance) only simple-sum M1 caused real GDP and only simple-sum M2 caused the GDP deflator.

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7. What Chrystal and McDonald call Divisia aggregates for Canada are actually Fisher ideal aggregates.

3.0 New Empirical Evidence on Short-Run Indicator Models

In this part of our study we examine empirical evidence up to 1989Q3 (the last date available for all the Fisher ideal aggregates) on the following:[8]

- the aggregates with the best fit in indicator model equations for inflation (as measured by the CPI, CPI excluding food and energy, and the GDP deflator), the growth of nominal GDP, and the growth of real GDP (section 3.2)

- the aggregates with the lowest root-mean-square errors for one-quarter-ahead out-of-sample forecasts for the five goal variables listed in the previous bullet (section 3.3)

- the aggregates with the lowest root-mean-square errors for multi-period-ahead forecasts for the five goal variables (section 3.4)

3.1 Definitions of the Monetary Aggregates

This study examines seven simple-sum aggregates and their Fisher ideal counterparts. The seven simple-sum aggregates include the three standard chartered-bank-based aggregates (M1, M2 and M3); two deposit-taking institutions aggregates (M2+ and M3+), which add to the bank aggregates the corresponding deposits from non-bank deposit-taking institutions; and two liquidity aggregates (LL and LL+) that add to M3 and M3+, respectively, bankers' acceptances, commercial paper, Canada Savings Bonds, treasury bills held by the non-financial public and 1- to 3-year government of Canada bonds held by the non-financial public. For

8. The Bank of Canada decided to stop constructing the Fisher ideal aggregates in mid-1990. This decision was motivated by three factors. First, the cost of constructing these aggregates, which relates to the cost of constructing some of the component parts and their rental prices was considered to be too high. Second, the failure of any of these weighted-sum aggregates to outperform simple-sum aggregates consistently as indicators argued against their maintenance. Third, the difficulties in translating theoretical concepts into empirical counterparts was thought to reduce the potential empirical gains from superlative aggregates.

comparability with the Fisher ideal indexes, which were constructed in the spring of 1990, these data are taken from the February 1990 issue of the *Bank of Canada Review* for the major aggregates (with the other aggregates constructed by summation). (In particular this means that M2+ does not include money market mutual funds and individual annuities at life insurance companies, items that were subsequently added to its definition.)

The comparable Fisher Ideal indexes, denoted by the suffix FI, are M1FI, M2FI, M3FI, M2+FI, M3+FI, LLFI and LL+FI. The benchmark rate, which by construction was forced to dominate all other own interest rates, is defined as:

R_{Bt} = Max (adjusted 10-year industrial bond rate, 90-day finance company paper rate, adjusted 3-year Canada rate).[9]

Note that the Fisher ideal aggregates are constructed, following one standard practice in the literature, by applying a different index number to official asset collections designated as aggregates by the central bank. Some research, however, has shown that these official groupings fail tests for weak separability, such that even weighted versions of them will perform poorly.[10]

3.2 Data and Summary Statistics

The data for the monetary aggregates are quarterly and are available from the first quarter of 1968 to the third quarter of 1989. Table 1 presents the mean and standard deviation of the year-over-year growth rates of the aggregates and selected macroeconomic variables. As shown in the table, with the exception of simple-sum M1, the simple-sum aggregates, on average, grew faster than their Fisher ideal counterparts. Also the standard deviations show that the growth rates of the simple-sum aggregates fluctuated more than their Fisher ideal aggregates.

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9. The 10-year industrial bond rate is the maturity-adjusted and liquidity-adjusted McLeod Young Weir index of rates on prime corporate issues, purged of any special features (e.g. low coupons, retractables, convertibles).

10. Belongia and Chrystal (1991) and Belongia (1995) find this problem to exist with the official monetary aggregates of the United Kingdom, the United States, Japan and Germany.

Table 1: Mean and Standard Deviation of the Four-Quarter Growth Rates of the Monetary Aggregates

Simple-Sum Aggregates	Mean	Standard Deviation	Fisher Ideal Aggregates	Mean	Standard Deviation
M1	7.49	4.54	M1FI	7.65	4.41
M2	11.07	3.78	M2FI	8.53	3.27
M2+	11.97	3.51	M2+FI	9.27	3.19
M3	11.12	5.87	M3FI	9.08	3.76
M3+	11.97	5.17	M3+FI	9.74	3.56
LL	12.39	3.64	LLFI	9.83	2.89
LL+	12.73	3.48	LL+FI	10.25	2.89

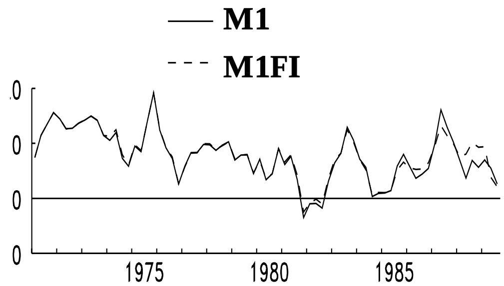
Mean and Standard Deviation of the Four-Quarter Growth Rates of the Goal Variables

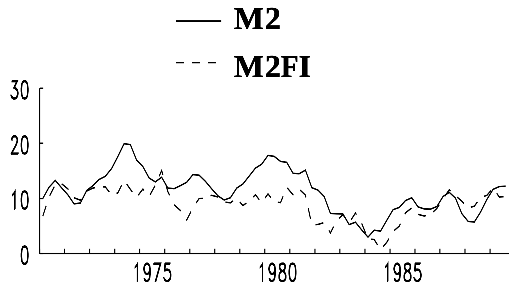
Goal Variables	CPI	CPIXFE	GDP Deflator	Nominal GDP	Real GDP
Mean	4.29	4.17	4.23	8.08	3.85
Standard Deviation	3.30	2.79	3.19	4.13	2.83

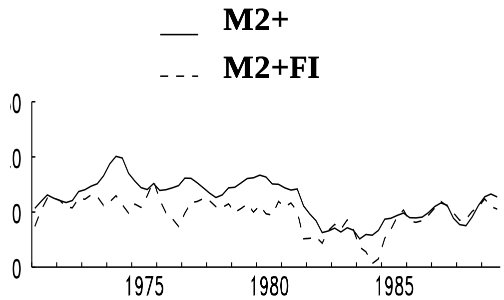
Plots of the four-quarter growth rates of the monetary aggregates are depicted in Figure 1. With the exception of the M1-aggregates, the figure shows the Fisher ideal aggregates to grow more slowly than their simple-sum counterparts.

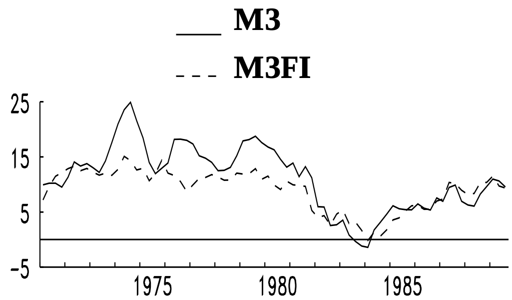
Deregulation in Canada was largely completed by 1967. However, as noted by Freedman (1983) and Fine (1990), major financial innovation — including the introduction of new deposit accounts — took place in the early 1980s. These developments led to a shift out of M1 deposits.

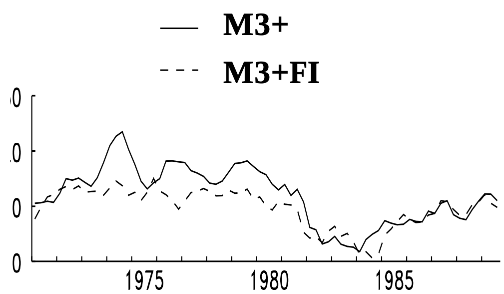
FIGURE 1: The Four-Quarter Growth Rates of the Monetary Aggregates

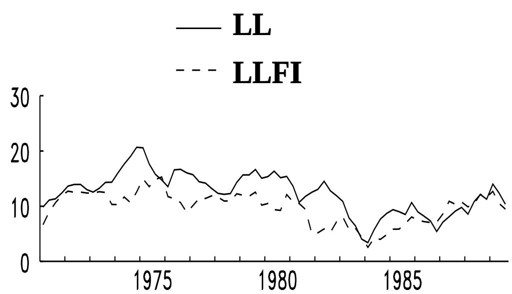

M1
M1FI
1975
1980
1985

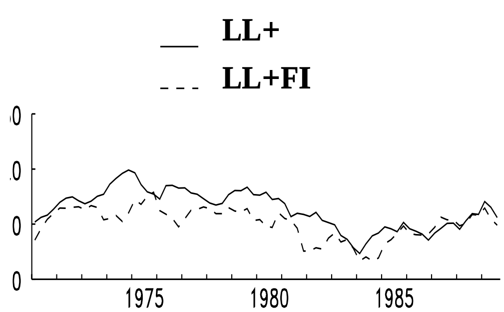

M2
M2FI
30
20
10
0
1975
1980
1985


M2+
M2+FI
1975
1980
1985


M3
M3FI
25
15
5
-5
1975
1980
1985


M3+
M3+FI
1975
1980
1985


LL
LLFI
30
20
10
0
1975
1980
1985


LL+
LL+FI
1975
1980
1985

3.3 Information Content for Five Goal Variables

In this section we use the Akaike information criterion (AIC) to choose the best bivariate indicator models using the growth rates of various monetary aggregates and lags on the dependent variable to explain the growth rates of five goal variables:[11] the CPI, the CPI excluding food and energy (CPIXFE), the GDP deflator, nominal GDP, and real GDP. The sample period is again 1971Q1-1989Q3. The general form of the indicator model is:

$$\Delta g_t = a + \sum_{i=1}^{k} \beta_i \Delta m_{t-i} + \sum_{j=1}^{h} \gamma_j \Delta g_{t-i} \tag{1}$$

where g and m are the logarithms of the goal and money variables, respectively. The parameters k and h are the optimal lag-lengths, chosen over the range 0 to 12 and according to minimum AIC.

Based on the minimum AIC, Table 2 displays the three best indicator models for each of the five goal variables. The models were ranked according to AIC so as to compare our results with those of Murray and Cockerline (1981) and Hostland, Poloz and Storer (1988).[12] For four of the five goal variables, a simple-summation aggregate fits best: M2+ for the CPI and CPIXFE, and M1 for nominal GDP and (expressed in real terms) for real GDP. Only in the case of the GDP deflator does a Fisher ideal aggregate, M3+FI, perform best.

In Table 3 we use Davidson and MacKinnon's (1981) J-test to determine whether the best Fisher ideal model adds explanatory power to a specification with the best summation aggregate and vice versa. For the three inflation models we find that neither the simple-sum model nor the

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11. We use the term "goal variable" to mean a variable that the monetary authorities are interested in, whether they have targets for them or not. As explained in the introduction, the Bank of Canada has inflation-control targets, but in that context it is very interested in the growth of output because of what that implies about the state of the output gap and thus about inflationary pressure.

12. Note that critical values for the differences between AICs do not exist in the literature.

Fisher ideal model can explain the goal variable on its own; both types of aggregates contain useful information. For nominal GDP the two models appear to be so highly collinear that both models fit well, and neither model adds to the other. For real GDP, the Fisher ideal aggregate adds nothing to the simple-sum aggregate, but the simple-sum aggregate adds significantly to the specification of the Fisher ideal model; hence the simple-sum model in this case is dominant. In the context of a St. Louis equation, Chrystal and MacDonald (1994) perform similar tests and find that simple-sum M1 has the greatest informational content for nominal income, but it is closely followed by Divisia M1. Chrystal and MacDonald also find that for broader aggregates, the Divisia measure has significant informational content for nominal income than its simple-sum equivalent.

Table 4 compares the results of our study with those of Cockerline and Murray (CM, 1981), Hostland, Poloz, and Storer (HPS, 1988), Serletis and King (SK, 1993) and Chrystal and MacDonald (CM, 1994). The results show fairly strong similarities. Both our study and HPS find that a fairly broad Fisher ideal monetary aggregate performs best for the GDP deflator (Fisher ideal privately held money supply in their case and M3+FI in ours), while SK found that the smallest tail area in their Granger-causality tests for the deflator occurred in the case of M2+ (our preferred simple-summation aggregate). All five studies found that M1 was the aggregate that performed the best in estimation for nominal spending, although in SK, M1 was ranked equally with M1FI. For real GDP, both HPS and we found that real M1 was the best aggregate, while SK, who restricted themselves to nominal aggregates, found that M1 and M1FI did equally as well.

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Table 2: The Three Best Indicator Models in Estimation, Based on the Minimum Akaike Information Criterion (AIC)

Rank	CPI	CPIXFE	GDP Deflator	Nominal GDP	Real GDP
1	M2+ (3.299)[a]	M2+ (2.842)	M3+FI (5.141)	M1 (11.530)	RM1[b] (11.598)
2	M2 (3.657)	M2 (2.851)	M3FI (5.196)	M1FI (11.596)	RM1FI (12.602)
3	M3 (3.769)	M3 (2.924)	M2+ (5.218)	LL+FI (12.926)	RLL+FI (13.839)

a. The AIC is recorded in parentheses.

b. Note that an "R" prefix indicates that money is in real terms, using the CPI.

Table 3: Davidson and MacKinnon (1981) J-Tests

Goal Variable	Money	**H_0**: $Goal = SUM\beta + \varepsilon$ **H_1**: $Goal = FI\gamma + \upsilon$ $Goal = (1-\alpha)SUM\beta + \alpha(FI\gamma) + \varepsilon$	**H_0**: $Goal = FI\gamma + \upsilon$ **H_1**: $Goal = SUM\beta + \varepsilon$ $Goal = (1-\alpha)FI\gamma + \alpha(SUM\beta) + \upsilon$	Conclusion
CPI	M2+ LLFI	Reject H_0 (3.04)	Reject H_0 (5.03)	Neither model provides complete specification
CPIXFE	M2+ LLFI	Reject H_0 (2.85)	Reject H_0 (3.83)	Neither model provides complete specification
GDP Deflator	M2+ M3+FI	Reject H_0 (3.02)	Reject H_0 (2.50)	Neither model provides complete specification
Nominal GDP	M1 M1FI	Do not reject H_0 (0.81)	Do not reject H_0 (1.09)	Cannot reject either specification
Real GDP	RM1 RM1FI	Do not reject H_0 (1.18)	Reject H_0 (2.23)	Summation model not rejected, but FI model is

Table 4: Best Aggregates in Estimation: A Comparison of Various Studies

Goal Variable	Money	Cockerline and Murray (1968Q2-1980Q4)[a]	Hostland, Poloz and Storer (1969Q1-1986Q4)[b]	Serletis and King (1968Q3-1989Q3)[c]	Chrystal and MacDonald (1968Q3-1987Q1)[d]	Longworth and Atta-Mensah (1971Q1-1989Q3)
GDP Deflator	Summation	—	M2	M2+	—	M2+
	Superlative	—	PHMS-FI	M3+FI	—	M3+FI
	Overall	—	PHMS-FI	M2+	—	M3+FI
Nominal GDP	Summation	M1	M1	M1	M1	M1
	Superlative	M1D	M1ALD-FI	M1FI	M1D	M1FI
	Overall	M1	M1	M1, M1FI	—	M1
Real GDP[e]	Summation	—	RM1	M1	—	RM1
	Superlative	—	RM1ALD-FI	M1FI	—	RM1FI
	Overall	—	RM1	M1, M1FI	—	RM1

a. Sample period in parentheses.

b. Note that M1ALD is defined as M1 plus non-personal notice deposits at banks. Also, PHMS is defined as M2 plus non-personal fixed-term deposits at banks.

c. Based on smallest tail area in Granger-causality tests.

d. Based on the St. Louis equation.

e. An “R” before an aggregate indicates that it is expressed in real terms.

3.4 One-Quarter-Ahead Out-of-Sample Forecasts for Five Goal Variables

The models estimated above were used to generate out-of-sample forecasts in the following way. First, the models were estimated over the sample period 1971Q1-1981Q4 and the value for the following quarter was forecast. The model was then re-estimated with the additional quarter of data and the next quarter was then forecast. This process was repeated for each quarter until 1989Q3. The root-mean-square errors were then calculated for the period 1982Q1-1989Q3.

For each goal variable the three best models by the root-mean-square-error criterion are shown in Table 5. Again, simple-sum aggregates dominate in the majority of cases. It is the liquidity aggregates LL+ and LL that provide the best forecasts for CPI and CPIXFE, respectively, while real M1 continues to do best for real GDP. M2+FI is marginally better than M2+ and M3+FI as a predictor of the GDP deflator, although the latter two variables did better as in-sample predictors. Finally, M1FI did better in predicting nominal GDP than M1, although the ordering was the reverse in estimation.

Table 6 compares the results from Table 5 with those obtained by HPS. In both studies, real M1 was the best out-of-sample predictor of real output. HPS found M1 to be the best predictor of nominal spending, whereas we found that M1FI was best. Finally, HPS found M2 to be the best predictor for the GDP deflator, whereas M2+FI was preferred over our sample period.

Table 5: Lowest RMSE for One-Quarter-Ahead Forecast (Annualized Growth Rates)

Rank	CPI	CPIXFE	GDP Deflator	Nominal GDP	Real GDP
1	LL+ (1.630)	LL (1.178)	M2+FI (1.889)	M1FI (2.812)	RM1 (3.199)
2	M3+ (1.670)	LL+ (1.208)	M2+ (1.903)	M1 (2.848)	AR Model (3.487)
3	M2 (1.671)	M3 (1.222)	M3+FI (1.909)	LL+FI (2.9359)	RLL (3.499)
Best FI Aggregate	M2FI (1.678)	M3FI (1.296)	—	—	RM1FI (3.517)

Table 6: Best Aggregates in Prediction: A Comparison of Various Studies (One-Quarter-Ahead Prediction)

Goal Variable	Money	Hostland Poloz and Storer (1975Q1-1986Q4)[a]	Longworth and Atta-Mensah (1982Q1-1989Q3)
GDP Deflator	Summation	M2	M2+
	Superlative	PHMS-FI	M2+FI
	Overall	M2	M2+FI
Nominal GDP	Summation	M1	M1
	Superlative	LLFI	M1FI
	Overall	M1	M1FI
Real GDP	Summation	RM1	RM1
	Superlative	Not Available	Worse than Autoregressive
	Overall	RM1	RM1

a. The forecasting period is in parentheses.

3.5 Multi-Period Out-of-Sample Forecasts for Five Goal Variables

We undertook further studies to find out which of the simple-sum and Fisher ideal aggregates would do best in multi-period forecasts using bivariate vector autoregressions, with the second equation explaining the monetary aggregate by its own lags and lags on the goal variable. The out-of-sample forecasts for 1982Q1-1989Q3 were performed in a way similar to that described above, except that multi-period forecasts were calculated in each case.

Table 7 presents the three best predictors for the 1-, 2-, 4-, 8- and 12-quarter horizons. With the exception of inflation measured by the GDP deflator, the broader simple-sum aggregates were the best predictors of inflation over the various horizons. In the case of the GDP deflator measure of inflation, M3+FI was seen as the best predictor over the forecast horizons beyond one quarter ahead.

With regard to nominal income, M1FI was the best predictor over the 1- and 2-quarter horizons; for 8- and 12-quarter horizons, M2+ was observed as the best predictor. Lastly, real M1 was found to be best predictor of real GDP for all horizons.

Following Chong and Hendry (1986), encompassing tests were conducted on the predictions of the best simple-sum and the best Fisher ideal models. The tests were conducted by regressing the following equation:

$$g = \phi_o + \phi_1 \hat{g}_{SS} + \phi_2 \hat{g}_{FI} + \zeta$$

where g is the observed goal variable, $\hat{g}_{SS}$ is the out-of-sample forecast of the goal variable obtained from the best simple-sum model, and $\hat{g}_{FI}$ is the prediction from the best Fisher ideal model. The test involves checking the statistical significance of the coefficients, ϕ_1 and ϕ_2. If one of the coefficients is significant and the other is not, the model with the significant coefficient encompasses the competing model. On the other hand, if both coefficients are significant, then both models are complementary, and an “optimal” forecast could be constructed on the basis of the two models. Finally, if none of the coefficients is significant, then neither model encompasses the other.

Table 7: The Three Best Aggregates in the Prediction of the Annualized Average Growth Rates of the Goal Variables (Based on Lowest RMSE)

Quarters	Rank	CPI	CPIXFE	GDP Deflator	Nominal GDP	Real GDP
1	1	LL+ (1.6301)[a]	LL (1.1778)	M2+FI (1.8890)	M1FI (2.8122)	RM1[b] (3.1988)
	2	M3+ (1.6704)	LL+ (1.2081)	M2+ (1.9034)	M1 (2.8482)	RLL (3.4985)
	3	M2 (1.6709)	M3 (1.2218)	M3+FI (1.9089)	LL+FI (2.9359)	RLL+ (3.5099)
2	1	M2+ (1.4607)	LL (1.1461)	M3+FI (1.6379)	M1FI (2.9541)	RM1 (2.6722)
	2	LL+ (1.5074)	LL+ (1.1471)	M2+FI (1.7052)	LL+FI (2.9644)	RM1FI (2.8942)
	3	M3+ (1.5138)	M3 (1.1819)	M2+ (1.7086)	M3+FI (2.9841)	RLL (3.0436)
4	1	M2+ (1.4285)	M3 (1.1110)	M3+FI (1.7183)	M3+FI (3.2171)	RM1 (2.1669)
	2	M3+ (1.6165)	M2+ (1.1286)	M3FI (1.7923)	LL+FI (3.2755)	RM1FI (2.3404)
	3	M3 (1.6990)	M2 (1.1382)	LLFI (1.7998)	LLFI (3.3285)	RLL (2.4502)
8	1	M2+ (1.8634)	M2+ (1.1770)	M3+FI (2.0550)	M2+ (3.7496)	RM1 (2.0757)
	2	M3+ (2.1701)	M3+ (1.3136)	M3FI (2.1360)	LL+ (3.7509)	RLL (2.1411)
	3	M2 (2.1761)	M3 (1.3279)	LL+FI (2.2457)	M3+FI (3.8054)	RLL+ (2.1577)
12	1	M2+ (2.5266)	M2+ (1.4677)	M3+FI (2.7282)	M2+ (4.2628)	RM1 (2.1054)
	2	M2 (2.6007)	M3+ (1.5948)	M3FI (2.7714)	LL+ (4.2877)	RLL (2.1191)
	3	M3+ (2.7926)	M3 (1.6164)	LL+ (2.8254)	M2 (4.3710)	RLL+ (2.1224)

a. RMSE in parentheses.

b. Note that an "R" prefix indicates that money is in real terms, using the CPI.

The results reported in Table 8 suggest that the Fisher ideal models encompass the simple-sum models in explaining most of the predictions for the growth rates of the GDP deflator and the CPI. Given the lower RMSE (Table 7) for the simple-sum models in the case of the CPI, this most likely indicates that the Fisher ideal projections have a greater bias. Except in a few cases, neither the Fisher ideal nor the simple-sum model encompasses the other in predictions of the growth rates of CPI excluding food and energy, and nominal GDP. The simple-sum model typically encompasses the Fisher ideal model in the case of real GDP.

able 8: Encompassing J-Test Results for the Dynamic Out-of-sample Forecasts (Annualized Average Growth Rate
(Forecast Horizon: 1982Q1 - 1989Q3)

Goal riable	Money	1Q		2Q		4Q		8Q		12Q	
		Money	**ϕ**	**Money**	**ϕ**	**Money**	**ϕ**	**Money**	**ϕ**	**Money**	**ϕ**
CPI	Best SS	LL+	0.225 (1.162)	M2+	0.219 (1.128)	M2+	0.053 (0.299)	M2+	0.060 (0.623)	M2+	0.07 (0.79
	Best FI	M2FI	0.598 (3.536)**	M2+FI	0.553 (1.850)	M2+FI	0.477 (2.104)*	M2+FI	0.362 (2.493)*	M2+FI	0.23 (2.198
PIXFE	Best SS	LL	0.743 (1.141)	LL	0.835 (2.541)*	M3	0.273 (1.488)	M2+	0.051 (0.371)	M2+	-0.06 (-0.58
	Best FI	M3FI	0.109 (0.181)	M2FI	0.004 (0.016)	LLFI	0.417 (2.572)*	LLFI	0.218 (1.326)	LLFI	0.12 (1.23
GDP eflator	Best SS	M2+	0.333 (1.599)	M2+	0.152 (1.072)	LL+	-0.035 (-0.232)	LL+	-0.173 (-1.537)	LL+	-0.25 (-2.51
	Best FI	M2+FI	0.434 (2.185)*	M3+FI	0.564 (3.273)**	M3+FI	0.552 (3.914)**	M3+FI	0.511 (5.347)**	M3+FI	0.28 (3.358
minal GDP	Best SS	M1	0.462 (1.598)	M1	0.516 (1.679)	M1	0.405 (3.036)**	M2+	-0.474 (-2.730)*	M2+	-0.21 (-1.88
	Best FI	M1FI	0.181 (0.662)	M1FI	0.058 (0.201)	M3+FI	0.010 (0.065)	M3+FI	0.515 (3.488)**	M3+FI	0.43 (3.620
Real GDP	Best SS	RM1	1.021 (2.378)*	RM1	0.935 (2.668)*	RM1	0.900 (1.719)	RM1	1.371 (3.050)**	RM1	0.90 (2.309
	Best FI	RM1FI	-0.129 (-0.300)	RM1FI	-0.088 (-0.203)	RM1FI	-0.114 (-0.263)	RM2FI	-2.566 (-4.232)**	RM2+FI	-1.50 (-2.09

4.0 Empirical Evidence on Long-Run Demand-for-Money Equations

In this section of the paper we apply the methodology of Johansen and Juselius (1990) to examine the long-run demand-for-money equations for each aggregate. The stability of the estimated demand functions are also examined.

4.1 Identifying the Cointegrating Vectors

This section discusses the cointegrating vectors estimated, using the Johansen and Juselius methodology, for the simple-sum and Fisher ideal monetary aggregates. The results are reported in Tables A4-A9.

Note that the Johansen and Juselius methodology is designed for variables that are integrated of order one. Unit-root tests were conducted on the monetary aggregates (simple-sum and Fisher ideal), user costs, R90, real income, and the three measures of inflation. The results are presented in Tables A1-A3. The ADF or Phillips-Perron tests suggest that all the variables used in this research are integrated of order one.

It has been argued by Fisher, Hudson and Pradhan (1993) that in estimating the demand-for-money function for a Fisher ideal monetary aggregate, the user cost of the aggregate should be used in the function, rather than an interest rate. The argument is that since the Fisher ideal aggregates contain interest-bearing assets, the relevant measure of the "opportunity cost of holding money" is the user cost rather than levels of one or more interest rates.

In this paper, we chose not to use the user costs in estimating the demand functions, because exclusion tests conducted did not find them to be statistically significant. The exclusion tests were carried out in two ways. First, for each of the Fisher ideal aggregates, we estimated the following equation and determined the statistical significance of the rental price index:

$$A(L)\Delta money_t = A_0 + B(L)\Delta income_t + C(L)\Delta R90_t + D(L)\Delta infl_t + \gamma ECM_{t-1} + E(L)RP_t + \varepsilon_t \quad (2)$$

where Δ is the difference-operator, RP is the user cost, the last term is an error term and money and income are in real terms. ECM is the error-correction term, which is derived from the estimated cointegrating vectors from money, income, R90 and inflation. If RP was an important variable, then E(L) would be statistically significant in the above equation.

The second method estimates the following alternative dynamic equation:

$$A(L)\Delta money_t = A_0 + B(L)\Delta income_t + C(L)\Delta R90_t + D(L)\Delta infl_t + \alpha money_{t-1} + \beta income_{t-1} + \delta R90_{t-1} + \theta infl_{t-1} + K(L)RP_t + \varepsilon_t \quad (3)$$

The purpose of the second method is to relax the constraint of imposing the cointegrating vectors. The importance of the RP depends on whether the K(L) terms are statistically significant. Note that in both regressions, the Akaike information criterion was used to select the lag lengths.

The results of the exclusion tests show that, under the two scenarios outlined above, the user costs are statistically insignificant at the 1 per cent significance level. Hence the RPs were dropped in the estimation of the money-demand functions.

In Table 9 we report the cointegrating vectors that fit the characteristics of a money-demand function — that is, where the income elasticity is positive, the semi-interest elasticity is negative, and inflation is also included in the equation. (For each cointegrating vector in Table 9, the first term is the coefficient of money initialized to unity; the second term is the income elasticity; the third term is the semi-interest elasticity and the fourth term is the semi-elasticity of inflation.)

Table 9: Estimated Cointegrating Vectors that fit the Characteristics of a Money-Demand Function

Money	System including CPI	System including CPIXFE	System including the GDP Deflator
M1	[1 -0.540 0.241 -0.009] [1 -0.607 0.040 -0.050]	[1 -0.457 0.149 -0.031] [1 -0.318 0.032 -0.052]	[1 -0.219 0.070 -0.003] [1 -0.440 0.023 -0.033]
M2	[1 -1.109 0.010 0.012]	[1 -1.061 0.006 0.021]	[1 -0.719 0.035 0.042]
M2+	[1 -1.431 0.012 0.009] [1 -1.890 0.014 -0.021]	[1 -1.500 0.023 0.008]	[1 -1.233 0.034 0.023]
M3	None	[1 -1.343 0.337 -0.120]	None
M3+	None	[1 -1.686 0.208 -0.072]	None
LL	None	[1 -1.750 0.009 -0.016]	[1 -1.274 0.001 0.019]] [1 -1.608 0.003 -0.006]
LL+	[1 -1.483 0.006 0.016]	[1 -1.559 0.003 0.016] [1 -1.812 0.003 -0.009]	[1 -1.374 0.009 0.018]
M1FI	[1 -0.590 0.230 -0.007] [1 -0.675 0.041 -0.051]	[1 -0.516 0.150 -0.028] [1 -0.371 0.032 -0.052]	[1 -0.243 0.079 0.003] [1 -0.489 0.025 -0.033]
M2FI	[1 -1.350 0.058 -0.060]	[1 -0.780 0.034 -0.042]	[1 -0.890 0.034 -0.032]
M2+FI	[1 -1.739 0.066 -0.067]	[1 -1.094 0.039 -0.046]	[1 -1.192 0.040 -0.037]
M3FI	[1 -1.806 0.077 -0.098]	[1 -1.021 0.046 -0.077]	[1 -1.318 0.052 -0.068]
M3+FI	[1 -2.015 0.078 -0.093]	[1 -1.263 0.049 -0.074]	[1 -1.487 0.053 -0.064]
LLFI	[1 -2.587 0.111 -0.119]	[1 -1.299 0.048 -0.065]	[1 -1.439 0.054 -0.053]
LL+FI	[1 -2.963 0.122 -0.132]	[1 -1.558 0.056 -0.074]	[1 -1.686 0.062 -0.061]

The Johansen and Juselius estimates obtained in this paper suggest the existence of cointegrating relationships among the monetary aggregates (in real terms), real income, R90 and inflation. However, not all of the vectors can be interpreted as long-run money-demand functions. In general, for the vectors which could be described as money-demand functions, Table 9 shows that, for both the simple-sum and the Fisher ideal aggregates, the semi-interest elasticity for the narrower aggregates is larger than that associated with the broader aggregates. As expected, we also found the income elasticities of the broader aggregates to be greater than those of the narrow aggregates.[13]

4.2 Estimating Dynamic Money-Demand Equations

Dynamic money-demand equations for all the aggregates were estimated in two ways. The first method uses the cointegrating vectors estimated by Johansen and Juselius methodology as an error-correction model (ECM). Thus, the following regression equation was used in estimating the demand function for all the monetary aggregates:

$$A(L)\Delta money_t = A_0 + B(L)\Delta income_t + C(L)\Delta R90_t + D(L)\Delta infl_t + \gamma ECM_{t-1} + \varepsilon_t \qquad (4)$$

where Δ is the difference-operator, the last term is an error term and money and income are in real terms. The results of the estimates are presented in Tables A10-A15. Within this framework, the ECM term acts as a measure of disequilibrium in any period.[14] Note that for dynamic stability of the functions, the estimated coefficients of the ECM terms must be negative. Also, if the ECM term is negative and statistically significant, then deviation of money from its long-run path (represented by the cointegrating vector) will lead to future changes in money holdings by economic agents, in order to move closer to their optimal long-run position.

In Table 10 we present the estimated coefficients of the ECM terms in the dynamic money-demand functions. The ECM term is generally negative and significant in the dynamic money-demand equations for the

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13. We also tested whether the long-run income elasticity of the demand for real balances is unity. Based on the likelihood-ratio statistic, the restriction on income was not rejected for any of the aggregates in the system that uses the GDP deflator. In the system that uses CPI, the restriction on the income elasticity is rejected for LL, M2FI, M2+FI, M3FI, M3+FI, LLFI and LL+FI. In the case of the system that uses CPI excluding food and energy, the restriction on the income elasticity is rejected for LL, LL+ and all the Fisher ideal aggregates except M1FI.

14. Note that in all cases only the cointegrating vectors that were found to be statistically significant in the regression equation were included.

simple-sum monetary aggregates, especially in the case where the CPI is the price index that is used. In the case of the Fisher ideal aggregates, other than M1FI, the ECM term was not very significant in the regression equations, thus casting doubt on the existence of a long-run money-demand equation in level form.

Table 10 : Estimated Coefficients of the ECM term in the Dynamic Money-Demand Functions

Money	System including CPI	System including CPIXFE	System including the GDP Deflator
M1	-0.021 (-6.921)**[a]	-0.039 (-7.508)**	-0.049 (-4.613)**
M2	-0.076 (-2.857)**	-0.026 (-1.808)	-0.006 (-1.561)
M2+	-0.049 (-3.632)**	-0.038 (-3.384)**	-0.017 (-3.566)**
M3	0.011 (3.493)**	-0.003 (-1.637)	0.002 (0.820)
M3+	0.012 (3.790)**	-0.005 (-2.262)*	0.001 (0.300)
LL	-0.096 (-4.048)** -0.073 (-2.834)**	-0.129 (-4.007)**	-0.077 (-3.648)** -0.066 (-2.461)*
LL+	-0.088 (-4.484)**	-0.097 (-4.473)**	-0.033 (-2.284)*
M1FI	-0.015 (-5.054)**	-0.036 (-7.935)**	-0.055 (-6.733)**
M2FI	-0.010 (-1.084)	-0.015 (-0.992)	-0.004 (-0.240)
M2+FI	-0.007 (-0.933)	-0.001 (-0.037)	-0.001 (-0.073)
M3FI	-0.007 (-1.293)	-0.019 (-2.378)*	-0.006 (-0.776)

Table 10 (Continued): Estimated Coefficients of the ECM term in the Dynamic Money-Demand Functions

Money	System including CPI	System including CPIXFE	System including the GDP Deflator
M3+FI	-0.005 (-1.037)	-0.006 (-0.734)	-0.002 (-0.291)
LLFI	-0.001 (-0.074)	-0.013 (-1.512)	0.001 (0.107)
LL+FI	0.002 (0.573)	-0.019 (-2.327)*	0.003 (0.441)

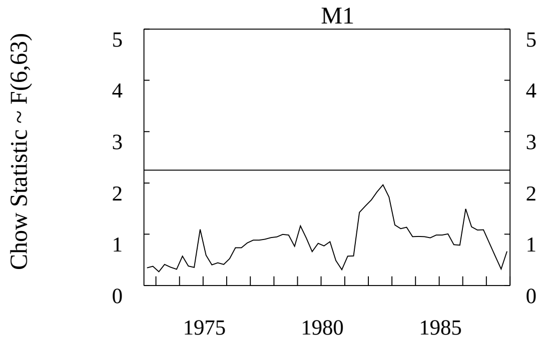
a. (t-statistics in parentheses)
* Significantly different from zero at the 5% level.
** Significantly different from zero at the 1% level.

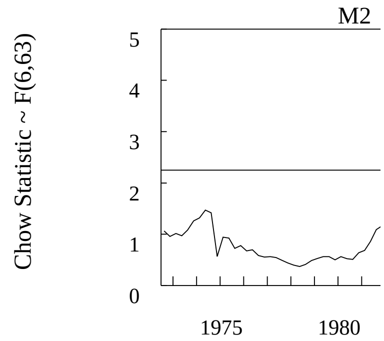
4.3 Stability of the Money-Demand Functions

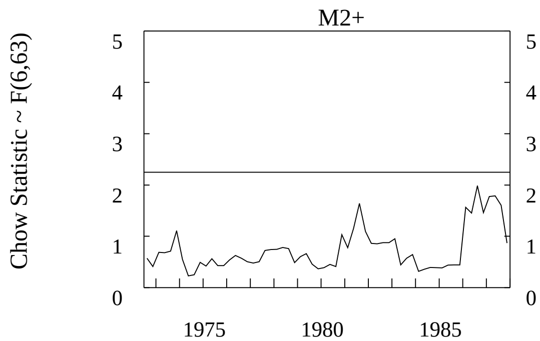
A rolling Chow test was used to assess the stability of the estimated dynamic money demand. Plots of the F-statistic for the various periods of the test are presented in Figures 2 and 3. Figure 2 shows that the dynamic money-demand equations for the simple-sum aggregates are very stable. However, as depicted in Figure 3, the demand function for the Fisher ideal aggregates is not stable.[15]

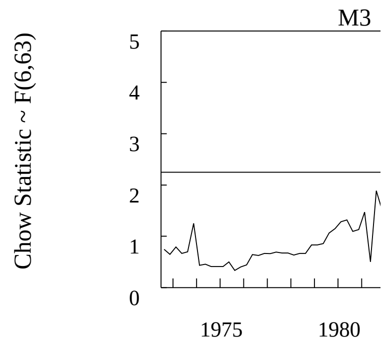
15. Note that figures 2 and 3 are based on the regression equations that use the CPI. Similar results are obtained for those involving the CPIXFE and the GDP delator.

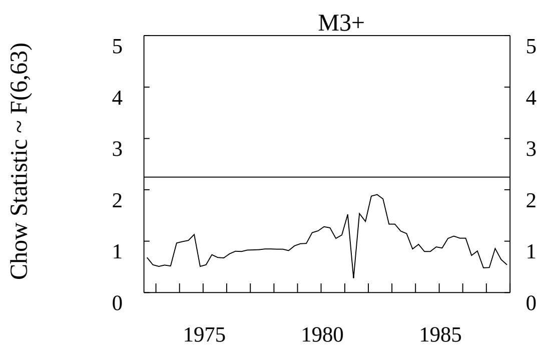
FIGURE 2: Rolling Chow Tests for Simple-Sum Monetary Aggregates

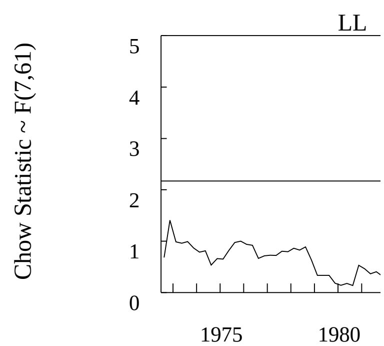


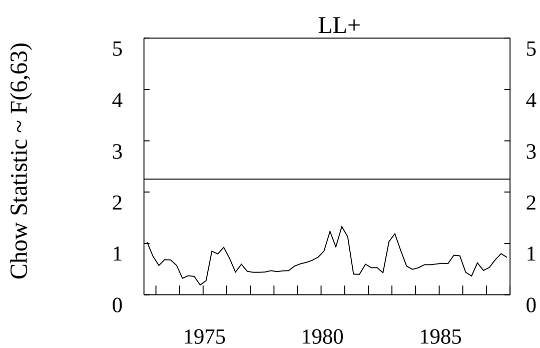








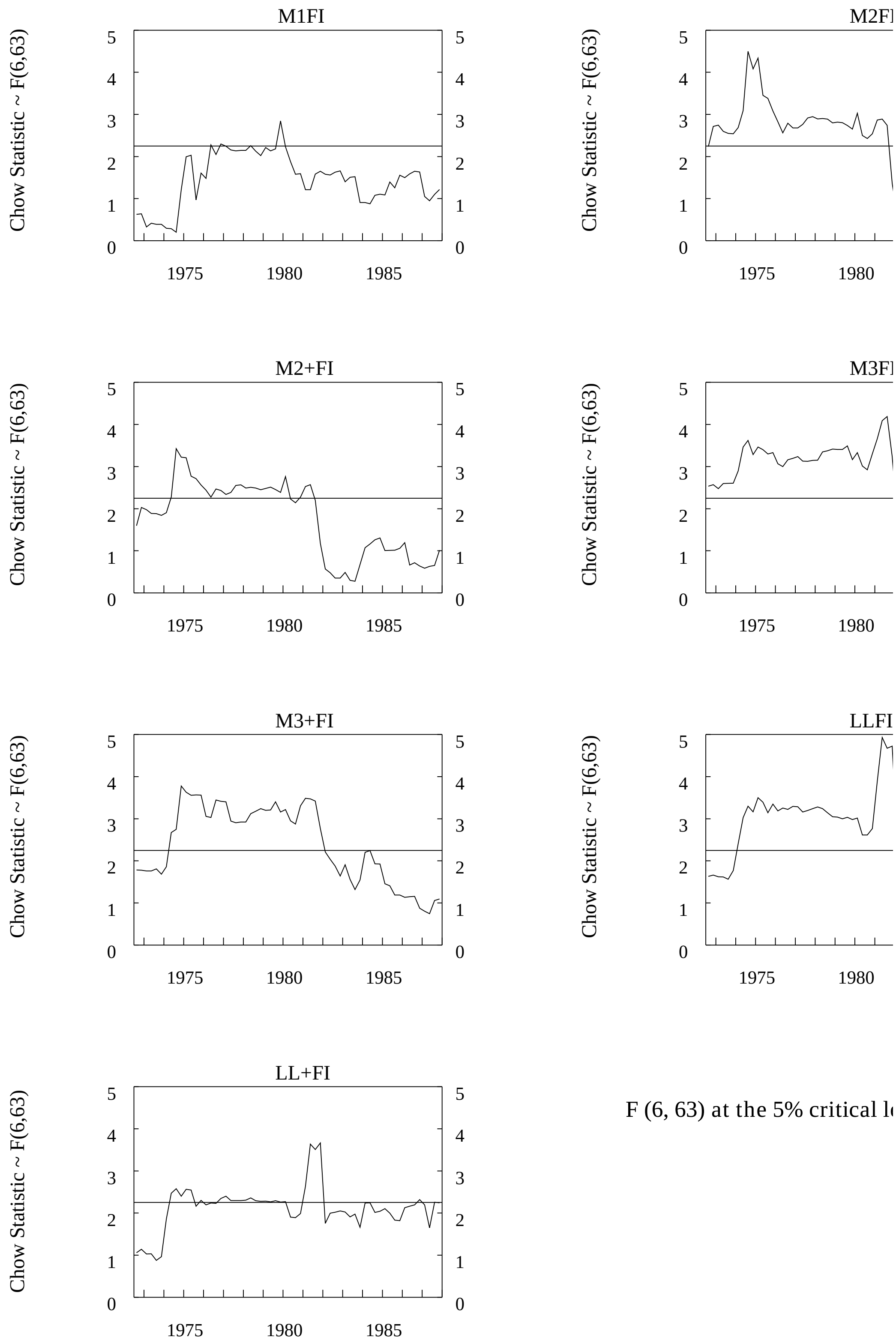




F (6, 63) at the 5% critical l

F (7, 61) at the 5% critical l

FIGURE 3: Rolling Chow Tests for Fisher Ideal Monetary Aggregates


M1FI
M2FI
M2+FI
M3FI
M3+FI
LLFI
LL+FI
Chow Statistic ~ F(6,63)
0
1
2
3
4
5
1975
1980
1985
F (6, 63) at the 5% critical l

5.0 Short-Run Causality Tests

In this section, we estimate a representation of a vector error-correction model (VECM) and subject it to exclusion tests on the lags of each of the first-differenced variables and on the lagged cointegrating terms. The VECM is of the form:

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \Gamma_2 \Delta X_{t-2} + \ldots + \Gamma_p \Delta X_{t-p+1} + \alpha ECM_{t-p} + \varepsilon_t \qquad (5)$$

where

$$ECM_{t-p} = \hat{\beta}' X_{t-p} \qquad (6)$$

and $\hat{\beta}$ denotes the statistically significant cointegrating vectors obtained by the Johansen and Juselius methodology. Note that the variables in the VECM are the same variables used in determining the cointegrating vectors. The exclusion tests are performed using F-statistics. The results are presented in Tables A16 - A21. [16]

A summary of the results is as follows:

- M1FI is the only aggregate that significantly influences real income in the short run.

- M2, M2+, M3 and M3+ are the only aggregates that significantly influence CPI in the short run.

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16. We also used the VECM and recursive regressions to compute the root-mean-square errors (RMSE) for forecasts of selected macroeconomic variables at 1- to 12-quarter horizons. Based on RMSE, the results show that the narrower measures of the Fisher ideal aggregates are best at predicting CPI at shorter horizons, while, at longer horizons, the broader measures of the simple-sum aggregates are the best. Also M3 and M3+ are best at predicting CPI excluding food and energy at shorter horizons, while, at longer horizons, the broader measures of the Fisher ideal aggregates are the best. For the GDP deflator, M2+ is the best predictor. Finally, the forecast results show that broader measures of the Fisher ideal aggregates are best at predicting real GDP at shorter horizons; M1 is the best at 8 quarters ahead, while M3 is the best at 12 quarters ahead. The out-of-sample results from the VECM model must be accepted with caution. Given that the money-demand functions were found to be unstable and the ECM terms insignificant for most of the Fisher ideal aggregates, one would have had little basis for choosing these aggregates for use in VECM models.

- M3, LLFI and LL+FI are the only aggregates that significantly influence CPIXFE in the short run.

- M2+ is the only aggregate that significantly influences the GDP deflator in the short run.

- R90 significantly influences all the Fisher ideal aggregates, M1, M2, M3, M3+ and real income in the short run.

6.0 Conclusion

In this paper we have compared the empirical performance of Canadian weighted (Fisher ideal) aggregates with the current summation aggregates in terms of their information content and forecasting performance for prices, real output and nominal spending for the period 1971Q1 to 1989Q3, at which point the data on the Fisher ideal aggregates end. As well, we have examined the properties of money-demand equations for these aggregates, including, importantly, their temporal stability.

"Indicator model" equations explaining the rates of change of real output, nominal spending and prices in terms of lagged own rates of growth and the rate of growth of one of the monetary aggregates generally reaffirm the conclusions of the earlier studies that weighted monetary aggregates rarely do better than simple-sum aggregates in predicting major Canadian macro-economic variables. In particular:

- in-sample estimation shows that the simple-sum aggregates M2+, M1, and real M1 provide the best explanation for the CPI (or CPI excluding food and energy), nominal spending, and real output, respectively, while Fisher ideal M3+ provides the best explanation for the GDP deflator;

- out-of-sample forecasts over horizons of 1, 2, 4, 8 and 12 quarters show that broad simple-sum aggregates such as M2+, M3, LL and LL+ provide the best forecasts for the CPI and CPI excluding food and energy (although in the context of J-tests, Fisher ideal models generally encompass simple-sum models for the CPI); Fisher ideal M3+ provides the best forecasts for the GDP deflator at forecast horizons greater than 1 quarter; Fisher ideal M1 provides the best forecast for nominal spending at short horizons while simple-sum M2+ provides the best forecast for the same variable at longer horizons; and real M1 provides the best forecasts for real GDP at all horizons.

Cointegrated money-demand functions are hardly ever found for Fisher ideal aggregates broader than Fisher ideal M1 (the two exceptions are M3FI and LL+FI when the CPI excluding food and energy is used as the price measure). In contrast, cointegrated money-demand functions are found for all the simple-sum aggregates, in particular when the CPI is used. Rolling Chow tests almost always reject the stability of the Fisher ideal money-demand functions, while they cannot reject the stability of the simple-sum money-demand functions when the CPI is used.

In the context of a vector error-correction model containing real money balances, a 90-day interest rate, inflation, and real output, only broad simple-sum monetary aggregates are found to cause CPI inflation and GDP deflator inflation in the short run, while the two Fisher ideal broad liquidity aggregates are found to cause inflation measured by the CPI excluding food and energy in the short run (as is simple-sum M3).

Over all, on the basis of in-sample fit of indicator models, out-of-sample forecasts by indicator models, the specification of money-demand functions, and the temporal stability of money-demand functions, one would conclude that Canadian simple-sum monetary aggregates are generally empirically superior to Fisher ideal aggregates. In particular, broad monetary aggregates are generally best in predicting inflation, M1 works well in predicting nominal spending, and real M1 is the best predictor of real output.

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Appendix Tables

Empirical Results

Table A1: ADF Test for Unit Roots - Simple-Sum Aggregates - (1971Q1-1989Q3)

Variable	k	t_{ρ}[a]	Outcome	Order of Integration
LRM1	1	-2.0886	Accept H_0	I(1)
ΔLRM1	3	-4.1282	Reject H_0	I(0)
LRM2	1	-1.8282	Accept H_0	I(2)[c]
ΔLRM2	4	-2.4275	Accept H_0	I(1)
LRM2+	1	-1.6535	Accept H_0	I(2)[c]
ΔLRM2+	4	-1.6952	Accept H_0	I(1)
LRM3	1	-1.6774	Accept H_0	I(2)[c]
ΔLRM3	4	-1.8948	Accept H_0	I(1)
LRM3+	1	-1.8326	Accept H_0	I(1)
ΔLRM3+	1	-2.9641[b]	Accept H_0	I(0)
LRLL	8	-1.6802	Accept H_0	I(2)[c]
ΔLRLL	8	-2.8672	Accept H_0	I(1)
LRLL+	8	-2.1642	Accept H_0	I(2)[c]
ΔLRLL+	8	-2.1089	Accept H_0	I(1)

a. The 95% critical value (with a trend variable) is -3.450, and that without a trend variable is -2.890.
b. The unit-root test conducted on these differences does not include a trend variable. Allowing for a trend variable made the variables integrated at a higher order.
c. These variables were found to be I(1) using the Phillips-Perron unit-root test.

Table A2: ADF Test for Unit Roots - Fisher Ideal Aggregates - (1971Q1-1989Q3)

Variable	k	t_ρ[a]	Outcome	Order of Integration
LRM1FI	1	-1.9659	Accept H_0	I(1)
ΔLRM1FI	2	-3.5356	Reject H_0	I(0)
LRM2FI	3	-0.7483	Accept H_0	I(1)
ΔLRM2FI	2	-3.5028	Reject H_0	I(0)
LRM2+FI	1	-0.8462	Accept H_0	I(1)
ΔLRM2+FI	2	-3.1439[b]	Reject H_0	I(0)
LRM3FI	3	-1.7995	Accept H_0	I(2)[c]
ΔLRM3FI	2	-2.3769	Accept H_0	I(1)
LRM3+FI	1	-1.5731	Accept H_0	I(2)[c]
ΔLRM3+FI	2	-2.4082	Accept H_0	I(1)
LRLLFI	3	-1.6723	Accept H_0	I(1)
ΔLRLLFI	2	-3.0743[b]	Reject H_0	I(0)
LRLLPFI	1	-1.2910	Accept H_0	I(2)[c]
ΔLRLL+FI	2	-2.8523	Accept H_0	I(1)

a. The 95% critical value (with a trend variable) is -3.450, and that without a trend variable is -2.890.
b. The unit-root test conducted on these differences does not include a trend variable. Allowing for a trend variable made the variables integrated at a higher order.
c. These variables were found to be I(1) using the Phillips-Perron unit-root test.

Table A3: ADF Test for Unit Roots - Selected Macroeconomic Variables - (1971Q1-1989Q3)

Variable	k	t_ρ[a]	Outcome	Order of Integration
LRY	1	-2.5376	Accept H_0	I(1)
ΔLRY	3	-3.8577	Reject H_0	I(0)
CPI	6	-3.0588	Accept H_0	I(1)
ΔCPI	5	-3.0456[b]	Accept H_0	I(0)
DEF	1	-2.9241	Accept H_0	I(1)
ΔDEF	3	-5.3703	Reject H_0	I(0)
CPIXFE	1	-2.7353	Accept H_0	I(1)
ΔCPIXFE	7	-3.2158[b]	Accept H_0	I(0)
R90	1	-2.6267	Accept H_0	I(1)
ΔR90	6	-3.7299	Reject H_0	I(0)

a. The 95% critical value (with a trend variable) is -3.450, and that without a trend variable is -2.890.
b. The unit-root test conducted on these differences does not include a trend variable. Allowing for a trend variable made the variables integrated at a higher order.

Table A4: Johansen Tests for Cointegration for the Simple-Sum Aggregates[a] (1971Q1 - 1989Q3)

System	Order	λ-max	Trace	Cointegrating Vectors (β)	Loadings (α)			
					Money	LRY	R90	DEF
LRM1, LRY,	1	79.45*	138.85*	[1 -0.219 0.070 -0.003]	-0.070	-0.026	-0.924	-0.960
R90, G1DEF		39.91*	59.41*	[1 -0.440 0.023 -0.033]	-0.047	-0.009	-2.483	20.336
		17.89^{+}	19.49^{+}	[1 0.068 -0.005 0.004]	-0.039	0.020	8.081	1.848
LRM2, LRY,	1	43.62*	85.64*	[1 -0.719 0.035 0.042]	-0.006	-0.018	-0.241	-3.501
R90, DEF		27.38*	42.02*	[1 -0.572 -0.055 0.048]	0.011	0.024	3.953	-5.678
		14.33^{W}	14.64	[1 -1.229 -0.002 0.001]	-0.096	0.039	-5.626	-4.415
LRM2+, LRY,	1	58.70*	99.63*	[1 -1.233 0.034 0.023]	-0.023	-0.026	-0.732	-3.990
R90, DEF		27.89*	40.93*	[1 -0.602 -0.063 0.062]	0.011	0.016	3.263	-4.950
LRM3, LRY,	2	42.08*	72.53*	[1 -0.606 -0.125 0.037]	0.019	0.019	2.234	0.349
R90, DEF		23.34^{+}	30.45^{W}	[1 0.866 -0.048 0.149]	0.003	-0.001	1.199	-1.633
LRM3+, LRY,	2	44.14*	74.02*	[1 -0.957 -0.106 0.022]	0.020	0.023	2.562	0.464
R90, DEF		23.07^{+}	29.88^{W}	[1 -0.075 -0.039 0.091]	0.002	-0.000	2.070	-2.490
LRLL, LRY,	1	54.07*	105.55*	[1 -1.274 0.001 0.019]	-0.041	-0.037	0.395	-15.172
R90, DEF		31.19*	51.48*	[1 -1.296 -0.027 0.017]	-0.015	0.076	6.615	-8.244
		19.33*	20.29^{+}	[1 -1.608 0.003 -0.006]	-0.057	0.008	-8.336	6.710
LRLL+, LRY,	1	63.42*	110.32*	[1 -1.374 0.009 0.018]	-0.043	-0.040	-0.452	-10.121
R90, DEF		29.21*	46.90*	[1 -1.241 -0.035 0.027]	0.005	0.050	6.374	-8.714
		17.03^{+}	17.68^{W}	[1 -1.649 -0.001 -0.003]	-0.058	0.042	-5.648	5.746

a. Monetary aggregates and income are deflated by PGDP.

Note: Critical values for λ-max and trace statistics are taken from Johansen and Juselius (1990, Table A2).

*: significant at 1% level

+: significant at 5% level

w: significant at 10% level

Table A5: Johansen Tests for Cointegration for Fisher Ideal Aggregates[a] (1971Q1 - 1989Q3)

System	Order	λ-max	Trace	Cointegration Vectors (β)	Loadings (α)			
					Money	LRY	R90	DEF
LRM1FI, LRY, R90, DEF	1	82.76* 39.06* 16.99+	140.58* 57.82* 18.76+	[1 -0.243 0.079 0.003] [1 -0.489 0.025 -0.033] [1 0.007 -0.004 0.003]	-0.056 -0.053 -0.040	-0.022 -0.015 0.018	-0.764 -2.995 8.258	-1.139 20.210 2.871
LRM2FI, LRY, R90, DEF	1	72.80* 30.58*	116.44* 43.64*	[1 -0.520 -0.038 -0.031] [1 -0.890 0.034 -0.032]	0.031 -0.036	0.023 -0.035	0.772 -6.030	4.291 12.694
LRM2+FI, LRY, R90, DEF	1	81.71* 29.48*	123.93* 42.22*	[1 -0.722 -0.049 -0.038] [1 -1.192 0.040 -0.037]	0.028 -0.025	0.019 -0.028	0.603 -5.217	3.285 10.362
LRM3FI, LRY, R90, DEF	1	86.15* 29.26*	125.37* 39.22*	[1 -0.479 -0.063 -0.025] [1 -1.318 0.052 -0.068]	0.030 -0.016	0.022 -0.012	1.064 -2.833	2.932 5.946
LRM3+FI, LRY, R90, DEF	1	94.93* 28.87*	133.83* 38.90*	[1 -0.678 -0.055 -0.026] [1 -1.487 0.053 -0.064]	0.034 -0.014	0.023 -0.013	1.047 -3.043	3.325 6.139
LRLLFI, LRY, R90, DEF	1	89.98* 28.15*	129.44* 39.46*	[1 -0.737 -0.041 -0.028] [1 -1.439 0.054 -0.053]	0.032 -0.014	0.024 -0.018	0.895 -3.577	4.103 6.568
LRLL+FI, LRY, R90, DEF	1	99.08* 27.86*	138.17* 39.09*	[1 -0.863 -0.050 -0.034] [1 -1.686 0.062 -0.061]	0.029 -0.010	0.020 -0.015	0.702 -3.150	3.270 5.616

a. Monetary aggregates and income are deflated by PGDP.

Note: Critical values for λ-max and trace statistics are taken from Johansen and Juselius (1990, Table A2).
*: significant at 1% level
+: significant at 5% level
w: significant at 10% level

Table A6: Johansen Tests for Cointegration for Simple-Sum Aggregates[a] (1971Q1 - 1989Q3)

System	Order	λ-max	Trace	Cointegrating Vectors (β)	Loadings (α)			
					Money	LRY	R90	CPI
LRM1, LRY, R90, CPI	1	81.30* 44.03* 16.91^{+}	143.63* 62.34* 18.31^{+}	[1 -0.540 0.241 -0.009] [1 -0.607 0.040 -0.050] [1 0.234 -0.006 0.004]	-0.020 -0.045 -0.044	-0.009 0.012 0.020	-0.407 -2.036 6.685	-0.360 11.248 1.001
LRM2, LRY, R90, CPI	2	37.34* 18.55	72.63* 35.30^{+}	[1 -1.103 -0.016 0.011] [1 -1.109 0.010 0.012]	-0.060 -0.048	0.017 -0.055	8.246 1.622	-8.163 2.152
LRM2+, LRY, R90, CPI	2	31.15* 21.28^{w} 14.20^{w}	68.04* 36.89^{+} 15.62^{w}	[1 -1.293 -0.021 0.016] [1 -1.431 0.012 0.009] [1 -1.890 0.014 -0.021]	-0.001 -0.066 -0.018	0.104 -0.051 0.018	9.858 2.243 -5.202	-8.781 7.313 4.692
LRM3, LRY, R90, CPI	2	34.26* 19.92^{w}	62.12* 27.86	[1 -0.111 -0.189 0.071] [1 1.668 -0.089 0.180]	0.015 -0.001	0.016 -0.003	1.309 1.057	-0.942 -0.563
LRM3+, LRY, R90, CPI	2	36.34* 20.16^{w}	64.89* 28.55^{w}	[1 -0.595 -0.153 0.051] [1 0.216 -0.055 0.100]	0.017 -0.003	0.019 -0.004	1.634 1.945	-1.262 -0.935
LRLL, LRY, R90, CPI	1	63.37* 29.66* 21.46*	115.69* 52.32* 22.66*	[1 -1.337 -0.000 0.019] [1 -0.926 -0.061 0.047] [1 -1.664 -0.001 -0.003]	-0.067 0.002 -0.055	-0.071 0.035 0.047	-1.514 4.300 -5.532	-13.246 -4.593 2.195
LRLL+, LRY, R90, CPI	1	71.94* 29.92* 18.75^{+}	121.39* 49.45* 19.53^{+}	[1 -1.483 0.006 0.016] [1 -0.505 -0.091 0.082] [1 -1.707 -0.005 0.001]	-0.074 0.007 -0.039	-0.072 0.015 0.076	-2.136 2.924 -2.416	-10.005 -3.501 -0.496

a. Monetary aggregates and income are deflated by PCPI.

Note: Critical values for λ-max and trace statistics are taken from Johansen and Juselius (1990, Table A2).

*: significant at 1% level

+: significant at 5% level

w: significant at 10% level

Table A7: Johansen Tests for Cointegration for Fisher Ideal Aggregates[a] (1971Q1 - 1989Q3)

System	Order	λ-max	Trace	Cointegrating Vectors (β)	Loadings (α)			
					Money	LRY	R90	CPI
LRM1FI, LRY, R90, CPI	1	86.25* 42.78* 16.19+	146.70* 60.45* 17.67	[1 -0.590 0.230 -0.007] [1 -0.675 0.041 -0.051] [1 0.157 -0.005 0.003]	-0.020 -0.041 -0.043	-0.009 0.011 0.022	-0.427 -2.237 6.850	-0.381 10.941 1.416
LRM2FI, LRY, R90, CPI	1	81.01* 33.80*	125.65* 44.64*	[1 -0.335 -0.036 -0.017] [1 -1.350 0.058 -0.060]	0.047 -0.022	0.043 -0.005	2.074 -3.559	3.537 6.392
LRM2+FI, LRY, R90, CPI	1	92.04* 32.89*	135.70* 43.66*	[1 -0.519 -0.042 -0.015] [1 -1.739 0.066 -0.067]	0.047 -0.016	0.039 -0.003	1.914 -3.112	2.953 5.436
LRM3FI, LRY, R90, CPI	1	93.86* 33.88*	135.05* 41.19*	[1 -0.281 -0.067 -0.012] [1 -1.806 0.077 -0.098]	0.039 -0.014	0.031 -0.001	1.711 -2.512	2.077 4.097
LRM3+FI, LRY, R90, CPI	1	105.67* 33.24*	146.63* 40.97*	[1 -0.499 -0.054 -0.012] [1 -2.015 0.078 -0.093]	0.047 -0.012	0.037 -0.001	1.995 -2.257	2.518 4.4147
LRLLFI, LRY, R90, CPI	1	101.72* 31.78*	142.25* 40.53*	[1 -0.561 -0.039 -0.011] [1 -2.587 0.111 -0.119]	0.050 -0.008	0.046 -0.002	2.359 -1.793	3.343 2.910
LRLL+FI, LRY, R90, CPI	1	113.49* 31.59*	154.02* 40.54*	[1 -0.670 -0.044 -0.011] [1 -2.963 0.122 -0.132]	0.048 -0.006	0.041 -0.001	2.102 -1.606	2.830 2.591

a. Monetary aggregates and income are deflated by PCPI.

Note: Critical values for λ-max and trace statistics are taken from Johansen and Juselius (1990, Table A2).

*: significant at 1% level

+: significant at 5% level

w: significant at 10% level

Table A8: Johansen Tests for Cointegration for Simple-Sum Aggregates[a] (1971Q1 - 1989Q3)

System	Order	λ-max	Trace	Cointegration Vectors (β)	Loadings (α)			
					Money	LRY	R90	CPIXFE
LRM1, LRY, R90, CPIXFE	1	75.90*	143.26*	[1 -0.457 0.149 -0.031]	-0.037	-0.015	-0.676	0.513
		52.26*	67.36*	[1 -0.318 0.032 -0.052]	-0.036	0.031	2.129	11.674
		13.42^{w}	15.10	[1 0.442 -0.020 0.021]	-0.021	0.009	4.970	-1.742
LRM2, LRY, R90, CPIXFE	1	42.71*	94.74*	[1 -1.061 0.006 0.021]	-0.038	-0.068	-2.892	-6.052
		33.08*	52.02*	[1 -0.807 -0.051 0.047]	0.017	0.025	2.872	-9.373
		18.18^{+}	18.95^{+}	[1 -1.310 -0.000 -0.004]	-0.084	0.044	-6.062	-0.571
LRM2+, LRY, R90, CPIXFE	1	56.75*	107.97*	[1 -1.500 0.023 0.008]	-0.061	-0.062	-3.344	-2.067
		35.11*	51.22*	[1 -1.099 -0.038 0.045]	0.010	0.006	2.804	-12.793
		15.64^{+}	16.12^{w}	[1 -1.594 -0.001 -0.002]	-0.038	0.071	-3.096	-1.555
LRM3, LRY, R90, CPIXFE	1	42.65*	88.08*	[1 -1.343 0.337 -0.120]	-0.007	-0.006	-0.342	0.204
		34.59*	45.43*	[1 22.671 -1.708 2.956]	0.000	-0.000	0.025	-0.162
LRM3+, LRY, R90, CPIXFE	1	49.71*	92.86*	[1 -1.686 0.208 -0.072]	-0.013	-0.010	-0.572	0.328
		33.50*	43.14*	[1 0.784 -0.154 0.237]	-0.000	-0.005	0.354	-2.044
LRLL, LRY, R90, CPIXFE	1	54.99*	115.98*	[1 -1.421 -0.007 0.021]	-0.061	-0.070	-4.421	-19.151
		36.82*	60.99*	[1 -1.270 -0.066 0.042]	0.013	0.037	2.086	-4.305
		22.12*	24.16*	[1 -1.750 0.009 -0.016]	-0.046	0.031	-4.307	4.413
LRLL+, LRY, R90, CPIXFE	1	58.43*	118.93*	[1 -1.559 0.003 0.016]	-0.081	-0.085	-4.860	-11.256
		39.29*	60.50*	[1 -1.348 -0.045 0.042]	0.022	0.037	2.958	-10.576
		19.85*	21.21^{+}	[1 -1.812 0.003 -0.009]	-0.032	0.055	-3.608	2.70

a. Monetary aggregates and income are deflated by PCPI.

Note: Critical values for λ-max and trace statistics are taken from Johansen and Juselius (1990, Table A2).

*: significant at 1% level

+: significant at 5% level

w: significant at 10% level

Table A9: Johansen Tests for Cointegration for Fisher Ideal Aggregates[a] (1971Q1 - 1989Q3)

System	Order	λ-max	Trace	Cointegration Vectors (β)	Loadings (α)			
					Money	LRY	R90	CPIXFE
LRM1FI, LRY, R90, CPIXFE	1	79.27* 51.30* 13.04[w]	145.41* 66.14* 14.84	[1 -0.516 0.150 -0.028] [1 -0.371 0.032 -0.052] [1 0.365 -0.018 0.019]	-0.035 -0.033 -0.021	-0.015 0.030 0.009	-0.674 1.973 5.161	0.457 11.908 -1.680
LRM2FI, LRY, R90, CPIXFE	1	69.37* 38.33*	119.89* 50.52*	[1 -0.120 -0.054 -0.011] [1 -0.780 0.034 -0.042]	0.040 -0.039	0.036 0.012	2.187 -1.046	0.709 13.707
LRM2+FI, LRY, R90, CPIXFE	1	78.97* 35.21*	125.86* 46.89*	[1 -0.306 -0.060 -0.010] [1 -1.094 0.039 -0.046]	0.039 -0.028	0.032 0.012	1.929 -1.281	0.518 11.596
LRM3FI, LRY, R90, CPIXFE	1	78.43* 34.81*	122.39* 43.96*	[1 0.033 -0.108 0.007] [1 -1.021 0.046 -0.077]	0.029 -0.012	0.022 0.012	1.396 -0.554	-0.137 7.379
LRM3+FI, LRY, R90, CPIXFE	1	87.42* 33.65*	130.34* 42.93*	[1 -0.265 -0.082 -0.000] [1 -1.263 0.049 -0.074]	0.037 -0.012	0.028 0.011	1.762 -0.785	0.025 7.480
LRLLFI, LRY, R90, CPIXFE	1	82.75* 33.31*	125.67* 42.92*	[1 -0.330 -0.061 0.001] [1 -1.299 0.048 -0.065]	0.042 -0.018	0.037 0.012	2.322 -1.012	0.046 7.974
LRLL+FI, LRY, R90, CPIXFE	1	92.31* 32.25*	133.98* 41.67*	[1 -0.442 -0.065 0.001] [1 -1.558 0.056 -0.074]	0.040 -0.013	0.034 0.011	2.073 -1.016	0.039 6.800

a. Monetary aggregates and income are deflated by PCPI.

Note: Critical values for λ-max and trace statistics are taken from Johansen and Juselius (1990, Table A2).

*: significant at 1% level

+: significant at 5% level

w: significant at 10% level

Table A10: Dynamic Money-Demand Equations for Simple-Sum Aggregates using the GDP Deflator (1971Q1 - 1989Q3)

Money (deflated by PGDP)	***Constant***	**Δ*Money***		**Δ*LRY***		**Δ*R90***		**Δ*DEF***		***ECM_{t-1}***		**$\bar{R}^2$**	**SEE**
		lag	**coef**	**lag**	**coef**	**lag**	**coef x 10^{-3}**	**lag**	**coef x 10^{-3}**	**term**	**coef**		
M1	0.403 (4.578)**	3	0.118 (1.367)	0	0.083 (0.448)	1	-6.363 (-4.957)**	4	-0.169 (-0.303)	1	-0.049 (-4.613)**	0.61461	0.012960
M2	0.024 (1.859)	1	0.457 (4.411)**	4	0.197 (2.267)*	1	-1.079 (-1.606)	0	-1.901 (-5.433)**	1	-0.006 (-1.561)	0.37563	0.00759
M2+	-0.050 (-3.370)**	1	0.491 (4.980)**	4	0.154 (2.128)*	4	-0.197 (-0.349)	0	-1.927 (-7.211)**	1	-0.017 (-3.566)**	0.57274	0.00580
M3	-0.054 (-0.790)	1	0.519 (5.189)**	4	0.350 (2.574)*	4	-2.687 (-2.651)**	0	-1.020 (-2.014)*	1	0.002 (0.820)	0.35100	0.01092
M3+	-0.011 (-0.241)	1	0.593 (6.335)**	4	0.286 (2.586)*	4	-2.305 (-2.843)**	0	-1.270 (-3.077)**	2	0.001 (0.300)	0.44388	0.00885
LL	-0.853 (-3.653)**	3	-0.067 (-0.653)	0	-0.008 (-0.053)	0	-1.979 (-1.901)	0	-1.690 (-3.126)**	1 3	-0.077 (-3.648)** -0.066 (-2.461)*	0.25587	0.01046
LL+	-0.152 (-2.141)*	3	0.111 (0.955)	0	0.108 (0.868)	4	-0.264 (-0.314)	1	-0.225 (-0.519)	1	-0.033 (-2.284)*	0.11046	0.00970

(t-statistics in parentheses)
* Significantly different from zero at the 5% level.
** Significantly different from zero at the 1% level.

Table A11: Dynamic Money-Demand Equations for Fisher Ideal Aggregates using the GDP Deflator (1971Q1 - 1989Q3)

Money (deflated by PGDP)	*Constant*	*ΔMoney*		*ΔLRY*		*ΔR90*		*ΔDEF*		*ECM_{t-1}*		$\bar{R}^2$	SEE
		lag	**coef**	**lag**	**coef**	**lag**	**coef x 10^{-3}**	**lag**	**coef x 10^{-3}**	**term**	**coef**		
M1FI	0.449 (6.663)**	3	0.054 (0.565)	0	0.024 (0.120)	0	-1.516 (-1.157)	2	-1.341 (-2.104)*	1	-0.055 (-6.733)**	0.50543	0.01416
M2FI	0.003 (1.553)	3	0.214 (2.044)*	1	0.251 (1.743)	1	-4.549 (-4.775)**	0	-1.235 (-2.361)*	2	-0.004 (-0.240)	0.31050	0.01066
M2+FI	0.002 (0.047)	3	0.288 (2.845)**	4	0.059 (0.459)	1	-4.379 (-4.875)**	0	-0.858 (-1.732)	2	-0.001 (-0.073)	0.30188	0.01065
M3FI	-0.027 (-0.714)	2	0.413 (3.969)**	4	0.128 (1.087)	1	-2.625 (-3.039)**	0	-0.810 (-1.675)	2	-0.006 (-0.776)	0.25625	0.01018
M3+FI	-0.012 (-0.237)	2	0.457 (4.607)**	4	0.164 (1.462)	1	-3.239 (-3.942)**	0	-0.785 (-1.718)	2	-0.002 (-0.291)	0.32160	0.00969
LLFI	0.009 (0.164)	3	0.319 (3.043)**	1	0.294 (2.172)*	1	-3.810 (-4.367)**	0	-1.335 (-2.840)**	2	0.001 (0.107)	0.33886	0.00980
LL+FI	0.031 (0.498)	3	0.369 (3.605)**	1	0.267 (2.071)*	1	-4.174 (-5.076)**	0	-1.261 (-2.851)**	2	0.003 (0.441)	0.39471	0.00925

(t-statistics in parentheses)
* Significantly different from zero at the 5% level.
** Significantly different from zero at the 1% level.

Table A12: Dynamic Money-Demand Equations for Simple-Sum Aggregates using CPI (1971Q1 - 1989Q3)

Money (deflated by PCPI)	***Constant***	**Δ*Money***		**Δ*LRY***		**Δ*R90***		**Δ*CPI***		***ECM*$_{t-1}$**		**$\bar{R}^2$**	**SEE**
		lag	**coef**	**lag**	**coef**	**lag**	**coef x 10^{-3}**	**lag**	**coef x 10^{-3}**	**term**	**coef**		
M1	0.122 (6.629)**	3	0.022 (0.242)	0	--0.033 (-0.166)	0	-2.196 (-1.683)	2	-2.491 (-3.116)**	1	-0.021 (-6.921)**	0.54364	0.01441
M2	-0.192 (-2.803)**	1	0.534 (4.997)**	0	0.275 (2.952)**	1	-2.097 (-2.935)**	0	-1.540 (-3.193)**	1	-0.076 (-2.857)**	0.36875	0.00779
M2+	-0.294 (-3.605)**	1	0.534 (5.040)**	0	0.072 (0.952)	1	-0.764 (-4.782)**	0	-1.806 (-4.782)**	2	-0.049 (-3.632)**	0.60873	0.00591
M3	-0.099 (-3.419)**	1	0.460 (4.677)**	3	0.287 (2.293)*	4	-1.198 (-1.274)	0	-1.267 (-2.107)*	1	0.011 (3.493)**	0.48219	0.01070
M3+	-0.039 (-3.583)**	1	0.522 (5.981)**	4	0.252 (2.507)*	4	-1.617 (-2.084)*	0	-1.467 (-3.049)**	1	0.012 (3.790)**	0.59166	0.00843
LL	-1.123 (-4.470)**	1	-0.016 (-0.154)	0	0.147 (1.043)	0	-2.135 (-2.151)*	0	-2.496 (-3.839)**	1 2	-0.096 (-4.048)** -0.073 (-2.834)**	0.35207	0.01028
LL+	-0.551 (-4.423)**	1	0.057 (0.513)	0	0.039 (0.365)	4	-0.036 (-0.047)	0	-2.388 (-4.519)**	1	-0.088 (-4.484)**	0.38062	0.00875

(t-statistics in parentheses)
* Significantly different from zero at the 5% level.
** Significantly different from zero at the 1% level.

Table A13: Dynamic Money-Demand Equations for Fisher Ideal Aggregates using CPI (1971Q1 - 1989Q3)

Money (deflated by PCPI)	***Constant***	***ΔMoney***		***ΔLRY***		***ΔR90***		***ΔCPI***		***ECM_{t-1}***		$\bar{R}^2$	**SEE**
		lag	**coef**	**lag**	**coef**	**lag**	**coef x 10^{-3}**	**lag**	**coef x 10^{-3}**	**term**	**coef**		
M1FI	0.449 (6.663)**	3	0.054 (0.565)	0	0.024 (0.120)	0	-1.516 (-1.157)	2	-1.341 (-2.104)*	1	-0.055 (-6.733)**	0.50543	0.01416
M2FI	0.003 (1.553)	3	0.214 (2.044)*	1	0.251 (1.743)	1	-4.549 (-4.775)**	0	-1.235 (-2.361)*	2	-0.004 (-0.240)	0.31050	0.01066
M2+FI	0.002 (0.047)	3	0.288 (2.845)**	4	0.059 (0.459)	1	-4.379 (-4.875)**	0	-0.858 (-1.732)	2	-0.001 (-0.073)	0.30188	0.01065
M3FI	-0.027 (-0.714)	2	0.413 (3.969)**	4	0.128 (1.087)	1	-2.625 (-3.039)**	0	-0.810 (-1.675)	2	-0.006 (-0.776)	0.25625	0.01018
M3+FI	-0.012 (-0.237)	2	0.457 (4.607)**	4	0.164 (1.462)	1	-3.239 (-3.942)**	0	-0.785 (-1.718)	2	-0.002 (-0.291)	0.32160	0.00969
LLFI	0.009 (0.164)	3	0.319 (3.043)**	1	0.294 (2.172)*	1	-3.810 (-4.367)**	0	-1.335 (-2.840)**	2	0.001 (0.107)	0.33886	0.00980
LL+FI	0.031 (0.498)	3	0.369 (3.605)**	1	0.267 (2.071)*	1	-4.174 (-5.076)**	0	-1.261 (-2.851)**	2	0.003 (0.441)	0.39471	0.00925

(t-statistics in parentheses)
* Significantly different from zero at the 5% level.
** Significantly different from zero at the 1% level.

Table A14: Dynamic Money-Demand Equations for Simple-Sum Aggregates using CPIXFE (1971Q1 - 1989Q3)

Money (deflated by PCPI)	***Constant***	**Δ*Money***		**Δ*LRY***		**Δ*R90***		**Δ*CPIXFE***		***ECM_{t-1}***		**$\bar{R}^2$**	**SEE**
		lag	**coef**	**lag**	**coef**	**lag**	**coef x 10^{-3}**	**lag**	**coef x 10^{-3}**	**term**	**coef**		
M1	0.228 (7.356)**	3	-0.001 (-0.011)	0	-0.036 (-0.193)	0	-2.101 (-1.622)	2	-2.217 (-2.464)*	1	-0.039 (-7.508)**	0.55484	0.01423
M2	-0.039 (-1.627)	1	0.402 (3.762)**	0	0.110 (1.036)	1	-0.728 (-0.974)	0	-1.252 (-2.166)*	1	-0.026 (-1.808)	0.29093	0.00825
M2+	-0.252 (-3.330)**	1	0.338 (3.197)**	0	0.129 (1.516)	4	0.227 (0.384)	0	-0.926 (-2.170)*	1	-0.038 (-3.384)**	0.50630	0.00664
M3	-0.005 (-1.181)	1	0.500 (5.077)**	0	0.246 (1.682)	4	-0.650 (-0.619)	3	-0.684 (-0.891)	1	-0.003 (-1.637)	0.42429	0.01128
M3+	-0.037 (-2.134)*	1	0.486 (5.270)**	0	0.233 (1.982)	4	-0.461 (-0.551)	3	-0.667 (-1.087)	1	-0.005 (-2.262)*	0.52735	0.00907
LL	-0.761 (-3.957)**	1	-0.032 (-0.310)	0	0.183 (1.398)	0	-2.433 (-2.425)*	0	-2.986 (-3.648)**	1	-0.129 (-4.007)**	0.24525	0.01109
LL+	-0.703 (-4.408)**	4	0.005 (0.532)	0	0.184 (1.607)	0	-1.736 (-2.108)*	0	-1.870 (-3.057)**	1	-0.097 (-4.473)**	0.32440	0.00914

(t-statistics in parentheses)
* Significantly different from zero at the 5% level.
** Significantly different from zero at the 1% level.

Table A15: Dynamic Money-Demand Equations for Fisher Ideal Aggregates using CPIXFE (1971Q1 - 1989Q3)

Money (deflated by PCPI)	***Constant***	***ΔMoney***		***ΔLRY***		***ΔR90***		***ΔCPIXFE***		***ECM_{t-1}***		**$\bar{R}^2$**	**SEE**
		lag	**coef**	**lag**	**coef**	**lag**	**coef x 10^{-3}**	**lag**	**coef x 10^{-3}**	**term**	**coef**		
M1FI	0.188 (7.778)**	3	0.045 (0.534)	0	-0.084 (-0.494)	0	-2.066 (-1.778)	2	-2.508 (-3.092)**	1	-0.036 (-7.935)**	0.59656	0.01279
M2FI	0.022 (1.044)	3	0.386 (4.155)**	0	0.290 (2.731)**	1	-3.707 (-4.070)**	2	-1.148 (-1.881)	2	-0.015 (-0.992)	0.41838	0.00962
M2+FI	0.002 (0.076)	3	0.399 (3.915)**	2	0.140 (1.150)	1	-4.603 (-4.812)**	2	-1.299 (-2.102)*	2	-0.001 (-0.037)	0.39045	0.00977
M3FI	-0.030 (-2.302)*	2	0.535 (5.163)**	2	0.216 (1.825)	2	-2.734 (-3.129)**	2	-0.359 (-0.578)	2	-0.019 (-2.378)*	0.36838	0.00981
M3+FI	-0.024 (-0.694)	2	0.525 (5.392)**	2	0.219 (1.940)	1	-3.596 (-4.071)**	2	-0.214 (-0.360)	2	-0.006 (-0.734)	0.41579	0.00931
LLFI	-0.064 (-1.452)	3	0.409 (4.035)**	2	0.206 (1.759)	1	-3.270 (-3.557)**	2	-0.989 (-1.604)	2	-0.013 (-1.512)	0.35078	0.00972
LL+FI	-0.143 (-2.258)*	3	0.485 (4.313)**	2	0.050 (0.416)	0	-1.157 (-1.224)	2	-1.308 (-2.027)*	2	-0.019 (-2.327)*	0.25422	0.01024

(t-statistics in parentheses)
* Significantly different from zero at the 5% level.
** Significantly different from zero at the 1% level.

Table A16 : Causality Tests Between Simple-Sum Aggregates and the VAR that uses CPI[a]

	M1	Real Income	R90	CPI
M1	1.42 (0.24)	1.29 (0.26)	10.29 (0.00)	0.35 (0.56)
Real Income	0.00 (0.96)	0.20 (0.65)	7.92 (0.01)	9.25 (0.00)
R90	15.34 (0.00)	1.65 (0.20)	0.08 (0.78)	3.20 (0.08)
CPI	5.75 (0.02)	0.22 (0.64)	1.99 (0.16)	0.32 (0.57)
ECM	11.12 (0.00)	3.46 (0.02)	6.23 (0.00)	4.61 (0.01)
	M2	**Real Income**	**R90**	**CPI**
M2	9.83 (0.00)	1.76 (0.18)	2.76 (0.07)	7.27 (0.00)
Real Income	2.64 (0.08)	8.45 (0.00)	6.43 (0.00)	9.19 (0.00)
R90	5.80 (0.00)	0.51 (0.60)	0.39 (0.68)	0.39 (0.68)
CPI	2.63 (0.08)	1.06 (0.35)	1.55 (0.22)	1.39 (0.26)
ECM	1.48 (0.23)	17.80 (0.00)	2.60 (0.11)	3.27 (0.08)
	M2+	**Real Income**	**R90**	**CPI**
M2+	7.26 (0.00)	0.67 (0.52)	1.32 (0.27)	8.63 (0.00)
Real Income	1.95 (0.15)	0.74 (0.48)	6.36 (0.00)	9.26 (0.00)
R90	1.94 (0.15)	1.18 (0.31)	0.26 (0.77)	0.87 (0.42)
CPI	2.74 (0.07)	0.90 (0.41)	0.19 (0.83)	0.91 (0.41)
ECM	16.30 (0.00)	13.01 (0.00)	5.45 (0.01)	1.55 (0.22)
	M3	**Real Income**	**R90**	**CPI**
M3	8.23 (0.00)	0.36 (0.70)	4.66 (0.01)	3.38 (0.04)
Real Income	0.44 (0.64)	1.03 (0.36)	2.81 (0.07)	11.86 (0.00)
R90	4.00 (0.02)	0.65 (0.52)	2.25 (0.11)	1.29 (0.28)
CPI	0.08 (0.93)	0.63 (0.53)	0.84 (0.43)	0.70 (0.50)
ECM	9.84 (0.00)	17.52 (0.00)	4.15 (0.05)	4.06 (0.05)

Table A16 (Continued): Causality Tests Between Simple-Sum Aggregates and the VAR that uses CPI[a]

	M3+	Real Income	R90	CPI
M3+	7.98 (0.00)	0. 36 (0.70)	4. 52 (0.01)	4.45 (0.02)
Real Income	0.74 (0.48)	0.70 (0.50)	3.11 (0.05)	11.35 (0.00)
R90	3.35 (0.04)	1.08 (0.34)	2.20 (0.12)	1.62 (0.21)
CPI	0.15 (0.86)	0.67 (0.52)	0.87 (0.42)	0.66 (0.52)
ECM	13.06 (0.00)	18.90 (0.00)	3.85 (0.05)	5.16 (0.02)
	LL	**Real Income**	**R90**	**CPI**
LL	0.09 (0.77)	0.32 (0.57)	0.50 (0.48)	0.13 (0.72)
Real Income	9.6 (0.00)	1.04 (0.31)	8.65 (0.00)	9.28 (0.00)
R90	0.14 (0.71)	4.64 (0.03)	0.60 (0.44)	0.81 (0.37)
CPI	0.39 (0.54)	0.18 (0.67)	0.00 (0.95)	1.68 (0.20)
ECM	8.52 (0.00)	11.32 (0.00)	4.74 (0.00)	2.05 (0.12)
	LL+	**Real Income**	**R90**	**CPI**
LL+	0.01 (0.94)	0.28 (0.60)	0.45 (0.51)	0.82 (0.37)
Real Income	9.30 (0.00)	1.19 (0.28)	9.15 (0.00)	9.88 (0.00)
R90	1.46 (0.23)	4.93 (0.03)	0.73 (0.40)	0.59 (0.45)
CPI	0.39 (0.53)	0.24 (0.63)	0.00 (1.00)	1.54 (0.22)
ECM	9.79 (0.00)	11.58 (0.00)	5.05 (0.00)	1.88 (0.14)

a. Note that the variable at the column head is the dependent variable. The numbers not in parentheses are F-statistics, while those in parentheses are the significance levels.

Table A17 : Causality Tests Between Fisher Ideal Aggregates and the VAR that uses CPI[a]

	M1FI	Real Income	R90	CPI
M1FI	1.66 (0.20)	2.99 (0.09)	8.98 (0.00)	0.24 (0.63)
Real Income	0.04 (0.83)	0.48 (0.49)	10.54 (0.00)	10.72 (0.00)
R90	15.30 (0.00)	2.00 (0.16)	0.20 (0.66)	2.56 (0.11)
CPI	4.59 (0.04)	0.378 (0.54)	1.61 (0.21)	0.43 (0.51)
ECM	16.74 (0.00)	3.40 (0.04)	4.99 (0.01)	6.44 (0.00)
	M2FI	**Real Income**	**R90**	**CPI**
M2FI	0.54 (0.46)	0.22 (0.64)	4.70 (0.03)	0.57 (0.45)
Real Income	1.13 (0.29)	0.54 (0.46)	8.42 (0.00)	10.57 (0.00)
R90	12.44 (0.00)	1.02 (0.32)	0.09 (0.76)	0.80 (0.37)
CPI	13.81 (0.00)	0.38 (0.54)	0.71 (0.40)	1.84 (0.18)
ECM	8.68 (0.00)	5.39 (0.00)	7.18 (0.00)	3.72 (0.03)
	M2+FI	**Real Income**	**R90**	**CPI**
M2+FI	1.28 (0.26)	0.02 (0.88)	3.78 (0.06)	0.83 (0.37)
Real Income	1.58 (0.21)	0.50 (0.48)	8.39 (0.01)	11.33 (0.00)
R90	15.93 (0.00)	1.33 (0.25)	0.10 (0.76)	0.84 (0.36)
CPI	13.14 (0.00)	0.27 (0.61)	0.87 (0.36)	1.99 (0.16)
ECM	9.28 (0.00)	5.29 (0.01)	7.03 (0.00)	3.50 (0.04)
	M3FI	**Real Income**	**R90**	**CPI**
M3FI	3.80 (0.06)	0.41 (0.52)	0.00 (1.00)	0.00 (0.95)
Real Income	0.10 (0.75)	0.10 (0.75)	5.19 (0.03)	9.76 (0.00)
R90	3.18 (0.08)	2.82 (0.10)	0.60 (0.44)	0.18 (0.67)
CPI	13.92 (0.00)	0.05 (0.83)	0.12 (0.73)	2.04 (0.16)
ECM	6.88 (0.00)	8.19 (0.00)	6.00 (0.00)	3.60 (0.03)

Table A17 (Continued): Causality Tests Between Fisher Ideal Aggregates and the VAR that uses CPI[a]

	M3+FI	Real Income	R90	CPI
M3+FI	4.05 (0.05)	0.81 (0.37)	0.01 (0.94)	0.01 (0.94)
Real Income	0.34 (0.56)	0.09 (0.76)	5.13 (0.03)	9.80 (0.00)
R90	5.65 (0.02)	3.11 (0.08)	0.60 (0.44)	0.25 (0.62)
CPI	13.99 (0.00)	0.02 (0.87)	0.17 (0.68)	2.15 (0.15)
ECM	7.23 (0.00)	8.04 (0.00)	6.03 (0.00)	3.45 (0.04)
	LLFI	**Real Income**	**R90**	**CPI**
LLFI	0.74 (0.39)	0.96 (0.33)	0.65 (0.42)	1.54 (0.22)
Real Income	0.20 (0.65)	0.15 (0.70)	6.41 (0.01)	7.73 (0.01)
R90	5.81 (0.02)	2.60 (0.11)	0.32 (0.57)	0.00 (0.97)
CPI	4.82 (0.03)	0.00 (0.96)	0.52 (0.47)	4.21 (0.04)
ECM	5.54 (0.01)	8.52 (0.00)	6.41 (0.00)	3.62 (0.03)
	LL+FI	**Real Income**	**R90**	**CPI**
LL+FI	1.48 (0.23)	1.23 (0.27)	0.67 (0.42)	0.90 (0.35)
Real Income	0.34 (0.56)	0.11 (0.74)	6.74 (0.01)	7.80 (0.01)
R90	8.91 (0.00)	2.88 (0.09)	0.24 (0.63)	0.01 (0.91)
CPI	5.69 (0.02)	0.00 (0.96)	0.59 (0.45)	4.02 (0.05)
ECM	5.63 (0.01)	8.04 (0.00)	6.59 (0.00)	3.10 (0.05)

a. Note that the variable at the column head is the dependent variable. The numbers not in parentheses are F-statistics, while those in parentheses are the significance levels.

Table A18 : Causality Tests Between Simple-Sum Aggregates and the VAR that uses CPIXFE[a]

	M1	Real Income	R90	CPIXFE
M1	2.58 (0.11)	1.54 (0.22)	7.44 (0.01)	0.24 (0.63)
Real Income	0.75 (0.39)	0.04 (0.84)	3.45 (0.07)	2.67 (0.11)
R90	9.79 (0.00)	0.83 (0.37)	0.37 (0.55)	0.22 (0.64)
CPIXFE	0.47 (0.50)	1.15 (0.29)	2.74 (0.10)	1.04 (0.31)
ECM	21.14 (0.00)	6.64 (0.00)	0.49 (0.61)	10.42 (0.00)
	M2	**Real Income**	**R90**	**CPIXFE**
M2	16.88 (0.00)	0.00 (0.96)	0.00 (0.97)	1.05 ((0.31)
Real Income	4.14 (0.05)	0.39 (0.54)	4.13 (0.05)	2.67 (0.11)
R90	2.85 (0.10)	4.08 (0.05)	2.78 (0.10)	0.38 (0.54)
CPIXFE	8.20 (0.01)	0.11 (0.75)	0.01 (0.94)	1.72 (0.19)
ECM	5.47 (0.010)	11.64 (0.00)	3.50 (0.04)	7.34 (0.00)
	M2+	**Real Income**	**R90**	**CPIXFE**
M2+	12.84 (0.00)	0.00 (0.99)	0.25 (0.62)	0.95 (0.33)
Real Income	4.88 (0.03)	0.41 (0.52)	5.01 (0.03)	2.37 (0.13)
R90	0.01 (0.91)	1.28 (0.26)	4.00 (0.05)	0.51 (0.48)
CPIXFE	4.24 (0.04)	0.02 (0.88)	0.04 (0.84)	1.48 (0.23)
ECM	11.94 (0.00)	7.35 (0.00)	4.38 (0.02)	7.43 (0.00)
	M3	**Real Income**	**R90**	**CPIXFE**
M3	9.98 (0.00)	0.50 (0.48)	9.71 (0.00)	2.89 (0.09)
Real Income	0.61 (0.44)	0.27 (0.61)	7.32 (0.01)	3.85 (0.05)
R90	11.93 (0.00)	2.89 (0.09)	6.80 (0.01)	0.42 (0.52)
CPIXFE	0.00 (0.97)	0.08 (0.78)	0.00 (0.97)	1.86 (0.18)
ECM	10.68 (0.00)	13.84 (0.00)	5.49 (0.01)	7.12 (0.00)

Table A18 (Continued): Causality Tests Between Simple-Sum Aggregates and the VAR that uses CPIXFE[a]

	M3+	**Real Income**	**R90**	**CPIXFE**
M3+	11.14 (0.00)	0.53 (0.47)	8.65 (0.00)	2.19 (0.14)
Real Income	1.25 (0.27)	0.30 (0.58)	7.72 (0.01)	3.68 (0.06)
R90	11.22 (0.00)	2.75 (0.10)	7.11 (0.01)	0.40 (0.53)
CPIXFE	0.05 (0.82)	0.03 (0.86)	0.01 (0.93)	2.07 (0.16)
ECM	14.19 (0.00)	12.41 (0.00)	6.31 (0.00)	6.91 (0.00)
	LL	**Real Income**	**R90**	**CPIXFE**
LL	0.16 (0.69)	0.72 (0.40)	2.62 (0.11)	0.19 (0.67)
Real Income	7.31 (0.01)	0.71 (0.40)	5.35 (0.02)	2.12 (0.15)
R90	0.01 (0.94)	3.91 (0.05)	1.53 (0.22)	0.03 (0.87)
CPIXFE	0.44 (0.51)	0.01 (0.91)	0.11 (0.74)	1.63 (0.21)
ECM	6.10 (0.00)	11.26 (0.00)	2.39 (0.08)	6.65 (0.00)
	LL+	**Real Income**	**R90**	**CPIXFE**
LL+	0.45 (0.51)	0.71 (0.40)	2.82 (0.10)	0.11 (0.75)
Real Income	7.71 (0.01)	0.92 (0.34)	5.97 (0.02)	2.12 (0.15)
R90	0.66 (0.42)	4.59 (0.04)	2.36 (0.13)	0.02 (0.88)
CPIXFE	0.34 (0.56)	0.00 (0.97)	0.14 (0.71)	1.44 (0.23)
ECM	8.54 (0.00)	11.61 (0.00)	2.81 (0.05)	6.07 (0.00)

a. Note that the variable at the column head is the dependent variable. The numbers not in parentheses are F-statistics, while those in parentheses are the significance levels.

Table A19 : Causality Tests Between Fisher Ideal Aggregates and the VAR that uses CPIXFE[a]

	M1FI	**Real Income**	**R90**	**CPIXFE**
M1FI	2.73 (0.10)	3.25 (0.08)	6.88 (0.01)	0.05 (0.83)
Real Income	0.65 (0.42)	0.08 (0.78)	3.88 (0.05)	2.85 (0.10)
R90	8.41 (0.01)	0.65 (0.42)	0.41 (0.53)	0.35 (0.56)
CPIXFE	0.54 (0.47)	1.38 (0.24)	2.18 (0.14)	0.71 (0.40)
ECM	21.69 (0.00)	5.01 (0.01)	0.56 (0.58)	10.96 (0.00)
	M2FI	**Real Income**	**R90**	**CPIXFE**
M2FI	0.44 (0.51)	0.20 (0.65)	1.07 (0.30)	0.64 (0.43)
Real Income	3.47 (0.07)	0.30 (0.59)	2.87 (0.09)	3.66 (0.06)
R90	9.16 (0.00)	0.46 (0.50)	2.45 (0.12)	0.57 (0.45)
CPIXFE	7.78 (0.01)	0.60 (0.44)	0.31 (0.58)	1.60 (0.21)
ECM	12.75 (0.00)	7.43 (0.00)	1.55 (0.22)	7.64 (0.00)
	M2+FI	**Real Income**	**R90**	**CPIXFE**
M2+FI	0.46 (0.50)	0.01 (0.92)	0.71 (0.40)	0.16 (0.69)
Real Income	4.23 (0.04)	0.35 (0.56)	3.06 (0.05)	3.98 (0.05)
R90	11.60 (0.00)	0.70 (0.41)	2.65 (0.11)	0.91 (0.34)
CPIXFE	4.48 (0.04)	0.48 (0.49)	0.22 (0.64)	2.07 (0.16)
ECM	14.42 (0.00)	7.2 (0.00)	1.90 (0.16)	6.63 (0.00)
	M3FI	**Real Income**	**R90**	**CPIXFE**
M3FI	0.93 (0.34)	0.28 (0.60)	1.10 (0.30)	0.06 (0.80)
Real Income	2.38 (0.13)	0.08 (0.78)	2.22 (0.14)	4.03 (0.05)
R90	0.26 (0.61)	1.93 (0.17)	4.33 (0.04)	1.19 (0.28)
CPIXFE	7.16 (0.01)	0.42 (0.52)	0.02 (0.89)	1.25 (0.27)
ECM	14.80 (0.00)	10.44 (0.00)	3.29 (0.04)	6.58 (0.00)

Table A19 (Continued): Causality Tests Between Fisher Ideal Aggregates and the VAR that uses CPIXFE[a]

	M3+FI	**Real Income**	**R90**	**CPIXFE**
M3+FI	0.89 (0.35)	0.66 (0.42)	0.97 (0.33)	0.00 (0.97)
Real Income	3.04 (0.09)	0.09 (0.76)	2.12 (0.15)	4.19 (0.04)
R90	1.44 (0.23)	2.12 (0.15)	4.31 (0.04)	1.13 (0.29)
CPIXFE	4.73 (0.03)	0.40 (0.53)	0.03 (0.86)	1.48 (0.23)
ECM	15.23 (0.00)	10.32 (0.00)	3.40 (0.04)	6.41 (0.00)
	LLFI	**Real Income**	**R90**	**CPIXFE**
LLFI	0.40 (0.53)	0.68 (0.41)	0.14 (0.71)	5.39 (0.02)
Real Income	1.52 (0.22)	0.19 (0.67)	2.49 (0.12)	3.53 (0.06)
R90	3.21 (0.08)	1.54 (0.22)	3.58 (0.06)	3.32 (0.07)
CPIXFE	5.14 (0.03)	2.1 (0.65)	0.02 (0.90)	4.13 (0.05)
ECM	10.46 (0.00)	10.99 (0.00)	3.00 (0.06)	6.16 (0.00)
	LL+FI	**Real Income**	**R90**	**CPIXFE**
LL+FI	0.53 (0.47)	0.93 (0.34)	0.16 (0.69)	3.49 (0.07)
Real Income	2.03 (0.16)	0.17 (0.68)	2.56 (0.11)	3.41 (0.07)
R90	5.26 (0.02)	1.79 (0.19)	3.59 (0.06)	3.15 (0.08)
CPIXFE	3.38 (0.07)	0.17 (0.69)	0.01 (0.92)	3.77 (0.06)
ECM	11.32 (0.00)	10.44 (0.00)	3.09 (0.05)	5.76 (0.00)

a. Note that the variable at the column head is the dependent variable. The numbers not in parentheses are F-statistics, while those in parentheses are the significance levels.

Table A20 : Causality Tests Between Simple-Sum Aggregates and the VAR that uses the GDP Deflator[a]

	M1	Real Income	R90	Deflator
M1	1.63 (0.21)	1.32 (0.25)	8.58 (0.00)	0.00 (0.98)
Real Income	0.28 (0.60)	0.83 (0.37)	6.83 (0.01)	0.33 (0.57)
R90	13.45 (0.00)	1.96 (0.17)	0.01 (0.91)	0.39 (0.54)
Deflator	0.69 (0.41)	0.01 (0.94)	0.00 (0.97)	4.23 (0.04)
ECM	9.96 (0.00)	3.24 (0.03)	6.75 (0.00)	5.02 (0.00)
	M2	**Real Income**	**R90**	**Deflator**
M2	11.73 (0.00)	0.02 (0.88)	0.49 (0.49)	1.70 (0.20)
Real Income	0.06 (0.81)	0.67 (0.42)	6.54 (0.01)	0.47 (0.50)
R90	1.92 (0.17)	0.91 (0.34)	0.40 (0.53)	0.21 (0.65)
Deflator	4.50 (0.04)	0.00 (0.99)	0.42 (0.52)	8.42 (0.01)
ECM	0.32 (0.73)	9.78 (0.00)	9.15 (0.00)	1.59 (0.21)
	M2+	**Real Income**	**R90**	**Deflator**
M2+	10.57 (0.00)	0.22 (0.64)	0.11 (0.74)	3.47 (0.07)
Real Income	0.20 (0.66)	0.71 (0.40)	6.54 (0.01)	0.52 (0.47)
R90	0.82 (0.37)	0.72 (0.40)	0.27 (0.61)	0.01 (0.93)
Deflator	3.31 (0.07)	0.00 (0.99)	0.61 (0.43)	7.82 (0.01)
ECM	3.03 (0.05)	7.72 (0.00)	9.69 (0.00)	0.85 (0.43)
	M3	**Real Income**	**R90**	**Deflator**
M3	6.49 (0.00)	2.13 (0.13)	3.96 (0.02)	0.46 (0.64)
Real Income	0.11 (0.90)	1.74 (0.18)	6.59 (0.00)	0.51 (0.60)
R90	3.16 (0.05)	2.56 (0.09)	2.28 (0.11)	0.24 (0.78)
Deflator	1.28 (0.29)	0.22 (0.80)	4.87 (0.01)	6.08 (0.00)
ECM	1.83 (0.17)	11.58 (0.00)	8.49 (0.00)	1.25 (0.29)

Table A20 (Continued): Causality Tests Between Simple-Sum Aggregates and the VAR that uses the GDP Deflator[a]

	M3+	Real Income	R90	Deflator
M3+	6.52 (0.00)	2.15 (0.12)	3.42 (0.04)	0.44 (0.65)
Real Income	0.24 (0.78)	1.67 (0.20)	7.16 (0.00)	0.51 (0.61)
R90	2.41 (0.10)	2.85 (0.07)	2.40 (0.10)	0.18 (0.84)
Deflator	0.83 (0.44)	0.21 (0.81)	5.04 (0.01)	6.01 (0.00)
ECM	2.20 (0.12)	11.38 (0.00)	8.64 (0.00)	0.92 (0.40)
	LL	**Real Income**	**R90**	**Deflator**
LL	0.10 (0.76)	0.88 (0.35)	1.17 (0.28)	0.14 (0.71)
Real Income	6.53 (0.01)	1.81 (0.18)	6.55 (0.01)	0.27 (0.61)
R90	0.01 (0.92)	2.92 (0.09)	0.71 (0.40)	0.24 (0.63)
Deflator	0.71 (0.40)	0.21 (0.65)	1.36 (0.25)	9.74 (0.00)
ECM	6.17 (0.00)	10.40 (0.00)	5.96 (0.00)	1.71 (0.17)
	LL+	**Real Income**	**R90**	**Deflator**
LL+	0.00 (0.96)	1.21 (0.28)	1.11 (0.30)	0.01 (0.92)
Real Income	5.29 (0.02)	1.79 (0.19)	6.97 (0.01)	0.40 (0.53)
R90	0.31 (0.58)	2.63 (0.11)	0.91 (0.34)	0.56 (0.46)
Deflator	0.62 (0.43)	0.17 (0.68)	1.47 (0.23)	10.04 (0.00)
ECM	6.75 (0.00)	10.55 (0.00)	6.45 (0.00)	1.30 (0.28)

a. Note that the variable at the column head is the dependent variable. The numbers not in parentheses are F-statistics, while those in parentheses are the significance levels.

Table A21 : Causality Tests Between Fisher Ideal Aggregates and the VAR that uses the GDP Deflator[a]

	M1FI	Real Income	R90	Deflator
M1FI	1.80 (0.18)	1.31 (0.26)	8.81 (0.00)	0.89 (0.35)
Real Income	0.27 (0.60)	0.78 (0.38)	6.83 (0.01)	0.52 (0.47)
R90	10.05 (0.00)	2.17 (0.15)	0.01 (0.93)	0.72 (0.40)
Deflator	1.22 (0.27)	0.00 (0.97)	0.03 (0.87)	3.09 (0.08)
ECM	10.02 (0.00)	3.35 (0.02)	6.97 (0.00)	5.15 (0.00)
	M2FI	**Real Income**	**R90**	**Deflator**
M2FI	0.75 (0.39)	0.21 (0.65)	5.97 (0.02)	0.26 (0.61)
Real Income	0.00 (0.97)	0.91 (0.34)	5.76 (0.02)	0.69 (0.41)
R90	11.74 (0.00)	0.59 (0.44)	0.35 (0.56)	0.19 (0.67)
Deflator	5.24 (0.03)	0.06 (0.80)	0.05 (0.83)	6.84 (0.01)
ECM	3.84 (0.03)	5.72 (0.01)	8.78 (0.00)	3.06 (0.05)
	M2+FI	**Real Income**	**R90**	**Deflator**
M2+FI	1.19 (0.28)	0.01 (0.91)	4.9 (0.03)	1.36 (0.25)
Real Income	0.00 (0.99)	0.75 (0.39)	5.77 (0.02)	0.95 (0.33)
R90	14.71 (0.00)	0.82 (0.37)	0.29(0.59)	0.10 (0.75)
Deflator	5.80 (0.02)	0.09 (0.77)	0.03 (0.85)	7.27 (0.01)
ECM	4.41 (0.02)	5.73 (0.00)	8.50 (0.00)	2.17 (0.12)
	M3FI	**Real Income**	**R90**	**Deflator**
M3FI	1.11 (0.30)	2.29 (0.13)	0.00 (0.98)	0.13 (0.72)
Real Income	0.23 (0.63)	0.22 (0.64)	3.73 (0.06)	0.41 (0.52)
R90	3.33 (0.07)	2.31 (0.13)	0.62 (0.43)	0.42 (0.52)
Deflator	2.71 (0.10)	0.00 (0.98)	0.41 (0.52)	8.09 (0.01)
ECM	5.18 (0.01)	8.70 (0.00)	6.63 (0.00)	2.66 (0.08)

Table A21 (Continued): Causality Tests Between Fisher Ideal Aggregates and the VAR that uses the GDP Deflator[a]

	M3+FI	Real Income	R90	Deflator
M3+FI	1.52 (0.22)	3.34 (0.07)	0.07 (0.80)	0.83 (0.37)
Real Income	0.25 (0.62)	0.17 (0.68)	3.86 (0.05)	0.68 (0.41)
R90	6.12 (0.02)	2.66 (0.11)	0.53 (0.47)	0.16 (0.69)
Deflator	3.73 (0.06)	0.00 (0.98)	0.32 (0.57)	7.82 (0.01)
ECM	4.85 (0.01)	9.35 (0.00)	6.60 (0.00)	2.15 (0.12)
	LLFI	**Real Income**	**R90**	**Deflator**
LLFI	0.27 (0.60)	2.69 (0.11)	0.92 (0.34)	0.00 (1.00)
Real Income	0.35 (0.56)	0.32 (0.58)	4.61 (0.04)	0.44 (0.51)
R90	7.55 (0.01)	1.59 (0.21)	0.51 (0.48)	0.68 (0.41)
Deflator	1.37 (0.25)	0.00 (0.97)	0.12 (0.73)	9.12 (0.00)
ECM	3.63 (0.03)	9.07 (0.00)	7.05 (0.00)	2.83 (0.07)
	LL+FI	**Real Income**	**R90**	**Deflator**
LL+FI	0.63 (0.43)	3.76 (0.06)	1.08 (0.30)	0.38 (0.54)
Real Income	0.40 (0.53)	0.19 (0.66)	4.87 (0.03)	0.68 (0.41)
R90	10.68 (0.00)	1.93 (0.17)	0.40 (0.53)	0.42 (0.52)
Deflator	2.09 (0.15)	0.01 (0.94)	0.10 (0.75)	8.55 (0.00)
ECM	3.77 (0.03)	9.64 (0.00)	7.27 (0.00)	1.99 (0.14)

a. Note that the variable at the column head is the dependent variable. The numbers not in parentheses are F-statistics, while those in parentheses are the significance levels.

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