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THE DYNAMIC BEHAVIOUR OF CANADIAN IMPORTS AND THE LINEAR-QUADRATIC MODEL

Evidence Based on the Euler Equation

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The Dynamic Behaviour of Canadian Imports and the Linear-Quadratic Model: Evidence Based on the Euler Equation

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CONTENTS

ABSTRACT

We examine the ability of the simple linear-quadratic model under rational expectations to explain dynamic behaviour of aggregate Canadian imports. In contrast to authors of previous studies who examine dynamic behaviour using the LQ model, we estimate the structural parameters using the Euler equation in a limited information framework that does not require an explicit solution for the model's control variables in terms of the exogenous forcing variables. In the first stage of our two-step methodology, we find statistically stable long-run elasticities of domestic activity and relative price to be about 1.5 and -0.5 over the sample period of estimation. In the second stage, we use the parameter estimates from the first stage and estimate the Euler equation. These empirical estimates imply that adjustment costs are about 9 to 13 times more important than disequilibrium costs. In sum, we find surprisingly encouraging evidence supporting the view that the LQ model is not inconsistent with the dynamic behaviour of Canadian aggregate imports.

RÉSUMÉ

Dans le présent article, les auteurs cherchent à établir si le modèle simple de forme quadratique linéaire peut, sous l'hypothèse de rationalité des attentes, expliquer le comportement dynamique de l'ensemble des importations canadiennes. Contrairement aux auteurs d'études antérieures, axées sur la méthode quadratique linéaire, ils estiment les paramètres structurels à l'aide de l'équation d'Euler en utilisant une méthode du maximum de vraisemblance à information limitée qui n'exige pas que les variables de contrôle du modèle soient explicitement résolues en fonction des variables d'impulsion exogènes. Dans la première des deux étapes de leur méthode, les auteurs trouvent des élasticités à long terme stables sur le plan statistique oscillant autour de 1,5 pour l'activité intérieure et autour de -0,5 pour les prix relatifs sur l'ensemble de la période considérée. Dans la deuxième étape, ils estiment l'équation d'Euler en se fondant sur les estimations des paramètres obtenues à la première étape. Ces estimations empiriques laissent supposer que les coûts d'ajustement sont de 9 à 13 fois environ plus importants que les coûts de déséquilibre. Somme toute, ils trouvent des résultats étonnamment encourageants validant l'hypothèse que le modèle de forme quadratique linéaire peut servir à formaliser le comportement dynamique de l'ensemble des importations canadiennes.

1 INTRODUCTION

One of the most important issues in international economics is the dynamic behaviour of imports. The conventional approach to examining the dynamic behaviour of imports has been to include a lagged dependent variable. However, this approach has been criticized for a number of reasons. These include the failure to incorporate forward-looking elements into the decision process, unduly restricting the response of imports, and econometric problems arising from nonstationary data and lagged dependent variables (see Gagnon 1989, Cuthbertson and Taylor 1987, and Amano and Wirjanto 1993, respectively). The purpose of this paper is to address these concerns using a simple linear-quadratic (LQ) model to explain the dynamic behaviour of Canadian aggregate imports when the forcing variables are nonstationary processes.

Three features of the LQ model allow us to address previously mentioned concerns. First, in the LQ framework agents are assumed to embody rational expectations and to minimize the expected discounted present value of quadratic costs of adjustment. As such, the LQ model incorporates forwarding-looking elements into the decision process and in addition provides some microfoundation. Second, despite its simplicity, the LQ model encompasses a variety of models often used in empirical studies. Examples include the standard partial-adjustment and error-correction models. Hence, the LQ model allows a wide range of possible dynamics. Third, the LQ model also gives rise to linear decision rules in the variables. This is an attractive feature, since the variables used in estimating aggregate import equations tend to be characterized by nonstationary processes, and the LQ model has well-understood properties for these nonstationary variables. LQ models are analysed in Sargent (1978), and identification and estimation with nonstationary forcing variables are studied by Gregory, Pagan and Smith (1990).

The LQ framework has been used to explain, *inter alia*, the demand for labour

(Sargent 1978, and Hansen and Sargent 1980), the demand for labour and capital (Meese 1980), the demand and supply of labour (Kennan 1988), natural resource extraction (Hansen, Epple and Roberds 1985), the supply of money (Mercenier and Sekkat 1988) and the demand for transaction balances (Cuthbertson and Taylor 1987).

In addition to examining the dynamics of a little-explored, but important macroeconomic variable, we use a different estimation approach and set of assumptions than those of most authors of early empirical studies who look at the LQ model. In our study we do not follow the usual assumption that the variables in the model are stationary in levels or contain deterministic trends. We instead assume that the variables are nonstationary due to the presence of stochastic trends or unit roots. The unit-root tests we present later in this paper suggest that it is important to examine the LQ model under the nonstationary assumption. As we show later, this assumption allows us to exploit the recently developed cointegration theory to estimate the parameters of the Euler equation. We also use a different estimation approach. The parameters of the LQ model are estimated using a limited-information procedure that is based on the model's Euler equation. In contrast, earlier empirical studies often estimated the parameters of the LQ model using a full-information approach that requires an explicit solution for the model's control variables in terms of the forcing processes. Under full-information maximum-likelihood (FIML) estimation, the process assumed to generate the forcing variables must be specified and estimated jointly with the law of motion and with certain cross-equation restrictions. Provided that the model is correctly specified, the FIML estimator will be more efficient than that based on the Euler equation approach. In contrast, the limited-information approach adopted in this paper provides us with consistent parameter estimates under more general conditions. We also note that in a Monte Carlo study based on stationary forcing variables, West (1986) finds that even under the assumption of no misspecification, full-information estimation is only moderately more efficient than limited-information estimation.

The organization of the paper is as follows. Section 1 describes the linear-quadratic model and derives some of its implications. Our estimation strategy is outlined in Section 2, while the empirical results are given in Section 3. Section 4 concludes the paper.

2 THE LINEAR-QUADRATIC MODEL

This section describes the LQ model and derives some of it implications. We generalize the static import demand equation by assuming that import demand is set according to an intertemporal loss function with quadratic costs of adjustment. These structures may be viewed as "realistic," as a result of aggregation over consumers or as providing local linearizations of the first-order conditions. The economic agent controls the import variable (m_t) and faces the problem of minimizing the expected present value of adjustment and disequilibrium costs, *viz*.,

$$\min_{\{m_i\}} \mathrm{E}_t \sum_{i=t}^{\infty} \beta^{i-t} [\gamma (m_i - m_i^*)^2 + (m_i - m_{i-1})^2] \tag{1}$$

for $i \geq t$, where E_t is the expectations operator conditional on the agent's information at time t (I_t), $\beta \in (0, 1)$ is the subjective discount rate and the parameter $\gamma > 0$ is a weighting factor that determines the relative size of the costs of adjustment. Note that γ is the inverse of the usual cost of adjustment.

The first-order necessary condition for the minimization of (1) is given by the following Euler equation:

$$\Delta m_t = \beta E_t \Delta m_{t+1} - \gamma (m_t - m_t^*) \tag{2}$$

and the corresponding transversality condition is

$$\lim_{T \to \infty} E_t [\beta^T \{\gamma (m_T - m_T^*) + \Delta m_T\}] = 0 \tag{3}$$

The forward solution to (3) is given by

$$m_t = \lambda m_{t-1} + (1-\lambda)(1-\beta\lambda) E_t \sum_{i=t}^{\infty} (\beta\lambda)^{i-t} m_i^* \tag{4}$$

where $\lambda < \beta^{-1/2}$ is the smallest stable root of the Euler equation obtained from the first-order condition and satisfies the condition

$$\beta\lambda^2 - (1+\beta+\gamma)\lambda + 1 = 0 \tag{5}$$

In order to complete the solution, it is necessary to specify a relationship between the target variable m_t^* and some observable economic variables. In general, we assume that the following law of motion for the target variable holds:

$$m_t^* = X_t^T \alpha + v_t \tag{6}$$

where v_t is a white noise process known to the agents, that is $v_t \in I_t$, but is unknown to the econometrican whose information set is $H_t \subset I_t$, X_t is a (*kx1*) row vector of forcing variables and α is an *(kx1)* column vector of unknown parameters.

It follows from equations (4) and (6) that the control variable m_t will inherit any stochastic trends in the forcing variables. For the purpose of illustration, assume that X_t is an independent random walk, that is,

$$(1-L) X_t = e_t \tag{7}$$

where $E_{t-1} e_t = 0$. Substituting equation (6) into (4) and (7) yields

$$(1-\lambda L) m_t = (1-\lambda) X_t^T \alpha + (1-\lambda) v_t \tag{8}$$

Since the root λ lies inside the unit circle, it follows from equation (8) that the endogenous variable m_t must be integrated of order one and that the white noise error term v_t is I(0). The latter implies that m_t and X_t are cointegrated with cointegrating vector $(1, \alpha)$.

Gregory, Pagan and Smith (1990) show that similar results also hold when the forcing variables follow more complicated I(1) processes. This result implies that if the forcing processes are I(1) variables, then the cointegration restriction between m_t and X_t is given by the LQ model.

In order to show that the LQ model encompasses other dynamic models, such as the standard partial adjustment and error-correction models, we use the Wiener-Kolmogorov prediction formula to replace the expectation in equation (4), given the law of motion for X_t, as in Sargent (1987). For the present analysis we focus on the case where the law of motion for X_t is given by the vector autoregressive process of order one:

$$X_t = \rho X_{t-1} + \zeta_t$$

where $|\rho| \leq 1$ and ζ_t is stationary and identically distributed. Given a stochastic process for X_t, equation (4) can be solved. For instance, if $|\rho| < 1$ then (4) becomes

$$\Delta m_t = (\lambda - 1)(m_{t-1} - X_{t-1}^T \alpha) + (1 - \lambda)\alpha \left[\frac{1 - \beta\lambda}{1 - \beta\rho\lambda} X_t - X_{t-1} \right]$$
$$+ (1 - \beta\lambda)(1 - \lambda)\mu_t$$

and if $\rho = 1$, so that the forcing variables are I(1), then equation (4) simplifies to an error-correction model:

$$\Delta m_t = (\lambda - 1)(m_{t-1} - X_{t-1}^T \alpha) + (1 - \lambda)\Delta X_t^T \alpha + (1 - \beta\lambda) 1 - \lambda\mu_t$$

or, rewritten in the form of the partial adjustment model,

$$m_t = \lambda m_{t-1} + (1 - \lambda) X_t^T \alpha + (1 - \beta\lambda)(1 - \lambda)\mu_t$$

To obtain an Euler equation that can be estimated, we first substitute equation (6) into (2) to obtain

$$\Delta m_t = \beta E_t \Delta m_{t+1} - \gamma(m_t - X_t^T \alpha) + \gamma v_t \tag{9}$$

and then we replace $E_t \Delta m_{t+1}$ by its realization $(\Delta m_{t+1} + u_{t+1})$, where u_{t+1} is a purely expectational error, such that $E_t u_{t+1} = 0$, and rewrite equation (9) as

$$\Delta m_t = \beta \Delta m_{t+1} - \gamma(m_t - X_t^T \alpha) + \eta_{t+1} \tag{10}$$

where $\eta_{t+1} = \beta u_{t+1} + \gamma v_t$ such that $E_t \eta_{t+1} = 0$. Thus, η_t is a composite error term that can be rewritten as a first-order moving average process, provided the structural error term v_t is a white noise process. Notice that equation (10) may be viewed as a "forward-looking" or error-correction model. Since, as noted earlier, the LQ model implies that m_t and the forcing variables X_t are cointegrated in the sense of Engle and Granger (1987), Dolado, Galbraith and Banerjee (1991) have suggested a two-step procedure for estimating the parameters in (10). We describe this estimation approach in the next section.

3 THE ESTIMATION STRATEGY

In this section we describe our two-stage estimation strategy for equation (10). In the first step, consistent estimates of the long-run parameter (α) may be obtained from a cointegrating regression:

$$m_t = X_t^T \alpha + \xi_t \tag{11}$$

where $\xi_t = (1 - \lambda L)^{-1} [\lambda \gamma v_t - \lambda \alpha e_t]$. Notice that since the smallest stable root λ satisfies the condition in (5), we see that as the adjustment cost gets large (that is, γ becomes small), the stable root approaches unity and ξ_t is nearly integrated, and hence highly persistent. Note also that since any bias in the ordinary least squares (OLS) estimates of equation (11) are of $O_p(T^{-1})$, it is possible to substitute these estimates into equation (10) and ignore any sampling uncertainty in the estimate of α when we estimate the remaining parameters in the Euler equation (see Stock 1987).

However, it is important to note that the rate T-convergence result does not, by itself, ensure that the parameters estimates of α will have good finite-sample properties. The reason is that the OLS estimates of α are not asymptotically efficient, in the sense that they have an asymptotic distribution that depends on nuisance parameters due to serial correlation in the error term and the endogeneity of the regressor matrix X_t induced by Granger-causation from innovations in m_t to innovations in X_t. This dependence on nuisance parameters obviously invalidates conventional inferential procedures. Therefore, it is desirable to use a procedure that is asymptotically optimal under more general conditions. In the estimation section of our paper we use three such procedures – Park's (1992) canonical cointegrating regression, Phillips and Hansen's (1990) prewhitened fully modified least squares and Stock and Watson's (1993) dynamic OLS. All three estimators are designed to eliminate nuisance parameter dependencies and possess the same limiting distribution as full-information maximum-likelihood estimates. The latter implies that the estimates are asymptotically optimal. The application of three different estimators also allows us to determine the robustness of the long-run parameter estimates.

We can use these approaches to estimate the forward-looking error-correction term $\hat{u}_t = m_t - X_t^T\hat{\alpha}$, where $\hat{\alpha}$ is a T-consistent and asymptotically efficient estimate of the long-run parameters. This in turn allows us to rewrite equation (10) as

$$\Delta m_t = \beta\Delta m_{t+1} - \gamma\hat{u}_t + \eta_{t+1} \tag{12}$$

Since all variables in (12) are I(0), Dolado, Galbraith and Banerjee (DGB) suggest estimating the discount rate β and the ratio of disequilibrium to adjustment cost γ by some type of generalized instrumental variable method. Therefore, in the second step we use Hansen's (1982) generalized method of moments (GMM) estimator. This estimator should allow us to control the effect of the MA(1) process in the composite error term on the standard errors. If the structural error term v_t is serially uncorrelated, then lags of Δm_t and

$\hat{u}_t$ at time *t-1* or earlier are valid instruments for GMM estimation. However, in order to allow for the possibility that v_t follows an MA(1) process, possibly due to the effects of time aggregation, we also estimate the model using lags of Δm_t and $\hat{u}_t$ at time *t-2* and earlier. To the extent that there are more instruments than parameters to be estimated the validity of the model is tested using Hansen's (1982) J-test for over-identifying restrictions.

In their recent paper, Gregory, Pagan and Smith (GPS) have observed a case where the above two-step method will fail. To see this, suppose that the process generating the forcing variable X_t is given by (7); then the covariance matrix between the instruments and the regressors is singular and only one of the two parameters β and γ is identifiable. If ΔX_t follows a higher-order (stationary) AR or VAR process, then the non-singularity condition is satisfied and both parameters will be identifiable. However, calculations in GPS indicate that even if ΔX_t follows a stationary AR(1) process, a joint estimation of β and γ may be difficult. The source of the problem arises from the estimation of β. This argument then provides some justification for the common practice of presetting the value of β. Accordingly, in the empirical analysis we also preset the value of β and estimate the value of γ. This allows us to check the sensitivity of the estimate of γ to different choices of the discount parameter.

4. THE EMPIRICAL RESULTS

4.1. Pretests for integration and cointegration

To implement the two-step procedure it is necessary to specify the forcing variables X_t that influence the economic agent's target level of aggregate imports (m_t^*). We follow the traditional approach and adopt the simplest possible specification of imports as

$$m_t^* = \alpha_0 + \alpha_1 y_t + \alpha_2 p_t + v_t \tag{13}$$

where y_t is a domestic activity measure constructed as the sum of consumption, and investment in machinery and equipment. Conventional import demand equations often use real income as a domestic activity measure. However, from the point of view of Canadian trade, real income is often not the most appropriate measure of economic activity. A large part of Canada's imports consists of consumption and capital goods. Therefore to isolate the influence of economic activity on imports, we construct a measure encompassing these goods. The variable p_t is the relative price of aggregate imports to the activity measure. We use quarterly data from 1960Q1 to 1993Q3 and use them in natural log form. Further details of the measures of m_t, y_t and p_t and their sources can be found in the Data appendix.

We begin by examining the time series properties of each series. To this end we use the augmented Dickey and Fuller (1979) and the non-parametric Phillips and Perron (1988) Z_α tests. These tests allow us to formally test the null hypothesis that a series is I(1) against the alternative that it is I(0). The test statistics are reported in Table 1 (p. 19). For all three variables, the null hypothesis of a unit root cannot be rejected even at the 10 per cent level of significance. Therefore, we conclude that the variables under consideration are well characterized as nonstationary or I(1) processes.

As we argued in the previous section, an implication of the LQ model is that if the forcing processes y_t and p_t are I(1), then these variables should form a cointegrating relationship with m_t. We test whether this implication is supported by the data by applying tests for cointegration.

To examine whether evidence consistent with cointegration exists, we use the two-step approach proposed by Granger (1983) and later refined by Engle and Granger (1987). Specifically, we employ the augmented Dickey-Fuller test suggested by Engle and Granger and the normalized bias version of the Phillips-Perron test proposed by Phillips and Ouliaris (1990). Using regression (13), we test for cointegration. Given the sample size used in this paper, the least-squares estimate of the cointegrating vector is likely to be substantially biased (see Banerjee, Dolado, Hendry and Smith 1986). Moreover, the simple least-squares estimation of equation (13) does not allow hypothesis testing to be carried out on the estimated parameters of the cointegrating vector. For these reasons, least-squares estimation of the cointegrating regression is carried out only for the purposes of testing the null hypothesis of no cointegration among the variables m_t, y_t and p_t. The test regressions include a constant, and a constant and a linear trend term. If we find cointegration in the mean-adjusted specification, this corresponds to "deterministic cointegration," which implies that the same cointegrating vector eliminates deterministic trends as well as stochastic trends. But if the linear stationary combinations of the I(1) variables have a non-zero linear trend, this then corresponds to "stochastic cointegration" (see Ogaki and Park 1989 for a discussion of stochastic and deterministic cointegration). The results of the cointegration tests for the variables m_t, y_t and p_t are reported in Table 2 (p. 19). Looking at the test statistics, we see that for the mean-adjusted case the augmented Engle and Granger (AEG) and Phillips and Ouliaris (PO) tests reject the null hypothesis of no cointegration at the 10 and 5 per cent levels respectively whereas for the detrended case, only the PO tests reject the null at conventional levels. Hence, we tentatively conclude that the variables under study form a valid cointegrating relationship.

Table 3 (p. 20) presents the OLS parameter estimates from the mean-adjusted cointegrating regression. As already mentioned, these estimates, even though they are consistent and converge to their true values at a faster rate T than the usual rate $T^{1/2}$, will not be efficient even asymptotically. They also have an asymptotic distribution that depends on nuisance parameters, thereby invalidating conventional inferential procedures. To control for these problems, we present, in Table 3, long-run parameter estimates using the procedures developed by Park (1992), Phillips and Hansen (1990) and Stock and Watson (1993). Looking at Table 3 we find the parameter estimates for y_t and p_t from all three estimators to be statistically significant and to have a priori expected signs. We also find that the estimates are not statistically different from each other. Specifically, we find the long-run elasticity of domestic activity with respect to imports to be about 1.5, while that for relative price is estimated to be about -0.5. We note that our long-run estimates fall well within the range of those estimated in earlier studies. Deyak, Sawyer and Sprinkle (1993) review previous empirical studies of Canadian import demand. From these studies they calculate the average activity elasticity to be about 1.5 and find relative price elasticities to range between -2.5 and -0.4.

For the purpose of interpreting the elasticities it is evidently crucial that the long-run parameter estimates be structurally stable over the sample period of estimation. Parameter stability is also required for the estimation of the Euler equation in the second stage of our procedure. To test for structural stability of the parameter estimates we use a series of parameter constancy tests for I(1) processes recently proposed by Hansen (1992) – the *Lc*, *MeanF* and *SupF* tests. All three tests have the same null hypothesis of parameter stability, but differ in their alternative hypothesis. Specifically, the *SupF* is useful if we are interested in testing whether there is a sharp shift in regime, while the *Lc* and *MeanF* tests are useful for determining whether or not the specified model captures a stable relationship. Unlike the stability tests often used in the empirical studies that assume stationary forcing variables, this test procedure treats the break point as unknown. This is an important advantage, since an ad hoc choice of the change point may adversely affect the power of the test, as the chosen break point may be misspecified for many alternatives of interest. Also, if an appropriate break point is chosen through inspection of the data, the size of the test will be incorrect even in large samples. The results presented in Table 4 (p. 20) suggest that we are unable to reject the null hypothesis for any of the tests even at the 20 per cent level. These results are reassuring, as they imply that our long-run parameter estimates are stable even though our sample period encompasses both changes in exchange rate regimes and implementation of free-trade agreements. We note that Hansen (1992) suggests that these tests may also be viewed as tests for the null of cointegration against the alternative of no cointegration. Thus, the test results also corroborate our previous conclusion of cointegration among the variables under study.

The evidence from this section suggests that our long-run parameter estimates are T-consistent, asymptotically efficient and stable. In the next section we use them to form a measure of the forward-looking error-correction term $\hat{u}_t$, which in turn will be used to estimate the Euler equation (12).

4.2. Results for the Euler equation

In this section we test whether the data are consistent with the LQ model using Hansen's (1982) GMM procedure to estimate the structural parameters in equation (12). The instruments include a constant, and lags of Δm_t and the constructed error-correcting variable $\hat{u}_t$. Two different sets of instruments are used and are denoted: I_4^1 and I_5^2, where I_j^i corresponds to the set $\{\text{constant}; \Delta m_{t-i}, ..., \Delta m_{t-j}; \hat{u}_{t-i}, ..., \hat{u}_{t-j}\}$. The instrument set lagged one period will yield consistent estimates of β and γ (subject to identification), given the assumption about the composite error term η_t, whereas the set lagged two periods will yield consistent estimates even if the structural error term v_t follows an MA(1) process, possibly owning to the effects of temporal aggregation.

Using the prewhitened Phillips and Hansen (PPH) parameter estimates from Table 3 to construct the error-correction term $\hat{u}_t$, we first attempt to estimate both the discount rate and the adjustment parameter by estimating, directly, the Euler equation. The results are reported in Table 5 (p. 21). Both discount rate parameter estimates are statistically significant at the 1 per cent level. The estimate of β corresponding to the instrument set I_4^1 is not in the expected range of 0.9 to 0.99, whereas the estimate using the instrument set I_5^2 lies within the expected range ($\hat{\beta}$ = 0.96). However, the point estimates are fairly imprecise, with large standard errors. For instance, if we consider the estimation results using the instrument set I_4^1 and subtract the standard error from the discount rate estimate, the latter would be in the neighborhood of 0.96. Turning to estimates of the adjustment parameter, we see that they lie within the range of 0.10 to 0.13, with the latter value being significant at the 10 per cent level. These estimates suggest that adjustment costs are about ninefold more important than disequilibrium costs in determining the demand for aggregate Canadian imports. Finally, we note that the J-tests are unable to reject the validity of the over-identifying restrictions imposed by the estimation for any of the instrument sets we consider, and that the estimates are not sensitive to the use of the other efficient cointegrating estimates presented in Table 3.

Although the previous results are relatively favourable, we next follow the standard practice of fixing the parameter β and then estimating the adjustment parameter from the Euler equation. We do this for two reasons. First, the results in GPS demonstrate the difficulties in identifying β when the forcing variables X_t are generated by an I(1) process. In the current study this is probably manifesting itself in the form of wide standard errors we noted earlier. Second, by estimating γ over a range of reasonable values for the discount parameter, we get an indication of the sensitivity of γ to different settings of β. Table 6 (p. 21) provides these results. For the instrument set I_4^1, we find estimates of the adjustment coefficient corresponding to β set equal to 0.99 and 0.975 to be significant at the 10 per cent level and to be about 0.8. In contrast, all the estimates corresponding to I_5^2 are significant (at the 5 per cent level) and range from 0.10 to 0.11. The former estimates imply that adjustment costs are about 13 times more important than disequilibrium costs, whereas the latter suggest about 9.5. We note two items here. First, the estimates of the adjustment parameter appear relatively insensitive to the fixed discount parameters we consider. That is, for both instrument sets the adjustment parameter estimates tend to lie within a relatively narrow range. Second, by fixing the discount parameter, the parameter γ's are estimated with greater precision. Specifically, the standard errors corresponding to the instrument sets I_4^1 and I_5^2 are about 33 and 44 per cent smaller, respectively, than the case where both β and γ are jointly estimated. Again, none of the J-tests reject the over-identifying restrictions, even at the 10 per cent level, for any of the instrument sets or fixed discount rates we consider.

5 CONCLUDING REMARKS

We have examined whether the simple linear-quadratic model under rational expectations is consistent with the dynamic behaviour of aggregate Canadian imports. In contrast to the authors of most previous studies examining the dynamic behaviour of imports, we incorporate a forward-looking element into the decision process and take careful consideration of the times-series properties of the data. Moreover, unlike the authors of other previous studies using the LQ model, we estimate the structural parameters using the Euler equation in a limited information framework that does not require a explicit solution for the model's control variables in terms of the exogenous forcing variables. We also make a different assumption about the data generation process of the variables which appears to be supported by the data.

Our results suggest that the behaviour of aggregate imports is consistent with the simple LQ model. In the first stage of our two-step methodology, we find the long-run elasticities of domestic activity and relative price with respect to imports to be about 1.5 and -0.5 over the 1960 to 1993 sample period. Not only are these estimates T-consistent and efficient, we find them to be stable over a sample period that encompasses both the implementation of free-trade agreements and changes in exchange rate regime. We find this stability of the long-run parameter estimates to be strong evidence in favour of our long-run specification. In the second stage, we use the parameter estimates from the first stage and estimate the Euler equation. When we estimate both discount and adjustment factors we find reasonable values for the adjustment parameter. However, the point estimates of the discount parameter are fairly imprecise, with large standard errors. The relatively imprecise nature of the point estimates and the desire to examine the sensitivity of the adjustment parameter to different discount settings led us to adopt the standard practice of presetting the discount parameter in estimating the Euler equation. These empirical estimates imply that adjustment costs are about 9 to 13 times more important than disequilibrium costs. In sum, we find surprisingly encouraging evidence supporting the view that the LQ model is not inconsistent with the dynamic behaviour of Canadian aggregate imports.

DATA APPENDIX

This appendix describes the variables used in the study. All series were drawn from CANSIM except nominal consumption, which was taken from the RDXF data base at the Bank of Canada. The data definitions and reference numbers (provided in parentheses) are as follows: domestic activity (D20488 + D20741), real aggregate imports (D20481 + D20482) and the relative price is constructed as the ratio between aggregate import and domestic activity price deflators. The deflators are constructed using nominal domestic activity (CON$ + IME$) and nominal aggregate imports (D20027 + D20328).

TABLES

Table 1:
Unit-Root Tests
Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) Tests[a]

Variable	ADF Lags[b]	ADF t-statistic	PP Z_{α}-statistic[c]
m_t	1	-2.692	-4.255
y_t	3	-1.157	-0.295
p_t	1	-1.542	-3.348

a. Henceforth, "***", "**", "*" indicate significance at the 1, 5 and 10 per cent levels, respectively. The ADF critical values are calculated from MacKinnon (1991), while the PP critical values are taken from Fuller (1976). All test regressions include a trend term.
b. We use the lag length selection procedure advocated by Hall (1989) and a 5 per cent critical value. The initial number of AR lags is set equal to the seasonal frequency plus 1 or 5.
c. The long-run variance is estimated using a VAR prewhitened quadratic kernel estimator with a plug-in automatic bandwidth parameter as suggested by Andrews and Monahan (1992).

Table 2:
Tests for Cointegration
Augmented Engle-Granger (ADF) and Phillips-Ouliaris (PO) Tests[a]

Regression	AEG Lags[b]	AEG t-statistic	PO Z_{α}-statistic[c]
Demeaned	0	-3.757*	-29.193**
Detrended	0	-3.743	-29.228*

a. AEG critical values are calculated from MacKinnon (1991). The PO Z_{α} critical values are taken from Haug (1992).
b. See footnote b, Table 1.
c. See footnote c, Table 1.

Table 3:
Estimation of the Static Import Equation

Variable	OLS	DOLS[a]	PPH[b]	CCR[c]
Constant	-7.051	-7.027*** (0.344)[d]	-7.244*** (0.369)	-6.808*** (0.362)
y_t	1.484	1.482*** (0.027)	1.500*** (0.030)	1.510*** (0.024)
p_t	-0.460	-0.552*** (0.093)	-0.522*** (0.099)	-0.556*** (0.086)

a. We use Newey and West (1987) standard errors as in Stock and Watson (1993). The truncation parameter is set equal to the seasonal frequency or 4. The estimates are based on fifth-order leads and lags.
b. The estimates are based on VAR(2) prewhitening procedure of Andrews and Monahan (1992), as this gave us serially uncorrelated residuals. The parameter estimates are not statistically different for non-prewhitened and VAR(1) to VAR(4) prewhitening.
c. As suggested by Park and Ogaki (1991), we report the third stage CCR estimates.
d. Standard errors are in parentheses.

Table 4:
Hansen Stability Tests of the Cointegrating Vector[a]

Lc	MeanF	SupF
0.168 (> 0.20)	3.361 (> 0.20)	7.791 (> 0.20)

a. We use the Phillips and Hansen estimates from Table 4 to calculate the test statistics.

Table 5:
Estimates of the Euler Equation[a]

Instruments	I_4^1	I_5^2
β	1.279*** (0.317)	0.957*** (0.341)
γ	0.128** (0.069)	0.096 (0.090)
J-test	4.777	7.915

a. The models are estimated using Hansen's (1982) GMM estimator. The second stage estimates of the weighting matrix are estimated using a lag length of 1 since this allows for an MA(1) error process.

Table 6:
Estimates of the Adjustment Term for Preset Values of Beta[a]

Instruments	I_4^1	I_5^2
β = 0.990 J-test	0.082* (0.046) 5.928	0.114** (0.050) 7.258
β = 0.975 J-test	0.079* (0.046) 6.048	0.110** (0.050) 7.334
β = 0.950 J-test	0.073 (0.047) 6.252	0.104** (0.050) 7.459

a. See footnote a, Table 5.

REFERENCES

Amano, Robert A. and Tony S. Wirjanto. 1993. "The Implications of Nonstationarity for the Stock-Adjustment Model." Working Paper No. 93-1. Bank of Canada, Ottawa.

Andrews, Donald W. K. and J. Christopher Monahan. 1992. "An Improved Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation." *Econometrica* 60:953-66.

Banerjee, Anindya, Juan J. Dolado, David F. Hendry and Gregor W. Smith. 1986. "Exploring Equilibrium Relationships in Econometrics Through Static Models: Some Monte Carlo Evidence," *Oxford Bulletin of Economics and Statistic*s 48:253-77.

Cuthbertson, Keith and Mark P. Taylor. 1987. "The Demand for Money: A Dynamic Rational Expectations Approach." *The Economic Journal* (Conference Papers) 87:65-76.

Deyak, Timothy A., W. Charles Sawyer and Richard L. Sprinkle. 1993. "The Adjustment of Canadian Import Demand to Changes in Income, Prices, and Exchange Rates." *Canadian Journal of Economics* 26:890-900.

Dickey, David A. and Wayne A. Fuller. 1979. "Distribution of the Estimator for Autoregressive Time Series with a Unit Root." *Journal of the American Statistical Association* 74:427-31.

Dolado, Juan J., John W. Galbraith and Anindya Banerjee. 1991. "Estimating Intertemporal Quadratic Adjustment Cost Models with Integrated Series." *International Economic Review* 32:919-35.

Engle, Robert F., and Clive W. J. Granger. 1987. "Cointegration and Error Correction: Representation, Estimation and Testing." *Econometrica* 55:251-76.

Fuller, Wayne A. 1976. *Introduction to Statistical Time Series*. New York: John Wiley and Sons.

Gagnon, Joseph E. 1989. "Adjustment Costs and International Trade Dynamics." *Journal of International Economics* 26:327-344

Granger, Clive W. J. 1983. "Co-Integrated Variables and Error-Correcting Models." Discussion Paper 83-13. Department of Economics, University of California at San Diego.

Gregory, Allan W., Adrian R. Pagan and Gregor W. Smith. 1990. "Estimating Linear Quadratic Models with Integrated Processes." In *Essays in Honour of Rex Bergstrom,* edited by Peter C. B. Phillips. Cambridge, MA: Blackwell Publishers.

Hall, Alastair. 1989. "Testing for a Unit Root in the Presence of Moving Average Parameters." *Biometrika* 76:49-56.

Hansen, Bruce E. 1992. "Tests for Parameter Instability in Regressions with I(1) Processes." *Journal of Business & Economic Statistics* 10:321-35.

Hansen, Lars P. 1982. "Large Sample Properties of Generalized Method of Moments Estimators." *Econometrica* 50:1029-54.

Hansen, Lars P., D. Epple and W. Roberds. 1985. "Linear-Quadratic Duopoly Models of Resource Depletion." In *Energy, Foresight and Strategy,* edited by Thomas J. Sargent. Washington D.C: Resources for the Future.

Hansen, Lars P. and Thomas J. Sargent. 1980. "Formulating and Estimating Dynamic Linear Rational Expectations Models." *Journal of Economic Dynamics and Control* 2:7-46.

Haug, Alfred A. 1992. "Critical Values for the $\hat{Z}_\alpha$-Phillips-Ouliaris Test for Cointegration." *Oxford Bulletin of Economics and Statistics* 54:473-80.

Kennan, John 1988. "An Econometric Analysis of Fluctuations in Aggregate Labor Supply and Demand." *Econometrica* 56:317-33.

MacKinnon, James G. 1991. "Critical Values for Cointegration Tests." In *Long-Run Economic Relationships: Readings in Cointegration*, edited by R. F. Engle and C. W. J. Granger. Oxford: Oxford University Press.

Meese, Richard. 1980. "Dynamic Factor Demand Schedules for Labor and Capital Under Rational Expectations." *Journal of Econometrics* 14:141-58.

Mercenier, Jean and Khalid Sekkat. 1988. "Money Stock Targeting and Money Supply: An Intertemporal Optimization Approach (with an Application to Canada)." *Journal of Applied Econometrics* 3:215-28.

Newey, Whitney K. and Kenneth D. West. 1987. "A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix." *Econometrica* 55:703-8.

Ogaki, Masao and Joon Y. Park. 1989. "A Cointegration Approach to Estimating Preference Parameters." Working Paper No. 209. Rochester Center for Economic Research, University of Rochester, Rochester.

Park, Joon Y. 1992. "Canonical Cointegrating Regressions." *Econometrica* 60:119-43.

Park, Joon Y. and Masao Ogaki. 1991. "Inference in Cointegrated Models using VAR Prewhitening to Estimate Short-Run Dynamics." Working Paper No. 281. Rochester Center for Economic Research, University of Rochester, Rochester.

Phillips, Peter C. B. and Bruce E. Hansen. 1990. "Statistical Inference in Instrumental Variables Regression with I(1) Processes." *Review of Economic Studies* 57:99-125.

Phillips, Peter C. B. and Sam Ouliaris. 1990. "Asymptotic Properties of Residual Based Tests for Cointegration." *Econometrica* 58:165-93.

Phillips, Peter C. B. and Pierre Perron. 1988. "Testing for a Unit Root in Time Series Regressions." *Biometrika* 75:335-46.

Sargent, Thomas J. 1978. "Estimation of Dynamic Labor Demand Schedules under Rational Expectations." *Journal of Political Economy* 86:1009-44.

Sargent, Thomas J. 1987. *Macroeconomic Theory*. Boston: Academic Press, Inc.

Stock, James H. 1987. "Asymptotic Properties of Least Squares Estimators of Cointegrating Vectors." *Econometrica* 55:1035-56.

Stock, James H. and Mark W. Watson. 1993. "A Simple Estimator of Cointegrating Vectors in Higher Order Integrated Systems." *Econometrica* 60:783-820.

West, Kenneth D. 1986. "Full versus Limited Information Estimation of a Rational Expectations Model: Some Numerical Comparisons." *Journal of Econometrics* 33:367-85.

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