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Speculative Behaviour, Regime-Switching and Stock Market Crashes

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Abstract

This paper uses regime-switching econometrics to study stock market crashes and to explore the ability of two very different economic explanations to account for historical crashes. The first explanation is based on historical accounts of "manias and panics." Its key features are that "overvaluation" increases the probability and expected size of a crash. Using U.S. data for 1926-89, we find considerable support for this model's predictions. The second explanation is based on switches in fundamentals. Simulations show that switching fundamentals can cause markets to mimic speculative behaviour. However, switches in fundamentals do not coincide with most stock market crashes.

Résumé

Les auteurs utilisent des méthodes de régression avec changement de régime pour étudier les krachs et examiner la validité de deux thèses très différentes qui pourraient expliquer ceux qui sont survenus dans le passé. La première repose sur une description des épisodes antérieurs de folie spéculative et de panique. Son argument central est que la «surévaluation» accroît la probabilité et l'ampleur attendue des krachs. L'analyse des données américaines couvrant la période 1926-1989 appuie dans une large mesure les prédictions obtenues. La deuxième thèse est fondée sur la présence de changements de régime dans les déterminants fondamentaux. Selon les simulations effectuées, les changements de régime peuvent amener les marchés à se comporter comme s'il existait une bulle. Toutefois, ils ne coïncident généralement pas avec des krachs.

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I. Introduction

Stock market crashes have presented a perennial challenge to our understanding of financial markets. The fact that there are sometimes abrupt changes in asset prices with little "news" about economic fundamentals is difficult to reconcile with simple models of asset pricing. Attempts to deal with this fact have tended to follow one of two approaches. The first allows asset prices to be determined by factors that do not affect their fundamental values. The second approach maintains that the market is "efficient," but suggests that the relationship between economic fundamentals and asset prices is highly non-linear, so that relatively minor pieces of news sometimes have exceptionally large effects.

Stock market crashes have long fascinated observers of financial markets, but they are hard to analyse using conventional econometric techniques. One key problem is that crashes may involve sharp breaks in the relationships among variables. This suggests using techniques that explicitly recognize such breaks. This paper demonstrates that models of stock market crashes have empirical implications that can be tested using switching-regression econometrics. We show that switching regressions reveal new patterns in the data that go beyond existing stylized facts.

The first model that we consider is designed to capture several common features of historical accounts of "manias and panics."[1] We refer to this as a model of speculative behaviour. The stylized pattern is an accelerating upswing in asset prices, followed by a precipitous reversal. Historical accounts suggest that an asset-price crash becomes more likely as the relationship between current prices and their fundamental value grows more extreme. In Section II, we show how the model of speculative behaviour has testable implications for the presence of regime-switching in stock market returns. In particular, we show that the model predicts a correlation between stock returns and the deviation from fundamental price, but that this correlation should vary across regimes. It also suggests that the size of the deviation from fundamental price should help predict the probability of a crash.

Regime-switching has been linked to speculative behaviour in a number of previous studies. Van Norden and Schaller (1993) and van Norden (1996) use similar methods to look

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[1] See Kindleberger (1989).

for evidence consistent with such behaviour in the Toronto Stock market and in the foreign exchange market. Hall and Sola (1993) and Funke, Hall and Sola (1994) use a univariate regime-switching framework to look for such evidence in other markets.[2] Several other studies (such as Turner, Startz and Nelson (1989) or Hamilton and Susmel (1994)) have applied regime-switching models to U.S. stock market data, but these typically feature a univariate framework and do not try to link regime-switching to speculative behaviour.

As pointed out by Flood and Hodrick (1990), any empirical evidence consistent with the presence of speculative behaviour can be re-interpreted as evidence of misspecified fundamentals. We examine this possibility in detail. In the case of switching regressions, evidence of the kind implied by the model of speculative behaviour could also be consistent with a model where agents rationally anticipate changes in fundamentals. Barsky and DeLong (1993) and Donaldson and Kamstra (1996) argue that investors try to estimate a dividend growth rate that may vary over time and that therefore even small changes in the dividend growth rate can lead to substantial price changes. If new information causes investors to lower their expectations of future dividend growth rates, the stock price might suddenly plunge. Cecchetti, Lam and Mark (1990; 1993) formalize a model of occasional shifts in the dividend growth rate between a high and a low growth regime. They have shown that such a model is capable of generating some of the previously documented features of U.S. stock market returns. To the best of our knowledge, we are the first to explore formally the possibility that stock market crashes are linked to regime changes in economic fundamentals.

In Section II of the paper, we develop the models of speculative behaviour and switching fundamentals. Section III shows the relationship between speculative behaviour and a switching-regression specification and tests the null hypothesis that stock market returns are unrelated to the deviation from fundamental price using parametric restrictions on the switching-regression specification. Section IV presents parameter estimates of the model of switching fundamentals and discusses to what extent shifts in fundamentals explain the switching-regression results. Section V analyses how well the probabilities of collapse

[2] Van Norden and Vigfusson (1996) compare these two testing approaches.

generated by each model accord with actual stock market crashes. Section VI offers conclusions and poses questions for future research.

II. Parametric Models of Stock Market Crashes

In this section, we develop the models of speculative behaviour and switching fundamentals, each of which has the potential to account for extraordinary movements in stock market prices. Both the model of speculative behaviour and the model of switching fundamentals are departures from the basic Lucas (1978) asset-pricing model, which is discussed in Part A. The distinctive feature of the model of speculative behaviour, which is described in Part B, is that it allows actual prices to diverge from the present value of future dividends. The distinctive feature of the model of switching fundamentals (outlined in Part C) is that there are regime shifts in the endowment process, which might correspond to booms or recessions in the aggregate economy.

A. Equilibrium Asset Prices In the Lucas Model

This section briefly reviews key features of the Lucas (1978) exchange-economy asset-pricing model; a more detailed derivation is left for the appendix. The model is deliberately simple; in particular, the process we assume for dividends implies that in this model the observed price-dividend ratio should be constant. The relationship between ex post returns and variations in the price-dividend ratio then becomes a key feature to be explained by either a model of switching due to speculative behaviour, or a model of switching in the growth of dividends. In the empirical work that follows, we show how the results are affected by relaxing the constant price-dividend ratio assumption in various ways. We find that, for the alternatives we examine, the conclusions are fairly robust. Therefore, we maintain this assumption here as a way of simplifying the exposition.

In the Lucas model, there are a large number of identical, infinitely lived agents and a fixed number of assets that produce units of the non-storable consumption good. Since agents are identical, per capita consumption (C) is equal to per capita dividends (D). This and the assumption of Constant Relative Risk Aversion (CRRA) utility gives the following stochastic difference equation for equilibrium prices

$$P_t \cdot D_t^{\gamma} = \beta \cdot E_t D_{t+1}^{\gamma} \cdot (P_{t+1} + D_{t+1}) \tag{1}$$

which in turn yields the following equation for fundamental price

$$P_t^* = D_t^{-\gamma} \cdot \sum_{k=1}^{\infty} \beta^k \cdot E_t D_{t+k}^{1+\gamma} \tag{2}$$

where P is the value-weighted stock market index adjusted for population size, β is the subjective discount factor, $0 < \beta < 1$, E_t is the mathematical expectation conditioned on information available at time t, and γ is the coefficient of relative risk aversion.

The expression for fundamental price depends on future dividends, which are not observable. One way to express the fundamental price in terms of observables is to make an assumption about the stochastic process for dividends. A common assumption (with some empirical justification) is that log dividends are a random walk with constant drift. This leads to a simple solution in which the fundamental price is a multiple of current dividends. Formally, dividends are

$$d_t = \alpha_0 + d_{t+1} + \varepsilon_t \tag{3}$$

where d_t is the logarithm of dividends, α_0 is the drift parameter, and ε_t is a sequence of independent, identically distributed normal random variables with mean zero and variance σ^2. This gives

$$\rho = \frac{\beta \cdot e^{\alpha_0 \cdot (1+\gamma) + (1+\gamma)^2 \cdot \sigma^2/2}}{1 - \beta \cdot e^{\alpha_0 \cdot (1+\gamma) + (1+\gamma)^2 \cdot \sigma^2/2}} \tag{4}$$

as the fundamental price-dividend ratio. The equilibrium gross return is:

$$\begin{aligned} R_t &\equiv \frac{P_t + D_t}{P_{t-1}} \\ &= \left(\frac{1+\rho}{\rho}\right) \cdot e^{\alpha_0 + \varepsilon_t} \end{aligned} \tag{5}$$

Note that the effect of an increase in the expected rate of dividend growth depends on whether γ is greater or less than -1. When $\gamma > -1$, increases in the dividend growth rate raise the price-dividend ratio; when $\gamma < -1$, the reverse is true.

B. A Model of Speculative Behaviour

The model presented in this subsection introduces speculative behaviour into the model of the previous subsection. Our goal is to develop a model which links switching-regression econometrics to some of the characteristics of speculative behaviour that have been noted in historical accounts, experimental economics, and surveys of market participants.[3] The main result of this section is a switching-regression econometric specification (derived from the model of speculative behaviour) that provides a framework for interpreting stock market crashes.

We define the size of the deviation from fundamental price as $B_t = P_t - P^*$. If the deviation from fundamental price is rational, then:

$$B_t \cdot D_t^{\gamma} = \beta \cdot E_t[D_{t+1}^{\gamma}] \cdot E_t[B_{t+1}] \tag{6}$$

where we use the fact that D_t will be independent of B_t (i.e., B_t is an extrinsic bubble). We can then use the assumed process for dividends given in (3) to show that

$$\begin{aligned} \frac{E_t[B_{t+1}]}{B_t} &= \frac{D_t^{\gamma}}{\beta \cdot E_t[D_{t+1}^{\gamma}]} \\ &= \beta^{-1} \cdot e^{-\gamma \cdot (\alpha_0 + \gamma \cdot \sigma^2/2)} \\ &\equiv M \end{aligned} \tag{7}$$

[3] In controlled experiments where all investors receive the same dividend from a known probability distribution at the end of a fixed number of trading periods, there is a marked tendency for prices to rise relative to fundamental value and then crash. (See Smith, Suchanek and Williams (1988); Van Boening, Williams and LaMaster (1993) provide references to recent work.) Using surveys of market participants, Shiller, Kon-Ya and Tsutsui (1991) find that during a speculative episode a large proportion of investors buy stocks because they think prices will continue to rise in the short term, even though they eventually expect prices to drop. Similarly, the experimental evidence suggests that capital-gains expectations influence price changes.

so the expected growth rate of the speculative component corresponds to the rate of return on the fundamental component. Blanchard (1979) and Blanchard and Watson (1982) propose a specific solution to (7) with two states of nature.[4] In one state (state C), the speculative component completely collapses, so $E_t[B_{t+1}] = 0$. In the other state (state S), the speculative component survives. The probability of being in state S next period is assumed to be some constant, q. In this case, (7) implies

$$B_t \cdot M \ = \ E_t[B_{t+1}] \ = \ q \cdot E_t[B_{t+1}|S] \ + \ (1-q) \cdot 0 \qquad \textbf{(8)}$$

$$\therefore E_t[B_{t+1}|S] \ = \ \frac{M \cdot B_t}{q} \qquad \textbf{(9)}$$

As noted by Kindleberger (1989) and others, historical accounts of “manias and panics” suggest that these episodes share many common features. One such feature is that a crash becomes more likely as the speculative component becomes a larger proportion of the price. To reflect the possibility that crashes are more likely to occur when the speculative component accounts for a large proportion of the price, we allow q, the probability of survival, to depend on the relative size of the speculative component

$$q \ \equiv \ q(b_t) \qquad \textbf{(10)}$$

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[4] There is an extensive literature noting restrictions on the admissibility of non-fundamental solutions, including Diba and Grossman (1988), Obstfeld and Rogoff (1983, 1986) and Tirole (1982, 1985). Blanchard and Fischer (1989, 238) argue that "[These restrictions] often rely on an extreme form of rationality and are not, for this reason, altogether convincing. Often bubbles are ruled out because they imply, with a very small probability and very far in the future, some violation of rationality, such as non-negativity of prices or the bubbles becoming larger than the economy. It is conceivable that the probability may be so small, or the future so distant, that it is simply ignored by market participants." Moreover, recent work by Allen and Gorton (1993), Allen, Morris and Postlewaite (1993) and Leach (1991) has shown that restrictions on non-fundamental solutions are not robust.

where $b_t = B_t/P_t$ and[5]

$$\frac{d\,q(b_t)}{d\,|b_t|} < 0 \tag{11}$$

Neither historical experience nor experimental evidence suggests that all deviations from fundamental price collapse in a single period. To take a prominent recent example, many observers interpreted the fall in the Tokyo stock exchange in the months following January 1990 as the gradual unwinding of a speculative episode. We therefore allow the expected value of b_{t+1} conditioned on collapse to be non-zero.[6] If we want to interpret state C as the collapse of a speculative component, then the expected value of b_{t+1} in state C must increase less than proportionately with b_t. To incorporate this, we define the expected value of b_{t+1} in state C as $u \cdot P_t$ and make u_t a function of b_t:

$$E_t[B_{t+1}|C] = u(b_t)\cdot P_t \tag{12}$$

where u(.) is a continuous and everywhere differentiable function, u(0)=0, and $1 \geq u' \geq 0$.[7] Imposing (7) then gives

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[5] Note the use of the absolute value of b_t, since we allow the speculative component to be positive or negative.

[6] Other empirical research has also considered the possibility that the speculative component may collapse only partially. See Evans (1991) and Hall and Sola (1993).

[7] The assumptions that u(0)=0 and $1 \geq u' \geq 0$ are added to ensure that the speculative component is expected to shrink in the collapsing state. To see this, draw a graph with $u(b_t)$ (which equals $E[B_{t+1}|C]/P_t$)on the vertical axis and b_t on the horizontal axis. The function $u(b_t)$ passes through the origin, since u(0)=0. The 45° line represents a situation where $E[B_{t+1}|C]/P_t = B_t/P_t$; i.e., where a "collapsing" speculative component is the same size as the previous period's speculative component. Since $0 \leq u' \leq 1$, $u(b_t)$ always lies on or below the 45° line. Thus, these assumptions ensure that a collapsing speculative component is no larger than the speculative component in the previous period.

$$E_t[B_{t+1}|S] = \frac{M}{q(b_t)} \cdot B_t - \frac{1-q(b_t)}{q(b_t)} \cdot u(b_t) \cdot P_t \tag{13}$$

The assumptions on $q(b_t)$ and $u(b_t)$ are intended to capture the spirit of speculative behaviour as an explanation of crashes in a way that can empirically be translated into a switching-regression econometric specification. These restrictions are not imposed on the data. Rather, they help to give the model of speculative behaviour empirically testable content.

As shown in the appendix, we can use the above equations to solve for the expected returns in each regime as a function of b_t. To guarantee that the estimates of q will be bounded between 0 and 1, we adopt the same approach used in Probit models by imposing the functional form $q = \Phi(\beta_{q0}+\beta_{qb}\cdot|b_t|)$ where Φ is the standard normal cumulative density function (CDF). After linearizing equations (12) and (13), we obtain the following three-equation model, which we estimate below.

$$\begin{aligned} E(R_{t+1}|S) &= \beta_{S0} + \beta_{Sb} \cdot b_t \\ E(R_{t+1}|C) &= \beta_{C0} + \beta_{Cb} \cdot b_t \\ q &= \Phi(\beta_{q0} + \beta_{qb} \cdot |b_t|) \end{aligned} \tag{14}$$

As shown in the appendix, the model of speculative behaviour has a number of testable implications. First, equation (11) implies that $\beta_{qb} < 0$, so that as the deviation from fundamental price grows, so does the probability of a collapse. Second, the model implies that β_{Cb} should be negative. Intuitively, this is because a larger (positive) speculative component implies a larger capital loss if it collapses. Third, the model implies that $\beta_{Sb} > \beta_{Cb}$. Intuitively, a larger speculative component means a larger difference between returns in the surviving and collapsing states.

C. Switching in Dividends

The preceding section has shown how speculative behaviour could generate regime-switching behaviour in stock returns. However, as we noted in the introduction, regime-switching in fundamentals might also lead to switching in stock market returns. In the remainder of this section, we consider a variation of the Lucas asset-pricing model that explicitly allows for switching in the dividend process. A dividend process that meets this requirement is:

$$d_t = d_{t-1} + \alpha_0 + \alpha_1 \cdot S_{t-1} + \nu_t \qquad \textbf{(15)}$$

where d_t is the logarithm of dividends, ν_t is a sequence of independent, identically normally distributed random variables with mean zero and variance σ^2, and S_t is the sequence of Markov random variables with state space {0,1} and transition matrix:

$$\boldsymbol{P} \equiv \begin{bmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \end{bmatrix} \qquad \textbf{(16)}$$

This is a Markov-switching model. As noted in Cecchetti, Lam and Mark (1990), this model is able to capture features of stock market returns not captured by other models, including ARIMA, ARCH and GARCH models. In particular, Cecchetti, Lam and Mark show that a Markov-switching process in dividends can account for the evidence of mean reversion found by Fama and French (1988a) and Poterba and Summers (1988). Cecchetti, Lam and Mark (1993) show that switching in the endowment process can also account for the first moment of the equity premium and the risk-free rate.[8]

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[8] See Bonomo and Garcia (1994) for more on this point.

Cecchetti, Lam and Mark (1990) show that the above equations lead to a relation between fundamental prices and dividends of the form:

$$P_t = \rho(S_t) \cdot D_t \tag{17}$$

In other words, the price-dividend ratio will take on one of two values, depending only on whether the economy is in the high or the low dividend growth state. Stock market crashes could then be caused by a transition from the high ρ to the low ρ state, implying large movements in stock prices.[9] The equilibrium gross return can be obtained from the relationship between current prices and dividends and the dividend process, so

$$R_t = \left(\frac{1 + \rho(S_t)}{\rho(S_{t-1})} \right) \cdot e^{(\alpha_0 + \alpha_1 \cdot S_{t-1} + \nu_t)} \tag{18}$$

III. Estimates of the Model of Speculative Behaviour

This section is divided into four subsections. Part A explains the link between the model of speculative behaviour presented in Section IIB and the switching-regression econometric specification, as well as how the model of speculative behaviour can be tested. Part B reviews the data series that we use and introduces our empirical measure of the deviation from fundamental price. Part C presents, tests and interprets parameter estimates for

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9 The derivation of the model may be found in the appendix. The high dividend growth state need not correspond to the high ρ state. In fact, it can be shown that

$$\left\{ \rho(0) \begin{matrix} > \\ = \\ < \end{matrix} \rho(1) \right\} \Leftrightarrow \left\{ \gamma \begin{matrix} > \\ = \\ < \end{matrix} -1 \right\}$$

where ρ(0) is defined to be the high-growth state.

the model of speculative behaviour. Part D explores the robustness of these results by re-estimating the model over various sub-periods and by using an alternative measure of the fundamental price that relaxes the assumption of a constant fundamental price-dividend ratio.

A. <u>Econometric Technique</u>

To estimate the model of speculative behaviour, we may rewrite (14) as:

$$R_{S,t+1} = \beta_{S0} + \beta_{Sb} \cdot b_t + \varepsilon_{S,t+1} \tag{19}$$

$$R_{C,t+1} = \beta_{C0} + \beta_{Cb} \cdot b_t + \varepsilon_{C,t+1} \tag{20}$$

$$q = \Phi(\beta_{q0} + \beta_{qb} \cdot |b_t|) \tag{21}$$

where $R_{S,t+1}$ and $R_{C,t+1}$ are the returns from period t to period t+1 conditional on survival and collapse respectively, and b_t is a measure of the deviation from fundamental price in period t. We assume $\varepsilon_{S,t+1}$ and $\varepsilon_{C,t+1}$ are mean-zero independent and identically distributed normal random variables.

The three equations (19), (20) and (21) form a standard switching-regression model of the type described by Goldfeld and Quandt (1976) and Hartley (1978). Given normality of ($\varepsilon_{S,t+1}$, $\varepsilon_{C,t+1}$), estimates of the β's can be found by maximizing the likelihood function

$$\prod_{t=1}^{T} \left[\Phi\left(\beta_{q0} + \beta_{q1} \cdot |b_t|\right) \cdot \phi\left(\frac{R_{t+1} - \beta_{S0} - \beta_{S1} \cdot b_t}{\sigma_S} \right) \cdot \sigma_S^{-1} \right.$$
$$\left. + \left\{ 1 - \Phi\left(\beta_{q0} + \beta_{q1} \cdot |b_t|\right) \right\} \cdot \phi\left(\frac{R_{t+1} - \beta_{C0} - \beta_{C1} \cdot b_t}{\sigma_C} \right) \cdot \sigma_C^{-1} \right] \tag{22}$$

where ϕ is the standard normal probability density function (PDF) and σ_S, σ_C are the standard deviations of $\varepsilon_{S,t+1}$, $\varepsilon_{C,t+1}$. Note that this estimation technique not only allows us to recover consistent and efficient estimates of the parameters in both states but it also does not require assumptions about which regime generated a given observation. Instead, it considers the probability that either regime may have generated a given observation and gives an optimal classification of observations into the underlying regimes. The probability of being in regime i at time t+1 is defined as the probability conditioning on all relevant information available at the end of period t, namely b_t. This is determined solely by the classifying equation (21) and is given by the formula $\Phi(1(i) \cdot (\beta_{q0} + \beta_{qb} \cdot |b_t|)) \equiv P_i^A$, where 1(i) equals 1 in the surviving state and -1 in the collapsing state.[10]

In Section IIB, we note that the model of speculative behaviour has three testable implications: $\beta_{Cb}<0$, $\beta_{Sb}>\beta_{Cb}$, and $\beta_{qb}<0$. Aside from the model of speculative behaviour, however, the switching regression that we estimate also nests a variety of stylized facts about stock market behaviour. To better understand the extent to which the data support or fail to support the model, it is useful to consider first whether the switching regression finds evidence of anything besides the existing stylized facts.

In particular, it would be interesting to know whether deviations from fundamental price have explanatory power for returns.[11] We therefore consider the extent to which our switching regression is able to reject three simpler specifications of return behaviour, each of which captures a different stylized fact.

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[10] For those familiar with Markov-switching models, it may be useful to note that our model has a related stochastic structure. A two-state Markov model has two state-dependent probabilities: $q(t) \equiv Pr(S_t=0 \mid S_{t-1}=0)$ and $p(t) \equiv Pr(S_t=1 \mid S_{t-1}=1)$. The switching model presented here has one state-independent probability $q(t) = Pr(S_t=0)$. This is the special case of the Markov model where $q(t) = 1-p(t)$; i.e., the probability of today's state is independent of the state yesterday. Evans and Lewis (1995) note that the standard Markov model is not compatable with the assumption that the expected rate of return is the same across the two states; our restriction is sufficient to ensure that this condition is satisfied.

[11] This can also be interpreted as a test of asset-pricing models, like that in Section IIA, which imply that variations in returns should not be predictable and that deviations from fundamental price should therefore be irrelevant "noise."

First, ARCH and related models have focused considerable attention on the changing volatility of stock market returns. Under the null hypothesis that b_t has no effect on returns, the switching regression can capture this fact (which we refer to as "volatility regimes") by imposing the restrictions $\beta_{S0} = \beta_{C0} = \beta_0$, $\beta_{Sb} = \beta_{Cb} = \beta_{qb} = 0$ but allowing $\sigma_S \neq \sigma_C$, so

$$R_{t+1} = \beta_0 + \varepsilon_{t+1} \tag{23}$$

where

$$\begin{aligned} \varepsilon_{t+1} &\sim N(0,\sigma_S) \;\; with\;\; prob\;\; q \\ \varepsilon_{t+1} &\sim N(0,\sigma_C) \;\; with\;\; prob\;\; 1-q \end{aligned} \tag{24}$$

A similar specification has been estimated by Schwert (1989), who finds evidence of volatility regimes.

One fact that the volatility regimes null fails to capture is that periods of high volatility are more likely to occur during stock market declines, while periods of low volatility tend to be associated with stock market increases. This is sometimes referred to as the "leverage effect." Therefore, we might wish to maintain the assumption that expected returns in each regime are constant but allow these constants to differ across regimes. This is the special case of the switching regression where $\beta_{Sb} = \beta_{Cb} = \beta_{qb} = 0$, so b_t has no effect. This implies that returns are well characterized by a mixture of normal distributions with different means and variances, which can be expressed as

$$\begin{aligned} R_{t+1} &\sim N(\beta_{S0},\sigma_S) \;\; with\;\; prob\;\; q \\ R_{t+1} &\sim N(\beta_{C0},\sigma_C) \;\; with\;\; prob\;\; 1-q \end{aligned} \tag{25}$$

for some constant q.

Another possibility we explore is that returns may be linearly predictable, but that mean returns do not differ across regimes. Testing this amounts to setting $\beta_{S0} = \beta_{C0} = \beta_0$, $\beta_{Sb} = \beta_{Cb} = \beta_1$, and $\beta_{qb} = 0$, giving

$$R_{t+1} = \beta_0 + \beta_1 \cdot b_t + \varepsilon_{t+1} \qquad \textbf{(26)}$$

where[12]

$$\begin{aligned} \varepsilon_{t+1} &\sim N(0,\sigma_s) \ \textit{with prob } q \\ \varepsilon_{t+1} &\sim N(0,\sigma_c) \ \textit{with prob } 1-q \end{aligned} \qquad \textbf{(27)}$$

We refer to this as the "mean-reversion" model, since it corresponds to the regression test for mean reversion in stock prices in Cutler, Poterba and Summers (1991), except that we allow more flexibility for volatility by allowing the variances of returns to be drawn from high and low volatility distributions.[13]

B. <u>Data and Measurement of Fundamental Price</u>

The data we examine for evidence of speculative behaviour is drawn from the stock price database of the Center for Research on Security Prices (CRSP). We use their monthly value-weighted price (P) and dividend (D) indices for all stocks from January 1926 to December 1989. We convert P and D to real per capita terms.[14] We use the all-items

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[12] Note that allowing $\sigma_s \neq \sigma_c$ simplifies the inference by allowing all parameters to be identified under the null.

[13] In contrast to most of the empirical work in Cutler, Poterba and Summers (1991), we use non-overlapping observations of one-period returns.

[14] The population growth adjustment uses annual population data from 1924-45 from Historical Statistics of the United States (1976) (series A29) and quarterly data from the BIS database (UBBA US01) from 1946 onwards. Monthly dates are linearly interpolated. Data from January 1960 onwards are divided by 1.0043 to correct for the inclusion of Alaska and Hawaii from that date onwards.

consumer price index (CPI) to deflate nominal returns. Since dividends display strong seasonal fluctuations, we follow Fama and French (1988b) in using an average over the twelve-month period ending in the given month.

The first measure of deviations from fundamental price that we use (b_t^A) is tied to the model of equilibrium asset prices outlined in Section IIA, where the fundamental price is $P_t^*=\rho\cdot D_t$. Under the hypothesis that actual prices may deviate from the fundamental price, we can measure the proportional deviation from fundamental price as:

$$b_t^A = \frac{B_t}{P_t} = \frac{P_t - P_t^*}{P_t} = 1 - \frac{\rho \cdot D_t}{P_t} \tag{28}$$

where we use the mean price-dividend ratio as ρ.[15] The solid line in Figure 1 shows how b_t moves over the sample period. Interestingly, two periods when b_t is rising and relatively large are 1929 and 1987. The value of b_t is unusually low in 1932, 1938 and 1942.

C. Estimation and Test Results

The estimated parameters for the model of speculative behaviour (19)-(21) are presented in the top half of Table I. The lower part of that table presents likelihood-ratio (LR) test statistics for the restrictions implied by the three "stylized fact" models of stock returns. We will begin by considering the latter.

As shown in the first column of the table, the LR statistic for the Volatility Regimes specification is 16.29 and has a p-value of 0.003. Our rejection of this null implies that the

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[15] Under the hypothesis that the actual price corresponds to the fundamental price in the model in Section IIA ($P_t = P_t^*$), the mathematical expectation of P_t/D_t is ρ.

regimes must differ in more than their variances. This means that either the information contained in b_t helps to determine which regime prevails, or that the regimes have different expected conditional returns, or both.

In the second column, we see the LR statistic of 16.21 for the Normal Mixture specification has an even lower p-value (0.001). From this we conclude that there is more in the data than just the "leverage effect." Put another way, this tells us that deviations from the fundamental price have significant predictive power for the distribution of stock returns.

The LR statistic for the mean reversion model (14.79) allows a similar rejection (p-value=0.002), implying that the previous rejections cannot be explained by simple linear predictability alone. It therefore seems that the relationship between b_t and R_t is highly significant, but non-linear.

We turn now to the three coefficient restrictions implied by the model of speculative behaviour. The first is $\beta_{Cb}<0$. As Table I illustrates, the point estimate of β_{Cb} is negative; the standard error implies that we can reject the hypothesis that $\beta_{Cb}=0$ at just above the .05 level in a one-sided test. Second, β_{Sb} should be greater than β_{Cb}. The point estimates of the two parameters are consistent with this implication (p- value = .145). Third, β_{qb} should be negative. Again, this implication is consistent with the estimates reported in Table I. Here, however, the statistical evidence is stronger: the p-value for the hypothesis that $\beta_{qb}=0$ is .001. Accordingly, we conclude that the estimated model is consistent with the restrictions implied by our model of speculative behaviour.

To better understand the behaviour that the estimated model is capturing, it is useful to consider the estimates in greater detail. Expected returns in the surviving regime for $b_t=0$ (the mean value of b_t) are given by β_{S0}. The point estimate of β_{S0} is 1.007, implying a monthly rate of return of .7 per cent (8.7 per cent on an annual basis). In contrast, the point estimate of β_{C0} is .976, implying a monthly rate of return of -2.4 per cent (-25.3 per cent on an annual basis) in the collapsing regime. The difference in returns is more dramatic in periods when b_t is relatively large. For example, in September 1929, b_t=.216; this implies an expected rate of return of 7 per cent per year in the surviving regime and -44.6 per cent in the collapsing regime.[16]

The parameter β_{q0} can be used to calculate the probability that the collapsing regime will occur. When b_t=0, the probability of the collapsing regime is $1\text{-}\Phi(2.098)$=.018. The point estimate of β_{qb} implies that in a period when $|b_t|$ is relatively large, the probability of the collapsing regime increases. For example, at the end of September 1929, the point estimates imply that the probability that the collapsing regime will occur in the following period is more than twice as large as when b_t=0.

D. Robustness

The results discussed in the previous section are generally supportive of the predictions of the model of speculative behaviour. However, such econometric results are sometimes sensitive to the time period over which the estimation is done. It would also be useful to know how sensitive the results are to the assumption of a constant fundamental price-dividend ratio. We now examine both of these questions in turn.

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[16] Expected returns are calculated using the point estimates in Table I. For example, the expected monthly return in the collapsing regime in September 1929 is .976 + (-.111) (.216).

We divide the sample into three subperiods: 1926-54, 1954-74, 1974-89. The first sub-period includes the Great Depression, World War II and the Korean War. The division between the second and third sub-periods coincides with the first OPEC shock.

Results are shown in Table II and are generally similar across periods. As in the estimates for the full period, we always find $\beta_{S0}>1.0$ and $\beta_{C0}<1.0$. Some tests based on U.S. stock market data (such as mean reversion tests) show more evidence of a potential departure from efficient markets in time periods which include the 1929 crash and the Great Depression.[17] In contrast, the likelihood-ratio tests presented in Table II actually show stronger rejection of the "stylized fact" null hypotheses in the periods 1954-74 and 1974-89 than in the period that includes the 1929 crash and the Great Depression. In all periods we reject the various null hypotheses at the .05 level.[18]

The evidence on the testable implications of the model of speculative behaviour also seems to be robust. In all sub-periods, the point estimate of β_{Cb} is negative; in most cases, the hypothesis that $\beta_{Cb}=0$ can be rejected at conventional levels. Except for the 1954-74 sub-period, $\beta_{Sb}>\beta_{Cb}$; the p-values for the first and third sub-periods are .156 and .000, respectively. In all sub-periods β_{qb} is negative; the hypothesis that $\beta_{qb}=0$ is strongly rejected in all cases.

Under the assumption that the expected dividend growth rate is constant, the Lucas asset-pricing model implies that the fundamental price-dividend ratio (P^*_t/D_t) is constant, as

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[17] See, for example, Fama and French (1988a) or Kim, Nelson and Startz (1991). Evidence of mean reversion does not necessarily imply that asset prices deviate from fundamentals; see, for example, Brock and LeBaron (1990), Cecchetti, Lam and Mark (1990), Fama (1991), Fama and French (1988a), or Jog and Schaller (1994).

[18] The marginal significance level of the volatility regime test statistic is .052 in the period 1926-54; in all other cases, the p-values are smaller.

shown in equation (A7). This means that the measure of deviation from fundamental price which we have used above (b^A_t) will attribute all of the variation in the price-dividend ratio to speculative behaviour. There are many ways to relax this assumption. Here we consider an approach that allows for flexible variation over time in expected dividend growth. Based on Campbell and Shiller (1987), the key idea is that (in the absence of speculative behaviour) stock market prices provide a way of capturing information about future dividend growth that is contained in the information set available to market participants. We define a second measure of apparent deviations from fundamental price as:

$$b_t^B = \frac{P_t - P_t^B}{P_t} \tag{29}$$

where

$$P_t^B = S_t' + \frac{1+r}{r} \cdot D_t \tag{30}$$

and the right-hand side of the last equation uses the notation of Campbell and Shiller (1987).[19] S_t' is an optimal linear forecast of future dividend changes based on a VAR representation that includes past prices and dividend changes.

Results for the second measure of apparent deviations form fundamental price are presented in Table III. The patterns are broadly similar to the first measure. The three likelihood-ratio statistics allow us to reject the various "stylized fact" models at the 1 per cent significance level in every case. We see that β_{Sb}, β_{Cb}, and β_{qb} are -0.008, -0.126, and -1.507, respectively (compared with -0.006, -0.111, and -1.560 for b^A_t) and that their significance levels are all roughly the same as before. Estimates of the other parameters are also similar to

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[19] We set r=1/(mean[P]/mean[D]-1), so that the spread between price and a multiple of current dividends (S_t in Campbell and Shiller's (1987) notation) is zero on average.

the estimates in Table I. It therefore appears that the results are robust to allowing for linearly predictable variation in dividend growth (and thus for variation in the fundamental price-dividend ratio).

IV. Switching Fundamentals

In this section, we examine the model of switches in fundamentals outlined in Section IIC. One way to compare this model with the model of speculative behaviour examined previously would be to estimate a more general model which nests both kinds of switching as special cases. In principle, this would allow each model to be tested against more general alternative. However, it would require the estimation of a four-state Markov-switching regression, subject to multiple non-linear restrictions. Furthermore, testing for the absence of switching due to either of our two sources would then be equivalent to testing whether the number of discrete states could be reduced from four to two. Inference in such cases would be complicated by the presence of numerous unidentified parameters under the null hypotheses. We chose not to pursue this avenue because of the difficulty of implementing it. Instead, we use a Monte Carlo experiment to compare the two models.

We begin by estimating the parameters of the switching process for dividend growth given in equations (15) and (16). We then choose reasonable values for the coefficient of relative risk aversion γ and the subjective discount rate β.[20] Next we generate data for

[20] We consider values of γ on both sides of -1 because the model of switching fundamentals implies that the high dividend growth rate state will yield high stock prices when γ>-1 and low stock prices when γ<-1. We choose (β,γ) pairs so that the expected value of ρ matches the

dividends using the Markov-switching model, derive the corresponding equilibrium stock prices using equations (17) and (A24) - (A27), and calculate the resulting returns.[21] This becomes artificial data with which we repeat the analysis of the preceding section; we estimate the switching regression and carry out the parametric tests. Repeating these steps with fresh draws of the random variables ε_t and S_t, we are able to estimate the distribution of the switching-regression parameters and likelihood-ratio statistics in a world with dividend-switching but no speculative behaviour.

One interesting feature of the simulations is the size of the apparent deviation from fundamentals, which is presented in Table IV.[22] As in Section III, this is constructed as the difference between the observed stock price and the "fundamental" price calculated under the (erroneous) assumption that there is no regime-switching in dividend growth. Therefore, the "apparent" deviation from fundamentals is due to misspecification rather than speculative behaviour. In the actual data, the mean absolute value of b_t is about .232. The model of switching fundamentals produces much smaller and more tightly distributed values of b_t. The median over 1,000 simulations (with β=.97 and γ=-1.585) of the mean absolute value of b_t is about .024 and its .95 confidence interval lies below .052. The standard deviation of b_t in the actual data is about .202, compared with the median standard deviation of about .033 and a .95 confidence interval lying below .033 for the simulations. Since b_t is defined as the actual

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mean price-dividend ratio in our sample. The particular values we consider are (β=.97, γ=-1.585) and (β=.95, γ=-.609).

21 The measure of apparent deviations from fundamental price (b^A_t) is calculated as before.

22 The results in the table are based upon the first definition of b_t, which we have referred to above as b^A_t.

price (P) minus the fundamental price (where the fundamental price is P^*, as defined in Section IIa) divided by the actual price, Table IV tells us that the stimulated prices stay much closer to the fundamental price than actual prices do. This implies that failing to account for regime-switching in dividend growth can explain relatively little of the volatility of b_t^A.

The parameter estimates and likelihood-ratio statistics from the Monte Carlo experiment with β=.97 and γ=-1.585 are shown in Table V. The likelihood-ratio statistics show that switching fundamentals are capable of accounting for the rejections of the null hypotheses of volatility regimes, a normal mixture and mean reversion that we found in Section IV. This is an extremely interesting result, since it shows that switching in fundamentals does more than simply induce regime-switching in returns; it also causes the apparent deviation from fundamental price to affect conditional returns.

The likelihood-ratio statistics suggest that the model of switching fundamentals does not correspond very precisely to the actual data. The problem is that the rejections of the null hypotheses of volatility regimes, a normal mixture and mean reversion are much stronger than what we find in the actual data. In each case, the likelihood-ratio statistics from the actual data are in the extreme left tail of the distribution of likelihood-ratio statistics generated by the model of switching fundamentals.

The parameter estimates from the simulations are also presented in Table V and are also somewhat different from those we obtain with the actual data. The most notable example is that the less-frequent state (which is labelled C in Table V) is associated with highly positive returns (the median of β_{C0} is 1.142), whereas the actual-data estimate shows negative returns in the less frequent regime (β_{C0}=.976). On the other hand, the point estimates of β_{Cb} and β_{qb} are negative and $\beta_{Sb}>\beta_{Cb}$.

Table VI reports Monte Carlo simulations using β=.95 and γ=-.609. The parameter estimates for this case correspond somewhat more closely to the actual data. The infrequent regime (which is again labelled C in Table VI) is now associated with negative returns (β_{C0}=.932). The point estimates of β_{Cb} and β_{qb} are again negative and $\beta_{Sb}>\beta_{Cb}$. As in the simulations for (β=.97,γ=-1.585), the likelihood-ratio statistics are much larger than in the actual data.

The Monte Carlo simulations show that the model of switching fundamentals is able to reproduce many of the basic stylized facts of regime-switching found in the data, if not their exact magnitudes. It accounts for the rejection of the volatility regimes, mixture of normals, and mean reversion nulls; in fact, the model of switching fundamentals leads to much stronger rejections than we find in the actual data. It also yields slightly positive returns in the surviving state balanced (when γ>-1) by large but infrequent losses in the collapsing state, a probability of collapse that increases with the absolute value of b_t, and a negative relation between b_t and returns in the collapsing regime.

In the Lucas model we presented in Section IIC, there is no conceptual distinction between dividends, consumption and income. In performing our simulations, it would therefore be reasonable to calibrate the switching process for fundamental to data for any one of these three variables. We have considered only the results based on calibrating the model to dividends. This suffices to show that the evidence of speculative behaviour like that found in Section III can be generated by a model of switching fundamentals. However, we will return to consider the possiblity of regime-switching in other measures of fundamentals in Section V.

V. Accounting for Historical Crashes

The evidence in Sections III and IV suggests that either the model of speculative behaviour or the model of switching fundamentals could account for the characteristics of U.S. stock market returns that are highlighted by the switching regression. In this section, we examine how well specific historical crashes are accounted for by each of the models. To do this, we use parameter estimates for each of the models to generate the probability of a stock market crash. For the model of speculative behaviour, we focus on the probability of a return that is two standard deviations below the mean return.[23] For the model of switching fundamentals, we focus on the probability of the low-growth regime.

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[23] In the switching regression model, the ex ante probability of observing a return equal to or less than some given value K is calculated according to the formula

$$Pr(R_{t+1} \leq K) = \Phi\left(\frac{K - \beta_{S0} - \beta_{Sb} \cdot b_t}{\sigma_S}\right) \cdot \Phi\left(\beta_{q0} + \beta_{qb} \cdot |b_t|\right) + \Phi\left(\frac{K - \beta_{C0} - \beta_{Cb} \cdot b_t}{\sigma_C}\right) \cdot \Phi\left(-\beta_{q0} - \beta_{qb} \cdot |b_t|\right) .$$

A. Speculative Behaviour

Figure 2 plots actual crashes versus the probability of a crash (generated from the empirical estimates of the model of speculative behaviour).[24] We begin by examining the two best-known crashes of this century. In both 1929 and 1987, there is an increase in the probability of a crash in advance of the actual crash. In both cases, the probability of a crash roughly doubles in the months preceeding the crash. There are also substantial increases in the probability of a crash around the time of the 1930, 1931, 1932, 1937, 1946, 1962 and 1974 crashes.[25] The 1970 crash is an exception, coming at a time when the probability of collapse had recently decreased and was low. To check the robustness of the results, we present a graph based on b_t^B, which allows for variation in the expected dividend growth rate, in Figure 3. The pattern is similar: a rise in the probability of a crash precedes most of the largest crashes.

The model of speculative behaviour also suggests that large undervaluations imply an increasing probability of a sharp movement towards fundamental price. Plots of the probability of a rally (Figure 4) show that it is less than 10 per cent for all but two periods between 1926 and 1989. The first exception is 1932, when the probability rose very sharply

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[24] Actual crashes were defined by using monthly returns to calculate the 20 largest three-month losses in our sample. Three-month rather than one-month losses were used both to capture more gradual (but large) price declines, as well as to exclude transitory losses that are almost immediately offset by subsequent price increases. Only 10 distinct crashes are shown since half of the three-month losses either overlapped or were very close to each other.

[25] A high probability of crash does not always coincide with one of the 10 biggest crashes, but this does not mean that it was not associated with a crash. For example, the probability of a crash was high in late 1941 and early 1942; in fact, the price declines in September-November 1941 and January-March 1942 were among the most severe in the sample but did not quite make the top 10 list.

to peak at about .35; this was followed immediately by one of the largest rallies in our sample. The second exception was in 1942, when the probability of a rally rose sharply to peak at about .20; this was followed (with a lag) by another of the largest rallies in our sample. The probability of a rally also rose before the 1938 rally. There was no increase in the probability of a rally in 1929 or 1987 because there was no apparent undervaluation at those times (see Figure 1).

B. Regime Switches in Fundamentals

In Figure 5, we examine how well switches in expected dividend growth account for actual stock market crashes. Figure 5 plots the ex ante probability of being in the high dividend growth state against actual market crashes. First, we note that the two most famous crashes (1929 and 1987) are marked by high and stable probabilities of high dividend growth. The 1946 and 1974 crashes are also marked by high and stable probabilities of being in the high dividend growth state. Therefore the dividend-switching model seems unable to explain these crashes. At the time of the 1930, 1931, 1937, 1962 and 1970 crashes, the probability of the high dividend growth state was falling rapidly, however. This is consistent with agents revising their expectations of future dividend growth and abruptly lowering stock prices.[26]

Intuitively, the ex ante probability (which is based on Hamilton's (1989) one-sided filter) uses only the information on past dividend growth available to investors in period t.

[26] In interpreting Figure 4, we have implicitly assumed that $\gamma>-1$ so the low-growth state is associated with lower stock prices. If $\gamma<-1$, crashes would be associated with transitions from low to high dividend growth states, rather than the reverse. We note that if we adopt this interpretation, then the dividend-switching model does much worse in explaining stock market crashes, with none of the crashes corresponding to such a transition.

However, investors may have information which is not reflected in past dividend growth and which helps them to forecast future dividend growth. One way to account for this is to use all information on dividend growth in the entire sample to assign probabilities. The result can be thought of as an ex post probability and corresponds to Hamilton's (1989) two-sided smoother. In Figure 6, we plot both the ex ante and ex post probabilities. Endowing investors with more information makes only a small difference in the results, essentially pushing back by a few months the date at which the regime changes in the 1930s and early 1940s could have been perceived.

Intriguingly, the 1932, 1933, 1938 and 1943 rallies correspond to periods in which the probability of the high dividend growth state was rising (see especially Figure 6). The probability was stable, however, at the time of the 1929, 1935, 1974 and 1987 rallies.

It has often been argued that dividends are a poor measure of either corporate cash flow or the endowment.[27] We therefore consider two other variables which might capture changes in economic regime: industrial production and interest rates. Figure 7 presents the ex ante probability of being in the high-growth state for industrial production. This probability is relatively low and variable in the early 1930s and 1937, but, with the exception of 1974, it does a poor job of explaining post-war crashes. For example, the probability of rapid growth in industrial production is rising in the months preceding the 1946 crash. Around the 1962, 1970 and 1987 crashes, the probability of rapid industrial growth is close to 1 and very stable. Switches in expected dividend growth do a better job of explaining stock market crashes than switches in expected industrial production growth. Figure 8 graphs the ex ante probability of

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[27] See, for example, Ackert and Smith (1993).

high real interest rates.[28] The graph is quite different than Figures 5 and 7; it shows relatively rare but abrupt and decisive regime shifts. The figure does not offer much encouragement to the view that changes in the real interest rate regime explain stock market crashes. Only the 1937 crash corresponds to a rise in the probability of the high real interest rate regime.

VI. Conclusion

Using the 1926-89 monthly Centre for Research in Securities Prices (CRSP) stock market data, our switching regressions find that there is non-linear predictability of returns based on the degree of apparent market overvaluation. We are careful to show that this cannot be accounted for by previously documented stylized facts, such as regime shifts in volatility, the "leverage effect" or the linear predictability of returns.

The model of speculative behaviour is consistent with this non-linear relationship. We find that the regime that corresponds to the collapse of a speculative component is significantly more likely when the degree of overvaluation in the previous period is high. We also find a substantial difference in expected returns between the regime corresponding to the survival of a speculative component and the regime corresponding to a speculative collapse. In the regime where the speculative component survives, the typical return is positive. In the regime where the speculative component collapses, the typical return is -25.3 per cent (on an

[28] The monthly interest rate series we use is Moody's Industrial Bond Index Annual Yield; we created an ex post real interest rate series by deflating by the year-over-year changes in the CPI (all-items urban consumer). We also examined an ex ante real interest rate series using the previous year's CPI to deflate; the results were very similar.

annual basis). The difference in expected returns between the two regimes increases with the size of the apparent overvaluation. In a period where the apparent overvaluation is relatively large, such as September 1929, the expected return if the speculative component collapses is -44.6 per cent (on an annual basis). The effects of apparent overvaluation are not restricted to the period that includes the Great Depression and World War II; in fact, the results are similar for each of three subperiods over which we estimate the model of speculative behaviour. The results are also robust to changes in the measure of fundamental price that try to capture the effects of anticipated changes in dividend growth rates. However, since we use linear forecasts of dividend growth rates, we might conceivably fail to capture important non-linearities in their behaviour.

We therefore calibrate a Markov-switching model of dividend growth to U.S. data for 1926-89, simulate the resulting asset-pricing model, and estimate the switching regression using the artificial data from the simulations. We find that switching fundamentals could account for much of the statistical evidence we find for speculative behaviour; there is strong evidence of regime-switching in the simulated returns and the degree of apparent "overvaluation" influences expected returns. However, we find that the degree and variability of the apparent market "overvaluations" are much smaller in the simulations than in the actual data.

The coefficient estimates and tests of parameter restrictions suggest that the model of speculative behaviour and the model of switching fundamentals are substitutes in the sense that either model is capable of accounting for the switching characteristics of actual returns we have highlighted. A different picture emerges when we examine specific stock market crashes. We find that the probability of a crash from the model of speculative behaviour rises in advance of several prominent stock market crashes (such as 1929 and 1987), but not in advance of all crashes. The probability of a crash calculated from the model of switching fundamentals fails to rise before the 1929 and 1987 crashes, although it does anticipate several major stock market crashes.

Overall, the evidence suggests the model of speculative behaviour and the model of switching fundamentals may be complements rather than substitutes. For example, the 1929 and 1987 crashes correspond well to the model of speculative behaviour but not to switches in dividend growth, while other crashes correspond more closely to the model of switching fundamentals.

The results presented in this paper raise many interesting questions for both theorists and empirical researchers. If one is inclined to believe that speculative behaviour is important, there are several areas in which an improved understanding would be helpful. First, why is it that actual prices sometimes begin to drift away from fundamental prices?[29] Second, are speculative episodes triggered by particular economic mechanisms, such as substantial increases in liquidity? If one is inclined to discount the importance of speculative behaviour,

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[29] Without any attempt to be exhaustive, two possibilities are learning and finance constraints. The key idea of the learning explanation is that agents may not have full information; the actual price may deviate from the full information fundamental price as agents attempt to draw inferences from the actions of the other agents (as they are reflected in prices, volume and other variables). For theoretical and empirical work see, for example, Grossman (1989) and Timmerman (1993). The key idea of the finance constraint story is that the inability to borrow may depress cash flows relative to the unconstrained case, leading to lower asset prices; in a sense, a present value model still holds, but the resulting returns behaviour may appear anomalous to the econometrician. For theoretical and empirical references, see Brock and LeBaron (1990), Jog and Schaller (1994) and Kiyotaki and Moore (1993).

then the empirical work presented in this paper poses other questions. Can a model based on economic fundamentals reproduce the magnitude and variability of the apparent "overvaluation" that we find in the actual data? How can a model based on economic fundamentals account for events like the 1929 and 1987 stock market crashes?

A variety of theoretical models have recently addressed the question of why stock markets might appear to display speculative behaviour, and specifically why prices might move dramatically in the absence of important news.[30] An interesting question is whether theories along any of these lines can account for the features of the data highlighted here.

[30] See, for example, Bikhchandani, Hirshleifer and Welch (1992), Bulow and Klemperer (1994), Gennotte and Leland (1990) and Romer (1993).

References

Ackert, Lucy and Brian Smith. 1993. "Stock Price Volatility, Ordinary Dividends and Other Cash Flows to Shareholders," *Journal of Finance*, Vol. 48, No. 4, 1147-1160.

Allen, Franklin and Gary Gorton. 1993. "Churning Bubbles," *Review of Economic Studies*, Vo. 60, 813-836.

Allen, Franklin, Stephen Morris and Andrew Postlewaite. 1993. "Finite Bubbles with Short Sale Constraints and Asymmetric Information," *Journal of Economic Theory*, Vol. 61, No. 2, 206-229.

Barsky, Robert and J. Bradford DeLong. 1993. "Why Does the Stock Market Fluctuate?" *Quarterly Journal of Economics*, Vol. CVIII, No. 2, 291-312.

Bikhchandani, Sushil, David Hirshleifer and Ivo Welch. 1992. "A Theory of Fads, Fashion, Custom and Cultural Change as Informational Cascades," *Journal of Political Economy*, Vol. 100, No. 5, 992-1026.

Blanchard, Olivier J. 1979. "Speculative Bubbles, Crashes and Rational Expectations," *Economics Letters*, Vol. 3, 387-389.

Blanchard, Olivier J. and Stanley Fischer. 1989. *Lectures on Macroeconomics*, (Cambridge: MIT Press. 1st edition.

Blanchard, Olivier and Mark Watson. 1982. "Bubbles, Rational Expectations and Financial Markets," in P. Wachtel (ed.), *Crises in the Economic and Financial Structure*, Lexington Books, Lexington MA.

Bonomo, Marco and Rene Garcia. 1994. "Can a Well-Fitted Equilibrium Assset-Pricing Model Produce Mean Reversion?" *Journal of Applied Econometrics*, Vol. 9, 19-29.

Brock, W. and B. LeBaron. 1990. "Liquidity Constraints in Production-Based Asset-Pricing Models," in *Asymmetric Information, Corporate Finance and Investment*, R. Glenn Hubbard, ed., Chicago: University of Chicago Press, 231-56.

Bulow, Jeremy and Paul Klemperer. 1994. "Rational Frenzies and Crashes," *Journal of Political Economy*, Vol. 102, No. 1, 1-23.

Campbell, John Y. and Robert J. Shiller. 1987."Cointegration and Tests of Present Value Models," *Journal of Political Economy*, Vol. 95, 1062-1088.

Cecchetti, Stephen G., Pok-Sang Lam and Nelson C. Mark. 1990. "Mean Reversion in Equilibrium Asset Prices," *American Economic Review*, Vol. 80, No. 3, 398-418.

Cecchetti, Stephen, Pok-sang Lam and Nelson Mark. 1993. "The Equity Premium and the Risk-Free Rate," *Journal of Monetary Economics*, Vol. 31, 21-45.

Cutler, David M., James M. Poterba and Lawrence H. Summers. 1991. "Speculative Dynamics," *Review of Economic Studies*, Vol. 58, 529-546.

Diba, Behzad T. and Herschel I. Grossman. 1988. "Explosive Rational Bubbles in Stock Prices?" *American Economic Review*, Vol. 78, No. 3, 520-528.

Donaldson, Glen and Mark Kamstra. 1996. "Using Dividend Forecasting Models to Reject Bubbles in Asset Prices: The Case of the 1929's Stock Crash," *Review of Financial Studies*, forthcoming.

Evans, George. 1991. "Pitfalls in Testing for Explosive Bubbles in Asset Prices," *American Economic Review*, Vol. 81, No. 4, 922-930.

Evans, Martin D. D. and Karen Lewis. 1995. "Do Long-Term Swings in the Dollar Affect Estimates of the Risk Premia?" *Review of Financial Studies*, Vol. 8, No. 3, 709-742.

Fama, Eugene F. 1991. "Efficient Capital Markets: II," *The Journal of Finance*, Vol. XLVI, No. 5, December 1991, 1575-1617.

Fama, Eugene F. and Kenneth R. French. 1988a. "Permanent and Temporary Components of Stock Prices," *Journal of Political Economy*, Vol. 96, No. 2, 246-274.

Fama, Eugene F. and Kenneth R. French. 1988b. "Dividend Yields and Expected Stock Returns," *Journal of Financial Economics*, Vol. 22, No. 1, 3-25.

Flood, Robert P. and Robert J. Hodrick. 1990. "On Testing for Speculative Bubbles," *Journal of Economic Perspectives*, Vol. 4, No. 2, 85-101.

Funke, Michael, Stephen Hall and Martin Sola. 1994. "Rational Bubbles During Poland's Hyperinflation: Implications and Empirical Evidence," *European Economic Review*, Vol. 38, 1257-1276.

Gennotte, Gerard and Hayne Leland. 1990. "Market Liquidity, Hedging and Crashes," *American Economic Review*, Vol. 80, 615-32.

Goldfeld, Stephen M. and Richard E. Quandt. 1976. *Studies in Nonlinear Estimation*., Ballinger, Cambridge, U.S.A., Grossman, Sanford. 1989. *The Informational Role of Prices*, Cambridge, Massachusetts: MIT Press.

Hall, Stephen and Martin Sola. 1993. "Testing for Collapsing Bubbles: An Endogenous Switching ADF Test," *London Business School*, Discussion Paper 15-93.

Hamilton, James B. 1989. "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle," *Econometrica*, Vol. 57, No. 2, 357-384.

Hamilton, James and Raoul Susmel. 1994. "Autoregressive Conditional Heteroskedasticity and Changes in Regime." *Journal of Econometrics* Vol. 64, No. 1-2, 307-33.

Hartley, Michael J. 1978. "Comment on 'Estimating Mixtures of Normal Distributions and Switching Regressions' by Quandt and Ramsey," *Journal of the American Statistical Association*, 738-741.

Jog, Vijay and Huntley Schaller. 1994. "Finance Constraints and Asset Pricing: Evidence on Mean Reversion," *Journal of Empirical Finance*, Vol. 1, 193-209.

Kim, M.J., Charles R. Nelson and Richard Startz. 1991. "Mean Reversion in Stock Prices? A Reappraisal of the Empirical Evidence," *Review of Economic Studies*, May, 515-528.

Kindleberger, Charles P. 1989. *Manias, Panics and Crashes: A History of Financial Crises*, New York: Basic Books, Inc., Revised Edition.

Kiyotaki, Nobuhiro and J.H. Moore. 1993. "Credit Cycles," paper presented at the NBER Economic Fluctuations Conference, July 1993.

Leach, John. 1991. "Rational Speculation," *Journal of Political Economy*, Vol. 99, 131-144.

Lucas, Robert E. Jr. 1978. "Asset Prices in an Exchange Economy," *Econometrica*, Vol. 66, 429-45.

Obstfeld, Maurice and Kenneth Rogoff. 1983. "Speculative Hyperinflations in Maximizing Models: Can We Rule Them Out?" *Journal of Political Economy*, Vol. 91, 675-687.

Obstfeld, Maurice and Kenneth Rogoff. 1986. "Ruling Out Divergent Speculative Bubbles," *Journal of Monetary Economics*, Vol. 17, 349-362.

Poterba, James M. and Lawrence H. Summers. 1988. "Mean Reversion in Stock Prices: Evidence and Implications," *Journal of Financial Economics*, Vol. 22, No. 1, 27-59.

Romer, David. 1993. "Rational Asset Price Movements Without News," *American Economic Review*, Vol. 83, No. 5, 1112-1130.

Schwert, G. William. 1989. "Business Cycles, Financial Crises and Stock Volatility," *Carnegie-Rochester Conference Series on Public Policy*, Vol. 31, 83-126.

Shiller, Robert, Fumiko Kon-Ya and Yoshiro Tsutsui. 1991. "Investor Behaviour in the October 1987 Stock Market Crash: The Case of Japan," *Journal of the Japanese and International Economies*, Vol. 5, 1-13.

Smith, Vernon, Gerry Suchanek and Arlington Williams. 1988. "Bubbles, Crashes and Endogenous Expectations in Experimental Spot Asset Markets," *Econometrica* Vol. 56, No. 5, 1119-1151.

Timmerman, Allan. 1993. "How Learning in Financial Markets Generates Excess Volatility and Predictability in Stock Prices," *Quarterly Journal of Economics*, Vol. CVIII, No. 4, 1135-1145.

Tirole, J. 1982. "On the Possibility of Speculation under Rational Expectations," *Econometrica,* Vol. 50, 1163-1181.

Tirole, J. 1985. "Asset Bubbles and Overlapping Generations," *Econometrica*, Vol. 53, 1071-1100.

Turner, Christopher M., Richard Startz and Charles R. Nelson. 1989. "A Markov Model of Heteroscedasticity, Risk and Learning in the Stock Market," *Journal of Financial Economics*, Vol. 25, 3-22.

Van Boening, Mark, Arlington Williams and Shawn LaMaster. 1993. "Price Bubbles and Crashes in Experimental Call Markets," *Economics Letters*, Vol. 41, 179-185.

Van Norden, Simon. 1996. "Regime Switching as a Test for Exchange Rate Bubbles." *Journal of Applied Econometrics,* forthcoming.

Van Norden, Simon and Huntley Schaller. 1993. "The Predictability of Stock Market Regime: Evidence from the Toronto Stock Exchange," *The Review of Economics and Statistics*, Vol. LXXV, No. 3, 505-510.

Van Norden, Simon and Robert Vigfusson. 1996. "Regime-Switching Models: A Guide to the Bank of Canada GAUSS Procedures," Bank of Canada Working Paper 96-3.

Appendix

A. Equilibrium Asset Prices

We begin with the Lucas (1978) exchange-economy asset-pricing model; readers familiar with the model may skip lightly over this part. Besides setting notation, there are two important points here. First, since we are going to discuss potential deviations from fundamental price in Part B, we must first derive the fundamental price. Second, in the empirical work in later sections, we will need to find an observable counterpart to the fundamental price. One way to do this is to specify a standard stochastic process for dividend growth (namely, a random walk with drift). This leads to a fundamental price which is a simple function of current dividends.[31]

In the Lucas model, there are a large number of identical, infinitely-lived agents and a fixed number of assets that produce units of the non-storable consumption good. The first-order necessary conditions for a typical agent's optimization problem are

$$P_{j,t}\cdot U'(C_t) = \beta\cdot E_t U'(C_{t+1})\cdot[P_{j,t+1} + D_{j,t+1}] \qquad j=1,2,...,N \tag{A1}$$

where P_j is the real price of asset j in terms of the consumption good, $U'(C_t)$ is the marginal utility of consumption, C_t, for a typical consumer/investor, β is the subjective discount factor, $0 < \beta < 1$, D_j is the pay-off or dividend from the jth productive unit and E_t is the mathematical expectation conditioned on information available at time t.

Since agents are identical, equilibrium per capita ownership of each asset is the reciprocal of the number of assets, so per capita consumption (C) is the sum over all assets of per capita dividends on each asset (D). Hence the equilibrium condition for economy-wide market prices and quantities is

$$P_t\cdot U'(D_t) = \beta\cdot E_t U'(D_{t+1})\cdot[P_{t+1} + D_{t+1}] \tag{A2}$$

where P is the portion of the market's value owned by a typical agent, which corresponds to the value-weighted stock market index adjusted for population size. We assume a Constant Relative Risk Aversion (CRRA) utility function

$$U(C_t) \equiv (1+\gamma)^{-1}\cdot C_t^{1+\gamma} \tag{A3}$$

where γ is the coefficient of relative risk aversion. Using this utility function in the market equilibrium condition gives the following stochastic difference equation for equilibrium prices

$$P_t\cdot D_t^{\gamma} = \beta\cdot E_t D_{t+1}^{\gamma}\cdot(P_{t+1} + D_{t+1}) \tag{A4}$$

This yields the following equation for fundamental price

$$P_t^* = D_t^{-\gamma}\cdot\sum_{k=1}^{\infty}\beta^k\cdot E_t D_{t+k}^{1+\gamma} \tag{A5}$$

---

[31] In the empirical work, we also consider a measure of fundamental price that is a more complicated function of current and lagged dividends. See Section IIIB.

The expression for fundamental price depends on future dividends, which are not observable. One way to express the fundamental price in terms of observables is to make an assumption about the stochastic process for dividends. A common assumption (with some empirical justification) is that log dividends are a random walk with constant drift. This leads to a simple solution in which the fundamental price is a multiple of current dividends. Formally, dividends are

$$d_t = \alpha_0 + d_{t+1} + \varepsilon_t \tag{A6}$$

where d_t is the logarithm of dividends, α_0 is the drift parameter, and ε_t is a sequence of independent, identically distributed normal random variables with mean zero and variance σ^2. To solve the model, first we conjecture a solution of the form

$$P_t^* = \rho \cdot D_t \ . \tag{A7}$$

To verify that this is a solution to the stochastic difference equation, we can substitute it into (A4) obtaining

$$\rho \cdot D_t^{\gamma+1} = \beta \cdot E_t[(\rho+1) \cdot D_{t+1}^{\gamma+1}] \tag{A8}$$

By rewriting the dividend process in levels, rather than logarithms

$$D_{t+1} = D_t \cdot e^{\alpha_0 + \varepsilon_{t+1}} \tag{A9}$$

and substituting into (A8) we obtain the following expression for the fundamental price-dividend ratio

$$\rho = \frac{\beta \cdot e^{\alpha_0 \cdot (1+\gamma) + (1+\gamma)^2 \cdot \sigma^2/2}}{1 - \beta \cdot e^{\alpha_0 \cdot (1+\gamma) + (1+\gamma)^2 \cdot \sigma^2/2}} \ . \tag{A10}$$

The effect of an increase in the expected rate of dividend growth depends on whether γ is greater or less than -1. When $\gamma > -1$, increases in the dividend growth rate raise the price-dividend ratio; when $\gamma < -1$, the reverse is true.[32]

Finally, the equilibrium gross return can be obtained from the relationship between current prices and dividends and the dividend process:

$$\begin{aligned} R_t &\equiv \frac{P_t + D_t}{P_{t-1}} \\ &= \left(\frac{1+\rho}{\rho}\right) \cdot e^{\alpha_0 + \varepsilon_t} \end{aligned} \tag{A11}$$

[32] The intuition is that there are offsetting income and substitution effects. Since $-1/\gamma$ is the intertemporal elasticity of substitution, $\gamma > -1$ means the substitution effect dominates, so a higher discount rate leads to a rise in saving and thus a rise in the demand for assets and in asset prices.

B. Derivation of the Switching Regression Form of the Model of Speculative Behaviour

We can solve for expected returns in each regime using the definitions $R_{t+1} \equiv (P_{t+1} + D_{t+1})/P_t$ and $B_t \equiv P_t - P^*_t$ to get

$$E_t[R_{t+1}] \ = \ \frac{E_t[P^*_{t+1} + D_{t+1}]}{P_t} \ + \ \frac{E_t[B_{t+1}]}{P_t} \tag{A12}$$

Using the definition of the dividend-generating process (A9) gives

$$\begin{aligned} \frac{E_t[P^*_{t+1} + D_{t+1}]}{P_t} \ &= \ \frac{E_t[(1+\rho)\cdot D_{t+1}]}{P_t} \\ &= \ (1+\rho)\cdot e^{\alpha_0 + \sigma^2/2}\cdot\frac{D_t}{P_t} \quad . \end{aligned} \tag{A13}$$

Substituting this and (13) in (A12) gives

$$E_t[R_{t+1}|S] \ = \ (1+\rho)\cdot e^{\alpha_0 + \sigma^2/2}\cdot\frac{D_t}{P_t} \ + \ \frac{M}{q(b_t)}\cdot b_t \ - \ \frac{1-q(b_t)}{q(b_t)}\cdot u(b_t) \ . \tag{A14}$$

However,

$$\frac{D_t}{P_t} \ \equiv \ \frac{1-b_t}{\rho} \tag{A15}$$

so (A14) becomes

$$E_t[R_{t+1}|S] \ = \ \frac{1+\rho}{\rho}\cdot e^{\alpha_0 + \sigma^2/2}\cdot(1-b_t) \ + \ \frac{M}{q(b_t)}\cdot b_t \ - \ \frac{1-q(b_t)}{q(b_t)}\cdot u(b_t) \ . \tag{A16}$$

Similarly, we can show that

$$E_t[R_{t+1}|C] \ = \ \frac{1+\rho}{\rho}\cdot e^{\alpha_0 + \sigma^2/2}\cdot(1-b_t) \ + \ u(b_t) \ . \tag{A17}$$

This model fits readily into the econometric framework of switching regressions. To see this, we can take first-order Taylor series approximations of (A16), (A17) and (10) around some arbitrary b_0 to obtain

$$\begin{aligned} E(R_{t+1}|S) &= \beta_{S0} + \beta_{Sb} \cdot b_t \\ E(R_{t+1}|C) &= \beta_{C0} + \beta_{Cb} \cdot b_t \quad . \\ q_{t+1} &= \beta_{q0} + \beta_{qb} \cdot |b_t| \end{aligned} \tag{A18}$$

This corresponds to a switching regression in which the size of the speculative component in the previous period helps to predict the probability of survival and influences the expected return conditional on survival or collapse.

We can also say something about the signs of the coefficients on b_t. By construction, $\beta_{qb} < 0$. The coefficient on b_t in the collapsing regime will be[33]

$$\left. \frac{dE_t[R_{t+1}|C]}{db_t} \right|_{b_t=b_0} = -\frac{1+\rho}{\rho} \cdot e^{\alpha_0+\sigma^2/2} + u^{\prime}(b_0) \ . \tag{A19}$$

Given that $\alpha_0 > 0$ (i.e., dividends tend to grow rather than shrink over time) we can show that the first term is < -1 while $u'(b) \leq 1$ by construction. Therefore, the whole expression (and therefore β_{Cb}) must be <0. Similarly, we can derive

$$\begin{aligned} &\left. \frac{dE_t[R_{t+1}|S]}{db_t} \right|_{b_t=b_0} \\ &= -\frac{1+\rho}{\rho} \cdot e^{\alpha_0+\sigma^2/2} + \frac{M}{q(b_0)} \cdot \left(1 - \frac{b_0 \cdot q^{\prime}(b_0)}{q(b_0)} \right) - u^{\prime}(b_0) \cdot \left(\frac{1-q(b_0)}{q(b_0)} \right) + \frac{q^{\prime}(b_0) \cdot u(b_0)}{q^2(b_0)} \\ &= \left. \frac{dE_t[R_{t+1}|C]}{db_t} \right|_{b_t=b_0} + \frac{M - u^{\prime}(b_0)}{q(b_0)} + \frac{q^{\prime}(b_0)}{q^2(b_0)} \cdot \left[u(b_0) - M \cdot b_0 \right] \ . \end{aligned} \tag{A20}$$

Under the assumption that M>1 (i.e., that the rate of return on the fundamental component is positive), M>u' since $u' \leq 1$, so the second term is positive. Since $q'(b_0) < 0$ iff $b_0 > 0$ by definition, the final term will always be positive. Since the last two terms of (A20) are both positive, $\beta_{Sb} > \beta_{Cb}$.

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[33] Note that the first-order Taylor expansion of f(x) around x_0 gives

$$\begin{aligned} f(x) &\approx f(x_0) + f^{\prime}(x_0) \cdot (x - x_0) \\ &= \left(f(x_0) - f^{\prime}(x_0) \cdot x_0 \right) + f^{\prime}(x_0) \cdot x \\ &= \lambda_0 + \lambda_1 \cdot x \end{aligned}$$

so we only need the derivative of conditional expected returns with respect to b_t in order to sign its coefficient.

C. Derivation of the Asset-Pricing Model with Markov-Switching in Dividend Growth[34]

To verify that (17) is a solution to the stochastic difference equation, substitute it into (1), obtaining:

$$\rho(S_t)\cdot D_t^{\gamma+1} = \beta\cdot E_t D_{t+1}^{\gamma+1}[\rho(S_{t+1}) + 1] \ . \qquad \textbf{(A21)}$$

By rewriting the dividend process in levels, rather than logarithms

$$D_{t+1} = D_t\cdot e^{(\alpha_0+\alpha_1\cdot S_t+\varepsilon_{t+1})} \qquad \textbf{(A22)}$$

and substituting into the previous equation, we obtain the following expression for the price-dividend ratio as a function of the state of the dividend process

$$\rho(S_t) = \beta\cdot e^{[\alpha_0\cdot(1+\gamma) + (1+\gamma)^2\cdot\sigma^2/2]} e^{[\alpha_1\cdot(1+\gamma)\cdot S_t]} E_t[\rho(S_{t+1}) + 1] \ . \qquad \textbf{(A23)}$$

Since the state space for the Markov-switching variable consists only of the states 0 and 1, this expression for the price-dividend ratio is effectively a system of two linear equations in the two unknown variables $\rho(0)$ and $\rho(1)$; the solution is

$$\rho(0) = \tilde{\beta}\cdot[1 - \tilde{\beta}\cdot\tilde{\alpha}_1\cdot(p + q - 1)]/\vartriangle \qquad \textbf{(A24)}$$

$$\rho(1) = \tilde{\beta}\cdot\tilde{\alpha}_1\cdot[1 - \tilde{\beta}\cdot(p + q - 1)]/\vartriangle \qquad \textbf{(A25)}$$

$$\textit{where } \tilde{\beta}=\beta\cdot e^{[\alpha_0\cdot(1+\gamma)+(1+\gamma)^2\cdot\sigma^2/2]},\ \tilde{\alpha}_1 = e^{\alpha_1\cdot(1+\gamma)}, \qquad \textbf{(A26)}$$

$$\vartriangle = 1 - \tilde{\beta}\cdot(p\cdot\tilde{\alpha}_1 + q) + \tilde{\beta}^2\cdot\tilde{\alpha}_1\cdot(p + q - 1) \ . \qquad \textbf{(A27)}$$

The equilibrium gross return can be obtained from the relationship between current prices and dividends and the dividend process, so

$$R_t = \left(\frac{1 + \rho(S_t)}{\rho(S_{t-1})}\right) e^{(\alpha_0 + \alpha_1\cdot S_{t-1} + \varepsilon_t)} \ . \qquad \textbf{(A28)}$$

[34] This follows Cecchetti, Lam and Mark (1990).

TABLE I - THE MODEL OF SPECULATIVE BEHAVIOUR	
Parameters	Estimates
β_{S0}	1.007 (0.002)
β_{Sb}	-0.006 (0.006)
β_{C0}	0.976 (0.039)
β_{Cb}	-0.111 (0.071)
β_{q0}	2.098 (0.294)
β_{qb}	-1.560 (0.510)
σ_S	0.044 (0.002)
σ_C	0.170 (0.028)
Likelihood-Ratio Tests	
Volatility Regimes	16.29 (0.003)
Mixture of Normals	16.21 (0.001)
Mean Reversion	14.79 (0.002)

Note: The model of speculative behaviour consists of equations (19) - (21) in the text. Figures in parentheses indicate standard errors for parameter estimates and p-values for likelihood-ratio tests. The volatility regimes test imposes the restrictions $\beta_{S0}=\beta_{C0}$, $\beta_{Sb}=\beta_{Cb}=\beta_{qb}=0$. The mixture of normals test imposes the restrictions $\beta_{Sb}=\beta_{Cb}=\beta_{qb}=0$. The mean-reversion test imposes the restrictions $\beta_{S0}=\beta_{C0}$, $\beta_{Sb}=\beta_{Cb}$, $\beta_{qb}=0$.

TABLE II - THE MODEL OF SPECULATIVE BEHAVIOUR (SUB-PERIODS)

Parameter Estimates	1926-54	1954-74	1974-89
β_{S0}	1.009 (0.004)	1.035 (0.005)	1.006 (0.003)
β_{Sb}	0.007 (0.014)	-0.140 (0.017)	0.011 (0.025)
β_{C0}	0.996 (0.035)	0.997 (0.003)	0.956 (0.021)
β_{Cb}	-0.150 (0.119)	-0.036 (0.016)	-0.532 (0.071)
β_{q0}	1.794 (0.355)	-0.483 (0.299)	4.600 (1.080)
β_{qb}	-1.673 (0.768)	-3.726 (1.689)	-13.289 (4.034)
σ_S	0.050 (0.003)	0.012 (0.003)	0.044 (0.003)
σ_C	0.187 (0.035)	0.037 (0.002)	0.041 (0.015)
Likelihood-Ratio Tests			
Volatility Regimes	9.40 (0.052)	19.02 (0.001)	14.98 (0.005)
Mixture of Normals	9.23 (0.026)	10.53 (0.015)	14.91 (0.002)
Mean Reversion	9.35 (0.025)	15.30 (0.002)	14.03 (0.003)

See Table I for notes.

TABLE III - THE MODEL OF SPECULATIVE BEHAVIOUR (Using b^B_t)

Parameters	Estimates
β_{S0}	1.006 (0.002)
β_{Sb}	-0.008 (0.006)
β_{C0}	0.972 (0.042)
β_{Cb}	-0.126 (0.079)
β_{q0}	2.103 (0.318)
β_{qb}	-1.507 (0.534)
σ_S	0.044 (0.002)
σ_C	0.172 (0.029)
Likelihood-Ratio Tests	
Volatility Regimes	15.21 (0.004)
Mixture of Normals	15.12 (0.002)
Mean Reversion	12.68 (0.005)

See Table I for notes.

TABLE IV

THE SIZE AND VARIABILITY OF THE APPARENT DEVIATION FROM FUNDAMENTAL PRICE IN THE MODEL OF SWITCHING FUNDAMENTALS

	Actual Data	2.5%	50%	97.5%
(β=.97, γ=-1.585)				
Mean of $\lvert b_t^A \rvert$	0.2321	0.0048942	0.024275	0.051050
Standard Deviation of $\lvert b_t^A \rvert$	0.2019	0.017435	0.032532	0.032886
(β=.95, γ=-.609)				
Mean of $\lvert b_t^A \rvert$	0.2321	0.0027203	0.014700	0.030615
Standard Deviation of $\lvert b_t^A \rvert$	0.2019	0.011565	0.023080	0.025291

Note: The entries in columns two, three and four report the 2.5 percentile, median and 97.5 percentile of the distribution of the mean and standard deviation of $\lvert b_t^A \rvert$ from Monte Carlo simulations of the model of switching fundamentals.

TABLE V
THE MODEL OF SWITCHING FUNDAMENTALS
(β=.97, γ=-1.585)

Parameter Estimates	Actual Data	2.5%	50%	97.5%
β_{S0}	1.007	1.0021	1.0050	1.0070
β_{Sb}	-0.006	-.17521	-.14199	-.11095
β_{C0}	0.976	1.0956	1.1419	1.1773
β_{Cb}	-0.111	-2.4257	-2.2629	-2.0800
β_{q0}	2.098	3.0489	2.6491	2.4342
β_{qb}	-1.560	-7.4315	-9.2958	-14.312
σ_S	0.044	0.014178	.014948	.015707
σ_C	0.170	.003514	.012315	.021518
Likelihood-Ratio Tests				
Volatility Regimes	16.29	56.30	154.49	282.48
Mixture of Normals	16.21	55.986	154.48	282.46
Mean Reversion	14.79	24.971	52.136	88.436

Note: The entries in columns two, three and four represent the probability distribution of the coefficient estimates and likelihood-ratio statistics of the model of equations (19) - (21) under the null hypothesis of switching fundamentals, based on a Monte Carlo simulation with 1000 replications.

TABLE VI
THE MODEL OF SWITCHING FUNDAMENTALS
(β=.95, γ=-.603)

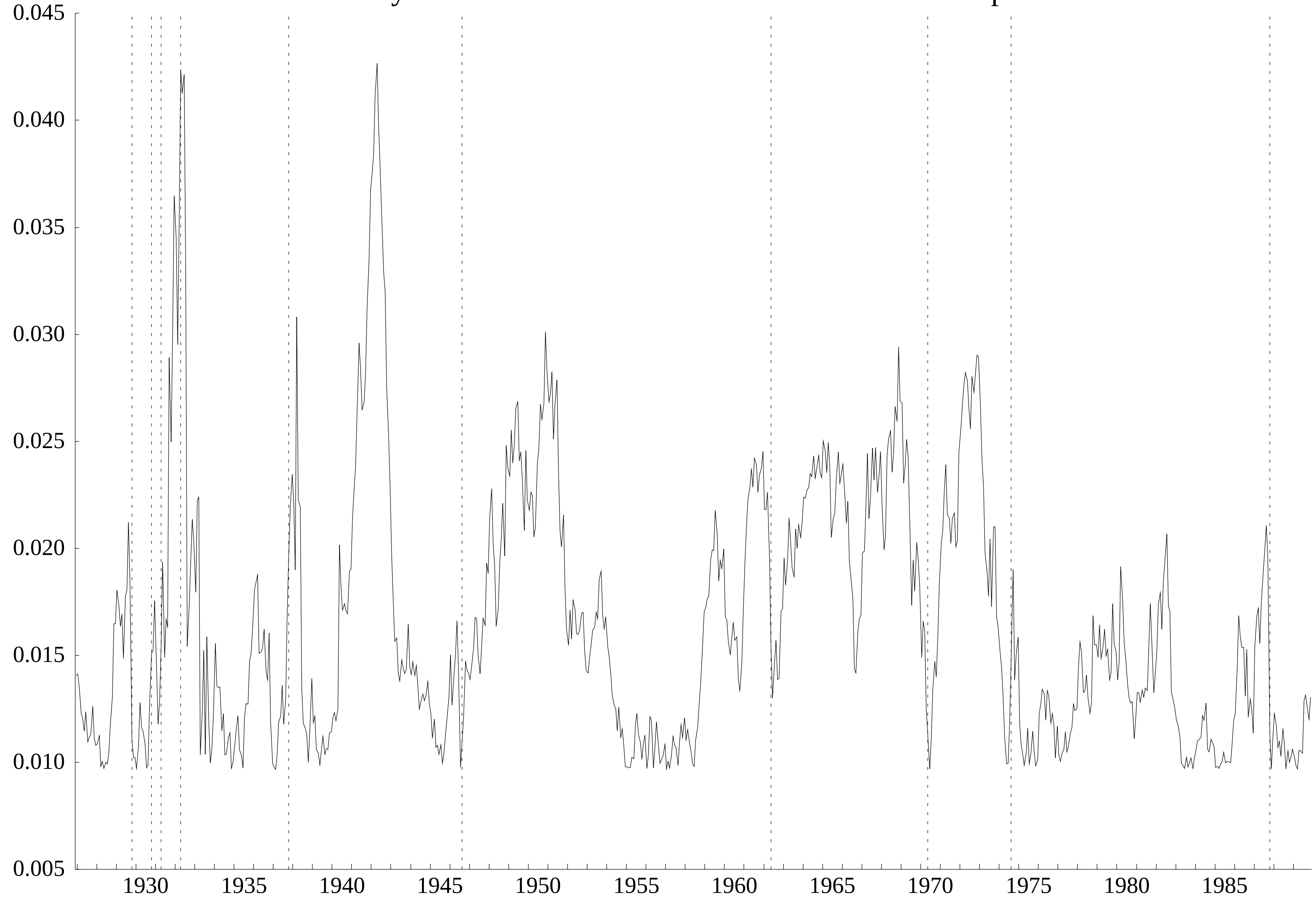
Parameter Estimates	Actual Data	2.5%	50%	97.5%
β_{S0}	1.007	1.0030	1.0054	1.0072
β_{Sb}	-0.006	0.15473	0.20442	0.24982
β_{C0}	0.976	0.91088	0.93230	0.99372
β_{Cb}	-0.111	-1.9307	-1.7001	-0.78587
β_{q0}	2.098	3.0233	2.6026	2.0395
β_{qb}	-1.560	-7.4679	-12.956	-20.412
σ_S	0.044	0.014096	0.014908	0.015729
σ_C	0.170	0.004490	0.012987	0.046817
Likelihood-Ratio Tests				
Volatility Regimes	16.29	43.153	123.57	232.04
Mixture of Normals	16.21	42.834	123.49	231.93
Mean Reversion	14.79	15.864	37.008	64.062

Note: The entries in columns two, three and four represent the probability distribution of the coefficient estimates and likelihood-ratio statistics of the model of equations (19) - (21) under the null hypothesis of switching fundamentals, based on a Monte Carlo simulation with 1000 replications.

Figure 1 - Deviation of Actual Price from Fundamental Price

Figure 2 - Probability of Stock Market Crash (from Model of Speculative Behaviour)

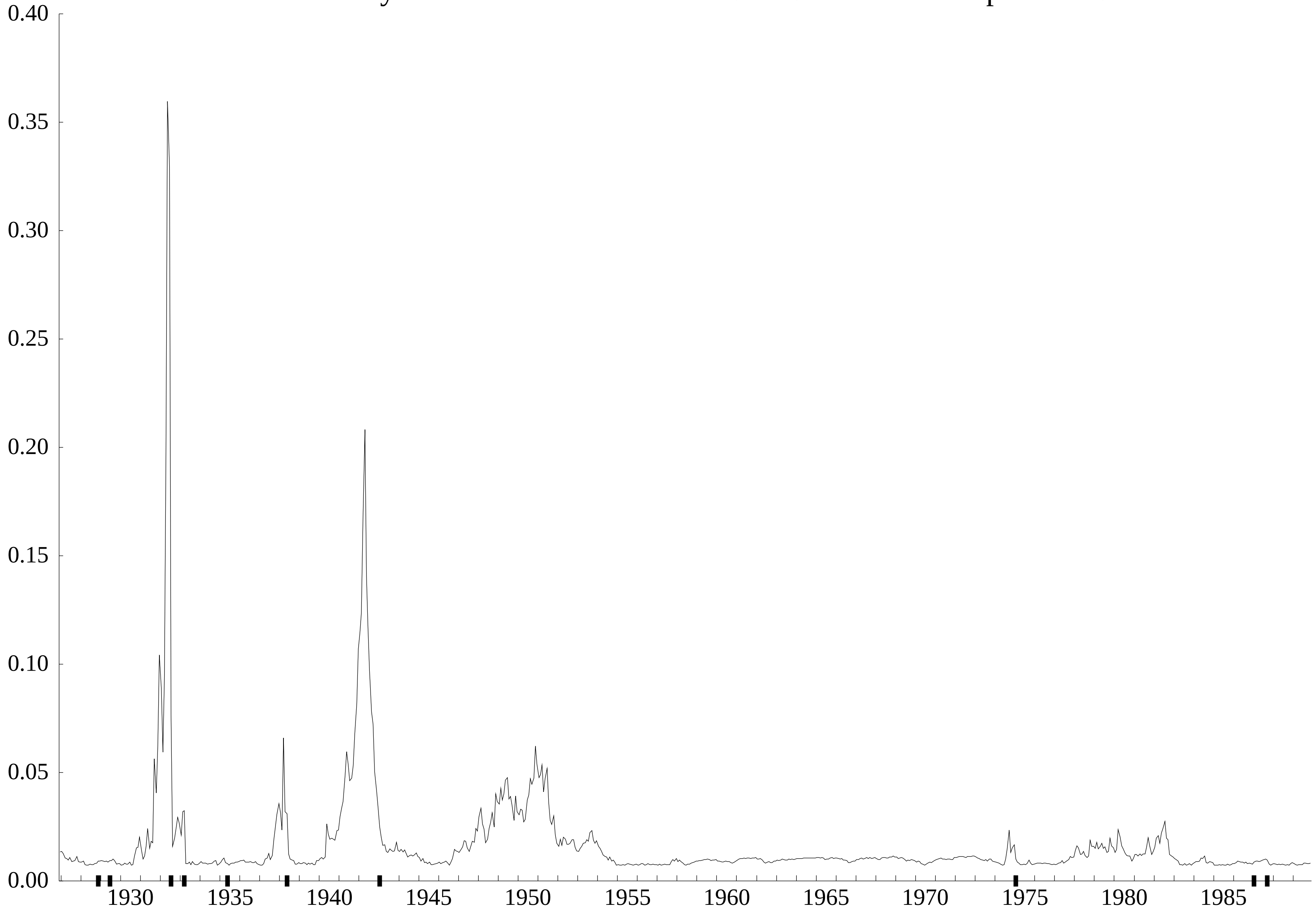
Probability of a return more than 2 std. dev. below the sample mean.



Dotted vertical lines indicate the 10 biggest 3-month stock market declines.

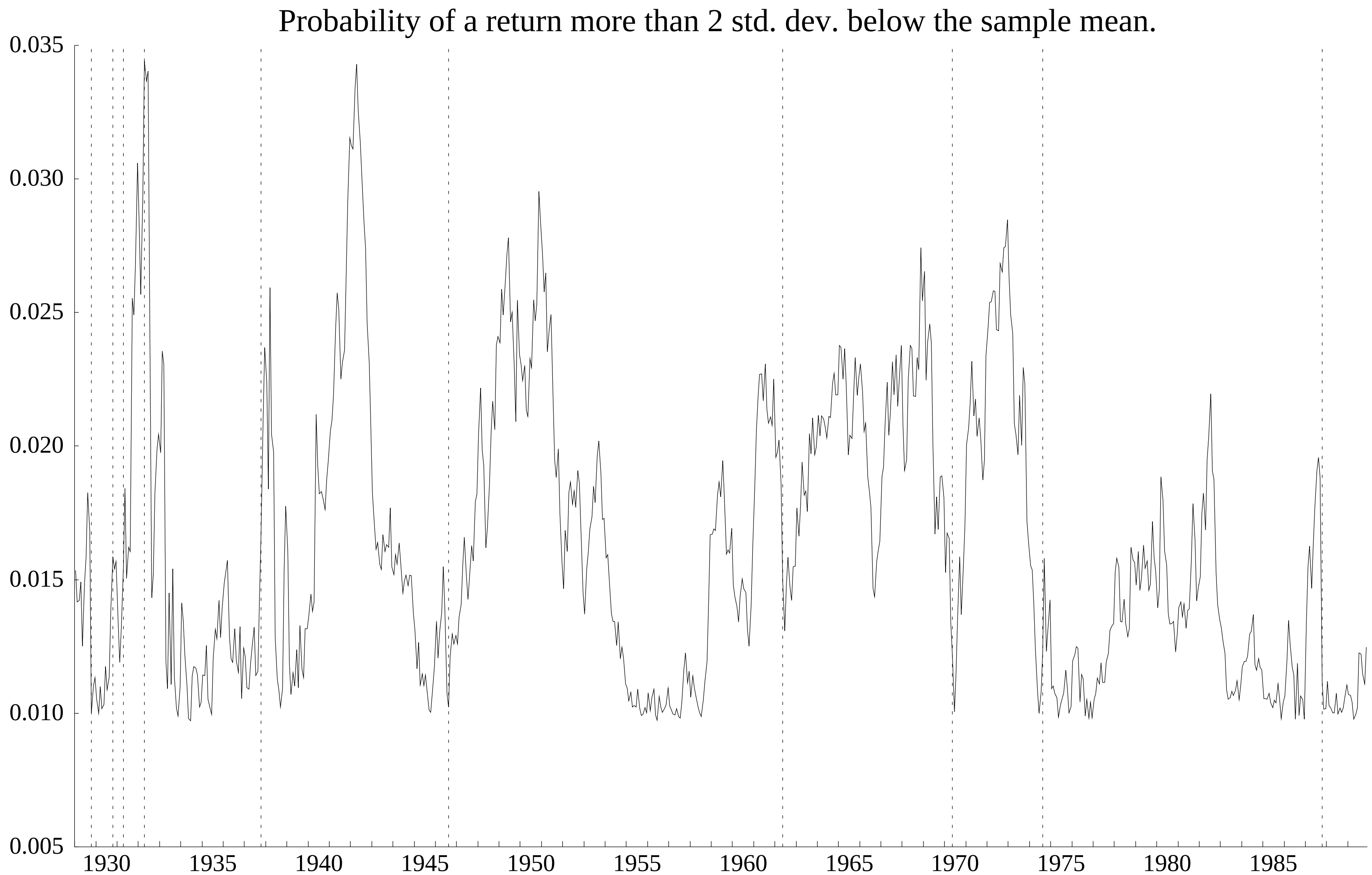
Figure 4 - Probability of Stock Market Rally (from Model of Speculative Behaviour)

Probability of a return more than 2 std. dev. above the sample mean.



Tick marks indicate the 10 biggest 3-month stock market increases.

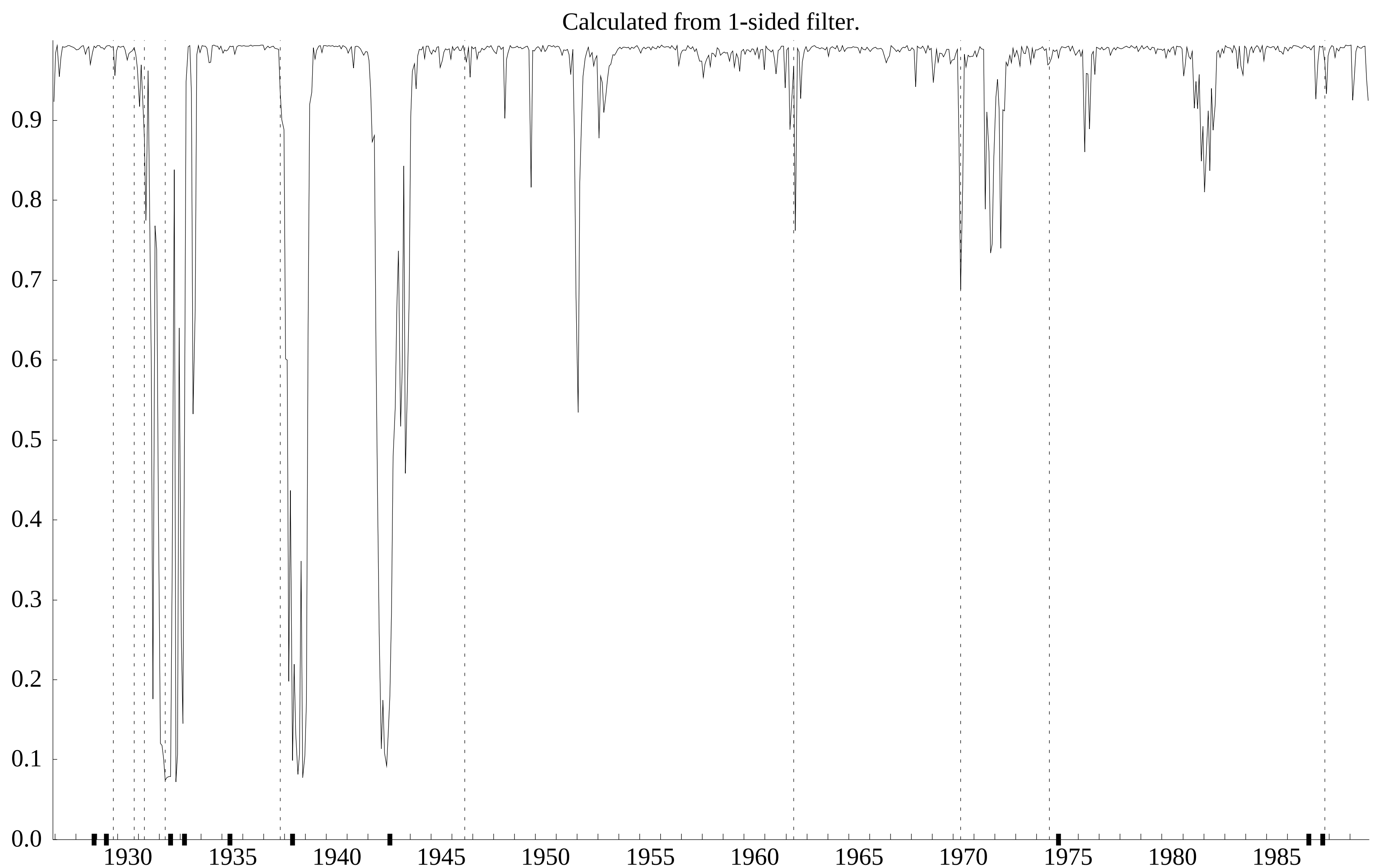
Figure 3 - Probability of Stock Market Crash (from Model of Speculative Behaviour) Using Measure B of Fundamental Price



Dotted vertical lines indicate the 10 biggest 3-month stock market declines.

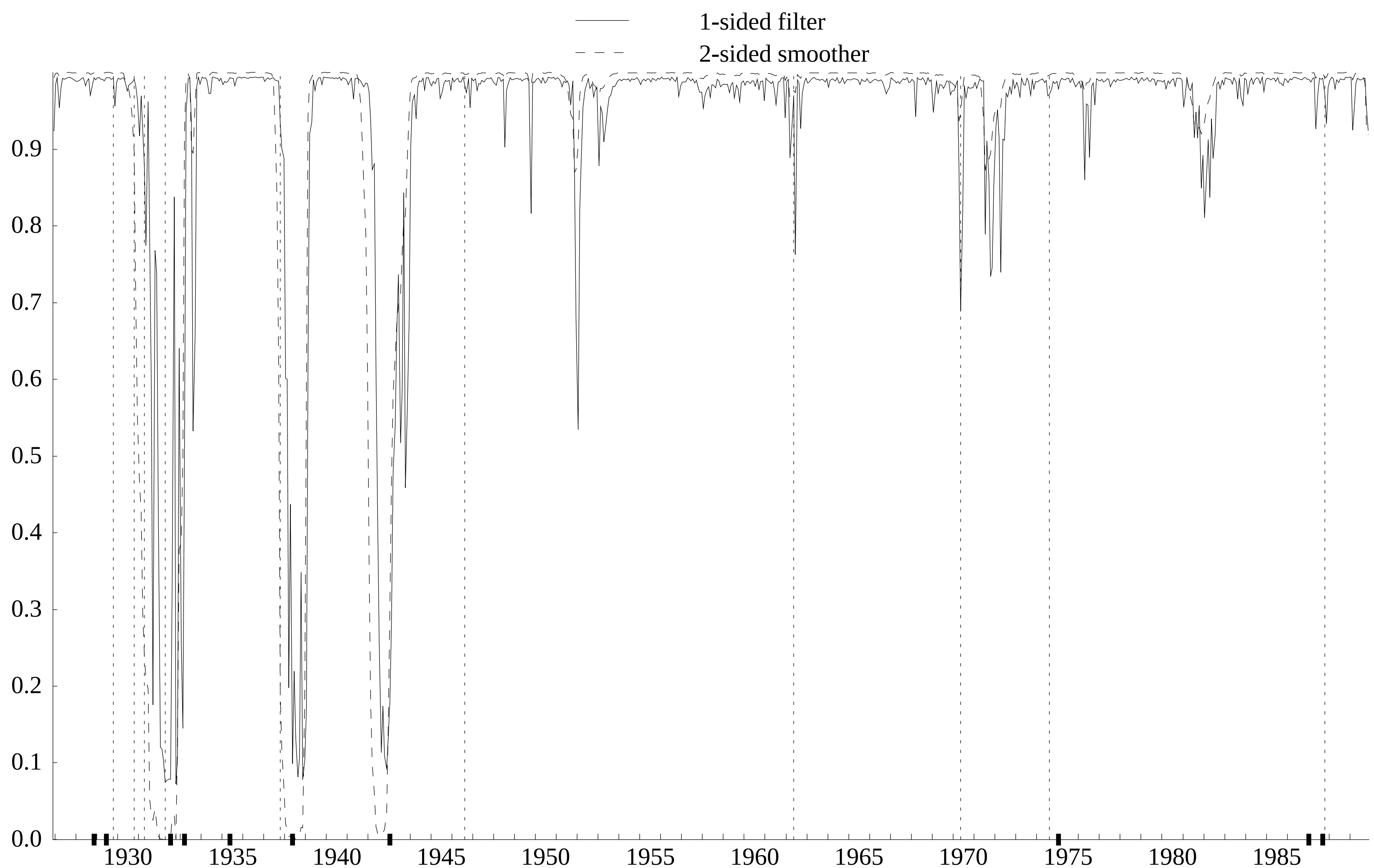
Measure B allows for variation in expected dividend growth based on Campbell and Shiller (1987)

Figure 5 - Real Per Capita Dividend Growth
Probability of Being in Low-Growth State



Dotted vertical lines indicate the 10 biggest 3-month stock market declines.
Tick marks indicate the 10 biggest 3-month stock market increases.

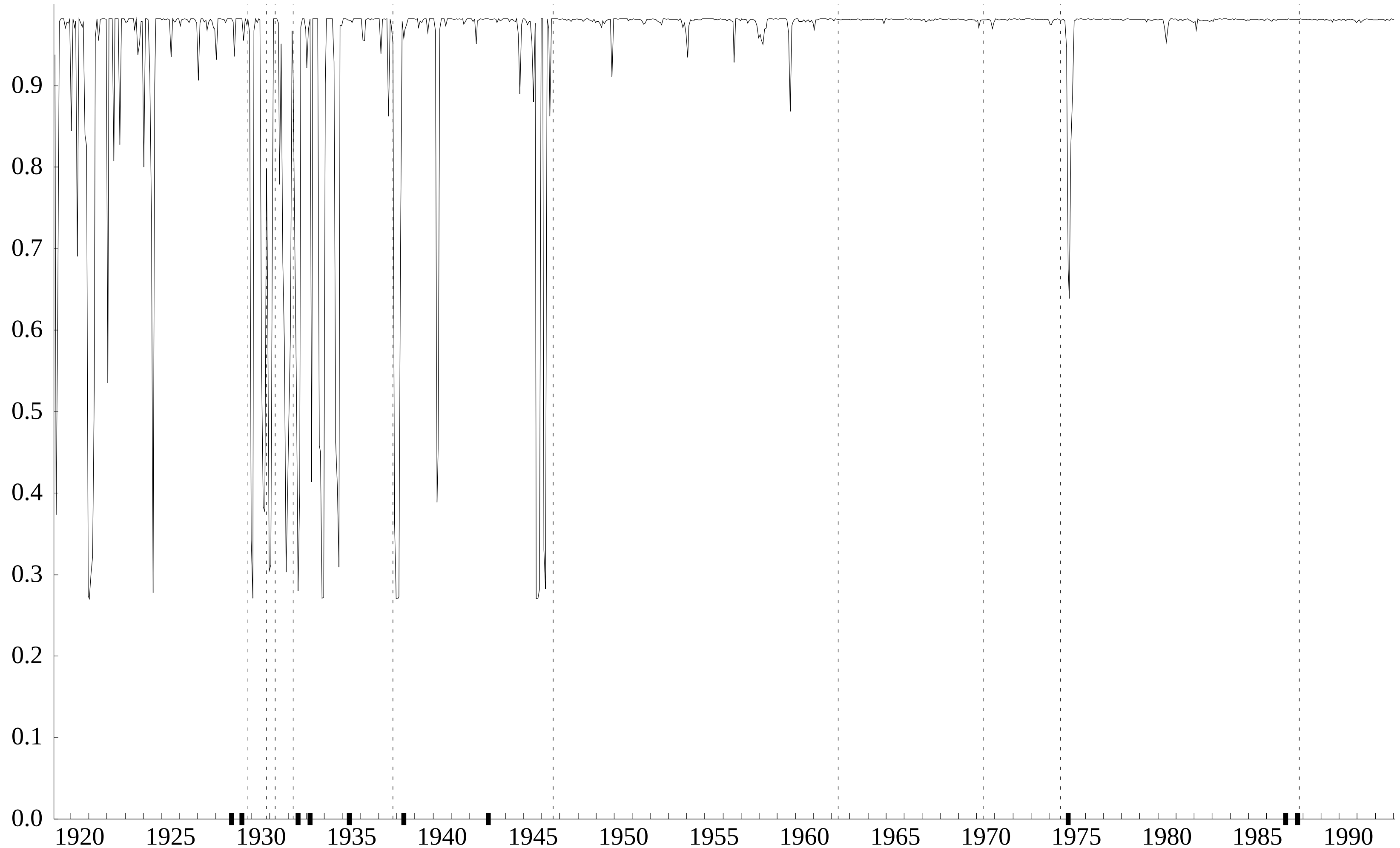
Figure 6 - Real Per Capita Dividend Growth Probability of Being in Low-Growth State



Dotted vertical lines indicate the 10 biggest 3-month stock market declines.
Tick marks indicate the 10 biggest 3-month stock market increases.

Figure 7 - Industrial Production Growth
Probability of Being in Low-Growth State

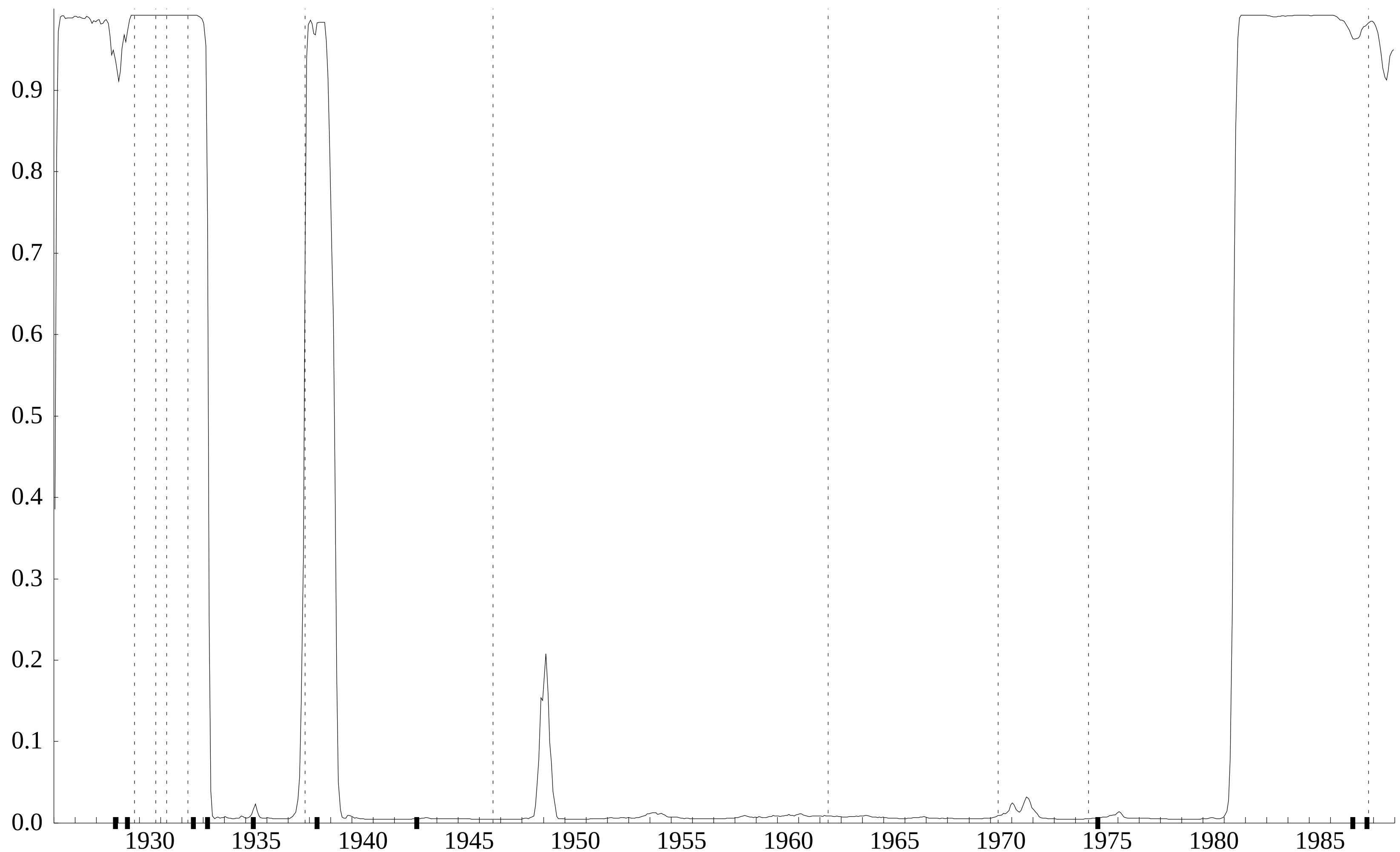
Calculated from 1-sided filter.



Dotted vertical lines indicate the 10 biggest 3-month stock market declines.
Tick marks indicate the 10 biggest 3-month stock market increases.

Figure 8 - Ex Ante Real Interest Rates
Probability of Being in High Real Interest Rate State

Calculated from 1-sided filter.



Dotted vertical lines indicate the 10 biggest 3-month stock market declines.
Tick marks indicate the 10 biggest 3-month stock market increases.

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