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Avoiding the Pitfalls:

Can Regime-Switching Tests Detect Bubbles?

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Abstract

Work on testing for bubbles has caused much debate, much of which has focussed on methodology. Monte Carlo simulations reported in Evans (1991) showed that standard tests for unit roots and cointegration frequently reject the presence of bubbles even when such bubbles are present by construction. Evans referred to this problem as the pitfall of testing for bubbles.

Since Evans' note, new tests for rational speculative bubbles that rely on regime-switching have been proposed. Van Norden and Schaller (1993) and van Norden (1996) use a switching regression to look for a time-varying relationship between returns and deviations from an approximate fundamental price. Hall and Sola (1993) and Funke, Hall and Sola (1994) test whether asset prices seem to switch between explosive growth and stationary behaviour.

Our paper reports on Monte Carlo experiments using Evans' data-generating process to gauge the performance of these two kinds of regime-switching tests. The experiments rely heavily on certain new, fast and robust programs developed at the Bank of Canada for the estimation of switching regression models that make Monte Carlo studies of such estimators practical. We find that for some (but not all) parameter values, regime-switching tests have a significant amount of power to detect periodically collapsing bubbles. We also compare and contrast the performance of the two different regime-switching tests.

Résumé

La mise au point de tests de détection des bulles spéculatives a occasionné bien des débats, principalement sur des points de méthodologie. Evans (1991) a démontré, au moyen de simulations de Monte-Carlo, que la présence de bulles est fréquemment rejetée par les tests standard de racine unitaire et de cointégration même quand des bulles ont été incorporées à la construction des données. Ce problème constitue, à ses yeux, la pierre d'achoppement de ce type de tests de détection des bulles.

Depuis la parution de l'article d'Evans, on a proposé de nouveaux tests de détection des bulles spéculatives rationnelles qui s'appuient sur un changement de régime. van Norden et Schaller (1993) et van Norden (1996) ont eu recours à une régression avec changement de régime afin d'établir s'il existe une relation, variable dans le temps, entre les rendements et les écarts observés par rapport à un prix fondamental approximatif. De leur côté, Hall et Sola (1993) et Funke, Hall et Sola (1994) ont cherché à déterminer si le prix des actifs oscille entre une croissance explosive et un état stationnaire.

Dans la présente étude, les auteurs évaluent la puissance de ces deux types de tests au moyen de simulations de Monte-Carlo; ils emploient pour cela le processus générateur de données qu'utilise Evans. Leurs simulations font appel aux nouveaux programmes rapides et éprouvés mis au point à la Banque du Canada pour l'estimation des modèles de régression avec changement de régime, lesquels rendent possible l'étude de tels estimateurs au moyen de simulations. Les auteurs constatent que pour certaines valeurs paramétriques (mais pas pour toutes), les tests de régression avec changement de régime sont suffisamment puissants pour déceler les bulles qui s'effondrent périodiquement. Enfin, ils comparent la performance des deux tests afin d'en faire ressortir les similarités et les différences.

Contents

1.0 Introduction

Work on testing for rational speculative bubbles in financial markets has prompted much debate, much of which has focussed on methodology. Early work used variance-bound tests, until various econometric problems with this approach were noted (see LeRoy 1989). The misspecification test suggested by West (1987) has fallen out of favour, since misspecified fundamentals should cause it to detect bubbles and there is little agreement on how to specify the fundamentals. (For example, see Flood and Hodrick 1990.) Diba and Grossman (1988) and Hamilton and Whiteman (1985) recommend the use of tests for stationarity and for cointegration to test for the absence of rational speculative bubbles. However, Monte Carlo simulations reported in Evans (1991) show that standard tests for unit roots and cointegration frequently reject the presence of bubbles even when such bubbles are present by construction.[1] Evans refers to this problem as the “pitfall” of testing for bubbles.

Since Evans' note, new tests for rational speculative bubbles that rely on regime-switching have been proposed. Van Norden and Schaller (1993a, 1993b) and van Norden (1996) use a switching regression to look for a time-varying relationship between returns and deviations from an approximate fundamental price. Hall and Sola (1993) and Funke, Hall and Sola (1994) test whether asset prices seem to switch between explosive growth and stationary behaviour.

A potential flaw of this new approach is that the regime-switching estimators may not be well behaved. There are two plausible grounds for concern.[2]

- The presence of rational speculative bubbles implies that the data are non-stationary, but the properties of regime-switching estimators in this instance are unknown. Since this non-stationarity exists only under the alternative hypothesis of bubbles, this raises the question of whether the regime-switching tests have the power to detect bubbles when they exist. This is similar to the pitfall that Evans (1991) found with the unit-root and cointegration tests.[3]
- Little is known about the finite-sample properties of regime-switching estimators. In particular, little has been done to determine whether the use of tests whose distribution is known only asymptotically leads to reliable inference. It is conceivable that asymptotically correct tests could experience size distortion in small samples, which would tend to produce evidence of speculative bubbles even when none is present.

1. Charemza and Deadman (1995) show that this problem extends to a broader range of processes than those considered by Evans (1991).

2. The problems of directly testing for the presence of regime-switching have recently become better understood. However, both of the above approaches circumvent these complications by testing a regime-switching alternative against a regime-switching null.

3. Both problems could lead us to conclude that bubbles are absent when they are in fact present. The difference is that, with the cointegration and unit-root tests, this result is caused by size distortion while with the regime-switching tests it is caused by a lack of power. This difference arises because the two kinds of tests reverse the null and alternative hypotheses.

Our paper is a first step in addressing these questions. We examine the power properties of regime-switching bubble tests by carrying out Monte Carlo experiments using Evans' data-generating process (DGP). Our work relies on certain new, fast and robust programs developed at the Bank of Canada (van Norden and Vigfusson 1996) for the estimation of switching regression models that make Monte Carlo studies of such estimators practical. We find that for some (but not all) parameter values, regime-switching tests have significant amount of power to detect periodically collapsing bubbles. We are also the first to compare and contrast the performance of the two different kinds of regime-switching tests.

In the following section, we explain the relationship between speculative bubbles and regime-switching, and then review the tests proposed by Hall and Sola and by van Norden. Section 3 explains the design of our Monte Carlo experiments, and their results are discussed in Section 4. Section 5 concludes and gives several suggestions for further research.

2.0 Tests for Rational Speculative Bubbles

This section has three goals. We first describe what a bubble is. We next describe the two regime-switching tests used in this paper to detect bubbles. Finally, we compare the two tests looking for similarities and differences.

2.1 Bubbles and Regime-Switching

Consider a simple asset-pricing model, which only requires that

$$p_t = f(X_t) + a \cdot E_t(p_{t+1}) \tag{EQ 1}$$

where p_t is the logarithm of the asset price, E_t is the operator for expectations conditional on information at time *t*, $0 < a < 1$, and X_t is a vector of other variables. Solving the equation forward gives the general result

$$p_t = \left(\sum_{j=0}^{T} a^j \cdot E_t(f(X_{t+j}))\right) + a^{T+1} \cdot E_t(p_{T+1}) . \tag{EQ 2}$$

One solution to equation (EQ 1), which we will denote p_t^*, occurs when

$$\lim_{T \to \infty} a^{T+1} \cdot E_t(p_{T+1}) = 0, \tag{EQ 3}$$

so

$$p_t^* = \sum_{j=0}^{\infty} a^j \cdot E_t(f(X_{t+j})) \ . \qquad \text{(EQ 4)}$$

We refer to (EQ 4) as the fundamental solution, since it determines the asset price solely as a function of the current and expected behaviour of other variables.

However, equation (EQ 4) is not the only solution to (EQ 1). We define bubble solutions to be any other set of asset prices and expected asset prices that satisfy equation (EQ 1) but where $p_t \neq p_t^*$. We define the size of the bubble B_t as

$$B_t \equiv p_t - p_t^* \ . \qquad \text{(EQ 5)}$$

Note that since p_t^* satisfies equation (EQ 1), it follows[4] from (EQ 1) and (EQ 5) that

$$B_t = a \cdot E_t(B_{t+1}) \ . \qquad \text{(EQ 6)}$$

Since $a < 1$, this means the bubble must be expected to grow over time.[5]

Nothing in the above model has any implications for regime-switching. Some of the early literature on rational speculative bubbles even considered purely deterministic bubbles. Regime-switching stems from the descriptions of asset market behaviour (for example, those surveyed in Kindleberger 1989) to which the above model of bubbles is often applied. The first example of regime-switching in the rational speculative bubble framework is Blanchard (1979), who proposes a bubble that moves randomly between two states, *C* and *S*. In state *C*, the bubble will collapse, so[6]

$$E_t(B_{t+1} | C) = 0 \ . \qquad \text{(EQ 7)}$$

4. Blanchard (1979) has a more complete derivation of this and subsequent steps found in this section.

5. A considerable literature exists on the conditions under which such bubbles are feasible rational-expectations solutions. Important contributions to this debate include Obstfeld and Rogoff (1983, 1986), Diba and Grossman (1987), Tirole (1982, 1985), Weil (1990), Buiter and Pesenti (1990), Allen and Gorton (1991), and Gilles and LeRoy (1992). In single-representative-agent models, a truly rational agent cannot expect to sell an over-valued asset (one with a positive bubble) before the bubble bursts. Therefore, bubbles should exist in such models only if they can be expected to grow without limit. Some researchers, such as Froot and Obstfeld (1991), have therefore suggested interpreting empirical tests for bubbles as tests of whether agents are fully rational, or whether they instead exhibit some form of myopia when considering events that are either very far in the future or of very low probability. An alternative interpretation would be to consider evidence of bubbles as suggesting that non-representative-agent models (such as those of De Long et al. 1990, Allen and Gorton 1991 or Bulow and Klemperer 1991) are required.

6. The notation $E_t(X_j | C)$ (or $E_t(X_j | S)$) denotes the expectation of X_j conditional on the fact that the state at *t* is *C* (or *S*) and on all other information available at time *t*.

State *S*, where the bubble survives and continues to grow, occurs with a fixed probability *q*. Since

$$E_t(B_{t+1}) = (1-q) \cdot E_t(B_{t+1}|C) + q \cdot E_t(B_{t+1}|S) \quad , \tag{EQ 8}$$

it follows from (EQ 6) that

$$E_t(B_{t+1}|S) = \frac{B_t \cdot (1+r)}{q} \tag{EQ 9}$$

This model was subsequently generalized by Evans (1991) and van Norden and Schaller (1993) to consider the case where both the size of collapses and their probability were functions of the size of the bubble.

The distinguishing feature of these regime-switching models is that the behaviour of the asset price is now state-dependent, and that the state itself is unobservable. However, these models may differ in the way the probability of observing a given regime varies over time. In Blanchard (1979), this is simply a constant. In the van Norden bubble test, the probability of observing the collapsing regime is assumed to be an increasing function of the size of the bubble. In the Hall and Sola test, this probability is assumed to follow a first-order Markov process, where the probability of remaining in a given regime is constant.[7] To distinguish these two kinds of switching models, we will refer to the case where the probability of observing a given state is independent of past states as “simple switching.” In the case of a two-state model, the simple switching model is simply the special case of the Markov-switching model where

$$Pr(S_t = 0|(S_{t-1} = 0)) = 1 - Pr(S_t = 1|(S_{t-1} = 1)) \tag{EQ 10}$$

where $Pr(S_t = k|S_{t-1} = k)$ is the probability of remaining in state *k* given that the last period’s state was *k*.

2.2 The Hall and Sola Test for Bubbles

As mentioned earlier, Diba and Grossman (1988) suggested using tests for stationarity to rule out the existence of bubbles. This method could be useful in the case of a non-collapsing bubble but, as shown in Evans (1991), these tests tend to reject the presence of bubbles when regime-switching bubbles are present. Hall and Sola (1993) address this problem by extending the standard Augmented Dickey-Fuller (ADF) test

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7. As noted by Evans and Lewis (1995), a two-state first-order Markov process is not compatible with (EQ 6). They reconcile this by modifying the usual two-state Markov model to allow for jumps in asset prices when the regime changes.

$$\Delta y_t = \alpha + \beta y_{t-1} + \sum_{k=1}^{n} \psi \Delta y_{t-k} + v_t$$

$$\text{where } v_t \sim N(0, \sigma) \text{ .} \qquad \text{(EQ 11)}$$

to allow the parameters to vary between two regimes, giving

$$\Delta y_t = \alpha_i + \beta_i y_{t-1} + \sum_{k=1}^{n} \psi_i \Delta y_{t-k} + v_{t,i}$$

$$\text{where } v_{t,i} \sim N(0, \sigma_i) \text{ .} \qquad \text{(EQ 12)}$$

The slope coefficients β_S and β_C are the basis of bubble test. Evidence that one regime is non-stationary (i.e. $\beta_S > 0$) while the other is stationary (i.e. $\beta_C < 0$) indicates the presence of a bubble. However, one property of switching regressions is that such models are identified only up to the particular relabeling of parameters that has the effect of swapping the names of the *S* and *C* regimes. This means that one should find either $\beta_S > 0, \beta_C < 0$ or $\beta_S < 0, \beta_C > 0$.

Our application of the Hall and Sola test (below) will be conducted on artificial data for the bubble, where the original authors tested asset prices (i.e., the bubble term plus the fundamental term). Since both must satisfy the same dynamic relationships, this change should be innocuous. Funke, Hall and Sola (1994) use the Markov-switching ADF test to find evidence for bubbles in the Polish economy in the late 1980s and early 1990s. Hall and Sola (1993) performed a brief study of the test's properties. However, they only estimated a single realization of each of five different data-generating processes, including Evans bubble process (described below) with the probability of continuing to grow, π, equal to 0.75. [8]

2.3 Van Norden Bubble Test

Van Norden (1993) and van Norden and Schaller (1993a, 1993b) modify the Blanchard model to allow for the possibility that the bubble is expected to collapse only partially in state *C* by replacing (EQ 7) with

$$E_t(B_{t+1} | C) = u(B_t) \qquad \text{(EQ 13)}$$

8. Note that Hall and Sola (1993) multiply the bubble term by twenty in constructing their simulated asset prices. This implies, unlike the case mentioned above, that their bubble will have a rate of return twenty times greater than that of the fundamental.

where *u(.)* is a continuous and everywhere differentiable function such that $u(0) = 0$ and $1 \geq u' \geq 0$. Hence, the expected size of collapse will be a function of the relative size of the bubble, B_t, and the bubble is also not expected to grow (and may be expected to shrink) in state *C*. They also suggest that the probability of the bubble's continued growth falls as the bubble grows, so that[9]

$$q = q(B_t) \qquad \frac{d}{d|B_t|}q(B_t) < 0 \tag{EQ 14}$$

van Norden (1993) and van Norden and Schaller (1993b) show that a first-order Taylor-series approximation of this process gives the following two-state switching regression system[10]

$$\begin{aligned} E_t(\Delta B_{t+1}|S) &= \alpha_S + \beta_S B_t \\ E_t(\Delta B_{t+1}|C) &= \alpha_C + \beta_C B_t \\ Pr(\text{State}_{t+1} = S) &= \Phi(\lambda + \eta B_t) \\ Pr(\text{State}_{t+1} = C) &= 1 - Pr(\text{State}_{t+1} = S) \end{aligned} \tag{EQ 15}$$

where the model implies that $\beta_S > 0$, $\beta_C < 0$ and $\eta < 0$, and $\Phi(x)$ is the Gaussian cdf function.[11] Again, one property of switching regressions is that such models are identified only up to a particular renaming of parameters that has the effect of swapping the names of the *S* and *C* regimes. In this case, this equivalence implies that

$$\text{llf}(\alpha_S, \beta_S, \alpha_C, \beta_C, \lambda, \eta, \sigma_S, \sigma_C)$$

$$= \text{llf}(\alpha_C, \beta_C, \alpha_S, \beta_S, -\lambda, -\eta, \sigma_C, \sigma_S) \tag{EQ 16}$$

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9. Since we will only consider positive bubbles in this paper, the use of the absolute value in the derivative in (EQ 14) is not strictly necessary.

10. The original model uses the exchange rate innovation R_{t+1} as the dependent variable. This variable in turn consists of innovations in fundamentals ε'_{t+1} and innovations in the bubble. Hence $R_{t+1} = \varepsilon'_{t+1} + B_{t+1} - E_t(B_{t+1})$. If we assume that in this model $\varepsilon'_{t+1} = 0$ and use (EQ 6) then $R_{t+1} = \Delta B_{t+1} - rB_t$. Since r is small, the use of ΔB_t as the dependent variable is a good approximation of the earlier model.

11. This model differs trivially from that considered in van Norden (1993) and van Norden and Schaller (1993). The former assumed that $\Phi(x)$ was the logistic cdf rather than the Gaussian. Both papers also used slightly different classifying equations (using either $|B_t|$ or B_t^2) to allow for the possibility of negative bubbles.

where llf() is the log-likelihood function, indicating that these alternative parameterizations cannot be distinguished without additional information. The van Norden bubble model implies that one should find either [$\beta_S > 0$, $\beta_C < 0$, $\eta < 0$] or [$\beta_S < 0$, $\beta_C > 0$, $\eta > 0$].

In addition to testing the above restrictions implied by the bubble model, van Norden (1993) and van Norden and Schaller (1993a,b) test whether the bubble-motivated switching regression model gives significantly more information about the behaviour of ΔB_{t+1}, than two simpler models.[12] Significant evidence of bubbles requires that the switching regression model can reject these simpler models. One of these is the normal-mixture model (NM)

$$\begin{aligned} \Delta B_{t+1} &\sim N(\alpha_S, \sigma_S) \qquad \text{when State}_{t+1} = S \\ \Delta B_{t+1} &\sim N(\alpha_C, \sigma_C) \qquad \text{when State}_{t+1} = C \; . \\ Pr(\text{State}_{t+1} &= S) = \Phi(\lambda) \end{aligned} \qquad \text{(EQ 17)}$$

which is simply the special case of (EQ 16) where $\beta_S = \beta_C = \eta = 0$. A rejection of this null hypothesis implies that there is a significant link between B_t and the behaviour of the mixing distributions, because it captures shifts either in their means, or in their mixing probabilities, or in both.[13]

(EQ 16) also nests the linear regression model as the special case where $\beta_S = \beta_C, \alpha_S = \alpha_C$ and $\eta = 0$, giving the error contamination model (EC) [14]:

$$\begin{aligned} \Delta B_{t+1} &= \alpha + \beta B_t + e_{t+1} \\ e_{t+1} &\sim N(0, \sigma_S) \text{with prob} \Phi(\lambda_q) \; . \\ e_{t+1} &\sim N(0, \sigma_C) \text{with prob} 1 - \Phi(\lambda_q) \end{aligned} \qquad \text{(EQ 18)}$$

Any rejection of this model can be interpreted as evidence of non-linear predictability in asset prices. Note that if the variances differ across the two regimes, all parameters will be identified under the null.

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12. van Norden (1993) also considers a third model. Since it nests within the normal-mixture model, rejections of the normal-mixture model imply a rejection of the third model.

13. van Norden (1993) also notes the relationship of the time-varying transition probabilities to Markov-mixture models. Schaller and van Norden (1994) consider generalizations of (EQ 16) to allow for Markovian state-dependent transition probabilities.

14. We also examined a similar model where we drop the restriction that $\eta = 0$. We found that this model was on average somewhat more problematic to estimate and more likely to statistically reject than the two models considered above.

Van Norden and Schaller (1993a) use this test framework to show evidence of bubbles in monthly returns from the Toronto Stock Exchange. Van Norden (1996) looks for evidence of bubbles in post-Bretton-Woods floating exchange rate data, and van Norden and Schaller (1993b) examine the behaviour of NYSE monthly stock returns from 1926 to 1989. The latter paper also presents extensive analysis on whether regime-switching in fundamentals can account for the evident regime-switching in stock returns.

2.4 Comparing Hall and Sola's Test with van Norden's Test

By comparing the last two sections, the reader can see that the Hall and Sola test and the van Norden test show some important similarities and differences in both parts of the regime-switching model: the level equations and the transition equations. Each of the two level equations gives the relationship between the observable dependent and explanatory variables for a particular regime. The transition equations give the probability of being in the current regime at a given period of time.

When both tests have the same dependent variable (i.e., $B_t = y_t$ for all t) the level equation of van Norden's test (EQ 15) is a simpler version of the level equation of Hall and Sola's test (EQ 11) where $\psi_{i,k} = 0$ for all i and k. In applications of these tests, several different kinds of dependent variable have been examined. Funke, Hall and Sola (1994) used the actual changes in asset prices and the residuals from a regression of fundamentals on the assets. The van Norden test has been applied to excess returns on exchange rates and the rates of returns on stock market indices. Thus the applied researcher can choose from a number of different transformations when using these switching models. To abstract from the difficulties of choosing the correct dependent variable, our Monte Carlo study uses only the bubble term as the dependent variable; therefore, the level equations of the two tests are similar.

The transition equations, however, are not necessarily the same. If van Norden's coefficient η equals zero, then the van Norden test becomes a constant probability simple switching model. Such a model is a special case of a Markov-switching model, implying that the van Norden test would then be nested inside of Hall and Sola's test. However, for a large majority of the bubbles examined below, estimates of η do not equal zero. Hence, the tests are not nested.

Not being nested doesn't mean that the tests are unrelated. For the Hall and Sola test, the probability of being in a given regime is dependent on an unobserved state variable that follows an AR(1) process with the autoregressive coefficient ρ equal to $Pr(S_t = S|S_{t-1} = S) + Pr(S_t = C|S_{t-1} = C) - 1$ (Hamilton 1989). In the van Norden test, the probability of being in a given regime is dependent on the level of the observed variable B_t. As B_t usually shows positive serial correlations, the dynamics of the two models can be quite similar.

The theory on bubbles is ambiguous on how the probability of collapse should be modelled. The degree of uncertainty on how to model these transition probabilities suggests that either model may be useful. One could test which model would be more appropriate by estimating a Markov-switching model where the transition probabilities were dependent on the size of the bubble. This non-constant transition probability Markov-switching model would encompass the other two models, but estimating such a model could prove difficult.[15]

3.0 Experimental Design

As we noted in the introduction, the purpose of this paper is to examine the behaviour of regime-switching bubble tests described in the preceding section. Specifically, we want to use Monte Carlo experiments to evaluate the power of the tests and to compare the two testing methodologies. This involves specifying a data-generating process that creates bubbles, generating multiple time series from this process, estimating the regime-switching models and applying the tests described above. All of our estimation is done by maximum-likelihood methods using the programs documented in van Norden and Vigfusson (1996).[16]

We decided to use various parameterizations of the Evans' (1991) bubble model as our data-generating process. This choice has several attractive features. First, the problems of unit-root and cointegration-based tests on this data set are well-documented, facilitating a comparison of the regime-switching tests with earlier tests.[17] Second, Charemza and Deadman (1995) study the performance of the earlier tests on other data-generating processes and reach conclusions broadly similar to those of Evans, suggesting that the Evans process might not produce atypical results. Third, as we explain below, the Evans model is not precisely nested within either the Hall and Sola or the van Norden bubble testing models. We think this introduces an interesting amount of misspecification into the experimen-

15. The Bank of Canada procedures likelihood function is written to handle a time-varying Markov-switching model, but the EM algorithm included in the procedures cannot estimate such a model. Diebold, Lee, and Weinbach (1994) describe an EM algorithm that could be used in such a case.

16. We made minor modifications to the code to improve its ability to find convergent solutions for hard-to-fit data sets. We improved the error-trapping in the original programs, and when both gradient-based and EM-based maximization strategies seemed to be failing, we used a few iterations of a simple simulated annealing procedure to get new starting values for maximum likelihood estimation.

17. Hooker (1996) uses the Evans DGP to examine a bubble test proposed by Durlaf and Hooker (1994) that differs from the regime-switching tests in testing both for specification error and for a bubble term separately and sequentially. Hooker conducts Monte Carlos for both the size and power of the tests. For the Evans DGP, the test performs well for all values of π with the percentage of correct detections ranging from 55 to 45 per cent, and decreasing slightly with π. Since the regime-switching tests are better with lower values of π, as shown in the next section, these two kinds of tests may be considered complementary.

Our results and Hooker's results, however, are not directly comparable. Here we test the bubble series alone. Hooker tests an I(2) series where fundamentals and bubble are combined. Furthermore, the parameter values used by Hooker differ from those used by Evans and by us.

tal design and may give a better indication of how the tests are likely to perform when confronted with real data that may not nest perfectly within either model. We also felt that it offered a neutral "middle ground" on which to compare the performance of the two tests.

As we noted in Section 2.0, the Evans model is a generalization of the Blanchard (1979) model where both the size of collapses and their probability are functions of the size of the bubble; it incorporates partial rather than total collapses and sets the probability of collapse equal to zero when $B_t \le \alpha$.

Initially the bubble grows at an average rate 1+r, but the realized rate of growth differs from the expected value by serially uncorrelated mean zero errors. We will refer to this phase of steady expected growth as Regime G. Once the bubble's size reaches a threshold level of α, its behaviour changes. It continues to grow at an expected rate of 1+r but there is now a probability $1-\pi$ of collapse to a level δ (Regime C). To compensate, if the bubble does not collapse (Regime E) it is expected to grow at a rate greater than $1+r$.

This model can be written as;

$$B_{t+1} = (1+r)B_t u_{t+1} \qquad \text{For} B_t \le \alpha$$
$$B_{t+1} = (\delta + \theta_{t+1}\pi^{-1}(1+r)(B_t - \delta(1+r)^{-1}))u_{t+1} \qquad \text{For} B_t > \alpha \qquad \text{(EQ 19)}$$

where α and δ are positive parameters with $\delta < (1+r)\alpha$, u_t is an exogenous independently and identically distributed strictly positive random variable with $E_t u_{t+1} = 1$ and θ_t is an exogenous independently and identically distributed Bernoulli process that takes the value 1 with probability π and 0 with probability $1-\pi$. Evans' bubble satisfies (EQ 6).

There are two points to note about this model. First, since u_t is strictly positive, the bubble will never change sign and will never entirely vanish. Second, Regime G is only distinguished from the mixture of the other two regimes by the distribution of innovations in the bubble. For a particular distribution of u_t, the innovations in the mixture of Regimes C and E will simply appear to be more volatile than in G.

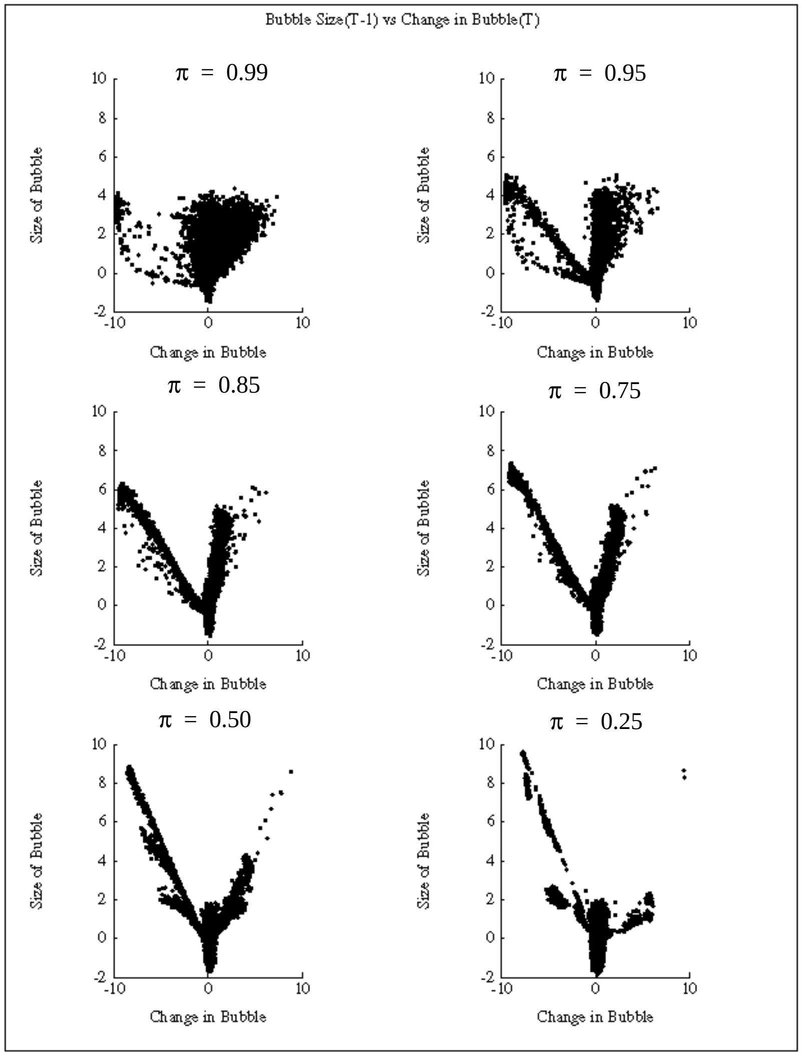
For estimation, we rewrite (EQ 19) in first differences as

$$\begin{array}{lll} G & \Delta B_t = \{(1+r)u_t - 1\}B_{t-1} & \text{For } B_{t-1} \le \alpha \\ E & \Delta B_t = \left\{\frac{(1+r)}{\pi}u_t - 1\right\}B_{t-1} + \frac{(\pi-1)\delta u_t}{\pi} & \text{For } B_{t-1} > \alpha \text{ and } \theta_t = 1 \\ C & \Delta B_t = \delta u_t - B_{t-1} & \text{For } B_{t-1} > \alpha \text{ and } \theta_t = 0 \end{array} \quad \text{(EQ 20)}$$

For our Monte Carlo experiments, we generate 5000 draws of the above process with 100 observations. We use the same parameter values as Evans, setting $r = 0.05$, $\alpha = 1$, $\delta = 0.5$, $B_1 = \delta$, and $u_t = exp\left(y_t - \frac{\tau^2}{2}\right)$ where $y_t \sim IIN(0, \tau^2)$ and $\tau = 0.05$. We allow the probability of the bubble continuation, π, to vary over the same interval as Evans: [0.999,0.25].

To simplify estimation, all data series were standardized to have a mean of zero and a variance of one. (For graphing, they were also centered at (0,0).) The relationship between ΔB_t and B_t can be seen in Figure 1. At high levels of π the graph appears to be composed of two branches. The left branch corresponds to State C, where the bubble collapses, and the right with States G and E, where the bubble continues to grow. As π decreases, State G becomes more distinct from State E. State G can be identified as the large mass centered at 0 on the horizontal axis. It is most prominent when $\pi = 0.25$. The decrease in π also causes a change in the slopes of the two branches. This is because the growth rate in State E increases as π decreases. This increase in the growth rate results in the decline in the slope of the right branch.

FIGURE 1.


Bubble Size(T-1) vs Change in Bubble(T)
π = 0.99
π = 0.95
π = 0.85
π = 0.75
π = 0.50
π = 0.25
Size of Bubble
Change in Bubble

4.0 Monte Carlo Results

4.1 Overview of Results

The two regime-switching tests frequently detect bubbles that the unit-root tests incorrectly reject. When comparing regime-switching with unit-root tests, one must remember that the nulls of the two kinds of tests are opposite. One might think that because critical values are chosen to minimize false rejections of the null that the unit-root tests should detect bubbles more often since their null hypothesis is that a bubble exists. As seen in Table 1, this is clearly not the case.

TABLE 1. Summary Table: Ability of Tests to Detect Bubbles

Test		**0.999**	**0.99**	**0.95**	**0.85**	**0.75**	**0.50**	**0.25**
Bhargava N1	rejection in favor of explosive alternative	78.5	32.5	0	0	0	0	0
	rejection in favor of stable alternative	0	1	65.5	94.6	98	100	100
Bhargava N2	rejection in favor of explosive alternative	95	58	15	4.5	2	1	0
	rejection in favor of stable alternative	0	1	18.5	90	94.5	97	97
van Norden	correct signs	8	89.5	82.5	96	99	89.5	95
	t-test	1	5	16	48.5	77	28.5	3
Hall & Sola	correct signs		39.5	67.5	78.5	83.5	83.5	71
	t-test		25	50	64	64	58	35

The relationship between the value of π and the ability to detect bubbles varies among the different tests. For values of π less than 0.99, the Bhargava (1986) N_1 and N_2 unit-root tests frequently and incorrectly reject the null of a bubble in favor of a stationary stable alternative.[18] The van Norden test does best when π equals 0.75 and does a poorer job for other values. The Hall and Sola test detects more bubbles than van Norden according to the t-tests for all values of π except when π equals 0.75.

The following sections give more details on our results. The next section discusses the difficulties experienced in trying to get the maximum-likelihood estimation methods to convergence. Following sections discuss each test individually. We then look at the added information each test provides, followed by conclusions.

18. The greatest difference between a percentage that we report and Evans is less than 5 per cent.

4.2 Convergence

A standard problem in performing Monte Carlo or other simulation experiments with iterative estimators is that some fraction of the estimates will typically fail to converge. This in turn puts limits on the confidence we should attach to our experimental results. Fortunately, this was not a serious problem in practice. Table 2 shows that for the Hall and Sola test, we achieved convergence for 90 per cent or more of the simulated data sets, regardless of the parameterization of the DGP considered. For the van Norden test, we needed to estimate as many as three switching models on each data sample. Fortunately, convergence rates were generally higher than for the Hall and Sola model, as shown in Table 2. We also occasionally have the problem that the restricted models give values of the higher likelihood function than the unrestricted model (which may reflect false convergence or the presence of multiple local maxima). As shown in Table 3, this problem was also rare, except when $\pi = 0.5$. (We explore the case where $\pi = 0.5$ in greater detail below.) In all subsequent tables, the reported fraction of cases in which bubbles were detected counts as non-detection cases whether some models failed to converge and cases where restricted models gave the highest values of the likelihood function.

TABLE 2. Percentage of Draws that Failed to Converge: Hall and Sola (HS) van Norden (vN,NM,EC)

π	0.999	0.99	0.95	0.85	0.75	0.50	0.25
Hall & Sola (HS)	NA[a]	7.60	10.78	4.80	3.40	3.46	10.42
van Norden (vN)	3.32	5.32	3.30	3.82	4.62	4.82	2.92
Normal Mixture (NM)	0.92	3.56	1.74	1.22	0.90	0.58	0.58
Error Contamination (EC)	3.4	4.42	3.78	2.78	3.38	2.92	2.76
vN & NM	0.14	0.62	0.06	0.18	0.12	0.08	0.02
vN & EC	0.52	0.46	0.36	0.64	0.50	0.18	0.22
NM & EC	0.06	0.22	0.32	0.18	0.18	0.08	0.06
HS ∪ vN	NA	11.98	13.78	8.32	7.880	8.160	13.06
HS & vN	NA	1.00	0.34	0.32	0.16	0.18	0.32

a. The Hall and Sola test has not been done for $\pi = 0.999$.

TABLE 3. Percentage of Draws with Restricted LLF Greater than Unrestricted: van Norden

π	**0.999**	**0.99**	**0.95**	**0.85**	**0.75**	**0.5**	**0.25**
NM > vN[a]	1.122	0.4862	0.04138	0.2080	5.789	88.33	2.514
EC > vN	6.594	3.974	1.179	1.414	8.578	87.87	2.267

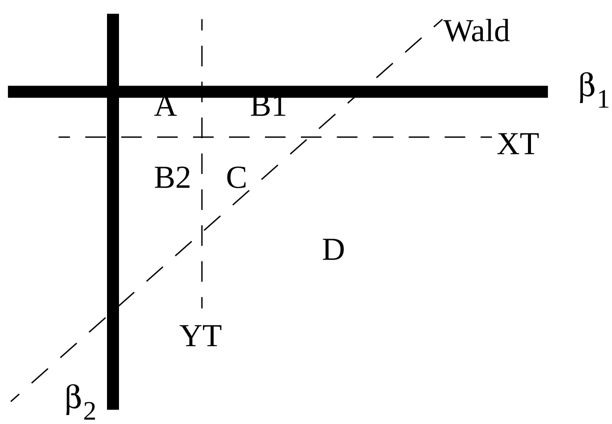
a. The NM > vN etc. are those draws that returned likelihood function values higher for the restricted model than the unrestricted case.The two restricted models are the Normal Mixture model (NM) and the Error Contamination model (EC).

4.3 Hall and Sola Test

We consider our results for the Hall and Sola test using two different levels of rigour. First, we examine whether the switching model gets the signs of the two autoregressive coefficients right. (Note that the two regimes were normalized by setting Regime 1 to be the regime with the greater slope coefficient.) Next, we test whether these estimates are significantly different from zero. We also do a Wald test to see whether the two coefficients are jointly different from each other.

The relationships between these tests can be seen in Figure 2. The test of the coefficient signs is equivalent to the entire area either right of the Y-axis or below the X-axis. The t-statistics restricts this area to right of the YT-line or below the XT-axis. Testing to see that they are both the right signs reduces the area to just the bottom quadrant below the X-axis and right of the Y-axis. Using the t-statistics reduces the area to the area of C+D. Finally using the Wald Test eliminates area C leaving only D. Results for these tests are reported in Table 4.

FIGURE 2.



The individual tests (Table 4) show considerable power to detect bubbles. The autoregressive coefficients are significantly different 40-70 per cent of the time, and the individual coefficients are significantly different from zero and have the correct sign even more frequently (with one exception.) The test seems to have the greatest power when the probability of the bubble surviving is around 80 per cent. At the highest probabilities, the estimated coefficient in the collapsing regime performs poorly, perhaps because so few collapses would be observed in a sample of 100 observations. At the lowest probabilities, the estimated coefficient in the surviving regime performs poorly (presumably for different reasons).

TABLE 4. Percentage of Draws Passing Single Test

	Regime 1 Explosive		Regime 2 Stable or Collapsing		Regimes not Equal
π	$\beta > 0$	t-stat > 2	$\beta < 0$	t-stat < –2	Wald Test
0.25	88.77	46.13	82.21	65.24	50.17
0.50	93.76	68.95	89.60	71.00	65.07
0.75	94.04	76.15	89.50	75.18	67.43
0.85	95.97	84.58	82.52	71.20	66.45
0.95	98.25	84.90	69.46	53.75	53.22
0.99	98.93	89.78	40.38	27.32	42.29

The joint test results (Table 5) also show that the bubble test is most successful for mid-range levels of π. If we simply require that the estimated autoregressive coefficients have the correct signs, then we find evidence of bubbles 40-80 per cent of the time. Even the most stringent tests, which require that all the coefficients are both statistically different from zero and from each other, find significant evidence of bubbles as much as 43 per cent of the time. However, it should be noted that this power again drops off considerably as π approaches 0 or 1.

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TABLE 5. Percentage of Draws Passing Multiple Tests

	Regime 1 Explosive and Regime 2 Stable or Collapsing		Regime 1 Explosive Regime 2 Stable or Collapsing and Regimes Not Equal	
π	$\beta > 0$ $\beta < 0$	t-stat > 2 t-stat < –2	$\beta > 0$ $\beta < 0$ Wald Test	t-stat > 2 t-stat < –2 Wald Test
0.25	70.98	34.78	35.72	17.26
0.50	83.36	57.97	54.63	37.79
0.75	83.54	63.79	55.94	42.51
0.85	78.49	64.37	52.48	42.82
0.95	67.70	50.16	36.44	27.03
0.99	39.30	24.91	15.96	10.10

4.4 Van Norden Bubble Test

Table 6 shows the results of the likelihood ratio tests for bubbles, which compare the fit of the regime-switching model (vN) (EQ 15) to that of the two simpler models NM (EQ 17) and EC (EQ 18). For both high and low values of π, the nulls are almost always rejected in favor of the switching model. (Remember that those draws where the restricted models had likelihood function values greater than the unrestricted models were included as non-rejections of the null! Including these draws did not have a great effect on the rejection rates except for $\pi = 0.5$.) However, the case where $\pi = 0.5$ stands out as an important exception. Here, there were very few rejections of the null, reflecting the very high (over 85 per cent) frequency with which the restricted models gave higher values of the likelihood function than the unrestricted model.

TABLE 6. Adjusted LR Tests.Percentage Rejections

Restriction	NM		EC	
π	**5%**	**1%**	**5%**	**1%**
.999	98.03	97.45	91.47	90.62
.99	99.24	98.90	94.97	93.81
.95	99.96	99.96	98.45	97.66
.85	99.73	99.73	98.02	97.44
.75	91.00	89.26	87.44	84.92
.5	11.46	11.40	12.01	11.96
.25	97.49	97.49	97.73	97.73

We next examine the parameters of our estimated model (Table 7). To avoid the identification problem noted in (EQ 16), the parameters are normalized by setting Regime 2 as the regime with the greater slope coefficient.The bubble model therefore implies that one should find [$\beta_1 < 0$, $\beta_2 > 0$, $\eta > 0$].

When $\pi = 0.999$ or $\pi = 0.99$ we may never observe a bubble collapse in our relatively small sample of 100 observations. This could cause the estimates to miss the behaviour of Regime C, so that the two regimes in the regime-switching model would be based upon the slight difference between Regimes E and G. This is consistent with the Monte Carlo results. Using the median of the distribution, we find $\beta_1 < 0$ and $\beta_2 > 0$ except for $\pi = \{0.999, 0.99\}$.

TABLE 7. Median Value of Parameters

π	**.999**	**.99**	**.95**	**.85**	**.75**	**.5**	**.25**
Parameter							
α_1	-0.0023	0.126	0.552	0.468	-0.541	-0.338	1.02
β_1	0.363	0.221	-0.667	-0.941	-1.09	-0.937	-1.03
α_2	0.0971	0.101	0.136	0.174	0.205	0.242	0.0897
β_2	0.749	0.628	0.221	0.339	0.470	0.854	0.0438
λ	0.211	-1.47	-2.69	-3.11	-3.23	-2.31	-185.
η	2.54	2.01	1.76	1.45	1.18	1.10	351.
σ_1	1.05	1.24	2.23	2.21	0.0987	0.0567	2.25
σ_2	0.154	0.0947	0.0643	0.0726	0.107	0.212	0.0914

As shown in Table 8, the t-statistics also provide evidence of bubbles. Independent of the level of π, Regime 2 usually has significantly positive slope and intercept terms, while the slope term in the equation for the probability of being in Regime 1 is usually (correctly) negative for all values of π. However the actual percentage varies greatly. The change in π greatly affects Regime 1's slope. At high levels of π the slope is often found to be significantly greater than zero. The lack of actual collapses at high levels of π may be responsible for this failure to detect a collapsing regime. As π decreases, the slope is found to be significantly less than zero. This corresponds well with the presence of a bubble.

TABLE 8. Percentage of Draws with Parameters Significantly Different From Zero[a]

π Parameter	**.999**	**.99**	**.95**	**.85**	**.75**	**.5**	**.25**
α_1	26.17	16.19	7.98	23.1	53.9	81.58	10.28
β_1	55.54	47.50	16.9	51.2	84.20	96.00	95.98
α_2	36.02	56.98	86.3	96.98	99.56	98.89	98.8
β_2	95.8	94.54	96.7	98.33	97.97	93.35	48.83
λ	16.58	44.1	76.9	90.52	94.3	52.35	8.02
η	47.7	60.4	83.6	91.88	90.7	29.24	6.76
σ_1	100	99.95	100	99.9	99.9	99.5	100
σ_2	99.8	99.97	100	100	100	99.9	100

a. Shaded cells are significantly less than zero and unshaded are significantly greater than zero. "Significantly" different means that the t-statistic is greater than 2 or less than -2. Both percentages were calculated for each parameter. The higher percentage is reported.

As mentioned earlier, the van Norden test for bubbles consists of testing three coefficients' signs: $\beta_1 < 0$, $\beta_2 > 0$, and $\eta < 0$. When all three coefficients have the correct sign, the series is classified as having a bubble. Requiring that the coefficients only have the correct signs implies that the van Norden test classifies almost all the series as being bubbles for values of π less than 0.99 (Table 9). Requiring statistical significance causes the level of detection to drop. The series for which π equals 0.25 sees the greatest decrease. This large drop is due to the transition equation's slope coefficient η being statistically significant only 6 per cent of the time (Table 8).

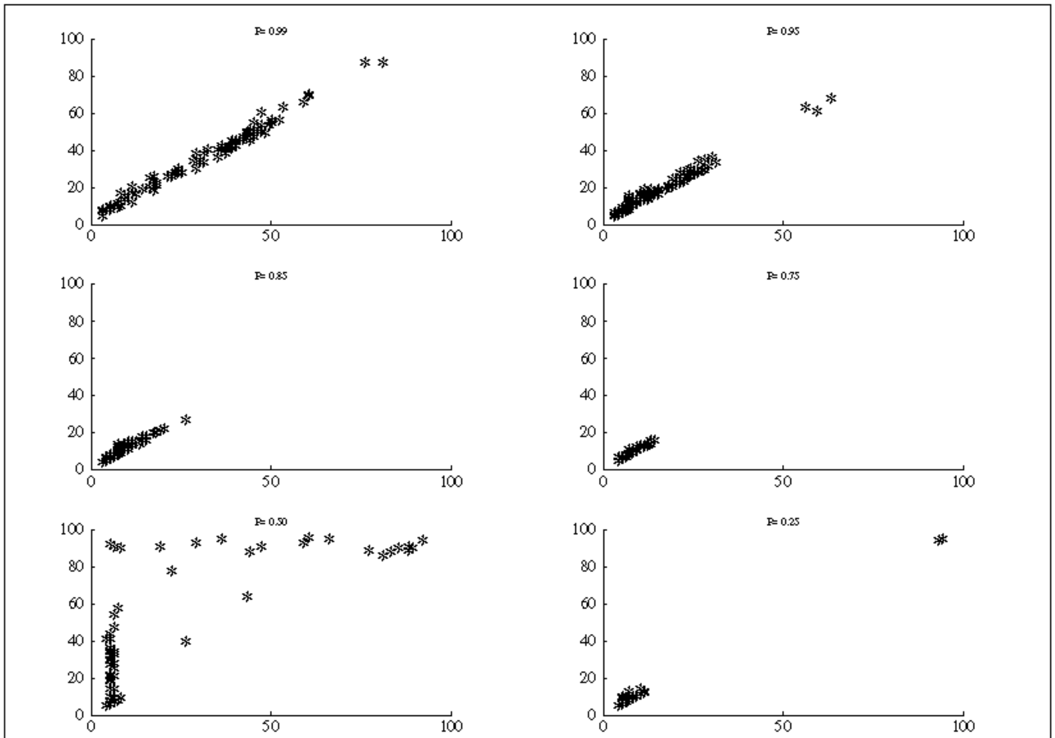
TABLE 9. Joint Bubble Tests $\beta_1 < 0$ $\beta_2 > 0$ and $\eta < 0$

	π	**0.999**	**0.99**	**0.95**	**0.85**	**0.75**	**0.50**	**0.25**
Percentage of all three coefficients being	of the correct sign	8.294	39.51	82.45	95.59	98.87	89.30	94.93
	statistically significant	0.7478	4.883	16.01	48.40	76.78	28.47	2.824

To better understand the behaviour of the test, we also examined the ex post probability that an observation was generated by Regime 1 (i.e., the probability conditional on

ΔB_{t+1}). We would expect to find many periods classified as growing and few classified as collapsing. If we identify the collapsing regime with Regime 1 then this is supported by Figure 3. The smaller the value on the horizontal axis, the smaller the number of draws that are classified as Regime 1. For π between 0.95 and 0.75, Regime 1 is somewhat infrequent, and at π equal to 0.50 there are a number of cases where Regime 1 is very frequent and Regime 2 is very infrequent. The majority of draws lie on the 45 degree line, implying that all the periods are clearly distinguished between Regimes 1 and 2. Each period has either a very low or very high probability of being in Regime 1. The exception is when π equals 0.50. Some draws contain a large number of periods whose probability of being in Regime 1 is neither very high (greater then 0.9), nor very low (less than 0.1). This may be due to the lack of a separate regime for State G, the growing state, in the specification.

FIGURE 3.



100-# of Periods with Probability <0.1 (Left Axis) VS # of Periods with Probability >0.9 (Bottom Axis)

4.5 Comparing van Norden and Hall and Sola

Having examined the individual performances of the two tests, we now use the two tests together. First, we examine how often the tests agree that a bubble is present. Second, we examine how often one test confirms the presence of a bubble already detected by the other.

In Table 10, using the least stringent condition that the coefficients are of the correct sign, bubbles are found over 50 per cent of the time for values of π equal to 0.85 and 0.75. Even the most stringent test that all the coefficients are statistically significant gives a positive result more than one third of the time for $\pi = 0.75$.

TABLE 10. Percentage of Draws that Agree with Both Tests

Tests		**π**					
Hall and Sola	**van Norden**	**0.99**	**0.95**	**0.85**	**0.75**	**0.50**	**0.25**
Correct signs and Wald test	correct signs	14.02	33.14	50.74	55.38	49.04	34.18
	t-test	1.545	7.172	26.27	43.36	15.05	0.9432
Correct t-tests and Wald test	correct signs	10.20	24.51	41.58	42.16	33.80	16.59
	t-test	1.318	5.661	22.01	33.56	10.34	0.5291

Table 11 shows the marginal benefit of running the second test: the percentage of bubbles found by test B given that test A has already found a bubble.

TABLE 11. Percentage of Times Test B Agrees with the Finding of a Bubble by Test A[a]

A	**B**	**0.99**	**0.95**	**0.85**	**0.75**	**0.50**	**0.25**
Hall and Sola	van Norden	10.94	21.00	51.61	79.00	27.36	3.071
van Norden	Hall and Sola	30.85	36.08	45.49	43.80	36.15	17.97

a. “Agrees” and “finding of a bubble” mean that all three slope coefficients are statistically significant for the van Norden test and that the t-tests and the Wald test are statistically significant for the Hall and Sola test.

Given a positive finding by the van Norden test, the Hall and Sola test is more likely to disagree rather than to agree for all values of π. The van Norden is more likely to agree rather than to disagree for a finding of bubble by the Hall and Sola test only for the cases where π is equal to 0.85 or 0.75.

5.0 Conclusions

We have examined the ability of two similar but not equivalent regime-switching tests to detect bubbles when they are present by construction. Both tests are shown to be substantially better than the previously used unit-root tests in detecting bubbles when they are present by construction. In particular, Evans (1991) showed that unit-root tests may inadvertently suggest the absence of bubbles when several regime switches are encountered in the sample; this seems to be when regime-switching methods have the most power to detect bubbles.

When used with similar dependent variables, the two regime-switching tests differ in three ways. First, the Hall and Sola test allows for more complicated dynamics within regimes by including lagged changes in the asset price. Second, the probability of being in a certain regime for Hall and Sola depends on last period's regime, while the same probability for van Norden's test depends on last period's value of the bubble. Third, to the extent that there are variations in the fundamental value of the asset, the tests use different explanatory variables; Hall and Sola use the asset's price while van Norden uses the difference between this price and the fundamental prices. These differences in structure result in the two tests having different abilities to detect bubbles. In this paper, we focussed on the second of these three differences, and found that the van Norden test tended to have more power than the Hall and Sola test for certain values of the probability of the bubble continuing to grow π, but that the power of the Hall and Sola test was less sensitive to the value of π.

Even though the tests are different, we currently cannot say that one test is superior to the other. The van Norden test did have higher rates of convergence, but convergence is likely to be more of an issue for a Monte Carlo study than for applied research. To establish which test, if either, is better would require more work.The power of each test when the bubble term is measured with serially correlated errors will be an important topic to examine. Serially correlated errors could likely be the case when applied to actual data. The size of the tests will also need to be examined. The ability of the regime-switching tests to detect bubbles when they are present will be of little use to the applied researcher if the tests also find many false positives.

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