SWITCHING BETWEEN CHARTISTS AND FUNDAMENTALISTS:
A Markov Regime-Switching Approach

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Abstract

Since the early 1980s, models based on economic fundamentals have been poor at explaining the movements in the exchange rate (Messe 1990). In response to this problem, Frankel and Froot (1988) developed a model that uses two approaches to forecast the exchange rate: the fundamentalist approach, which bases the forecast on economic fundamentals, and the chartist approach, which bases the forecast on the past behaviour of the exchange rate. This was an innovation, as only the fundamentalist approach had been used before.

A feature of the chartist-and-fundamentalist (c&f) model is that these two approaches' relative importance varies over time. Because this weighting is unobserved, the c&f model can not be estimated or tested using standard techniques. To overcome these difficulties and to test the model, the author uses Markov regime-switching techniques. He defines the two groups' different methods of forecasting as regimes and rewrites the c&f model as a regime-switching model.

The model is then used to test for c&f behaviour in the Canada-U.S. daily exchange rate between 1983 and 1992. The author finds favourable though inconclusive evidence for the c&f model and accordingly makes suggestions for further research.

Résumé

Depuis le début des années 80, les modèles fondés sur les facteurs économiques fondamentaux n'ont guère contribué à expliquer les variations du taux de change (Messe 1990). Devant ce problème, Frankel et Froot (1988) ont élaboré un modèle dans lequel ils ont utilisé deux approches de prévision du taux de change : l'approche fondamentaliste, dans laquelle la prévision repose sur des facteurs économiques fondamentaux, et l'approche technique, où la prévision s'appuie sur le comportement passé du taux de change. Il s'agissait là d'une innovation car, auparavant, seule l'approche fondamentaliste était utilisée.

Une caractéristique propre au modèle fondamentaliste et technique (f et t) est que le poids relatif des deux composantes varie avec le temps. Comme cette pondération n'est pas observable, le modèle ne peut être estimé ni testé à l'aide des techniques standard. Pour surmonter ces difficultés et tester le modèle, l'auteur fait appel aux techniques de changement de régime de Markov. Il définit chacune des deux approches comme un différent régime de génération d'informations et transforme le modèle f et t en un modèle avec changement de régime.

Le modèle est ensuite utilisé pour tester le comportement du taux de change au jour le jour Canada-États-Unis pour la période 1983-1992. Les résultats favorables quoique non concluants du modèle amènent l'auteur à suggérer la poursuite des recherches.

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1.0 Introduction

Economists have been dissatisfied with the explanatory power of fundamental exchange rate models since the early 1980s (Messe 1990). For example, the large appreciation of the U.S. dollar in the early 1980s is difficult to explain from a fundamentalist perspective. In response to this problem, Frankel and Froot (1988) developed an exchange-rate forecasting model that incorporates both fundamentalist and chartist techniques. Chartists base their expectations of future changes in the exchange rate not on economic fundamentals, but solely on the rate's past behaviour.

The chartist-and-fundamentalist (c&f) model, when simulated, can mimic the behaviour of an exchange rate. With the right parameters, the simulated c&f exchange rate behaves like the U.S. dollar in the 1980s: it appreciates, it plateaus, and then it depreciates (Frankel and Froot 1988). Like actual exchange rates, the simulated rate tests positive for a unit root and the forward rate is a biased predictor of future changes (De Grauwe and Dewachter 1992).

Anecdotal evidence also supports the c&f model. Taylor and Allen (1992) survey market participants in the London foreign exchange market. They report that 90 per cent of those surveyed used some chartism on the short-term horizon (up to a week away) and about 60 per cent viewed chartism as "at least as important as fundamentals." When forecasting a year in advance, most respondents (85 per cent) thought fundamentals were more important than charts.

While the c&f model shows promise, direct empirical testing has been absent. The main difficulty has been that the relative importance of each technique varies over time and is unobservable, which makes it difficult to estimate this model with standard statistical methods. I attempt to overcome this difficulty by estimating a Markov regime-switching model[1] for the exchange rate. This switching model approximates the c&f model in two key respects. First, the switching model has two forecasting equations corresponding to the two elements in the c&f model. Second, it places a time-varying weight on each of these two forecasts, as does the c&f model.

Markov regime switching has been applied to exchange rates in Engel and Hamilton (1990), Kaminsky (1993) and Engel (1994). However, none of these researchers consider the possibility of chartists, and the models they estimate differ from ours.

The rest of the paper is organized along the following lines. Section 2 surveys the c&f literature. Section 3 rewrites the c&f model as a Markov regime-switching model. This section also details the forecasting models used in the empirical section. The fundamentalists base their forecasts on two models: purchasing power parity (PPP) and a terms-of-trade (TOT) model (Amano and van Norden 1995). Likewise, the chartists base their forecasts

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1. Hamilton (1994) has a chapter on the Markov regime-switching models. The statistical/signal processing literature has a similar model called the Hidden Markov Model (Elliot, Aggoun and Moore 1995).

on two different approaches. In one, they use recent lagged values of changes in the exchange rate. In the other, they use a pair of moving averages. Section 4 contains the empirical work. I estimate the model developed in Section 3 on the daily Canada-U.S. exchange rate from 1983 to 1992. Section 5 provides conclusions.

2.0 Literature Survey

The general c&f model was first suggested by Frankel and Froot (1988). They assume two types of agents forecasting asset prices: chartists and fundamentalists. Chartists extrapolate from trends, while fundamentalists forecast a return to the long-run equilibrium price. De Grauwe and Dewachter (1992, 1993) and De Grauwe (1994) extend the model and offer some simplifications. De Long et al. (1990a) explain why chartists (identified as "noise traders") are not driven out of the market by fundamentalists (identified as "sophisticated traders"). Youssefmir, Huberman and Hogg (1994) extend the model to continuous time and link the degree of chartism to the frequency of trading. Each of these ideas will be considered in turn.

2.1 The Frankel and Froot Model

Frankel and Froot start with a general model of the exchange rate.

$$y_t = cE\Delta y_{t+1} + H_t \tag{EQ 1}$$

where y_t is the log of the spot rate, Δy_t is the change in the exchange rate, $E\Delta y_{t+1}$ the expected rate of depreciation, and H_t other determinants. The expected rate of depreciation is a weighted average of the fundamentalist and the chartist expectations.

$$E\Delta y_{t+1} = \omega_t E\Delta y_{t+1}^f + (1 - \omega_t)E\Delta y_{t+1}^c \tag{EQ 2}$$

The fundamentalist forecast of Δy_{t+1} is $E\Delta y_{t+1}^f = \theta(\tilde{y} - y_t)$ where $\tilde{y}$ is the log of the fundamental value of the exchange rate. This fundamental value is the forecast of the exchange rate based on the economic model used by the fundamentalists. The coefficient θ is the rate at which the exchange rate returns to fundamentals. The chartist forecast is assumed to be a random walk $E\Delta y_{t+1}^c = 0$.[2]

The change in the weight placed on the fundamentalist forecast is determined by

$$\Delta\omega_t = \delta(\hat{\omega}_{t-1} - \omega_{t-1}) \tag{EQ 3}$$

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2. To make the model easier to manipulate, Frankel and Froot's forecasting equation is simpler than what chartists normally use. The assumption is relaxed in subsequent work.

where $\hat{\omega}_{t-1}$ is the ex-post computed weight that would have accurately predicted the change in the exchange rate in the previous period. This weight $\hat{\omega}_{t-1}$ is defined as

$$\hat{\omega}_{t-1} = \frac{\Delta y_t}{\theta(\tilde{y} - y_{t-1})} \qquad \text{(EQ 4)}$$

The value of δ is important for the system's stability. It determines the importance of the most recent information on updating the weights. If δ is large, then the system is stable. If it is small, then the system is unstable. Exactly how large δ needs to be for stability depends on c and θ.

By substituting the definition of $\hat{\omega}_{t-1}$ back into (EQ 3), the change in weight becomes

$$\Delta\omega_t = \frac{\delta\Delta y_t}{\theta(\tilde{y} - y_{t-1})} - \delta\omega_{t-1} \qquad \text{(EQ 5)}$$

Frankel and Froot impose a restriction that ω_t be bounded between one and zero. The expected change can at most adopt only one view exclusively.

2.2 De Grauwe and Dewachter

De Grauwe's model (1994) is similar to Frankel and Froot's, with two exceptions. First, the chartist equation has an autoregressive specification for the expectation of future exchange rate change. Frankel and Froot's random walk is nested in this model, since a random walk is an autoregressive equation with no lags. Second, the weight assigned to the fundamentalist element is $\omega_t = 1 - \dfrac{1}{1 + b \cdot (\tilde{y} - y_{t-1})^2}$, where $b > 0$. Hence the weight on the fundamentalist element is an increasing function of the deviation from economic fundamentals.

De Grauwe's rationale for this weighting function appeals to a model of heterogeneous beliefs. Suppose that among a group of fundamentalists, each makes a forecast of the exchange rate. Each forecast is the true fundamental value common to all fundamentalists plus a normally distributed mean-zero error term. When the actual value of the exchange rate equals the fundamental value, half the fundamentalists view the exchange rate as overvalued and half as undervalued. This difference in opinion makes fundamentalists' net demand zero, so they have no net effect on the market. When the exchange rate is far from the true fundamental value, the majority of fundamentalists can now agree that there is this large difference. This agreement results in the fundamentalists' working together to move the exchange rate back towards fundamentals.

2.3 Noise Traders

Many economists regard any discussion of chartists and fundamentalists with skepticism because of their belief in the efficient market hypothesis. This hypothesis implies that chartists, who do not concentrate on underlying economic fundamentals, will be eliminated from the market by fundamentalists who do concentrate (Taylor and Allen 1992, 305). De Long et al. (1990a) provide an alternative model where the fundamentalists do not drive the chartists out of the market.

De Long et al. present an overlapping generations model with two groups: "noise traders" (chartists) and "sophisticated traders" (fundamentalists). In each period the noise traders price the asset as its true fundamental value plus an error term. Such incorrect pricing generates additional risk in holding the asset. This risk limits the willingness of the sophisticated traders to go against the noise traders, even if these traders know the true value of the asset. Hence, arbitrageurs do not drive the asset's price toward fundamental values. In addition, the risk generated by the noise traders results in their earning higher average returns than those of the sophisticated traders. However, this is also accompanied by a higher variance in returns.[3]

With higher average returns than the fundamentalists, the chartists are difficult to drive out of the market. How difficult depends on how successive generations choose their investment strategy. If they base their choice on last period's relative return, then the noise traders are never eliminated. The noise traders are much easier to eliminate if the next generation considers both returns and volatility in returns. De Long et al. argue that basing a decision on returns is more believable, considering the behaviour of stock market investors.

Another paper by the same authors (De Long et al. 1990b) presents a model with positive feedback traders, those who buy when prices rise and sell when prices fall, and rational speculators. The rational speculators modify their investment strategies to anticipate the feedback traders' actions. After this change in strategies, the rational speculators, who were thought to have been a stabilizing influence, actually result in increased volatility in the market.

2.4 Youssefmir, Huberman and Hogg

Youssefmir, Huberman and Hogg (1994) present a continuous time version of the c&f model. This version differs from the earlier models in that each agent individually places a time-varying weight on the relative importance of the chartist and fundamentalist perspectives. The frequency of market participation affects their behaviour. The more often the

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3. Some recent empirical work calculates the relative returns of the two strategies. Pilbeam (1995) found that over his sample period both chartists and fundamentalists had similar average returns. Neither group was more volatile than the other. The result of being equal in returns and volatilities lies in between the opposing results of the noise trader model and the efficient market hypothesis.

agents transact in the market, the more chartist-like their behaviour, just as real-world survey results (Taylor and Allen 1992) would suggest. The difference between an asset's price and the agent's notion of its fundamental value also influences how fundamentalist the agent acts. The agents believe that the further the price diverges from the fundamental value, the more likely the price will revert back to fundamentals. In other words, agents become more fundamentalist as the deviations from fundamentals increase, which is very similar to De Grauwe (1994).

2.5 Summary

The surveyed models have several common features. Each model assumes there are chartists and fundamentalists. The chartists look at past trends, while the fundamentalists expect the mispriced asset to return to some exogenously determined value. Each group has a time-varying influence on the asset's price, which is explained differently in different papers. For Frankel and Froot, and De Long et al., past returns determine the dominant group. This can result in explosive currency movements. De Grauwe and Dewachter and Youssefmir et al. place less weight on the chartists as the size of the divergence from fundamental values grows. This implies an eventual return to the fundamental value.

3.0 Chartists and Fundamentalists as Regimes

There has been no direct empirical test of the c&f model, even partially, because components of the model are unobservable. I now show how a two-regime Markov switching model can capture c&f behaviour and how it leads naturally to some direct tests.

A Markov regime-switching model consists of two sets of equations. Level equations forecast the dependent variable conditional on a particular regime. The two-level equations represent the exchange rate forecasts made by the fundamentalists and the chartists. Transition equations give the probability of being in a regime in the current period given the previous period's regime. The probability of being in the fundamentalist regime approximates the weight placed on the fundamentalist forecast.

The Markov switching model differs from the c&f models found in Section 2. The Markov model assumes in each period that the market is *either* chartist *or* fundamentalist. The c&f model assumes that there is a mixture of the two groups in the market at any one time. In spite of this, I argue below that a Markov switching model is still a good approximation of a c&f model.

In Frankel and Froot's model, the chartists and fundamentalists do not participate directly in the market. Rather these two groups are limited to making forecasts while portfolio managers transact in the market. The managers base their decisions on a weighted average of the two forecasts. This weighting is based on the two groups' relative success at forecasting market developments.

The Markov switching model has a lot in common with the portfolio manager. The maximization of the log likelihood function (EQ 6) approximates what managers do. The log-likelihood function (LLF) is the following:

$$LLF = \sum_{t=1}^{T} \sum_{s_t} p(s_t) d(y_t | s_t) \qquad \text{(EQ 6)}$$

where $d(y_t | s_t)$ is the normal density function of the regime's residual and $p(s_t)$ is the probability of being in a particular regime. Similar to what a portfolio manager does, the two forecasts are weighted to maximize the fit to the data. So while the Markov switching model may not be the same as the c&f model, the approximation is close.

The forecasting equations for the fundamentalists and chartists are similar to those of Frankel and Froot. For estimation reasons, the model is expressed in differences rather than levels.

$$\Delta y_t = \theta(\tilde{y}_{t-1} - y_{t-1}) + \beta F_t + \varepsilon_t^f \qquad \varepsilon_t^f \sim N(0, \sigma_f^2)$$
$$\Delta y_t = \psi(y_t) + \Gamma C_t + \varepsilon_t^c \qquad \varepsilon_t^c \sim N(0, \sigma_c^2) \qquad \text{(EQ 7)}$$

where $\tilde{y}_t$ is the log of the fundamental exchange rate, F_t is a K x 1 vector of other exogenous variables used by fundamentalists to forecast exchange rate movements, and C_t is the chartists' corresponding vector. The chartists' equation nests the random-walk case in a more general specification, a function ψ of lagged values of y_t.

From (EQ 7), the density for the fundamentalists is

$$d(y_t | s_t) = \frac{1}{\sqrt{\pi}\sigma_f} \exp\left(\frac{(y_t - (\theta(\tilde{y}_{t-1} - y_{t-1}) + \beta F_t))^2}{\sigma_f^2} \right)$$

and for the chartists the density is $\quad d(y_t | s_t) = \dfrac{1}{\sqrt{\pi}\sigma_c} \exp\left(\dfrac{(y_t - (\psi(y_t) + \Gamma C_t))^2}{\sigma_c^2} \right)$

Different ways have been suggested to update the weight placed on each forecast in the c&f model. This updating corresponds with the Markov regime-switching model's transition equations. So, it will be useful to estimate some different transition equations. The simplest functional form will be a stationary Markov chain, where the probability of being in a regime given last period's regime is constant over time.

$$p(s_t = f | s_{t-1} = f) = \Phi(\alpha_f)$$
$$p(s_t = c | s_{t-1} = c) = \Phi(\alpha_c)$$

(EQ 8)

where Φ is the normal cumulative density function.

The specifications by De Grauwe and Youssefmir are modelled as a non-constant transition-probability Markov chain, where the deviation from the fundamentals determines the probability of transitions.

$$p(s_t = f | s_{t-1} = f) = \Phi(\alpha_f + \gamma_f | s_{t-1} - \tilde{s}_t |)$$
$$p(s_t = c | s_{t-1} = c) = \Phi(\alpha_c + \gamma_c | s_{t-1} - \tilde{s}_t |)$$

(EQ 9)

The transition equations calculate only the conditional probability of a regime given the last period's regime. However, the probability of a regime *unconditional* of last period's regime is what is really interesting. In this paper, smoothed probabilities are used to examine the probability of being in a certain regime. Smoothed probabilities use information on the level of the dependent variable in both the future and the past. They were calculated using Kim's algorithm found in Hamilton (1994, 695).

The transition probabilities also provide information about the long-run properties of the model if the transition probabilities are constant over time. The first step is to put the transition probabilities into the transition matrix K, which is equal to

$$K = \begin{bmatrix} p(s_t = f | s_{t-1} = f) & 1 - p(s_t = c | s_{t-1} = c) \\ 1 - p(s_t = f | s_{t-1} = f) & p(s_t = c | s_{t-1} = c) \end{bmatrix}$$

(EQ 10)

Given a vector ρ of the probability of being in the different regimes at time *t*-1, then the probability of being in the different regimes at time *t* is equal to $K\rho$. The stationary probabilities that give the long-run probability of being in the different regimes is equal to π, where $K\pi = \pi$. For a two-regime system, π is the following:

$$\pi = \frac{1}{2 - p(s_t = f | s_{t-1} = f) - p(s_t = c | s_{t-1} = c)} \begin{bmatrix} 1 - p(s_t = f | s_{t-1} = f) \\ 1 - p(s_t = c | s_{t-1} = c) \end{bmatrix}$$

(EQ 11)

3.1 Fundamentals

Before attempting to estimate a c&f model, one must specify a set of fundamentals. For ease of estimation, the model should be fairly simple. Two models of fundamentals will be used in this study: PPP and a TOT model.

These models are long-run models of the exchange rate. As such, they often fail to explain changes in daily data. For the purpose of this paper, this flaw may not be serious since the only requirement is a long-run equilibrium value for the exchange rate that represents the fundamentalists' beliefs. Furthermore if a regime-switching model is the correct specification, it is unnecessary for the model of fundamentals to forecast accurately. The influence of the other regime would cause persistent deviations.

3.1.1 Purchasing Power Parity

PPP arises from the law of one price. For any good i, the log of the domestic price $p_t(i)$ minus the log of the foreign price $p_t'(i)$ equals the log of the domestic currency price of foreign exchange s_t. For empirical work, this is often assumed to be true for comparison across country specific "baskets" such as the consumer price index (CPI). Froot and Rogoff (1994) provide a good review of the theory and history of the empirical work done on PPP.

Daily series for domestic and foreign price levels were constructed using the monthly CPI number for each day in the month.

3.1.2 Terms of Trade (TOT)

Amano and van Norden (1995) have estimated a TOT model for the real exchange rate between Canada and the United States, where the change in the real exchange rate y_t is a function of integrated long-run and stationary short-run explanatory variables. The price of exported energy and the price of exported non-energy commodities, each divided by the price of imported manufactured goods, are the long-run variables. The difference between Canadian and U.S. spreads between short-term and long-term interest rates is the short-run variable.

A cubic spline transformed the forecasted fundamental value of the nominal exchange rate into a daily series.

3.2 Chartist Function ψ

The expectations of the chartists are modelled in two ways. First, lagged values of the change of the exchange rate are used to predict future values. This approach has been used in several earlier papers (e.g. De Grauwe 1994; Cutler, Poterba and Summers 1990). Sec-

ond, a pair of moving-averages of the exchange rate is used. A short-term moving average used in conjunction with a long-term average is a common technical measure in asset markets. If the shorter-term average exceeds the longer-term average, then one buys the currency. The assumption is that if the exchange rate is increasing, it is expected to further increase. In this paper the time over which the moving averages are calculated is arbitrarily chosen as 14 and 200 days. Brock, Lakonishok and LeBaron (1992) used a moving-average rule in a study of the profitability of different technical trading rules on the Dow Jones, while Taylor (1992) did the same for currency futures.

For the sake of simplicity, the first model will be referred to as the autoregressive or AR model and the second as the moving-average or MA model.

4.0 Empirical Results

My sample covers the daily movements in the Canada-U.S. exchange rate from January 1983 to December 1992. Amano and Gable (1994) examined the statistical properties of the Canada-U.S. daily exchange rate over this period. They found that the daily exchange rate variations are leptokurotic (higher peak and fatter tails than in a normal distribution). This could be evidence of the exchange rate being a mixture of distributions, which supports the idea that a Markov regime-switching model, which is a special case of a mixture of distributions, would be a good characterization of the data.

The models in the following sections are estimated by a combination of an EM algorithm and maximum likelihood (Gauss's maxlik package.) The EM algorithm comes close to a maximum quickly, but it is slow to converge. Hence, the other method is used to achieve final convergence. The maxlik package also provides diagnostic statistics. Both the EM algorithm and the likelihood function are described in Hamilton (1994).

The likelihood function for regime-switching models is infamous for having a large number of local maximums (Hamilton 1994). Two methods were used to reduce the probability that the reported results are only a local maximum. The model of the TOT fundamentals and moving-average chartists was estimated 800 times with different starting values, using the EM algorithm. The reported estimate had the greatest log likelihood score. Simulated annealing (Geoffe, Ferrier and Rodgers 1992) was used to test the model of PPP fundamentals and AR chartists. Using a fast workstation (SPARC 10) for over 26 hours, the simulated annealing procedure was unable to find a point on the surface with a greater LLF value than the reported parameters. While neither of these tests is conclusive, the results are reassuring.

This section is divided into three subsections. The first two deal with the two different ways of modelling the chartists' expectations. The first specification uses a pair of moving averages of the exchange rate to forecast future changes. The second specification forecasts using lagged values of the change in the exchange rate. These chartist equations are

paired with the two models of fundamentals: PPP and a TOT model. The final section compares the results.

4.1 Chartists MA

I first estimate the regime-switching model where the chartists' equation has the 14-day and 200-day moving averages of the exchange rate (ma_{14} and ma_{200} respectively). Additionally, the chartists' forecasting equation uses the interest rate spread and a constant term. The model is estimated using the PPP fundamentals and then the TOT fundamentals.

$$\Delta y_t = f + \theta(\tilde{y}_{t-1} - y_{t-1}) + \beta i_{t-1} + \varepsilon_t^f \qquad \varepsilon_t^f \sim N(0, \sigma^f)$$

(EQ 12)

$$\Delta y_t = c + \psi_{14} ma_{14} + \psi_{200} ma_{200} + \Gamma i_{t-1} + \varepsilon_t^c \qquad \varepsilon_t^c \sim N(0, \sigma^c)$$

where $\tilde{y}_t$ is the exchange rate's fundamental value, f and c are constants, and i is the spread between Canadian and U.S. 30-day interest rates.

TABLE 1. Estimated Coefficients

$\tilde{y}_t$		f	θ	β	σ_f	α_f
PPP	coefficient	0.001	0.0051	-0.0004	0.0018	1.2598
	t-statistic	2.729	2.243	-0.933	26.341	10.037
TOT	coefficient	0.0001	0.0119	0.0002	0.0018	1.2656
	t-statistic	0.369	2.243	0.381	26.371	10.076

$\tilde{y}_t$		c	ψ_{14}	ψ_{200}	Γ	σ_c	α_c
PPP	coefficient	0.0002	0.0070	-0.0078	-0.0007	0.0007	1.6771
	t-statistic	1.541	2.361	-2.645	-4.030	33.538	17.680
TOT	coefficient	0.0002	0.0070	-0.0079	-0.0007	0.0007	1.6784
	t-statistic	1.573	2.381	-2.677	-4.000	33.634	17.704

Strong t-statistics are reported for virtually all the coefficients except for the interest rate term in the fundamentalist regime. With the TOT fundamentals, the reversion to fundamentals coefficient θ is twice as large as for PPP. The constant term f is also about a tenth as large. The moving-average coefficients are of the correct size and signs, as their real world use suggests. The chartist regime has a lower variance.

The coefficients α_f and α_c indicate the degree of persistence for each regime. The normal cumulative density function evaluated at the value of the coefficient gives the probability of continuing in the same regime in the next period. With this probability, it is seen that

the chartist regime has a higher persistence than the fundamentalist regime. The long-run stationary probabilities π, such that $K\pi = \pi$, are approximately $\pi \cong \begin{bmatrix} 0.31 & 0.69 \end{bmatrix}$. Hence, chartists dominate the market about twice as often as fundamentalists.

4.1.1 Specification Testing

White's information matrix test has already been applied to Markov regime-switching models (Hamilton 1996). In essence, one tests whether the derivative of the likelihood function with respect to a given parameter of the model is serially correlated. The results presented here are calculated with the analytic gradients developed by Gable, van Norden, and Vigfusson (1995).

TABLE 2. White's Misspecification Tests

	PPP Fundamentals		TOT Fundamentals	
	Test Statistic	**P value**	**Test Statistic**	**P value**
AR(1) Test for regime 1	0.3012	0.5832	0.4798	0.4885
AR(1) Test for regime 2	0.0018	0.9659	0.0007	0.9789
ARCH(1) Test for regime 1	0.4407	0.5068	0.4030	0.5256
ARCH(1) Test for regime 2	1.6348	0.2010	1.5745	0.2095
Test for higher-order Markov effects in regime 1	193.4723	5.55e-44	192.6158	8.53e-44
Test for higher-order Markov effects in regime 2	443.5130	1.86e-98	441.6598	4.71-98

The test statistics indicate that the model does not suffer from ARCH errors. This is remarkable, as several studies have found ARCH effects in high-frequency exchange-rate data.

The only misspecification appears to be a need to include higher-order Markov effects. Information on the regime at a time earlier than the last period is useful for predicting the current regime. Several possible solutions for this problem are discussed further in the appendix (Section 6).

The results appear to be independent of the model of fundamentals used. Hence the following results will focus on the model with the TOT fundamentals.

4.1.2 Likelihood-Ratio Tests

One must be careful when doing likelihood-ratio tests on the Markov regime-switching model. Standard likelihood-ratio tests are only valid when the assumption that there are two regimes is maintained. Testing the two-regime model versus a one-regime model is

unfortunately very difficult. If the one-regime model is taken as a restricted two-regime model, then the transition equations' parameters are undefined. Standard likelihood-ratio tests do not apply in this situation.[4] The model must be tested against simpler two-regime models.

TABLE 3. Likelihood Ratio Tests

	Mean LLF	Test Statistic	χ^2 Degrees of Freedom	P value
Full Model	5.45348		# of Restrictions	
Restriction				
$\sigma_f = \sigma_\iota$	5.3785	343.77	1	9.6515e-77
$\theta = 0$	5.4525	4.5401	1	0.033109
$\psi_{14} = -\psi_{200}$	5.4532	1.1924	1	0.27485
$\psi_{14} = 0 = \psi_{200}$	5.4509	11.832	2	0.0026961
$\theta = 0$ $\psi_{14} = 0 = \psi_{200}$	5.4510	11.419	3	0.0096625
$\beta = 0 = \Gamma$	5.4502	15.226	2	0.00049411
Mixture of Normals	5.4460	34.120	5	2.2537e-06

The likelihood-ratio tests support only one restriction, the restriction that the moving-average coefficients are of equal magnitude but opposite sign. Evidence of this restriction is compatible with the MA model's real world use. The statistic for θ is small, but the restriction that θ equals zero at the 5 per cent level is rejected. These tests do not support excluding any of the explanatory variables. This provides support for the c&f model. The strongest test statistic is found for the regime-varying variance.

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4. Hansen (1990) has developed a test; however, it is computationally intensive and requires a grid search over the parameter space. Hence, it was not done in this paper.

4.1.3 Graphical Examination

As seen in the lower half of Figure 1a, the probability of being in the fundamentalist regime has varied greatly over the sample period. Periods of stability in the exchange rate correspond with a high probability of being in the chartist regime. The movement between the two groups is interesting in that the weight gradually moves from one group to the other. Markov switching papers do not typically find intermediate steps between regimes. They tend to find abrupt changes, so that the regime probabilities are almost always very close to either zero or one.

Figure 1a

* Probability of Being in Fundamentalist Regime

14
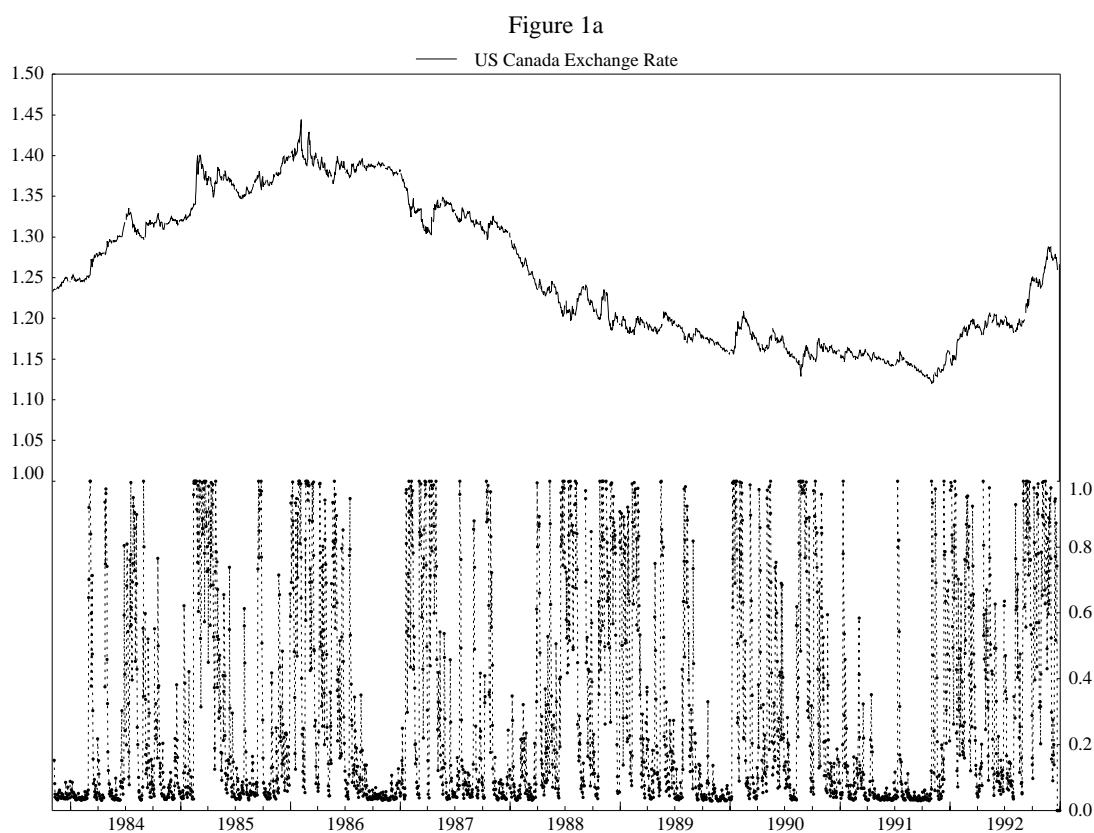
The distribution of the probability of being in the fundamentalist regime is shown in Figure 1b. Most observations have either a very low or a very high probability of being in a given regime. Relatively few observations are between these two extremes.

Figure 1b: Histogram of Smoothed Probabilities
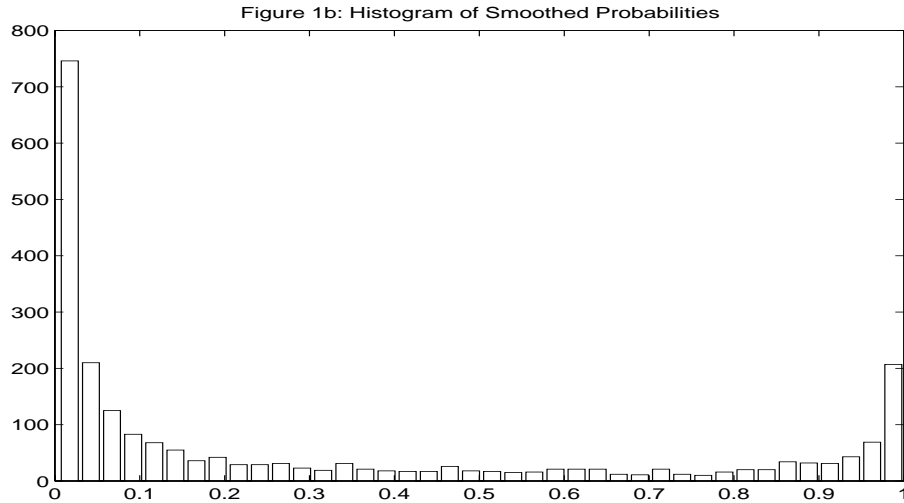
Figures 2a and 2b clarify the relationship between the fundamentalist regime and the change in the exchange rate. Figure 2a plots the actual change in the exchange rate against the probability of being in the fundamentalist regime. The sample is centered around zero with a few large outliers. A large change in the exchange rate is associated with a high probability of being in the fundamentalist regime. In Figure 2b, exchange rate changes were sorted by size and placed in bins of uniform size (25 observations each). For each bin, the average probability of being in the fundamentalist regime was calculated. The results are plotted with the bins ranked in ascending order of size of change. The fundamentalists dominate only in the tails of the distribution. Elsewhere, the chartists are more important, although they never seem to dominate as completely as the fundamentalists do.

Figure 2a: dy vs p smooth

Figure 2b : p smooth bins
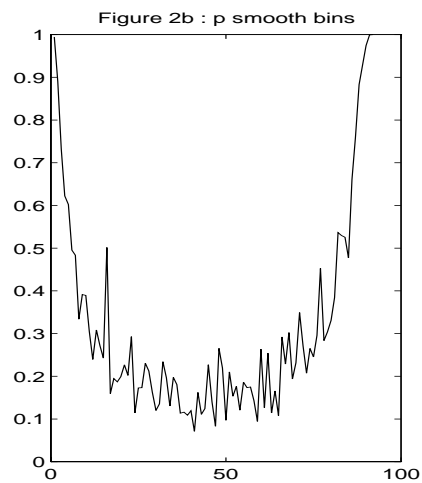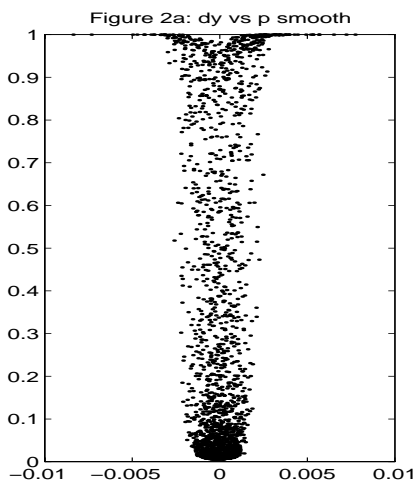
4.2 Chartists: AR

For the chartist forecasting equation, the moving-average terms are replaced by lagged values of the change in the exchange rate (four lags).[5] Also, the interest-rate spread is dropped from the chartist regime.

$$\Delta s_t = f + \theta(\tilde{s}_{t-1} - s_{t-1}) + \beta i_{t-1} + \varepsilon_t^f \qquad \varepsilon_t^f \sim N(0, \sigma^f)$$

$$\Delta s_t = c + A(L)\Delta s_t + \varepsilon_t^c \qquad \varepsilon_t^c \sim N(0, \sigma^c)$$

(EQ 13)

TABLE 4. Estimated Coefficients

$\tilde{s}_t$		f	θ	β	σ_f	α_f
PPP	coefficient	.0001	0.0002	-0.0006	0.0007	1.7499
	t-statistic	0.726	0.283	-4.609	34.27	18.51
TOT	coefficient	.0001	-0.0008	-0.0006	0.0007	1.7533
	t-statistic	1.490	-0.489	-4.529	34.41	18.51

$\tilde{s}_t$		c	L(1)	L(2)	L(3)	L(4)	σ_c	α_c
PPP	coefficient	0.0002	0.0377	-0.0153	-0.0283	0.0242	0.0017	1.4101
	t-statistic	2.445	1.041	-0.409	-0.738	0.62	28.13	11.53
TOT	coefficient	0.0002	0.0374	-0.0155	-0.0284	0.0238	0.0017	1.4150
	t-statistic	2.441	1.035	-0.415	-0.743	0.61	28.26	11.59

When PPP forms the fundamental variables, θ is the correct sign, but statistically insignificant, which is consistent with the difficulty in finding evidence for PPP in short samples. β is statistically significant. The chartist regime's autoregressive coefficients are all statistically insignificant. A likelihood-ratio test presented later finds that they are also jointly insignificant. Only the intercept and the variance are significant in the chartist regime. The variance in the fundamentalist regime is less than half of the variance in the chartist regime. Re-estimation using TOT fundamentals provides similar results.

Using the Markov chain analysis, it is found that the fundamentalist regime is in effect twice as often as the chartist regime, the opposite of what occurred for the MA models (Section 3.1).

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5. The number of lags was chosen arbitrarily. Because of the low persistence found in exchange rates, it is doubtful that more lags would be necessary. The test for no lags is reported in the next section.

As the last section emphasized use of the TOT variables for fundamentals, PPP will be used as the model of fundamentals for subsequent analysis.

4.2.1 Specification Testing: Likelihood Ratio Tests

Likelihood-ratio tests do not support the AR version of the c&f model. Restrictions on the explanatory variables, except for the interest rate spread, are not rejected. Restricting the autoregressive terms to be equal to zero would result in the random-walk forecast in the original Frankel and Froot model. The regime-varying variances are found to be very important. Restricting the variances to be equal across regimes lowers the likelihood-function value more than any other restriction.

TABLE 5. Likelihood Ratio Tests

	Mean LLF	Test Statistic	χ^2 Degrees of Freedom	P value
Full Model	5.49600		# of Restrictions	
Restriction				
$\sigma_f = \sigma_c$	5.41264	416.63	1	1.3190e-92
$\beta = 0$	5.49190	20.49	1	5.9887e-06
$\theta = 0$,	5.49598	0.0999	1	0.75187
No Lags in Chartist Regime	5.49557	2.1491	4	0.7083
$\theta = 0$ A(L)=0	5.49556	2.1991	5	0.8209
$\theta = 0$ A(L)=0 $\beta = 0$	5.49054	27.2890	6	0.00012
$\sigma_f = \sigma_c$ $\theta = 0$ A(L)=0 $\beta = 0$	5.40779	440.8735	7	4.046e-91

4.2.2 Information Matrix Test

Specification testing gives similar results to the chartist MA models. As in the MA models, strong evidence is found for missing higher-order Markov effects. Now there is also evidence of ARCH in the fundamentalist regime.

TABLE 6. White's Misspecification Tests

	Unrestricted Model (Uses PPP as fundamentals.)		Restricted Model (Only has constants and interest rate spread.)	
	Test Statistic	P value	Test Statistic	P value
AR(1) Test for regime 1	0.0051	0.9427	0.0005	0.9811
AR(1) Test for regime 2	2.2180	0.1364	0.6024	0.4376
ARCH(1) Test for regime 1	5.2262	0.0222	5.1606	0.0231
ARCH(1) Test for regime 2	0.3652	0.5456	0.5037	0.4779
Test for higher-order Markov effects in regime 1	412.2288	1.19e-91	412.3299	1.14e-91
Test for higher-order Markov effects in regime 2	178.6202	9.69e-41	178.5114	1.02e-40

Based upon these tests, the restricted model that has neither AR terms nor a PPP term performs as well as the unrestricted model. The variances being different in the two regimes dominates all other explanatory variables.

### 4.2.3 Time-Varying Transition Probabilities

I also estimated a model of the PPP fundamentalist equation and the AR chartist equation with time-varying transition probabilities. The transition probabilities are dependent on the distance between the actual and fundamental value as in (EQ 9). However, the results are very similar to the constant transition probability case. The estimated coefficients are nearly identical to those in the constant transition probabilities model and the likelihood-ratio test does not reject the restriction of constant transition probabilities.

I tried to estimate a time-varying transition probabilities model with the TOT fundamentalist equation and the moving-average chartist equation. However, I was unable to find a point of convergence with a log likelihood greater than the restricted model and hence, certainly have not yet found a global maximum. Without the global maximum, I can not make any inferences about this time-varying transition probabilities model.

4.3 Comparing the Results

In the MA chartist model, the chartist regime has a lower variance and is more persistent than the fundamentalist regime. In the autoregressive chartist model, the opposite is true. A case could be made for associating chartists with either the high- or low-variance regimes. Both the De Grauwe model and the Youssefmir, Huberman and Hogg model would have the chartists dominating periods when the exchange rate is close to its fundamental value. This would suggest that small changes in the exchange rate (low variances) would typically reflect chartists' behaviour, while large changes (high variance) would indicate that the market was under the sway of fundamentalists. On the other hand, the anecdotal literature ascribes more volatile markets to the chartists' influence. Hence, the chartists would be associated with a high-variance regime.

An alternative interpretation of my results would be that the difference between the two sections is only one of labelling. From Figure 3, it is apparent that the smoothed probabilities of being in the fundamentalist regime under the MA model are approximately one minus the same probabilities of the autoregressive model. In other words, the probability of being in the low-variance regime in the first instance is almost always equal to that of being in the low-variance regime in the second instance. These probabilities are equal even though the low-variance regime is the chartist regime in the first case and the fundamentalist regime in the second. This suggests that the labels "chartist" and "fundamentalist" may be inappropriate. It might be more accurate to describe the results as switching between periods of high and low variances.

Figure 3: Probability of Being In Fundamentalist Regime: Section 4.2 vs 4.3
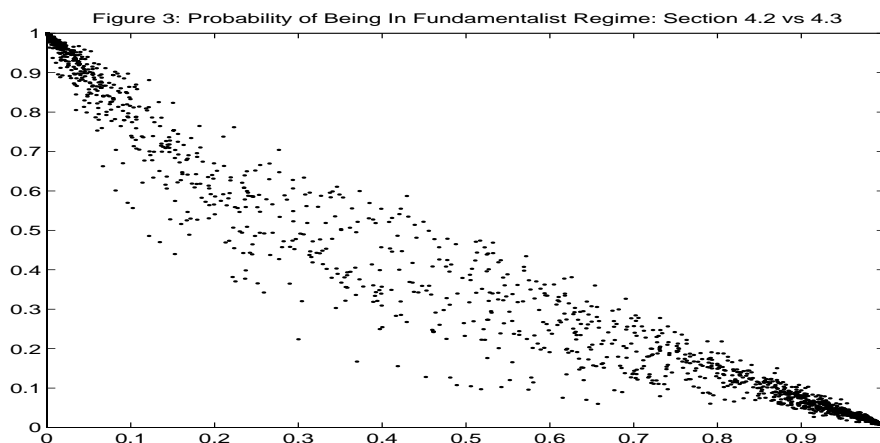
5.0 Conclusion

The c&f model has shown promise in explaining exchange rate models, but it was difficult to test empirically. This paper makes a contribution by approximating the c&f model as a Markov regime-switching model and then estimating this switching model. The results from the Canada-U.S. daily exchange rate can provide evidence for the c&f model.

There are four main findings of the empirical work. First, the two models of fundamentals used give similar results. Second, the MA chartist models appear to do much better than the AR chartist models. Third, the MA model gives some support for the c&f model. Fourth, the chartist regime, when estimated with the MA model, is more common and has a lower variance than the fundamentalist regime.

Having different variances in the two regimes appears more important than any chartist and fundamentalist variables. This is a serious problem, since classification may be driven by high- and low-variance regimes, rather than by c&f elements. To avoid this problem, one could reestimate using a two-regime Markov-switching model where the two regimes share a common ARCH effect. Ideally, this would allow one to rule out variance-induced switching and isolate the c&f influences on the exchange rate.

My results may also have implications for time-series models of the exchange rate. The large number of switches between the two regimes suggests that there is something different from structural breaks or "long swings" in the dollar. The finding that the regime-switching model eliminates ARCH effects also suggests the need for further research. One could move away from testing the c&f model and instead focus on the ability of Markov regime-switching to eliminate ARCH effects. If one were to do this, one would also want to model the higher-order Markov effects found in this study.

In conclusion, based upon these results, the c&f model deserves further research. Research could now either look at new variables for the c&f forecasts or use new techniques to reexamine the forecasting equations used here.

6.0 Appendix I: Persistence in Regime-Switching Models

In modelling the exchange rate, there appeared to be higher-order Markov effects that were not taken into account by the regime-switching model. There are a few possible solutions to this problem. The first is to rewrite the system as a Markov chain with more regimes. For example, a two-regime model that depends on the last two periods' regimes can be rewritten as a four-regime model. Of course, estimation becomes more difficult and the number of parameters in the model increases rapidly. A parsimonious alternative would be to consider Durland and McCurdy's (1994) duration-dependent model. In this model, the transition probabilities are dependent on the time spent in a given regime D_t.

$$p(s_t = i | s_{t-1} = i, D_{t-1} = d) = \begin{array}{ll} \dfrac{exp(a_i + b_i d)}{1 + exp(a_i + b_i d)} & d \leq \lambda \\[3mm] \dfrac{exp(a_i + b_i \lambda)}{1 + exp(a_i + b_i \lambda)} & d > \lambda \end{array} \qquad \text{(EQ 14)}$$

A more general extension of the above would be a semi-Markov process. It is more persistent than a Markov chain. Upon entering a given regime i, one remains there for an integer valued random variable number μ_i of periods. Afterwards, the process switches to another regime. The transition probabilities p_{ij} determine the probability of going to the other regimes.

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