# The Structure and Dynamics of RDXF

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## ABSTRACT

Technical Reports 25 and 26 deal with the September 1980 version of the RDXF model, which is used to generate short-term projections of the Canadian economy. The model was developed by the analytic departments of the Bank of Canada and is based on the foundations of the Bank's RDX2 model.

Technical Report 25 contains a full documentation of the equations and exogenous variables in RDXF. Technical Report 26 presents an analysis of the equations and their dynamic properties, both at a sectoral level and in a full model context. There is some emphasis on issues related to the supply side of the model.

## RÉSUMÉ

Le rapport technique n° 25, comme le n° 26 qui l'accompagne, a pour objet de présenter le modèle RDXF dans la version qui a été mise au point en septembre 1980. RDXF permet de faire des projections de courte durée sur l'économie canadienne. Construit aux départements d'analyse de la Banque du Canada, il est dérivé du modèle RDX2.

Le rapport n° 25 contient une description complète des équations et variables exogènes de RDXF. Le rapport technique n° 26, de son côté, analyse ces équations et leurs propriétés dynamiques, tant du point de vue de l'ensemble du modèle que de chacun de ses différents secteurs. Il décrit également quelques aspects de RDXF se rapportant à la structure de l'offre.

## INTRODUCTION

Jean-Pierre Aubry

RDXF est un modèle économétrique qui se situe bien dans la lignée des modèles RDX développés à la Banque du Canada, non seulement à cause de ses dimensions mais à cause du cadre théorique qui lui est sous-jacent. Si les premières versions de RDXF ont été développées et utilisées pour faire des prévisions sur un horizon d'environ deux ans, l'utilisation récente du modèle pour générer différents scénarios pouvant cerner plus adéquatement l'effet de différentes hypothèses de prévision nous a obligés à reserrer les contraintes théoriques et à revenir davantage à l'approche des constructeurs de RDX2. Il ne faut donc pas s'étonner de la grande ressemblance des deux modèles et de l'accent théorique du texte qui suit.

Ce rapport technique est une présentation du cadre théorique utilisé pour développer les principaux secteurs du modèle et une analyse de la dynamique de ces secteurs. Il faut également noter le caractère critique de cette analyse, comme ce fut le cas pour le Rapport technique 6. Nous sommes bien conscients des limites de notre modèle et voulons les cerner davantage, tout en espérant susciter de nombreux commentaires afin d'améliorer le modèle. Comparativement au Rapport technique 5, nous avons poussé davantage notre étude sur l'interaction entre les différents sous-groupes de secteurs pour mieux comprendre les implications macro-économiques de la spécification des différents secteurs du modèle. Nous nous permettons d'attirer ici l'attention du lecteur particulièrement sur l'analyse de l'"offre" dans le modèle.

### RDXF ET LES IMPERATIFS DES PROJECTIONS TRIMESTRIELLES*

Lorsqu'on a pensé à utiliser RDX2 pour générer différentes projections trimestrielles, on s'est vite rendu compte de

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* Voir le rapport technique 24.

l'existence de plusieurs obstacles de taille. Le plus évident était certes celui de la lenteur et du coût de la mise à jour de la base de données. Il a donc fallu réduire la base de données à des séries chronologiques rapidement disponibles, ce qui a eu pour effet de limiter, plus que dans le cas de RDX2, le choix des variables explicatives dictées par la théorie lors de la spécification des différents secteurs. Il a également fallu fortement automatiser la production des bandes de données. Dans cette optique visant à simplifier et à accélérer la remise à jour du modèle, on a même tenté d'obtenir un modèle ayant la moitié moins de variables endogènes que RDX2 (325).

Avant l'utilisation de RDXF, l'exercice de prévision était assumé à la Banque par plusieurs groupes de spécialistes suivant au jour le jour l'évolution de l'activité des différents secteurs de l'économie. Si l'utilisation d'un modèle économétrique pouvait permettre d'accélérer l'exercice de prévision et surtout d'obtenir une plus grande cohérence dans les diverses composantes d'une prévision, la capacité limitée des modèles à expliquer l'évolution de la conjoncture, à court terme notamment, nous a poussés à amalgamer la dynamique du modèle au jugement des spécialistes. Cette tâche nous a obligés à élargir considérablement le groupe responsable du développement du modèle. Il a donc fallu rendre beaucoup plus accessible la technologie qui était utilisée pour estimer et simuler le modèle RDX2, d'où d'importants développements du programme d'informatique TSP. On a également intégré les travaux déjà terminés par différents groupes de spécialistes, d'où l'introduction de fréquences multiples (trimestrielle et mensuelle). Mais cette intégration nous a malheureusement éloignés de notre objectif quant à la taille du modèle.

**BREVE DESCRIPTION DE RDXF**

RDXF est un modèle économétrique d'environ 400 équations décrivant les principaux secteurs de l'économie canadienne. La fréquence de la plupart des équations est trimestrielle; les autres équations, principalement groupées dans les secteurs énergétiques et financiers, utilisent la fréquence mensuelle. La période d'estimation diffère d'une équation à l'autre; la grande majorité des équations ont été estimées sur la période 1961-1978.

RDXF, tout comme RDX2, se situe dans la lignée des modèles keynésiens. Les contraintes d'offre étant relativement faibles, un accroissement de la demande est accommodé en général par un accroissement de la production. Cette caractéristique est renforcée par la linéarité de la réponse de RDXF à un choc de la demande. Un des mécanismes importants pour soulager les pressions de la demande sur la production intérieure est la forte propension marginale à importer, qui exerce également des pressions importantes sur les prix intérieurs par le biais des pressions sur le taux de change.

Les dépenses de consommation sont principalement déterminées par différentes structures de délais affectant le revenu réel disponible et par des variations des prix relatifs. Si l'on a abandonné les concepts de revenu permanent salarial et non-salarial utilisés dans RDX2, on a inclus un mécanisme qui diminue la variable de revenu introduite dans les fonctions de consommation dans le cas d'un accroissement du taux d'inflation. Il est important de noter que la propension marginale à consommer est beaucoup plus élevée dans RDXF que dans RDX2.

Les mises en chantier ne sont plus déterminées, comme dans RDX2, par la disponibilité des prêts hypothécaires; l'activité dans le secteur de la construction est déterminée directement par les pressions de l'offre, sur la base du concept de rentabilité. L'équation de prix est spécifiée en termes d'une demande excédentaire.

La demande des facteurs de production découle des conditions de maximisation des profits dans le cadre de l'utilisation d'une fonction de production Cobb-Douglas ayant des rendements constants à l'échelle. N'ayant pas imposé, comme dans RDX2, les conditions d'homogénéité pour les facteurs de production, la productivité à court terme doit combler les écarts entre l'offre et la demande, et ce, pour des périodes pouvant être très longues. Les dépenses en investissements non énergétiques sont déterminées principalement par l'écart entre la demande attendue et le stock actuel de capital (accélérateur). L'emploi est à la fois déterminé par les impératifs de la fonction de production comme un facteur résiduel pour assurer la production demandée et par la valeur réelle de la production marginale du travail.

Les importations et les exportations en termes réels sont toutes les deux déterminées par des fonctions de demande où les principales variables explicatives sont des variables de dépenses, de prix relatifs et de déséquilibre entre l'offre et la demande intérieure. Le secteur gouvernemental se caractérise surtout par des dépenses en biens et services exogènes en termes réels, par la modélisation de l'indexation de l'impôt personnel et par une attention toute spéciale apportée à la spécification des différentes taxes imposées au secteur énergétique.

Les prix intérieurs sont déterminés par les prix à l'étranger, par les prix de l'énergie (en dollars canadiens) et par les coûts unitaires en main d'oeuvre et en capital. La plupart des équations sont tout près des conditions d'homogénéité entre les prix et la somme des coûts. Le déséquilibre entre l'offre et la demande sur le marché des biens entre également dans plusieurs équations de prix, mais son effet est faible. Les prix à l'importation sont déterminés principalement par les prix à l'étranger et il en est de même pour les prix à l'exportation, les coûts canadiens ayant relativement peu d'effets sur eux. On s'est donc considérablement éloigné de la spécification de RDX2, où les coûts canadiens étaient la variable dominante dans les équations des prix à l'exportation. Les salaires ne sont plus déterminés par un ajustement vers un salaire réel désiré comme dans RDX2, mais par la formulation d'une courbe de Phillips ajustée pour tenir compte des attentes inflationnistes. La variable de déséquilibre qui entre dans cette spécification est la différence entre le taux de chômage et un taux de chômage ajusté pour tenir compte des variations cycliques et des déplacements structuraux. Le taux de change est déterminé à long terme par la parité des pouvoirs d'achat et, à court terme, par la différence entre les taux d'intérêt canadiens et américains et par les variations de différentes composantes de la balance des paiements.

Pour sa part, le secteur financier se réduit essentiellement à une demande de monnaie, qui, une fois inversée, sert à déterminer les taux d'intérêt à court terme, ce, compte tenu d'une cible monétaire déterminée de façon exogène. Les taux d'intérêt à long terme qui entrent dans la spécification des équations des autres secteurs du modèle sont déterminés par les taux à court terme et par les attentes inflationnistes. Disons finalement que, dans l'ensemble du modèle, les attentes sont définies par une somme pondérée des valeurs passées de la variable attendue.

**PROJET DE DEVELOPPEMENT**

Après l'analyse du texte, le lecteur verra clairement quels sont les secteurs du modèle que nous désirons tout particulièrement améliorer. En tête de liste, il y a certes les canaux par lesquels la politique monétaire influence la formation des prix. Comme nous refusons d'imposer un lien direct entre les prix et le stock de monnaie, il nous faut améliorer les liens par lesquels les taux d'intérêt nominaux et surtout les taux réels influencent la demande et l'offre réels. A ce chapitre, il faudra développer des concepts de richesse et évaluer leur impact sur le secteur réel; cet effort impliquera beaucoup de travail au niveau de la formation des attentes. Il faudra améliorer les liens entre la formation des prix et des salaires et les variables de déséquilibre entre l'offre et la demande, et ce, tant au niveau du marché des biens que du marché de la main-d'oeuvre.

Nous croyons fermement que la pierre angulaire de cette recherche est une spécification plus rigoureuse de l'offre dans le modèle. Actuellement, nous avons plusieurs spécialistes engagés dans ce projet qui consiste à reformuler l'offre dans RDXF. Nous travaillons sur l'estimation de fonctions de production qui incluent non seulement le capital et le travail mais également les matières premières et l'énergie. Ce travail nous obligera à estimer des fonctions de production pour l'énergie et pour les matières premières. De plus, il en découlera des variables nouvelles de déséquilibre entre l'offre et la demande et des fonctions de prix incluant de façon plus systématique le coût unitaire de tous les facteurs de production. Il faudra finalement inclure dans les prochaines versions de RDXF beaucoup plus de non-linéarité afin que les contraintes d'offre se répercutent davantage sur la formation des prix.

Ce développement devrait accroître de beaucoup la taille du modèle et afin d'éviter cette conséquence nous devrons nous orienter vers un modèle de base de plus faible dimension et ayant des secteurs (satellites) plus ou moins désaggrégés qui pourront, au gré de l'utilisateur, être joints au modèle de base. Nous devrons donc construire pour la spécification du modèle de base des équations représentant la forme réduite de différents secteurs du modèle (i.e. les secteurs gouvernemental, énergétique ou financier).

Depuis la publication de la dernière version de RDX2, nous avons fait des progrès sur plusieurs plans. A cet égard, il faut d'abord mentionner le développement du programme TSP qui facilite de beaucoup l'utilisation de RDXF. Nous avons développé une méthodologie [R.T. 24] pour faire rapidement différentes projections avec le modèle tout en y introduisant l'expertise d'un groupe important de spécialistes sur l'évolution à court terme de l'économie. Au niveau de la dynamique du modèle, nous avons accru, par rapport à RDX2, la réponse du modèle aux chocs monétaires, mais il y a encore beaucoup de chemin à parcourir à ce niveau et au niveau du traitement de l'offre et du déséquilibre entre l'offre et la demande. Dans ce contexte, l'étude qui suit reflète bien la dynamique du modèle, les problèmes qui en découlent et les solutions possibles.

# INTRODUCTION

Jean-Pierre Aubry

The econometric model RDXF, documented in Technical Report 25, is a model of the Canadian economy and a successor to the RDX models developed in the Bank of Canada during the late 1960s and the 1970s. RDXF was developed to forecast economic developments over a two-year period and to generate alternative scenarios over a medium-term forecast horizon to aid in evaluating different forecasting hypotheses. Thus, in developing the model, consideration was given both to its forecasting properties and to its longer run dynamics.

This technical report presents the theoretical framework and dynamic properties of the main sectors in RDXF. In the report the theory and dynamics are presented in tandem, sector by sector. As well, some sectors are grouped into submodels so as to pinpoint intersectoral linkages and thus aid in assessing dynamic problems, especially in the supply side and in the wage-price sectors. Finally, the full model properties are analyzed in a series of simulation experiments. The limitations of the model have been pointed out to give direction to future model development and research.

## HISTORICAL ANTECEDENTS

The RDXF model evolved through time from the efforts of many people. Since it was to be used for forecasting, it had to be more concise and easily manipulated than RDX2 but yet retain the many desirable characteristics of the previous Bank of Canada models. With these stipulations in mind, Bill Alexander and Tom Maxwell formulated the theoretical basis for the development and use of a forecasting model at the Bank of Canada. Lloyd Kenward developed and tested a forecasting version of RDX2 during 1976 and 1977. Leo deBever then supervised a small group of economists in the conversion of the model from seasonally unadjusted to seasonally adjusted data, tested its dynamics, and developed the TSP software system required to produce the model forecast. By March of 1978 the model was sufficiently developed to begin a model-based forecasting exercise and to allow input by the sectoral specialists at the Bank.

Since 1978 the development of individual sectors of the model has been the primary responsibility of the sectoral specialists, whereas the overall dynamics and model development have been overseen by a model coordination group. Specialists have devoted a considerable amount of time to incorporating into RDXF aspects of individual forecasting models that were in use prior to the model-based forecasting exercise.

## SUMMARY DESCRIPTION OF THE MODEL

RDXF is an econometric model of approximately 400 equations which describe the main sectors of the Canadian economy. Most of the equations use quarterly data; those remaining, mainly concentrated in the energy and financial sectors, use monthly data. The estimation period is different from one equation to another, but the majority of equations were estimated over the 1961-78 period.

As with most large econometric models, RDXF is in the Keynesian tradition with considerable emphasis on demand-side responses. Since supply constraints are relatively weak, an increase in demand is generally accommodated in the model by an increase in production. This characteristic is reinforced by the linearity of the response to a demand shock. A major mechanism to relieve the pressure of demand on domestic production, however, is the high marginal propensity to import, which in turn puts substantial pressure on domestic prices through the exchange rate.

Consumption expenditures are determined by a lagged function of real disposable income, relative prices, and financial variables. To account for the positive bias on income during inflationary periods, the income term in the consumption function is defined to exclude the inflation premium. The marginal propensity to consume is close to unity. Housing is determined over the short term by supply conditions, based on profit maximization. Over the longer term demand conditions influence investment through variations in housing prices.

The demand for factors of production is derived from the conditions of profit maximization in the context of a Cobb-Douglas production function with constant returns to scale. Over the longer run, factor demands do not conform to the underlying production technology, and therefore short-run productivity fills the gap between demand and supply over long periods of time.

Import volumes are a function of domestic activity and relative prices, and a term that measures the degree of slack in the product market. Export volumes are functions of U.S. and overseas activity, and of the export price relative to foreign prices that proxy demand conditions and supply variables represented by domestic price terms. Government expenditures are exogenous in real terms. There is a considerable amount of institutional detail in the government revenue sectors. Government financing requirements are met through cash and treasury bills.

The level of interest rates, the monetary aggregates and the assets and liabilities of various financial intermediaries are determined in a monthly submodel of the financial sector. Long-term interest rates that enter the specification of other sectors of the model are determined by a term structure of short rates. The nominal long rate, less inflationary expectations, defines the real supply price of capital. Finally, in the model as a whole, expectations are defined as a weighted summation of historical values of the expected variable.

**PROJECTED DEVELOPMENT**

The preparation of this report has highlighted development needs in two major areas. First are the linkages through which monetary policy influences price formation. Since we have not established any direct relationship between prices and the stock of money, we should improve the linkages through which nominal and, especially, real interest rates influence real demand and supply. In this connection, we plan to develop wealth concepts and evaluate their impact on the real sector; this implies considerable effort in modeling expectations formations. Furthermore, the links between price formation and wages and between the variables of supply and demand disequilibrium have to be reinforced in both the goods and the labour markets.

The second, and more important, development need is a stricter specification of the supply side of the model. Several staff members are currently working on this project, which involves estimating a production function that will include not only capital and labour but also primary resources and energy. In addition, price functions will include the unit cost of all the factors of production in a more systematic manner. The resulting system of equations will account for disequilibrium between supply and demand. Finally, subsequent versions of RDXF will be more non-linear so that supply constraints may better reflect price formation.

These developments are expected to increase the size of the model substantially. To avoid such a result we will try to construct a simpler model with sectors that are more or less self-contained, yet able to be incorporated into the basic model as a user requires. We will therefore have to build into the specification of the basic model some equations representing the reduced form of individual sectors of the model (e.g., the government, energy or financial sectors).

The following chapters describe RDXF as developed up to September, 1980. Strengths and limitations of various parts of the model are pointed out as the sectors are presented.

# Chapter 1

## CONSUMPTION AND SAVINGS
(Sector 1)

Heather Robertson

The consumption sector of the RDXF model will be discussed from two points of view: the theory and dynamics underlying the consumption functions, and the savings rate that is determined residually from these functions. The first section describes the theoretical framework employed in the consumption functions, emphasizing the constraints on own- and cross-price elasticities, the exclusion of the "inflation premium" from the definition of income, problems associated with wealth evaluation, and real-financial linkages. Next is a description of the dynamics of this sector. The third section focuses on the historical pattern of the savings rate, relating in some detail how the savings rate implied by the estimated consumption functions in the RDXF model explains the positive trend and the cyclical swings in the actual savings rate. In particular, the contribution of the inflation premium, income, interest rates, and relative prices to the variation in the savings rate will be discussed. The next section presents the response of the consumption and the savings rate to an increase of 100 basis points in interest rates when interest income and real effective purchasing power are allowed to change. Finally, possibilities for improvement are pointed out.

### 1 THEORETICAL FRAMEWORK OF CONSUMPTION*

Consumer expenditure is disaggregated into nine components: household durables (CHSHD), motor vehicles (CMV), miscellaneous

* Recent developmental work in the consumption sector was done by Bruce Rayfuse. Various developments were described in Gyori [34], O'Reilly and Rayfuse [65] and Rayfuse [69].

durables (CDMIS), semi-durables (CSD), food (CFOOD), energy (CENERG), other non-durables (CNDO), services excluding rent (CS), and rent (CRENT). The consumption function for each component is of the following general specification:

$$\log(C_i/N) = \alpha_{i0} + \alpha_{i1}\log(U/N) + \alpha_{i2}\log(P_i/P) + \alpha_{i3}R, \qquad (1)$$

where $\alpha_{i0}$-$\alpha_{i3}$ represent dynamic elasticities, $\Sigma C_i = C$.

Real consumption per capita ($C_i/N$) is a function mainly of a current income term, real effective purchasing power per capita (U/N), a relative price term ($P_i/P$), and interest rates (R). Most of the consumption equations have lagged dependent variables; exceptions are the CS and CNDO equations, which have distributed lags on income. In the relative price and income terms the consumer price index (CPI) is used rather than the consumption deflator because it is assumed to be the variable perceived by workers as relevant for wage bargaining and for price comparisons.

The consumption equations have not been derived explicitly from a theory of consumer behaviour. The inclusion of the current income term, plus the lagged dependent variable in each consumption equation, could describe either a permanent income or a habit persistence hypothesis.* And, conforming with either hypothesis, the income elasticity for total consumption is close to unity. However, because coefficients on the lagged dependent variables are different in each consumption category, a different measure of permanent income is implied within each equation, and income elasticities vary widely among categories.

**Current Income Term**

The income term in the consumption equations, real effective purchasing power (U/N), is defined as real personal disposable income excluding the inflation premium and current transfers to

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* See Evans [27], p. 24, for a proof of this.

corporations.* The inflation premium is the portion of current interest receipts that individuals save in order to maintain their perceived real stock of wealth, which is eroded during inflationary periods. If total interest income is defined as:

$$INT = rW_M \qquad (2)$$

$$= (i+P^e)W_M,$$

where r is the nominal rate of interest, i is the real rate of interest, $P^e$ is the expected rate of inflation, and $W_M$ is the market value of nominal stock of personal financial wealth, the actual amount that must be saved in order to maintain the existing stock of wealth is $P^eW_M$. The portion that consumers perceive as the requisite amount is YINFL = $\alpha P^eW_M$, $0<\alpha>1$, where $\alpha$ is the recognition coefficient; YINFL, rather than $P^eW_M$, is excluded from personal disposable income. Current transfers to corporations (YTCORP) are excluded from income because they are not a discretionary component of income; rather they are payments for goods purchased in previous periods.

The inflation premium in RDXF is defined as:

$$YINFL = P^eW_M = .5PCPICE/100[(LGFCSB+LGFTB+LGBF)\beta_1$$

$$+ (LPPMB)\beta_2 + (ABBCD+LGBCN)_{-1}]$$

$$\beta_1 = c/r + (1-c/r)\ 1/(1+r)^5;\ \beta_2 = c/r$$

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* In previous versions of RDXF, where real personal disposable income was the income argument in the consumption equations, the marginal propensity to consume was significantly less than one (.8 in the long run). This problem, and the observation by Davidson and McKinnon [19] that high savings rates (or a low marginal propensity to consume) were correlated with high levels of inflation because personal disposable income overestimates effective income during inflationary periods, led to a re-estimation of the consumption equations with the adjusted income term.

where $\beta_1$ and $\beta_2$ are the formulas for converting assets from market value to book value, c is the coupon rate, proxied by a five-year moving average of the interest rate on ten provincial bonds (RL10P) and r, the market rate, is simply RL10P. The inflation premium, which is an average of $642 million in 1972, rises steadily to $3.23 billion in 1979, representing as much as 2.1% of total personal disposable income and 24.4% of personal investment income net of dividends in the first quarter of 1975. In 1979, the inflation premium is 1.9% of personal disposable income.

The three determining variables of the inflation premium, $\alpha$ , $P^e$ and W, have obvious measurement difficulties, which created problems in estimating consumption functions with a real effective purchasing power term. O'Reilly and Rayfuse [66] tested various price expectations series under a number of assumptions about the recognition coefficient and finally imposed values for $\alpha$ and $P^e$. The recognition coefficient, following the example of Davidson and McKinnon [19] is set at .5. The expected rate of inflation, PCPICE in RDXF, is defined as the year-over-year per cent change in the CPI lagged over eight quarters. Financial wealth is defined as the sum of government and Bank of Canada liabilities, which approximate assets in the personal sector. Components of wealth are Canada savings bonds (LGFCSB), treasury bills (LGFTB), direct market issues (LGBF), personal sector holdings of provincial and municipal bonds (LPPMB), Bank of Canada deposits (ABBCD) and notes outstanding (LGBCN). (As O'Reilly and Rayfuse have pointed out, this definition of wealth is understated because it does not include foreign assets held by the personal sector.) Federal and provincial bonds, which are longer term assets, are converted from market to book value. Federal bonds are assumed to have an average term to maturity of five years, whereas provincial bonds are assumed to be of sufficiently long term that their market value is the product of the book value and the ratio of the coupon rate to the market rate of interest.*

In the calculation of real effective purchasing power (U/N), an asymmetry exists between the wealth revaluation that occurs during inflationary periods and the lack of revaluation during periods when only real interest rates are fluctuating. The reason is that not all the various wealth measurements are calculated at market value. While the inflation premium and total interest receipts, including the inflationary component in (2), are determined with wealth at market value, the measure of investment income (YPI) included in U/N is simply the current stream of interest receipts from assets at their book value. In actual fact, an interest rate increase means that interest income is higher but the market value of assets declines. And, in parallel fashion to the inflation premium effect, real effective purchasing power should change by the amount individuals perceive their wealth to have changed.

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* The market value (MV) is calculated from the present-value formula:

$$PV_t = \frac{R}{(1+r_t)} + \frac{R}{(1+r_t)^2} + \cdots \frac{R}{(1+r_t)^n} + \frac{B}{(1+r_t)^n} = MV_t$$

where R is the coupon value, $r_t$ is the market rate of interest. This series can be rewritten:

$$MV_t = \frac{R}{r_t} [1 + \frac{1}{(1+r_t)^n}] + \frac{B}{(1+r_t)^n} ,$$

where n is the remaining term to maturity. Since the coupon value R is the product of the coupon rate c and the book value B:

$$MV_t = \frac{c}{r_t} B + (1-\frac{c}{r_t}) \frac{1}{(1+r_t)^n} B$$

For $n \to \infty$, $MV \simeq \frac{c}{r}B$.

Personal investment income net of the perceived wealth revaluations would then be:

$$\begin{aligned} INT_M &= iW_M \\ &= rW_B + \beta r(W_M-W_B) - \alpha P^e W_M \\ &= YPI + \Delta mkt - YINFL \ , \end{aligned} \qquad (3)$$

where $\beta$ is a recognition coefficient similar to that for the inflation premium. The exclusion of the term $\Delta$mkt implies an overstatement of the positive influence of interest rate increases on real effective purchasing power.

**Relative Price Term**

The consumption equations include explicit own-price effects. Because the CPI is modeled as a weighted geometric average (weights $\beta_i$ sum to unity) of the individual consumption deflators, equation (1) can be expanded to:

$$\log(C_i/N) = \alpha_{i0} + \ldots + \alpha_{i2}\log P_i - \alpha_{i2}\Sigma\beta_i \log P_i + \ldots, \qquad (1')$$
$$\eta_{ii} = \alpha_{i2}(1-\beta_i), \eta_{ij} = -\alpha_{i2}\beta_j.$$

Own-price elasticities are the product of the relative price coefficient ($\alpha_{i2}$) and one minus the expenditure weight ($\beta_i$) of the own-consumption component $C_i$. Cross-price elasticities are constrained to be the product of the (negative) coefficient $\alpha_{i2}$ and the expenditure weight ($\beta_j$) of the cross-consumption component $C_j$. The constraints imply that all consumption goods are substitutes ($\eta_{ij}>0$). Exceptions are the CHSHD and CRENT equations, which are modeled as complementary to residential construction. Other cases where complementarity may exist have been impossible to verify empirically.

Because the consumption sector is modeled on a disaggregated basis, the own- and cross-price elasticities have not been constrained explicitly across equations, and thus aggregate consumption (and hence the savings rate) may be affected by relative price changes. Total consumption should be unchanged with a relative price shift within the consumption sector that did not affect the rate of time preference or total income.* It will become apparent in the next section that these constraints are not violated over the longer run, although total consumption may vary considerably over the short run with relative price changes.

**Interest Rate Term**

Financial market conditions influence consumption through the interest rate terms in the rent, household durables, and motor vehicles equations as well as through changes in real effective purchasing power. The rent and household durables equations show an indirect response to movements in the prime interest rate (RPRIME) and the mortgage rate (RMC), which influence the housing market. The rent equation, instead of following the general specification in (1), is a function only of the stock of residential construction and a lagged dependent variable. Household durables are modeled as complementary to investment in residential construction as well as to the demand determinants in (1). In the motor vehicles equation a Jorgensonian imputed rental price, which is constructed using the deflator for motor vehicles, the real supply price of capital (RHOR) and the rate of depreciation on the stock of motor vehicles and parts, characterizes the cost of automobile ownership. The supply price of capital is, in turn, a function of the average rate on the ten industrial bonds (R10IND).

**Other Terms**

Services (CS) are estimated net of the rent component. The equation for services includes the annual rate of inflation as an

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* For total consumption to be unaffected, own- and cross-price elasticities would conform to constraints similar to a series of Slutsky equations.

explanatory variable, as well as the terms in (1). This "money illusion" term has the effect of reducing services consumption and increasing the savings rate during periods of high inflation. Of course, its inclusion contradicts the original motive for removing the inflation premium from income, which was to model rational consumer behaviour towards the effects of inflation. However, the term significantly improves the estimation results in the CS equation, and repeated attempts to remove it have not produced good results.

The consumption of energy, which to a large extent is made up of home heating, includes a term describing climatic changes (QQHEAT).

## 1.2 DYNAMICS OF THE CONSUMPTION FUNCTIONS

The first four columns of Table 1 present the dynamic partial elasticities of the consumption components with respect to real effective purchasing power, population, interest rates, and prices. The final two columns show the simulated marginal propensity to consume and actual average propensity to consume for each consumption category. The consumption model used to derive these results includes all consumption equations (Sector 1), the interest rate sector (Sector 17), and the housing sector (Sector 2).

With a **1% increase in real effective purchasing power,** total consumption increases .4% on impact, rising to a sustained .9% above control after one year.* Because the income elasticity is slightly less than unity, there is a gradual increase in the real savings rate as real income increases. The impact elasticities are highest in the three durables categories; the response in each of these equations peaks after about one year. Categories with elasticities exceeding unity after eight years are the "luxury" goods: CDMIS (mainly recreational equipment) and

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* The implied lagged dependent variable for total consumption is (1-.4)/.94 = .57.

## Table 1

### DYNAMIC PARTIAL ELASTICITIES OF CONSUMPTION COMPONENTS*

| | Real Effective Purchasing Power | Population | Interest Rates | All Prices | MPC | APC |
|---|---|---|---|---|---|---|
| Household | 1.0 | -.1 | .0 | .0 | .04 | |
| durables (CHSHD) | 2.0 | -.3 | -.7 | .1 | .08 | .04 |
| | 1.2 | -.2 | -.2 | .0 | .05 | |
| Motor Vehicles | 1.0 | .1 | -.1 | .0 | .07 | |
| (CMV) | 1.5 | .1 | -.2 | .0 | .11 | .07 |
| | .9 | .1 | -.1 | .0 | .06 | |
| Misc. durables | .8 | .2 | .0 | .0 | .03 | |
| (CDMIS) | 1.8 | -.6 | .0 | .0 | .07 | .05 |
| | 1.7 | -.7 | .0 | .0 | .09 | |
| Semi-durables | .4 | .6 | .0 | .0 | .05 | |
| (CSD) | .8 | .3 | .0 | .0 | .09 | .13 |
| | .8 | .2 | .0 | .0 | .11 | |
| | .4 | .6 | .0 | .0 | .05 | |
| Food (CFOOD) | .6 | .4 | .0 | .0 | .09 | .12 |
| | .7 | .4 | .0 | .0 | .07 | |
| | .3 | .7 | .0 | .0 | .02 | |
| Energy (CENERG) | .7 | .3 | .0 | .0 | .04 | .06 |
| | .8 | .2 | .0 | .0 | .04 | |
| Other non- | .3 | .7 | .0 | .0 | .03 | |
| durables (CNDO) | .7 | .3 | .0 | .0 | .07 | .09 |
| | .7 | .3 | .0 | .0 | .07 | |
| Services (incl. | .3 | .7 | .0 | -.6 | .10 | |
| rent (CS) | .7 | .1 | -.0 | -.6 | .28 | .37 |
| | 1.0 | -.1 | -.5 | .0 | .40 | |
| | .0 | .9 | -.0 | .0 | .00 | |
| Rent (CRENT) | .0 | .6 | -.0 | .0 | .00 | |
| | .8 | .0 | -1.4 | .1 | .11 | |
| Total consumption | .4 | .6 | -.0 | -.2 | .38 | |
| (CON) | .9 | .2 | -.0 | -.2 | .83 | .93 |
| | .9 | .0 | -.2 | +.0 | .89 | |

* Elasticities reported are impact, 1-year and 8-year, in that order.

CHSHD (furniture, carpets, and household appliances). Lowest longer run elasticities (-.7) are CNDO (alcohol, beer, tobacco, and sundry household items) and CFOOD.

The partial elasticity of consumption with respect to a 1% **increase in all levels of population** is inversely related to the income elasticity.* The long-run elasticity of total consumption with respect to population is close to zero; because the income elasticity is close to unity, the partial effects of a population increase merely change the composition of consumption. Miscellaneous durables show the most substantial decline relative to control, and household durables and services fall somewhat. The remaining categories show an increase, in particular food and other non-durables. The rent category, because of the slow speed of adjustment, sustains an increase over a longer period of time.

There is virtually no demographic detail in the consumption sector. Minor exceptions are the motor vehicles and rent equations, which are estimated in proportion to the population 15 years and over rather than to total population. Thus, arguments to describe variations in the savings rate through variations in the age distribution of population (life-cycle hypothesis) are limited in RDXF.

The partial response (assuming income is unchanged) of total consumption to an increase of 100 basis points in exogenous short-term **interest rates** is quite weak, with a slow speed of adjustment. The entire effect stems from the household durables, rent, and motor vehicles equations. Both household durables and rent are influenced indirectly through changes in the housing market. An increase of 100 basis points in exogenous domestic short rates--the bank rate (RBANK) and the 90-day paper rate

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* From (1) $\log C - \log N = .4(\log Y - \log N) + .57(\log N)_{-1}$, implying that, in the long run:

$$\log C = .4/(1-.57)\log Y + (1-.4/(1-.57))\log N$$

or the long-run income and population elasticities should sum to unity. The values reported in Table 1 do not obtain this result because CRENT does not follow this general specification, and has not attained its long-run value after eight years.

(R90)--influences other domestic rates through the term structure equations in the interest rate sector. Interest rates affecting the consumption sector are RPRIME and RMC, which influence the housing sector and therefore household durables and rent, and R10IND, which determines the supply price of capital and the consumption of motor vehicles. An increase of 100 basis points in RBANK and R90 implies increases of 29 basis points in RMC, 75 basis points in RPRIME, and 17 basis points in R10IND.

Total consumption declines gradually to .2% below control after eight years, attaining only 50% of this long-run value after three-and-a-half years. The relatively slow speed of adjustment stems from a very gradual response in the rent equation. After one year 56%, 31%, and 13% of the consumption declines stem from the CHSHD, CMV, and CRENT components respectively. After eight years the response is divided 4%, 4%, and 92% respectively among these categories.

**A 1% increase in the level of prices** induces only a temporary decline in the level of consumption. However, a 1% increase in the rate of inflation will create a permanent .2% reduction in total consumption. The entire negative response originates from the money illusion term in the services equation, which lowers this category by .6%. The rent and household durables equations display a slight positive response to the overall price level because housing starts, which indirectly affects their categories, is not homogeneous of degree zero in all prices and costs.

The final two columns of Table 1 show the **marginal propensity to consume** (MPC) for each consumption category and the actual average propensity to consume for the 1972-79 period. The MPC with respect to income adjusted for the inflation premium is .38 on impact and .89 after eight years for total consumption. (The measured average propensity to consume is .93 during 1972-79.) After one year, 94 per cent of the long-run value obtains. The fastest speeds of adjustment are in the three durables equations, which exceed their long-run elasticities before the end of one year, then gradually decline through the stock adjustment effect. On impact, 37% of the contribution to the MPC is from durables, declining to 22% after eight years. Thus the more transitory the income shock, the greater the variability in durables expenditure. (The portion of the imputed stream of services from durables expenditure not currently consumed can be considered as savings.)

Table 2 shows the own-price elasticities for the individual consumption components and the elasticity of total consumption. All consumption categories except miscellaneous durables, which has an own-price elasticity of unity, exhibit long-run own-price elasticities of .6 or less. The three durables categories have price elasticities that overshoot their long-run values; the adjustment to durables expenditure in more flexible over the short term. Services display the full own- price effect on impact, plus a response through the money illusion term lasting four quarters. The CRENT equation is not a function of its own price and therefore shows no price response. Other than rent, the lowest own-price elasticities are in energy and household durables.

As was mentioned in the previous section, cross-equation contraints that would guarantee that the elasticity of total consumption with respect to any individual deflator is zero in the long run* have not been imposed. However, total consumption deviates less than .1% from control after eight years in response to a change in a particular deflator. For all deflators the elasticity is negative on impact, reflecting in part time lags in adjusting to the optimal consumption stream and in part the money illusion term in the services equation. Both rent and

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* Since $\eta_i = \sum_{j=i} \frac{C_i}{C}\eta_{ij}$, from (1'), the required cross-equation constraint would be:

$$\frac{C_i}{C}\alpha_{ii}(1-\beta_i) = \sum_{j\neq i}\frac{C_j}{C}\alpha_{ji}\beta_j$$

Table 2

PRICE ELASTICITIES IN THE CONSUMPTION SECTOR

| Price | OWN-PRICE ELASTICITY | | | TOTAL CONSUMPTION ELASTICITY | | |
|---|---|---|---|---|---|---|
| | Impact | Long-run | Max/Min Value | Impact | Long-run | Max/Min Value |
| Household durables (PHSHD) | -.27 | -.29 | -.47 | -.01 | .00 | -.01 |
| Motor vehicles (PCMV) | -.45 | -.39 | -.72 | -.02 | .0 | -.04 |
| Misc. durables (PCDMIS) | -.47 | -1.01 | -1.17 | -.02 | -.05 | -.05 |
| Semi-durables (PCSD) | -.29 | -.61 | -.61 | -.02 | -.04 | -.04 |
| Food (PFOOD) | -.29 | -.53 | -.53 | -.02 | .00 | .03 |
| Energy (PENERG) | -.13 | -.33 | -.33 | .00 | .01 | .02 |
| Other non-durables (PCNDO) | -.52 | -.54 | -.54 | -.04 | -.01 | -.04 |
| Services (incl. rent)* (PCS) | -.54 | -.44 | -.54 | -.16 | -.08 | -.16 |
| Rent (PRENT) | .0 | .0 | .0 | .02 | .07 | .08 |

* Consumption component reported is CS.

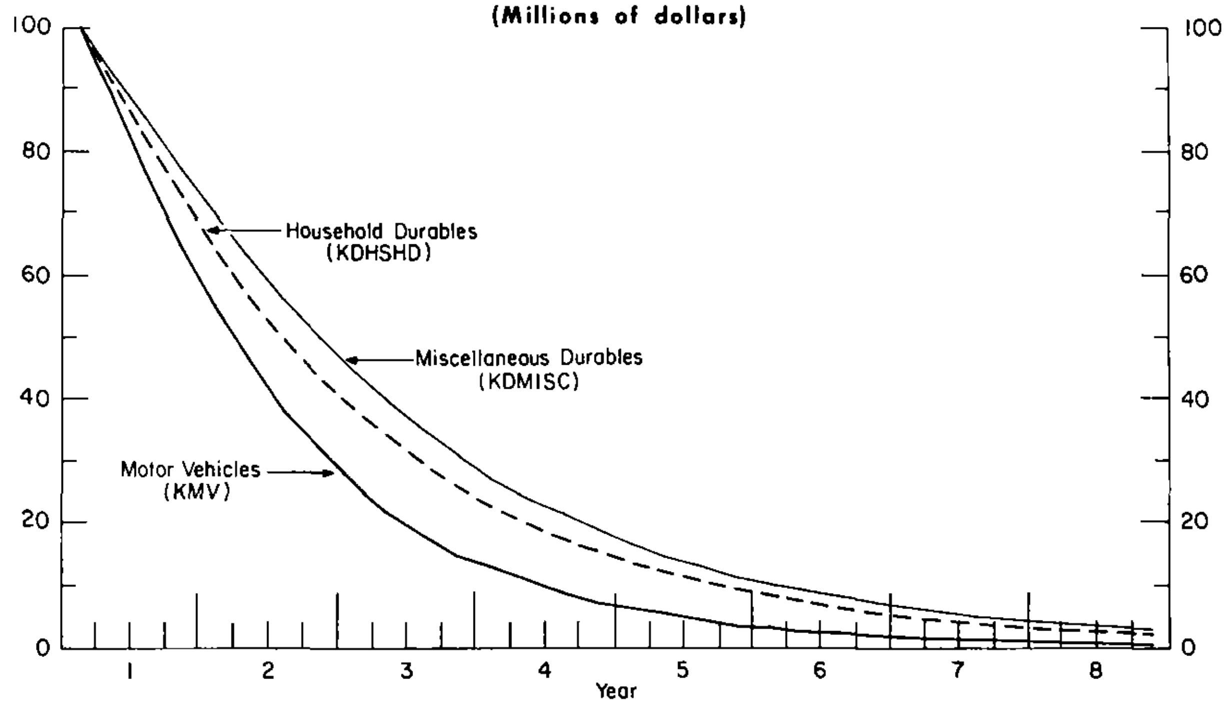
miscellaneous durables deflators produce a response that is more pronounced over the long run than on impact.

Figure 1 presents the stock adjustment in the three durables categories to an exogenous one-quarter **increase of $100 million in each of these stocks.** All stocks are less than $3 million above control after eight years. Ninety per cent of the adjustment has occurred after four years for motor vehicles, five years for household durables, and six years for miscellaneous durables.

Figure 1

STOCK ADJUSTMENT FOR THE DURABLES CONSUMPTION CATEGORIES

(Millions of dollars)



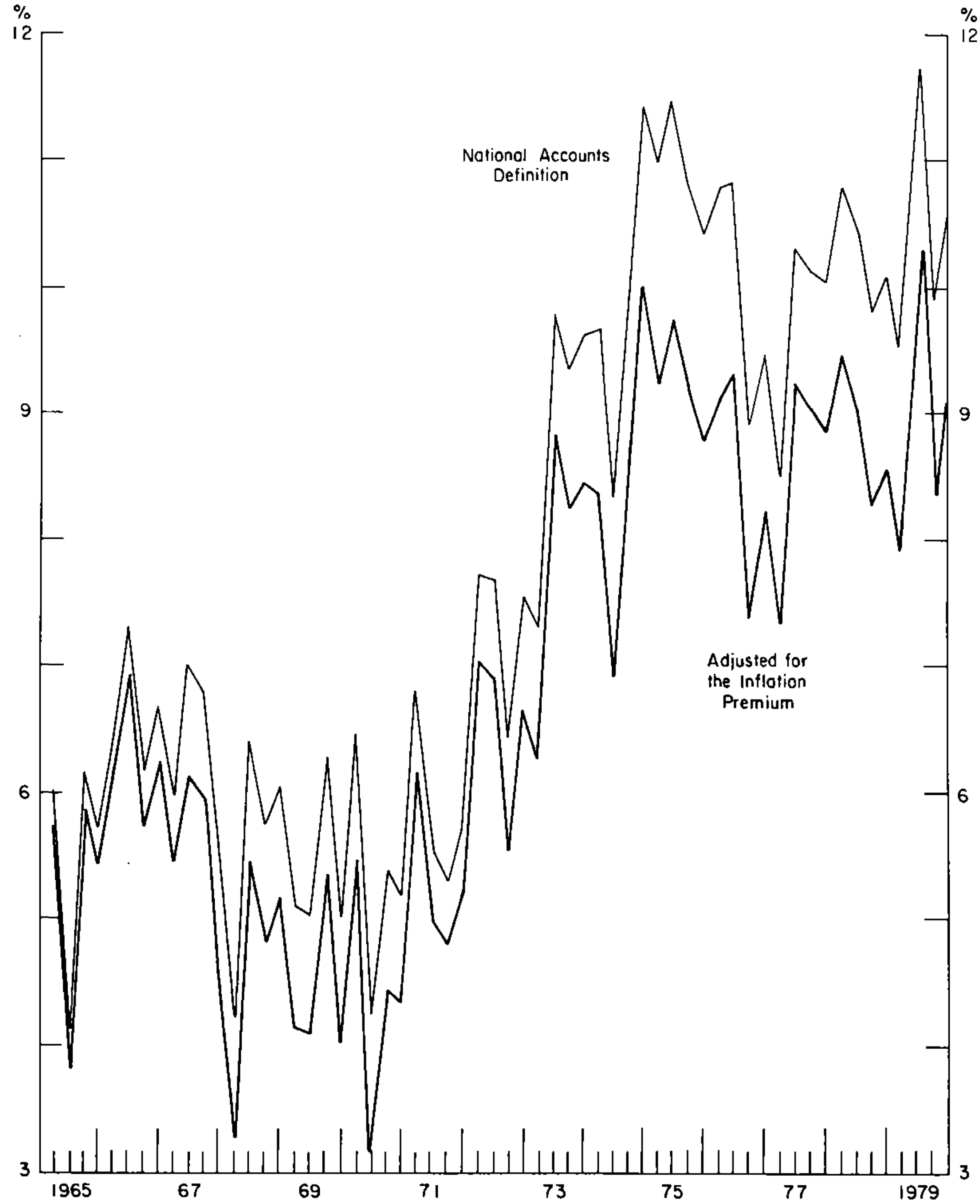

1.3 THE SAVINGS RATE

Throughout the 1970s the measured savings rate showed a strong upward trend, increasing from an average of 5.3% in 1970 to over 11% in 1974Q4, 1975Q2 and in 1979Q2 (Figure 2, upper line). This pattern has been combined with large swings of several quarters in length and sharp one-quarter swings of over 2% in amplitude. This section attempts to explain these phenomena, first by adjusting the savings rate to remove the positive inflationary bias, and then relating in some detail the factors that implicitly influence the pattern of the savings rate through the consumption equation.

The bottom line in Figure 2 shows the savings rate adjusted for the inflation premium. The peak value of this series is 10.3% in 1979Q2; remaining observations are less than 10%. The inflation premium explains over 100 basis points of the savings rate after 1973, and as much as 170 basis points in the first half

**Figure 2**

**THE SAVINGS RATE**
**1965-79**



of 1975. However, a strong upward trend remains in the adjusted savings rate series over the 1970-75 period.*

To isolate the factors underlying the remaining variation in the adjusted savings rate, a series of simulations were performed to control for fluctuations of the determining variables in the consumption sector.** Each simulation successively removed the effect of the growth in prices and real income, the cycle in income, and the growth in trend income and structural variables such as population. The simulations were run over the 1970-79 period, on a model composed of the consumption and housing sectors plus the adjusted savings rate equation. Figure 3 demonstrates the cumulative effect of the determining variables on the adjusted savings rate. When inflation and nominal interest rate increases are controlled, virtually all of the upward trend in the savings rate is removed. The elimination of the cycle in income reduces some of the short saw-toothed movements in the adjusted savings rate, and also removes some of the amplitude in the savings cycle. The trend in income and growth in other structural variables creates only a very slight increase in the adjusted savings rate. When all fluctuations in explanatory variables are removed, the adjusted savings rate stabilizes at 4.6% throughout the 1970s.

In the first simulation, price inflation is zero and real interest rates are constant. Furthermore, all relative prices remain at their 1970s level, and the inflation premium is zero.

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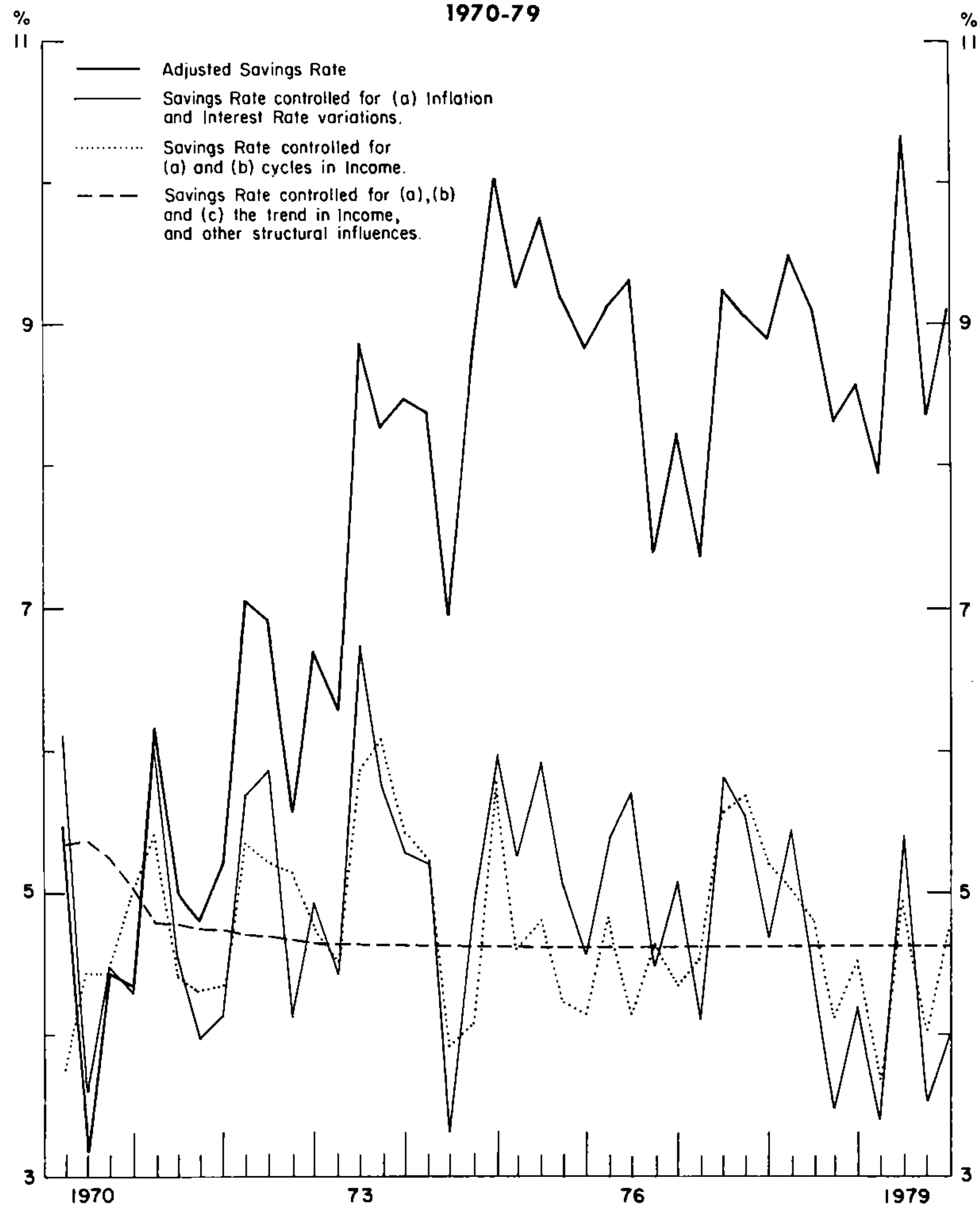
* The adjusted savings rate is defined as:
(YDP-CON$-YTCORP-YINFL)/(YDP-YINFL-YTCORP)·100, or:
(UDPADJ·PCPI-CON$-MTRP -YINFL)/(UDPADJ·PCPI)·100;
whereas the National Accounts definition is:
(YDP-CON$-MTRP-YTCORP)/YDP·100.
The adjusted definition excludes the inflation premium (YINFL) and transfers to corporations (YTCORP) from the denominator as well as from the numerator to conform with the income term in the consumption functions.

** The simulation methodology is similar to that employed by Aubry and Fleurent [2]. Technical details are discussed in the appendix to this chapter.

**Figure 3**

**SOURCES OF VARIATION IN THE ADJUSTED SAVINGS RATE 1970-79**



The reduction in the inflation premium implies that real effective purchasing power is 1%-2% higher over the simulation period, which creates a temporarily higher savings rate in 1970--the short-run marginal propensity to consume is only .38. However, there is only a slight increase in the adjusted savings rate over the longer term, as the income elasticity for total consumption is close to one. Holding relative prices constant does induce a significant change in the composition of consumption. Those categories showing a relative price decline over the seventies--miscellaneous durables (CDMIS), household durables (CHSHD), semi-durables (CSD), and other non-durables (CNDO)--are significantly lower by the end of 1979, whereas the food and energy categories are much higher. Services are considerably higher, both because rent consumption increases as a result of stable nominal interest rates, and because the money illusion term in the services equation does not create downward pressure on services. The change in the services component alone is sufficient to account for an increase of over 5% in real consumption by the end of the 1970s, which translates into an adjusted savings rate that is lower by 5 percentage points.

Removing the cycle in income reduces the short-term fluctuations and the amplitude in the adjusted savings rate. The effect on the savings rate is relatively slight, however; inflation and interest rates are by far the most significant determining factors. The simulated series peaks at 6% in 1973, and is 4.7% lower than the actual in 1979.

Finally, setting income and population growth at zero* and removing the estimation error produce an adjusted savings rate that is a sustained 4.6% throughout the 1970s. The trends in income and population create only a very slight upward drift in the adjusted savings rate.

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* For this shock the housing sector is exogenous and the stock of residential construction is set at its 1970Q1 value. This was required because the housing sector continues to show positive growth even when income and prices are at zero growth.

In conclusion, there is still considerable weight on inflation (and, to some extent, on interest rate increases) in the underlying consumption equation specifications as an explanation for increases in the adjusted savings rate. Although some of the upward drift is the result of a low recognition coefficient in the inflation premium, (recall that the recognition coefficient is only .5) a considerable part stems from the money illusion term in the services equation. Other hypothesized explanations of the pattern in the savings rate--such as changes in the age structure of population and family size, the after-tax rate of return on various assets, and the existence of social security and pension funds--are not explicit arguments in the consumption functions and thus are ruled out in this particular exercise. And, to this point, there has not been sufficient empirical evidence to include these arguments in the consumption equations.

### 1.4 RESPONSE TO AN INCREASE IN REAL INTEREST RATES

This section reports the response of consumption and savings to an increase of 100 basis points in exogenous real interest rates when the components of real effective purchasing power are endogenous. In this example, the interest portions of income and debt respond directly through the interest rate terms, as do durables and rent. Equations in the simulation include the consumption, housing, interest rate, and wealth sectors, and the interest and debt components of personal income. The average coupon rates on government bonds (EACR, EACRCS), federal investment income (YGIF), and interest on the provincial public debt (GTPINF), which are exogenous in the RDXF model, are assumed to respond as a 20-quarter moving average of the yield on the ten provincial bonds (RL10P).

As shown in Table 3, real effective purchasing power increases from .2% above control on impact to .5% above control after eight years; the increase stems from higher interest

Table 3

RESPONSE OF CONSUMPTION AND SAVINGS TO AN INCREASE OF 100 BASIS POINTS IN EXOGENOUS INTEREST RATES
(Shock minus control, per cent)

| | Impact | 1 Year | 8 Years |
|---|---|---|---|
| Personal investment income | 2.0 | 2.5 | 3.8 |
| Real effective purchasing power | .2 | .2 | .5 |
| Consumption | .06 | .13 | .18 |
| Durables consumption | .11 | .09 | .50 |
| Adjusted savings rate (level) | .10 | .08 | .26 |

payments on the federal debt (GTPINF) and on the provincial debt (GTPINP), and from interest and miscellaneous investment income, which after eight years are 4.3%, 2.8%, and 1.9% above control respectively. These are offset somewhat by an increase in payments on consumer debt (YTCORP) which is 4.6% higher after eight years. The inflation premium is below control for the first half of the simulation because of a temporary decline in the market value of assets. As the coupon rate adjusts, the inflation premium returns to control.*

Total consumption is .2% higher after eight years. (Recall from the second section that the partial response of consumption was -.2% after eight years.) All components increase except those that are linked to the housing market; household durables are below control for the first few years, as is rent for the entire simulation period. The durables categories, which exhibit the greater (negative) interest sensitivity when income is exogenous, are strongly positive in this simulation, because the household

* There would be no wealth revaluation through this term if the $\Delta$mkt term in equation (3) were included in effective purchasing power.

durables and miscellaneous durables categories exhibit strong income elasticities.*

The adjusted savings rate increases 30 basis points after eight years. The savings rate would be somewhat higher if the wealth effect were incorporated into the consumption equation.

## 1.5 CONCLUSION

In terms of its forecasting performance and tracking ability, the consumption sector is one of the better sectors in the RDXF model. A number of improvements could be made, particularly along the lines of incorporating a single measure of permanent income, and correctly distinguishing the effects on consumption of variations in wage and non-wage income. Furthermore, the channels of influence for real interest rates could be strengthened. It should be borne in mind, however, that considerable efforts have been made along these lines since the development of the RDX2 model (most recently Williamson [83], Rayfuse [69] and [70], and O'Reilly and Rayfuse [65] and [66]), and the benefits of these proposals were considered to be marginal.

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* In fact, a constructed "durables real savings rate", shows a decline over the entire simulation period. This term is defined as the ratio of expenditures on durables, less the imputed services derived from these durables, to effective purchasing power. The imputed term is a two-quarter moving average of the stock of motor vehicles, household durables, and miscellaneous durables, multiplied by the real supply price of capital and the respective depreciation rates.

## Appendix to Chapter 1

### TECHNICAL DESCRIPTION OF CONSUMPTION SHOCKS
(Sector 1)

**Table 1 Partial Elasticities of Consumption Components**

Equations: Sectors 1, 2 and 17; EQP13

Time Period: 1972Q1 - 1979Q4

Shocks:
1. UDPADJ·1.01
2. NPOP·1.01, NPOPT·1.01
3. R90+1, RBANK+1
4. (PCMV, PCSD, PFOOD, PENERG, PCNDO, PCSD, PMLS, PHSHD, PCDMIS, PCSXR, PRENT)·1.01
5. UDPADJ+100

**Table 2 Price Elasticities in the Consumption Sector**

Equations: Sector 1; EQP13, EQP52

Time Period: 1972Q1 - 1979Q4

Shocks:
1. PCDMIS·1.01
2. PENERG·1.01
3. PFOOD·1.01
4. PHSHD·1.01
5. PCMV·1.01
6. PCNDO·1.01
7. PCS·1.01
8. PCSD·1.01
9. PRENT·1.01

**Figure 1 Stock Adjustment for the Durables Categories**

Equations: CHSHD, KDHSHD, CMC, KMV, CDMIS

Shocks:
1. KDHSHD = 9.44·J1L(KDHSHD) + CHSHD/4 + 100(NPER.EQ.19721)
2. KMV= 9.33·J1L(KMV) + CMV/4 + 100(NPER.EQ.19721)

**Figure 3 Sources of Variations in the Savings Rate**

Equations: Sectors 1 and 2; EQ478, EQP66, EQ where EQ1 is SAV2 = UDPADJ·PCPI-MTRP$-CON$)/(UDPADJ·PCPI)·100)

Time Period: 1970Q1 - 1979Q4

Shocks:

1. Add back estimation errors in first three simulations.
   Set PCSD, RHOR, PCPI, PFOOD, PENERG PCNDO, PCSXR, PHSHD, PCMV, PCSD, PRENT, PCS, WNICIR, PXLUM, MTRP$, at 70Q1 values.
   $YDP = YDP_C/PCPI_C \cdot PCPI$.
   $YINFL = 0$, $YTCORP = YTCORP_C/PCPI_C \cdot PCPI$.
   Set RPRIME and RMC at nominal value in 1970Q1, less 4th/4th rate of inflation
2. As in 1, plus exogenize EQV78.
   Generate UDPADJ from the equation
   log UDPADJ = 9.18(NPER.LT.1974) + .02·QQTIME(NPER.LT.19741) + (9.18+.07(.02-.008))(NPER.GE.19741) + .008QQTIME (NPER.GE.19741), estimated from 1970Q1 to 1979Q4.
3. As in 1, plus exogenize EQY78, EQP66.
   Set NPOP, KRESD, NPER, UDPADJ, NPOPT, QQHEAT, QREB, ULC, HSSPLI, ESIRC, EIRCM, EHST, QSTR, QHELP, at 70Q1 values; set CDRES, CDRES$ = 0.
4. As in 3, plus remove estimation errors.

Figure 3 shows results from 1, 2 and 4.

**Table 3 Interest Rate Shock Including Income Effect**

Equations: Sectors 1, 2, 8, 17 and 18; EQQ03, EQG02, EQP53

Time Period: 1972Q1 - 1979Q4

Shock: R90+1; RBANK+1

# Chapter 2

## INVESTMENT IN RESIDENTIAL CONSTRUCTION
(Sector 2)

Heather Robertson

The residential construction series is derived separately from business fixed investment (Chapter 5) and is based on a different model of investment behaviour. Whereas business fixed investment is part of an interrelated factor demand network that determines private sector output in RDXF, investment in residential construction (IRCA) is derived in a separate model of the housing market. The reason for distinguishing between them is that IRCA does not display the same cyclical pattern as business fixed investment, being more interest sensitive and more responsive to variations in income.

A theoretical model of the housing sector* is described in the first part of this chapter, which includes a discussion of how the RDXF housing model deviates from the hypothetical model. Next is a description of housing sector dynamics; any problems can usually be attributed to the fact that the RDXF model does not strictly adhere to the assumptions in the hypothetical model. The third section isolates the variation in housing starts by source of deviation in the explanatory variables. Suggestions for future research are presented in the conclusion.

### 2.1 THEORETICAL MODEL OF THE HOUSING SECTOR

The two central equations in the housing sector describe housing starts (HST), which is a supply equation, and the multiple listing price per unit of housing (PMLS), which is an excess demand equation. Housing starts, derived on a seasonally unadjusted basis, are a function of the multiple listing price, an index of

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* The most recent version of the housing model was developed by S. Williamson [89-91], who built on earlier work by R. Khemani [48, 50] and the RDX2 housing model developed by L. Smith.

per unit construction costs (PHCC), financing costs, represented by the prime rate of interest (RPRIME), plus a vector of seasonal dummies (QSEAS) and a dummy variable to capture the effects of multiple unit residential building subsidies on housing starts in the first half of 1978 (QMURB). Another dummy variable ($Q^*=1$ in 1967Q2, 1970Q3, 1970Q4, and 1975Q4) supposedly captures increases in starts financed by public funds under federal legislation. Since the data on these series are available only on an annual basis, the quarters to be dummied out were identified by the residuals.

$$\log HST = \log H^{s} = \alpha_1 \log PMLS - \alpha_2 \log PHCC - \alpha_3 RPRIME_{-1} + \alpha_4 QSEAS + \alpha_5 QMURB + \alpha_6 Q^* \quad (1)$$

Interest rates are lagged one quarter because of the delay in timing between loan approvals and actual housing starts. The relative magnitude of coefficients on the price and cost terms are such that general price increases have a positive influence on housing starts.*

The estimated MLS housing price adjusts to reduce excess demand for housing per capita. In the PMLS equation, housing demand is represented by arguments that would appear in an implicit demand for housing function ($H^d$); housing supply is simply the explanatory variable in the HST equation:

$$\Delta \log PMLS = \beta_1 (\log H^{d}/N - \log HST/N). \quad (2a)$$

The implicit demand for new housing per capita ($H^d/N$) is a function of the deviation between the desired stock of new housing

---

* If price increases not affecting profitability did not influence supply, i.e., if supply were homogeneous °0 in prices and costs, the coefficients would be constrained such that $\alpha_1 = \alpha_2 + \alpha_3/RPRIME$.

per capita (Sd/N) and the previous quarter's actual stock per capita:

$$\log H^d/N = \gamma_0(\log S^d/N - \log S_{-1}/N).$$

The desired stock per capita is a function of real effective purchasing power per capita (UDPADJ/N), the selling price (PMLS), *and the mortgage rate (RMC). Thus:*

$$\log H^d/N = \gamma_0(\gamma_1 \log UDPADJ/N - \gamma_2 \log PMLS^4 - \gamma_3 RMC_{-1}) - \gamma_0 \log S_{-1}/N. \qquad (2b)^*$$

Population (N) is defined as all persons 15 years of age and over.** The UDPADJ term, defined as current real personal disposable income net of the inflation premium and transfers to corporations, is the same income measure as in the consumption functions (see Chapter 1). The equation that exactly incorporated the specification in (1), (2a) and (2b) would be:

$$\log PMLS = \frac{1}{(1+\beta_1\gamma_0\gamma_2+\beta_1\alpha_1)}[\gamma_0((\gamma_1 \log UDPADJ/N - \gamma_3 RMC_{-1}) - \log \frac{S_{-1}}{N}) - (-\alpha_2 \log PHCC - \alpha_3 RPRIME_{-1} + \alpha_4 QSEAS + \alpha_5 QMURB + \alpha_6 Q^*) - \log N + \frac{\log PMLS_{-1}}{\beta_1}] \qquad (2)$$

The equation for the multiple listing price (PMLS) in RDXF follows the specification in (2) only in a very general form. Because the PMLS and HST equations are estimated individually, PMLS by an OLS technique and HST with an adjustment for autocorrelation,

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* The PMLS term was not scaled by the CPI (i.e. in terms of relative prices) in the September 1980 version of RDXF. This modification was made in the March 1981 version.

** Defining population in terms of households was felt by Williamson to introduce simultaneity bias.

there are no cross-equation constraints such that the supply coefficients in (1) match those in (2), nor are there constraints imposed such that ΔlogPMLS equals zero when excess demand (as defined) is zero. Furthermore, the PMLS equation does not include a separate population term (-logN in (2)) that would be required to scale housing supply. These differences are reflected in the housing sector dynamics discussed in the next section.

Interest rate arguments in the housing sector are expressed in nominal terms; and, to the extent that high nominal interest rates imply high inflationary expectations, the nominal interest rate arguments tend to impart a downward bias on housing starts relative to real rates during inflationary periods. Attempts to model real interest rate effects have met with little success.

The stock adjustment in the housing sector comes into play through the PMLS equation. If the lagged stock exceeds the desired stock (represented by the income and price terms), PMLS falls, thereby reducing profitability and creating some offsetting decline in housing starts. The adjustment ($\gamma_0$=.8) is extremely slow. If, for example, the stock of housing increases exogenously by 100,000 units, the weak increase in PMLS produces only a gradual offsetting decline in housing starts; the actual stock still exceeds desired by 49,000 units* after eight years.

The remaining equations in the housing sector comprise construction costs (PHCC), and equations required to convert housing starts to seasonally adjusted investment expenditures (IRCA)** and to the stock of residential construction (KRESD). The mortgage rate is determined independently from the housing market through a term structure of domestic interest rates, and therefore will not be discussed in this chapter.

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* This result is derived from a simulation experiment that includes the complete housing model. The stock of housing is increased exogenously by 100,000 units for one quarter, allowing the model to work off the excess. These results are approximately linear in the model.

** The equations determining IRCA are:

Construction costs (PHCC) are a function of seasonal dummies, normalized unit labour costs (ULC), the export price of lumber (PXLUM), and a lagged dependent variable. In determining the stock and flow of residential investment, the distinction is made between starts and completions on the one hand, and singles and multiples on the other. The singles-multiples distinction is required because of the marked difference in the timing of completions in each of these categories. Total housing starts are split between singles (HSTS) and multiples (HSTM) by an exogenous variable measuring the historical share. Completions of singles (HCOMPS) and multiples (HCOMPM) are Almon lag functions of their respective starts components. Investment expenditure on an unadjusted basis (IRCU) is determined by a lagged function of singles and multiples housing starts* plus a scaling factor measuring the proportion of total residential investment to investment in housing (IRCMIS). The ratio of IRCU to a seasonal factor (ESIRC) determines IRCA. The sum of completions and a proportion of the lagged stock determine the stock of housing (SHT). The coefficient on the lagged stock term exceeds unity in order to reconcile the flow of completion data with the housing stock numbers, which are available from census data. The greater-than-unity value for depreciation

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$$HSTS = HST - HSTM \qquad HSTM = HST \cdot (HSSPLI)$$

$$HCOMPS = \varepsilon_1(L)\, HSTS \qquad HCOMPM = \varepsilon_2(L) HSTM$$

$$IRCU = [19.7(Z_1(L)HSTS \cdot QSEAS) + 13.6(Z_2(L)HSTM \cdot QSEAS)]IRCMIS$$

$$IRCA = IRCU/ESIRC$$

$$SHT = \delta_1 SHT_{-1} + HCOMPS + HCOMPM$$

* Lagged values of starts are scaled by the "percentage put in place" coefficients and the average 1971 value of single units ($19,700) and multiple units ($13,640) respectively.

could reflect the fact that conversions exceed demolitions, as well as the errors in the data series.

Although the real portion of investment expenditure in residential construction is derived in the housing sector, the deflator for this category (PIRC) is not. The PIRC equation (see Chapter 7) is modeled on the assumption that firms follow a flexible markup pricing scheme, and is a function of wage costs, the export price of lumber and energy prices. Thus there is no modeled relationship between PIRC and prices within the housing sector (PHCC, PMLS).* For this reason, the deflator will not be discussed further in this chapter.

Residential construction investment comprises only a small portion of GNE, about 5% in 1980. The importance of this component, however, lies in its large cyclical swings and its interest sensitivity, and therefore its potential channel for monetary linkages. Figure 4 presents the year-over-year per cent change in the simulated and actual values of HST (upper panel) and IRCA (lower panel).** As a reference, the cycle in real GNE is included in each graph. The variability in the HST series is considerable, ranging from -41.7% 1975Q1/1974Q1 to 79% 1976Q1/1975Q1. The IRCA series shows about half the variation because of the smoothing effect of the lags between starts and completions.

The simulated HST series is fairly successful in capturing the swings in the actual series, particularly from 1978 onward. The QMURB dummy variables explain a considerable amount of this success in the first half of 1978. The simulated series for IRCA is considerably less successful in tracking the growth of the actual; the changes in the two series are out of phase by about two quarters. Possible explanations for the poor track record in the _

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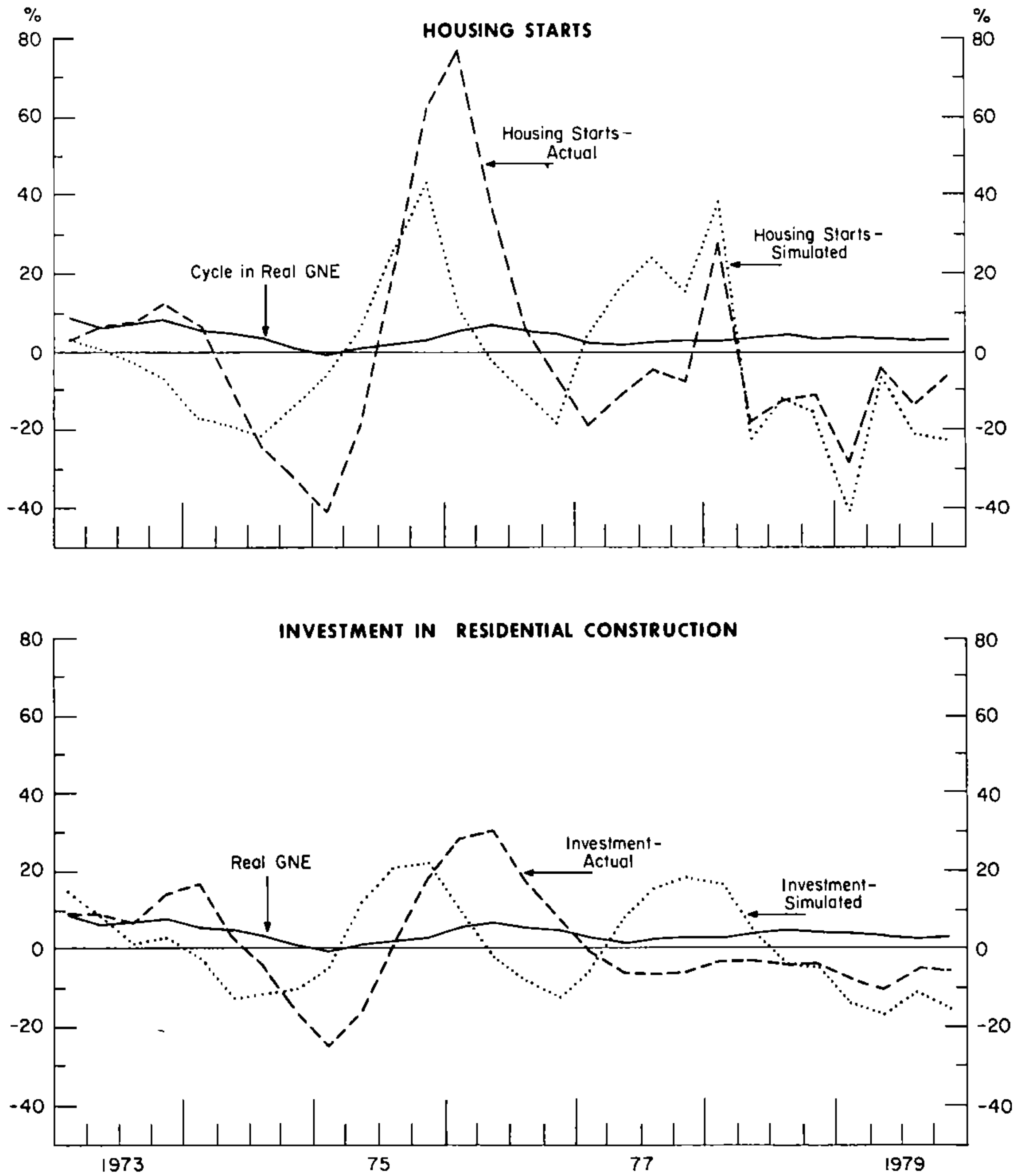
* This can create some problems. For example, an increase in energy prices, which causes an increase in PIRC, does not directly influence housing starts and, to the extent that unit labour costs follow a different pattern than wage costs, PMLS and PIRC could behave quite differently. The merits of modeling these deflators in a consistent fashion will be examined at a future date.

** The simulation includes the entire housing sector.

**Figure 4**

**CYCLES IN HOUSING STARTS AND INVESTMENT IN RESIDENTIAL CONSTRUCTION**

**(Per cent change, year over year)**



simulated IRCA series are the percentage-put-in-place coefficients and the dollar value of singles and multiples, all of which are constant in the equation that converts housing starts to investment in residential construction; these series actually vary considerably over time.

## 2.2 DYNAMICS OF THE HOUSING SECTOR

The analysis of the housing sector dynamics is divided into two parts. First, the dynamic elasticities of the four key variables in the housing sector, housing starts (HST), multiple listing price per unit (PMLS), construction costs (PHCC), and investment expenditure (IRCA) are presented in Table 4, to highlight the different responses in the housing sector to supply-side and demand-side shocks. Elasticities were calculated by increasing each component exogenous to the housing sector by 1%. The response to interest rates are reported as semi-elasticities. The appendix to this chapter provides a more detailed description of the simulation results.

Table 4

DYNAMIC ELASTICITIES OF VARIABLES IN THE HOUSING SECTOR

| | | UDPADJ | R90 | ULC | NPOP | PXLUM |
|---|---|---|---|---|---|---|
| 1. Housing starts (HST) | Impact | 1.4 | 0 | -.6 | -.0 | .0 |
| | 1 year | 2.9 | -5.9 | -.3 | -.0 | .1 |
| | 8 years | 1.4 | -2.3 | .0 | -.0 | .0 |
| 2. Multiple listing price (PMLS) | | .8 | .0 | .2 | -.0 | .0 |
| | | 1.6 | 1.0 | .7 | -.0 | .1 |
| | | 1.0 | 2.0 | .8 | -.0 | .2 |
| 3. Construction costs (PHCC) | | 0 | 0 | .8 | -.0 | 0 |
| | | 0 | 0 | 1.1 | -.0 | .2 |
| | | 0 | 0 | 1.1 | -.0 | .2 |
| 4. Investment in residential construction (IRCA) | | .3 | 0 | -.1 | -.0 | .0 |
| | | 2.4 | -5.3 | -.3 | -.0 | -.1 |
| | | 1.6 | -2.7 | -.0 | -.0 | -.0 |

Income and interest rate variables exert a strong influence on housing starts, whereas population and construction costs produce very little response.

The second part of this section analyzes the sources of variation in the housing cycle, construction costs, income, population and interest rates, and their relative contributions over the 1972-79 period.

**Construction Costs**

The supply of housing is virtually unaffected by construction and materials costs over the longer period. Recall that the equation was estimated with unconstrained profitability terms; i.e., general price increases have a positive effect on starts, and the HST coefficients and PMLS equations are not constrained such that the supply terms in HST are of the same magnitude as those representing supply in PMLS. (Financing costs do influence starts, as will be discussed later.) A 1% increase in normalized unit labour costs (ULC) produces an increase of .8% in PHCC on impact, rising to a sustained 1.1% above control after one year. The elasticity of PHCC with respect to the export price of lumber (PXLUM) is zero on impact--PXLUM enters PHCC with a one-quarter lag--rising to .2% after one year. Thus PHCC increases 1.3% with respect to all costs. The MLS price of housing has an elasticity of .31 with respect to PHCC, rising to .74 after eight years, implying an elasticity with respect to ULC of .8 and to PXLUM of .2. Because PMLS responds with a lag to PHCC, a 1% increase in ULC produces a .6% decline in HST on impact, the effect gradually dissipating over time. Moreover, because the induced PMLS increase is sufficient to offset the PHCC increase,* labour and materials costs exert virtually no influence on housing starts over the longer term. This results from not imposing the constraints defined in equations (1) and (2).

---

* The elasticity of HST with respect to PMLS is 1.6, and with respect to PHCC is -1.2. Then the net effect of PHCC on HST, including the indirect effect through PMLS is 1.6·(.74-1.2), approximately zero.

Because the split between singles and multiples is exogenous, the ratio of singles to multiples remains unchanged in this and in following shocks.

**Income and Population**

In the housing model the response to a change in income per capita that stems from an increase in real effective purchasing power is markedly different from one that stems from a change in population.

A 1% increase in real effective purchasing power (UDPADJ) induces upward pressure in PMLS with higher excess demand for new housing, thereby encouraging housing starts as profitability rises. The elasticities of PMLS and HST peak after one year, with a subsequent tempering of the increase from the stock adjustment effect. Investment in residential construction is 2.4% above control after one year and 1.6% above control after eight years, thereby demonstrating a very strong income elasticity.

The population term exerts two separate and opposing influences in the PMLS equation, each of about equal strength. Therefore a 1% increase in population produces no response in any of the variables in the housing model.* A population increase reduces income per capita, thereby reducing the demand for new housing, and produces a decline in the stock of housing, thus increasing the deviation between the desired and actual stock and increasing PMLS. Note that if population entered into the equation as a scaling factor for the housing supply terms as shown in equation (2), there would be a stronger positive influence on PMLS with an increase in population.

**Interest Rates**

This section reports the response in the housing sector to various interest rate shocks.

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* This is not the case in the March 1981 version of PMLS, where the stock adjustment coefficient outweighs the income coefficient.

Table 4 presented the reduced form elasticities in the housing model with respect to a 100-basis-point increase in exogenous domestic interest rates, the 90-day paper rate (R90) and the bank rate (RBANK). The mortgage rate and the prime rate respond to R90 and RBANK through the term structure. RMC increases 29 basis points over the longer term (16 on impact) and RPRIME increases 75 basis points (5 on impact). The supply effect through RPRIME dominates in the PMLS equation; the decline in supply stemming from higher financing costs serves to increase PMLS 1% after one year and 2% after eight years. Housing starts are 5.9% below control after one year and are 2.3% below control after eight years. Real investment in residential construction, responding with a lag to housing starts, declines 2.7% after eight years.

Table 5 isolates the response of HST and PMLS to a 100-basis-point increase in the mortgage rate (RMC) and the prime rate (RPRIME). The results ignore any interaction of interest rate variables through the term structure. The mortgage rate affects housing starts via the decline in PMLS, due to a reduction in excess demand. The decline in PMLS reduces profitability, therefore reducing the supply of housing; HST is 4% below control after one year. The stock adjustment effect reduces downward pressure in PMLS, moderating the decline in HST to -2.2% after eight years. A 100-basis-point increase in RPRIME reduces housing starts by 7.3% with a lag of one quarter as financing costs increase. The reduction in supply creates upward pressure in PMLS which, by influencing profitability, offsets the decline in HST, so that starts are 2.2% below control after eight years.

The response is particularly unsatisfactory because housing prices <u>increase</u> in response to higher interest rates. The effect on PMLS of a decline in supply dominates the effect of a decline in demand. Furthermore, there is no explicit term in the PMLS equation to capture capital gains or losses on the stock of housing due to interest rate changes.*

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* Of course, the problem would be less pronounced if the term structure were such that an increase in RMC was as strong as that of RPRIME, as it has been over the recent historical period.

Table 5

PARTIAL RESPONSE OF HOUSING STARTS AND THE MLS PRICE TO AN INCREASE IN MORTGAGE AND PRIME RATES
(Shock minus control, per cent)

| | | Mortgage Rate (RMC) | Prime Rate (RPRIME) |
|---|---|---|---|
| Housing starts (HST) | Impact | 0 | 0 |
| | 1 year | -4.0 | -6.2 |
| | 8 years | -2.2 | -2.2 |
| Multiple listing price (PMLS) | | 0 | 0 |
| | | -2.3 | 2.5 |
| | | -1.6 | 4.6 |

The interest rate response in the housing model, including the induced change in real effective purchasing power resulting from higher interest income, is presented in Table 6. The model includes the consumption, housing and the interest rate sectors plus the equations explaining personal interest and investment income.* As in the example above, a 100-basis-point increase in R90 and RBANK creates an increase in RPRIME of 75 basis points and in RMC of 29 basis points over the longer term. After one year real effective purchasing power is .2% above control, climbing to .5% above control after eight years. Higher income induces an even stronger positive PMLS response than in Table 2, which in turn lessens the decline in HST and IRCA somewhat as profitability is further increased. Residential construction investment is 1.9% below control (-$104 million) after eight years.

Although the decline in IRCA is significant, the magnitude relative to total GNE is actually quite small. Recall from Chapter 1 that the consumption response to a similar shock is positive over the entire simulation, reaching .18% above control ($148.2 million) after eight years. The positive consumption response is sufficient to offset the decline in IRCA. Real GNE is above control by the final year of the simulation, reaching .03% ($44.2 million) relative to control in the final quarter. Therefore, although the housing sector displays a large degree of sensitivity to interest

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* This is the same model used to report the interest sensitivity in the consumption sector in Chapter 1.

Table 6

EFFECT OF AN INCREASE IN EXOGENOUS INTEREST RATES:
HOUSING, CONSUMPTION & INTEREST INCOME MODEL
(Shock minus control, per cent)

| | End of year | | |
|---|---|---|---|
| | 1 Year | 2 Years | 8 Years |
| UDPADJ | .22 | .24 | .45 |
| PMLS | 1.3 | 1.8 | 3.5 |
| HST | -5.4 | -4.2 | -1.6 |
| IRCA | -4.9 | -4.4 | -1.9 |
| CON | .13 | 1.4 | .18 |
| UGNE | -.18 | -1.6 | .05 |

rates, this is overridden by a weak interest elasticity combined with a fairly strong income elasticity in the consumption sector.

**Miscellaneous Items**

The effect of the MURB dummy (1 in 1978Q1, -1 in 1978Q2) is to increase the stock of housing (SHT) by 4,800 units by the end of 1980. This implies a $109 million increase in the stock of residential dwellings (KRESD). The MLS price displays a very slight decline through the stock adjustment effect.

A 1% increase in the ratio of multiples to total starts (HSSPLI) creates a decline in the value of IRCA by .3% over the longer term because the value of multiples units in 1941 dollars ($13,640) is only about 70% that of singles units ($19,699). However, because the split between singles and multiples is made outside the system of equations determining housing behavior, HST, PMLS, and PHCC are unaffected.

## 2.3 DETERMINANTS OF THE HOUSING CYCLE

The purpose of this section is to analyze the relationship between the housing cycle and variations in the explanatory variables in the housing sector. For this exercise, explanatory variables were grouped into four categories: income (UDPADJ) and interest rates (RPRIME, RMC); costs (PXLUM, ULC); and population (NPOP). As well, there were three set of dummies: seasonal dummies (QSEAS); the MURB dummy (QMURB); and the dummy explaining starts financed by public funds (Q*). In a series of successive simulations, these groups of variables were in turn set at their trend values such that in the final simulation all components were at trend.* The difference between the final simulation solution (shock) and the control solution, in which all explanatory variables are at their actual level, isolates the total variation in the housing sector stemming from variations in all the explanatory variables. Because this exercise is partial in nature, it does not provide an explanation of the cause of cycles in the housing market which, in a full model context, exert cyclical influences on the rest of the model. It does, however, highlight the transmission of variations in the explanatory variables to the housing sector.

Figure 5 shows the cumulative variation in the series HST. Line A, which is the variation related to real effective purchasing power (UDPADJ), rises steadily to a peak in 1975, then declines to its minimum value by the end of 1979. The combined variation in income and interest rates (line B) results in a cycle in housing starts peaking in early 1973, late 1975 and late 1977. The additional variability in costs does not markedly affect the cycle in housing (line C). Seasonality, however, adds considerably to the variation in HST (line D).

The UDPADJ series exhibits a peak in 1975Q2, 4.5% above trend, declining to 7.4% below trend by the end of 1979. The income cycle creates a more magnified cycle in HST, which is 13% above control in 1975Q2, dipping to 17% below control by the end of 1979.

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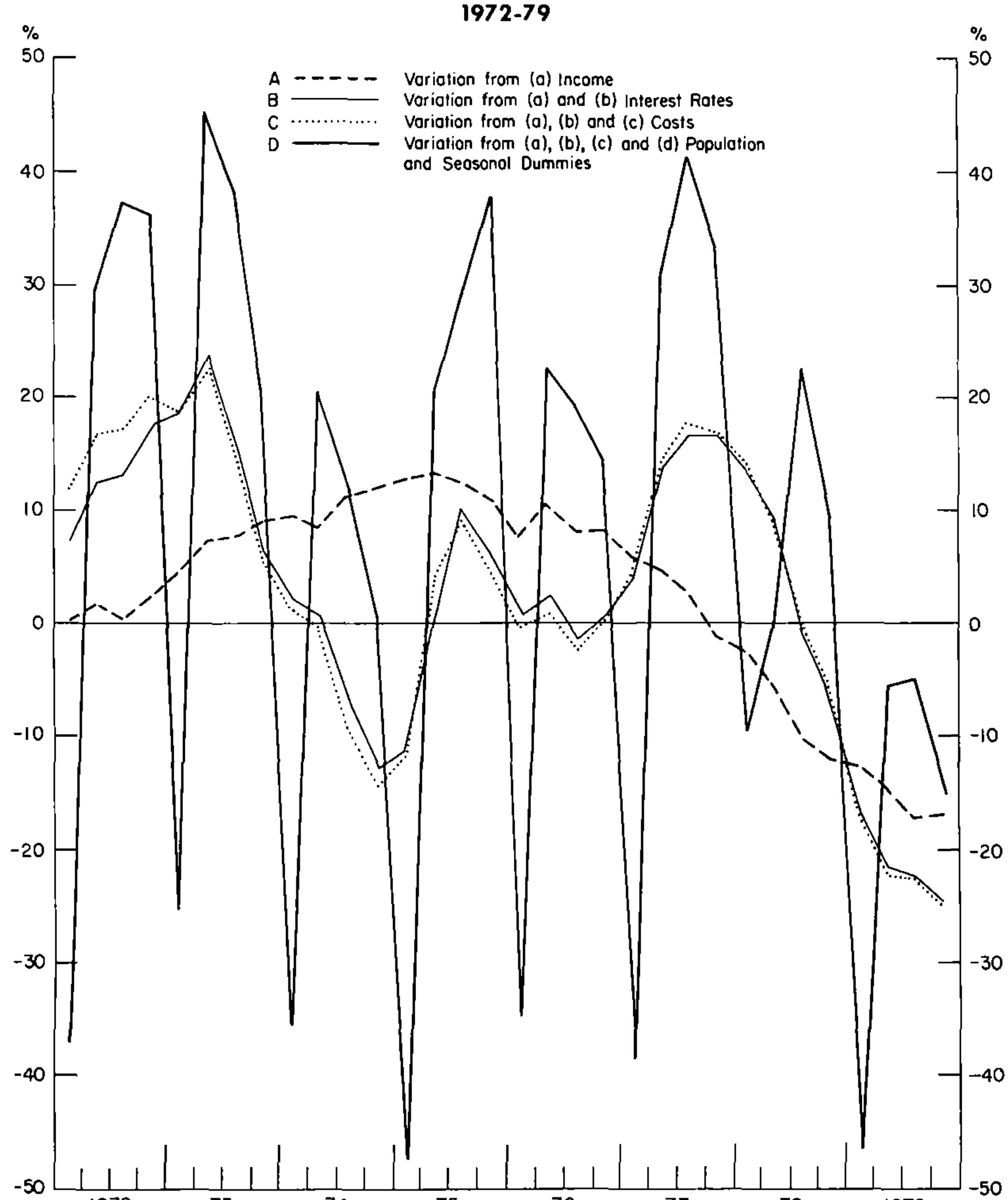
* Trend values were calculated simply by regressing a log-linear transformation of the variables on a constant and a time trend. The results are discussed in detail in the appendix to this chapter.

**Figure 5**

**DECOMPOSITION OF CYCLICAL VARIATIONS IN HOUSING STARTS 1972-79**



However, the major determinant of the pattern in the housing series is not, with one exception, the linkage through income but that through interest rates. For example, the prime rate is above trend from 73M09 to 76M11, and from 78M10 to 79M12, coinciding with weak growth in housing starts. In fact, in early 1975, the cumulative effect of interest rates and income reduces HST 12%, whereas income alone increases HST 13%. Both income and interest rates combine to explain particularly weak housing in 1979. In 1979Q1, HST declines 13% in association with weak income growth, and a further 4% as a result of high interest rates. In the first quarter of 1978 the strength in housing is explained by interest rates and the MURB dummy.

The above sources of variation do not fully explain the cycle in housing. Other determinants are estimation errors and the percentage-put-in-place coefficients, which create fluctuations in the stock of housing.

## 2.4 CONCLUSION

Improvements in the housing sector could be made along the following lines. Housing starts and the MLS price of housing, which represent the supply and excess demand equations for housing respectively, should be estimated simultaneously so that the supply terms in each equation display consistent elasticities. Further work should also be done to model the effects of demographic changes on the demand for housing. Another problem is the interest rate response in the PMLS equation, which should be negative rather than positive. And, the benefits of including real interest rate terms in the housing sector could be reexamined. Finally, the influence of the MLS price on the response to the investment and rent deflators should be examined. If there were linkages from PMLS to the investment deflator for residential construction, or possibly to the stock of wealth, the payoff to improving the structure of the PMLS equation would increase.

## Appendix to Chapter 2

## TECHNICAL DESCRIPTION OF THE SIMULATION IN THE HOUSING SECTOR

**Figure 4 Simulation of the Housing Sector**

Equations: Sector 2
Time Period: 1972Q1 - 1979Q4

**Table 4 Reduced Form Elasticities**

Equations: Sector 2, Sector 17

Time Period: 1972Q1 - 1979Q4

Shocks: a) 1% UDPADJ·1.01

b) R90+1

c) ULC·1.01

d) NPOP·1.01

e) PXLUM·1.01

**Table 5 Partial Interest Rate Elasticities**

Equations: Sector 2

Time Period: 1972Q1 - 1979Q4

Shocks: a) RMC+1

b) RPRIME+1

**Table 6 Interest Rate Response Including Income Effect**

Equations: Sectors 1, 2, 17, 18, EQY05-EQY07, EQY50-EQY79

Simulation Rules: $EACR_S = Z \cdot EACRC_C$

$EACRCS_S = Z \cdot EACRC_C$

$GTPINP_S = Z \cdot GTPINP_C$

$YGIF_S = Z \cdot YGIFC$

where $Z = [J20A(J8COL(RL10P_S))/J20A(J8COL(RL10P_C))]$

**Figure 5 Decomposition of Cycle in Housing**

Equations: Sector 2

Sample Period: 1972Q1 - 1979Q4

Shocks:
1. Set UDPADJ at trend
2. 1 + set interest rates at trend
3. 2 + set costs at trend
4. 3 + set population and dummies at trend.

**Generation of Trend Series**

Sample Period: 1979Q1 - 1979Q4

1. log(UDPADJ) = 9.83 + .013QQTIME
   $R^2$ = .96 DW = .15

2a. log(RPRIME) = .99 + .012QQTIME
   $R^2$ = .44 DW = .04

2b. log(RMC) = 1.90 + .0046QQTIME
   $R^2$ = .36 DW = .08

3a. log(PXLUM) = -2.3 + .028QQTIME
   $R^2$ = .90 DW = .24

3b. log(ULC) = -2.1 + .020QQTIME
   $R^2$ = .99 DW = .00

4a. log(NPOP) = 2.2 + .006QQTIME
   $R^2$ = .99 DW = .06

4b. QQ1 = .25 = QQ2 = QQ3 = QQ4

4c. HSSPLI = HSSPLI 721-794

4d. AH0106 = 0, AH0108 = 0

# Chapter 3

## FOREIGN TRADE
(Sectors 4 and 7)

Heather Robertson

The foreign trade sector in RDXF is critical in shaping the overall model dynamics. For example, a strong marginal propensity to import is the force that counterbalances the strength of the domestic expenditure multiplier. As well, the sensitivity of the exchange rate to variations in the Canada-United States interest rate differential determines, for the most part, the linkage between money and inflation. Finally, trade prices are the channel for transmitting foreign inflation shocks to domestic inflation.

Within the trade sector trade volumes and prices are modeled in considerable detail. However, most of the linkages from the trade sector to the rest of the model are mainly through aggregates; interaction at a disaggregated level is less well developed. In particular, domestic prices are for the most part a function of one import deflator, namely the import price of other consumer goods, and imports of producers' goods do not directly influence the supply side of RDXF.

The purpose of this chapter is to describe the detail in the trade sector and its dynamics, and to provide some suggestions for improving the linkages from the trade sector to the rest of the RDXF model. The discussion is divided into five parts: import volumes and prices; export volumes and prices; the exchange rate dynamics; the interaction of the full trade sector; and a conclusion in which suggestions for improving the trade sector are proposed.

### 3.1 IMPORTS

This section contains a discussion of the theory underlying the equations for imports of goods and services, and for import prices, followed by an analysis of dynamic elasticities of those components.

**Imports of Goods***

Imports of goods (MG) equations, which are modeled on a 1971 dollar volume basis, are disaggregated into nine categories: industrial materials (MIM, 21.4%), producers' equipment (MME, 29.2%); motor vehicles and parts from the U.S. (MMVP2, 22.2%); motor vehicles and parts from other countries excluding the U.S. (MMVP3, 2.6%); construction materials (MCM, 2.3% of MG in 1980); food (MFOOD, 5.4%); other consumer goods excluding food (MOCG, 12.7%); crude petroleum (MCRPT, 1.6%); and other fuels (MOTFL, .8%). About 75% of goods imports are made up of the MIM, MME, and MMVP2 categories.

In general, the import equations are modeled as the final demand for the respective categories, based on consumers' utility functions that include a variety of domestic and imported goods.**

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* The most recent developmental work by Bank of Canada staff on the imports volumes and prices was documented by Raynald Lētourneau based on earlier work by R. Hannah, L. Kenward, H.-H. Lau, and M. McDougall. A. London developed the services equations.

** Suppose the aggregate utility function includes both domestic ($d_i$) and imported ($M_j$) goods and is of a Cobb-Douglas functional form

$$U = \Pi d_i^{\alpha i} \Pi M_j^{\beta j}.$$

Consumers face a budget constraint

$$Y - \Sigma p_i d_i - \Sigma p_j M_j = 0.$$

Demand functions for domestic and imported goods respectively would be:

$$d_i = \frac{\alpha_i Y}{\Sigma(\alpha_i + \beta_j) P_i} \qquad (1)$$

$$M_j = \frac{\beta_j Y}{\Sigma(\alpha_i + \beta_j) P_j} \qquad (2)$$

Rearranging (1) in terms of Y and substituting into (2):

$$M_i = \frac{\beta_j d_i}{\alpha_i} \frac{P_i}{P_j}$$

The producers' goods categories (MME, MIM and MCM) have a slightly different underlying structure. The producers' equipment (MME) and construction materials equations (MCM), modeled as factor demands, are expressed as a constant proportion of the related total investment goods category, plus a capacity term to allow for short-term demand fluctuations.* Industrial materials imports (MIM), on the other hand, are expressed as a derived demand, and are functions of final sales of categories requiring industrial materials in their production processes. The general functional form of the imports equations is:

$$\log M_i = \alpha_{1i} \log PM_i/P_i + \alpha_{2i} \log U_i + \alpha_{3i} \log CAPU,$$

where $P_i = \Sigma p_j U_j$, $U_i = \Sigma u_j$

and $\alpha_{1i} - \alpha_{3i}$ are dynamic elasticities.

Imports are a function of the own price relative to a domestic price ($PM_i/P_i$), a measure of domestic activity ($U_i$), and, in the case of producers' goods, an aggregate capacity measure (CAPU). Because the domestic expenditure categories do not conform exactly to the level of disaggregation on the import side, various expenditure categories and their deflators ($U_j, P_j$) are grouped to approximate the definition of the relevant import category, as listed on the right-hand side of Table 7. Capacity (CAPU), measured as the ratio of real gross private business product to trend output (UGPP/UGPPD) is an explanatory variable in the producers' goods categories (MIM, MCM, MME) and imports of other consumer goods (MOCG).** Because this

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which is the general functional form of the import equations. Thus consumption of domestic goods and imports are determined recursively in RDXF.

* Thus the imports of producers' goods equations, following arguments similar to the footnote above, could be based on a Cobb-Douglas technology with both domestic and imported goods as factors of production.

** The capacity term in the MME equation is constrained to have the same coefficient as on the domestic activity variable.

## Table 7

### DYNAMIC ELASTICITIES FOR THE IMPORTED GOODS EQUATIONS

| | | Relative Price | Domestic Activity | (of which: CAPU) | Exports | Category of Domestic Activity |
|---|---|---|---|---|---|---|
| Industrial materials (MIM) | Impact | -.21 | 1.26 | (.77) | .06 | Household & miscellaneous durables (CHSHD, CDMIS); total gov't (GGST) less gov't inventories (IIG); CAPU |
| | 1 year | -.39 | 2.36 | (1.42) | .11 | |
| | 8 years | -.42 | 2.48 | (1.52) | .16 | |
| Producers' equipment (MME) | | -.18 | 1.21 | (.61) | -0- | Investment in machinery and equipment (IME); CAPU |
| | | -.47 | 1.74 | (.87) | | |
| | | -.54 | 2.06 | (1.02) | | |
| Motor vehicles & parts U.S. (MMVP2) | | -.86 | .48 | -0- | .56 | Consumption of motor vehicles (MV) |
| | | -.86 | .76 | | .56 | |
| | | -.86 | .76 | | .56 | |
| Construction materials (MCM) | | -.58 | 1.79 | (1.13) | -0- | Investment in residential construction (IRCA) & non-residential construction (INRC); CAPU |
| | | -.94 | 2.92 | (1.83) | | |
| | | -.96 | 2.99 | (1.89) | | |
| Food (MFOOD) | | -.38 | .60 | -0- | | Real personal disposable income (YDP/(NPOPT*PCPI)) |
| | | -.65 | 1.04 | | | |
| | | -.68 | 1.08 | | | |
| Other consumer goods excluding food (MOCG) | | -.78 | 2.22 | (.75) | -0- | Household & miscellaneous durables (CHSHD, CDMIS); other non-durables (CNDO); semi-durables (CSD), CAPU |
| | | -1.0 | 2.90 | (.98) | | |
| | | -1.0 | 2.94 | (.98) | | |
| Total goods (MG) | | -.43 | 1.06 | (.44) | .15 | |
| | | -.60 | 1.60 | (.67) | .17 | |
| | | -.70 | 1.80 | (.75) | .18 | |

MG = MCM + MCRPT* + MFOOD + MIM + MME + MMVP2 + MMVP3** + MOTFL** + MOCG + EMGRES**

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* Determined in the energy sector.

** Energy categories

aggregate measure has limited explanatory power for capacity bottlenecks at an individual industry level, dummies have been added to individual equations to proxy interindustry capacity constraints. The MIM and MCM equations include dummy variables equal to unity over the 1973Q4-1974Q4 period that represent the capacity constraint resulting from large inventory rebuilding in the manufacturing sector over this period. The dummies account for a 25% increase in MCM and an 11% increase in MIM in 1974 in excess of increases emanating from overall capacity tightness. Another dummy for 1978Q2 in the MME equation proxies the recent large increases in energy investment projects requiring imported equipment, implying a 10% increase in MME. The capacity term enters linearly in the import equations; attempts to estimate a non-linear capacity term have been unsuccessful.

The import and export motor vehicles and parts (MMVP2, XMVP2) form an interdependent system of equations; their interaction is the result of terms outlined in the auto pact agreement. Imports are a function of exports as well as domestic consumption of motor vehicles (CMV). Exports are a function of a relative price term, U.S. consumption of motor vehicles (CMV2), and CMV, the latter reflecting the effect of the auto pact. The XMVP2 equation includes two dummies, one in 1971Q1 to proxy the extensive stockpiling as a result of an auto strike, and one during the 1974-75 energy crisis when Canadian auto exports were reduced disproportionately because they generally comprised large, fuel inefficient cars. Because domestic energy prices were somewhat insulated by world price fluctuations, imports of motor vehicles were less directly affected by world energy price increases than exports were. In order to counteract the negative effect of the dummy through the XMVP2 equation, MMVP2 includes a positive dummy over the 1974-75 period, which alone explains 9.3% of the increase in MMVP2 in this period. The long-run elasticities of imports and exports of motor vehicles with respect to domestic and U.S. demand for motor vehicles are:

| | CMV | CMV2 |
|---|---|---|
| MMVP2 | 1.15 | .52 |
| XMVP2 | .7 | .92 |

Imports of motor vehicles and parts from countries other than the U.S. (MMVP3) and imports of other fuels (MOTFL) are exogenous in RDXF. Imports of crude petroleum, which do not follow the general specification in (1), are the residual item, defined as consumption, less production and exports, plus inventories in the energy sector.* Because a number of determinants of the energy sector are defined only over the forecast period, MCRPT will be assumed exogenous in the remaining analysis of this chapter.

Table 7 presented the dynamic price, domestic activity, and export elasticities for the various import categories. In general, imports display moderate longer term price elasticities over the longer term, ranging from -.4 for industrial materials to (negative) unity for other consumer goods. The price elasticity for total goods is -.7 over the longer term. The extremely strong response to domestic activity is noteworthy: the elasticity for total goods imports exceeds unity on impact, rising to 1.8 after eight years. The strongest categories are those with capacity terms, all of which have an output elasticity exceeding unity on impact. In fact, the capacity term alone produces an elasticity of .75 in the imports of goods equations over the long run. The particularly strong propensity to import plays an important role in reducing the long-run effect of domestic expenditure shocks; this is especially apparent in the government expenditure shock discussed in Chapter 10. The elasticity of MG with respect to domestic activity only is about unity over the longer run,

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* Inventories and exports of crude petroleum are exogenous. Net domestic production of crude oil, up to a policy-determined maximum level, is a function of net sales of petroleum products in Canada (COIL) and crude petroleum exports (XPET). The COIL equation includes a relative price term which is the Alberta wellhead price relative to the domestic energy deflator (POILAL/PENERG), domestic energy consumption (CENERG) and non-energy prediction (UGPP-CENERG).

implying that, for a given relative price, imports maintain a constant share of total expenditure.

The response of investment, inventories and imports of producers' equipment to a **$100 million increase in domestic production** (UGPP) is presented in Table 8, which highlights the supply response of domestic versus imported factors to short-term and longer term demand fluctuations.* The increase in UGPP creates an increase in total investment expenditures through the accelerator mechanism and an increase in imports, as both capacity utilization and investment increase.

**Table 8**

**RESPONSE OF PRODUCER CATEGORIES TO AN INCREASE IN DOMESTIC DEMAND**
(Shock minus control, $ millions)

| | Impact | 1 Year | 8 Years |
|---|---|---|---|
| Investment - machinery & equipment | 4.4 | 19.5 | 6.6 |
| Investment - non-residential construction | 1.9 | 5.1 | .2 |
| Inventories | 4.7 | 14.3 | -2.3 |
| Imports - industrial materials | 7.5 | 16.4 | 9.4 |
| Imports - construction materials | .8 | 1.7 | .6 |
| Imports - machinery & equipment | 5.3 | 16.4 | 8.7 |

Imports show a particularly strong increase on impact; in fact, the import response exceeds that for total investment. One reason for this is that imports generally are channeled directly into inventories for a period of time; only subsequently do they show up as investment goods. Inventories are $4.7 million above control on impact in this example; however, after eight years

* The simulation, which is described in detail in the appendix to this chapter includes Sector 3 and Sector 4 equations. UGPP is endogenous. The shocked component is government expenditure (GGST).

when inventories are negative, the import response of machinery and equipment continues to exceed the total investment response for that category. Imports of investment goods can exceed the total investment response because the interaction between these categories is incomplete. First of all, investment expenditure is determined by production, rather than by expected sales. Higher imports imply lower required production, which reduces total investment expenditure. However, because imports increase only temporarily because of capacity constraints, expected future sales would not be influenced by these import fluctuations, and investment expenditure should respond more strongly. Secondly, there is no direct linkage from imports to the production function; imports of machinery and equipment and construction materials are subsumed in the investment categories, and industrial materials are not an argument in the production function at all. Thus imports have no influence on capacity through increases in supply output.

**Imports of Services**

Imports of services (MSV$), which are modeled on a nominal basis, are subdivided into six categories: interest payments to foreigners (MINT$ = 17.8% of services in 1980Q1); dividend payments (MDIV$ = 11.6%); travel payments (MTR$ = 18.2%); freight and shipping (MFS$ = 13.6%); other services (MOS$ = 33.3%) and withholding taxes payables by nonresidents (TWF = 5.5%). Interest payments to foreigners, measured in Canadian dollars, are modeled as proportional to their U.S. dollar equivalent (MIDCM$), which is exogenous, and to the exchange rate. Dividend payments are a function of the Statistics Canada definition of dividends to foreigners (YDIVF), which is a component of personal income. Travel payments respond to the current value of domestic services consumption (CS$), and to the ratio of the U.S. services deflator converted into Canadian dollars to the domestic services deflator. Freight and shipping imports are a proportion of total nominal imported goods. Withholding taxes are a function of interest and dividend payments. Other services, which are composed of some short-term interest payments plus commissions, are a function of the 90-day paper rate (R90) times total short-term liabilities, nominal domestic expenditure in machinery and equipment, and the exchange rate. The reduced form dynamic elasticities for the MSV$ equation are:

| MSV$ = | PFX | DOMAC | MG$ | MIDCM$ | YDIVF | R90 | |
|---|---|---|---|---|---|---|---|
| | .54 | .10 | .07 | .11 | .12 | .0 | Impact |
| | .48 | .27 | .13 | .11 | .14 | .0 | 1 year |
| | .59 | .32 | .10 | .13 | .12 | .04 | 8 years |

where DOMAC is domestic activity.

**Import Prices**

The level of disaggregation for import prices is the same as for import volumes. In general import prices are modeled on a flexible markup basis, and are a function of U.S. costs converted to Canadian dollars, and a U.S. capacity term. Domestic market conditions are represented only in the deflator for imports of other consumer goods, which includes the domestic CPI (excluding food and energy) as well as U.S. costs. U.S. costs and prices include unit labour costs and the import price of crude petroleum plus other U.S. prices, such as the wholesale price of durables (PWDFD2), which appears in the deflators for imports of machinery and equipment and for other consumer goods, the U.S. food CPI (PFCPI2) , and the unit value index of U.S. imported foods (PMFFB2), which are arguments in the deflator for food imports (PMFOOD). The PMFOOD equation also includes a dummy equal to one in 1974 representing the influence of a commodity price boom on imported prices. The deflator for industrial materials includes a term describing the deviation of the rate of growth of the GNE deflator from its trend growth rate. This creates a strong overshoot in the response of industrial materials prices to a U.S. price shock (the impact elasticity is 1.7%, and the long-run elasticity is .8). The U.S. capacity utilization in manufactured durables represents demand pressure in the import price equations.

The reduced form equation of the deflator for total goods imports (PMG)* displays the following dynamic elasticities:

| PMG = | PUS | + PFX | + PCPIFE | + UCAPD2 | |
|---|---|---|---|---|---|
| | .8 | .9 | 0 | .1 | Impact |
| | .8 | .9 | .1 | .1 | 1 Year |
| | .9 | 1.0 | .1 | .1 | 8 Years |

where PUS represents all U.S. prices and costs, PFX is the exchange rate, PCPIFE is the CPI excluding food and energy, and UCAPD2 is U.S. capacity utilization in manufactured durables. Reported elasticities are on impact, after one year and after eight years. The PCPIFE term is an argument in the equation determining the deflator for imports of other consumer goods (PMOCG). This category includes a number of protected sectors (e.g., textiles). The inclusion of the domestic price term is based on the hypothesis that competitors in a protected market follow domestic price movements in order to maintain their market share.

With the exception of the deflator for imports of other consumer goods (PMOCG) all import deflators display an elasticity with respect to U.S. costs exceeding .8, i.e., thus are close to obtaining homogeneity in costs. The deflator for imports of other consumer goods increases only .2% on impact and .5% after

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* PMG = (PMCM·MCM+PMCRPT·MCRPT+PMFOOD·MFOOD+PMIM·MIM+PMME·MME +PMMVP2·MMVP2+PMMVP3·MMVP3+PMOTFL·PMOCG·MOCG+EMGRE$)/ (MCM+MCRPT+MFOOD+MIM+MME+MMVP2+MMVP3+MOTFL+MOCG+EMGRES). Reduced form elasticities were derived by performing a series of price and capacity shocks on a model composed of the individual import prices and PMG.

eight years. The import price elasticity with respect to the **exchange rate** is constrained to unity in all categories except PMOCG, which shows the same response to PFX as to U.S. prices. With this exception, the PFX passthrough is virtually instantaneous and one-for-one for the imported goods deflators. The response of the deflator for other consumer goods to the **domestic consumer price index excluding food and energy** (PCPIFE) is .5 over the longer term; thus PMOCG is homogeneous in Canadian and U.S. prices. The industrial materials and construction materials prices exhibit a greater degree of volatility through the U.S. capacity term. The total imported goods deflator increases .1% with a 1% increase in capacity.

In the RDXF model the import deflators directly influence import goods and domestic prices. Because there is a wide range of responses of the individual import deflators to U.S. price changes, the overall domestic price response, through variations in import prices, depends on the choice of import deflators in the domestic price equations. Most domestic prices are a function of PMOCG, which displays a very low elasticity with respect to U.S. prices, so that the transmission of U.S. inflation to domestic prices is quite weak. And, because domestic prices show a relatively weak response to U.S. prices, import volumes exhibit a relatively stronger U.S. price response.

### 3.2 EXPORTS

As in the previous section, the discussion of exports begins with the theory that underlies the equations for exports of goods and services and for export prices, and then presents the dynamic elasticities of the components.*

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* Bank economist R. Dion developed the export volume (excluding automobiles) and price equations and the U.S. price equations. Earlier work based on the RDX2 model was done by R. Hannah, L. Kenward, and M. McDougall, also of the Bank staff.

**Exports of Goods**

Exports of goods (XG) are divided into eleven categories: motor vehicles and parts to the U.S. (XMVP2 = 20% of XG in 1980Q1) and to the rest of world (XMVP3 = 19%); farm and fish products (XFFO = 7%); wheat (XW = 3%); metals and minerals (XMMO = 15.5%); chemicals and fertilizers (XCF = 5%); lumber (XLUM = 5%); forest products (XFPO = 12%); petroleum (XPET = 1%); natural gas (XNG = 1%); and other manufactured goods (XOTHER = 27%). Generally exports are modeled as loglinear demand equations, and are a function of U.S. activity terms, usually on an aggregate basis, and the own export price relative to a related U.S. price converted to Canadian dollars. All equations excluding XMVP2 are a function of an overseas activity variable, which is the weighted average of the indices of industrial production for the five major overseas countries (U.K., France, Italy, Japan, Germany), whose weights represent the volume of trade by each country in each particular export item.

A number of equations (XOTHER, XFFO, XCF, XLUM, XFPO) also include supply terms, which are expressed as the ratio of a domestic deflator to the product of the export deflator and trend output (UGPPD). The supply terms enter negatively into the export equations because (a) if domestic prices are high relative to export prices, firms prefer to supply the domestic market and exports decline, and (b) for a given relative price, if trend output is high, firms are able to supply more to foreign markets. In four of the five export equations that include a supply term, the representative domestic price is the deflator for real gross private business output (PGPP). The domestic deflator in the equation for farm and fish products exports is a weighted average of the deflators for domestic crops and livestock excluding wheat, which are in the farm sector. Also in XOTHER, an exchange-rate-adjusted U.S. price is used instead of the export price, because the latter partly depends on domestic unit labour costs.

In some cases, United States activity in the export equations is represented by U.S. real GNE. Exceptions are the exports of motor vehicles, a function of U.S. consumption of motor vehicles; exports of farm and fish products, a function of U.S. non-durables consumption; exports of lumber, a function of U.S. housing starts as well as U.S. real GNE; and exports of other manufactured goods, a function of both U.S. non-residential investment and U.S. real GNE.

The exports of crude petroleum (XPET), exports of natural gas (XNG), exports of wheat (XW), and exports of motor vehicles other than to the U.S. (XMMVP3) categories are exogenous.

Table 9 presents the dynamic elasticities for each category of exports of goods. With the exception of chemicals and fertilizers (XCF), all export categories show a negative response to their **own price**. The response is positive for chemicals and fertilizers because the own price enters only in the supply term, the only category displaying an elasticity exceeding (negative) unity is fish and farm products. **Domestic prices and costs** (the domestic output deflator (PGPP) and domestic unit labour costs (ULC)) exert a stronger influence on XCF (elasticity = -1.3) than on any other category. In fact, the response of XCF to domestic prices outweighs the (positive) supply response so that substitution effects on the demand side remain much stronger than on the supply side. Because the domestic prices argument is in first-difference form in the exports of lumber, an increase in domestic prices is only transitory. The XFFO and XFPO categories have very weak domestic price elasticities.

The **U.S. activity variables** influence total exports of goods with an elasticity of near unity (.9), implying that exports have maintained almost a constant share of U.S. demand. Individual components differ widely, however, as low as .7 for metals and minerals and as high as 2.1 for farm and fish. The **overseas activity elasticity** is generally quite weak. For XG the elasticity is .3, implying a large erosion of the share of Canadian exports in overseas demand. This in part reflects the fact that auto exports a large component of total exports, do not react to overseas activity. The only component near unity are

Table 9

**DYNAMIC ELASTICITIES IN THE EXPORT SECTOR**

| | | Own Price | Domestic Price | U.S. Price | U.S. Activity | Foreign Activity | Domestic Capacity | Domestic CMV |
|---|---|---|---|---|---|---|---|---|
| Chemicals & fertilizers (XCF) | Impact | .3 | -1.3 | .9 | 1.5 | 1.0 | .3 | .0 |
| | 1 year | .3 | -1.3 | .9 | 1.5 | 1.0 | .3 | .0 |
| | 8 years | .3 | -1.3 | .9 | 1.5 | 1.0 | .3 | .0 |
| Fish & farm (XFFO) | | -1.5 | -.1 | 1.4 | 2.1 | .4 | .0 | .0 |
| | | -1.5 | -.1 | 1.4 | 2.1 | .4 | .0 | .0 |
| | | -1.5 | -.1 | 1.4 | 2.1 | .4 | .0 | .0 |
| Forest products (XFPO) | | -.6 | -.2 | .8 | .0 | .0 | .2 | .0 |
| | | -.6 | -.2 | .8 | .7 | .3 | .2 | .0 |
| | | -.6 | -.2 | .8 | .7 | .3 | .2 | .0 |
| Lumber (XLUM) | | -.3 | -.4 | .7 | 1.3 | 1.1 | .4 | .0 |
| | | -.7 | .0 | .7 | 1.3 | 1.1 | .0 | .0 |
| | | -.7 | .0 | .7 | 1.3 | 1.1 | .0 | .0 |
| Metals & minerals (XMMO) | | -.6 | .0 | .6 | .7 | .5 | .0 | .0 |
| | | -.6 | .0 | .6 | .7 | .5 | .0 | .0 |
| | | -.6 | .0 | .6 | .7 | .5 | .0 | .0 |
| Motor vehicles & parts - U.S. (XMVP2) | | -.4 | .0 | .4 | .4 | .0 | .0 | .0 |
| | | -.8 | .0 | .8 | .8 | .0 | .0 | 4.0 |
| | | -.9 | .0 | .9 | .9 | .0 | .0 | 7.0 |
| Other goods (XOTHER) | | -.3 | -.2 | .5 | 1.1 | .4 | .2 | .0 |
| | | -.3 | -.2 | .5 | 1.3 | .4 | .2 | .0 |
| | | -.3 | -.2 | .5 | 1.3 | .4 | .2 | .0 |
| Total goods (XG) | | | -.1 | .6 | .7 | .3 | .1 | .0 |
| | | | -.1 | .6 | .9 | .3 | .1 | .1 |
| | | | -.1 | .6 | .9 | .3 | .1 | .1 |

XG = XFFO + XW* + XMMO + XPET* + XNG* + XCF + XFPO + XLUM + XOTHER + XMVP2 + XMPV3* + EXGRES*

* Exogenous categories

lumber (1.1) and chemicals and fertilizers (1.0). With the exception of exports of chemicals and fertilizers, which includes a domestic unit labour cost term as well as the capacity-scaled supply price term, trend output enters with a coefficient in inverse proportion to domestic prices.

**Exports of Services**

The four exports of services categories are modeled on a nominal basis: travel (XTR$ = 26.6% of total services in the 1980Q1); freight and shipping (XFS$ = 29.1%); other services (XOS$ = 34.9%) and interest and dividends (XID$ = 9%). Exports of travel payments are a function of U.S. consumption of services and the ratio of the U.S. and Canadian services deflators. Freight and shipping is proportional to the nominal exports of goods. Other services, which comprise short-term interest payments and commissions, are a function of the U.S. Treasury bill rate and U.S. nominal GNE converted to its Canadian dollar value. Interest and dividends are a function of direct investment to the rest of the world (IDROW) Canadian foreign exchange reserves (in Canadian dollars), and the U.S. long-term interest rate. The reduced form elasticities for services are:

| XSV$ = | USACT | + IDROW | + R2 | + PCS | + PFX | |
|---|---|---|---|---|---|---|
| | .16 | .01 | .05 | .27 | .3 | Impact |
| | .94 | .00 | .10 | .07 | .7 | 1 year |
| | 1.45 | .02 | .51 | -.05 | 1.0 | 8 year |

where USACT is U.S. activity and R2 is U.S. interest rates.*

**Export Prices**

Export prices are, in general, modeled as a function of a related U.S. price converted to its Canadian dollar equivalent. Exceptions are the deflators for metals and minerals exports

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* In this example, interest rates are reported as elasticities rather than semi-elasticities. The response of XO$$ to the U.S. Treasury bill rate is extremely strong. This has been modified in the June 1981 version of RDXF.

(PXMMO) and for exports of other goods (PXOTHE), which are modeled directly as a function of U.S. costs and capacity, and the deflator for motor vehicles and parts exports to the U.S. (PXMVP2), which is a function of domestic costs and the related import prices (PMMVP2).* The U.S. prices, in turn, are modeled as a function of U.S. unit labour costs (ULC2), unit capital costs (UKC), the U.S. import price of crude petroleum (PMCR2) and U.S. demand pressure variables. The U.S. price equations approach linear homogeneity in the U.S. costs. The two measures of demand pressure are capacity utilization in total manufacturing (UCAPT2), which enters as a non-linear function in some equations (1/(1-UCAPT2/100)) and the ratio of unfilled orders to shipments (UORDS2). Most deflators include a correction for autocorrelation. The deflators for wheat exports (PXW), natural gas exports (PXNG) and motor vehicles and parts to the rest of world (PXMVP3) are exogenous.

The deflator for total exports of goods (PXG)** displays the dynamic elasticities below.

**Table 10**

DYNAMIC ELASTICITIES OF THE EXPORT SECTOR

| Elasticity with respect to: | Impact | 1 Year | 8 Years |
|---|---|---|---|
| U.S. wage costs (ULC2) | .20 | .35 | .43 |
| U.S. import price of crude petroleum (PCMR2) | .07 | .12 | .13 |
| U.S. GNE deflator (PGNE2) | .01 | .01 | .01 |
| U.S. capital costs (UCC2) | .01 | .03 | .04 |
| U.S. capacity (UCAPT2) | .17 | .38 | .54 |
| Exchange rate - Cdn.$/U.S.¢ (PFX) | .58 | .75 | .79 |
| Domestic unit labour costs (ULC) | .12 | .13 | .15 |

* The PMMVP2 term proxies materials costs, as a large portion of imports in this category is parts and engines.

** PXG = XG$/XG = (XM•PXW+XFFO•PXFFO•PXFFO+XCF•PXCF+XMMO•PXMMO+XFPO•PXFPO+XOTHER•PXOTHE+XPET•PXPET+XNGXNG+XLUM•PXLUM+XMVP2•PXMVP2+XMVP3•PXMVP3+EXGRE$)/(XFFO+XW+XMMO+XPET+XNG+XCF+XFPO+XLUM+XOTHER+XMVP2+XMVP3+EXGRES). Reduced form elasticities for PXG were calculated by a series of simulations of a model composed of the individual export deflators and PXG.

The elasticity of the deflator for goods exports with respect to **U.S. wage costs** is .4 after eight years. The individual components display a large variation, however, ranging from .15 for the deflator for exports of other manufactured goods to as high as .93 for the export price of lumber. The overall elasticity with respect to **energy prices** is .13; most categories display an elasticity of about .21, with the exception of the deflator for petroleum exports, in which the elasticity is .9. (The aggregate is lower because exogenous prices show no response.) An increase in **the GNE deflator** produces a .26% increase in the export price of petroleum products after 8 years. The aggregate deflator is only .01% higher. **Unit capital costs**, through their influence on some U.S. prices, create a .1%-.2% increase in the deflators for forest products exports (PXFPO) and other goods exports (PXOTHE). Because these are the only prices that respond, the net effect on the total export deflator is only .04. The elasticity of the deflator for goods exports with respect to all U.S. costs is .6 after eight years.

Export deflators are close to having a unitary elasticity with respect to the **exchange rate.** Deviations from long-run unitary elasticity are the deflator for chemicals and fertilizers exports (.7) and for the exports of other goods (.3).

### 3.3 THE FOREIGN EXCHANGE SECTOR*

The essence of the exchange rate sector is in the following three equations:

$$PFX = f(R90-RCP2, ULS, PGNE/PGNE2, XBAL\$/YGNE-XBAL\$2/YGNE2) \quad (1)$$

$$\begin{aligned} ULS &= FXO+BRESB-UBAL \\ &= FXO+BRESB-(XBALR\$+F1L) \end{aligned} \quad (2)$$

$$FXO = g(PFX) \quad (3)$$

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* Bank of Canada economist M. Kennedy developed the exchange rate equations, basing some of his work on results by R. D. Haas and W. E. Alexander [37]. A more recent system of equations has been proposed by K. McPhail and D. Longworth, also on the staff of the Bank.

The exchange rate (PFX) is a function of the differential between domestic and U.S. interest rates (R90-RCP2), outstanding net international short-term liabilities between Canada and the rest of the world (ULS), an absolute purchasing power parity term, which is the ratio of the domestic to the U.S. GNE deflators (PGNE/PGNE2) and the difference between the Canadian and the U.S. ratio of the current account to nominal GNE. The change in net international short-term liabilities is defined as the sum of the official excess demand for spot exchange (FXO) plus Government of Canada net borrowings in international capital markets (BRESB), less the basic balance (UBAL), which in turn is the sum of the current account balance (XBALR$) and long-term capital flows (FIL). Both BRESB and FIL are exogenous.

An increase of 100 basis points in the **Canadian-U.S. short-term interest rate differential** creates a 1.3% appreciation in the exchange rate (-.5% on impact) when the exchange rate is around unity. Because the exchange rate equation is expressed in levels (rather than a loglinear function) the elasticity varies depending on the value of PFX. By the end of 1979 the exchange rate appreciates only 1.1%.

A $1 billion reduction in the **stock of outstanding debt** (ULS) per quarter (stemming from an increase in long-term capital flows) creates a 4% appreciation per annum, when the exchange rate is unity, and a 2.3% appreciation after eight years. The **purchasing power parity term** has no influence on the exchange rate on impact. The long-run effect of a 1% increase in the domestic deflator relative to the U.S. is a 1 percentage point depreciation. The speed of adjustment is very slow.

## 3.4 DYNAMICS OF THE FULL TRADE SECTOR

This section describes the interaction in the full trade sector in response to two simulation experiments: a 1% depreciation of the exchange rate and a $1 billion improvement in the current account. Simulations were performed over an eight-year period, from 1972 to 1979, on a model composed of all export and import volumes and prices, domestic wages and prices, and, for the second simulation, the exchange rate sector.

The dynamic response to an **exchange rate depreciation** is presented in Table 11. Import prices increase .9% on impact and 1% after eight years; recall that most categories of import deflators are constrained to have a unit elasticity with respect to the exchange rate. Export prices, which respond directly to the exchange rate also show a further increase as a result of higher domestic unit labour costs. The export goods deflator increases .82% after eight years. The domestic consumer price index increases .3% after eight years; because a number of domestic deflators are a function of import prices of other consumer goods (PMOCG), which increases only .69% after eight years, the response of the domestic deflators to foreign price shocks is tempered somewhat.* The GNE deflator responds more strongly--.21% after one year and .37% after eight years--because the investment deflators, as well as the export and import deflators, are more sensitive to exchange rate changes.

Real imports of goods decline .3% over the entire simulation. The reduction is tempered somewhat by the increase in domestic prices. The weakest categories are industrial materials (MIM), which has a low price elasticity, and machinery and equipment (MME) and motor vehicles and parts (MMVP2), where the domestic deflators in the relative price terms, PIME and PCMV, display a strong response to an exchange rate devaluation.

Real exports of goods, which increase .3 on impact, show a weaker response over the longer term for two reasons: the offsetting supply response in the XOTHER, XFFO, XCF, XLUM, and XFPO categories and the rise in domestic unit labour costs, which reduces the competitiveness of XOTHER, a large component of total

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* The CPI response is considerably understated because energy, farm gate and capital costs are exogenous. If these prices were linked to the exchange rate, the elasticity of the CPI with respect to the exchange rate would approach unity.

Table 11

**RESPONSE TO AN 1% EXCHANGE RATE DEPRECIATION**
**FULL TRADE MODEL WITH WAGES AND PRICES ENDOGENOUS**
(Shock minus control, per cent)

| | Impact | 1 Year | 2 Years | 8 Years |
|---|---|---|---|---|
| Volumes | | | | |
| Imports-Total | -.23 | -.32 | -.33 | -.28 |
| -Goods | -.28 | -.33 | -.34 | -.32 |
| Exports-Total | .25 | .25 | .26 | .27 |
| -Goods | .28 | .21 | .18 | .17 |
| Prices | | | | |
| Imports-Total | .86 | .93 | .96 | .99 |
| -Goods | .92 | .94 | .97 | 1.0 |
| Exports-Total | .51 | .67 | .71 | .74 |
| -Goods | .58 | .75 | .79 | .82 |
| CPI | .05 | .13 | .18 | .29 |
| G.N.E. deflator | .14 | .21 | .25 | .37 |
| Current account (level) | 27.3 | 83.7 | 106.6 | 277.2 |

exports. As domestic prices increase, firms reduce their supply abroad; exports of goods are only .17% above control after eight years. The current account shows a $84 million improvement after one year (real net exports are $144 million higher) rising steadily to $277 million after eight years.

A $1 billion permanent increase in the current account (Table 12) was implemented by increasing nominal exports of wheat, which is an exogenous component of total exports, by $1 billion. The initial impact is on the exchange rate, which appreciates as outstanding net international short-term liabilities (ULS) decline, reinforced initially by an increase in the difference between the Canadian and U.S. rates of the current account to nominal GNE (XBAL$/YGNE-XBAL$2/YGNE2) and subsequently by the induced improvement in purchasing power parity. The exchange rate appreciates 1.1% after one year, rising steadily to 5% after eight years.

With the exchange rate appreciation, the competitive position

**Table 12**
**RESPONSE TO A $1 BILLION EXOGENOUS INCREASE IN THE CURRENT ACCOUNT**

| | Impact | 1 Year | 2 Years | 8 Years |
|---|---|---|---|---|
| | Level | | | |
| Exports - Nominal | | | | |
| Goods (XG$) | 830.1 | 654.6 | 424.2 | -2529.5 |
| Services (XSV$) | 34.9 | 25.5 | -16.6 | -689.4 |
| Total (X$) | 864.9 | 680.2 | 407.6 | -3218.9 |
| Total excluding wheat | -135.1 | -319.8 | -592.4 | -4218.9 |
| Imports - Nominal | | | | |
| Goods (MG$) | -50.7 | -136.4 | -266.8 | -1997.0 |
| Services (MSV$) | -18.8 | -78.1 | -203.7 | -1493.8 |
| Total (M$) | -69.6 | -214.5 | -470.5 | -3490.8 |
| Current account (XBAL$) | 934.5 | 894.6 | 878.2 | 271.9 |
| Basic balance (UBAL)* | 274.9 | 242.1 | 237.2 | 73.4 |
| Short-term liabilities* (ULS) | -198.3 | -700.3 | -1435.1 | -4971.7 |
| Exports - Real | | | | |
| - Goods | 888.1 | 687.4 | 255.4 | 7.1 |
| - Total | 925.6 | 722.2 | 263.9 | -190.2 |
| Total excluding wheat | -75.3 | -107.7 | -160.7 | -504.2 |
| Imports - Real | | | | |
| - Goods | 20.1 | 59.2 | 105.1 | 375.7 |
| - Total | 26.8 | 49.3 | 26.3 | 208.0 |
| Net exports | 898.8 | 672.9 | 237.6 | -398.2 |
| | Per cent | | | |
| Exchange rate Cdn¢/US$ | -45. | -1.09 | -1.61 | -4.95 |
| Export deflator - Total | -.43 | -.48 | -.18 | -3.2 |
| - Goods | -.51 | -.53 | -.29 | 3.6 |
| Import deflator - Total | -.40 | -.96 | -1.40 | -4.76 |
| - Goods | -.42 | -1.00 | -1.55 | -5.07 |
| Consumer Price Index | -.02 | -.11 | -.22 | -1.14 |

* unadjusted

of domestic firms is eroded, thereby creating a reduction in real exports net of the wheat component and an increase in real imports. Total real exports net of wheat decline $504 million after eight years; this is such a substantial decline that total exports are $190 million below control after eight years. The major contributions to the decline are exports of other manufactured goods (XOTHER) and exports of motor vehicles and parts (XMVP2). The major portion of the $689 million decline in nominal services exports after eight years is other services (XOS$), which are linked directly to the exchange rate with a long-run elasticity of 1.5.

Import prices decline 4.8%, whereas export prices are only 3.2% lower after eight years. Because of the overall improvement in the terms of trade the current account shows an improvement over the entire simulation period, and is $272 million above control after eight years.

### 3.5 CONCLUSION

Since the September 1980 version of RDXF, the exchange rate has been modified. The new equation is of the same general specification but has improved forecasting ability. Also, the U.S. price equations have been removed from the RDXF model and are now calculated directly in the U.S. forecasting model. To complement the proposed revisions to the supply side of RDXF, the equations for imports of producers' goods may be tied in more closely to the investment sector.

## Appendix to Chapter 3

## TECHNICAL DESCRIPTION OF SIMULATIONS IN FOREIGN TRADE
(Sectors 4 and 7)

**Table 1 Dynamic Elasticities for the Imported Goods Equations**

Equations: Sector 4
Time Period: 1972Q1 - 1979Q4
Shocks: 1% increase to:
1. PMME, PMFOOD, PMCM, PMIM, PMOCG, PMCRPT, PMOTFL, PMMVP2, PMMVP3
2. YDP, IRCA, INRC, CHSHD, CDMIS, UGPD, IIG, IME, IIG$, GGST, GGST$, CNDO, CSD, CDMIS, CMV, CENERG
3. UGPPD/1.01
4. XMVP2, XOTHER

**Imports and Exports of Motor Vehicles**

Equations EQF06, EQF13
Time Period: 1972Q1 - 1979Q4
Shocks: 1. CMV•1.01
2. CMV2•1.01

**Table 2 Response of Producer Categories...**

Equations: Sector 3; EQF01, EQF02, EQF04, EQ459, EQF52
Time Period: 1972Q1 - 1979Q4
Shocks: GGST + 100

**Dynamic Elasticities of Services Imports**

Equations: Sector 4
Time Period: 1972Q1 - 1979Q4
Shocks: 1. PFX • 1.01
2. (CS, IME) • 1.01
3. MG$ • 1.01
4. MIDCM$ • 1.01
5. YDIVF • 1.01
6. R90 • 1.01

**Dynamic Elasticities of the Import Goods Deflator**

Equations: EQP20 - EQP25, EQP28, EQP31, EQP32, EQP56 - EQP58, EQ1, PMG = MG$/MG
Time Period: 1972Q1 - 1979Q4
Shocks: 1. (PCS2, PWDMD2, PMFFB2, PFCPI2, ULC2, PMCR2, YGNE2, PMM3US) • 1.01
2. PFX • 1.01
3. PCPIFE • 1.01
4. UCAPD2 • 1.01

**Dynamic Elasticities of the Exports Sector**

Equations: EQF07-EQF14, EQF16, EQF18, EQF22, EQF54-EQF58, EQF60, EQF62
Time Period: 1972Q1 - 1979Q4
Shocks: <u>Goods</u>

1. (PXFF, PXW, PXMMO, PXPET, PXCF, PXFPO, PXLUM, PXOTHE, PXMVP2, PXMVP3, EXGRE$, PXNG) • 1.01
2. (PGPP, PFPIC, PFPIL, UCCR, ULCR, ULC) • 1.01
3. (PMCGA2, PDUR2, PWFF2, PWMM2, ULC2, PWLUM2, PWPP2) • 1.01
4. (IPENR2, UGNE2, CMV2, CND2, HSTT2) • 1.01
5. (UINUK, UINJA, UINIT, UINFR, UINGE) • 1.01
6. UGPPD • 1.01
7. CMV • 1.01

<u>Services:</u>

8. (CS$2, XG$, YGNE2, IPENR2, IPDER2) • 1.01
9. IDROW • 1.01
10. (RTB2, RL2) • 1.01
11. PCS • 1.01
12. PFX • 1.01

**Dynamic Elasticities of Export Prices**

Equations: EQP14-EQP18, EQP26, EQP28-EQP30, EQP34-EQP38, EQP40, EQP55-EQP57, EQP63
Time Period: 1972Q1 - 1979Q4
Shocks:

1. (ULC2, ULCAG2, WMD2) • 1.01
2. PMCR2 • 1.01
3. YGNE2 • 1.01
4. UKC2 • 1.01
5. UCAPT2 • 1.01
6. PFX • 1.01
7. ULC • 1.01

**Dynamics in the Full Trade Model**

Equations: Use Sectors 4, 7 and 6, EQY52, EQY53
Time Period: 1972Q1 - 1979Q4
Shocks:

1. PFX • 1.01
2. XW$ + 1000

# Chapter 4

## GOVERNMENT
## (Sectors 9, 10, 11 and 14)

Michael McDougall

The government sector of the model* consists of a variety of equations that determine the revenues and expenditures of the federal and provincial-municipal levels of government, along with some exogenous variables describing the Canada and Quebec pension plans, investment income of the three levels of government and hospitals, and various governmental transfers and subsidies. Of the levels of government the federal level is by far the most detailed.

The first section of this chapter describes the revenue side of the balance while the second section deals with expenditures, highlighting the operation of the unemployment insurance fund. The final section describes the federal government financing rule and the equation for the interest payments on the federal debt.

### 4.1 GOVERNMENT REVENUE

In this section the categories that make up government revenue are described: personal income taxes, corporate taxes and indirect taxes (Sectors 9 and 10 in the model). Included in the income tax section is an example of the effect on government revenues of indexing personal exemptions.

#### Personal Income Taxes

Personal income taxes are a major source of revenue for the federal and provincial governments and, of course, are an important determinant of personal disposable income. Many of the equations in the sector are identities that reflect the structure

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* Much of the developmental work was done by G. Stuber [82-86], P. Dattels and S. Wiseman [17 and 18].

of the tax system. Where stochastic equations are used, the estimation period starts in 1973 since substantial changes were made in the tax system in 1972.

The calculation of total federal income tax collections makes use of the federal tax table and requires the number of taxable returns, total taxable income and the indexing factor.

The **number of taxable returns** (NTFT) is a function of the level of employment in the economy (NE) with a coefficient of 1.03,* thus assuming under simulation that all employed persons submit a tax return. Correct calculation of NTFT would require a model of the distribution of income. However RDXF, unlike RDX2, does not model income distribution because of the cost involved and because the distribution remains fairly stable over the short-run forecasting range of one to five years.

**Total taxable income** (YPTAX) is defined as assessed income less deductions and personal exemptions. Assessed income (YPASS) is composed of the taxable components of personal income: wages and salaries, personal investment income, non-farm unincorporated business income, farm income, unemployment insurance benefits, family allowances, old age security pensions and Canada and Quebec pension plan pensions. These personal income components are then scaled by assessment ratios in order to determine the portion that actually appears on taxable returns. The 1979Q4 value of the assessment ratio for each category is presented in Table 13. The ratios remain relatively stable over time for most categories and range from a high of 92% for wages and salaries to a low of 17% for old age security pensions.

Deductions are exogenous to the model while personal exemptions are a function of the basic personal exemption, the number of basic exemptions claimed per return (both of which are

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* The number of taxable returns is published only annually and a regression was run over an annual sample range to estimate the relationship between NTFT and NE.

exogenous), the number of taxable returns, and the indexing factor (RTI). Since 1974, personal exemptions have been indexed to increase by an inflation factor that is equal to the annual percentage change in the CPI for the twelve months ending September.

**Table 13**

**ASSESSMENT RATIOS FOR THE CATEGORIES OF ASSESSED INCOME**
(1979Q4)

| | |
|---|---|
| Wages and salaries | 92 |
| Personal investment income | 80 |
| Non-farm business income | 68 |
| Farm income | 50 |
| Unemployment insurance benefits | 68 |
| Family allowances | 78 |
| Old age security pensions | 17 |
| Canada and Quebec pension plans | 39 |

In order to use the federal tax tables, total taxable income is converted to the taxable income of the representative taxpayer (YPTRTA). Although YPTRTA is not exactly equal to per capita taxable income due to skewing of income distribution, the following relationship is used as a close approximation:

$$YPTRTA = \alpha(YPTAX/NTFT) \qquad (1)$$

Given the taxable income of the representative taxpayer, the basic federal income tax paid (TPYFBA) is calculated from an equation representing the federal tax table where the individual tax brackets are adjusted each year by **the indexing factor.**

Finally, total federal income tax collections (TPYF) are determined by multiplying the number of taxable returns by the basic tax paid by the representative taxpayer. As well, account is taken of the Quebec income tax abatement and other reductions.

Provincial income tax collections excluding Quebec (TPYPXQ) are determined by applying a weighted average provincial personal income tax rate to total federal collections (TPYF) with allowance made for special provincial tax reductions.

Quebec income tax collections are modeled in a similar fashion to the federal counterpart where use is made of the tax table. The number of Quebec taxpayers (NTFTQ) and the level of assessed income (YPASSQ) are determined by taking a fixed proportion, 25.5% and 24.5% respectively, of NTFT and YPASS to reflect Quebec's share of the Canadian totals.* Taxable income (YPTAXQ) is assessed income less Quebec deductions and personal exemptions. Personal exemptions only became indexed in 1980, and the indexing factor is exogenous in the model. The taxable income of the representative taxpayer (YPTRTQ), which is related to per capita Quebec taxable income, is then run through the tax table equation to calculate the Quebec personal income tax payable by the representative taxpayer (TPYQA); the tax brackets of the tax table are not indexed. Finally, Quebec personal income tax collections are determined by multiplying the representative taxpayer's tax payable by the number of Quebec tax returns.

Total personal income tax collections (TP) are the sum of the federal, Quebec and other provincial collections.

The elasticities of the personal income tax model with respect to a 1% increase in assessed income are presented in Table 14. The first column shows the response to a **1% increase in real assessed income.** Taxable income increases 1.4%. Although deductions increase in proportion to real or nominal assessed income, the elasticity exceeds unity because personal exemptions, which are a function of the indexing factor, increase only with the rate of inflation and remain unchanged in this shock. Total personal tax collections increase nearly 1.9%, an increase of over $500 million in 1979. The federal share is $300 million, while

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* It is recognized that these scaling factors will change over time.

Quebec and the other provinces contribute about $100 million each.

Table 14

**RESPONSE OF PERSONAL INCOME TAX COLLECTIONS TO AN INCREASE IN ASSESSED INCOME**
(Shock minus control, per cent, 1979)

| | 1% increase in real assessed income | 1% increase in nominal assessed income |
|---|---|---|
| Taxable income | 1.41 | 1.0 |
| Federal tax collections | 1.87 | 1.2 |
| Provincial ex. Quebec collections | 1.5 | 1.08 |
| Quebec collections | 2.18 | 2.18 |
| Total personal tax collections | 1.87 | 1.34 |

The second column in the table shows the response to a 1% increase in assessed income combined with a 1% rise in the indexing factor, implying a **1% increase in nominal assessed income.** Exemptions and deductions increase 1%, producing a 1% increase in taxable income. Thus the indexing factor distinguishes real income shocks from nominal income shocks, i.e., those associated with an increase in the rate of inflation. For 1979, total personal collections increase 1.3% or $365 million; Quebec tax collections, which are not indexed automatically, increase 2.2% in both shocks.

To estimate the direct cost of indexing on the government balance, a simulation of the tax sector alone was performed eliminating the indexing factor. Without indexing, federal income tax collections would have been $6.3 billion more in 1979 and provincial, excluding Quebec collections, would have been $2.4 billion more.

**Corporate Taxes**

Total corporate tax accruals (TCA) are the sum of federal

(TCAF) and provincial (TCAPM) accruals. Accruals are calculated by scaling taxable corporate profits by a weighted average corporate tax rate. In order to approximate taxable profits, various deductions and exemptions must be applied to before-tax profits calculated on a National Accounts basis (YC). We do not employ a separate equation for taxable corporate profits in the model but rather estimate a relationship between taxable profits and YC of the following form over the sample range 1965-75,

$$YCTAX = -681 + .87\ (YC - QCORP \cdot IME \cdot PIME + TAXROY), \qquad (2)$$

and insert it into the TCAF and TCAPM equations. The term QCORP·IME·PIME measures the accelerated depreciation allowances for investment in manufacturing and processing activity, which are deducted from before-tax profits, while TAXROY captures non-deductible provincial royalties less resource allowances, which are added to profits. To this expression is added a further deduction available to corporations: beginning in March 1977, firms are allowed to deduct 3% of the opening value of nominal inventories from profits. The weighted average federal corporate tax rate (RTCAF) is then applied to the resulting total.

In the third quarter of 1975 an investment tax credit for new investment in manufacturing and resource industries was instituted. The tax credit base is determined by scaling total nominal business investment by the proportion accounted for by manufacturing and resource industries. The base is then scaled by the weighted tax credit rate (RITC). The investment tax credit is used only if the corporation pays tax. If corporate taxes are not paid in a certain year the tax credit may be carried forward for a maximum of five years. The carry-forward portion of the investment tax credit is embodied in the endogenous variable TRMITC.

Included in the equation as an explanatory variable is the year-over-year change in corporate profits which indicates that when corporate profits are increasing corporations use various tax measures to reduce taxable income.

The tax base used in the calculation of provincial corporate tax accruals is similar to that used in the federal case, except that there is no investment tax credit and corporations do not add non-deductible provincial royalties less the resource allowance to profits. A weighted average provincial corporate tax rate is then applied to the calculated level of taxable profits to produce TCAPM. As in the federal case, the change in corporate profits enters as an explanatory variable.

**Indirect Taxes**

The indirect tax sector is disaggregated both at the federal and at the provincial-municipal levels. Indirect taxes are specified as an appropriate tax base multiplied by a weighted average tax rate. Care has been taken to introduce various tax law changes into the relevant equations.

At the federal level six indirect taxes are modeled, and a seventh miscellaneous category, (TIFMIS), is exogenous. The most important of these is the **manufacturers sales tax** (TISF), which contributes almost 40% of the total indirect tax revenue of the federal government. The tax base is composed of three major classes of goods, each with its own average tax rate: production machinery, construction materials and finally consumer expenditure on goods excluding services. The basic manufacturers sales tax is applied to the final category where account has been taken of tax exemptions on clothing, footwear and food as well as the change from a specific sales tax on gasoline to an ad valorem tax.

Federal **customs duties** (TICUSF), 24% of indirect tax revenue, are represented by a tax base composed of nominal imports of goods excluding imports of crude oil and motor vehicles and parts from the United States. The estimated tariff rate on this broad group of goods, after taking account of general tariff reductions introduced in 1969 and 1973, is slightly over 9%.

The federal **oil export tax** (TIOILF) was introduced in

1973 and is represented by a tax base composed of nominal exports of crude petroleum less exports under the swap agreements with the U.S. The export tax rate is calculated as the difference between the export price of crude petroleum in Canadian dollars and the domestic Alberta well-head price.

Federal **excise tax on gasoline** (TIGASF) was imposed in 1975 at a rate of 10¢ per gallon as a means of conserving gasoline and to help finance the federal oil import subsidy. Real consumer expenditure on energy proxies for gasoline consumption. Federal **excise duties** (TIEXDF) are applied to domestic alcoholic beverages and tobacco products, and thus real consumer expenditure on non-durables excluding food and energy serves as the tax base. **Miscellaneous excise duties** (TIOEXF) compose the final federal indirect tax.

On the provincial-municipal side three indirect taxes are modeled. The provincial **retail sales tax** (TISP) is modeled to include two tax bases and rates. A weighted average of the provincial retail sales taxes (RTISPM) is applied to most consumer goods excluding food and energy and to most investment goods, while a second rate (RTISPA) is levied on consumption of alcohol and tobacco, an important source of provincial revenue. Other **provincial indirect taxes** (TIOP), a miscellaneous category including gasoline taxes, liquor commission profits, motor vehicle fees and various natural resource taxes, are represented by a tax base composed of nominal consumption of goods. Each of these provincial taxes (TISP and TIOP) accounts for about 30% of total indirect tax collected by the provincial-municipal governments. **Municipal indirect taxes** (TIM) consist mainly of property taxes and are a function of nominal provincial and municipal expenditure on goods and services on the assumption that municipal indirect taxes are raised to meet any required financing.

Federal and provincial-municipal indirect taxes combine with government subsidies to produce total indirect taxes less subsidies (TILGS) which is used in the calculation of nominal gross private business product at factor cost.

## 4.2 GOVERNMENT EXPENDITURE

This section contains a brief description of government expenditure and includes transfers to persons (Sector 11) excepting employment insurance, which is described separately.

### Government Expenditure on Goods and Services

The various categories of government expenditure on goods and services are exogenous in real terms, with price determining the nominal level of expenditure. Total real government expenditure (GGST) consists of expenditure on goods and services by the federal government (GGSF), the provincial-municipal government (GGSPM), hospitals (GGGSH), the Canada and Quebec pension plans (GGSCP, GGSQP), as well as aggregate inventory holdings of all levels of government (IIG) and investment expenditure by hospitals (IH). A single price (PGGS, described in Chapter 7) is employed to construct the nominal level for these six expenditure categories, implying that in a shock-minus-control experiment all nominal categories grow at the same rate. Government investment expenditure is not modeled explicitly but is contained within the totals.

The aggregate level of expenditure is proportioned between wage (GW) and non-wage expenditure excluding inventories (GNW) according to the historical share, and within the wage category between the military and non-military (GWXM). Real government non-military wage expenditure is constructed by subtracting constant-dollar military wage expenditure (determined as the number of persons in the armed forces scaled by the average 1971 wage in the military) from total government wage expenditure (GW). The deflator used to convert real non-military wage expenditure to nominal expenditure (PGWXM) is a function of wages and salaries in the non-commercial sector (WOTH) with a near unitary coefficient. Nominal military wage expenditure (GMPF$) is exogenous.

Modeling expenditure in this manner assumes that real government expenditure is a policy choice of the levels of government and does not react under simulation to changing economic conditions. This is a useful construction in a short-run forecasting context but can pose problems in longer run policy simulations.

**The Unemployment Insurance Fund**

The operation of the unemployment insurance fund is modeled through equations explaining unemployment insurance revenues (TUIRF) and unemployment insurance benefits (GTPUIB). Contributions to the fund depend on the number of employed persons (NE), the rate of employer-employee contributions to the fund (RUIF), and the maximum level of insurable earnings per employee (QYIUF), which is related by legislation to changes in average weekly wages in the private sector. Benefits paid, on the other hand, are a function of the number of claimants, proxied by total unemployed persons (NU), and the weighted average rate of maximum weekly unemployment benefits (ERUIB); the latter is equal to 60% of weekly maximum insurable earnings.

In order to examine the balance of the unemployment insurance fund, an employment shock was performed on a model that contained the revenue and benefit equations. The shock is a 1 **million increase in employment**, i.e., in contributors to the fund. The participation rate equation (Chapter 6) indicates that a 1 million increase in employment leads to a .7 million increase in the labour force, implying a .3 million decrease in the number of unemployed (claimants). In 1979 the 1 million increase in contributors and .3 million decrease in claimants leads to a $250 million increase in fund revenues and a $1.2 billion decline in payments.*

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* As indicated in Chapter 7 the response of the labour force to employment shocks appears high, and the equation has been respecified in a more recent version of the model. Using the new equation, a 1 million increase in employment leads to a .3 million increase in the labour force and payments decline $2.8 million.

## 4.3 THE FEDERAL GOVERNMENT FINANCING RULE

This section describes the series of identities that determines the surplus or deficit position of the federal government. The size of the deficit determines the financing requirements of the government, and the degree to which cash or bonds are the source of financing has implications for the real and financial sectors of the economy. Most of the equations make use of seasonally unadjusted data and are of monthly frequency in order to facilitate the link into the monthly financial model (see Chapter 9).

The national accounts position of the federal government (GBALF) is determined as the difference between revenue and expenditure on a national accounts basis. Federal fiscal framework cash requirements (GFFFCN), however, differ from the national accounts deficit mainly for two reasons. First, corporate income tax is treated on a collections basis in the public accounts rather than on the accruals basis in the national accounts; second, other adjustments must be made to reflect the accounting practices of the government on a cash basis. In order to account for these differences, federal corporate tax accruals are translated to collections (TCCF), and the difference between collections and accruals plus a variable that accounts for miscellaneous asset and liability changes (GAMIS) are added to the national accounts deficit position to derive the fiscal framework requirements.

Total Canadian dollar cash requirements (GFTRCN), which ultimately determine the necessary amount of financing, are the sum of fiscal framework cash requirements determined above and total federal foreign exchange transactions (GFEXN) which include: the Government's regular transactions for purchases of goods and services in foreign currency; official intervention in the exchange market; and Bank of Canada-Exchange Fund swap transactions, which assist in the management of cash reserves.* In the model the major liabilities of the Bank of Canada, chartered bank deposits at the Bank (ABBCD) and notes outstanding (LGBCN), are endogenous while the main assets, Bank holdings of bonds (LGBFBC) and bills (LGFTBA), are exogenous.** Thus, changes in the size of the financial system have an influence on the financing requirements.

In order to satisfy its cash requirements, the federal government can issue treasury bills or bonds of various maturities, draw down cash balances held at chartered banks, or do both. The issuance of long bonds by the federal government, Canada Savings Bonds (LGCSBC) and other marketable bonds (LGCMBC), is exogenous as are the planned treasury bill program (ELGFTB) and the planned change in cash balances (EDDGFB). Under simulation the difference between the amount of exogenous financing and the endogenously calculated deficit (GFTRCN) requires a further source of financing. Such unanticipated financing is satisfied by further drawing down cash balances at the chartered banks (DDGFBT) and by issuing more treasury bills (LGFTBC). In the historical period the variable determining the proportion of unanticipated financing met through cash or treasury bills (RLGBF) has been set so that all financing is accomplished through cash balances. This has implications for the size of the financial system, as chartered banks rearrange their portfolios in response to exogenous inflows and outflows of cash. The parameter however may be set at any value desired in order to examine various financing schemes, and in the full model the shock presented in Chapter 10 has been set at .5, so that half the marginal financing would be

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* The Bank of Canada has an arrangement with the federal government whereby any excess of liabilities over assets of the central bank are swapped for foreign exchange assets. This transaction has the effect of increasing the cash balances of the government and thus reducing its cash requirements.

** There are other assets and liabilities but, for the purpose of modeling, these have been assumed to be offsetting.

from cash and half from treasury bills.

A drawback of modeling the federal government financing rule in this manner is that the planned debt program is exogenous and there is no mechanism by which it can be restored to a historical balance within the model. When there is a sustained increase in the deficit, for example, the ratio of the stock of treasury bills to marketable bonds keeps increasing, while there is no minimum desired level of cash balances held by the federal government.*

Interest payments on the federal debt (GTPINF) are a function of three distinct classes of outstanding debt, each weighted by an average interest rate.** The largest class of debt, all unmatured federal debt excluding Canada Savings Bonds and treasury bills (DEBXTC), consists of the outstanding stock of domestic (DRCCBT) and foreign currency (DRTLFT) marketable bonds and outstanding drawings on domestic (LGMBCT) and foreign (LGMBFT) chartered bank lines of credit. This stock is weighted by an average coupon rate (EACR) and denominated in Canadian dollars, and thus linked to the exchange rate. The remaining two outstanding stocks of debt, Canada Savings Bonds and treasury bills, are weighted by an average coupon rate (EACRCS) and the 90-day finance paper rate (R90) respectively.

In the GTPINF equation the first two stocks of debt are basically exogenous although changes in the exchange rate have a small effect on the magnitude of the foreign debt. The two coupon rates and the 90-day paper rate are also exogenous. Under simulation, therefore, GTPINF is driven by the change in treasury bills, which is in turn determined by the size of the federal debt and the proportion that is financed by treasury bill issues.

The provincial-municipal deficit, calculated from National

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* These problems have been addressed by P. Dattels and S. Wiseman [18].

** This equation is an approximation for the federal government's total liabilities against which interest is charged, and it should be noted that some liabilities such as government pension funds are not included.

Accounts revenue and expenditure, is financed through the sale of bonds; interest payments on the outstanding debt are exogenous.

## Appendix to Chapter 4

## TECHNICAL DESCRIPTION OF THE GOVERNMENT SHOCKS

**Table 14 Response of Personal Income Tax Collections to an Increase in Assessed Income**

Equations: Sector 9 excluding EQT78
Time Period: 1975Q1 - 1979Q4
Shocks:
1. Real Assessed Income
   YPASS•1.01
2. Nominal Assessed Income
   YPASS•1.01
   PCPI•1.01

**Direct Cost of Indexing**

Equations: Sector 9 excluding EQT50
Time period: 1976Q1 - 1979Q4
Shocks: RTI = 1

**Unemployment Insurance Fund Balance**

Equations: EQQ01, EQQ02, EQN53
Time Period: 1972Q1 - 1979Q4
Shock: NE + 1
NL + .7

# Chapter 5

## BUSINESS FIXED INVESTMENT AND INVENTORIES
(Sector 3)

Michael McDougall

This chapter begins by describing the theoretical framework underlying the equations for the two business investment categories in the model, machinery and equipment and non-residential construction.* The investment sector is then subjected to various shocks in order to analyze its dynamic response to increases in business product and rental prices. Next the equation for the stock of business inventories is examined, with particular attention paid to the buffering role of inventories when demand increases. Finally, the tracking ability of the flow of inventories equation is highlighted.

### 5.1 BUSINESS FIXED INVESTMENT

Business fixed investment is modeled as the sum of investment in machinery and equipment and investment in non-residential construction. In each of these categories, total investment comprises an energy and a non-energy component, with the energy component exogenous due to the particularly lumpy nature of energy investment projects.**

Stochastic equations determine total investment in each category; the dependent variable is purged of the energy component and non-energy investment is then defined residually as total investment less the energy portion. Four capital stock equations are constructed, a total and a non-energy stock for each investment category. The non-energy stocks are used in the production function.

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* Much of the recent developmental work in the investment sector was done by R. Khemani [49] and F. Scotland [75].

** From 1972 to 1980 energy comprised, on average, 16% of investment in machinery and equipment and 40% of investment in non-residential construction.

**Theoretical Framework of Investment**

The investment equations embody the standard flexible accelerator neoclassical theory. Given a Cobb-Douglas production function and a zero profit condition on the part of firms, the desired capital stock is:*

$$K_t^d = \alpha_K U_t^e P_t^e / R_t^e \tag{1}$$

where $\alpha_K$ is the capital share from the production function,
$U^e$ is expected real output,
$P^e$ is the expected price of output, and
$R^e$ is the expected rental price of capital.

Gross investment at time 't' is defined as

$$I_t = \beta(K_t^d - K_{t-1}) + \delta K_{t-1} \tag{2}$$

where $\beta$ is the speed of adjustment in closing the gap between the desired and lagged actual stock of capital (K) and
$\delta$ is the rate of depreciation.

Substituting (1) into (2) and dividing by $K_{t-1}$ yields the general specification for investment equations:**

$$I_t/K_{t-1} = \delta + \beta \log(\alpha_K \cdot U_t \cdot P_t^e / R_t^e) - \beta \log K_{t-1} \tag{3}$$

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* See Scotland [75], from Brechling, for the following derivation.

** For values of $K_t^d/K_{t-1}$ close to unity,

$$\log(K_t^d/K_{t-1}) \cong (K_t^d/K_{t-1}) - 1$$

In the model, investment in machinery and equipment (IME) and investment in non-residential construction (INRC) take the following form where both equations are estimated with a correction for first-order autocorrelation.

$$(\text{IME-IMENRG})/\text{KMEXE}_{-1} = \gamma_0 + \gamma_1 \cdot \text{J10W}(\log(\text{UGPP})) - \gamma_2 \cdot \text{J4W}(\log(\text{RCME/PGPP})) - \gamma_3 \cdot \log(\text{KMEXE}_{-1}) \quad (4)$$

$$(\text{INRC-INRCNR})/\text{KNRCXE}_{-1} = \varepsilon_0 + \varepsilon_1 \cdot \text{J13W}(\log(\text{UGPP})) - \varepsilon_2 \cdot \text{J6W}(\log(\text{RCNR/PGPP})) - \varepsilon_3 \cdot \log(\text{KNRCXE}_{-1}) \quad (5)$$

where KMEXE, KNRCXE are the relevant non-energy capital stocks,
IMENRG, INRCNR are the exogenous energy investment flows,
RCME, RCNR are the relevant imputed rental prices,
UGPP is real gross private business product, and
PGPP is the price of gross private business product,
JNW represents an Almon lag structure of length N.

The rate of depreciation, $\delta$, in the capital stock identities

$$K = (1-\delta)K_{-1} + I/4 \quad (6)$$

is 8% per annum for machinery and equipment and 3.5% per annum for non-residential construction. The rate of depreciation is the same for the total and non-energy capital stocks within each category.

Expectations of future output are captured by Almon lag terms on real gross private business product. Different lag lengths on UGPP in the two equations represent the planning horizons for investment projects. The planning horizon for investment in machinery and equipment (M & E) is ten quarters, while on non-residential construction the horizon is 13 quarters. The Almon lag on the rental-price/output-price ratio is much shorter than the lag on output, four and six quarters respectively in the two equations. Note, however, that the lag in the non-residential category is still longer than that in the M & E category.

The two imputed rental prices of capital, RCME and RCNR, are Jorgensonian rates of return [59] equal to the discounted present value of the price of capital goods net of depreciation and taxes. RCME is expressed as follows:*

RCME = PIME(RHOR+.02)(1-RITC)(1-CPVME·RTCA)/(1-RTCA)

where PIME is the deflator for investment in machinery and equipment,

RHOR is the real supply price of capital or the discount rate,

.02 is the quarterly rate of depreciation, implicit in the capital stock equation,

RITC is the rate of investment tax credit,

CPVME is the present value of tax-deductible depreciation allowances, and

RTCA is the corporate tax rate.

The imputed rental prices are constructed using the contemporaneous value of the investment deflator, an assumption that capital goods involve no commitment and can be bought and sold easily. A more realistic price is the expected price when the capital good is in place, which would require a lag structure on the investment deflator. This problem does not affect the investment equations since the lag structure on the rental-price/output-price ratio implicitly includes forward-looking prices. However, when the rental prices enter into normalized unit capital costs, which are used in the price sector, it should

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* RCNR is expressed similarly.

be noted that contemporaneous investment deflators are embodied in the rental prices. RCME and RCNR are calculated for total investment including energy, whereas the investment equations are estimated excluding energy.

The real supply price of capital, described in Chapter 9, is a function of the real interest rate on ten long industrial bonds where inflation expectations are determined by a simple adaptive process on past changes in the consumer price index.

We modify the standard formulation to take account of the investment tax credit that was introduced in the third quarter of 1975. The tax credit is deducted from the market cost before the capital cost allowance of an asset is determined, thus the inclusion of (1-RITC).

The imputed rental price is the channel through which monetary and fiscal policies influence investment. Monetary policy affects investment behaviour through the real supply price of capital and to some extent through depreciation allowances. Fiscal policy changes are felt directly through the corporate tax rate, the investment tax credits and depreciation allowances. Over the past decade, the corporate tax rate has ranged from a high of 44.4% in 1972 to a trough of 41.4% in 1975, and has eased up slowly to 43.4% in 1980. The investment tax credit, on the other hand, has increased steadily from 5% in 1975 to 8.5% in 1980.

When the investment equations are estimated the coefficients on the rental-price/output-price ratios are insignificant; nevertheless the ratios are retained in order to conform to our theoretical priors.* The coefficients are sufficiently small, however, that the equations are driven essentially by the accelerator.

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* In the alternative specification (used in the current version of the model), where cost minimization is assumed, the coefficient on the wage/rental-price ratio is insignificant and small as well.

**Dynamics of Investment**

Figure 6 presents the adjustment of the two non-energy stocks of capital to an exogenous temporary $100 million increase in these stocks in the first quarter of the simulation. This shock demonstrates the speed of adjustment in closing the gap between the desired capital stock and the actual capital stock in the investment equations. The desired stock remains unchanged in the simulation while the actual stock increases above control for one quarter. Investment responds to the opening of a gap between the actual and desired capital stock by declining in the second quarter of the simulation; investment in M & E falls $23 million while non-residential investment falls $40 million. The decline in investment then reduces the increase in the actual capital stock, and in the second quarter KMEXE is only $92 million above control while KNRCXE is $89 million above control. In the long run the interaction of the stock and flow equations reduce the actual stock of capital to the control value. The stock of non-residential construction returns to control before the stock of machinery and equipment, reflecting a faster speed of adjustment. The non-energy stock of non-residential construction is $10 million above control after five years while the non-energy stock of machinery and equipment is still $10 million above control after seven years.* We would, however, expect a faster speed of adjustment in machinery and equipment, since this category of investment may be augmented by imports, and investment projects are not as time consuming as those in non-residential construction.

The response of the non-energy investment flows and stocks to various shocks is presented in Table 15. The top half of the table reports the response of the flow of investment, and the lower half reports the response of the stock.

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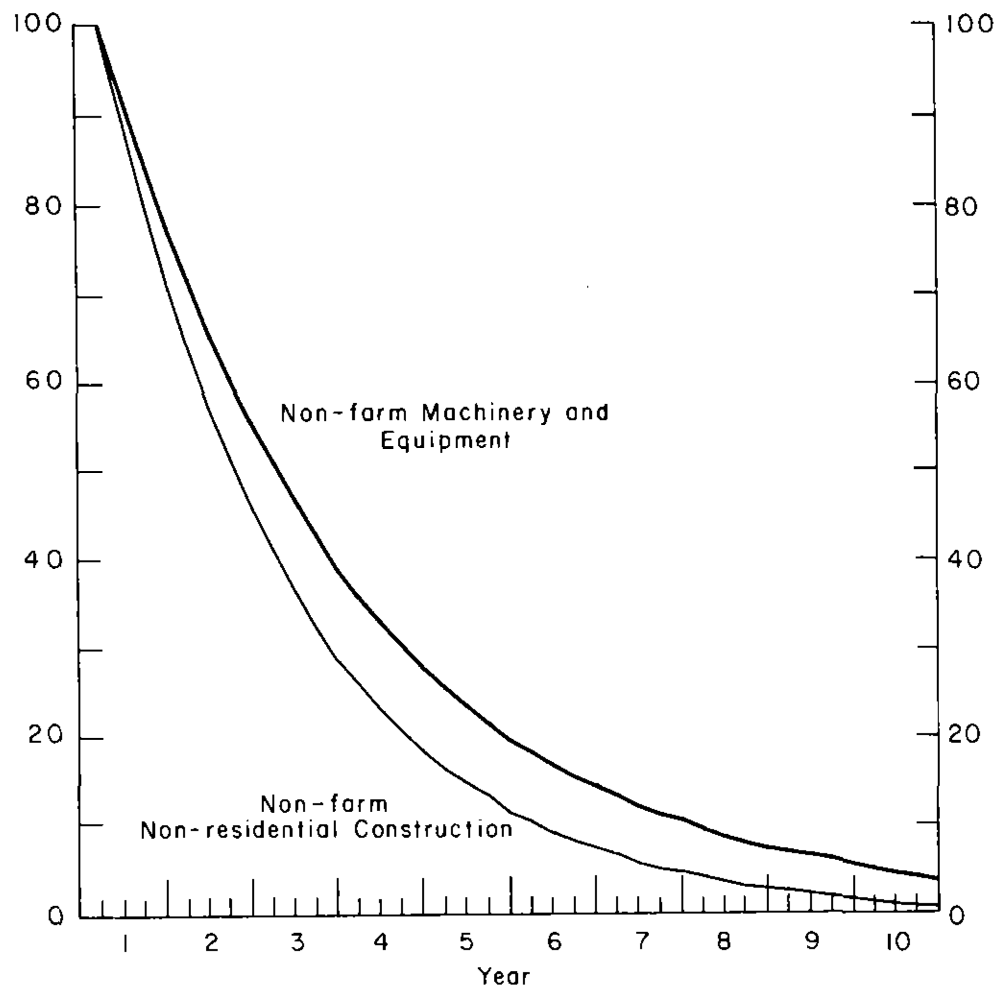
* The speed of adjustment in investment is relatively slow compared to that in the equations for the consumption of durables. (See Chapter 1.)

**Figure 6**

**STOCK ADJUSTMENT FOR THE INVESTMENT CATEGORIES**
**(Millions of dollars)**




**Increase in Business Product.** The first shock is a sustained 1% increase in real gross private business product. This increase in output leads to a new higher equilibrium level of the desired capital stock, which is implicit in each investment equation (see equation (1)) where the path of adjustment is given by the lag structure on UGPP. The desired stock of machinery and equipment rises rapidly to an equilibrium of 1.1% above control after nine quarters. Due to the longer expectations in non-residential construction the desired capital stock is reached much more slowly, after twelve quarters, at .73% above control.

## Table 15

### DYNAMIC RESPONSE OF BUSINESS INVESTMENT IN MACHINERY & EQUIPMENT AND NON-RESIDENTIAL CONSTRUCTION
(Shock minus control, per cent)

### INVESTMENT

| | Quarters | | | |
|---|---|---|---|---|
| | 1 | 4 | 12 | 32 |
| 1% increase in: | | | | |
| Business product | | | | |
| IMEXE | .51 | 1.9 | 1.9 | 1.1 |
| INRCXE | .40 | .91 | 2.9 | .76 |
| Rental price | | | | |
| IMEXE | -.11 | -.15 | -.10 | -.06 |
| INRCXE | 0 | -.18 | -.12 | -.04 |
| Supply price of capital | | | | |
| IMEXE | -.05 | -.07 | -.06 | -.04 |
| INRCXE | 0 | -.10 | -.09 | -.03 |
| Corporate tax rate | | | | |
| IMEXE | -.04 | -.05 | -.03 | -.02 |
| INRCXE | 0 | -.10 | -.07 | -.02 |

### CAPITAL STOCKS

| | Quarters | | | |
|---|---|---|---|---|
| | 1 | 4 | 12 | 32 |
| 1% increase in: | | | | |
| Business product | | | | |
| KMEXE | .02 | .16 | .70 | 1.0 |
| KNRCXE | 0 | .04 | .34 | .70 |
| Rental price | | | | |
| KMEXE | 0 | -.02 | -.04 | -.06 |
| KNRCXE | 0 | 0 | -.03 | -.04 |
| Supply price of capital | | | | |
| KMEXE | 0 | 0 | -.02 | -.03 |
| KNRCXE | 0 | 0 | -.02 | -.03 |
| Corporate tax rate | | | | |
| KMEXE | 0 | 0 | -.01 | -.02 |
| KNRCXE | 0 | 0 | -.01 | -.02 |

Thus the increase in output opens the gap between the desired and actual capital stock and results in an increase in investment. The increase in investment in time period t then leads to an increase in the actual capital stock in time period t + 1. In the short run, as long as the desired capital stock is increasing the gap grows, since the effect of rising lagged actual capital is not sufficient to outweigh the increase in desired capital. Once the desired capital stock attains an equilibrium point the gap no longer increases, the flow of investment reaches a peak, and downward pressure is exerted on investment until the desired and actual capital stocks are equalized.

In more detail, non-energy investment in machinery and equipment (IMEXE) increases .51% on impact and reaches a peak of 2.2% above control after seven quarters; at this point the stock of machinery and equipment is .36% above control. The peak is attained a few quarters before the desired capital stock stabilizes due to the relatively large increase in the actual capital stock. From the peak IMEXE declines slowly to about 1.1% above control. The response of non-energy investment in non-residential construction (INRCXE) is much slower, increasing only .4% on impact, and is 1.6% above control after seven quarters when the peak in IMEXE is reached. Although the desired stock of capital does not increase as much as in the case of IMEXE, the gap increases for a longer period of time and is sufficient to enable INRCXE to reach a peak of almost 3% above control after twelve quarters. At the peak the actual stock of capital is .34% above control. Again, once the desired capital stock reaches an equilibrium the gap begins to close and investment is reduced. We have seen that the coefficient $\beta$, the speed of adjustment highlighted in Figure 6, is greater for INRCXE than for IMEXE and thus the reduction in the flow of investment from the peak is much more rapid in the case of INRCXE. From the peak of 3% after twelve quarters INRCXE falls to .79% above control after eight years while the stock is .7% above control. After eight years the system has just about stabilized and the desired and actual capital stocks are in fact equal after eleven years.

Combining IMEXE and INRCXE, total non-energy business investment and the associated capital stock increase almost 1% in response to the output shock. The response of total business investment including energy investment (IBUS) is lower, however, due to the exogenous nature of energy investment. IBUS increases only .71% due to a .95% increase in investment in machinery and equipment and a .43% increase in non-residential construction investment.

**Increase in Rental Costs.** The second shock is a sustained 1% increase in the imputed rental prices of capital. The increase in rental price relative to output price leads to a decline in the desired stocks of capital. The response of investment to this shock follows a similar but inverted pattern to the output shock. IMEXE reaches a trough of .16% after three quarters, and by the end of the simulation is .06% below control, while INRCXE reaches a trough of .22% below control after six quarters and is eventually .04% below control. The capital stocks decline .06% and .04% respectively, very much less than after the output shock. Thus changes in the rental-price/output-price ratio have very little effect on investment. Such a weak response has implications for the interrelationships in the factor market examined in Chapter 8.

**Monetary and Fiscal Shocks.** The weak response also limits the influence of monetary and fiscal policy on investment. In the final two shocks, the real supply price of capital and the corporate tax rate are increased 1% in order to examine the effect of first monetary and then fiscal policy changes on the implicit rental prices and thus on investment. As can be seen in Table 16, the rental price of non-residential construction is more responsive than the rental price of M & E to the two shocks. The stocks of capital remain virtually unchanged (Table 15).

Table 16

RESPONSE OF IMPLICIT RENTAL PRICES TO MONETARY AND FISCAL SHOCKS
(Shock minus control, per cent)

| | RCME | RCNR |
|---|---|---|
| 1% increase in: | | |
| Real supply price of capital | .51 | .70 |
| Corporate tax rate | .30 | .55 |

## 5.2 BUSINESS INVENTORIES

This section describes business inventories in RDXF and reports their dynamics.

### General Description

Non-farm business inventories are modeled as a stock-to-sales ratio, where sales are defined as total non-service consumer expenditure plus private fixed investment expenditure plus exports of goods. The inventory-holding behaviour of firms is described by four demand categories: consumer expenditure on durables and semi-durables plus private fixed investment; consumer expenditure on non-durables; imports of goods; and exports of goods. The secular decline in the ratio is captured by a time trend.*

### Dynamics of Inventories

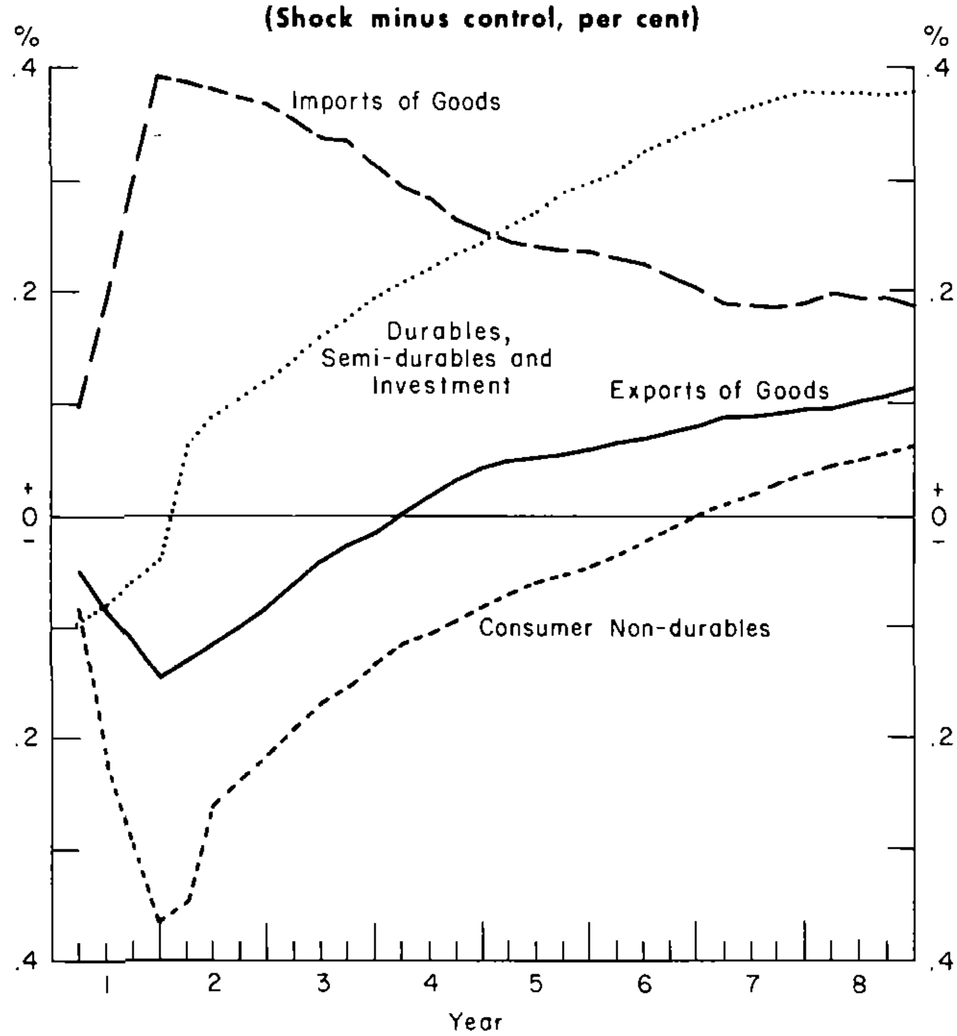
Figure 7 presents the dynamic response of the stock of inventories to a sustained 1% increase in each of its determinants, which may be considered as an increase in sales for each variable. The response of the stock of inventories to domestic demand and exports of goods shows an initial decumulation of inventories followed by a gradual accumulation to a new desired stock-to-sales ratio. The length of time of decumulation and the

* The value of physical change in government-held inventories is exogenous, while the change in farm inventories is modeled in the farm sector.

**Figure 7**

**RESPONSE OF THE STOCK OF BUSINESS INVENTORIES TO A 1% INCREASE IN EXPLANATORY VARIABLES**

**(Shock minus control, per cent)**

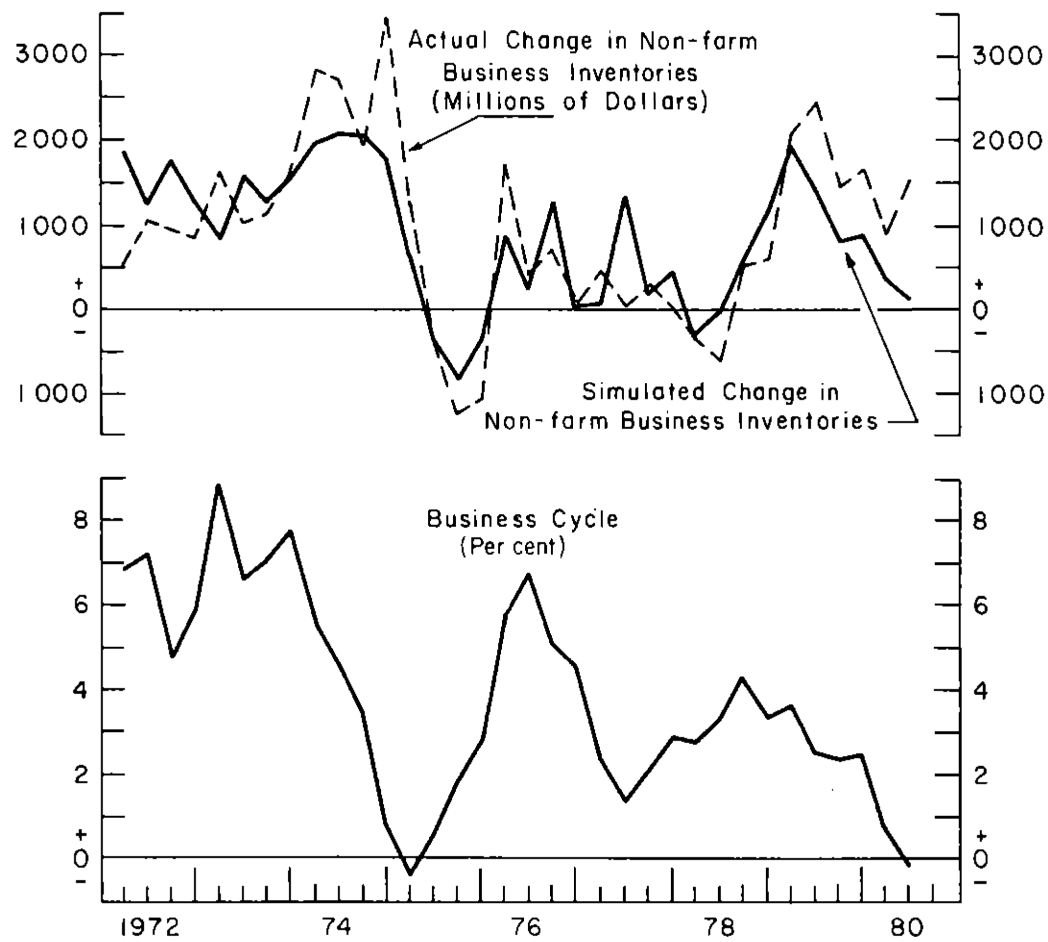



pattern of response varies widely among categories. The buffering response reverses most quickly, after four quarters, for consumer expenditure on durables and semi-durables plus investment. A shock to exports of goods produces a rapid decumulation of inventories which reaches a trough after one year and a subsequent accumulation after twelve quarters. A shock to consumer expenditure on non-durables produces the same pattern of response as exports of goods but an accumulation of inventories is reached only after 23 quarters. A shock to imports of goods induces an immediate accumulation which peaks after four quarters, and then dissipates. A 1% increase in all sales yields a less than unitary increase in inventories in the long run, indicating that an increase in sales produces a decline in the stock-to-sales ratio.

A comparison of the ex post prediction of the change in inventories and the actual series is presented in Figure 8.*

**Figure 8**

**TRACKING ABILITY OF THE INVENTORIES EQUATION 1972-80**




* The simulation included the equations for the stock and flow of inventories.

Inventories are notoriously difficult to model since the actual series exhibits a very high variance. For instance, in 1974Q4 there was an inventory accumulation of $3.5 billion followed by a decumulation of $1.2 billion in 1975Q3--a swing of $4.7 billion. However, the inventory equation is fairly successful in predicting the pattern of inventories; e.g., over the period 1972Q1 to 1980Q1 the equation correctly predicted whether inventories were accumulating or decumulating.

## Appendix to Chapter 5

## TECHNICAL DESCRIPTION OF INVESTMENT AND INVENTORIES SHOCKS
(Sector 3)

**Figure 6 Stock Adjustment for Inventories Categories**

Equations: EQI01, EQI02, EQI60-EQI63

Time Period: 1972Q1 - 1979Q4

Shocks: $KMEXE = .97965\ KMEXE_{-1} + IMEXE/4 + 100(NPER.EQ.19721)$

$KNRCXE = .99125\ KNRCXE_{-1} + INRCXE/4 + 100(NPER.EQ.19721)$

**Table 15 Dynamic Response of Business Investment**

Equations: EQI01, EQI02, EQI53, EQI54, EQI60-EQI63

Time Period: 1972Q1 - 1979Q4

Shocks:
1. UGPP·1.01
2. PGPP·1.01
3. RHOR·1.01
4. RTCA·1.01

**Table 16 Response of Implicit Rental Prices to Monetary and Fiscal Shocks**

Equations: EQI53, EQI54

Time Period: 1972Q1 - 1979Q4

Shocks: RHOR·1.01

RTCA·1.01

**Figure 7 Response of the Stock of Business Inventories**

Equations: EQI03, EQC50

Time Period: 1972Q1 - 1979Q4

Shocks:
1. CSD·1.01
   CS·1.01
   CMV·1.01
   CHSHD·1.01
   CDMIS·1.01
   IME·1.01
   INRC·1.01
   IRCA·1.01
2. CNDO·1.01
   CFOOD·1.01
   CENERG·1.01
3. MG·1.01
4. XG·1.01

# Chapter 6

## THE LABOUR MARKET
(Sector 5)

Heather Robertson

This chapter begins with a description of the theory underlying the demand and supply equations for labour, and the labour market gap. The demand for labour (employment) in the private sector is modeled as an adjustment to desired employment; in the public sector, employment is simply a proportion of real government wage expenditure. The labour supply equation, expressed as the labour force participation rate, is a function of a cyclical term and time trends proxying structural changes in the labour force. The amount of slack in the labour market, termed the labour market gap, is determined residually as the difference between the unemployment rate and the "unemployment rate at trend output".

The second section of the chapter contains a description of the dynamic elasticities in a quasi-reduced form employment model. The final section reports on recent improvements in the labour supply equations and on proposed changes that will improve the dynamics in the model to various shocks.

### 6.1 THEORETICAL MODEL OF THE LABOUR MARKET*

The discussion of labour demand describes the transmission of changes in domestic activity to employment, hours, and short-run productivity. In addition, labour hoarding is analyzed by comparing the relationship between the actual business cycle and the simulated cycle in private sector employment. The labour supply discussion emphasizes the relative importance of the

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* The employment and labour force equations were developed at the Bank by G. Bilkes, L. deBever, H. Robertson, and G. Schaefer. More recent developments are by G. Meredith. Work on the unemployment rate at trend output was mainly done by M. Gosselin, updating the work of J.-P. Aubry.

cyclical and trend components in the labour force participation rate. The determinants of the gap between demand and supply of labour are then analyzed.

**Labour Demand**

The labour demand model includes equations for paid employment in the private sector or industrial composite (NIC) and employment in community services, government and agriculture (NOTH), and for average weekly hours in mining and manufacturing (HAWMM). Labour demand for the private sector is not modeled in terms of total man-hours, but rather hours and employment are expressed in separate equations in order to isolate the different short-run and long-run behaviour in the two categories.*

The specification for private sector employment assumes a dynamic adjustment to desired employment, which is derived from the classical theory of labour demand. Desired employment can be expressed by two equivalent functional forms that are based on the same technological and behavioural assumptions. The first is the dynamic analogue to the classical static derivation, where man-hours are hired to the point at which wages equal the marginal revenue product of labour.** Assuming a Cobb-Douglas production function, desired employment is directly proportional to the output elasticity for labour ($\alpha_L$), expected prices ($P^e$), and expected output ($U^e$), and inversely proportional to expected wages ($W^e$):

$$N_a^d = \alpha_L P^e U^e / W^e. \qquad (1a)$$

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* Furthermore, there is a difference in coverage for the NIC and HAWMM series. First of all mining and manufacturing (the coverage for the HAWMM series) comprises only about 40% of the workers in the total industrial composite. Second, NIC is derived from the Statistics Canada All Establishments Survey, whereas HAWMM is from the Large Establishment Survey.

** This formulation is derived from profit-maximizing behaviour on the part of firms. As will be discussed in more detail in Chapter 8, formulation (1a) does not have a determinate level of output if it is based on a Cobb-Douglas production function with constant returns to scale.

The second formulation treats desired employment as a residual, given the expected level of output and the desired capital stock ($K^d$):

$$N_b^d = [U^e/K^{d^{\alpha_k}}]^{1/\alpha_L} \qquad (1b)$$

where the desired stock of capital is:

$$K^d = \alpha_K P^e U^e/R^e;$$

therefore factor and output prices influence $N^d$ indirectly through $K^d$.

The variable in the RDXF model that is similar to the formulation of desired employment in (1b) is required employment (NICD). Required employment is defined as the employment needed to produce a trend level of output, given the capital stock actually in place (K), exogenous trend hours ($\bar{h}$) and trend factor productivity (ETFP).* The loglinear transformation of NICD is:

$$NICD = 1/.66[\beta_1(L)UGPP-ETFP-.34K_{-1}] - \bar{h}, \qquad (1)$$

where $\beta_1(L)$ is the lag structure (a four-quarter moving average) that determines a trend level of output. The NICD formulation has a slightly different interpretation than (1b) for two reasons. First, NICD includes the actual stock of capital at the beginning of the period rather than the desired stock; the greater the difference between actual and desired stock, the more employment

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* The Cobb-Douglas production function in the RDXF model that is the basis for the NICD formulation is:

$$UGPPS = J1L(KMEXE)^{.18}J1L(KNRCXE)^{.16}(NIC \cdot HAWMM \cdot 52)^{.66}ETFP,$$

where KMEXE is the stock of machinery and equipment excluding energy, and KNRCXE is the stock of non-residential construction excluding energy. (The $.34K_{-1}$ in (1) is a shortened representation of the capital stocks in UGPPS).

is required. Second, the use of an annual trend on output rather than on expected output implies an adjustment of required employment to medium-term output fluctuations that may or may not be reflected in expected output.*

The private sector employment (NIC) equation is modeled as an adjustment to required employment (NICD), and, as well, includes price (PGPP), wage (WNIC) and output (UGPP) terms which represent in a very general sense desired employment in (1a). Long-run coefficients are not, however, constrained such that actual employment equals desired employment as defined above. The loglinear functional form of the employment equation is:

$$NIC = \frac{1}{(1-\gamma_5 L)} [\gamma_1 NICD + \gamma_2 PGPP - \gamma_3 WNICIR + \gamma_4 UGPP]. \quad (2)$$

The lagged dependent variable ($\gamma_5$), represents both the intertemporal adjustment to the desired level of employment and an adaptive mechanism on price and wage expectations. Substituting (1) into (2):

$$NIC = \frac{1}{(1-\gamma_5 L)} [\gamma_1/.66\ \beta_1(L)UGPP + \gamma_4 UGPP] \quad (2')$$

$$- \gamma_1/.66(ETFP+.34K_{-1}+\bar{h}) + \gamma_2 PGPP - \gamma_3 WNICIR.$$

In the RDXF model, as in RDX2, factors of production adjust in a hierarchy to output changes. In terms of flexibility, employment is quasi-fixed, adjusting more readily than capital but more slowly than hours or productivity. In fact, employment counterbalances the inflexibility of capital through the NICD

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* Required output could be rewritten,

$$NICD = N_b^d [K^d/K_{-1}]^{\alpha K/\alpha L} \beta_1(L)UGPP/U^e,$$

or required employment is the product of desired employment, the deviation of desired from actual capital scaled by the output elasticities, and the ratio of average to expected output.

term. However, there is still a certain degree of labour hoarding because of lags in adjustment, the rationale being that hiring and training costs limit the possibility for laying off workers in the event of temporary demand fluctuations. And the greater the labour hoarding, the more productivity and hours must adjust, since they are determined residually.

The degree of labour hoarding implied by the employment equation is demonstrated in Figure 9, where the simulated employment cycle is superimposed on the actual cycle in business output.* As the figure shows, simulated private sector employment exhibits a cycle damped in amplitude relative to actual output, with turning points lagged one to two quarters. The detrended employment series is consistently below detrended output in upswings in the business cycle, and above it for downswings, and does not exhibit the short spiked pattern in the output series. Whereas the maximum trough in output is 7.5% below trend in 1970Q4, the corresponding trough in employment one quarter later is 4.3% below trend.

Employment in community services, government, and agriculture (NOTH) is modeled as a proportion ($\alpha$ = .66) of the ratio of nominal government wage expenditure excluding the military (PGWXM·GWXM) to wage costs in government and agriculture, including supplementary labour payments (WOTHIR(1+RYWSLP)). The adjustment to actual output, prices and wages is instantaneous. The value of $\alpha$ is less than unity because the output definition includes only the government sector, whereas NOTH includes other categories of employment as well as government. The specification implies that

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* The output cycle is the per cent deviation between actual and trend UGPP, where trend output is generated from the following equation, estimated from 70Q1 to 79Q4:

$$\log UGPP = \underset{(174.9)}{9.91} + \underset{(21.2)}{.012}QQTIME \qquad \bar{R}^2 = .93 \qquad DW = .2$$

The time period was chosen so that beginning and end points are mid-cycle. The simulated employment cycle is the per cent deviation between simulated employment at the actual level of output and simulated employment at the trend level of output. Equations in the simulation are NIC and NICD.

Figure 9

CYCLES IN OUTPUT AND EMPLOYMENT

1970-79

the government maintains its wage bill in fixed proportion to nominal government expenditure.

The NIC and NOTH series, calculated from Statistics Canada Establishment Survey information, sum approximately to total paid employment (NEPD) from the Labour Force Survey.* The series are linked through a near-identity, expressed in loglinear form:

$$\log NEPD = .01 + 1.01 \cdot \log(NIC+NOTH) - .65 \cdot \log((NIC+NOTH)/NEPD)_{-1}$$

Over the 1961-79 period, the mean deviation between the series is 3.9%. NEPD is systematically higher than (NIC+NOTH) by as much as 5.7%. Attempts by Statistics Canada to reconcile this discrepancy have not met with success.

Paid plus unpaid employment (NEUNPD) sum to total employment:

$$NE = NEPD + NEUNPD \qquad (3)$$

where NEUNPD is modeled as an (exogenous) proportion of total employment.

There are a number of linkages from NIC and NE to the rest of the RDXF model. Private sector employment is used in the calculation of the various output concepts (UGPPD, UGPPS) and in normalized unit labour costs (ULC). The sum of NIC and NOTH determine total wage income (YW). Total employment (NE) determines the unemployment rate and labour market tightness variables.

The demand for hours is modeled separately from employment

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* The original reason for linking these series was that the commercial-noncommercial breakdown was not available from the Labour Force Survey, but was required to model commercial output in the private sector. Total employment, however, was required in order to determine the unemployment rate. The paid-unpaid distinction may not be necessary as it does not add to the explanatory power of the labour sector.

because of its assumed rapid adjustment to general demand pressure. Average weekly hours (HAWMM) are a function of a time trend that captures the secular decline in hours, and the ratio of contemporaneous gross private business product (UGPP) to the output that could be produced by the Cobb-Douglas technology;

$$HAWMM = \delta_1(UGPP/UGPPS) - \delta_2 QQTIME \quad (4)$$

$$= \delta_1/(1-.66\delta_1)[UGPP-.66NIC-.34K_{-1}-ETFFP]-\delta_2 QQTIME \quad (4')$$

The split of total man-hours between hours and employment, which are fully substitutable in the Cobb-Douglas technology, depends on the deviation between UGPP and UGPPS. Any positive deviation is met partially by an immediate increase in hours, which slowly declines as the gap diminishes and employment increases. If the gap were closed, average weekly hours would follow the exogenous trend, and the increase in man-hours would emanate entirely from employment.

The coefficient ($\delta_1$=.1) determines the role that hours play in easing short-term fluctuations. Because the coefficient is less than unity, total man-hours do not fully accommodate fluctuations in demand for a given capital stock. To the extent that capital, employment and man-hours do not fully adjust, higher output is satisfied by utilizing existing factors more intensively. The variation in productivity is measured by the gap between production (UGPP) and output implied by the production function (UGPPS). The weakness in the response of hours to variations in demand is one of the factors underlying a persistent productivity gap in demand shocks. This will be discussed further in Chapter 8.

**Labour Supply**

In RDXF, labour supply is approximated by the total labour force (NL), which is derived from Labour Force Survey data. The dependent variable in the NL equation is actually the labour force participation rate (NL/NPOP). Based on assumptions similar to those of Strand and Dernberg [115] the participation rate is a positive function of a cyclical term, the employment ratio (NE/NPOP), and a time trend (QQTIME):

$$\log NL/NPOP = \varepsilon_1 \log NE/NPOP + \varepsilon_2 QQTIME \qquad (5)$$

Wages influence the participation rate only indirectly, and through the employment term. Therefore the partial effect of an increase in wages is to <u>reduce</u> labour supply, because employment responds negatively to wage increases. The positive relationship between the participation rate and the employment ratio ($\varepsilon_1$=.7) represents the discouraged worker effect: secondary workers choose the optimal time in the business cycle to search for a job, which is at peaks in labour demand, and they drop out of the labour force in periods of low labour demand, when their decision wage falls below their reservation wage. Secular increases in the participation rate that are represented in the time trend are attributed to changes in household technology, institutional factors (e.g., the increase in unemployment insurance benefits), sociological factors (e.g., the women's movement) and the changing age structure in the population. Given the specification of equation (5), the following relationship exists between total employment, the participation rate, and the unemployment rate:

| Shock | Response | |
|---|---|---|
| Total Employment | Participation Rate | Unemployment Rate |
| 1.0% | .7% | - 27 basis points |
| 3.7% | 2.6% | -100 basis points |

The decline in the unemployment rate accompanying an increase in employment is quite small, or, conversely, a large participation rate response is associated with changes in unemployment. This cyclical effect is considerably larger than in other similarly specified equations.*

The discouraged worker effect may be overstated for two reasons. One is the probable spurious correlation between employment and the labour force. In the RDXF specification the time trend explains only 4 points of the 7-basis-point increase in the participation rate in the 1970s, implying that a large portion of the explained increase in employment emanates from the "cyclical" variable. The second reason is that intermediate swings in the participation rate may originate from variables not included in the participation rate equation, such as changes in age structure and fertility rates (Wachter [87]). If so, an upward bias could be created in the short-term cyclical coefficient, with a resulting overstatement of the discouraged worker effect.**

Note that both the labour demand and the labour supply equations exhibit a linear response to fluctuations in aggregate demand, and there are no upward constraints in either equation. The demand for labour is assumed to fall within a range where there is a sufficiently large pool of unemployed to draw upon at any level of capacity. The labour supply specification assumes the same proportion of the population willing to enter the labour force at very high levels of capacity as at low levels for a given per cent increase in employment. Although this may be true on aggregate, labour markets are sufficiently segmented and nonhomogeneous that individual sectors could face hiring bottlenecks at reasonably high levels of capacity.

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* Bowen and Finegan estimate a .66% increase, and Tella and Strand and Dernberg estimate a .6% increase in the labour force accompanying a 100-basis-point decline in the unemployment rate (81). These studies were done with U.S. participation rates; the discouraged worker effect in Canada is generally believed to be lower.

** In the light of these comments, the participation rate equation was modified in the December 1980 version of RDXF. The discouraged worker term is represented by the deviation of unemployment from trend. The equation includes demographic terms such as the portion of the population over 55, and a non-linear trend term. The discouraged worker effect is then less than one-half as strong as reported here.

Employment is determined by the demand for labour at all levels of capacity: there are no constraints such that the short side of the market dominates. In fact, there are extended periods when the labour market gap is negative as employment exceeds the supply of labour at the going wage rate. In such periods individuals misperceive their real wage over the short run, while firms do not.

**The Labour Market Gap**

The unemployment rate (RNU), defined residually as the per cent difference between the labour force and total employment,

$$RNU = (1-NE/NL)100 \qquad (6)$$

does not represent the actual degree of slack in the labour market. Rather, the labour market gap (RU) is defined as the difference between the actual unemployment rate and a constructed variable, the unemployment rate at trend output (RNUTO). Both private and public sector wages equations, which are modeled as expectations-augmented Phillips curves, include the labour market gap term as an argument.

The RNUTO series is calculated by a two-step procedure. First RNU is regressed on the output gap (GAP) and two structural variables; then, the cyclical variation caused by GAP is eliminated and RNUTO is generated from the estimated equation. The output gap is defined as the ratio of real GNE to its trend. The two structural variables are: the ratio of the weighted annual average of maximum unemployment insurance benefits (ERUIB) to average annual weekly wages (WNICAV), which proxies the opportunity cost of being unemployed; and the proportion of the working population aged 15 to 24 (NP1524/NPOP), an age group with historically high unemployment rates. A regression of RNU on the three arguments yields the following relationship:

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* GAP = UGNE/$\widehat{UGNE}$, where $\widehat{UGNE}$ = EXP (a+bQQTME). More recent versions of RDXF include trend factor productivity in calculating GAP.

RNU = 32.0 - 43.4·GAP + 6.2·ERUIB/WNICAV + 54.8·NP1524/NPOP

Once the cyclical component is removed and GNE is set at its trend value (GAP=1), the unemployment rate at trend output is defined as:

RNUTO = -11.4 + 6.2·ERUIB/WNICAV + 54.8·NP1524/NPOP.

The distinction has been made between the unemployment rate at trend output and a natural or non-accelerationist rate of unemployment. Usually the natural rate is defined in a system with a Phillips curve, where prices are homogeneous of degree one in wages; in this type of system the natural rate of unemployment is achieved when wage and price growth are equal and non-accelerating. In the RDXF model, however, prices are a function not only of wages, but also of other costs, such as trade prices, energy prices and capital costs, as well as of productivity. Thus a zero labour market gap is not the only condition required for a non-accelerationist rate of inflation.

As the following table shows, the unemployment rate at trend output increased from 5.4% in 1970 to a maximum value of 7.5% in 1973Q4, declining to 6.4% by the end of 1979. (Aubry calculated a constant trend unemployment rate as 7% after 1971; the Fortin-Phaneuf series was as high as 6.8% in 1978).

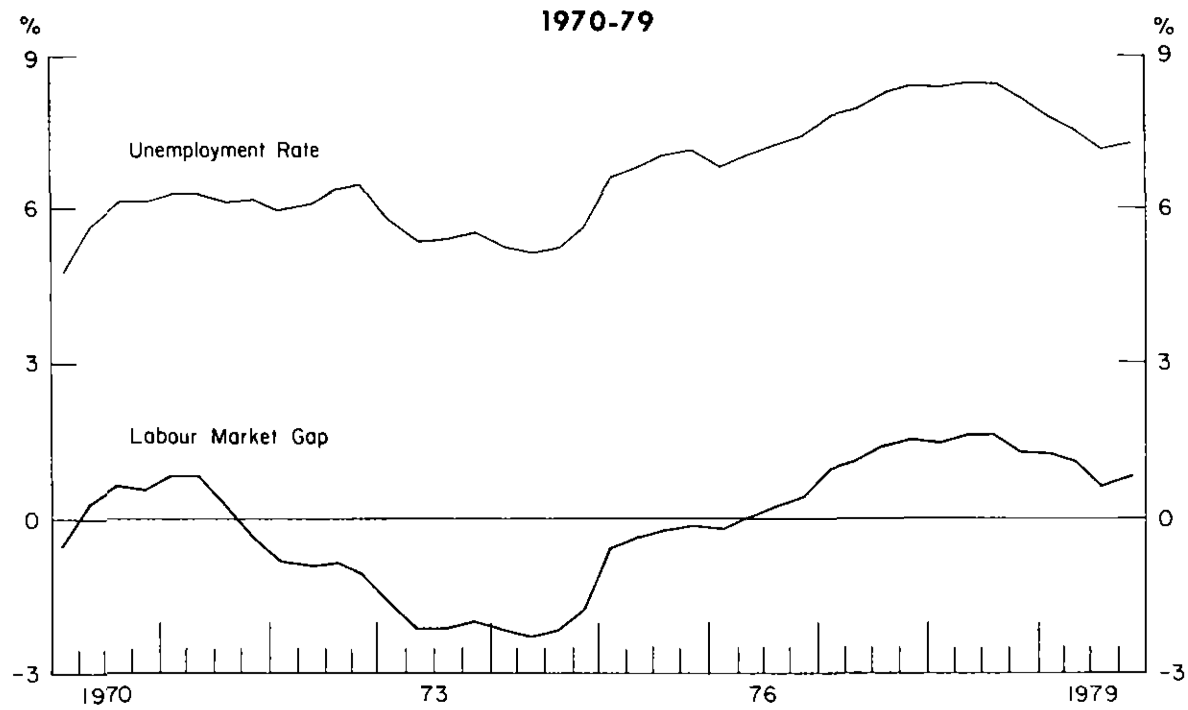
| | RNUTO | Change in Basis Points | Point Contribution UIC | Point Contribution Age Structure |
|---|---|---|---|---|
| 1970Q1 | 5.4 | - | - | - |
| 1973Q4(peak) | 7.5 | 210 | 198 | 12 |
| 1979Q4 | 6.4 | -110 | -73 | -37 |

Of the 210-point increase in RNUTO between 1970 and 1973, 198 points are attributable to an increase in the benefit-wage ratio.

(Aubry estimated that 179 basis points emanated from UIC benefits.) A decline in both the benefit-wage ratio and the portion of the working population aged 15 to 24 years contributed 73 and 37 basis points respectively to the decline in the unemployment rate at trend output by the end of 1979.

The relationship between the level of unemployment and the labour-market tightness variable is shown in Figure 10. Both series exhibit the same cycles; however, a substantial portion of the upward trend in the actual unemployment rate has been removed from the labour market gap variable. The RU series is negative from 1971Q4 to 1976Q1, falling to a trough of -2.2% for most of 1973 and 1974.

**Figure 10**
**THE UNEMPLOYMENT RATE AND THE LABOUR MARKET GAP**
**1970-79**




Another labour market tightness term* implicit in the supply output (UGPPS) and normal output (UGPPD) identifies:

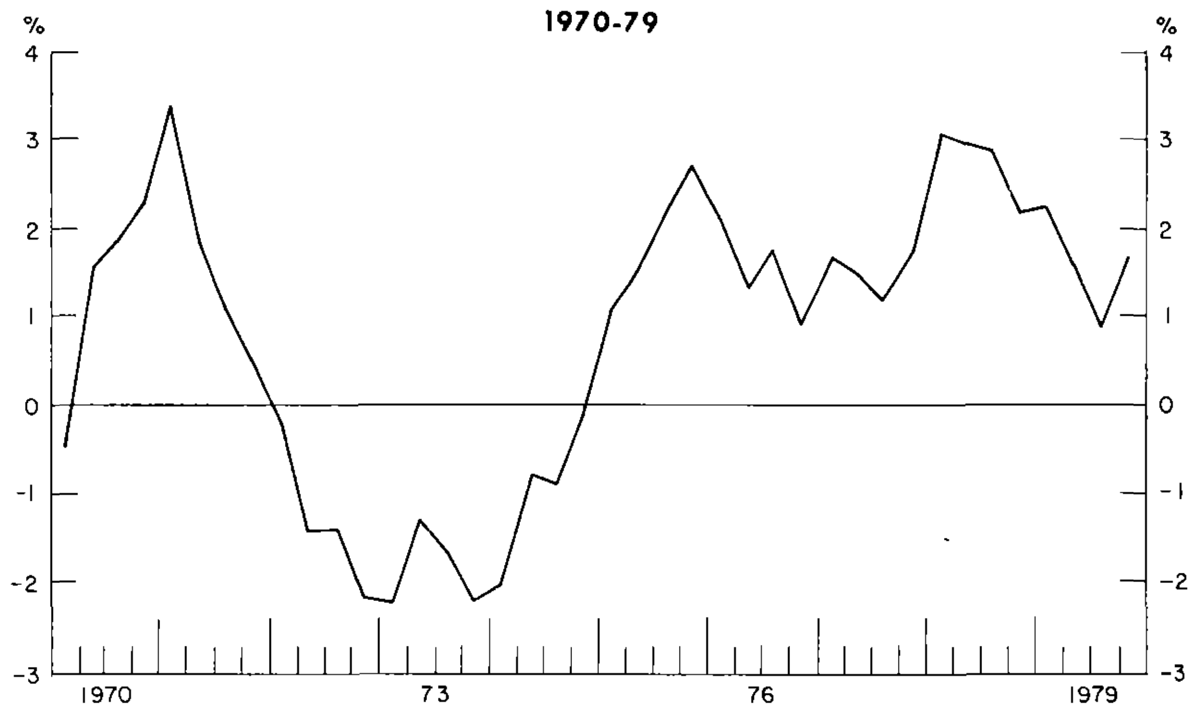
* Since RPRIV is a measure of the labour market gap in the private sector, this may be the appropriate explanatory variable in the private sector wage equation.

$$RPRIV = 1 - UGPPS/UGPPD = 1 - L/L^*$$
$$= 1 - \frac{(NIC)HAWMM}{NL(1-RNUTO/100) - J8A(NE-NIC)\bar{h}}$$

RPRIV is the ratio of one minus the proportion of man-hours worked in the private sector to private sector man-hours that would exist at the trend level of output. The L* term is defined as the product of trend hours and the total labour force (NL), less the trend non-private sector employment [J8A(NE-NIC)] less the number of trend unemployed (NL·RNUTO/100). Figure 11 shows the historical pattern of RPRIV. The pattern is quite similar to the labour market slack variable for the overall economy (RU in lower line of Figure 10). However, this measure displays a shorter period over which the gap is negative (72Q1-74Q4 for RPRIV and 74Q1-76Q4 for RU). Deviations in the RPRIV and RU series stem from two sources. First, the labour slack in the public sector may move differently from that in the private sector. Second, problems in reconciling the (NL and NE) and (NIC) data create measurement errors in the L* variable.

**Figure 11**

**THE LABOUR MARKET GAP IN THE PRIVATE SECTOR 1970-79**



Within the labour market inconsistency among the equations for labour demand, labour supply, and unemployment at trend output is also a problem. If employment and the labour force are interpreted as the demand and supply of labour respectively, together with exogenous or trend arguments representing functional and structural forces in the economy, then the unemployment rate at trend output would simply be the per cent difference between the trend labour force and trend employment. The trend labour force would be a function of those arguments in the equation for the trend rate of unemployment that proxy structural changes in supply, whereas trend employment would be a function of the arguments that proxy structural changes in demand. In the model, when output, capital, the output deflator, government expenditure and wages are at their trend values, or trend private sector employment is a function of trend factor productivity and trend hours, and trend public sector employment is a function of trend government expenditure, the trend labour force is represented as:

$$NIC_T = f(ETFP, \overline{h})$$

$$NOTH_T = f(\overline{G})$$

$$NL_T = f(QQTIME).$$

This would imply an unemployment rate at trend output with the following arguments:

$$RU_T = f(ETFP, \overline{h}, QQTIME, \overline{G}),$$

in contrast to the RNUTO equation in the RDXF model. To obtain a consistent system of equations in the labour market, either RNUTO should be defined residually, or the labour market should be estimated simultaneously with cross-equation constraints on the trend variables in the NL, NIC and RNUTO equations.

## 6.2 DYNAMICS OF THE LABOUR MARKET

The dynamic elasticities of the quasi-reduced form employment equations 1, 2', 3, 4', 5 and 6 are presented in Table 17. The unemployment rate response is reported in basis points.

Table 17

**PARTIAL ELASTICITIES IN THE LABOUR SECTOR**
(Shock minus control, per cent)

| | | UGPP | KMEXE & KNRCXE | ETFP | NPOP | WNIC | PGPP |
|---|---|---|---|---|---|---|---|
| | Impact | .31 | 0 | -.14 | | -.13 | .13 |
| NIC | 1 year | .77 | -.09 | -.29 | 0 | -.27 | .26 |
| | 8 years | .90 | -.11 | -.31 | | -.29 | .28 |
| | | .38 | 0 | -1.5 | | | |
| NICD | | 1.49 | -.51 | -1.5 | 0 | 0 | 0 |
| | | 1.50 | -.51 | -1.5 | | | |
| | | .23 | 0 | -.11 | | -.10 | .10 |
| NE | | .59 | -.07 | -.22 | 0 | -.21 | .20 |
| | | .70 | -.08 | -.24 | | -.23 | .22 |
| | | .10 | 0 | -.11 | | .01 | -.01 |
| HAWMM | | .06 | -.03 | -.10 | 0 | .02 | -.02 |
| | | .05 | -.03 | -.10 | | .02 | -.02 |
| | | .16 | 0 | -.07 | .31 | -.07 | .07 |
| NL | | .41 | -.05 | -.15 | .31 | -.14 | .14 |
| | | .49 | -.06 | -.17 | .31 | -.16 | .15 |
| | | -.07 | 0 | .03 | .29 | .03 | -.03 |
| RNU* | | -.17 | .02 | .06 | .29 | .06 | -.06 |
| | | -.20 | .02 | .07 | .29 | .06 | -.06 |
| | | .73 | -0 | -.83 | | .08 | -.08 |
| SRP | | .45 | -.25 | -.74 | 0 | .16 | -.16 |
| | | .37 | -.24 | -.73 | | .18 | -.17 |

* Basis point change

Because the employment and labour force equations exhibit a linear response to excess demand variations, these elasticities are approximately constant over wide ranges of capacity.

A **1% increase in private business output** affects NIC directly through the UGPP term and indirectly through an induced change in desired employment as expected output increases. After one year expected output has adjusted fully to actual, and desired employment increases in proportion to the inverse of the output elasticity for labour. Twenty-four per cent of the increase in man-hours originates in HAWMM on impact--HAWMM increases .1% compared to a .31% increase in NIC. After eight years NIC is .9% above control, providing 95% of the man-hours adjustment. The majority of the employment adjustment is within the first year, when NIC is .8% above control. Because of the lags in adjustment in employment and the weak response in hours, productivity (or the deviation between production and output implied by the production function) increases .73% on impact. After eight years, SRP is .37% above control. Productivity variations would be smaller if capital were endogenous; however as is evident in Chapter 8, the productivity gap is large and persistent even when all factors respond to output changes. Total employment increases according to the share that makes up private sector employment.

The response of required employment is about 70% stronger than that of actual employment after eight years, partly because NICD is defined to accommodate any increase in expected output not met by the existing capital stock. If, for example, the actual stock of capital were to increase 1% in response to the increase in UGPP, NICD would increase exactly 1% and NIC .8%, thereby narrowing the gap between the response of required and actual employment to 20% after eight years.

Because of the overstated discouraged worker effect the unemployment rate declines only 20 basis points after eight years. By way of comparison, if the labour supply curve showed no response to labour market changes and only employment responded, the unemployment rate would be 65 basis points below control after eight years for the same increase in output.

The employment response to supply factors is substantially different from the response to demand factors. **A 1% increase in the non-energy stock of machinery and equipment and non-residential construction** (KMEXE, KNRCXE) increases UGPPS after one year by .34%, which is the sum of the output elasticities for capital. NICD shows a sustained decline of .51% after one quarter. Although the magnitude of the required employment response is about one-third as strong as in the UGPP shock, the NIC response is only one-tenth as strong.

Results are similar for a **1% increase in trend factor productivity (ETFP)**. The NICD response is, after one year, equal in absolute terms to the UGPP shock. However, NIC shows only one-third the response (-.31%) after eight years. Hours, which fall .1%, and short-run productivity, which declines .73% below control after eight years, close the remaining output gap.

The weak negative response of employment to ETFP affects both income distribution and prices. Because wages respond directly to productivity growth,* and employment with respect to wages is less than unity, total nominal wage income and therefore labour's share of income increase with higher productivity. However, with a Cobb-Douglas production function, labour's share should be unchanged with a productivity increase, implying a proportional decline in employment as wages rise. In response to the slowdown in productivity growth in the mid-1970s, the RDXF model would, ceteris paribus, show a decline in labour's share because of the weak offsetting employment response. Prices, which are a function of normalized unit labour costs (WN1CIR·NIC·52/UGPPS) are not neutral to productivity changes partly because of the weak employment response as well.

A **1% increase in population** affects only the labour force and unemployment rate, which show an immediate and sustained increase of .31% (or a .7% decline in the participation rate) and 29 basis points respectively. Although it has been argued that the change in the age distribution of population affects the participation rate (an argument that is not explicit in the labour force equation), such a strong decline in the participation rate accompanying a <u>general</u> population increase is questionable.

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* If all productivity gains accrued to labour, then wages would increase by the inverse of the output elasticity for labour ($1/\alpha L \cong 1.5$).

The size of the wage and price coefficients determines the influence of profitability on hiring decisions. The wage and price elasticities in the RDXF employment equation are about one-third as large as the output elasticity, and all are nearly identical in magnitude, implying that only real wages affect the demand for labour. (This is quite different from the original specification of desired employment in (1a) where the elasticity of desired employment with respect to real wages is negative unity.) On impact, a 1% increase in private sector wages (WNIC) reduces employment by .13%. Employment declines gradually to -.29% after eight years. The weakness of the employment response has implications for the effect of excess demand shocks in the full model. Because real wages show a strong response to excess demand pressure, and because the offsetting decline in employment is small, real income shows a strong increase, thereby refueling demand pressure.

Because the labour force is a function of employment, an increase in real wages implies a reduction in the labour force in this partial simulation. In the full model, the direction of the labour supply response depends on the source of the real wage change. For example, a real wage increase induced by higher aggregate demand (and hence higher employment) increases the labour force. A real wage increase resulting from higher productivity (which induces a decline in labour demand) creates a decline in the labour force.

Public sector employment responds somewhat differently to wage increases, so that distribution between the two sectors is affected in the event of general wage inflation. Essentially NOTH depends only on the level of real government wage expenditure (GWXM) because the government wage deflator exhibits near homogeneity with respect to WOTH (.982). The reduced form NOTH equation (substituting the PGWXM equation into the NOTH equation) is:

$$\log(NOTH) = .003 + .66 \log(GWXM) - .018 \log(WOTHIR) + Z$$

In Chapter 5, the partial elasticity of PGPP with respect to wages is calculated to be .7%. Therefore, given the elasticity of private sector employment with respect to WNIC and PGPP, private sector wage income increases .9% with a 1% increase in WNIC. Wage inflation, assuming no change in relative wage changes, creates a slight erosion of the share of private sector income relative to the share of the government non-commercial sector; the government sector is less inclined to substitute away from labour with public sector wage increases.

## 6.3 CONCLUSION

In the course of the next several months, we hope to improve the labour market equations along the following lines. First, private sector employment will be derived from cost minimization behaviour and will conform with the underlying production technology. This should improve the dynamic response in the model to several shocks. For instance, if the response of employment to real wages is stronger, there will be less of an induced increase in real income with a demand shock, and therefore less productivity fluctuation as well. In addition, a stronger response to ETFP will improve the income distribution and price response to a change in productivity. A unitary elasticity of employment in the long run with respect to output will allow labour demand to conform to the production technology.

Improvements have been made recently to the labour supply equations (Meredith [56, 57]). The participation rate equation now includes the labour market gap term (RU) to describe the discouraged worker effect, a non-linear time trend, and some demographic detail such as the proportion of population over 55. The result is a much weaker discouraged worker effect, about one-half as strong as reported here. Much progress can still be made in integrating the labour force, employment, and labour market gap equations into a more coherent and consistent system.

## Appendix to Chapter 6

### TECHNICAL DESCRIPTION OF SIMULATIONS IN THE LABOUR MARKET
Time Period 1972Q1 - 1979Q4

### Simulated Employment Cycle

Trend Output: $\log UGPP_T = 10.234 + .0091 \cdot QQTIME$

Simulation 1: EQN01, NIC with actual output (UGPP)

Simulation 2: EQN02, NIC with trend output ($UGPP_T$)

### Partial Elasticities in the Labour Sector

Equations:

| | | | | | |
|---|---|---|---|---|---|
| EQI64 | UGPPAV | EQN60 | NEUNPD | EQN53 | NU |
| EQN54 | NICD | EQN03 | NL | EQN55 | RNU |
| EQN01 | NIC | EQN58 | RNUTO | EQN59 | RU |
| EQN04 | NOTH | EQI56 | UGPPD | EQQ06 | QYIUF |
| EQN05 | NEPD | EQI57 | UGPPS | EQQ52 | ERUIB |
| EQN52 | NE | EQN02 | HAWMM | EQQ02 | GTPUIB |

Shocks:

1. UGPP·1.01
2. KMEXE·1.01
   KNRCXE·1.01
3. ETFP·1.01
4. NPOP·1.01
   NPOPT·1.01
   NP1524·1.01
5. WNIC·1.01
   WNICIR·1.01
6. PGPP·1.01

# Chapter 7

## WAGES AND PRICES (Sectors 6 and 7)

Michael McDougall

In this chapter the wage-price nexus of the model is examined. The chapter begins with a description of the expectations-augmented Phillips curve wage equations and the short-run difference between them for the private and the public sectors. In the second section, the consumer expenditure deflators that make up the consumer price index (CPI) are described, and an equilibrium equation is derived with special attention paid to the degree of homogeneity of the CPI with respect to costs. A reduced-form CPI equation and the private sector wage equation are then linked and a real wage equation is derived. Finally, the wage-price model is expanded to include all domestic prices as well as trade prices, thereby allowing three aggregate prices to be examined: the final domestic demand deflator, the deflator for gross private business product and the gross national expenditure deflator.

## 7.1 WAGES*

This section discusses the theory underlying the wage measures in RDXF, and then presents the dynamics of the wage equations.

### Description of Wage Measures

Two wage measures are employed in the model, average weekly wages in the industrial composite (WNIC) and average weekly wages in community services, government, and agriculture (WOTH). Both of these measures are constructed to exclude special payments and to include losses due to strikes. The wage earners receiving WOTH represent a group approximately one-third as large as the group receiving WNIC.

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* The wage sector is described in detail in Gosselin and Staranczak [32, 33].

WNIC is modeled as an expectations-augmented Phillips curve, which can be represented in loglinear terms as follows:

$$\Delta WNIC = \alpha_0 \Delta ETFP - \alpha_1 RU + \alpha_2 \Delta PCPI_{-1} - \alpha_3 QAIB. \qquad (1)$$

The first term measures the effect of productivity on the growth rate of wages, since ETFP is the measure of trend factor productivity derived from the Cobb-Douglas production function (see Chapter 8). This specification allows the impact of changing productivity growth, which slowed considerably after the mid-1970s, to be explicitly modeled. Productivity growth enters the equation as a twelve-quarter moving average because changes in productivity are only slowly incorporated into wage bargaining.

The trade-off between wages and the unemployment rate is incorporated through the labour-market gap variable (RU), which measures the difference between the observed unemployment rate for the whole economy (RNU) and the rate of unemployment at trend output (RNUTO); the distinction is not made between the private and the public sectors. RU appears in level terms in the equation and indicates that a sustained labour market gap results in a permanent reduction in the growth rate of wages. Price expectations are captured through an eight-quarter Almon lag on the one-quarter lagged growth rate of the CPI. The lag coefficients follow the pattern of a second degree polynominal the sum of which has been constrained to unity ($\alpha_2=1$).* Thus, in the long run, a 1% rise in the CPI leads, ceteris paribus, to a 1% rise in nominal wages.

The dummy variable QAIB measures the downward pressure on the growth rate of wages due to the Anti-Inflation Board wage and price control programs that were in effect from October 1975 to December 1978. It is estimated that the controls reduced wage growth by 1.6% at annual rates during this period.

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* When this lag structure was freely estimated the sum was not significantly different from 1.

WOTH is also modeled as an expectations-augmented Phillips curve modified by the inclusion of a relative wage term (WNIC/WOTH).

The specification in loglinear terms is as follows:

$$WOTH = \beta_0 RU + \beta_1 \Delta PCPI_{-2} + \beta_3 (WNIC-WOTH)_{-1} \quad (2)$$

In the short run, in response to various shocks, WOTH and WNIC may exhibit different responses. However, in the long run the relative wage term ensures that the growth rate of WOTH is identical to the growth rate of WNIC. The sum of the Almon lags on PCPI in the WOTH equation is not constrained to unity since it does not affect the long-run condition that a 1% increase in prices results in a 1% rise in wages. The coefficient takes on a value of 1.45 and allows a different speed and path of adjustment to equilibrium under price shocks than occurs in the WNIC equation.

Although total factor productivity does not explicitly enter the WOTH equation, its influence is felt through the relative wage term. Productivity increases that occur in the industrial sector of the economy are passed on to the public sector.

**Dynamics of Wages**

In the following shocks WOTH and WNIC are simulated together because of the relative wage term in WOTH. The partial elasticities of the wage equations are presented in Table 18.

A **1% increase in total factor productivity** leads to a 1.3% increase in the level of WNIC after 12 quarters, which then remains unchanged throughout the remainder of the simulation. The consequent rise in the level of WOTH is much slower, reaching only 1.0% above control after 32 quarters. The relative wage effect, however, continues to put upward pressure on WOTH.

In response to a **1% increase in the level of the consumer price index**, the percentage increase in the level of WNIC follows the cumulation of the Almon lags on the price expectations

Table 18

RESPONSE OF WAGES TO INCREASES IN
PRODUCTIVITY, THE CPI AND THE UNEMPLOYMENT RATE
(Shock minus control, per cent)

Response of Wages in the Industrial Composite (WNIC)

| 1% increase in: | Quarters 1 | 4 | 12 | 32 |
|---|---|---|---|---|
| Total factor productivity | | | | |
| Per cent difference | .11 | .44 | 1.31 | 1.30 |
| Growth rate difference** | .11 | .47 | .47 | 0 |
| Consumer price index | | | | |
| Per cent difference | 0 | .25 | .99 | 1.0 |
| Growth rate difference** | 0 | .27 | 0 | 0 |
| Unemployment rate* | | | | |
| Per cent difference | -.20 | -.78 | -2.3 | -6.1 |
| Growth rate difference** | -.21 | -.85 | -.85 | -.90 |

Response of Wages in Community Services,
Government and Agriculture (WOTH)

| 1% increase in: | Quarters 1 | 4 | 12 | 32 |
|---|---|---|---|---|
| Total factor productivity | | | | |
| Per cent difference | 0 | .04 | .36 | 1.02 |
| Growth rate difference** | 0 | .04 | .23 | .09 |
| Consumer price index | | | | |
| Per cent difference | 0 | .11 | 1.39 | 1.08 |
| Growth rate difference** | 0 | .12 | .43 | -.03 |
| Unemployment rate* | | | | |
| Per cent difference | -.30 | -1.17 | -3.2 | -7.4 |
| Growth rate difference** | -.33 | -1.28 | -1.13 | -.97 |

* Increase of 100 basis points

** Quarterly

term, reaching 1% above control after nine quarters. No response is felt in the first quarter because WNIC does not react to contemporaneous price changes. The response of WOTH is quite different in the short run. No effect is felt in the first two quarters of the shock. By the eleventh quarter, however, the level of WOTH is 1.4% above control. The greater-than-unit coefficient on price expectations results in an overshooting of WOTH to its long-run value. The relative wage term then reduces the percentage increase in WOTH from the peak, and by the end of the simulation WOTH is 1.08% above control. The increase in the level of factor productivity and prices has no long-run effect on the growth rate of private sector wages; however, the growth rate of public sector wages is still adjusting after eight years.

The final shock to the wage sector is an **increase of 100 basis points in the unemployment rate,** which does affect the long-run growth rate of wages. The annual growth rate of WNIC stabilizes at about .9% below the growth rate in the control after three quarters. The growth rate response of WOTH is much larger in the short run, -.3% compared to -.21% for WNIC. The response reaches a trough after one year at 1.28% below control and then gradually increases towards the growth rate increase in WNIC. The growth rate decrease translates to a 6.1% decrease in the level of WNIC and 7.4% decrease in the level of WOTH after eight years.

## 7.2 PRICES

This section first describes how the deflators that make up the consumer price index (CPI) are constructed and then presents the results of various shocks to a partial model of the price sector. Prices are restricted here to the CPI so as to derive real wages in a price/wage interaction later on in the chapter.

### Determinants of the Consumer Price Index*

The CPI is a near identity derived from the aggregation of

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* Domestic prices are described in detail in Staranczak [77-80].

nine consumer expenditure deflators using fixed expenditure weights that sum to unity where the weights are the share of the components in total consumption in a given base year. To capture shifting consumer expenditures the weights change in 1973. The nine deflators, which correspond to the detail in the consumption sector, are the prices of: motor vehicles and parts (PCMV), food (PFOOD), energy (PENERG), other non-durables (PCNDO), services excluding rent (PCSXR), household durables (PHSHD), miscellaneous durables (PCDMIS), semi-durables (PCSD) and, finally, gross rent (PRENT). Of these, only the price of energy is exogenous in recognition of its special role as a policy variable after late 1973.

These deflators are specified as producer prices and reflect a flexible mark-up theory of pricing behaviour. The typical price equation in loglinear form can be written:

$$
\begin{aligned}
(P-(1+RT)) = {} & \gamma_0 + \gamma_1 ULC + \gamma_2 UKC + \gamma_3 PT + \gamma_4 PENERG \\
& + \gamma_5 POTH + \gamma_6 CAPU + \gamma_7 (P-(1+RT))_{-1} \qquad (3)
\end{aligned}
$$

where RT is an indirect sales tax rate,
ULC is normalized unit labour costs
$= (WNICIR \cdot NIC \cdot 52)(1+RYWSLP)/UGPPS$,
UKC is normalized unit capital costs
$= (.01 \cdot RCME \cdot KMEXE + .01 \cdot RCNR \cdot KNRCXE)/UGPPS$,
PT are trade prices,
PENERG is the price of energy,
POTH are other costs including costs, property taxes, farm gate prices and financial costs,
CAPU is the model's measure of capacity utilization.

Generally, the most important cost faced by a firm is labour cost. Producers are assumed to respond to standard or normalized unit labour costs, which are represented by the ratio of the private sector wage bill plus supplementary payments to supply output (UGPPS). Supply output rather than actual output is used in the denominator to reduce the effect of short-run productivity on price determination; the use of actual output could lead to a price decline in the short run when demand rises. Unit capital costs are expressed similarly to unit labour costs where RCME·KMEXE and RCNR·KNRCXE are the imputed rental cost of the stock of machinery and equipment and of non-residential construction respectively. Only in the semi-durables (PCSD) equation were normalized unit capital costs found to be statistically significant.

Trade prices enter most of the price equations as a cost, usually as an import price reflecting the import content of the domestic price.* The most widely used import price is the price of other consumer goods excluding food (PMOCG). The price of autos, however, is a function of the export price of motor vehicles and parts to the U.S. since it is assumed that through the auto pact exported cars are more indicative of domestic consumption than imported cars. The import price of food is an argument in the food price equation.

The price of energy enters as an intermediate cost in most domestic deflators. Imposed coefficients on the energy cost terms (PENERG) are calculated from the Statistics Canada 1976 input-output tables. In spite of several well known drawbacks to the use of input-output models--fixed coefficients, non-substitutability, timeliness, etc.--our results appear reasonably satisfactory and work is continuing along this line.

All prices except food and rent, which are not subject to tax, are estimated net of indirect sales taxes. Such sales taxes are assumed to be passed fully and immediately through to the price. The most widely used sales tax measure is the weighted average rate of provincial sales tax (RTISPM).

The price influence of excess demand in the product market is

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* Consumer services and rent are not traded and thus are not directly a function of trade prices.

captured by the ratio of real gross private business product to trend output, UGPP/UGPPD. This capacity utilization term enters the PCSXR and PHSHD equations in level terms, allowing for a mild non-linearity in response due to the semi-log nature of the specification, with respectively an eight- and four-quarter moving average to reflect the fact that prices respond slowly to demand pressures.

Exogenous farm gate prices (PFA) enter the PFOOD equation, and PFOOD is in turn a determinant of the price of services excluding rent. The price of rent (PRENT) reflects a different specification from the other deflators, being a function of the price of residential construction (PIRC), the conventional mortgage rate (RMC) and unit property taxes (TIM/(KRESD+KNRC)). The mortgage rate is the only direct source of interest cost in this partial model and has a positive influence on the CPI.

**Dynamics of the Consumer Price Index**

In the following shocks, investment in residential construction (PIRC) and the eight endogenous determinants of PCPI are included in a partial model of the price sector. The response of PCPI to a 1% increase in various costs is presented in Table 19. As can be seen, in the long run PCPI is nearly homogeneous with respect to costs: 1% increase in all costs results in a .92% increase in PCPI.*

**A 1% increase in wages** leads to a .39% increase in PCPI in the long run and accounts for under half of the .92% increase in PCPI when all costs are increased. The service deflator excluding rent exhibits the largest response to the wage shock, increasing .69% in the long run. Wage costs enter the equation directly as well as indirectly through PFOOD. On the other hand, the price of rent is not directly a function of wages but is influenced through PIRC and increases only .04%.

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* The aggregate response masks varying responses of the component. PCDMIS increases only .73% in response to a 1% increase in all costs while PRENT increases 1.03%.

Table 19

DYNAMIC ELASTICITIES OF THE CONSUMER PRICE INDEX
(Shock minus control, per cent)

| 1% Increase in: | Quarters 1 | 4 | 12 | 32 |
|---|---|---|---|---|
| Wage costs | .08 | .24 | .37 | .39 |
| Trade prices | .08 | .18 | .21 | .21 |
| Energy prices* | .08 | .11 | .12 | .13 |
| Farm gate prices | .03 | .07 | .07 | .08 |
| Unit capital costs | .01 | .02 | .03 | .03 |
| Unit property taxes | 0 | .02 | .03 | .03 |
| Mortgage rate | 0 | .01 | .02 | .05 |
| Total | .28 | .65 | .85 | .92 |
| Capacity utilization rate | .01 | .06 | .20 | .28 |
| Indirect sales taxes | .03 | .03 | .03 | .03 |
| Productivity | -.04 | -.12 | -.19 | -.19 |

* Energy and farm gate prices may be considered trade prices that move in tandem with the exchange rate. If this assumption is made then the response of the CPI to a 1% trade price increase is .42% in the long run.

Most prices are specified in terms of unit labour costs and thus an increase in productivity, UGPPS/NIC, has an influence on prices that is equal and opposite to the wage increase. However, in aggregate an **increase in productivity of 1%** lowers the CPI by only .19% in the long run since the service deflators are a function of wage costs, not unit labour costs.

Trade prices are the second most important determinant of PCPI with a **1% increase in all traded goods prices** resulting in a .21% increase in PCPI. The price of motor vehicles (PCMV) exhibits the largest response, increasing .72% in the long run, with service deflators responding very little since they respond only through movements in PFOOD and PIRC.

PCPI increases .13% in response to a **1% increase in the exogenous energy price** (PENERG). PENERG has a weight of .063 in the PCPI equation, so that just over half of the response of PCPI comes through the effect of energy prices as an intermediate cost in the other deflators.

The weak positive response of the CPI to an **interest rate increase** is due to the partial nature of the shock. In a full model context the upward pressure on the CPI from the price of rent is dominated by output and exchange rate influences that lead to a decline in the CPI.

**A 1% sustained increase in the rate of capacity utilization** results in a .28% increase in PCPI with the entire effect coming through increases of PCSXR and PHSHD. The weakness of excess demand pressure influences on the CPI has implications for the flexible-price factor market model examined in Chapter 8 and the full model shocks in Chapter 10.

## 7.3 INTERACTION OF WAGES AND PRICES

The long-run properties of the interaction of wages and prices may be examined from the partial simulation results presented in the previous sections of this chapter. An equilibrium PCPI and WNIC equation may be expressed as follows:*

$$PCPI = .39WNIC + .21PT + .33POTH + .18CAPU - .19PROD \quad (4)$$

$$WNIC = 1.3ETFP - .9RU + PVPI \quad (5)$$

where POTH is expanded to include energy prices and PROD is UGPPS/NIC from the unit labour cost expression.

Substitution yields the following reduced form equations for PCPI and WNIC:

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* All the coefficients represent the percentage increase in the dependent variable except the coefficient on RU, which represents the annual growth rate increase.

PCPI = .83ETFP - .58RU + .34PT + .54POTH (6)
+ .3CAPU - .32PROD

WNIC = 2.13ETFP - 1.48RU + .34PT + .54POTH (7)
+ .3CAPU - .32PROD

The quasi-reduced form price and wage equations exhibit near homogeneity with respect to costs PT and POTH: the coefficients sum to .88. They are not homogeneous with respect to costs because of the lack of homogeneity in the CPI equation reported in section 7.2.

In the long run the productivity measure UGPPS/NIC will equal long-term productivity from the production function ETFP. In this reduced form model a 1% increase in productivity (ETFP and PROD) will lead to a rise in prices of over .5%. This result occurs because not all prices are specified in terms of unit labour costs, in which case wages and productivity would have equal and opposite effects on prices, and because the coefficient on long-term productivity in the wage equation is greater than unity.

A real wage equation may be derived by subtracting (6) from (7):

WNIC - PCPI = 1.3ETFP - .9RU (8)

In the long run, trade prices, other costs, and the rate of capacity utilization do not have an influence on the real wage. If the labour market is in equilibrium, only factor productivity determines the real wage, with a coefficient of 1.3. The short run is a different matter altogether, since it takes time for price expectations to feed through into nominal wage increases and the labour market may not be in equilibrium. The response of the real wage in the short run is of crucial importance to the dynamics of the model over the medium term.

## 7.4 AN EXTENDED PRICE MODEL

In this section the price model* is expanded to include all domestic prices as well as import and export prices. The larger model allows the dynamics of four important aggregate price deflators to be examined:

(a) the gross private business product deflator (PGPP), which measures the value of gross private business product at factor costs. (The construction of this price is presented in Chapter 8.)

(b) the gross national expenditure deflator (PGNE);

(c) the final domestic demand deflator (PFDD), which is a weighted average of the total consumption deflator,** the investment deflators and the government goods and services deflator;

(d) the consumer price index (PCPI).

### Components of Aggregate Prices

Before the dynamics of the aggregate deflators are reported, three main components, the deflators for investment, government and inventories, are examined below; trade prices were analyzed in Chapter 3.

For **investment**, three deflators are modeled: business investment in machinery and equipment (PIME), business investment in non-residential construction (PINRC) and investment in residential construction (PIRC). All are specified in a manner similar to the consumption deflators described above.

$$PI = \varepsilon_0 + \varepsilon_1 ULC + \varepsilon_2 PT + \varepsilon_3 PENERG + \varepsilon_4 CAPU + \varepsilon_5 PI_{-1} \quad (9)$$

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* Wages are exogenous.

** The consumption deflator moves similarly to PCPI; the difference between the two is that PCPI uses fixed expenditure weights whereas the consumption deflator uses flexible weights.

PIME is a function of normalized unit labour costs while PINRC and PIRC are a function of wage costs. The last two deflators are very labour intensive and exhibit high elasticities with respect to wages. The import price of machinery and equipment (PMME) strongly influences PIME and reflects the fact that almost half of machinery and equipment is imported. The U.S. housing market influences PIRC through the export price of lumber (PXLUM) while trade prices have little effect on PINRC. Demand pressures in the form of the capacity utilization rate affect PIME and PINRC. All three deflators are specified with an imposed energy coefficient to reflect the intermediate cost of energy price increases and are estimated net of indirect sales taxes.

Table 20 presents the dynamic elasticities of the total investment deflator. The deflator is homogeneous with respect to costs, with wages representing by far the largest component and excess demand having a slightly greater effect on the investment deflator than on the CPI.

Table 20

**DYNAMIC ELASTICITIES OF THE TOTAL INVESTMENT DEFLATOR**
(Shock minus control, per cent)

| | Quarters | | | |
|---|---|---|---|---|
| 1% increase in: | 1 | 4 | 12 | 32 |
| Wage costs | .21 | .61 | .66 | .68 |
| Trade prices | .07 | .23 | .25 | .26 |
| Energy prices | .02 | .05 | .05 | .05 |
| Total | .30 | .89 | .96 | .99 |
| Capacity utilization rate | .01 | .10 | .22 | .22 |
| Indirect sales taxes | 0 | .01 | .01 | .01 |
| Productivity | -.06 | -.20 | -.20 | -.20 |

The deflator for total **government expenditure** on goods and services including inventories (PGGST) is a weighted average of the government non-wage and wage deflators and the exogenous price of government inventories. The non-wage deflator (PGG) is a function of the deflators for business investment in machinery and equipment (PIME) and total consumption of services (PCS) with coefficients of respectively .77 and .23. The deflator for government non-military wage expenditure (PGWXM) is a function of public sector wages (WOTH) with a near unit coefficient. Nominal military pay and allowances (GMPF$), the final component of the government wage deflator, is exogenous. Non-wage expenditure accounts for about 40% of total government expenditure.

The value of sales, used in the inventories equation (Chapter 5), determines the price of the stock of business **inventories.**

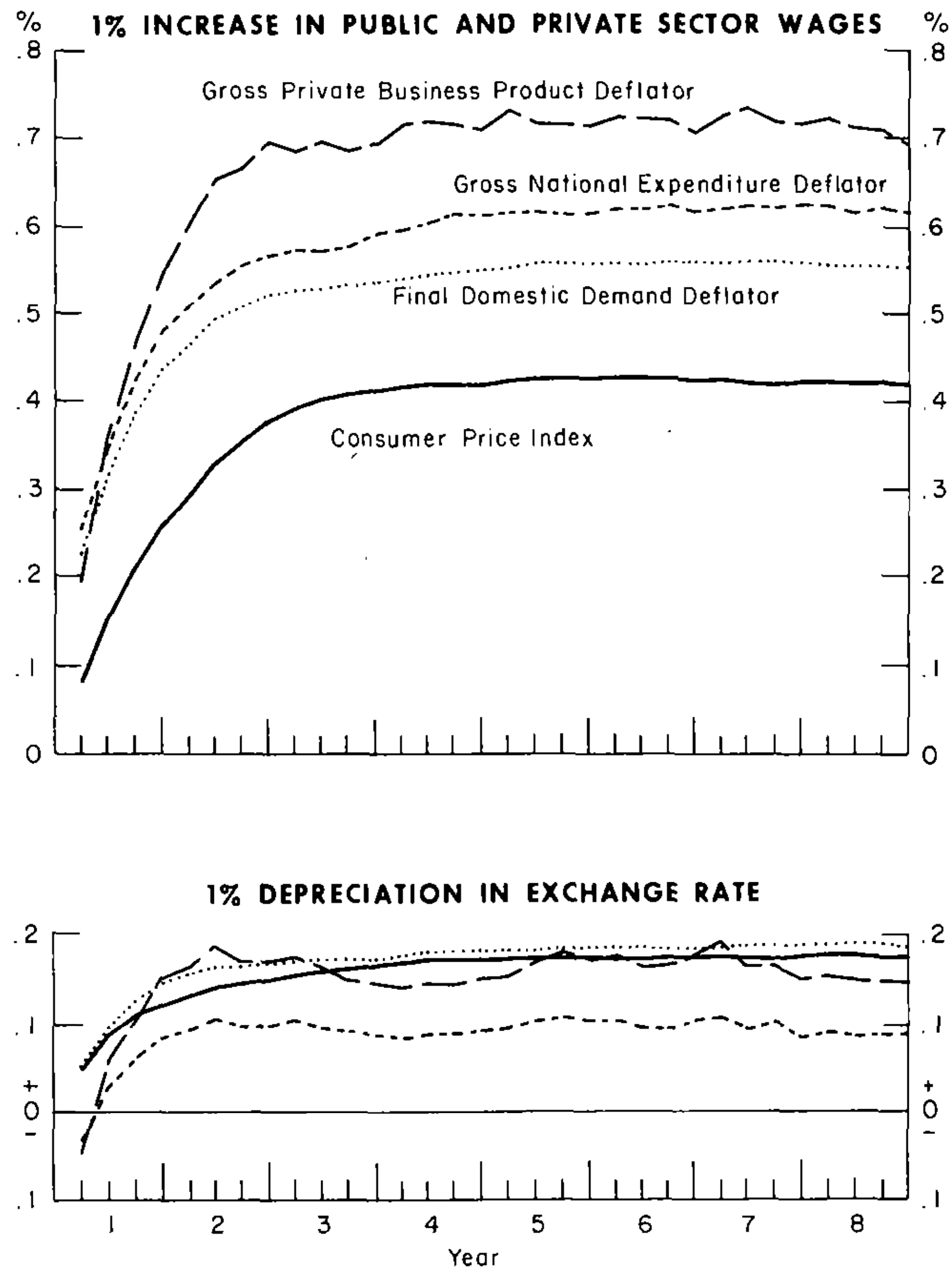
**Dynamics of the Aggregate Prices**

The dynamics of the aggregate prices (wages exogenous) are analyzed through wage and exchange rate shocks to the full price model. The first shock to the model is a 1% wage rate increase, divided into two parts: a 1% increase in both private and public sector wages, and a 1% increase in only private sector wages with the government sector deflators exogenous. The first case indicates the response of the aggregate deflators to general wage pressure while the second highlights the response of the aggregate deflators to private sector wage increases, and in particular estimates the elasticity of the gross private business deflator (PGPP) with respect to private sector wages. The second shock to the price model is a 1% depreciation of the external value of the Canadian dollar where import and export prices are allowed to respond to the depreciation.

The response of the four aggregate deflators to a **1% increase in both private and public sector wages** is shown in the upper panel of Figure 12. The percentage increase in the CPI is slightly higher than that reported in Section 7.2, .42% above

**Figure 12**

**DYNAMIC RESPONSE OF AGGREGATE PRICE DEFLATORS**
**(Shock minus control, per cent)**



control compared to .39%. The increased response does not emanate from the public sector wage increase (in this partial model WOTH has no influence on consumer expenditure deflators) but rather is the result of (a) the positive influence of unit labour costs on the export price of motor vehicles, which in turn results in higher domestic auto prices, and (b) the influence of investment deflators on imputed rental prices and thus on unit capital costs, leading to twice the response in the price of semi-durables.

The final domestic demand deflator (PFDD) exhibits a faster and larger response to the wage shock than the CPI. On impact PFDD increases almost three times as much as the CPI, and after eight years the deflator is .55% above control. As mentioned in the previous section, the non-consumption components of final domestic demand are more sensitive to wage increases than are the consumption components. The business investment deflator increases .66% in the long run due in large part to the strength of the response in the non-residential construction deflator. The combination of a private and public sector wage increase leads to a .75% increase in the total government deflator, .6% on impact. The government wage deflator is a significant contributor to the rise, increasing almost 1% since it is driven by increases in WOTH. The non-wage deflator rises .53%, responding to private sector wages through increases in the investment deflator and the consumer total service deflator, which is one of the most labour intensive of all consumer expenditure deflators.

Improvement in the terms of trade and an increase in the price of inventories combined with the response of PFDD leads to a .62% rise in the gross national expenditure deflator by the end of the simulation. The terms of trade improve throughout the simulation, since the price of imports remains virtually unchanged while the price of exports increases .21% by the end of the simulation. Although a few export prices of goods are a function of domestic unit labour costs, the main contributor to the improvement is the export price of services excluding interest and dividends. This export price depends on the domestic price of services excluding rent which again is very responsive to wage increases.

The gross private business product deflator is the most responsive of the aggregate deflators, increasing .7% in the long run.

In order to isolate the response of the aggregate deflators to private sector wage rate increases, the shock was rerun increasing only WNIC and holding the government deflators exogenous. A **1% increase in private sector wages** leads to the same response in the consumption, investment and inventories deflators and in the terms of trade as in the previous shock; however, with the government sector exogenous the response of the final domestic demand deflator and GNE deflator is lowered considerably--the long run values are .39% and .45% respectively. In response to the private sector wage shock the business output deflator exhibits a .67% increase.

The response of the four deflators to a **1% depreciation in the exchange rate** is presented in the lower panel of Figure 12. By the end of the simulation PCPI is .18% above control compared to a .21% increase reported in section 7.2 when all trade prices were increased 1%. The reduced response is explained by the movement of two trade prices: the import price of other consumer goods and the export price of motor vehicles to the U.S. increase only slightly more than half as much as the exchange rate. Thus, the upward pressure on most of the deflators comprising PCPI is reduced through the endogenization of trade prices.

The final domestic demand deflator increases only slightly more than PCPI throughout the simulation; however, the similarity of the response masks very different movements in the components of PFDD. The machinery and equipment (M & E) investment deflator increases .44% relative to control in the long run and is the main contributor to a .27% increase in the total business investment deflator. Although business investment accounts for only about 15% of final domestic demand, the relatively large increase in price puts considerable upward pressure on PFDD. The government non-wage deflator increases .30% relative to control due mainly to the large increase in the price of M & E. However, because wages do not move, a large portion of the total government deflator remains unchanged and the deflator increases only .14%. This relatively modest increase works to dampen the upward pressure from the investment deflator with the result that the movement in PFDD is very similar to PCPI.

In the first quarter of the simulation, the depreciation of the exchange rate results in a .35% deterioration in the terms of trade because import prices react faster than export prices. Since domestic prices respond relatively slowly to the depreciation, the terms of trade deterioration cause PGNE to fall slightly relative to control. However, by the second quarter of the simulation, domestic price increases outweigh the depressing effect of the terms of trade and PGNE rises above control. The deterioration in the terms of trade eases slightly in the long run but PGNE is still only .09% above control by the end of the simulation, well below the increase in PFDD.

In the long run the business product deflator responds in much the same way as the CPI and the final domestic demand deflator, reaching .15% above control. However, in the short run the terms of trade deterioration lead to a decline in the deflator, which reverses after one quarter.

Assumptions made concerning the response of the farm gate deflator and especially the price of energy are crucial when calculating the magnitude of the response of the aggregate deflators to an exchange rate depreciation. The shock was rerun including a simulation rule in which the farm deflator and the price of energy respond fully to the depreciation. The farm deflator influences the food equation, while the price of energy enters all the consumer expenditure deflators and investment deflators directly as an intermediate cost. When these added costs are filtered throughout the price model, the CPI increases .41%, the final domestic deflator .32%, the GNE deflator .23% and PGPP .37% in the long run. The price response doubles with respect to a depreciation of the exchange rate.

## Appendix to Chapter 7

### *TECHNICAL DESCRIPTION OF THE WAGE & PRICE SHOCKS*

**Table 18 Response of Wages to Increases in Productivity, The CPI and the Unemployment Rate**

Equations: EQN59; EQW01, EQW02

Time Period: 1972Q1 - 1979Q4

Shocks:
1. ETFP = ETFP·1.01
2. PCPI = PCPI·1.01
3. RNU = RNU+1

**Table 19 Dynamic Elasticities of the Consumer Price Index**

Equations: EQP01-EQP06, EQP08, EQP12, EQP13, EQP66

Time Period: 1972Q1 - 1979Q4

Shocks:
1. WNICIR = WNICIR·1.01
2. PMOCG = PMOCG·1.01
   PXLUM = PXLUM·1.01
   PXMMO = PXMMO·1.01
   PXMVP2 = PXMVP2·1.01
3. PENERG = PENERG·1.01
4. PFA = PFA·1.01
5. RCME = RCME·1.01
   RCNR = RCNR·1.01
6. TIM = TIM·1.01
7. RMC = RMC·1.01
8. UGPP = UGPP·1.01
9. TCNDO = TCNDO·1.01
   RTISFM = RTISFM·1.01
   RTISFR = RTISFR·1.01
   RTISFS = RTISFS·1.01
   RTISPA = RTISPA·1.01
   RTISPM = RTISPM·1.01
10. UGPPS = UGPPS·1.01

**Table 20 Dynamic Elasticities of the Total Investment Deflator**

Equations: EQ1, EQI59, EQP07-EQP09, where
EQ1 = (IBUS$+PIRC·IRCA)/(IBUS+IRCA)

Time Period: 1972Q1 - 1979Q4

Shocks:
1. WNICIR = WNICIR 1.01
2. PMCM = PMCM·1.01
   PMME = PMME·1.01
   PXLUM = PXLUM·1.01
3. PENERG = PENERG·1.01
4. UGPP = UGPP·1.01
5. RTISFM = RTISFM·1.01
   RTISFR = RTISFR·1.01
   RTISFS = RTISFS·1.01
   RTISPM = RTISPM·1.01
6. UGPPS = UGPPS·1.01

**Figure 12 Dynamic Response of Aggregate Price Deflators**

Equations: Sector 7; EQC53, EQC55
EQF50, EQF51, EQF53-EQF55,
EQF57, EQF58, EQF60-EQF62
EQI04-EQI08, EQI53-EQI55, EQI59,
EQY05, EQY52, EQY53, EQY62, EQY63, EQY71
plus
EQ1XID$ = XA·(MG$/MA)
EQ2XTR$ = XB·PXSEID
EQ3XFS$ = XC·PXSEID
EQ4XOS$ = XD·PXSEID
EQ5MID$ = MA·(XG$/XG)
EQ6MTR$ = MB·PMSEID
EQ7MFS$ = MC·PMSEID
EQ8TWF = MD·PMSEID
EQ9MOS$ = ME·PMSEID

where

XA = XID$/(MG$/MA)

XB = XTR$/PXSEID

XC = XFS$/PXSEID

XD = XOS$/PXSEID

MA = MID$/(XG$/XG)

MB = MTR$/PMSEID

MC = MFS$/PMSEID

MD = TWF/PMSEID

ME = MOS$/PMSEID

Time Period: 1972Q1 - 1979Q4

Shocks:

1. WNICIR = WNICIR·1.01
   WOTHIR = WOTHIR·1.01
2. Delete from the model:
   EQP11, EQP33, EQP61, EQP62, EQY62, EQY71.
   WNICIR = WNICIR·1.01
3. PFX = PFX·1.01
4. Add to the model:
   EQ10, PENERG = APENER·PFX/APFX
   EQ11, PFA = ADFA·PFX/APFX
   where: APENER, APFX and APFA are the historical values of PENERG, PFX and PFA respectively.
   PFX = PFX·1.01

# Chapter 8

# AGGREGATE SUPPLY

Heather Robertson

The economic developments of the mid-1970s have raised a number of supply-related issues and brought into question the capabilities of the traditional large macroeconomic models in addressing such issues. In particular, the models have had difficulty in explaining such 1970s phenomena as large increases in energy and materials prices, a severe and protracted downturn in the business cycle accompanied by record rates of inflation, declines in both labour and capital productivity, and lower levels of potential output. The difficulty stems from the fact that large structural models of the Keynesian tradition are demand-oriented and lack a well developed supply bloc.

The RDXF model is similarly limited for various reasons. First of all, the structure of RDXF is one in which demand determines output at any given time, and deviations in productivity close the gap between actual output and supply output, i.e., the output that could be produced according to the production process. In principle, price, wage, and factor adjustments should, over the longer term, guarantee that actual output equals supply output. In fact, because long-run consistency has not been imposed on prices, factors or the production process, productivity can deviate significantly from trend for long periods of time. Second, the RDXF model employs an aggregate measure of supply output, whereas prices and demand components have been developed on a disaggregated basis. Although the model is more parsimonious as a result of this structure, its ability to explain such phenomena as supply bottlenecks and capacity constraints within an industry is somewhat limited. Finally, because the production technology includes capital and labour inputs only, the model does not adequately capture factor substitution effects that should result from materials and energy price shocks, nor can it explain effective resource constraints in the economy.

The purpose of this chapter is to describe the supply bloc in RDXF and to present its advantages and limitations in the light of recent economic developments. Section 8.1 discusses the general supply variables in the model, starting with a description of the various concepts of production: supply output, which is defined by a Cobb-Douglas production function; normal output, which is supply output defined at trend hours and employment; and real gross private business product, which determines the actual level of output in RDXF. Next is an analysis of the factor shares and trend factor productivity in the production function. The section concludes with a comparison of the level of disaggregation in the production function to that in output prices and the real expenditure components.

Section 8.2 describes the specification and dynamic properties of a fixed-price supply model. The dynamic interaction of factor demands and output are highlighted by two series of shocks: a simulated increase in business product and simulated increases in factor and output prices. Section 8.3 outlines a flexible-price supply model, which is composed of the domestic employment, investment, factor price, and output price equations and includes real-price linkages through capacity utilization and labour market tightness terms. The discussion focuses on the behaviour of firms implicit in the specification of factor demands and prices. At the end of the section two simulations, one with a demand-side shock and the other with a supply-side shock illustrate: the source of short- and long-term adjustments, particularly the role of short-run productivity in filling the excess demand gap; the asymmetry between demand-side and supply-side shocks; the price-quantity trade-off; and the question of non-neutral responses among factors of production. The discussion of the real wage response to the two shocks will preview the implications for second-round demand responses that would occur in a full model context.

## 8.1 CONCEPTS OF PRODUCTION AND PRODUCTIVITY

Following is a description of the methodology in the construction of supply output, normal output and real gross private business product. Supply output (UGPPS), defined as a synthetic Cobb-Douglas production function, is a measure of production capabilities with current capital stock and current man-hours. 'Normal' output (UGPPD) is a measure of production capabilities with the current capital stock, trend hours, and employment at trend output. Real gross private business product (UGPP) is an argument in equations for the three factors of production--man-hours, the non-energy stock of machinery and equipment and the non-energy stock of nonresidential construction--thereby providing the link between factor demands and the demand for goods and services. Various issues and problems that were confronted in constructing these production concepts, such as the inclusion and exclusion of various sectors in the definition of output; the choice of factors of production; the definition of productivity; and the factor shares implied by the production technology are discussed in some detail.

**Supply output** in RDXF (UGPPS) is based on a Cobb-Douglas production function with three inputs: the stock of machinery and equipment excluding energy (KMEXE), the stock of non-residential construction excluding energy (KNRCXE)* and total man-hours, which is the product of private sector employment (NIC) and average weekly hours in mining and manufacturing (HAWMM). Trend factor productivity (ETFP) is the augmenting factor in the production function, which converts factor quantities (measured in 1971 dollars for capital and in man-hours for labour) to the trend flow

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* The stocks of capital, which include agriculture, do not correspond exactly to UGPP, which excludes agriculture. Although agriculture is a very small component of business product (approximately 3.7%) it could affect the productivity measure if there were substantial variations in the average product of capital in the agricultural sector.

of services that the factors provide. Productivity is assumed to be Hicks neutral, and is therefore embodied both in labour and in capital. The production function is defined as:

$$UGPPS = J1L(KMEXE)^{.18} J1L(KNRCXE)^{.16} (NIC \cdot HAWMM \cdot 52)^{.66} ETFP$$

Energy-related capital is excluded as a factor of production because of the downward bias that energy capital imparts to productivity, particularly in the post-1975 period. Bilkes [10] calculated a .3% deviation between total factor productivity growth* including the energy-related capital stock and excluding this category. The differential has widened over time because energy investment is an increasing proportion of total investment: the energy component in non-residential construction investment increased from an average of 39% in 1961 to 48% in 1979; for machinery and equipment, the proportion increased from 12% in 1961 to 17% in 1979. Energy investment typically exhibits lower productivity growth over the short run because of long gestation periods and because of large energy price increases since 1973, which rendered profitable the less efficient energy projects.**

**'Normal' output** (UGPPD) is defined as the output from the Cobb-Douglas production technology, using the existing capital stocks, trend hours and private sector employment at trend output ($\bar{N}$). Private sector employment at trend output is the product of the total labour force (NL) and the employment rate at trend

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* Bilkes' non-energy productivity measure was defined in terms of RDP for manufacturing, KMEXE, KNRCXE, and man-hours. The measure including energy was calculated using KME and KNRC. Bilkes calculated 1.3% productivity growth per annum when energy was included and 1% when energy was excluded in the 1975-78 period.

** Note that energy is not excluded in the calculation of UGPP. The reason for this is the difficulty in isolating the energy portion of value added from each component of GNE. Private sector employment includes employment in the energy sector as well.

output (1-.01 RNUTO),* less trend employment in the government and non-commercial sector (J8A(NE-NIC)), which is an eight-quarter moving average of the difference between total employment and private sector employment:

$$UGPPD = J1L(KMEXE)^{.18} J1L(KNRCXE)^{.16} (H \cdot \bar{N} \cdot 52)^{.66} ETFP,$$

where $\bar{N} = NL(1-.01\ RNUTO) - J8A(NE-NIC)$

$H = a_0 - a_1 QQTIME.$

Trend output is the benchmark level of production determining capacity utilization in RDXF, and, in some sense, defines the equilibrium level of output; a deviation of actual output from trend (variations in capacity utilization) triggers price adjustment, whereas deviations of supply output from trend (representing disequilibrium in the labour market) trigger wage adjustment. However, trend output does not necessarily represent the non-accelerationist level of production; a number of conditions other than equilibrium in the labour market are required in order that inflation be constant in the RDXF model.**

**Real gross private business product** (UGPP) is calculated by deflating each component of nominal business product and summing the components. Nominal gross private business product (YGPP) represents output in the commercial non-agricultural sector, and therefore is nearly equivalent to gross domestic product (GDPC) for this sector. The main difference in definition between the

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* Note that the actual rather than the "trend" labour force is used in calculating UGPPD. Because NL shows cyclical variations, the movement in UGPPD is overstated, and capacity utilization understated. There is a certain degree of circularity in the calculation of RNUTO and UGPPD. RNUTO, which is defined independently of UGPPD, is the level of unemployment given the institutional structure of the labour market at which real GNE is at a trend level, where the trend is defined exogenously. Actually this trend would be influenced by the "trend" utilization of factors which creates a simulaneity problem in calculating these series.

** Such as a constant rate of growth in (exogenous) energy and materials prices, purchasing power parity, homogeneity of prices with respect to costs. See Gosselin and Staranczak [33].

two series is that YGPP does not include rent. (The exclusion of rent conforms with the choice of factors in the Cobb-Douglas production function, where the stock of residential construction is not part of the capital stock.) However, rather than building the series from its industry components as with GDP, the methodology for calculating YGPP is to convert nominal GNE to gross domestic product at factor cost, and then to subtract the income components that most closely correspond to value-added in the industries excluded from GDPC, as follows:

| | |
|---|---|
| YGPP = YGNE | Gross national expenditure |
| - (XID$-MID$+TWF+ENARS$) | Converted to total GDP at market prices |
| - TILGS | Converted to total GDP at factor cost |
| - (YWOTH+GMPF$) | Less wage income in the non-commercial sector |
| - (CCAGF$+CCAGH$+CCAGPM$) | Less non-commercial CCA |
| - (YFA+IIF$)·1.6 | Less accrued farm income |
| - CRENT$ | Less paid plus imputed rent |

Subtracting net receipts of interest and dividend payments (XID$-MID$), withholding taxes payable by non-residents (TWF), and the national accounts expenditure residual (ENARS$) converts GNE to GDP at market prices. The elimination of indirect taxes (TILGS) converts GDP to factor cost. Wage income in the non-commercial and government sectors (YWOTH) and from the military (GMPF$), plus federal (CCAGF$), provincial and municipal (CCAGPM$), and hospital (CCAGH$) capital consumption allowances, correspond to non-commercial value-added, and therefore are excluded from YGPP. Accrued farm income (YFA) plus farm inventories (IIF$)* proxy agricultural GDP; the 1.6 scaling factor

* Farm inventories should not be added to YFA because the latter component already includes farm inventories. This "double counting" problem has been corrected in the March 1980 version of RDXF.

provides the link between accrued farm income and GDP in the farm sector.

Real gross private business product in 1971 dollars is therefore defined as:

UGPP = (UGNE-XID$/(MG$/MG) + (MID$-TWF)/(XG$/XG)
- 7121.5•NOTH - 10282.9•NARMY
- (CCAGF$+CCAGPM$+CCAGH$)/PGI-URDPAG-CRENT)•EPGPP.

The export and import goods deflators (MG$/MG, XG$/XG) convert interest receipts and withholding taxes to 1971 dollars. Note that an item comparable to the 1971 value of TILGS* is not excluded from UGPP. Such a term is not necessary because the final goods deflators include indirect taxes; the scaling by these deflators automatically converts each term to factor costs. The product of paid employment for those other than in the industrial composite (NOTH) and for the armed forces (NARMY) and their respective 1971 average annual wages is the real income for the government and noncommercial sectors. The constant wage assumes zero productivity growth in these sectors as does the calculation in the National Accounts. The government investment deflator (PGI) converts the capital consumption categories to 1971 dollars. Agricultural real domestic product (UGDPAG) measures real farm output. A scaling factor (EPGPP=.8)** is required to guarantee that the sum of the deflated components produce an overall deflator (PGPP=YGPP/UGPP) of unity in 1971.

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* There is a problem with the TILGS component in YGPP. TILGS should not include municipal indirect taxes because UGPP does not include rent. Work is underway to correct this problem.

** The conversion factor is required mainly because the TILGS component is not excluded from UGPP. Thus EPGPP is defined as:

$$\frac{(YGPP_{71}-TILGS_{71})}{YGPP_{71}}$$

**Factor Shares and**
**Trend Factor Productivity**

Nominal gross private business product (YGPP) is divided between labour and capital as follows.

The average of labour's share* ($\alpha_L$) from 1961 to 1980 determines the output elasticity for labour in the production function. The sum of the two output elasticities for capital, i.e., for machinery and equipment and non-residential construction, are constrained to be one minus the output elasticity for labour ($\alpha_K=1-\alpha_L$), with the proportions of $\alpha K$ being determined by the respective average shares of the returns to capital.**

The upper line in Figure 13 shows labour's share over the 1961-80 period. Although labour's share ranged from .62 in 1961Q4 to .70 in 1971Q1, the mean in each half-decade has been very close to the average share over the entire period ($\alpha_L$=.66). Therefore, it seems reasonable to impose a constant output elasticity for labour over this range. The lower lines in the figure show capital's total share of YGPP ($a_K$)*** and one minus labour's share ($a_K{}^* = 1-a_L$)).

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* $aL=WNICIR \cdot (1+RSPIC) \cdot NIC \cdot 52/YGPP$. Critical to deriving output elasticities from shares is the assumption of constant returns to scale. From Euler's theorem: $K\partial Q/\partial K+L\partial Q/\partial L=kQ$, where k is the degree of homogeneity. Dividing by Q, $\alpha_K+\alpha_L=k$, where $\alpha_i=MP_i/AP_i$, i=K,L. From the static profit maximizing decision of the firm: $\alpha_L=WL/PQ$, $\alpha_K=RK/PQ$, or $WL/PQ+RK/PQ=k=1$, or the shares exhaust output produced.

** $\alpha_{K1}=\alpha_K \cdot RCME \cdot KMEXE/(RCME \cdot KMEXE+ RCNR \cdot KNRCXE)$, and $\alpha_{K2}=\alpha_K \cdot RCNR \cdot KNRCXE /(RCME \cdot KMEXE+RCNR \cdot KNRCXE)$. The term 'returns to capital' is imprecise because rental rates are generally defined in terms of the expected future stream of returns, rather than current returns. RCME and RCNR are an approximation to current returns in the definition of returns to capital.

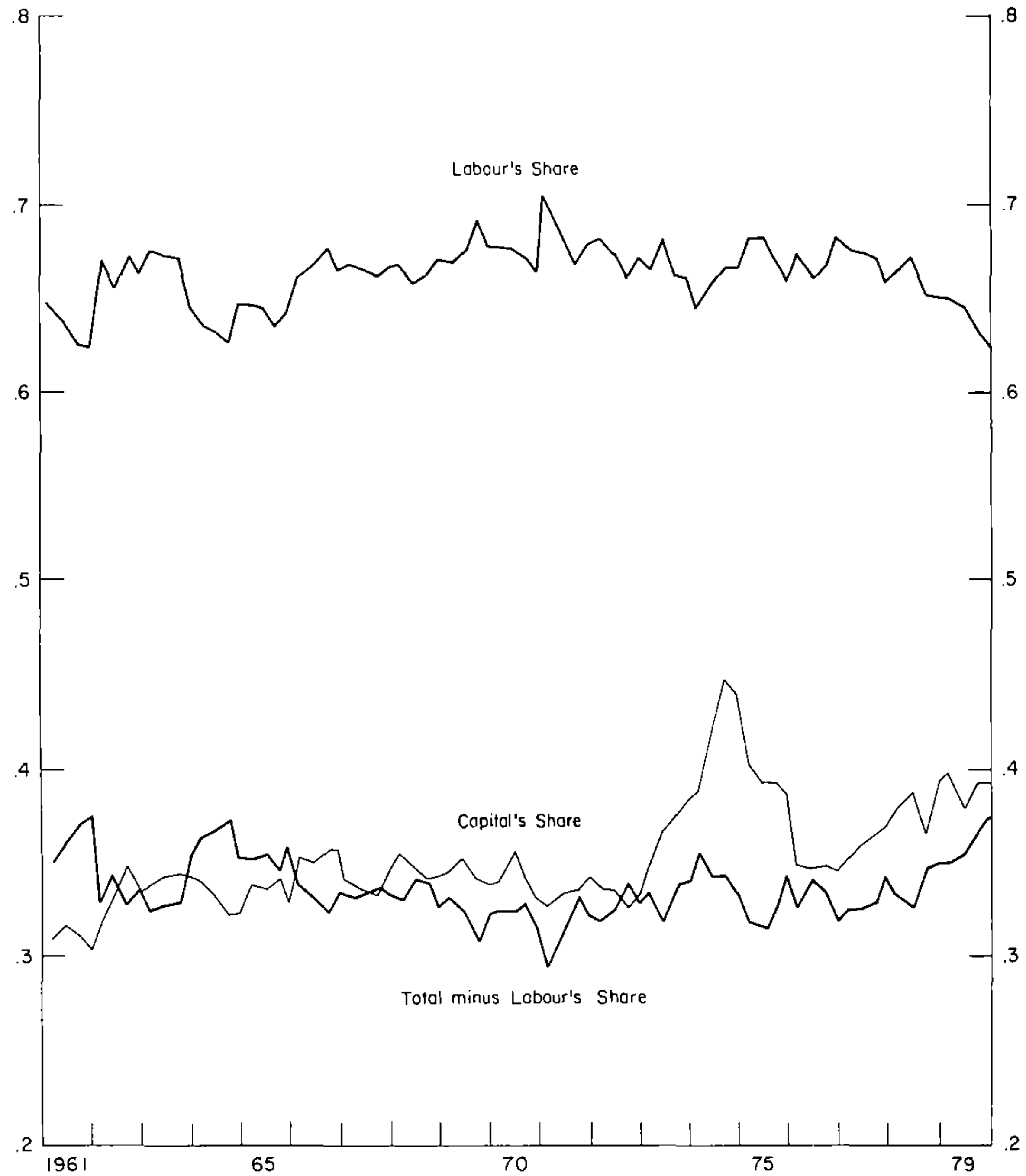
*** $a_K$ = $(RCME \cdot KMEXE+RCNR \cdot KNRCXE)/YGPP/[\bar{a}_{K(6+7)}/a^*{}_{K(61-71)}]$

The means of the series are constrained to be the same from 1961

**Figure 13**

**FACTOR SHARES OF OUTPUT**

**1961-79**



If returns to factors exactly make up the value of output (recall that YGPP is measured on a value-added basis) then the $a_K$ and $a_K$* series should be identical. Any variation between the two series could stem from a number of sources. First of all, the actual return on capital does not equal the desired return at any given time; the actual exceeds the desired when the stock of capital is too large, implying that investment should fall. This would create short-run deviations in the two series only insofar as the investment sector is in disequilibrium. Second, factors of production could be omitted from the production function. Only under very restrictive assumptions about the production function would this not create a bias in the calculation of shares.* The greater the deviation between relative factor prices, the more pronounced the bias would be.

Another source of deviation has been measurement error in the factor price, factor, and YGPP series. For example, variations in the utilization of the existing capital stock over the business cycle are not measured in KMEXE and KNRCXE, and thus short-run variations between $a_K$ and $a_K$* occur. In the 1961-72 period, the two series exhibit the same (constant) trend although the short-run fluctuations are negatively correlated. From 1973 onward, however, the series show substantial deviation from each other. Capital's share, measured using the existing stocks and

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to 1971, because RCME and RCNR are measured as indices, and therefore are not the same units of measurement as annual wages. Furthermore, if there are excluded factors of production, the sum of $a_K$* and $a_L$ should be less than unity.

* With the production function Q=f(K,L,E) where E is an omitted factor, if Q/E=k, a constant, then the production function can be rewritten Q=g(K,L,k). One way of meeting the constant average product condition would be with the Leontief technology. Also, in the case of a nested production function, where the nested factors are used in constant proportion, the production function can be rewritten in terms of two factors of production: Q=f(g(K,E)L), E=kK, Q=f(g(K,rK),L)=h(K,L,k).

rental existing stocks and rental rates ($a_K$), is considerably higher than the residual $a_K$*, particularly in the 1973-74 period. The deviation narrows after about three years and remains approximately constant after 1976. The divergence is coincident with large energy and materials price increases after 1973. If these price increases rendered a larger amount of the capital stock obsolete, then $a_K$, where capital is measured at purchase value with a constant depreciation rate, would be temporarily overestimated. If the capital measurements overestimated the actual capital being utilized since 1973, then the measurement of trend productivity (ETFP) might have a downward bias as well.

Trend total factor productivity (ETFP) is calculated by estimating the trend growth rates of the total factor productivity series (TFP).* TFP is defined as the ratio of real gross private business product (UGPP) to the factor quantities scaled by their factor shares:

$$TFP = UGPP/(KMEXE_{-1}{}^{.18}(KNRCXE_{-1}{}^{.16})(NIC\cdot HAWMM\cdot 52)^{.66}$$

TFP was regressed on two time trends, one from 1961Q1 to 1970Q3, and the other from 1974Q1 onward, with the structural breaks determined by a rolling regression technique.** Over the interim

* As a result of some of the criticisms stated here, ETFP was constructed on a judgmental basis. The revised ETFP series grows at 2.2% per annum from 1961Q1 to 1972Q4 and .5% from 1975Q1 thereafter. In the 1973Q1 to 1975Q1 period the growth rate is an arithmetic weighted average of those in the initial and latter period. The ETFP series and TFP series are designed to intersect in 1961Q4 rather than in the trough period of 1961Q1.

** TIMVAR program. The formulation of the equation is:
ETFP = TFP where log(TFP) = (A1+A2·QQTIME)·(NPER.GE.19611)·
(NPER.LE.19703)+(A3+A4·QQTIME)·J13A((NPER.GE.19704)·
(NPER.LE.19734)).+(A1+A2·QQTIME)·(1-J13A((NPER.GE.19704)·
(NPER.LE.19734))+(A3+A4·QQTUNE)·(NPER.GE.19741)·(NPER.LE.19854).

period (1970Q4 to 1973Q4), the estimated value from the first period was gradually phased out using arithmetic declining weights while the estimated value from the second value was phased in. The ETFP series was then generated from the fitted values of the TFP regression.

The deviation of TFP from ETFP describes the cyclical swings in productivity* as shown in Figure 14, which presents the ETFP series superimposed on the TFP series. To the extent that there are short-run costs of adjustment in factors of production, TFP varies positively with the business cycle, because existing factors would be temporarily utilized above trend in an upswing and underutilized in a downswing. The annual growth in ETFP is 1.97% in the first period and .16% in the later period; over the interim period the annual rate of growth jumps to 4.7% in 1970Q4 and is above the 1961-70 growth rate for 10 of the 13 transitional quarters. Note that TFP is below trend from 1971Q1 to 1972Q4, in a period of strong growth in UGPP. Also in the post-1974 period, although UGPP growth was considerably below average, there are regular swings of TFP around trend.

Difficulties in modeling ETFP are (a) a questionable relationship between ETFP and the total variation in productivity, (b) a possible overestimation of the growth in the interim period, and (c) an underestimation of the growth in the final period. These problems stem mainly from the rolling regression methodology for choosing structural breaks in a series, which is biased towards choosing cyclical turning points as 'true' breaks in the trend series. This can be seen from the choice of 1970Q4, a trough, and 1973Q4, a peak, as structural breaks; because the final period runs peak to trough, ETFP growth will be

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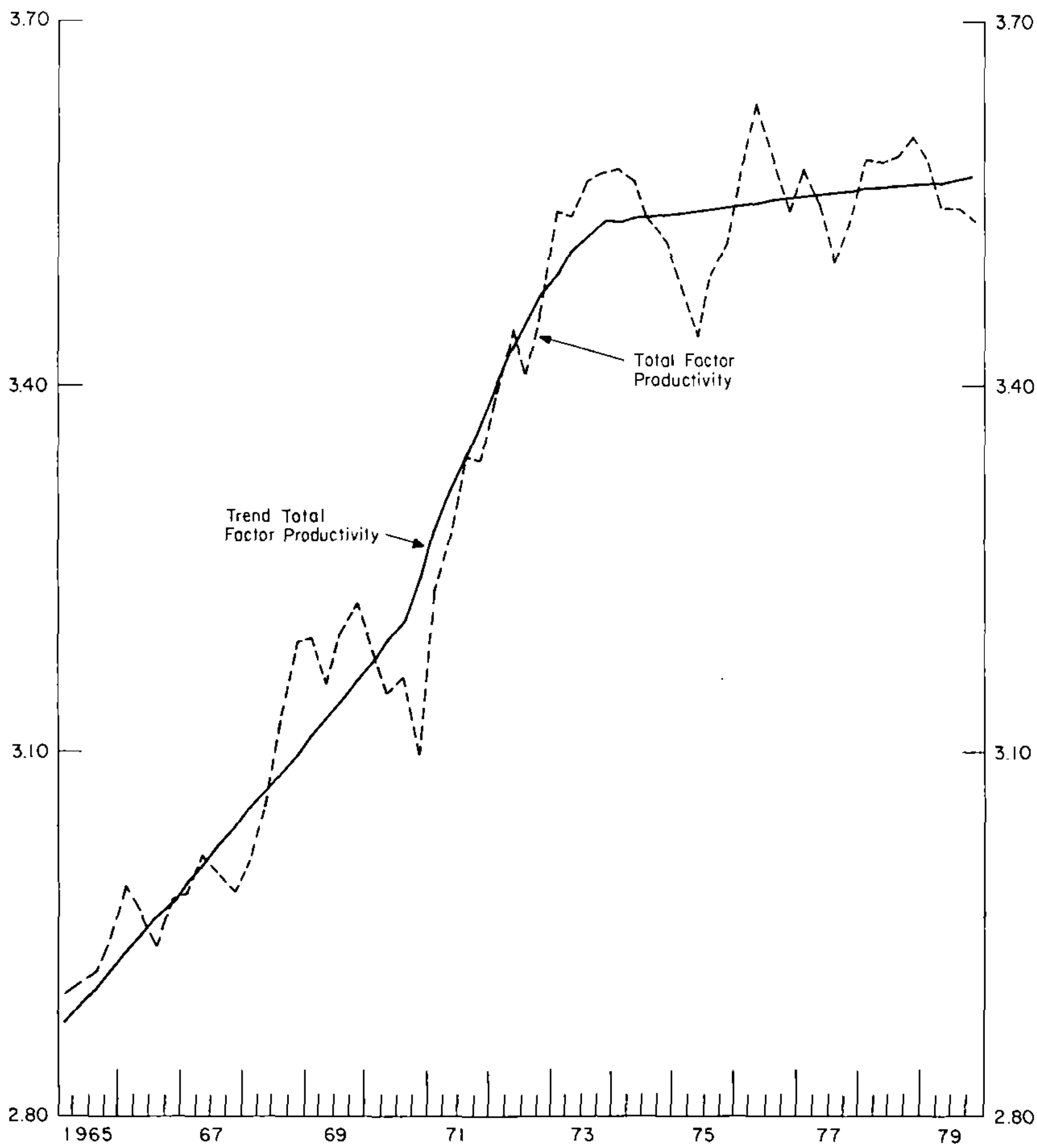
* Decomposing TFP into its cyclical and trend components gives logTFP = logETFP + log(UGPP/UGPPS) = logETFP + logSRP, where SRP is 'short-run' productivity, a measure of the variation in the utilization of factors over the business cycle.

**Figure 14**

**CYCLICAL SWINGS IN PRODUCTIVITY**
**1965-79**



underestimated over this period.

Aside from the problems of the choice in structural breaks cited above, the low productivity growth in the post-1973 period could stem from measurement problems in the capital stock series. (Recall from Figure 13 that the returns to capital deviate considerably from their share, particularly in the 1973-75 period.) Over periods in which the capital stock is overestimated (i.e., because of an increase in obsolescence due to large energy price increases) trend factor productivity would be underestimated. It would return to its original level after a period of time during which the obsolete stock was replaced.

There is some difficulty in reconciling the timing and magnitude of the change in ETFP growth with recent empirical research. The decline in Canadian productivity growth in the 1970s has been well documented: growth in real GNE per worker was 2.3% per annum from 1956-74 and only .6% from 1974-78; real domestic product per man-hour grew 2.6% from 1967 to 1973 but 1.1% thereafter. The decline in overall productivity has been attributed both to the cyclical downturn in the economy that persisted for a large part of the latter half of the 1970s and to structural changes in the economy, such as lower productivity in energy-related industries, higher income and dividend payments abroad, declining capital-labour ratios in a number of sectors, and the rapid increase in energy and materials prices since 1973.

Variations in the business cycle, should not, however, affect trend productivity. Furthermore, some of the structural reasons for lower productivity growth listed above would have less of a negative influence on ETFP than on the labour-related measures. For instance, the energy-related capital stock is not included as a factor in the production function, and therefore lower productivity in the energy sector would create less of a negative influence on ETFP.* As for the capital-labour ratio, its decline

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* Bilkes [10].

would induce less of a decline in ETFP than in a measurement of labour productivity because ETFP includes capital as well as labour productivity.

The causes of changing patterns in ETFP and methods for modeling this series merit further investigation.

## 8.2 LEVELS OF DISAGGREGATION

The demand and supply sectors in RDXF are not equally disaggregated. While output prices, the level of real output and the import component of real output are modeled at a disaggregated level, the production function and factors of production (and their respective factor prices) are represented as composite commodities. Such partial disaggregation, although it provides structural detail for certain purposes, may create problems because individual behaviour does not always conform to the aggregate.

Demand equations are represented by disaggregated real domestic expenditure components ($u_i$) and are a function of real income and relative prices. Imports are disaggregated in categories similar to the domestic expenditure components. They respond directly to specific activity variables, but only to general excess demand pressure in the short term, although changes in the ratio of disaggregated import prices to their equivalent domestic prices can act as an indirect channel for demand pressure in the individual markets. Although inventories are modeled at an aggregate level, they show a different response to each individual expenditure component.

Deflators for the disaggregated expenditure components ($P_i$) are modeled as supply equations for each market:*

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* See Nordhaus [64]. With firms facing the following Cobb-Douglas production function $U_i=L^{\alpha_L}K^{\alpha_K}M^{\alpha_C}\cdot ETFP$, where $\alpha_L+\alpha_K+\alpha_C=1$, and the demand equation $U=Y^{\alpha_1}P^{\alpha_2}$, the firm's price function is:

$$\log P_i = \log(A(\alpha_2/(1-\alpha_2)))+\alpha_K \log R+\alpha_L \log W+\alpha_c \log C.$$

Since $\log U_i = \alpha_K \log K+ \alpha_L \log L+ \alpha_L \log M+\log ETFP$,

$$\log P_i = \beta_{i0} + \beta_{iL}\log ULC + \beta_{iK}\log UKC + \beta_{iC}\log C^* + \beta_{i4}CAPU,$$

where ULC = unit labour costs
UKC = unit capital costs
C* = other costs
CAPU = capacity utilization.

The individual final deflators respond to aggregate factor costs and to aggregate capacity utilization. The expression of costs at an aggregate level implies that factors of production are assumed to be homogeneous, and therefore all factors implicit in the individual price equations should be included in the aggregate production function (UGPPS). Further, coefficients on the cost terms represent the output elasticities related to the respective technology i, and the weighted sum of the output elasticities for all individual deflators (where the weights are the expenditure shares of $P_i$ in P) should equal the output elasticities in UGPPS.*

However, as presented in Chapter 7, costs in the model do not correspond to the technology represented in UGPPS because costs of production in the actual price equations include energy,

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$$\log P_i + \log U_i = \log(A(\alpha_2/(1-\alpha_2)) + \alpha_K \log(R\cdot K) + \alpha_L \log(W\cdot L) + \alpha_C \log(C\cdot M) + \log ETFP$$

or $\log P_i = \log(A(\alpha_2/(1- a)) + \alpha_L \log ULC + \alpha_K \log UKC + \alpha_C \log UMC + \log ETFP$

where R, W, and C are costs for capital, labour, and materials and UKC, ULC and UMC are normalized unit costs.

* Since $\Sigma\gamma_i \log P_i = P$, $\Sigma\gamma_i\beta_{iL} = \alpha_L$, $\Sigma\gamma_i\beta_{iK} = \alpha_K$, and $\Sigma\gamma_i\beta_{iC} = \alpha C$, thus the (implicit) aggregate price equation is

$$\log P = \alpha_L \log ULC + \alpha_K \log UKC + \alpha_C \log M,$$

where P represents the output deflator PGPP. Because UGPP is measured on a value-added basis, the coefficients $\beta_i$ should be scaled by the domestic portion of $u_i$, such that the import component is excluded.

farm-gate, and import prices as well as wage and capital costs. In fact, a 1% increase in wage costs produces a .7% increase in PGPP over the longer term (the production function implies .66). A 1% increase in capital costs produces a .04% increase in PGPP (.34% from the production function).* When all costs are increased 1%, PGPP increases .89%. Therefore the aggregate cost function implied by the individual price deflators in RDXF is not correctly specified as the dual to the production function, both because it is not homogeneous of degree one with respect to costs, and because the factors in the production function are not the same as the ones implied in the cost function. The implications in a full model context are that the steady state properties of RDXF are less desirable than they could be and that the ability to explain factor substitution as a result of cost variations is limited.

Thus, while the representation of production and factors as composite commodities is a convenient method of reducing the size and the complexity of the RDXF model,** some important linkages are omitted. The magnitude of this problem is seen in the dynamics of a model and depends on the origin of a shock to the system. For example, suppose there is an exogenous increase in the demand for a particular commodity. This would be translated into general excess demand pressure through an increase in UGPP, which, in turn, induces an increase in aggregate factors of production and factor prices. Higher factor prices, plus generally higher capacity utilization, would cause upward pressure in all price components, but whether the relative price of the 'high demand' component actually increases depends on the

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* This number may be biased downward to the extent that capital in the production function includes that required in the production and processing of materials. Thus the inclusion of materials costs may be collinear with capital costs in the individual price equations.

** For example, if the same domestic consumption components had fully disaggregated factor demands, production functions, and inventory and import responses, about 120 equations would be required. Currently, RDXF has about 30 equations (18 for volumes and deflators) to describe the consumption sector.

magnitude of the capacity and factor price elasticities in the particular equation. Its relative price could, indeed, decline. Now suppose there is a shock on the supply side, say higher energy prices caused by a reduction in the supply of this factor. In this example, each disaggregated output price increases according to the (implicit) factor intensity of energy in the production process of the particular commodity. Individual demand components would respond to relative price changes in each component. Aggregate output and price changes would then influence factors of production and other factor prices; the response unique to factors in the energy sector would not be captured, nor would the resulting bottlenecks that might develop through various stages of processing.

The transmission of energy and materials price shocks through the supply bloc have been a topic of particular interest in recent literature on supply-side economics. One way to create linkages would be to include energy and materials as factors of production directly in the production process. In the RDXF model, labour inputs include employment at all stages of processing, but capital inputs do not include employment at the energy production stage. Since UGPP is a value-added concept, total capital and labour would represent the value added at each stage of processing, but exclude imported energy and materials. Only if output were a gross measure would raw materials as well as the factors required to produce them be included, and problems would still occur from representing non-homogeneous factors of production in an aggregate production function. For example, the energy-related capital stock would respond very differently to an energy price increase than would non-energy related capital.

To summarize, the RDXF model is equipped to answer certain questions at a disaggregated level, but not others. The payoff to adding a greater amount of detail in factor quantities, factor prices and production depends on the focus of analysis. And, the less homogeneous the markets are, the more the analytical detail will be lost by using aggregate measures to proxy disaggregated ones.

## 8.3 A FIXED-PRICE SUPPLY MODEL

This section describes the specification and dynamic properties of a fixed-price supply model, which is used to examine the partial dynamic elasticities in the supply equations with respect to changes in production, factor prices, and output prices. The generalized loglinear structural form of the fixed-price model is as follows:

| | | |
|---|---|---|
| $K_1 = \alpha_{11}U + \alpha_{12}(P-R_1)$ | Stock of machinery and equipment | (1a) |
| $K_2 = \alpha_{21}U + \alpha_{22}(P-R_2)$ | Stock of non-residential construction | (1b) |
| $K = \alpha_{K1}K_1 + \alpha_{K2}K_2$ | Total stock of capital* | (1') |
| $= \alpha_1 U + \alpha_2(P-R)$, where $\alpha_1 = (\alpha_{K1}\alpha_{11}+\alpha_{K2}\alpha_{21})$ and $\alpha_2 = (\alpha_{K1}\alpha_{12}\cdot\frac{R}{R1} + \alpha_{K2}\alpha_{22}\cdot\frac{R}{R2}$ | | |
| $h = \beta_1(U-U_s) + Z2$ | Average weekly hours | (2) |
| $N = \gamma_1 N^* + \gamma_2 U + \gamma_3(W-P)$ | Private sector employment | (3) |
| $\bar{U} = \alpha_K K + \alpha_L NL + E + Z_4$ | Normal output | (4) |
| $U_s = \alpha_K K + \alpha_L(h+N) + E$ | Production function | (5) |
| $N^* = (\delta_1 U - \alpha_K K)\alpha L$ | Required employment | (6) |
| $NL = \varepsilon_1 N + Z_1$ | Labour force participation rate | (7) |

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* These equations are determined as flows; substitution of the flow equation into the comparable stock equation yields the above relationship.

where U is the real gross private business output

P is the output deflator

R is the rental rate on capital ($R_1$ = rate on M & E, $R_2$ = rate on non-residential construction)

W are private sector wages

E is trend productivity

Z are variables exogenous within the context of this discussion.

Coefficients represent dynamic elasticities, and therefore incorporate the lag structure in the equations.

The three investment equations are a function of the desired capital stocks, which in turn are a function of expected output, the output deflator and the rental price. The gestation period of the particular type of investment determines the time horizon on the expectations terms. Average weekly hours are a function of exogenous terms and the deviation between actual output and the output that would be produced, given current man-hours and capital, with a Cobb-Douglas production function. Employment is a function of required employment, actual output, prices and wages. Required employment is a technical relationship determining the amount of employment needed to produce an average level of output, given the actual capital stock, trend productivity, and the production technology. Average weekly hours are a function of exogenous terms and the production function. The labour force participation rate is determined by the level of employment and exogenous variables.

Substituting 5), 6), and 7) into 2), 3), and 4) gives the following quasi-reduced form model:

$$K = \alpha_1 U + \alpha_2 (P-R) \qquad (1')$$

$$h = \frac{1}{(1+\alpha_L \beta_1)} [U - \alpha_K K - \alpha_L N + E] + Z_2' \qquad (2')$$

$$N = (\frac{\gamma_1 \delta_1}{L} + \gamma_2) \cdot U - \gamma_1 \frac{\alpha K}{\alpha_L} \cdot K + \gamma_3 (W-P) \qquad (3')$$

$$U = \alpha_K K + \alpha_L \varepsilon_1 N + Z_4' \qquad (4')$$

Inventories will not be studied in the context of the supply model because the supply relationships in RDXF do not include any direct linkages between inventories and factors of production. The inventories equation, which is modeled as a stock-to-sales ratio, is a function of the ratio of various demand components--durables, non-durables, imports, and exports--to total sales. No distinction is made in the equation between response to transitory or unexpected demand changes and response to permanent changes, nor is the response used as a direct signal by firms in determining their productive capacity or hiring decisions. In fact, because lagged values of output rather than expected sales determine factor demands, an inventory decumulation or an increase in imports as a result of higher demand pressure creates an offsetting reduction in UGPP, thereby signalling less of an increase in prices and factor demands. However, because imports increase and inventories decline only temporarily, expected future sales would not be influenced by these short-run fluctuations; the use of an expected sales argument in the factor demand equations would imply more forward-looking behaviour by firms.*

In the *RDXF* model, two signals of excess demand pressure exist: a change in aggregate capacity utilization (the ratio of UGPP to normal output), which appears as an argument in the price equations; and a change in expected output (modeled on past values of UGPP), which determines factor demands.

**Dynamics of the Fixed-Price Supply Model**

In this section we examine the dynamics of a fixed-price supply model, in which factor and output prices are constant, by applying two shocks, an increase in real gross private business product and an increase in factor and output prices.

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* In fact, the linkages between imports and investment are such that an increase in demand is met by a larger increase in investment imports than in total investment. This was discussed in detail in Chapter 3.

**1% Increase in Real Gross Private Business Product**

In this simulation, the fact that factors of production do not conform to the production technology in the long run is highlighted. This is one of the reasons for the persistent productivity gap in the adjustment process in RDXF.

Figure 15 presents the interaction among factors of production in the fixed-price supply model. Below each variable is the impact and "longer term"* response to a 1% sustained increase in real gross private business product (UGPP).** As can be seen, an increase in UGPP induces different speeds of adjustment in the response of the various factors of production. Implicit in the timing of response of each factor of production is the short-run adjustment cost associated with the respective factor. In the RDXF model the hierarchy of adjustment (from fastest to slowest) is, first, short-run productivity (SRP) and hours, then employment, and finally capital. (An increase in short-run productivity is defined as an increase in the intensity of existing factor utilization.) Of the 1% increase in UGPP, .2% is met through higher employment and .07% through hours.*** On impact the stock of capital is unchanged. The remainder of the

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* "Longer term" denotes the response after eight years.

** In shocks to UGPP in this chapter the increase is assumed to be <u>net</u> of any inventory response. Therefore the shock can be interpreted as a 1% increase in production. A 1% increase in all components of UGPP produces an inventory decumulation for two quarters. The stock of inventories is below control for five quarters, and then rises to .7% above control after eight years. The speed of adjustment is extremely slow.

*** Contributions are the product of the percent change of each factor and the factor's share. Rearranging the production function:

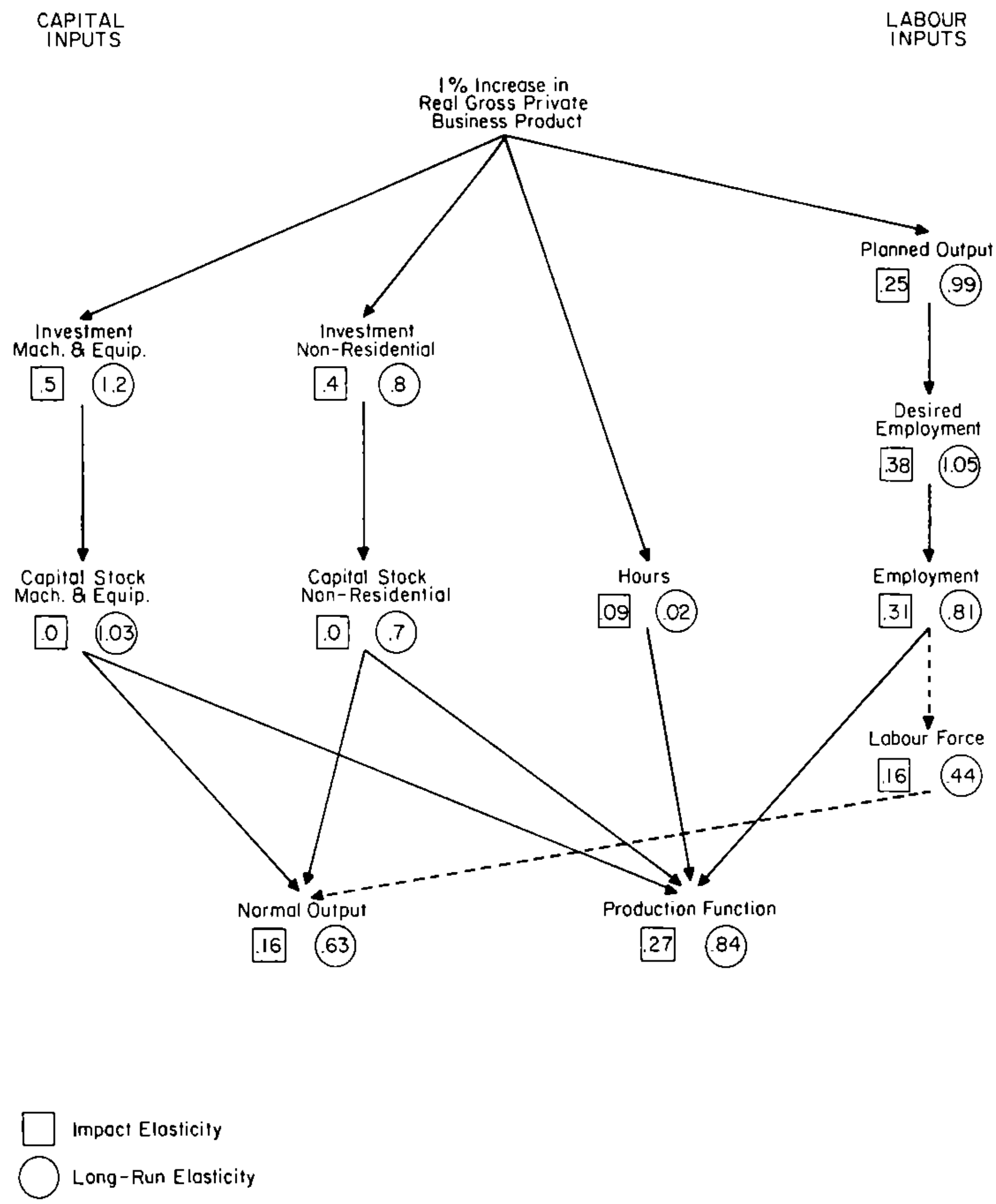
$$\log UGPP = .66 \cdot \log NIC + .66 \cdot \log HAWMM + .18 \cdot \log KMEXE_{-1} + .16 \cdot \log KNRCXE_{-1} + [\log UGPP - \log UGPPS]$$

therefore contributions are approximately:

$$\% UGPP = .66\% \Delta NIC + .66\% \Delta HAWMM + .18\% \Delta KMEXE + .16\% \Delta KNRCXE + \% \Delta SRP.$$

**Figure 15**

**LINKAGES BETWEEN AGGREGATE DEMAND AND AGGREGATE SUPPLY**



increase (.73%) is met through the residual, higher short-run productivity.

Figure 16 shows the relationship between the responses of actual and desired factors of production.* Required employment overshoots its longer term value of 1.05%, reaching a maximum of 1.5% (the inverse of labour's share) after one year, because more employment is required in the shorter term to compensate for the slower adjustment of the capital stocks. The desired stocks of machinery and equipment and of non-residential construction reach their steady state values after nine quarters and twelve quarters respectively. Required employment (NICD), on the other hand, has not reached a steady state after eight years; its response is slower because this variable is a function of the actual capital stock rather than the desired stocks. The response of actual employment is only 77% of the NICD response after eight years, and displays only a slight overshoot. Correspondence between the actual and desired stocks of capital is much closer, because investment is modeled only as a function of the desired stock and lag terms, whereas employment includes other arguments, such as output, prices and wages as well. In both machinery and equipment and non-residential construction, the adjustment of actual stocks exceeds 90% of that for the desired stocks.

After eight years, the contributions to a 1% increase in UGPP are .53% for employment, .01% for hours, .19% for the stock of machinery and equipment, and .11% for the stock of non residential construction. This implies a .16% increase in "short-run" productivity, and factors utilized significantly above their trend level of efficiency for a sustained period of time. Factor demands, therefore, are not constrained to conform with the

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* Desired capital stocks are calculated directly from the investment equations:

$$I_t/K_{t-1} = a + b\cdot\log(\alpha_K U_t^e \cdot P_t^e/R_t^e) - b\cdot\log K_{t-1}$$

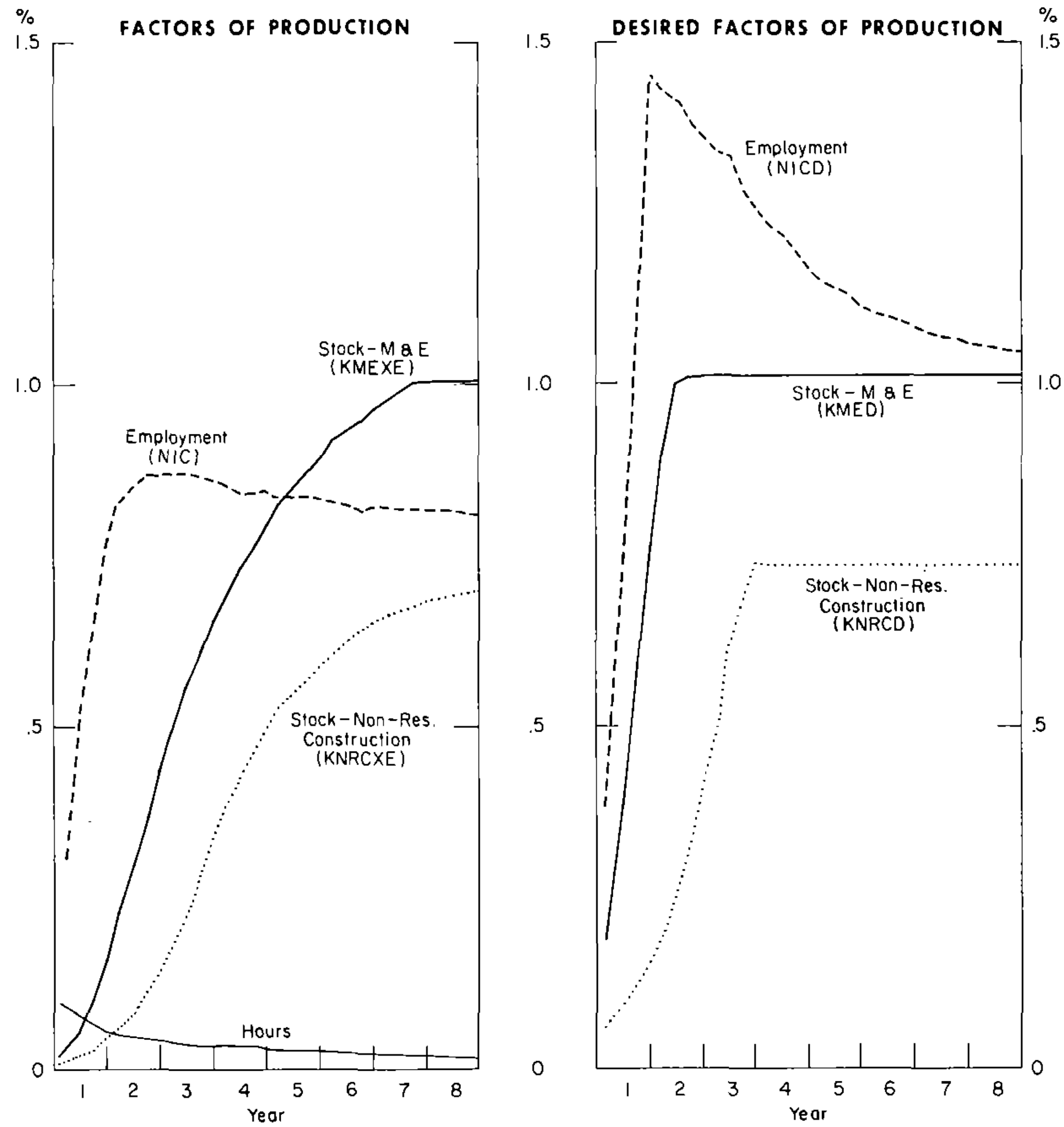
where

$$\alpha_K U_t^e \cdot P_t^e/R_t^e = \text{the desired stock of capital.}$$

Figure 16

**RESPONSE OF ACTUAL AND DESIRED FACTORS OF PRODUCTION TO A 1% INCREASE IN OUTPUT: FIXED-PRICE SUPPLY MODEL**

(Shock minus control, per cent)



production function in the long run. To satisfy the increase in production with the existing factors of production utilized at their trend levels of productivity, the factors would increase 1%. (The exception is hours, which are unchanged in the long run; hours deviate from their trend value only when UGPP does not equal UGPPS.) The increase would guarantee that the capital-labour ratio remained unchanged, which is an implication of the fixed shares in the Cobb-Douglas production function. However, the fixed-price supply model factors do not show a neutral response; the stock of machinery and equipment (1.03% above control after eight years) has a relatively higher share, and employment (.8%) and non-residential construction (.7%) a relatively lower share.

Normal output (UGPPD) increases .63% after eight years. Therefore capacity utilization, (the ratio of UGPP to UGPPD) is .37% higher. The magnitude of the increase in capacity utilization is determined to a large extent by the labour force response, which increases .44% after eight years.*

To summarize, the supply response is a long way from meeting the increase in output after eight years, because the factor responses are not sufficiently strong. Given that employment makes up such a large share of output, its weak response is the main cause of the persistent short-run productivity gap in the model.

**Response to a Change in Factor and Output Prices****

These simulations show the bias in the supply model that

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* The deviation between the response of UGPPD and UGPPS is determined by the deviation of the labour force (NL) response from the private sector employment response. The employment at trend output term in UGPPD is the product of NL and one minus the unemployment rate at trend output.

** Theoretically, prices should not be shocked in the supply model, as they are supply determined. However the purpose of this section is to report the channels through which prices changes are transmitted.

occurs with various relative price movements. The fact that the various factors do not respond consistently to price shocks is, again, the direct result of not imposing the production technology on factors.

Figure 17 summarizes the response of the factors of production in the fixed-price supply model to changes in selected factor and output prices. The top line represents the response of the capital-labour ratio to a 1% increase in private sector wage; the middle and lower lines show the response to a 1% increase in rental rates and the business output deflator respectively.

The increase in wages results in a .3% higher capital-labour ratio, which stems entirely from a decline in employment because there is no investment response.* The reason is that the factor-demand equations are expressed in terms of the ratio of own-factor prices to output prices, and thus they do not include direct cross-price responses. The employment equation, however, exhibits some cross-price response through the desired employment (NICD) term.** As reported in the employment and investment chapters, the own-factor price elasticities are -.3 for employment, -.06 for the stock of machinery and equipment, and -.04 for the stock of non-residential construction in the longer term.

The 1% increase in imputed rental prices creates a slight decline in capital stocks. The induced increase in employment

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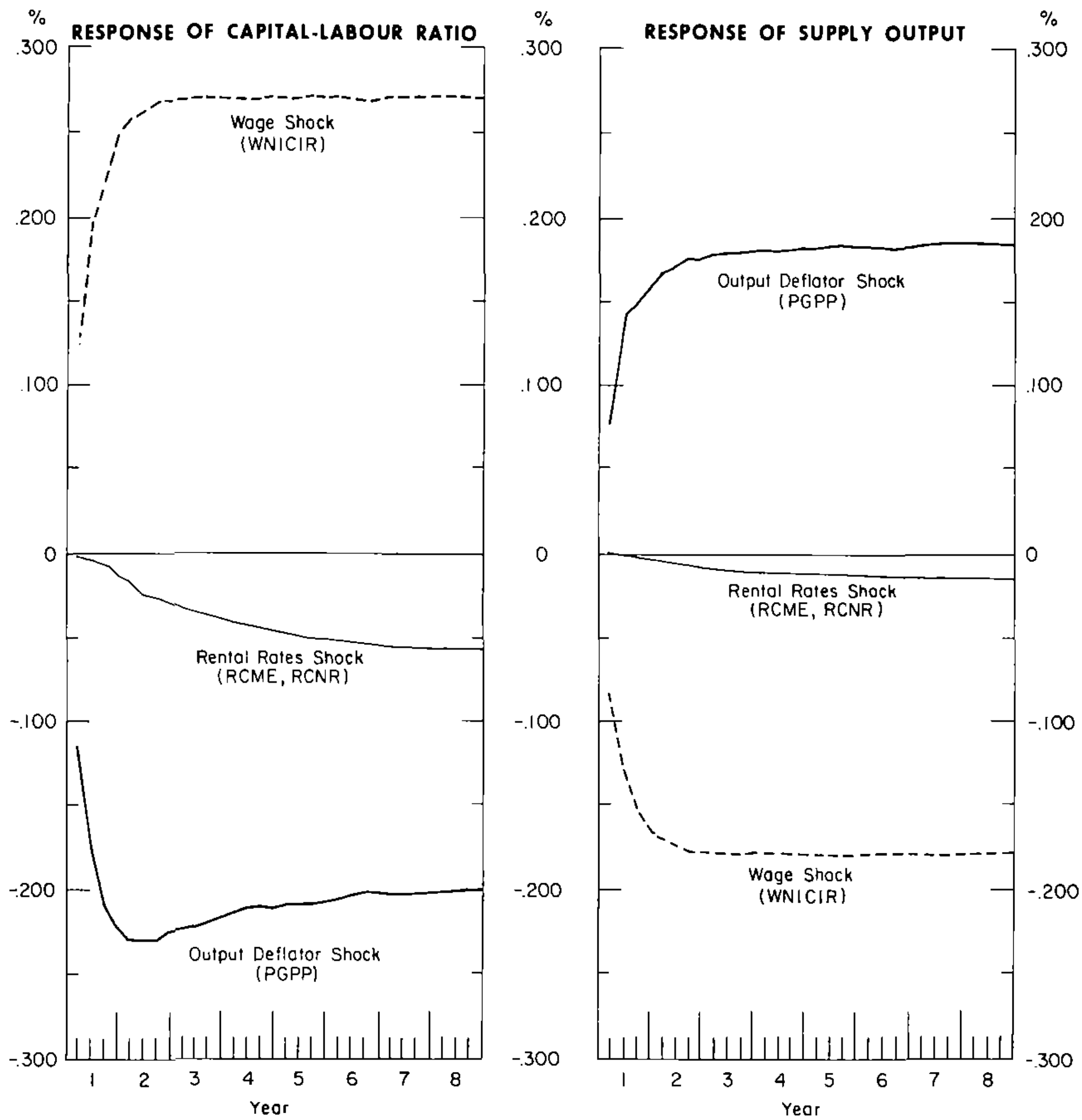
* If prices were endogenous, an increase in wages would induce an increase in the investment deflators (thereby increasing rental rates) and in the business output deflator. The elasticities of RCME, RCNR, and PGPP with respect to wages are .5, .8, and .7 respectively. In this case, an increase in wages would imply an increase in the stock of M&E and a <u>decline</u> in the stock of non-residential construction.

** The elasticity of employment with respect to rental rates $\varepsilon r$, depends positively on the elasticity of NIC with respect to NICD ($\eta$), on capital's shares ($\alpha k_1, \alpha k_2$), and on the rental rate elasticities in the capital equations ($\eta_1$, $\eta_2$), and inversely on labour's share ($\alpha_L$):
$\varepsilon r = (\alpha k_1 \eta_1 + \alpha k_2 \eta_2) ./ \alpha_L$.

Figure 17

RESPONSE OF FACTORS AND SUPPLY OUTPUT
TO A 1% INCREASE IN VARIOUS PRICES:
FIXED-PRICE SUPPLY MODEL

(Shock minus control, per cent)



through the NICD term reinforces the decline in the capital-labour ratio, which is .06 below control after eight years.

The 1% increase in the output deflator causes a decline in the capital-labour ratio because of the differing strengths of the relative price response in the factor demand equations. The maximum decline of .23% occurs after one year. A slight offset in the long run stems from the relatively slow increase in the capital stocks and from the small induced decline in desired employment.

Because cross-price terms in the factor demand equations are weak (or nonexistent), supply output (UGPPS) responds to a change in relative factor prices (Figure 17). A 1% increase in wages induces a .18% decline in UGPPS; there is no increase in the stocks of capital to offset the decline in employment. With a 1% increase in rental rates there is virtually no UGPPS response because the own-price effect in the investment equations is so weak. The .18% increase in UGPPS induced by a 1% increase in the output deflator (PGPP) stems almost entirely from higher employment.

## 8.4 A FLEXIBLE-PRICE SUPPLY MODEL

This section will provide an analysis of a flexible-price supply model that includes the RDXF equations (1') through (4') of the previous section for capital, employment and normal output, plus the wage, rental rate, and domestic price equations (8') through (10'). The capacity utilization and labour market tightness equations (11') and (12') provide the link between factor and output quantities, and factor and output prices.

$$W = \xi_1 RU + \xi_2 \dot{CPI} + \xi_3 \dot{E} \qquad (8')$$

$$= \xi_1 RU + \dot{CPI}^e + \xi_e E$$

$$R = P_K + Z_6 \qquad (9')$$

$$P_i = \eta_{i1}(W+N-Us)+\eta_{i2}(R+K-Us)+\eta_{i3}C^*+\eta_{i4}CAPU \quad i=1,\ldots k \qquad (10a')$$

$$CPI = \sum_{1}^{j} \theta_i P_i, \ \sum_{1}^{j} \theta_i = 1 \qquad (10b')$$

$$P = Y/U$$

$$= (\sum_{i=1}^{K} U_i P_i + Z_7)/U \qquad (10'c)$$

$$CAPU = U - \bar{U} \qquad (11')$$

$$= U-\alpha_K K-\alpha_L NL-E-Z_8$$

$$RU = (NL-N-Z_9)/NL-RNUTO \qquad (12')$$

where U,P,R,N,E are defined as for the fixed-price model and

CPI is the consumer price index

$P_1 \ldots P_j$ are deflators comprising the CPI

$P_k$ is the price of capital

$C^*$ are farm gate, energy and import prices

CAPU is capacity utilization

RU is the labour market, gap variable

RNUTO is the unemployment rate at trend output

$Z_1$-$Z_9$ are variables exogenous within the context of this discussion

Y is the nominal gross private business product

$CPI^e$ is the expected rate of inflation for the CPI.

All of the above equations except (12') and (10') are loglinear transformations (the RU term in (8') is in levels), and (·) represents the first difference of the variable. The growth in nominal private sector wages (W) is a function of the level of the labour market gap, which is the difference between the actual unemployment rate and the unemployment rate at trend output; the expected rate of change in the CPI (with a coefficient of unity); and the growth in trend factor productivity. Rental rates (R) are a function of the current price of capital and other terms exogenous to the supply model. Disaggregated prices ($P_i$) are a function of aggregate normalized unit labour costs, normalized unit capital costs, miscellaneous costs, and the output gap. The CPI is a function of the weighted sum (fixed weights summing to one) of the consumption deflators. The output deflator (P) is the ratio of nominal gross private business product (Y) to its 1971 dollar values; Y is the sum of the products of disaggregated expenditure components and their respective deflators, plus terms exogenous within the context of this discussion.

The factor demand equations and price equations are somewhat at odds in terms of their underlying behaviour. Factor demand equations are solved by the standard profit maximizing problem, where firms choose output, given factor and output prices. However, assuming constant returns to scale (CRTS) the level of output is indeterminate (firms have a linear, positively sloped profit function). The constrained cost-minimization problem, which assumes that firms take output as given but choose prices, could apply and would conform to the behaviour underlying the price equation specifications where firms' prices are determined on a mark-up basis.

In the price equations, the CAPU terms imply that firms enjoy a certain degree of market power in the short run, as a result of (temporarily) imperfect information and adjustment costs. The CAPU terms enter linearly* evoking the same proportionate response to a given increase in excess demand, regardless of the slackness or tightness in the economy. In the long run, output and normal output are equal, and prices are a function only of normalized costs.

The demand for labour determines the level of employment at any point in time. The supply of labour (which determines the labour force participation rate), however, determines the level of employment in the long run; any gap between labour supply and labour demand that exceeds the 'unemployment at trend output' is met by upward wage pressure through the labour market gap variable.

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* Inventories, imports, and the growth in nominal wages exhibit a loglinear response to an increase in demand as well.

Investment is demand-determined over both the shorter and the longer term, and is extremely price inelastic. The supply of capital is (implicitly) assumed to be perfectly elastic.

An increase in excess demand will, in this model, create upward price and wage pressure as well as an increase in factors of production and short-run productivity. Higher prices will induce a greater factor response (assuming wages are unchanged) through an increase in profitability that reduces the output gap.* Wage increases, on the other hand, serve to reduce factor demands by increasing costs, thereby widening the output gap. The adjustment of the output gaps in the flexible-price versus the fixed-price supply model depends on the relative strengths of the wage and price responses and their elasticities in the factor demand equations.

The steady state, if it were attained, would be characterized by the equality of output (UGPP), output implied by the production function (UGPPS), and 'normal' output (UGPPD) so that employment and productivity would return to their trend values after a shock. The gap between UGPP and UGPPS measures short-run productivity (SRP), or the deviation of productivity from trend, which implies that firms can move off their production function in the short run. The gap between UGPPS and UGPPD measures the deviation of man-hours from trend. The output gap can be decomposed into:

| | | |
|---|---|---|
| (UGPP/UGPPD) = | (UGPP/UGPPS) | (UGPPS/UGPPD) |
| | Short-run productivity | Deviation of man-hours from trend |

Upward wage pressure would continue if labour market tightness were above trend, while upward price pressure would persist with higher wages or the existence of an output gap.

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* In a full model context the output gap would narrow further through a decline in real personal disposable income, thereby inducing a reduction in aggregate demand.

The final level of employment is determined by the elasticity of supply of labour with respect to changes in output. If the labour force did not respond to changes in output, or if it did so only temporarily, then the ultimate increase in demand would be met only through a higher stock of capital. Because the supply of labour is less than perfectly elastic with respect to employment in the RDXF model, a higher level of aggregate demand in the steady state produces a higher capital-labour ratio.

The final output-price tradeoff is determined by the labour supply and investment demand elasticities with respect to output, the factor price elasticities in the factor demand equations, and the price response to factor costs. The greater the elasticity of supply with respect to prices, the larger the output effect of a demand shock and the weaker the output effect of a supply shock.

The actual attainment of a steady state depends on the values of critical long-run elasticities in the factor demand and price equations. In particular, the elasticities in the factor-quantity and factor-price equations must be constrained so that factors are employed at their trend level of productivity and receive their trend shares of nominal business output in the long run.* As was evident in the discussion of the fixed-price model dynamics, these long-run conditions are not met; a 1% increase in business output produces a continued increase in short-run productivity. In the flexible-price model, the persistence of the output gap could be exacerbated, depending on the relative magnitudes of the labour market tightness and the output gap terms in the wage-price equations, and on the wage-price interactions.** If real wages

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* In the fixed-price model, a steady state depends on the condition $\overline{N}=\overline{K}=\overline{U}$. In the flexible-price model, $\overline{N}+\overline{W}=\overline{R}+\overline{K}=\overline{P}+\overline{U}$.

** In a full model context, induced aggregate demand effects would even worsen this problem. If the elasticity of employment with respect to profitability did not exceed unity, real personal disposable income would increase, thereby increasing aggregate demand and UGPP. This would serve to further increase the output gap, thereby creating a further upward wage and price pressure, and so on. This effect would be more destabilizing, the smaller the difference between the slopes of the demand and supply curves.

increase with a demand shock (the labour market gap dominates), the decline in profitability induces a reduction in employment, creating an offsetting decline in UGPPD and UGPPS. The greater the price elasticities are in the factor demand equation, the more pronounced this decline will be. For prices, the analogue to the long-run constraints in the factor demand equations is that prices are homogeneous of degree one* with respect to factor costs, where factors are the same as those in the production function. The key prices are the CPI, the price individuals use in forming price expectations, and the deflator for business output (PGPP).**

### Dynamics of the Flexible-Price Supply Model

This section reports the dynamics of the flexible-price supply model in light of the relative strengths of various channels in resolving excess demand pressure. The first shock is a 1% increase in real gross private business product (UGPP). Comparing the results from the flexible-price and the fixed-price models highlights the role of output and factor prices, particularly the role of real wages, in resolving excess demand pressure. The second shock is a 1% increase in trend factor productivity (ETFP). Of particular interest in this example is the asymmetry of response in the model to excess demand pressure

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* Gosselin and Staranczak [33] emphasize the importance of homogeneity of the CPI with respect to wage costs for the attainment of accelerationist results. In fact, homogeneity is the requirement for the existence of a steady-state solution in the context of RDXF.

** From equations 10',

$$\sum_{i=1}^{j} \theta_i \eta_{iL} + \sum_{i=1}^{j} \theta_i \eta_{iK} = 1 \qquad (1)$$

for homogeneity of the CPI, and (1) plus, $\eta_{K1} + \eta_{K2} = 1$, for homogeneity of PGPP.

originating from the demand-side versus the supply-side of the model.

### Increase in Real Gross Private Business Product

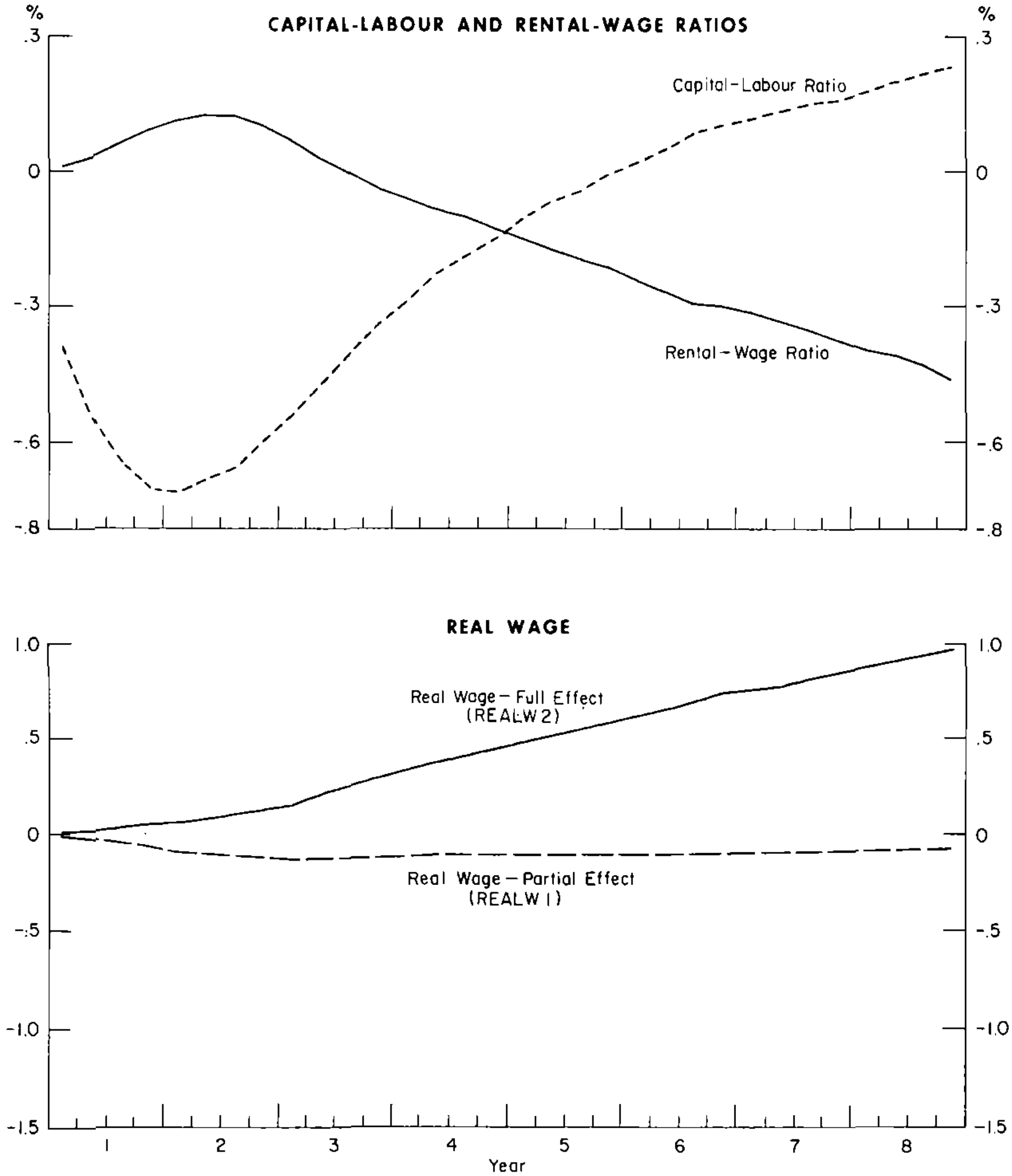
Table 21 presents the response of key variables to a 1% increase in the real gross private business product (UGPP). Employment (NIC) displays a considerably different adjustment in the flexible-price model than in the fixed-price model. After an overshoot of .9% above control after two years, NIC is .6% above control (.8% in the fixed-price model) after eight years because of lower profitability. By the end of the simulation period the employment response is only 57% (versus 77%) of the required employment response. The response of the stocks of capital are virtually identical in both models because the price response in the investment equations is so weak. By the end of the simulation period, the stock of machinery and equipment is 1.0% higher and the stock of non-residential construction .7% higher.

Figure 18 shows the per cent change in the capital-labour ratio and in the rental-wage ratio (upper panel). The capital-labour ratio declines initially because the employment response is more immediate. The subsequent increase stems from the non-neutral response of factors, biased toward greater capital intensity (recall that this occurred in the fixed-price supply model), and a stronger increase in real wage rates than in real rental rates, combined with a relatively higher factor price elasticity in the employment equation. The capital-labour ratio is .23% above control after eight years. The rental-wage ratio shows an initial increase because the response of investment deflators to higher wage costs plus a higher output gap is sufficient to increase the deflators (and therefore the rental rates) more than wages for ten quarters. After this, the wage response dominates and the ratio declines to .47% below control after eight years. Thus, labour's share is higher than control by the end of the simulation period.

**Figure 18**

**RESPONSE OF FACTORS AND COSTS TO A 1% INCREASE IN OUTPUT: FLEXIBLE-PRICE SUPPLY MODEL**

(Shock minus control, per cent)




REALW1 = WNIC1/PCPI, WNIC1 = $\alpha_0 ETFP - \alpha_1 RU_s + \alpha_2 P^e_s$;
REALW2 = WNIC2/PCPI, WNIC2 = $\alpha_0 ETFP - \alpha_1 RU_c + \alpha_2 P^e_s$,
where s - shock and c - control.

Table 21

EFFECT OF A 1% INCREASE IN GROSS PRIVATE BUSINESS PRODUCT
SUPPLY MODEL WITH FLEXIBLE PRICES
(Shock minus control, per cent)

| | Quarters 1 | 2 | 3 | 4 | 8 | 32 |
|---|---|---|---|---|---|---|
| Response of Factors: | | | | | | |
| Capital - Machinery & equipment (KMEXE) | .02 | .05 | .10 | .16 | .43 | 1.04 |
| - Non-residential construction (KNRCXE) | .01 | .02 | .03 | .04 | .15 | .68 |
| construction (KNRCXE) | | | | | | |
| Employment (NIC) | .31 | .51 | .66 | .77 | .86 | .60 |
| Man-hours (HAWMM) | .10 | .08 | .07 | .06 | .04 | .04 |
| Short-run productivity (SRP) | .73 | .60 | .51 | .43 | .32 | .29 |
| Labour market tightness (RU) level | -.07 | -.11 | -.15 | -.17 | -.20 | -.14 |
| Output gap (UGPP/UGPPD) | .84 | .73 | .66 | .60 | .54 | .47 |
| CPI (PCPI) | .01 | .02 | .04 | .06 | .18 | .67 |
| Business output deflator (PGPP) | .01 | .03 | .06 | .10 | .27 | .92 |
| Private sector wages (WNICIR) | .01 | .04 | .06 | .10 | .31 | 1.63 |
| Normalized unit labour costs (ULC) | .05 | .15 | .23 | .30 | .49 | 1.51 |

The lower panel of Figure 18 also presents a decomposition of the change in real wages into the total change (upper line) and that emanating only from a change in capacity utilization (lower line). Nominal wages respond to demand pressure directly through the labour market gap term and indirectly through price expectations as capacity utilization increases.

Higher capacity utilization increases the price level which, because of a lag in expectations, creates initial downward pressure on real wages (lower line). Subsequently, real wages gradually increase as nominal wages catch up to prices and as the output gap closes in the later stages of the demand shock, thereby reducing upward price pressure. In the upper line, the labour market gap variable creates initial pressure on the growth in wages, which would be somewhat less pronounced as labour market tightness eased in the long run. In the RDXF model, because this variable clearly dominates the effect of capacity utilization, real wages are higher throughout the simulation, and reach .96% above control by the end of the simulation period.

Figure 19 shows the response of the output deflator after one year and eight years, as well as the output gap and short-run productivity at these points in time. The slope of the aggregate supply curve (in PGPP-UGPP space) is essentially linear because all demand-pressure terms enter linearly into the equations of the flexible-price model. The output deflator increases steadily to .9% above control after eight years.* Because productivity provides 30% of the increase in production and because labour market tightness is considerably higher (RU is 14 basis points below control after eight years), the upward price pressure would continue well past the end of the simulation period.

Noteworthy is the very slow reduction in both the output gap and productivity. From the second to the eighth year of the simulation, productivity declines only .14%, and the output gap narrows only .13%. At the end of the simulation, the output gap is .47% (versus .37% in the fixed-price supply model), and productivity, which increases .29% above control, is twice as large as in the fixed-price model.

**Increase in Trend Factor Productivity**

Table 22 presents the response to a 1% increase in trend factor productivity (ETFP). The direct impacts are: a 1% increase in UGPPS and UGPPD as existing factors are utilized at a higher level of efficiency; a 1% decline in desired employment as

---

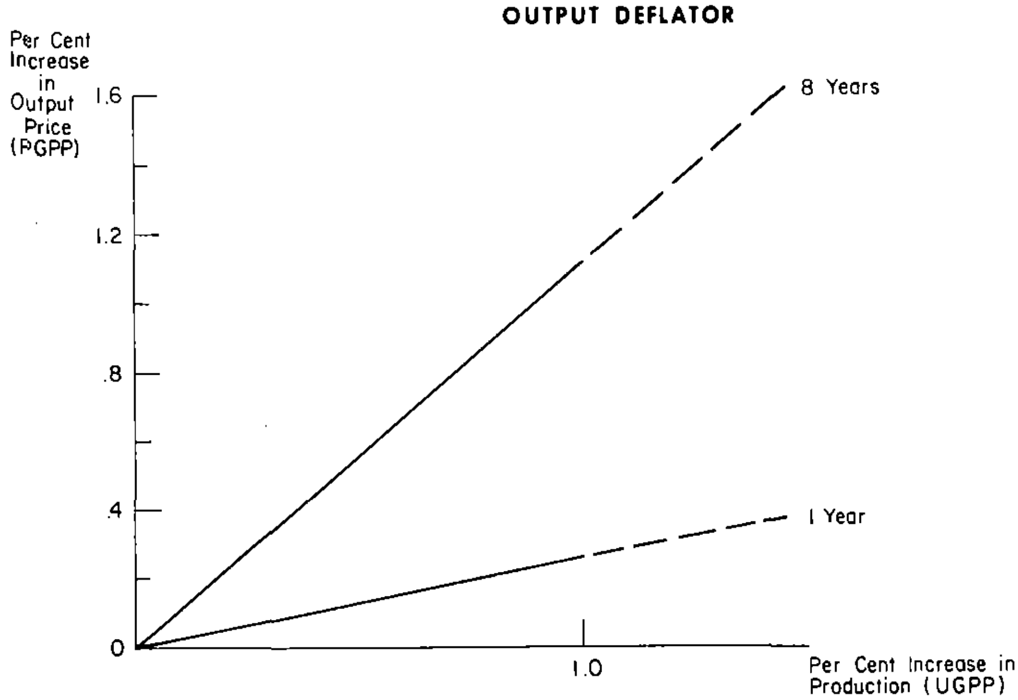
* The output deflator (PGPP) shows a stronger response than the CPI because of the relatively strong increases in the investment deflators. The deflator for M&E is .83% above control. The deflator for non-residential construction is 1.5% above control after eight years.

**Figure 19**

**RESPONSE OF SUPPLY VARIABLES TO A 1% INCREASE IN OUTPUT: FLEXIBLE-PRICE SUPPLY MODEL**

**(Shock minus control, per cent)**

**OUTPUT DEFLATOR**




**OUTPUT GAP AND SHORT-RUN PRODUCTIVITY**

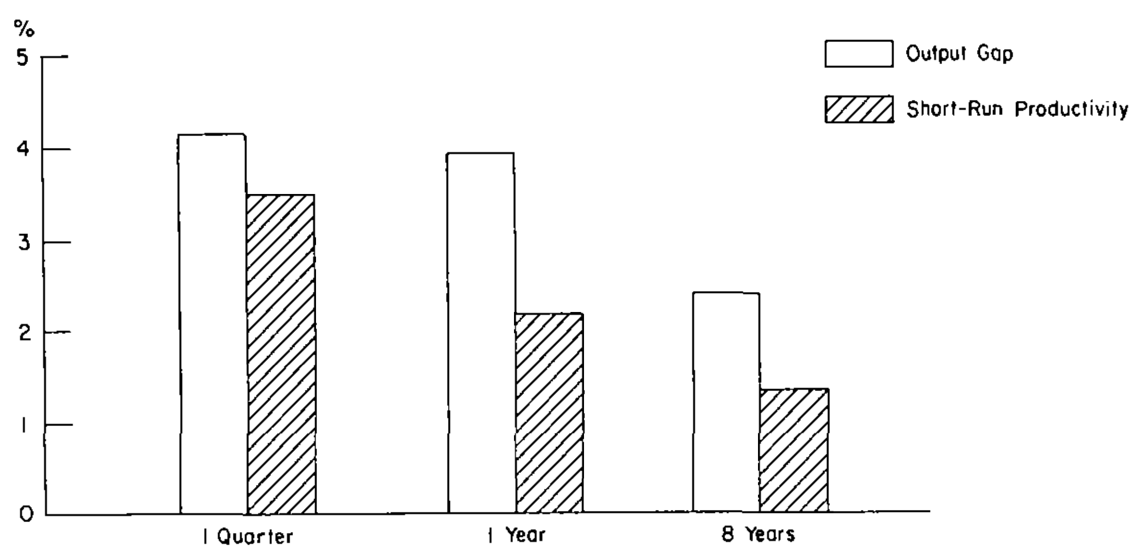


Table 22

EFFECTS OF A 1% INCREASE IN TOTAL FACTOR PRODUCTIVITY
SUPPLY MODEL WITH FLEXIBLE PRICES
(Shock minus control, per cent)

| | Quarters | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 8 | 32 |
| Response of Factors: | | | | | | |
| Capital - Machinery & equipment (KMEXE) | .00 | .00 | .00 | .00 | .01 | .02 |
| - Non-residential construction (KNRCXE) | -.00 | -.00 | -.00 | -.00 | -.00 | -.01 |
| Employment (NIC) | -.16 | -.27 | -.35 | -.40 | -.51 | -.48 |
| Man-hours (HAWMM) | -.11 | -.10 | -.09 | -.09 | -.08 | -.08 |
| Short run productivity (SRP) | -.82 | -.75 | -.70 | -.67 | -.61 | -.62 |
| Labour market gap (RU) (level difference) | .04 | .06 | .08 | .09 | .13 | .11 |
| Output gap (UGPP/UGPPD) | -.91 | -.86 | -.82 | -.80 | -.78 | -.81 |
| CPI (PCPI) | -.03 | -.06 | -.09 | -.10 | -.14 | -.16 |
| Business output deflator (PGPP) | -.05 | -.10 | -.12 | -.13 | -.11 | -.19 |
| Private sector wages (WNICIR) | .10 | .20 | .29 | .38 | .65 | .38 |
| Normalized unit labour costs (ULC) | -.87 | -.82 | -.76 | -.69 | -.46 | -.73 |

less employment is required to produce a given level of output with the existing capital stock; and a 1.3% increase in private sector wages after twelve quarters--productivity growth is modeled explicitly in the wage equations. Note that the channels for transmitting the productivity shock are as if productivity accrued to labour only; there is no comparable change in the desired capital stock in the investment equations, nor a change in rental rates. (If desired stocks of capital were modeled analogously to desired employment, then the expected output term in the equations would be scaled by productivity).

As shown in the table, employment declines .16% on impact as a result both of higher wages and of lower desired employment. After eight years employment is .48% below control. This creates some offsetting decline in UGPPS and UGPPD; UGPPS is .62% above control (SRP .62% below control), and UGPPD is .81% above control (output gap .81% below control). One reason for the large persistent excess supply gap is the lack of investment response. If investment and therefore the capital stock were to decline, this would create a larger reduction in UGPPS and UGPPD, thereby reducing the SRP gap.

The decline in normalized unit labour costs (-.7% after eight years), which stems from higher UGPPS (reinforced by lower employment and offset by higher wages) reduces domestic prices. The CPI declines .16% and the deflator for gross private business product declines .19% relative to control after eight years. Private sector wages are .38% above control after eight years. The 1.3% increase directly attributable to higher productivity is eroded considerably by an easing of labour market tightness.

Figure 20 shows the per cent change in the capital-labour and rental-wage ratios. The capital-labour ratio is .6% above control after eight years; virtually all of the increase emanates from the decline in employment. The large change in factor intensity stems from the lack of channels in the investment equation for transmitting productivity shocks.

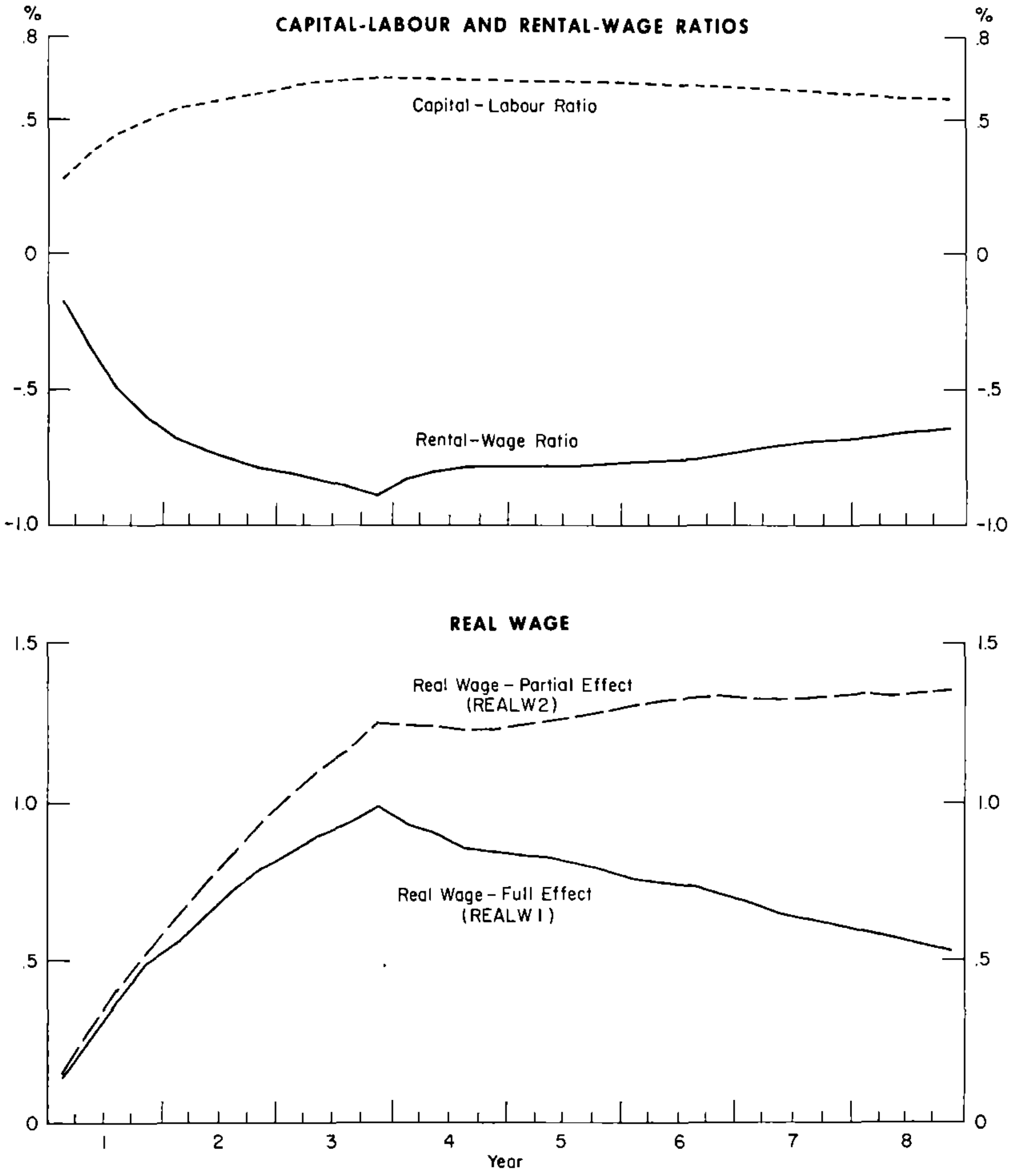
The figure also shows the relationship between the full real wage response (lower line), and the response attributable only to the change in prices and productivity (upper line). Real wages would be 1.4% higher after eight years if the labour market gap were unchanged (upper line). The response is slightly higher than the direct impact of productivity because nominal wages have not adjusted fully to the expected decline in prices. The .54% increase in real wages from all sources (lower line) includes a .9% decline stemming from an easing of labour market tightness.

The rental-wage ratio is .6% below control after eight years, mainly from the particularly strong wage response to the increase in productivity. Although the rental price of machinery and equipment declines, the rental price for non-residential construction increases. This is because the wage costs are

**Figure 20**

**RESPONSE OF FACTORS AND COSTS TO A 1% INCREASE IN TOTAL FACTOR PRODUCTIVITY: FLEXIBLE-PRICE SUPPLY MODEL**

**(Shock minus control, per cent)**



expressed in terms of unit labour costs (which are declining) in the first deflator and in terms of the level of wages (which are increasing) in the second deflator.

CONCLUSION

As of a result of the limitations in the current version of the RDXF supply bloc, much of the supply sector will be redesigned in the near future. The production process will be disaggregated into three separate sectors--commercial output, energy, and materials--and the respective factor demands and factor prices will be modeled. The technology will incorporate variable elasticities of substitution among factors, possibly with nested CES or translog production functions. The dynamic price and output response of firms will be integrated into a more consistent framework. The adjustment to excess demand pressure will be addressed by modeling (a) an expectations mechanism for output and prices; (b) inventory and import responses interrelated with factor demands, which vary at different levels of capacity; and (c) supply bottlenecks in particular expenditure functions. Steady state properties in the final price and factor demand equations will be imposed.

## Appendix to Chapter 8

### Equations in the Fixed-Price Supply Model

| | | | |
|---|---|---|---|
| EQI64 | UGPPAV | EQI50 | IIB |
| EQN54 | NICD | EQI51 | KME |
| EQN01 | NIC | EQI52 | KNRC |
| EQN02 | HAWMM | EQI56 | UGPPD |
| EQI57 | UGPPS | EQI58 | IBUS |
| EQN05 | NEPD | EQI59 | IBUS$ |
| EQN52 | NE | EQI60 | IMEXE |
| EQN60 | NEUNPD | EQI61 | INRCXE |
| EQN03 | NL | EQI62 | KMEXE |
| EQN53 | NU | EQI63 | KNRCXE |
| EQN55 | RNU | EQI01 | IME |
| EQN59 | RU | EQI02 | INRC |

Shocks:

1. UGPP•1.01
2. RCME•1.01
   RCNR•1.01
3. PGPP•1.01
4. WNICIR•1.01

### Equations in the Flexible-Price Supply Model

| | | | | | |
|---|---|---|---|---|---|
| EQN01 | NIC | EQP08 | PIRC | EQI62 | KMEXE |
| EQN02 | HAWMM | EQP09 | PINRC | EQI63 | KNRCXE |
| EQN03 | NL | EQY05 | YIVA | EQP66 | ULC |
| EQN05 | NEPD | EQP10 | PKIB | EQI53 | RCME |
| EQN52 | NE | EQP12 | PRENT | EQI54 | RCNR |
| EQN53 | NU | EQP13 | PCPI | EQI56 | UGPPD |

| | | | | | |
|---|---|---|---|---|---|
| EQN54 | NICD | EQP27 | PFOOD | EQI57 | UGPPS |
| EQN55 | RNU | EQP50 | PGNE | EQI58 | IBUS |
| EQN59 | RU | EQP51 | PGPP | EQI59 | IBUS$ |
| EQN60 | NEUNPD | EQP52 | PCS | EQI60 | IMEXE |
| EQW01 | WNIC | EQW52 | WNICIR | EQI61 | INRCXE |
| EQI64 | UGPPAV | EQW53 | WOTHIR | EQY59 | UGNE |
| EQI55 | UGPP | EQI01 | IME | EQP01 | PCSD |
| EQC53 | CRENT$ | EQI02 | INRC | EQP02 | PCNDO |
| EQY53 | YGPP | EQI51 | KME | EQP03 | PCSXR |
| EQY52 | YGNE | EQI52 | KNRC | EQP04 | PCMV |
| EQP05 | PHSHD | EQP06 | PCDMIS | EQP07 | PIME |

```
EQ1  RATIO+(((KMEXE**EKME)·(KNRCXE**EKNR))**(1/(EKME+EKNR)))/
     (NIC·HAWMM·52.)

EQ3  RENTAL+(((RCME**EKME)·(RCNR**EKNR))**(1/(EDME+EKNR)))/
     (WNICIR·(1+RYWSLP))

EQ4  SRP+UGPP/UGPPS

EQ5  REALW1=WNIC/PCPI

EQ6  (WNIC2)·J1D(LOG(WNIC2))=AW0100·J12A(J1D(LOG(ETFP)))+AW0101·
     RUCON+AW0102·(NPER.GE.19761.)·(NPER.LE.19784.)+
     J8W(AW0103,J1D(LOG(J1L(PCPI))))

EQ7  REALW2=WNIC2/PCPI
     UGPP Shock:

     replace EQI55, EQY52, EQY59,

     with    EQI55A  UGPP = (UGNE-...)·EPGPP+.01·UGPPA·(1-EPGPP)

             EQY52A  YGNE = CSD·PCSD...+.01·UGPPA·PGPP

             EQY59A  UGNE = CSD+CFOOD...+.01·UGPPA

     ETFP Shock:

             ETFP = ETFP·1.01
```

# Chapter 9

## THE FINANCIAL SECTOR
(Sectors 15, 16, 17 and 18)

Michael McDougall

The RDXF model contains a detailed account of the financial sector of the economy.* This sector is composed of 72 monthly equations, 47 of which are stochastic, and it models variables such as interest rates, financial assets, holdings of the non-bank public (including various monetary aggregates), as well as assets and liabilities of chartered banks, trust and mortgage loan companies (TMLs) and the Bank of Canada. A quarterly equation links the real supply price of capital to interest rates and federal government debt.

A close link exists between the financial and government sectors of the model, since the federal government is able to influence the operation of the chartered banks and TMLs through government deposits. As indicated in Chapter 5, the federal government financing rule is specified in monthly frequency to facilitate this link.

Since our TSP software system allows for the estimation and simulation of mixed frequency models,** we have been able to incorporate the Bank's monthly financial model, originally developed by Clinton and Masson [15], into RDXF. The use of monthly data to estimate and simulate financial models allows for a good representation of the lag structure in the estimated equations, which is desirable because of the rapid response of financial variables to changes in, say, interest rates.

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* R. Parker [67, 68] and F. Scotland [104] did much of the work of developing and testing a quarterly financial sector for RDXF before it was decided to use the monthly model developed in the Monetary and Financial Analysis Dept. [15]. P. Duguay was very helpful in the preparation of the chapter.

** For a technical description of mixed frequency estimation and simulation see [24].

Although the sector is detailed, allowing a careful representation of the financial system, the feedbacks from the financial sector to the real sector are weak. The chief linkages are through movements in interest rates, the most important of which is the influence of the Canadian-U.S. short-term interest rate differential on the exchange rate. Under an assumption of a fixed exchange rate, the response of the real side of the economy to monetary policy is weak.

Instead of discussing the make-up of the financial sector, this chapter focuses primarily on those areas that interact with the real side of the economy, i.e., interest rates, financial assets of the general public and the real supply price of capital.

## 9.1 INTEREST RATES

Sixteen interest rates are modeled, nine deposit rates of the financial intermediaries, six bond rates, and the conventional mortgage rate. Exogenous to the system are two short-term Canadian rates, the 90-day finance paper rate (R90) and the Bank of Canada rate (RBANK),* as well as two U.S. rates, the 90-day commercial paper rate (RCP2, the U.S. equivalent of R90) and the rate on recently offered AAA utility bonds (RAAA2). In their reduced form, interest rates are essentially predetermined since most are a function solely of the exogenous rates.**

The theory underlying the specification of the deposit rates is that the rate-setting decisions of the chartered banks and the TMLs are based on the rates on (a) their primary earning asset

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* Alternatively, one may want to invert the money demand equation to yield short-term paper rates endogenously, given an exogenously determined money supply. In the full model shocks reported in Chapter 10, nominal interest rates were endogenized to yield unchanged real rates, i.e., short rates are increased with the shock-minus-control change in expected inflation.

** The limiting case of this is the treasury bill rate which, since the adoption in March 1980 of a floating bank rate, is derived from the identity RBANK = RTB + .25. The 30- and 60-day finance company paper rates, which enter the demand equation for M1, are also exogenous.

and (b) the competing intrument available to the depositor. A direct supply effect appears, however, in the determination of the Canadian dollar 90-day deposit rate (RNPT) as it is assumed that, excepting the periods when it was subject to a ceiling under the Winnipeg agreement,* this rate is set in the wholesale deposit market to meet the chartered banks' demand for funds. The RNPT equation can thus be regarded as an inverted demand function for non-personal term and notice deposits (DNPTB); the two major competing rates are the covered U.S. commercial paper rate and the Canadian paper rate.

The rate on day loans (RDAY) and the prime business loan rate (RPRIME) are both affected by changes in the liquidity of the banking system. Being an administered rate, the prime rate adjusts only sluggishly to market rates, i.e., the 90-day finance company paper rate; this adjustment can be speeded up, however, if changes in the short-term paper rates are accompanied by changes in the bank rate, signaling the intentions of the central bank. Furthermore, an increase in available funds (AVFUN) coming from larger inflows of deposits or a slowing demand for bank loans, (and resulting in an increase in chartered banks' free liquid assets), will induce the banking system to lower its prime lending rate. This will also feed through to deposit rates.

All equations are estimated from January 1968 to September 1978 using rational distributed lags to capture the adjustment lags. A recursive structure is imposed among administered rates in which, for example, the chartered banks' rate on savings deposits influences but is not influenced by the rate set on similar deposits by TMLs, and the TMLs' GIC rate is influenced by the chartered banks' personal term deposit rates, rather than vice versa.

The five-year conventional mortgage rate (RMC) is specified

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* The Winnipeg agreement dummy, effective July 1969 through May 1970, and June 1972 through January 1975, represents the period when the chartered banks, with the agreement of the Minister of Finance, imposed interest rate ceilings on certificates of deposit.

as a function of the rate on 3- to 5-year Canadas (RMS) and the change in the bank rate.

Six bond rates are determined in the model. The representative long-term bond yield, from which others are fed, is taken to be McLeod Young Weir's average yield on 10 provincal bonds, (RL10P) which is related to the U.S. bond rate and the short-term paper rates in both U.S. and Canada. The whole term structure of Government of Canada bond yields is then anchored to this representative rate and the 90-day paper and treasury bill rates by the following simultaneous system:

$$RS = \alpha(RTB, R90, RMS) \quad (1)$$

$$RMS = \beta(RS, RML) \quad (2)$$

$$RML = \gamma(RMS, RL) \quad (3)$$

$$RL = \delta(RTB, R90, RL10P) \quad (4)$$

The rates on 1- to 3-year, 3- to 5-year and 5- to 10-year bonds are truly simultaneously determined, with each rate being a function of adjacent rates on the term structure; the over 10-year rate, RL, is derived from the 10 provincials. This simultaneity was found to present some convergence problems in simulations when the demand for money was inverted to yield interest rates endogenously, and this prompted us to rewrite equation (2) as a reduced form equation of (1), (2) and (3).

To demonstrate the response of the interest rate sector two shocks are presented. In the first, the two exogenous short Canadian rates, R90 and RBANK, are increased by 100 basis points throughout the simulation; in the second, these rates together with the two exogenous U.S. rates, RCP2 and RAAA2, are increased 100 basis points. In the first shock (Table 23), which may be considered a tightening of domestic monetary policy, the response of the interest rates ranges from an increase of 96 basis points*

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* The treasury bill rate, due to the nature of the specification, increases 100 basis points.

Table 23

**RESPONSE OF INTEREST RATES TO AN INCREASE OF 100 BASIS POINTS IN EXOGENOUS CANADIAN RATES**
(Shock minus control, level)

| Variable* | Months 1 | 2 | 3 | 36 |
|---|---|---|---|---|
| RTB | 100 | 100 | 100 | 100 |
| RDAY | 48 | 96 | 96 | 96 |
| RNPT | 88 | 91 | 91 | 91 |
| RT1TL | 69 | 83 | 85 | 86 |
| RBPFTQ | 57 | 67 | 74 | 85 |
| RPRIME | 51 | 70 | 70 | 75 |
| RPD | 35 | 57 | 63 | 68 |
| RSDTL | 18 | 40 | 49 | 54 |
| RPFT5B | 34 | 42 | 46 | 38 |
| RMC | 19 | 28 | 32 | 30 |
| RGIC | 26 | 34 | 37 | 30 |
| RS | 75 | 65 | 55 | 40 |
| RMS | 52 | 48 | 40 | 26 |
| RML | 38 | 39 | 35 | 25 |
| RL | 19 | 23 | 25 | 29 |
| RL10P | 8 | 14 | 18 | 28 |
| R10IND | 16 | 17 | 12 | 17 |

* See Appendix B to this chapter (page 212) for the definitions of the mnemonics.

for the day rate to an increase of 17 basis points for the rate on ten industrials. For most of the rates the long-run value is reached quickly, generally after four or five months. The use of monthly data enables us to capture the short lags in interest-setting behaviour. In the case of the simultaneously determined government bond rates RS, RMS and RML, there is an intitial overshooting, unlike the pattern exhibited by the other rates, and the long-run values appear suspiciously small.* In all cases the path to equilibrium is fairly smooth.

In Table 24, where both Canadian and U.S. exogenous rates are increased, there is little additional effect on most of the short rates; but now most of the long rates increase almost 100 basis points. The notable exceptions are the medium-term rates coming from the simultaneous system, which in turn affect the mortgage rate and the rates on longer term deposits. The other exception is the prime lending rate and the savings deposits rates which depend on it.

## 9.2 LIQUID ASSETS OF THE GENERAL PUBLIC

The most important of the assets of the general public in terms of linkages to the real sector are those that enter the interest and miscellaneous investment income equation (YIM). Five assets are involved: two pertain primarily to the corporate sector and three to the household sector.

In general, asset demands are specified as a function of interest rates and wealth. Interest rates appear as the differential between the own rate and the rate on competing assets. Net wealth (V) is the sum of the replacement cost of the real capital stock, including the stock of business capital, residential construction and business inventories, government debt (net of Government of Canada deposits), plus net foreign assets

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* This result may be due to simultaneous equation estimation bias since the equations were estimated using ordinary least squares rather than a simultaneous estimation technique.

Table 24

RESPONSE OF INTEREST RATES TO AN INCREASE OF
100 BASIS POINTS IN EXOGENOUS CANADIAN AND U.S. RATES
(Shock minus control, level)

| Variable* | Months 1 | 2 | 3 | 36 |
|---|---|---|---|---|
| RTB | 100 | 100 | 100 | 100 |
| RDAY | 48 | 96 | 96 | 96 |
| RNPT | 86 | 90 | 91 | 91 |
| RT1TL | 69 | 83 | 85 | 86 |
| RBPFTQ | 57 | 69 | 74 | 85 |
| RPRIME | 51 | 70 | 70 | 75 |
| RPD | 35 | 57 | 63 | 68 |
| RSDTL | 18 | 40 | 49 | 54 |
| RPFT5B | 34 | 42 | 46 | 52 |
| RMC | 21 | 33 | 40 | 56 |
| RGIC | 27 | 38 | 43 | 50 |
| RS | 110 | 96 | 84 | 60 |
| RMS | 94 | 85 | 75 | 52 |
| RML | 87 | 83 | 78 | 56 |
| RL | 69 | 73 | 77 | 85 |
| RL10P | 64 | 72 | 77 | 89 |
| R10IND | 54 | 64 | 57 | 103 |

* See Appendix B to this chapter (page 212) for definitions of the mnemonics.

(measured on a book value basis as the cumulated value of the current account balance).

On the corporate side, the assets included in YIM are commercial paper outstanding (ANPAP) and TML time deposits of one year and under (DT1TL). In the model these two assets plus non-personal term and notice deposits at the chartered banks (DNPTB) make up the liquid assets of the corporate sector and are substitutes for each other.

ANPAP and DT1TL are functions of contemporaneous wealth since the business sector adjusts its liquid portfolio rapidly, i.e., within a month, to exogenous inflows and outflows of wealth. The response of the portfolios to relative interest rate differentials is much longer and is captured by a 13-month Almon lag on the difference between the own rates, R90 and RT1TL respectively, and the rate on non-personal term and notice deposits. Thus the relative level of R90, RT1TL and RNPT determines the corporate sector holdings of ANPAP and DT1TL. As mentioned above, the supply of non-personal term and notice deposits is determined by the chartered banks' model of liability management except when the Winnipeg agreement is in effect. When the agreement is operative the equation takes the form of a straightforward demand function with contemporaneous wealth and lagged interest rate differentials. The interest differential term is the own rate less R90 and RT1TL with weights of .7 and .3 respectively.

In periods when the agreement is not in effect, DNPTB takes the form of a supply function where it responds to movements in predetermined items in the chartered banks' balance sheets. An increase in business loans and other less liquid assets results in an increase in DNPTB. On the other hand, increases in liabilities of the chartered banks, demand deposits (DDB), personal savings deposits (DPB), and Government of Canada deposits (DDFGB) lead to a reduction in DNPTB. A $1 million increase in assets leads to about a $.75 million increase in DNPTB while a $1 million increase in liabilities results in a similar decline in DNPTB.

On the household side, the assets included in YIM are personal savings deposits at chartered banks (DPB), TML savings deposits (DSTL) and TML time deposits of over one year (DT2TL).

Personal savings deposits at chartered banks include both savings and term deposits. For this reason there are three interest rate differential terms appearing in the equation for DPB; RPD - RSDTL, RPFT5B - RGIC and RBPFTQ - RT1TL.* A 13-month distributed lag on wealth is used, on the assumption that it takes the personal sector longer than the corporate sector to adjust its portfolio when wealth changes. To capture the substitution between CSBs and personal savings accounts, the exogenously determined stock of Canada Savings Bonds (LGFCSB) appears directly as an argument of the DPB equation. This solution was prefered to the alternative of endogenizing the demand for CSBs and including its determinants in the deposit equation.

Savings deposits at TMLs compete with chartered bank savings deposits and Canada Savings Bonds. At chartered banks, the deposit competition is captured by the interest rate differential RSDTL - RPD, and by the outstanding amount of CSBs. There is some chequing allowed in savings accounts, and in order to capture the transactions element, nominal gross national expenditure instead of wealth is used as the scale variable. Trust and mortgage loan company term deposits of more than one year are a function of the difference between the rate on guaranteed investment certificates and the average of the rate on 10 long provincial bonds and 10 industrials. The scale variable is a distributed lag on wealth.

## 9.3 REAL SUPPLY PRICE OF CAPITAL

The direct link from the financial sector to the real sector occurs through the real supply price of capital (RHOR), which is an argument in the implicit rental price of capital (see Chapter 5). RHOR is calculated as the ratio of dividends from Canadian corporations paid to domestic residents (YDIV11) and those paid to

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* See Appendix B to this chapter (page 212) for mnemonic definitions.

foreign shareholders (YDIVF), plus a proportion (Q) of retained corporate profits (YCR) to the value of equity.

$$RHOR = (YDIV11 + YDIVF + Q \cdot YCR)/VEQ \qquad (1)$$

The value of equity is constructed by multiplying overall earnings of Canadian corporations by the Toronto Stock Exchange price earnings ratio (RTSEPE):

$$VEQ = RTSEPE(YDIV11 + YDIVF + YCR) \qquad (2)$$

while Q, representing the ratio of market value of capital to its replacement value, is defined as in Scotland [75] as the ratio of VEQ to the replacement value of capital (KB$). Thus, RHOR can be viewed as a corrected earnings/price ratio, where the correction consists of discounting reinvested profits by Q. Only when Q = 1 is the shareholder indifferent between paid-up dividends and reinvested earnings. When Q is greater than 1, the stock market values the prospective earnings of a dollar of fixed investment at more than a dollar, and inversely when Q is less than 1. With this definition of RHOR, a regressor is run relating the change in RHOR to the change in the rate on ten industrial bonds (R10IND), the expected rate of inflation (PCPICE) and the rate of change in the supply of government debt (LGD). Inflationary expectations are determined by a simple adaptive process in which geometric declining weights that sum to unity are applied to past changes in the consumer price index. The results of the regression are presented below:

$$\Delta RHOR = -.02 + .57 \cdot \Delta R10IND - .06 \cdot \Delta PCPICE$$
$$-.008(\%\Delta(LGD) - PCPICE)) \qquad (4)$$

The fit of the equation is poor, reflecting the great volatility in quarterly changes in RHOR; the corrected coefficient of determination is only .19.

The results indicate that a 1% increase in nominal long interest rates leads to a .57% change in RHOR, while a 1% increase in inflationary expectations leads to only a small decline in RHOR. It does not appear that the equation is capable of distinguishing very well between changes in nominal rates and changes in real rates. Alternatively, it can be hypothesized that quarterly changes in inflationary expectations are not adequately captured by the proxy ΔPCPICE and are included in the residual error.

## Appendix A to Chapter 9

## TECHNICAL DESCRIPTION OF SHOCKS IN THE FINANCIAL SECTOR

**Table 1 Response of Interest Rates to An Increase of 100 Basis Points in Exogenous Canadian Rates**

Equations: Sector 17

Time Period: 1977M1 - 1979M12

Shocks: R90 = R90+1

RBANK = RBANK+1

**Table 2 Response of Interest Rates to An Increase of 100 Basis Points in Exogenous Canadian and U.S. Rates**

Equations: Sector 17

Time Period: 1977M1 - 1979M12

Shocks: R90 = R90+1

RBANK = RBANK+1

RCP2 = RCP2+1

RAAA2 = RAAA2+1

## Appendix B to Chapter 9

### INTEREST RATE MNEMONIC DEFINITIONS

| | |
|---|---|
| RBPFTQ | Rate on 90-day chartered bank personal fixed term deposits. |
| RDAY | Rate on day loans. |
| RGIC | Rate on trust and mortgage loan company 5-year guaranteed investment certificates. |
| RL | Rate on 10-year Canadas. |
| RL10P | Rate on 10 provincial long bonds. |
| RMC | Conventional mortgage rate. |
| RML | Rate on 5- to 10-year Canadas. |
| RMS | Rate on 3- to 5-year Canadas. |
| RNPT | Chartered bank Canadian dollar 90-day deposit rate. |
| RPD | Rate on chartered bank personal savings deposits. |
| RPFT5B | Rate on chartered bank 5-year fixed personal term deposits. |
| RPRIME | Chartered bank prime business loan rate. |
| RS | Rate on 1- to 3-year Canadas. |
| RSDTL | Rate on trust and mortgage loan company savings deposits. |
| RTB | Treasury bill rate at Thursday tender. |
| RT1TL | Rate on trust and mortgage loan company time deposits of under one year. |
| R10IND | Rate on 10 industrial bonds. |

# Chapter 10

## THE RESPONSE OF THE RDXF MODEL TO AN INCREASE IN GOVERNMENT EXPENDITURE

Heather Robertson
Michael McDougall

This chapter presents an analysis of the dynamic response of the RDXF model to a sustained increase of $100 million in government real non-wage expenditure.* To highlight the relative importance of the exchange rate, trade, and supply influences, the shock is performed on three versions of the model. The first is the full RDXF model. Next is the full model with the exchange rate exogenous, which will be referred to as the fixed exchange rate model. Finally all trade variables are exogenized in the closed economy model. The simulations are performed over a six-year period commencing in 1972. In all three cases the change in federal government financing requirements is satisfied by drawing down government cash balances at the chartered banks and selling treasury bills, in equal proportions. Exogenous domestic short-term interest rates, the 90-day paper rate and the bank rate are assumed to increase by the shock-minus-control change in the expected rate of inflation (PCPICE), implying that administered rates are constant in real terms. Energy and farm gate prices, which are exogenous in the RDXF model, are assumed to move in proportion to the annual average shock-minus-control change in the exchange rate.

The government multipliers in the three models are presented in Figure 21. In the first simulation exercise the government multiplier is smallest, peaking at 1.6 in the fifth and sixth

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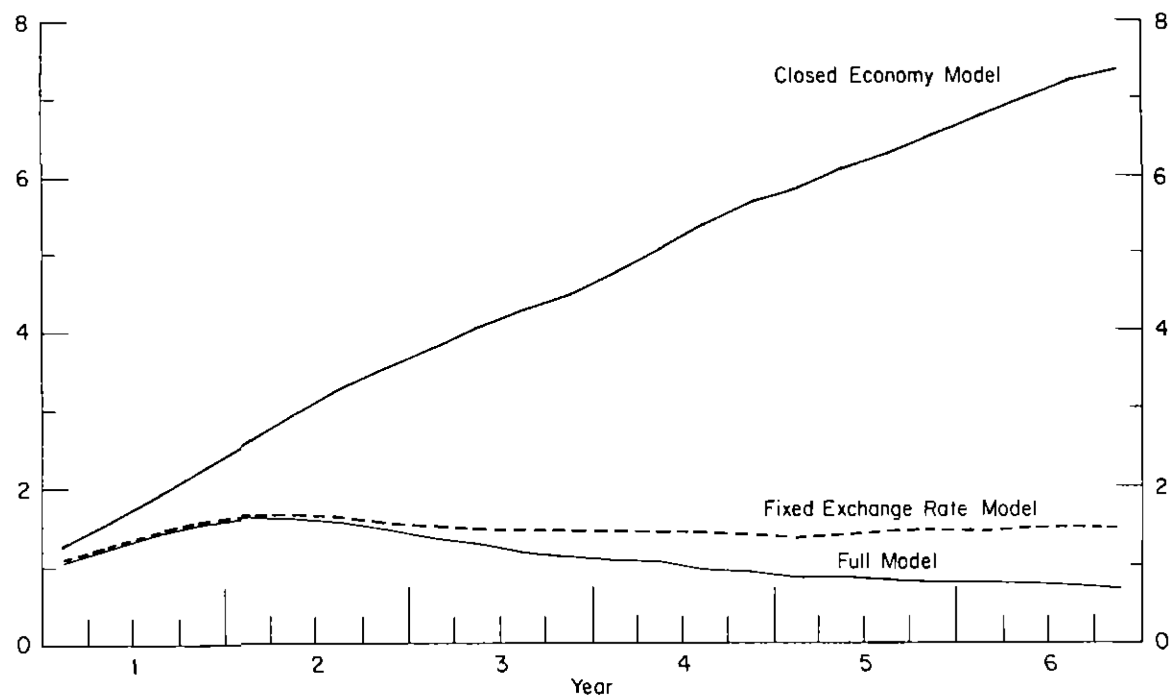
* Only non-wage expenditure is shocked, in order that the labour market will not be affected directly. If government wage expenditure were shocked, both private and public sector wages would increase as the degree of slack in the labour market was reduced. This in turn would create greater upward wage pressure, personal disposable income, and induced demand.

Figure 21

GOVERNMENT EXPENDITURE MULTIPLIERS




quarters and declining to .7 after six years. In the fixed exchange rate case, the multiplier follows a similar path to the first example up to six quarters, peaking at 1.6, but maintains a considerably higher value for the remainder of the simulation. Because the exchange rate is fixed, there is substantially less domestic price pressure and therefore a greater increase in personal disposable income and private domestic expenditure. A higher level of imports resulting from increased domestic activity counteracts the strength in final domestic demand somewhat; however, the multiplier exceeds unity over the entire period. In the final example the multiplier increases continuously over the sample period, reaching 7.3 after six years. The distinctly different pattern stems from a strong increase in real wages and private domestic production. This pattern can be traced to the problems in the supply bloc that were discussed in Chapter 8.

## 10.1 RESPONSE OF THE FULL MODEL

The transmission of a government expenditure shock through the various sectors in the RDXF model can be summarized by the following hierarchy of responses. First, the higher level of government expenditure creates an expansion in production, met initially by higher levels of productivity. As expected output increases, private sector employment rises. The subsequent increase in total wage income leads to higher consumption and housing expenditure, and firms gradually increase investment expenditure in order to meet the higher desired stock of capital. Demand pressure then moderates as import penetration increases. The deterioration of the current account produces an exchange rate depreciation, which increases domestic prices and creates an offsetting decline in personal disposable income and domestic demand. The effect of the higher level of imports and the exchange rate depreciation are sufficient to reverse the upward trend in the multiplier after five quarters.

Figure 22 shows the response of each expenditure component to the government shock. The major contributor to the upswing in the multiplier is consumption. The slower response of business investment, which peaks after two years, adds more to the medium-term strength of GNE. After this, investment growth falls off at the same pace as consumption. After three and a half years, induced private expenditure is below control and the government multiplier is less than unity. By the final year of the simulation, business investment, residential construction and net exports are below control while consumption remains slightly above control.

### How the Increase in Demand is Accommodated

The increase in demand resulting from the government expenditure shock is met in varying proportions by inventories, imports, output, and short-run productivity as shown in Table 25. In the first quarter, total domestic and foreign demand increases

**Figure 22**

**RESPONSE OF GNE COMPONENTS TO THE INCREASE IN GOVERNMENT EXPENDITURE**
**(Shock minus control, $ millions)**

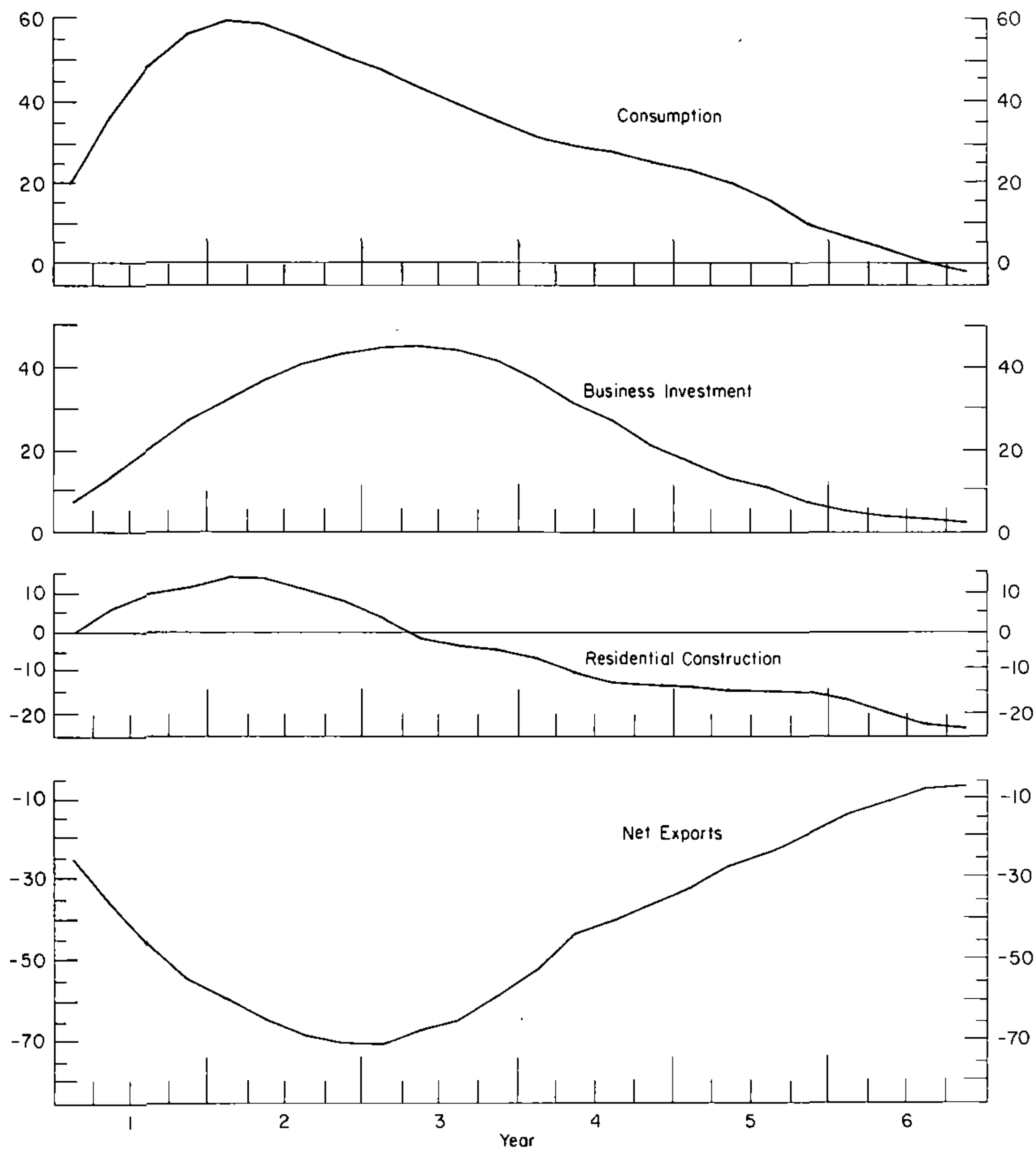


Table 25

**SOURCES OF EXPENDITURE INCREASES: FULL MODEL**
(Shock minus control, $ millions)

| | Quarters 1 | 4 | 6 | 8 | 24 |
|---|---|---|---|---|---|
| Government expenditure | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| Induced private expenditure* | 29.2 | 100.3 | 118.7 | 111.3 | -9.4 |
| Total domestic and foreign demand | 129.2 | 200.3 | 218.7 | 212.3 | 90.6 |
| How demand is satisfied | | | | | |
| Inventory decumulation** | -2.8 | -16.4 | -16.2 | -11.2 | +2.3 |
| Imports | 25.7 | 58.8 | 72.4 | 80.0 | 20.2 |
| Private sector output, of which | 90.7 | 130.0 | 134.5 | 120.0 | 60.9 |
| (Short-run productivity) | (65.0) | (59.9) | (48.4) | (30.7) | (6.4) |
| Other production | 16.0 | 27.9 | 28.0 | 23.2 | 7.2 |
| Total sales | 129.2 | 200.3 | 218.7 | 212.3 | 90.6 |

* Induced private expenditure = Final domestic demand plus exports mimus government
Private sector production = Gross private business product
Other production = Gross national expenditure minus gross private business product.

** The changes in inventories are reported with the reverse sign; i.e., a decumulation is positive.

$129.2 million. About 20% of this increase is met through imports and 70% through private sector output. Three-quarters of the output increase stems from short-run fluctuations in capacity and about 10% through induced government output and the housing and farm sectors. Private sector production continues to meet 60%-70% of the demand increase over the entire simulation period, increasing to a peak of $134 million after six quarters, with short-run fluctuations in production accounting for almost 50% of the increase. After six years, productivity increases account for 10% of the change in private production. Imports provide 37% of the increase in expenditure, or 47% of the change in real GNE after eight quarters, falling to about 20% after six years. The strong increase in imports initially occurs to satisfy capacity constraints, and subsequently in response to higher domestic consumption and investment. Tightened capacity has the strongest influence on imports of industrial materials and imports of machinery and equipment (M & E), which increase respectively $10 million and $5 million on impact.* Imported M & E accounts for 70% of the increase in total M & E after one year, 78% after two years, and 42% after six years. Inventory decumulation plays no role in accommodating the increase in demand. Business inventories are above control for the first two-and-a-half years of the simulation period because the stock-to-sales ratio responds positively to imports and negatively to exports.

The factors of production in supply output (UGPPS), which are hours, employment and the stocks of capital, respond directly to higher levels of private sector output. Hours show a very slight increase on impact, gradually moving towards control for subsequent periods in the simulation. As desired employment, which responds to the higher level of expected private output, and production increase, actual private sector employment (NIC) rises. After one year, NIC is .15% or 8,000 workers above control. The peak in employment, .18% above control, occurs after seven quarters, delayed somewhat in relation to production because of labour hoarding. The effect of increases in total employment (NE) on tightness in the labour market is offset considerably by an increase in the labour force (NL); the elasticity of NL with

* These increases are not consistent with the change in investment and inventories. Total investment in machinery and equipment increases $4.7 million on impact, implying some crowding out of domestic investment. This is because the imports respond much faster than investment in general. Furthermore, because materials are not an input into the production process, M1M has no influence on supply output (UGPPS).

respect to NE is about .7. The degree of slack in the labour market, measured by RU, falls 4 basis points below control after seven quarters, then eases back to less than 1 basis point below control after six years. Total business fixed investment is .21% ($27 million) above control after one year. Almost 80% of the increase originates in the machinery and equipment category, which exhibits a much faster output response than non-residential construction. Total investment peaks after ten quarters; M & E peaks after seven quarters ($30 million) and non-residential construction after twelve quarters ($22 million). After six years total business investment is only $2 million above control with non-residential construction slightly below control. The stock of machinery and equipment increases smoothly to $69 million above control, then eases off slightly. The stock of non-residential construction increases $51 million after five years and then stabilizes at this level. As was evident in Chapter 8, factor intensity leans toward labour for the first two years of the simulation; however, because employment exhibits an output elasticity in the long run that is considerably less than unity, the capital-labour ratio is higher than control in subsequent years of the simulation.

**Personal Income and Personal Expenditure**

Changes in the personal expenditure components are influenced mainly by movements in income and real effective purchasing power. This section will first analyze the response of real effective purchasing power, and then relate the consequent adjustments in the housing and consumption sectors. The responses of the components of real effective purchasing power over the simulation period are shown in Table 26. Labour income is $21.5 million above control on impact, with the increase stemming primarily from higher private sector employment. Public sector employment is mainly a function of the real government wage bill, which is exogenous. The increase in average weekly hours does not affect income because wages, which are calculated on a weekly basis, do not respond to hours.

Table 26

RESPONSE OF COMPONENTS OF REAL EFFECTIVE PURCHASING POWER TO THE INCREASE IN GOVERNMENT EXPENDITURE: FULL MODEL
(Shock minus control, $ millions)

| | Impact | Quarter 4 | Quarter 24 |
|---|---|---|---|
| Labour income | 21.5 | 79.7 | 464.1 |
| Other income* | 46.2 | 41.8 | 145.1 |
| Personal income | 67.7 | 121.4 | 609.2 |
| Personal direct taxes | 10.3 | 19.4 | 127.5 |
| Personal disposable income | 57.6 | 102.5 | 473.4 |
| Inflation premium | 0 | 1.9 | 27.6 |
| Transfers to corporations | 0 | .3 | 9.0 |
| Real personal disposable income | 54.6 | 81.3 | 35.3 |
| Real effective purchasing power | 54.6 | 79.5 | 20.4 |

* Includes personal investment income, farm income, and non-farm unincorporated income.

A far more substantial source of increase in the first quarter is in the non-wage income components, such as non-farm unincorporated income ($27.1 million) and personal investment income ($18.3 million), which is composed of interest and miscellaneous investment income and domestic dividends. Over $40 million of the increase in these components is attributable to the particular way which the income side of the model is constructed. A discrepancy term, YDISCR, which is the residual item between the income and expenditure components, is allocated arbitrarily between interest and miscellaneous investment income (YIM), non-farm unincorporated business income (YNFNC) and corporate profits (YC) in proportion to their share of non-wage income. The value of the discrepancy is $79 million on impact. This arbitrary allocation method is critical to the size of the government multiplier in the short run. If, for example, the whole discrepancy were allocated to corporate profits, the increase in personal income would be considerably less pronounced.

Real personal disposable income increases $54.6 million on impact; there is very little short-run price response. Real effective purchasing power, which determines consumption and housing expenditure, increases by the same amount because the inflation premium and current transfers to corporations are unchanged on impact. For the first two years of the simulation the slight decline in the real wage exerts some downward pressure on real personal disposable income. However, this effect is small, and is dominated by the increase in employment. Real disposable income peaks at $83.2 million above control after six quarters. In the latter part of the simulation real wage increases exert some upward pressure on disposable income; however, the decline in employment growth causes real income to decline steadily to $37 million above control. The increase in labour income accounts for 34.8% of the increase in total personal income on impact and rises to 76.2% after six years. Over the course of the simulation the discrepancy between the income and expenditure sides is reduced, declining from $68 million in the first year to $22 million in the final year, and the allocation problem is not as severe.

Real effective purchasing power peaks a little earlier and shows a weaker pattern in the final years of the simulation compared to real disposable income. By the final year of the simulation, real effective purchasing power is $15 million lower than real disposable income. This is mainly because of the increase in the consumer price index, which affects the inflation premium, and higher transfers to corporations as a result of higher nominal interest rates.

Investment in residential construction exhibits a very pronounced humped pattern, increasing $14.5 million after five quarters and then declining to $23 million below control by the end of the simulation, the largest decline relative to control of all components. In the initial period, higher real effective purchasing power increases the demand for new housing, thereby increasing the multiple listing service price (PMLS), and higher profitability induces an increase in housing starts. However, interest rate increases and higher construction costs induce downward pressure on housing starts; investment in residential construction is below control after ten quarters.

The rise in administered interest rates--the 90-day paper rate is 70 basis points above control by the final quarter of the simulation period--induces a 53-point increase in the prime rate and a 16-point increase in the conventional mortgage rate. The prime rate, which is a cost component in the housing supply equation (HST), and the mortgage rate, which reduces the MLS price as ownership costs increase, both reduce housing starts. A higher export price of lumber (PXLUM) and unit labour costs (ULC) serve to increase construction costs. However, because construction costs are a determinant of the MLS price, and both these terms enter into the housing starts equation, neither PXLUM nor ULC exert any influence on starts: the elasticities are offsetting.

The pattern of total consumption reflects the path of real effective purchasing power. Total real consumption reaches a peak of $60 million above control after five quarters, coinciding with the peak in real purchasing power, and then falls continuously to $1.5 million below control in the last quarter of the simulation. Although real effective purchasing power is above control throughout the simulation, only miscellaneous durables, other non-durables, and semi-durables are above control after six years. These components show the weakest increase in their own prices relative to the CPI. For this reason, the response of these categories reaches a peak seven to nine quarters into the simulation period, an average of two quarters later than the other categories. The consumption of motor vehicles declines below control for the final year of the simulation as a result of a higher real supply price of capital* which is used in calculating the imputed rental rate of motor vehicles, an argument in the motor vehicles equation. The turnaround in the consumption of household durables is the result of strong relative price increases in the second and third years; the longer term decline is induced from the substantially weakened housing activity. The sustained increase in food and energy prices relative to the CPI induce a decline in these categories below control; both categories exhibit relatively low income elasticities. Consumption of services declines dramatically to $8 million below control, reflecting the influence of the money illusion term which increases with the CPI throughout the simulation.

The savings rate increases 5 basis points on impact, reflecting the habit persistence behaviour in the consumption functions. Coincident with the peak in consumption the savings rate response is moderated to .025 basis points above control. Subsequently the savings rate rises to 4 basis points above control as a consequence of higher levels of inflation. The savings rate adjusted for the inflation premium exhibits this response as well, because the inflation premium does not remove the total inflationary basis from measured income.

**The Trade Sector**

The trade sector response contains certain elements that are vital in moderating induced expenditure over the longer period; in fact, as will be demonstrated in the closed economy shock, these elements are required for a stable solution. The first is the strong import response, which reduces the upward pressure on domestic production and factor demands. The second is the consequent exchange rate depreciation, which creates a general domestic price pressure that serves to moderate real wage increases. Total imports peak at $80 million above control after

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* Note that although administered real rates are constant in real terms at the short end, they are higher in real terms at the long end because of the term structure modeled in the financial sector.

two years, declining to $20 million above control by the end of the simulation. As indicated earlier, tighter capacity constraints increase the imports of industrial materials and machinery and equipment, which contribute a major portion of the import increase for the first year. As employment and investment respond to the demand increase, capacity pressure eases thereby reducing the role of these imports in satisfying production requirements. Imports of all categories of goods peak after two years; downward pressure thereafter is the result of both weaker domestic demand and a substitution away from foreign goods as import prices increase relative to domestic prices because of the exchange rate depreciation. By the end of the simulation all categories of imported goods except industrial materials and machinery and equipment are below control. At the peak of the import response, goods constitute 78% of the total increase in imports; however, by the end of the simulation services dominate, with 78% of the increase. Imports of services are sustained for three reasons: higher corporate profits increase foreign dividends; higher nominal interest rates increase interest payments abroad; and the exchange rate depreciation increases Canadian dollar payments for a given U.S. dollar debt.

Real exports grow slowly throughout the simulation to $13 million above control. The major source of strength is in exports of motor vehicles and parts, which are $8.3 million above control. For the initial five years of the simulation, higher domestic auto consumption increases exports through the terms of the auto pact. Although domestic consumption eventually falls below control, motor vehicle exports remain positive because the export price of motor vehicles (PXMVP2)* declines relative to the Canadian dollar

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* PXMVP2 is a geometric weighted average of the import price of motor vehicles (PMMVP2) and domestic unit labour costs (ULC). The weak response of ULC produces a low response of PXMVP2.

equivalent of the U.S. price. The remaining increase in exports is broadly based in the goods categories; services exhibit virtually no response after six years.

The worsened current account and the increase in domestic prices lead to a depreciation in the exchange rate of .43% by the end of the simulation. A slightly offsetting influence is the increase in the Canadian-U.S. interest rate differential. The deflator for total imports increases slightly less than the exchange rate (.43%). The export deflator increases only .37%, reflecting the relatively slow growth in the domestic costs that determine some export prices. The terms of trade deteriorate .06% after six years.

**Domestic Wages and Prices**

The response of wages and prices in the full RDXF model to increased government expenditure is shown in Table 27. Over the short run the major contributors to wage and price increases are labour market tightness and capacity utilization respectively. Greater labour market tightness results in an increase in private sector wages of .02% after one year, moving gradually to .32% after six years. It exerts a stronger influence on public sector wages, which are almost .4% above control after six years. The capacity utilization term exerts its major influence on the price of household durables (PHSHD) which reaches .13% above control after two years, contributing most of the .07% increase in the consumer price index. After this point, however, the upward pressure stemming from capacity utilization is diminished.

Another contributor to price increases is higher normalized unit labour costs, which are .06% above control after one year; they increase faster than wages because of the strong increase in employment. The price of semi-durables, the only category to show a decline, is below control for the first five quarters because of a decline in unit capital costs. As the exchange rate depreciates energy and farm gate prices increase, along with import and export

Table 27

RESPONSE OF DOMESTIC PRICES AND WAGES AND THEIR DETERMINANTS TO THE INCREASE IN GOVERNMENT EXPENDITURE: FULL MODEL
(Shock minus control, per cent)

| | Impact | 1 Year | 8 Years |
|---|---|---|---|
| Prices | | | |
| Consumption | 0 | .02 | .28 |
| Government | 0 | .02 | .34 |
| Housing | 0 | .01 | .30 |
| Business investment | 0 | .04 | .35 |
| Final domestic demand | 0 | .03 | .30 |
| Gross national expenditure | 0 | .02 | .29 |
| Consumer price index | 0 | .02 | .28 |
| Business output deflator | .02 | .04 | .30 |
| Wages | | | |
| Private sector wages | .00 | .02 | .32 |
| Public sector wages | .00 | .03 | .39 |
| Determinants of Prices and Wages | | | |
| Unit labour costs | .00 | .06 | .31 |
| Exchange rate | .01 | .06 | .43 |
| Energy and farm gate prices | .00 | .04 | .41 |
| Export prices | .01 | .05 | .37 |
| Import prices | .01 | .05 | .43 |
| Terms of trade | -.00 | -.01 | -.06 |
| Capacity utilization | .13 | .13 | .02 |
| Labour market gap | -.01 | -.03 | -.00 |

prices. After two years the energy and food deflators are the major contributors to the growth in the CPI.

The upward pressure on the CPI stemming from tighter capacity utilization and the exchange rate depreciation is sufficient to reduce real wages slightly below control for the first nine quarters of the simulation. Thereafter, labour market tightness exerts a stronger influence on wages, and real wages increase to .03% above control by the end of the simulation period.*

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* The real wage response changes substantially when the labour force is exogenous. Under this assumption, real wages are above control on impact, reaching .09% above control after six years. Because the labour force response is considered to be too strong in the September 1980 version of the RDXF model, the stronger real wage response (and hence higher government multipliers) is more likely to occur.

The business investment deflators increase rapidly in the initial periods of the simulation because of their sensitivity to capacity utilization. After six years the investment deflators exhibit the largest increase of all the domestic deflators because they are labour intensive, and therefore respond more readily to wage increases.

For the first ten quarters of the simulation the ratio of the private sector wage to the output deflators is below control, reaching a trough of -.3% after ten quarters and creating a slight incentive to increase employment. After this point, the ratio increases to .09% above control, adding to the downward influence of output on employment. In the long run the wage-output price ratio increases less than the real wage because the output deflator embodies investment, which exhibits a relatively strong response. The rental-output price ratios increase steadily to .15% above control by the end of the simulation period. The strength in the rental rates for machinery and equipment and non-residential construction stems from the combined effects of rapidly growing investment deflators and a higher real supply price of capital, which is driven by the nominal long lending rate and the expected rate of inflation. The downward pressure on investment from this source is weak, however, because of the relatively low price elasticities in the investment equations.

**The Government Deficit**

The federal government deficit increases $82 million on impact, improves slightly to $58 million after six quarters, and then increases to $130 million by the end of the simulation. The temporary improvement is the result of a reduction in unemployment insurance payments resulting from the peak in employment. The largest nominal government expenditure component after six years is, of course, non-wage which increases $225 million, reflecting a .27% increase in the government non-wage deflator. Interest payments on the federal debt increase $29 million, as a result of higher payments on the cumulative stock of treasury bills, which by assumption finance 50% of the growing deficit. Direct taxes increase $64 million in response to personal income growth, corporate taxes increase $21 million, and indirect taxes increase $30 million, mainly as a result of higher revenues from sales taxes and customs duties.

The provincial balance improves $18 million after six years; provincial revenues increase in line with their federal counterpart while nominal expenditure increases only $83 million due solely to the rise in the deflator.

## 10.2 RESPONSE OF THE FIXED EXCHANGE RATE MODEL

The response in a fixed exchange rate model* to a $100 million increase in government non-wage expenditure differs from the full model response by inducing less upward price pressure. Thus real wages are increased and, in turn, so is domestic expenditure. However, because the exchange rate is fixed, the trade balance is worse than in the first simulation experiment. Higher domestic expenditure dominates, however, and the multiplier is higher in this model.

In the previous simulation considerable domestic price pressure was felt through the depreciation of the exchange rate. With a fixed exchange rate the total export and import deflators show very little response since most of the component prices are specified in terms of the Canadian dollar equivalent of United States prices and costs. Export prices increase only .09% (vs. .37% in the previous example) by the end of the simulation; their increase stems from the influence of domestic unit labour costs on some exports of goods prices and the influence of domestic service prices on the service receipts deflator. The import price deflator responds only marginally to a slight increase in the other goods deflator, which is influenced by the domestic consumer

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* The authorities are assumed to possess a sufficiently large source of foreign exchange that sterilization transactions do not have any direct influence on the domestic economy.

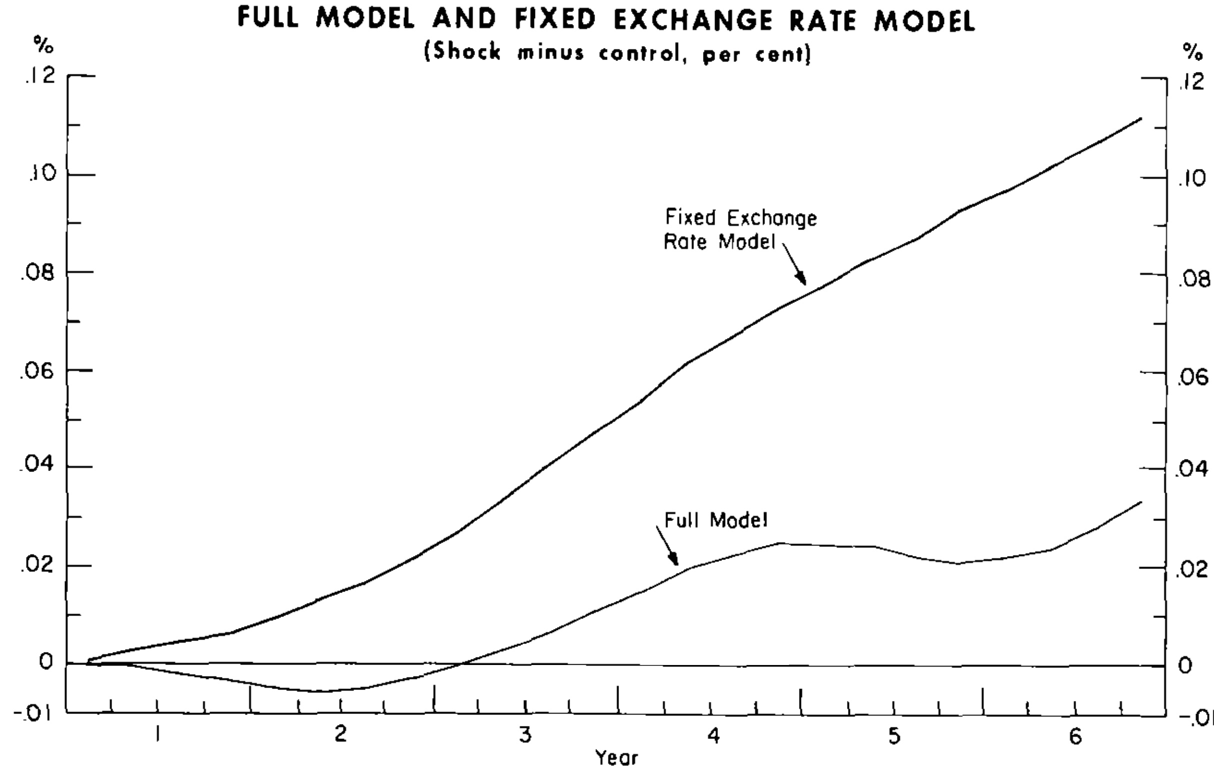
price index excluding food and energy. The terms of trade improve throughout the simulation (vs. a .06 deterioration by the end of the simulation period in the first shock). Both energy and farm gate prices, which are linked directly to the exchange rate, are unchanged. Thus, because trade, farm gate, and energy prices constitute over 40% of total costs in the consumer price index, the CPI is considerably weaker, .10% above control after six years versus .28% in the first simulation. The more labour-intensive components of the CPI such as services show the strongest response. Similarly, the relatively labour-intensive government and investment deflators push the final demand deflator .21% above control, versus .30% in the previous shock. With the improvement in the terms of trade the GNE deflator increases .23% versus .29%.

Figure 23 shows the response of the private sector real wage in the full model and in the fixed exchange rate model. Because the labour market gap shows a similar response in both simulations, nominal wages are almost as strong in the second simulation, increasing .33%. Real wages, therefore, are considerably higher in the fixed exchange rate model, rising continuously to .11% above control (vs. .03% in the full model). In response to the real wage increase, real effective purchasing power increases steadily to $124 million above control after six years. Real consumption, following the pattern in income, increases $90 million after six years.

The depressing effect of nominal interest rates on investment in residential construction is lessened in the fixed exchange rate model because of reduced upward price pressure; the 90-day rate increases only 30 basis points. The interest rate effect begins to bite after a year and residential construction is reduced from a peak of $10 million to about control after ten quarters. However, as real effective purchasing power continues to grow, investment picks up and increases steadily throughout the remainder of the simulation, reaching $15 million above control.

Figure 23

RESPONSE OF PRIVATE SECTOR REAL WAGES
TO THE INCREASE IN GOVERNMENT EXPENDITURE:
FULL MODEL AND FIXED EXCHANGE RATE MODEL
(Shock minus control, per cent)




The response of real net exports is very different in this example, declining continuously to $110 million below control. The relatively higher domestic activity and capacity, complemented by lower import prices relative to domestic prices, induce a stronger import response than in the first example; real imports are $90 million above control by the end of the simulation. Real exports decline slowly throughout the simulation, reaching $20 million below control. Most nominal service receipts do not change since they are a function of foreign variables and the exchange rate. However, because of an increase in the service receipts deflator, real service receipts are $15 million below control after six years.

Although real business output exhibits a similar response over the first two years in both examples, in subsequent years output is considerably higher in the fixed exchange rate model. The continuous decline in net exports reduces output from its peak after a year but the large increase in consumption is sufficient to keep output about $20 million above the previous value in the final four years. Business investment, driven primarily by the accelerator, exhibits a similar pattern to the previous shock; however, the response is a sustained $10 million higher in the latter part of the simulation. Employment exhibits the familiar humped response and is higher than in the previous shock. A strong increase in the wage/output-price ratio, .3% above control after six years, creates considerably more downward pressure on employment than in the previous shock, counteracting the effect of higher output.

A major change on the income side is the response of corporate profits. Wage costs that are relatively higher than nominal expenditure and an unchanged exchange rate lead to a .45 percentage point decline in profits relative to the previous case. In contrast to the marked decline in profits, net national income is only .1 percentage point below its value in the previous shock. As a result of lower profits, corporate tax accruals are reduced leading to a relatively large decline in government revenue. By the end of the simulation the federal government balance is $10 million worse than before.

## 10.3 RESPONSE OF THE CLOSED ECONOMY MODEL

The closed economy model yields a government multiplier of 7.30 after six years. Because imports do not play a moderating role in siphoning demand pressure, domestic private production increases continually, thereby spurring factor demands. Greater labour market tightness increases real wage pressure to such an extent that real effective purchasing power is $530 million above control after six years, versus $30 million in the first simulation. Personal and corporate taxes are sufficiently strong that the federal government obtains a surplus position relative to control after three years.

The accommodating role of production in the closed economy model is illustrated in Table 28. On impact, private output increases $101.6 million, 72% of which is an increase in short-run productivity. Inventory decumulation accounts for $7 million of the increase in sales. After one year, business output has climbed to $183 million, 50% of which is short-run productivity. Throughout the simulation short-run productivity plays a strong role in accommodating excess demand pressure, maintaining 30% of the increase after six years.

**Table 28**

**SOURCES OF EXPENDITURE INCREASES: CLOSED ECONOMY MODEL**
(Shock minus control, $ millions)

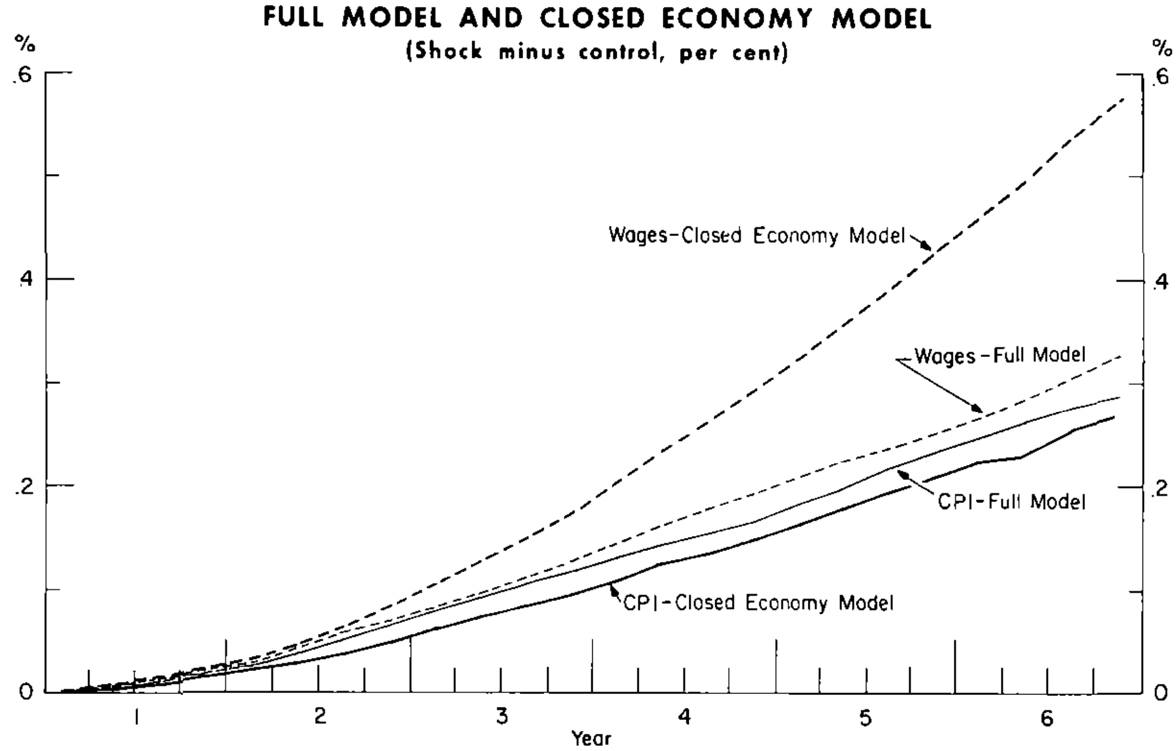
| | Impact | Quarters 4 | Quarters 8 | Quarters 24 |
|---|---|---|---|---|
| Government | 100.0 | 100.0 | 100.0 | 100.0 |
| Private final domestic demand | 131.9 | 136.3 | 245.6 | 611.3 |
| Total demand | 231.9 | 236.3 | 345.6 | 711.3 |
| How demand is satisfied | | | | |
| Inventory decumulation | 7.0 | 8.9 | -7.4 | -19.0 |
| Private output, of which | 101.6 | 182.8 | 282.7 | 582.7 |
| (Short-run productivity) | (73.0) | (90.0) | (111.7) | (174.0) |
| Other | 23.3 | 44.7 | 70.3 | 47.6 |
| Total sales | 231.9 | 236.3 | 345.6 | 711.3 |

The source of instability in the closed economy model is on the supply side; as mentioned in Chapter 8, there is no steady state imposed in this part of the model. Because prices are not homogeneous with respect to factor costs, and because capacity exerts a very weak influence in the price equations whereas labour market tightness plays a strong role in the determination of wages, a real wage spiral can develop that continuously refuels demand pressure. Figure 24 shows the response of private sector wages and the CPI in the full and closed economy models. It is noteworthy that the CPI moves similarly in both simulations, whereas private sector wages are considerably higher in the closed economy version. This results chiefly from a labour market tightness gap that falls steadily to .11 basis points below control after six years. Furthermore, this increased demand pressure is not checked by any supply constraints; at any level of output demand can be accommodated by increasing the productivity of the existing factors of production. And, this so-called "short-run" productivity gap may persist for extended periods of time because there is no cost associated with productivity variations in the model, nor are there constraints on factor demands such that they conform with the production technology in the long run.

**Figure 24**

**RESPONSE OF PRIVATE SECTOR WAGES AND THE CPI TO THE INCREASE IN GOVERNMENT EXPENDITURE: FULL MODEL AND CLOSED ECONOMY MODEL**

**(Shock minus control, per cent)**



The instability in the closed economy model is apparent in the change in the federal government deficit position relative to control. Table 29 shows the annual average of the main federal government revenues and expenditures in the first, third, and sixth years of the simulation.

Table 29

**RESPONSE OF FEDERAL GOVERNMENT REVENUES AND EXPENDITURES TO THE INCREASE IN GOVERNMENT EXPENDITURE: CLOSED ECONOMY MODEL**
(Shock minus control, $ millions)

| | Quarters | | |
|---|---|---|---|
| | 4 | 12 | 24 |
| Direct Taxes | | | |
| Persons | 18.1 | 75.1 | 193.7 |
| Corporations | 14.1 | 33.3 | 30.1 |
| Indirect Taxes | 8.1 | 26.2 | 53.9 |
| Total Revenue | 42.4 | 145.5 | 311.8 |
| Expenditures on goods and services | 112.5 | 159.1 | 260.8 |
| Transfers | -12.4 | -27.4 | -43.2 |
| Interest on debt | 3.2 | 12.0 | 7.5 |
| Total expenditures | 103.3 | 143.4 | 224.3 |
| Surplus | -60.8 | 2.1 | 87.5 |

After four quarters, the federal government deficit increases by $60 million dollars. The $112 million increase in nominal government expenditure is offset by a $12 million decline in UIC benefits as a result of lower levels of unemployment, and a $40 million increase in tax revenues. By the fourth year of the simulation, the revenue increase is sufficient to place the government in a surplus position; that is, a $100 million increase in real government expenditure is sufficiently expansionary that it pays for itself after three years. By the final year of the simulation, the government is in a net surplus position of $87 million; the $193 million increase in personal direct taxes, $30 million increase in indirect taxes, and $54 increase in corporate taxes are the main source of revenue increase.

All components of net national income display strong increases over the simulation period. By the end of the year, labour income is $1,247 million above control, corporate profits $154 million and interest and miscellaneous investment income $237 million. Corporate profits capture a greater share of nominal private business product for four years. Subsequent to this, the strong output increases are dominated by an expanding wage bill. Labour's share declines slightly for the first five years, after which it gains a greater than historical share of nominal business product.

## 10.4 CONCLUSION

The purpose of this chapter was to illustrate that the RDXF model exhibits a quite reasonable response on aggregate to a government expenditure shock, but also that this may be the result of offsetting influences. At a more disaggregated level RDXF displays much less desirable properties. In the full model, the strong marginal propensity to import is sufficient to choke off the demand for domestically produced goods and services and reduce the multiplier to .7 after six years. When this influence is not at play, as in the closed economy model, the increase in government demand is accommodated by higher production and productivity increases, and the subsequent increases in employment and real wages further fuel domestic demand. The result is a spiral in real wages and domestic output that is unchecked throughout the simulation. And, because the model is linear, this effect would occur at all levels of output.

In future, the aspects of the model that will be addressed to improve the model's response to a demand shock are (a) restrictions that produce a steady state such that short-run fluctuations in productivity disappear and factors are at their desired level in conformation to the production function, and (b) a non-linear response, so as to reflect supply constraints at tight levels of capacity.

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