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Certainty of Settlement and Loss Allocation with a Minimum of Collateral

by
Walter Engert

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Walter Engert

Department of Monetary and Financial Analysis

Bank of Canada

Ottawa, Ontario

Canada

KIA OG9

Telephone: (613) 782-8194

Fax: (613) 782-7508

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Abstract

To assure liquidity needs and the robustness of loss allocation in a clearing and settlement system based on netting, collateral requirements and net debit caps could be set in a specific, interrelated way. For instance, it has been advocated that system participants collateralize a predetermined fraction of the largest credit line that they extend within the system, and that net debit caps be set equal to the sum of the credit lines that participants receive multiplied by the same fraction. This paper shows how this approach will assure sufficient liquidity for daily settlement to occur in the event of the default of any participant in the system. The paper also examines the degree to which the collateral would cover each participant's obligations under the loss allocation rule.

The paper then shows how this approach to collateral requirements and net debit caps can lead to excess collateral in the system. It suggests ways to reduce collateral to a minimum, while securing liquidity needs and the robustness of loss allocation. In addition, an arrangement is outlined that would allow a given stock of collateral to simultaneously secure a survivors-pay net debit cap and a defaulter-pays net debit cap in a clearing and settlement system. Finally, the paper shows that maximizing the capacity of a clearing and settlement system with both defaulter-pays and survivors-pay segments requires that a participant use its survivors-pay segment before using its defaulter-pays segment.

Résumé

Pour garantir la satisfaction des besoins de liquidités et la solidité du mode de répartition des pertes dans un système de compensation et de règlement axé sur l'établissement de la position nette, on peut envisager une approche où les exigences en matière de nantissement et le plafond du montant net des débits de paiement seraient établis de manière précise et interdépendante. Par exemple, on a préconisé que les participants au système garantissent sous forme de nantissement une fraction pré-établie de la ligne de crédit la plus élevée que chacun d'eux accorde et que le plafond du montant net des débits des participants égalise le produit qui est obtenu lorsque la somme des montants des lignes de crédit octroyées à chacun est multipliée par cette fraction. La présente étude montre comment cette approche peut assurer un apport de liquidités suffisant pour que les règlements quotidiens se continuent, même en cas de défaillance de n'importe lequel des participants. L'auteur examine aussi dans quelle mesure cette garantie permettrait à chaque participant d'honorer ses obligations en vertu de la règle de répartition des pertes.

L'étude montre ensuite que cette façon d'aborder la question du nantissement et du plafonnement du montant net des débits peut se solder par un excès de sûreté dans le système. Elle propose des façons de réduire le nantissement à un niveau minimum tout en satisfaisant les besoins de liquidités et en assurant la solidité du mode de répartition des pertes. En outre, l'auteur suggère un arrangement qui permettrait à un bloc de titres donné de garantir le plafonnement du montant net des débits selon deux approches, à savoir celle où la perte est supportée par l'établissement défaillant et celle où la perte est supportée par le reste du groupe. Enfin, l'étude fait ressortir que, pour maximiser l'efficacité d'un système de compensation et de règlement dans lequel interviennent ces deux approches, il faut qu'un participant utilise la première avant la seconde.

1. Introduction

In recent years, a considerable amount of activity in Canada has aimed at improving the arrangements for clearing and settling various kinds of transactions. An important example is the Large Value Transfer System (LVTS) that is being developed to clear and settle large Canadian dollar payments.[1] Efforts have been directed toward increasing the efficiency of clearing and settlement systems and lowering the risks that can arise in these arrangements. More specifically, a central objective has been to ensure that the transactions that have entered the system will be finalized, even in the event of a default of any participant in the system – that is, there would be certainty of settlement.

The approach to clearing and settlement systems that has been followed in Canada is based on netting: the systematic setting-off of multiple participants' positions against one another.[2] Risk management will require a variety of features, such as limits on the exposure that any participant can bring to the system, that is, net debit caps. A procedure would also have to be in place to allocate losses to participants in a predetermined way in the event of the failure of other system participants, that is, a legally binding loss allocation rule would have to apply.

Collateralizing exposures to assure the liquidity needs of the clearing house and to guarantee participants' performance under the loss allocation rule would play an important role in risk containment as well. For instance, it has been advocated that collateral requirements and net debit caps be set in a specific, interrelated way. That is, participants could collateralize a predetermined fraction of the largest credit line that they extend within the system. To complement these collateral requirements, participants' net debit caps would be limited to the sum of the credit lines that they receive, multiplied by the same fraction. For simplicity, we can call this the "basic approach" to collateral requirements and net debit caps.

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1. For more on the developing LVTS, see *Characteristics of the Proposed Large Value Transfer System (LVTS)*, (Ottawa: Canadian Payments Association, June 1993).

2. Instead of netting, a payments system could be based on a gross settlement model. In this arrangement, each payment instruction would result in an immediate debit for the payer and an immediate credit for the receiver in their settlement accounts, provided that the payer has sufficient funds in its account to make the payment. Otherwise, payment orders could be queued until sufficient funds have accumulated from incoming payments to allow the payments to be executed. To avoid queuing payment orders, collateralized overdrafts could provide the funds needed to allow settlement to proceed.

Although important technical differences exist between netting and gross systems, ultimately, they both rely on different combinations of the same basic techniques to ensure settlement: net debit caps, prefunding, collateral and loss allocation rules. (For gross systems, debit caps and loss allocation rules would be required if overdrafts were permitted.)

This paper shows how the basic approach will ensure that the clearing house will be able to arrange sufficient liquidity for daily settlement to occur in the event of the default of any single participant in the clearing and settlement system. That is, it satisfies Lamfalussy Standard No. 4.[3] This paper also shows how the basic approach will provide sufficient collateral to cover each participant's possible loss allocation in the event of any single failure.

However, in the case of two defaults summing to less than the collateral pool, loss allocations can in certain cases exceed a participant's collateral contribution. The significance of this result needs to be qualified by a few considerations. First, whether a participant's loss allocation can exceed its collateral contribution depends largely on the pattern of credit lines that the participants extend to one another within the system, and the distributions of lines that can lead to this result appear to be unusual ones. Second, to some observers, two failures on the same day may seem an unreasonable contingency to safeguard against. Third, survivors' net credits (if any) can be used along with collateral to absorb loss allocations, thereby helping to ensure the robustness of loss allocation. In sum, therefore, the basic approach to collateral requirements and net debit caps seems to be a reasonable way to secure liquidity needs and loss allocation in a survivors-pay clearing and settlement system.

Lamfalussy Standard No. 4 essentially calls for ensuring certainty of settlement in the event of the default of the participant with the largest net debit cap. This paper shows that the basic approach to collateral and net debit caps can lead to a collateral pool that is greater than any single net debit cap in the system. As a result, the basic approach can be seen to lead to excess collateralization. Thus, collateral rebates could be used to lower the collateral pool to the lowest acceptable level – the largest net debit cap – and thereby still secure the liquidity needs of the system. The paper also describes how to minimize excess collateral while securing liquidity needs and ensuring the robustness of loss allocation (for any one default).

The preceding is concerned with the properties of survivors-pay clearing and settlement systems. The penultimate section of the paper considers a set-up in which a given stock of collateral both secures survivors-pay net debit caps (as above) and simultaneously provides for defaulter-pays net debit caps. This approach would increase the capacity of the system for a given

3. In November 1990, the G-10 central banks issued a report that analyses the impact of netting on credit, liquidity and systemic risks and that puts forward a set of minimum standards for the design and operation of netting schemes: *Report of the Committee on Interbank Netting Schemes of the Central Banks of the Group of Ten Countries*, (Basel: Bank for International Settlements, November 1990). This report is commonly referred to as the Lamfalussy Report. Lamfalussy Standard No. 4 stipulates that multilateral netting systems should, at a minimum, be capable of ensuring the timely completion of daily settlement in the event of a default by the participant with the largest single net-debit position.

pool of collateral, or put differently, this would lower collateral needs for a given capacity of the system. However, the system's ability to handle multiple defaults is more limited under this approach to net debit caps and collateral than under the basic approach. Finally, in a clearing and settlement system with both defaulter-pays and survivors-pay segments, maximizing the capacity of the system requires that a participant use its survivors-pay segment before using its defaulter-pays segment.

2. Ensuring certainty of settlement

Lamfalussy Standard No. 4 requires that a clearing house have sufficient liquidity to ensure daily settlements in the event of the default of the participant with the largest net debit cap. To satisfy this constraint, each participant could post collateral equal to the "scaled-down" largest bilateral credit line that it extends. That is, each participant x would pledge collateral, C_x, equal to the largest bilateral credit line that it has extended, E_x^{λ}, multiplied by θ.[4] (Each participant x could extend its largest bilateral line to any other participant so that λ *could* indicate a different participant for each x.)

More formally, $C_x = \theta E_x^{\lambda}$, so that the total collateral posted by all participants $(1...n)$ is

$$C^T = \sum_{x=1}^{n} C_x = \theta \sum_{x=1}^{n} E_x^{\lambda}.$$

Each participant j's net debit cap is a function of the bilateral credit lines received from all other $(n-1)$ participants.[5] Participant j's net debit cap is defined as

$$\delta_j = \theta \sum_{x=1}^{y_j} E_x^j, \; x \neq j,$$

where y_j is the number of lines received by participant j and $y_j \leq (n-1)$.

4. The multilateral netting of a given set of bilateral transactions would lead to a multilaterally netted balance that is a fraction of the underlying bilateral positions. The specific value chosen for θ would reflect the "power of netting" in the system. That is, the more powerful the netting in the system, the lower would be θ. (For example, θ might equal 0.30, or in a more powerful netting system, θ might equal 0.20.) For more on the rudiments of netting and on risk management in multilateral netting arrangements, see Walter Engert, "An Introduction to Multilateral Foreign Exchange Netting," Working Paper 92-5 (Ottawa: Bank of Canada, September 1992).

5. That is, participant j cannot extend a line to itself.

For simplicity, we can call the preceding the "basic approach" to collateral requirements and net debit caps.[6] It follows that the collateral pool is (more than) sufficient to cover *any* single default:

$$C^T = \theta \sum_{x=1}^{n} E_x^{\lambda} > \delta_j = \theta \sum_{x=1}^{y_j} E_x^j, \tag{1}$$

for any j, since $E_x^{\lambda} \geq E_x^j$, and $n > y_j$.

With collateral contributions calculated in this way, the collateral pool will always exceed the largest net debit cap.[7] In addition, the collateral pool obviously will ensure sufficient liquidity to cover multiple defaults up to C^T.

3. Ensuring the robustness of the loss allocation rule

A second motivation for having participants collateralize their largest bilateral line, one that has gained appeal in some public policy circles, is to secure loss allocation, that is, to ensure that participants will honour their contingent obligations. In the event of default(s) summing to no more than the collateral pool, relying on collateral to cover a participant's loss allocation resulting from these defaults implies that the loss allocation would not exceed its own contribution to the collateral pool. That is, this criterion is concerned with the distribution among participants of loss allocations resulting from defaults to the clearing house relative to participants' own collateral contributions.

More formally, in the event that

$$\sum_{x=1}^{d} D_i \leq C^T,$$

we should have

$$C_j \geq \sum_{i=1}^{d} L_j^i, \tag{2}$$

6. This approach is followed in the Clearing House Interbank Payments System (CHIPS) in the United States, and it has been proposed by the Canadian Payments Association for the survivors-pay (tranche II) portion of the emerging LVTS in Canada. See *Characteristics of the Proposed Large Value Transfer System (LVTS)*, (Ottawa: Canadian Payments Association, June 1993), pp. 2-3.

7. Excess collateral is considered in more detail in Section 7 below.

for each participant $j = (1...n)$, $j \neq i$,

where D_i = the net debit of participant i in default;

d = the number of defaulters;

C_j = the collateral posted to the clearing house by participant j; and

L_j^i = the loss allocation to participant j from the default of participant i.

We can rewrite equation (2) as

$$\left(C_j - \sum_{i=1}^{d} L_j^i\right) \geq 0, \tag{3}$$

where $L_j^i = \left(\dfrac{E_j^i}{\sum_{x=1}^{y_i} E_x^i}\right)(D_i - C_i)$.

That is, losses are allocated to surviving participants in proportion to their bilateral credit lines to the defaulters. In addition, the collateral posted by the defaulter is set off against the net debit in default, thereby lowering the loss allocations to the surviving participants.

The second term of equation (3), the sum of loss allocations to participant j, is a function of a number of variables:

$$\sum_{i=1}^{d} L_j^i = f\left[\overset{(+)}{\sum_{i=1}^{d}\left(\frac{E_j^i}{\sum_{x=1}^{y_i} E_x^i}\right)}, \overset{(+)}{\sum_{i=1}^{d} D_i}, \overset{(-)}{\sum_{i=1}^{d} C_i}\right].$$

Loss allocations to participant j are positively related to participant j's share of the losses to be allocated and to the size of the net debits in default. On the other hand, loss allocations to participant j are negatively related to the collateral posted by the defaulters.

Since a net debit in default can be as large as a participant's net debit cap, we can substitute the defaulter's net debit cap (δ_i) for the net debit in default (D_i), so that we have

$$C_j - f\left[\overset{(+)}{\sum_{i=1}^{d}\left(\frac{E_j^i}{\sum_{x=1}^{y_i} E_x^i}\right)}, \overset{(+)}{\sum_{i=1}^{d}\delta_i}, \overset{(-)}{\sum_{i=1}^{d}C_i}\right] \geq 0. \tag{4}$$

That is, for each participant j, the loss allocation rule is less likely to be guaranteed under the following conditions: the greater is participant j's share of the losses to be allocated, the larger are the net debit caps of the defaulters and the less collateral that is posted by participant j and by the defaulters.

Finally, we can rewrite condition (4) as a function of the bilateral lines that the participants have extended to one another:

$$\theta E_j^{\lambda} - f\left[\overset{(+)}{\sum_{i=1}^{d}\left(\frac{E_j^i}{\sum_{x=1}^{y_i} E_x^i}\right)}, \overset{(+)}{\sum_{i=1}^{d}\left(\theta\sum_{x=1}^{y_i} E_x^i\right)}, \overset{(-)}{\sum_{i=1}^{d}(\theta E_i^{\lambda})}\right] \geq 0. \tag{5}$$

4. Loss allocation is guaranteed by collateral in the event of one default

The loss allocation to participant j from the default of any other participant i is

$$L_j^i = \left(\frac{E_j^i}{\sum_{x=1}^{y_i} E_x^i}\right)(D_i - C_i),$$

where all variables are as defined above.

The largest possible value for the net debit in default (D_i) is participant i's net debit cap:

$$\delta_i = \theta \sum_{x=1}^{y_i} E_x^i.$$

Therefore, substituting δ_i for D_i gives us the largest possible loss allocation to participant j:

$$L_j^i = \theta E_j^i - \left(\frac{E_j^i}{\sum_{x=1}^{y_i} E_x^i} \right) C_i.$$

The collateral posted by participant j exceeds this possible loss allocation, that is,

$$C_j = \theta E_j^{\lambda} > \theta E_j^i - \left(\frac{E_j^i}{\sum_{x=1}^{y_i} E_x^i} \right) C_i.$$

As a result, compliance with the loss allocation rule is guaranteed in the event of a single default.

5. Loss allocation is not guaranteed by collateral in the event of two defaults

Whether the robustness of the loss allocation rule is guaranteed by collateral in the event of two defaults is an empirical question, depending largely on the relative sizes of the relevant bilateral lines – see equation (5). Therefore, to assess this question, consider a survivors-pay clearing house with 5 participants, where the participants extend bilateral lines to one another as shown in Table 1. Thus, for example, participant B has extended a credit line of 8 to C which has received bilateral lines totalling 32 from all of the participants. Participant E has received the largest bilateral lines from the others; for example, the largest line extended by participant C is to E, a line of 12. The largest line extended by E is to participant D, 15.

Table 1: Bilateral lines in a 5-participant clearing house

	To A	To B	To C	To D	To E
From A	not applicable	10	10	5	10
From B	2	not applicable	8	8	15
From C	3	9	not applicable	10	12
From D	3	7	6	not applicable	18
From E	5	9	8	15	not applicable
Sum of lines	13	35	32	38	55

Net debit caps and collateral requirements depend on the power of netting in the system, that is, on the scaling-down factor, θ. So, for example, with $\theta = 0.20$, participant C's net debit cap is $0.20(32) = 6.4$, and its collateral requirement is $0.20(12) = 2.4$. Table 2 presents the net debit caps and collateral requirements for two different values of θ.

Table 2: Net debit caps and collateral for two values of θ

Participant	$\theta = 0.20$		$\theta = 0.30$	
	Debit cap	Collateral	Debit cap	Collateral
A	2.60	2.00	3.90	3.00
B	7.00	3.00	10.50	4.50
C	6.40	2.40	9.60	3.60
D	7.60	3.60	11.40	5.40
E	11.00	3.00	16.50	4.50
Collateral pool		14.00		21.00

(a) Defaults of participants B and C

In this section, we look at the defaults of two participants whose net debits sum to less than the total collateral pool. While the clearing house will have sufficient collateral to meet its liquidity needs to ensure settlement, each participant's collateral will not be sufficient to absorb its loss allocation.

Case 1: $\theta = 0.20$

Suppose that participants B and C both default, and that at the time of default they are at their net debit caps, so that the total net debit in default is 7.0 + 6.4 = 13.4, an amount less than the collateral pool (14.0). As a result, the clearing house can easily meet its liquidity needs to ensure settlement on the day of default.

How do the loss allocations to the surviving participants, A, D and E, compare with the collateral that they have posted to the clearing house? The loss to be allocated from the defaults of participants B and C equals the sum of their net debits less their collateral, which the clearing house seizes to minimize loss allocations; that is, (7.0 – 3.0) + (6.4 – 2.4) = 8.0. As shown in Table 3, the loss allocation to participant A is 60 per cent more than the amount of the collateral that it has posted to the clearing house.[8]

Table 3: Loss allocations and collateral for $\theta = 0.20$

Participant	Loss allocation	Collateral	Is collateral sufficient?
A	3.21	2.00	no – a 60% shortfall
D	2.08	3.60	yes
E	2.71	3.00	yes
Total	8.00	8.60	not applicable

8. Appendix I provides details on the loss allocation calculations.

Case 2: θ = 0.30

In this case, the sum of the defaults of participants B and C is 20.1, less than the collateral pool (21.0), so that the clearing house's liquidity needs would be easily met. Again, the loss to be allocated from the defaults of participants B and C equals the sum of their net debits less the collateral that they have posted; that is, (10.5 – 4.5) + (9.6 – 3.6) = 12.0. Again, as shown in Table 4, the loss allocation to participant A significantly exceeds its collateral.

Table 4: Loss allocations and collateral for θ = 0.30

Participant	Loss allocation	Collateral	Is collateral sufficient?
A	4.81	3.00	no – a 60% shortfall
D	3.11	5.40	yes
E	4.08	4.50	yes
Total	12.00	12.90	not applicable

(b) Survivors' net credits also can absorb loss allocations

Any multilateral netting arrangement is a zero-sum system. That is, the sum of the net debits in the system must equal the sum of the net credits in the system. As a result, in the event of multiple defaults, at least one (and probably more) of the survivors would have net credit, or receivable, positions in the clearing house. These balances also could be used to absorb loss allocations and thereby help to ensure the robustness of the loss allocation mechanism. For instance, in case 2 of the preceding example, that is, with θ = 0.30, *if* participant A had a net credit position in the clearing house of 1.81, then the loss allocation would be fully covered by the combination of A's collateral and its net credit.[9] The right to set off loss allocations against net credits – if incorporated into the rules of the clearing house in a legally effective way – would enhance the robustness of the loss allocation mechanism.

9. This is complicated somewhat if the clearing and settlement system is the payments system that includes the central bank, which implements monetary policy through interventions in the system. For example, monetary policy could be implemented through the transfer of government balances between the government's account at the central bank and its accounts with participants in the payments system, as is the case in Canada. These transfers can alter the size of each participant's net (credit or debit) position in the system, so that these positions could be less than or greater than the amount implied by the clearing and settlement process among the private-sector participants alone. (The precise effect would depend on the particular monetary policy operating system in force.)

6. Discussion

Under the basic approach used here to calculate collateral requirements, certainty of settlement and loss allocation are assured for a single default. However, two defaults that do not exceed the collateral pool can result in a loss allocation to a surviving participant that exceeds its own collateral contribution.[10] As a result, although the clearing house would have sufficient collateral to satisfy its liquidity needs, compliance with the loss allocation rule is not necessarily assured by the collateral pledged by each participant. However, the significance of this result needs to be qualified by a few considerations.

First, whether in practice this result can occur depends largely on the pattern of bilateral lines that the participants extend to one another, and the distributions of bilateral lines that can lead to loss allocations exceeding collateral pledges seem to be unusual ones. Second, while multiple "technical" defaults (on the same day or over a short period of time) are a reasonable consideration, to some observers two insolvencies are not a reasonable possibility. In other words, while it may seem reasonable to safeguard against multiple technical defaults, that is, against those related to an institution's liquidity or communications/information processing problems, some may feel that two bankruptcies on the same day are not a reasonable contingency to safeguard against. Third, survivors' net credits (if any) can be used along with collateral to absorb loss allocations, thereby helping to ensure the robustness of loss allocation.

In sum, in spite of the preceding analysis, the basic approach to collateral requirements and net debit caps seems to be a reasonable way to secure liquidity needs and loss allocation in a survivors-pay clearing and settlement system.

7. The basic approach to collateral requirements can lead to excess collateral

As shown above (Section 2), if system participants collateralize a predetermined fraction of the largest bilateral line that they extend, the collateral pool would be *more than* sufficient to provide liquidity to cover any possible single default, including the default of the participant with the largest net debit cap. Again,

10. Two defaults summing to just the largest single net debit cap can also lead to loss allocations in excess of collateral requirements for some of the surviving members. See Appendix II.

$$C^T = \theta \sum_{x=1}^{n} E_x^{\lambda} > \delta_j = \theta \sum_{x=1}^{y_j} E_x^j, \tag{6}$$

for any j, since $E_x^{\lambda} \geq E_x^j$, and $n > y_j$.

The amount of excess collateral is, by definition,

$$\varepsilon = C^T - \delta_n,$$

where δ_n is the largest net debit cap,[11] and δ_n is defined as

$$\delta_n = \theta \sum_{x=1}^{y_n} E_x^n. \tag{7}$$

As a result, the amount of excess collateral can be expressed as a function of the largest bilateral line that each participant provides, which determines the collateral pool, and of the bilateral lines extended to the participant with the largest net debit cap:

$$\varepsilon = \theta \sum_{x=1}^{n} E_x^{\lambda} - \theta \sum_{x=1}^{y_n} E_x^n. \tag{8}$$

We can provide an alternative expression for the amount of excess collateral by redefining the largest net debit cap, δ_n, as

$$\delta_n = \theta \sum_{x=1}^{n-1} E_x^n.$$

Comparing this expression to equation (7), we have allowed the summation across E_x^n to go up to $(n-1)$, where it is understood that some participants may not have extended a bilateral line to participant n; that is, for some x's, $E_x^n = 0$.

Again, we define the amount of excess collateral as

$$\varepsilon = C^T - \delta_n,$$

so that we have

11. Again, the objective here is to provide for a collateral pool sufficient to cover only the largest net debit cap.

$$\varepsilon = \theta \sum_{x=1}^{n} E_x^{\lambda} - \theta \sum_{x=1}^{n-1} E_x^{n}$$

and therefore

$$\varepsilon = \theta E_n^{\lambda} + \theta \sum_{x=1}^{n-1} (E_x^{\lambda} - E_x^{n}) . \tag{9}$$

Equation (9) shows that excess collateral is a function of two broad components: the largest bilateral line extended by participant n, multiplied by θ, and an amount that depends on whether any participant x has extended its largest line to a participant other than the participant with the largest net debit cap, participant n. The amount of excess collateral increases with θ, with the largest line extended by participant n, with the number of participants that extend their largest line to a participant other than participant n, and with the difference between these largest lines and the lines extended to participant n.

8. Eliminating excess collateral

(a) Lowering collateral to the minimum acceptable level

To lower total collateral to the amount of the largest net debit cap, each member's collateral requirement could be reduced simply by, in effect, giving a proportional rebate on its calculated collateral requirement.[12] More specifically, the collateral requirement of each participant x could be credited with a proportion (α_x) of the excess collateral. In this case, each member's collateral contribution would be

$$C_x = \theta E_x^{\lambda} - \alpha_x \varepsilon .$$

Total collateral would be

$$C^T = \theta \sum_{x=1}^{n} E_x^{\lambda} - \sum_{x=1}^{n} \alpha_x \varepsilon ,$$

where $\sum_{x=1}^{n} \alpha_x = 1$.

12. In this paper, the largest net debit cap is seen as the minimum acceptable level of the collateral pool. Of course, depending on the preferences of the participants or the authorities, the collateral pool could be set at a higher level.

As a result, $C^T = \theta \sum_{x=1}^{n} E_x^{\lambda} - \varepsilon$.

Substituting from equation (8) for ε, we have

$$C^T = \theta \sum_{x=1}^{n} E_x^{\lambda} - \left(\theta \sum_{x=1}^{n} E_x^{\lambda} - \theta \sum_{x=1}^{y_n} E_x^{n} \right).$$

As a result, $C^T = \theta \sum_{x=1}^{y_n} E_x^{n} = \delta_n$ – and over-collateralization is eliminated.

One unavoidable drawback is that participants must establish their bilateral lines before knowing precisely their collateral requirements, C_x. The collateral reductions are a function of the amount of excess collateral (ε), which is known only after all participants determine all of their bilateral lines. However, an upper limit would be provided by θE_x^{λ}, and as argued in the next section, ε might not be that important.

(b) Collateral haircuts

To reflect the volatility of their market value (for example, credit and interest rate risk), all contributions to the collateral pool would be discounted – that is, given a "haircut." As a result, ε might overstate the extent of excess collateral. For instance, suppose that the haircut was equal to ε/C^T. Given that $\varepsilon = (C^T - \delta_n)$, the excess would be

$$C^T - (\varepsilon/C^T)\,C^T - \delta_n = 0.$$

In sum, depending on ε and on the haircut for collateral, "over-collateralization" might not be that important.

(c) The weighting scheme to allocate the collateral reductions: α_x

The weighting scheme used to allocate the collateral reductions is an important consideration; different α's will provide different collateral requirements and different incentives to the participants. One possibility would be to allocate the rebates in proportion to the largest bilateral line that each member has extended. That is,

$$\alpha_x = E_x^{\lambda} / \left(\sum_{j=1}^{n} E_j^{\lambda} \right).$$

9. Minimizing excess collateral and ensuring the robustness of loss allocation

Providing collateral rebates as outlined above will result in a collateral pool that would enable the clearing house to arrange sufficient liquidity so that daily settlements can occur in the event of the default of any participant (Lamfalussy Standard No. 4). However, as argued above – see equation (4) – the less collateral that participants post, the less likely it is that the loss allocation rule would be secured. As a result, if ensuring the robustness of loss allocation (say, for a single default) is a necessary condition, then the collateral calculation would need to ensure that any participant's collateral requirement was not less than its largest possible loss allocation from any single default.

Minimizing excess collateral while securing liquidity needs and ensuring the robustness of loss allocation requires solving a constrained minimization problem like the following one.

Choose C_x to minimize $\varepsilon = C^T - \delta_n$ for a given distribution of bilateral lines (that is, the E_x^i's) subject to

$$(1)\ C^T \geq \delta_i = \theta \sum_{x=1}^{y_i} E_x^i$$

$$(2)\ C_x \geq L_x^i = \left(\frac{E_x^i}{\sum_{j=1}^{y_i} E_j^i} \right) (\delta_i - C_i) \text{ and}$$

$$(3)\ C_x \leq \theta E_x^{\lambda}$$

where L_x^i is the loss allocation to participant x from the default of participant i.

The first constraint ensures that the collateral pool will be sufficient to cover any single net debit cap in the system and thus satisfy Lamfalussy Standard No. 4. The second constraint ensures that each participant's collateral requirement will cover any loss allocation from any single default. Finally, the third constraint, which might be seen as optional, ensures that each participant is no worse off than in the case where collateral is simply a predetermined fraction of the largest bilateral line that it provides. That is, the collateral that each participant x pledges, C_x, should be no larger than θE_x^{λ} – the amount of collateral that would be required under the basic approach.

10. Providing for both survivors-pay and defaulter-pays net debit caps with the same collateral

The preceding sections are concerned with the properties of survivors-pay clearing and settlement systems. In this section, we consider a set-up in which a given stock of collateral both secures survivors-pay net debit caps (as above) and provides for defaulter-pays net debit caps.

(a) A system with both defaulter-pays and survivors-pay aspects

Any clearing and settlement system can include both defaulter-pays and survivors-pay aspects.[13] A participant can obtain defaulter-pays net debit space equal to the amount of collateral that it pledges to the clearing house to secure its own net debit. A participant's survivors-pay net debit space would be provided for by the extension of bilateral credit lines from other participants (as outlined above). If the collateral that a participant pledges against the bilateral line that it extends in the survivors-pay portion could also *simultaneously* provide for a defaulter-pays cap, then the capacity of the system would increase compared to the basic approach considered above.[14] Alternatively, this would lower collateral needs for a given capacity. For simplicity, we can call this approach, where the same collateral both secures survivors-pay net debit caps and provides for defaulter-pays net debit caps, the "double-duty" approach to collateral and net debit caps.

In this case, the net debit cap of a participant i is the sum of its defaulter-pays cap (δ_i^d) and its survivors-pay cap (δ_i^s). That is,

$$\delta_i = \delta_i^d + \delta_i^s \tag{10}$$

where

$$\delta_i^s = \theta \sum_{x=1}^{y_i} E_x^i \tag{11}$$

and

$$\delta_i^d = \theta E_i^\lambda + \Omega_i. \tag{12}$$

13. For a discussion of defaulter-pays and survivors-pay clearing houses, see Walter Engert, "An Introduction to Multilateral Foreign Exchange Netting," Working Paper 92-5 (Ottawa: Bank of Canada, September 1992).

14. Greater net debit space would reduce the probability of "gridlock," where the inability of some transfer or payment instructions to be executed because of binding net debit caps prevents other instructions from being executed, with the cumulative effect that a substantial volume of transfers cannot be completed as scheduled.

Equation (12) says that participant i's defaulter-pays cap is equal to a minimum of the collateral pledged by participant i in respect of the survivors-pay portion, θE_i^{λ}, plus an amount equal to whatever additional collateral that participant i pledges in the defaulter's-pay portion, Ω_i.[15]

Therefore, substituting equations (11) and (12) into equation (10), the total net debit cap is

$$\delta_i = \Omega_i + \theta E_i^{\lambda} + \theta \sum_{x=1}^{y_i} E_x^i. \tag{13}$$

The collateral pledged by participant i is equal to δ_i^d, that is,

$$C_i = \theta E_i^{\lambda} + \Omega_i. \tag{14}$$

Thus, in the event of the default of any participant i, the collateral available to the clearing house is

$$C^T = \Omega_i + \theta E_i^{\lambda} + \theta \sum_{x=1}^{n-1} E_x^{\lambda}, \tag{15}$$

where $x \neq i$, so that we have

$$C^T = \Omega_i + \theta \sum_{x=1}^{n} E_x^{\lambda}. \tag{16}$$

Comparing equation (13) with equation (15) or (16), we see that $\delta_i \leq C^T$ for every participant i. As a result, in the event of the default of any single participant, the clearing house will have sufficient collateral to ensure settlement.

However, compared with the basic approach to collateral requirements and net debit caps, the ability of the clearing house to handle multiple defaults is more limited under this double-duty approach. We can see this by noting first that the stock of collateral is the same under the two approaches (ignoring Ω_i). That is, under both approaches, the collateral pool is

$$C^T = \theta \sum_{x=1}^{n} E_x^{\lambda}.$$

15. As the system started up, Ω_i might be zero, and it might remain at zero. However, if other participants reduced their bilateral lines to participant i, thereby decreasing i's survivors-pay net debit cap, participant i could pledge collateral (Ω_i) to increase its defaulter-pays net debit cap to compensate for the decline in its survivors-pay net debit cap.

However, the net debit cap of any participant *i* is greater under the double-duty approach than under the basic approach. That is,

$$\theta E_i^{\lambda} + \theta \sum_{x=1}^{y_i} E_x^i > \theta \sum_{x=1}^{y_i} E_x^i .$$

As a result, the same amount of collateral secures larger exposures under the double-duty approach, so that multiple defaults are more likely to exhaust and exceed the collateral pool under the double-duty approach than under the basic approach.

(b) An illustration

Table 5 presents an illustration of the net debit caps that would result under the double-duty approach to collateral requirements and net debit caps for a given collateral pool. For simplicity, Ω is set to zero. (Table 5 is based on the example from Section 5, where $\theta = 0.30$; see Table 2.) The total collateral pool is 21.0, and as shown in the table, the collateral that each participant pledges in the survivors-pay portion is also used to provide for a defaulter-pays net debit cap.

Note that each participant's total net debit cap has been increased considerably. For example, under the basic approach to collateral requirements and net debit caps, participant B has a net debit cap of 10.50 (column 1); under the double-duty approach, it has a net debit cap of 15.0. Similarly, the total capacity of the system has increased. Again, under the basic approach, the sum of the net debit caps is 51.90; under the double-duty approach, the sum of the net debit caps is 72.90.

Table 5: Survivors-pay and defaulter-pays net debit caps secured by the same collateral

Participant	Survivors-pay cap: δ_i^s	Defaulter-pays cap: $\delta_i^d = \theta E_i^{\lambda}$	Total net debit cap: $\delta_i^s + \delta_i^d$
A	3.90	3.00	6.90
B	10.50	4.50	15.00
C	9.60	3.60	13.20
D	11.40	5.40	16.80
E	16.50	4.50	21.00
Total	51.90	21.00	72.90

In addition, each participant's total net debit cap is less than or equal to the collateral pool of 21.0, so that in the event of any single default, the clearing house would be able to arrange sufficient liquidity to ensure daily settlements. However, as discussed above, the clearing house's ability to handle multiple default scenarios is more limited than under the basic approach to collateral and net debit caps. For example, under the basic approach, the system could handle the liquidity needs associated with the simultaneous defaults of participants C and D at their net debit caps; that is, 9.60 + 11.40 = 21.0, which is the amount of the collateral pool. Under the double-duty approach to collateral and net debit caps, the defaults of these participants would significantly exceed the amount available in the collateral pool; that is, 13.20 + 16.80 = 30.0 > 21.0.

(c) Loss allocation is guaranteed for one default under this approach

The largest possible loss allocation to participant j from the default of any other participant i is

$$L_j^i = \left(\frac{E_j^i}{\sum_{x=1}^{y_i} E_x^i}\right)(\delta_i - C_i)\,. \tag{17}$$

Substituting for δ_i and C_i from equations (13) and (14), we can rewrite equation (17) as

$$L_j^i = \left(\frac{E_j^i}{\sum_{x=1}^{y_i} E_x^i}\right)\left(\theta E_i^{\lambda} + \Omega_i + \theta \sum_{x=1}^{y_i} E_x^i - (\theta E_i^{\lambda} + \Omega_i)\right). \tag{18}$$

Thus, the largest possible loss allocation to surviving participant j is $L_j^i = \theta E_j^i$. However, this loss allocation does not exceed the collateral posted by participant j:

$$L_j^i = \theta E_j^i \le C_j = \theta E_j^{\lambda} + \Omega_j.$$

Even if $\Omega_j = 0$, or if it were not available to secure the loss allocation procedure, this loss allocation would not exceed participant j's available collateral, that is, $L_j^i = \theta E_j^i \le C_j = \theta E_j^{\lambda}$.

In sum, compliance with loss allocation is guaranteed in the event of a single default.

(d) Sequencing the usage of defaulter-pays and survivors-pay portions to maximize system capacity

The clearing and settlement system considered in this section has both defaulter-pays and survivors-pay features. If participants have both defaulter-pays and survivors-pay net debit caps, the sequence in which these portions are used by the participants can affect the capacity of the system. For instance, consider a system in which a participant's net debits first consume its defaulter-pays space before starting to use its survivors-pay space. Suppose a participant had transactions with others from which it also had bilateral credit lines (and therefore related survivors-pay net debit space) which led to net debits. Suppose also that these transactions fully consume its defaulter-pays net debit space. If the same participant then wanted to do the same amount of net transactions with other participants from which it did *not* have bilateral credit lines, it could not accommodate these transactions, since all of its defaulter-pays space has already been consumed.

Now consider a system in which the sequence of net debit space usage is the reverse, that is, a participant's net debits first consume its survivors-pay space before starting to use its defaulter-pays space. In our example, the participant's transactions with others from which it also had bilateral credit lines would now consume its survivors-pay space and it then would be able to accommodate the subsequent transactions in its defaulter-pays space. This sequencing, that is, survivors-pay before defaulter-pays, would allow all of the desired transactions to be accommodated, thereby increasing the capacity of the system.

In sum, maximizing the capacity of a clearing and settlement system that has both defaulter-pays and survivors-pay aspects (such as the double-duty approach developed above), requires ensuring that the participants that provide bilateral credit lines to another participant do not use its defaulter-pays space. So, as each transaction comes in, survivors-pay space would be used if possible; if there is not enough survivors-pay space, then the transaction (or part of it) would use the participant's defaulter-pays space.[16]

11. Conclusion

This paper has shown how collateralizing the largest bilateral credit line multiplied by θ and setting net debit caps as the sum of all bilateral lines received multiplied by θ – what we have called the basic approach to collateral and net debit caps – will ensure certainty of settlement in the event of any single default and in the event of multiple defaults up to the amount of the collateral pool.

16. In spite of this sequencing, a defaulter's collateral would be seized to reduce loss allocations in the event of a default.

The basic approach will also ensure the robustness of loss allocation for one insolvency, but not necessarily for two insolvencies. However, the significance of the latter result was qualified by a few considerations, so that in sum, the basic approach to collateral requirements and net debit caps seems to be a reasonable way to secure liquidity needs and loss allocation in a survivors-pay clearing and settlement system.

However, the basic approach can lead to a collateral pool that is greater than any single net debit cap. Lamfalussy Standard No. 4 essentially calls for ensuring certainty of settlement in the event of the default of the participant with the largest net debit cap. As a result, the basic approach can be seen to lead to excess collateral in the system. Thus, each participant's collateral requirement could be credited with a proportion of the excess collateral to reduce the collateral pool to the amount of the largest net debit cap in the system and thereby still secure the liquidity needs of the system. The paper also described how to minimize excess collateral, while simultaneously securing liquidity needs *and* ensuring the robustness of loss allocation (for any one default).

Most of the analysis here has been concerned with the properties of survivors-pay clearing and settlement systems. However, the paper also considered a set-up in which a given stock of collateral both secures survivors-pay net debit caps (as above) and simultaneously provides for defaulter-pays net debit caps. This approach would increase the capacity of the system for a given pool of collateral, or put differently, this would lower collateral needs for a given capacity of the system. However, the ability of the system under this approach to handle multiple defaults is more limited compared with the basic approach. Finally, in a clearing and settlement system with both defaulter-pays and survivors-pay segments, maximizing the capacity of the system requires that a participant use its survivors-pay segment before using its defaulter-pays segment.

Appendix I
Loss allocations from the defaults of participants B and C

In these examples, losses are allocated in proportion to the survivors' bilateral credit lines to the defaulter. In the cases of multiple defaults considered here, a loss allocation to a participant that has also defaulted is shifted to participants with bilateral lines to the original defaulter.

Case 1: θ = 0.20

The loss to be allocated from the default of participant B is 7.0 – 3.0 = 4.0. The loss allocation to C is $L_C^B = (9/35)4 = 1.03$. However, since C has defaulted as well, its loss allocation is shifted to the surviving participants in proportion to their bilateral lines to participant B. Thus, the loss allocations to the surviving participants, A, D and E are

$$L_A^B = (10/35)4 + (10/26)1.03 = 1.54$$

$$L_D^B = (7/35)4 + (7/26)1.03 = 1.08$$

$$L_E^B = (9/35)4 + (9/26)1.03 = 1.38.$$

The loss to be allocated from the default of participant C is 6.4 – 2.4 = 4.0. The loss allocation to B is $L_B^C = (8/32)4 = 1.0$. However, since B has defaulted as well, its loss allocation is shifted to the surviving participants in proportion to their bilateral lines to participant C. Thus, the loss allocations to the surviving participants, A, D and E are

$$L_A^C = (10/32)4 + (10/24)1.0 = 1.67$$

$$L_D^C = (6/32)4 + (6/24)1.0 = 1.00$$

$$L_E^C = (8/32)4 + (8/24)1.0 = 1.33.$$

Therefore, total loss allocations are

$$L_A^{B+C} = 1.54 + 1.67 = 3.21$$

$$L_D^{B+C} = 1.08 + 1.0 = 2.08$$

$$L_E^{B+C} = 1.38 + 1.33 = 2.71.$$

Case 2: θ = 0.30

The loss to be allocated from the default of participant B is 10.5 – 4.5 = 6.0. The loss allocation to C, which must be shifted to the surviving participants, is $L_C{}^B = (9/35)6 = 1.54$. Thus, the loss allocations to the surviving participants, A, D and E are

$$L_A{}^B = (10/35)6 + (10/26)1.54 = 2.31$$

$$L_D{}^B = (7/35)6 + (7/26)1.54 = 1.61$$

$$L_E{}^B = (9/35)6 + (9/26)1.54 = 2.08.$$

The loss to be allocated from the default of participant C is 9.6 – 3.6 = 6.0. The loss allocation to B, which must be shifted to the surviving participants, is $L_B{}^C = (8/32)6 = 1.5$. Thus, the loss allocations to the surviving participants, A, D and E are

$$L_A{}^C = (10/32)6 + (10/24)1.5 = 2.50$$

$$L_D{}^C = (6/32)6 + (6/24)1.5 = 1.50$$

$$L_E{}^C = (8/32)6 + (8/24)1.5 = 2.0.$$

Therefore, total loss allocations are

$$L_A{}^{B+C} = 2.31 + 2.50 = 4.81$$

$$L_D{}^{B+C} = 1.61 + 1.50 = 3.11$$

$$L_E{}^{B+C} = 2.08 + 2.0 = 4.08.$$

A comment on the approach to the allocation of defaulted loss allocations

When there are multiple defaults, say defaults of both B and C as above, someone has to pick up C's share of B's default, and someone has to pick up B's share of C's default. In these calculations, the loss allocation to participant C resulting from the default of B is shifted to participants A, D and E in proportion to their bilateral lines to B. (Similarly, the loss allocation to participant B resulting from the default of C is shifted to the surviving participants in proportion to their bilateral lines to C.)

An alternative approach would be to shift the loss allocation to participant C resulting from the default of B to participants A, D and E in proportion to their bilateral lines to C (instead of in proportion to their lines to B). Yet another approach would be to allocate the defaulted loss allocations in equal proportions to the survivors.

However, in this example, a different approach to the allocation of defaulted loss allocations would not change the basic results. For instance, with $\theta = 0.20$, the "first-round" loss allocations to A alone exceed its collateral: that is, $(10/35)4 + (10/32)4 = 2.39 > 2.0$ – participant's A's collateral contribution.

Appendix II
Loss allocations for two defaults summing to the largest net debit cap

The illustrations in the text considered two defaults that sum to less than the collateral pool so that the clearing house could meet its liquidity objectives, although loss allocation broke down. However, in the cases considered, the two defaults exceeded the largest single net debit cap. Here we consider two defaults summing to only the largest net debit cap, and the following presents an illustration of loss allocation breaking down in this case as well. Consider a 5-participant survivors-pay clearing house, in which the participants extend bilateral lines to one another as shown in Table II.1.

Table II.1: Bilateral lines in a 5-participant clearing house

	To A	To B	To C	To D	To E
From A	not applicable	10	10	5	10
From B	2	not applicable	8	8	15
From C	3	9	not applicable	10	12
From D	5	4	5	not applicable	18
From E	3	7	2	15	not applicable
Sum of lines	13	30	25	38	55

Table II.2 presents the net debit caps and collateral requirements

Table II.2: Net debit caps and collateral for two values of θ

Participant	θ = 0.20		θ = 0.30	
	Debit cap	Collateral	Debit cap	Collateral
A	2.60	2.00	3.90	3.00
B	6.00	3.00	9.00	4.50
C	5.00	2.40	7.50	3.60
D	7.60	3.60	11.40	5.40
E	11.00	3.00	16.50	4.50
Collateral pool		14.00		21.00

(a) Defaults of participants B and C

Case 1: $\theta = 0.20$

Suppose that participants B and C both default, and that at the time of default they are at their net debit caps, so that the total net debit in default is 6.0 + 5.0 = 11.0, which is also the single largest cap – of participant E – and much less than the collateral pool, 14.0. As a result, the clearing house can easily meet its liquidity needs to ensure settlement on the day of default.

How do the loss allocations to the surviving participants, A, D and E, compare with the collateral that they have posted to the clearing house? The loss to be allocated from the defaults of participants B and C equals the sum of their net debits less the collateral that they have posted; that is, (6.0 – 3.0) + (5.0 – 2.4) = 5.6. As shown in Table II.3, the loss allocation to participant A is almost 50 per cent more than the amount of the collateral that it has posted to the clearing house.[1]

Table II.3: Loss allocations and collateral for θ=0.20

Participant	Loss allocation	Collateral	Is collateral sufficient?
A	2.96	2.00	no – a 48% shortfall
D	1.33	3.60	yes
E	1.31	3.00	yes
Total	5.60	8.60	not applicable

Case 2: $\theta = 0.30$

In this case, the sum of the defaults of participants B and C is 16.5, which again equals the single largest cap and is much less than the collateral pool, 21.0. As a result, the clearing house can easily meet its liquidity needs to ensure settlement on the day of default.

Again, the loss to be allocated from the defaults of participants B and C equals the sum of their net debits less the collateral that they have posted: (9.0 – 4.5) + (7.5 – 3.6) = 8.4.

1. Losses are allocated as in Appendix I. Allocating losses in the alternative ways described in Appendix I would not change the basic results; again, the "first-round" loss allocations to A alone exceed its collateral.

And again, as shown in Table II.4, the loss allocation to participant *A* significantly exceeds its collateral.

Table II.4: Loss allocations and collateral for θ=0.30

Participant	Loss allocation	Collateral	Is collateral sufficient?
A	4.44	3.00	no – a 48% shortfall
D	2.00	5.40	yes
E	1.96	4.50	yes
Total	8.40	12.90	not applicable

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