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The Dynamic Demand for Money in Germany, Japan and the United Kingdom

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The Dynamic Demand for Money in Germany, Japan and the United Kingdom

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CONTENTS

Abstract

This paper examines the ability of the linear-quadratic model under rational expectations to explain the dynamic behaviour of broad money aggregates in Germany, Japan and the United Kingdom. In contrast with other previous studies, we estimate the structural parameters by means of the Euler equation, using a limited-information approach that does not require an explicit solution for the model's control variables in terms of the exogenous forcing variables. The empirical results from Japanese and U.K. data provide support for the model, whereas the results from German data do not.

Résumé

Les auteurs de la présente étude cherchent à établir si le modèle quadratique linéaire peut, sous l'hypothèse de rationalité des attentes, expliquer le comportement dynamique des agrégats monétaires au sens large en Allemagne, au Japon et au Royaume-Uni. Contrairement aux auteurs d'études antérieures, ils estiment les paramètres structurels **à** l'aide de l'équation d'Euler, en utilisant une méthode du **maximum** de **vraisemblance à information limitée qui n'exige pas** que **les variables** de contrôle du modèle soient explicitement résolues en fonction des variables d'impulsion exogènes. Les résultats empiriques obtenus **à** l'aide des données sur le Japon et le Royaume-Uni confirment la validité du modèle, tandis que les résultats obtenus dans le cas de l'Allemagne ne **le** font pas.

¹ INTRODUCTION

The purpose of this paper is to examine the ability of the simple linear-quadratic (LQ) model to explain the dynamic behaviour of broad real monetary aggregates in Germany, Japan and the United Kingdom when the forcing variables are nonstationary processes. The linear-quadratic model is a popular framework for investigating the dynamic behaviour of economic agents. The model has been used to explain, *inter alia,* the demand for labour (Sargent 1978, and Hansen and Sargent 1980), the demand for labour and capital (Meese 1980), the demand and supply of labour (Kennan 1988), natural resource extraction (Hansen, Epple and Roberds 1985), the supply of money (Mercenier and Sekkat 1988) and the demand for transaction balances (Cuthbertson and Taylor 1987, and Otto and Wirjanto 1992). One of the advantages of the LQ model is that it gives rise to linear decision rules in the variables. This is an attractive feature, since the variables used in estimating money demand equations tend to be characterized by nonstationary processes and the LQ model has well-understood properties for these nonstationary variables. Despite its simplicity, the LQ model encompasses a class of models typically used to estimate money demand. Examples include the standard partial-adjustment model, the permanent-income model, and the error-correction model.

There are basically two estimation strategies that one can adopt for the LQ model. In our paper we estimate the parameters of the LQ model using a limited-information procedure that is based on the model's Euler equation. An alternative procedure would be to use a full-information approach that requires an explicit solution for the model's control variables in terms of the forcing variables. In a full-information maximum-likelihood (FIML) estimation procedure, the process assumed to generate the forcing variables must be specified and estimated jointly with the law of motion and with certain cross-equation restrictions. Provided that the model is correctly specified, the FIML estimator will be

more efficient than one based on the Euler equation approach.¹ In contrast, the limitedinformation approach adopted in this paper provides us with consistent parameter estimates under more general conditions.

Most early empirical studies that used the LQ model assumed that the variables in the model are stationary in levels or contain deterministic trends. In this paper, we assume that the variables are nonstationary due to the presence of stochastic trends or unit roots. The unit root tests we present later suggest that it is important to examine the LO model under this assumption.

In this paper we study the LQ model, focussing on a broader measure of money across three different countries. Specifically, we examine the dynamic behaviour of M3 in Germany, M2 plus certificates of deposit in Japan and M4 in the United Kingdom. These broader aggregates allow us to shift the focus from a transaction-based interpretation towards a portfolio-based interpretation, since these broader aggregates compete with other financial instruments as stores of wealth. Perhaps more importantly, these broader monetary aggregates may have internalized many of the structural instabilities often associated with the narrower definitions of money. We investigate this hypothesis in Section 4.1.

The organization of the paper is as follows. Section 2 describes the linear-quadratic model and derives some of its implications. Our estimation strategy is outlined in Section 3 while the empirical results are given in Section 4. Section 5 concludes the paper.

^{1.} In a Monte Carlo study (based on stationary forcing variables) West (1986) finds that even under the assumption of no misspecification, full-information estimation is only moderately more efficient than limited-information estimation.

2 THE LINEAR-QUADRATIC MODEL

Assume that a representative agent's money holdings are determined by an intertemporal loss function with quadratic costs of adjustment. More specifically, the money holder is assumed to control real balances (m_r) and solves the problem of minimizing the expected present value of the adjustment and disequilibrium costs, that is,

$$
\min_{\{m_i\}} \mathbf{E}_i \sum_{i=1}^{\infty} \beta^{i-1} [\gamma (m_i - m_i^*)^2 + (m_i - m_{i-1})^2]
$$
\n(1)

for $i \geq t$, where E_t is the expectations operator conditional on the agent's information at time t (I_t), $\beta \in (0, 1)$ is the subjective discount rate and the parameter $\gamma > 0$ is a weighting factor that determines the relative size of the costs of adjustment. Note that γ is the inverse of the usual cost of adjustment.

The static equilibrium relationship describing the law of motion for the target variable is assumed to be given by

$$
m_t^* = \alpha_0 + \alpha_1 y_t + \alpha_2 r_t + e_t
$$

= $x_t^T \alpha + e_t$ (2)

where e_t is a white noise process known to the agents, that is, $e_t \in I_t$, but which is unknown to the econometrician whose information set is $H_t \subset I_t$; x_t is a *(kxl)* row vector of forcing variables, 1, y_r , r_t , where y_t is real income and r_t is a nominal interest rate.

The first-order necessary condition for the minimization of (1) is given by the following Euler equation:

$$
\Delta m_t = \beta E_t \Delta m_{t+1} - \gamma (m_t - m_t^*)
$$
 (3)

and the corresponding transversality condition is

$$
\lim_{T \to \infty} E_t \left[\beta^T \{ \gamma (m_T - m_T^*) + \Delta m_T \} \right] = 0 \tag{4}
$$

The forward solution to (3) is given by

$$
m_t = \lambda m_{t-1} + (1 - \lambda) (1 - \beta \lambda) E_t \sum_{i=t}^{\infty} (\beta \lambda)^{i-t} m_i^*
$$
 (5)

where $\lambda < \beta^{-1/2}$ is the smallest stable root of the Euler equation obtained from the firstorder condition and which satisfies the condition

$$
\beta \lambda^2 - (1 + \beta + \gamma) \lambda + 1 = 0 \tag{6}
$$

The Wiener-Kolmogorov prediction formula can be used to replace the expectation in (5), given the law of motion for x_t , as in Sargent (1987). In this paper we focus on the case where the law of motion for x_t is given by the vector autoregressive process of order one

$$
x_t = \rho x_{t-1} + \varepsilon_t \tag{7}
$$

where $|\rho| \leq 1$ and ε , is stationary and identically distributed. Given a stochastic process for x_t , equation (5) can be solved. For instance if $|p| < 1$ then (5) becomes

$$
\Delta m_t = (\lambda - 1) (m_{t-1} - x_{t-1}^T \alpha) + (1 - \lambda) \alpha \left[\frac{1 - \beta \lambda}{1 - \beta \rho \lambda} x_t - x_{t-1} \right]
$$

$$
+ (1 - \beta \lambda) (1 - \lambda) e_t
$$
(8)

and if $\rho = 1$, so that the forcing variables are integrated processes of order one, denoted 1(1), then (5) simplifies to an error-correction model:

$$
\Delta m_t = (\lambda - 1) (m_{t-1} - x_{t-1}^T \alpha) + (1 - \lambda) \Delta x_t^T \alpha + (1 - \beta \lambda) (1 - \lambda) e_t
$$
 (9)

or, rewritten in the form of the partial adjustment model:

$$
m_t = \lambda m_{t-1} + (1 - \lambda) x_t^T \alpha + (1 - \beta \lambda) (1 - \lambda) e_t
$$
 (10)

which can be reparameterized into Bewley's (1979) form as^2

$$
m_t = (-\lambda / (1 - \lambda)) \Delta m_t + x_t^T \alpha + (1 - \beta \lambda) e_t
$$
 (11)

3 ESTIMATION STRATEGY

3.1 The Euler equation approach

In this subsection we describe a methodology that allows us to consistently estimate the Euler equation (3). To obtain a form for equation (3) that can be estimated, we replace $E_t \Delta m_{t+1}$ by its realization $(\Delta m_{t+1} + \eta_{t+1})$, where η_{t+1} is a purely expectational error, such that $E_t \eta_{t+1} = 0$ and rewrite equation (3) as

$$
\Delta m_t = \beta \Delta m_{t+1} - \gamma (m_t - x_t^T \alpha) + v_t
$$
\n(12)

where $v_t = \beta \eta_{t+1} + \gamma e_t$, such that $E_t v_t = 0$. Thus, v_t is a composite error term that can be rewritten as an MA(1) process, provided the structural error term e_t is a white noise process.

Since the LQ model implies that m_t and the forcing variables x_t are cointegrated in the sense of Engle and Granger (1987), a two-step procedure for estimating the parameters in (12) has been suggested by Dolado, Galbraith and Banerjee (1991). In the first step, consistent estimates of the long-run parameter (α) may be obtained from a

^{2.} In general, when there is persistence in ε , the Bewley transformation takes the following form: In general, when there is persistence in ε_i , the Bewley transformation takes the following form:
 $m_i = (-\lambda/(1-\lambda))\Delta m_i + x_i^T\alpha + h_1\varepsilon + (1-\beta\lambda)e_i$, where the form of h_1 is determined by the **stochastic** process generating ϵ_i . For instance, if ϵ_i follows a stationary autoregressive (AR) process of order one-AR(1), $\varepsilon_t = a\varepsilon_{t-1} + w_t$, where $|a| < 1$ and w_t is a white-noise process, then $h_1 = \alpha a \beta \lambda / (1 - \alpha \beta \lambda) (1 - aL)$. If ε_t is assumed to be independently distributed then $h_1 = 0$ and **we obtain equation (11).**

cointegrating regression:

$$
m_t = x_t^T \alpha + u_t \tag{13}
$$

where, under the assumption that ε , is independently distributed, the cointegrating errors are given by

$$
u_{t} = [1/(1 - \lambda)] \Delta m_{t} + (1 - \beta \lambda) e_{t}
$$

= $(1 - \lambda L)^{-1} [(1 - \lambda) (1 - \beta \lambda) e_{t} + \lambda \alpha \varepsilon_{t}]$ (14)
= $(1 - \lambda L)^{-1} [\lambda \gamma e_{t} - \lambda \alpha \varepsilon_{t}]$

which are serially correlated: $u_t = \lambda u_{t-1} + \varphi_t$, where $\varphi_t = (\lambda \gamma e_t - \lambda \alpha \varepsilon_t)$ is a white noise process.³ Since any bias in the OLS estimates of equation (13) are of $O_p(T^{-1})$, it is possible to substitute these estimates into equation (12) and ignore any sampling uncertainty in the estimate of α when we estimate the remaining parameters in the Euler equation (12). However, it is important to note that the rate T-convergence result does not, by itself, ensure that the parameter estimate of α will have good finite-sample properties. The reason is that OLS estimates of α are not asymptotically efficient, in the sense that they have an asymptotic distribution that depends on nuisance parameters. This problem is due to serial correlation in the error term and the endogeneity of the regressor matrix x_t that is induced by Granger causation from innovations in m_t to innovations in x_t .

We also note that since the smallest stable root λ satisfies the condition in (6), that as the adjustment cost gets large (that is, γ becomes small), the stable root approaches unity and *u_t* is nearly integrated. It follows that tests for cointegration, such as those in Engle

^{3.} In general, when e(is serially correlated, the cointegrating regression errors are given by In general, when ε_i is serially correlated, the cointegrating regression errors are given by $u_i = (1 - \lambda L)^{-1} [\lambda h_2 e_i - \lambda h_3 \varepsilon_i]$, where h_2 and h_3 depend upon the stochastic process generating ϵ_i . For example, if ϵ_i is a stationary AR(1) process with the AR parameter given by a, then $h_2 = \gamma$ and $h_3 = -\alpha (1 - a\beta) / (1 - a\beta\lambda) (1 - \lambda L)$. If ε_t is a white noise process, then $h_2 = \gamma$ and h_3 = $-\alpha$ and we obtain equation (14).

and Granger (1987) and Phillips and Ouliaris (1990), will encounter difficulty in detecting a cointegrating relationship in the data, even when one is present. In other words, despite the fact that such tests are asymptotically appropriate with serially correlated residuals, finite sample evidence in Gregory (1991) for cointegration tests in LQ models suggests that these tests will lack power when u_t is nearly integrated.

In light of the previous arguments, we use the "dynamic" OLS (DOLS) regression recently proposed by Stock and Watson (1993) to control for the endogeneity of the regressors parametrically by including leads and lags of the first difference of the regressors, and to correct for serial correlation in the residuals by using a non-parametric procedure.⁴ The correction suggested by Stock and Watson also allows us to draw inferences about α from the t-statistics associated with the regression.

Depending on whether the inverse of the adjustment cost parameter (y) or the stable root (λ) is assumed to be known, we can obtain two different "modified" cointegrating regressions with less serial correlation in the error term. If the adjustment cost parameter were known, equation (12) would imply the following modified cointegrating regression:

$$
m_t - \tilde{\gamma}^{-1} (\Delta m_t - \beta \Delta m_{t+1}) = x_t^T \alpha + \tilde{\gamma}^{-1} v_t
$$
 (15)

Since equation (15) uses prior information about the error structure in (14) we would expect the DOLS estimates of α from (15) to have better finite sample properties than those from (13).

On the other hand, if the stable root λ were known then the Bewley (1979) transformed equation in (11) would yield the following modified cointegrating regression:

$$
m_t + (\lambda / (1 - \lambda)) \Delta m_t = x_t^T \alpha + \phi_t
$$
 (16)

^{4.} A Monte Carlo study by Inder (1993) finds cointegration estimates that include dynamics are much more reliable than simple OLS estimates.

where $\phi_t = (1 - \beta \lambda) e_t$, under the assumption that ε_t is independently distributed. Notice that in this case the Bewley transformation completely eliminates the serial correlation in the error term, since $(1 - \beta \lambda) e_t$ is a white noise process by assumption. Even in the general case of serially correlated ε , the degree of serial correlation in ϕ , is expected to be less than that of *ut* in (13), and we would therefore expect improved performance from standard tests for no cointegration based on (13).

In general the value of λ is unknown and needs to be estimated. Methods to estimate λ have been discussed in Dolado, Galbraith and Banerjee (1991) and Gregory, Pagan and Smith (1990). In particular, Gregory, Pagan and Smith have observed that the discount factor (β) is difficult to estimate, and in some cases it is not even identified. Accordingly, the value of β will be preset at 0.99, 0.975, 0.95 and 0.9 in subsequent estimation. We can then proceed to estimate (13) by DOLS (or simple OLS) to obtain T-consistent $\tilde{\alpha}$ and form $\tilde{u}_t = m_t - x_t^T \tilde{\alpha}$, and estimate γ using Hansen's (1982) generalized method of moments (GMM) from

$$
\Delta m_t - \beta \Delta m_{t+1} = -\gamma \tilde{u}_t + v_t \tag{17}
$$

using instruments $\{\Delta m_{t} = i, i = 1, \dots\}$. Finally, the stable root λ can be estimated $T^{-1/2}$ -consistently using the estimate $\tilde{\alpha}$ and the condition in (6).

Note that the above estimation procedure requires no knowledge of the forcing process. However, the results of the Monte Carlo experiment in Gregory, Pagan and Smith suggest that the stable root λ should be estimated from the error-correction model in (9) by solving for the unknown expectation. This estimation procedure requires prior knowledge of a unit root in the forcing process. We now turn to the discussion of this method.

3.2 The error-correction model approach

Having solved for the expectation, we can estimate the cointegrating parameter α and the stable root λ satisfying the condition in (6) from the error-correction model (ECM) in (9), provided that the adjustment proposed by Gregory, Pagan and Smith is made. Once an estimate of λ has been obtained, the inverse of the adjustment parameter (γ) can be recovered using the condition in (6).

This approach consists of linearizing equation (9) around initial estimates of the parameter α and λ ($\tilde{\alpha}$ and $\tilde{\lambda}$):

$$
m_t - x_t^T \tilde{\alpha} \tilde{\lambda} = (\tilde{u}_{t-1} - \Delta x_t^T \tilde{\alpha}) \lambda + [(1 - \tilde{\lambda}) x_t^T] \alpha + \omega_t
$$
 (18)

where $\omega_t = x_t^T(\alpha - \tilde{\alpha}) (\lambda - \tilde{\lambda}) + (1 - \beta \lambda) (1 - \lambda) e_t$. Since $(\tilde{u}_{t-1} - \Delta x_t^T \tilde{\alpha})$ is an I(0) process, while x_t are I(1) processes, the regressors in equation (18) are (asymptotically) orthogonal to each other. Therefore, we can use a sequential procedure to estimate (18). More specifically, we first estimate the equation

$$
(m_t - x_t^T \tilde{\alpha} \tilde{\lambda}) - (\tilde{u}_{t-1} - \Delta x_t^T \tilde{\alpha}) \hat{\lambda} = [(1 - \tilde{\lambda}) x_t^T] \alpha + \vartheta_t
$$
 (19)

to obtain an estimate of $\hat{\alpha}$; then we estimate the regression

$$
(m_t - x_t^T \tilde{\alpha} \tilde{\lambda}) + [(1 - \tilde{\lambda}) x_t^T] \hat{\alpha} = (\tilde{u}_{t-1} - \Delta x_t^T \tilde{\alpha}) \lambda + \zeta_t
$$
 (20)

which permits us to obtain an estimate of *X.*

The preliminary estimator of α (that is, $\tilde{\alpha}$) can be obtained by estimating equation (13) by OLS, whereas the preliminary estimator of λ (that is, λ) can be obtained from OLS estimation of the ECM (9) with α replaced by $\tilde{\alpha}$; that is,

$$
\Delta m_t = (\lambda - 1) \tilde{u}_{t-1} + (1 - \lambda) \Delta x_t^T \tilde{\alpha} + f_t
$$
 (21)

Notice that equation (18) can be rewritten as

$$
m_t + \left[\tilde{\lambda} / \left(1 - \tilde{\lambda}\right)\right] \Delta m_t = x_t^T \alpha + \omega_t / \left(1 - \tilde{\lambda}\right)
$$
 (22)

where

$$
m_t + \left[\tilde{\lambda} / (1 - \tilde{\lambda})\right] \Delta m_t = (m_t - \tilde{\lambda} m_{t-1}) / (1 - \tilde{\lambda})
$$

=
$$
\left[(m_t - x_t^T \tilde{\alpha} \tilde{\lambda}) - (\tilde{u}_{t-1} - \Delta x_t^T \tilde{\alpha}) \tilde{\lambda} \right] / (1 - \tilde{\lambda}) \quad (23)
$$

which would be identical to (11), except that λ has been replaced by its estimate λ . A comparison between (22) and (14) suggests that the estimator of α differs from α in that the error term in (14) has been purged of a known component. The factor extracted from the estimate of α in (15) occurs due to different assumptions made about the conditioning set. If the stochastic process generating the forcing variables (x) is exactly known, the cointegrating errors can be reduced to a function of the econometrician's error alone. Without this prior knowledge, we have a remaining component that depends on "expectational error" as well.

4 EMPIRICAL RESULTS 4.1 Pre-tests for integration and cointegration

We use quarterly data from 1972Q1 to 1990Q4, which we truncate as necessary to compensate for leads and lags in the test procedures. For Germany we use M3 deflated by the GNP price deflator as the monetary aggregate (GYRM), real GNP asthe measure ofreal income (GYY) and a 90-day money market interest rate to represent the short-term interest rate (GYR). For Japan we use the M2 and certificates of deposits aggregate deflated by the GNE deflator as the measure of money (JPRM), real GNE as real income (JPY), and a 90 day money market rate as the short-term interest rate (JPR). For the United Kingdom, the broad measure of money we use is M4 divided by the GDP deflator (UKRM), real GDP is used to proxy real income (UKY), while a 90-day money market rate is used to measure opportunity cost (UKR) .⁵ All variables, except the interest rate measures, are expressed in natural logs.

Prior to estimation of the Euler equations, the properties of each series are examined using the parametric augmented Dickey-Fuller (ADF) test as suggested by Dickey and Fuller (1979) and Said and Dickey (1984), and a non-parametric test proposed by Phillips and Perron (1988). The ADF test is based on a t-test for H_0 ($a_3 = 0$) against H_1 ($a_3 < 0$) in the following test regression:

$$
\Delta x_t = \hat{a}_0 + \hat{a}_1 t + \hat{a}_3 x_{t-1} + \sum_{i=1}^k \hat{\eta}_i \Delta x_{t-i} + \hat{f}_t
$$
 (24)

where x_t is the variable under examination, *t* is a linear time trend and f_t is independently and identically distributed $(0, \sigma_f^2)$. If the null cannot be rejected at a chosen level of significance, then x_t is said to be nonstationary. Note that a time trend is included to make the distribution of the coefficient a_3 free of the unknown intercept a_0 .

The limiting distribution for this test has been obtained by Dickey and Fuller (1979) for the case when the disturbances are independent and $\eta_i = 0$ for all $i = 1,...,k$. If the disturbances are serially correlated or heterogeneously distributed, the asymptotic distribution obtained by Dickey and Fuller becomes non-standard and depends on nuisance parameters. To eliminate this dependency, lags of the first difference of the data are included in the regression as additional regressors. The number of additional regressors should increase with the sample size at a controlled rate as specified in Said and Dickey (1984). Thus, with this procedure, the residuals are restricted to the class of $ARMA(p,q)$ processes.

However, the Said and Dickey (1984) theoretical rule for determining *k* does not

^{5.} See the Appendix for precise data definitions.

provide clear guidance in finite samples. As a result, the choice ot the additional regressors in the ADF test regression is determined with a simple data-dependent rule advocated by Hall (1989). We begin by arbitrarily choosing an upper bound on k , say k^* , and estimate an *AR(k*)* model. If the last included regressor is statistically significant, we then select $k = k^*$; if not, we omit the last regressor from the regression. This process is continued until the coefficient on the last included lag is statistically significant. If none are significant, we choose $k = 0$, which leaves us with the simple Dickey-Fuller test.

The Phillips and Perron (1988) normalized bias test is based on the test regression

$$
x_{t} = \hat{a}_{4} + \hat{a}_{5}t + \hat{a}_{6}x_{t-1} + \hat{g}_{t}
$$
 (25)

The residual g_t can be serially correlated and/or conditionally heteroscedastic. To determine whether the process under examination contains a unit root, we form the test statistic

$$
Z_{\alpha} = T(\hat{a}_6 - 1) - \frac{1}{2} [\hat{\omega}_g^2 - \hat{\sigma}_g^2] \left(T^2 \sum_{t=2}^T (x_{t-1} - \bar{x}_{-1})^2 \right)^{-1}
$$
(26)

where $\hat{\sigma}_{g}^{2} = T^{-1} \sum g_{t}^{2}$ and $\hat{\omega}_{g}^{2}$ is an estimator of the spectrum of *g* at frequency zero (the long-run variance). We use the vector-autoregressive prewhitened estimator developed by Andrews (1991) and Andrews and Monahan (1992) to estimate the long-run variance. As in the ADF test, inability to reject the null hypothesis suggests that the data are consistent with a unit-root process. The limiting distribution for the Phillips-Perron (PP) test is nonstandard and is taken from Fuller (1976).⁶

It is well known that the power of unit-root tests increases with the span of the data.

^{6.} We use the normalized bias version of the Phillips and Perron (1988) test, since this test is found **to** be more powerful than its t-statistic counterpart (see Campbell and Perron 1991, and Gregory **1991).**

Therefore, even though we are limited to the 1972Q1 to 1990Q4 period by our interest rate and money measures, we perform the unit-root tests for real income over a longer horizon beginning in 1960Q1 (see Perron 1991). The results of the ADF and PP tests are reported in Table 1. Both tests fail to provide evidence for rejecting the unit-root null for any of the variables even at the 10 per cent level. This result suggests that the variables under consideration are well characterized as 1(1) processes.

As we argued in the previous section, an implication of the LQ model is that if the forcing processes y_t and R_t are all I(1), then these variables should form a cointegrating relationship with m_t . We test whether this implication is supported by the data by applying tests for cointegration and no-cointegration.

The first test we use is the two-step approach proposed by Granger (1983) and later refined by Engle and Granger (1987). The test regressions include a constant, and a constant and a linear trend term. If we find cointegration in the mean-adjusted specification, this corresponds to "deterministic cointegration," which implies that the same cointegrating vector eliminates deterministic trends as well as stochastic trends. But if the linear stationary combinations of the $I(1)$ variables have a non-zero linear trend, this then corresponds to "stochastic cointegration."⁷

The first step of the Engle and Granger two-step approach involves using least squares (LS) to estimate a static long-run regression:

$$
W_t = \hat{a}_7 + \hat{a}_8 t + Z_t^T \hat{\beta} + \hat{h}_t
$$
\n(27)

where Z_t is a matrix of regressors postulated to have a long-run relationship with W_t , (in the current case, $W_t = m_t$ and $Z_t = [y_t, R_t]$). The second step consists of determining whether \hat{h}_t is stationary or nonstationary. To this end, we employ the ADF test suggested

^{7.} See Ogaki and Park (1989) for a discussion of stochastic and deterministic cointegration.

by Engle and Granger (1987) and the normalized bias version of the PP test proposed by Phillips and Ouliaris (1990). If \hat{h}_t is found to be stationary, the null hypothesis of no-cointegration is then rejected in favour of the cointegration alternative. The limiting distributions for both tests are non-standard and depend on the number of regressors in (27). We calculate the augmented Engle and Granger (AEG) critical values from the response surface estimates of MacKinnon (1991) and the Phillips and Ouliaris (PO) values from Haug (1992). Table 2 (p. 23) presents these cointegration test results. Overall, we find very little evidence consistent with cointegration for any of the countries.

Since the cointegrating regression for the LQ model is shown to be serially correlated, it is difficult to discern whether the inability to reject the no-cointegration null really reflects a non-cointegrated system or simply the weak power of these cointegration tests in the presence of persistence roots. Hence, we also apply a residual-based test recently proposed by Shin (1992), which has cointegration as its null hypothesis. One requirement of this test is that the residuals come from a test regression that admits parameter estimates that are efficient as well as consistent. Although the parameter estimates from regression (27) are T-consistent, they are not asymptotically efficient. In order to obtain efficient estimates, we use the previously mentioned DOLS approach and add leads and lags of the first differences of the regressors to regression (27) :⁸

$$
W_{t} = \hat{a}_{9} + \hat{a}_{10}t + Z_{t}^{T}\hat{\beta} + \sum_{i=-k}^{k} \Delta Z_{t-i}^{T}\hat{\pi}_{i} + \hat{\zeta}_{t}
$$
 (28)

The residuals from this regression are then used to calculate the Shin test statistic

$$
\hat{\eta} = \frac{T^{-2} \cdot \sum_{i=1}^{T} S_i^2}{s^2(k)}
$$
\n(29)

^{8.} We chose the number of leads and lags to equal *INT* $(T^{1/3})$ or 4, since this is consistent with the simulation results in Stock and Watson (1993). The conclusions are not sensitive to this choice.

where $S_t = \sum_{i=1}^{t} \hat{\zeta}_i$ and $s^2(k)$ is the Newey and West (1987) estimate of the long-run variance of $\zeta_t^{i=1}$ and *k* is equal to the truncation parameter. Note that as in the PP test, the residual term ζ , can be serially correlated and/or heteroscedastic. The limiting distribution of the test statistic is non-standard and critical values are tabulated in Shin (1992).

The results of the Shin test reported in Table 3 (p. 24) suggest the presence of cointegration for all countries under examination. In fact, for both Germany and Japan the results strongly suggest the presence of both stochastic and deterministic cointegration, while for the United Kingdom the evidence suggests stochastic cointegration among the variables. These results suggest that the conclusions drawn from the AEG and PO tests may be due to persistent serial correlation in the residuals rather than a lack of cointegration. These results are consistent with the Monte Carlo evidence reported by Gregory (1991). We discuss this issue further in Section 4.3. For now we assume that the variables under consideration are cointegrated.

Table 4 (p. 24) presents parameter estimates obtained from a tested-down Stock and Watson (1993) DOLS regression. In addition to providing parameter estimates that are consistent as well as efficient, the DOLS approach also allows us to perform hypothesis testing using conventional asymptotic methods. We note that all regressors have the expected signs and are statistically significant at conventional levels. Furthermore, using Wald tests we are able to reject the homogeneity hypothesis for real income. We note that this latter result is consistent with the cross-country analysis of Boughton (1991), who also rejects the homogeneity assumption of income for German, Japanese and U.K. broad monetary aggregates.

Given the importance of these parameter estimates for the estimation of the Euler

^{9.} We use the Newey and West (1987) long-run variance estimator, as it can be shown that the test statistic for cointegration using a prewhitened kernel estimator with the plug-in bandwidth parameter is not consistent against the alternative of no-cointegration (see Shin 1992).

equation, we examine the stability of the cointegrating vectors using a parameter constancy test for 1(1) processes recently proposed by Hansen (1992). Specifically, we apply Hansen's Lc test to determine whether the estimates of the cointegrating vector are unstable over our sample period.¹⁰ The Lc test results (available from the authors upon request) suggest that we are unable to reject the null hypothesis of parameter stability at the 5 per cent level. This result has two important implications: (i) since the Lc test can also be viewed as a test with the null of cointegration, the test statistics corroborate the conclusions drawn from the Shin tests; and (ii) the stability of the estimates suggests that by looking at these broader aggregates, we may have been able to avoid some of the structural instability often associated with narrower monetary aggregates. In particular, these broader aggregates may have internalized many of the instabilities associated with the narrower definitions.

Hence, we tentatively conclude that our long-run parameter estimates are stable and in the next section, we use them (in Table 4) to form a measure of \tilde{u}_t . This in turn will be used to estimate the Euler equation (17).

4.2 Results for the Euler equation

In this section we test whether the data are consistent with the LQ model using Hansen's (1982) GMM procedure to estimate the parameters in equation (17). The instruments include a constant as well as lags of Δm_t , and the constructed variable \tilde{u}_t . Two different sets of instruments are used and are denoted: I_4^1 and I_5^2 , where I_j^i corresponds to the set {constant; $\Delta m_{t-j},...,\Delta m_{t-j}; u_{t-j},...,u_{t-j}\}$. When lagged one period the instrument set will yield consistent estimates of β and γ (subject to identification), given the assumption about the composite error term v_t . Lagged two periods, the set will yield

^{10.} Hansen (1992) actually proposes three parameter instability tests. We apply the Lc test because it requires no arbitrary decision for trimming.

consistent estimates even if the structural error term *e,* follows an MA(1) process, possibly due to the effects of temporal aggregation.

Using the parameter estimates from Table 4 (p. 24) to construct \tilde{u}_t , we attempt to estimate both the discount rate and the adjustment parameter by estimating the Euler equation directly. The results in Table 5 (p. 25) show that the Japanese and U.K. data provide reasonable and statistically significant estimates of γ . For Japan the adjustment parameter estimate lies in the neighborhood of 0.07 to 0.09 while that for the United Kingdom lies between 0.03 and 0.04. In contrast, the German data admit evidence against the LQ model because the estimates of γ , although statistically insignificant, are both negative. This finding is inconsistent with the prediction of the LQ model. Turning to the discount rates, we find that for all countries the discount rate estimates are not in the expected range of 0.9 to 0.99. For Germany the estimates appear too small, ranging between 0.15 and 0.25, whereas for the United Kingdom both estimates are too large, being greater than 1.0. For Japan, the estimates range from too small (0.72) for instrument set I^1_A to too large (1.14) for set I_5^2 . These conclusions are not entirely surprising, given the results in Gregory, Pagan and Smith (1990), which points out the difficulties in identifying β when the forcing variables x_t are generated by an $I(1)$ process. Hence, in the forthcoming analysis we follow the standard practice of fixing β and then estimating the adjustment parameter. Finally, we note that the J-tests are unable to reject the validity of the overidentifying restrictions imposed by the estimation for any of the equations we consider.

Given the difficulties in identifying both β and γ from the data, we examine the sensitivity of the results by fixing the discount rates to a value that ranges from 0.99 to 0.90 and then re-estimating the model. Table 6 (p. 25) provides these results. Again, the German data offer very little support for the LQ model. Using the instrument set I_4^1 , we find the estimates of the adjustment coefficient to be positive but insignificant. With the set I_5^2 the estimates, although insignificant, are negative. As previously mentioned, the LQ

model requires this parameter to be non-negative. The results for Japan and the U.K. are more encouraging. For Japan, the estimates of γ are statistically significant and lie within the narrow range of 0.07 and 0.08 for the values of the discount rate β we consider. This suggests that adjustment costs are about 13 times more important than disequilibrium costs in determining the demand of broad money in Japan. Similarly, the adjustment parameter estimates for the United Kingdom are significant and range between 0.037 and 0.041. This suggests that the importance of adjustment relative to disequilibrium costs is about 25. Finally, the J-tests again do not allow us to reject the over-identifying restrictions, even at the 10 per cent level, for any of the instrument sets or fixed discount rates we consider.

In sum, the results indicate that the Japanese and U.K. data provide substantially more support for the version of the forward-looking model examined in this paper than do the German data. In the next section we proceed to estimate the stable root.

4.3 Results for the error-correction model

In this section we use the methodology laid out in Section 3.2 to estimate the stable root λ . The results can be easily summarized. For German data, we find a point estimate for λ of about 0.79 and a 0.09 standard error (henceforth given in parentheses). For Japanese and U.K. data, we get estimates in the neighborhood of 0.88 (0.10) and 0.99 (0.02), respectively.

It is instructive to compare these results with those reported in a recent paper by Lane and Poloz (1992). They estimate Ml error-correction models for several countries under various assumptions and find that, although the estimates of the error-correction term vary over the different assumptions, the ranking of the error-correction term for Germany, Japan and the United Kingdom always remains the same. As we do, Lane and Poloz find that the United Kingdom has the largest term, followed by Japan and then Germany.

These results also shed some light on the cointegration results from Section 4.1. Recall that standard cointegration tests were unable to reject the no-cointegration null. The evidence here suggests that this result is an artifact of these tests' (the AEG and PO tests') inability to reject the null in the presence of persistent alternatives or large λ .

5 **CONCLUSIONS**

We have examined whether the simple linear-quadratic model under rational expectations is consistent with the dynamic behaviour of money in Germany, Japan and the United Kingdom. In contrast with previous studies, we estimate the structural parameters with the Euler equation, using a limited-information approach that does not require an explicit solution for the model's control variables in terms of the exogeneous forcing variables.

Our results suggest that the behaviour of Japanese and U.K. money is consistent with the simple LQ model. The empirical estimates for Japan imply that adjustment costs are about 13 times greater than disequilibrium costs, while the evidence for the United Kingdom suggests that the importance of adjustment relative to disequilibrium costs is about 25. In contrast to these encouraging results, those for Germany seem to provide evidence against the equilibrium condition implied by the model over the 1972Q1 to 1990Q4 sample period.

Appendix 1: Data definitions

For Germany, the measure of broad money we use is end-of-quarter M3. The opportunity cost is the end-of-month 90-day representative money market rate. The real income measure is real GNP, while aggregate prices are constructed as a ratio of nominal GNP to real GNP. All German series, except for the interest rate measure, are taken from the *Monthly Report* of the Deutsche Bundesbank and are seasonally adjusted. The interest rate measure is drawn from *World Financial Markets* (Morgan Guaranty Trust).

For Japan, we use end-of-month M2 plus certificates of deposit as the money aggregate. The interest rate measure is the end-of-quarter, 90-day representative money market interest rate. Real income is measured by real GNE, where aggregate prices are simply the ratio of nominal to real GNE. All Japanese series are seasonally adjusted and have been retrieved from the *Economic Statistics Monthly* (Bank of Japan). As for Germany, the interest rates are taken from *World Financial Markets* (Morgan Guaranty Trust).

The U.K. measure of broad money is end-of-month M4. The interest rate measure is an end-of-quarter, 90-day representative money market rate and is obtained from *World Financial Markets* (Morgan Guaranty Trust). The real income measure is real GDP, while aggregate prices are constructed as a ratio of nominal GDP to real GDP. Unless otherwise specified, the data are seasonally adjusted and taken from the U.K. Central Statistical Office's *Economic Trends.*

| Variable | Sample begins | ADF lags | ADF t-statistic | PP Z_{α} -statistic |
|-------------|---------------|----------------|--------------------|-------------------------------|
| GYRM | 1972Q1 | $\overline{0}$ | -2.399 | $-8,829$ |
| GYY | 1960Q1 | 4 | -1.950 | $-7,221$ |
| GYR | 1972Q1 | 3 | -2.952 | -14.335 |
| JPRM | 1972Q1 | 4 | -2.895 | -2.300 |
| JPY | 1960Q1 | $\overline{2}$ | -2.103 | $-2,067$ |
| JPR | 1972Q1 | $\overline{2}$ | -3.091 | -12.505 |
| UKRM | 1972Q1 | $\overline{2}$ | -1.715 | -0.212 |
| UKY | 1960Q1 | 7 | -2.811 | -11.650 |
| UKR | 1972Q1 | 6 | -2.795 | -13.596 |

Table 1: Unit-root tests Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests

Note: * indicates significance at the 10 per cent level.

Table 2: Tests for the null hypothesis of no-cointegration for the demand for money equation augmented Engle-Granger (AEG) and Philiips-Ouliaris (PO) tests

Notes: * indicates significance at the 10 per cent level and AEG lag lengths are in parentheses.

Table 3:

Notes: * indicates significance at the 10 per cent level. The truncation parameter is chosen according to *INT(Tr/2)* or 9. This rate is usually satisfactory under both the null and the alternative (see Andrews 1991).

| Variable | Germany | Japan | United Kingdom | |
|---------------------|----------|----------|-----------------------|--|
| Constant | -3.048 | -6.278 | 2.877 | |
| | (0.147) | (0.305) | (0.483) | |
| Real income | 1.619 | 1.493 | 2.114 | |
| | (0.024) | (0.024) | (0.111) | |
| (Interest rate)/100 | -0.586 | -0.362 | -0.789 | |
| | (0.089) | (0.219) | (0.468) | |

Table 4: Dynamic OLS estimation of the demand for money equation

Note: We use Newey and West (1987) standard errors as in Stock and Watson (1993). The truncation parameter is set equal to one.

 \bar{z}

| | Germany | | Japan | | United Kingdom | |
|----------|----------|----------|---------|----------|----------------|----------|
| | I_4^1 | I_5^2 | I_4^1 | I_5^2 | I_4^1 | 1_{5} |
| β | 0.153 | 0.242 | 0.729 | 1.135 | 1.186 | 1.312 |
| | (0.581) | (0.776) | (0.144) | (0.130) | (0.124) | (0.133) |
| γ | -0.077 | -0.122 | 0.071 | 0.091 | 0.034 | 0.041 |
| | (0.103) | (0.109) | (0.033) | (0.048) | (0.014) | (0.017) |
| constant | 0.009 | 0.005 | 0.005 | -0.001 | -0.002 | -0.003 |
| | (0.004) | (0.005) | (0.003) | (0.002) | (0.002) | (0.002) |
| J-test | 6.839 | 1.545 | 10.162 | 9.346 | 4.453 | 3.512 |

Table 5: Estimates of the Euler equation

Notes: Henceforth, the values in parentheses corresponding to the parameter estimates are asymptotic standard errors. The degree of freedom corresponding to the J-test in this Table is 6.

| | Germany | | Japan | | United Kingdom | |
|---------------------------|---------------------------|------------------------------|----------------------------|---------------------------|---------------------------|---------------------------|
| | I_4^1 | I_5^2 | I_4^1 | I_5^2 | I_4^1 | I_5^2 |
| $\beta = 0.990$ J-test | 0.054 (0.056) 3.425 | -0.025 (0.087) 1.413 | 0.072 (0.038) 11.516 | 0.081 (0.038) 8.959 | 0.037 (0.011) 5.950 | 0.041 (0.012) 6.678 |
| $\beta = 0.975$ J-test | 0.052 (0.056) 3.443 | -0.028 (0.086) 1.409 | 0.072 (0.038) 11.445 | 0.081 (0.038) 8.889 | 0.037 (0.011) 6.078 | 0.041 (0.012) 6.847 |
| $\beta = 0.950$ J-test | 0.049 (0.055) 3.476 | -0.031 (0.085) 1.403 | 0.072 (0.037) 11.321 | 0.080 (0.037) 8.772 | 0.037 (0.011) 6.292 | 0.041 (0.012) 7.119 |
| $\beta = 0.900$ J-test | 0.042 (0.054) 3.549 | -0.038 (0.084) 1.394 | 0.071 (0.036) 11.046 | 0.079 (0.035) 8.541 | 0.038 (0.011) 6.729 | 0.039 (0.011) 7.615 |

Table 6: Estimates of the adjustment term for pre-set values of beta

Note: The degree of freedom corresponding to the J-test in this Table is 8.

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