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Stockout Avoidance Inventory Behaviour with Differentiated Durable Products

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Abstract

Stockout avoidance inventory models imply that firms maintain inventory stocks that are low – too low to be justified by the data. The reason is that these models are based on the representative agent paradigm. Thus, if one firm experiences a stockout then all firms do, and the cost of the stockout is simply the delay of the marginal sale by one period. In contrast, this study shows that, with heterogeneity, stockouts need not be universal, and that the cost of a stockout is therefore the permanent loss of a marginal sale to the competition. This suggests that firms may maintain large inventory stocks, a conclusion that has implications for the dynamics of prices, output and sales.

In this paper a model is developed in which each firm sells a single, differentiated, durable product. A distribution over consumers' preferences for the differentiated products exists. The stochastic element is a fad shock that affects the consumers' preference for the differentiated attribute. Dynamic programming is used to derive decision rules for consumers and producers, and a closed-form solution is found. The results suggest that in the aggregate market, inventory stocks, which may be large, always exist, and yet the average price is always influenced by stockouts.

Résumé

Dans les modèles d'évitement des ruptures de stocks, les entreprises maintiennent des stocks à un niveau inférieur à celui qui est observé dans les données. La raison en est que ces modèles reposent sur le schéma de l'agent représentatif. Ainsi, si une entreprise subit une rupture des stocks, l'ensemble des entreprises la subit aussi, et le coût de la rupture équivaut simplement aux frais qu'entraîne le report de la vente marginale à la période suivante. L'auteur de la présente étude montre au contraire que, sous l'hypothèse d'hétérogénéité, les ruptures de stocks n'ont pas à être universelles; par conséquent, le coût d'une rupture des stocks est la perte permanente d'une vente marginale au profit de la concurrence. Cela implique que les entreprises peuvent détenir des stocks importants, ce qui a des conséquences pour la dynamique des prix, de la production et des ventes.

La présente étude décrit un modèle où chaque entreprise vend un bien durable différencié. On observe une distribution des préférences des consommateurs à l'égard des produits différenciés. L'élément stochastique est un choc attribuable à un engouement pour un produit particulier, lequel influe sur les préférences des consommateurs pour la caractéristique de différenciation. L'auteur se sert de la programmation dynamique pour élaborer les règles de comportement des consommateurs et des producteurs et parvient à une solution analytique complète de son modèle. Les résultats indiquent que dans un marché global il existe toujours des stocks, qui peuvent être importants, et pourtant les ruptures de stocks ont toujours une incidence sur le prix moyen.

1 Introduction

Macroeconomists' interest in inventory behaviour stems from the growing realization that business cycles are, to a large extent, inventory cycles (Blinder and Maccini 1990). Much of this literature, since the early 1980s, has debated the applicability of the Production-Smoothing Inventory Model to modern economies. The production-smoothing model suggests that if demand is subject to stochastic shocks and production is subject to a convex technology, adjustment costs, or both, then firms will minimize costs by using inventories as a buffer stock to smooth output over time. Thus, the motivation for firms to hold inventories would be to avoid bearing the costs of peak production levels and adjustment costs. Evidence shows however, that output is more volatile, over time, than sales – the opposite of what this model predicts (Blinder 1982).

A number of alternative explanations of inventory behaviour have been proposed, which produce more realistic results. The cost-smoothing approach (Eichenbaum 1983, 1988) suggests that inventories are held as a buffer stock to exploit cost fluctuations. Another approach (Ramey 1991) suggests that production technologies may be concave – inventories are held to make production more volatile and hence to minimize cost. Finally, the stockout avoidance approach (Blanchard 1983) suggests that firms hold inventories in order to avoid the lost sales when stockouts occur.

Recently, the development of the Stockout Avoidance Inventory Model has been formalized through the use of the inventory non-negativity constraint (Abel 1985; Kahn 1987; Thurlow 1992). The inventory non-negativity constraint simply reflects the fact that inventories must be positive quantities. The inclusion of this constraint, with positive autocorrelation in the stochastic demand process, implies production-countersmoothing. However, these models also imply very small inventory stocks when, in fact, they are often quite large (Blanchard and Melino 1986). These small inventory stocks in turn suggest frequent stockouts, an event that the data rarely, if ever, indicate.

The standard explanation for this difference between the model and observations has been aggregation. Suppose markets are defined very narrowly on the basis of geography, time and attributes. Then, even if inventory stocks in any one market are small, aggregate inventories may be very large. Also, although stockouts may exist in individual markets, the aggregate data would never record them as long as at least one market did not experience a stockout.

The reason that stockout avoidance models based on the inventory non-negativity constraint generate very small inventory stocks is that the cost of a stockout is small. In these models, firms are identical, so if one firm experiences a stockout, then all do. Thus, the cost of a stockout, which is the loss of the marginal sale in this period, is only the cost of delaying that sale into the next period.

In many inventory models, the role played by inventories, with respect to the business cycle, is that of a propagation mechanism. That is, given stochastic demand shocks, inventory behaviour conveys and amplifies the effect of these shocks on production. If, as the stockout avoidance models based on the inventory non-negativity constraint suggest, stockout costs really are small, then it is difficult to see how stockout avoidance could play a major role in the cycle. Yet, anecdotal and empirical evidence seems to suggest that stockout avoidance really is significant (Blanchard and Melino 1986).

A possible explanation would start with the assumption of differentiated, durable goods, with each firm offering a single, differentiated product. Suppose also that some distribution of consumers' preferences over the differentiated products exists. Then, if one firm experiences a stockout, consumers may substitute across attributes. (Note that in that case an individual firm may experience a stockout, while other firms continue to maintain inventory stocks.) Thus, the cost of a stockout to an individual firm is the permanent loss of a sale to the firm's competitor.[1] As a result, the stockout costs are much greater and the size of inventory stocks maintained by each firm is likely to be realistic. It is important to get the size of the inventory stocks right, since this determines the frequency of stockouts and hence the dynamics of price, output and sales. This paper involves a formalization of such a model, with the aim of drawing out the implications and dynamics.

2 The model

Consider a market for a durable product that has one variable attribute. This attribute may take one of two discrete values (red or blue, for example). The market is competitive. Each firm offers only one type of product. A large number of consumers exist. Consumers are distinguished by their preferences over the two types of product, which are distributed according to a continuous uniform distribution. In addition, individual consumers may start with differing stock of the durable good. For simplicity, the model is set up and solved for the two-period problem. Consider first the consumers' problem in the second period.

1. In fact, a market failure may exist. The private cost of a stockout (permanent loss of the marginal sale) may be greater than the social cost (consumers getting products with a set of attributes slightly different from what they may optimally desire). This may suggest that inventory stocks are larger than what is socially optimal.

2.1 The consumers' problem (period 2)

Individual consumers (superscripted i) are endowed with the utility function for period t,

$$U(C_t^i, q_{1,t}^i, q_{2,t}^i) = \alpha_0 C_t^i - \frac{\alpha_1}{2}[\bar{Q} - \gamma^i(1+v_t)q_{1,t}^i - (1-\gamma^i)(1-v_t)q_{2,t}^i]^2, \tag{2.1}$$

where C_t^i is individual i's consumption of all other goods in period t, $\bar{Q}$ is the desired consumption of the durable good (with either attribute), $q^i_{1,t}$ and $q^i_{2,t}$ are the consumer's stock of the durable product at the end of period t, (the subscripts 1 and 2 refer to the attribute of the product), γ^i is a uniformly distributed random variable with $0 \le \gamma^i \le 1$, which describes the individual's attribute preference, $v_t \in (-1,1)$ is a stochastic taste shifter with mean zero, and α_0 and α_1 are parameters. Thus, preferences are separable between consumption and the durable good. The stochastic term, v_t, may be considered to be a "fad" shock, since all individuals are subject to the same shock. This utility function is similar to that used by Thurlow (1992) to generate a linear demand function for durable goods.

The individual consumer maximizes his or her utility, given by equation (2.1), through the choices of C_t^i, $q^i_{1,t}$ and $q^i_{2,t}$, subject to

$$q_{1,t}^i = (1-\Delta)q_{1,t-1}^i + s_{1,t}^i, \tag{2.2}$$

$$q_{2,t}^i = (1-\Delta)q_{2,t-1}^i + s_{2,t}^i, \tag{2.3}$$

$$A_t^i = (1+r)(A_{t-1}^i + Z_t^i - C_t^i - P_{1,t}\cdot s_{1,t}^i - P_{2,t}\cdot s_{2,t}^i), \tag{2.4}$$

$$q_{1,t}^i \ge 0, \tag{2.5}$$

$$q_{2,t}^i \ge 0, \tag{2.6}$$

where Δ is the depreciation rate of the consumer's stock, $s^i_{1,t}$ and $s^i_{2,t}$ are the purchases of each type of the durable product, A_t^i is the stock of financial assets owned by the consumer at the end of period t, r is the interest rate, Z_t^i is the employment income earned by the consumer during period t, and $P_{1,t}$ and $P_{2,t}$ are the prices of new sales of each type of the durable good. Equations (2.2) and (2.3) represent the consumer's stock accumulation. Equation (2.4) is the intertemporal budget constraint. It is assumed that $\beta = 1/(1+r)$, where β represents the consumer's rate of time preference. The latter two constraints are non-negativity constraints on the consumer's stock.

The consumer's decision is made subject to the information set

$$\Omega_t^{i+} = \{q_{1,\tau-1}^i, q_{2,\tau-1}^i, K_{1,\tau}, K_{2,\tau}, P_{1,\tau}, P_{2,\tau}, v_\tau, \gamma^i\}_{-\infty}^{\tau=t}, \tag{2.7}$$

where $K_{1,t}$ and $K_{2,t}$ are the stocks of each type of durable good available for sale in period t. The superscript "+" indicates that this information set includes information that becomes available during period t, which includes the realizations of prices and the fad shock v_t.

In order to solve this problem, set the Lagrangean as follows:

$$
\begin{aligned}
L = {} & \alpha_0 C_t^i - \frac{\alpha_1}{2}\left[\bar{Q} - \gamma^i(1+v_t)q_{1,t}^i - (1-\gamma^i)(1-v_t)q_{2,t}^i\right]^2 \\
& + \lambda\beta^{-1}\left[A_{t-1}^i + Z_t^i - C_t^i - P_{1,t}\cdot s_{1,t}^i - P_{2,t}\cdot s_{2,t}^i\right] \\
& + \zeta_1 q_{1,t}^i + \zeta_2 q_{2,t}^i ,
\end{aligned}
\tag{2.8}
$$

where λ is the Lagrangean multiplier for the first three constraints, and ζ_1 and ζ_2 are the Lagrangean multipliers for the non-negativity constraints. Note that since this is the second of the two periods, financial wealth at the end of the period A_t^i is zero. The first-order Kuhn-Tucker conditions for an optimum are

$$\alpha_0 - \lambda\beta^{-1} = 0, \tag{2.9}$$

$$\alpha_1\gamma^i(1+v_t)\left[\bar{Q} - \gamma^i(1+v_t)q_{1,t}^i - (1-\gamma^i)(1-v_t)q_{2,t}^i\right] - \alpha_0 P_{1,t} - \zeta_1 = 0, \tag{2.10}$$

$$\alpha_1(1-\gamma^i)(1-v_t)\left[\bar{Q} - \gamma^i(1+v_t)q_{1,t}^i - (1-\gamma^i)(1-v_t)q_{2,t}^i\right] - \alpha_0 P_{2,t} - \zeta_2 = 0, \tag{2.11}$$

$$\zeta_1 \ge 0, \quad q_{1,t}^i \ge 0, \quad \zeta_1 q_{1,t}^i = 0. \tag{2.12}$$

$$\zeta_2 \ge 0, \quad q_{2,t}^i \ge 0, \quad \zeta_2 q_{2,t}^i = 0. \tag{2.13}$$

The first condition implies that the marginal utility of consumption in this period is equal to the marginal value of wealth. The following equations indicate that the marginal utility that is derived from the stock of the durable good of each type is equal to the expected marginal utility of foregone consumption.

There are four possible types of consumer, a condition that leads to four possible individual demand functions for the durable good.

Case 1: $q_{1,t}^i = q_{2,t}^i = 0$, which occurs if

$$\alpha_1\gamma^i(1+v_t)\bar{Q} \le \alpha_0 P_{1,t} \tag{2.14}$$

and

$$\alpha_1(1-\gamma^i)(1-v_t)\bar{Q} \le \alpha_0 P_{2,t} . \tag{2.15}$$

Note that in all other cases (where some purchase is made), the following is true:

$$MRS \equiv \frac{U_{q_{1,t}}}{U_{q_{2,t}}} = \frac{\gamma^i(1+v_t)}{(1-\gamma^i)\,(1-v_t)}$$

and $$MRT \equiv \frac{P_{1,t}}{P_{2,t}}.$$

Thus, both the marginal rate of substitution and the marginal rate of transformation are constant over $q_{1,t}$ and $q_{2,t}$. This means that, as long as the constraints (equations 2.14 and 2.15) are not binding, the consumer will, in most cases, purchase only q_1 or q_2. The exception occurs when $MRS = MRT$, in which case the consumer is indifferent to q_1 and q_2.

Case 2: $q^i_{1,t} > 0,\ q^i_{2,t} = 0$, or more specifically,

$$q^i_{1,t} = \frac{1}{\gamma^i(1+v_t)}\left[\bar{Q} - \frac{\alpha_0}{\alpha_1\gamma^i(1+v_t)}P_{1,t}\right], \tag{2.16}$$

which occurs if condition (2.14) is not satisfied and

$$\frac{P_{1,t}}{\gamma^i(1+v_t)} < \frac{P_{2,t}}{(1-\gamma^i)\,(1-v_t)}.$$

Case 3: $q^i_{1,t} = 0,\ q^i_{2,t} > 0$, or more specifically,

$$q^i_{2,t} = \frac{1}{(1-\gamma^i)\,(1-v_t)}\left[\bar{Q} - \frac{\alpha_0}{\alpha_1(1-\gamma^i)\,(1-v_t)}P_{2,t}\right], \tag{2.17}$$

which occurs if condition (2.15) is not satisfied and

$$\frac{P_{1,t}}{\gamma^i(1+v_t)} > \frac{P_{2,t}}{(1-\gamma^i)\,(1-v_t)}.$$

Case 4: $q^i_{1,t} > 0,\ q^i_{2,t} > 0$,

which occurs if neither (2.14) nor (2.15) are satisfied and

$$\frac{P_{1,t}}{\gamma^i(1+v_t)} = \frac{P_{2,t}}{(1-\gamma^i)\,(1-v_t)}.$$

Note that the set of consumers who have a multivalued demand correspondence has measure zero and can be ignored (unless $P_{1,t} = P_{2,t} = 0$).

2.1.1 Individual demand properties

Figure 1 shows an individual's demand for the durable good in price space. Important quantities on the diagram are the individual's reservation prices, $P^R_{1,t}$ and $P^R_{2,t}$, and the price ratio of indifference, *PI*. The reservation prices are derived from the conditions (2.14) and (2.15) and are

$$P^R_{1,t} = \frac{\alpha_1 \gamma^i (1+v_t) \bar{Q}}{\alpha_0} \quad ,$$

$$P^R_{2,t} = \frac{\alpha_1 (1-\gamma^i)(1-v_t) \bar{Q}}{\alpha_0} .$$

The price ratio of indifference can be derived from the condition associated with Case 4, and is given by

$$P_{1,t} = \frac{\gamma^i (1+v_t)}{(1-\gamma^i)(1-v_t)} P_{2,t} \quad .$$

The figure indicates the areas in which the individual's demand for either product will be positive or zero. In order for an individual to purchase a certain type of the durable product, two conditions must be satisfied. First, the price of the particular type of the durable product must be less than the reservation price. Second, the individual prefers the type of durable being purchased, given the existing prices. Note that the lines in Figure 1 show the individual's decision rules, so that the lines will vary across individuals. The price realization is a point in the diagram, and this determines the demands of all individuals.

2.1.2 The aggregation problem

The aggregate demand function for the first type of durable good can be derived from

$$Q_{1,t} = \int_{A_1} \left(\frac{\bar{Q}}{\gamma^i (1+v_t)} - \frac{\alpha_0 P_{1,t}}{\alpha_1 [\gamma^i (1+v_t)]^2} \right) d\gamma^i, \tag{2.18}$$

where $Q_{1,t}$ is the aggregate stock of the first type of the durable good that consumers desire to maintain, and

$$A_1 = \left\{ \gamma^i : 1 > \gamma^i > max \left[\frac{\alpha_0 P_{1,t}}{\alpha_1 (1+v_t) \bar{Q}}, \quad \left[\frac{P_{1,t}}{(1+v_t)} + \frac{P_{2,t}}{(1-v_t)} \right]^{-1} \frac{P_{1,t}}{(1+v_t)} \right] \right\} .$$

This condition indicates that γ^i must be sufficiently large for a consumer to decide to purchase a q_1

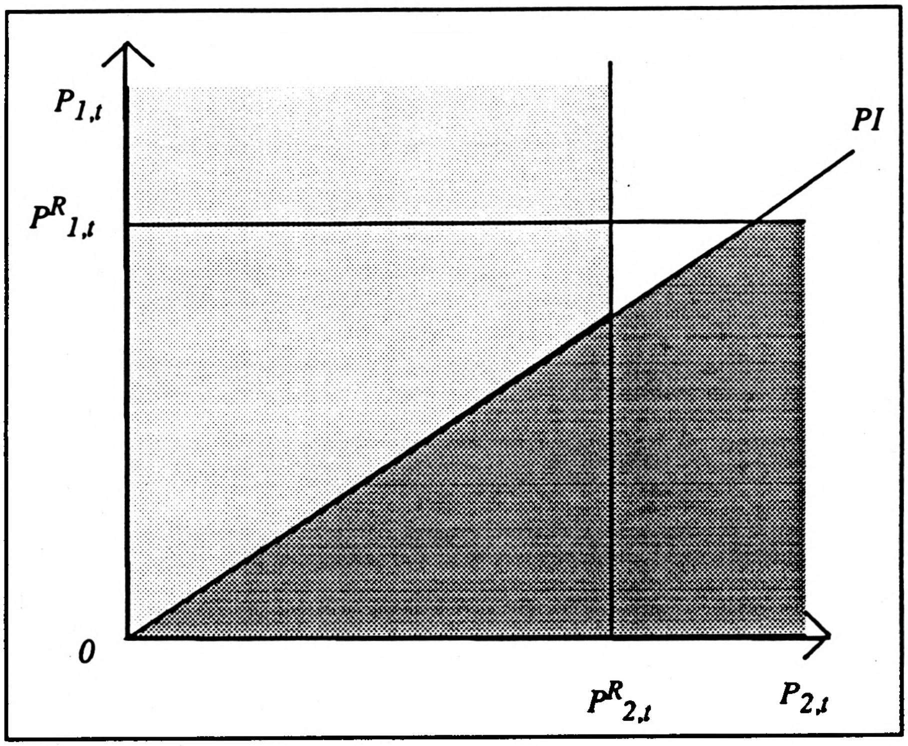

Figure 1: Consumer *i*'s demand in price space

$P^R_{1,t}$ - Reservation $P_{1,t}$
$P^R_{2,t}$ - Reservation $P_{2,t}$
PI - Price ratio of indifference

Region where $q^i_{1,t} = q^i_{2,t} = 0$

Region where $q^i_{1,t} > 0$ and $q^i_{2,t} = 0$

Region where $q^i_{1,t} = 0$ and $q^i_{2,t} > 0$

type of durable product.[2]

Similarly, the aggregate demand function for the second type of durable good can be derived from

$$Q_{2,t} = \int_{A_2} \left(\frac{\bar{Q}}{(1-\gamma^i)(1-v_t)} - \frac{\alpha_0 P_{2,t}}{\alpha_1 [(1-\gamma^i)(1-v_t)]^2} \right) d\gamma^i, \tag{2.19}$$

where $Q_{2,t}$ is the aggregate stock of the first type of the durable good that consumers wish to maintain, and

$$A_2 = \left\{ \gamma^i : 0 < \gamma^i < min \left[1 - \frac{\alpha_0 P_{2,t}}{\alpha_1 (1-v_t) \bar{Q}}, \quad \left[\frac{P_{1,t}}{(1+v_t)} + \frac{P_{2,t}}{(1-v_t)} \right]^{-1} \frac{P_{1,t}}{(1+v_t)} \right] \right\}.$$

This condition indicates that γ^i must be sufficiently small for a consumer to decide to purchase a q_2 type of durable good.

Solving the integration problem yields the aggregate demand functions for the two types of consumer durable,

$$Q_{1,t} = -\frac{\bar{Q}}{(1+v_t)} ln(F_{1,t}) + \frac{\alpha_0 P_{1,t}}{\alpha_1 (1+v_t)^2} (1 - F_{1,t}^{-1}), \tag{2.20}$$

where $$F_{1,t} = max \left[\frac{\alpha_0 P_{1,t}}{\alpha_1 (1+v_t) \bar{Q}}, \left[\frac{P_{1,t}}{(1+v_t)} + \frac{P_{2,t}}{(1-v_t)} \right]^{-1} \frac{P_{1,t}}{(1+v_t)} \right],$$

and $$Q_{2,t} = -\frac{\bar{Q}}{(1-v_t)} ln(F_{2,t}) + \frac{\alpha_0 P_{2,t}}{\alpha_1 (1-v_t)^2} (1 - F_{2,t}^{-1}), \tag{2.21}$$

where $$F_{2,t} = max \left[\frac{\alpha_0 P_{2,t}}{\alpha_1 (1-v_t) \bar{Q}}, \left[\frac{P_{1,t}}{(1+v_t)} + \frac{P_{2,t}}{(1-v_t)} \right]^{-1} \frac{P_{2,t}}{(1-v_t)} \right].$$

2. Note that if $P_{2,t} = 0$, then the second term is unity, which implies that all consumers who buy the durable purchase type 2, and no sales of type 1 exist.

2.1.3 Aggregate demand properties

Consider the market for the first type of consumer durable. The demand function displays a kink. There are three possible regions on the demand curve, depending on which expression $F_{1,t}$ takes. In particular, let the expression for $F_{1,t}$ be $F_{1,t} = max\ [B_{1,t}, D_{1,t}]$.

Consider Case 1, where $B_{1,t} > D_{1,t}$. In this situation, the markets for the two durable goods appear to be quite unrelated. This situation will tend to appear when the prices in both markets are high. The demand function can be rewritten as

$$Q_{1,t} = \frac{\bar{Q}}{(1+v_t)}\,[X_{1,t} - 1 - lnX_{1,t}]\ ,$$

where $\quad X_{1,t} = \dfrac{\alpha_0 P_{1,t}}{\alpha_1 \bar{Q}\,(1+v_t)} \quad$ and $\quad 0 < X_{1,t} < 1.$

The slope of this demand curve is given by

$$\frac{\partial Q_{1,t}}{\partial P_{1,t}} = \frac{\bar{Q}}{(1+v_t)}\left[\frac{\alpha_0}{\alpha_1 \bar{Q}\,(1+v_t)} - \frac{1}{P_{1,t}}\right] < 0\ ,$$

and this can be shown to be negative.

Consider Case 2, where $B_{1,t} < D_{1,t}$. This situation, where the two types of durable good interact with each other, tends to occur when the prices are low. The demand function can be written as

$$Q_{1,t} = -\frac{\bar{Q}}{(1+v_t)}\,ln\left[\frac{(1-v_t)P_{1,t}}{(1-v_t)P_{1,t} + (1+v_t)P_{2,t}}\right] - \frac{\alpha_0 P_{2,t}}{\alpha_1\,(1+v_t)\,(1-v_t)}\ .$$

The slope of the demand curve is

$$\frac{\partial Q_{1,t}}{\partial P_{1,t}} = \left(-\frac{\bar{Q}}{(1+v_t)P_{1,t}}\right)\left[\frac{(1+v_t)P_{2,t}}{(1-v_t)P_{1,t} + (1+v_t)P_{2,t}}\right] < 0\ ,$$

which is clearly negative. The cross partial is given by

$$\frac{\partial Q_{1,t}}{\partial P_{2,t}} = \frac{\bar{Q}}{(1-v_t)P_{1,t} + (1+v_t)P_{2,t}} - \frac{\alpha_0}{\alpha_1\,(1+v_t)\,(1-v_t)} > 0\ ,$$

which can be shown to be positive.

This case is illustrated by the three-dimensional diagrams of the aggregate demand for Q_{1t}, which are given in Figures 2 and 3.[3] These diagrams are based on the parameter values of $\alpha_0 = 0.1$, $\alpha_1 = 0.9$, $\overline{Q} = 20$ and v_t takes the values of 0.5 and -0.5 in Figures 2 and 3 respectively. The diagrams indicate that Q_{1t} is a negative function of P_{1t} and is positively related to P_{2t}. Although it has been verified that all points on both diagrams are consistent with Case 2 ($B_{1t} < D_{1t}$), it appears that there would be a smooth transition between the two cases. Notice that in Figure 2, as P_2 gets large, the diagram gets flat in the P_2 direction.

At the kink, $B_{1,t} = D_{1,t}$, which implies

$$P_{1,t} = \frac{\alpha_1 \overline{Q} (1 + v_t)}{\alpha_0} - \frac{(1 + v_t)}{(1 - v_t)} P_{2,t} \quad .$$

Moreover, it can be shown that Case 1, the situation of separate markets, exists above the kink.

2.2 The producers' problem (period 2)

It is assumed that the market for each type of durable good is competitive. In particular, each firm offers either durable good type *1* or *2*. The cost of production is constant across firms, durable good types and over time. The cost of production is set at w. The costs are linear in output.

Firms have two decisions to make at different points in time within a period (and hence, the decisions are subject to different information sets). The first decision is how much output, $Y_{j,t}$, to produce (where j refers to the durable good type). Firms make this decision prior to knowing what the fad shock, v_t, is for the period. The second decision, which they make after the fad shock is observed, is how much product to sell.

More formally, the representative firm that produces the durable good of type j, where $j = 1,2$, maximizes profit, given by

$$\Pi_{j,t} = E_t^- (P_{j,t} \cdot S_{j,t}) - w \cdot Y_{j,t} + \underline{P} E_t^- (I_{j,t}) \quad , \qquad (2.22)$$

with respect to output, $Y_{j,t}$, where $P_{j,t}$ has finite support, which is greater than or equal to a positive scrap value, $\underline{P}$. Note that the scrap value is common to both durable types, and the scrap value is less than the price that is realized in the inventory state, thus,

$$\underline{P} < \beta (1 - \delta) w \quad .$$

3. Actually, the inverse demand functions are shown.

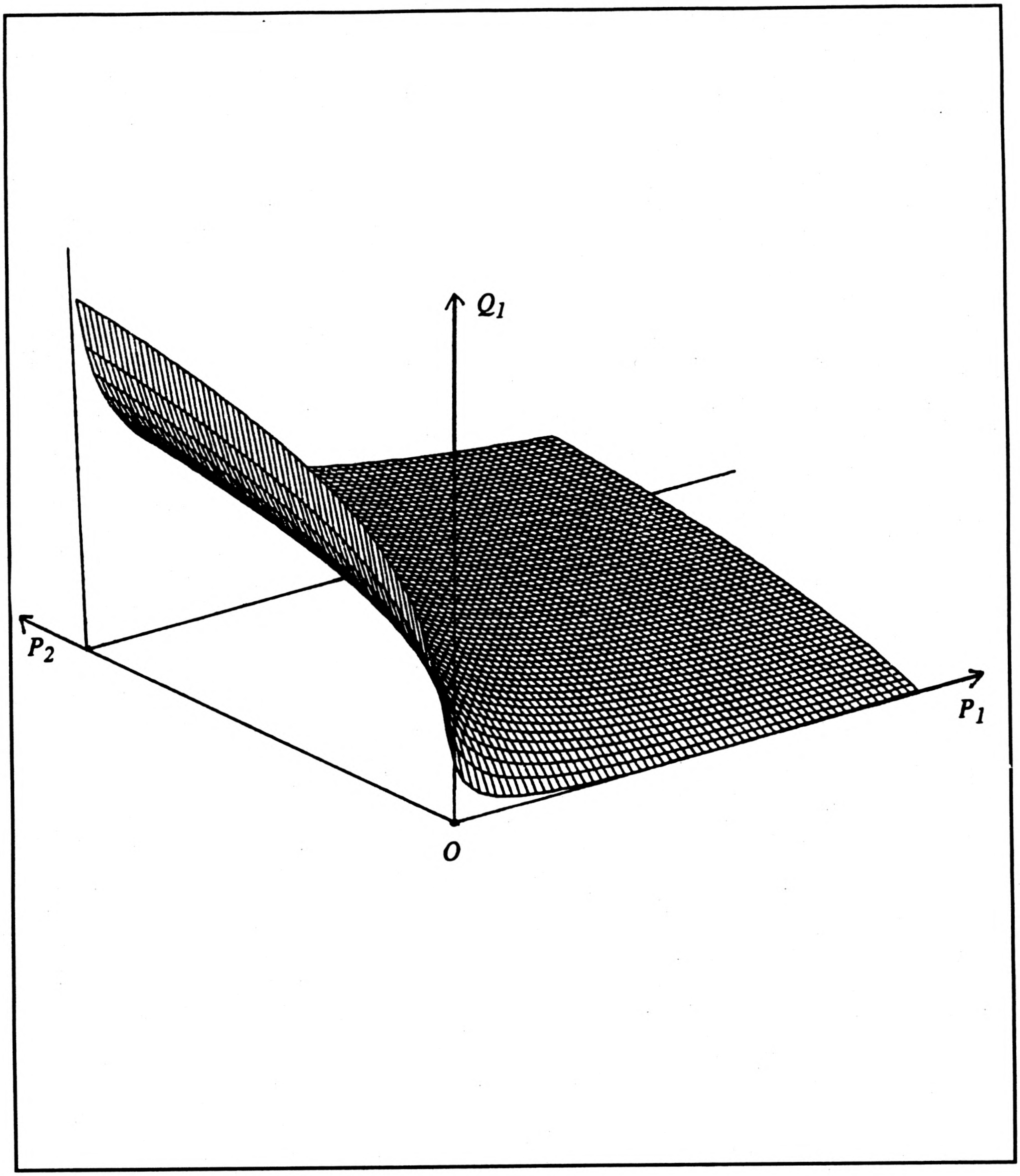

Figure 2: Aggregate demand for Q_{1t}, when $v_t = 0.5$

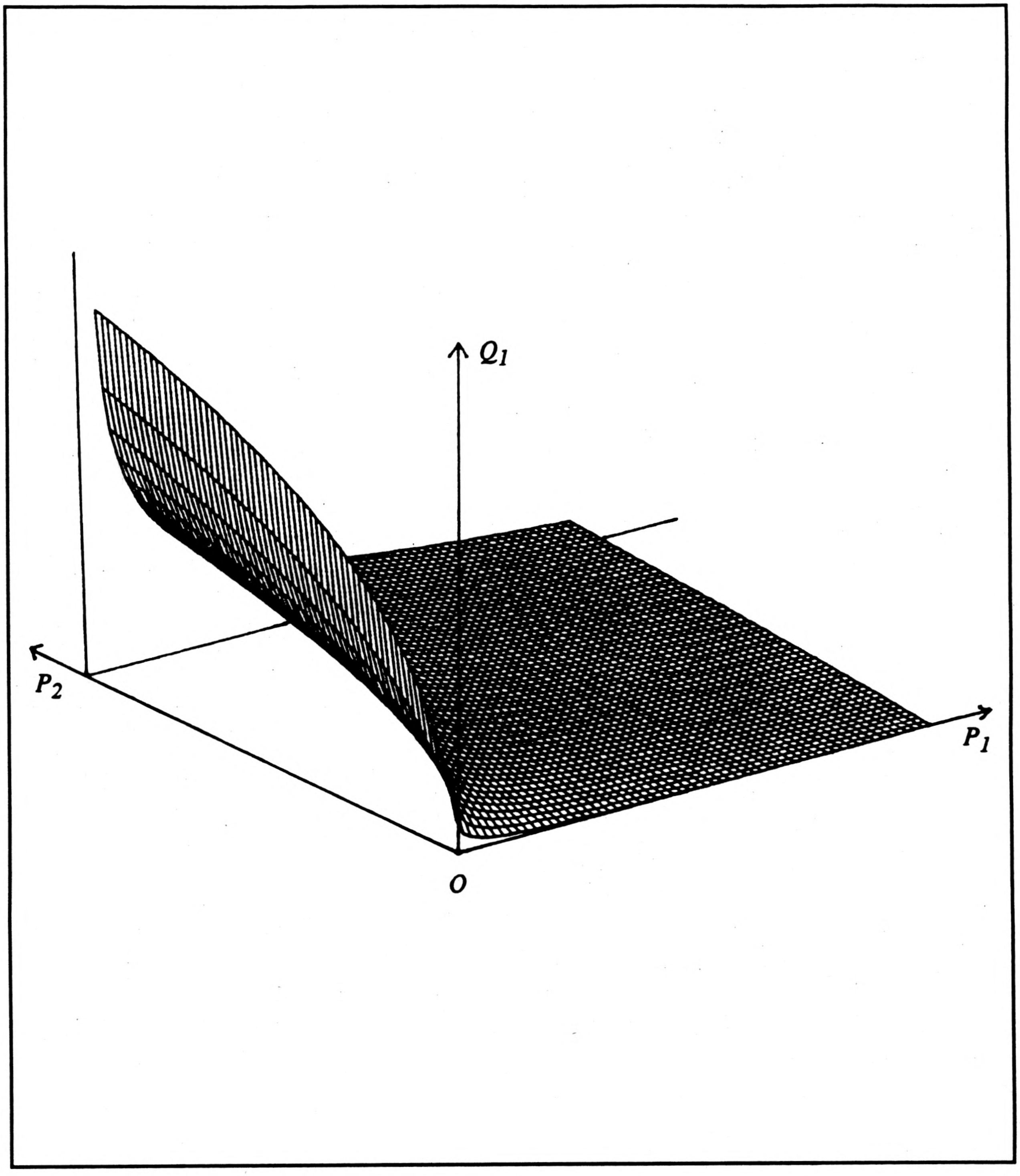

Figure 3: Aggregate demand for $Q_{1t,}$ when v_t = -0.5

Equation (2.22) is maximized subject to

$$S_{j,t} = (1-\delta)I_{j,t-1} + Y_{j,t} - I_{j,t} \quad , \tag{2.23}$$

$$I_{j,t} \geq 0 \, , \tag{2.24}$$

where δ is the depreciation rate of inventory stocks. The depreciation of $I_{j,t}$ is assumed to be less rapid than that of the consumer's stock, thus $\Delta > \delta$.

Firms of each type make the output decision conditional upon the common information set

$$\Omega_t^- = \{Q_{1,\tau-1}, Q_{2,\tau-1}, I_{1,\tau-1}, I_{2,\tau-1}, P_{1,\tau-1}, P_{2,\tau-1}, v_{\tau-1}, w\}_{-\infty}^{\tau=t} . \tag{2.25}$$

The representative firm's solution is

$$Y_{j,t} = \begin{cases} \infty & \text{if } E_t^- P_{j,t} > w \\ [0,\infty) & \text{if } E_t^- P_{j,t} = w \\ 0 & \text{if } E_t^- P_{j,t} < w. \end{cases} \tag{2.26}$$

Once the fad shock, v_t, is known, the information set available to the firm becomes

$$\Omega_t^+ = \{Q_{1,\tau-1}, Q_{2,\tau-1}, I_{1,\tau-1}, I_{2,\tau-1}, P_{1,\tau-1}, P_{2,\tau-1}, v_\tau, w\}_{-\infty}^{\tau=t} . \tag{2.27}$$

Thus, the only difference between the "+" and "–" information sets is that in the "+" set firms have access to the current fad shock v_t. The firm's optimal sales decision, based on the Ω_t^+ information set, is

$$S_{j,t} = \begin{cases} [0, (1-\delta)I_{j,t-1} + Y_{j,t}) & \text{if } P_{j,t} = \underline{P} \; , \\ (1-\delta)I_{j,t-1} + Y_{j,t} & \text{if } P_{j,t} > \underline{P} \end{cases} \tag{2.28}$$

which implies

$$I_{j,t} = \begin{cases} [0, (1-\delta)I_{j,t-1} + Y_{j,t}] & \text{if } P_{j,t} = \underline{P} \\ 0 & \text{if } P_{j,t} > \underline{P} \end{cases} . \tag{2.29}$$

2.3 Equilibrium (period 2)

In the time t^- subperiod, the equilibrium condition for an interior solution is

$$E_t^- P_{j,t} = w \; . \tag{2.30}$$

If firms make output decisions such that this expectation holds, and if we assume that the ex post price exceeds the scrap value in both states (that is, $P_{1,t} > \underline{P}$ and $P_{2,t} > \underline{P}$), then sales are $S_{j,t} = (1-\delta)I_{j,t-1} + Y_{j,t}$ and $I_{j,t} = 0$. From the firm's accumulation identity, we see that equilibrium output is determined, such that the following holds:

$$Y_{j,t} = E_t^{-}S_{j,t} - (1-\delta)I_{j,t-1} \quad (2.31)$$

If we assume that $E_t^{-}(P_{jt}) = w$, the equilibrium output equations can be rewritten as

$$\begin{aligned} Y_{1,t} = & -(1-\delta)I_{1,t-1} - (1-\Delta)Q_{1,t-1} \\ & - \frac{\bar{Q}}{(1+E_t^{-}v_t)} ln(E_t^{-}F_{1,t}) + \frac{\alpha_0 w}{\alpha_1 (1+E_t^{-}v_t)^2}(1-E_t^{-}F_{1,t}^{-1}) , \end{aligned} \quad (2.32)$$

where $$E_t^{-}F_{1,t} = max\left[\frac{\alpha_0 w}{\alpha_1 \bar{Q}(1+E_t^{-}v_t)}, \frac{1-E_t^{-}v_t}{2}\right] ,$$

$$\begin{aligned} Y_{2,t} = & -(1-\delta)I_{2,t-1} - (1-\Delta)Q_{2,t-1} \\ & - \frac{\bar{Q}}{(1-E_t^{-}v_t)} ln(E_t^{-}F_{2,t}) + \frac{\alpha_0 w}{\alpha_1 (1-E_t^{-}v_t)^2}(1-E_t^{-}F_{2,t}^{-1}) , \end{aligned} \quad (2.33)$$

where $$E_t^{-}F_{2,t} = max\left[\frac{\alpha_0 w}{\alpha_1 \bar{Q}(1-E_t^{-}v_t)}, \frac{1+E_t^{-}v_t}{2}\right] .$$

In this case, $S_{j,t} = (1-\delta)I_{j,t-1} + Y_{j,t}$ and prices clear both of the submarkets.[4]

4. In the simple case where v_t is identically and independently distributed, the price equations are intractable. This problem does not arise in the period 1 problem.

These results are those of an atemporal model, since none of the agents consider the next period. The intertemporal choice problem involves expectations of the next period, and is given below as the period 1 problem.

2.4 The consumers' problem (period 1)

In the first period consumers decide on an optimal plan for consumption of the durable goods of each type and of the non-durable product, for this and the following period. Thus, consumers are maximizing the utility function

$$U(C_t^i, q_{1,t}^i, q_{2,t}^i) + \beta U(C_{t+1}^i, q_{1,t+1}^i, q_{2,t+1}^i) \,, \tag{2.34}$$

through the choices of C_t^i, C_{t+1}^i, $q_{1,t}^i$, $q_{1,t+1}^i$, $q_{2,t}^i$ and $q_{2,t+1}^i$. This decision is made subject to the following constraints:

$$q_{1,t}^i = (1-\Delta) q_{1,0}^i + s_{1,t}^i, \quad q_{1,t+1}^i = (1-\Delta) q_{1,t}^i + s_{1,t+1}^i \,, \tag{2.35}$$

$$q_{2,t}^i = (1-\Delta) q_{2,0}^i + s_{2,t}^i, \quad q_{2,t+1}^i = (1-\Delta) q_{2,t}^i + s_{2,t+1}^i \,, \tag{2.36}$$

$$A_t^i = (1+r)\left(A_0^i + Z_t^i - C_t^i - (P_{1,t} \cdot s_{1,t}^i) - (P_{2,t} \cdot s_{2,t}^i)\right) \,, \tag{2.37}$$

$$0 = (1+r)\left(A_t^i + Z_{t+1}^i - C_{t+1}^i - (P_{1,t+1} \cdot s_{1,t+1}^i) - (P_{2,t+1} \cdot s_{2,t+1}^i)\right) \,, \tag{2.38}$$

$$q_{1,t}^i \geq 0, \quad q_{1,t+1}^i \geq 0 \,, \tag{2.39}$$

$$q_{2,t}^i \geq 0, \quad q_{2,t+1}^i \geq 0 \,, \tag{2.40}$$

This set of constraints is essentially the same as that of the one-period problem (equations 2.2 to 2.6), except that now they must apply in both periods.[5]

Again there are four possible types of consumer, a situation which leads to four possible individual demand functions for the durable good.

Case 1: $q_{1,t}^i = q_{2,t}^i = 0$, which occurs if

$$\frac{\alpha_1 \gamma^i (1+v_t) \bar{Q}}{P_{1,t} - \beta (1-\Delta) E_t^+ P_{1,t+1}} < \alpha_0 \tag{2.41}$$

5. The one-period problem is the same as the period 2 problem given in sections 2.1, 2.2 and 2.3, since agents are only concerned with the current period.

$$\text{and} \qquad \frac{\alpha_1 (1-\gamma^i)(1-v_t)\bar{Q}}{P_{2,t} - \beta(1-\Delta)E_t^+ P_{2,t+1}} < \alpha_0 \quad . \tag{2.42}$$

Case 2: $q^i_{1,t} > 0, q^i_{2,t} = 0,$ or more specifically,

$$q^i_{1,t} = \frac{1}{\gamma^i(1+v_t)}\left[\bar{Q} - \frac{\alpha_0}{\alpha_1\gamma^i(1+v_t)}(P_{1,t} - \beta(1-\Delta)E_t^+ P_{1,t+1})\right] , \tag{2.43}$$

which occurs if condition (2.41) is not satisfied and

$$\frac{P_{1,t} - \beta(1-\Delta)E_t^+ P_{1,t+1}}{\gamma^i(1+v_t)} < \frac{P_{2,t} - \beta(1-\Delta)E_t^+ P_{2,t+1}}{(1-\gamma^i)(1-v_t)} \quad .$$

Case 3: $q^i_{1,t} = 0, q^i_{2,t} > 0,$ or more specifically,

$$\frac{1}{(1-\gamma^i)(1-v_t)}\left[\bar{Q} - \frac{\alpha_0}{\alpha_1(1-\gamma^i)(1-v_t)}(P_{2,t} - \beta(1-\Delta)E_t^+ P_{2,t+1}\right. \tag{2.44}$$

which occurs if condition (2.42) is not satisfied and

$$\frac{P_{1,t} - \beta(1-\Delta)E_t^+ P_{1,t+1}}{\gamma^i(1+v_t)} > \frac{P_{2,t} - \beta(1-\Delta)E_t^+ P_{2,t+1}}{(1-\gamma^i)(1-v_t)} \quad .$$

Case 4: $q^i_{1,t} > 0, q^i_{2,t} > 0,$

which occurs if neither condition (2.41) nor condition (2.42) is satisfied and

$$\frac{P_{1,t} - \beta(1-\Delta)E_t^+ P_{1,t+1}}{\gamma^i(1+v_t)} = \frac{P_{2,t} - \beta(1-\Delta)E_t^+ P_{2,t+1}}{(1-\gamma^i)(1-v_t)} \quad .$$

Note again that the set of consumers who have a multivalued demand correspondence has measure zero and can be ignored.

2.4.1 The aggregation problem

The aggregate demand function for the first type of durable good can be derived from

$$Q_{1,t} = \int_{A_1}\left(\frac{\bar{Q}}{\gamma^i(1+v_t)} - \frac{\alpha_0(P_{1,t} - \beta(1-\Delta)E_t^+ P_{1,t+1})}{\alpha_1[\gamma^i(1+v_t)]^2}\right)d\gamma^i , \tag{2.45}$$

where $$A_1 = \left\{\gamma^j; 1 > \gamma^j > max\left[\frac{\alpha_0 (P_{1,t} - \beta(1-\Delta) E_t^+ P_{1,t+1})}{\alpha_1 \bar{Q} (1+v_t)}\right],\right.$$

$$\left.\left[\frac{P_{1,t} - \beta(1-\Delta) E_t^+ P_{1,t+1}}{(1+v_t)} + \frac{P_{2,t} - \beta(1-\Delta) E_t^+ P_{2,t+1}}{(1-v_t)}\right]^{-1} \frac{P_{1,t} - \beta(1-\Delta) E_t^+ P_{1,t+1}}{(1+v_t)}\right\}$$

This condition indicates that γ^j must be sufficiently large for a consumer to decide to purchase a q_1 type of durable product.

Similarly, the aggregate demand function for the second type of durable good can be derived from

$$Q_{2,t} = \int_{A_2} \left(\frac{\bar{Q}}{(1-\gamma^j)(1-v_t)} - \frac{\alpha_0 (P_{1,t} - \beta(1-\Delta) E_t^+ P_{2,t+1})}{\alpha_1 [(1-\gamma^j)(1-v_t)]^2} \right) d\gamma^j , \qquad (2.46)$$

where $$A_2 = \left\{\gamma^j; 1 > \gamma^j > max\left[\frac{\alpha_0 (P_{2,t} - \beta(1-\Delta) E_t^+ P_{2,t+1})}{\alpha_1 \bar{Q} (1-v_t)}\right],\right.$$

$$\left.\left[\frac{P_{1,t} - \beta(1-\Delta) E_t^+ P_{1,t+1}}{(1+v_t)} + \frac{P_{2,t} - \beta(1-\Delta) E_t^+ P_{2,t+1}}{(1-v_t)}\right]^{-1} \frac{P_{2,t} - \beta(1-\Delta) E_t^+ P_{2,t+1}}{(1-v_t)}\right\}.$$

This condition indicates that γ^j must be sufficiently small for a consumer to decide to purchase a q_2 type of durable good.

Solving the integration problem yields the aggregate demand functions for the two types of consumer durable:

$$Q_{1,t} = -\frac{\bar{Q}}{(1+v_t)} ln(G_{1,t}) + \frac{\alpha_0 (P_{1,t} - \beta(1-\Delta) E_t^+ P_{1,t+1})}{\alpha_1 (1+v_t)^2} (1 - G_{1,t}^{-1}) , \qquad (2.47)$$

where $$G_{1,t} = max[B_{1,t}, D_{1,t}] ,$$

and $$B_{1,t} = \frac{\alpha_0 (P_{1,t} - \beta(1-\Delta) E_t^+ P_{1,t+1})}{\alpha_1 (1+v_t) \bar{Q}} ,$$

and $$D_{1,t} = \frac{(1-v_t)[P_{1,t} - \beta(1-\Delta) E_t^+ P_{1,t+1}]}{(1-v_t)[P_{1,t} - \beta(1-\Delta) E_t^+ P_{1,t+1}] + (1+v_t)[P_{2,t} - \beta(1-\Delta) E_t^+ P_{2,t+1}]}$$

and
$$Q_{2,t} = -\frac{\bar{Q}}{(1-v_t)} ln(G_{2,t}) + \frac{\alpha_0 (P_{2,t} - \beta(1-\Delta) E_t^+ P_{2,t+1})}{\alpha_1 (1-v_t)^2} (1 - G_{2,t}^{-1}) \quad , \tag{2.48}$$

where
$$G_{2,t} = max[B_{2,t}, D_{2,t}] \ ,$$

$$B_{2,t} = \frac{\alpha_0 (P_{2,t} - \beta(1-\Delta) E_t^+ P_{2,t+1})}{\alpha_1 (1-v_t)\bar{Q}} \ ,$$

$$D_{2,t} = \frac{(1+v_t)\,[P_{2,t} - \beta(1-\Delta) E_t^+ P_{2,t+1}]}{(1+v_t)\,[P_{2,t} - \beta(1-\Delta) E_t^+ P_{2,t+1}] + (1-v_t)\,[P_{1,t} - \beta(1-\Delta) E_t^+ P_{1,t+1}]} \ .$$

Clearly, the equations (2.47) and (2.48) indicate that two different demand regimes exist. When $B > D$, "Separate Demands," the demand functions appear to be independent of each other, and when $B < D$, "Substitution Demands," the demand functions are interrelated.

2.5 The producers' problem (period 1)

The producers' problem is set up as a dynamic programming problem, similar to that in Thurlow (1992). The problem of the risk-neutral representative firm producing in industry number 1, which does not face liquidity constraints, is outlined and solved below. Profits in the period are given by

$$\Pi^i_{1,t} = P_{1,t} \cdot S^i_{1,t} - w \cdot Y^i_{1,t} \ . \tag{2.49}$$

The first term is revenues and the last term is costs. The constraints that the firm faces are

$$S^i_{1,t} \leq (1-\delta) I^i_{1,t-1} + Y^i_{1,t} \ , \tag{2.50}$$

$$I^i_{1,t} = (1-\delta) I^i_{1,t-1} + Y^i_{1,t} - S^i_{1,t} \ , \tag{2.51}$$

$$Q_{1,t} = (1-\Delta) Q_{1,t-1} + \sum_{i=1}^{m} S^i_{1,t} \ , \tag{2.52}$$

and $Q_{1,t}$ is given by the demand equation (2.47). The first constraint is the inventory non-negativity constraint, where inventories depreciate at the rate δ. The second constraint is the inventory accumulation equation and the third constraint is the consumers' stock accumulation equation. The superscript i refers to the ith firm, and a total of m firms service each submarket.

Again, the producers' problem can be broken into two parts. The first problem is to decide on output, without current information about v_t, and the second problem, which is faced after v_t becomes apparent, is to decide on sales. Before proceeding, let us define the stock of the good available for sale as $K_{1,t}$. Thus $K_{1,t} = (1-\delta)I_{1,t-1} + Y_{1,t}$, where δ is the depreciation rate of inventory stocks (note that $\Delta > \delta$ is also assumed), $I_{1,t-1}$ is the inventory stock of durable good type 1 at the start of period t, and Y_{1t} is the output of the first durable good type during period t. A similar equation determines K_{2t}.

To solve this problem, the technique of dynamic programming with alternating value functions is employed. Let $J^i_1(I^i_{1,t-1}, Q_{1,t-1})$ and $J^i_2(K^i_{1,t}, v_t)$ be the value functions for the t^- and t^+ subperiods, respectively. The Bellman's equations are

$$J^i_1(I^i_{1,t-1}, Q_{1,t-1}) = max[-w \cdot Y^i_{1,t} + E^-_t(J^i_2(K^i_{1,t}, v_t))] \quad , \tag{2.53}$$

$$J^i_2(K^i_{1,t-1}, v_t) = max[P_{1,t} \cdot S^i_{1,t} + \beta E^+_t(J^i_1(I^i_{1,t}, Q_{1,t}))] \quad , \tag{2.54}$$

subject to the constraints (2.50), (2.51) and (2.52) given previously.

An interior solution of (2.53) implies that

$$w = \left[\frac{\partial(J^i_2(K^i_{1,t}, v_t))}{\partial K^i_{1,t}}\right] \tag{2.55}$$

is true. The interior solution gives the envelope equation

$$\frac{\partial J^i_1(I^i_{1,t-1}, Q_{1,t-1})}{\partial I^i_{1,t-1}} = (1-\delta)w \; . \tag{2.56}$$

Equation (2.55) indicates that in an interior solution, the marginal cost, w, is equal to the expected marginal benefit of having an extra unit of stock for sale, which is given by the term on the right-hand side. The envelope equation states that the value of an extra unit of starting inventory is the expected evaluation in the next subperiod. This, in turn, is equal to the expected marginal benefit of output, which is also equal to the marginal cost of output. This equation indicates that the value of an extra unit of inventory at the start of the period is the cost saving of not producing that marginal unit of output now.

The solution of (2.54) is

$$S^i_{1,t} = \begin{cases} [0, (1-\delta)I^i_{1,t-1} + Y^i_{1,t}) & if\ P_{1,t} = \beta(1-\delta)w \\ (1-\delta)I^i_{1,t-1} + Y^i_{1,t} & if\ P_{1,t} > \beta(1-\delta)w \end{cases} \; . \tag{2.57}$$

In subperiod t^+ there are two possible states in which sales are strictly positive: the "Inventory" and "Stockout" regimes.[6] These regimes are distinguished by the presence of end-of-period inventories in the former regime, and their absence in the latter.

In regime 1, Inventory, the envelope equation is

$$\frac{\partial J^i_2(K^i_{1,t}, v_t)}{\partial K^i_{1,t}} = \beta(1-\delta)^2 w \ . \tag{2.58}$$

The solution in this state indicates that if $\beta < 1$ and/or $\delta > 0$, then price falls below cost. The reason is that output has been decided upon (and built!) at the time of this decision, and the cost of production is a sunk cost. Thus, if the firm does not sell the marginal unit, it is carried as an inventory, and the extra unit of inventory implies that a cost saving can accrue in the next period. The envelope equation indicates that the value of an extra unit of inventory is the expected cost saving of not producing the marginal unit in the next period.

In regime 2, Stockout, the envelope equation is

$$\frac{\partial J^i_2(K^i_{1,t}, v_t)}{\partial K^i_{1,t}} = (1-\delta) P^S_{1,t} \ , \tag{2.59}$$

where the superscript S refers to the market-clearing Stockout price. This envelope equation indicates that an extra unit of start-of-period inventory leads to an extra sale, with its benefit being denoted by the price.

In subperiod t^- the state is unknown, but if we let the probability that regime 1 (Inventory) will occur be Φ_t,[7] then the condition for the interior solution of (2.53) is

$$0 = -w + \beta(1-\delta) w\Phi_t + (1-\Phi_t) E^-_t P^S_{1,t} \ . \tag{2.60}$$

This equation states that the cost of an extra unit of output is w, while the benefit depends upon whether or not a stockout occurs. If a stockout does not occur, then the extra unit of inventory means that a cost saving resulting from not producing the marginal unit next period is forthcoming. If a stockout does occur, then the extra unit of output is sold, which generates the market-clearing price, and this is the last term of equation (2.60).

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6. The other possible solution, which involves the collapse of the market, $S_{1,t} = 0$ if $P_{1,t} < \beta(1-\delta)w$, is not an equilibrium solution.

7. Note that Φ_t is potentially a complex function of many variables.

2.6 Equilibrium (period 1)

Thus far, two different demand regimes, Separate Goods and Substitution Goods, and two different supply regimes, Stockout and Inventory, have been identified. It is not possible to prove that a certain demand regime always corresponds to a certain supply regime; that would depend on the values of the parameters and the shock. It is possible to show the conditions that are necessary to get the correspondence, and this is outlined in the next subsection. The following subsections derive equilibrium solutions in two states of the world.

2.6.1 Supply-demand regime correspondences

Recall that in order for the Separate Goods demand regime to be realized, the necessary condition was $B_t > D_t$. This implies the general condition

$$P_{1,t} > \beta(1-\Delta)w + (1+v_t)\left[\frac{\alpha_1\bar{Q}}{\alpha_0} - \frac{P_{2,t} - \beta(1-\Delta)w}{(1-v_t)}\right] . \tag{2.61}$$

It should be noted that this condition incorporates the assumption (that will be proved below) that $E_tP_{1,t+1} = w$. When industry number 1 experiences a stockout, and industry number 2 maintains inventory, then this condition becomes

$$P^S_{1,t} > \beta(1-\Delta)w + (1+v_t)\left[\frac{\alpha_1\bar{Q}}{\alpha_0} - \frac{\beta(\Delta-\delta)w}{(1-v_t)}\right] , \tag{2.62}$$

where expressions for $P^S{}_{1,t}$ are derived as indicated below. The latter condition incorporates the result that $P^v{}_{2,t} = \beta(1-\delta)w$, where the superscript v refers to the inventory state. Obviously this condition changes somewhat if both industries experience a stockout or maintain inventories at the same time, but the focus of this paper is the situation where the two industries experience different supply conditions.

2.6.2 The solution

In order to derive the solution, some structure is placed on the stochastic process, v_t. In particular, v_t, which is assumed to be identically and independently distributed (iid), is given a symmetric two-point support. Thus, $v_t \in \{\underline{v}, \bar{v}\}$, where $-1 < \underline{v} < 0$, $0 < \bar{v} < 1$, and $|\underline{v}| = \bar{v}$, and the magnitudes of $\underline{v}$ and $\bar{v}$ are known to all agents in the t^- subperiod. It can be shown that this stochastic process is consistent with the two industries being faced with differing supply situations in all periods (except for the last period.) Consider now the two possible states:

Case A: Separate Goods ($B > D$)

In this state, industry number 1's demand function can be rewritten as

$$Q_{1,t} = -\frac{\bar{Q}}{(1+v_t)}[1 + lnX_t - X_t] \quad , \tag{2.63}$$

where
$$X_t = \frac{\alpha_0 [P_{1,t} - \beta(1-\Delta)w]}{\alpha_1 \bar{Q}(1+\bar{v})} \quad .$$

This equation can be solved for the stockout price, which is

$$P^S_{1,t} = \beta(1-\Delta)w + \frac{\alpha_1 \bar{Q}(1+\bar{v})}{\alpha_0 [e^{(H_{1,t}+1)} - 1]} \quad , \tag{2.64}$$

where $H_{1,t} = \dfrac{(1+\bar{v})Q_{1,t}}{\bar{Q}}$ and $Q_{1,t} = (1\text{-}\Delta)Q_{1,t\text{-}1} + (1\text{-}\delta)I_{1,t\text{-}1} + Y_{1,t}$.

Equation (2.64) can now be substituted into equation (2.60) and solved for $Y_{1,t}$. Note that the stochastic process implies that $\Phi_t = 0.5$ in equation (2.60). This expression is

$$Y_{1,t} = \frac{\bar{Q}}{(1+\bar{v})}[lnT^P_t - 1] - (1-\Delta)Q_{1,t-1} - (1-\delta)I_{1,t-1} \ , \tag{2.65}$$

where
$$T^P_t = 1 + \frac{(0.5)\alpha_1 \bar{Q}(1+\bar{v})}{\alpha_0 [1 - \beta(1 - 0.5(\delta+\Delta))]w} \quad ,$$

Finally, the expression for the sales of industry number 1 in the inventory state is

$$S_{1,t} = -\frac{\bar{Q}}{(1-\bar{v})}[1 + lnX_t] + \frac{\alpha_0 \beta(\Delta-\delta)w}{\alpha_1 (1-\bar{v})^2} - (1-\Delta)Q_{1,t-1} \ , \tag{2.66}$$

where
$$X_t = \frac{\alpha_0 [\beta(\Delta-\delta)w]}{\alpha_1 \bar{Q}(1+\bar{v})} \quad .$$

Case B: Substitute Goods ($B< D$)

The expression for the stockout price is

$$P^S_{1,t} = \beta(1-\Delta)w + \frac{(1+\bar{v})\beta(\Delta-\delta)w}{(1-\bar{v})[e^{(H_{1,t}+M_t)} - 1]} \quad . \tag{2.67}$$

where $M_t = \dfrac{\alpha_0 \beta(\Delta-\delta)w}{\alpha_1 \bar{Q}(1-\bar{v})}$ and again, $H_{1,t} = \dfrac{(1+\bar{v})Q_{1,t}}{\bar{Q}}$,

and $Q_{1,t} = (1\text{-}\Delta)Q_{1,t\text{-}1} + (1\text{-}\delta)I_{1,t\text{-}1} + Y_{1,t}$.

Equation (2.67) can now be substituted into equation (2.60) and solved for $Y_{1,t}$. Again, note that the stochastic process implies that $\Phi_t = 0.5$ in equation (2.60). This expression is

$$Y_{1,t} = \frac{\bar{Q}}{(1+\bar{v})}\left[lnT_t^B - \frac{\alpha_0\beta(\Delta-\delta)w}{\alpha_1\bar{Q}(1-\bar{v})}\right] - (1-\Delta)Q_{1,t-1} - (1-\delta)I_{1,t-1}, \tag{2.68}$$

where $$T_t^B = 1 + \frac{(1+\bar{v})(0.5)\beta(\Delta-\delta)}{(1-\bar{v})[1-\beta(1-0.5(\delta+\Delta))]}.$$

Finally, the expression for the sales of industry number 1 in the inventory state is

$$S_{1,t} = -\frac{\bar{Q}}{(1-\bar{v})}[lnG_{1,t}] - \frac{\alpha_0[P_{2,t}^S - \beta(1-\Delta)w]}{\alpha_1(1+\bar{v})(1-\bar{v})} - (1-\Delta)Q_{1,t-1}, \tag{2.69}$$

where $$G_{1,t} = \frac{(1+\bar{v})\beta(\Delta-\delta)w}{(1+\bar{v})\beta(\Delta-\delta)w + (1-\bar{v})[P_{2,t}^S - \beta(1-\Delta)w]}.$$

3 Calibration of the iid model

To illustrate the implications of the model, consider the following numerical example. Given the parameter values

$\beta = 0.995$ $\quad\Delta = 0.05$ $\quad\delta = 0$ $\quad w = 1.0$

$\bar{Q} = 20$ $\quad\bar{v} = 0.5$ $\quad\alpha_1 = 0.1$ $\quad\alpha_0 = 0.9$

the solution can be characterized numerically.

Consider first the implications for the average individual ($\gamma^i = 0.5$), when $\bar{v}$ is positive. The reservation prices are $P^R{}_{1,t} = 1.6667$ and $P^R{}_{2,t} = 0.5556$. The price ratio at which this consumer is indifferent to the two types of the good is $P_{1,t}/P_{2,t} = 3.0$.

It is important to determine whether these parameter values imply demand state A (Separate Goods, where $B > D$) or demand state B (Substitute Goods, where $B < D$). The prices are required in order to make these calculations; thus, assume state B. Below it is proved that this is the implied state. Given a positive realization of v_t, industry number 1 experiences a stockout. (Assume $Q_{j,t-1} = I_{j,t-1} = 0$.) Thus, output and sales for this industry are $Y_{1,t} = S_{1,t} = 16.0985$, and the price is $P^S{}_{1,t} = 1.0050$. In industry number 2, output is the same at $Y_{2,t} = 16.0985$, but sales are only $S_{2,t} = 12.7515$, and the price is $P^V{}_{2,t} = 0.9950$. Therefore in industry number 2, an inventory of $I_{2,t} = 3.3470$ is maintained. Both industries always offer the same quantity for sale, and inventories always exist in one industry. Note that even though profits are zero, the sales weighted price is 1.0006. This and another example are shown in Table 1 (p. 29).

Now let us confirm that demand state B is consistent with the parameter values. If industry number 1 experiences a stockout, then $B_{1,t} = 0.18$, and $D_{1,t} = 0.28$, and if this industry maintains an inventory, then $B_{1,t} = 0.04$, and $D_{1,t} = 0.71$. Thus, the demand state is confirmed.

At the aggregate level in this example, inventories are always maintained. Note that total industry demand is constant, and that a standard stockout avoidance inventory model applied at the aggregate level would show zero inventories being maintained. Moreover, the magnitude of the inventory could be quite large. If $\bar{v}$ were 0.7 instead of 0.5, then the inventory-output ratio would rise from 11 per cent to 88 per cent.

Note also that at the aggregate level, the overall industry would appear to be characterized by tranquillity and profitability, but this is by no means the case when the subindustries are considered. This example, and indeed this version of the model, are completely acyclical. The next step is to build persistence into the stochastic process v_t.

4 Persistence

Perhaps the simplest way to build persistence into the stochastic process v_t is to leave it with the symmetric two-point support given above, but to make the probabilities of the state of the world subject to a Markov chain. In particular, when $v_{t-1} = \underline{v}$, then $\Phi_t = q$, and when $v_{t-1} = \bar{v}$, then $\Phi_t = (1-q)$, where Φ_t is the probability of the inventory state being realized (see equation 2.60). If $q > 0.5$ then the fad shock exhibits positive serial correlation. The price and sales equations given above are appropriate here, as only the output equation changes. Again, there are two possible states:

Case A: Separate Goods ($B > D$)

Recall the expression for the stockout price:

$$P^S_{1,t} = \beta(1-\Delta)w + \frac{\alpha_1 \bar{Q}(1+\bar{v})}{\alpha_0 [e^{(H_{1,t}+1)} - 1]}, \tag{4.1}$$

where $H_{1,t} = \dfrac{(1+\bar{v})Q_{1,t}}{\bar{Q}}$ and $Q_{1,t} = (1-\Delta)Q_{1,t-1} + (1-\delta)I_{1,t-1} + Y_{1,t}$.

(i) Assume $v_{t-1} = \underline{v}$.

Equation (4.1) can now be substituted into equation (2.60) and solved for $Y_{1,t}$. Note that the stochastic process implies that $\Phi_t = q$ in equation (2.60). This expression is

$$Y^{pl}_{1,t} = \frac{\bar{Q}}{(1+\bar{v})}[\ln T^{pl}_t - 1] - (1-\Delta)Q_{1,t-1} - (1-\delta)I_{1,t-1}, \tag{4.2}$$

where $$T^{pl}_t = 1 + \frac{(1-q)\alpha_1\bar{Q}(1+\bar{v})}{\alpha_0[1-\beta((1-\Delta)-q(\Delta-\delta))]w},$$

the superscript *pl* refers to the Separate Goods state, and the lower demand state is expected.

(ii) Assume $v_{t-1} = \bar{v}$.

Equation (4.1) can still be substituted into equation (2.60) and solved for $Y_{1,t}$. Note that the stochastic process now implies that $\Phi_t = (1-q)$ in equation (2.60). This expression is

$$Y^{pu}_{1,t} = \frac{\bar{Q}}{(1+\bar{v})}[\ln T^{pu}_t - 1] - (1-\Delta)Q_{1,t-1} - (1-\delta)I_{1,t-1}, \tag{4.3}$$

where $$T^{pu}_t = 1 + \frac{q\alpha_1\bar{Q}(1+\bar{v})}{\alpha_0[1-\beta((1-\delta)-q(\Delta-\delta))]w},$$

the superscript *pu* refers to the Separate Goods state, and the upper demand state is expected.

Case B: Substitute Goods ($B < D$)

The expression for the stockout price is

$$P^S_{1,t} = \beta(1-\Delta)w + \frac{(1+\bar{v})\beta(\Delta-\delta)w}{(1-\bar{v})[e^{(H_{1,t}+M_t)} - 1]}, \tag{4.4}$$

where $M_t = \dfrac{\alpha_0\beta(\Delta-\delta)w}{\alpha_1\bar{Q}(1-\bar{v})}$ and again, $H_{1,t} = \dfrac{(1+\bar{v})Q_{1,t}}{\bar{Q}}$

and $Q_{1,t} = (1-\Delta)Q_{1,t-1} + (1-\delta)I_{1,t-1} + Y_{1,t}$.

(i) Assume $v_{t-1} = \underline{v}$.

Equation (4.4) can now be substituted into equation (2.60) and solved for $Y_{1,t}$. Again, note that the stochastic process implies that $\Phi_t = q$ in equation (2.60). This expression is

$$Y_{1,t}^{Bl} = \frac{\bar{Q}}{(1+\bar{v})}\left[lnT_t^{Bl} - \frac{\alpha_0\beta(\Delta-\delta)w}{\alpha_1\bar{Q}(1-\bar{v})}\right] - (1-\Delta)Q_{1,t-1} - (1-\delta)I_{1,t-1}, \tag{4.5}$$

where $T_t^{Bl} = 1 + \dfrac{(1+\bar{v})(1-q)\beta(\Delta-\delta)}{(1-\bar{v})[1-\beta((1-\Delta)-q(\Delta-\delta))]}$,

the superscript Bl refers to the Substitute Goods state, and the lower demand state is expected.

(ii) Assume $v_{t-1} = \bar{v}$.

Equation (4.4) can still be substituted into equation (2.60) and solved for $Y_{1,t}$. Again, note that the stochastic process implies that $\Phi_t = (1-q)$ in equation (2.60). This expression is

$$Y_{1,t}^{Bu} = \frac{\bar{Q}}{(1+\bar{v})}\left[lnT_t^{Bu} - \frac{\alpha_0\beta(\Delta-\delta)w}{\alpha_1\bar{Q}(1-\bar{v})}\right] - (1-\Delta)Q_{1,t-1} - (1-\delta)I_{1,t-1}, \tag{4.6}$$

where $T_t^{Bu} = 1 + \dfrac{(1+\bar{v})q\beta(\Delta-\delta)}{(1-\bar{v})[1-\beta((1-\delta)-q(\Delta-\delta))]}$,

the superscript Bu refers to the Substitute Goods state, and the upper demand state is expected.

5 Calibration of the Markov chain model

In order to keep this numerical example consistent with that given in Section 3, the parameter values are maintained at

$\beta = 0.995$	$\Delta = 0.05$	$\delta = 0$	$w = 1.0$
$\bar{Q} = 20$	$\bar{v} = 0.5$	$\alpha_1 = 0.1$	$\alpha_0 = 0.9$

In addition, the probability that current preferences remain unchanged, q, is set at 0.8.

The results are presented in Tables 2, 3 and 4 (pp. 30-32). At the aggregate level in this example, inventories are always maintained. As before, industry demand is constant, thus a standard stockout avoidance inventory model applied at the aggregate level would show zero inventories being maintained. Again, the magnitude of the inventory could be quite large. If $\bar{v}$ were 0.9 instead of 0.5, then the inventory-output ratio would rise from 9 per cent to 46 per cent.

There is no aggregate cycle in this model, but shocks do occur. However, at an aggregate level the shocks resemble supply shocks, since under the initial impulse, sales fall and the average price rises. Cycles do however show up at the subindustry level. At this level, the variance of output exceeds that of sales due to the inventory dynamics. The subindustry level volatility is offsetting, so that it does not appear at the aggregate level. Under certain circumstances the subindustry level volatility may not be offsetting. This could be the case, for example, if subindustry number 1 and subindustry number 2 were based in different countries.

6 Conclusions

The closed-form solution of the model built in this paper has shown that competitive behaviour, together with fad shocks and stockout avoidance inventory behaviour can cause firms to maintain large inventory stocks, even in the absence of additive aggregate demand shocks. Moreover, this paper has shown that fad shocks can appear, at the aggregate level, to be the same as aggregate supply shocks. Thus, the average price is influenced by the fad shock and the presence of stockouts. Finally, it has been shown that the dynamics at a subindustry level can be much more extreme than at an aggregate level. This accords well with the results of Bresnahan and Ramey (1992) who show that aggregation hides heterogeneous shocks and responses in the automobile industry.

The next step in this research would seem to be to include additive aggregate demand shocks with the fad shocks that producers face. Clearly, if producers can distinguish between the two types of shocks, then two different types of reaction will occur. The aggregate demand shocks may have large effects, however, if producers are not initially able to distinguish between the two types of shock.

Table 1: The iid model
Subindustry and aggregate quantities and prices

Period	v_t	Subindustry #1						Subindustry #2						Aggregate					
		I_{t-1}	Y_t	K_t	S_t	I_t	P_t	I_{t-1}	Y_t	K_t	S_t	I_t	P_t	I_{t-1}	Y_t	K_t	S_t	I_t	P_t
(i) $v_t = 0.5$																			
1	1.5	0	16.1	16.1	16.1	0	1.005	3.3	12.8	16.1	12.8	3.3	0.995	3.3	28.9	32.2	28.9	3.3	1.0006
2	0.5	0	16.1	16.1	12.8	3.3	0.995	3.3	12.8	16.1	16.1	0	1.005	3.3	28.9	32.2	28.9	3.3	1.0006
3	0.5	3.3	12.8	16.1	12.8	3.3	0.995	0	16.1	16.1	16.1	0	1.005	3.3	28.9	32.2	28.9	3.3	1.0006
(ii) $v_t = 0.7$																			
1	1.7	0	19.6	19.6	19.6	0	1.005	18.4	1.2	19.6	1.2	18.4	0.995	18.4	20.8	39.2	20.8	18.4	1.0044
2	0.3	0	19.6	19.6	1.2	18.4	0.995	18.4	1.2	19.6	19.6	0	1.005	18.4	20.8	39.2	20.8	18.4	1.0044
3	0.3	18.4	1.2	19.6	1.2	18.4	0.995	0	19.6	19.6	19.6	0	1.005	18.4	20.8	39.2	20.8	18.4	1.0044

Table 2: The Markov chain model

Subindustry and aggregate quantities and prices

$v_t = 0.5 \quad q = 0.8$

Period	v_{t-1}	vt	Subindustry #1						Subindustry #2						Aggregate					
			I_{t-1}	Y_t	K_t	S_t	I_t	P_t	I_{t-1}	Y_t	K_t	S_t	I_t	P_t	I_{t-1}	Y_t	K_t	S_t	I_t	P_t
1	0.5	0.5	1.9	12.1	14.0	12.1	1.9	0.995	0	16.7	16.7	16.7	0	1.001	1.9	28.8	30.7	28.8	1.9	0.9986
2	0.5	1.5	1.9	12.1	14.0	14.0	0	1.020	0	16.7	16.7	12.1	4.6	0.995	1.9	28.8	30.7	26.1	4.6	1.0084
3	1.5	1.5	0	16.7	16.7	16.7	0	1.001	4.6	9.4	14.0	12.1	1.9	0.995	4.6	26.1	30.7	28.8	1.9	0.9986
4	1.5	1.5	0	16.7	16.7	16.7	0	1.001	1.9	12.1	14.0	12.1	1.9	0.995	1.9	28.8	30.7	28.8	1.9	0.9986
5	1.5	0.5	0	16.7	16.7	12.1	4.6	0.995	1.9	12.1	14.0	14.0	0	1.020	1.9	28.8	30.7	26.1	4.6	1.0084
6	0.5	0.5	4.6	9.4	14.0	12.1	1.9	0.995	0	16.7	16.7	16.7	0	1.001	4.6	26.1	30.7	28.8	1.9	0.9986
7	0.5	0.5	1.9	12.1	14.0	12.1	1.9	0.995	0	16.7	16.7	16.7	0	1.001	1.9	28.8	30.7	28.8	1.9	0.9986

Table 3: The Markov chain model

Subindustry and aggregate quantities and prices

$v_t = 0.7 \quad q = 0.8$

Period	v_{t-1}	vt	Subindustry #1						Subindustry #2						Aggregate					
			I_{t-1}	Y_t	K_t	S_t	I_t	P_t	I_{t-1}	Y_t	K_t	S_t	I_t	P_t	I_{t-1}	Y_t	K_t	S_t	I_t	P_t
1	0.3	0.3	6.4	11.1	17.5	11.1	6.4	0.995	0	20.3	20.3	20.3	0	1.001	6.4	31.4	37.8	31.4	6.4	0.9990
2	0.3	1.7	6.4	11.1	17.5	17.5	0	1.020	0	20.3	20.3	11.1	9.2	0.995	6.4	31.4	37.8	28.6	9.2	1.0103
3	1.7	1.7	0	20.3	20.3	20.3	0	1.001	9.2	8.3	17.5	11.1	6.4	0.995	9.2	28.6	37.8	31.4	6.4	0.9990
4	1.7	1.7	0	20.3	20.3	20.3	0	1.001	6.4	11.1	17.5	11.1	6.4	0.995	6.4	31.4	37.8	31.4	6.4	0.9990
5	1.7	0.3	0	20.3	20.3	11.1	9.2	0.995	6.4	11.1	17.5	17.5	0	1.020	6.4	31.4	37.8	28.6	9.2	1.0103
6	0.3	0.3	9.2	8.3	17.5	11.1	6.4	0.995	0	20.3	20.3	20.3	0	1.001	9.2	28.6	37.8	31.4	6.4	0.9990
7	0.3	0.3	6.4	11.1	17.5	11.1	6.4	0.995	0	20.3	20.3	20.3	0	1.001	6.4	31.4	37.8	31.4	6.4	0.9990

Table 4: The Markov chain model

Subindustry and aggregate quantities and prices

$v_t = 0.9 \quad q = 0.8$

Period	v_{t-1}	vt	Subindustry #1						Subindustry #2						Aggregate					
			I_{t-1}	Y_t	K_t	S_t	I_t	P_t	I_{t-1}	Y_t	K_t	S_t	I_t	P_t	I_{t-1}	Y_t	K_t	S_t	I_t	P_t
1	0.1	0.1	16.4	8.8	25.2	8.8	16.4	0.995	0	29.9	29.9	29.9	0	1.001	16.4	38.7	55.1	38.7	16.4	0.9996
2	0.1	1.9	16.4	8.8	25.2	25.2	0	1.020	0	29.9	29.9	8.8	21.1	0.995	16.4	38.7	55.1	34.0	21.1	1.0225
3	1.9	1.9	0	29.9	29.9	29.9	0	1.001	21.1	4.1	25.2	8.8	16.4	0.995	21.1	34.0	55.1	38.7	16.4	0.9996
4	1.9	1.9	0	29.9	29.9	29.9	0	1.001	16.4	8.8	25.2	8.8	16.4	0.995	16.4	38.7	55.1	38.7	16.4	0.9996
5	1.9	0.1	0	29.9	29.9	8.8	21.1	0.995	16.4	8.8	25.2	25.2	0	1.020	16.4	38.7	55.1	34.0	21.1	1.0225
6	0.1	0.1	21.1	4.1	25.2	8.8	16.4	0.995	0	29.9	29.9	29.9	0	1.001	21.1	34.0	55.1	38.7	16.4	0.9996
7	0.1	0.1	16.4	8.8	25.2	8.8	16.4	0.995	0	29.9	29.9	29.9	0	1.001	16.4	38.7	55.1	38.7	16.4	0.9996

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