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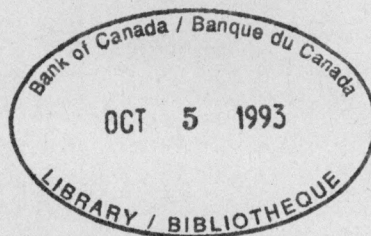
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**The Demand for M2+, Canada Savings Bonds  
and Treasury Bills**

by  
Kim McPhail



Bank of Canada



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September 1993

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and Treasury Bills**

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The views expressed here are those of the author.  
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## ABSTRACT

This paper uses the Johansen methodology to test for the existence of a stable long-run demand function for certain broad measures of liquidity in Canada. We find strong evidence of cointegration over the 1970-90 period between real balances, output, an own-rate of interest, a competing rate of interest and the rate of inflation. The strong evidence in favour of cointegration presented in this paper contrasts sharply with previously published research on the demand for the broad monetary aggregates in Canada. This difference can be attributed to three main factors. First, allowing inflation to have long-run effects on the demand for these broad aggregates appears to help in obtaining a money-demand function that is stable through the 1980s. Second, the Johansen tests for cointegration appear to be more powerful than the residual-based tests which have generally been used in previous research. Finally, allowing for portfolio shifts between a broad measure of money (M2+), treasury bill holdings and Canada Savings Bonds appears to explain part of the downward shift in M2+ during the 1982-84 period.

## RÉSUMÉ

Dans la présente étude, l'auteur met à contribution la méthode de Johansen pour vérifier l'existence d'une fonction de demande à long terme stable de certains agrégats monétaires au sens large au Canada. Les résultats obtenus indiquent une forte cointégration entre les encaisses réelles, la production, un taux d'intérêt propre, un taux d'intérêt concurrent et le taux d'inflation au cours de la période 1970-1990. Ces résultats se démarquent fortement de ceux générés par les travaux sur la demande des agrégats monétaires au sens large au Canada publiés antérieurement. La divergence peut être attribuée à trois grands facteurs. Premièrement, le fait que l'auteur permet à l'inflation d'exercer une incidence à long terme sur la demande de ces agrégats semble faciliter l'obtention d'une fonction de demande de monnaie stable tout au long des années 80.

Deuxièmement, les tests de cointégration de Johansen semblent être plus puissants que les tests, fondés sur des résidus d'estimation, qui ont été généralement utilisés dans les recherches antérieures. Enfin, le fait que l'auteur autorise des mouvements de portefeuille entre un agrégat monétaire au sens large (M2+), les bons du Trésor et les obligations d'épargne du Canada semble expliquer en partie la baisse de M2+ de 1982 à 1984.

## 1. INTRODUCTION

In the mid-1970s, the Bank of Canada adopted a policy of setting targets for the growth of M1 (which includes currency and net demand deposits). A consideration in the choice of M1 was that it appeared to be a relatively stable function of a small number of variables — a price variable, an output measure and a short-term interest rate. In the early 1980s, however, financial institutions began to offer daily interest deposits to the Canadian public. At the same time, high and variable interest rates had created an incentive for individuals and businesses to economize on their non-interest bearing deposits and hold their liquid assets in deposits that earned interest. The spread of new, higher-yielding accounts caused a shift out of M1 (which excluded these accounts) and into these new instruments. Partly as a result of the problems associated with trying to model and predict such shifts, the targets for M1 were dropped.

Since that time, greater focus has been placed on modelling the demand for the broader aggregates, which internalize many of the shifts described above and which appear less volatile. However, in addition to providing transactions services, which are the focus of the demand for narrow monetary aggregates, the broader aggregates provide major forms in which wealth can be held. Consequently, rates of return on substitute vehicles for storing wealth can be expected to influence the demand for money.

Recent research confirms the difficulty of finding evidence of stable, long-run relationships in Canada between a broad aggregate and real income, prices and interest rates — in the sense that money demand is cointegrated — unless a dummy variable allowing for some shift in the early 1980s is included.<sup>1</sup> One hypothesis that has been advanced to explain this shift is that money holders, when confronted with a severe recession, very high interest rates and a wider spread between the interest rate paid on debt and interest earned on financial assets, opted to consolidate their balance sheets, using deposits from M2 or M2+ (henceforth M2P)<sup>2</sup> to pay down debt. While this hypothesis is consistent with anecdotal evidence, it has been difficult to find strong statistical support for it. An M2P equation which includes the spread between consumer lending rates and

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1. See Ambler and Paquet (1992), Caramazza, Hostland and McPhail (1992), Caramazza, Hostland, and Poloz (1990) and McPhail and Caramazza (1990) for evidence of such a shift.
  2. M2 consists of M1 plus notice and personal-sector fixed-term deposits held at chartered banks. M2P adds to M2 those deposits held at other major financial institutions - trust companies, credit unions, etc. The data for M2P used in this paper do not incorporate the broadening of the definition of M2P in January 1992 to include money market mutual funds and annuities of life insurance companies.

deposit rates<sup>3</sup> does not remove the evidence of a shift, for example. Although non-linear effects of interest rates and income cannot be ruled out, other explanations for the downward shift in the 1982-84 period have been explored.

One alternative explanation focusses on portfolio shifts. Previous studies<sup>4</sup> have shown that Canada Savings Bonds are highly substitutable with M2P. In those studies, this substitution effect is captured by including the quantity of Canada Savings Bonds (CSB) as a regressor in the M2P equation. Another margin of substitution may have become increasingly important over the last decade. Until the 1980s, treasury bill holdings of households and non-financial corporations accounted for less than 1 per cent of combined holdings of M2P, CSB and treasury bills (Tbill); at the end of 1990, they represented 9 per cent of the three-asset portfolio (Figure 1, p. 38).

The break in trend of M2P in the 1982-84 period that has been identified in previous research is clearly evident in Figure 3 (p. 40). Figure 2 (p. 39) illustrates that over this same period, there was a sharp increase in combined holdings of CSB and Tbill. Indeed, in nominal terms, it is difficult to distinguish any break in trend for the combined aggregate M2P+CSB+Tbill over this period. It may be, therefore, that the shift out of M2P during this period was caused by a portfolio reallocation towards CSB and Tbill rather than by a scaling-down of balance sheets (beyond that associated with normal economic downturns).

One complication in modelling the links between M2P, CSB and Tbill is suggested by previous research (Caramazza, Hostland and McPhail 1990), which finds that the relationship between M2P, CSB and Tbill is asymmetric:<sup>5</sup> purchases of Canada Savings Bonds in the fourth quarter of each year tend to be funded by M2P balances, whereas redemptions of Canada Savings Bonds during the remainder of the year have been found to have no statistically significant impact on M2P but rather have tended to be added to Tbill holdings.

It is not surprising that Canada Savings Bonds give rise to such asymmetries, since they can be purchased only in the fourth quarter but can be redeemed (with interest) during any time of the

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3. The differential R90-B90, used later in this paper and also in other Canadian money demand studies, may be capturing this intermediation spread, to the extent that R90 (the commercial paper rate) moves in line with other *lending* rates, while B90 (the rate on 90-day term deposits) moves with *deposit* rates.
  4. McPhail and Caramazza (1990), Caramazza, Hostland and McPhail (1992), Caramazza, Hostland and Poloz (1990).
  5. Much of the initial research on the asymmetry of CSB effects is based on unpublished research at the Bank of Canada by Patrice Muller.



year. This asymmetry suggests that major realignments of portfolios may occur during the Canada Savings Bond sales period, whereas during other times of the year (denoted "redemption periods"), a more gradual response to variables that shift the demand for various assets may occur. Incorporating such asymmetries into a fully articulated dynamic model of each of the three assets would be difficult.

It may also be the case that the statistical insignificance of CSB effects on M2P during redemption periods reflects a difficulty in picking up the effects of more marginal and gradual portfolio adjustments that may occur during those periods — the signal-to-noise ratio being far stronger during the Canada Savings Bond sales period than during the rest of the year.

There are other difficulties in modelling the substitutability between M2P and Tbill. Treasury bill holdings of households and non-financial firms were negligible during the 1970s, but grew very rapidly during the early 1980s. One explanation for this rapid growth was the desire of individuals to obtain market rates of return on their liquid assets at a time when nominal interest rates were very high.<sup>6</sup> At the same time, government deficits grew rapidly, resulting in a sharp increase in issues of government debt, including treasury bills, while private-sector demand for credit was weak from 1982 to 1986. Anecdotal evidence suggests that, at that time, "commission rates"<sup>7</sup> charged by financial institutions or investment dealers for the retail purchase of treasury bills fell sharply. In the absence of any time series on commission fees, it would be difficult to capture these effects in an econometric model.

This paper models the links between M2P, CSB and Tbill in a different way, by considering whether the combined aggregates M2P+CSB and M2P+CSB+Tbill exhibit cointegration. Previous research<sup>8</sup> has already examined whether the failure to capture portfolio shifts between M2P and CSB can account for the lack of cointegration of M2P demand and the downward shift in the early 1980s. This hypothesis was evaluated by including CSB as a regressor along with output, prices and interest rates in a static model of M2P demand. Residual-based tests found little evidence of cointegration. If, in this study, we find that M2P+CSB is cointegrated (with prices, real income and rates of return, for example), then the lack of evidence in the previous studies may be due to the

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6. High-balance daily interest savings accounts, which pay a rate of return very close to the Tbill rate on the top tier of balances, did not become common until the second half of the 1980s. See Fine (1992).

7. By commission rates, I refer to the spread between the rate at which dealers or banks acquired treasury bills and the rate at which they offered them to the retail public.

8. Caramazza, Hostland and McPhail (1992).

different long-run functional relationship between M2P and CSB in the two cases (linear versus log-linear). If, instead, we find that M2P+CSB+Tbill is cointegrated but M2P+CSB is not, this would suggest that portfolio shifts between Tbill and the other two assets may have been an important factor in accounting for long-term trends in M2P in the 1980s. Since, for broad measures of liquidity, there is potential for substitution with "real" assets, we also include the inflation rate as a possible determinant of demand.

This method of analysis is open to a number of criticisms. The major criticism might be that the "simple-sum" method of aggregation of M2P, CSB and Tbill is not appropriate.<sup>9</sup> It is certainly different from some previous specifications, where the substitution between M2P and CSB has been modelled by including CSB as a regressor in an otherwise conventional money-demand function (McPhail and Caramazza 1990; Caramazza, Hostland and McPhail 1992; Caramazza, Hostland and Poloz 1990). Over the years, however, as M2 and M2P equations that include CSB as a regressor have been updated with additional data, problems with the original specification of CSB became apparent. The original *log-linear* relationship between CSB and M2P no longer produced credible estimates in that a \$1 change in CSB produced a change of *more than \$1* in M2P; a change in specification was introduced to incorporate a *linear* relationship between CSB and M2P. The simple-sum aggregation of M2P and CSB that is used in this paper assumes that the linear specification is appropriate and that, moreover, the substitution of M2P for CSB is dollar-for-dollar. This is not very different from a specification where CSB enters linearly, which finds, using recent data, that a \$1 increase in CSB during the Canada Savings Bond sales campaign results in a reduction of \$0.90 in M2P.<sup>10</sup>

The inclusion of the *quantity* of Canada Savings Bonds in equations for M2 and M2P that has been used in previous studies has always represented an uneasy compromise between the clear, empirical importance of abnormal Canada Savings Bond campaigns on the M2 and M2P aggregates, the specialized nature of the Canada Savings Bond instrument (it can be purchased only once a year) and theoretical and econometric considerations, which generally indicate that the

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9. Divisia indexes are the most theoretically appealing alternative. However, there is little recent evidence in Canada that using Divisia indexes improves the properties of the broad money equations. See Cockerline and Murray (1981) for a thorough study of the properties of Divisia indexes in Canada.

10. This is based on a first-difference log-linear specification which, apart from the CSB effect (which is converted from log-linear to linear form), is similar to that in McPhail and Caramazza (1990) or to the first-difference specifications in Caramazza, Hostland and McPhail (1992).

appropriate way to model portfolio substitution is to include the *rate of return* of competing assets as the explanatory variable. Aggregating M2P with CSB is more appealing in that it avoids treating an asset whose demand is determined jointly with that of M2P as an exogenous variable in M2P regressions. The asymmetric substitution between treasury bills and CSB mentioned above, as well as the difficulty in modelling the reduction in the cost of acquiring treasury bills, motivates modelling substitution between Tbill and M2P in the same way. The fact that the definition of M2P has been broadened recently to include money-market mutual funds (which can be thought of as “ownership shares” in treasury bills) is an added reason to “simple-sum” Tbill with M2P.<sup>11</sup>

A previous study (McPhail 1991) pursued the approach that is proposed here. The aggregates M2P+CSB and M2P+CSB+Tbill were regressed on prices, output and various rates of return. For M2P+CSB, residual-based tests for cointegration found some evidence of cointegration—augmented Dickey-Fuller (ADF) statistics usually significant at the 10 per cent level. However, the strongest evidence in favour of cointegration came from the aggregate M2P+CSB+Tbill, which when deflated by the consumer price index (Pcpi), generally produced ADF statistics that were significant at the 5 per cent level.

Since the power of residual-based tests for cointegration is known to be low, and since substantial bias can occur in the estimates of the cointegration vectors identified by static regressions, this paper uses the Johansen procedure to examine this issue. The Johansen procedure is a systems rather than a single-equation method. In addition to providing what one hopes are more powerful tests for the existence of cointegration and more efficient (and hence reliable) estimates of cointegration vectors,<sup>12</sup> the Johansen methodology has two important advantages over the Engle-Granger single-equation approach. It allows hypotheses about the set of cointegration vectors to be tested using asymptotically valid standard distribution theory (which cannot be done using the Engle-Granger approach). In addition, as part of the procedure, it specifies the number of cointegrating vectors that exist among a set of variables.

Finding a stable, long-run relationship between measures of liquidity and prices and output (and possibly a small number of other variables) would be useful for several reasons. First, such a

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11. The measure Tbill used in this paper includes only direct holdings of treasury bills. It excludes indirect holdings through money market mutual funds.

12. Gonzalo (1991) compares several estimators of cointegration vectors and concludes that the Johansen approach has the best properties.

relationship would provide a basis for assessing the rate of growth of a monetary aggregate that is consistent with price stability. Second, to the extent that a stable long-run relationship exists between the *levels* of the monetary aggregate, prices and output, deviations of money from this equilibrium relationship may provide leading information that is useful for forecasting future output or prices. This is the approach used in the P\* models of inflation developed at the Board of Governors of the Federal Reserve System by Hallman, Porter and Small (1989, 1991). It is different from typical monetary indicator models, which use rates of money *growth* as indicators for inflation and output growth (as in Muller, 1992, or Laidler and Robson, 1991).

## 2. DATA

The M2P series used in this paper is constructed slightly differently from published M2P data. M2 is based on "last Wednesday of month" data and the "plus" component is measured as of the "end-of-month." CSB is measured at end-of-month.<sup>13</sup> The treasury bill series is available only quarterly on an end-of-quarter basis.<sup>14</sup> To reduce the measurement problem that would arise from aggregating average-of-month-ends M2P+CSB with end-of-quarter treasury bills, two-quarter moving averages of the latter are used. Raw data are averaged to form quarterly series, and the raw series M2P+CSB and M2P+CSB+Tbill are then seasonally adjusted (to avoid residual seasonality that might result from seasonally adjusting the three components individually). Real balances are obtained by dividing nominal balances either by the consumer price index, P<sub>cpi</sub>, or the GDP deflator, P<sub>gdp</sub>.

The treasury bill data comprise holdings of (resident) households and non-financial corporations. Assessment of several alternative definitions of treasury bills in McPhail (1991) indicated that this measure produced the strongest evidence in favour of cointegration.<sup>15</sup>

Various possibilities exist for proxying the own rate of return (ROWN) on the aggregates. The one that is theoretically most appealing is to construct a weighted average of rates payable on each of their components. Since previous research cited in Caramazza, Hostland and McPhail (1992) for M2 and M2P (where a much more basic average of deposit rates was used) indicated little payoff to such an approach, the procedure of using a single interest rate to proxy for the own rate was adopted here. For the aggregate M2P+CSB, the 90-day term deposit rate at chartered banks (B90) was used, as it is the own rate proxy which has been found to provide the highest explanatory power for M2P in previous research. For the aggregate M2P+CSB+Tbill, B90 and the 90-day Tbill rate (RTB) were considered as possible proxies for the own rate. RTB is in some senses closer to a *marginal rate*, since it represents the yield earned on the highest-paying

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13. Canada Savings Bonds are valued at par. An alternative definition of CSB might be one that included accrued interest earnings.

14. The end-of-quarter treasury bill series is interpolated at the Bank of Canada from year-end data on stocks and quarterly data from the flow-of-funds accounts.

15. Other measures tested included treasury bill holdings of households only, and the CANSIM series treasury bill holdings of the non-bank public (which includes holdings of non-bank financial institutions and of non-residents). The CANSIM series has the advantage that it is available on a monthly basis. Given that the components of M2P (other than M1 and notice deposits) are limited to personal sector holdings, households' holdings of treasury bills might be the preferred measure on theoretical grounds.

component of the aggregate. Given the low quantities of treasury bills held by households and firms in the 1970s, however, it may not be very useful in the first half of the sample period. Competing rates are proxied by the 90-day rate on commercial paper (R90). The monetary aggregate, output (measured by real GDP,  $y$ ) and both prices are measured in logarithms. Interest rates are used in percentage points. Inflation is defined as:

$$\text{infl} = 400 * \Delta \log(p_t), \quad \text{where } p = P_{\text{cpi}} \text{ or } P_{\text{gdp}}$$

Figure 3 (p. 40) illustrates the relative movements in M2P, M2P+CSB and M2P+CSB+Tbill as a ratio of nominal GDP over the 1968 to 1990 period.<sup>16</sup> Each of the three series is dominated by an upward trend (that is, velocity is trending down over the sample period). However, the swings in the broader measures M2P+CSB and M2P+CSB+Tbill appear to be shorter — particularly between mid-1976 and 1984 and after the end of 1987. The detrended data<sup>17</sup> illustrate several sharp swings in the two aggregates (Figures 4 and 5, pp. 41 and 42): two declines in the constant dollar aggregates associated with downturns in real output in 1970 and 1981-82 and a third one that coincides with the rapid increase in inflation and the oil price shock of October 1973 and the accompanying sharp rise in the differential between the commercial paper rate and the treasury bill rate in 1974.

The behaviour of the interest rate differential RTB-R90 is also of interest, as it exhibits clearly different behaviour over the 1968-1975 period than over the latter part of the sample period. The earlier period is characterized by substantial volatility, whereas the later period is dominated by a progressive narrowing in the gap between the two rates. The B90-R90 differential is dominated by a negative trend over the entire sample period.

Table 1 (p. 27) presents the results of ADF unit root tests applied to the series used in this research. The aggregate M2P+CSB is I(1) when deflated by either  $P_{\text{gdp}}$  or  $P_{\text{cpi}}$ . The aggregate M2P+CSB+Tbill appears to be I(1) when deflated by  $P_{\text{gdp}}$  but it appears to contain two unit roots when deflated by the  $P_{\text{cpi}}$ . Interest rates appear to be I(1). The  $P_{\text{cpi}}$  appears to be I(2) and the GDP deflator is borderline between I(1) and I(2). The interest rate differential RTB-R90 is found to contain one unit root, whereas the differential B90-R90 appears to be stationary around a

16. Treasury bill data are available only from 1969Q1. In these graphs, Tbill holdings, which were negligible before 1975, are assumed to equal 0 in 1968.

17. This detrending is not meant to isolate accurate cycles in the variables, merely to indicate overall movements in the series.

deterministic trend. Since the small-sample properties of the ADF test can vary greatly with the dynamics of the processes involved, this test can only be considered to be a rough indication of the properties of the series.<sup>18</sup>

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18. Some of these results differ from those of McPhail (1991) because the ADF test is used in this paper whereas the Phillips-Perron test was used in the earlier research. Monte Carlo simulations illustrate that the Phillips-Perron test is very unreliable in samples of the size used here.

### 3. THE JOHANSEN METHODOLOGY

The Johansen test for cointegration begins by considering the unrestricted reduced form of a system of variables which, by assumption, can be represented as a finite-order vector autoregressive model (VAR):

$$Y_t = \mu + A(L)Y_{t-1} + U_t \quad (1)$$

where:<sup>19</sup>

$$Y_t = [Y_{1t} \dots Y_{nt}]^T$$

$$A(L) = \sum_{i=1}^k A_i L^i$$

$$E(U_t U_t^T) = \Sigma$$

The key requirement for this methodology to be useful is that none of the variables in the system be integrated of an order higher than 1 (that is, I(2) variables must be ruled out).<sup>20</sup> This approach also assumes that the disturbances of the model,  $U_t$  are distributed normally.<sup>21</sup>

Equation ( 1 ) can be recoded into an equivalent vector error-correction model (VECM) form:

$$\Delta Y_t = \mu + B(L)\Delta Y_{t-1} + \Pi Y_{t-k} + U_t \quad (2)$$

If there is no cointegration among the set of  $n$  variables included in  $Y$ , then the rank of  $\Pi$  is 0. In this case equation ( 2 ) becomes a simple VAR in first differences. On the other hand, if the rank of  $\Pi$  is  $n$ , then there are  $n$  linearly independent combinations of  $Y_t$  that are I(0). Since *any* linear combination of I(0) variables must also be I(0), this means that the  $n$  variables in  $Y_t$  must

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19.  $T$  refers to transpose.

20. Given a possible second unit root in (M2P+CSB+Tbill)/Pcpi, as indicated by the ADF test, Johansen tests which use this aggregate may be somewhat suspect.

21. However, Gonzalo (1991) finds that the Johansen approach is as robust to non-normality of the disturbances as other methods of estimating cointegration vectors.



themselves be  $I(0)$ , and it is appropriate to estimate an unrestricted VAR in *levels*, equation ( 1 ) (though it may be coded in error-correction form as in equation 2).

In the case which is of most interest in the present analysis, some, but probably fewer than  $n$ , stationary linear combinations may exist. In terms of equation ( 1 ), this means that there are cross-equation restrictions among the coefficients  $A(L)$ . The advantage of coding the system in VECM form is that, in equation ( 2 ), all of the cross-equation restrictions are concentrated in the matrix  $\Pi$ . In the Johansen approach, the number of such restrictions is determined by, first, estimating  $\Pi$  without restrictions from equation ( 2 ) and then determining the rank of  $\Pi$ . The restrictions on  $\Pi$  are then imposed and a restricted VECM is estimated.

In practice, this process is achieved in two steps. First,  $\Delta Y_t$  and  $Y_{t-k}$  are each regressed on a constant and lagged  $\Delta Y$ , yielding residuals:

$$RO_t = \Delta Y_t - \hat{a} - \hat{b}(L) \Delta Y_{t-1} \quad (3)$$

$$Rk_t = Y_{t-k} - \hat{c} - \hat{d}(L) \Delta Y_{t-1} \quad (4)$$

The ordinary least squares (OLS) estimate of  $\Pi$  from equation ( 2 ), with no restriction on the rank of  $\Pi$ , would then be obtained by regressing  $RO_t$  on  $Rk_t$ .

$\Pi$  can always be recoded in a variety of ways, as:

$$\Pi = \alpha\beta \quad \text{where } \alpha, \beta \text{ are } nxn$$

However, under the hypothesis that  $\Pi$  is of reduced rank, the dimensions of  $\alpha$  and  $\beta$  can be reduced to  $n \times r$ ,  $r < n$ . The Johansen procedure involves choosing the first column of  $\beta$  as the maximum likelihood estimate of the linear combination of  $Rk_t$ , (that is,  $\beta_1 Rk_t$ ) that is most correlated with  $RO_t$ . The second column of  $\beta$  is the linear combination, orthogonal to  $\beta_1$ , which has the next highest correlation, and so on. A representation of the VECM which is equivalent to equation ( 2 ) is then given by

$$\Delta Y_t = \mu + B(L) \Delta Y_{t-1} + \alpha ECM_{t-k} \quad (5)$$

$$\text{where } ECM_{t-k} = \beta Y_{t-k} \quad \beta = [\beta_1 \dots \beta_n]$$

Johansen and Juselius (1988)<sup>22</sup> provide two test statistics, denoted the “trace” and “ $\lambda$ -max” statistics, to determine the rank of the matrix  $\Pi$ . If, for example, the tests indicate that two cointegrating vectors exist, maximum likelihood estimates of those vectors are given by the first two columns of  $\beta$  and the VECM, equation (2), is given under the restriction of two cointegrating vectors by

$$\Delta Y_t = \mu + B(L)\Delta Y_{t-1} + \alpha_1 ECM_{1,t-k} + \alpha_2 ECM_{2,t-k} \quad (6)$$

$$\text{where: } ECM_i = \beta_i Y_{t-k} \quad i=1,2$$

All of the variables in (6) are now stationary and inference on any of the parameters in (6) can proceed using asymptotically valid standard distribution theory. The coefficients  $\alpha_1$  and  $\alpha_2$  represent the effects of the stationary linear combinations ECM1 and ECM2 on the system and are referred to as loadings (in a single-equation approach, they are frequently referred to as “speeds of adjustment”).

It should be noted that no structural interpretation can be placed on the error-correction terms without making further identifying assumptions. However, in the present research, if there were only one cointegration vector in the system, and if it included the financial aggregate, one might wish to interpret it as a “long-run” demand function. If more than one cointegration vector exists among the data, any structural interpretation is difficult to place on it, though examining the effect of each cointegration vector on future changes of the variables in the system may provide indirect evidence as to the nature of the relationship that has been identified.

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22. See their paper for technical details of how the tests are applied.

#### 4. THE JOHANSEN ESTIMATES OF THE COINTEGRATION VECTORS

##### (i) Identification of the cointegration vectors in the basic model

For the aggregate M2P+CSB, regressions were run over the period 1971Q1 (to allow for lags) to 1990Q4. Owing to availability of treasury bill data, regressions for the aggregate M2P+CSB+Tbill end in 1989Q4. The orders for the models were selected on the basis of Akaike's final prediction error criterion.

Table 2 (p. 28) reports the results of the Johansen tests for the aggregate M2P+CSB deflated by Pgdg (denoted rm1); Table 3 (p. 29) deflates the aggregate by Pcpj (rm2). The tests for cointegration were first applied to the bivariate system given by real balances and output (line 1). These show little evidence of cointegration, even at a 10 per cent significance level. In the next set of regressions (lines 2 and 3), an interest rate variable (either B90 or R90) was added to the VAR. This resulted in much stronger evidence of cointegration (all statistics are significant at least at the 5 per cent level). Output has the expected positive long-run effect on the aggregate, though in systems using Pgdg, this effect is very large (elasticity of 1.6 versus 1.2 for real balances deflated by the Pcpj).

In the system using Pgdg, interest rates have a positive and very large effect in the long run on the aggregate. For example, a 100-basis-point increase in B90 is associated with a 22 per cent increase in real balances in the long run. It is difficult to interpret any of these cointegration vectors as long-run money demand relationships. When both interest rates are added to the VAR (line 4), similar problems of interpretation arise.

Finally, the inflation rate was considered as a possible determinant of the long-run demand for real balances. In a system consisting of real balances, output and inflation (line 5 of Table 2) there is strong evidence of cointegration. The coefficient on income is more reasonable in this specification (1.2) and a 1 percentage point increase in the annual rate of inflation is associated with a 1.4 per cent decline in real balances in the long run. In line 6, this system is augmented by B90 and R90, the proxies for the own and competing rates of return on the aggregate. The coefficients on output and inflation are almost unchanged and each interest rate has the expected sign. In this system, there is also evidence of a second cointegration vector with a similar income coefficient (significant at the 5 per cent level).

Deflating the aggregate by the Pcp (Table 3) leads to broadly similar conclusions. The cointegration vector that has the most natural interpretation as a long-run money demand function comes from the system that includes own and competing interest rates as well as the rate of inflation. In this system, there is also evidence of a second cointegration vector. The income and inflation effects are similar to those from Table 2 and, although the interest-rate coefficients are somewhat different, they have the expected signs.

Graphs of the cointegration vectors identified by the Johansen methodology for the system [rm y B90 R90 infl] are presented in Figure 6 (p. 43).<sup>23</sup> Also shown are the error-correction terms corrected for the short-run dynamics. These are the variables  $\beta Rk$  where  $Rk$  is taken from equation 4). Since, over the sample period, the error-correction terms can exhibit considerable persistence and some drift because of variations in the dynamics of the variables in the system, correcting the error-correction terms for the effects of those dynamics gives a better visual impression of whether the error-correction terms are mean-reverting.

The Johansen tests described here provide much stronger indication of cointegration than the residual-based tests described in McPhail (1991), where evidence of cointegration was generally found at a 10 per cent level of significance. This may well reflect the better power of the Johansen tests, particularly in systems that have more than two or three variables.<sup>24</sup> Still, the small-sample properties of the Johansen tests have yet to be well understood.

Tables 4 and 5 (pp. 30 and 31) present the results of the Johansen tests for the aggregate M2P+CSB+Tbill. For the bivariate system given by real balances and output, there is some evidence of cointegration at the 10 per cent level, according to the  $\lambda$ -max statistic). When an interest rate is added to the VAR (lines 2 to 4), there is stronger evidence of cointegration (at the 5 per cent level, according to the  $\lambda$ -max statistics). In these systems, interest rates have a positive effect in the long run on the aggregate, regardless of which interest rate is used (B90, R90 or RTB) to represent the general movement in interest rates, or which price variable is used to deflate the aggregate.

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23. The graphs include two of the "potential" cointegration vectors (that is, two of the columns of the ECM matrix in equation 5). The first and occasionally the second (according to Tables 2 and 3) of these vectors are actually found to be stationary.

24. The critical values of Engle and Yoo (1988) for residual-based tests for cointegration increase dramatically as the number of variables in the system grows. Consequently, in a system with more than two or three variables, the power of residual-based tests is very low.

When both an own rate and the competing interest rate are included in the regression (lines 5 and 6), there is evidence of cointegration at the 1 per cent significance level in specifications using RTB as a proxy for the own rate (there is weaker evidence of cointegration when B90 is used as the own rate). In regressions using RTB, however, each interest rate has the opposite sign to what one would predict given a money demand interpretation to the cointegration vector. In addition, when PgdP is used as the price variable, the size of the coefficient on income is much larger than in the previous cases and seems excessively large (close to 2) for a money demand interpretation.

In a system consisting of real balances, output and inflation (line 7), there is strong evidence of cointegration using either PgdP or PcpI as a measure of prices (in regressions with PgdP, there is some evidence of a second cointegration vector). Inflation is negatively related to real balances in these specifications. Adding inflation to the model tends to reduce the long-run income elasticity compared to earlier specifications. When own and competing interest rates are added to this system (lines 8 and 9), there is again strong evidence of cointegration, although models proxying the own rate by B90 have the wrong sign on this variable. Using RTB as the own rate produces consistent evidence of a second cointegration vector.<sup>25</sup> Graphs of the cointegration vectors for this system are presented in Figures 7 and 8 (pp. 44 and 45).

In summary, for both aggregates M2P+CSB and M2P+CSB+Tbill, the Johansen tests find strong evidence of cointegration. Cointegration vectors from systems that include income, inflation and a proxy for own and competing interest rates have reasonable interpretations as long-run money demand functions, although as in any econometric study, further identification restrictions are necessary in order for such an interpretation to be valid. While these identification restrictions can never be tested conclusively, I argue below that more indirect evidence makes this interpretation reasonable.

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25. The second cointegration could be the inflation rate itself.

The Johansen tests were also run allowing for a linear trend in the cointegration vector from the system [rm y ROWN R90 infl]. There was again strong evidence of cointegration in the five-variable system. The coefficient on real output is reduced sharply, not surprisingly, since the trend in the cointegration vector is mainly picking up the effect of growth in potential output. The coefficient on inflation is also lower, as the average drift in inflation is now also being picked up by the trend in the cointegration vector. The fit of the system is not much improved by the inclusion of the trend.

## (ii) Tests of restrictions on the cointegration vectors of the system

The Johansen procedures allow one to test linear restrictions on the cointegration vectors. Finding that only one stationary linear combination exists in the system is very convenient, since in that case, the test can be interpreted as testing restrictions on the cointegration vector of interest. When more than one cointegration vector exists, however, the test developed in Johansen and Juselius' 1988 paper is less useful, as it was designed to evaluate whether the linear restriction holds across the whole *set* of cointegration vectors.<sup>26</sup>

In the present research, two hypotheses are of particular interest: (1) whether homogeneity exists between real balances and real income and (2) whether the interest rate differential alone matters for real balances or whether the level of nominal interest rates is also a factor. Tables 6 and 7 (p. 32) summarize the likelihood-ratio tests of these restrictions. Test statistics are distributed asymptotically as  $\chi^2$ .

Table 6 tests these two hypotheses for the aggregate M2P+CSB. In the system using Pgdp, the interest rate restriction is rejected, although the rejection is clearly coming from the second cointegration vector in the system (see Table 2, p. 28). The restriction on income is not rejected in the two cointegration vectors jointly but is rejected for the first cointegration vector considered alone.<sup>27</sup> In systems using Pcpri, the income restriction is not rejected but the interest rate restriction is (although the joint restriction is not rejected). Johansen estimates incorporating the restrictions which are not rejected are shown in Table 8 (p.33).

For the aggregate M2P+CSB+Tbill (Table 7), the restriction that interest rates enter with equal size but opposite sign coefficients in the cointegration vectors is rejected strongly in all models except the model [rm2, y, B90, R90, inf], where both B90 and R90 have very small coefficients. A long-run income coefficient of one is rejected when Pgdp is used as a deflator and RTB is used as the own rate, but not in the other systems. Maximum likelihood estimates of the systems with the income constraint imposed are presented in Table 9 (p. 33). It is still the case that models using B90 to proxy for the own rate have the wrong sign for this variable.

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26. A later paper, Johansen and Juselius (1990), develops tests for restrictions on a *particular* cointegration vector.

27. This is evaluated by applying the likelihood ratio test on the assumption that only the first potential error-correction term (which is the one that is significant in the equation for real balances) is cointegrated. Since, in fact, the tests suggest that two cointegration vectors exist, this test may not strictly be valid.

On balance, then, there is only mixed support for either of these hypotheses, although for the aggregate M2P+CSB+Tbill deflated by Pgdp, the interest rate restriction appears to be acceptable.

### (iii) The reduced-form VAR and the issue of weak exogeneity

When a "long-run money demand function" is estimated by means of a static cointegrating regression, additional problems to the potential for substantial finite-sample bias in the estimate of long-run money demand are the risks of multiple cointegration vectors (which can destroy the identifiability of the long-run money demand parameters by OLS) and the problem of normalization: there are as many super-consistent estimates of long-run money demand as there are variables in the money demand function. The systems approach addresses the normalization issue by restricting the *same* linear combinations to appear in all equations and the issue of multiplicity of cointegration vectors by estimating the number of cointegration vectors in the system.

Long-run money demand functions are frequently identified as the steady-state solutions to single-equation error-correction models for the monetary aggregate. Following the typical Hendry methodology, the key requirement for such a methodology to be successful is that of weak exogeneity<sup>28</sup> of the regressors for the parameters of interest. In the context of the parameters of a long-run (cointegrated) money demand function, this requires that the deviation of money from its long-run demand not affect any of the other variables in the system; if it did, there would be a loss of information by restricting the data used to estimate the parameters of interest to the dynamics of the money demand specification. The Johansen methodology can provide evidence as to whether the weak exogeneity assumptions required to justify typical single-equation approaches are valid.

Tables 2 and 3 (pp. 28 and 29) include the estimates of the loadings, that is, the components of  $\alpha$  from equation (3), estimated for each system with their corresponding t-statistics. These estimates are useful for two reasons: they assess the issue of weak exogeneity, and they provide an indication of the direction and strength of the effect of the corresponding error-correction term on the dynamics of the system. While the cointegration vectors identified by the Johansen methodology cannot be given a structural interpretation in the absence of additional identifying

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28. See Engle, Hendry, and Richard (1983) for a precise definition of weak exogeneity.

restrictions, finding, for example, that a loading has a “wrong sign” may warn against giving a particular structural interpretation to a specific cointegration vector. Similarly, finding that a particular cointegration vector induces no subsequent change in a particular variable in the system may rule out a structural interpretation. However, since the Johansen procedure is estimating reduced forms, this information cannot be conclusive.

For the aggregate M2P+CSB+Tbill, deflated by Pgdp, Table 4 (p. 30) indicates, for example, that in the bivariate system [rm1 y] and in the systems where one or both interest rates are added, the effect of the cointegration vector has the “wrong” sign, if one wishes to interpret the cointegration vector that has been identified as a long-run money demand relationship and if one accepts that disequilibrium will tend to be eliminated by changes in real balances rather than by changes in real income or interest rates.<sup>29</sup> Indeed, in all of these systems, the strongest effect of the cointegration vector is on real output. If a “long-run money-demand” interpretation is given to the cointegration vector, this effect would indicate that excess supply of money tends to increase real output as well as real balances. In the system in which RTB and R90 as well as inflation are included, however, the first cointegration vector has its strongest effect on real balances (now with an appropriate sign) and on the two interest rates, whereas the second cointegration vector again has its main effect on real output. When B90 is used to proxy the own rate, the single cointegration vector again has its main effect on real balances (with the “correct” sign on the loading) and on interest rates. The results of Table 5 (p. 31), where the aggregate is deflated by Pcp<sub>i</sub>, suggest a fairly similar pattern.

For the aggregate M2P+CSB deflated by Pgdp (Table 2, p. 28), in the system that includes inflation as well as both interest rates, the first cointegration vector also has its main influence in the equation for real balances and for the two interest rates, whereas the second cointegration vector has its main effect on output and is insignificant in the real balances equation. When this aggregate is deflated by Pcp<sub>i</sub>, on the other hand, both cointegration vectors appear to influence real balances. In this case, there is less justification in interpreting either cointegration vector as a potential long-run money demand function. However, the income and inflation effects in both of these vectors are similar.

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29. This will of course depend on the monetary regime and the extent to which the central bank and financial system accommodate a shock to money demand at prevailing interest rates.



These results concerning weak exogeneity are somewhat surprising when compared to the studies in other countries using a similar methodology. In contrast to studies using U.S., U.K., and Scandinavian data,<sup>30</sup> they suggest that the cointegration vector that looks most like a money demand function (that is, which is similar to the steady state implied by estimates of single-equation error-correction money demand models) Granger-causes not only money but also RTB, B90 and R90. This suggests that a gain in efficiency may be achieved when cointegration vectors are estimated by means of the Johansen "systems" approach rather than single-equation methods.

#### **(iv) Alternative specifications and the role of inflation in the cointegration vectors**

The results presented here have assessed whether portfolio shifts between M2P, CSB and Tbill can explain the previously reported lack of empirical support for cointegration of M2P demand. The results indicate strong evidence in favour of cointegration, though they do not clarify whether the stronger evidence here than in previous studies results from the different methodology used (Johansen versus residual-based tests), the different set of explanatory variables considered (in particular, the inclusion of inflation as a long-run determinant of money demand in the present study) or the different methodology to account for substitutability of M2 and M2P with other financial assets.

It would be interesting to know, in addition, whether the Johansen methodology adopted here provides any empirical evidence in favour of the "consolidation period" hypothesis. In previous research, I attempted to test the view that the severity of the recession and high borrowing rates contributed to the downward shift in M2P demand in 1982-84 by including a proxy for the cost of financial intermediation (the spread between interest rates on financial assets and liabilities) in the M2P equation (McPhail and Caramazza 1990). To test for this effect here, the 5-year mortgage rate and the 5-year rate on GICs were included as variables in the Johansen system. Little or no gain in explanatory power of the system occurred when these proxies were included, and the resulting cointegration vectors had no obvious interpretation. In addition, for the aggregate M2P+CSB, the Johansen systems were estimated with a dummy variable equal to 0 before 1982Q3 and 1 afterwards in order to evaluate how sensitive estimates of the long-run relationships between the economic variables of interest might be to such a shift. Estimation results are shown in line 7

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30. For example, Hendry and Mizon (1993) and Johansen and Juselius (1988).

of Tables 2 and 3 (pp. 28 and 29). Inclusion of the dummy variable does not alter the conclusions with respect to the number of cointegration variables in the system, the main properties of these vectors or issues of weak exogeneity. The main effect of including the dummy variable is on the estimates of the interest rate coefficients in the cointegration vectors.

Previous Bank of Canada research (McPhail and Caramazza 1990, Caramazza, Hostland and McPhail 1990) has also considered whether portfolio substitution between M2P and equity can help stabilize M2P demand and account for the downward shift in M2P demand in 1982-84. Evidence in favour of this hypothesis has been mixed. Including the earnings-price ratio (EP) in growth-rate equations for M2 and M2P does little to increase the explanatory power of those models.<sup>31</sup> However, at a more disaggregated level, adding the price-earnings ratio to a model of demand for *personal* M2 appeared to stabilize the equation and eliminate the need for the consolidation period dummy.<sup>32</sup>

The empirical results of this study suggest that the price-earnings ratio is picking up the same type of effects as the inflation rate. This should not come as a surprise, given the similarity in the low-frequency movements of the two series, as can be seen in Figure 9 (p. 46). If, for example, estimates of the cointegration vectors from the system [rm1 y RTB R90 EP] for M2P+CSB+Tbill are compared with those of the system [rm1 y RTB R90 infl], a number of similarities come to light. In both systems there is evidence of two cointegration vectors and the graphs of these vectors have the same general shape (Figures 7 and 10, pp. 44 and 47 respectively). The systems with inflation, however, have a somewhat higher explanatory power for real balances than the system with the earnings-price ratio.

There are several reasons to think that inflation might affect real balances. First, the opportunity cost of holding financial assets (such as the broad aggregates considered here) relative to "real" assets (for example, durable goods and housing) is the real interest rate. An increase in inflation, given the nominal interest rate, would reduce the relative return to holding financial assets and induce substitution towards "real" assets. In "after-tax" terms, these effects may be more pronounced, since wealth that is held in the form of principal dwellings escapes tax on capital

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31. McPhail and Caramazza 1990.

32. Caramazza, Hostland and McPhail (1992).

gains,<sup>33</sup> whereas the inflation premium on the rate of return to financial assets is taxed.<sup>34</sup> In addition, to the extent that the rate of inflation increases the after-tax spread between borrowing and deposit rates, higher inflation could create an incentive to consolidate balance sheets and contribute to a reduction in financial assets.

Abstracting from theoretical considerations and thinking about the usual dynamics of money demand models makes it clear that if nominal money balances adjust only gradually (and monotonically) to a permanent price level shock, then a change in the rate of inflation (a growth-rate shock) will have permanent effects on the level of real balances. The slower the adjustment of money to a change in prices, the larger the reduction of real balances will be in response to an inflation shock. In previously estimated M2 and M2P growth rate equations (McPhail and Caramazza 1990; Caramazza, Hostland and McPhail 1990), since the growth rate of nominal balances adjusts only gradually to changes in the rate of growth of prices, the level of real balances will be lower after the adjustment is complete (although the long-run effect of inflation in growth rate equations is quite small).

A recent IMF study by Boughton and Tavlas (1991) gives some idea of the range of estimated long-run effects of inflation on real balances in the G5 countries. The authors estimate partial adjustment and buffer stock models of demand for M1 and for a measure of broad money in the United States, Japan, Germany, the United Kingdom and France and find that inflation has negative effects on money demand in the United States, Japan and the United Kingdom. The effects of inflation on U.S. and Japanese M2 are estimated to be quite small. The effects on U.S. M1 and M3 in the United Kingdom are of similar magnitude to those reported here.

#### **(v) Dynamic specifications of money demand**

The Johansen procedure estimates a dynamic equation for money demand (one of the equations in the VECM of equation 6). However, this is a reduced-form equation containing no contemporaneous values of the variables in the system as explanatory variables. In this section of the paper, single-equation error-correction models are estimated for the monetary aggregates that allow for such contemporaneous effects.

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33. More recently, legislation allows for a lifetime capital gains exemption of \$500,000 dollars on most types of Canadian equity.

34. For most of the period, there was a \$1,000 interest income deduction, which partially reduced these effects.

Two different approaches to estimating the single-equation error-correction model are pursued. The first approach uses the typical single-equation methodology in which the steady state of the equation is determined according to the single-equation OLS estimates of:

$$A(L) \Delta rm_t = a_0 + B(L) \Delta y_t + C(L) \Delta (ROWN - R90)_t + D(L) \Delta R90_t \quad (7)$$

$$+ E(L) \Delta infl_t + \gamma rm_{t-1} + a y_{t-1} + b ROWN_{t-1} + c R90_{t-1} + d infl_{t-1}$$

The estimated coefficients on the lagged levels can then be recombined to yield the error-correction term:

$$ECM_t = rm_t - \frac{\hat{a}}{\hat{\gamma}} y_t - \frac{\hat{b}}{\hat{\gamma}} ROWN_t - \frac{\hat{c}}{\hat{\gamma}} R90_t - \frac{\hat{d}}{\hat{\gamma}} infl_t \quad (8)$$

The second method uses the cointegration vectors identified by the Johansen procedure to anchor the steady state of the error-correction model:

$$A(L) \Delta rm_t = a_0 + B(L) \Delta y_t + C(L) \Delta (ROWN - R90)_t + D(L) \Delta R90_t \quad (9)$$

$$+ E(L) \Delta infl_t + \gamma ECM_{t-1}$$

where ECM = cointegration vectors from Tables 2 and 3 (pp. 28 and 29).

Table 10 (p. 34), where Pgdg is used as the price variable, and Table 11 (p. 35), which uses Pcp, present the results of estimates of the error-correction model for the financial aggregate M2P+CSB. Third-order models were estimated. Column (1) anchors the single-equation error-correction model by the Johansen estimates from the system [rm y B90 R90 infl] from Tables 2 and 3. Since the Johansen procedure had identified two cointegration vectors in each system, both vectors were initially included in the original error-correction model specification.

In systems using Pgdg (Table 10), the second cointegration vector was found to be insignificant (this agrees with the reduced-form results of the Johansen model) and was therefore

dropped. The first cointegration vector is significant. About 70 per cent of the variation in real balances is explained by the model. In column (2) of Table 10, the error-correction model is estimated using equation (7). The resulting estimates are very similar to those of Column (1), though in column (2), the long-run interest rate effects are smaller. When a dummy variable allowing for a permanent shift beginning in 82Q3 was added to the specification in column (2) to test for consolidation period effects, it was insignificant. When a transitory dummy was included, it was not significant at the 5 per cent level (t-statistic of 1.5) but produced a minor reduction in the standard error of estimate of the equation.<sup>35</sup>

Column (3) of Table 10 drops the error-correction or lagged "levels" terms and consists, therefore, of a simple first-difference specification. The long-run income elasticity is 0.8, considerably larger than in recently estimated Almon lag monthly growth rate equations for M2P (Caramazza, Hostland and McPhail 1992) but smaller than when an anchor is provided to the level of real balances, as in columns (1) and (2). In column (3), the constant term would be contributing about 1.6 percentage points to annualized growth in the aggregate. Column (4) of Table 10 adds to the growth rate specification the same consolidation period dummy used in McPhail and Caramazza (1990) and Caramazza, Hostland and McPhail (1992). This dummy takes the value 1 from 82Q3 to 83Q4. It is insignificant.

Table 11 (p. 35) repeats the dynamic specifications when the aggregate is deflated by P<sub>cpi</sub>. In general, the regressions have somewhat less explanatory power than those in Table 10.

Table 12 (p. 36), where P<sub>gdp</sub> is used as the price variable, and Table 13 (p. 37), which uses P<sub>cpi</sub>, present the results of estimates of the error-correction model for M2P+CSB+Tbill. Third-order models were again used as a starting point. The contemporaneous price variable was found to have no effect on nominal money balances (or equivalently, since inflation is measured at annual rates, contemporaneous inflation had a coefficient of -0.0025 in equations 7 and 9). Hence, it was convenient to recode the dependent variable in the dynamic specifications as  $\log [(M+C+Tbill)_t/P_{t-1}]$ .

Column (1) of Tables 12 and 13 anchors the single equation error-correction model by the Johansen estimates from the system [rm y RTB R90 infl] shown in Table 2 and Table 3 respectively. Since the Johansen procedure had identified two cointegration vectors in each system, both vectors

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35. Including transitory changes in the mortgage-rate GIC differential also produced a modest improvement in the fit of the equations.

were initially included in the original error-correction model specification. The second cointegration vector was found to be insignificant (again agreeing with the reduced-form results of the Johansen model) and was dropped. The first cointegration vector is significant. About 75 per cent of the variation in real balances is explained by the model. Diagnostic tests find no evidence of autocorrelation, ARCH effects or non-normality of the residuals. Column (1a) of Table 13 incorporates the restricted Johansen vector from Table 9, line 4, where the income elasticity is constrained to one. (Again, the second cointegration vector was insignificant and was dropped.)

Column (2) of Tables 12 and 13 presents the single-equation estimates of the error-correction model. Both long-run interest rate semi-elasticities are larger than in the Johansen estimates, although the net long-run effect of interest rates is similar. The long-run effect of inflation is somewhat smaller as well.

Columns (3) to (4) use B90 to proxy for the own rate. Column (4), which contains the single-equation estimates, show that the long-run effect of B90 is still negative, as it was for the Johansen estimates. In general, then, using RTB to proxy for the own rate produces more satisfactory results than using B90 for this aggregate.

Finally, the PgdP models were re-estimated allowing for transitory effects from changes in mortgage-rate versus GIC differentials (since, despite the negative findings for these variables in the Johansen cointegration vectors, the dynamics of the error-correction models could be affected by these variables). There was no increase in the explanatory power of the regressions from the inclusion of these variables.

To assess the stability of various models, CUSUM and CUSUMSQ series were calculated. The PcpI models for M2P+CSB+Tbill showed some evidence of instability (moving outside of their approximate 5 per cent confidence intervals). The residuals as well as CUSUM and CUSUMSQ plots for several of the remaining models are shown in Figures 11 to 15 (pp. 48-52) along with the corresponding 5 per cent confidence intervals. The models deflating the aggregates by PgdP are generally stable.<sup>36</sup>

The residuals from the M2P+CSB models with error-correction terms show some evidence of "consolidation period" effects over several quarters (Figures 11 and 12). However, these effects

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36. The CUSUM series corresponding to the Johansen long-run estimates with an income elasticity constrained to one exhibit borderline evidence of instability.

appear to be temporary and are not important enough to move the recursive residuals outside of their confidence intervals. On the other hand, for the growth rate equation for M2P+CSB (Figure 13, p. 50), there is evidence of a more prolonged overestimate of money demand during the 1982-84 period. For the aggregate M2P+CSB+Tbill, there is little evidence from the ordinary residuals or from the CUSUM or CUSUMSQ plots of any consolidation-period effect.

The simulation properties of these models are shown in Figures 16 to 20 (pp. 53-55). The response of the error-correction models to real output shocks and to inflation shocks are generally similar (Figures 16, 17, 19, 20). The adjustment profiles are in general fairly smooth but very slow. The main difference between the models is in the response to changes in interest rates. The growth-rate model for M2P+CSB (Figure 18, p. 54) has faster speeds of adjustment but the long-run effects of changes in income and inflation are much smaller. In the growth-rate model, a considerable amount of the historical variation in real income, for example, is attributed to the constant term, leaving a smaller role for cyclical responses.

## 5. CONCLUSION

The Johansen methodology appears to be useful for modelling exercises of the kind pursued here. In general, the Johansen tests for the existence of cointegration appear to produce far more frequent evidence of cointegration than do augmented Dickey-Fuller tests or Phillips-Perron tests based on the residuals of a static cointegrating regression. However, Monte Carlo evidence indicating whether this reflects greater power of the Johansen tests in finite samples or whether the size (probability of Type I error) of the Johansen tests in finite samples may be underestimated is only now beginning to appear.

The results of this study confirm those of McPhail (1991), which suggested that cointegration existed between a "monetary" aggregate consisting of M2P+CSB+Tbill, real output and possibly interest rates and inflation. The evidence in favour of cointegration for the slightly narrower aggregate M2P+CSB is much stronger here than in the 1991 study. No shift variable is required in the early 1980s in order to find evidence of cointegration. The dynamic specifications estimated here appear more stable when the implicit price deflator for GDP is used as the measure of prices.

The estimated income and inflation responses of various models estimated here are reasonably stable across alternative specifications. The long-run income elasticity identified here is much higher than in previously estimated growth-rate equations and appears more in line with theoretical priors for broad aggregates, which are one of the major forms in which wealth can be held (assuming reasonable stability in the long-run relationship between income and wealth).

In the present research, the significance of the cointegrating vector, which resembles typical single-equation estimates of long-run money demand in explaining future interest rates, is an important feature of the data over the last twenty years. It reveals the potential for the Johansen systems approach to provide more efficient estimates of the long-run cointegrating vector of interest than can be obtained by restricting attention to a single-equation approach, as is typically done.



## TABLES

**TABLE 1**  
**Univariate unit root tests**  
**(70Q1-90Q4)**

	First differences t-statistics <i>b</i>	Levels t-statistics <i>b</i>
log(Pgdp)	3.35	1.41
log(Pcpi)	2.79	0.90
log(ugdp)	5.23*	2.34
log(M2P+CSB)	3.25	1.06
log(rm1)	4.45*	1.94
log(rm2)	4.18*	1.91
log(M2P+CSB+Tbill)	5.18*	0.16
log(rm1)	4.54*	2.31
log(rm2)	2.85	2.31
R90	6.21*	2.79
B90	6.19*	2.34
RTB	6.59*	2.08
R90-RTB	5.88*	2.85
RTB-B90	7.74*	4.55*
5% critical values	3.45	3.45

Notes: 5% critical values are from Fuller (1976, 373).

Regressions  $\Delta y_t = a + by_{t-1} + C(L)\Delta y_{t-1} + dt$

The order of  $C(L)$  is chosen by the FPE.

\* indicates significance at 5% level.

rm1 - aggregate deflated by Pgdp

rm2 - aggregate deflated by Pcpi

**TABLE 2**  
**Johansen tests for cointegration for M2P+CSB (using PgdP)**

System	Order	$\lambda$ -max	Trace	Cointegration vectors ( $\beta$ )	Loadings ( $\alpha$ ) <sup>1</sup>				
1) rml,y	2	11.5	15.9*	[1 -1.5]	0.024 (0.9)	0.102 (3.4)			
2) rml, y, R90	3	25.3†	40.2†	[1 -1.6 -0.14]	0.023 (1.4)	0.089 (4.7)	0.718 (0.3)		
3) rml, y, B90	2	26.3†	38.8‡	[1 -2.7 -0.22]	0.004 (3.0)	0.008 (4.8)	0.182 (1.0)		
4) rml, y, B90, R90	2	27.2*	50.8†	[1 -4.1 -0.50 0.14]	(0.003) (3.7)	0.005 (4.6)	0.094 (0.8)	0.146 (1.0)	
5) rml, y, infl	3	39.4‡	51.4‡	[1 -1.2 0.014]	-0.169 (5.4)	0.015 (0.4)	9.3 (0.9)		
6) rml, y, B90, R90, infl	2	45.2‡	97.0‡	[1 -1.2 -0.007 0.007 0.016]	-0.091 (3.7)	0.028 (0.9)	9.1 (2.7)	14.6 (3.5)	-9.3 (1.1)
		27.7†	51.8†	[1 -1.2 0.019 -0.044 0.008]	0.007 (0.5)	0.059 (3.9)	1.8 (1.1)	4.0 (2.0)	3.3 (0.8)
7) rml, y, B90, R90, infl, D82	2	49.7‡	123.8‡	[1 -1.3 0.008 -0.005 0.020 0.067]	-0.074 (3.3)	0.041 (1.6)	4.4 (1.6)	9.1 (2.6)	-10.7 (0.7)
		31.0†	51.8†	[1 -1.4 0.034 -0.049 0.006 0.055]	0.010 (0.7)	0.070 (4.2)	0.6 (0.4)	2.8 (1.3)	3.2 (1.6)

Note: Critical values for trace and  $\lambda$ -max statistics are taken from Johansen and Juselius (1988, Table D2.).

\* significant at 10% level

† significant at 5% level

‡ significant at 1% level

1. t-statistics in brackets

**TABLE 3**  
**Johansen tests for cointegration for M2P+CSB (using Pcp)**

System	Order	$\lambda$ -max	Trace	Cointegration vectors ( $\beta$ )	Loadings ( $\alpha$ ) <sup>1</sup>				
1) rm2,y	2	10.2	14.5	[1 -1.2]	-0.119 (0.9)	0.058 (3.4)			
2) rm2, y, R90	2	28.3‡	48.5‡	[1 -1.2 0.023]	-0.054 (5.0)	-0.032 (2.3)	-1.5 (0.7)		
		18.6†	20.2†	[1 -1.3 -0.006]	-0.052 (1.8)	0.139 (3.8)	11.5 (2.0)		
3) rm2, y, B90	2	29.2‡	48.2‡	[1 -0.6 0.115]	-0.012 (4.5)	-0.010 (3.3)	-0.311 (0.3)		
		17.7†	19.1†	[1 -1.3 -0.004]	-0.081 (2.4)	0.137 (3.4)	6.93 (1.4)		
4) rm2, y, B90, R90	2	32.8‡	62.2‡	[1 0.4 0.226 0.064]	-0.005 (5.0)	-0.004 (3.2)	-0.089 (0.6)	-0.156 (0.9)	
		16.0	29.5*	[1 -1.5 -0.049 0.046]	-0.019 (0.6)	0.100 (2.7)	0.758 (0.2)	-1.555 (0.3)	
5) rm2, y, infl	2	35.6‡	48.4‡	[1 -1.1 0.014]	-0.089 (5.1)	-0.003 (0.1)	-1.0 (0.2)		
6) rm2, y, B90, R90, infl	2	39.4‡	94.7‡	[1 -1.1 -0.022 0.017 0.017]	-0.073 (4.0)	-0.000 (1.6)	8.0 (3.1)	11.5 (3.6)	-6.3 (1.1)
		28.6†	55.4†	[1 -1.3 -0.036 0.008 0.012]	0.041 (2.7)	0.078 (4.0)	4.48 (2.1)	7.2 (2.7)	-7.6 (1.6)
7) rm2, y, B90, R90, infl, D82	2	51.4	131.7	[1 -1.3 0.000 -0.003 0.020 0.1131]	-0.060 (2.8)	0.056 (2.2)	4.2 (1.4)	9.2 (2.3)	-15.0 (2.3)
		33.1	80.3	[1 -1.5 -0.006 -0.018 0.007 0.097]	0.040 (3.3)	0.068 (4.7)	4.0 (0.3)	1.8 (0.8)	-4.0 (1.1)

Note: Critical values for trace and  $\lambda$ -max statistics are taken from Johansen and Juselius (1988, Table D2).

\* significant at 10% level

† significant at 5% level

‡ significant at 1% level

1. t-statistics in brackets

**TABLE 4**  
**Johansen tests for cointegration for M2P+CSB+Tbill (using PgdP)**

System	Order	$\lambda$ -max	Trace	RB <sup>2</sup>	Cointegration vectors ( $\beta$ )	Loadings ( $\alpha$ ) <sup>1</sup>				
1) rm1,y	2	14.2*	14.9	[0.154 0.263]	[1 -1.6]	0.042 (1.6)	0.107 (3.8)			
2) rm1, y, RTB	3	22.4†	29.9*	[0.179 0.308 0.218]	[1 -1.6 -0.010]	0.031 (1.5)	0.099 (4.5)	-0.743 (0.3)		
3) rm1, y, R90	2	22.1†	31.4*	[0.185 0.328 0.141]	[1 -1.6 -0.023]	0.020 (1.9)	0.056 (4.8)	2.200 (1.3)		
4) rm1, y, B90	2	22.9†	31.7†	[0.185 0.331 0.242]	[1 -1.7 -0.032]	0.021 (2.4)	0.044 (4.7)	0.945 (0.8)		
5) rm1, y, RTB, R90	3	36.2‡	56.1‡	[0.145 0.330 0.281 0.265]	[1 -2.0 0.191 -0.233]	0.010 (1.50)	0.031 (4.5)	1.028 (1.2)	2.047 (2.2)	
6) rm1, y, B90, R90,	2	23.6*	45.3*	[0.286 0.324 0.269 0.142]	[1 -1.9 -0.094 0.058]	0.023 (3.2)	0.037 (4.5)	0.573 (0.6)	0.794 (0.6)	
7) rm1, y, infl	2	43.2‡	59.8‡	[0.261 0.384 0.267]	[1 -1.3 0.021]	-0.060 (2.3)	0.098 (3.4)	-13.8 (1.6)		
		16.6†	16.7*		[1 -1.6 0.001]	0.018 (0.7)	0.096 (3.4)	4.70 (0.6)		
8) rm1, y, RTB, R90, infl	3	50.0‡	99.8‡	[0.316 0.409 0.392 0.451 0.258]	[1 -1.2 0.000 0.013 0.035]	-0.053 (3.6)	-0.015 (0.9)	6.10 (3.0)	6.961 (3.3)	-2.072 (0.4)
		28.4†	50.0†		[1 -1.5 0.048 -0.057 0.016]	-0.019 (0.7)	0.118 (3.8)	6.381 (1.7)	9.611 (2.4)	6.890 (0.7)
9) rm1, y, B90, R90, infl	3	50.1‡	90.4‡	[0.353 0.302 0.372 0.346 0.275]	[1 -1.2 0.012 -0.003 0.027]	-0.072 (3.6)	-0.015 (0.7)	6.494 (2.1)	9.335 (2.9)	-1.817 (0.3)

Note: Critical values for trace and  $\lambda$ -max statistics are taken from Johansen and Juselius (1988, Table D2).

\* significant at 10% level

† significant at 5% level

‡ significant at 1% level

1. t-statistics in brackets

**TABLE 5**  
**Johansen tests for cointegration for M2P+CSB+Tbill (using Pcp)**

System	Order	$\lambda$ -max	Trace	RB <sup>2</sup>	Cointegration vectors ( $\beta$ )	Loadings ( $\alpha$ ) <sup>1</sup>				
1) rm2,y	4	13.8*	14.8	[0.385 0.344]	[1 -1.4]	-0.26 (0.5)	0.196 (3.5)			
2) rm2, y, RTB	3	23.2†	29.8*	[0.387 0.428 0.163]	[0 -1.4 -0.009]	0.055 (1.9)	0.165 (4.7)	3.82 (0.7)		
3) rm2, y, R90	3	23.7†	31.0*	[0.405 0.412 0.136]	[1 -1.4 -0.014]	0.043 (2.1)	0.115 (4.5)	6.41 (1.5)		
4) rm2, y, B90	3	24.6†	31.5†	[0.410 0.421 0.308]	[1 -1.5 -0.015]	0.055 (2.6)	0.115 (4.4)	2.25 (0.7)		
5) rm2, y, RTB, R90	3	33.7‡	55.0‡	[0.420 0.388 0.176 0.162]	[1 -1.6 0.095 -0.132]	0.020 (2.5)	0.041 (4.0)	1.96 (1.4)	3.38 (2.1)	
6) rm2, y, B90, R90,	3	25.7*	44.6*	[0.440 0.428 0.246 0.115]	[1 -1.5 -0.018 0.001]	0.065 (3.1)	0.098 (3.7)	1.78 (0.5)	4.94 (1.1)	
7) rm2, y, infl	3	31.1‡	41.8‡	[0.466 0.263 0.296]	[1 -1.2 0.016]	-0.104 (3.9)	0.025 (0.6)	-3.98 (0.5)		
8) rm2, y, RTB, R90, infl	3	45.2‡	95.2‡	[0.489 0.382 0.312 0.322 0.308]	[1 -1.0 -0.017 0.027 0.040]	-0.037 (4.2)	-0.018 (1.5)	3.50 (2.2)	3.32 (2.0)	-1.04 (0.4)
		28.9†	50.0†		[1 -1.3 0.045 -0.062 0.016]	0.003 (0.2)	0.086 (3.6)	8.91 (2.9)	12.27 (3.6)	-4.54 (0.8)
9) rm2, y, B90, R90, infl	3	39.2‡	77.1‡	[0.458 0.336 0.295 0.212 0.290]	[1 -1.1 0.001 0.001 0.025]	-0.58 (3.6)	-0.011 (0.5)	4.85 (1.9)	7.51 (2.3)	-3.98 (0.8)

Note: Critical values for trace and  $\lambda$ -max statistics are taken from Johansen and Juselius (1988, Table D2).

\* significant at 10% level

† significant at 5% level

‡ significant at 1% level

1. t-statistics in brackets

**TABLE 6**  
**Likelihood ratio tests of restrictions on cointegration vectors**  
**M2P+CSB**

System	Restriction	Restriction is imposed on both cointegration vectors			Restriction is imposed on first cointegration vector		
		LR stat	MSL (%)	df	LR stat	MSL (%)	df
rm1, y, B90, R90, infl	1, -1, x, x, x	4.9	8.0	2	4.3	3.9	1
	x, x, 1, -1,x	10.6	0.5	2	0.0	97.5	1
	1, -1,1, -1,x	36.9	0.5	4	4.3	11.5	1
rm2,y,B90,R90,infl	1, -1, x, x, x	0.6	37.3	2			
	x, x, 1, -1,x	13.9	0.1	2			

LR stat likelihood ratio statistic  
MSL marginal significance level  
df degrees of freedom  
x unrestricted

**TABLE 7**  
**Likelihood ratio tests of restrictions on cointegration vectors**  
**M2P+CSB+Tbill**

System	Restriction	Restriction is imposed on both cointegration vectors		
		LR stat	MSL (%)	df
rm1, y, RTB, R90, infl	1, -1, x, x, x	8.0	1.8	2
	x, x, 1, -1,x	12.7	0.2	2
rm1, y, B90, R90, infl	1, -1, x, x, x	1.9	16.8	1
	x, x, 1, -1,x	7.7	5.5	1
rm2,y,RTB,R90,infl	1, -1, x, x, x	1.9	38.7	2
	x, x, 1, -1,x	15.4	0.4	2
rm2,y,B90,R90,infl	1, -1, x, x, x	0.2	65.5	1
	x, x, 1, -1,x	0.2	65.5	1

LR stat likelihood ratio statistic  
MSL marginal significance level  
df degrees of freedom  
x unrestricted

**TABLE 8**  
**Restricted estimates of cointegration vectors**  
**M2P+CSB**

System	Cointegration vectors	Loadings				
1) rm1, y, B90, R90, infl	[1 -1 <sup>c</sup> 0.027 -0.010 0.043]	-0.037 (3.5)	-0.015 (1.1)	3.18 (2.2)	5.04 (2.9)	-1.14 (0.3)
2) rm2, y, RTB, R90, infl	[1 -1 <sup>c</sup> -0.016 0.025 0.035]	-0.042 (4.2)	-0.019 (1.3)	4.01 (2.3)	3.82 (2.0)	-1.19 (0.4)
	[1 -1 <sup>c</sup> 1.36 -1.74 0.299]	0.000 (0.4)	0.003 (2.6)	0.044 (3.1)	0.060 (3.9)	0.233 (0.9)
3) rm2, y, B90, R90, infl	[1 -1 <sup>c</sup> 0.011 -0.003 0.034]	-0.041 (3.7)	-0.013 (0.8)	3.15 (1.8)	4.80 (2.2)	2.46 (0.7)

**TABLE 9**  
**Restricted Johansen estimates of cointegration vectors**  
**M2P+CSB+Tbill**

System	Cointegration Vectors ( $\beta$ )	Loadings ( $\alpha$ ) <sup>1</sup>				
1) rm1, y, B90, R90, infl	[1 -1 <sup>c</sup> 0.011 -0.010 0.027]	-0.047 (3.7)	-0.004 (0.3)	4.74 (2.7)	7.49 (3.5)	-3.70 (0.8)
	[1 -1 <sup>c</sup> 0.055 -0.091 0.013]	0.001 (0.1)	0.034 (3.3)	1.64 (1.4)	3.41 (2.5)	2.63 (0.9)
2) rm1, y, B90, R90, infl	[1 -1.2 -0.007 0.007 <sup>c</sup> 0.016]	-0.091 (3.7)	0.027 (0.8)	9.04 (2.6)	14.46 (3.4)	-9.31 (1.1)
	[1 -0.95 0.13 -0.13 <sup>c</sup> -0.005]	-0.015 (1.5)	0.007 (0.5)	-1.08 (0.8)	-0.31 (0.2)	8.71 (2.6)
3) rm1, y, B90, R90, infl	[1 -1 <sup>c</sup> 0.011 -0.011 <sup>c</sup> 0.027]	-0.050 (3.7)	-0.001 (0.1)	5.03 (2.8)	7.97 (3.5)	-3.92 (0.9)
	[1 -1 <sup>c</sup> 0.012 -0.012 <sup>c</sup> 0.005]	-0.016 (1.5)	0.009 (0.6)	-1.22 (0.8)	0.28 (0.1)	9.41 (2.6)
4) rm2, y, B90, R90, infl	[1 -1 <sup>c</sup> -0.014 0.010 0.023]	-0.053 (4.1)	0.011 (0.7)	5.2 (2.8)	7.5 (3.4)	-3.9 (1.0)
	[1 -1 <sup>c</sup> -0.022 -0.039 0.030]	0.017 (2.6)	0.027 (3.0)	2.1 (2.2)	3.5 (3.0)	-3.4 (1.6)

**TABLE 10**  
**Dynamic specifications for  $(M2P+CSB)_t/Pgdp_t$**   
**(71Q1-90Q4)**

	(1)		(2)		(3)		(4)	
	lag	coef.	lag	coef.	lag	coef.	lag	coef.
$\Delta(B90-R90)$ ( $\times 10^{-3}$ )	1	3.796 (3.3)	1	3.294 (2.6)	1	5.060 (3.4)	1	5.218 (3.5)
$\Delta R90$ ( $\times 10^{-3}$ )					2	2.641 (1.7)	2	2.775 (1.8)
$\Delta y$	0	0.113 (2.0)			0	-1.914 (3.8)	0	-2.081 (4.0)
$\Delta infl$					0	0.160 (2.2)	0	0.182 (2.5)
$\Delta rm$					1	0.195 (2.4)	1	0.211 (2.6)
constant					2	0.151 (2.3)	2	1.494 (2.3)
D82	0	-2.081 (9.2)	0	-2.186 (9.7)	0	-1.823 (6.7)	0	-1.805 (6.7)
ECM					1	0.336 (3.3)	1	0.305 (2.9)
		-0.389 (9.3)		-0.402 (2.5)		0.004 (3.4)		0.005 (3.6)
		-0.112 (9.6)		-0.106 (4.6)				-0.002 (1.3)

#### Long-run elasticities

Source:	Johansen (Table 2, line 6)	Single-equation	Single-equation First-difference	Single-equation First-difference
y	1.2	1.3	0.8	0.8
B90	0.007	0.001	0.012	0.012
R90	-0.007	-0.002	-0.015	-0.014
infl	-0.016	-0.016	-0.003	-0.003

RB <sup>2</sup>	0.717	0.717	0.625	0.628
SEEx10 <sup>-2</sup>	0.4394	0.4391	0.5053	0.5033
DW	1.93	1.95	2.06	2.09
LM4 ( $\chi^2(4)$ )	4.3	4.9	6.6	8.0
ARCH4 ( $\chi^2(4)$ )	5.2	3.6	2.9	3.2
JB ( $\chi^2(2)$ )	3.2	2.2	2.1	1.1
order	3	3	3	3

Notes: t statistics in brackets

c constrained from Johansen estimates

LM4 Lagrange multiplier test for temporal dependence of residuals of up to 4th order

ARCH4 Lagrange multiplier test for up to 4th order ARCH model

JB Jarque-Bera test for normality of residuals



**TABLE 11**  
**Dynamic specifications for  $(M2P+CSB)_t/Pcpi_t$**   
**(71Q1-90Q4)**

	(1)		(1a)		(2)		(3)		(4)	
	lag	coef.	lag	coef.	lag	coef.	lag	coef.	lag	coef.
$\Delta(B90-R90)$ ( $\times 10^{-3}$ )	1	2.988 (2.2)	0	2.650 (1.8)	1	2.845 (1.9)	1	3.359 (2.3)	1	3.450 (2.4)
$\Delta R90$ ( $\times 10^{-3}$ )							0	-1.642 (3.1)	0	-1.788 (3.3)
$\Delta y$	0						0	0.176 (2.4)	0	0.207 (2.6)
$\Delta infl$	0	1.951 (6.8)	0	-1.800 (6.2)	0	-1.906 (6.3)	0	-1.410 (4.4)	0	-1.393 (4.4)
$\Delta rm2$	1	0.192 (1.9)	1	0.215 (2.1)	1	0.198 (1.9)	1	0.525 (5.8)	1	0.490 (5.2)
constant		-0.069 (1.2)		-0.008 (0.1)		-0.190 (1.1)		0.002 (1.8)		0.003 (2.1)
D82										-0.002 (1.1)
ECM1		-0.085 (6.1)		-0.053 (5.0)		-0.086 (2.9)				
ECM2		0.020 (1.9)		0.013 (2.4)						

#### Long-run elasticities

Source:	Johansen (Table 3, line 6)	Johansen (Table 8, line 2)	Single-equation	Single-equation First-difference	Single-equation First-difference
y	1.0	1.0 <sup>c</sup>	1.2	0.9	0.9
B90	0.018	0.010	0.017	0.007	0.007
R90	0.000	-0.033	-0.019	-0.011	-0.010
infl	-0.019	-0.020	-0.013	-0.003	-0.003

RB <sup>2</sup>	0.679	0.686	0.670	0.601	0.601
SEEx10 <sup>-2</sup>	0.4818	0.4765	0.4890	0.5377	0.5368
DW	1.98	2.02	2.00	2.3	2.3
LM4 ( $\chi^2(4)$ )	6.3	4.2	6.2	8.7	10.1*
ARCH4 ( $\chi^2(4)$ )	2.0	1.3	1.4	3.6	4.0
JB ( $\chi^2(2)$ )	3.1	2.8	2.3	1.8	1.5
order	3	3	3	3	3

Notes: t statistics in brackets

c constrained from Johansen estimates

LM4 Lagrange multiplier test for temporal dependence of residuals of up to 4th order

ARCH4 Lagrange multiplier test for up to 4th order ARCH model

JB Jarque-Bera test for normality of residuals

\* significance at 5 per cent level

**TABLE 12**  
**Dynamic specifications for  $(M2P+CSB+Tbill)_t/Pgdp_{t-1}$**   
**(71Q1-89Q4)**

	(1)		(2)		(3)		(3a)		(4)	
	lag	coef.	lag	coef.	lag	coef.	lag	coef.	lag	coef.
$\Delta(\text{ROWN-R90})$ ( $\times 10^{-3}$ )	0	3.208 (2.4)	0	3.975 (2.4)	1	2.912 (3.2)	1	4.633 (3.9)	1	2.557 (2.4)
$\Delta R90$ ( $\times 10^{-3}$ )			2	3.106 (2.1)					2	2.281 (2.2)
constant	2	1.138 (3.0)	2	0.836 (2.2)			1	0.909 (2.0)		
ECM		-0.131 (13.5)		-0.192 (1.3)		-0.212 (14.0)		0.011 (21.0)		-0.338 (2.3)
		-0.053 (15.1)		-0.065 (3.3)		-0.069 (15.0)		-0.039 (14.3)		-0.082 (4.4)

#### Long-run elasticities

Source:	Johansen (Table 4, line 8)	Single-equation	Johansen (Table 4, line 9)	Johansen (Table 9, line 1)	Single-equation
y	1.20	1.23	1.23	1.00 <sup>c</sup>	1.31
ROWN	0.002	0.0233	-0.0125	-0.0110	-0.0033
R90	0.0126	-0.0331	0.0003	0.0101	-0.0071
infl	-0.0349	-0.0281	-0.0272	-0.0270	-0.0242

RB <sup>2</sup>	0.771	0.758	0.764	0.759	0.754
SEEx10 <sup>-2</sup>	0.3688	0.3788	0.3739	0.3782	0.3815
DW	1.76	1.84	1.85	1.81	1.82
LM4 ( $\chi^2(4)$ )	4.3	6.3	3.9	4.7	4.7
ARCH4 ( $\chi^2(4)$ )	4.1	5.2	3.5	2.8	3.0
JB ( $\chi^2(2)$ )	3.1	1.4	1.2	0.6	2.3
order	3	3	3	3	3

Notes: Columns (1) and (2) use RTB as the own rate; columns (3) and (4) use B90 as the own rate.  
t statistics in brackets

c constrained from Johansen estimates

LM4 Lagrange multiplier test for temporal dependence of residuals of up to 4th order

ARCH4 Lagrange multiplier test for up to 4th order ARCH model

JB Jarque-Bera test for normality of residuals

**TABLE 13**  
**Dynamic specifications for  $(M2P+CSB+Tbill)_t/Pcpi_{t-1}$**   
**(71Q1-89Q4)**

	(1)		(1a)		(2)		(3)		(3a)		(4)	
	lag	coef.	lag	coef.	lag	coef.	lag	coef.	lag	coef.	lag	coef.
$\Delta(\text{ROWN-R90})$ ( $\times 10^{-3}$ )	0	3.208 (2.1)	0	2.979 (2.0)	0	3.319 (2.0)	1	2.066 (2.0)	1	2.373 (2.3)		
$\Delta R90$ ( $\times 10^{-3}$ )	2	1.029 (2.8)	2	1.001 (2.7)	2	1.022 (2.3)					2	0.746 (1.8)
$\Delta(m20)$	1	0.238 (2.4)	1	0.237 (2.4)	1	0.236 (2.3)	1	0.240 (2.4)	1	0.224 (2.4)	1	0.228 (2.2)
constant		0.031 (7.5)		0.006 (5.6)		-0.050 (0.3)		-0.109 (6.4)		0.004 (3.7)		-0.208 (1.1)
ECM		-0.036 (7.0)		-0.041 (7.0)		-0.052 (1.7)		-0.061 (6.6)		-0.041 (6.5)		-0.067 (2.2)

#### Long-run elasticities

Source:	Johansen (Table 5, line 8)	Johansen (Table 9, line 4)	Single-equation	Johansen (Table 9, line 3)	Single-equation	Johansen (Table 5, line 9)
y	0.95	1.00 <sup>c</sup>	1.08	1.13	1.00 <sup>c</sup>	1.23
ROWN	0.0170	0.0158	0.0245	-0.0015	-0.0101	-0.0169
R90	-0.0273	-0.0247	-0.0331	-0.0014	0.0032	-0.0201
infl	-0.0398	-0.0355	-0.0264	-0.0250	-0.0338	-0.0239

RB <sup>2</sup>	0.756	0.756	0.743	0.747	0.744	0.737
SEEx10 <sup>-2</sup>	0.4060	0.4056	0.4160	0.4134	0.4158	0.4212
DW	1.99	1.98	2.00	1.90	2.00	1.90
LM4 ( $\chi^2(4)$ )	2.7	2.7	3.4	2.6	3.0	2.0
ARCH4 ( $\chi^2(4)$ )	5.3	5.1	4.3	3.6	3.9	3.5
JB ( $\chi^2(2)$ )	1.0	0.9	2.2	0.5	0.6	0.1
order	3	3	3	3	3	3

Notes: Columns (1) and (2) use RTB as the own rate; columns (3) and (4) use B90 as the own rate.  
t statistics in brackets

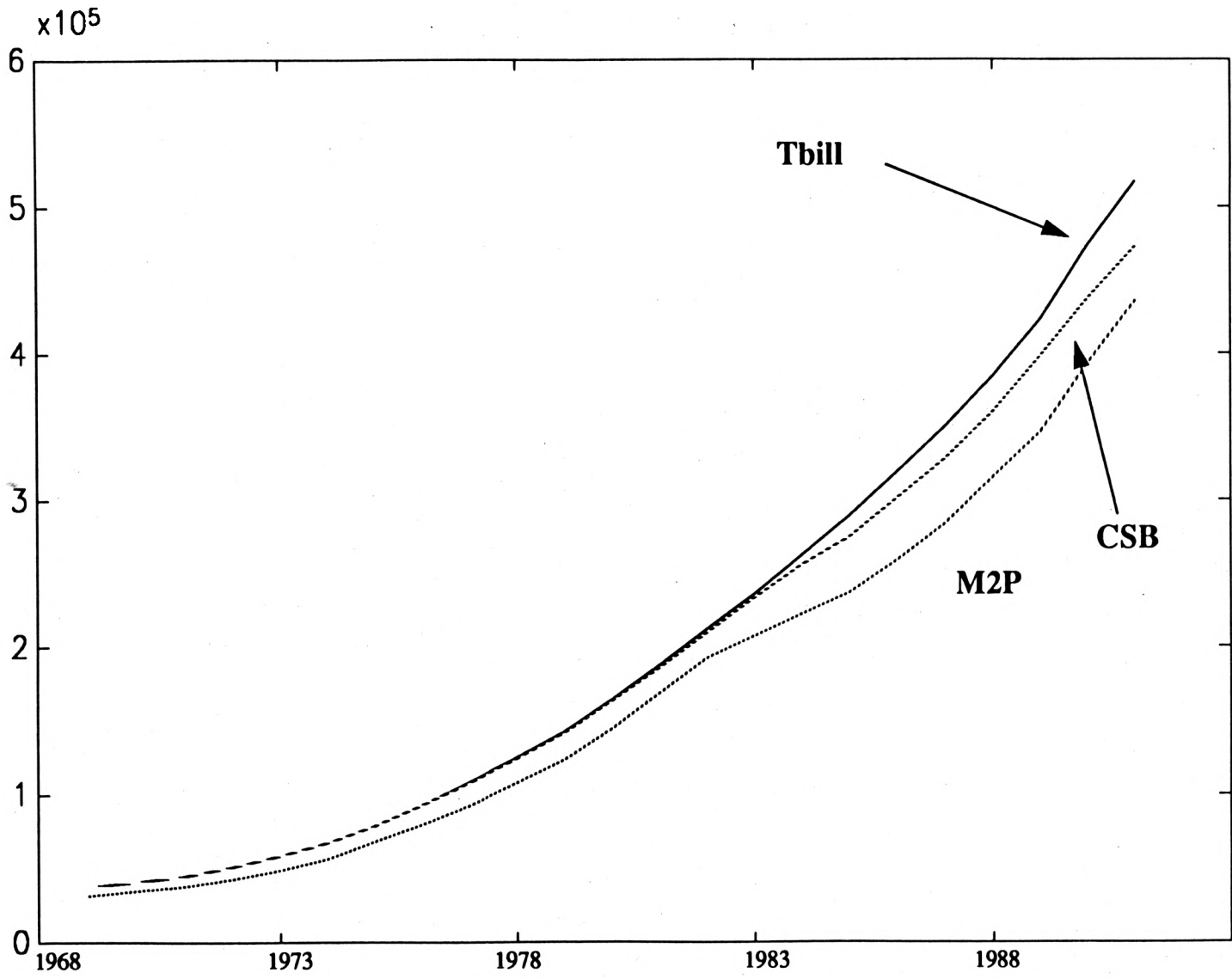
c constrained from Johansen estimates

LM4 Lagrange multiplier test for temporal dependence of residuals of up to 4th order

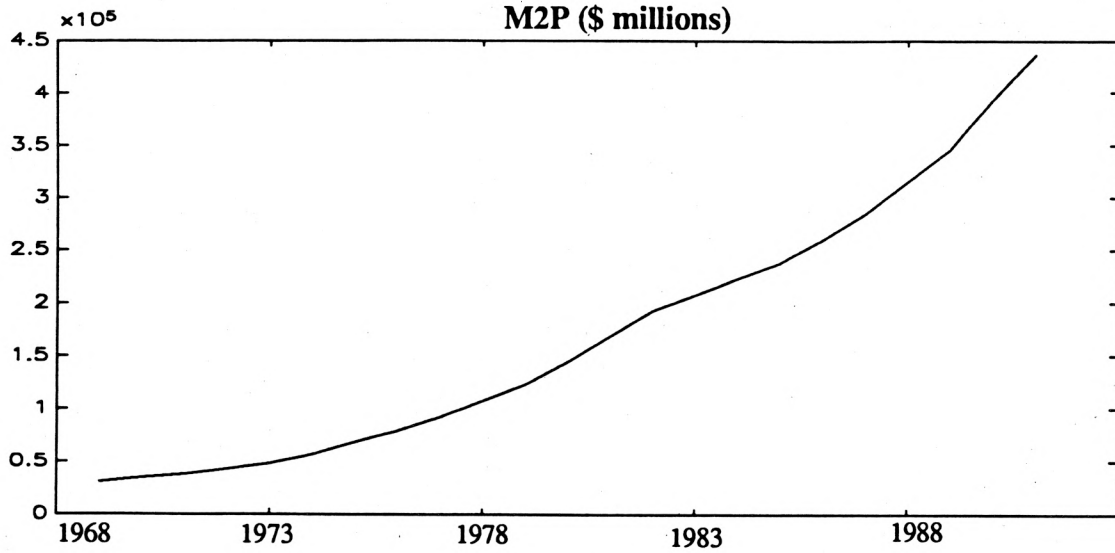
ARCH4 Lagrange multiplier test for up to 4th order ARCH model

JB Jarque-Bera test for normality of residuals

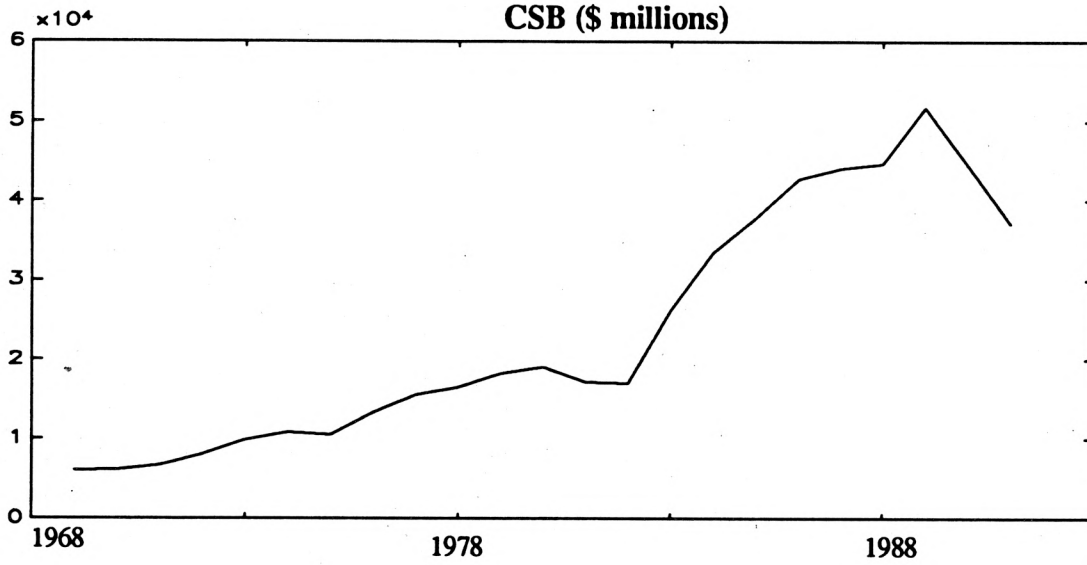
**Figure 1**  
**Composition of M2P+CSB+Tbill (\$ millions)**



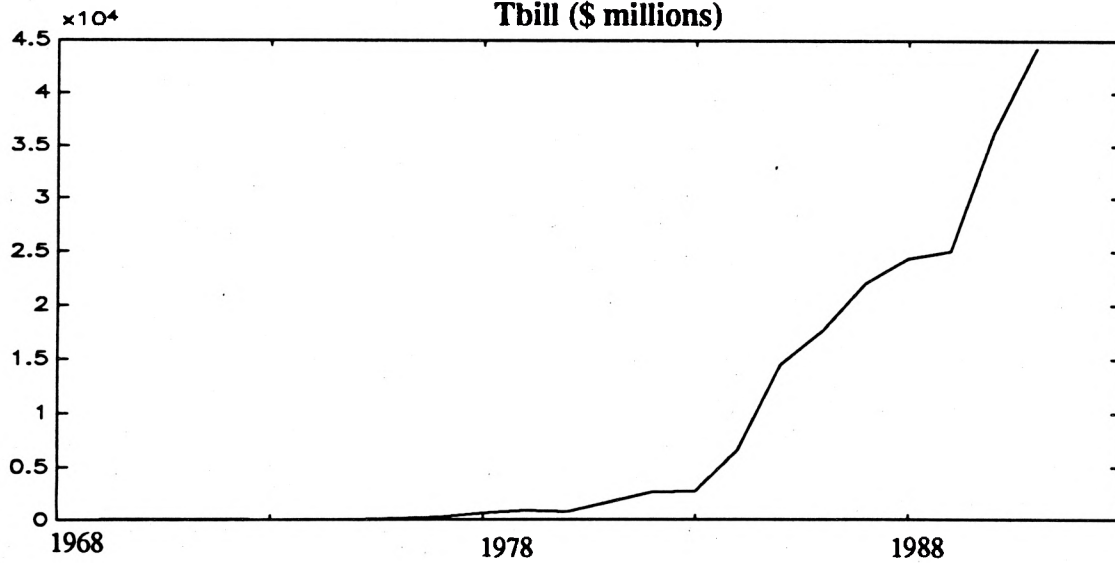
**Figure 2**  
**M2P (\$ millions)**



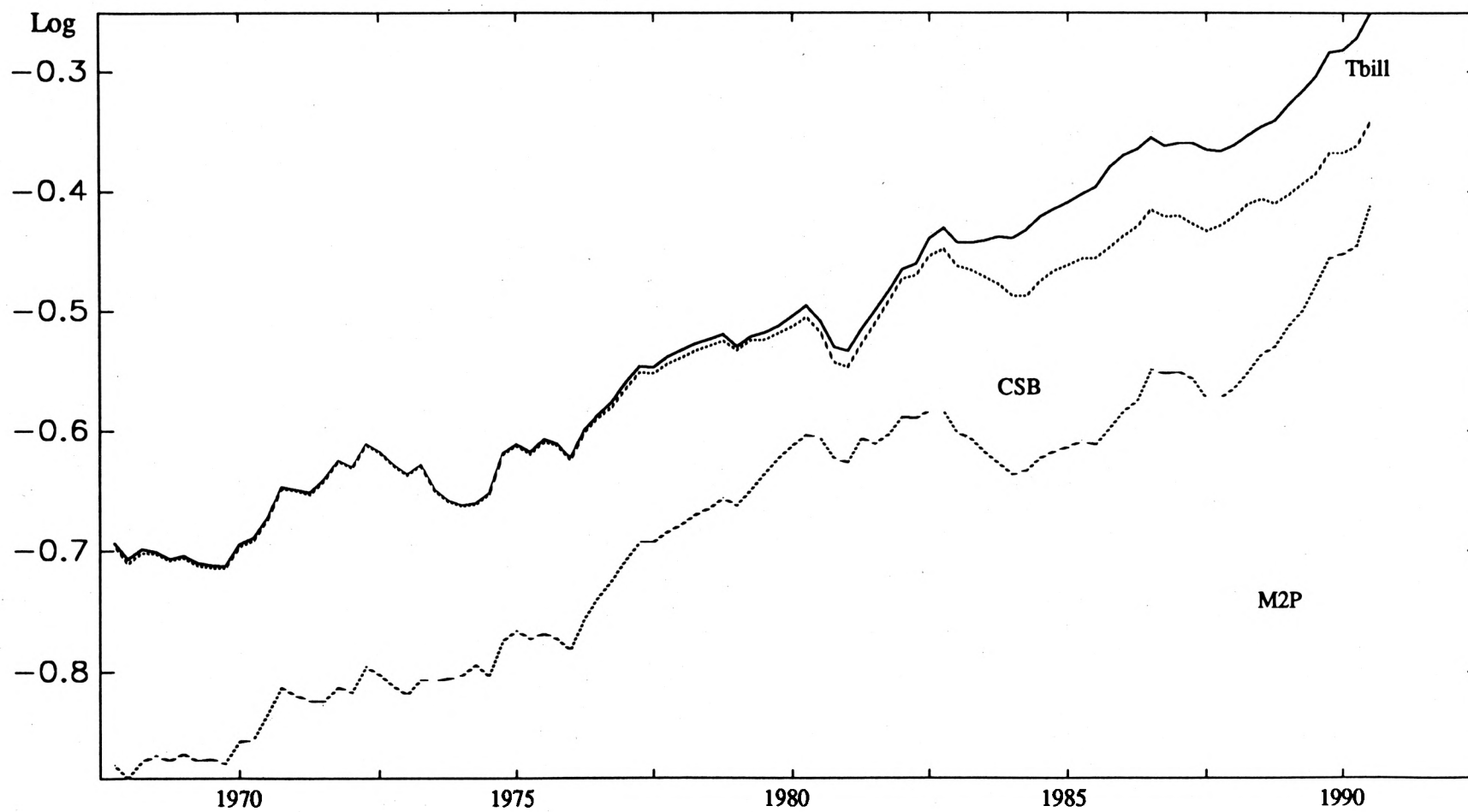
**CSB (\$ millions)**



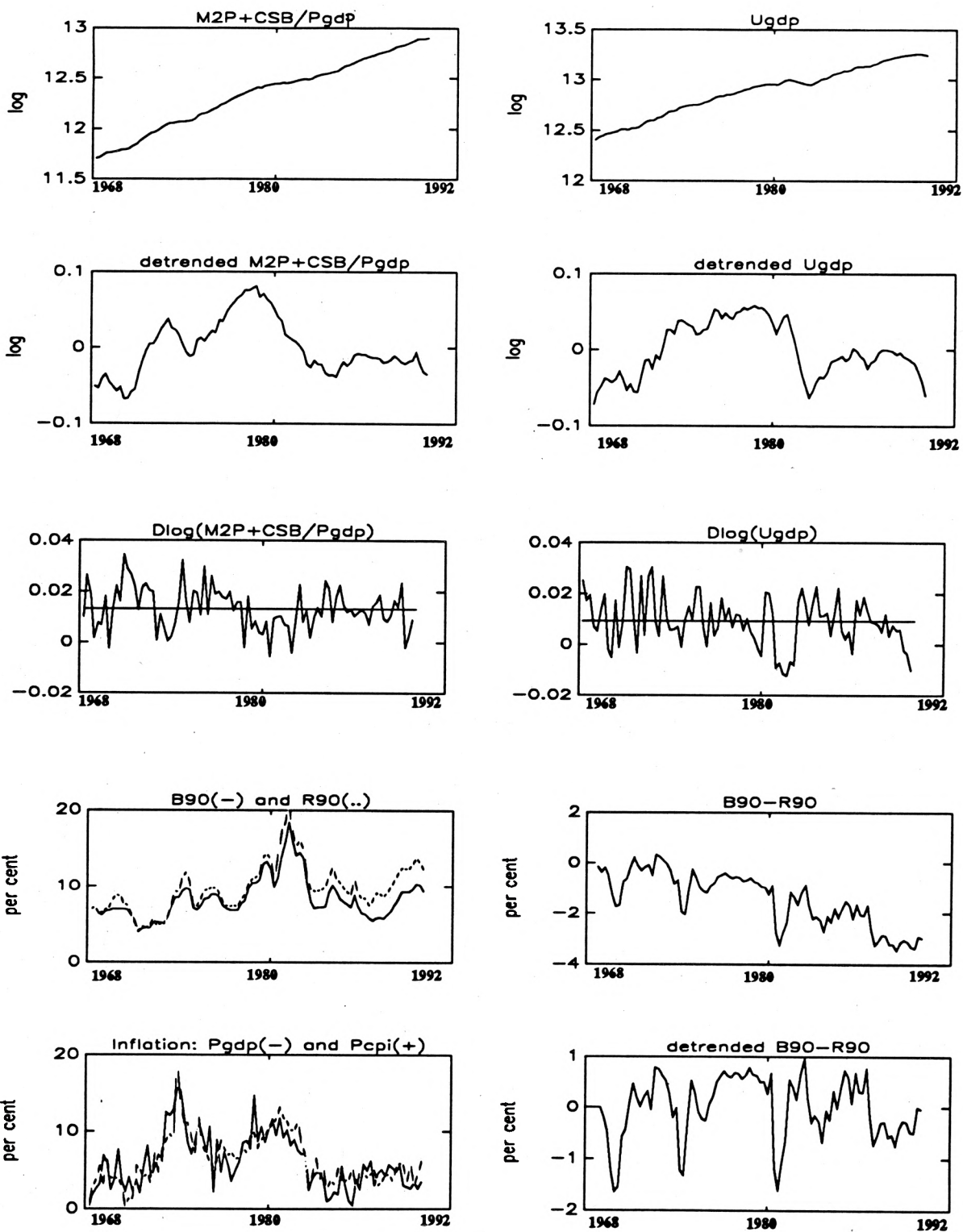
**Tbill (\$ millions)**



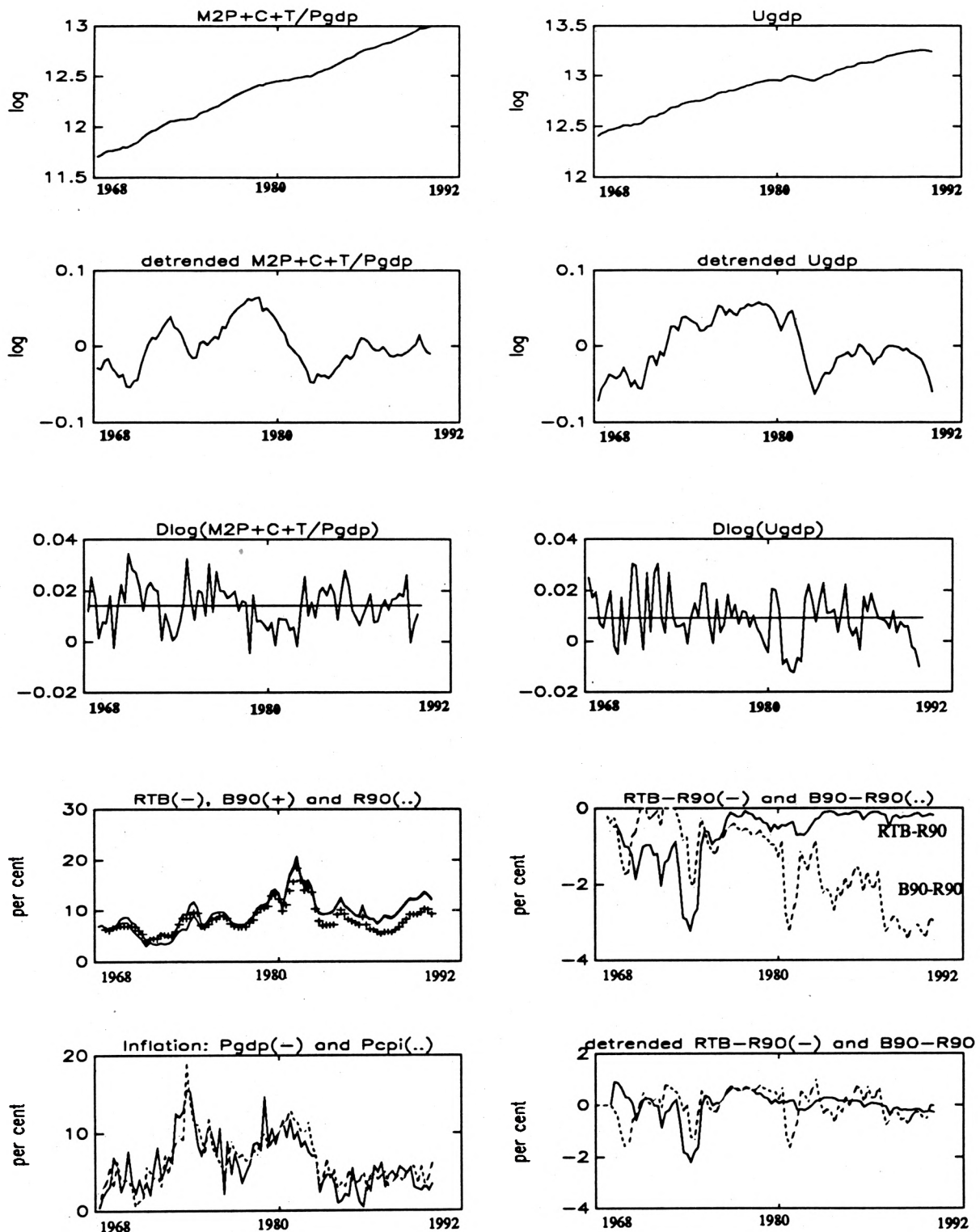
**Figure 3**  
**M2P, M2P+CSB and M2P+CSB+Tbill as a ratio of nominal GDP**



**Figure 4**  
**M2P+CSB**



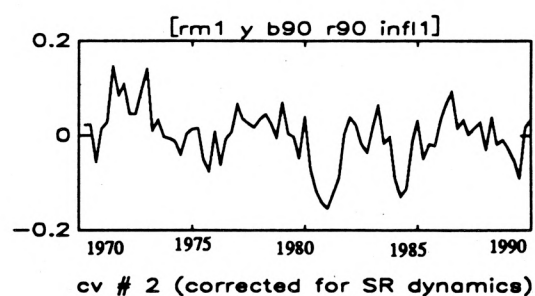
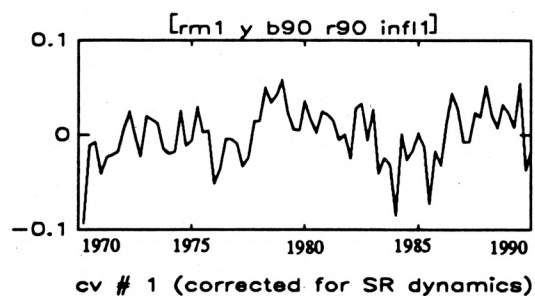
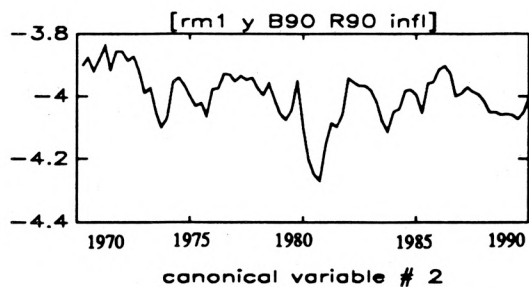
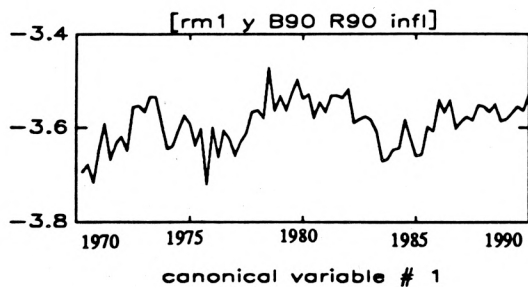
**Figure 5**  
**M2P+CSB+Tbill**



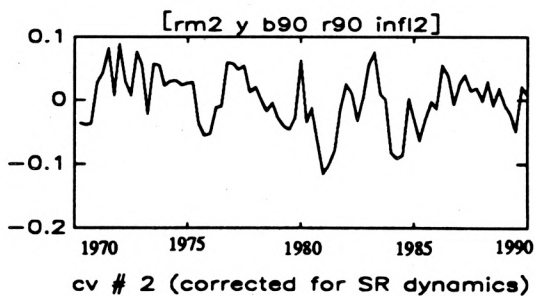
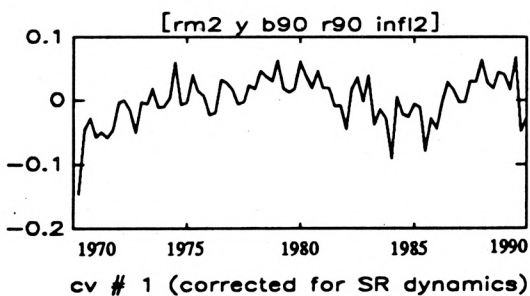
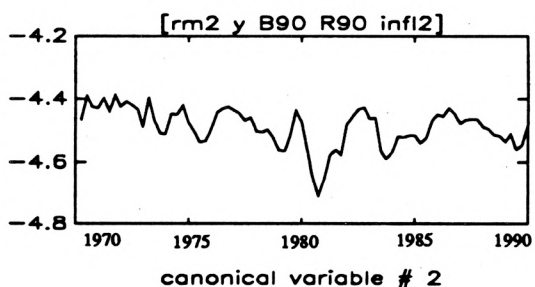
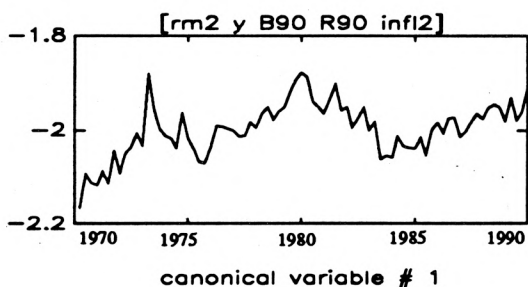


**Figure 6**  
**M2P+CSB**

**System: [rm1 y B90 R90 infl1]**

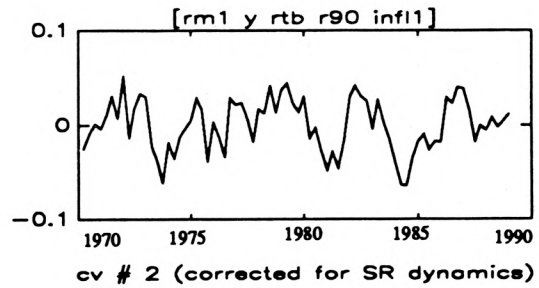
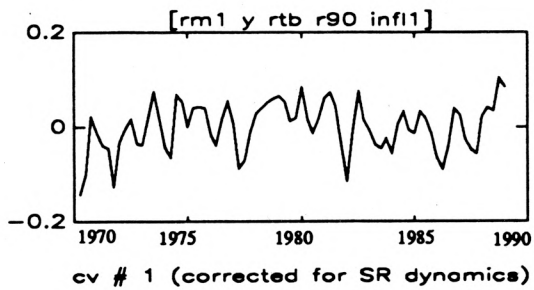
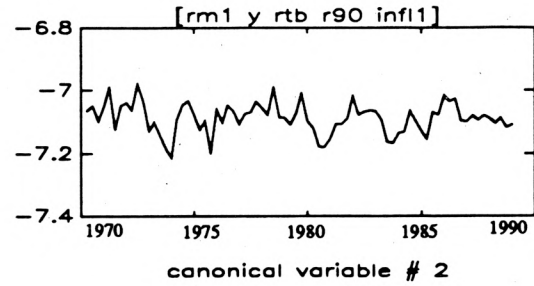
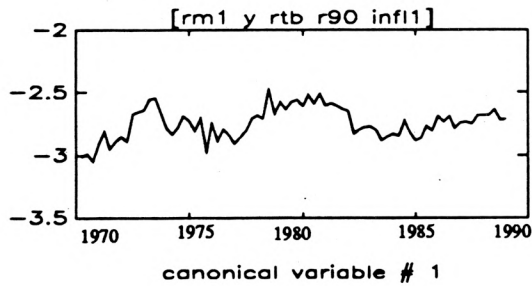


**System: [rm2 y B90 R90 infl2]**

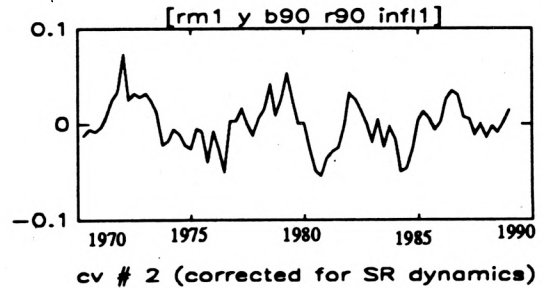
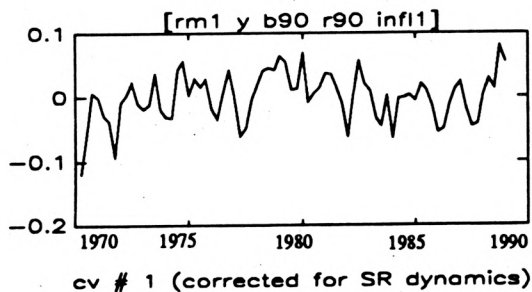
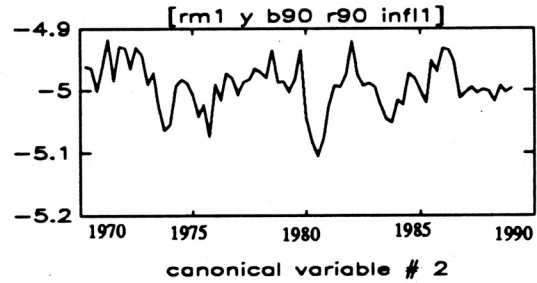
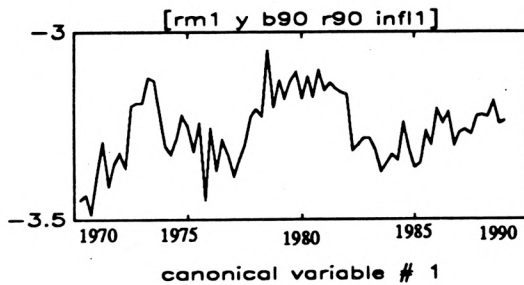


**Figure 7**  
**M2P+CSB+Tbill**

**System: [rm1 y RTB R90 infl1]**

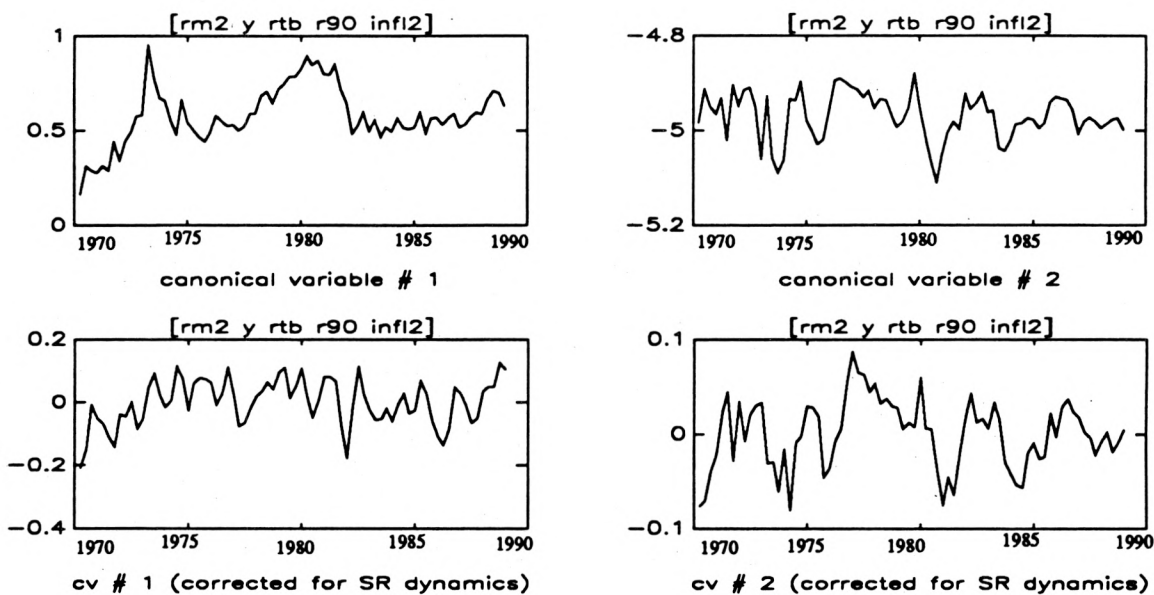


**System: [rm1 y B90 R90 infl1]**

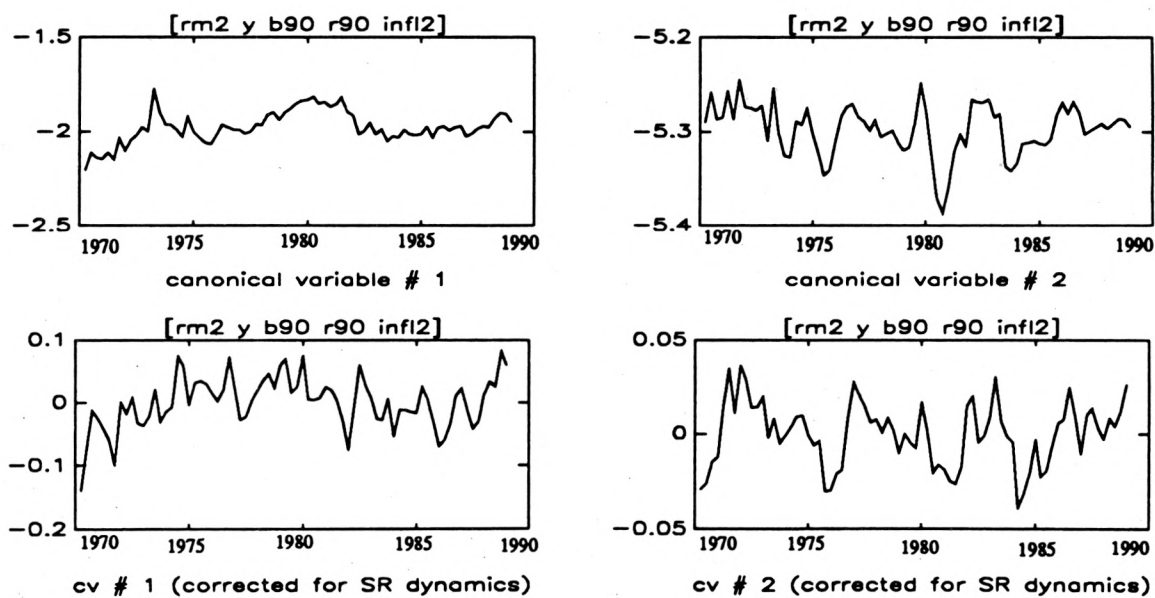


**Figure 8**  
**M2P+CSB+Tbill**

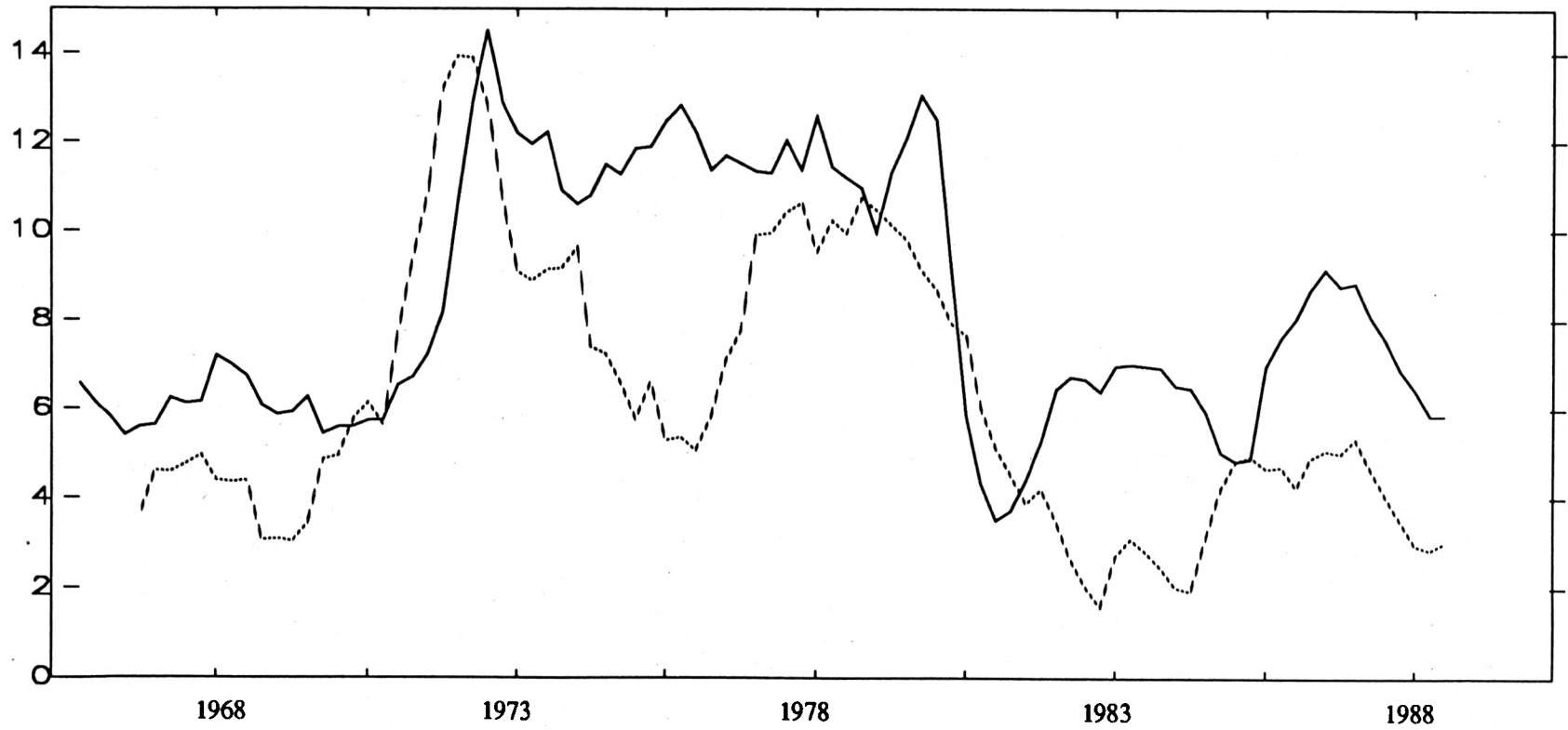
**System: [rm2 y RTB R90 infl2]**



**System: [rm2 y B90 R90 infl2]**

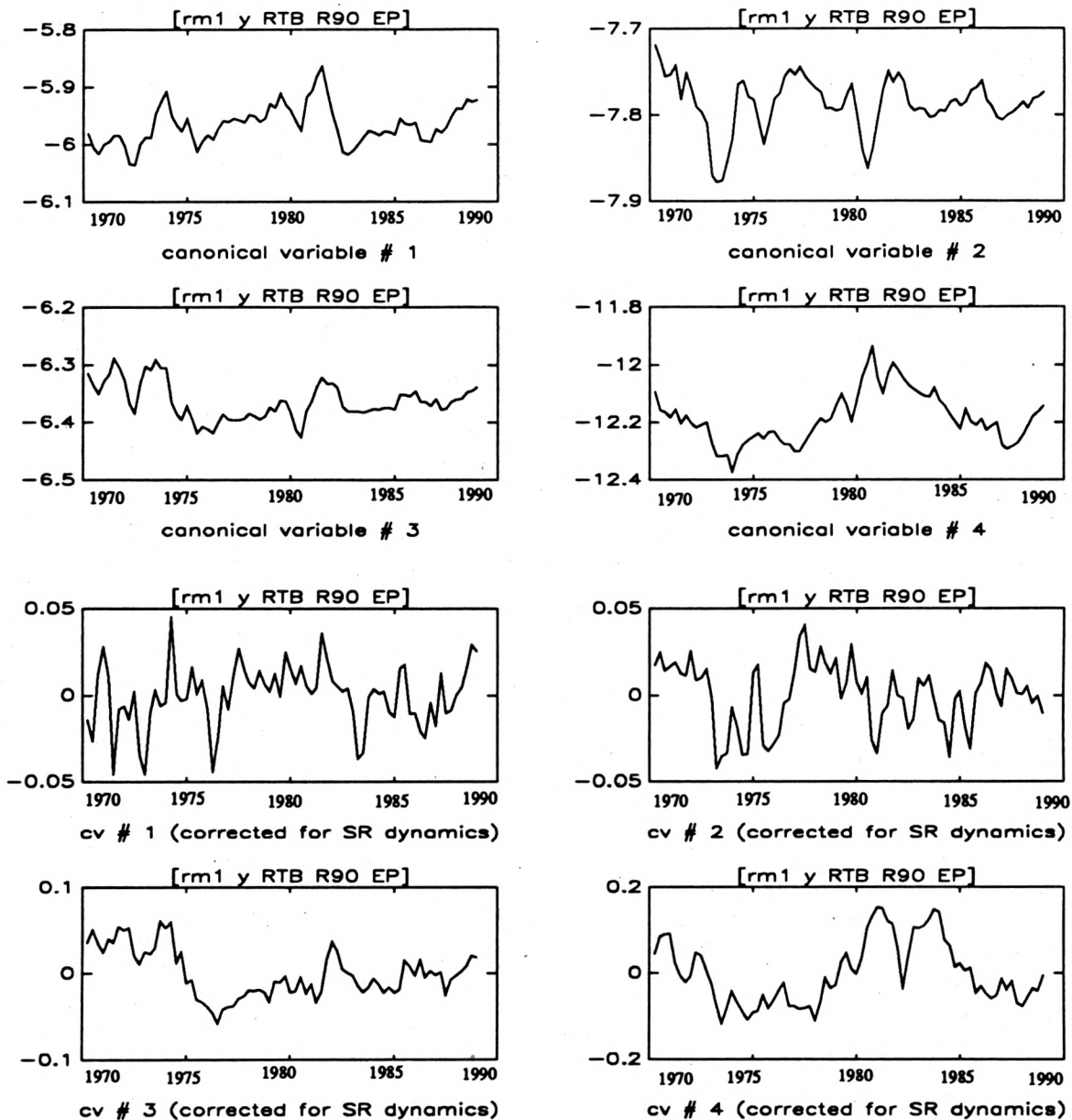


**Figure 9**  
**Earnings-price ratio (-) and inflation rate (GDP deflator) (...)**



**Figure 10**  
**M2P+CSB+Tbill**

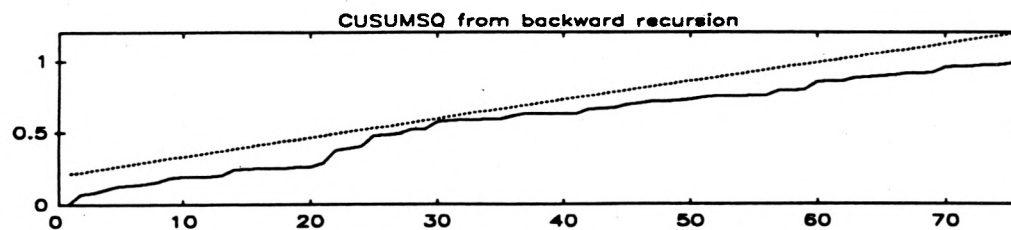
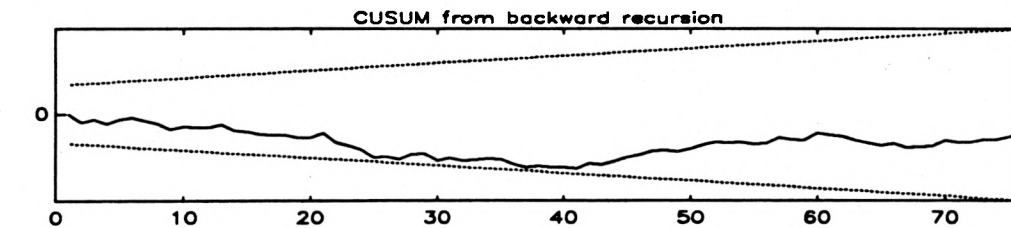
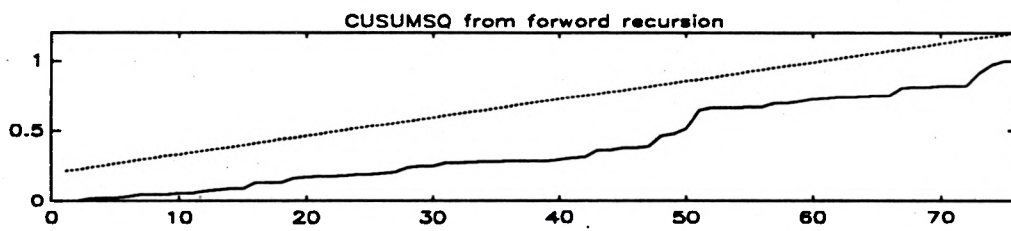
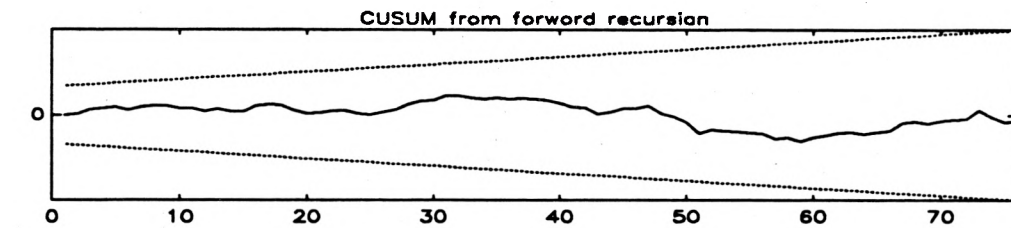
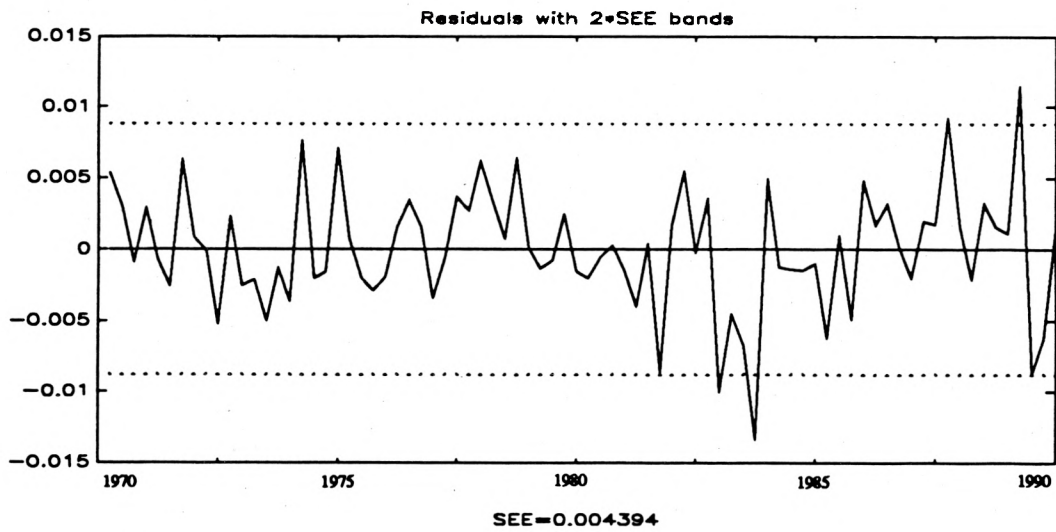
**Cointegration vectors in system [rm1 y RTB R90 EP]**



# Figure 11

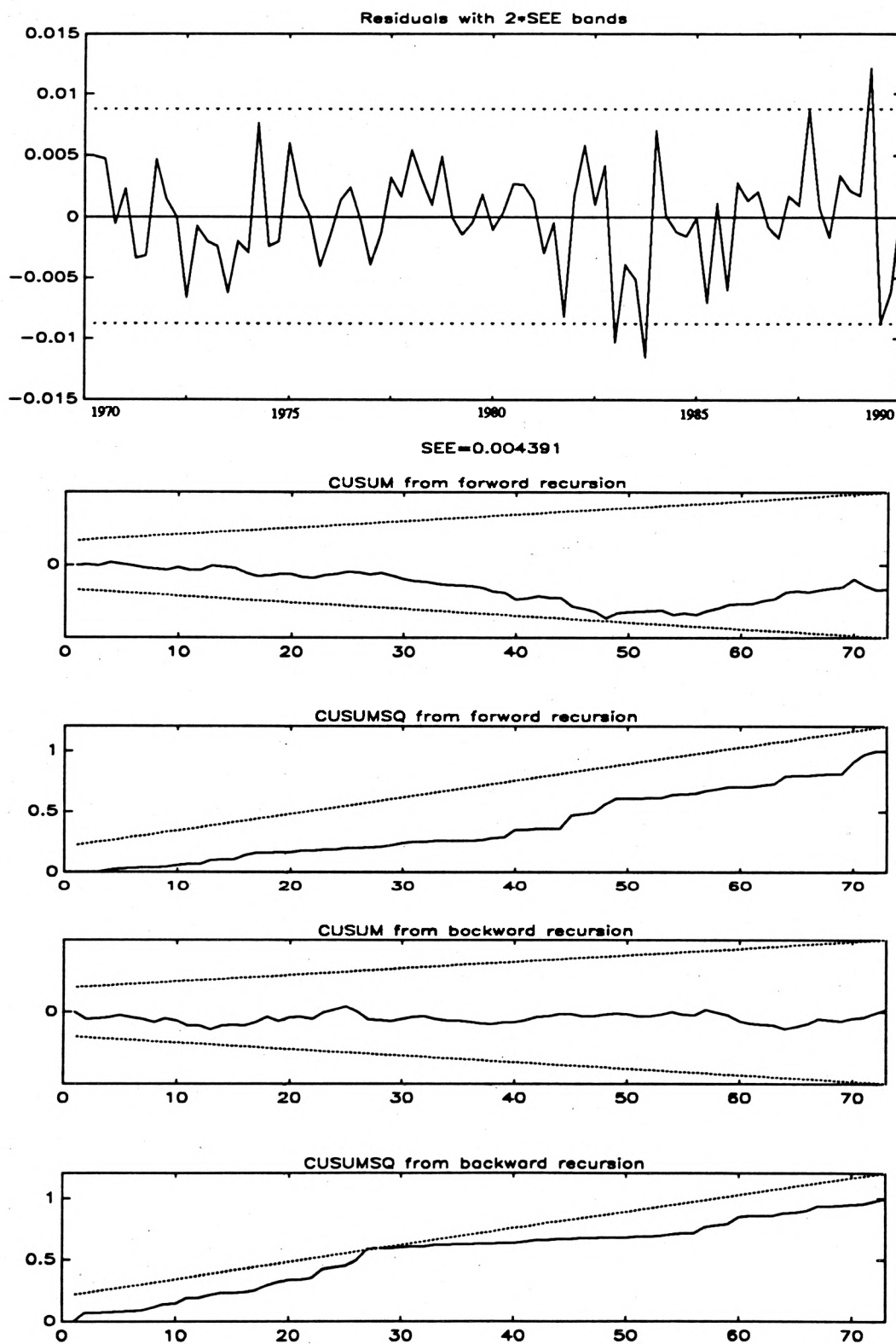
## M2P+CSB

Table 10, Equation 1



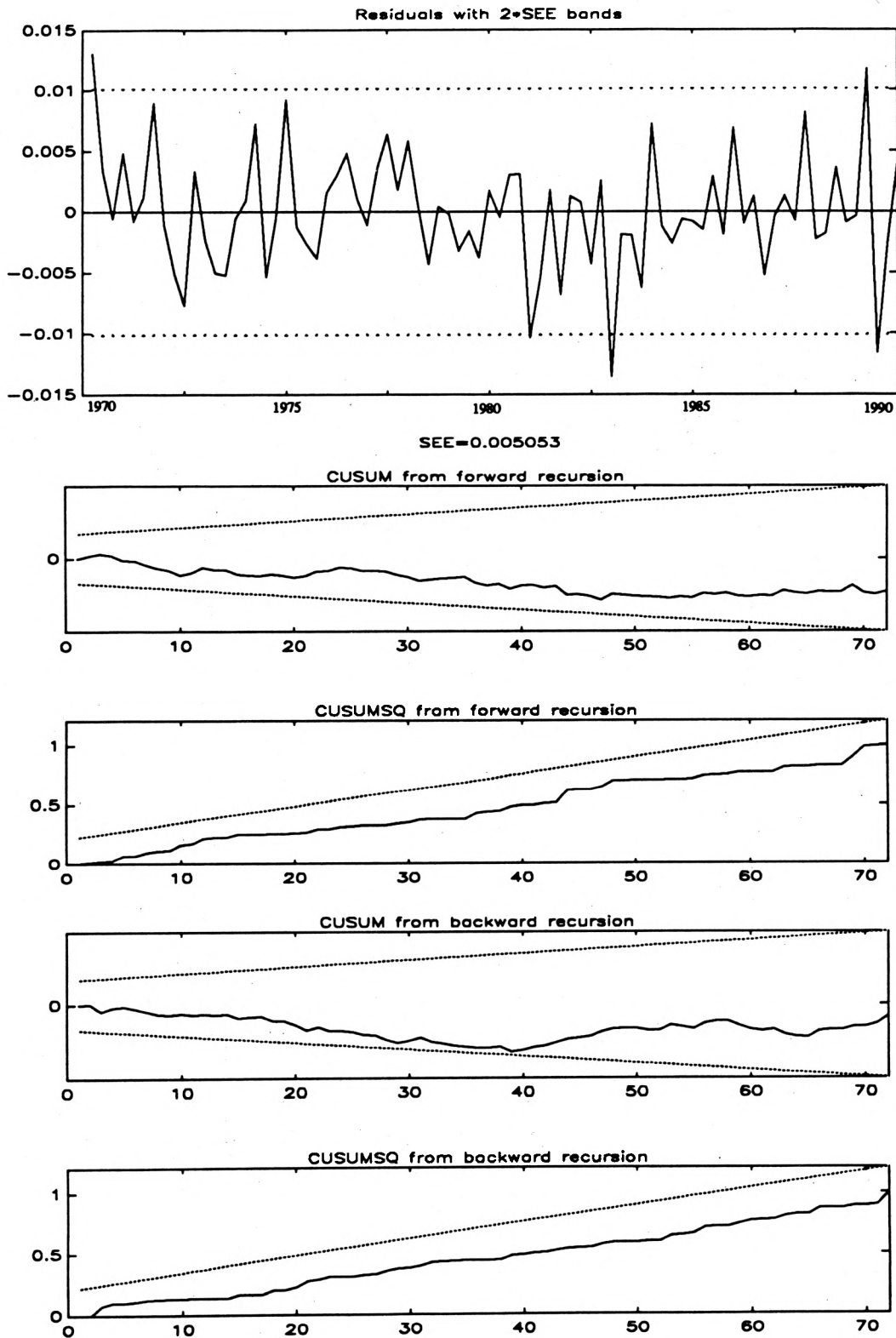
**Figure 12**  
**M2P+CSB**

**Table 10, Equation 2**



### Figure 13 M2P+CSB

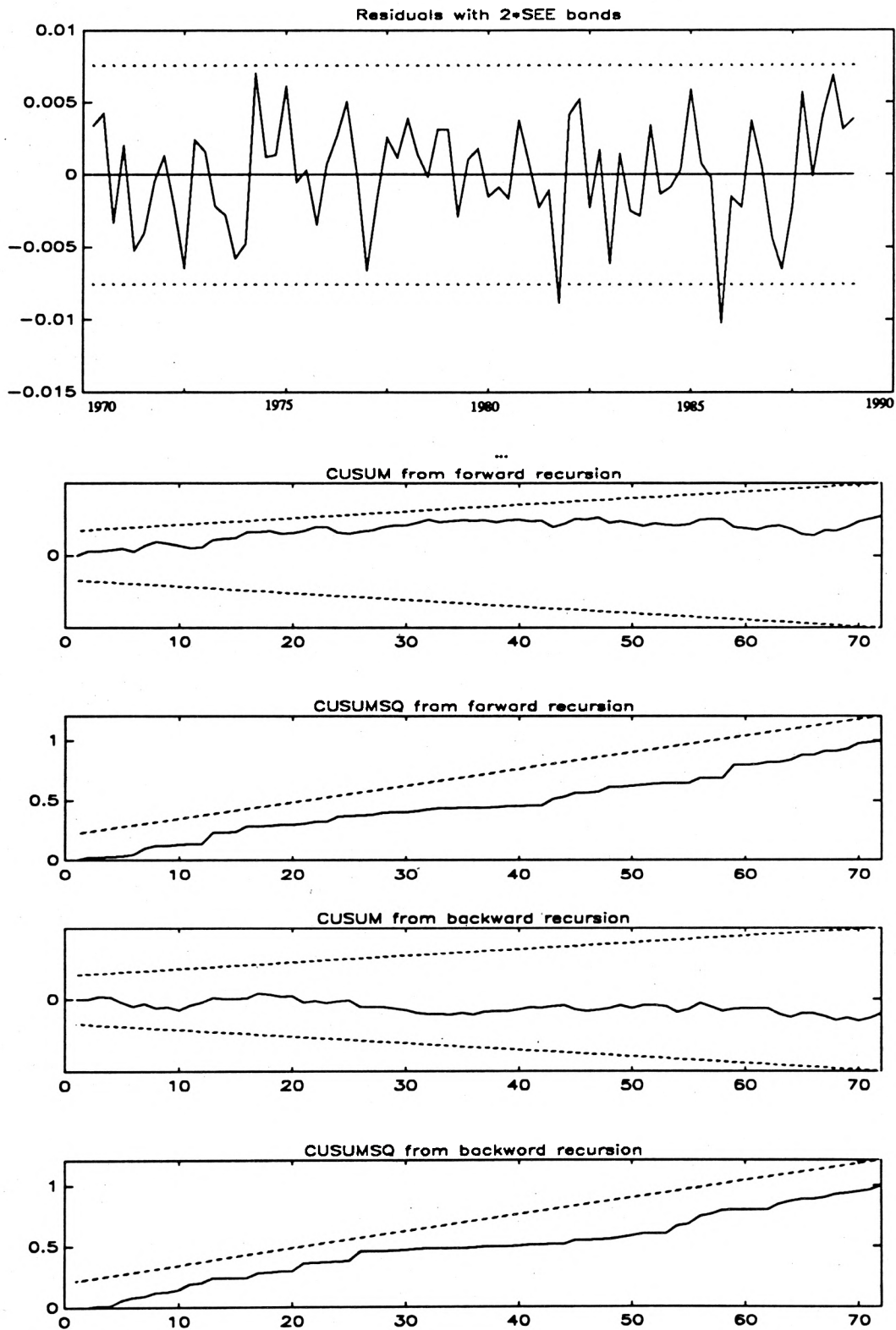
Table 10, Equation 3





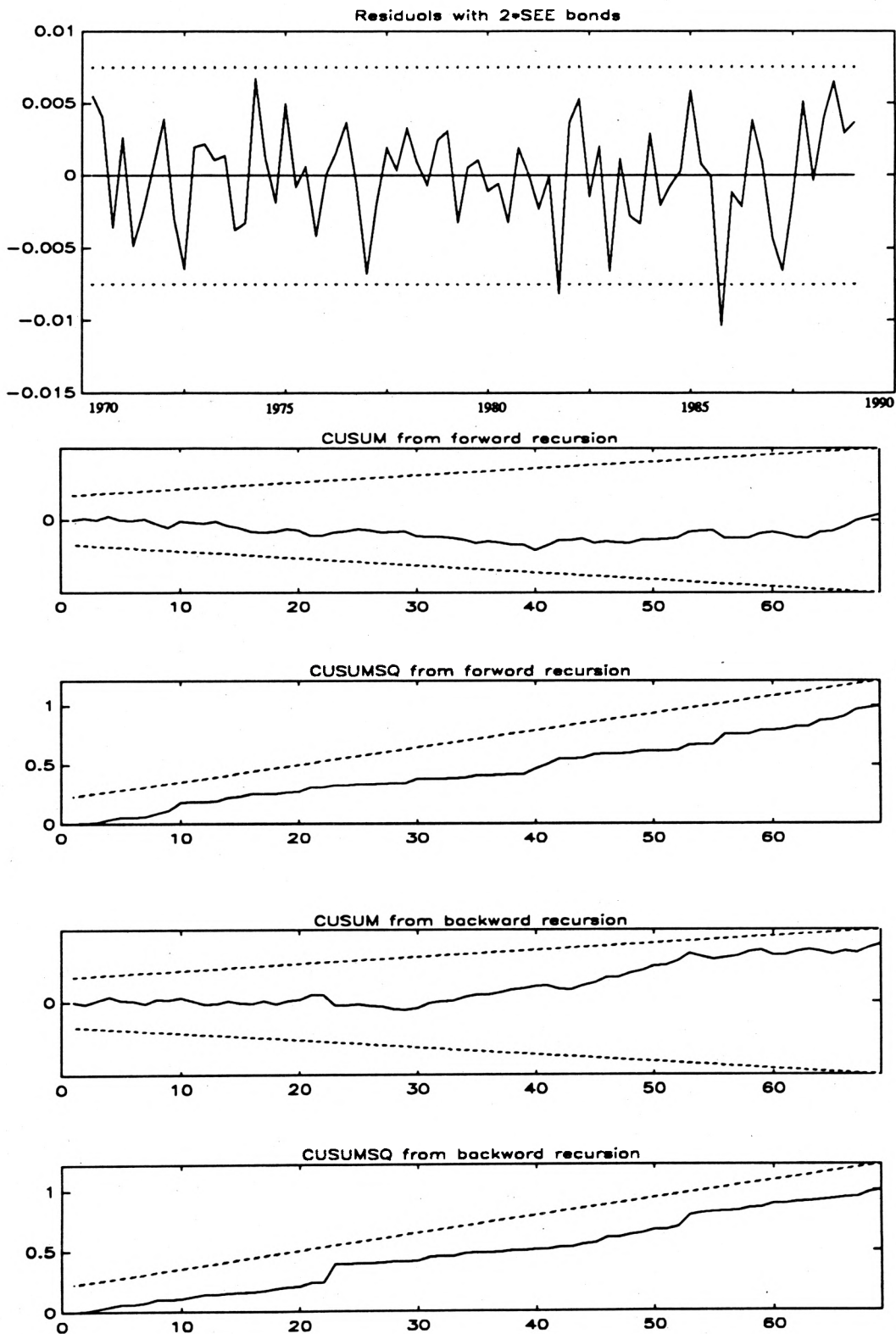
**Figure 14**  
**M2P+CSB+Tbill**

**Table 12, Equation 1**



**Figure 15**  
**M2P+CSB+Tbill**

**Table 12, Equation 2**



## Figure 16 M2P+CSB

### Table 10, Equation 1

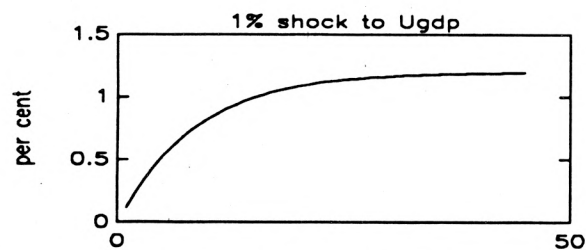


Table 6 Eq. 1

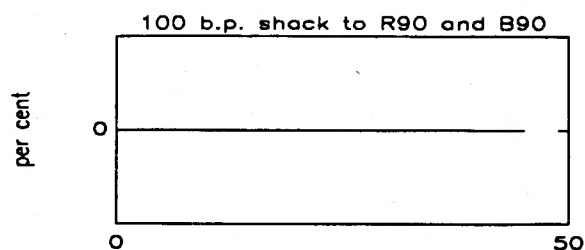


Table 6 Eq. 1

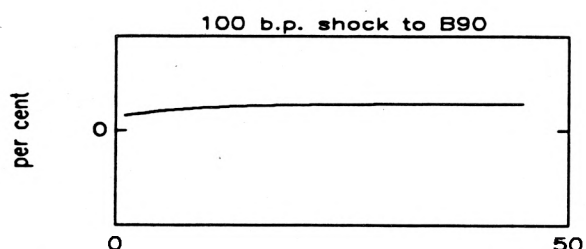


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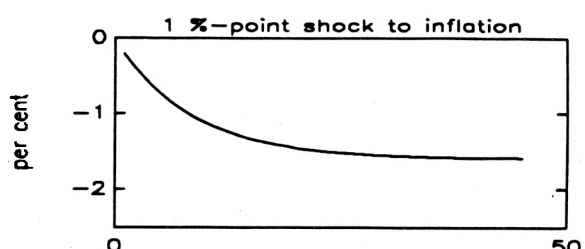


Table 6 Eq. 1

## Figure 17

### Table 10, Equation 2

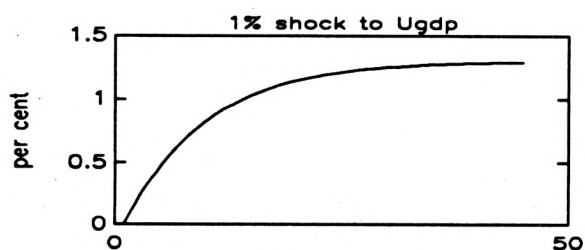


Table 6 Eq. 2

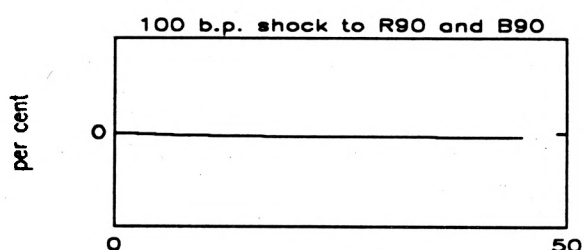


Table 6 Eq. 2

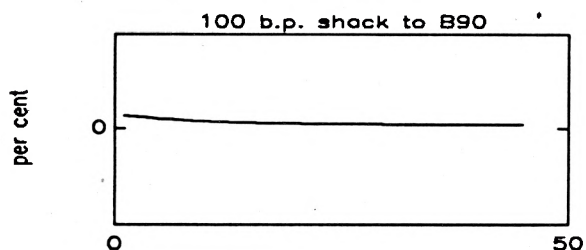


Table 6 Eq. 2

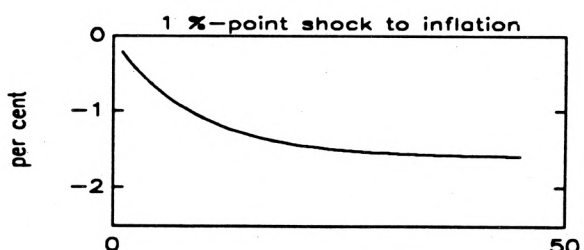


Table 6 Eq. 2

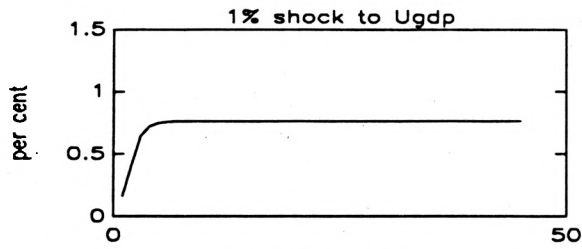
**Figure 18**  
**M2P+CSB****Table 10, Equation 3**

Table 6 Eq. 5

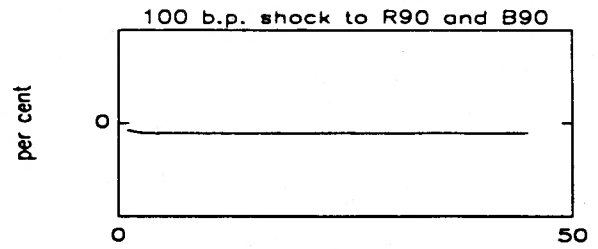


Table 6 Eq. 5

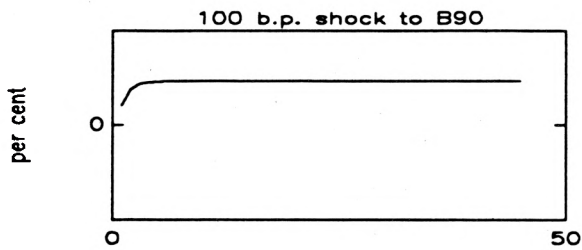


Table 6 Eq. 5

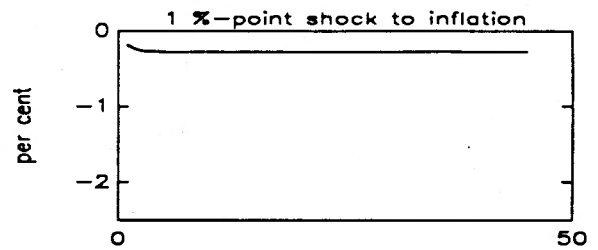
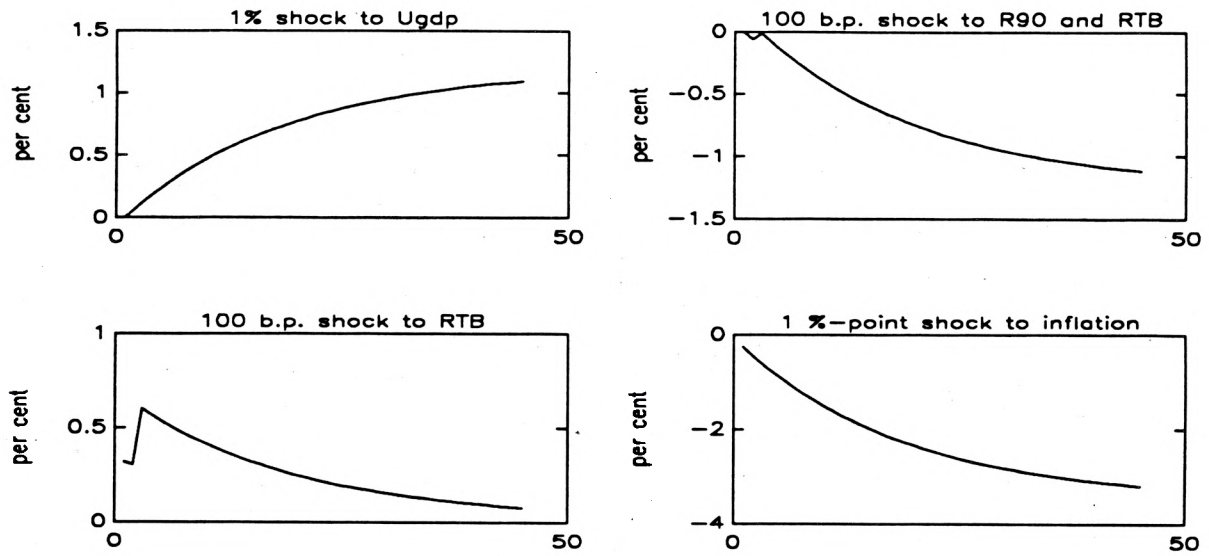


Table 6 Eq. 5

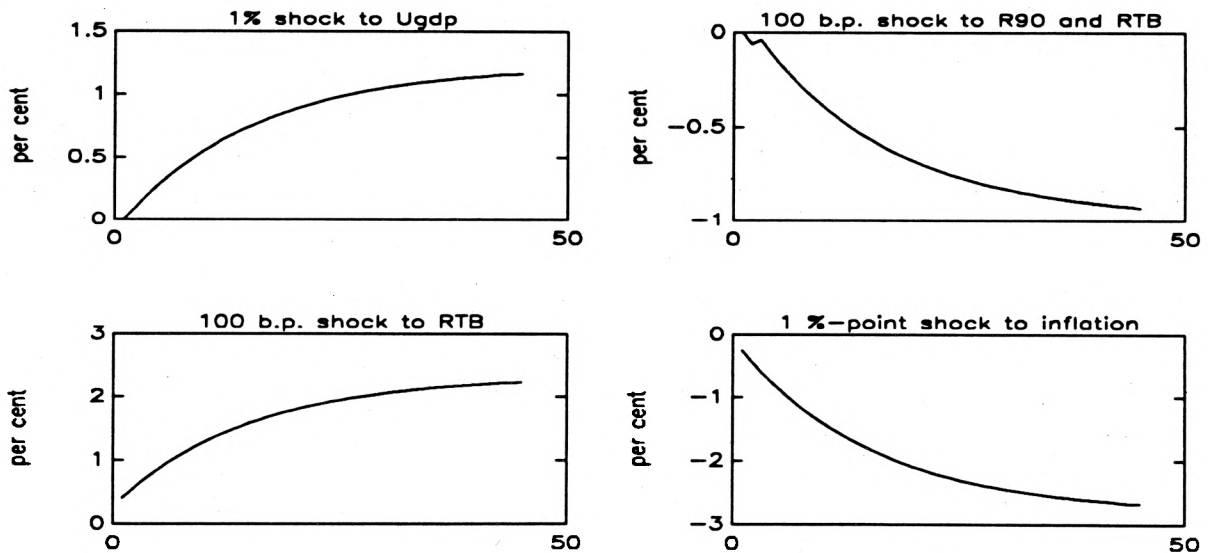
**Figure 19**  
M2P+CSB+Tbill

**Table 12 Equation 1**



**Figure 20**

**Table 12, Equation 2**



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