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Simon van Norden

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Simon van Norden

International Department
Bank of Canada
234 Wellington Street
Ottawa, Ontario
K1A 0G9

Tel.: (613)782-7234
Fax: (613)782-7658

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## Abstract

This paper develops a new test for speculative bubbles which is applied to data for the exchange rate of the Japanese yen, the Deutsche mark and the Canadian dollar against the U.S. dollar from 1977 to 1991. The test assumes that bubbles display a particular kind of regime-switching behaviour, which is shown to imply coefficient restrictions on a simple switching-regression model of exchange rate innovations. Test results are sensitive to the specification of exchange rate fundamentals and other factors. An overshooting model of the Canadian dollar and a purchasing-power parity model of the Japanese yen give the most consistent evidence of bubbles.

## Résumé

Dans la présente étude, l'auteur met au point un nouveau test visant à déterminer si les taux de change incorporent des bulles spéculatives et l'applique aux données du taux de change du yen, du mark allemand et du dollar canadien par rapport au dollar É.-U. pour la période 1977-1991. L'auteur postule l'existence de deux régimes, ce qui suppose quelques restrictions sur les coefficients d'un modèle simple de régression avec changement de régime formalisant les variations non anticipées de taux de change. Les résultats des tests varient selon la spécification des déterminants fondamentaux du taux de change et selon d'autres facteurs. Des différents modèles mis à contribution, ce sont le modèle de surréaction pour le dollar canadien et le modèle de parité des pouvoirs d'achat pour le yen qui donnent les résultats les plus favorables à la présence de bulles.

## 1. Introduction

This paper develops a new test for speculative bubbles in exchange rates and then applies this test to data for three bilateral exchange rates over the 1977-91 period.

Recent work in testing for bubbles has shifted from general tests that should detect any kind of bubble to those that test for a particular form of bubble.[1] An advantage of the latter is that such tests give more information about the kind of behaviour that produces significant evidence of bubbles. The test introduced below follows this newer approach. In particular, it focusses on a kind of stochastic bubble, which in each period is expected either to continue growing or to collapse (partially or completely). Given assumptions about the expected probability and size of these collapses, one can show that such behaviour should lead to a particular kind of regime-switching behaviour in exchange rate innovations. Tests for such behaviour can be conducted using switching-regression techniques. These tests are applied to data for the exchange rates of the Japanese yen, the Deutsche mark and the Canadian dollar against the U.S. dollar and use various specifications of the underlying "true" model of exchange rate fundamentals.

It is well understood that a bubble model will be observationally equivalent to a model without bubbles, but with a different specification of fundamentals.[2] In the case studied below, the switching regression motivated by bubbles could also be motivated by the presence of regime switching in fundamentals. One example of such an alternative model would be the "peso problem" considered by Krasker (1980). Therefore, as with all tests for bubbles, the results presented here should be interpreted as evidence of the kind of behaviour predicted by the bubble model, and not as definitive proof of the presence or absence of bubbles.

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1. Examples of the former include Meese (1986), West (1987) and Gros (1989). Examples of the latter are Froot and Obstfeld (1991) and van Norden and Schaller (1993a, 1993b).
2. For a discussion of these issues and the value of empirical tests for bubbles, see Flood and Hodrick (1990).

Nonetheless, such a qualified conclusion should be interesting for a number of reasons.

First, any results consistent with bubbles will have implications for research on the efficiency of foreign exchange markets. If one interprets such results as evidence of bubbles, then this violates some definitions of market efficiency. If one instead interprets the results as evidence of switching in fundamentals, then this implies that empirical models of risk premiums need to take such regime-switching behaviour into account. In addition, the empirical link between regime-switching behaviour in exchange rates and other macroeconomic series then becomes another stylized fact that a satisfactory model of risk premiums needs to explain.

Second, the evidence presented below adds to the work on the univariate properties of exchange rate changes. In addition to recent research on autoregressive conditional heteroscedasticity models and semi-parametric estimators,[3] there has been particular interest in mixture of distribution models.[4] One of the key attractions of such models is their ability to capture the occasional occurrence of large, discrete exchange rate changes by appealing to a secondary data-generating process that is observed only infrequently. By placing the mixture of distribution model in a multivariate context, the switching regression may offer new explanations of large abrupt exchange rate movements by linking them to other macroeconomic time series. Furthermore, the econometric techniques needed to estimate the simple switching-regression models used here are well established and may be easier to compute than some univariate estimation methods, such as those proposed by Hamilton (1989).[5]

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3. For example, see Diebold (1988) for work with ARCH processes and Gallant, Hsieh and Tauchen (1988) for semi-non-parametric methods.
4. See Akgiray and Booth (1988), Bates (1988), Boothe and Glassman (1987), Engle and Hamilton (1990), Jorion (1988), or Tucker and Pond (1988).
5. Seminal papers in this area include Quandt (1972), Goldfeld and Quandt (1973), Quandt and Ramsey (1978) and Hartley (1978).

Finally, tests of the switching-regression model also contribute to the growing literature on the predictability of returns in asset markets.[6] In particular, these papers show that simple measures of the deviation of asset prices from their fundamental values help to predict future returns. For example, Cutler, Poterba and Summers (1991) show that such relationships exist across a wide range of asset markets, although the evidence for exchange rate markets is quite weak. Since the switching-regression model nests the linear regression they used, one can test for evidence of a more complex relationship and consider its sensitivity to a variety of assumptions about fundamental exchange rates.

The following section introduces a simple regime-switching model of bubbles that generalizes the model first suggested by Blanchard (1979). Section 3 discusses the econometric issues involved in the estimation and testing of such models, while section 4 explains the data and various models of fundamental exchange rates used. Sections 5 and 6 review the empirical results, while the final section offers conclusions.

## 2. A regime-switching model of stochastic bubbles

We begin with a general model of exchange rate determination, which requires only that

$$s_t = f(X_t) + a \cdot E_t(s_{t+1}), \tag{1}$$

where $s_t$ is the logarithm of the spot exchange rate, $E_t$ is the operator for expectations conditional on information at time $t$, $0 < a < 1$, and $X_t$ is a vector of other variables. The variables included in $X_t$ and the form of $f(.)$ will vary from model to model, and several alternative formulations will be considered below. For the time being, it is sufficient to note that equation (1) is general enough to encompass both fixed- and flexible-price monetary models, as well as models with

6. For example, see Fama and French (1988), Poterba and Summers (1988), Cecchetti, Lam and Mark (1990).

imperfect international asset substitutability. Solving the equation forward gives the general result

$$s_t = \sum_{j=0}^{T} a^j \cdot E_t(f(X_{t+j})) + a^{T+1} \cdot E_t(s_{T+j+1}). \tag{2}$$

If

$$\lim_{T \to \infty} a^{T+1} \cdot E_t(s_{T+j+1}) = 0, \tag{3}$$

then one solution to equation (1), which we will denote $s_t^*$, is

$$s_t^* = \sum_{j=0}^{\infty} a^j \cdot E_t(f(X_{t+j})). \tag{4}$$

We refer to (4) as the fundamental solution, since it determines the exchange rate solely as a function of the current and expected behaviour of other macroeconomic variables. In what follows below, we will specify a particular form for $f(.)$ and an expected future path of $X$, and calculate explicit values for the fundamental exchange rate that result from various exchange rate models.

However, equation (4) is not the only solution to (1). We define bubble solutions to be any other set of exchange rates and exchange rate expectations that satisfy equation (1) but where $s_t \neq s_t^*$. We define the size of the bubble $b_t$ as

$$b_t \equiv s_t - s_t^* . \tag{5}$$

Note that since $s_t^*$ satisfies equation (1), from (1) and (5) it follows that

$$b_t = a \cdot E_t(b_{t+1}). \tag{6}$$

Since $a < 1$, the bubble must be expected to grow over time.

A considerable literature exists on the conditions under which such bubbles are feasible rational expectations solutions.[7] One of the key insights of this literature is that in single-representative-agent models, a truly rational agent cannot expect to sell an over-valued asset (one with a positive bubble) before the bubble bursts. Therefore, bubbles should exist only if they can be expected to grow without limit. Some researchers, such as Froot and Obstfeld (1991), have therefore suggested interpreting empirical tests for bubbles as tests of whether agents are fully rational, or whether they exhibit some form of myopia when events are considered that are either very far in the future or that occur with only very low probabilities. An alternative interpretation would be to consider evidence of bubbles as suggesting that non-representative-agent models are required.[8]

Blanchard (1979) proposed a particular example of a process that satisfies (6) and captures some of the important features that have historically been attributed to bubbles. In particular, his process can generate large abrupt movements in $s_t$ that are unrelated to news about the future of $X_t$. He considers a bubble process that moves randomly between two states, $C$ and $S$. In state $C$, the bubble will collapse, so

$$E_t(b_{t+1} | C) = 0 . \tag{7}$$

State $S$, where the bubble survives and continues to grow, occurs with a fixed probability $q$. Since

$$E_t(b_{t+1}) = (1-q) \cdot E_t(b_{t+1} | C) + q \cdot E_t(b_{t+1} | S) , \tag{8}$$

(7) and (6) imply

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7. Important contributions to this debate include Obstfeld and Rogoff (1983, 1986), Diba and Grossman (1987), Tirole (1982, 1985), Weil (1988), Buiter and Pesenti (1990), Allen and Gorton (1991), and Gilles and LeRoy (1992).
8. Recent examples of such models include Allen and Gorton (1991) and De Long et al. (1990).

$$E_t(b_{t+1}|S) = \frac{b_t}{a \cdot q} . \tag{9}$$

Note that the lower the probability $q$ of the bubble's survival, the faster the bubble must be expected to grow in the surviving state. The potentially large difference in the expected asset price between $S$ and $C$ implies that such bubble collapses could cause sudden and large price changes.

While it is a tractable and suggestive solution to (6), the Blanchard process seems unrealistically restrictive in at least two ways. First, it assumes that in state $C$, the bubble is expected to collapse fully. There is no obvious theoretical reason for such an assumption; there is a continuum of bubble paths that will satisfy (6), so choosing any single path must be an arbitrary, if convenient, assumption. Furthermore, there may be institutions in the real world that would tend to work against an instantaneous and complete collapse. (For example, central banks may have a policy of trying to smooth sudden exchange rate changes as part of an effort to maintain orderly foreign exchange markets.) Finally, historical exchange rate movements that are sometimes attributed to bubbles, such as the rise and fall of the U.S. dollar in 1984-85, tend to be reversed over a period of several months rather than in a single day. It is therefore reasonable to allow for the possibility that the bubble is expected to collapse only partially in state $C$. In particular, (7) could be replaced with

$$E_t(b_{t+1}|C) = u(b_t), \tag{10}$$

where $u(.)$ is a continuous and everywhere differentiable function such that $u(0) = 0$ and $1 \geq u' \geq 0$. This means that the expected size of collapse will be a function of the relative size of the bubble, $b_t$, and that the bubble is not expected to grow (and may be expected to shrink) in state $C$.

Another restrictive feature of the Blanchard bubble process is the assumption of a constant probability of collapse, whereas one might otherwise expect the probability to vary over time. For example, Kindleberger (1989) describes the typical life-cycle of a "bubble" or of "speculative mania." He notes that as the bubble in the price of a particular asset grows, the prices of close substitutes become affected by the bubble, and that "collapses" or "panics" usually follow shortly thereafter.[9] One might therefore expect that the probability of the bubble's continued growth falls as the bubble grows, so that

$$q = q(b_t), \ \frac{d}{d|b_t|}q(b_t) < 0 \ . \tag{11}$$

Note that the derivative of $q$ is defined using the absolute value of $b_t$, since we wish to consider cases where $b_t$ may be positive or negative. If we now use (10) and (11) with (6), we derive the revised counterpart to (9):

$$E_t(b_{t+1}|S) = \frac{b_t}{a \cdot q(b_t)} - \left(\frac{1-q(b_t)}{q(b_t)} \cdot u(b_t)\right) . \tag{12}$$

We can see that in addition to replacing $q$ with $q(b_t)$, the expected value of the bubble in state $S$ is now lower by an additional factor that reflects its greater expected value in state $C$.

An interesting feature of the bubble model given in equations (10), (11) and (12) is the structure it implies in exchange rate innovations. If we consider the unexpected change in the log exchange rate, $s_{t+1} - E_t(s_{t+1})$, this must be uncorrelated with all the information used to form $E_t(s_{t+1})$. Since $b_t$ must be part of this information, it too will be uncorrelated with these innovations.[10]

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9. This is a very stylized description of Kindleberger's much richer narrative. The interested reader is referred to Kindleberger (1989) for more details. One interpretation of this effect is that as the bubble begins to distort relative prices, demand switches to close substitutes. At some point, this switch in demand begins to depress expected price changes, shifting the system into a new expectational equilibrium.
10. That is not to say that the innovations will be independent of $b_t$. For example, one can show that $\mathrm{Var}(s_{t+1} - E_t(s_{t+1}))$ will generally be increasing in $b_t$.

However, if we could separate these innovations into those drawn from state $C$ and those drawn from state $S$, this would no longer be the case. To see this, note that (5) implies we can decompose the exchange rate innovation into that arising from fundamentals and that arising from the bubble

$$\begin{aligned} s_{t+1} - E_t(s_{t+1}) \equiv R_{t+1} &= [s^*_{t+1} - E_t(s^*_{t+1})] - [b_{t+1} - E_t(b_{t+1})] \\ &= \varepsilon^*_{t+1} + [b_{t+1} - E_t(b_{t+1})], \end{aligned} \tag{13}$$

where $\varepsilon^*_{t+1}$ is the innovation in the fundamental exchange rate. If the bubble collapses at $t+1$, we observe state $C$ and

$$R_{t+1}| C = \varepsilon^*_{t+1} + u(b_t) + \varepsilon^C_{t+1} - b_t/a \,, \tag{14}$$

where $\varepsilon^C_{t+1}$ is the expectational error term associated with (10), and $E_t(b_{t+1})$ is replaced using (6). Since $a < 1$, $u(b_t) < b_t/a$ if $b_t > 0$ and $u(b_t) > b_t/a$ if $b_t < 0$. If we assume that $E(\varepsilon^*_{t+1}| C) = E(\varepsilon^*_{t+1}| S) = 0$, then, conditional on a bubble collapse, the expected innovation in the exchange rate will be non-zero.[11] This expected value will itself be a decreasing function of $b_t$ since

$$\frac{d}{db_t} E(R_{t+1}| C) = u'(b_t) - \frac{1}{a} < 0 \,. \tag{15}$$

Similarly, it can be shown that in the surviving state, $S$,

$$\begin{aligned} R_{t+1}| S &= \varepsilon^*_{t+1} + \frac{b_t}{a \cdot q(b_t)} - \left( \frac{1 - q(b_t)}{q(b_t)} \cdot u(b_t) \right) + \varepsilon^S_{t+1} - b_t/a \\ &= \frac{1 - q(b_t)}{a \cdot q(b_t)} \cdot [b_t - (a \cdot u(b_t))] + \varepsilon^*_{t+1} + \varepsilon^S_{t+1} \quad , \end{aligned} \tag{16}$$

11. The assumption is equivalent to assuming that $b_t$ is an extrinsic bubble. For a discussion of intrinsic bubbles and their relationship to non-linearity, see Froot and Obstfeld (1991).

where $\varepsilon_{t+1}^{S}$ is the expectational error term associated with (12). This time, the expected innovation conditional on state $S$ will be positive if $b_t > 0$ and negative if $b_t < 0$. The expectation will be an increasing function of $b_t$ since

$$\frac{d}{db_t}E(R_{t+1}|S) =$$
$$\frac{[1-q(b_t)] \cdot [1-a \cdot u'(b_t)]}{a \cdot q(b_t)} + \frac{-q'(b_t) \cdot [b_t - a \cdot u(b_t)]}{a \cdot q(b_t)^2}, \tag{17}$$

which is unambiguously positive.[12]

There are several points to note about the results in (14) and (16). First, they imply the existence of a particular type of non-linear relationship between exchange rates and fundamentals. In particular, they predict that the relationship between exchange rate innovations and deviations from fundamentals should be state-dependent if bubbles are present. Second, they provide a rationale for a mixture of distributions to be present in exchange rate innovations, since $\varepsilon_{t+1}^{*} + \varepsilon_{t+1}^{S}$ and $\varepsilon_{t+1}^{*} + \varepsilon_{t+1}^{C}$ will generally have different distributions. Third, testing for evidence of the relationships predicted by this bubble model will be difficult, since they depend on the regime generating the observation, which is not directly observed. They will also depend on the use of a measure of $b_t$, which requires an explicit model of fundamental exchange rates. The next section of the paper takes up the question of how one can reasonably test for these relationships and suggests various measures of $b_t$. The remainder of this section considers how one should interpret such test results.

If we test for the effects predicted by a model of bubbles, can we conclude whether or not bubbles are present? The answer is no, not without additional assumptions. There are two key problems. First, suppose we fail to find evidence to support the model. While this could be because the type of bubble described

12. To see this, note that both denominators and both numerators are always positive.

above does not exist, it could also be due to misspecification of $b_t$, which might then prevent us from finding the expected relationship between $b_t$ and exchange rate innovations. Second, suppose we find evidence to support the model. While this might occur because the above type of bubble does exist, it might also be the result of other phenomena. For example, Flood and Hodrick (1986) argue that bubbles will be observationally equivalent to process switching in fundamentals. The bubble process specified here is no exception. Consider the following example.

Suppose there are no bubbles, but we misspecify the fundamental exchange rate $s_t^*$ so that the actual exchange rate is given by

$$\tilde{s}_t = \sum_{j=0}^{\infty} a^j \cdot E_t(g(X_{t+j})). \tag{18}$$

If $E_t(g(X_{t+j})) > E_t(f(X_{t+j})) \forall j$, then $\tilde{s}_t > s_t^*$, so we would think a positive bubble was present. Furthermore, because of the possibility of changes in fiscal, monetary or trade policies, $X_{t+j}$ may be generated by distinct regimes. For example, it might be the case that fiscal policy can switch between a "tight" and a "loose" stance, and that the greater the government debt, the lower the probability that the loose stance will continue and the greater the expected change in stance. This would lead to a model of regime switching that is completely isomorphic to that described above, except that the size of the bubble, $b_t$, would now be replaced by some measure of the deviation of fiscal policy from its sustainable path. By misspecifying the fundamentals, however, any purported measure of bubbles could conceivably be correlated with such a deviation from the sustainable path. We could therefore find all the evidence suggested by the bubble model, even in the absence of bubbles.

As noted by Flood and Hodrick (1990), this kind of problem occurs in all bubble tests. They conclude that while this makes the interpretation of bubble test results difficult, it adds value to them as a diagnostic test of models of fundamentals. One interpretation of any evidence of bubbles found by the tests proposed below might be that bubbles are indeed present and that agents therefore seem to exhibit some form of myopia, or perhaps that non-representative-agent models are needed. An alternate interpretation would be that exchange rate fundamentals exhibit switching behaviour, which would be an important factor in modelling foreign exchange market risk premiums.

## 3. Estimation and hypothesis-testing issues

As shown in the previous section, while the innovation in the exchange rate $R_{t+1}$ should be uncorrelated with information available at time $t$, which therefore includes $b_t$, there may be a non-linear relationship between these variables that takes the form of state-dependency; that is, the relationship between $R_{t+1}$ and $b_t$ exists, but varies across states. If we knew with certainty which regime generated each observation of $R_{t+1}$, we could estimate these relationships using standard least-squares techniques on equations (15) and (17). Given uncertainty about the classification of $R_{t+1}$ into these regimes, however, standard estimation techniques will give biased and inconsistent estimates.[13] Nonetheless, consistent, efficient, asymptotically normal parameter estimates of such systems can still be obtained, provided that the equations are estimated simultaneously and that explicit account is taken of classification uncertainty.[14]

To understand the estimation procedure, suppose that in regime $C$

$$R_{t+1} = R^C_{t+1} = h_C(b_t) + e^C_{t+1} , \tag{19}$$

and that in regime $S$

$$R_{t+1} = R^S_{t+1} = h_S(b_t) + e^S_{t+1} , \tag{20}$$

13. See Lee and Porter (1984) for a proof.
14. See Goldfeld and Quandt (1973) and Kiefer (1978) for proofs.

where $e^C_{t+1} \equiv \varepsilon^*_{t+1} + \varepsilon^C_{t+1}$ and $e^S_{t+1} \equiv \varepsilon^*_{t+1} + \varepsilon^S_{t+1}$. This implies that we can write the probability density function of an observation conditional on its being generated by a given regime as:

$$\phi_C(e^C_{t+1}) = \phi_C(R_{t+1} - h_C(b_t)) \tag{21}$$

and

$$\phi_S(e^S_{t+1}) = \phi_S(R_{t+1} - h_S(b_t)) \; . \tag{22}$$

If we have no information on which regime generates each observation, we may denote the average probability that an observation comes from regime $S$ as $q$. More generally, if we have a set of variables $M_t$ that contain imperfect classifying information, we can write the probability that $R_{t+1} = R^S_{t+1}$ as $q(M_t)$. Therefore, the unconditional probability density function of each observation is

$$q(M_t) \cdot \phi_S(R_{t+1} - h_S(b_t)) + [1 - q(M_t)] \cdot \phi_C(R_{t+1} - h_C(b_t)) \tag{23}$$

and the likelihood function for a set of $T$ observations is

$$\prod_{t=1}^{T} \{ q(M_t) \cdot \phi_S(R_{t+1} - h_S(b_t)) + [1 - q(M_t)] \cdot \phi_C(R_{t+1} - h_C(b_t)) \} \; . \tag{24}$$

Maximizing this likelihood function therefore estimates both (19) and (20) simultaneously with a set of parameters for $q(M_t)$ and can be shown to lead to consistent and efficient estimates without the need for *a priori* restrictions on which observations correspond to a given regime.[15]

15. A variety of other estimation approaches have been suggested, with various strengths and weaknesses. Hartley (1978) suggests using the EM algorithm, which is equivalent to maximizing the likelihood function but which may be computationally easier. Mehta and Swamy (1975) propose a Bayesian approach, and Quandt and Ramsey (1978) suggest using a moment-generating-function method. It is well understood in this literature that the likelihood function is unbounded, but that a local maxima exists that has the desirable properties claimed above.

Transforming the bubble model from section 2 into a form that can be estimated requires several additional pieces of information: functional forms for $h_S(b_t)$ and $h_C(b_t)$, a functional form and explanatory variables for $q(M_t)$, and distributional assumptions for $e^S_{t+1}$, $e^C_{t+1}$ (which will imply functional forms for $\phi_S$, $\phi_C$). To keep the computational difficulty of estimation manageable, one can take a first-order Taylor series expansion of (14) and (16) around some arbitrary value $b_0$ to obtain

$$h_S(b_t) = \beta_{S0} + \beta_{Sb} b_t \tag{25}$$

$$h_C(b_t) = \beta_{C0} + \beta_{Cb} b_t \tag{26}$$

where the model implies that $\beta_{Sb} > 0$, $\beta_{Cb} < 0$.[16] Furthermore, equation (9) implies that $b_t$ should give information on the probability of observing $S$ or $C$, so one can use $M_t = b_t$. In choosing a form for $q(.)$, it is important to ensure that $q(.)$ will be bounded between 0 and 1, and that it is not monotonic in $b_t$ (since it should be decreasing as either positive or negative bubbles grow in absolute value). One such candidate would be a logit function of the form $q(b_t) = \Phi(\beta_{q0} - \beta_{qb} \cdot b_t^2)$ where $\beta_{qb} > 0$ and $\Phi(x) \equiv (1 + e^x)^{-1}$ is the logistic cumulative distribution function. However, if the fundamental exchange rate $s_t^*$ is misspecified by some constant amount $k$ so that the measured size of the bubble is $\hat{b}_t = b_t + k$, this would force $q$ to have its maximum at $b_t = -k$ instead of $b_t = 0$. This can be avoided by using the more general functional form

$$q(b_t) = \Phi(\beta_{q0} - \beta_{qb1} b_t - \beta_{qb2} \cdot b_t^2)\ , \tag{27}$$

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16. The sign restrictions are those implied by (15) and (17).

which still has the testable implication that $\beta_{qb2} > 0$.[17] Finally, we will assume that $e^S_{t+1}$ and $e^C_{t+1}$ follow independent and identically distributed normal distributions with mean 0 and standard deviations $\sigma_S$ and $\sigma_C$. This means that the log-likelihood function llf(.) for the bubble model can be written as

$$\mathrm{llf}(\beta_{S0}, \beta_{Sb}, \beta_{C0}, \beta_{Cb}, \beta_{q0}, \beta_{qb1}, \beta_{qb2}, \sigma_S, \sigma_C)$$

$$= \sum_{t=1}^{T} ln\left(\left(\Phi(\beta_{q0} - \beta_{qb1}b_t - \beta_{qb2} \cdot b_t^2) \cdot \phi(\frac{R_{t+1} - \beta_{S0} - \beta_{Sb}b_t}{\sigma_S}) / \sigma_S\right)\right.$$

$$\left.+\left([1 - \Phi(\beta_{q0} - \beta_{qb1}b_t - \beta_{qb2} \cdot b_t^2)] \cdot \phi(\frac{R_{t+1} - \beta_{C0} - \beta_{Cb}b_t}{\sigma_C}) / \sigma_C\right)\right), \qquad (28)$$

where $\phi(.)$ is the standard normal probability density function.

After estimating the model, the results should be checked for evidence of misspecification that might in turn lead to inconsistent estimates or invalid inferences. Fortunately, White (1987) presents a general score-based test for misspecification in maximum-likelihood models that leads to several immediately useful tests in the switching-regression context considered here.[18] White's test uses the fact that each element of gradient of the likelihood function at $t$ should be uncorrelated with its own lags. By choosing different elements of the gradient, we can derive tests with power against various kinds of misspecification. Furthermore,

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17. $\beta_{qb2} > 0$ is required to satisfy (11). The reader may verify that misspecifying the fundamental exchange rate by any constant amount $k$ will simply alter the estimates of $\beta_{S0}, \beta_{C0}, \beta_{q0}$ and $\beta_{qb1}$ without affecting $\beta_{Sb}, \beta_{Cb}$ or $\beta_{qb2}$, since the functional forms of (25), (26) and (27) will be unchanged.
18. See Hamilton (1990) for a discussion of these tests in the context of a Markov-mixture-of-normal-distributions model. Hamilton also presents Monte Carlo evidence which suggests that White's test tends to over-reject the null hypothesis in small samples. Accordingly, all the tests statistics presented below are interpreted using 1 per cent significance levels, as Hamilton suggests.

White's test is general enough to allow us to test for these effects either individually or jointly.

The results presented below test for three kinds of misspecification; omitted serial correlation, omitted heteroscedasticity, and more complex state dependence. Specifically, first-order serial correlation in the derivative of the likelihood function with respect to $\beta_{S0}$ and $\beta_{C0}$ would indicate the presence of an AR(1) error process in regimes $S$ and $C$ respectively. Similar correlations in the derivatives with respect to $\sigma_S$ and $\sigma_C$ would indicate the presence of ARCH(1) effects in their respective regimes. The presence of such first-order serial correlation in the derivative with respect to $\beta_{q0}$ would be evidence of state dependence in the classification probabilities and imply that a Markov switching regression would be more appropriate. Testing for omitted ARCH or Markov switching effects would seem to be particularly important given the popularity of ARCH and Markov mixture models of financial time series.

Having estimated the switching regression and tested for misspecification, we can then test the restrictions implied by the stochastic bubble model from section 2. One way to do so would seem to be by testing whether $\beta_{Sb} > 0$, $\beta_{Cb} < 0$ and $\beta_{qb2} > 0$. However, one property of switching regressions is that such models are identified only up to a particular renaming of parameters that has the effect of swapping the names of the $S$ and $C$ regimes. In this case, this equivalence implies that

$$\text{llf}(\beta_{S0}, \beta_{Sb}, \beta_{C0}, \beta_{Cb}, \beta_{q0}, \beta_{qb1}, \beta_{qb2}, \sigma_S, \sigma_C)$$

$$= \text{llf}(\beta_{C0}, \beta_{Cb}, \beta_{S0}, \beta_{Sb}, -\beta_{q0}, -\beta_{qb1}, -\beta_{qb2}, \sigma_C, \sigma_S) \qquad (29)$$

so these alternative parameterizations cannot be distinguished without additional information. Therefore, the bubble model implies that one should find either $[\beta_{Sb} > 0, \beta_{Cb} < 0, \beta_{qb2} > 0]$ or $[\beta_{Sb} < 0, \beta_{Cb} > 0, \beta_{qb2} < 0]$.

In addition to testing the restrictions implied by the bubble model, this paper also aims to test a more general set of hypotheses. In particular, we wish to see whether the estimated switching-regression model gives additional information about the behaviour of mixtures of distributions in $R_{t+1}$, whether it gives evidence of a particular kind of non-linearity in exchange rate behaviour, and whether it indicates that the distribution $R_{t+1}$ is predictable. We will now consider these points in turn.

As noted in the previous section's discussion of (14) and (16), one implication of the stochastic bubble model is that the errors generating $R_{t+1}$ will generally be from a mixture of distributions, which was assumed above to be a mixture of normals. This means that the switching regression embodied by (25), (26) and (27) will nest a general normal-mixture model as the special case where $\beta_{Sb} = \beta_{Cb} = \beta_{qb1} = \beta_{qb2} = 0$. This gives the model:

$$
\begin{aligned}
R^S_{t+1} &\sim N(\beta_{S0}, \sigma_S) \\
R^C_{t+1} &\sim N(\beta_{C0}, \sigma_C) \\
Pr(R_{t+1} = R^S_{t+1}) &= \Phi(\beta_{q0}) \, . \qquad (30)
\end{aligned}
$$

Note that (30) is more general than the restricted normal-mixture model estimated by Boothe and Glassman (1987), who also imposed the assumption of identical means, so $\beta_{S0} = \beta_{C0} = \beta_0$. It therefore seems logical to test both of these null hypotheses against the general switching-regression alternative, which can be done using standard likelihood-ratio (LR) tests.[19] A rejection of these null hypotheses would imply that there is a significant link between $b_t$ and the

19. A number of authors have noted that while Lagrange Multiplier and Wald tests should be asymptotically equivalent to the LR tests, they can give quite different results. The LR tests are thought to be the most reliable. For example, see Engle and Hamilton (1990).

behaviour of the mixing distributions, either because it captures shifts in their means, or in their mixing probabilities, or both.[20]

The switching-regression model also nests the linear regression model as the special case where $\beta_{S0} = \beta_{C0}, \beta_{Sb} = \beta_{Cb}, \beta_{qb1} = \beta_{qb2} = 0$, giving:[21]

$$\begin{aligned} R_{t+1} &= \beta_0 + \beta_b b_t + e_{t+1} \\ e_{t+1} &\sim N(0, \sigma_S) \text{ with prob } \Phi(\beta_q) \\ e_{t+1} &\sim N(0, \sigma_C) \text{ with prob } 1 - \Phi(\beta_q) \ . \end{aligned} \tag{31}$$

While equation (31) is a linear equation, it is not just the standard linear exchange rate model since it implies

$$R_{t+1} = \beta_0 + \beta_b \cdot (s_t - f(X_t)) \ , \tag{32}$$

where $f(X_t)$ is the linear model of exchange rate fundamentals defined in section 2. This means that fundamentals may be linear, but also that $R_{t+1}$ may be predictable using information available at time $t$.

One interpretation of (32), if $R_{t+1}$ is measured as the return to holding foreign exchange, would be that it represents a linear model of exchange rate risk premiums. Alternatively, this regression has the same form as that used in Cutler,

---

20. The Markov switching model of Engle and Hamilton (1990) does not nest within this general switching model since it introduces state-dependent switching probabilities. Nonetheless, the switching regression can capture very similar effects. In the Markov switching model, the probability of observing a given regime will vary over time depending on an unobserved state variable. In the switching regression, this probability varies as a function of the observed variable $b_t$. Given that $b_t$ usually shows positive serial correlation, the dynamics of the two models can be quite similar. Formal tests of these two models could be done by estimating a Markov switching regression. The bubble model would imply that the Markov behaviour should collapse to simple switching, while the Markov mixture model would imply that all the coefficients on $b_t$ should be insignificant. Instead, we simply test whether the switching-regression model incorrectly omits Markov switching effects that are present in the data.
21. In principle, one could also impose $\sigma_S = \sigma_C$, but this greatly complicates testing the null hypothesis against any switching alternative. Furthermore, given the possibility of heteroscedasticity in the data, it seems advisable to allow for time variation in $\sigma$ under the null hypothesis.

Poterba and Summers (1991) to describe non-rational speculative dynamics in a variety of asset markets. Any rejection of the restrictions implied by (31) would therefore not only be evidence of non-linearities in exchange rate behaviour, but also evidence of a more complex form of predictability than that considered by Cutler, Poterba and Summers (1991).

To summarize, maximization of the likelihood function in (28) allows estimation of the switching-regression system consistent with a model of stochastic bubbles. The bubble model implies testable coefficient restrictions on the switching-regression estimates. Furthermore, the switching regression can be tested against several simpler, nested models that include both normal-mixture models and linear models of return predictability. All that remains is to specify appropriate measures of $R_{t+1}$ and $b_t$ before estimating the model. This will be done in the next section.

## 4. Data and models of fundamental exchange rates

The empirical work presented below focusses on four of the most widely traded currencies in the world -- the Deutsche mark (DM), the Japanese yen (¥), and the Canadian dollar (Can.$) -- all measured relative to the U.S. Dollar (U.S.$). Unless specified otherwise, all data are in natural logarithms and are measured monthly. The series cover most of the post-Bretton Woods floating exchange rate period, from September 1977 to October 1991. More details on all the data series used may be found in the Appendix.

The first series to be defined is the innovation in the exchange rate $R_{t+1}$. The most straightforward measure would be to assume that covered and uncovered interest parity hold, so that the log of the one-period forward exchange rate, $f_t$, is equal to the expected value of the log spot rate next period, $s_{t+1}$.[22] This would suggest using

22. The literature on uncovered interest parity varies between assuming equality in levels or equality in logs, which differ slightly because of Jensen's inequality. The formulation used above has the advantage of avoiding Seigel's paradox. See Hodrick (1987) for details.

$$R_{t+1} = s_{t+1} - f_t \,. \tag{33}$$

While tests of covered interest parity suggest it holds quite closely, there is considerable evidence rejecting uncovered interest parity.[23] If this were simply due to the presence of a constant risk premium, then only the mean of $R_{t+1}$ would be affected, which would not affect any of the tests or restrictions proposed in section 3. Certainly, most of the empirical evidence implies that the predictable component of $s_{t+1} - f_t$ accounts for only a small fraction of its total variance at the monthly frequency considered here. Furthermore, the fraction of this variance that represents risk premiums is disputed. The standard, rational, representative-agent paradigm implies that all of the predictability should be the result of risk premiums. On the other hand, empirical models of the determinants of these premiums have had limited success, and studies using survey data on agents' forecasts of the spot rate imply that only a small fraction of the variance of the predictable component is due to risk premiums.[24]

The above discussion suggests that equation (33) will give a reasonable measure of the exchange rate innovation. Nonetheless, to check the robustness of the results below, an alternative measure of innovations was also used. This simply uses survey data on agents' expectations at $t$ of the exchange rate at $t+1$, ${}_t s^e_{t+1}$, to define the innovation as

$$R_{t+1} = s_{t+1} - {}_t s^e_{t+1} \,. \tag{34}$$

The disadvantage of this alternative measure is that data are only available for the DM/U.S.$ and ¥/U.S.$ exchange rates, and only from November 1982 to January 1988. Note that both measures are strongly correlated with the raw change

23. See Hodrick (1987) for an excellent summary and for a discussion of the empirical points raised in the remainder of the paragraph.
24. See Frankel and Froot (1987, 1990) and Ito (1990).

in the log spot exchange rate, $s_{t+1} - s_t$, which is the variable most commonly used in studies of mixtures of distributions.

The other series to be defined is the size of the bubble $b_t$. Because of the lack of a widely agreed-upon empirical model of fundamental exchange rates, several different models were used to construct $b_t$. Note that because the tests and restrictions suggested in section 3 are invariant to changes in the mean of $b_t$, the log of the fundamental exchange rate $\tilde{s}_t$ need only be defined up to an additive constant.

The simplest exchange rate model tested uses the assumption of relative purchasing power parity (PPP), which implies that the real exchange rate should be constant.[25] Therefore, the measured real exchange rate will move one-for-one with the size of the bubble. The real exchange rate measure used here is the Morgan Guaranty real effective exchange rate index. This is a multilateral index based on general and wholesale price indices for 40 nations. To provide a benchmark for the bilateral exchange rate against the U.S. dollar, indices for Canada, Germany and Japan are divided by that for the United States.[26]

An alternative assumption, common in the international trade literature, is that the fundamental real exchange rate is that rate which equilibrates the external sector of the economy. The deviation from this rate, $b_t$, should then be a function of the degree of external imbalance. The current account balance was therefore used as another measure of deviation from fundamentals, with an increase indicating a more undervalued (or less overvalued) exchange rate. Current accounts for all four nations are divided by GNP or GNE to provide a scale-free measure of imbalance, and the series for Canada, Germany and Japan were again measured relative to those for the United States.[27]

25. To fit this into the framework of equation (4), one can define $E_t(X_{t+1}) = (1-a) \cdot (p_t - p_t^f)$.
26. The use of a bilateral index based on normalized unit labour costs gave similar results.
27. Note that this measure is available on a quarterly basis only. Since it is already scale-free, this series is used in levels, not logarithms.

While these models of fundamental exchange rates may be quite simple, they have the advantage of being highly visible economic indicators. More sophisticated models of fundamentals are required to take account of realistic macroeconomic dynamics, however. One common approach is to use uncovered *real* interest parity, which requires that the real interest differentials correspond to expected changes in real exchange rates, or

$$E_t(Q_{t+k} - Q_t) = r_t - r_t^f \tag{35}$$

$$Q_t \equiv s_t - p_t + p_t^f , \tag{36}$$

where $Q_t$ is the logarithm of the real exchange rate at time $t$, $p_t$ and $p_t^f$ are the logarithms of the domestic and foreign price levels, $r_t$ is the logarithm of the sum of 1 plus the k-period real interest rate at time t, and $r_t^f$ is the corresponding transformation of the k-period foreign real interest rate.[28] If one assumes that no changes are expected in the long-run real exchange rate, then

$$E_t(Q_{t+k}) \rightarrow \overline{Q} \qquad \text{as} \qquad k \rightarrow \infty , \tag{37}$$

where $\overline{Q}$ is this long-run value. This model can be used to determine fundamental exchange rates by using a suitably large $k$.[29] This implies that the $k$-period real interest rate differential gives us an index of the fundamental real exchange rate, since (35) then gives

$$\overline{Q} - (r_t - r_t^f) = Q_t . \tag{38}$$

---

28. Note that nominal uncovered interest parity implies real uncovered interest parity. Therefore, the discussion of the validity of the uncovered interest parity assumption in the context of defining $R_{t+1}$ applies here also.
29. See Shafer and Loopesko (1983), Campbell and Clarida (1987), Meese and Rogoff (1987), and Edison and Pauls (1991) for a discussion of this class of models.

To convert this into a measure of bubble size simply requires[30]

$$b_t = s_t - (p_t - p_t^f) + (r_t - r_t^f) - \bar{Q} \, . \tag{39}$$

Earlier studies of bubbles in foreign exchange markets, such as those by Meese (1986), Woo (1985), West (1987), and Gros (1989), have used variants of the sticky-price monetary (or "overshooting") model of exchange rates. All begin with nominal uncovered interest parity

$$E_t(s_{t+1} - s_t) = i_t - i_t^f \tag{40}$$

and substitute out the interest rates using money-demand equations of the general form[31]

$$m_t - p_t = -a_0 \cdot i_t + a_1 \cdot y_t + a_2 \cdot (m_{t-1} - p_{t-1}) \ , \tag{41}$$

where $m_t, p_t, y_t$ represent the log of relative money supplies, prices and output, respectively, and $0 < a_2 < 1$. If we then assume that relative prices adjust slowly to their PPP values:

$$s_t - p_t = a_2 \cdot (s_{t-1} - p_{t-1}) \ . \tag{42}$$

West (1987) shows we can solve for the exchange rate, obtaining

$$\tilde{s}_t = \gamma s_{t-1} + E_t\left(\sum_{j=0}^{\infty} \lambda^j \cdot z_{t+j}\right) , \tag{43}$$

where

30. Note that the addition of a constant risk premium to (35) would have the same effect as a shift in $\bar{Q}$, shifting the intercept term but maintaining a one-to-one relationship between $(r_t - r_t^f)$ and $Q_t$, and therefore maintaining the validity of (39).

31. Woo (1985) uses a slightly more general form, allowing for different coefficients on foreign and domestic output.

$$z_t = (\lambda / a_0) \cdot (m_t - a_2 m_{t-1} - a_1 y_t) \ ; \tag{44}$$

$$\gamma = \{1 + a_0 - \sqrt{(1+a_0)^2 - 4a_0 a_2}\} / (2a_0) \ ; \tag{45}$$

and

$$\lambda = (2a_0) / \{1 + a_0 + \sqrt{(1+a_0)^2 - 4a_0 a_2}\} \ . \tag{46}$$

Experimentation with alternative estimates of $a_0$, $a_1$ and $a_2$ showed that reasonable estimates of the fundamental exchange rate could be obtained using $a_0 = 0.5$, $a_1 = 1.0$ and $a_2 = 0.9$ on monthly data.[32]

## 5. Estimation and test results

Tables 1 through 3 (pp. 36 to 38) summarize the results of the estimated switching-regression model using the 1977-1991 data on forward and spot exchange rates. As one would expect, the results are sensitive to the model of fundamentals used to construct $b_t$ and to the particular exchange rate considered.

The results for Canada in Table 1 (p. 36) show that the overshooting model of fundamentals gives the highest values of the likelihood function and the most support for the bubble model. This bubble measure gives LR statistics that allow us to reject the three simpler models of regime switching in favour of the switching-regression alternative predicted by the bubble model. We also find that although two of the four bubble measures have a significant influence on $R_{t+1}$ in state $S$, only the overshooting measure gives evidence that the size of the bubble also affects $R_{t+1}$ in state $C$. Furthermore, the opposite signs on $\beta_{Sb}$ and $\beta_{Cb}$ that the overshooting measure gives are consistent with the predictions of the bubble

32. Instability of the money-demand functions in the 1980s could potentially cause this model to undergo structural shifts. Attempts to estimate parameters directly and allow for these shifts gave very poor results, with fundamental exchange rates that differed from actual rates by factors of 10 or more. In contrast, use of the constant parameter values given above gave fundamental rates that seemed reasonable over the full sample.

model. There is no significant evidence for any of the bubble measures that the classification probabilities are affected by the square of the bubble's size, nor is there any evidence of misspecification.

The results in Table 2 (p. 37) for Germany show that the external balance measure of the bubble has the best fit (as judged by the value of the likelihood function) and is the only measure that can reject either the unrestricted or the restricted normal-mixture models. However, it is unable to reject the null hypothesis of linear predictability. None of the bubble measures had more than one coefficient in the switching regression that was significant at the 5 per cent level. The external balance measure comes close, with a significant $\beta_{Sb}$ and a $\beta_{Cb}$ with a marginal significance level of 5.2 per cent (that is, with a t-ratio of 1.94 versus a 5 per cent critical value of 1.96). It should be noted, however, that both parameter estimates are negative, while the bubble model predicts that they should have opposite signs. Only the PPP measure produces evidence of misspecification, apparently due to the presence of both serial correlation and ARCH effects in regime $S$.

The Japanese results in Table 3 (p. 38) show that the PPP measure fits best, followed by the external balance measure. Both allow us to reject all three simpler switching models, while the other bubble measures fail to reject the null hypothesis of an unrestricted mixture of normal distributions. The estimate of $\beta_{Sb}$ is negative, significant and quite similar for all four measures of $b_t$. Additional coefficients are significant for the PPP and external balance measures, which therefore gives us another binding restriction on the bubble model's predictions. These are satisfied in the case of the PPP measure, but not in the case of the external balance measure, where increases in the bubble are found to decrease expected returns in both regimes. There is no significant evidence of misspecification, except in the case of the PPP measure. Even there the evidence is weak, with a significant joint test (with an LR statistic of 15.8 versus a critical value of 15.1) but no significant individual tests. The joint test presumably reflects some weak evidence of serial correlation, as the test statistics for first-order autocorrelation in each regime are closest to their critical values (4.7 and 3.6, compared to a critical value of 6.6).

To summarize these results, we can see that measures of apparent deviations of exchange rates from their fundamental values have some descriptive power for subsequent excess returns in a number of cases. Furthermore, for Canada and Japan there is evidence of significant non-linearities in this relationship. Diagnostic tests also generally suggest that the simple switching regression adequately captures several aspects of the data. In particular, there is almost no evidence of omitted ARCH effects or Markov state dependencies. None of the bubble measures produced evidence that supported all three of the parameter restrictions predicted by the bubble model. On the other hand, in two cases (the overshooting measure for Canada and the PPP measure for Japan) two of these three parameters were significant and had the predicted signs.

To check the sensitivity of these results to the definition of exchange rate innovations $R_{t+1}$, Tables 4 and 5 (pp. 39 and 40) show the results of re-estimating the switching regression using the survey data on exchange rate expectations rather than the forward rate. Unfortunately, this greatly reduces the number of observations (from 170 to 63) and the time span of the data (November 1982 to January 1988 instead of September 1977 to October 1991), which makes it difficult to determine whether any changes are due to the way expectations are measured, or to the change in sample size and period. To control for the latter, Tables 6 and 7 (pp. 41 and 42) present the switching-regression results estimated on the forward rate data, but using only the observations corresponding to the survey data.

Table 4 shows that the survey data give somewhat different results for Germany. The external balance and PPP measures are still the best fitting, but both now reject all three nested models in favour of the general switching regression. The real interest parity measure also now rejects the normal mixture null (and therefore implicitly rejects the volatility regimes null as well), but still fails to reject the null hypothesis of linear predictability. None of the bubble measures produce more than one significant bubble coefficient, as before, so there are still no binding restrictions on the bubble model's parameters. Finally, the misspecification tests give results similar to those in Table 2 (p. 37), except that the PPP measure no longer gives significant evidence of misspecification.

Table 5 shows that the survey data also give somewhat different results for Japan. The overshooting measure still gives the highest value of the likelihood function, but now it is the only model that gives significant LR test statistics, and now all three statistics are significant. The LR statistics for the PPP model are still weakly significant, however, with each one rejecting its null hypothesis at around the 10 per cent significance level. There are fewer significant coefficients on the bubble measures, and no longer more than one at a time, although simple Wald tests still reject the hypothesis that $\beta_{Sb} = \beta_{Cb}$ for all but the real interest parity measure of $b_t$. Finally, the weak evidence of misspecification found in the forward rate data is now absent.

Clearly, results differ somewhat depending on which measure of $R_{t+1}$ (and therefore which sample period) we consider. By comparing the above results with those in Table 6 and Table 7 (pp. 41 and 42), we can see that neither the measure nor the sample period by themselves are responsible for the differences. The German results in Table 6 are similar to the survey data results for the overshooting measure of $b_t$, but they differed somewhat for the external balance and real parity measures. Furthermore, while the statistics that were significant for the PPP measure in the survey data are still significant, the estimate of $\beta_{Sb}$ now has the opposite sign. Accordingly, the relative importance of the sample period and the definition of $R_{t+1}$ seem to vary across bubble measures.

The results for Japan in Table 7 lead to the same conclusion. While the results for the overshooting measure are very similar to those for the survey data (particularly if we allow for a relabelling of regimes $S$ and $C$). There is also some similarity in the results for the PPP measure (again allowing for a relabelling of regimes $S$ and $C$) if we consider the survey data's LR statistics to be weak evidence against their respective null hypotheses. However, results for the other two measures simply seem to be quite different across the survey and forward rate data, even after one controls for the effects of the sample period.

## 6. Further implications of the bubble model

While the above results give formal statistical tests of the bubble model, they furnish little insight into the kind of behaviour that the model captures. In particular, to help assess the reasonableness of the bubble hypothesis, it would be useful to see whether such behaviour is consistent with popular accounts of speculative episodes. This could also help in determining whether the results are due to speculative bubbles or to process switching in fundamentals.

For that reason, we now examine more closely the behaviour of the switching regression in two of the cases above that most consistently support the bubble model. The first case we consider is the Canada-U.S. exchange rate using the overshooting model of fundamentals with $a_2 = 0.9$.[33] The second case is the Japan-U.S. exchange rate with the PPP model of fundamentals. Because none of the results we considered above gave very strong support to the bubble model for the Germany-U.S. exchange rate, particularly in the full forward-rate sample, we do not present further results for that exchange rate.

Figure 1 (p. 45) compares the spot Canada-U.S. exchange rate with the calculated size of the bubble. (Note that the bubble size is identified only up to an arbitrary constant and is graphed with a mean of zero.) Both series are dominated by the depreciation of the Canadian dollar over the 1983-86 period, which causes a rise in the bubble measure of 0.12-0.15. Therefore, if we believe the exchange rate was close to fundamentals at the beginning of this period, by early 1986 the

33. In the graphs beginning on page 45, the data cover a slightly longer sample period, ending in January 1992. While the addition of these three extra observations has no noticeable effect on any of the test results reported above, it allows us to see the apparent collapse of a bubble in January 1992.

Canadian dollar appeared to be 12 to 15 per cent undervalued. In contrast, the currency is most overvalued from 1977-81 and in 1991, where the same benchmark suggests an overvaluation of about 2 to 5 per cent.

The top of Figure 2 (p. 46) shows the *ex ante* probability of a collapse, calculated as $1 - q(b_t)$ from (27). This suggests two or three distinct bubble episodes. Not surprisingly, the most prominent one covers the undervaluation of the Canadian dollar from early 1985 to the end of 1986, and corresponds to the peaks in the size of the bubble. At its most extreme, the *ex ante* probability of a collapse exceeded 20 per cent per month, suggesting the bubble had become quite fragile. A second episode is the apparent overvaluation of the Canadian dollar in 1991, which produces a short-lived but distinct surge in the probability of a collapse. Note that this bubble is apparently almost totally unwound by the sharp depreciation of the Canadian dollar in January 1992. The third episode is less distinct and corresponds to a possible overvaluation of the Canadian dollar in the late 1970s. The peaks here are lower than in the other two episodes and there is greater month-to-month variability in the results, however, so the evidence is less definitive. Finally, we can also note that the probability of collapse is lowest in the 1982-83 and 1988-89 periods, suggesting that the exchange rate was close to its fundamental values at these times.

To understand what this implies for the expected effects of bubble collapse, the bottom of Figure 2 shows the difference in expected returns on U.S. dollars across states S and C. (As with $b_t$ in Figure 1, p. 45, this series is graphed with a mean of zero.) Since it is just a linear transformation of $b_t$, we find it has a similar shape, with peaks in the 1984-87 period and lows during 1977-81 and 1991. Relative returns in the two regimes change by less than 2 per cent per month (27 per cent per annum) from the peak of the undervaluation to the maximum overvaluation. This implies only a fraction of the bubble can be expected to be reversed in any given month, so bubbles may take some time to be eliminated, even conditional on a collapse.

While the timing of the bubble episode in the late 1970s may be somewhat surprising, particularly in light of its apparent longevity, the first two episodes identified above seem to fit with the view of other market participants. The undervaluation of the Canadian dollar in early 1986 led the Canadian government and the Bank of Canada to take several steps to correct what they believed to be undue speculation against the Canadian dollar. This included intensive exchange market intervention in support of the Canadian currency, whose exchange rate they felt "did not reflect the fundamentals of our economic and financial situation. . . ."[34] The overvaluation in 1991 also corresponds to a period where several Canadian observers felt the currency was overvalued.[35] However, it should be noted that the overvaluation covers a shorter period of time and was smaller than some of these observers suggested.[36]

The last two graphs present the same evidence for the Japanese data. Figure 3 (p. 47) shows a pattern somewhat similar to that for Canada. The bubble size is dominated by an apparent undervaluation of the yen starting in 1982 and declining rapidly after the Plaza Accord in September 1985. In addition, there appear to be two periods of yen overvaluation before and after the undervaluation, consisting of a brief period in the latter half of 1978 and a more prolonged episode centred in 1988. The variation in the bubble's size is much more pronounced than in the Canadian data, with a peak-to-trough change of roughly 70 per cent, compared to a range of only 18 per cent for Canada.

The probabilities of collapse in Figure 4 (p. 48) tend to confirm the timings of these bubble episodes, although they suggest that the undervaluation of the yen in the mid-1980s may be divided into two episodes, with a distinct peak and decline in 1982 followed by an even larger peak in 1985. This corresponds to a

34. Bank of Canada (1986, 15).

35. For example, see Courchene (1991) and Harris (1991).

36. The overshooting model finds a bubble that does not begin until 1990 at the earliest, and never exceeds Can.$0.05 in size, whereas some observers suggested an overvaluation starting in 1987 and ranging from Can.$0.10-$0.15 in size.

period when, probably more so than in any other period since the breakdown of the Bretton Woods system, many observers felt the U.S. dollar was overvalued.[37] The probability of collapse shows even higher peaks for Japan than for Canada, reaching 44.6 per cent per month in March 1985 as the U.S. dollar peaked against most overseas currencies. Variations in expected returns across regimes again reflect the movements of the bubble measure. Given the larger apparent size of the bubble in the ¥-U.S. dollar exchange rate, however, it is perhaps surprising to see that differences in returns across regimes are expected to be roughly the same size as those in the Canadian data.

Overall, it appears that the econometric evidence and the historical record can be consistent with the implications of the bubble model, given particular assumptions about fundamentals. As argued in the introduction, however, the switching regression can be useful in its own right as a descriptive device to characterize the behaviour of risk premiums and the distribution of excess returns. For that reason, we now turn briefly to consider two aspects of the switching-regression estimates that are not directly related to the bubble model.

The first of these is behaviour of expected excess returns, or risk premiums. It is straightforward to show that the expected value of $R_{t+1}$ conditional on $b_t$ (which is contained in agents' information sets) is given by

$$E(R_{t+1}|b_t) = q(b_t) \cdot (\beta_{S0} + \beta_{Sb} b_t) + (1 - q(b_t)) \cdot (\beta_{C0} + \beta_{Cb} b_t) \qquad (47)$$

The bubble model derived in section 2 simply assumed that excess returns are unpredictable. The switching regression does not impose this restriction, however. Figures 5 and 6 (pp. 49 and 50) graph $E(R_{t+1}|b_t)$ alongside $b_t$ for the Canadian and Japanese models discussed above.

We see that for Canada, expected excess returns are almost perfectly correlated with $b_t$. This reflects the fact that $\beta_{qb1}$, $\beta_{qb2}$ are not significantly different from zero and have quite small estimates. This in turn means that $q(b_t)$ is

37. See Krugman (1985).

roughly constant, in which case $E(R_{t+1}|b_t)$ effectively collapses to a linear function of $b_t$, which is what we observe in this case. If we compare Figure 5 to Figure 2, we see that the variation in expected excess returns is only a fraction of the variation in the difference in expected returns across regimes. The former has a range of just under 0.0045, while the latter's range is more than three times larger.

Figure 6 shows that results for Japan look quite different. The correlation between $b_t$ and $E(R_{t+1}|b_t)$ is weaker and negative. This reflects both the non-linearity arising from a larger $\beta_{qb2}$ and the negative estimates of $\beta_{Sb}$ and $\beta_{Cb}$. Furthermore, the variation in expected excess returns over time is both larger than that for Canada, and is almost as great as the variation in expected returns across regimes. (See Figure 4.) The latter suggests that the bubble model may omit an important source of predictable variation in excess returns, that is itself correlated with deviations from the fundamental (PPP) exchange rate.

The second interesting aspect of the distribution of excess returns is the predictable variation in their dispersion. One way to quantify this is to consider the conditional probability of observing an outlier of a given size, say two standard deviations from the sample mean. It can be shown that

$$Pr(R_{t+1} < x) = \\ = \varphi\left(\frac{x - \beta_{S0} - \beta_{Sb}b_t}{\sigma_S}\right) \cdot q(b_t) + \varphi\left(\frac{x - \beta_{C0} - \beta_{Cb}b_t}{\sigma_C}\right) \cdot (1 - q(b_t)) \tag{48}$$

and that

$$Pr(R_{t+1} > x) = \\ = \varphi\left(\frac{-x + \beta_{S0} + \beta_{Sb}b_t}{\sigma_S}\right) \cdot q(b_t) + \varphi\left(\frac{-x + \beta_{C0} + \beta_{Cb}b_t}{\sigma_C}\right) \cdot (1 - q(b_t)) \tag{49}$$

where $\varphi$ is the standard normal cummulative distribution function.[38]

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38. Note that changes in $x$ just give a monotonic transformation of the probabilities, so the conclusions below are robust to changes in $x$.

Figures 7 and 8 (pp. 51 and 52) below show the probabilities of observing an excess return either two standard deviations above or below the sample mean. If the expected value of returns $E(R_{t+1}|b_t)$ is fairly constant but its variance around this expection is not, the probablilities of observing high and low outliers should be positively correlated. However, if $E(R_{t+1}|b_t)$ is quite variable and the variance around this expectation is stable, then the probabilities of observing high and low outliers should be negatively correlated. The figures show that for both Canada and Japan, the probabilities of observing outliers two standard deviations above or below the mean are strongly negatively correlated, implying that variations in the conditional distribution of returns are dominated by shifts in its mean rather than its dispersion. This strengthens the conclusion drawn from the graphs of $E(R_{t+1}|b_t)$ that there is an important source of predictable variation in expected excess returns that is correlated with deviations from the fundamental exchange rate.

## 7. Conclusions

This paper has described a two-regime model of speculative bubbles. We have seen how simple restrictions on the behaviour of bubbles lead to a new set of testable predictions about the behaviour of exchange rate innovations. These predictions have implications for a variety of current areas of research. They suggest generalizing univariate mixture-distribution models of innovations to a switching-regression framework and linking mixing behaviour to other macroeconomic variables. They also suggest a relationship between current macroeconomic variables and future exchange rate innovations that nests other empirical models of asset price dynamics.

The results presented above for the Canada-U.S., Germany-U.S. and Japan-U.S. exchange rates show that support for the predictions of the bubble model may be found. However, results are generally sensitive to changes in the definition of the fundamental exchange rate and to the measurement of exchange rate innovations. Evidence supporting the bubble model is strongest when using excess returns data and an overshooting model of fundamentals for the Canada-U.S.

exchange rate or a PPP model for the Japan-U.S. exchange rate. Furthermore, the bubble episodes identified by the switching-regression model in these cases correspond well to periods that others have associated with deviations from fundamentals. Additional evidence also suggests that deviations from fundamental exchange rates in these cases have an important influence on expected excess returns, a fact which is not predicted by the bubble model.

Regardless of whether one accepts the bubble interpretation of the above results or not, the findings should be of interest to those studying the efficiency of foreign exchange markets. Obviously, the implication that bubbles may be present is of direct interest. However, even the interpretation that there are other, fundamental causes of switching implies that the distribution of exchange rate innovations varies over time in a manner not previously considered. Since this time variation may be predicted by other macroeconomic variables, such variation should affect exchange rate risk premiums. A satisfactory, fundamental model of this switching behaviour and of the relationship between deviations from fundamentals and expected returns should therefore be an important ingredient in future research on risk premiums.

Distinguishing between switching in returns due to bubbles and that caused by process switching in fundamentals will be difficult and is beyond the scope of this paper. A useful way to proceed would be to specify a particular model of switching in fundamentals and to examine whether actual switches in fundamentals correspond to apparent switches in exchange rate innovations. In van Norden and Schaller (1993a), we use such an approach to study historical crashes in U.S. stock prices, and conclude that both bubbles and switches in fundamentals seem to play a role. Work by Lewis (1989) with monetary models of exchange rate determination has shown that while switches in monetary policy can explain the apparent predictability of forward-rate prediction errors in certain periods, they do not seem to fit during the significant U.S. dollar appreciation in 1984-85. In contrast, the results here seem to fit well during this period, again suggesting that both factors may be at work.

## Appendix: Data definitions

The spot exchange rates and the one-month forward premium/discount rates used to generate our measures of exchange rate innovations are taken from the Bank of International Settlements data base. The DM-U.S.$ spot rate is the official fixing at 13:00 Frankfurt time, while its forward rate is the middle market rate around noon Swiss time. The ¥-U.S.$ spot and forward rates are the Tokyo market closing middle rate and the London middle market rate at around noon Swiss time, respectively. Both spot and forward Can.$-U.S.$ exchange rates are London middle market rates at around noon, Swiss time. Excess returns are calculated based on the following conventions:

i) The spot settlement date is two business days after the trading date except for the Can.$-U.S.$, for which the settlement date is one business day after the trading date.

ii) The one-month maturity date is the same day as the spot settlement date moved forward to the next month, unless the maturity date is not a valid business date in either of the two home markets. In this case, the maturity date is delayed until the next business day valid in both countries. However, if the previous conventions take us out of the month, we move backwards to the first suitable one-month maturity date.

iii) However, if the spot settlement date is the last valid business day of the current month, then the one-month maturity date is the last business day of the next month.

The procedure used to calculate the innovations also requires variables to represent days when the markets were closed in the respective home countries. These "holiday" variables are the DM-U.S.$ spot rate for Germany, the ¥-U.S.$ spot rate for Japan, the noon Can.$-U.S.$ spot as recorded by the Bank of Canada for Canada, and the 90-Day U.S. Treasury bill interest rate for the United States.

The alternative measure of innovations is based on the one-month ahead Money Market Services survey data and the spot rate 30 days later, as documented in Frankel and Froot (1990).

PPP exchange rates are Morgan Guaranty's real effective exchange rate indices, which are based on wholesale price indices for 18 industrial countries and 22 less-developed countries.

The real long interest rates are constructed by using year-over-year CPI inflation as our proxy of expected inflation. The interest rate differential measures are then constructed by using the formula:

$$diff = \left(\frac{1 + r^{US}}{1 + r^{other}}\right)^{10} \tag{50}$$

where $r^{US}$ is the U.S. real long-term interest rate and $r^{other}$ is the real rate for the differential country. The long-term interest rates are based on 7- to 10-year government bonds for each nation.

Finally, the overshooting-model exchange rates were calculated using data from a variety of sources. Money supplies were taken to be national measures of M1, prices were national CPIs, output measures were indices of industrial production, and interest rates were 30-day money market or commercial paper rates.

## Tables

**Table 1: Results for Canada using forward rates**
**September 1977 - October 1991 (170 observations)**

| | PPP | External balance | Real interest parity | Over-shooting |
|---|---|---|---|---|
| Ave. llf | 2.931 | 2.926 | 2.909 | 2.957 |
| Restricted normal mix. | 10.05 (7.3%) | 8.22 (14.4%) | 2.47 (78%) | **18.85** (0.2%) |
| Unrestricted normal mix. | 8.47 (7.5%) | 6.63 (15.7%) | 0.88 (92%) | **17.24** (0.2%) |
| Linear regression | 9.02 (6.0%) | 6.29 (17.8%) | 2.34 (67.7%) | **15.99** (0.3%) |
| $\beta_{Sb}$ | **0.0045** (3.68) | 0.003 (1.95) | -0.001 (-0.5) | **0.002** (4.47) |
| $\beta_{Cb}$ | -0.0008 (-0.487) | 0.0007 (0.36) | -0.006 (-0.439) | **-0.003** (-1.98) |
| $\beta_{qb2}$ | -0.011 (-0.028) | 0.338 (-1.132) | 0.0753 (0.214) | -0.139 (-0.45) |
| Joint | 3.01 | 7.04 | 1.56 | 1.44 |
| AR(1):S | 0.809 | 0.891 | 0.332 | 0.171 |
| AR(1):C | 0.330 | 0.191 | 0.239 | 0.067 |
| ARCH:S | 0.761 | 4.906 | 0.446 | 0.048 |
| ARCH:C | 0.345 | 0.520 | 0.412 | 0.785 |
| Markov | 0.454 | 0.237 | 0.204 | 0.140 |

Note: See the explanatory notes on page 43.
**Boldface** indicates significance at the 5 per cent level for likelihood-ratio tests and parameter estimates, and significance at the 1 per cent level for diagnostic tests.

**Table 2: Results for Germany using forward rates**
**September 1977 - October 1991 (170 observations)**

| | PPP | External balance | Real interest parity | Over-shooting |
|---|---|---|---|---|
| Ave. llf | 1.918 | 1.925 | 1.916 | 1.906 |
| Restricted normal mix. | 8.86 (11.5%) | **11.38** (4.4%) | 8.47 (13.2%) | 4.89 (42.9%) |
| Unrestricted normal mix. | 8.85 (6.5%) | **11.37** (2.2%) | 8.46 (7.5%) | 4.88 (29.9%) |
| Linear regression | 8.83 (6.5%) | 4.86 (30.1%) | 5.15 (27.3%) | 3.89 (41.9%) |
| $\beta_{Sb}$ | 0.0008 (0.02) | **-0.044** (-3.29) | **-0.010** (5.18) | 0.012 (1.80) |
| $\beta_{Cb}$ | -0.0014 (-0.46) | -0.006 (-1.94) | -0.003 (-0.75) | -0.008 (-0.62) |
| $\beta_{qb2}$ | 14.43 (-0.19) | 3.97 (1.24) | -0.127 (-0.32) | 0.069 (0.12) |
| Joint | 11.0 | 2.73 | 7.68 | 9.83 |
| AR(1):S | **8.272** | 0.033 | 5.767 | 4.277 |
| AR(1):C | 0.005 | 0.657 | 0.080 | 2.025 |
| ARCH:S | **8.272** | 0.220 | 0.200 | 0.054 |
| ARCH:C | 0.756 | 1.389 | 0.777 | 1.931 |
| Markov | 0.880 | 0.876 | 0.000 | 1.723 |

Note: See the explanatory notes on page 43.
**Boldface** indicates significance at the 5 per cent level for likelihood-ratio tests and parameter estimates, and significance at the 1 per cent level for diagnostic tests.

**Table 3: Results for Japan using forward rates**
**September 1977 - October 1991 (170 observations)**

| | PPP | External balance | Real interest parity | Over-shooting |
|---|---|---|---|---|
| Ave. llf | 1.925 | 1.915 | 1.891 | 1.894 |
| Restricted normal mix. | **28.73** (0.0%) | **25.28** (0.0%) | **17.15** (0.4%) | **18.26** (0.2%) |
| Unrestricted normal mix. | **18.66** (0.2%) | **15.18** (0.4%) | 7.04 (13.4%) | 8.15 (8.6%) |
| Linear regression | **26.15** (0.0%) | **13.83** (0.7%) | **16.27** (0.2%) | **16.65** (0.3%) |
| $\beta_{Sb}$ | **-0.001** (-6.32) | **-0.002** (-3.02) | **-0.002** (-5.05) | **-0.002** (-12.7) |
| $\beta_{Cb}$ | -0.006 (-1.60) | **-0.008** (-2.52) | -0.003 (-0.89) | -0.004 (-1.17) |
| $\beta_{qb2}$ | **-1.139** (-2.52) | 0.897 (1.26) | -0.240 (-0.84) | -0.562 (-0.91) |
| Joint | **15.8** | 10.1 | 9.81 | 11.0 |
| AR(1):S | 4.704 | 2.292 | 1.039 | 1.525 |
| AR(1):C | 3.623 | 2.224 | 3.548 | 4.101 |
| ARCH:S | 2.701 | 0.122 | 0.668 | 0.952 |
| ARCH:C | 0.433 | 0.035 | 0.394 | 0.269 |
| Markov | 0.022 | 0.619 | 0.008 | 2.803 |

Note: See the explanatory notes on page 43.
**Boldface** indicates significance at the 5 per cent level for likelihood-ratio tests and parameter estimates, and significance at the 1 per cent level for diagnostic tests.

**Table 4: Results for Germany using survey data**
**November 1982 - January 1988 (63 observations)**

| | PPP | External balance | Real interest parity | Over-shooting |
|---|---|---|---|---|
| Ave. llf | 2.242 | 2.264 | 2.199 | 2.178 |
| Restricted normal mix. | **15.94** **(0.7%)** | **18.66** **(0.2%)** | 10.52 (6.2%) | 7.77 (16.9%) |
| Unrestricted normal mix. | **15.83** **(0.3%)** | **18.55** **(0.1%)** | **10.41** **(3.4%)** | 7.66 (10.4%) |
| Linear regression | **15.91** **(0.3%)** | **10.98** **(2.6%)** | 4.15 (38.0%) | 7.68 (10.4%) |
| $\beta_{Sb}$ | **0.014** **(3.62)** | **-0.009** **(-3.15)** | **-0.008** **(-2.84)** | 0.003 (1.21) |
| $\beta_{Cb}$ | -0.011 (-0.47) | 0.035 (0.58) | 0.003 (0.10) | -0.069 (-0.97) |
| $\beta_{qb2}$ | 3.09 (1.55) | -3.402 (-0.84) | -1.832 (-0.72) | -2.94 (-0.48) |
| Joint | 4.28 | 13.4 | 4.59 | 10.8 |
| AR(1):S | 1.756 | 1.402 | 1.382 | 4.951 |
| AR(1):C | 0.155 | 1.726 | 0.617 | 0.829 |
| ARCH:S | 0.644 | 2.362 | 0.336 | 2.461 |
| ARCH:C | 1.989 | 5.843 | 2.034 | 0.571 |
| Markov | 0.105 | 2.619 | 0.073 | 1.698 |

Note: See the explanatory notes on page 43.
**Boldface** indicates significance at the 5 per cent level for likelihood-ratio tests and parameter estimates, and significance at the 1 per cent level for diagnostic tests.

**Table 5: Results for Japan using survey data**
**November 1982 - January 1988 (63 observations)**

| | PPP | External balance | Real interest parity | Over-shooting |
|---|---|---|---|---|
| Ave. llf | 2.320 | 2.290 | 2.314 | 2.370 |
| Restricted normal mix. | 9.20 (10.1%) | 5.36 (37.2%) | 8.42 (13.5%) | **15.54** (0.8%) |
| Unrestricted normal mix. | 8.82 (6.6%) | 4.97 (29.0%) | 8.01 (9.1%) | **15.11** (0.4%) |
| Linear regression | 8.58 (7.2%) | 4.76 (31.3%) | 4.60 (32.6%) | **13.64** (0.9%) |
| $\beta_{Sb}$ | **-0.010** (-4.17) | **-0.014** (-5.07) | -0.008 (-1.82) | **-0.008** (-5.31) |
| $\beta_{Cb}$ | 0.008 (1.44) | 0.004 (0.66) | -0.001 (-0.01) | 0.005 (0.86) |
| $\beta_{qb2}$ | -4.22 (-0.94) | 0.292 (0.49) | 0.458 (0.44) | -2.976 (-1.39) |
| Joint | 8.39 | 7.70 | 12.9 | 11.1 |
| AR(1):S | 0.042 | 2.635 | 6.111 | 2.015 |
| AR(1):C | 2.715 | 2.346 | 0.245 | 4.053 |
| ARCH:S | 1.268 | 0.214 | 1.276 | 4.456 |
| ARCH:C | 0.053 | 0.532 | 3.566 | 0.027 |
| Markov | 2.697 | 1.444 | 1.577 | 0.223 |

Note: See the explanatory notes on page 43.
**Boldface** indicates significance at the 5 per cent level for likelihood-ratio tests and parameter estimates, and significance at the 1 per cent level for diagnostic tests.

**Table 6: Results for Germany using forward rates**
**November 1982 - January 1988 (63 observations)**

| | PPP | External balance | Real interest parity | Over-shooting |
|---|---|---|---|---|
| Ave. llf | 1.988 | 1.967 | 1.963 | 1.942 |
| Restricted normal mix. | **15.31** (0.9%) | **12.60** (2.7%) | **12.17** (3.3%) | 9.46 (9.2%) |
| Unrestricted normal mix. | **12.65** (1.3%) | **9.96** (4.1%) | **9.52** (4.9%) | 6.81 (14.6%) |
| Linear regression | **15.25** (0.4%) | 5.58 (23.3%) | **9.88** (4.3%) | 8.54 (7.4%) |
| $\beta_{Sb}$ | **-0.021** (-21.9) | 0.011 (0.78) | **-0.021** (-2.57) | -0.009 (-1.06) |
| $\beta_{Cb}$ | 0.003 (0.76) | 0.009 (0.76) | -0.005 (-1.13) | -0.010 (-1.40) |
| $\beta_{qb2}$ | 0.120 (0.21) | 0.063 (0.05) | 1.359 (1.27) | 0.727 (1.79) |
| Joint | 5.45 | 6.30 | 4.20 | 3.21 |
| AR(1):S | 1.058 | 0.752 | 2.335 | 0.294 |
| AR(1):C | 0.019 | 4.994 | 0.132 | 2.820 |
| ARCH:S | 1.058 | 0.177 | 0.069 | 0.040 |
| ARCH:C | 0.166 | 0.002 | 0.081 | 0.452 |
| Markov | 5.761 | 0.228 | 1.736 | 0.022 |

Note: See the explanatory notes on page 43.
**Boldface** indicates significance at the 5 per cent level for likelihood-ratio tests and parameter estimates, and significance at the 1 per cent level for diagnostic tests.

**Table 7: Results for Japan using forward rates November 1982 - January 1988 (63 observations)**

| | PPP | External balance | Real interest parity | Over-shooting |
|---|---|---|---|---|
| Ave. llf | 2.133 | 2.139 | 2.118 | 2.146 |
| Restricted normal mix. | **19.37** (0.1%) | **20.05** (0.1%) | **17.47** (0.4%) | **20.91** (0.1%) |
| Unrestricted normal mix. | 8.45 (7.6%) | 9.13 (5.8%) | 6.55 (16.1%) | **9.99** (4.0%) |
| Linear regression | **19.34** (0.1%) | **17.63** (0.1%) | **16.74** (0.2%) | **20.72** (0.1%) |
| $\beta_{Sb}$ | -0.003 (-0.72) | 0.009 (0.52) | -0.003 (-0.64) | -0.003 (-0.49) |
| $\beta_{Cb}$ | **-0.008** (-2.58) | 0.004 (0.85) | **-0.007** (-2.64) | **-0.007** (-2.60) |
| $\beta_{qb2}$ | 0.441 (0.95) | -0.553 (-1.13) | -0.138 (-0.62) | 0.614 (1.63) |
| Joint | 12.7 | 2.67 | 4.59 | 11.7 |
| AR(1):S | 4.227 | 2.222 | 2.135 | 6.196 |
| AR(1):C | 0.005 | 0.006 | 2.257 | 0.304 |
| ARCH:S | 0.124 | 0.378 | 0.222 | 0.135 |
| ARCH:C | 4.084 | 0.036 | 0.050 | 3.321 |
| Markov | 0.199 | 0.064 | 0.199 | 0.443 |

Note: See the explanatory notes on page 43.
**Boldface** indicates significance at the 5 per cent level for likelihood-ratio tests and parameter estimates, and significance at the 1 per cent level for diagnostic tests.

## Notes to tables

The first row of the tables presents the average log-likelihood of each model, which allows a comparison of the model's fit using different measures of the bubble.

The second section of the tables presents the likelihood-ratio test statistics for various restrictions of the switching-regression model

$$(R^S_{t+1} - \beta_{S0} - \beta_{Sb} b_t) \sim N(0, \sigma_S)$$
$$(R^C_{t+1} - \beta_{C0} - \beta_{Cb} b_t) \sim N(0, \sigma_C)$$
$$\Pr(R_{t+1} = R^S_{t+1}) = \Phi(\beta_{q0} - \beta_{qb1} b_t - \beta_{qb2} \cdot b^2_t) \ ,$$

where $\Phi(.)$ is the logistic cumulative distribution function. The first row tests the restricted normal-mixture model, which implies that

$$\beta_{Sb} = \beta_{Cb} = \beta_{qb1} = \beta_{qb2} = 0$$

and that $\beta_{S0} = \beta_{C0}$. The second row tests the unrestricted normal-mixture model, which drops the restriction that $\beta_{S0} = \beta_{C0}$, so the model becomes

$$R^S_{t+1} \sim N(\beta_{S0}, \sigma_S)$$
$$R^C_{t+1} \sim N(\beta_{C0}, \sigma_C)$$
$$Pr(R_{t+1} = R^S_{t+1}) = \Phi(\beta_{q0}) \ .$$

The third rows tests the linear regression model, which imposes $\beta_{S0} = \beta_{C0}$, $\beta_{Sb} = \beta_{Cb}$, $\beta_{qb1} = \beta_{qb2} = 0$, giving

$$R_{t+1} = \beta_0 + \beta_b b_t + e_{t+1}$$
$$e_{t+1} \sim N(0, \sigma_S) \text{ with probability } \Phi(\beta_q)$$
$$e_{t+1} \sim N(0, \sigma_C) \text{ with probability } 1 - \Phi(\beta_q)$$

Under the null hypotheses that each of these restrictions are satisfied, the test statistics should be $\chi^2$ distributed with 5, 4 and 4 degrees of freedom respectively. The figures shown in parentheses under each test statistic are the marginal significance levels at which we are able to reject the null. Test statistics shown in boldface are those that allow us to reject the null hypothesis at the 5 per cent significance level.

The next section of the tables presents estimates of those parameters whose signs are predicted by the bubble model. In particular, we should expect either $\beta_{Sb} > 0$, $\beta_{Cb} < 0$ and $\beta_{qb2} > 0$, or $\beta_{Sb} < 0$, $\beta_{Cb} > 0$ and $\beta_{qb2} < 0$. The figures shown in parentheses below the parameter estimates are t-ratios, based upon the inverse of the Hessian.[39] Parameter estimates shown in boldface indicate those that are significantly different from zero at the 5 per cent level using a two-tailed test. Note that each measure of $b_t$ is standardized to have a mean of 0 and a variance of 1 before estimation.

The final section of the tables displays White's (1987) score-based tests for misspecification. The first row is the joint test for omitted serial correlation, ARCH effect and Markov switching. The remaining rows test for each specific type of misspecification, where the AR and ARCH tests are specific to a given regime. All test statistics are distributed $\chi^2(1)$ under the null hypothesis of no misspecification, except for the joint test, which is distributed $\chi^2(5)$. Based on the Monte Carlo results of Hamilton (1990), a 1 per cent critical value is used to determine the significance of the test results, as the test is prone to reject the null too frequently in finite samples. Test statistics that allow a rejection of the null at this significance level are shown in boldface.

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39. Limited work with White's (1980) heteroscedasticity-robust standard errors produced similar results. However, White's standard errors are not valid in switching-regression models, since parameter estimates may be inconsistent in the presence of heteroscedasticity.

# Figures

Figure 1
Spot exchange rates and bubble size
Canada - overshooting model

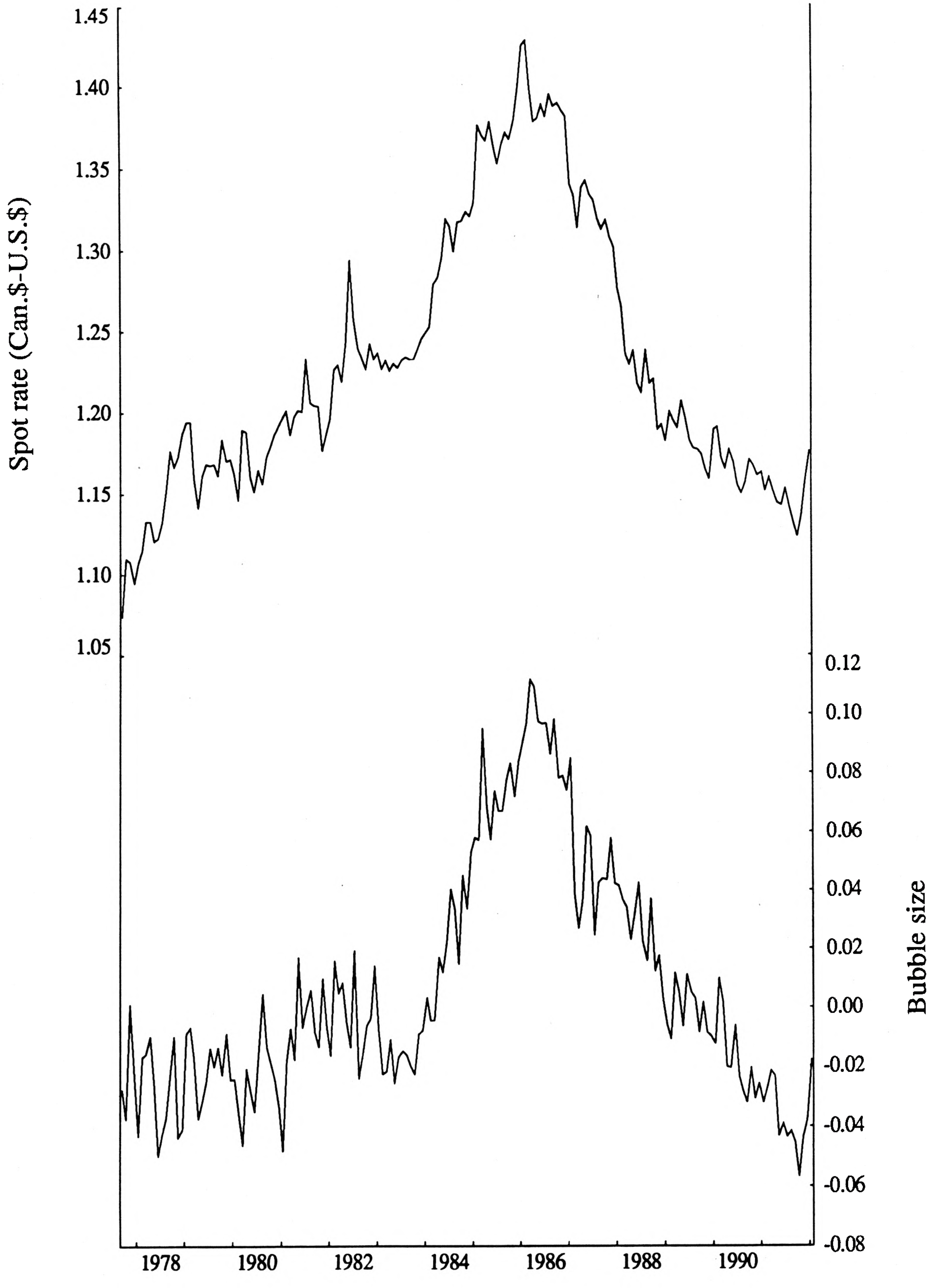


Figure 2
Ex ante probability of bubble collapse and
expected difference in returns across regimes
Canada - overshooting model

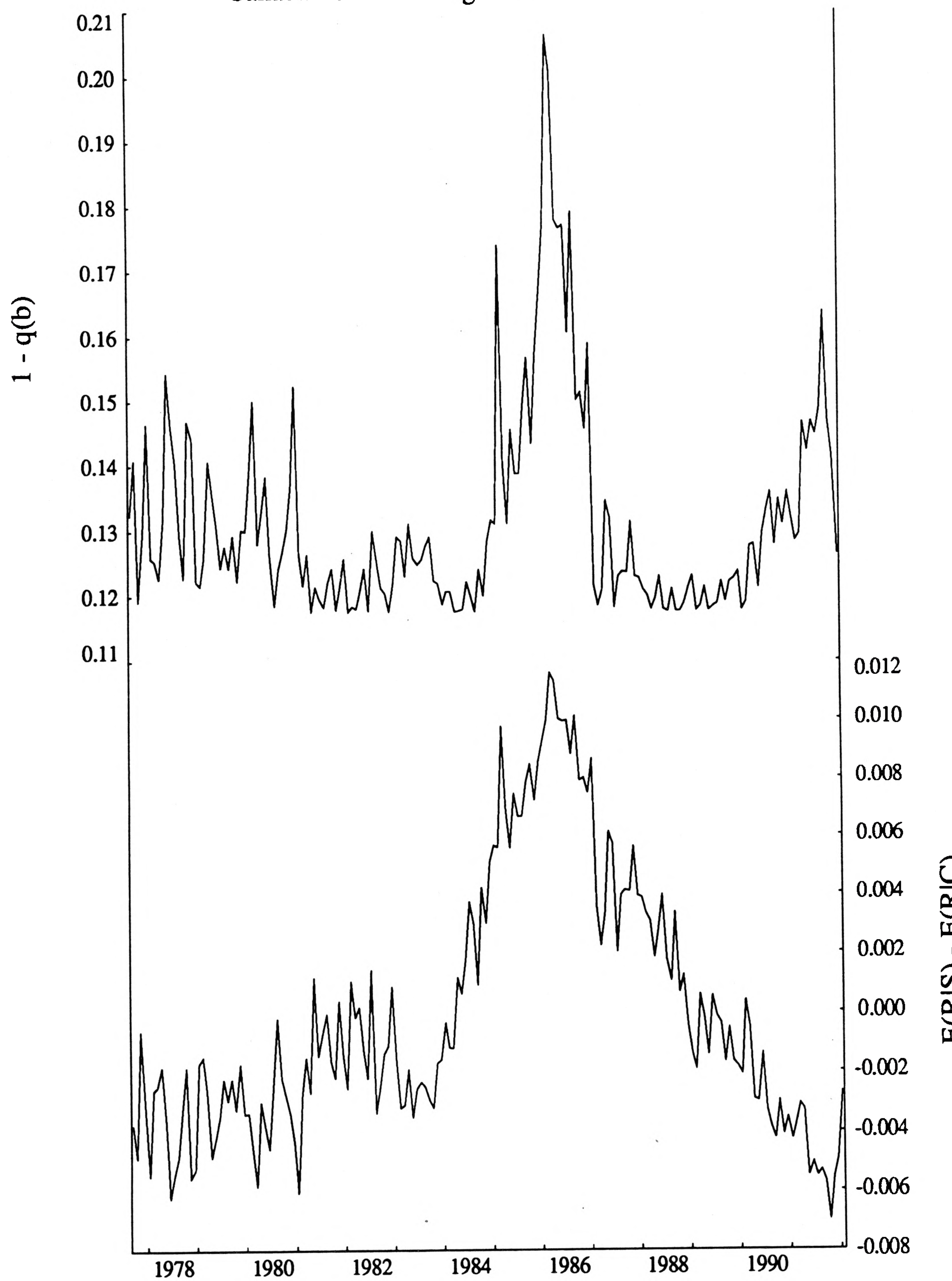


Figure 3
Spot exchange rates and bubble size
Japan - PPP model

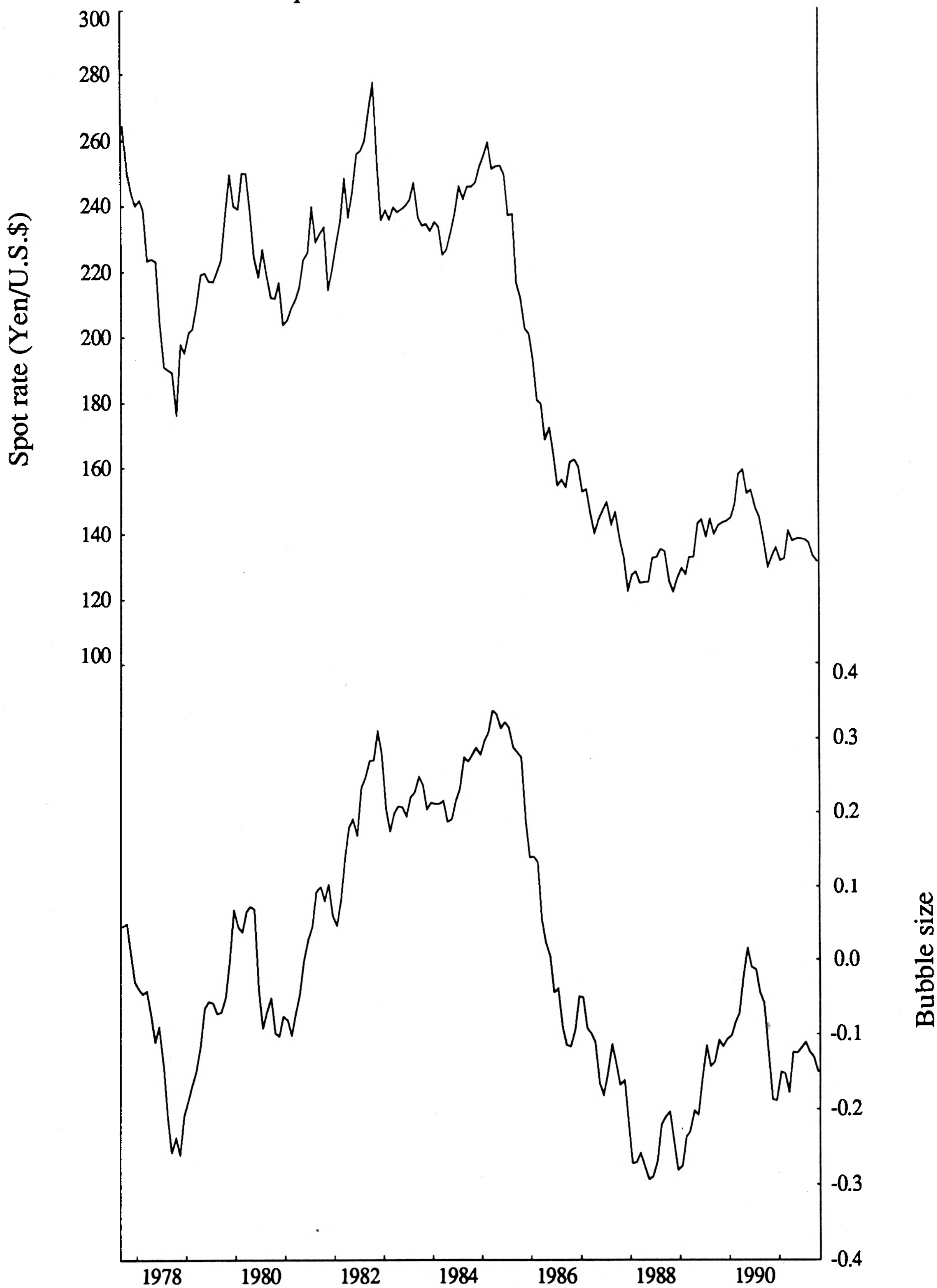


Figure 4
Ex ante probability of bubble collapse and
expected difference in returns across regimes
Japan - PPP model

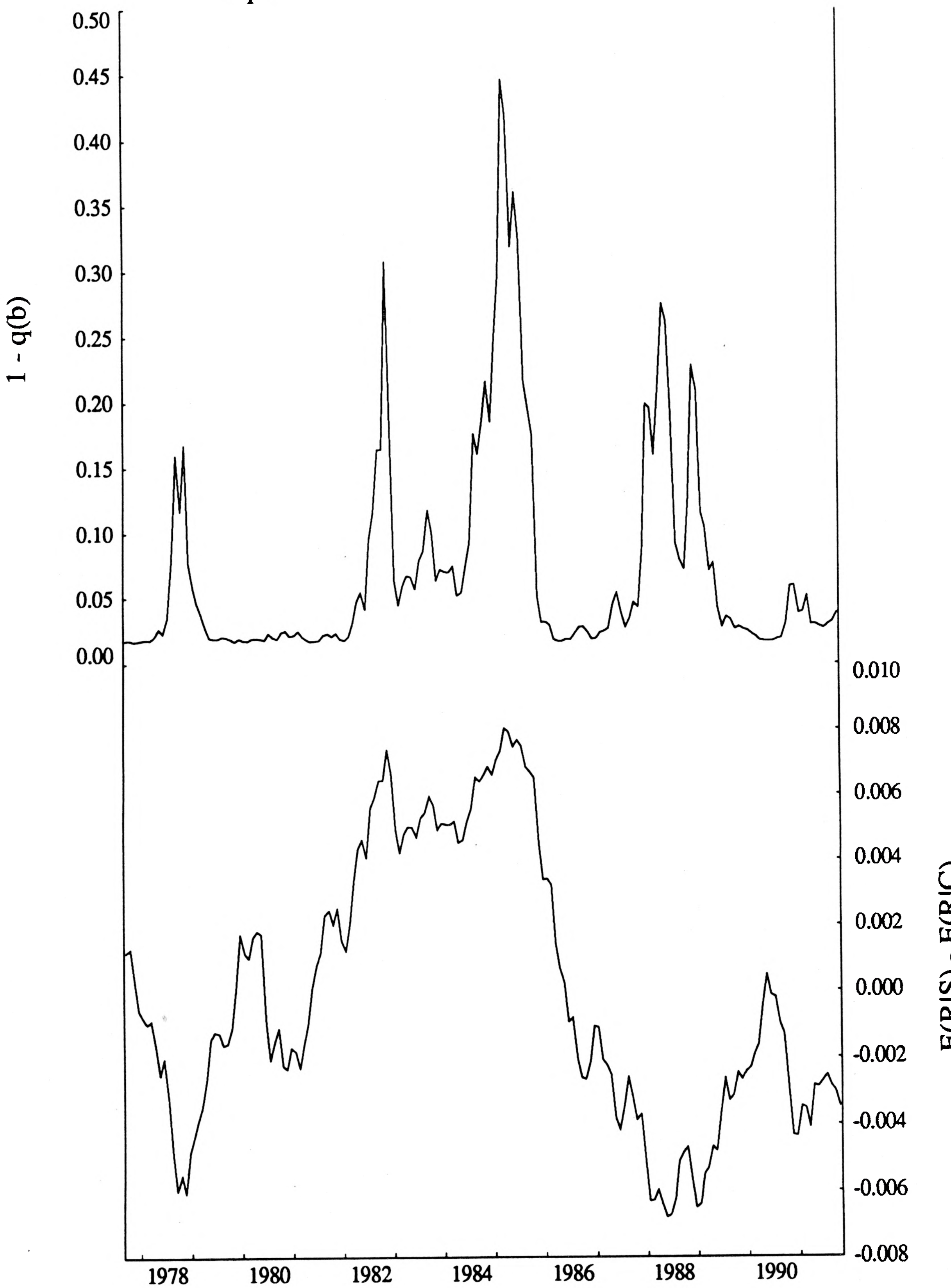


Figure 5
Expected excess returns
Canada - overshooting model

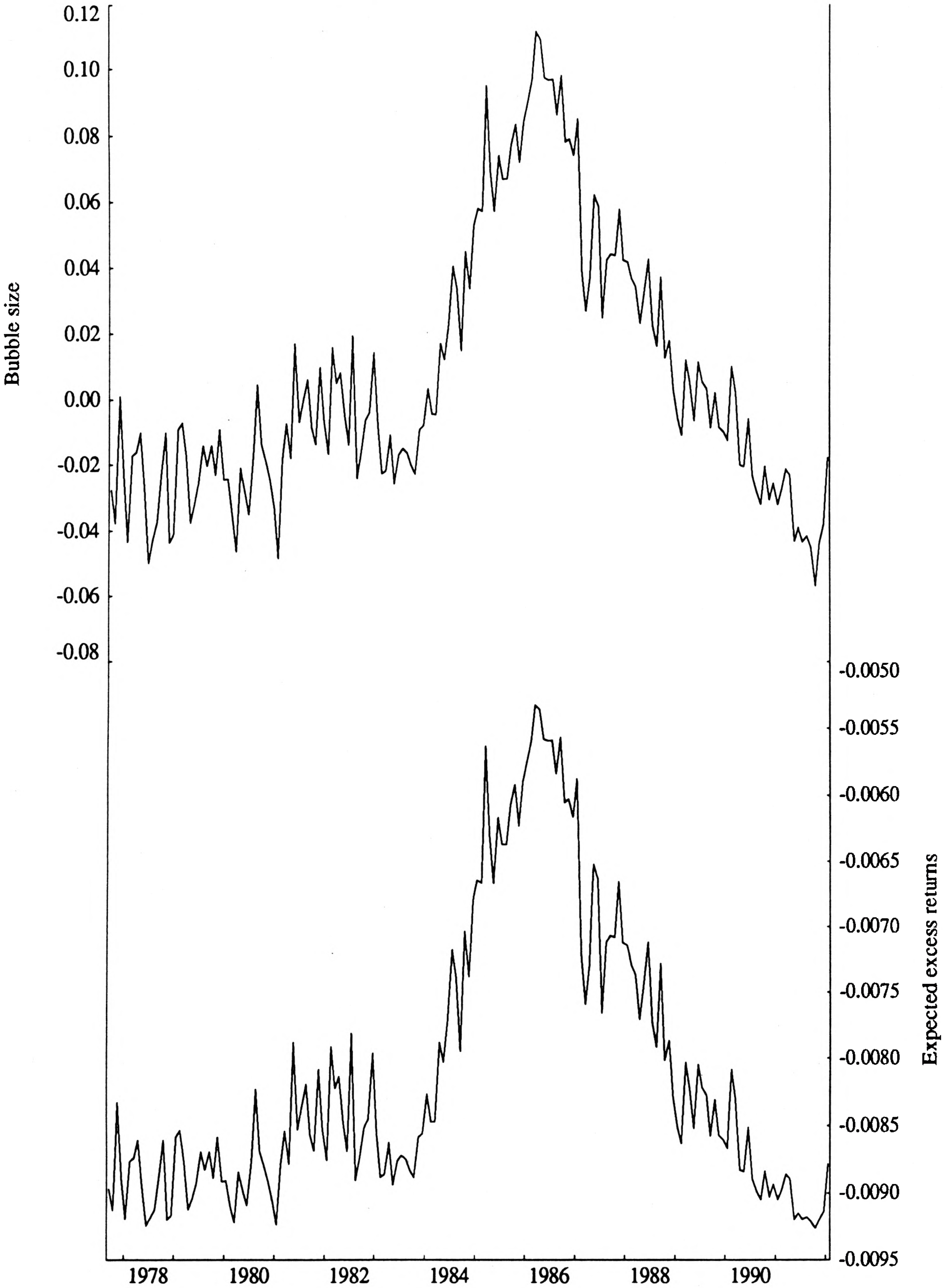


Figure 6
Expected excess returns
Japan - PPP model

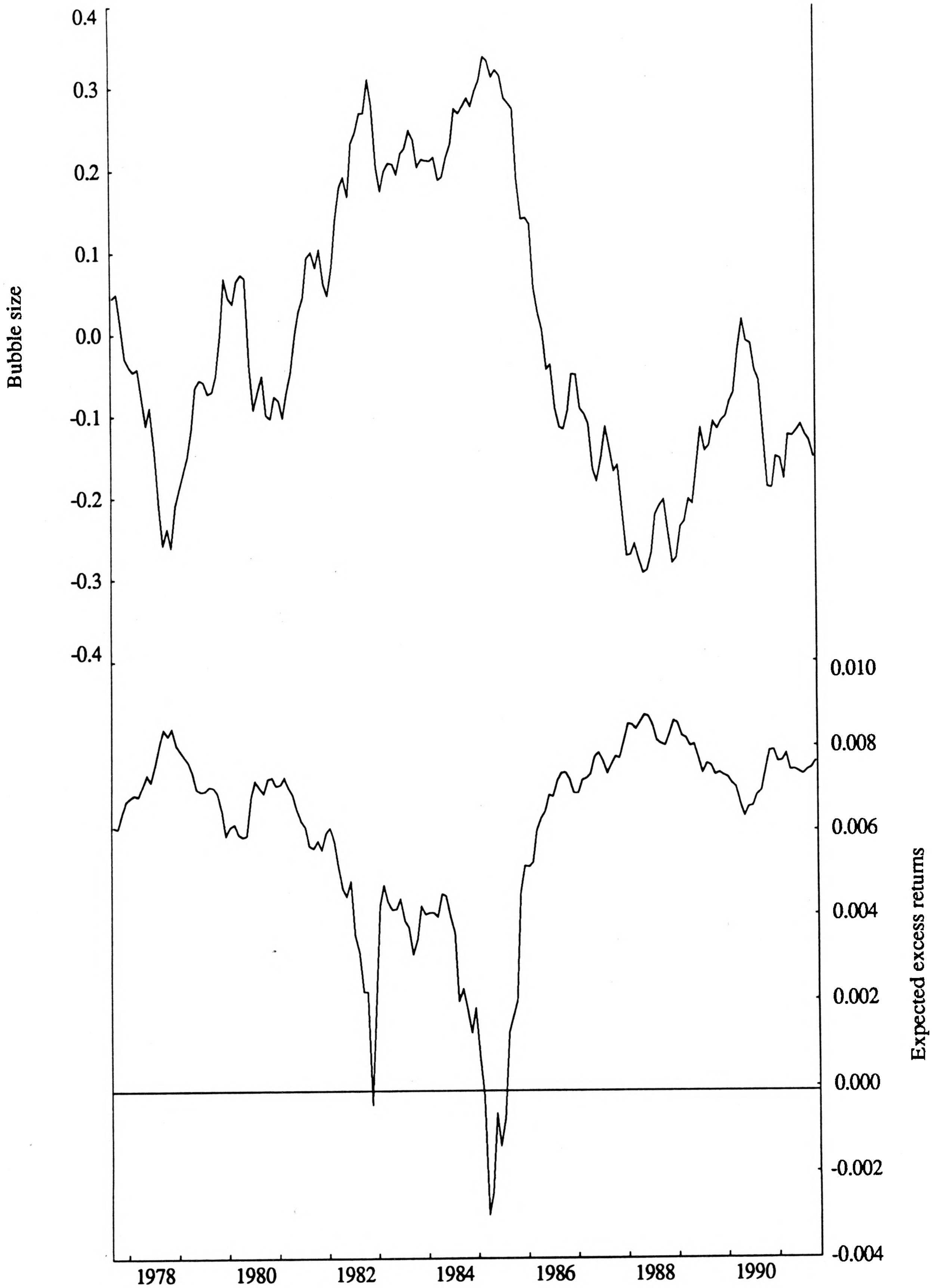


Figure 7
Probability of a two-standard-deviation outlier
Canada - overshooting model

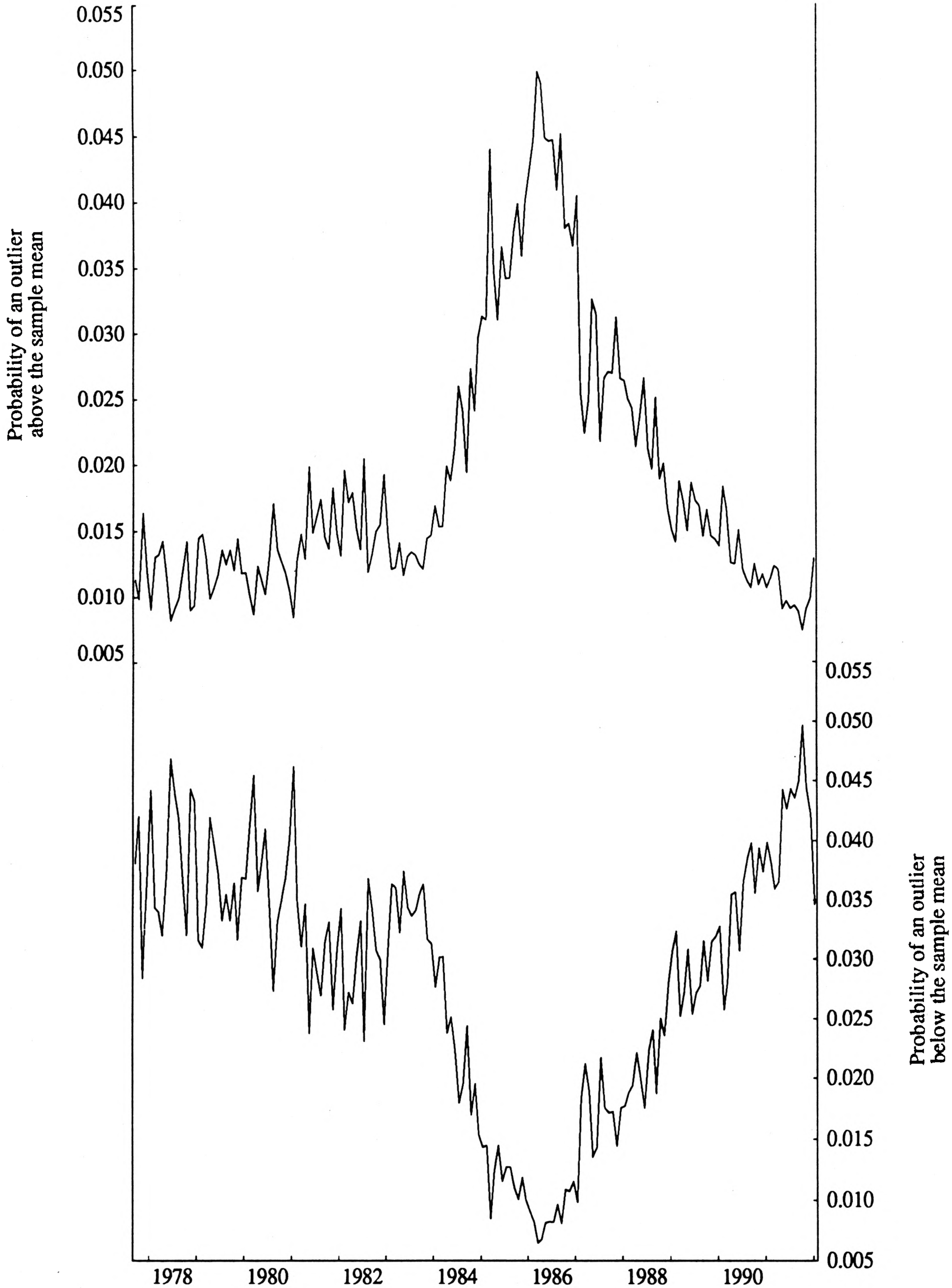
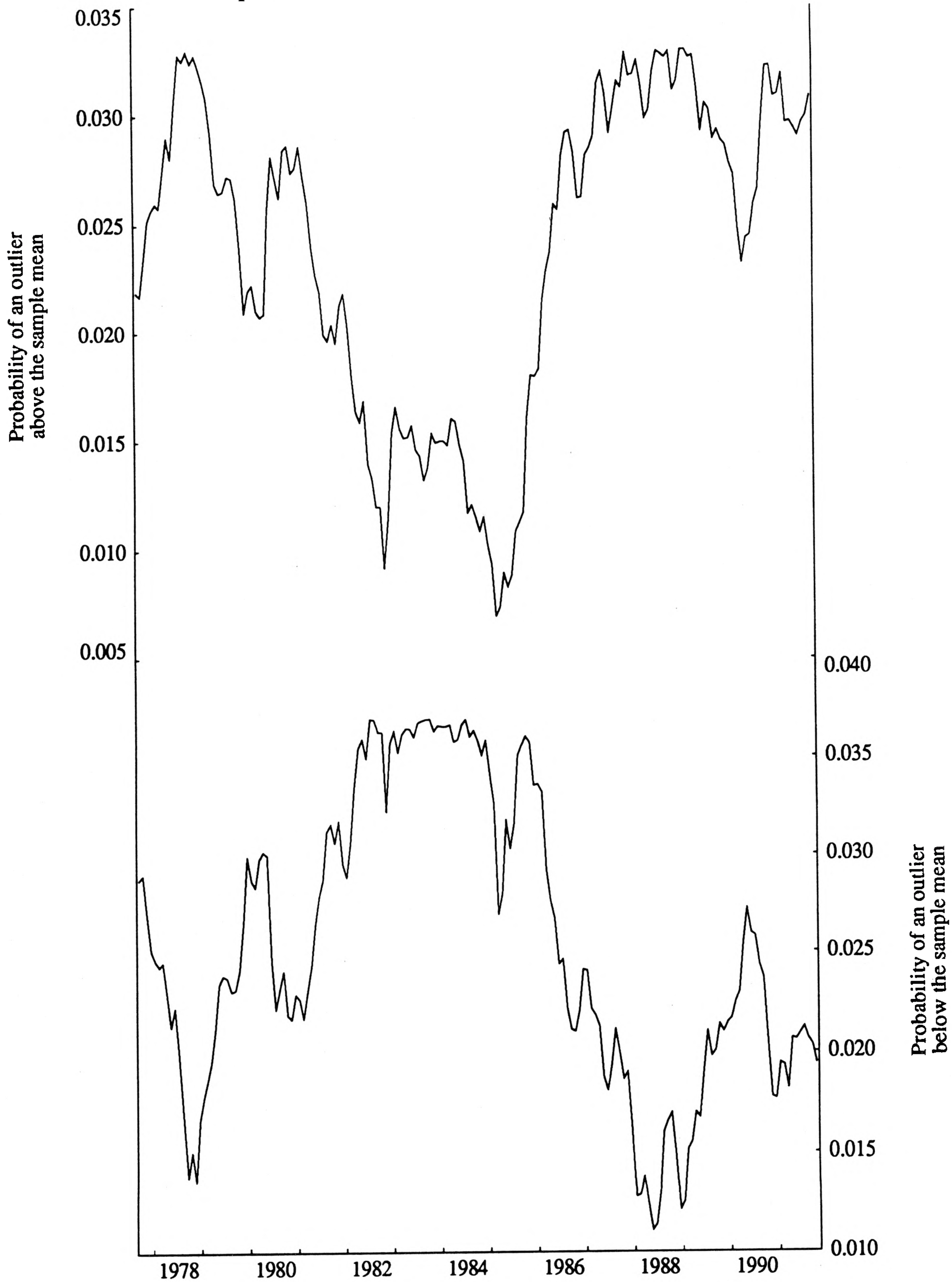


Figure 8
Probability of a two-standard-deviation outlier
Japan - PPP model



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