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Speculative Behaviour, Regime Switching and Stock Market Fundamentals

by
Simon van Norden and Huntley Schaller

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Simon van Norden
International Department
Bank of Canada

Huntley Schaller
Economics Department
Carleton University

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ABSTRACT

Based on historical accounts of "manias and panics," we explore a model of speculative behaviour in which apparent overvaluation increases the probability of a stock market crash. Using U.S. data for 1926-89, we find that the deviation of actual prices from those implied by a simple Lucas asset-pricing model predicts regime switches in the stock market. The probability of a collapse (calculated from switching-regression estimates of the model of speculative behaviour) rises sharply before the 1929 and 1987 crashes but fails to rise before several other actual crashes. Monte Carlo simulations show that switching fundamentals could also account for much of the statistical evidence we find for speculative behaviour. The probability of a price collapse (calculated from estimates of the model of switching fundamentals) rises before several market crashes that are not predicted by the speculative behaviour model, but fails to rise before the 1929 and 1987 crashes. These results suggest that news about both fundamentals and speculative behaviour may play a role in explaining stock market crashes.

RÉSUMÉ

En s'appuyant sur des études historiques des mouvements de panique qui ont secoué les marchés boursiers dans le passé, les auteurs analysent un modèle de comportement spéculatif, dans lequel une surévaluation manifeste des actifs accroît la probabilité d'un krach. Ils constatent, sur la base de données allant de 1926 à 1989, que l'écart des cours observés par rapport aux prix obtenus à l'aide d'un modèle simple de détermination des prix des actifs à la Lucas devrait donner lieu à des changements de régime sur le marché boursier. La probabilité d'un effondrement des prix calculée à l'aide d'une régression avec changements de régime appliquée au modèle de comportement spéculatif augmente fortement avant les krachs de 1929 et de 1987, mais pas avant plusieurs autres. Des simulations de Monte-Carlo montrent que le changement de perception des agents relativement aux déterminants fondamentaux des cours des actions peut aussi éclairer en grande partie les résultats statistiques confirmant l'hypothèse de comportement spéculatif. En effet, si l'on calcule la probabilité d'un effondrement des prix à l'aide du modèle de déterminants fondamentaux, celle-ci croît avant certains krachs qui ne sont pas prévus par le modèle de comportement spéculatif, mais elle ne le fait pas avant les krachs de 1929 et de 1987. À en juger par ces résultats, tant le comportement spéculatif que les déterminants fondamentaux contribuent à expliquer les krachs.

I INTRODUCTION

For many years, observers of financial markets have described phenomena, such as overtrading, speculative frenzy, mania or bubbles, that suggest that asset prices may depart from their "fundamental" values. If this suggestion were true, it would have serious economic implications. First, like taxes and subsidies, such deviations could distort the intertemporal allocation of saving and investment and the cross-sectional allocation of capital among industries.[1] Second, the sharp rises and falls in asset prices associated with speculative behaviour could contribute to macroeconomic fluctuations, as some economists believe occurred with the October 1929 Wall Street crash and the Great Depression. Finally, deviations from fundamentals could affect the volatility and riskiness of asset markets.

There are a number of common features in historical accounts of "speculative frenzies."[2] The most important is that there are sharp rises and falls in asset prices which are hard to relate to news about fundamentals. The stylized pattern is an accelerating upswing in asset prices, followed by a precipitous reversal. A second common feature is that a growing proportion of trades are made for short-term capital gains rather than for the long-term dividend stream they represent.[3] Finally, historical accounts suggest that an asset price crash becomes more likely as the relationship between current prices and dividends grows more extreme.

In this paper, we attempt to formalize these notions in an economic model and to devise an empirical framework consistent with the model. Our theoretical framework

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[1]Blanchard, Rhee and Summers (1990), Chirinko and Schaller (1991), Galeotti and Schiantarelli (1990), and Rhee and Rhee (1991) investigate the potential effects of deviations from fundamentals on investment. See Morck, Schleifer and Vishny (1990) for a discussion of the effects of such distortions across firms.

[2]Kindleberger (1989) provides a summary of some of the common features of these historical accounts.

[3]Shiller et al. (1991) highlight a key element of the traditional notion of a speculative bubble: as a positive bubble grows, increasing numbers of investors buy stocks because they think prices will continue to rise for a while longer before dropping. Using survey evidence on market participant's beliefs, Shiller et al. find that a large proportion of investors seem to think in these terms.

is an extension of the Blanchard and Watson (1982) model. By extending their model, we are able to capture several of the most universal and striking features of historical accounts of "manias" and "overtrading." In Section II, we show how our model leads to a new type of empirical analysis based on a switching-regression specification.

We use the estimated parameters from the switching regression to calculate the probability of a stock market collapse in each period. Using 1926-89 monthly stock market data from the Centre for Research in Securities Prices, we find that the probability of collapse is greatest in periods like September 1929 and September 1987. We also test the null hypothesis that stock market returns depend exclusively on fundamentals by imposing parameter restrictions on the switching regression. We find evidence that apparent deviations from fundamentals influence the distribution of stock market returns.

While we show that regime switching in returns is consistent with speculative behaviour, it could also arise from switching fundamentals. For example, Cecchetti, Lam and Mark (1990) (CLM) have shown that an equilibrium model with regime switching in dividends is capable of generating some of the previously documented features of U.S. stock market returns, such as skewness and mean reversion. To help distinguish between these explanations, we simulate a model of switching fundamentals and then estimate our switching regression using the artificial data from the simulations. We also compare the timing of actual stock market crashes with that of switches in dividend growth. The results suggest that both speculative behaviour and switching fundamentals contribute to understanding large swings in stock prices. For example, the 1929 and 1987 crashes correspond well to the model of speculative behaviour but not to switches in dividend growth, while crashes in the early 1930s correspond more closely to the switching fundamentals model.

Section II of the paper provides the relevant theory. In subsection A, we review the Lucas asset-pricing model in order to derive the fundamental stock price under standard assumptions. In subsection B, we develop the model of speculative behaviour which provides the structure for our econometric work. Finally, subsection C introduces the model of switching fundamentals. Section III shows how a switching-regression specification captures speculative behaviour and reports the parameter estimates. Section IV tests the null hypothesis that stock market returns are unrelated to our measure of deviations from fundamentals using parametric restrictions on the switching-regression model. Section V presents parameter estimates of the model of switching fundamentals and discusses how well shifts in fundamentals explain the switching-regression results and actual market crashes. Section VI analyses how well the probabilities of collapse generated by either model accord with actual stock market crashes. Section VII offers conclusions.

II MODELS OF FUNDAMENTALS AND SPECULATIVE BEHAVIOUR

In this section, we consider three different models of stock price determination, noting their implications for the relationship between stock prices and dividends, and for the behaviour of returns. In part A, we review the standard Lucas (1978) exchange economy model, where dividend growth is assumed to be log-normally distributed. In part B, we loosen the assumptions of this model by assuming prices may deviate from their fundamental value. In part C, we alter the model from part A in a different way, this time by assuming that log dividend growth follows a Markov mixture of normal distributions.

A. Equilibrium asset prices

We begin with a Lucas (1978) exchange-economy, asset-pricing model in which there are a large number of identical, infinitely lived agents and a fixed number of assets that produce units of the non-storable consumption good. The first-order necessary conditions for a representative agent's optimization problem are

$$P_{j,t}U'(C_t) = \beta E_t U'(C_{t+1})[P_{j,t+1} + D_{j,t+1}] \qquad j=1,2,\dots,N \tag{1}$$

where P_j is the real price of asset j in terms of the consumption good, $U'(C_t)$ is the marginal utility of consumption, C_t, for a typical consumer/investor, β is the subjective discount factor, $0 < \beta < 1$, D_j is the payoff or dividend from the jth productive unit and E_t is the mathematical expectation conditioned on information available at time t.

Since agents are identical, equilibrium per capita ownership of each asset is the reciprocal of the number of assets, so per capita consumption (C) is the sum over all assets of per capita dividends on each asset (D). Hence the equilibrium condition for economy-wide market prices and quantities is

$$P_t U'(D_t) = \beta E_t U'(D_{t+1})[P_{t+1} + D_{t+1}] \tag{2}$$

where P is the portion of the market's value owned by a typical agent, which corresponds to the value-weighted stock market index adjusted for population size. We assume a constant relative risk-aversion utility function

$$U(C_t) \equiv (1 + \gamma)^{-1} C_t^{1-\gamma} \tag{3}$$

where γ is the coefficient of relative risk aversion. Using this utility function in the market equilibrium condition gives the following stochastic difference equation for equilibrium prices

$$P_t \cdot D_t^{\gamma} = \beta \cdot E_t D_{t+1}^{\gamma} \cdot (P_{t+1} + D_{t+1}) \tag{4}$$

This yields the familiar equation for equilibrium price

$$P_t = D_t^{\gamma} \cdot \sum_{k=1}^{\infty} \beta^k \cdot E_t D_{t+k}^{1-\gamma} \tag{5}$$

In order to obtain a relationship between current prices and current dividends, we must make some assumptions about the dividend process. In the first case we consider, we assume log dividends are a random walk with drift. This will lead to a simple solution in which equilibrium asset prices are a multiple of current dividends. Formally, dividends are

$$d_t = \alpha_0 + d_{t-1} + \varepsilon_t \tag{6}$$

where d_t is the logarithm of dividends, α_0 is the drift parameter, and ε_t is a sequence of independent, identically distributed normal random variables with mean zero and variance σ^2. To solve the model, first we conjecture a solution of the form

$$P_t = \rho \cdot D_t \tag{7}$$

To verify that this is a solution to the stochastic difference equation, we can substitute it into (4), obtaining

$$\rho \cdot D_t^{\gamma+1} = \beta \cdot E_t[(\rho+1) \cdot D_{t+1}^{\gamma+1}] \tag{8}$$

By rewriting the dividend process in levels, rather than logarithms,

$$D_{t+1} = D_t \cdot e^{\alpha_0 + \varepsilon_{t+1}} \tag{9}$$

and substituting into (8) we obtain the following expression for the price-dividend ratio

$$\rho = \frac{\beta e^{\alpha_0(1+\gamma) + (1+\gamma)^2\sigma^2/2}}{1 - \beta e^{\alpha_0(1+\gamma) + (1+\gamma)^2\sigma^2/2}} \tag{10}$$

The effect of an increase in the expected rate of dividend growth depends on whether γ <> -1. When $\gamma > -1$, increases in the dividend growth rate α_0 raise the price-dividend ratio; when $\gamma < -1$, the reverse is true.

Finally, the equilibrium gross return can be obtained from the relationship between current prices and dividends and the dividend process:

$$\begin{aligned} R_t &\equiv \frac{P_t + D_t}{P_{t-1}} \\ &= \left(\frac{1+\rho}{\rho}\right) e^{\alpha_0 + \varepsilon_t} \end{aligned} \tag{11}$$

B. A model of speculative behaviour

The model presented in this subsection introduces speculative behaviour, which allows deviations from the fundamental asset price that occasionally collapse. This possibility implies that there are two regimes generating stock market returns, one where the deviation from fundamentals collapses and one where it survives. Rational investors take this into account when deciding whether or not to hold an asset. The arbitrage condition between assets with and without deviations from fundamentals allows us to impose some structure on asset returns in the surviving and collapsing regimes: in a surviving regime returns should grow sufficiently rapidly to compensate the investor for the possibility that the deviation may collapse. Combined with the historical observation that larger overvaluations are more likely to collapse, this provides us with the essential elements for a regime-switching specification for asset returns. In the remainder of this subsection, we translate this verbal description into a model that can be linked with the Lucas asset-pricing model of fundamentals developed in Section II.A.[4]

In the preceding subsection, we noted that any equilibrium price P must satisfy (4). If we now distinguish between the fundamental price P^*_t given in (5) and other possible solutions to (4), we may define the size of the deviation from fundamental price as $B_t \equiv P_t - P^*_t$. Since both P_t and P^*_t satisfy (4), it follows that

$$B_t \cdot D_t^\gamma = \beta \cdot E_t[D_{t+1}^\gamma] \cdot E_t[B_{t+1}] \tag{12}$$

where we use the fact that D_t will be independent of B_t in an endowment economy. We can then use the assumed process for D_t given in (9) to show that

$$\begin{aligned} \frac{E_t[B_{t+1}]}{B_t} &= \frac{D_t^\gamma}{\beta \cdot E_t[D_{t+1}^\gamma]} \\ &= \beta^{-1} e^{\gamma(\alpha_0 + \gamma\sigma^2/2)} \\ &\equiv M \end{aligned} \tag{13}$$

[4]There is an extensive literature noting restrictions on the admissibility of non-fundamental solutions, including Diba and Grossman (1988), Obstfeld and Rogoff (1983, 1986), and Tirole (1982, 1985). Blanchard and Fischer (1989, 238) argue that "[These restrictions] often rely on an extreme form of rationality and are not, for this reason, altogether convincing. Often bubbles are ruled out because they imply, with a very small probability and very far in the future, some violation of rationality, such as non-negativity of prices or the bubbles becoming larger than the economy. It is conceivable that the probability may be so small, or the future so distant, that it is simply ignored by market participants." Moreover, recent work by Allen and Gorton (1991) and Leach (1991) has shown that restrictions on non-fundamental solutions are not robust to minor changes in assumptions, such as heterogeneous agents and either continuous time or more than two periods in an overlapping generations model. Our motivation for building the sort of model of speculative behaviour presented in this section is the same as Solow (1957, 323-24) in using an aggregate production function, which was controversial at the time: "Either this kind of [approach] appeals or it doesn't. . . . If it does, I think one can draw some . . . useful conclusions from the results."

so the expected growth rate of the speculative component is the same as that of the fundamental price.

Blanchard (1979) and Blanchard and Watson (1982) propose a specific solution to (13) with two states of nature. In one state (state C), the speculative component always collapses, so $E_t[B_{t+1} | C] = 0$. In the other state, (state S), the speculative component always survives. The probability of being in state S next period is assumed to be some constant, q. In this case, (13) implies

$$\begin{aligned} B_t \cdot M &= E_t[B_{t+1}] \\ &= q \cdot E_t[B_{t+1} | S] + (1-q) \cdot 0 \end{aligned} \tag{14}$$

$$\therefore E_t[B_{t+1} | S] = \frac{M \cdot B_t}{q} \tag{15}$$

It is easy to see that the rate of growth of the speculative component is sufficiently large in the state where it survives (S) to compensate the investor for the expected loss in the state of the world where it collapses (C), giving an expected rate of return of M on the speculative component.

As noted above, historical accounts suggest that the probability of a speculative component surviving decreases as the speculative component grows. We therefore allow the probability of survival q to depend on the relative size of the speculative component

$$q \equiv q(b_t) \tag{16}$$

where $b_t \equiv B_t / P_t$ and

$$\frac{dq(b_t)}{d|b_t|} < 0 \tag{17}$$

Note the use of the absolute value of b_t, since the speculative component may be positive or negative.

Second, while some notable market crashes have occurred in a single day, in other cases a collapse may occur over several months.[5] To model this, we allow the expected

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[5]The fall in the Tokyo stock exchange in the months following January 1990 is an example.

value of the bubble conditioned on collapse to be non-zero, thereby allowing for partial collapses. We assume the expected size of a bubble in state C, which we define as $u_t \cdot P_t$, depends on the relative size of the bubble in the previous period, so

$$E_t[B_{t+1}|C] = u(b_t)\cdot P_t \tag{18}$$

We further assume that u(·) is a continuous and everywhere differentiable function and that u(0)=0 and $1 \geq u' \geq 0$, which ensures that a collapse means that the speculative component is expected to shrink.[6] Imposing (13) then gives

$$E_t[B_{t+1}|S] = \frac{M}{q(b_t)}\cdot B_t - \frac{1-q(b_t)}{q(b_t)}\cdot u(b_t)\cdot P_t \tag{19}$$

This shows that the expected value of the speculative component in the surviving state will decline as its expected value in the collapsing state $u(b_t) \cdot P_t$ and the probability of survival $q(b_t)$ increase.

As shown in Appendix 1.A, we can use the above to solve for the expected returns in each regime as a function of b_t. After linearizing, we obtain the following three-equation model which we estimate below.

$$\begin{aligned} E(R_{t+1}|S) &= \beta_{S0} + \beta_{Sb} b_t \\ E(R_{t+1}|C) &= \beta_{C0} + \beta_{Cb} b_t \\ q_{t+1} &= \beta_{q0} + \beta_{qb} |b_t| \end{aligned} \tag{20}$$

Appendix 1.A shows our assumptions imply that β_{Cb} should be negative, so that as the speculative component grows, expected returns in the collapsing state decrease, reflecting the larger expected size of collapse. While β_{Sb} may be positive or negative, our assumptions on $q(b_x)$ and $u(b_x)$ imply that $\beta_{Sb} > \beta_{Cb}$. Finally, equation (17) implies that $\beta_{qb} < 0$, so that the larger the deviation from fundamentals, the greater the probability of a collapse. We will see in Section III that these predictions are consistent with the data.

[6]As with the assumptions on $q(b_t)$, the assumptions on $u(b_t)$ are not imposed on the data. Instead, they allow us to determine the expected signs and relative magnitudes of the parameters. These predictions turn out to be consistent with the data.

C. Switching in dividends

The preceding section has shown how speculative behaviour could generate regime-switching behaviour in stock returns. However, fundamental explanations for regime switching also exist. In the remainder of this section, we consider a variation of the Lucas asset-pricing model which explicitly allows for switching in the dividend process. A dividend process that meets this requirement is:

$$d_t = d_{t-1} + \nu_t + \alpha_0 + \alpha_1 S_{t-1} \tag{21}$$

where d_t is the logarithm of dividends, ν_t is a sequence of independent, identically normally distributed random variables with mean zero and variance σ^2, and S_t is the sequence of Markov random variables with state space {0,1} and transition matrix:

$$P \equiv \begin{bmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \end{bmatrix} \tag{22}$$

This is a Markov switching model. As noted in Cecchetti, Lam and Mark, it is able to capture features of stock market returns not captured by other models, including ARIMA, ARCH and GARCH models. In particular, the Markov switching model is able to reproduce the skewness and kurtosis of actual U.S. stock market returns, and CLM show that a Markov switching process in dividends can account for the evidence of mean reversion found by Fama and French (1988b) and Poterba and Summers (1988).

CLM show that the above equations lead to a relation between fundamental prices and dividends of the form

$$P_t = \rho(S_t)D_t \tag{23}$$

In other words, the price-dividend ratio will take on one of two values, depending only on whether the economy is in the high- or the low-dividend growth state. Stock market crashes (or bull markets) could then be caused by a transition from the high ρ to the low ρ state (or from low to high), implying large movements in stock prices. The derivation of the model may be found in Appendix 1.B. One important point to note is that the high-dividend growth state need not correspond to the high ρ state. In fact, it can be shown that

$$\rho(0) \gtreqless \rho(1) \Leftrightarrow \gamma \gtreqless -1 \tag{24}$$

where $\rho(0)$ is defined to be the high-growth state.[7] The equilibrium gross return can be obtained from the relationship between current prices and dividends and the dividend process, so

$$R_t = \left(\frac{1 + \rho(S_t)}{\rho(S_{t-1})} \right) e^{(\alpha_0 + \alpha_1 S_{t-1} + v_t)} \tag{25}$$

III ESTIMATES OF A MODEL OF SPECULATIVE BEHAVIOUR

In this section, we estimate the parameters of the model of speculative behaviour presented in Section II.B. From (20), the form of our linearized switching model is

$$R_{S,t+1} = \beta_{S0} + \beta_{Sb} b_t + \varepsilon_{S,t+1} \tag{26}$$

$$R_{C,t+1} = \beta_{C0} + \beta_{Cb} b_t + \varepsilon_{C,t+1} \tag{27}$$

$$q_{t+1} = \beta_{q0} + \beta_{qb} |b_t| \tag{28}$$

where $R_{S,t+1}$ and $R_{C,t+1}$ are the returns from period t to period t+1 conditional on survival and on collapse respectively, and b_t is our measure of the deviation from fundamentals in period t. For the purposes of estimation we assume $\varepsilon_{S,t+1}$ and $\varepsilon_{C,t+1}$ are mean zero i.i.d. normal random variables. However, in this form there is no guarantee the resulting estimates of q will be bounded between 0 and 1. We adopt the same solution used in Probit models by imposing the functional form

$$q = \Phi(\beta_{q0} + \beta_{qb} \cdot |b_t|) \tag{29}$$

where Φ is the standard normal cumulative density function (CDF).

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[7]See Cecchetti, Lam and Mark (1990, 407) for a proof.

The three equations (26), (27) and (29) form a standard switching-regression model of the type described by Goldfeld and Quandt (1976) and Hartley (1978). Given normality of $(\varepsilon_{S,t+1}, \varepsilon_{C,t+1})$, estimates of the β's can be found by maximizing the likelihood function

$$\prod_{t=1}^{T}\left[\Phi\left(\beta_{q0}+\beta_{q1}|b_t|\right)\cdot\phi\left(\frac{R_{t-1}-\beta_{S0}-\beta_{S1}b_t}{\sigma_S}\right)\cdot\sigma_S^{-1} + \left\{1-\Phi\left(\beta_{q0}+\beta_{q1}|b_t|\right)\right\}\cdot\phi\left(\frac{R_{t-1}-\beta_{C0}-\beta_{C1}b_t}{\sigma_C}\right)\cdot\sigma_C^{-1}\right] \tag{30}$$

where ϕ is the standard normal probability density function (pdf) and σ_S, σ_C are the standard deviations of $\varepsilon_{S,t+1}$, $\varepsilon_{C,t+1}$. Note that this estimation technique not only allows us to recover consistent and efficient estimates of the parameters in both states but does not require assumptions about which regime generated a given observation. Instead, it considers the probability that either regime may have generated a given observation and gives an optimal classification of observations into the underlying regimes. The probability of being in regime i at time t+1 is defined as the probability conditioning on all relevant information available at the end of period t, namely b_t. This is determined solely by the classifying equation (29) and is given by the formula $\Phi(1(i) \bullet (\beta_{q0} + \beta_{qb} \bullet |b_t|)) \equiv P_i^A$, where 1(i) equals 1 in the surviving state and -1 in the collapsing state.

The data we examine for evidence of speculative behaviour are drawn from the Center for Research in Security Prices data base and are described in more detail in Appendix 2. Briefly, we use their monthly value-weighted price (P) and dividend (D) indices for all stocks from January 1926 to December 1989. To be consistent with the model in Section II.A, P and D are converted to real per capita terms. The measure of deviations from fundamentals that we use (b_t) is tied to the model of equilibrium asset prices outlined in Section I.A, where the fundamental price is $P_t=\rho D_t$. Under the null hypothesis that actual prices correspond to fundamentals, the mean price-dividend ratio is a consistent estimate of ρ. Under the alternative hypothesis, the proportional deviation from fundamental price is $b_t = 1 - \rho D_t/P_t$. As noted in Appendix 2, evidence suggests ρ may have shifted over time. To deal with this possibility, we use three approaches. The first uses the mean over the full sample; the second sets ρ equal to the mean of P/D in three distinct periods to construct b_t; and the third divides the sample into subperiods, using the mean of P/D to construct b_t and estimating the parameters separately for each subperiod.

The estimated parameters for the model of speculative behaviour (26), (27) and (29) are presented in Table I (p. 33). The first column presents estimates assuming ρ is constant, while the second allows for shifts in ρ. The two give very similar results. Looking first at the classifying equation, we see that $\hat{\beta}_{q0}$ is large and positive, implying that the average probability of a crash in the coming month is low (only 2.4 per cent, or $1-\Phi(1.982)$).[8] The estimate $\hat{\beta}_{qb}$ is negative and statistically significant, suggesting that the larger the apparent speculative component, the more likely a crash, which accords with historical accounts of speculative behaviour. For example, at the end of September 1929, the value of b_t is 0.37, which implies the probability of a crash in October 1929 is 15.1 per cent, more than six times larger than average.

Turning now to the behaviour of returns within each regime, we see that the average returns in the surviving regime ($\hat{\beta}_{S0}$) are always greater than 1.0, while those in the collapsing regime ($\hat{\beta}_{C0}$) are always less than 1.0. Furthermore, the estimates of $\hat{\beta}_{Sb}$ and $\hat{\beta}_{Cb}$ accord with the restrictions suggested by historical accounts, that $\hat{\beta}_{Sb}>\hat{\beta}_{Cb}$ and $\hat{\beta}_{Cb}<0$. Interestingly, the variance of ε is about four times as high in the collapsing regime as in the surviving regime. We can calculate expected returns conditional on a given state, using $\hat{\beta}_{S0}$, $\hat{\beta}_{C0}$, $\hat{\beta}_{Sb}$, and $\hat{\beta}_{Cb}$. The estimate $\hat{\beta}_{S0}$ implies an average annual rate of return of 8.7 per cent in the surviving state, while $\hat{\beta}_{C0}$ implies an average annual loss rate of 7.0 per cent in the collapsing state. Contrast this with a period when b_t was large, such as in September 1929, and the difference between the annual rates of return in the surviving and collapsing regimes rises from 15.7 per cent to 47.1 per cent.

Time series econometric results are sometimes sensitive to the time period over

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[8]Since the mean of our measure of b_t is zero by construction, we can interpret the constants in each equation of the switching model as descriptions of average behaviour. In all the examples given in this section, we use the parameter estimates from column 2 of Table 1 for our calculations.

which the estimation is done. To test the robustness of our results, we divide the sample into three subperiods: 1926-54, 1954-74, 1974-89.[9] As Table II (p. 34) shows, the parameter estimates are generally similar across time periods. As in the estimates for the full period, we always find $\hat{\beta}_{S0}$>1.0 and $\hat{\beta}_{C0}$<1.0. In each subperiod, $\hat{\beta}_{Cb}$<0, so larger deviations from fundamentals imply more strongly negative returns in the collapsing regime; in the two later subperiods, the effect of b_t is statistically significant. In all subperiods, the apparent deviation from fundamentals significantly influences the stock market regime.

IV PARAMETRIC TESTS OF THE SPECULATIVE BEHAVIOUR MODEL

Having demonstrated above that our parameter estimates of the speculative behaviour model are generally consistent with restrictions implied by historical accounts and are robust to small changes in our measure of deviations from fundamentals and to variations in sample periods, we now test the role of deviations from fundamental value by imposing parameter restrictions on the switching regression. These tests can be interpreted in a number of ways. At the simplest level, they test whether our measure of deviations from fundamentals has any explanatory power for returns. They also implicitly test simple models of asset pricing, like that in Section II.A, which imply that returns should not be predictable and that our measure of deviations from fundamentals should therefore be irrelevant "noise." Finally, by using a variety of parameter restrictions we show how our switching regression nests a variety of stylized facts about stock market behaviour. For example, some parameter restrictions allow us to capture the fact that stock market returns are characterized by periods of high and low stock market volatility. Under another set of restrictions, our switching regression corresponds to a special case of the Cutler, Poterba and Summers (1991) regression test

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[9] These subperiods correspond to the apparent shifts in ρ.

for mean reversion. Therefore, our tests of these restrictions determine whether our model merely mimics well-known facts, or whether it captures some new facet of return behaviour.

The popularity of ARCH and related models has focussed considerable attention on the changing volatility of stock market returns. Under the null hypothesis that b_t has no effect on returns, our switching regression can capture this fact (which we refer to as "volatility regimes") by imposing the restrictions $\beta_{S0} = \beta_{C0} = \beta_0$, $\beta_{Sb} = \beta_{Cb} = \beta_{qb} = 0$ but allowing $\sigma_S \neq \sigma_C$ so

$$R_{t+1} = \beta_0 + \varepsilon_{t+1} \tag{31}$$

where

$$\begin{aligned} \varepsilon_{t+1} &\sim N(0,\sigma_S) \textit{ with prob } q \\ \varepsilon_{t+1} &\sim N(0,\sigma_C) \textit{ with prob } 1-q \end{aligned} \tag{32}$$

The statistics for the likelihood ratio (LR) test of this null against the alternative of our equations (26), (27) and (29) are presented in the lower portion of Table I (p. 33). The LR statistic has a $\chi^2(4)$ distribution under the null. As shown in the first column of the table, the actual LR statistic is 16.29 and has a p-value of 0.003. Since we reject this null, we conclude that the regimes differ in more than just their variances. This implies either that the information contained in the measure of deviations from fundamentals helps to determine which regime prevails, or that the regimes have different expected conditional returns, or both.

One fact our volatility regimes null fails to capture is that periods of high volatility are more likely to occur during stock market declines, while periods of low volatility tend to be associated with stock market increases. Therefore, we might wish to maintain the assumption that expected returns in each regime are constant but allow these constants to differ across regimes. This is the special case of the switching regression where $\beta_{Sb} = \beta_{Cb} = \beta_{qb} = 0$, so b_t has no effect. This implies that returns are well characterized by a mixture of normal distributions with different means and variances, which can be expressed as

$$R_{t+1} \sim N(\beta_{S0}, \sigma_S) \text{ with prob } q$$
$$R_{t+1} \sim N(\beta_{C0}, \sigma_C) \text{ with prob } 1-q \tag{33}$$

for some constant q. The LR statistic testing this restriction against equations (26), (27) and (29) has a $\chi^2(3)$ distribution under the null, which implies a 99 per cent critical value of 11.345. As shown in the first column of Table I, the actual LR statistic is 16.21 and has a p-value of 0.001.

Another possibility we explore is that returns are predictable, but that mean returns do not differ across regimes. To test this, we compare equations (26), (27) and (29) with the case where deviations from fundamentals help predict returns but mean returns are the same across regimes and deviations have no predictive power for the probability of a given regime. It therefore sets $\beta_{S0} = \beta_{C0} = \beta_0$, $\beta_{Sb} = \beta_{Cb} = \beta_1$, and $\beta_{qb} = 0$, giving

$$R_{t+1} = \beta_0 + \beta_1 b_t + \varepsilon_{t+1} \tag{34}$$

where

$$\varepsilon_{t+1} \sim N(0, \sigma_s) \text{ with prob } q$$
$$\varepsilon_{t+1} \sim N(0, \sigma_c) \text{ with prob } 1-q \tag{35}$$

We refer to this as the "mean-reversion" model, since it corresponds to the regression test for mean reversion in stock prices in Cutler, Poterba and Summers (1991), except that we allow more flexibility for volatility by allowing the variances of returns to be drawn from high and low volatility distributions.[10] The LR statistic again has a $\chi^2(3)$ distribution under the null. As shown in the first column of Table I, the actual LR statistic is 14.79, which has a p-value of 0.002, so we again reject the null, suggesting that there is more in the data than simple mean reversion.[11]

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[10]We also use non-overlapping observations of one-period returns.

[11] Rejection of this null also implies a rejection of the single regime null, since the latter is just the special case where $\sigma_S = \sigma_C$. However, we cannot test directly whether we have one regime or two, because the parameters of our alternative hypothesis are not identified under the null of only one regime. See Lee and Chesher (1986) for details.

The second column of the table presents LR statistics for the same tests, allowing ρ to vary. The test statistics are 21.10 for volatility regimes, 21.03 for a mixture of normal distributions, and 20.26 for mean reversion. Their p-values are always less than 0.0005. These all imply stronger rejection of the null hypothesis than do the corresponding tests in the first column.

Table II (p. 34) presents the LR statistics for the 1926-54, 1954-74, and 1974-89 subperiods. Some tests for deviations from fundamentals in U.S. stock market data (such as mean reversion tests) show more evidence of a departure from efficient markets in time periods that include the 1929 crash and the Great Depression.[12] In contrast, the LR statistics presented in Table II actually show stronger rejection of the null hypothesis in the periods 1954-74 and 1974-89 than in the period that includes the 1929 crash and the Great Depression. In all periods we reject the various null hypotheses at the 5 per cent significance level.[13]

In summary, the LR tests all reject the null hypothesis that apparent deviations from fundamental price have no influence on returns. In the next section, we explore the possibility that this apparent evidence of speculative behaviour is actually the result of regime switches in fundamentals.

V SWITCHING FUNDAMENTALS

Either news about fundamentals or a shift in speculative sentiment could lead to sudden changes in asset prices. In Sections III and IV, we compared a simple Lucas asset-pricing model with a model of speculative behaviour and found evidence that was

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[12]See, for example, Fama and French (1988a) or Kim, Nelson and Startz (1990). Evidence of mean reversion does not necessarily imply that asset prices deviate from fundamentals; see, for example, Brock and LeBaron (1989); Cecchetti, Lam and Mark (1990); Fama (1990); Fama and French (1988a); or Jog and Schaller (1991).

[13]The marginal significance level of the volatility regime test statistic is 0.052 in the period 1926-54; in all other cases, its significance level is smaller.

inconsistent with the null hypothesis that stock market prices correspond to fundamental values. In this section, we examine the model of switches in fundamentals outlined in Section II.C that has the potential to account for dramatic changes in asset prices. Since the equations describing stock market returns in the models of speculative behaviour and switching fundamentals do not nest, we use a Monte Carlo experiment to compare the two models.

We begin by estimating the parameters of the switching process for dividend growth given in equations (21) and (22).[14] We then choose a reasonable value for the coefficient of relative risk aversion γ and the subjective discount rate β. Next we generate data for dividends using the Markov switching model, derive the corresponding equilibrium stock prices using equations (23), (A13) and (A14), and calculate the resulting returns and price-dividend ratio for each period. This becomes artificial data with which we repeat the analysis of the preceding section; we re-estimate the switching-regression model and test it against the various alternatives. Repeating these last two steps with fresh draws of the underlying normal random variables, we are able to estimate the distribution of our parameters and LR statistics in a world with dividend switching but no deviations from fundamental price.

While the parameters of the dividend-switching process can be estimated directly, the choice of appropriate values for β and γ are less clear-cut. We can remove one degree of freedom by requiring that any acceptable pair (β,γ) generate an expected ρ that matches the mean price-dividend ratio in our sample. Noting that $\gamma=0$ implies risk-neutrality and $\gamma>0$ implies risk-loving agents, we restricted our attention to negative values of γ. In addition, the relationship is not monotonic, so that for $\beta \geq .975$ there is no value of γ that matches the mean price-dividend ratio. Limited experiments showed that the Monte Carlo results are sensitive to the choice of (β,γ). We present simulations

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[14]Given the results in the appendix, which imply that this process undergoes structural breaks in 1954 and 1974, we estimate these parameters separately over each subperiod.

for (β=0.97, γ=-1.767) and (β=0.95, γ=-0.603). We chose these two sets of values for two reasons. First, β=0.97 is the case considered by CLM and represents a reasonable estimate of both the discount rate and the degree of risk aversion. Second, values of γ above and below -1 lead to qualitatively different results: when γ>-1, the model of switching fundamentals implies that an increase in the dividend growth rate will increase stock prices, but when γ<-1, the model yields the somewhat counter-intuitive result that an increase in the dividend growth rate decreases stock prices.[15]

One interesting feature of the simulations is the size of the deviation from fundamentals. In the actual data, the mean absolute value of b_t is 0.1579. The model of switching fundamentals produces much smaller and more tightly distributed values of b_t. The median mean absolute value of b_t is 0.0241 and its 0.99 confidence interval lies below 0.0341. The standard deviation of b_t in the actual data was 0.2056, compared to the median standard deviation of 0.0321 and a 0.99 confidence interval lying below 0.0384 for the simulations.

The parameter estimates and LR statistics from the Monte Carlo experiment with β=0.97 and γ=-1.767 are shown in Table III (p. 35).[16] The parameter estimates are somewhat different from those we obtain from the actual data. The less-frequent state (which is labelled C in Table III) is associated with highly positive returns (β_{co}=1.058), whereas the actual data show negative returns in the less frequent regime. The LR statistics show that switching fundamentals are capable of accounting for the rejections of the null hypotheses of volatility regimes, a normal mixture, and mean reversion, which we found in Section IV. In fact, the CLM model generates much stronger evidence of mean reversion than the actual data; it is only when we test the null of mean reversion against the alternative of equations (25), (26) and (28) that the actual likelihood ratio statistic falls within the (0.01 to 0.99) bounds of the simulated distribution.

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[15]When γ=-1, $\rho(0)=\rho(1)$, so regime switches will be unable to explain stock market crashes.

[16]Given that each trial estimated five maximum likelihood models with 756 observations, this proved to be computationally intensive. Computing 1000 trials took about five days on a 16MHz 386 PC.

Table IV (p. 36) reports Monte Carlo simulations using β=0.95 and γ=-0.603. The parameter estimates for this case correspond somewhat more closely to the actual data. The infrequent regime (which is again labelled C in Table IV) is now associated with negative returns (β_{C0}=0.89). An increase in b_t increases the probability of the C regime (β_{qb} is negative). The variance of ε_t in the C regime is high compared to the S regime. As in the previous simulation, the actual likelihood ratio statistics for volatility regimes and a mixture of normal distributions lie below the 95 per cent confidence interval from the simulation, so switching fundamentals lead to clear evidence of mean reversion. The model of switching fundamentals yields weaker evidence (than the actual data) of a returns process in which deviations from fundamentals influence the stock market regime or in which the effect of deviations differs across regimes: the mean reversion test statistic from the actual data is close to the upper boundary of the 95 per cent confidence interval of the simulations.[17]

We are inclined to conclude from the Monte Carlo simulations that the model of switching fundamentals is able to reproduce the basic stylized facts of regime switching found in the data, if not their exact magnitudes. It accounts for the rejection of the volatility regimes, mixture of normals, and mean reversion nulls (even if it rejects the first two more strongly than the actual data do). The model also gives slightly positive returns in the surviving state, balanced (when γ>-1) by large but infrequent losses in the collapsing state, a probability of collapse that increases with b_t, and a collapsing state that is more volatile than the surviving state.

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[17]The LR statistic for the mean reversion null is 20.26 in the actual data; the 0.975 value for the simulations is 22.68.

VI ACCOUNTING FOR HISTORICAL CRASHES

The evidence in Sections III, IV and V suggests that either the model of speculative behaviour or a model with regime switches in fundamentals could account for the characteristics of U.S. stock market returns that are highlighted by our switching regression. In this section, we examine how well specific historical crashes are accounted for by each of the models. To do this, we use parameter estimates for each of the models to generate the probability of a stock market crash. For the model of speculative behaviour, we focus on the probability of a return which is two standard deviations below the mean return.[18] For the model of switching fundamentals, we focus on the probability of a switch in the dividend regime.

Figure I plots actual crashes versus the probabilities of collapse (generated from the empirical estimates of the model of speculative behaviour) for positive speculative components.[19] We begin by examining the two best-known crashes of this century. In both 1929 and 1987, we see an accelerating upward movement in the probability of a collapse which peaks just before the actual crash. This pattern accords well with historical descriptions of speculative behaviour. These sharp increases in the probability of collapse could reflect the explosive growth associated with a surviving speculative component. As the speculative component grows, the probability of collapse increases. A higher probability of collapse increases the required rate of return on the speculative

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[18]In the switching regression model, the *ex ante* probability of observing a return equal to or less than some given value K is calculated according to the formula

$$Pr(R_t \leq K) = \Phi\left(\frac{K-\beta_{S0}-\beta_{Sb}\cdot b_t}{\sigma_S}\right)\Phi\left(\beta_{q0}+\beta_{qb}\cdot|b_t|\right) + \Phi\left(\frac{K-\beta_{C0}-\beta_{Cb}\cdot b_t}{\sigma_C}\right)\Phi\left(-\beta_{q0}-\beta_{qb}\cdot|b_t|\right)$$

[19] Actual crashes were defined by using monthly returns to calculate the 20 largest three-month losses in our sample. Three-month rather than one-month losses were used both to capture more gradual (but large) price declines and to exclude transitory losses that are almost immediately offset by subsequent price increases. Only 10 distinct crashes are shown, because half of the three-month losses overlapped.

component in the survival state. This high rate of return implies that the speculative component grows still more rapidly, further increasing the probability of collapse. In addition to the well-known 1929 and 1987 events, there are episodes in 1932 and 1946 with a very similar pattern. The probability of collapse rises rapidly to a high level, peaks at the time of the actual crash, and declines rapidly thereafter.[20]

The 1930 and 1931 crashes came at a time when the probability of collapse had recently decreased and was very low.[21] These episodes might be consistent with a fundamental interpretation, perhaps occurring as the banking crises of the early 1930s led market participants to dramatically revise their forecasts of future economic performance, bringing about stock market crashes. To summarize, the most famous stock market crashes of the twentieth century seem to be well captured by the model of speculative behaviour, but there are other crashes which do not seem to be the result of speculative behaviour.

In Figure II, we examine how well switches in fundamentals account for actual stock market crashes. Figure II plots the *ex post* probability of being in the low-dividend growth state against actual market crashes.[22] First, we note that the two most famous crashes (1929 and 1987) are marked by small and stable probabilities of low-dividend

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[20]The model of speculative behaviour in the equations (26), (27) and (29) is symmetric in the sense that large undervaluations imply an increasing probability of a sharp movement towards fundamental price. We focus on crashes rather than rallies because historical accounts have given us a set of famous crashes but no corresponding set of famous rallies. Plots of the probability of a rally show that it is less than 10 per cent for all but two periods between 1926 and 1989. The first exception is 1932, when the probability rose very sharply to peak at over 40 per cent; this was followed immediately by one of the largest rallies in our sample. The second exception was in 1942, when the probability of a rally rose sharply to peak at about 25 per cent; again, this was quickly followed by one of the largest rallies in our sample. As an additional test of the robustness of the model, we checked to see whether overvaluations and undervaluations entered symmetrically; a likelihood ratio test failed to reject the hypothesis that the coefficients were the same for $b_t>0$ and $b_t<0$.

[21] The 1937 , 1962, 1970 and 1974 crashes also occur at times when the model of speculative behaviour yields a relatively low probability of collapse.

[22]The *ex post* probability of being in the low dividend growth state is calculated using Hamilton's (1989) full-sample smoother.

growth. The 1932 and 1946 crashes, which, like the two more famous crashes, seem to accord well with the model of speculative behaviour, are also marked by stable probabilities of being in the collapsing dividend growth state. Therefore, the dividend-switching model seems unable to explain these crashes. At the time of the 1930, 1931 and 1937 crashes, the probability of the low-dividend growth state was rising rapidly, however. This is consistent with agents revising their expectations of future dividend growth and abruptly lowering stock prices.[23] The model of switching fundamentals therefore seems to do a better job of explaining these crashes than the model of speculative behaviour.

Thus, when we compare the probabilities of a switch in fundamentals with actual crashes, we find that switching fundamentals do a good job of accounting for stock market crashes such as those of 1930, 1931 and 1937. There is little evidence, however, of a switch in fundamentals around the time of the 1929, 1932, 1946 or 1987 stock market crashes. The most plausible economic interpretation of these results seems to be that important news about expected future dividends can cause sharp changes in asset prices, but that some of the most famous market crashes are difficult to explain in terms of switches in fundamentals.

VII CONCLUSION

The model of speculative behaviour outlined above implies that stock market returns should evidence switching behaviour and that larger deviations from fundamentals will increase the probability of a crash. Our coefficient estimates of this model are consistent with these predictions. We find a frequently occurring regime in

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[23]In interpreting Figure II, we have implicitly assumed that $\gamma > -1$, so the low-growth state is associated with lower stock prices. If $\gamma < -1$, crashes would be associated with transitions from low- to high-dividend growth states, rather than the reverse. We note that if we adopt this interpretation, then the dividend-switching model does much worse in explaining stock market crashes, with only one crash (in 1962) corresponding to such a transition.

which stocks earn a positive return and the variance of shocks to expected conditional returns is smaller. In the alternative regime, returns are negative, more variable and much more strongly influenced by apparent deviations from fundamentals. The probability that the collapsing regime will occur increases significantly with the size of the apparent deviation from fundamentals. In this broad sense, the model of speculative behaviour seems to capture some aspects of actual returns data. A more formal test is to impose parameter restrictions which apply under the null hypothesis that stock market prices are unaffected by apparent deviations from fundamentals. Restrictions of this type are strongly rejected by the data. These results are consistent with the hypothesis that speculative behaviour influences stock prices.

To determine whether the characteristics of stock market returns highlighted by our switching-regression econometric techniques can be explained by an asset-pricing model in which prices correspond to fundamentals, we focus on a model with regime switches in dividend growth. For some choices of the unobservable taste parameters, simulations of the model of switching fundamentals capture the major features of actual stock market returns.

The coefficient estimates and tests of parameter restrictions seem to suggest that the two models are substitutes in the sense that either model is capable of accounting for the characteristics of actual returns we have highlighted. We therefore examine specific stock market crashes in Figures I and II. We find that the probability of a crash generated from estimates of the model of speculative behaviour rises sharply before the two best-known stock market crashes of this century. This is not true for all crashes; in the early 1930s, there are crashes at a time when the model of speculative behaviour generates a very low probability of a collapsing regime. If we consider the evidence in Figures I and II together, it suggests that the two models may be complements rather than substitutes. The best-known stock market crashes of the last 70 years (1929 and 1987) seem to be captured quite well by the model of speculative behaviour but not by a model of switching fundamentals. Several other crashes, such as those in 1930 and

1931 seem to be relatively well captured by the model of switching fundamentals. There is no a priori reason to consider models of speculative behaviour and switching fundamentals to be mutually exclusive. We are inclined to conclude that some crashes may be related to speculative behaviour while others occur in response to important news about fundamentals.

APPENDIX 1: MATHEMATICAL DERIVATIONS

A. Derivation of the switching-regression form of the model of speculative behaviour

We can solve for expected returns in each regime using the definitions $R_{t+1} \equiv (P_{t+1} + D_{t+1})/P_t$ and $B_t \equiv P_t - P^*_t$ to get

$$E_t[R_{t+1}] = \frac{E_t[P^*_{t+1}+D_{t+1}]}{P_t} + \frac{E_t[B_{t+1}]}{P_t} \tag{A1}$$

Using the definition of the dividend generating process (9) gives

$$\begin{aligned}\frac{E_t[P^*_{t+1}+D_{t+1}]}{P_t} &= \frac{E_t[(1+\rho)D_{t+1}]}{P_t} \\ &= (1+\rho)\cdot e^{\alpha_0+\sigma^2/2}\cdot\frac{D_t}{P_t}\end{aligned} \tag{A2}$$

Substituting this and (19) in (A1) gives

$$E_t[R_{t+1}|S] = (1+\rho)\cdot e^{\alpha_0+\sigma^2/2}\cdot\frac{D_t}{P_t} + \frac{M}{q(b_t)}\cdot b_t - \frac{1-q(b_t)}{q(b_t)}\cdot u(b_t) \tag{A3}$$

However,

$$\frac{D_t}{P_t} \equiv \frac{1-b_t}{\rho} \tag{A4}$$

so (A3) becomes

$$E_t[R_{t+1}|S] = \frac{1+\rho}{\rho}\cdot e^{\alpha_0+\sigma^2/2}\cdot(1-b_t) + \frac{M}{q(b_t)}\cdot b_t - \frac{1-q(b_t)}{q(b_t)}\cdot u(b_t) \tag{A5}$$

Similarly, we can show that

$$E_t[R_{t+1}|C] = \frac{1+\rho}{\rho}\cdot e^{\alpha_0+\sigma^2/2}\cdot(1-b_t) + u(b_t) \qquad \textbf{(A6)}$$

This model fits readily into the econometric framework of switching regressions. To see this, we can take first-order Taylor series approximations of (A5), (A6) and (17) around some arbitrary b_0 to obtain

$$\begin{aligned} E(R_{t+1}|S) &= \beta_{S0} + \beta_{Sb}b_t \\ E(R_{t+1}|C) &= \beta_{C0} + \beta_{Cb}b_t \\ q_{t+1} &= \beta_{q0} + \beta_{qb}|b_t| \end{aligned} \qquad \textbf{(A7)}$$

This corresponds to a switching regression in which the size of the speculative component in the previous period helps to predict the probability of survival and influences the expected return conditional on survival or collapse.

We can also say something about the signs of the coefficients on b_t.[24] By construction, $\beta_{qb} < 0$. The coefficient on b_t in the collapsing regime will be

$$\left.\frac{dE_t[R_{t+1}|C]}{db_t}\right|_{b_t=b_0} = -\frac{1+\rho}{\rho}\cdot e^{\alpha_0+\sigma^2/2} + u'(b_0) \qquad \textbf{(A8)}$$

Given that $\alpha_0>0$ (that is, dividends tend to grow rather than shrink over time) we can show that the first term is < -1 while $u'(b)\leq 1$ by construction. Therefore, the whole expression (and therefore β_{Cb}) must be <0. Similarly, we can derive

[24]Note that the 1st order Taylor expansion of f(x) around x_0 gives

$$\begin{aligned} f(x) &\approx f(x_0) + f'(x_0)\cdot(x-x_0) \\ &= \left(f(x_0) - f'(x_0)\cdot x_0\right) + f'(x_0)\cdot x \\ &= \lambda_0 + \lambda_1\cdot x \end{aligned}$$

so we only need the derivative of conditional expected returns with respect to b_t in order to sign its coefficient.

$$\left.\frac{dE_t[R_{t+1}|S]}{db_t}\right|_{b_t=b_0} = -\frac{1+\rho}{\rho}\cdot e^{\alpha_0+\sigma^2/2} + \frac{M}{q(b_0)}\cdot\left(1-\frac{b_0\cdot q'(b_0)}{q(b_0)}\right) - u'(b_0)\cdot\left(\frac{1-q(b_0)}{q(b_0)}\right) + \frac{q'(b_0)\cdot u(b_0)}{q^2(b_0)} \quad \textbf{(A9)}$$

$$= -\frac{1+\rho}{\rho}\cdot e^{\alpha_0+\sigma^2/2} + \frac{M}{q(b_0)} + u'(b_0)\left(1-q^{-1}(b_0)\right) + \frac{q'(b_0)}{q^2(b_0)}\left[u(b_0)-M\cdot b_0\right]$$

Since we know that ρ, M and q are always positive, it follows that the first term must always be <0 while the second must be >0. Furthermore, we know $u'>0$ and $q<1$ so the third term is <0. Finally, under the assumption that M>1 (that is, that the expected value of the speculative component grows over time), we can show $u(b_0) - Mb_0 > 0$ if and only if $b_0>0$. Since $q'(b_0)>0$ if and only if $b_0>0$ by definition, the final term will always be >0. Therefore, we have the sum of two positive and two negative terms, so the sign of β_{Sb} is indeterminate. However, since the sum of the first term and third terms must be > β_{Cb}, $\beta_{Sb} > \beta_{Cb}$.

B. Derivation of the asset-pricing model with Markov switching in dividend growth[25]

To verify that (23) is a solution to the stochastic difference equation, substitute it into (4), obtaining

$$\rho(S_t)D_t^{\gamma+1} = \beta E_t D_{t+1}^{\gamma+1}[\rho(S_{t+1}) + 1] \quad \textbf{(A10)}$$

By rewriting the dividend process in levels, rather than logarithms

$$D_{t+1} = D_t e^{(\alpha_0+\alpha_1 S_t+\varepsilon_{t+1})} \quad \textbf{(A11)}$$

and substituting into the previous equation, we obtain the following expression for the price-dividend ratio as a function of the state of the dividend process

[25]This follows Cecchetti, Lam and Mark (1990).

$$\rho(S_t) = \beta e^{[\alpha_0(1+\gamma) + (1+\gamma)^2\sigma^2/2]} e^{[\alpha_1(1+\gamma)S_t]} E_t[\rho(S_{t+1}) + 1] \quad \textbf{(A12)}$$

Since the state space for the Markov switching variable consists only of the states 0 and 1, this expression for the price-dividend ratio is effectively a system of two linear equations in the two unknown variables $\rho(0)$ and $\rho(1)$; the solution is

$$\rho(0) = \tilde{\beta}[1 - \tilde{\beta}\tilde{\alpha}_1(p + q - 1)]/\Delta \quad \textbf{(A13)}$$

$$\rho(1) = \tilde{\beta}\tilde{\alpha}_1[1 - \tilde{\beta}(p + q - 1)]/\Delta \quad \textbf{(A14)}$$

$$\textit{where } \tilde{\beta} = \beta e^{[\alpha_0(1+\gamma)+(1+\gamma)^2\sigma^2/2]},\ \tilde{\alpha}_1 = e^{\alpha_1(1+\gamma)}, \quad \textbf{(A15)}$$

$$\Delta = 1 - \tilde{\beta}(p\tilde{\alpha}_1 + q) + \tilde{\beta}^2\tilde{\alpha}_1(p + q - 1) \quad \textbf{(A16)}$$

The equilibrium gross return can be obtained from the relationship between current prices and dividends and the dividend process, so

$$R_t = \left(\frac{1 + \rho(S_t)}{\rho(S_{t-1})}\right) e^{(\alpha_0 + \alpha_1 S_{t-1} + \varepsilon_t)} \quad \textbf{(A17)}$$

APPENDIX 2: THE CONSTRUCTION OF b_t

The data we examine for evidence of speculative behaviour is drawn from the data base of the Center for Research in Security Prices. We use their monthly value-weighted price (P) and dividend (D) indices for all stocks from January 1926 to December 1989. As outlined in Section I.A, P and D should be in real per capita terms. The population growth adjustment uses annual population data from 1924 to 1945 from Historical Statistics of the United States (1976) (series A29) and quarterly data from the U.S. Department of Commerce *Survey of Current Business* (Table 2.1, line 34) from 1946 onwards. Monthly dates are linearly interpolated. Data from January 1960 onwards are divided by 1.0043 to correct for the inclusion of Alaska and Hawaii from that date onwards. To deflate nominal returns, Cecchetti, Lam and Mark (1990), Poterba and Summers (1988) and Fama and French (1988a) use the all items consumer price index (CPI), while Campbell and Shiller (1988) argue the all commodity producer price index (PPI) is superior. We found our results to be robust to this distinction and present results using the CPI. Since dividends display strong seasonal fluctuations, we follow Fama and French (1988b) in using an average over the twelve-month period ending in the given month.

The measure of deviations from fundamentals that we use (b_t) is tied to the model of equilibrium asset prices outlined in Section I.A, where the fundamental price is $P_t = \rho D_t$ and the price-dividend ratio is

$$\rho = \frac{\beta e^{[\alpha_0(1+\gamma) + (1+\gamma)^2\sigma^2/2]}}{1-\beta e^{[\alpha_0(1+\gamma) + (1+\gamma)^2\sigma^2/2]}}$$

Under the null hypothesis that actual prices equal fundamental prices, the mean price-dividend ratio is a consistent estimate of ρ. Under the alternative of speculative behaviour the proportional deviation from the fundamental price is $b_t \equiv 1 - \rho D_t/P_t$. We estimate the drift parameter α_0 as the mean of the change in log dividends and the variance parameter σ^2 as the variance of the residuals. The subjective discount factor β and the coefficient of risk aversion γ are not observable but the actual price-dividend ratio is. Column one of Table A1 (p. 37) shows a grid of possible βs and γs consistent with our estimates of α_0 and σ^2 and the mean of P/D over our sample. We see that γs in the vicinity of -1 are consistent with plausible estimates of the subjective discount factor.

The actual price-dividend ratios over our sample are also presented in Table AI. This suggests that there may be subperiods with distinct means. In particular, P/D seems consistently higher in the 1960s and early 1970s than in the rest of the sample. This impression is confirmed by a series of Chow tests which are also graphed in Figure A1 (p. 40). The dotted line shows the Chow test statistics for the null hypothesis of a constant mean against the alternative of a shift in mean occurring at a given date. This gives a very strong rejection of the null of a constant mean, with the most likely break point coming after March 1974. The dashed line then repeats the Chow tests for each of the two new subsamples. It shows that while the mean after 1974 looks stable, there is again a significant shift in mean somewhere in the mid-1950s, with the most likely point being after November 1954.

We set ρ to the mean price-dividend ratio for the relevant subperiod. Of course, this is equivalent to picking a value for either β or γ and allowing the other parameter to be pinned down using the mean price-dividend ratio and the estimated values of α_0 and σ^2. The relationship this implies between β and γ in each subperiod is shown in columns two, three and four of Table A1. The ρs are shown as the horizontal lines in Figure A1, and the gap between them and P/D shows the sign and the size of b_t.

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TABLES

Table I: The model of speculative behaviour (full sample)

Parameter Estimates	Constant ρ	Variable ρ
β_{S0}	1.007 (0.002)	1.007 (0.002)
β_{Sb}	-0.006 (0.006)	-0.009 (0.010)
β_{C0}	0.976 (0.039)	0.994 (0.026)
β_{Cb}	-0.111 (0.071)	-0.106 (0.071)
β_{q0}	2.098 (0.294)	1.982 (0.266)
β_{qb}	-1.560 (0.510)	-2.568 (0.774)
σ_S	0.044 (0.002)	0.043 (0.002)
σ_C	0.170 (0.028)	0.153 (0.023)
Likelihood Ratio Tests		
Volatility Regimes	16.29 (0.003)	21.10 (0.000)
Mixture of Normals	16.21 (0.001)	21.03 (0.000)
Mean Reversion	14.79 (0.002)	20.26 (0.000)

The model of speculative behaviour is equations (26), (27) and (29) in the text. Figures in parentheses indicate standard errors for parameter estimates and p-values for likelihood ratio tests. The latter impose the following restrictions:

Volatility Regimes:

$$\beta_{S0}=\beta_{C0}=\beta_0,\ \beta_{Sb}=\beta_{Cb}=\beta_{qb}=0$$

Mixture of Normals:

$$\beta_{Sb}=\beta_{Cb}=\beta_{qb}=0$$

Mean Reversion:

$$\beta_{S0}=\beta_{C0}=\beta_0,\ \beta_{Sb}=\beta_{Cb}=\beta_1,\ \beta_{qb}=0$$

In column two, we calculate ρ separately for 1926-54, 1954-74 and 1974-89.

Table II: The model of speculative behaviour (subperiods)			
Parameter Estimates	1926-1954	1954-1974	1974-1989
β_{S0}	1.009 (0.004)	1.035 (0.005)	1.006 (0.003)
β_{Sb}	0.007 (0.014)	-0.140 (0.017)	0.011 (0.025)
β_{C0}	0.996 (0.035)	0.997 (0.003)	0.956 (0.021)
β_{Cb}	-0.150 (0.119)	-0.036 (0.016)	-0.532 (0.071)
β_{q0}	1.794 (0.355)	-0.483 (0.299)	4.600 (1.080)
β_{qb}	-1.673 (0.768)	-3.726 (1.689)	-13.289 (4.034)
σ_S	0.050 (0.003)	0.012 (0.003)	0.044 (0.003)
σ_C	0.187 (0.035)	0.037 (0.002)	0.041 (0.015)
Likelihood Ratio Tests			
Volatility Regimes	9.40 (0.052)	19.02 (0.001)	14.98 (0.005)
Mixture of Normals	9.23 (0.026)	10.53 (0.015)	14.91 (0.002)
Mean Reversion	9.35 (0.025)	15.30 (0.002)	14.03 (0.003)

The model of speculative behaviour is equations (26), (27) and (29) in the text. Figures in parentheses indicate standard errors for parameter estimates and p-values for likelihood-ratio tests. These tests impose the following restrictions:

Volatility Regimes
$$\beta_{S0}=\beta_{C0}=\beta_0,\ \beta_{Sb}=\beta_{Cb}=\beta_{qb}=0$$

Mixture of Normals
$$\beta_{Sb}=\beta_{Cb}=\beta_{qb}=0$$

Mean Reversion
$$\beta_{S0}=\beta_{C0}=\beta_0,\ \beta_{Sb}=\beta_{Cb}=\beta_1,\ \beta_{qb}=0$$

Table III: The model of switching fundamentals (β=0.97, γ=-1.767)				
Parameter Estimates	Actual Data	2.5%	50%	97.5%
β_{s0}	1.007	1.008	1.011	1.014
β_{sb}	-0.009	-.160	-.131	-.101
β_{C0}	0.994	1.026	1.058	1.110
β_{Cb}	-0.106	-2.25	-1.70	-.72
β_{q0}	1.982	1.77	2.12	2.51
β_{qb}	-2.568	-11.32	-6.87	-.73
σ_s	0.043	.014	.015	.016
σ_C	0.153	.084	.113	.161
Likelihood Ratio Tests				
Volatility Regimes	21.10	46.9	140.6	224.3
Mixture of Normals	21.03	46.4	139.9	223.9
Mean Reversion	20.26	7.29	20.12	40.20

The entries in columns two, three and four represent the probability distribution of the coefficient estimates and likelihood ratio statistics of the model of equations (26), (27) and (29) under the null hypothesis of switching fundamentals, based on a Monte Carlo simulation with 1000 replications.

Table IV: The model of switching fundamentals (β=0.95, γ=-0.603)				
Parameter Estimates	Actual Data	2.5%	50%	97.5%
β_{s0}	1.007	1.001	1.003	1.005
β_{sb}	-0.009	0.110	0.173	0.217
β_{C0}	0.994	0.76	0.89	0.97
β_{Cb}	-0.106	-3.75	-2.29	1.77
β_{q0}	1.982	2.29	2.51	2.87
β_{qb}	-2.568	-20.99	-13.50	-9.57
σ_s	0.043	0.014	0.015	0.016
σ_C	0.153	0.215	0.277	0.398
Likelihood Ratio Tests				
Volatility Regimes	21.10	24.0	96.3	172.9
Mixture of Normals	21.03	22.9	95.6	172.6
Mean Reversion	20.26	5.06	12.94	22.68

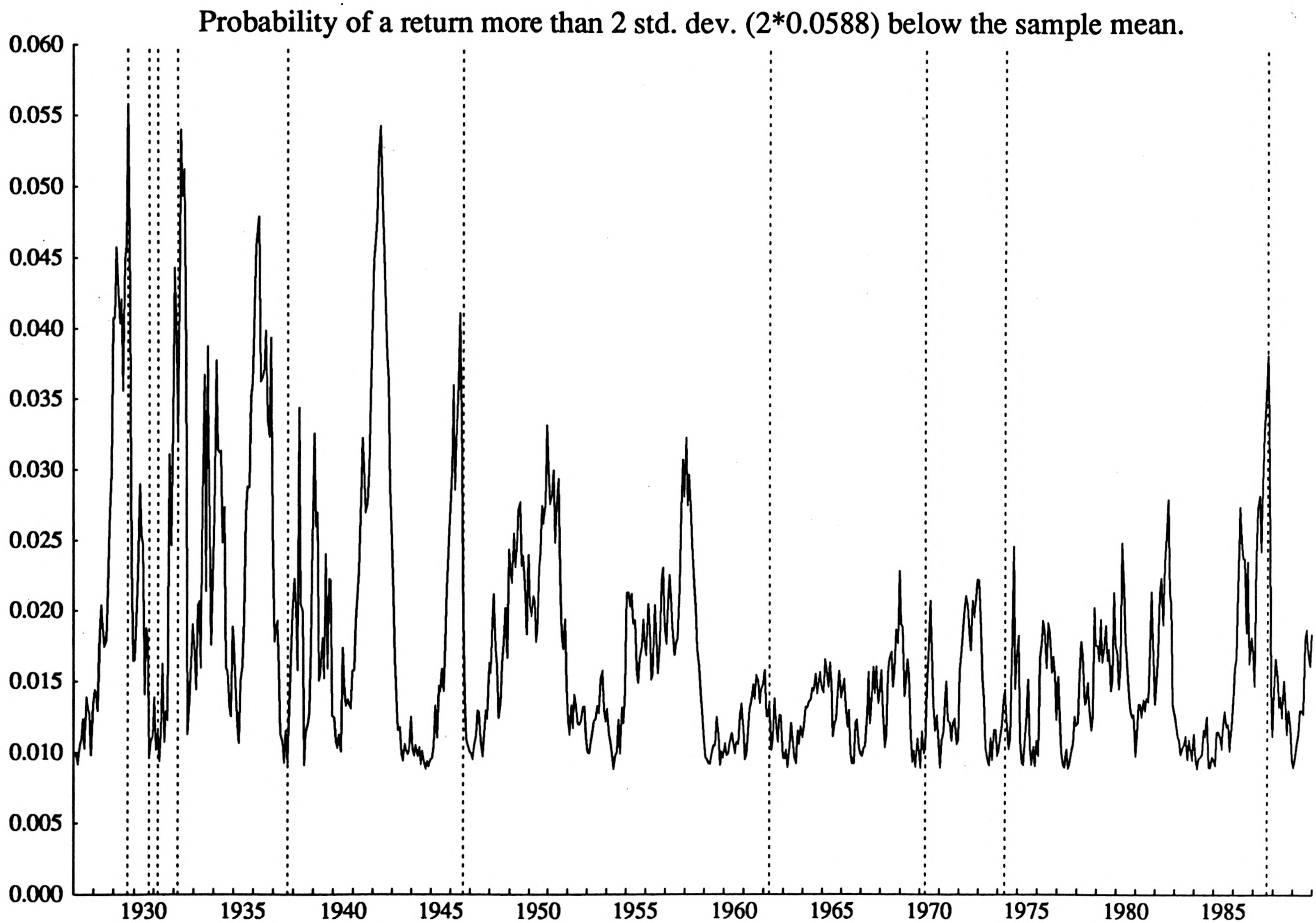
The entries in columns two, three and four represent the probability distribution of the coefficient estimates and likelihood ratio statistics of the model of equations (26), (27) and (29) under the null hypothesis of switching fundamentals, based on a Monte Carlo simulation with 1000 replications.

Table A1 - Relationship between β and γ implied by the model of switching fundamentals

γ	β full sample	β 1926-54	β 1954-74	β 1974-89
0.00	0.93658	0.92911	0.95513	0.92252
-0.50	0.94820	0.94087	0.96165	0.93993
-1.00	0.95923	0.95179	0.96773	0.95712
-1.50	0.96966	0.96184	0.97338	0.97406
-2.00	0.97946	0.97098	0.97857	0.99073
Mean P/D =	287.80014	242.35926	365.37655	273.28103
Mean ΔD =	0.00187	0.00184	0.00101	0.00297
Std. Dev. ΔD =	0.01587	0.01861	0.01283	0.01383

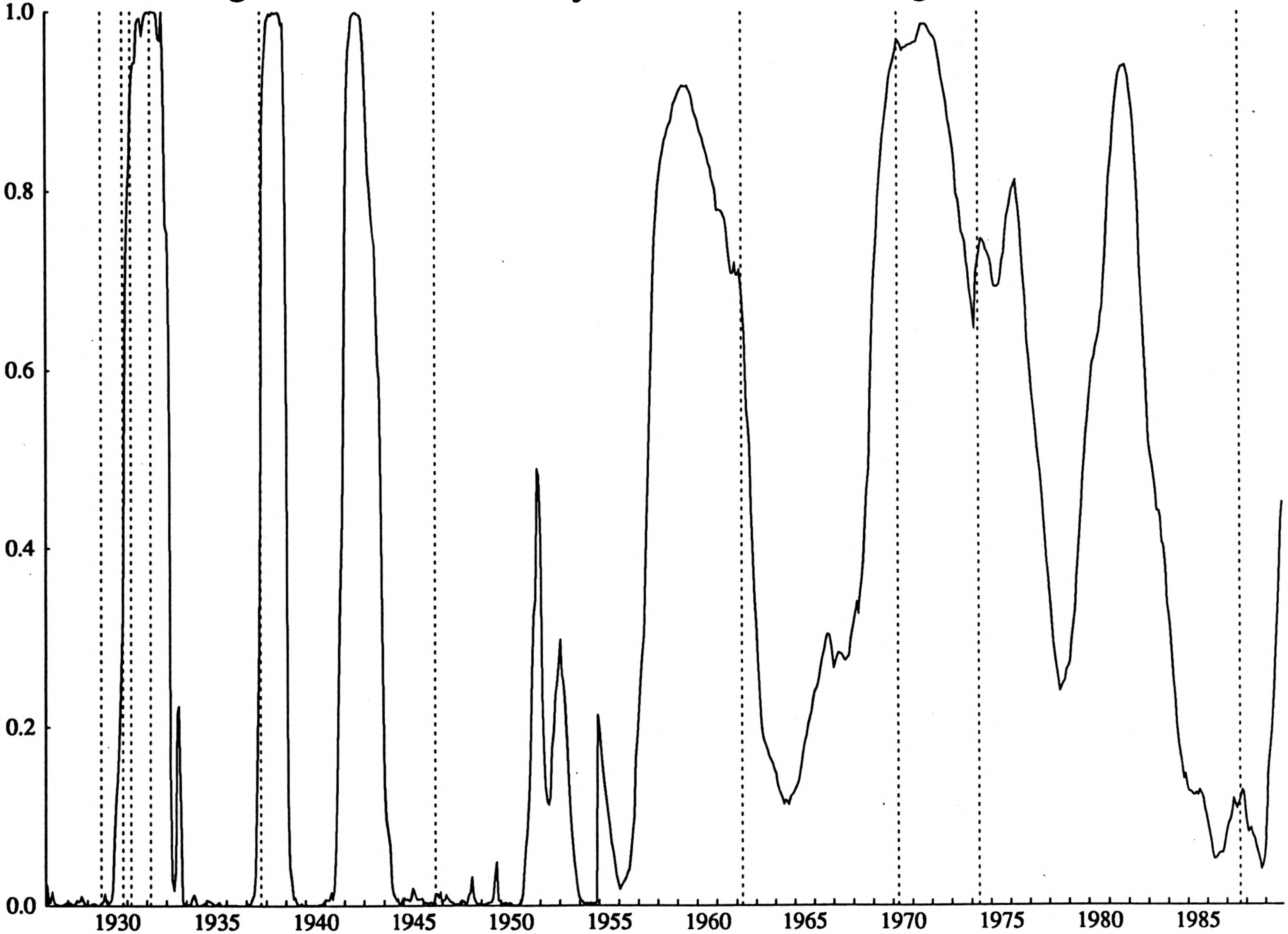
Note that β is expressed as an annual rate for ease of comparison.

Figure I - Probability of stock market crash



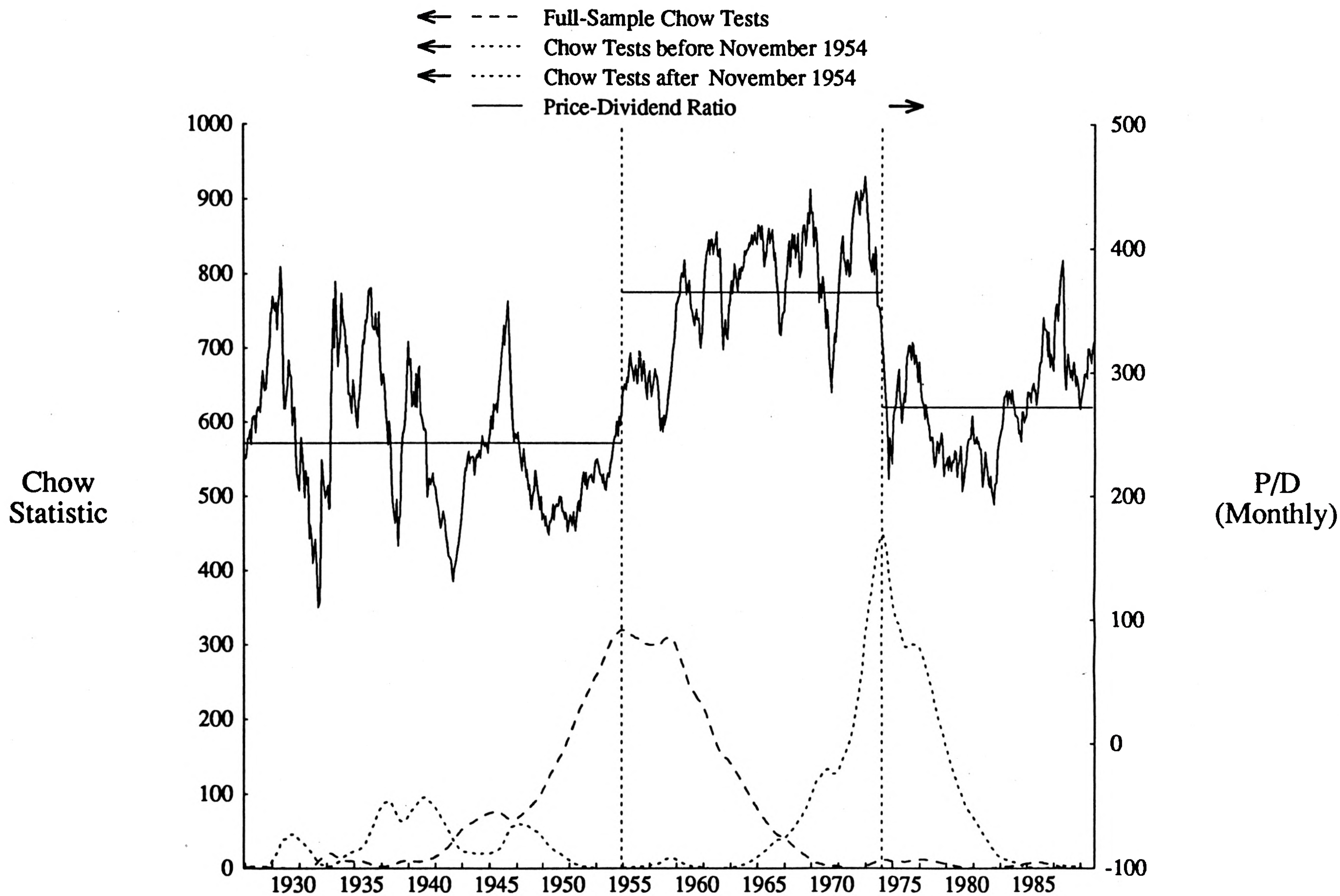
Vertical lines indicate the 10 worst three-month stock market declines.

Figure II - Probability of low dividend growth state



Vertical lines indicate the 10 worst three-month stock market declines.

Figure A1 - Recursive Chow tests for shifts in P/D



Peaks shown are at 1954M11 and 1974M3.

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