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by
Robert A. Amano and Tony S. Wirjanto

Bank of Canada Banque du Canada

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Robert A. Amano
International Department
Bank of Canada
Ottawa, Ontario, Canada
K1A 0G9

Tony S. Wirjanto
Department of Economics
University of Waterloo
Waterloo, Ontario, Canada
N2L 3G1

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CONTENTS

Abstract

This paper studies the asymptotic behaviour of least squares estimates in a stock-adjustment model when the variables are nonstationary. The paper first considers the case in which the variables are not cointegrated, and then examines the case under cointegration. In the case of no cointegration, we find that the least squares estimate of the adjustment coefficient is close to zero, independent of its true value. In the case of cointegration, it is shown that a transformation can be applied to the model so that the resulting instrumental variable estimates will have standard distributions. Economic examples are used to illustrate each case and Monte Carlo simulations are performed in order to gauge the relevance of the asymptotic results for finite samples. The simulation evidence suggests that the asymptotic results are useful approximations in finite samples.

Résumé

La présente étude traite du comportement asymptotique des paramètres que l'on obtient en estimant par la méthode des moindres carrés un modèle d'ajustement des stocks doté de variables non stationnaires. Les auteurs examinent d'abord le cas où les variables ne sont pas cointégrées, puis celui où elles le sont. En l'absence de cointégration des variables, l'estimation du coefficient d'ajustement obtenue par la méthode des moindres carrés avoisine zéro, peu importe la valeur véritable du coefficient. Dans le cas contraire, il est possible de transformer le modèle de manière que les estimations obtenues au moyen de variables instrumentales aient une distribution connue. Les auteurs illustrent chaque cas au moyen d'exemples de nature économique et effectuent des simulations de Monte-Carlo afin d'évaluer la pertinence des résultats asymptotiques pour des échantillons finis. À en juger par ces simulations, les résultats asymptotiques obtenus constituent des approximations utiles lorsque l'échantillon est fini.

1 INTRODUCTION

The stock-adjustment process is frequently used to model the dynamic behaviour of economic agents in the presence of costs that do not allow full adjustment within a single period. However, when researchers estimate a stock-adjustment model, they often find that the estimated adjustment coefficient is "implausibly low" or inconsistent with economic theory. For example, stock-adjustment models of money demand, inventory investment and consumer durables have often produced very low speeds of adjustment.[1] Recent attempts to resolve this issue have, in general, involved applying different estimation methods.[2]

In this paper, we explore the implications of nonstationary data for the stock-adjustment model. In particular, we study this issue under both the no cointegration and cointegration cases. In the case of no cointegration, we find that the least squares (LS) estimate of the adjustment coefficient is close to zero, independent of its true value. We also show that the LS estimate of the nonstationary regressor in the model tends to zero, but that its t-statistic increases with the sample size. The former result differs from the spurious regression model first examined by Granger and Newbold (1974) using a set of Monte Carlo experiments and more recently investigated by Phillips (1986) using the functional central limit theorem. In the case of a spurious regression, Phillips showed that the LS estimate of a nonstationary regressor converges in probability to the functionals of Brownian Motion, while in this paper we show that it converges in probability to zero.[3] In the cointegration case, it is shown that the transformation proposed by Wickens and Breusch (1988) can be applied to the model so that the resulting instrumental variables (IV) estimates have standard distributions and statistical inference can proceed in the usual manner. We also show that this approach can reduce parameter estimation bias relative to simple LS estimation.

The paper is organized as follows. Section 2 examines the asymptotic behaviour of LS estimates when the variables in the model are nonstationary and not cointegrated, and then

1. See, for example, Blinder (1986) for inventory investment, Bernanke (1985) for consumer durables and Caramazza, Hostland and Poloz (1990) for money.
2. For example, see Hall and Rossana (1991).
3. In both situations, however, the t-statistic has a divergent limiting distribution.

considers the case in which the variables in the model are cointegrated in the sense of Engle and Granger (1987). Section 3 provides empirical examples and simulation evidence which allows us to gauge the usefulness of the asymptotic results for finite samples. Concluding remarks follow.

2 ANALYTICAL RESULTS

2.1 The case of no cointegration

This subsection considers the implications of nonstationary data for the stock-adjustment model when the data are not cointegrated. We begin by briefly introducing the stock-adjustment model. For simplicity, the version of the model used in this section contains only one regressor (x_t). Suppose that y_t^*, the desired level of y_t, is related to x_t by the equation

$$y_t^* = \beta x_t + u_t. \tag{1}$$

The stock-adjustment process assumes that in any given period the actual value of y_t may not adjust completely to the desired level, that is

$$(1-L)\,y_t = \lambda\,(y_t^* - Ly_t)\,; \quad \lambda \in (0, 1] \tag{2}$$

where λ is the so-called adjustment parameter and L is a lag operator such that $L^i y_t = y_{t-i}$. Equation (2) specifies that the change in current y_t will respond only partially to differences between the desired level and the previous period's value of y_t. Substituting (1) into (2) and solving for y_t admits

$$y_t = \gamma_1 x_t + \gamma_2 y_{t-1} + \eta_t \tag{3}$$

where $\gamma_1 = \lambda\beta$, $\gamma_2 = (1-\lambda)$ and $\eta_t = \lambda u_t$.

Now, let y_t and x_t be I(1) variables and u_t an I(0) variable such that the data generation process (DGP) for y_t is a cointegrated system of the form

$$y_t = \beta x_t + u_t; \quad u_t = \rho u_{t-1} + g_t, \ |\rho| < 1 \tag{4}$$

$$\Delta x_t = d_t \tag{5}$$

where d_t and g_t are i.i.d$(0,\sigma_d^2)$ and i.i.d$(0,\sigma_g^2)$ respectively. In this context the above DGP for y_t can be thought of as a stock-adjustment model with complete adjustment in any given period, that is $\lambda = 1$.

Suppose that the model to be estimated is given by

$$y_t = ay_{t-1} + bz_t + v_t \qquad (t=1,2,\ldots, T) \tag{6}$$

where $\Delta z_t = e_t$, $\Delta y_t = f_t$, e_t is i.i.d$(0,\sigma_e^2)$, f_t is i.i.d$(0,\sigma_f^2)$ and $\text{cov}(f_t, e_{t+i}) = 0\ \forall\ i$; in other words, y_t and z_t are each I(1), but they are not cointegrated. Alternatively, equation (6) can be viewed as a misspecified stock-adjustment model.

Theorem 1 below provides weak convergence results for the LS estimates of equation (6) when the DGP is given by (4) and (5). The following lemma is useful in the derivation of this and other theorems. For notational convenience we denote $\sum_{t=1}^{T}$ as $\sum$ and $\int_0^1$ as $\int$.

LEMMA 1. Let " $\Rightarrow$ " denote weak convergence of the relevant probability measures as the sample size (T) tends to infinity. Define W(r) and V(r) as independent Wiener processes on the function space C[0,1], where C[0,1] is the space of all real valued continuous functions on the interval [0,1]. Then as $T \to \infty$,

(a) $T^{-2}\sum x_t^2 \Rightarrow \sigma_d^2 \int W(r)^2 dr$

(b) $T^{-2}\sum z_t^2 \Rightarrow \sigma_e^2 \int V(r)^2 dr$

(c) $T^{-2}\sum z_t x_t \Rightarrow \sigma_e \sigma_d \int V(r)\, W(r)\, dr$

(d) $T^{-2}\sum y_{t-1} u_t \Rightarrow 0$

(e) $T^{-2}\sum x_t u_t \Rightarrow 0$

(f) $T^{-2}\sum z_t u_t \Rightarrow 0$

(g) $T^{-2}\sum y_{t-1}^2 \Rightarrow \beta^2\sigma_d^2\int W(r)^2 dr$

(h) $T^{-2}\sum y_{t-1}x_t \Rightarrow \beta\sigma_d^2\int W(r)^2 dr$

(i) $T^{-2}\sum y_{t-1}z_t \Rightarrow \beta\sigma_d\sigma_e\int V(r)\,W(r)\,dr$

THEOREM 1. Suppose that equation (6) is estimated by LS and the conditions of Lemma 1 are satisfied. Then as $T \to \infty$,

(i) $\hat{a} \Rightarrow 1$

(ii) $\hat{b} \Rightarrow 0$

PROOF: See Appendix.

Theorem 1 tells us that if a non-cointegrated stock-adjustment model is estimated by LS, then the parameter estimate of the lagged dependent variable (LDV) will tend to unity asymptotically, while the parameter estimate of the I(1) regressor will tend, asymptotically, to zero. These results imply that in the non-cointegrated model the LS estimate of the adjustment coefficient (λ) will be low even though the true value of λ is unity.

While Theorem 1 summarizes the behaviour of the first-sample moments of the variables in equation (6), the limiting distribution of the LS estimates of $(a,b)^T$ is given by Theorem 2 below. Useful results for Theorem 2 are summarized in the following lemma.

LEMMA 2. Define P(r) as a Wiener process that is independent of W(r) and V(r) on C[0,1]. Then as $T \to \infty$,

(a) $T^{-2}\sum y_t^2 \Rightarrow \sigma_f^2\int P(r)^2 dr$

(b) $T^{-2}\sum y_{t-1}z_t \Rightarrow \sigma_f\sigma_e\int P(r)\,V(r)\,dr$

(c) $T^{-1}\sum y_{t-1}v_t \Rightarrow (1/2)\,\sigma_f^2\,[P(1)^2 - 1]$

(d) $T^{-1}\sum z_t v_t \Rightarrow \sigma_f\sigma_e\int V(r)\,dP(r)$

THEOREM 2. Suppose that equation (6) is estimated by LS and the conditions of Lemma 1 and 2 are satisfied. Then as $T \to \infty$,

(i)

$$T(\hat{a}-1) \xrightarrow{d} \{\sigma_e^2 \int V(r)^2 dr\,(1/2)\,\sigma_f^2 [P(1)^2 - 1] - \sigma_f \sigma_e \int P(r)\,V(r)\,dr$$

$$\sigma_f \sigma_e \int (V(r)\,dP(r)\})/\{\sigma_f^2 \int P(r)^2 dr\, \sigma_e^2 \int V(r)^2 dr - (\sigma_f \sigma_e \int P(r)\,V(r)\,dr)^2\}$$

(ii)

$$T(\hat{b}-0) \xrightarrow{d} \{\sigma_f \sigma_e \int P(r)\,V(r)\,dr - (1/2)\,\sigma_f^2 [P(1)^2 - 1]\}/$$

$$\{\sigma_f^2 \int P(r)^2 dr\, \sigma_e^2 \int V(r)^2 dr - (\sigma_f \sigma_e \int P(r)\,V(r)\,dr)^2\}$$

PROOF: See Appendix.

The results in Theorem 2 tell us that the limiting distribution of the LS estimates of $\hat{a}$ and $\hat{b}$ are functionals of Wiener processes. These results imply that the limiting distribution of the t-statistic ($T^{-1/2} t_{\hat{b}}$) converges in distribution to functionals of Wiener processes. In other words, the limiting distribution of the t-statistic for the hypothesis $b = 0$ diverges as the sample size tends to infinity. Thus, even though the LS estimator of b converges to zero in probability, the t-statistic for the null hypothesis of $b = 0$ will be significant for large samples if one uses conventional asymptotic critical values. Moreover, the frequency of rejections will increase with the sample size.

It should be stressed that the case investigated in this paper differs from the case of a spurious regression examined by Phillips (1986). Phillips showed that the coefficient of the I(1) regressor converges in probability to Brownian Motion. In the present case, we show that the coefficient converges in probability to zero even though the limiting distribution of the t-statistic test for $b = 0$ is divergent in both cases. The spurious regression problem does not arise in our model because there is an LDV in the regression which serves to offset the nonstationary component of the stochastic process generating the dependent variable. Thus regression (6) can be loosely viewed as regressing the stationary component of the dependent variable on an I(1) regressor. Interestingly, the fact that the present model is not a spurious regression implies that a regression with I(1) variables does not necessarily generate I(1) residuals, even though they fail to be cointegrated; rather, this result depends on whether there is an LDV in the regression.

Although the analytical results in this section are obtained under the assumption of i.i.d. errors, Wirjanto (1992) uses a multivariate generalization of Corollary 1 in Herrndorf (1984a) to show that relaxing this assumption to allow the disturbances to be weakly dependent and heterogeneously distributed does not change the results in Lemmas 1 and 2 and Theorems 1 and 2. The framework of investigation for this general case is similar to that of Phillips (1986, 1987, 1989) and is based on the earlier works of Billingsley (1968), Herrndorf (1983, 1984a, 1984b), McLeish (1975a, 1975b, 1977) and Pollard (1984).

2.2 The case of cointegration

In this subsection we examine the stock-adjustment model when the nonstationary data are cointegrated. Consider the DGP given by (4) and (5) and suppose that the model to be estimated is given by

$$y_t = ay_{t-1} + bx_t + v_t \qquad (t=1, 2, \ldots, T) \tag{7}$$

where x_t and y_t are cointegrated. That is, there is no misspecification regarding the choice of the regressor (x_t) in equation (7). For simplicity the regression error term is assumed to be i.i.d$(0,\sigma_v^2)$.

It is straightforward to demonstrate that the LS estimates of the coefficients of equation (7), although consistent, will have non-standard limiting distributions which make statistical inference difficult to conduct. It is thus desirable to find a transformation of the data that yields I(0) variables and allows us to perform inference on the coefficients of the transformed variables using standard distributional theory. One such procedure is proposed by Wickens and Breusch (1988), which allows us to estimate both the short-run and long-run dynamics in the same model. The Wickens-Breusch procedure uses the same idea as that proposed by Bewley (1979) by transforming equation (7) into a form where the only variables in levels are those in the cointegrating regression, and all other variables are in first differences. The resulting equation can then be estimated using instrumental variables (IV), and inference may proceed using standard asymptotic theory.

The Wickens-Breusch transformation of equation (7) results in

$$y_t = g_1 \Delta y_{t-1} + g_2 x_t + n_t \tag{8}$$

where $g_1 = -a(1-a)^{-1}$, $g_2 = b(1-a)^{-1}$ and $n_t = (1-a)^{-1} v_t$. The IV estimate of g from equation (8) using the instrument set $z_t = \left[x_t\, y_{t-1}\right]$ is given by

$$\begin{bmatrix} \tilde{g}_1 - g_1 \\ \tilde{g}_2 - g_2 \end{bmatrix} = \begin{bmatrix} \sum y_{t-1} \Delta y_t & \sum x_t y_{t-1} \\ \sum x_t \Delta y_t & \sum x_t^2 \end{bmatrix}^{-1} \begin{bmatrix} \sum y_{t-1} y_t \\ \sum x_t y_t \end{bmatrix}. \tag{9}$$

The resulting estimate is identical to the LS estimate $\hat{g}^T = \left[\hat{g}_1\ \hat{g}_2\right]$, where $\hat{g}_1 = -\hat{a}(1-\hat{a})^{-1}$ and $\hat{g}_2 = \hat{b}(1-\hat{a})^{-1}$, and $\hat{a}$ and $\hat{b}$ are the LS estimates of a and b from equation (7).

To formally derive the limiting distribution of the Wickens-Breusch IV estimates, we use the following lemma.

LEMMA 3. Let $M(r)$ be a Wiener process independent of $W(r)$ on the function space $C[0,1]$. As $T \to \infty$ we have

(i) $T^{-1/2} \sum_{t=1}^{[Tr]} d_t \Rightarrow \sigma_d^2 W(r)$

(ii) $T^{-1/2} \sum_{t=1}^{[Tr]} v_t \Rightarrow \sigma_v^2 M(r)$

where [Tr] is the integer part of Tr. Note that equation (7) can be expressed as $y_t = (1-aL)^{-1}(bx_t + v_t)$; then as $T \to \infty$,

(a) $T^{-2} \sum x_t^2 \Rightarrow \sigma_d^2 \int W(r)^2 dr$

(b) $T^{-2} \sum x_t y_{t-1} = T^{-2} \sum x_t (1-aL)^{-1}(bx_t + v_t)$

$$\Rightarrow g_2 \sigma_d^2 \int W(r)^2 dr$$

(c) $T^{-1}\sum x_t v_t \Rightarrow \sigma_v \sigma_d \int W(r)\, dM(r)$

(d) $T^{-1}\sum y_{t-1} v_t = T^{-1}\sum (1-aL)^{-1}(bx_{t-1}+v_{t-1})\, v_t$

$$\Rightarrow g_2 \sigma_v \sigma_d \int W(r)\, dM(r)$$

(e) $T^{-1}\sum x_t \Delta y_t = T^{-1}\sum x_t (1-aL)^{-1}(b\Delta x_{t-1}+\Delta v_{t-1})$

$$\Rightarrow g_2 \sigma_d^2 \int [W(r)\, dW(r) + 1]$$

(f) $T^{-1}\sum y_{t-1}\Delta y_t = T^{-1}\sum (1-aL)^{-2}(bx_{t-1}+v_{t-1})\,(b\Delta x_{t-1}+\Delta v_{t-1})$

$$\Rightarrow g_2^2 \sigma_d^2 \int W(r)\, dW(r) + (1-a)^{-2}\sigma_d^2$$

THEOREM 3. Suppose that equation (8) is estimated by LS and the conditions of Lemma 3 are satisfied, then as $T \to \infty$

(i) $T^{1/2}(\tilde{g}_1 - g_1) \Rightarrow \{g_2 \sigma_v \sigma_d \int W(r)\, dM(r)\} /$

$$\{(1-a)\, g_2^2 \sigma_d^2 \int W(r)\, dW(r) + [\sigma_v^2 (1-a)^{-2}]\} \tag{10}$$

(ii) $$T(\tilde{g}_2 - g_2) \Rightarrow \{\sigma_v \sigma_d \int W(r)\, dM(r)\} / \{(1-a)\, \sigma_d^2 \int W(r)^2 dr\} \tag{11}$$

PROOF: See Appendix.

Equations (10) and (11) show that the IV estimator $\tilde{g}_1$ is asymptotically normal, while the IV estimator $\tilde{g}_2$ is asymptotically a mixture of normal distributions. This implies that inference about the parameters g_1 and g_2 may be carried out using standard distribution theory. Following the steps in Wirjanto (1992), the previous results can be shown to hold when the i.i.d. errors are replaced by weakly dependent and heterogeneously distributed errors.

3 EMPIRICAL EXAMPLES AND SIMULATION RESULTS

3.1 The case of no cointegration

As an example of the no cointegration case, we consider a conventional stock-adjustment model of money demand, which may be written as

$$m_t = k + b_1 y_t + b_2 i_t + a m_{t-1} + v_t \tag{12}$$

where $a = (1 - \lambda)$ and λ is the adjustment coefficient from (2), m_t represents money, y_t is a measure of income and i_t is an opportunity cost measure.

To estimate (12), we use seasonally adjusted quarterly Canadian time series data over the 1965Q1 to 1992Q1 sample period. Specifically, the variable m_t is the real value of M1 (currency plus demand deposits), y_t is real gross domestic product and i_t is the 90-day Treasury bill rate.[4] All data are retrieved from CANSIM and, except for the interest rate measure, are expressed in logarithmic form.

Prior to estimation of equation (12), the properties of each series are examined using the parametric augmented Dickey and Fuller test as suggested by Dickey and Fuller (1979) and Said and Dickey (1984), and the nonparametric test proposed by Phillips and Perron (1988).[5] The results are reported in Table 1 (p. 25). The tests fail to provide evidence for rejecting the unit-root null even at the 10 per cent level of significance, suggesting that the data are nonstationary.

Since each variable appears to be an I(1) process, we attempt to determine whether these variables are cointegrated using the two-step approach proposed by Granger (1983) and later refined by Engle and Granger (1987). The test regressions include a constant, and a constant and a trend term. If we find cointegration in the demeaned specification, this corresponds to "deterministic cointegration," which implies that the same cointegrating vector eliminates deterministic trends as well as stochastic trends. But if the linear stationary combinations of the I(1) variables have non-zero linear trends, this corresponds to "stochastic cointegration."[6] To test the null hypothesis of no cointegration, we employ the parametric augmented Dickey and Fuller test suggested by Engle and Granger (1987) and a nonparametric test proposed by Phillips and

4. The GDP price deflator is used to deflate M1 and GDP.
5. A time trend is included in the augmented Dickey-Fuller regressions to make the distribution of the test statistic free of the unknown intercept term.
6. See Ogaki and Park (1989) for a discussion of stochastic and deterministic cointegration.

Ouliaris (1990). The results presented in Table 2 (p. 25) show no evidence of cointegration even at the 10 per cent level of significance.

Some may argue that our inability to reject the null hypothesis of no cointegration may in part reflect the low power of standard unit-root tests against persistent alternatives. Thus we also apply a residual-based test recently proposed by Shin (1992), which has cointegration as its null hypothesis. One difficulty with this test is that the residuals must come from a test regression that admits parameter estimates that are efficient as well as consistent. Although the parameter estimates from the Engle-Granger static test regression are super-consistent, they are not efficient. In order to get efficient estimates, we follow the suggestion of Stock and Watson (1992) and add leads and lags of the first differences of the regressors to the Engle-Granger test regression.[7] The results, reported in Table 3 (p. 25), corroborate our previous conclusion by rejecting the null of cointegration in favour of the no cointegration alternative. Hence, there is strong evidence against cointegration between real M1, real output and nominal short-term interest rates.[8]

Given the pretesting results, we proceed to estimate equation (12) by LS. From the results in Section 2.1, we expect the parameter estimate corresponding to the lagged dependent variable to be close to unity and the parameter estimates of the independent variables to be close to zero but highly significant. The estimation results presented in Table 4 (p. 26) confirm our priors. The estimated coefficient for the LDV is 0.954, while the parameter estimates of the independent variables are close to zero and highly significant. A striking feature of these results is the fact that the adjustment parameter (λ) is only 0.046, which implies that only 17.2 per cent of the adjustment towards the desired level is completed within one year.

Since the analytical results established in Section 2.1 are asymptotic, the question arises whether they can provide a useful guide in explaining the empirical results obtained in finite

7. We chose the number the leads and lags to equal $INT(T^{1/3})$ or 4 since this is consistent with the simulation results in Stock and Watson (1992). The conclusions are not sensitive to this choice.

8. Amano and van Norden (1992), using Monte Carlo experiments, find that a joint unit-root - stationary testing procedure allows researchers to be more confident about their conclusions when both tests indicate that the data are stationary.

samples, such as the foregoing results for the money demand equation. To investigate this issue, a set of simple Monte Carlo experiments is conducted. The simulated data are generated by DGPs of the form

$$y_t = k + y_{t-1} + f_t \tag{13}$$

$$z_t = k + z_{t-1} + e_t \tag{14}$$

where $k = 1$, f_t is i.i.d$(0, \sigma_f^2)$ and e_t is i.i.d$(0, \sigma_e^2)$. In the simulation, f_t and e_t are drawn from an independent N(0, 0.5) population, such that $\mathrm{cov}(f_t, e_t) = 0$. Thus the variates y_t and z_t are each I(1) but are not cointegrated by design. The stock-adjustment model

$$y_t = k + ay_{t-1} + bz_t + v_t \tag{15}$$

is estimated by LS. For each experiment we perform 5,000 replications with the sample size T set equal to 25, 50, 100, 200 and 500 and record the parameter values at selected percentiles and the rejection frequency of z_t using standard asymptotic critical values.[9]

The results are presented in Tables 5 and 6 (p. 26) and 7 (p. 27). Tables 5 and 6 report the parameter estimates for the LDV and the independent variable at selected percentiles, respectively. As one would expect, given the asymptotic results established earlier, as the number of observations increases, the estimated coefficient on the LDV appears to be approaching unity, while that on the independent variable approaches zero. The results in Table 7 provide evidence supporting our conjecture that the rejection frequency of the null hypothesis of $b = 0$ should increase with the sample size. It is apparent that if we use standard normal asymptotic critical values, we would reject the null far too often. For example, at the 10 per cent level the number of rejections begins at 41.4 per cent when $T = 25$ and increases monotonically to 46.0 per cent when $T = 500$. These false rejections help explain the empirical results we obtained earlier; that is, even though the coefficients of real output and the interest rate are close to zero, their t-statistic suggests

9. We begin data generation at T = -50 and discard the first 51 observations to minimize any problems associated with starting values. The simulations were performed on a Sun SPARCstation using RATS version 3.1.

that they are statistically significant. Hence, it appears that the asymptotic results established in Section 2.1 are useful approximations in finite samples.

It is instructive to compare the above results to the results reported by Granger and Newbold (1974) for the case of a spurious regression. In their experiment the DGPs are given by (13), (14) and (15), with the coefficient on the LDV set equal to zero; that is, with complete adjustment within each period. Using 1,000 replications and the 5 per cent normal asymptotic critical value, Granger and Newbold report that the t-statistic for $b = 0$ rejects the null hypothesis falsely at a rate of 75 per cent when $T = 50$. In contrast, our result for $T = 50$ suggests that the rejection rate is only 30.4 per cent, which is considerably lower than that of a spurious regression. However, it is still much larger the 5 per cent level, suggesting that the t-statistic is not a valid test for the I(1) variable in regression (15).

3.1 The case of cointegration

As an example of the cointegration case, we consider the U.S. aggregate consumption equation. Specifically, the log of U.S. real consumption of nondurables and services per capita (c_t) and the log of U.S. real disposable income per capita (yd_t) are tested for cointegration over the 1948Q1 to 1991Q3 sample period. The data, obtained from Data Resources Inc., are seasonally adjusted.

We start by testing the time-series properties of the data. The results, reported in Table 8 (p. 28), suggest that both series are nonstationary. Next we determine whether our measures of consumption and real disposable income are cointegrated. Using the simple Engle and Granger (1987) two-step approach, we find significant evidence of cointegration at the 5 per cent level (as shown in Table 9 (p. 27)).[10] Since we are able to reject the null of no cointegration using tests with weak power, we do not apply the test with the cointegration null hypothesis. The results of estimating the static cointegration regression

10. Since we are able to reject the null hypothesis of no cointegration, we do not apply the test with the null of cointegration.

$$c_t = \alpha + \beta yd_t + u_t \tag{16}$$

are presented in the upper panel of Table 10 (p. 28).

Since the variables c_t and yd_t are found to be cointegrated, as explained in Section 2.2, the Wickens-Breusch transformation is applied to the stock-adjustment model for aggregate consumption,

$$c_t = k + ac_{t-1} + byd_t + v_t, \tag{17}$$

to yield the following equation:

$$c_t = g_0 + g_1 \Delta c_t + g_2 yd_t + n_t. \tag{18}$$

This equation can be estimated using IV, using the instrument set $z_t = \left[\mu\ c_{t-1}\ yd_t\right]$, where μ is a vector of ones. In the present context, the coefficient of interest g_2 is the long-run elasticity of income; in other words, the IV estimate of g_2 in equation (18) provides us with an alternative estimate of the cointegrating vector β in (16). The results from estimating equation (18) are given in the lower panel of Table 10. The IV estimate of the long-run elasticity of income is 0.865, which is smaller than the LS estimate (0.896) from equation (16). Given the IV estimates of g_1 and g_2, the coefficient on the LDV is 0.870, while that on the income variable is 0.113.

The Monte Carlo results in Banerjee, Dolado, Hendry and Smith (1986) suggest that the LS estimate of β in equation (16) has substantial finite sample bias because the equation ignores the short-run dynamics. The more correlated yd_t and u_t are in equation (16), the greater the bias. This will occur when the coefficient of the LDV in equation (17) is close to unity. The IV estimate of g_2, on the other hand, is likely to have better finite sample properties than the LS estimate of β, because it estimates both the short-run and long-run dynamics in the same equation. Moreover, the least square estimate of β, although $O_p(T^{-1})$, has a non-standard limiting distribution. In contrast, the IV estimate of g_2 is also $O_p(T^{-1})$ and has a mixture of normal limiting distribution that permits conventional asymptotic theory to be used for inferential purposes.

In order to investigate whether the IV estimator has better finite sample properties than the simple LS estimator, we again use Monte Carlo simulations. The DGP of the variates is

$$y_t = k + a y_{t-1} + (1-a) x_t + u_t \tag{19}$$

and

$$\Delta x_t = \rho \Delta x_{t-1} + e_t \tag{20}$$

where $k = 1$, u_t is i.id$(0,\sigma_u^2)$ and e_t is i.id$(0,\sigma_e^2)$. The simulated data are used in a simple static equation estimated by LS and then in a Wickens and Breusch transformed equation estimated by IV. The parameter ρ is set equal to 0.4. We consider various values of a (0.95, 0.75, 0.50 and 0.25), sample sizes (25, 50, 100, 200 and 500) and $s = \sigma_e/\sigma_u$ (5.0, 1.0 and 0.2) to determine the robustness of the results. For each experiment, 5,000 replications are performed and the bias of the LS and IV estimators are calculated and compared.

The simulation results, presented in Table 11 (p. 28), can easily be summarized.[11] As expected, the bias increases with the coefficient on the LDV and decreases with the number of observations. When we compare the two estimators, we find that the IV estimator dominates the LS estimator with regard to parameter estimation bias over the sample sizes and parameter space we consider. The reduction in bias is most notable for large samples and large LDV coefficients. For example, when $T = 100$ and the coefficient on the LDV is set equal to 0.95, the parameter bias corresponding to the LS estimator is about -0.458, whereas the bias corresponding to the IV estimator is only -0.083. These results support our conjecture that the IV estimator will have better finite sample behaviour than the LS estimator, especially when the LDV coefficient is close to unity.

11. We only report simulation results for $s = 0.2$, as the conclusions do not change for the other values of s. The omitted results are available from the authors upon request.

4 CONCLUSIONS

The aim of this paper has been to study the implications of nonstationarity for the stock-adjustment model. We found that in a non-cointegrated model, the estimate of the adjustment coefficient will tend to zero, independent of its true value. Moreover, the LS estimate of the I(1) regressor will tend to zero, but the t-statistic for its insignificance will reject the null if conventional asymptotic critical values are used. This then provides *one* explanation for the typically observed low estimate of the adjustment coefficient in stock-adjustment models.

In the case of cointegration, we showed that IV estimation can be used to estimate the transformed model and that statistical inference can proceed using standard distributional theory. We also demonstrated that the IV estimator can reduce parameter estimation bias relative to the LS estimator.

There are two important implications emerging from this study: (i) the empirical estimates from a stock-adjustment model do not provide an economic measure of the adjustment coefficient if the I(1) variables in the model are not cointegrated; and as a result, (ii) a pre-testing for cointegration between the I(1) variables in stock-adjustment models should be carried out prior to estimation.

APPENDIX: THEOREM PROOFS

Proofs of Theorems 1 and 2. Again, for notational convenience we denote $\sum_{t=1}^{T}$ as $\sum$ and $\int_0^1$ as $\int$.

1. Some results for Lemma 1

$$T^{-2}\sum y_{t-1}^2 = T^{-2}\sum(\beta x_{t-1}+u_{t-1})^2$$

$$= T^{-2}\sum(\beta^2 x_{t-1}^2 + 2\beta x_{t-1}u_{t-1} + u_{t-1}^2)$$

$$= \beta^2 T^{-2}\sum x_{t-1}^2 + 2\beta T^{-2}\sum x_{t-1}u_{t-1} + T^{-2}\sum u_{t-1}^2$$

$$T^{-2}\sum y_{t-1}x_t = T^{-2}\sum(\beta x_{t-1}+u_{t-1})x_t$$

$$= T^{-2}\sum(\beta x_{t-1}x_t + u_{t-1}x_t)$$

$$=\beta T^{-2}\sum x_{t-1}x_t + T^{-2}\sum x_t u_{t-1}$$

$$T^{-2}\sum y_{t-1}z_t = T^{-2}\sum(\beta x_{t-1}+u_{t-1})z_t$$

$$= T^{-2}\sum(\beta x_{t-1}z_t + u_{t-1}z_t)$$

$$= \beta T^{-2}\sum x_{t-1}z_t + T^{-2}\sum z_t u_{t-1}$$

2. Proof of Theorem 1

The least squares estimates of a and b are given by

$$\begin{bmatrix}\hat{a}\\ \hat{b}\end{bmatrix} = \begin{bmatrix}\sum y_{t-1}^2 & \sum y_{t-1}z_t\\ \sum y_{t-1}z_t & \sum z_t^2\end{bmatrix}^{-1}\begin{bmatrix}\sum y_{t-1}y_t\\ \sum z_t y_t\end{bmatrix}$$

$$= \frac{1}{D}\begin{bmatrix}\sum z_t^2 & \sum y_{t-1}z_t\\ \sum y_{t-1}z_t & \sum y_{t-1}^2\end{bmatrix}\begin{bmatrix}\sum y_{t-1}(\beta x_t+u_t)\\ \sum z_t(\beta x_t+u_t)\end{bmatrix} = \begin{bmatrix}F_1/D\\ F_2/D\end{bmatrix} \tag{A.1}$$

where

$$D = \sum y_{t-1}^2\sum z_t^2 - (\sum y_{t-1}z_t)^2$$

$$F_1 = \sum z_t^2\sum y_{t-1}(\beta x_t+u_t) - \sum y_{t-1}z_t\sum z_t(\beta x_t+u_t)$$

$$= \beta\sum z_t^2\sum y_{t-1}x_t + \sum z_t^2\sum y_{t-1}u_t - \beta\sum y_{t-1}z_t\sum z_t x_t - \sum y_{t-1}z_t\sum z_t u_t$$

$$F_2 = -\sum y_{t-1}z_t\sum y_{t-1}(\beta x_t+u_t) + \sum y_{t-1}^2\sum z_t(\beta x_t+u_t)$$

$$= -\beta\sum y_{t-1}z_t\sum y_{t-1}x_t - \sum y_{t-1}z_t\sum y_{t-1}u_t + \beta\sum y_{t-1}^2\sum z_t x_t + \sum y_{t-1}^2\sum z_t u_t$$

Then, as $T \to \infty$,

(i) $T^{-1/2}\sum_{t=1}^{[Tr]} d_t \Rightarrow \sigma_d^2 W(r)$

(ii) $T^{-1/2}\sum_{t=1}^{[Tr]} e_t \Rightarrow \sigma_e^2 V(r)$

where [Tr] is the integer part of Tr. Then we obtain the convergence results in Lemma 1, reproduced below as

(a) $T^{-2}\sum x_t^2 \Rightarrow \sigma_d^2\int W(r)^2 dr = B_{xx}$

(b) $T^{-2}\sum z_t^2 \Rightarrow \sigma_e^2\int V(r)^2 dr = B_{zz}$

(c) $T^{-2}\sum z_t x_t \Rightarrow \sigma_e\sigma_d\int V(r)\,W(r)\,dr = B_{zx}$

(d) $T^{-2}\sum y_{t-1}u_t \Rightarrow 0$

(e) $T^{-2}\sum x_t u_t \Rightarrow 0$

(f) $T^{-2}\sum z_t u_t \Rightarrow 0$

(g) $T^{-2}\sum y_{t-1}^2 \Rightarrow \beta^2\sigma_d^2\int W(r)^2 dr = \beta^2 B_{xx}$

(h) $T^{-2}\sum y_{t-1}x_t \Rightarrow \beta\sigma_d^2\int W(r)^2 dr = \beta B_{xx}$

(i) $T^{-2}\sum y_{t-1}z_t \Rightarrow \beta\sigma_d\sigma_e\int V(r)\,W(r)\,dr = \beta B_{xz}$

Hence as $T \rightarrow \infty$ we obtain the following results:

$$T^{-4}D \Rightarrow \beta^2 B_{xx}B_{xz} - (\beta B_{xz})^2 = \beta^2 (B_{xx}B_{xz} - B_{xz}^2) \tag{A.2}$$

$$T^{-4}F_1 \Rightarrow \beta B_{zz}\beta B_{xx} - \beta(\beta B_{xz}B_{xz}) = \beta^2 (B_{zz}B_{xx} - B_{xz}^2) \tag{A.3}$$

$$T^{-4}F_2 \Rightarrow -\beta(\beta B_{xz}\beta B_{xx}) + \beta(\beta^2 B_{xx}B_{xz}) = \beta^3(-B_{xx}B_{xz} + B_{xx}B_{xz}) = 0 \tag{A.4}$$

which implies that

$$\hat{a} = (T^{-4}F_1/T^{-4}D) \Rightarrow 1 \tag{A.5}$$

$$\hat{b} = (T^{-4}F_2/T^{-4}D) \Rightarrow 0 \tag{A.6}$$

Q.E.D.

3. Proof of Theorem 2

To prove Theorem 2, equation (12) is reproduced as

$$\begin{aligned} y_t &= ay_{t-1} + bz_t + v_t \\ &= \begin{bmatrix} y_{t-1} & z_t \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} + v_t \\ y &= W\alpha + v \end{aligned} \tag{A.7}$$

Then the LS estimate of α is

$$\hat{\alpha} = (W^T W)^{-1} W^T y \tag{A.8}$$

which can be expressed as

$$\hat{\alpha} - \alpha = (W^T W)^{-1} W^T v. \tag{A.9}$$

Let H be a (2x2) matrix of the form

$$H = \begin{bmatrix} T & 0 \\ 0 & T \end{bmatrix}. \tag{A.10}$$

Then (A.9) can be rewritten as

$$\begin{aligned} H(\hat{\alpha} - \alpha) &= (H^{-1} W^T W H^{-1})^{-1} H^{-1} W^T v \\ &= \begin{bmatrix} T^{-2}\sum y_{t-1}^2 & T^{-2}\sum y_{t-1} z_t \\ T^{-2}\sum y_{t-1} z_t & T^{-2}\sum z_t^2 \end{bmatrix}^{-1} \begin{bmatrix} T\sum y_{t-1} v_t \\ T\sum z_t v_t \end{bmatrix} \\ &= \frac{1}{D^*} \begin{bmatrix} T^{-2}\sum z_t^2 & -T^{-2}\sum y_{t-1} z_t \\ -T^{-2}\sum y_{t-1} z_t & T^{-2}\sum y_{t-1}^2 \end{bmatrix} \begin{bmatrix} T^{-1}\sum y_{t-1} v_t \\ T^{-1}\sum z_t v_t \end{bmatrix} = \frac{1}{D^*} \begin{bmatrix} F_1^* \\ F_2^* \end{bmatrix} \end{aligned} \tag{A.11}$$

where

$$D^* = T^{-2}\sum y_{t-1}^2 \; T^{-2}\sum z_t^2 - (T^{-2}\sum y_{t-1} z_t)^2$$

$$F_1^* = T^{-2}\sum z_t^2 \; T^{-1}\sum y_{t-1} v_t - T^{-2}\sum y_{t-1} z_t \; T^{-1}\sum z_t v_t$$

$$F_2^* = T^{-2}\sum y_{t-1} z_t \; T^{-1}\sum y_{t-1} v_t + T^{-2}\sum y_{t-1}^2 \; T^{-1}\sum z_t v_t.$$

Since as $T \rightarrow \infty$,

(i) $T^{-1/2} \sum_{t=1}^{[Tr]} e_t \Rightarrow \sigma_e^2 V(r)$

(ii) $T \sum_{t=1}^{[Tr]} f_t \Rightarrow \sigma_f^2 P(r)$

we obtain the convergence results in Lemma 2, which are reproduced as

(a) $T^{-2}\sum y_{t-1}^2 \Rightarrow \sigma_f^2 \int P(r)^2 dr = B_{yy}$

(b) $T^{-2}\sum y_{t-1} z_t \Rightarrow \sigma_f \sigma_e \int P(r)\, V(r)\, dr = B_{yz}$

(c) $T^{-1}\sum y_{t-1} v_t \Rightarrow (1/2)\, \sigma_f^2 [P(1)^2 - 1] = B_{yv}$

(d) $T^{-1}\sum z_t v_t \Rightarrow \sigma_f \sigma_e \int V(r)\, dP(r) = B_{zv}$

Thus as $T \to \infty$,

$$D^* \Rightarrow (B_{yy}B_{zz} - B_{yz}^2) \tag{A.12}$$

$$F_1^* \Rightarrow B_{zz}B_{yv} - B_{yz}B_{zv} \tag{A.13}$$

$$F_2^* \Rightarrow B_{yz}B_{yv} + B_{yy}B_{zv} \tag{A.14}$$

such that

$$T(\hat{a} - 1) = (F_1^*/D^*) \xrightarrow{d} \{B_{zz}B_{yv} - B_{yz}B_{zv}\} / \{B_{yy}B_{zz} - B_{yz}^2\} \tag{A.15}$$

$$T(\hat{b} - 0) = (F_2^*/D^*) \xrightarrow{d} \{-B_{yz}B_{yv} + B_{yy}B_{zv}\} / \{B_{yy}B_{zz} - B_{yz}^2\} \tag{A.16}$$

Q.E.D.

Proof of Theorem 3

To proceed with the proof, substitute the definition of y_t into equation (7) and premultiply by the matrix Diag $[T^{-1/2}, T]$ yields

$$\begin{bmatrix} T^{1/2}(\tilde{g}_1 - g_1) \\ T(\tilde{g}_2 - g_2) \end{bmatrix} = (1-a)^{-1} \begin{bmatrix} T^{-1}\sum y_{t-1}\Delta y_t & T^{-3/2}\sum xy \\ T^{-3/2}\sum x_t \Delta y_t & T^{-1}\sum x_t^2 \end{bmatrix}^{-1} \begin{bmatrix} T^{-1/2}\sum y_{t-1} v_t \\ T^{-1}\sum x_t v_t \end{bmatrix}$$

$$= (1-a)^{-1} \begin{bmatrix} F_1^+/D^+ \\ F_2^+/D^+ \end{bmatrix} \tag{A.17}$$

where

$$D^+ = T^{-1}\sum y_{t-1}\Delta y_t\, T^{-2}\sum x_t^2 - T^{-3/2}\sum x_t y_{t-1}\, T^{-3/2}\sum x_t \Delta y_t$$

$$F_1^+ = T^{-2}\sum x_t^2 \; T^{-1/2}\sum y_{t-1}v_t - T^{-3/2}\sum x_t y_{t-1} \; T^{-1}\sum x_t v_t$$

$$F_2^+ = -T^{-3/2}\sum x_t \Delta y_t \; T^{-1/2}\sum y_{t-1}v_t + T^{-1}\sum y_{t-1}\Delta y_t \; T^{-1}\sum x_t v_t$$

Using the convergence results (a) to (f) from Lemma 3 yields, as $T \rightarrow \infty$

$$D^+ \Rightarrow [g_2^2\sigma_d^2\int WrdWr + (1-a)^{-2}\sigma_v^2]\,\sigma_d^2\int W(r)^2 dr \tag{A.18}$$

$$F_1^+ \Rightarrow \sigma_d^2\int W(r)^2 dr g_2\sigma_v\sigma_d\int W(r)\,dM(r) \tag{A.19}$$

$$F_2^+ \Rightarrow [g_2^2\sigma_d^2\int W(r)\,dW(r) + (1-a)^{-2}\sigma_v^2]\,\sigma_v\sigma_d\int W(r)\,dM(r) \tag{A.20}$$

and the intended results follow immediately.

Q.E.D.

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Table 1:
Tests for Unit Roots
Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) Tests
Sample: 1965Q1 to 1992Q2

Variables	ADF Lags[a]	ADF t-statistic	PP t-statistic[b]	10 Per Cent Critical Value[c]
Real M1	4	-1.980	-2.560	-3.156
Real GDP	1	-1.077	-2.349	-3.156
90 day T-bill Rate	1	-2.397	-2.617	-3.156

a. Henceforth, we use the data-dependent lag length selection procedure advocated by Hall (1989).
b. Unless otherwise specified, the long-run variance is estimated using a VAR prewhitening procedure suggested by Andrews and Monahan (1992).
c. Henceforth, unit-root and cointegration critical values are calculated using the response-surface estimates reported in Table 1 of MacKinnon (1991) for the actual sample size used in computing the test statistics.

Table 2:
Tests for the Null Hypothesis of No Cointegration for the Demand for Money Equation
Augmented Dickey-Fuller (ADF) and Phillips-Ouliaris (PO) Tests
Sample: 1965Q1 to 1992Q2

Regression	ADF Lags	ADF t-statistic	PO t-statistic	10 Per Cent Critical Value
Constant	4	-1.748	-1.882	-3.509
Constant & Trend	4	-1.425	-2.260	-3.922

Table 3:
Test for the Null Hypothesis of Cointegration for the Demand for Money Equation
Sample: 1965Q1 to 1992Q2

Regression	Truncation Parameter[a]	Test statistic[b]	10 Per Cent Critical Value
Constant	10	0.252	0.121
Constant & Trend	10	0.087	0.069

a. The truncation parameter is chosen according to $INT(T^{1/2})$. This rate is usually satisfactory under both the null and the alternative (see Andrews 1991).
b. Unlike the Phillips-Perron test, we use the Newey and West (1987) long-run variance estimator, as it can be shown that the test statistic for cointegration using a prewhitened kernel estimator with the plug-in bandwidth parameter is not consistent against the alternative of no cointegration.

Table 4:
OLS Estimation of the Stock-Adjustment Demand for Money Equation
Sample: 1965Q1 to 1992Q2

Variable	Parameter Estimate	Newey-West Standard Errors[a]	t-statistic
Constant	0.093	0.142	0.653
Real GDP	0.033	0.007	4.466
90 day T-bill Rate	-0.004	0.001	-7.813
LDV	0.954	0.018	51.867

a. We use Newey and West (1987) standard errors, as we found evidence of autocorrelation and autoregressive conditional heteroscedasticity in the residuals. The truncation parameter is set equal to the seasonal frequency (4).

Table 5:
Finite Sample Parameter Estimates of the Independent Variable

	Empirical Percentiles		
Sample	0.25	0.50	0.95
25	0.252	0.157	0.019
50	0.132	0.085	0.011
100	0.069	0.043	0.005
250	0.034	0.022	0.003
500	0.014	0.009	0.001

Table 6:
Finite Sample Parameter Estimates of the Lagged Dependent Variable

	Empirical Percentiles		
Sample	0.25	0.50	0.95
25	0.746	0.841	0.985
50	0.867	0.916	0.999
100	0.932	0.957	0.999
250	0.965	0.978	0.999
500	0.986	0.991	1.000

Table 7:
Percentage of Rejections of the Null Hypothesis ($b = 0$)

	Asymptotic Critical Values		
Sample	1.645	1.960	2.576
25	41.42	29.44	11.54
50	42.76	30.36	11.70
100	43.94	31.10	12.24
200	44.82	32.66	12.26
500	46.00	33.04	12.28

Table 8:
Tests for Unit Roots
Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) Tests
Sample: 1948Q1 to 1991Q3

Variables	ADF Lags	ADF t-statistic	PP t-statistic	10 Per Cent Critical Value
Consumption	1	-1.238	-2.382	-3.142
Disposable Income	4	-1.855	-2.354	-3.142

Table 9:
Tests for the Null Hypothesis of No Cointegration for the Consumption Equation
Augmented Dickey-Fuller (ADF) and Phillips-Ouliaris (PO) Tests
Sample: 1948Q1 to 1991Q3

Regression	ADF Lags	ADF t-statistic	PO t-statistic	5 Per Cent Critical Value
Constant	1	-4.056	-4.918	-3.070
Constant & Trend	1	-3.933	-4.809	-3.537

Table 10:
Estimation of the Consumption Equation
Sample: 1948Q2 to 1991Q3

Variable	Parameter Estimate	Newey-West Standard Errors[a]	t-statistic
Least Squares Estimation			
Constant	0.746	0.061	12.245
Disposable Income	0.896	0.007	136.428
Instrumental Variables Estimation			
Constant	1.065	0.236	4.514
Disposable Income	0.865	0.024	35.494
ΔConsumption	-6.717	3.211	-2.092

a. We use Newey and West (1987) standard errors because we found evidence of autocorrelated residuals for both regressions. The truncation parameter is set equal to the seasonal frequency (4).

Table 11:
Finite Sample Bias of the Long-run Parameter
Least Squares versus Instrumental Variables Estimation
$s = 0.2$

	Least Squares Estimates				Instrumental Variables Estimates			
$a \Rightarrow$	0.95	0.75	0.50	0.25	0.95	0.75	0.50	0.25
T=25	-0.768	-0.308	-0.107	-0.043	0.600	-0.217	-0.020	-0.012
T=50	-0.778	-0.202	-0.062	-0.018	0.429	-0.044	-0.008	-0.002
T=100	-0.458	-0.105	-0.035	-0.011	-0.083	-0.010	-0.005	-0.002
T=200	-0.339	-0.061	-0.014	-0.006	-0.037	-0.005	0.001	-0.001
T=500	-0.172	-0.025	-0.007	-0.003	-0.007	-0.002	-0.001	-0.001

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