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ABSTRACT

In an economy where both supply and demand shocks are at work, there will be considerable uncertainty associated with any estimates of potential output and the natural rate of unemployment. Laxton and Tetlow have shown that the detrending techniques that have been used in the past to provide estimates of excess demand conditions are likely to suffer from measurement errors that persist over time. This paper reports Monte Carlo evidence that suggests that such errors in measuring the extent of excess demand can result in significantly biased estimates of the coefficients of Phillips curves and incorrect inferences in hypothesis tests. In particular, it is shown that tests of whether or not the *change* in the level of excess demand contributes to the explanation of inflation via a Phillips curve are likely to lead to incorrect conclusions. For the type of Phillips curve considered in this paper, the estimated effect on inflation of the *level* of excess demand tends to be too small and the estimated effect of the *change* of excess demand tends to be too large. It is shown that researchers using standard techniques would often falsely conclude that the *change* term was necessary to explain the data, when in fact there was no such effect. It is noted that this is likely to lead to overestimates of the degree of hysteresis in output and hence of the output costs of reducing inflation. The Monte Carlo experiments are carried out using an unobserved components model of output, structured and calibrated to reflect Canadian data. The results appear to be robust to a variety of changes in the assumptions.

RÉSUMÉ

Dans une économie soumise à la fois à des chocs d'offre et de demande, les estimations de la production potentielle et du taux de chômage naturel comportent une forte dose d'incertitude. Laxton et Tetlow ont montré que les méthodes utilisées par le passé pour purger les données de leur tendance, et ce dans le but d'estimer la demande excédentaire, impliquent probablement des erreurs de mesure systématiques et persistantes. Les simulations de Monte Carlo dont fait état la présente étude indiquent que ces erreurs peuvent biaiser considérablement les estimations des coefficients des courbes de Phillips et mener à des inférences statistiques erronées. Par exemple, des tests d'hypothèses basés sur ces estimations pourraient conduire les chercheurs à attribuer indûment un rôle à la *variation* du niveau de la demande excédentaire. Compte tenu de la spécification de la courbe de Phillips retenue dans l'étude, l'effet estimatif du *niveau* de la demande excédentaire sur l'inflation tend à être trop faible, tandis que celui de la *variation* de la demande excédentaire tend à être trop élevé. L'étude fait ressortir que les chercheurs ayant recours aux méthodes habituelles seraient souvent amenés, à tort, à conférer à cette *variation* un rôle explicatif. Ils risquent ainsi de surestimer le degré d'hystérèse de la production et, par conséquent, la production à laquelle il faut renoncer pour réduire l'inflation. Les simulations de Monte Carlo portent sur un modèle qui formalise les composantes non observées de la production et dont la structure a été adaptée aux caractéristiques principales des données canadiennes. Les résultats semblent robustes même lorsqu'on modifie les hypothèses.

1. INTRODUCTION

Various researchers in Canada and the United States have observed that the *change* in the output gap or the unemployment gap seems to help explain variations in the inflation rate, contrary to the simplest versions of the price Phillips curve, which use only the level of the gap.[1] Robert Gordon has argued that the level-gap specification "obscures the fact that price change has been much more closely related to the contemporaneous rate of change of detrended output" and that the literature to that point "has shown no awareness of the importance of *the rate-of-change phenomenon* and has continued to specify equations based on the unadorned expectational Phillips curve." (Gordon, 1980, p. 243, emphasis added). Gordon has devoted considerable effort to justifying a specification that includes a change-in-the-gap term. On various occasions he has used hysteresis and production bottlenecks as reasons why such a term may be needed. At other times he has justified it simply on the observation of serially correlated errors from the simpler model.[2] Fortin (1990, 1991) has argued for a change-in-the-gap specification for Canadian data, appealing to a hysteresis argument.

There is considerable recent empirical evidence that the change in the gap does help improve the fit of a price Phillips curve. See, for example, Duguay (1984) and Cozier and Wilkinson (1990), as well as the work of Gordon and of Fortin noted above. Cozier and Wilkinson find that both the level and the change in the gap enter with statistically significant coefficients in a price Phillips curve, estimated using Canadian data, and that this specification is preferable to other specifications based on an $\overline{R}^2$ criterion. They reject Fortin's pure hysteresis restriction whereby only the change in the gap affects inflation.

An important potential econometric problem arises, however, because all these estimates are based on proxies for the main explanatory variables, the output gap and its change, which are not directly observable. Measurement errors may bias the estimates of parameters that are central to important policy conclusions, such as the size of the sacrifice ratio faced by the monetary authorities in reducing inflation. This brief note builds on the work of Laxton and Tetlow to offer evidence that

1. By "output gap" we mean the difference between the log of GDP and the log of potential GDP. The change in the gap, then, is also the gap between growth in potential and actual output. The unemployment gap would be the difference between the unemployment rate and the natural rate of unemployment.

2. Selected exampies of Gordon's work, from which these observations are drawn, are listed in the references.

the posited relationship between inflation and the change in the gap may be an illusion caused by errors in gauging potential output and the natural rate of unemployment. The evidence comes from a Monte Carlo exercise using an unobserved components model of output. Implicit in this model is the assumption that the economy is driven by both demand and supply shocks. We show that when standard approaches to measuring potential output are used, the resulting estimates of the parameters of the Phillips curve are substantially biased towards too low an estimated coefficient on the level gap and too high an estimated coefficient on the change in the gap. These results for price Phillips curves are robust over a variety of sensitivity checks. Since the natural rate of unemployment is also likely to be affected by a stochastic process that displays substantial persitence, these arguments will also apply to wage Phillips curves that are driven by unemployment gaps.

The model and the Monte Carlo results are described in section 2. In section 3, we offer some further thoughts on the specification, measurement and estimation issues involved and some concluding remarks on the implications of the results for current policy debates.

2. Stochastic Potential and Deterministic Trends

Consider an economy that generates a path for output from three sources: deterministic upward drift over time, a shock that raises (or lowers) actual and potential output simultaneously, and a shock that creates or changes the gap between actual and potential output. The former shock, which is often called a supply shock, is usually thought of as permanent, as it is in Blanchard and Quah (1989) and Dea and Ng (1990) for instance, but need not be so. All that is required is that it have high persistence. The latter shock, which is often called a demand shock, must be temporary (output gaps cannot persist indefinitely), but it can have quite persistent effects. Indeed, most empirical estimates based on Canadian data do suggest substantial persistence of output gaps. Finally, suppose that inflation is generated according to a price Phillips curve, i.e. the course of inflation is influenced by the output gap. The econometrician observes output but cannot observe directly whether the shock is a supply shock or a demand shock or some combination of the two. In other words, the econometrician cannot observe the true output gap. Moreover, the problem is not limited to levels. Since not all fluctuations in output imply fluctuations in the gap, the econometrician also cannot know how the gap has changed. To estimate the Phillips curve, the econometrician must provide a proxy for the gap and, at least implicitly, a proxy for potential output or the natural rate of unemployment.

This characterization of output is often called an unobserved components decomposition.[3] Following Laxton and Tetlow, we write down such a decomposition and calibrate it to Canadian data. In accordance with time-series evidence, equation (2) posits that potential output (Y_t^P) evolves according to a process with a unit root and drift.[4] The degree of drift is chosen to approximate historical Canadian data. The data also tell us that the cyclical (demand) portion of total output (Y_t^C) can be fairly well represented by the AR(2) process shown in equation (3).[5] We complete the model with a Phillips curve,[6] using parameter estimates taken from Cozier and Wilkinson (1990, Model 3, Table 1, p. 15). Note that we assume that the inflation path is influenced by the (lagged) level output gap but not its change.

$$Y_t = Y_t^P + Y_t^C \quad (1)$$

$$Y_t^P = Y_{t-1}^P + 0.010949 + \varepsilon_{P,t} \quad (2)$$

$$Y_t^C = 1.21599Y_{t-1}^C - 0.31306Y_{t-2}^C + \varepsilon_{C,t} \quad (3)$$

$$\Pi_t = 0.42\Pi_{t-1} + 0.27\Pi_{t-2} + 0.17\Pi_{t-3} + 0.14\Pi_{t-4} + 0.32Y_{t-1}^C + \varepsilon_{\Pi,t} \quad (4)$$

The data cannot provide direct measures of the variances of $\varepsilon_{C,t}$ and $\varepsilon_{P,t}$ since the two are not independently observable. For these values we turn to the literature on the stochastic properties of output. Christiano and Eichenbaum (1990) estimate the

3. See Blanchard and Fischer (1989, pp. 5-12) for a discussion of the decomposition of output into trends and cycles.
4. See Macklem (1991) where this formulation is tested and not rejected, using Canadian data.
5. This simple AR propagation mechanism can be thought of as a reduced-form approximation to a general, dynamic, linear, stochastic model of the economy. Higher order processes might be necessary, according to tests of whether the process is exhausting the exploitable correlations in the data, but the results of this paper depend only on the dominant root of the process, and the AR(2) representation is adequate for our purposes.
6. This model is sometimes referred to as the *accelerationist* model. We prefer Summers'(1988) characterization of this model as the *integral-gap* model. If the change in inflation is specified as a function of the gap, inflation today is a function of the sum or integral of all past gaps. Once the desired rate of inflation is attained, the authorities must ensure that the integral of all gaps from that time on tends to zero. Any shocks that create a cumulative excess demand gap must be offset by an equal policy-induced excess supply gap. As Sargent (1971) demonstrated, the natural-rate hypothesis model and the accelerationist hypothesis can be correct, without there being a unit-sum restriction on the coefficients on lagged inflation in the Phillips curve. This unit-root restriction imposes the extreme view that expectations of inflation are formed as if the monetary authorities do not maintain any control over the equilibrium level of inflation.

proportion of total output variation in the U.S. data that is attributable to permanent shocks, $\sigma^2_{\Delta YP}/(\sigma^2_{\Delta YP}+\sigma^2_{\Delta YC})$, at anywhere between 20 and 80 per cent. Cogley (1990) places his best guess for both Canada and the United States at about 40 per cent. Dea and Ng (1990) find a value of 80 per cent for Canada. We ran Monte Carlo tests using all three of these figures and found no qualitative differences in the results. Accordingly, we report detailed results only for the intermediate case of 40 per cent. This implies the following standard deviations for the error terms in equations (2) and (3): $\{\sigma_P, \sigma_C\} = \{0.006325, 0.007219\}$. Five hundred draws were taken from these distributions to generate hypothetical quarterly data for output over a sample of 33 years' duration (i.e., 132 quarters). Corresponding inflation data were generated using equation (4), with random disturbances drawn from a distribution with standard deviation 0.35 (derived from the Cozier and Wilkinson results and measured in percentage terms).

The problem here comes from inferring the output gap (Y_t^C) when only aggregate output (Y_t) is observable. In accordance with current common practice, we measure trend output using the Hodrick-Prescott (1980) filter. With these trend values as estimates of potential output, we compute the implied output gaps. We then estimate the extended Phillips curve -- i.e., including the change-in-the-gap term.[7] Do we tend to retrieve the true data-generating process for inflation from these regressions using estimated output gaps?

The results are reported in graphical form. Figures 1 and 2 show the histograms for the estimated coefficients on the level-gap term and on the change-in-gap term, respectively, for the case where we assume a conventional value (1600) for the smoothness parameter in the H-P filter. The histogram in Figure 1, which has a mean of 0.097 and a standard deviation of 0.035, is far from the true value of the level-gap coefficient, 0.32. Even the largest estimated coefficients in the 500 trials is over 35 per cent too small. Tests that the true value is 0.32 are generally falsely rejected, even at very high confidence levels. More importantly, the false hypothesis that the level coefficient is zero is not rejected at the 95 per cent confidence level in 22 per cent of the trials. Figure 2 shows the histogram for the estimated coefficient on the change in the gap. By construction, this variable does not belong in the model and the true coefficient is zero, but the estimated coefficient tends to be rel-

7. To avoid any confounding influences stemming from simultaneity problems, we ensure that both the level-gap and change-in-the-gap coefficients enter with a lag.

atively large (the mean of the estimates is 0.137). The estimated test statistic falsely rejects the zero restriction in 76 per cent of the trials.[8]

Figures 3 and 4 show the results when we use a very large smoothness parameter in the H-P filter, which amounts to replacing the curvilinear detrending of the standard H-P filter with a linear time trend. Not surprisingly, this makes matters worse; the histograms are even further away from the true values. Moreover, if we dramatically lower the smoothness parameter, so that the estimates of potential output are close to the actual output, the bias results do not improve, even if we assume that this is close to the truth by setting a high value for the relative importance (variance) of supply shocks. This seems to happen because estimates of the gap become relatively less reliable as actual and potential output become highly correlated, implying that the actual gap tends to stay close to zero.

We experimented with several other specifications for the data-generating process and with different proportions of total shocks generated from potential disturbances. When we replaced the unit root in potential output with a stationary but long-lived process, the results were unchanged. This should not be surprising since Christiano and Eichenbaum (1990) and others have noted that there is very little difference in the short-run properties of these two models. When the proportion of supply shocks is set at 20 per cent, the results are essentially the same. The mean of the estimated coefficients is 0.11 and the standard deviation is 0.04. The bias is reduced by only 0.01 (3 per cent). The same pattern holds for the 80 per cent case, but the bias is somewhat worse. We tried introducing some autocorrelation in the permanent shock, changing the AR process in the cyclical component to order 1 (retaining about the same persistence), and changing the pattern of coefficients on the lagged inflation terms in the Phillips curve. We also tried a quadratic-time-trend proxy for potential output in place of the H-P filter. In all cases, the results indicated either similar or worse bias. The bias is reduced, but not eliminated, if we lower substantially the degree of persistence in the effects of demand shocks. In sum, the negative bias in the estimated coefficient on the level of the gap and the spurious inclusion of the change in the gap with a strong positive coeffi-

8. If the correct model is estimated (that is the zero restriction is imposed on the change-in-the-gap term) using the measures of the output gap derived from the filter, the bias is reduced but not eliminated. The mean of the estimates of the level coefficient is 0.14 and the standard deviation of the estimates is 0.033. The bias is reduced by 14 per cent, but it is still substantial. Thus, even if the true structure is known, errors in measuring potential output with this filter appear to generate underestimates of the effect of excess demand on inflation.

cient appear to be robust results.[9]

3. Concluding Remarks

We have used a simple unobserved components model of output to demonstrate the possible effects of mismeasuring potential output on the estimation of Phillips curves. Our conclusion is that there can be biases that are quite large. When the true model is the pure Phillips curve (i.e., no change-in-the-gap term), but the excess demand gap is mismeasured, one will tend to conclude falsely that the change in the gap influences inflation and one will tend to underestimate the effect of the level of excess demand on inflation. In our experiments with univariate filters, we have been unable to find anything but such negative bias in the estimated coefficient of the level gap and positive bias in the estimated coefficient on the change in the gap. If the stochastic process that generates the natural rate of unemployment also contains substantial persistence, exactly the same problem would arise in wage Phillips curves.

Intuitively, the reason for the problem is as follows. Whether there is a unit root in potential output or just strong persistence, the use of an H-P trend (or any univariate filter we have tried) fails to pick up the propagation in supply shocks that truly exists in the data. The propagation in supply shocks is instead incorrectly attributed to a demand disturbance. Because the gap measure therefore tends to be too big in absolute value, the econometric estimator reduces the supposed size of the gap coefficient. Similarly, the estimator assigns a weight to the change in the gap because the first difference tends to eliminate some of the error in the gap measure itself. This spurious volatility in the measured output gap only disappears as the smoothness parameter is relaxed and the trend begins to mimic the actual path of output. However, in this case, the estimated output gap itself tends to degenerate and the consequences of missing the propagation of demand shocks become important.

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9. It is well known that one cannot generally sign the bias in an "errors-in-variables" problem like this one. However, for univariate filters to get measures of potential output and for this model calibrated to respect the historical data, we were unable to find any other pattern than the one reported here.The estimates we obtain from our base-case Monte Carlo experiment are qualitatively similar to those found in Canadian time-series data in that the estimated coefficient on the change in the gap is significantly bigger than the coefficient on the level of the gap. Moreover, the t-statistics on the two coefficients are in the two to two-and-a-half range obtained by Cozier and Wilkinson. This suggests that our decomposition is not unrepresentative of the data. Therefore, subject to the caveats above, it is reasonable to think that our bias results are more than illustrative.

Quite often, researchers are interested in the coefficients of the Phillips curve for what they tell us about the sacrifice ratio -- the output cost of reducing inflation. Cozier and Wilkinson (1990) show that the sacrifice ratio, in the extended model that includes the change in the gap, is a negative function of the coefficient on the level gap but does not depend on the coefficient on the change in the gap. That being the case, our results suggest that sacrifice ratios could be overstated when estimated from the class of linear, integral-gap Phillips curve models.[10] Moreover, arguments that there would be permanent output losses from an attempt to reduce inflation (i.e., hysteresis) based on such models, as in Fortin (1990, 1991), and tests of this idea, as in Cozier and Wilkinson (1990), are likely to err towards accepting the existence of hysteresis and towards overstating the costs of reducing inflation.[11] Our results suggest that this bias can be substantial.

None of this means that the change in the gap does not belong in the Phillips curve. What it does mean is that there is insufficient information in typical single-equation estimation results to infer whether more than the level of the output gap belongs in the model. All methods of estimating output gaps that we have considered yield biased estimates of the parameters in the Phillips curve and systematically incorrect inferences from hypothesis tests. We see no way to improve the results other than to use the information, if not the structure, of the rest of the macro model, in an attempt to provide more powerful identification of the parameters. One way to do this might be to extend the work of Ford and Rose (1989) to include a consistent estimate of equilibrium total factor productivity.

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10. Note that this result carries over to the case where the correct model is estimated. See footnote 8 for details.

11. It is important to stress that these results are particular to the class of models with linear Phillips curves. In work with non-linear Phillips curves, we have generated examples where the sacrifice ratio, for example, tends to be underestimated. Also, if the true model were such that output gaps enter the Phillips curve contemporaneously, the possibility exists that the simultaneity problem in the single-equation Phillips curve would bias the estimated coefficient on the change-in-the-gap variable in the opposite direction of the mismeasurement problem discussed here. This possibility is discussed by Duguay (1984).

Figure 1: Frequency distribution of coefficient on level of gap
(H-P smoothing parameter = 1600)

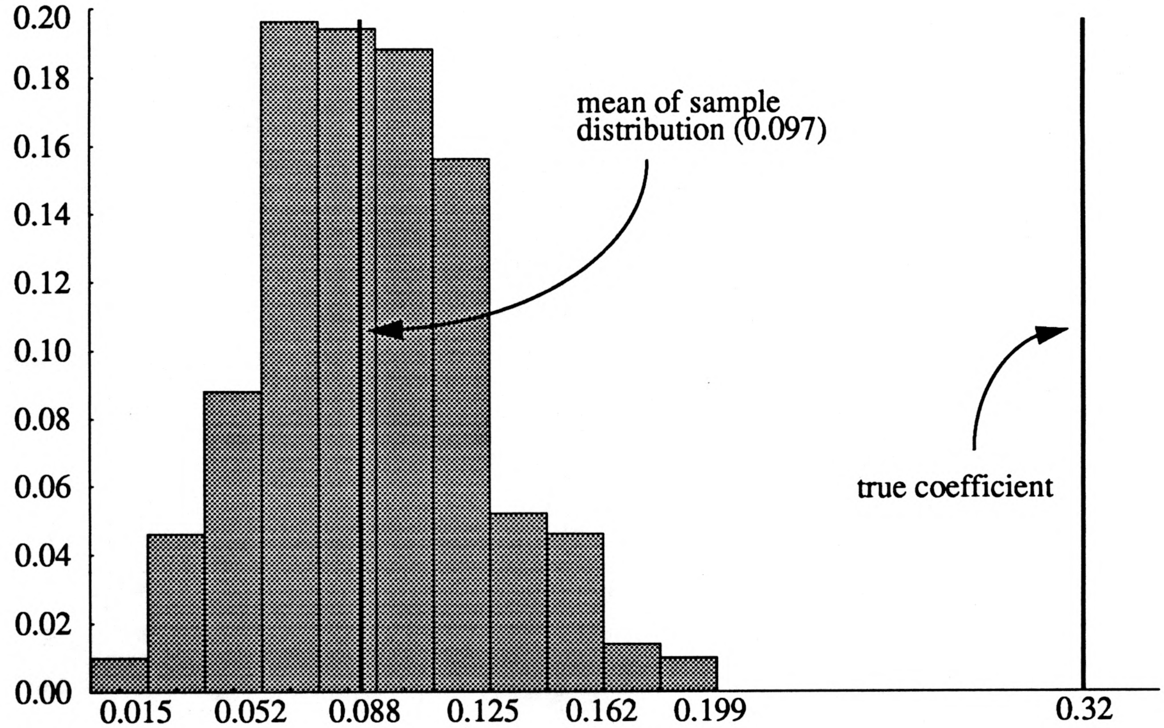


Figure 2: Frequency distribution of coefficient on change in gap
(H-P smoothing parameter = 1600)

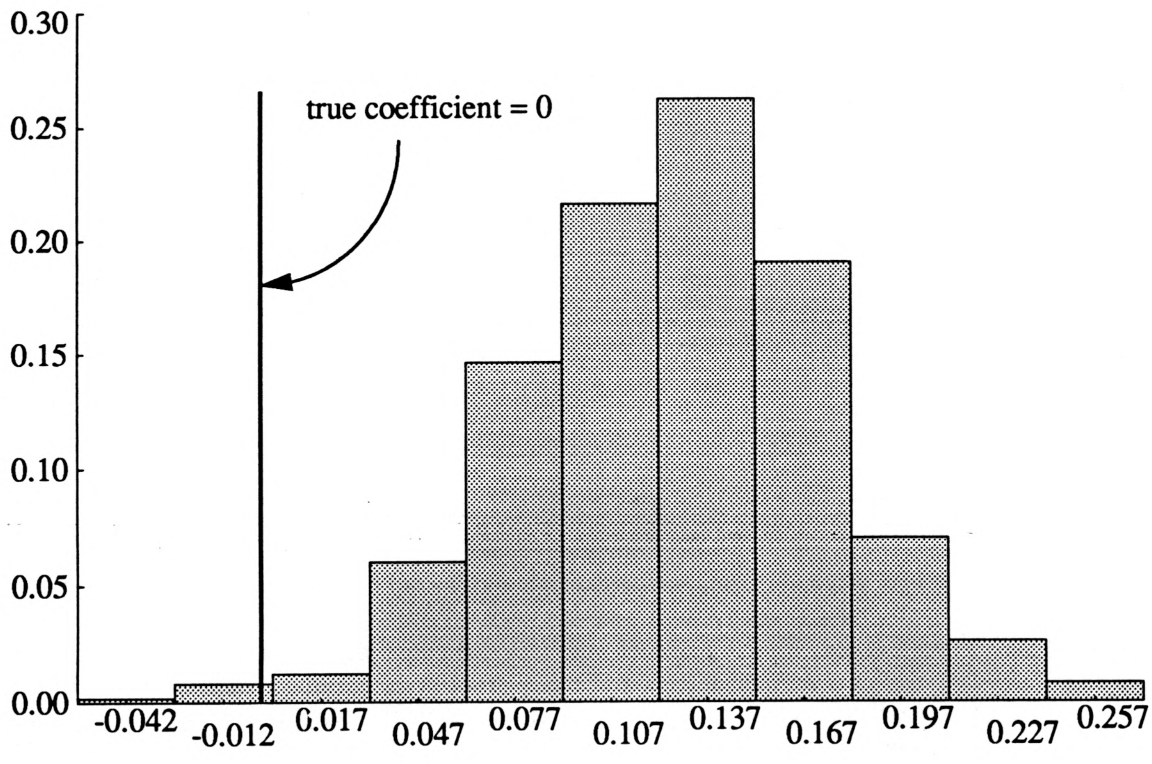


Figure 3 : Frequency distribution of coefficient on level of gap
(H-P smoothing parameter very large)

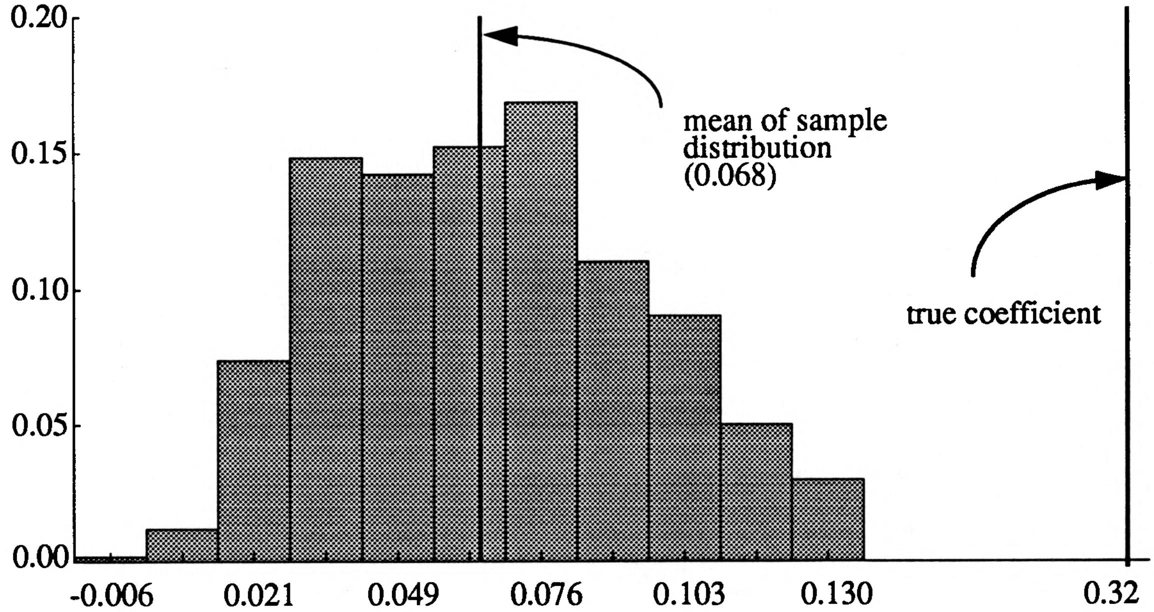
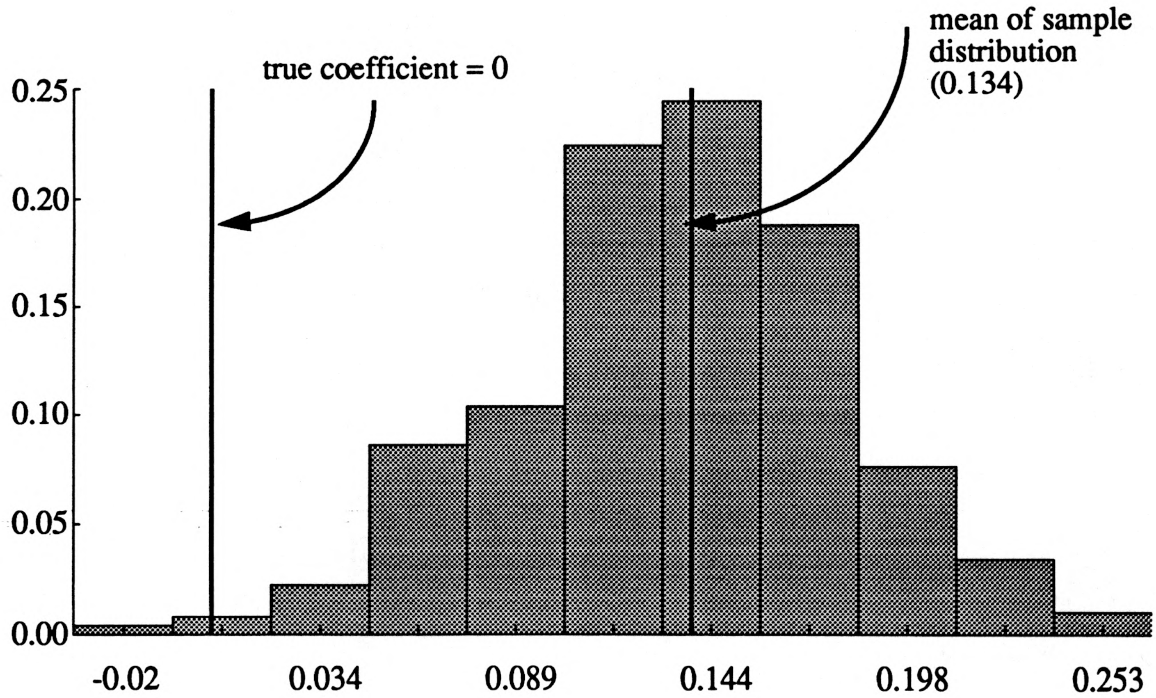


Figure 4: Frequency distribution of coefficient on change in gap
(H-P smoothing parameter very large)



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