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The Predictability of Stock Market Regime: Evidence from the Toronto Stock Exchange

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THE PREDICTABILITY OF STOCK MARKET REGIME:

EVIDENCE FROM THE TORONTO STOCK EXCHANGE

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ABSTRACT

Are stock market crashes related to deviations from the apparent fundamental share price? Using a switching-regression framework, we test whether apparent deviations help to predict the regime from which the next period's stock market return is drawn and the magnitude of returns in that regime. We find that the ex ante probability of a collapse rises before most actual crashes. Likelihood ratio tests confirm that regime switches are influenced by apparent deviations.

RÉSUMÉ

Les krachs sont-ils liés aux écarts que les cours des actions affichent par rapport à leur valeur fondamentale? Nous testons un modèle de régression avec changement de régime pour voir si les écarts ainsi observés permettent de prédire le régime de la période subséquente et l'ampleur du rendement des actions dans ce dernier. Nous constatons que la probabilité anticipée d'un krach augmente la plupart du temps avant que celui-ci ne se produise. Les tests du rapport de vraisemblance confortent l'hypothèse que les changements de régime sont influencés par les écarts observés.

I. INTRODUCTION

The past decade has seen several large fluctuations in asset prices that are not easily explained by news about market fundamentals. The increase in the U.S. dollar in 1984-85, the rise of the price-earnings ratio on the Tokyo stock market in 1985-86, and the October 1987 world stock market crash are a few recent examples. One possible explanation for these fluctuations is the existence of speculative bubbles in asset prices. In a bubble model, asset prices may consist of two components, the market fundamental and a speculative bubble. A rational risk-neutral investor will hold an asset that is overvalued relative to its market fundamental so long as its expected rate of return equals the rate of return on a non-bubbly asset. If there is a chance that the bubble will collapse, then the expected return on a "bubbly" asset in the states where the bubble survives must be higher to compensate the investor for the possibility of collapse. This means the bubble component must be growing at a more rapid rate than the fundamental. Thus the existence of bubbles would not only account for occasional asset price crashes but also rapid run-ups in asset prices before a crash.

The question we address is whether or not stock market crashes and the booms that precede them are related to apparent deviations from fundamentals as a model of bubbles would predict. We begin by showing how a simple bubble model leads to regime-switching behaviour in stock prices. In one regime, an apparent deviation from fundamentals grows from one period to the next; in the second, the apparent deviation shrinks. Linearizing this model gives us a system of three equations (one for returns in each regime and one for the probability of the bubble collapsing) that we can estimate using standard switching-regression techniques.

We estimate the switching regression suggested by the bubble model on data for the Toronto Stock Exchange from 1956 to 1989. We then test for the influence of deviations from fundamentals by imposing three sets of parameter restrictions on the switching-regression model derived in section II. To allow for the apparent regime changes in stock market volatility, the first set of restrictions allows for different volatilities but no difference in expected returns between the two regimes and no role for deviations from fundamentals. The second set of restrictions allows for differences in both volatility and expected returns between regimes along the lines of the mixture of normal distributions model estimated by Akgiray and Booth (1987), but provides no role for apparent deviations from fundamentals. The third allows apparent deviations to generate mean reversion in asset prices, but does not allow this effect to differ across regimes as our model of stochastic bubbles predicts. This should capture the apparent mean reversion in asset prices noted by Fama and French (1988), Poterba and Summers (1988), and Culter, Poterba and Summers (1991). The data reject all three sets of alternatives at conventional significance levels.

In addition, we use the coefficient estimates from the switching regression to calculate the ex ante and ex post probabilities of a market crash. The ex post probability of a crash is based on the current period return; the ex ante probability is based on the apparent deviation from fundamentals in the previous period. We find that the ex post probability shows dramatic spikes which correspond to actual crashes. More surprisingly, we find that the ex ante probability of a crash typically rises before a crash, suggesting that deviations from fundamentals have some predictive ability for stock market regimes.

As Flood and Hodrick (1986), among others, have shown, there is generally a way in which tests for bubbles can be reinterpreted in terms of the process driving fundamentals. While we provide a stochastic bubble example as a motivation for how stock market regimes might arise and why both the regime and the returns conditional on the regime would be predictable, we believe that our primary contribution is a new type of data description. If the stylized facts that emerge in our empirical work are not idiosyncratic to our data, then later studies may wish to offer and test competing explanations.

Section II explains our model of stochastic bubbles and how it generates regime switching in stock returns. It also shows how such a model fits into a switching-regression framework. Section III describes the data we use and the measure of fundamental stock prices used in the switching regression. Section IV reports the estimated switching model and summarizes several diagnostic tests of its fit. It also reports the ex ante and ex post probabilities of collapse. Section V tests the switching-regression model against the nested alternatives described above. Section VI discusses the interpretation of our empirical results and suggests directions for further research.

II. A STOCHASTIC BUBBLE INTERPRETATION OF REGIME SWITCHING

This section provides an economic interpretation for the switching-regression relationships we estimate below. We will motivate them with a model of stochastic bubbles, using a generalization of the bubble model in Blanchard (1979) and Blanchard and Watson (1982). After describing its behaviour, we show how a first-order approximation of the model leads to the switching-regression model we estimate. Since we do not claim that bubbles are the only mechanism which could account for this kind of regime-switching behaviour and we do not explicitly consider other mechanisms, we do not consider the evidence of regime switching that we find to be definite evidence of the existence of bubbles. However, we feel it may give insight into why explanations involving bubbles recur over the centuries; the bubble model seems to describe some empirical regularities in asset prices that are not predicted by standard asset-pricing models.

We begin by considering a simple asset-pricing model where risk-neutral investors choose between holding a risk-free asset that yields (1+r) in period t and a risky stock.[1] For both assets to be held in equilibrium, it must be true that

$$P_t = (1+r)^{-1} \cdot E_t(P_{t+1}) + D_t \tag{1}$$

where P_t and D_t are the stock's price and dividend at time *t* and E_t denotes the expectation conditional on information available at time *t*. One possible solution to this equation defines the fundamental price

[1]There is an extensive literature noting restrictions on the admissibility of bubble solutions in this class of models, including Diba and Grossman (1987,1988) and Obstfeld and Rogoff (1983,1986). More recent work by Weil (1988) and Allen and Gorton (1991) has shown that these restriction are not robust to minor changes in the model. As our purpose in this section is to provide intuition for the presence of switching behaviour and not to contribute to the theoretical bubble literature, we have chosen to use this simpler model as an expository device.

$$P_t^* \equiv \sum_{k=0}^{\infty} (1+r)^{-k} \cdot E_t(D_{t+k}) \tag{2}$$

All other prices are said to be "bubbly," with the size of the bubble defined as

$$B_t \equiv P_t - P_t^* \tag{3}$$

Since all asset prices, bubbly or not, must satisfy (1), this implies that the bubble must satisfy the condition

$$E_t(B_{t+1}) = B_t \cdot (1+r) \tag{4}$$

Blanchard (1979) and Blanchard and Watson (1982) consider a particular stochastic solution to (4), which they feel captures what is usually meant by a speculative bubble. They suppose there are two states of nature, one where the bubble survives (state S) and one where it collapses (state C). If the probability of being in state S is some constant q and being in state C implies $B_t=0$, then (4) implies

$$E_t(B_{t+1} \mid S) = \frac{(1+r)}{q} \cdot B_t \tag{5}$$

The intuition here is that (if $B_t>0$) agents expect capital losses of B_t in state C, which must be balanced by expected capital gains in state S in order to have the required rate of return on the bubble. Equation (5) simply notes that the less likely state S becomes, the higher these expected capital gains must be.

We now generalize the Blanchard-Watson model in two ways. First, we wish to allow the probability q of being in state S to vary. We do this by positing that the probability of survival may depend on the relative size of a bubble (defined as $b_t \equiv B_t/P_t$), with relatively large (in absolute value) bubbles less likely to survive. This can be written as

$$\frac{d\,q(b_t)}{d\,|b_t|} < 0. \tag{6}$$

Second, while some notable market crashes have occurred in a single day, in other cases a collapse may occur over several months.[2] To model this, we allow the expected value of the bubble conditioned on collapse to be non-zero, thereby allowing for partial collapses. We assume the expected size of a bubble in state C, which we define as $u_t \cdot P_t$, depends on the relative size of the bubble in the previous period, so

$$E_t(B_{t+1}\,|\,C) = u(b_t)\cdot P_t \tag{7}$$

We further assume that u(·) is a continuous and everywhere differentiable function and that

$$u(0) = 0 \tag{8}$$

$$1 \geq \frac{d\,u(b_t)}{d\,b_t} \geq 0 \tag{9}$$

This ensures that $|B_t| \geq |u(b_t)P_t|$, so a collapse means that the bubble is expected to shrink. Together with (4) it implies that the expected value of the bubble in state C cannot be larger than that in state S.

Using these two generalizations together with (4) allows us to write our new bubble model as

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[2]The fall in the Tokyo stock exchange in the months following January 1990 is an example.

$$E_t(B_{t+1}) = \begin{cases} \dfrac{1+r}{q(b_t)}B_t - \dfrac{1-q(b_t)}{q(b_t)}u(b_t)\cdot P_t & \text{with probability} \quad q(b_t) \\ u(b_t)\cdot P_t & \qquad\qquad 1-q(b_t) \end{cases} \tag{10}$$

We can see that when $q(b_t)\equiv q$ and $u(b_t)\equiv 0$, this reduces to the Blanchard-Watson process. As before, the more likely S, the smaller expected capital gains in S need to be. Also, as the expected value of the bubble in state C ($u(b_t)P_t$) rises, agents require smaller expected capital gains in state S to satisfy (4).

It is straightforward to derive the expected excess returns R in each regime, where excess returns are the rate of return on the bubbly asset less the rate of return on the riskless asset

$$E_t(R_{t+1}|S) = \frac{1-q(b_t)}{q(b_t)}\left[(1+r)b_t - u(b_t)\right] \tag{11}$$

$$E_t(R_{t+1}|C) = u(b_t) - (1+r)b_t \tag{12}$$

Noting that conditional expected excess returns are a function of b_t, we can take first-order Taylor series approximations of $E_t(R_{t+1}\,|\,S)$ and $E_t(R_{t+1}\,|\,C)$ with respect to b_t around some arbitrary value $\bar{b}$ to obtain:

$$E(R_{t+1}|S) = \gamma_{S0} + \gamma_{S1}b_t \tag{13}$$

$$E(R_{t+1}|C) = \gamma_{C0} + \gamma_{C1}b_t \tag{14}$$

where

$$\gamma_{S1} \equiv \left(-\frac{1}{q(\bar{b})^2} \cdot \frac{\mathrm{d}q(\bar{b})}{\mathrm{d}b_t} \left[(1+r)\cdot\bar{b} - u(\bar{b}) \right] + \frac{1-q(\bar{b})}{q(\bar{b})} \left[1 + r - \frac{\mathrm{d}u(\bar{b})}{\mathrm{d}b_t} \right] \right) \tag{15}$$

$$\gamma_{C1} \equiv \left[\frac{\mathrm{d}u(\bar{b})}{\mathrm{d}b_t} - (1+r) \right]. \tag{16}$$

Assuming $r \geq 0$, we can then prove that $\gamma_{S1} \geq 0$ and $\gamma_{C1} \leq 0$.[3]

By dropping the expectations operator E_t in equations (13) and (14), we can rewrite them as

$$R_{S,t+1} = \gamma_{S0} + \gamma_{S1} \cdot b_t + \varepsilon_{S,t+1} \tag{17}$$

$$R_{C,t+1} = \gamma_{C0} + \gamma_{C1} \cdot b_t + \varepsilon_{C,t+1} \tag{18}$$

To complete the switching-regression model, we need only combine (17) and (18) with an equation that describes the probability that an observed excess return R_t corresponds to regime S (or regime C). The simplest way to do this would be to linearize $q(b_t)$, but then there is no guarantee that the resulting probabilities will be bounded between zero and one. We adopt the same solution used in Probit models by imposing the functional form

$$Pr(\text{regime S}) = \Phi(\gamma_{q0} + \gamma_{q1} \cdot |b_t|) \tag{19}$$

where Φ is the standard normal cumulative distribution function. Note that (6) implies $\gamma_{q1} \leq 0$.

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[3]The proof for γ_{C1} follows directly from (9). For γ_{S1}, we can use this condition and the fact that $1 \geq q(b_t) \geq 0$ to show that the second term in the expression is always non-negative. (6), (8) and (9) then together imply that the first expression is also non-negative, so their sum will be non-negative.

The three equations (17), (18) and (19) form a standard switching-regression model of the type described by Goldfeld and Quandt (1976) and Hartley (1978). Once the assumption of normality is imposed, estimates of the γ's can be found by maximizing the likelihood function

$$\prod_{t=1}^{T}\left[\Phi\left(\gamma_{q0}+\gamma_{q1}|b_t|\right)\phi\left(\frac{R_{t+1}-\gamma_{S0}-\gamma_{S1}b_t}{\sigma_S}\right)\sigma_S^{-1}+\left\{1-\Phi\left(\gamma_{q0}+\gamma_{q1}|b_t|\right)\right\}\phi\left(\frac{R_{t+1}-\gamma_{C0}-\gamma_{C1}b_t}{\sigma_C}\right)\sigma_C^{-1}\right] \tag{20}$$

where ϕ is the standard normal probability density function and σ_S, σ_C are the standard deviations of $\varepsilon_{S,t+1}$, $\varepsilon_{C,t+1}$. Note that this estimation technique not only allows us to recover consistent estimates of the parameters in both states, but it does not require assumptions about which regime generated which observation. Instead, it considers the probability that either regime may have generated a given observation and gives an optimal classification of observations into the underlying regimes, as we will see below.

III. DATA

The basic data used below to test for bubbles are monthly prices and dividends for the Toronto Stock Exchange. They cover the period from January 1956 to November 1989 and are month-end figures. Stock prices (P_t) are measured by the Toronto Stock Exchange Composite (300, weighted) Index, not seasonally adjusted, from *Bank of Canada Review*, series B4237. Dividends are calculated from the above prices and the corresponding dividend yield from *Bank of Canada Review*, series B4245. Excess returns on stocks are calculated as $R_t = P_{t+1}/P_t + (DY_t - i_t)/1200 - 1.0$; where DY_t is the above dividend yield series and i_t is the Canadian interest rate on 90-day prime corporate paper in per cent per annum from *Bank of Canada Review*, series B14017. The results were not very sensitive to the choice of interest rate series as excess returns are dominated by the change in stock prices.

Choosing a measure of deviations from fundamentals for bubble tests must always be controversial, since the test result will hinge upon the measure chosen and there is no way to test the model of fundamentals under the alternative hypothesis that bubbles are present. However, we do not aim to prove rigorously the presence or absence of bubbles; our primary goal is data description. We therefore use an intuitive measure of deviations from fundamentals; we regress the log of the stock price index on the log dividend index and a constant and use the residuals as our measure of deviations from fundamentals.[4] This specification has the advantage of being very similar to the log price-dividend ratio, a widely used measure of stock market valuation. This makes it interesting to relate our results to work on the relationship between price-dividend ratios and predictable asset market returns, such as Fama and French (1988).

Our measure of apparent deviations from fundamentals is shown in Figure 1. This measure indicates that apparent deviations from fundamentals were greatest in 1987, just prior to the October crash, but were also large in 1973. The most negative apparent deviations came in mid-1982, near the trough of a severe recession, although the period from late 1974 to early 1980 also saw a large negative apparent deviation.

IV. ESTIMATION OF SWITCHING MODELS

Table I presents the estimated coefficients from the general switching model. Since the measure of deviations is, by construction, equal to zero on average, γ_{S0} and γ_{C0} give us the average behaviour of stock returns in each regime. We see that, conditional on the bubble surviving, we expect an excess return of 0.55% per month (6.4% per annum). Conditional on its collapse, however, we expect losses of 4.9% per

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[4]In a Lucas (1978) asset-pricing model where dividends follow a random walk with drift, the fundamental price can be expressed as $P^*_t = \rho D_t$, where ρ is a function of tastes and the parameters of the stochastic process for dividends. Since $P_t - \rho D_t$ is the deviation from fundamentals, we regress P_t on D_t (taking logs to reduce heteroscedasticity) and use the residuals as our measure of b_t.

month (45.4% per annum). On average, the probability of survival is $\Phi(\gamma_{q0})$ = 97.6%, where Φ is the standard Gaussian cumulative density function (cdf). Thus the estimates from our switching regression reproduce the stylized fact that returns are characterized by large but infrequent crashes. Our estimates imply that the expected average excess return is 0.976*.0055 - .024*.0480 = 0.42% per month (5.2% per annum). The results therefore show an equity premium in our sample close to the 6% found in U.S. data by Mehra and Prescott (1985). It is also interesting to note that the volatility of returns is more than twice as large (measured by the standard deviation of the error term) in states where the bubble collapses as in states where it survives.[5]

The ex ante probability of being in regime i at time t is defined as the probability conditioning on b_{t-1}, which is the size of the apparent deviation in the previous period. This probability is given by the formula $\Phi(1(i) \bullet (\gamma_{q0} + \gamma_{q1} \bullet |b_{t-1}|)) \equiv P_i^A$ where 1(i) is 1 or -1, depending on the regime i. The ex post probability of regime i at time t also conditions on the realized excess return R_t and is given by

$$\frac{P_i^A \cdot \phi\left(\dfrac{R_t - \gamma_{i0} - (\gamma_{i1} \cdot b_{t-1})}{\sigma_i}\right)\sigma_i^{-1}}{P_i^A \cdot \phi\left(\dfrac{R_t - \gamma_{i0} - (\gamma_{i1} \cdot b_{t-1})}{\sigma_i}\right)\sigma_i^{-1} + (1-P_i^A)\cdot\phi\left(\dfrac{R_t - \gamma_{j0} - (\gamma_{j1} \cdot b_{t-1})}{\sigma_j}\right)\sigma_j^{-1}} \equiv P_i^X \tag{21}$$

where ϕ is the standard normal probability density function (pdf) and $j \neq i$. Note that these definitions imply that $P_i^A + P_j^A = 1$ and that $P_i^X + P_j^X = 1$.

Figure 2 plots the ex ante and ex post probabilities of a collapse. The ex post probability of collapse is marked by distinct spikes which generally seem to coincide with actual market crashes. For example, we see spikes coinciding with crashes in 1957, 1962, 1969, 1973, 1974, 1979, 1981, 1982 and 1987.

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[5]In a Markov mixture of normals model of monthly U.S. stock returns from 1834 to 1987, Schwert (1989) finds that the variance of returns is about 2.3 times higher in the regime with negative mean returns.

What may be more interesting is the behaviour of the ex ante probability of collapse. Periods when the ex ante probability of collapse is highest end with a spike in the ex post probability, as can be seen in 1972, 1974-75, 1982 and 1987. However, the converse is not always true. In some cases, such as 1969 or 1979-80, there is a spike in the ex post probability of collapse without a corresponding rise in the ex ante probability. Of course, 1979-80 was a period when news arrived about oil shocks, so it would not be surprising if this was a period in which news about fundamentals was responsible for dramatic changes in stock prices.

With few exceptions, the ex ante probability of collapse in any given month is relatively small (less than 20%). This tends to obscure the fact that large apparent deviations from fundamentals are associated with large cumulative probabilities of collapse. As Figure 3 shows, the cumulative ex ante probability of collapse rose above 90% before the 1962, 1969, 1973, 1979 and 1987 collapses.[6] Again, there are several spikes in the ex post probability of collapse clustered around the 1979-80 oil shock which were not preceded by high cumulative probabilities of collapse.

In the appendix, we present a variety of diagnostic statistics, the highlights of which are briefly summarized here. The raw returns show very strong evidence of negative skewness which is eliminated in the ex post residuals. The raw returns also show kurtosis; this is reduced in the ex post residuals but is still statistically significant. Finally, the Tukey Box plots in the appendix show a dramatic reduction in outliers, particularly negative outliers, from the raw returns to the ex post residuals. The common thread appears to be that the switching model does a good job of capturing crashes.

To focus on an episode that may be of particular interest, we evaluate expected returns on the eve of the October 1987 crash when $b_t = 0.437$. For this value of the apparent deviation from fundamentals,

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[6]In Figure 3, we calculate the cumulative probability of collapse as

$$1 - \prod_{k=\tau}^{t} P_k^A$$

where τ is the last period in which $P_\tau^X < .75$ and P_t^X and P_t^A are the ex post and ex ante probabilities of survival in period t.

our estimates predict a monthly excess return of -12% conditional on a collapse[7] and an ex ante probability of collapse of 29%. The cumulative ex ante probability of a collapse in the three-month period ending in October 1987 was 67%.[8]

V. TESTS OF THE SWITCHING MODEL

We can formally test how well the stochastic bubble model corresponds to the data by imposing parameter restrictions on the switching regression. Under the null hypothesis of no bubbles, the bubble measure should have no effect on which regime occurs, nor on the magnitude of returns in a given regime. By imposing different sets of parameter restrictions on the switching regression, we can mimic a variety of stylized facts about market returns. For example, by setting all the bubble coefficients equal to zero and imposing equal constants in the two regimes, we can reproduce the stylized fact of high and low volatility regimes described in Schwert (1989). A second set of parameter restrictions would apply if returns were generated by a simple mixture of normal distributions, as estimated by Akgiray and Booth (1987). Under a third set of parameter restrictions, our switching regression corresponds to the Cutler, Poterba and Summers (1991) regression test for mean reversion.

Volatility regimes can be characterized as a situation in which mean returns are identical across regimes, but there are periods of high and low volatility in the market. Formally, this would be the special case of the general switching model where $\gamma_{S0} = \gamma_{C0} = \gamma_0$, $\gamma_{S1} = \gamma_{C1} = \gamma_{q1} = 0$ but we allow $\sigma_S \equiv \mathrm{Var}(\varepsilon_{S,t+1}) \neq \sigma_C \equiv \mathrm{Var}(\varepsilon_{C,t+1})$, so

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[7]This compares to the actual monthly excess return of -23% for the period from the end of September to the end of October 1987.

[8]It is interesting to compare these results to those of Friedman and Laibson (1989), who look at the macroeconomic implications of a model similar to ours in that it involves infrequent large movements in stock prices. Using a diffusion-jump model applied to U.S. data, for example, they estimate the probability of a crash in the quarter before October 1987 as about 4%. The key difference between their approach and ours is that they do not incorporate the information contained in a measure of deviations from fundamentals.

$$R_{t+1} = \gamma_0 + \varepsilon_{t+1} \tag{22}$$

where

$$\begin{aligned} \varepsilon_{t+1} &\sim N(0,\sigma_S) \textit{ with prob } q \\ \varepsilon_{t+1} &\sim N(0,\sigma_C) \textit{ with prob } 1-q \end{aligned} \tag{23}$$

Coefficient estimates for the volatility regimes case are presented in column two of Table I. The LR statistic should have a χ^2 distribution under the null with 4 degrees of freedom. As shown in the table, the actual LR statistic is 14.40. Since we reject this null, we conclude that the regimes differ in more than just their variances.[9] This implies either that the information contained in the measure of deviations from fundamentals helps to determine which regime prevails or that the regimes have different expected values, or both. Both are implied by the bubble model presented in Section II.

The second possibility we consider is that returns are well characterized by a mixture of normal distributions with different means and variances, but are unrelated to deviations of stock prices from fundamentals, which can be expressed as

$$\begin{aligned} R_{t+1} &\sim N(\gamma_{S0},\sigma_S) \textit{ with prob } q \\ R_{t+1} &\sim N(\gamma_{C0},\sigma_C) \textit{ with prob } 1-q \end{aligned} \tag{24}$$

for some constant q. This is the special case of the general model where $\gamma_{S1} = \gamma_{C1} = \gamma_{q1} = 0$. Coefficient estimates for the normal mixture case are presented in column three of Table I. The LR statistic should have a χ^2 distribution under the null with 3 degrees of freedom. As shown in the table, the actual LR statistic is 11.24. The rejection of the null of a normal mixture implies that apparent deviations from

[9]Rejection of this null also implies a rejection of the single regime null since the latter is just the special case where $\sigma_S = \sigma_C$. However, we cannot test directly whether we have one regime or two since the parameters of our alternative hypothesis are not identified under the null of only one regime. See Lee and Chesher (1984) for details.

fundamentals help to determine expected returns in each regime and/or which regime prevails.

The third possibility we explore is that returns are predictable, but mean returns do not differ across regimes. To test this, we compare the general switching framework with the restricted case where deviations from fundamentals help predict returns but mean returns are the same across regimes and deviations have no predictive power for the probability of a given regime. The "mean-reversion" case therefore sets $\gamma_{S0} = \gamma_{C0} = \gamma_0$, $\gamma_{S1} = \gamma_{C1} = \gamma_1$, and $\gamma_{q1} = 0$. It corresponds to the regression test for transitory components in stock prices in Cutler, Poterba and Summers (1991), except that we allow more flexibility for volatility by allowing the variances of returns to be drawn from high and low volatility distributions

$$R_{t+1} = \gamma_0 + \gamma_1 b_t + \varepsilon_{t+1} \tag{25}$$

where

$$\begin{aligned} \varepsilon_{t+1} &\sim N(0,\sigma_s) \ \textit{with prob} \ \ q \\ \varepsilon_{t+1} &\sim N(0,\sigma_c) \ \textit{with prob} \ 1-q \end{aligned} \tag{26}$$

Coefficient estimates for the volatility regimes case are presented in column four of Table 1. The LR statistic should have a χ^2 distribution under the null with 3 degrees of freedom. As shown in the table, the actual LR statistic is 12.64. The coefficient estimates and the LR test suggest that there is more in the data than simple mean reversion. In particular, γ_{Q1} is significantly different from 0, which implies that a large deviation from fundamentals makes a collapse more likely.

VI. CONCLUSION

We investigate whether stock market booms and crashes are linked to apparent deviations from fundamentals. We consider a model in which stock market returns arise from two possible states of the world. In the first, an apparent deviation from fundamentals survives from one period to the next; in the second, the apparent deviation collapses. Econometrically, this corresponds to a switching regression in which deviations influence both the probability of collapse and the magnitude of returns conditional on survival or collapse.

The data suggest that stock market booms and crashes are related to apparent deviations from fundamentals. We find evidence of regime switches in stock market returns that are influenced by apparent deviations from fundamentals. Our estimates are consistent with the stylized fact of large but infrequent crashes. The size of the apparent deviations from fundamentals has an important influence on both the probability and expected magnitude of a collapse. For example, using the apparent deviation just before the October 1987 crash, our estimates suggest a cumulative probability of collapse of 67% for the three-month period ending in October and a one-month loss conditional on collapse of 12%. Collapse probabilities tend to be followed by actual crashes. While some crashes are not predicted by our model, these include cases such as the 1979, when news about fundamentals may have strongly influenced prices.

We believe there are two important directions for further research. The first is to determine whether our stylized fact -- that apparent deviations from fundamentals influence both the regime that prevails in the stock market and the size of expected returns -- generalizes to other countries, time periods and asset markets. The second is to see whether a plausible model of equilibrium asset pricing could explain our results in terms of switching fundamentals. Whatever the ultimate interpretation, our paper highlights an aspect of the behaviour of stock market returns that has not previously received attention.

APPENDIX: DIAGNOSTIC STATISTICS

To give a better picture of the switching-regression model's ability to explain excess returns, we run a series of tests for non-normality and serial dependence on the return series and the residuals of the switching model. The excess returns predicted by the model are just the weighted sum of the fitted values for the surviving and collapsing regime equations, where the weights are the probabilities that the observation came from that regime. Since these probabilities depend on the information we condition on, we constructed them in two different ways; once using the ex ante probabilities and once using the ex post. Since the ex post condition on all available information, this should reflect more accurately the model's explanatory power. Results are shown in Table A1.

The first set of statistics test for the presence of serial correlation or persistence in the data. The Durbin-Watson and the Q-test(1) test for first-order serial correlation while the Q-test(12) tests all orders up to 12. The Durbin Watson tests find no significant serial correlation for any of the series. The Q-tests find weak evidence of serial correlation for all the series; the test statistics are always significant at the 10% level but never at 1%. Clearly, the switching model does not reduce the evidence of serial correlation; if anything, it slightly increases it. Note, however, that these conclusions are based on the assumption of normally distributed errors required for the validity of these tests. As an alternative, we also report the standardized runs statistic, which is a non-parametric test for serial persistence and is robust to deviations from normality. It gives results quite different from those for the Q-statistic; there is weak evidence (i.e., significance between the 10 and 1% levels) of persistence for the raw returns series and the ex ante residuals, but this vanishes for the ex post residuals.

The next section of Table A1 tests for deviations from normality. As is common with financial data, there is very strong evidence of skewness in the raw returns series. While the ex ante residuals show almost as much skewness, the ex post series have none, showing that the model can effectively explain all the skewness in the data. A non-parametric analogue of the skewness test is the sign test, which tests whether the median of the distribution is equal to the sample mean. It gives a similar conclusion, finding weak evidence of skewness in returns and the ex ante series, but none in the ex post.

Tests of kurtosis also confirm the general finding that financial data tend to be "fat-tailed" relative to the normal distribution. Both model residual series have significant evidence of excess kurtosis, although the test statistics do show a steady decline from the raw series to the ex ante to the ex post. As a result, the Jarque-Bera test for normality, which simply combines the skewness and kurtosis tests, just fails to reject the hypothesis of normality at the 5% significance level for the ex post residuals, while very strongly rejecting it for the other series. We conclude that the switching regression model can explain most, but not quite all, of the deviations from normality found in our returns series.

The final section of the table contains the results of Engle (1982)'s tests for first-order Autoregressive Conditional Heteroscedasticity (ARCH). ARCH models have been a popular way to model the observed "clustering" of volatility in financial data. Surprisingly, however, we find no evidence of ARCH in our raw returns, but some weak (not significant at the 5% level) evidence of it in our ex post residuals. This suggests either spurious evidence of ARCH, or that it is a statistical artifact of the model. Since there is no evidence of ARCH in the returns, it does not imply that the switching model has failed to capture an important feature of the data.

Another view of the model's fit can be seen from the Tukey Box plots shown in Figure A1. The height of the rectangle drawn for each residual and returns series shows its inter-quartile range and the line bisecting the rectangle indicates the median. The T-bars on each side indicate 2.5 times the inter-quartile range,[10] and all observations falling outside this band are indicated with a '-'. We see that the switching model moves the median closer to zero, but has little effect on the inter-quartile range. Much of the difference comes in the tail of the distributions, where the ex post residuals show fewer outliers, especially negative ones. This suggests that the bubble model has more power to explain crashes than bull markets, and is also consistent with the large reduction in skewness we noted above.

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[10]For a normal distribution, this should cover all but about 9% of the data. Given our 406 observations, we would expect about 35 observations to lie outside this range.

Table I: Estimates of The General Switching Model and Nested Null Hypotheses

	General Switching Model	Volatility Regimes	Normal Mixture	Mean Reversion
γ_{S0}	0.0055 (.0024)	0.0041 (.0021)	0.0063 (.0025)	0.0043 (.0021)
γ_{S1}	-0.0122 (.0142)			-0.0168 (.0118)
γ_{C0}	-0.0492 (.0352)		-0.0189 (.0149)	
γ_{C1}	-0.1749 (.0923)			
γ_{q0}	1.9704 (.4500)	1.1144 (.3683)	1.0412 (.3288)	1.0298 (.3802)
γ_{q1}	-3.2317 (1.4547)			
σ_S	0.0381 (.0019)	0.0366 (.0027)	0.0360 (.0026)	0.0360 (.0028)
σ_C	0.0811 (.0266)	0.0832 (.0149)	0.0775 (.0115)	0.0795 (.0136)
Log - Likelihood	700.10	692.90	694.48	693.78
Likelihood Ratio Test Against General Switching Model		14.40	11.24	12.64
p - Value		0.0061	0.0115	0.0055

TABLE A1 - Switching Regression Diagnostics

Test Statistic	Returns	Ex Ante	Ex Post
Durbin-Watson	1.80	1.76	1.78
Q-test (1)	4.00 (.0455)	5.91 (.0151)	4.82 (.0281)
Q-test (12)	20.11 (.0650)	21.81 (.0397)	22.47 (.0326)
Runs Test	-1.84 (.0656)	-1.90 (.0571)	-1.41 (.1580)
Skewness	-0.59 (.0000)	-0.57 (.0000)	-0.08 (.5322)
Sign Test	2.03 (.0419)	2.23 (.0255)	0.74 (.4566)
Kurtosis	2.68 (.0000)	1.88 (.0000)	0.59 (.0154)
Jarque-Bera	115.92 (.0000)	73.30 (.0000)	5.93 (.0515)
ARCH (1)	0.13 (.7234)	0.88 (.3491)	3.11 (.0778)

NOTES:

- Figures in () are significance levels. Values close to 0 imply rejection of the null.

- The Q-test is the Ljung-Box portmanteau test for serial correlation up to the *nth* order. It is distributed $\chi^2(n)$ under the null of no serial correlation.

- The Runs test is a non-parametric test for persistence and has a standard Gaussian distribution under the null.

- The Skewness and Kurtosis statistics are those suggested by Kendall and Stuart (1958). Significance levels are based on transformations of these statistics which should have a standard normal distribution under the null.

- The Sign test is a non-parametric test of whether the median is zero and has a standard normal distribution under the null.

- The ARCH test is Engle (1981)'s test for a first order ARCH process and is $\chi^2(1)$ under the null.

- The Jarque-Bera test for normality combines both skewness and kurtosis and is $\chi^2(2)$ under the null of normality.

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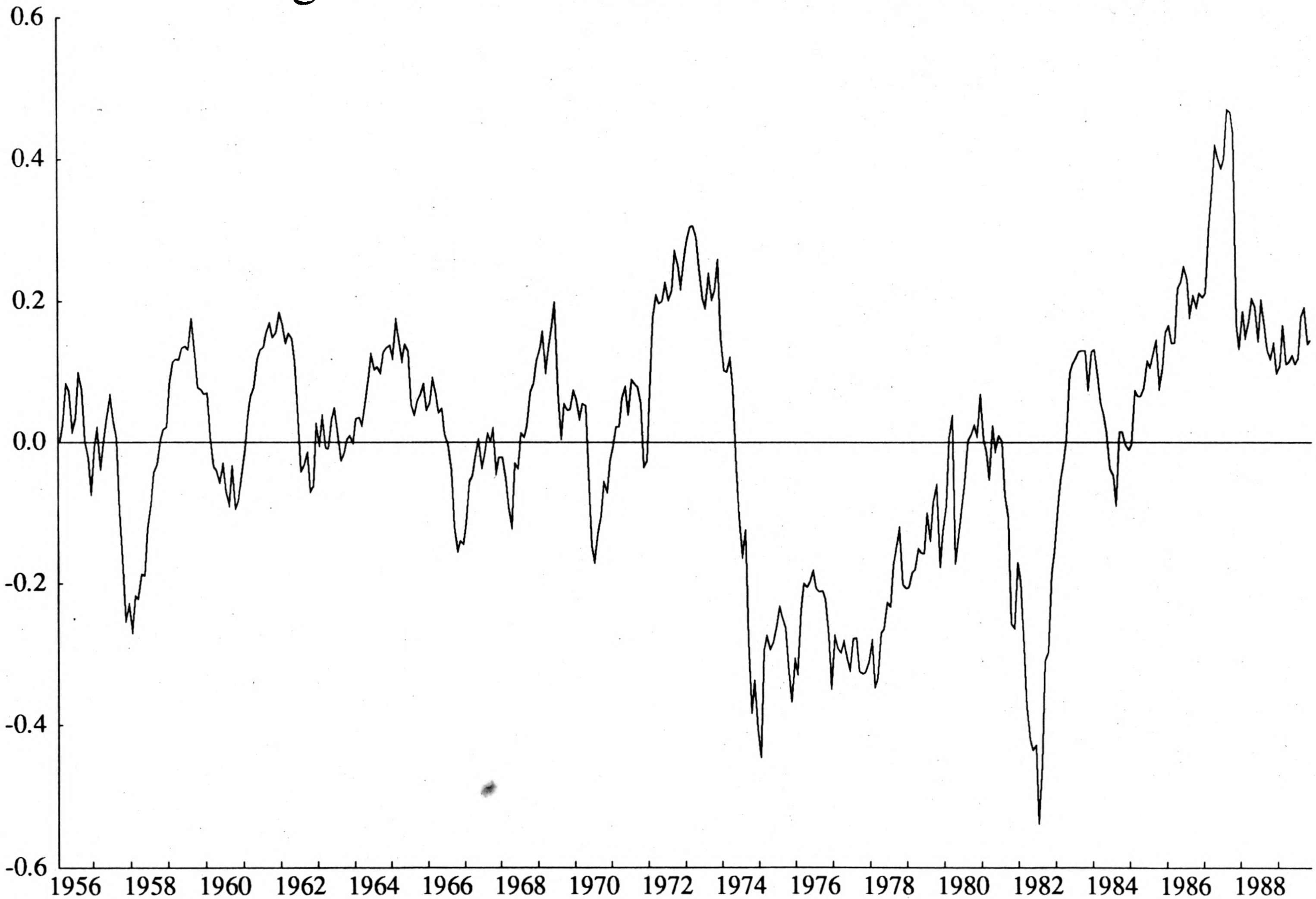
Figure 1 - Deviations from Fundamentals



Residuals from Regression of TSE log Prices on log Dividends

Figure 2 - Probabilities of Bubble Collapse

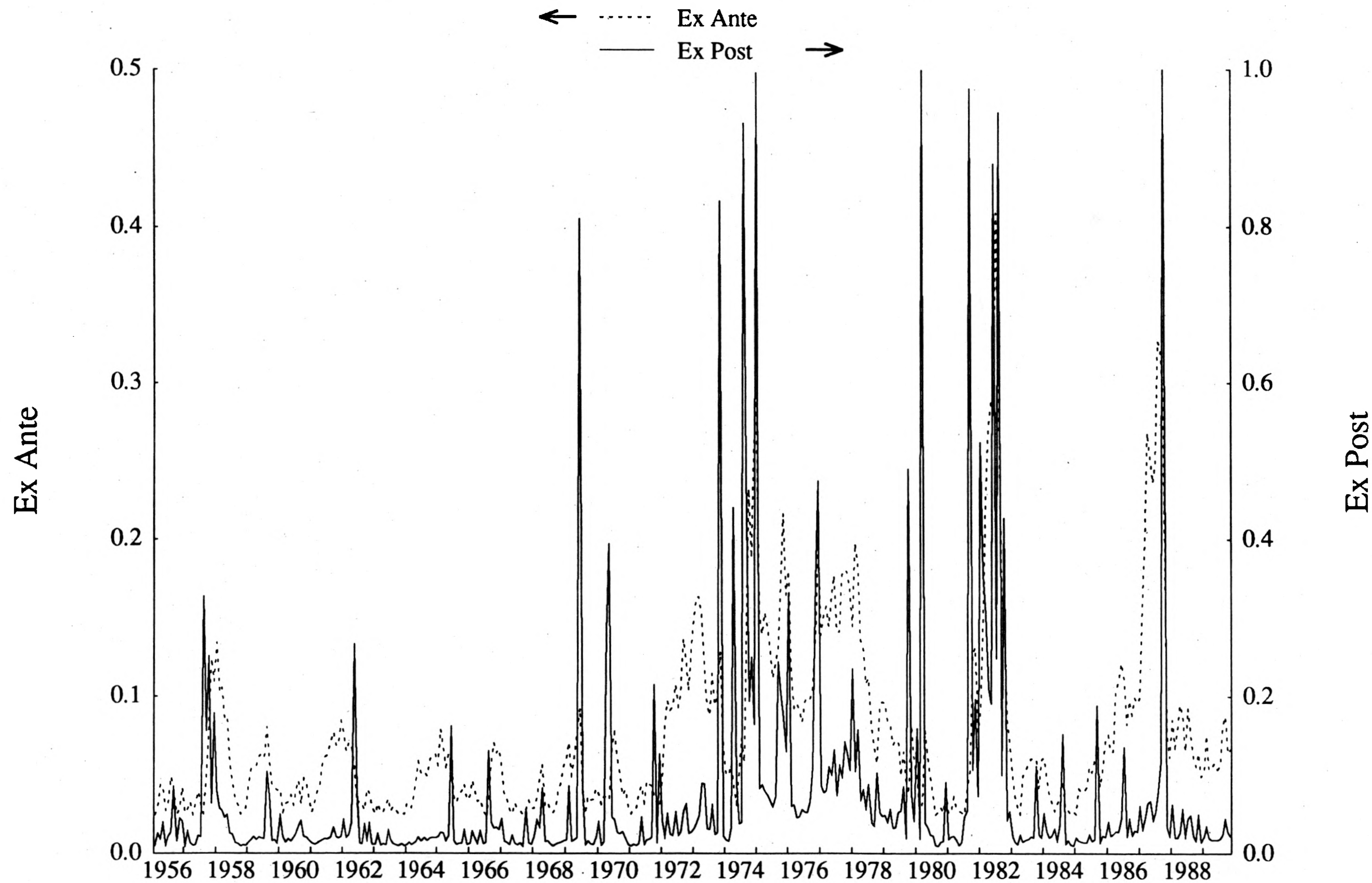


Figure 3 - Cumulative Ex Ante Probabilities of Collapse

Critical Value = .75

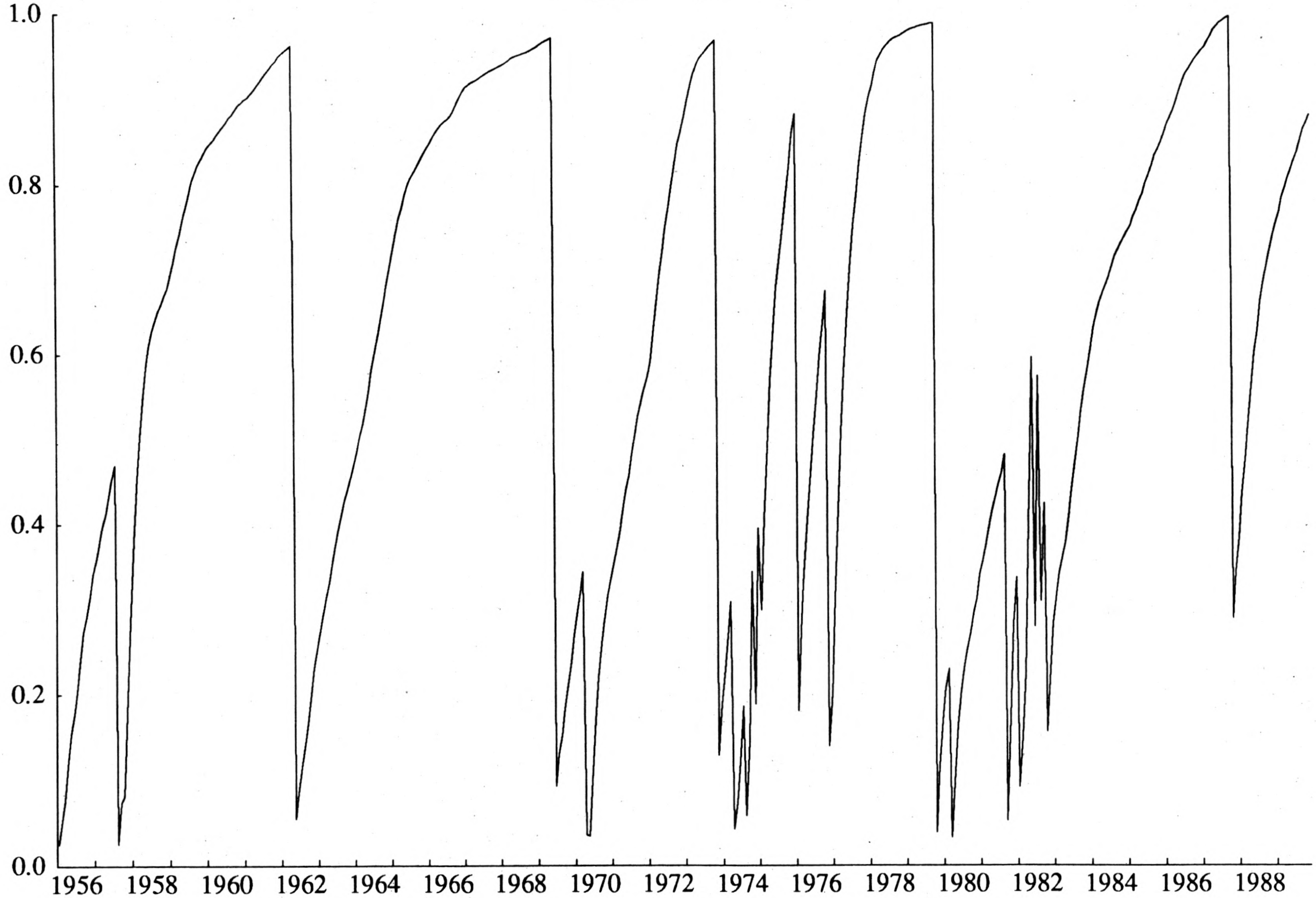
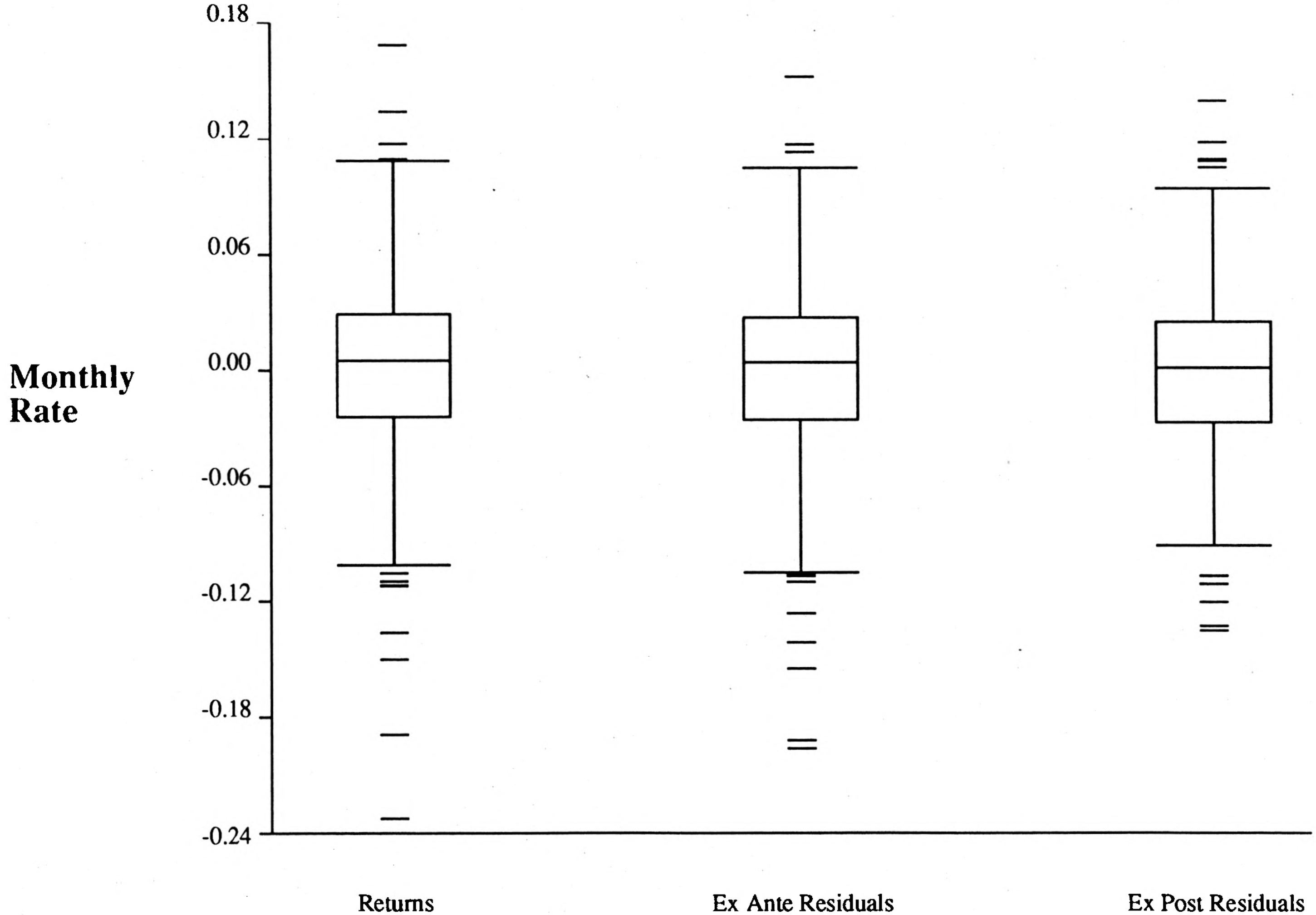


Figure A1: Distribution of TSE Stock Returns
Comparision of Raw Returns and Switching Model Residuals



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