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Abstract

This paper develops a measure of private-sector wealth which includes financial, physical and human wealth. In an attempt to capture important observed differences between individuals, consumers are modelled as being of two types — liquid and illiquid. Liquid consumers can borrow against expected future earnings, while illiquid consumers cannot. This approach has the attractive feature that the resulting measure of aggregate wealth is flexible enough to encompass a range of views on the importance of the forward-looking component of wealth and the validity of the Ricardian equivalence proposition. Human wealth, or more specifically, the expected discounted value of future earnings net of government expenditures, is evaluated using a time-series approach. A numerical solution for this expectation is obtained by approximating a real-valued vector process as a discrete-valued Markov chain. To the extent possible, non-human wealth is measured at market value. The paper concludes by comparing alternative historical series for wealth which embody different assumptions regarding Ricardian equivalence.

Résumé

Dans cette étude, l'auteur élabore une mesure de la richesse du secteur privé, qui inclut le patrimoine et le capital humain. Afin de saisir les différentes caractéristiques des consommateurs, ils les divise en deux groupes: ceux qui ont des liquidités et ceux qui n'en ont pas. Les premiers peuvent contracter des emprunts sur la base de leurs revenus anticipés, tandis que les seconds n'ont pas cette possibilité. L'avantage de cette approche est qu'elle permet à l'auteur d'en arriver à une mesure assez souple de la richesse, c'est-à-dire qui intégre divers points de vue concernant l'impact du revenu anticipé sur la richesse et la validité de l'hypothèse d'équivalence ricardienne. L'auteur détermine, à l'aide des techniques des séries chronologiques, la richesse humaine ou plus spécifiquement la valeur actuelle anticipée des revenus, déduction faite des dépenses publiques . Il obtient une solution numérique pour cette valeur escomptée en calculant une approximation d'un processus vectoriel à l'aide de chaînes de Markov évaluées avec les valeurs discrètes. Le patrimoine est calculé dans la mesure du possible au cours du marché. L' étude se termine par une comparaison des différentes séries chronologiques de la richesse obtenues à partir de divers postulats concernant la validité de l'hypothèse d'équivalence ricardienne.

1 Introduction

This paper develops a method for measuring wealth and implements this method using Canadian data. The study is motivated by the many important economic questions connected with the behaviour of wealth. These include the wealth effects on aggregate demand of fluctuations in interest rates and equity prices, the economic impact of changes in the size of the government debt, and estimates of permanent-income consumption and money demand functions. With these questions in mind, the goal is to measure household wealth at market value, including human wealth and household claims on business-sector assets.[1]

The measurement of wealth is organized into four sections. Section 2 presents a definition of wealth that incorporates financial, physical and human wealth. In recognition of important observed differences between individuals, consumers are modelled as being of two types — liquid and illiquid. Liquid consumers can borrow against expected future earnings, while illiquid consumers cannot. Aggregate wealth is the sum of the wealth of both groups. The resulting definition of aggregate wealth has the attractive feature that it is flexible enough to incorporate a range of views on the importance of forward-looking behaviour and validity of the Ricardian equivalence proposition. In general, wealth in this two-consumer-type model has an important forward-looking component, but permits departures from Ricardian equivalence by recognizing that some consumers cannot borrow against their future income. If all consumers are liquid, this general definition reduces to a strict Ricardian concept of wealth, while a traditional "myopic" definition emerges if everyone is illiquid.[2]

Section 3 focuses on how to measure the expected discounted value of future earnings — human wealth. The approach adopted models the variables over which expectations must be taken using time series techniques, and then uses the resulting time series model to form expectations of the future. The first step in this approach is to examine the stochastic properties of the relevant time series. Tests for nonstationarity are used to determine the appropriate method of detrending. The resulting stationary series are then modelled as a vector autoregression (VAR) and approximated as a discrete-valued Markov chain. Using the discrete approximation to the VAR, a closed-form solution for the expected present value of

[1]It is worth noting that "household" in this sense is not the same as the household or personal sector as defined in the national accounts. In the national accounts, the household sector is one of the sectors which comprise the private sector. In this study, household wealth refers to the consolidated wealth of the entire private sector.

[2]Consumers in this environment appear myopic as a result of the liquidity constraints that they face.

future net income is obtained, and a historical series for this present value is created.

Section 4 shifts attention to the measurement of more tangible assets such as equities, bonds and housing. The general approach is to consolidate the assets and liabilities of the various sectors of the economy in an effort to "see through" the financial structure of the economy and to measure only the net worth of the ultimate owners of private-sector wealth — households. This approach is implemented by combining annual data from the national balance sheet accounts with quarterly data from the financial flow accounts. An attractive feature of this "balance sheet" approach is that assets can be measured at market value so the resulting wealth measure will fluctuate in response to relative price changes, such as the 1987 stock market crash.

In section 5 the various components of wealth are combined using the flexible definition of wealth developed in section 2. Alternative series for wealth which embody different views on Ricardian equivalence are generated and compared. Brief comments conclude the paper.

2 A flexible definition of wealth

In a competitive economy with perfect capital markets, an individual agent's wealth is usually defined as the sum of his financial and physical assets, plus the expected discounted value of his current and expected future after-tax earnings. The inclusion of expected future earnings in wealth reflects the fact that consumers in this frictionless environment can borrow against their future earnings. As a result, current expenditures are not constrained by tangible assets and current disposable income, but by the sum of tangible assets and the expected present value of lifetime after-tax earnings. If we add the assumptions of intergenerational altruism and lump-sum taxes, wealth in this economy is characterized by the well-known Ricardian equivalence proposition. For a given path for government expenditures and the foreign debt, wealth is invariant to the timing of taxes and the size of the government debt. Domestically held government debt nets out of wealth, since forward-looking consumers realize that the value of the government debt they hold is offset by future tax liabilities.

While the Ricardian concept of wealth provides a useful benchmark, both casual empiricism and more formal econometric evidence suggest that some individuals are unable to borrow against the entire value of their discounted future earnings.[3] For these liquidity-

[3]For econometric evidence of the importance of liquidity constraints for some consumers, see Hall and

constrained individuals, wealth is the sum of tangible assets, current disposable income and the proportion of their human capital against which they can borrow. Ricardian equivalence no longer holds since, in the absence of perfect capital markets, the value of the government debt is now only partially offset by future tax liabilities. For some individuals the proportion of their future earnings against which they can borrow may be zero. For these consumers, wealth includes only tangible assets and current disposable income, so the government debt they hold is viewed entirely as net wealth.[4]

In an effort to incorporate forward-looking expectations into wealth while permitting departures from strict Ricardian equivalence, the assumption adopted in this study is that the economy can be modelled as if there were two types of consumers — liquid and illiquid. Liquid (or Ricardian) consumers are individuals who can borrow against the entire value of their expected discounted future earnings at the competitive discount rate. In addition, liquid consumers are assumed to care about their children and to be always sufficiently liquid to make positive bequests. As shown by Barro (1974) these assumptions imply that liquid consumers can be modelled as if they had infinite horizons. The wealth of the representative liquid consumer is therefore defined as

$$W_t^L = A_t^L + e_t F_t^L + D_t^L + (Y_t^L - T_t^L) + E_t \left\{ \sum_{i=1}^{\infty} \left[\prod_{j=1}^{i} \left(\frac{1}{1 + r_{t+j}} \right) \right] (Y_{t+i}^L - T_{t+i}^L) \right\} \tag{1}$$

where all quantity variables are measured in real terms and

A_t^L = liquid consumer's net domestic physical and financial assets
F_t^L = liquid consumer's net foreign assets
e_t = exchange rate (defined as the price of foreign exchange)
D_t^L = liquid consumer's holdings of government bonds
Y_t^L = liquid consumer's labour income
T_t^L = liquid consumer's taxes net of transfers
r_t = real interest (discount) rate
E_t = expectations operator conditioned on information available at time t.

The wealth of the liquid consumer comprises tangible assets, current disposable income and the expected discounted value of future disposable income.

Mishkin (1982), Hayashi (1985), Campbell and Mankiw (1989) and Zeldes (1989).

[4]It may also be the case that even some tangible assets have limited marketability, but this should be only a short-run problem. The effects of this type of short-run illiquidity of some tangible assets are not considered in this study.

Illiquid consumers are individuals who are unable to borrow against their future labour earnings. As a result, illiquid consumers discount the future entirely. The wealth of the representative illiquid agent therefore includes only tangible assets and current disposable income

$$W_t^I = A_t^I + e_t F_t^I + D_t^I + (Y_t^I - T_t^I) \tag{2}$$

where variables are defined as above with the superscript I for illiquid replacing L for liquid.

Aggregate per capita wealth is the proportion of the population which is liquid times the wealth of the representative liquid consumer, plus the proportion of the population that is illiquid times the wealth of the representative illiquid consumer. Let η be the proportion of the population that is liquid and $1-\eta$ the proportion that is illiquid.[5] Aggregate per capita wealth may then be written as

$$W_t = A_t + e_t F_t + D_t^d + (Y_t - T_t) + \eta E_t \left\{ \sum_{i=1}^{\infty} \left[\prod_{j=1}^{i} \left(\frac{1}{1+r_{t+j}} \right) \right] (Y_{t+i}^L - T_{t+i}^L) \right\} \tag{3}$$

where A_t, F_t, D_t^d, Y_t and T_t are aggregate per capita quantities. The superscript d on D_t^d denotes that the sum of the government debt held by liquid and illiquid consumers equals domestically held government debt. Total government debt is the sum of the debt held by domestic consumers plus the debt held by foreigners. In order to express (3) entirely in terms of aggregate per capita quantities, it is convenient to assume that the after-tax income of the representative liquid consumer is proportional to aggregate per capita after-tax income. If the factor of proportionality is γ, aggregate per capita wealth may be written as

$$W_t = A_t + e_t F_t + D_t^d + (Y_t - T_t) + \theta E_t \left\{ \sum_{i=1}^{\infty} \left[\prod_{j=1}^{i} \left(\frac{1}{1+r_{t+j}} \right) \right] (Y_{t+i} - T_{t+i}) \right\} \tag{4}$$

where $\theta = \eta\gamma \leq 1$. If liquid consumers have a larger after-tax labour income than illiquid consumers, this implies $\gamma > 1$ so $\theta > \eta$. If this is the case, the human wealth of the liquid consumers will feature in aggregate wealth in greater proportion than the fraction of the population that is liquid.[6]

An attractive feature of the general definition of wealth given in (4) is that it is flexible enough to incorporate a range of views on the validity of the Ricardian equivalence proposition. If all consumers are liquid, strict Ricardian equivalence holds. In this case the entire

[5] In general we might expect η to be time-varying, and, more specifically, procyclical. As is commonly done, however, η is assumed to be constant. The algebra which follows does not depend on the constancy of η. The difficulty in allowing η to vary comes in specifying a time path for η.

[6] The empirical evidence on η and θ is briefly reviewed in section 5.

value of the domestically held government debt nets out of wealth since the value of the debt held by liquid consumers is entirely offset by their expected future tax liability.[7] If, on the other hand, all consumers are illiquid, then wealth as defined in (4) reduces to the traditional myopic definition which includes government debt but excludes expected future net earnings. Between these two extremes, the definition of wealth given in (4) includes some portion of government debt as net wealth, reflecting the fact that some fraction of the population is illiquid.

This flexibility of the general definition of wealth is demonstrated by combining (4) with the government's budget constraint. The government's flow budget constraint requires that expenditures on goods and services equal taxes net of transfers plus the change in the debt. If we assume that the infinitely-lived government has the same discount rate as the liquid consumers, then the government's per capita flow budget constraint is

$$G_t = T_t + \left(\frac{D^d_{t+1} + e_{t+1}D^f_{t+1}}{1+r_{t+1}}\right) - (D^d_t + e_t D^f_t) \tag{5}$$

where D^f_t is the per capita government debt held by foreigners. Provided that the government debt grows at a rate less than the rate of interest on average, repeated substitution in (5) yields the lifetime government budget constraint

$$T_t + E_t\left\{\sum_{i=1}^{\infty}\left[\prod_{j=1}^{i}\left(\frac{1}{1+r_{t+j}}\right)\right]T_{t+i}\right\} = D^d_t + e_t D^f_t + G_t + E_t\left\{\sum_{i=1}^{\infty}\left[\prod_{j=1}^{i}\left(\frac{1}{1+r_{t+j}}\right)\right]G_{t+j}\right\}. \tag{6}$$

Substituting (6) into (4) aggregate per capita wealth becomes

$$W_t = A_t + e_t F_t + D^d_t - \theta(D^d_t + e_t D^f_t) + (Y_t - (1-\theta)T_t - \theta G_t) + \theta E_t\left\{\sum_{i=1}^{\infty}\left[\prod_{j=1}^{i}\left(\frac{1}{1+r_{t+j}}\right)\right](Y_{t+i} - G_{t+i})\right\} \tag{7}$$

Note that the weights attached to the components of wealth all depend on θ, where θ is the income adjusted proportion of the population that is liquid. In this respect, θ may be viewed as an index of how Ricardian the economy is.[8] As θ is varied from zero to unity, the weight on the forward-looking component increases and wealth approaches the Ricardian ideal. If

[7] We are implicitly assuming that government finance is stable and that the government will not default on its debts.

[8] Formally, this interpretation of θ is based on the two-consumer-type abstraction, but as an *approximation* it may also be possible to view θ as a summary statistic which combines the effects of a number of sources of non-Ricardian equivalence such as imperfect capital markets, finite horizons, immigration and myopia.

all consumers are liquid so $\theta = 1$, then (7) reduces to

$$W_t = A_t + e_t F_t - e_t D_t^f + (Y_t - G_t) + E_t \left\{ \sum_{i=1}^{\infty} \left[\prod_{j=1}^{i} \left(\frac{1}{1 + r_{t+j}} \right) \right] (Y_{t+i} - G_{t+i}) \right\}. \tag{8}$$

Liquid consumers fully internalize the governments budget constraint, so wealth has the Ricardian property that it does not depend on taxes or the size of the domestic debt. What matters for wealth is the level of government expenditures and the size of the foreign debt. If, on the other hand, all consumers are illiquid so $\theta = 0$, wealth reduces to the myopic definition

$$W_t = A_t + e_t F_t + D_t^d + (Y_t - T_t). \tag{9}$$

In the absence of a forward-looking component, the entire value of the government debt is now net wealth. A government policy of substituting current taxes for debt will affect the wealth of the illiquid consumer by increasing both current disposable income and the value of the outstanding government debt.

3 Human wealth

The expected discounted value of future net earnings or *human wealth* is not directly measurable because it depends on unobservable expectations. This study measures expected variables by modelling the behaviour of variables over which expectations must be taken using time series techniques, and then using the resulting time-series model to form expectations of the future. This approach is implemented using human wealth defined as the expected discounted value of labour income net of government expenditures (as in (7)). Under the conditions described above, this definition of human wealth is equivalent to the expected present value of after-tax labour income (as in (4)), but for purposes of measurement the definition given in (7) is preferred. Recent experience with deficit reduction in Canada and elsewhere suggests that governments have more flexibility on the tax side than on the expenditure side. This suggests expenditures are probably more exogenous than taxes and thus the definition of human wealth in terms of government expenditures is better suited to a time-series approach to evaluating expectations.

This time-series approach proceeds in two steps. Step one determines suitable time-series models to describe the stochastic processes generating income net of government expenditures (hereafter, net income) and the discount rate. Tests for different types of nonstationarity are used to determine suitable detrending methods, and the resulting stationary processes are modelled as a parsimonious VAR process. In step two the VAR process is approximated as a discrete-valued vector Markov chain, and a closed-form solution for the expected discounted value of future net income is obtained.[9] This solution is then used to generate a historical time series for the expected present value.

3.1 Time series models for net income and the discount rate

The three variables that determine human wealth are labour income, government expenditures on goods and services, and the discount rate. The measurement of these and other variables is described in detail in appendix A. A few words of explanation are nonetheless in order. To control for population growth, labour income and government expenditures are both measured on a per capita basis. Both variables are also deflated by the CPI to put them in real terms. The published labour income series is augmented to include labour income in the farming and unincorporated business sectors by assuming that the share of labour in these two categories is the same as for the overall economy. Government expenditures on goods and services are measured as the reported quarterly government expenditure series less the fraction of the reported series which historically has been paid for by corporations, non-residents and government interest income. The decision to subtract these components from the reported government expenditure series reflects an attempt to obtain a measure of government expenditures which accurately reflects the tax liabilities of households. The resulting government expenditure series was then smoothed using an eight-quarter moving average because the raw quarterly data exhibited large high-frequency movements which have more to do with timing considerations than with the future tax liabilities of households.[10] Net income (X_t) is defined as labour income less government expenditures: $X_t = Y_t - G_t$.

The discount rate is more difficult to measure. In the abstract world in which wealth is defined above, consumers, firms and governments all borrow and lend at the same interest rate r. In actual economies, however, there is typically a spread between borrowing and lending rates, and different sectors typically face different borrowing rates. Moreover, even

[9]This finite-state Markov chain approach to evaluating expected values has been widely used in both qualitative and quantitative studies of business cycle fluctuations and asset prices. See, for example, Lucas (1982), Mehra and Prescott (1985), Greenwood, Hercowitz and Huffman (1988), and Macklem (1991).

[10]The smoothing of the government expenditure series might not have been necessary if government expenditures had been modelled as a separate variable in the VAR process describing the behaviour of the variables over which expectations are taken. However, in order to keep the state space of the resulting discrete-valued Markov chain approximation manageable, it was necessary to combine Y_t and G_t to form a single variable X_t.

agents within the same sector may face different borrowing rates. For example, rates for consumers range from credit card rates, through consumer loan rates obtainable at banks, down to mortgage loan rates. This variety of interest rates in the market place makes it difficult to measure *the* real interest rate. Fortunately, market interest rates tend to move together, so the problem of choosing the discount rate is largely a level problem.

The discount rate chosen for this study is the real interest rate on 90-day corporate paper plus a constant premium. This premium is chosen so that human wealth as a proportion of total wealth is in line with labour's share of income. Historically, labour income as a proportion of gross national output excluding government expenditures on goods and services has been reasonably stable at about 70 per cent. If this situation is expected to continue and labour and capital are expected to be taxed at about the same rate, then human wealth should be about 70 per cent of total wealth, with the remaining 30 per cent representing non-human wealth. Accordingly, the average level of the discount rate is chosen to deliver this 70 per cent ratio on average. The required premium is 4.5 per cent at annual rates. The real interest rate on 90-day corporate paper is measured as the nominal rate less the expected rate of inflation. Expected inflation is measured as the one-period-ahead forecast of a univariate time-series model for inflation. Using standard Box-Jenkins techniques, the inflation rate is identified as an MA(1) process in first differences.[11]

Figures 1 and 2 plot net income and the real interest or discount rate over the sample 1956Q1-1990Q1. A cursory glance at Figure 1 reveals that net income is nonstationary in its mean and must therefore be detrended. The real interest rate, in contrast, appears stationary, although more formal tests are required before any firm conclusions may be drawn. The appropriate method of detrending a nonstationary time series has recently received a great deal of attention in macroeconometrics.[12] The two principal classes of nonstationary processes that have been studied are trend stationary and difference stationary processes. The trend stationary process models the nonstationarity as a deterministic function of time. An appropriate detrending procedure is therefore to regress the series on time. The difference stationary model maintains that the series contains unit roots, and appropriate differencing of the data is therefore the correct way to remove nonstationarities.

[11]I also experimented with alternative time-series models for the inflation process, such as an AR(4) model in the level of inflation, and found that the alternatives produced very similar one-period-ahead forecasts to the MA(1) growth-rate specification.

[12]See, for example, Nelson and Plosser (1982), Watson (1986), Wasserfallen (1986), Durlauf and Phillips (1988), and Perron (1987).

The choice of assumptions concerning the form of nonstationarity is very important for the evolution of the expected present value of future net income. If current net income is modelled as the sum of a linear trend and a covariance stationary process, then innovations to net income do not affect the long-run outlook for net income. As a result the expected present value of future net income changes by less than the innovation so, other things being equal, this present value will be smoother than net income. If, however, net income is modelled as a difference stationary process, an innovation to net income changes the long-run outlook for net income because now the trend level shifts. This will result in larger revisions in the expected present value of future income.

Two types of tests are used to determine the appropriate method of detrending — augmented Dickey-Fuller and Phillips-Perron. Both tests entail estimating the regression

$$z_t = \alpha + \beta t + \rho z_{t-1} + \sum_{i=1}^{k} \delta_i (z_{t-i} - z_{t-i-1}) + \epsilon_t. \tag{10}$$

Under the hypothesis that the series z_t has a unit root, $\rho = 1$. On the other hand, if $|\rho| < 1$ and $\beta = 0$, z_t is stationary in levels, while $|\rho| < 1$ and $\beta \neq 0$ suggests trend stationarity. The Dickey-Fuller tests evaluate these alternatives using t- and F-ratios which are compared to the appropriate critical values tabulated in Fuller (1976) and Dickey and Fuller (1981). Since the tests rely on white-noise residuals, enough lagged differences are included to remove any residual autocorrelation. Unit-root tests are also performed using the statistics suggested by Phillips and Perron (1986). The Phillips-Perron statistics are robust to autocorrelated and heteroskedastic residuals, and therefore the estimated equation (10) does not require the inclusion of lagged differences. Since the Phillips-Perron statistics have the same asymptotic distribution as the Dickey-Fuller statistics, they should be compared to the Dickey-Fuller critical values. Both sets of tests are performed with net income measured in logs and the real interest rate in levels. Estimation and test results are reported in Table 1.

Focussing first on (the log of) net income, the results reported in Table 1 provide convincing evidence in favour of the difference stationary model. The estimates of ρ of 0.98 and 0.99 are very close to unity and the hypothesis $\rho = 1$ cannot be rejected at the 95 per cent confidence level on the basis of either the Dickey-Fuller or the Phillips-Perron tests. In addition, tests of the joint hypothesis $\rho = 1$ and $\beta = 0$ also fail to reject this null. In order to test for the presence of additional unit roots, (10) was also estimated with z_t defined as the first difference of the log of net income. The results (not reported) uniformly rejected the presence of a unit root in this first difference specification, suggesting that net income is appropriately modelled as stationary in growth rates.

Turning next to the real interest rate, the statistics reported in Table 1 suggest that the real interest rate is stationary in levels. The estimates of ρ of about 0.8 are more noticeably less than unity, and the hypothesis $\rho = 1$ is rejected at the 95 per cent confidence level. Examining estimates of β we see that the time trend is not significant, so the real interest rate exhibits neither trend nor stochastic growth.

It is worth noting that these findings are generally consistent with the research reported by others. In particular, Nelson and Plosser (1982) and Wasserfallen (1986) both report that real wages in the U.S. are a difference stationary process which will produce a unit root in labour income. Wasserfallen (1986) also finds that *ex post* real interest rates in the U.S., Great Britain, France, Italy, West Germany and Switzerland do not contain unit roots. On Canadian data, Dea and Ng (1989) report that over 80 per cent of output fluctuations are the result of permanent shocks, which is generally consistent with the presence of a unit root in net income.

Having modelled the nonstationarities in the data, the next step is to model the stationary components of net income and the discount rate. The growth rate of net income and the level of the real interest rate are modelled as a vector autoregressive process. Due to computational constraints stemming from the approximation of this vector process as a discrete-valued Markov chain, we are restricted to the case of a first-order VAR.[13] Fortunately this constraint does not seem too serious since the first-order model captures most of the predictive content of past net incomes and real interest rates. Table 2 reports estimated coefficients as well as selected diagnostics for the first-order VAR. Both net income growth and the real interest rate exhibit positive first-order serial correlation, with the interest rate being the more serially dependent of the two. Lagged net income growth is positively signed and significant at conventional levels in the real interest rate equation. The lagged discount rate does not enter significantly into the net income growth equation, but is included nonetheless since it is sensibly signed and of plausible magnitude.

[13]In principle there is no reason why the techniques developed in this paper could not be used in the case of a higher order VAR, except that the resulting state space of the discrete Markov chain approximation outstrips my available computer memory.

3.2 A historical series for human wealth

If we assume, consistent with the data, that net income growth and the real interest rate are stationary random variables, human wealth (H_t) can be expressed as the product of a stationary component and a nonstationary or trend component

$$H_t = E_t \left\{ \sum_{i=1}^{\infty} \left[\prod_{j=1}^{i} \left(\frac{1}{1 + r_{t+j}} \right) \right] X_{t+i} \right\} = \Gamma_t X_t \tag{11}$$

where

$$\Gamma_t = E_t \left\{ \sum_{i=1}^{\infty} \left[\prod_{j=1}^{i} \left(\frac{1 + x_{t+j}}{1 + r_{t+j}} \right) \right] \right\} \qquad \text{and} \qquad x_t = (X_t - X_{t-1})/X_{t-1} \tag{12}$$

The nonstationary or trend component of human wealth is simply the current level of net income. The stationary component Γ_t of human wealth captures the cumulative effect of expected future growth. In a deterministic model with net income growth and the discount rate at their long-run values $\bar{x}$ and $\bar{r}$, this cumulative growth factor may be solved using the geometric series formula

$$\Gamma = \frac{1 + \bar{x}}{\bar{r} - \bar{x}} \quad . \tag{13}$$

To take a concrete example, if $\bar{r} = 1.91$ per cent and $\bar{x} = 0.5$ per cent at quarterly rates, the expected present value of future net income in this deterministic economy is 72 times the current level of quarterly net income (X_t).[14]

Evaluating Γ_t in a stochastic environment is more difficult. Since the expected value of a non-linear function is not the same as the function of the expectation, we cannot solve for Γ_t by separately evaluating the expectations of future x and r. Therefore, we cannot simply replace x_{t+j} and r_{t+j} in (12) with their j-period ahead forecasts and then drop the expectations operator. Instead we are forced to evaluate the expectation in Γ_t directly. The approach to evaluating Γ_t pursued in this study is to approximate the variables x_t and r_t as sets of discrete points and to model their dynamic behaviour as a finite-state, discrete-valued Markov chain. By discretizing net income growth, the discount rate, and their joint distribution, the expected discounted value of future net income growth is solved as a probability weighted sum over possible outcomes, rather than an intractable integral.

The discretization procedure is due to Tauchen (1986) and its application to human wealth is described in detail in Appendix B. The procedure is implemented using 16 point

[14] If per capita net income is say \$10,000 a year or \$2,500 each quarter, per capita human wealth will be \$180,000.

grids for the variables x and r, which yields a system with $16^2 = 256$ possible states. Discrete-valued variables are distinguished from continuous variables with a hat — $\hat{x}$ and $\hat{r}$. The lower panel of Table 2 reports the slope and intercept coefficients of the discrete-valued Markov chain.[15] Comparing these to the regression coefficients of the underlying VAR in the top panel of Table 2, it is apparent that the discrete-valued system closely mimics the statistical properties of the VAR.

Using the discrete-valued system, a closed-form solution for the cumulative growth factor Γ can be obtained which expresses Γ as a function of last period's realization of $\hat{x}$ and $\hat{r}$ and their conditional distribution. From the closed-form solution, the cumulative growth factors associated with each of the 256 states of the system can be computed. A flavour of the results is given below and the details of the closed-form solution are relegated to Appendix C. The cumulative growth factors in every state of the system are conveniently reported in a matrix.

Cumulative Growth Factors in Every State of the System

state $\hat{x}\backslash\hat{r}$		1	2	3	$\cdots$	15	16
	value $\hat{x}\backslash\hat{r}$	0.0005	0.0030	0.0055	$\cdots$	0.0351	0.0376
1	-0.0325	81.11	80.36	79.42	$\cdots$	67.23	66.36
2	-0.0274	81.17	80.38	79.42	$\cdots$	67.23	66.36
3	-0.0224	81.21	80.41	79.43	$\cdots$	67.22	66.37
4	-0.0174	81.27	81.42	79.43	$\cdots$	67.22	66.38
5	-0.0124	81.31	80.44	79.42	$\cdots$	67.22	66.39
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
15	0.0377	81.51	80.45	79.35	$\cdots$	67.31	66.65
16	0.0427	81.51	80.44	79.34	$\cdots$	67.32	66.70

The values for $\hat{x}$ and $\hat{r}$ in the second column and row respectively are the discrete values which net income growth and the discount rate (both measured as quarterly rates) can take on. The states for $\hat{x}$ and $\hat{r}$ are given in the first column and row respectively. For example, suppose $\hat{x}$ is in state 5 and $\hat{r}$ is in state 3. Then current net income growth is -1.24 per cent at quarterly rates, the current discount rate is 0.55 per cent at quarterly rates, and the cumulative growth factor is 79.42.

Comparing the results in different states of the system we see that the higher the discount

[15] These moments are not sample estimates obtained by running a regression. Rather, they are computed directly from the finite-state system since all possible realizations of the discrete variables $\hat{x}$ and $\hat{r}$, as well as their conditional and unconditional distributions, are known.

rate in the current state, the lower the cumulative growth factor. This largely reflects the fact that the discount rate exhibits marked positive serial correlation in the estimated VAR reported in Table 2. Therefore an above-average realization of r_t raises the probability of an above-average realization of r_{t+1}, and thus lowers the expected present value of future net income growth. Reinforcing this own effect is the cross effect stemming from the negative relationship between the current discount rate and future net income growth. Thus, in addition to raising expected future interest rates, an above-average realization of r_t also lowers expected future net income growth, and this reduces the expected present value of future net income growth further. The quantitative response of the cumulative growth factor to a rise in the real interest rate depends on the initial state of the system, which reflects the non-linear nature of the problem. To be more concrete, the response of the cumulative growth factor to a one per cent increase in the real interest rate ranges from -0.9 per cent to -1.4 per cent, with the larger responses coming when r and x are near their means.

The effects of innovations in net income growth on the cumulative growth factor are small compared with the effects in innovations in the discount rate. This reflects the fact that in the case of innovations in net income growth, the own and cross effects work in opposite directions. The bi-variate VAR for x and r indicates that an above-average realization of x_t raises the probability of an above-average realization of x_{t+1} and raises the likelihood of an increase in the discount rate. The rise in expected net income growth raises the cumulative growth factor, but the increase in the expected discount rate lowers it. Quantitatively, the two effects largely offset each other so the net impact of innovations in x_t on the cumulative growth factor is very small and its sign depends on the initial state of the system.

A historical series for the expected present value of future net income is constructed by determining what state of the system the economy was in at each point in time, and then forming the product $\Gamma_t X_t$. The state of the economy is determined by picking the $\hat{x}$ and $\hat{r}$ which are closest to the observed x_t and r_t in each period. In Figure 3 the year-over-year growth rates of the resulting human wealth series and net income are plotted against time. Comparing the growth rates of the expected present value and net income, two features of the human wealth series stand out. First, the broad movements in the growth rate of human wealth follow the cyclical pattern of net income growth. This reflects the fact that the stochastic trend component of human wealth is the level of net income X_t. Second, the growth rate of human wealth exhibits important departures from its trend path. These fluctuations about trend are the result of changes in the cumulative growth factor, and stem largely from movements in the real interest rate. Two episodes in particular highlight the impact of interest rate changes on human wealth. In 1971-72 the growth rate of human wealth was noticeably above the growth rate of net income, while in 1981 the situation was reversed. Comparing Figures 2 and 3 we see that the above-trend growth of human wealth in 1971-72 is coincident with a sharp drop in the real interest rate. Symmetrically, the below-trend growth of human wealth a decade latter coincides with the high real interest rates of 1981.

4 Net assets, government debt and taxes

Having measured the expected present value of future net income, the principal remaining task is to measure domestic and foreign net assets, the domestic and foreign government debt, and current disposable income. The general approach to measuring stocks is to build up from the balance sheets of the various sectors in the economy to obtain the stocks of economic interest. This involves consolidating the assets and liabilities of the various sectors of the economy to obtain the net worth of the ultimate holders of wealth in the economy — households. The quarterly balance sheet data used to measure the stocks of financial and physical assets and debt is obtained by combining the annual stock data reported in the national balance sheet accounts with the quarterly flow data reported in the financial flow accounts. Unfortunately the annual stock and the quarterly flow data do not match up in the sense that the year-to-year change in the stock does not equal the sum of the flows over the year. Accordingly, to obtain a consistent quarterly stock series, the flow data were first adjusted so as to reconcile the flow and the stock data. The adjustment involves first calculating the difference between the four-quarter cumulated flow and the year-to-year change in the stock, and then allocating this difference to each of the four quarters in proportion to the size of the flow in each quarter. Having obtained a consistent quarterly flow series, the variables A_t, e_tF_t, D_t^d and $e_tD_t^f$ are constructed on a real per capita basis by dividing by the CPI and the population. The construction of these variables is described in detail in appendix A. An overview is provided below.

Net domestic and foreign assets $(A_t + e_tF_t)$ is defined as the sum of non-financial and financial assets held by persons and unincorporated business, less the liabilities of this sector, plus the government's capital stock, plus the value of the Canadian and Quebec Pension plans, less the value of domestically held outstanding government debt. Non-financial assets includes residential and non-residential structures, machinery and equipment, consumer durables, inventories and land. Financial assets is defined as the sum of currency and deposits, government debt, corporate bonds, life insurance and pensions, foreign investments, and equity. The principal liabilities of persons and unincorporated business are consumer, mortgage and other loans.

To understand this definition of net assets, it is important to distinguish between the components of $A_t + e_tF_t$ and their sum. This point is well illustrated by the treatment of deposits and government debt. Deposits are a component of $A_t + e_tF_t$, implying that this variable includes inside money. This, however, should not be the case, since inside money should be offset by consumer and business loans. Consumer and business loans are a liability to consumers, either directly or indirectly via their equity holdings in firms. In the case of government debt, $A_t + e_tF_t$ includes the government debt held directly by persons and unincorporated business, and then subtracts the total outstanding stock of domestically held government debt. As a result, both the government debt held directly by households and the government debt held by firms (and thus indirectly by households via their equity holdings) nets out.

For the most part, the national balance sheet data used to measure these assets and liabilities is market value data, but two important exceptions are equity and bonds. Equity in the national balance sheet accounts is measured at "current" value, which is defined as the sum of book value and cumulated retained earnings. Since it is the market value of equity which is the desired variable, the current value of equity reported in the national balance sheet accounts is replaced with a market value measure constructed by scaling the book value of equity by the growth rate of the TSE300 composite stock price index. Bonds are reported in the national balance sheet accounts at book value. In the case of treasury bills, this is not a serious problem since book and market values do not differ substantially for these short-term bonds. In the case of longer maturity federal, provincial and municipal bonds and CSBs, the book value series reported in the national balance sheet accounts is replaced with a market value series which is constructed by multiplying the original book value series by a constructed bond price index. This bond price index is constructed using the present value approach adopted by Rose and Selody (1985).[16] In the case of corporate

[16]The pricing of non-marketable debt such as CSBs is a particularly thorny problem. The approach adopted here is to price CSBs "as if" a market existed for this asset. While this approach to pricing non-marketable

bonds, no comparable market adjustment was made since direct holdings of corporate bonds by persons and unincorporated business is relatively small.

The total outstanding stock of government debt (D_t) is defined as the sum of CSBs, treasury bills, federal, provincial and municipal government bonds, less government bonds held by nonresidents, the various levels of government, public enterprises, and the Bank of Canada. Again, treasury bills are measured at book value since the book value is a good measure of the market value, while the remaining stock of debt is scaled by the constructed government bond price index. Foreign debt ($e_t D_t^f$) is the sum of treasury bills and federal, provincial and municipal government bonds held by non-residents, with a parallel adjustment applied to the reported book value series to obtain a market-value measure. The market value of domestically held debt is simply the market value of the total debt less the market value of foreign debt.

Disposable income depends on labour income (Y_t), government expenditures on goods and services (G_t), and taxes net of transfers (T_t). Since Y_t and G_t are defined above, this leaves only T_t, which is defined as the sum of income, sales, and other taxes from persons less transfers to persons.

5 Alternative time series for wealth

With all the ingredients now in place, creating a time series for wealth is simply a matter of specifying how Ricardian the economy is (i.e., choosing θ), and combining the various components of wealth using the flexible definition given in (7). Unfortunately, it is not obvious what value of θ is appropriate. The degree of truth in the Ricardian equivalence proposition remains very much an open question. At an empirical level, the results emerging from the growing literature testing the predictions of the Ricardian model are very mixed.[17] The essential problem is that the number of historical episodes in which the national debt has changed substantially is small, and most of these are associated with wars, cyclical fluctuations in the level of economic activity, or changes in government expenditures. Since each of these factors can affect wealth and therefore consumption and investment, it has

government debt is popular (see Seater (1981), Cox and Hirschhorm (1983), and Cox and Haslag (1986)) it is not without its drawbacks. See Poitras (1989) and Boothe *et al.* (1989) for a discussion of the potential biases of this approach as well as a possible alternative.

[17]See Barro (1989) and Bernheim (1987,1989) for two very different interpretations of the empirical literature testing the Ricardian equivalence proposition.

proven extremely difficult to evaluate the separate impact of changes in the timing of taxes. My own view is that a value of θ of about 0.75 is appropriate, which may be roughly interpreted as suggesting that the economy is 75 per cent Ricardian. This view is based on my interpretation of the stylized facts regarding liquidity constraints and bequests as they apply to the two-consumer-type model of wealth developed in section 2.

Recall that, literally interpreted, θ is the income-weighted proportion of the population that is liquid, and $1 - \theta$ is the similarly weighted proportion that is liquidity constrained or illiquid. More specifically, $\theta = \eta\gamma$, where η is the proportion of the population that is liquid and γ scales η by the ratio of the average income of liquid consumers to total average income. Since liquidity-constrained consumers are presumably at the bottom of the income distribution, γ is probably greater than one. Attempts to estimate either η or θ in the literature have produced a variety of estimates implying values for θ of between about 0.5 and 1.0. Using U.S. panel data on automobile expenditures, Bernanke (1984) finds no evidence of liquidity constraints, and this despite the lumpy nature of automobiles. On the other hand, Hall and Mishkin (1982), using similar data on food consumption, find 20 per cent of consumers to be liquidity constrained. Hayashi (1985), based on more comprehensive Japanese panel data, estimates that 15 per cent of consumers are liquidity constrained. If we assume that $\gamma > 1$, these estimates of $1 - \eta$ put an upper limit on $1 - \theta$ of 0.20, or a lower limit on θ of 0.80. More recently, Campbell and Mankiw (1989), using aggregate U.S. time series data, estimate θ to be about 0.50. Our own preliminary estimates of Campbell and Mankiw's model using Canadian data suggest a value of θ for Canada between 0.50 and 0.75. My preferred value for θ of 0.75 reflects a compromise between the higher values for θ obtained from micro cross-sectional data and the lower values for θ emerging from aggregate time series data.

In addition to being able to borrow against their future income, liquid agents as defined in the two-consumer-type model are also assumed to make altruistic bequests to their children. A value for θ of 0.75 therefore implies that at least 75 per cent of the population does in fact make positive planned bequests to their offspring. As yet nobody has estimated the proportion of the population that makes altruistically motivated bequests, so there is no firm evidence in favour of this 75 per cent specification. At the same time, the stylized facts that we do have regarding bequests are broadly consistent with the view that a large proportion of the population does plan to make positive bequests, but not the entire population. These stylized facts are as follows.

First, intergenerational transfers are important. Kotlikoff and Summers (1981) present evidence indicating that intergenerational transfers have financed 80 per cent of aggregate U.S. capital accumulation. The remainder is financed by life-cycle savings. Second, counter to the predictions of standard versions of the life-cycle model, the elderly do not appear to run down their wealth during retirement.[18] This stylized fact is consistent with a bequest motive, although bequests are not the only possible explanation.[19] Third, while these features of the data suggest that bequests are both nontrivial and planned, there is also evidence that some consumers cannot make bequests. Evidence from the balance sheets of U.S. citizens reported by Diamond and Hausman (1984) suggests that about 20 per cent of the population arrives at retirement with essentially no bequeathable assets. Comparable data for Canada puts the proportion of the population in this country that arrives at retirement with less than \$1000 of bequeathable assets also at 20 per cent.[20] Since it is likely that liquidity and bequest constraints both apply principally to low-income individuals, this evidence is broadly consistent with my choice of $\theta = 0.75$.

In addition to the $\theta = 0.75$ case, results are also reported for the pure Ricardian case of θ equal to unity and the myopic case of θ equal to zero. To characterize the behaviour of the resulting measures of wealth, selected series are plotted against time and various summary statistics are reported. In addition, in order to get a better understanding of the behaviour of the alternative wealth series, each alternative is broken down into its human and non-human components. The human component is defined as labour income net of taxes and government expenditures weighted by θ and $(1-\theta)$ respectively — $(Y_t - ((1-\theta)T_t + \theta G_t))$ — plus the expected present value of net income weighted by θ.[21] The non-human component is the sum of net assets $A_t + e_t F_t$ and the net wealth component of government debt $D_t^d - \theta(D_t^d + e_t D_t^f)$. To provide a frame of reference, the wealth series are compared to per capita real output.

Figure 4 plots the level of total wealth for my preferred specification of $\theta = 0.75$; a

[18] See Mirer (1979) for a review of the U.S. evidence. For some Canadian evidence, see King and Dicks-Mireaux (1982), and Robb and Burbidge (1989). For a contrary view see Hurd (1987).

[19] Davies (1981) presents evidence suggesting that the failure of the elderly to dissave during retirement is consistent with a life-cycle model with uncertainty regarding the length of life and risk averse agents.

[20] This statistic is obtained from Table 8, p.73 of *The Distribution of Income and Wealth*, Statistics Canada, 1977.

[21] Note that there is a difference between the human component of total wealth and human wealth. Human wealth is defined to be the expected present value of future net income. The human component of total wealth is human wealth weighted by θ plus current labour income less a weighted average of current taxes and government expenditures $(Y_t - ((1-\theta)T_t + \theta G_t))$.

complete listing of the series and its components is provided in Appendix D. In Figure 5 the year-over-year growth rate of this total wealth series is plotted together with the year-over-year growth rate of per capita real output. Comparing these growth rates, we see that wealth tracks real output reasonable closely but exhibits larger cyclical fluctuations than real output. From Figures 6 and 7 it is apparent that this observed relationship between total wealth and real output stems from both the human and non-human components of wealth. The growth rates of the human and non-human components both follow the broad movements in real output growth. In addition, the growth rates of both components are more variable than is real output growth.[22] The variability of the human component stems largely from fluctuations in the expected present value of future net income, the sources of which are discussed in section 3. The principal source of volatility in the non-human component comes from the market value of equities, which makes up about 22 per cent of non-human wealth on average. The dramatic decline in non-human wealth during the 1981-82 recession largely reflects a large and prolonged drop in stock prices over this period. The importance of equity is also illustrated by the sudden but less severe drop in non-human wealth which results from the October 1987 stock market crash. Other large components of non-human wealth which exhibit important cyclical fluctuations are the value of residential housing, deposits and debt. Returning to Figure 5, notice that in addition to the larger cyclical movements, the growth rate of total wealth also exhibits important high frequency movements as compared to the growth rate of real output. These high frequency fluctuations come from the expected present value of future net income and are the result of fluctuations in the real interest rate.

Table 3 reinforces the conclusions drawn from Figures 4-7. Focussing still on the case of $\theta = 0.75$, note that the standard deviations of the growth rates of total wealth and its components exceed the standard deviation of per capita real output growth by more than

[22]The finding that wealth is more variable than current income has important implications for consumption behaviour. Friedman's (1957) Permanent Income Hypothesis (PIH) was invented to explain the fact that consumption is smoother than income, but if wealth or permanent income is more variable than current income, the PIH is turned on its head. With a unit root in the income process, the problem of "excess" variability of consumption relative to income (eg., Flavin, 1981) is replaced with the problem of "insufficient" variability of consumption relative to income (see Mankiw and Shapiro, 1985, and Deaton, 1986). Christiano (1986) offers a potential solution to this insufficient-variability puzzle by demonstrating that in an artifical economy the smoothness of consumption relative to income can be squared with the PIH and a unit root in the income process once the associated general equilibrium movements in real interest rates are taken into account. Whether or not the wealth measure constructed in the current study is useful in explaining consumption is largely an empirical question.

50 per cent. In addition the standard deviations of the growth rates of the human and non-human components are almost equal, so both components of wealth exhibit about the same variability.

Table 3 also provides statistical information on wealth and its components as measured using the Ricardian ($\theta = 1$) and myopic ($\theta = 0$) definitions. The means, standard deviations, and correlations with real output growth reported in the table reveal that wealth for $\theta = 1$ and $\theta = 0.75$ exhibit very similar statistical properties, whereas the behaviour of wealth in the case of $\theta = 0$ is somewhat different. This conclusion is confirmed by the last column of Table 3, which reports the correlations of the growth rates of wealth for $\theta = 0$ and $\theta = 0.75$ with the Ricardian case of $\theta = 1.0$. The cross correlation between the Ricardian definition of total wealth and the $\theta = 0.75$ case is 0.997 as compared to a correlation of only 0.523 between the Ricardian and myopic definitions. The *very* high correlation between the $\theta = 1.0$ and 0.75 wealth measures suggests that, as a practical matter, whether we assume 70, 80, 90 or 100 per cent of agents are liquid, the growth rates of wealth as measured in this study will not be greatly affected. At the same time, the much lower correlations between the myopic and Ricardian wealth measures suggest that the behaviour of definitions of wealth which take very different views on the proportion of the population that is liquid will be markedly different.

6 Concluding remarks

This paper defines wealth as a weighted sum of net domestic and foreign assets, domestic and foreign government debt, labour income, taxes, government expenditures, and the expected present value of future net income. The expected present value of future net income is measured using a time-series approach. Due to the non-linear nature of this present value, expectations are evaluated by approximating the time series model as a finite-state, discrete-valued Markov chain. Non-human wealth is measured using a balance-sheet approach which permits physical and financial assets to be measured at market value.

The measures of wealth have several interesting and plausible features. First, the failure to reject the presence of a unit root in the net income process implies that shocks to the level of net income are expected to be permanent. As a result the broad movements in human wealth are dominated by their trend component (net income), which is strongly procyclical. At the same time, fluctuations in real interest rates are found to produce important departures of human wealth from its stochastic trend. Non-human wealth is also procyclical. As a result, total wealth tracks the business cycle closely with smaller high-frequency fluctuations due to the affect of changes in the discount rate applied to future net income. The importance of both income and interest rate effects on wealth suggests that the methods developed in this paper provide a promising approach to measuring wealth. The litmus test will be the ability of human and non-human wealth to explain behaviour.

Our immediate plans are to examine the ability of wealth to explain consumption behaviour. Of particular interest will be the effects of interest changes on consumption through their impact on human wealth. The estimation of permanent-income consumption functions may also provide an opportunity to obtain additional evidence regarding the value of θ. In particular, it may be possible to estimate θ by including the various components of wealth as separate explanatory variables in a consumption equation and imposing the restrictions implied by the flexible definition of wealth. In addition, the results of the consumption study may well suggest possible improvements to the measurement of wealth. In particular, many difficult decisions have had to be made regarding the choice of data and the appropriate filter with which to adjust the raw data.

With this in mind, it would certainly be premature to draw any firm inferences from the wealth series constructed in this study. Nonetheless, it is difficult to resist the temptation to put them to work. In the week from October 16 to October 23, 1987, the TSE300 stock price index fell 17 per cent, and the 90-day treasury bill interest rate fell 157 basis points. Longer maturity interest rates also fell sharply, with the size of the drop depending on the term to maturity. These events no doubt had important effects on wealth. Using the $\theta = 0.75$ specification for wealth, the 17 per cent drop in the TSE300 is predicted to have caused a 5.6 per cent drop in non-human wealth. If interest rates had remained unchanged, this would have resulted in a 1.7 per cent drop in total wealth. The principal impact on wealth of lowering interest rates is to raise the expected present value of future earnings.[23] A 157 basis point decline in the treasury bill rate is predicted to increase human wealth by about 1.6 per cent.[24] This translates into a 1.1 per cent increase in total wealth so the net effect

[23]Falling interest rates also raise bond prices, thereby increasing non-human wealth. However, with $\theta = 0.75$ this effect is small since only 25 per cent of government debt is considered net wealth, and because much of the increase in the value of domestic debt is offset by a rise in the value of foreign debt.

[24]At least two caveats are in order here. First this predicted response holds expected inflation constant. Second, the analysis in this study is based on quarterly data. Some care must therefore be exercised in attempting to use the predictions at a weekly frequency.

on wealth is a decline of 0.6 per cent. Over the week of the crash, the drop in interest rates therefore offset about two-thirds of the impact of falling stock prices on total wealth. To put a 0.6 per cent drop in wealth in historical perspective, with the onset of recession in 1981 wealth fell by 8.1 per cent over the year beginning in 1980Q3 and in a single quarter it dropped by as much as 3.6 per cent. Viewed in this light, perhaps it is not surprising that the stock market crash of October 1987 did not have significant real effects.

Table 1

Tests for Unit Roots and Deterministic Trends

$$z_t = \alpha + \beta t + \rho z_{t-1} + \sum_{i=1}^{k} \delta_i(z_{t-i} - z_{t-1-i}) + \epsilon_t$$

	Augmented Dickey-Fuller		*Phillips-Perron*	
	log of net income 57:2-89:4	*discount rate* 56:4-89:4	*log of net income* 56:2-89:4	*discount rate* 56:2-89:4
	coefficient estimates and diagnostics			
α	0.200	0.006	0.112	0.006
β	1.3×10^{-4}	5.9×10^{-5}	5.0×10^{-5}	5.7×10^{-5}
ρ	0.975	0.797	0.986	0.805
DW	1.98	1.99	1.40	1.94
Q(33)	23.0	33.2	49.8	35.3
k	3	1	0	0
hypothesis	test statistics			
$\rho = 1$ [-3.45]	-1.86	-3.65	-2.28	-3.47
$\beta = 0$ [2.79]	1.67	1.85	1.56	1.84
$\rho = 1, \beta = 0$ [6.49]	1.76	6.76	1.20	6.83

Terms in square brackets are the Dickey-Fuller 95% confidence critical values for samples of 100 observations. DW is the Durbin-Watson statistic and Q(33) is the Box-Ljung Q statistic calculated with 33 lagged residuals.

Table 2

Estimated VAR for Net Income Growth and the Discount Rate and its Vector Markov Chain Approximation

Estimated VAR: 56:2 - 89:4

$$x_t = \underset{(1.52)}{0.0053} + \underset{(3.74)}{0.3046x_{t-1}} - \underset{(-0.51)}{0.0896r_{t-1}}$$

$$r_t = \underset{(3.01)}{0.0026} + \underset{(2.31)}{0.0492x_{t-1}} + \underset{(17.51)}{0.8502r_{t-1}}$$

x-equation	*r-equation*
$\bar{R}^2 = 0.097$	$\bar{R}^2 = 0.709$
DW = 2.15	DW = 1.93
Q(33) = 30.6	Q(33) = 33.2
st.dev. resid. = 0.011924	st.dev. resid. = 0.003206

Statistical Properties of Discrete-Valued Vector Markov Chain

$$\hat{x}_t = 0.0051 + 0.3038\hat{x}_{t-1} - 0.0916\hat{r}_{t-1}$$

$$\hat{r}_t = 0.0022 + 0.0492\hat{x}_{t-1} + 0.8697\hat{r}_{t-1}$$

Bracketed terms below coefficients in estimated VAR are t-statistics.

Table 3

Selected Moments for the Growth Rates of Alternative Measures of Wealth

variable(year-over-year growth)	*mean*	*st.dev.*	*corr. real output*	*corr.* $\theta = 1$
real output per capita	2.23	2.37	1.00	-
labour income (y)	2.25	2.78	0.66	-
net income (x)	2.27	3.62	0.63	-
total wealth				
$\theta = 1.00$	2.21	3.84	0.43	1.000
$\theta = 0.75$	2.27	3.69	0.45	0.997
$\theta = 0$	2.81	3.71	0.48	0.523
human component				
$\theta = 1.00$	2.03	4.32	0.33	1.000
$\theta = 0.75$	2.03	4.31	0.33	1.000
$\theta = 0$	2.01	3.35	0.30	0.274
non-human component				
$\theta = 1.00$	2.95	4.64	0.49	1.000
$\theta = 0.75$	2.92	4.37	0.49	0.997
$\theta = 0$	2.84	3.80	0.48	0.951

The first three columns report the means, standard deviations and cross correlations with per capita year-over-year real output growth. The last column reports the correlation between the Ricardian wealth measure in each category ($\theta = 1$) and the $\theta = 0.75$ and $\theta = 0$ wealth measures.

Figure 1
Quarterly Net Income at Annual Rates
(per capita, constant 1986 dollars)

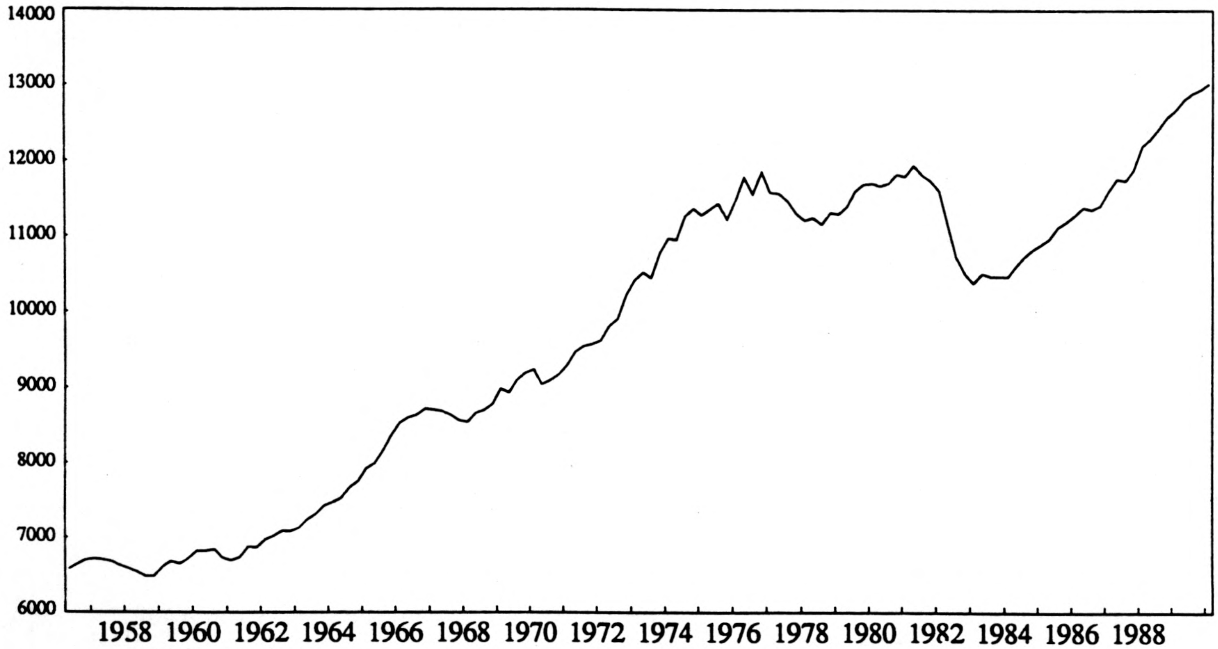


Figure 2
Discount Rate at Annual Rates

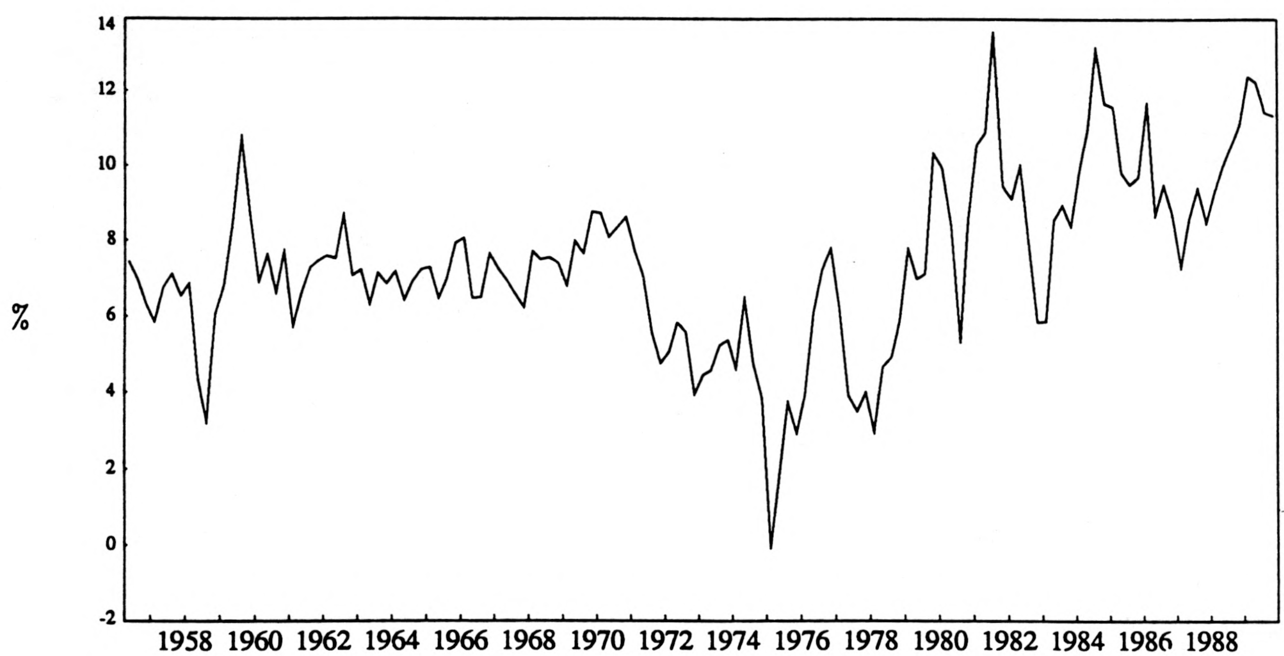


Figure 3
Year-Over-Year Growth Rates of Net Income and Human Wealth

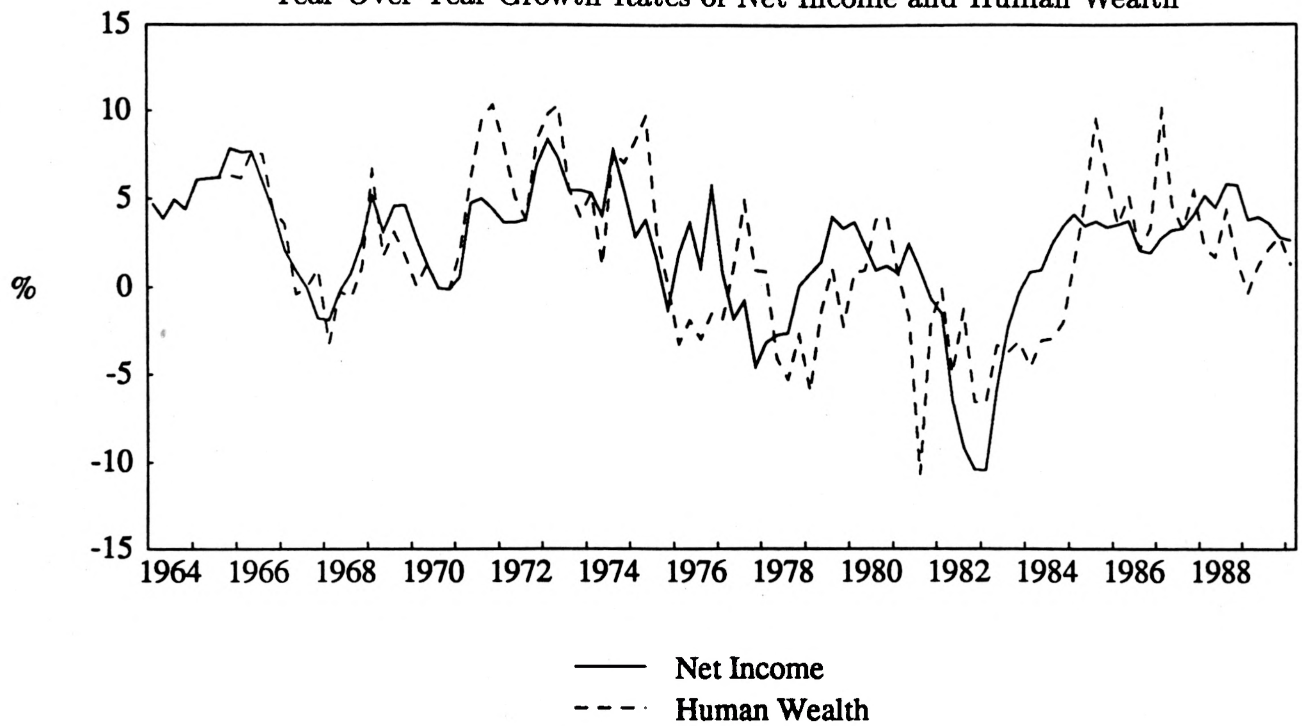


Figure 4
Total Wealth
(per capita, constant 1986 dollars)

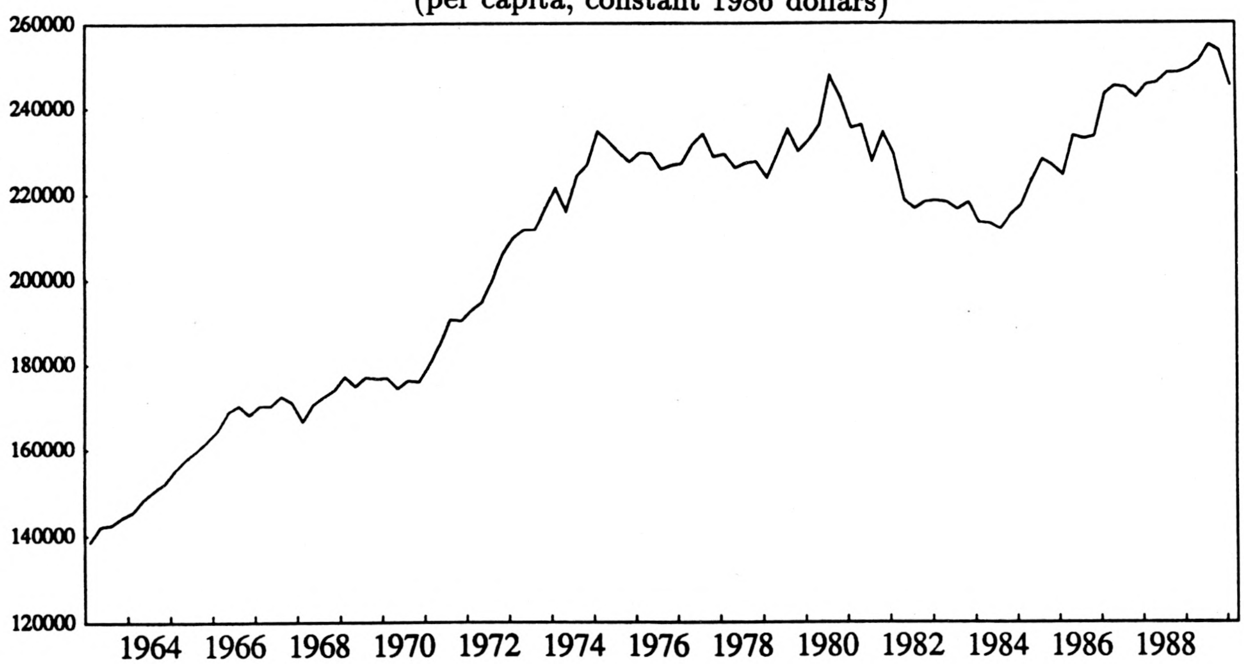


Figure 5
Year-Over-Year Growth Rates of Real
Per Capita Output and Total Wealth

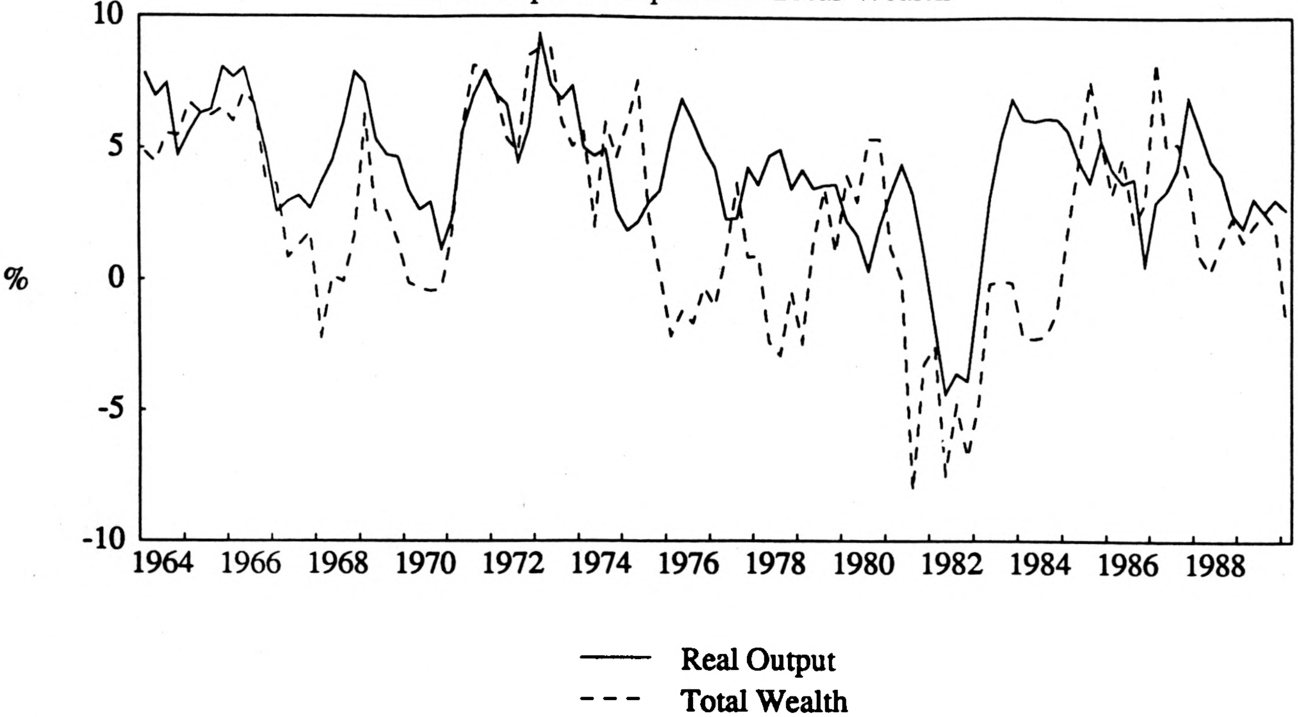


Figure 6
Year-Over-Year Growth Rates of Real
Per Capita Output and the Human Component of Wealth

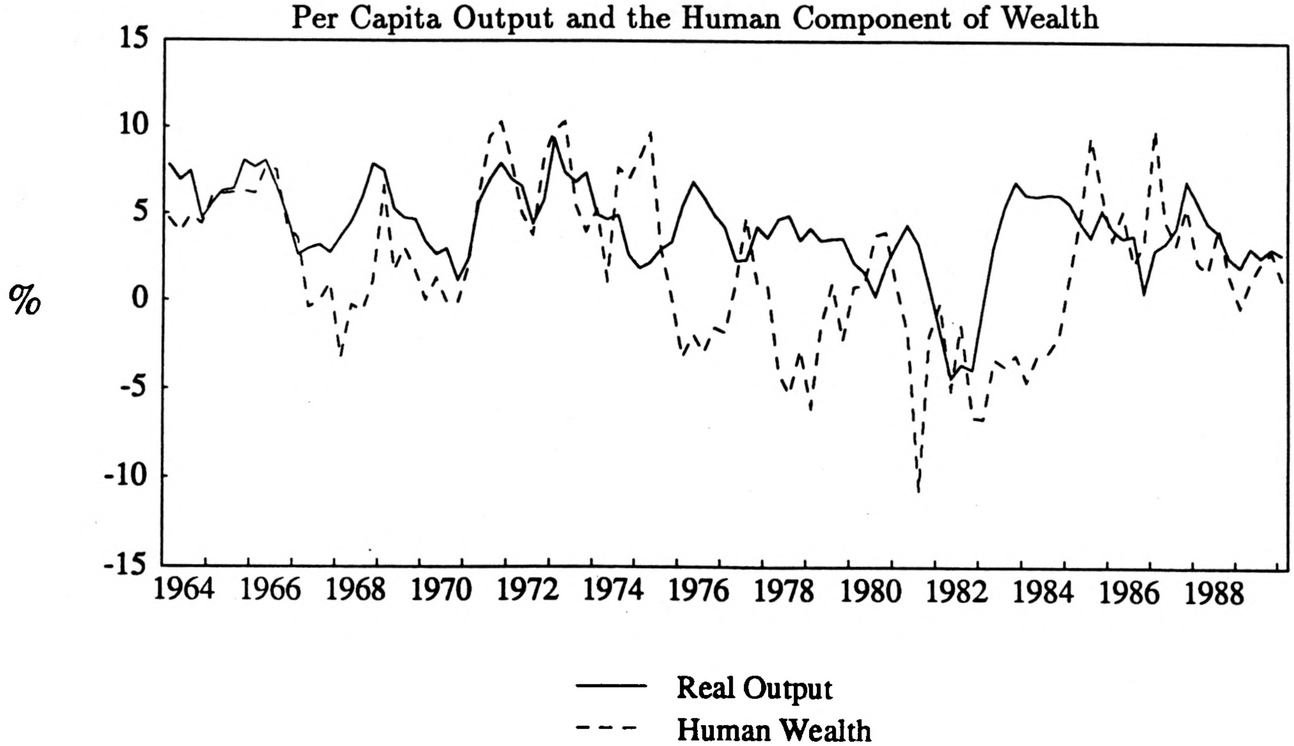
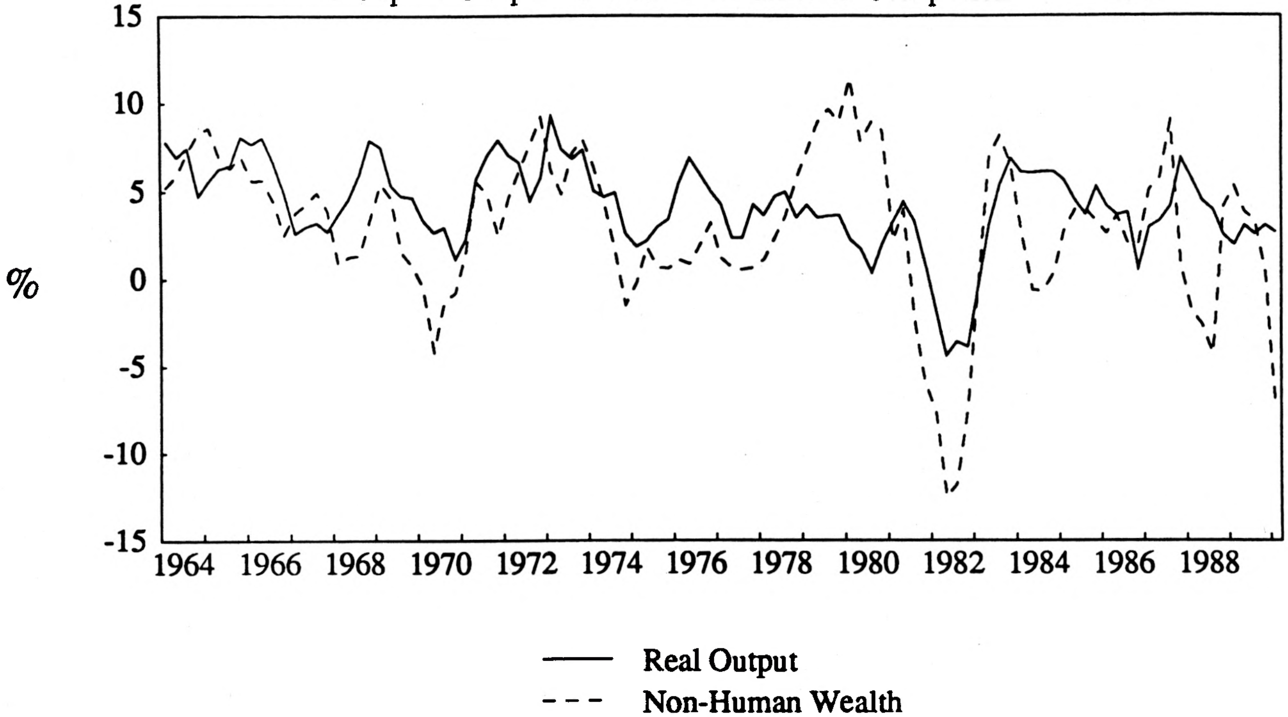


Figure 7
Year-Over-Year Growth Rates of Real
Per Capita Output and the Non-Human Component of Wealth



Appendix A
The Data

1. Basic Series

CPI = consumer price index, all items, average of seasonally adjusted monthly index. CANSIM B820000.

$NPOP$ = population in millions. RDXF. (RDXF refers to the data base for the Bank of Canada's large-scale macroeconometric model RDXF.)

2. Labour Income

Y = real per capita labour income, quarterly, seasonally adjusted.
$= (100 \times LBINC)/(POP \times CPI \times 4)$

$LBINC$ = labour income in millions of current dollars, seasonally adjusted, quarterly at annual rates.
$= YW + [YW/(YGDP - YENAR)] \times [YFA + YNFNC]$

YW = labour income (wages, salaries and supplementary labour income) in millions of current dollars. RDXF

$YGDP$ = gross domestic product in millions of current dollars. RDXF.

$YENAR$ = residual error. RDXF

YFA = farm income in millions of current dollars. RDXF.

$YNFNC$ = unincorporated business income in millions of current dollars. RDXF.

3. Discount Rate

r = discount rate.
$= (R90 + PREMIUM)/400 - E_t(\pi_{t+1})$

$R90$ = 90-day interest rate on prime corporate paper.

π_t = rate of inflation.
$= \ln(CPI_t) - \ln(CPI_{t-1})$

$E_t(\pi_{t+1})$ = one period ahead forecast of an ARIMA process for π_t. On the basis of standard Box-Jenkins techniques, π_t is found to be well described by an MA(1) process in first differences. The estimated model is $(\pi_t - \pi_{t-1}) = -0.46\epsilon_{t-1} + \epsilon_t$.

4. Taxes Net of Transfers

$T =$ real per capita taxes net of transfers.
$= (100 \times NETAX)/(CPI \times NPOP \times 4)$

$NETAX = INCTAX + SALTAX + OTHTAX - TRANF$

$INCTAX =$ direct taxes from persons. CANSIM D20155.

$SALTAX =$ indirect taxes. CANSIM D20158.

$OTHTAX =$ other current transfers to persons. CANSIM D20159.

$TRANF =$ transfer payments to persons. CANSIM D20163.

5. Government Expenditures

$G =$ real per capita government expenditures on goods and services.
$= (100 \times GOVS)/(CPI \times NPOP \times 4)$

$GOVS =$ 4 quarter moving average of GOV.
$= [GOV + GOV_{-1} + GOV_{-2} + GOV_{-3}]/4$

$GOV =$ reported government expenditures series less historical proportion of government expenditures which are not paid for by households.
$= GOVEXP(1 - \Omega)$.

$GOVEXP =$ government expenditures on goods and services. CANSIM D20162.

$\Omega =$ rolling historical average proportion of government expenditures that are paid for by corporations, non-residents and government interest income.
$= [\omega + \omega_{-1} + \omega_{-2} + \omega_{-3} + \omega_{-4} + \omega_{-5} + \omega_{-6} + \omega_{-7}]/8$

$\omega = (CPTAX - CPSUB - CPTRAN) - (NRSTAX - NRSTR) - GOVIN$

$CPTAX =$ direct taxes from corporations and government enterprises. CANSIM D20156.

$CPSUB =$ transfer payments to business (subsidies). CANSIM D20164.

$CPTRAN =$ transfer payments to business (capital assistance). CANSIM: D20165.

$NRSTAX =$ direct taxes from nonresidents. CANSIM D20157.

$NRSTR =$ transfer payments to nonresidents. CANSIM D20166.

$GOVIN =$ government's investment income. CANSIM: D20160.

6. Debt

$D^d =$ real per capita government debt at market value held by Canadians.
$= (100 \times DDEBT\$)/(CPI \times NPOP)$

$DDEBT\$ =$ domestically held government debt at market value.
$= TBILL - FORTBQ + PBG \times (OTHDEBT - FORGBQ - FORPBQ - FORMBQ)$

$TBILL =$ net Treasury Bills outstanding at book value.
$= TTBQ - GTBQ - PTBQ - PFITBQ - GETBQ - BCTBQ$

$OTHDEBT =$ net other debt outstanding at book value.
$= TGBQ + TPBQ + TMBQ - GGBQ - GPBQ - GMBQ - PGBQ - PPBQ - PMBQ - PFIGBQ - PFIPBQ - PFIMBQ - GEGBQ - GEPBQ - BCGBQ$

$eD^f =$ real per capita government debt at market value held by nonresidents.
$= (100 \times FDEBT\$)/(CPI \times NPOP)$

$FDEBT\$ =$ market value of government debt held by nonresidents.
$= FORTBQ + PBG \times (FORGBQ + FORPBQ + FORMBQ)$

$DEBT =$ net total government debt at book value.
$= TBILL + OTHDEBT$

$\_GBQ =$ Government of Canada bonds.

$\_PBQ =$ provincial government bonds.

$\_MBQ =$ municipal government bonds.

$\_TBQ =$ treasury bills.

$T\_ =$ total outstanding stock.

$FOR\_ =$ holdings by non-residents.

$G\_ =$ holdings of the Government of Canada.

$P\_ =$ holdings of other levels of government and hospitals.

$M\_ =$ holdings of municipal governments.

$PFI\_ =$ holdings of public financial institutions.

$GE\_ =$ holdings of government enterprises.

$BC\_ =$ holdings of the Bank of Canada.

$PBG =$ price index for government bonds. (This price index is constructed following the present value approach adopted in SAM with a slight modification in recognition of the quarterly frequency of the data. See Rose and Selody (1985) pp. 239-241 for the motivation of this price index.)
$= RAC/RG + (1 - RAC/RG)\exp[-RG \times (1 - RATAX) \times TM]$

$RG =$ average yield to maturity on 3-5 year Government of Canada Bonds.

$TM =$ average yield to maturity of debt (set to 19.6 quarters which is 4 times the average term to maturity of 4.9 years used in by Rose and Selody, 1985).

$RAC =$ average coupon rate.

$= RAC_{-1}$ if $(DIFDEBT + 1/TM) < 0$

$= RAC_{-1} \times [(DEBT_{-1}/DEBT) - 1/TM] + RG \times [((DEBT - DEBT_{-1})/DEBT) + 1/TM]$ otherwise

$RATAX =$ average tax rate $= INCTAX/LBINC$.

$DIFDEBT = (DEBT - DEBT_{-1})/(DEBT + DEBT_{-1})/2$

CANSIM #'s

$TGBQ$: D162898,D151980	$TPBQ$: D162899,D151981
$TMBQ$: D162900,D151982	$TTBQ$: D162894,D151976
$FORGBQ$: D162749,D151814	$FORPBQ$: D162750,D151815
$FORMBQ$: D162751,D151816	$FORTBQ$: D162745,D151811
$GGTBQ$: D162185,D151482	$GPBQ$: D162190,D151549
$GMBQ$: D162191,D151488	$GTBQ$: D162185,D151482
$PGBQ$: D162259,D151548	$PPBQ$: D162260,D151549
$PMBQ$: D162261,D151550	$PFIGBQ$: D161979,D151330
$PFIPBQ$: D161980,D151331	$PFIMBQ$: D161981,D151332
$PFITBQ$: D161975,D151326	$GEGBQ$: D160159,D150151
$GEPBQ$: D160160,D150152	$GEMBQ$: D160161,DD15053
$GETBQ$: D160155,D150147	$BCGBQ$: D160439,D150353
$BCTBQ$: D160435,D150350	RG: B14010

7. Net Domestic and Foreign Assets

$A + eF =$ net domestic and foreign assets excluding government debt.

$=(100 \times NETWORTH)/(CPI \times NPOP)$

$NETWORTH = NFINAQ + FINAQ - EQUITY + EQUITY\$ - HDEBT + HDEBT\$ - HLIABQ + KGGQ + KGPQ + KGMQ + NNWPFIQ + NNWGEQ + CPPQ + QPPQ - DDEBT\$$

$NFINAQ =$ Non-financial assets of persons and unincorporated business. CANSIM D160063, D150041. (Includes residential and non-residential structures, machinery and equipment, consumer durables, inventories and land.)

$FINAQ =$ Financial assets of persons and unincorporated business. CANSIM D160000, D150046. (Includes currency and deposits, consumer credit, treasury bills, finance and other short-term paper, federal, provincial and municipal Government bonds, other Canadian bonds, life insurance and pensions, shares ($EQUITY$), foreign investments and other financial assets.

$EQUITY =$ "current" value of shares held by persons and unincorporated business. "Current" value is measured as the sum of book value and cumulated retained earnings. CANSIM D160027,D150067.

$EQUITY\$$ = market value of equity held by persons and unincorporated business.
$=EQUITY\$_{-1} \times TSE/TSE_{-1} + [BEQUITY - BEQUITY_{-1}]$
$EQUITY\$$ was cumulated up starting in 1962Q4 with $EQUITY\$$ set equal to $BEQUITY$ for the initial observation.

TSE = TSE 300 Composite stock price index. CANSIM B4237.

$BEQUITY$ = book value of equity held by persons and unincorporated business.
$= EQUITYQ - YCR/4 \times (EQUITYQ/TEQUITYQ)$
$BEQUITY$ adjusts $EQUITYQ$ by removing the cumulated retained earnings accruing to persons and unincorporated business in order to get back to book value.

$TEQUITY$ = total outstanding stock of equity. CANSIM D162906,D151988.

YCR = total retained earnings. RDXF

$EQUITYQ$ = total shares outstanding. CANSIM D162906,D151988.

$HDEBT$ = book value of government debt held by persons and unincorporated business. CANSIM: D160015,D150035; D160019,D150062; D160020,D150063; D160021, D150064.
$= HTBQ + HGBQ + HPBQ + HMBQ$

$HDEBT\$$ = market value of government debt held by persons and unincorporated business.
$= HTBQ + PBG \times (HDEBT - HTBQ)$

$HLIABQ$ = total liabilities of persons and unincorporated business. CANSIM D160031,D150050. (Includes consumer credit, trade payables, bank loans, other loans, finance and other short-term paper, other Canadian bonds and mortgages.)

KG_Q = quarterly capital stock series for the Government of Canada (G), provincial governments (P), and municipal governments (M). These variables are linear interpolations of annual series. CANSIM D883476,D883508, D883540.

$NWGEQ$ = networth of government enterprises. CANSIM: D160172, D150131.

$NNWGEQ = NWGEQ - GEGBQ - GEPBQ - GEMBQ - GETBQ$

$NWPFIQ$ = networth of public financial institutions. sourcs:CANSIM D161992,D151310.

$NNWPFIQ = NWPFIQ - PFIGBQ - PFIPBQ - PFIMBQ - PFITBQ$

$CPPQ$ = Canadian Pension Plan. CANSIM D162590,D151760

$QPPQ$ = Quebec Pension Plan CANSIM D162660,D151784.

Appendix B

Approximating a Vector Autoregression as a Finite-State, Discrete-Valued Markov Chain

This appendix describes the implementation of Tauchen's (1986) procedure for approxiating a vector autoregression as a finite-state, discrete-valued Markov chain. Write the bi-variate VAR for x and r in matrix notation as

$$\boldsymbol{z}_t = \boldsymbol{\beta} + \boldsymbol{A}\boldsymbol{z}_{t-1} + \boldsymbol{\epsilon}_t \tag{B.1}$$

where $\boldsymbol{z}_t$ is the 2×1 vector $[x_t \quad r_t]'$, $\boldsymbol{\beta}$ is the 2×1 vector of intercept terms, $\boldsymbol{A}$ is the 2×2 matrix of slope coefficients, and $\boldsymbol{\epsilon}_t$ is a 2×1 vector white noise process. Assume that the elements of $\boldsymbol{\epsilon}_t$, denoted as $\epsilon_{i,t}$ for $i = x, r$, are mutually independent so the variance-covariance matrix for $\boldsymbol{\epsilon}_t$, call it $\boldsymbol{\Sigma}_\epsilon$, is a diagonal matrix: $\boldsymbol{\Sigma}_\epsilon = \text{diag}[\sigma_{\epsilon,x}, \sigma_{\epsilon,r}]$. In addition, let F denote the standard normal distribution function for the normalized disturbances $\epsilon_{it}/\sigma_{\epsilon,i}$.

Consider a discrete-valued vector process $\hat{\boldsymbol{z}}_t$ which approximates the real-valued process in $\boldsymbol{z}_t$. The components of $\hat{\boldsymbol{z}}_t$, denoted by $\hat{z}_{it}$, each take on one of N_i values in the grid $\hat{Z}_i = \{\hat{z}_i^1, \hat{z}_i^2, \ldots, \hat{z}_i^{N_i}\}$. To select a suitable grid Tauchen suggests the following procedure. Let the last point in the grid for variable $i = x, r$ equal a multiple m of the unconditional standard deviation of z_i plus the unconditional mean of z_i

$$\hat{z}_i^{N_i} = m\sigma_{iz} + \mu_{iz}.$$

Symmetrically, the first point in the grid is defined to be

$$\hat{z}_i^1 = -m\sigma_{iz} + \mu_{iz}.$$

The σ_{iz} are the square roots of the diagonal elements of $\boldsymbol{\Sigma}_z$, the variance-covariance matrix for z_t. Since $\boldsymbol{\Sigma}_z$ satisfies $\boldsymbol{\Sigma}_z = \boldsymbol{A}\boldsymbol{\Sigma}_z\boldsymbol{A}' + \boldsymbol{\Sigma}_\epsilon$, the variance-covariance matrix $\boldsymbol{\Sigma}_z$ can be found by iterating on $\boldsymbol{\Sigma}_z(r) = \boldsymbol{A}\boldsymbol{\Sigma}_z(r-1)\boldsymbol{A}' + \boldsymbol{\Sigma}_\epsilon$. The μ_{iz} are the unconditional means of the variables z_{it}, and may be obtained as the elements of the vector $\boldsymbol{\mu}_z = (\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{\beta}$. To obtain the remaining points in the grid, define a step $s_i = (\hat{z}_i^{N_i} - \hat{z}_i^1)/(N_i - 1)$ and let $\hat{z}_i^j = \hat{z}_i^1 + (j-1)s_i$. This amounts to spacing the $\hat{z}_i^j$ evenly between the two end points of the grid, $\hat{z}_i^1$ and $\hat{z}_i^{N_i}$.

Having made the state space for the random vector $\boldsymbol{z}_t$ discrete, the next step is to reduce the distribution for $\boldsymbol{z}_t$ to a set of discrete points. There are $N^* = N_x \times N_r$ possible states of the system. Index these states by $k = 1, \ldots, N^*$. For each i let

$$p_i(k,j) = \text{prob}[\hat{z}_{it+1} = \hat{z}_i^j | \hat{\boldsymbol{z}}_t = \hat{\boldsymbol{z}}(k)]$$

where $\hat{\boldsymbol{z}}(k)$ is the vector $\hat{\boldsymbol{z}}$ when the system is in state k. Since there are N^* states of the system and N_i possible states for $\hat{z}_i$, these p_i's form an $N^* \times N_i$ matrix. To obtain these probabilities, Tauchen suggests the following. For each j from 1 to N_i and each k from 1 to N^*, let

$$p_i(k,j) = \begin{cases} F\left(\frac{\hat{z}_i^j - [\beta_i + \boldsymbol{A}_i \hat{\boldsymbol{z}}(k)] + s_i/2}{\sigma_{i\epsilon}}\right) - F\left(\frac{\hat{z}_i^j - [\beta_i + \boldsymbol{A}_i \hat{\boldsymbol{z}}(k)] - s_i/2}{\sigma_{i\epsilon}}\right) & \text{if } 2 \le j \le N_i - 1 \\ F\left(\frac{\hat{z}_i^1 - [\beta_i + \boldsymbol{A}_i \hat{\boldsymbol{z}}(k)] + s_i/2}{\sigma_{i\epsilon}}\right) & \text{if } j = 1 \\ 1 - F\left(\frac{\hat{z}_i^{N_i} - [\beta_i + \boldsymbol{A}_i \hat{\boldsymbol{z}}(k)] - s_i/2}{\sigma_{i\epsilon}}\right) & \text{if } j = N_i \end{cases} \tag{B.2}$$

where $\boldsymbol{A}_i$ and β_i are the i-th rows of $\boldsymbol{A}$ and $\boldsymbol{\beta}$ respectively. In order to gain some intuition as to how Tauchen's procedure works, focus on the case of $2 \le j \le N_i - 1$. Note that $\hat{z}_i^j - [\beta_i + \boldsymbol{A}_i \hat{\boldsymbol{z}}(k)]$ is the difference between the value of $\hat{z}_i$ in state j and the value of $\hat{z}_i$ that the VAR model predicts for next period. If state j is close to the state for $\hat{z}_i$ predicted by the VAR, the difference between $\hat{z}_i^j$ and $\beta_i + \boldsymbol{A}_i \hat{\boldsymbol{z}}(k)$ will be small so the distribution function will be evaluated near the mean of z_i. More specifically, the distribution function will be evaluated at *two* points near the mean of z_{it}. These two points correspond to half a step ($\frac{s_i}{2}$) above and below $\hat{z}_i^j - [\beta_i + \boldsymbol{A}_i \hat{\boldsymbol{z}}(k)]$. Since the normal distribution function is steepest near its mean, the difference between the value of F at these two points will be large, thereby producing a large conditional probability ($p_i(k,j)$) of going from state k to state j. If, on the other hand, state j is not at all close to the state predicted by the VAR, the difference between $\hat{z}_i^j$ and $\beta_i + \boldsymbol{A}_i \hat{\boldsymbol{z}}(k)$ will be large. The distribution function will therefore be evaluated at two points which are far away from the mean of z_i. Since the distribution function is relatively flatter far away from the mean, the difference between the value of F at the two points will be small, so the conditional probability of going from state k to state j will now be small.

In the univariate case the $p(k,j)$'s are the transition probabitities since there is only one variable in the system. In the multivariate case one more step is required to obtain the joint probabilities associated with all the possible combinations of the points in each grid. Since the ϵ_{it}'s are mutually independent, the transition probabilities for the state vector $\hat{\boldsymbol{z}}$ are the products of the appropriate p_i's.

$$\begin{aligned} \phi_{k,l} &= \text{prob}[\hat{\boldsymbol{z}}_{t+1} = \hat{\boldsymbol{z}}(l) | \hat{\boldsymbol{z}}_t = \hat{\boldsymbol{z}}(k)] \\ &= \prod_{i=1}^{M} p_i(k,j) \end{aligned}$$

where l indexes the N^* states in period $t+1$. These $\phi_{k,l}$'s form an $N^* \times N^*$ matrix $\boldsymbol{\Phi}$ of transition probabilities. Given some initial probability distribution for $\hat{\boldsymbol{z}}$, call it $\boldsymbol{\rho}^0$ where $\boldsymbol{\rho}^0$ is a $1 \times N^*$ row vector, the probability distribution governing $\hat{\boldsymbol{z}}_{t+1}$ is given by $\boldsymbol{\rho}^1 = \boldsymbol{\rho}^0\boldsymbol{\Phi}$. Probabilities of states more than one period ahead may be computed from powers of $\boldsymbol{\Phi}$. Assuming the Markov chain is ergodic, the long-run or unconditional distribution is given by $\boldsymbol{\rho}^* = \boldsymbol{\rho}^0\boldsymbol{\Phi}^*$ where $\boldsymbol{\Phi}^* = \lim_{t\to\infty}\boldsymbol{\Phi}^t$. Note that since $\boldsymbol{\Phi}^* = \boldsymbol{\Phi}^*\boldsymbol{\Phi}$, we have $\boldsymbol{\rho}^* = \boldsymbol{\rho}^0\boldsymbol{\Phi}^*\boldsymbol{\Phi} = \boldsymbol{\rho}^*\boldsymbol{\Phi}$. The unconditional distribution is therefore a fixed point. To find this fixed point, let $\boldsymbol{\rho}^0$ be a $1 \times N^*$ vector with all its elements equal to $1/N^*$, and calculate $\boldsymbol{\rho}^1 = \boldsymbol{\rho}^0\boldsymbol{\Phi}$, $\boldsymbol{\rho}^2 = \boldsymbol{\rho}^1\boldsymbol{\Phi}$, and so on until $\boldsymbol{\rho}^{m+1} = \boldsymbol{\rho}^m\boldsymbol{\Phi}$, at which point the unique fixed point has been found.[25]

In practice m is set equal to three so the discrete probability density function spans three standard deviations of the underlying continuous probability density function. In addition, grids of 16 points for the $x-$ and $r-$processes are used so there are N^* =256 states in the system.

Appendix C

Calculating the Cumulative Growth Factors

This appendix describes the closed-form solution obtained for the stationary component of wealth Γ. Expanding out (12), Γ may be written as:

$$\begin{aligned}\Gamma_t &= E_t\left[\left(\frac{1+x_{t+1}}{1+r_{t+1}}\right) + \left(\frac{1+x_{t+1}}{1+r_{t+1}}\right)\left(\frac{1+x_{t+2}}{1+r_{t+2}}\right) + \left(\frac{1+x_{t+1}}{1+r_{t+1}}\right)\left(\frac{1+x_{t+2}}{1+r_{t+2}}\right)\left(\frac{1+x_{t+3}}{1+r_{t+3}}\right) + \cdots\right] \\ &= E_t[q_{t+1}] + E_t[q_{t+1}q_{t+2}] + E_t[q_{t+1}q_{t+2}q_{t+3} + \cdots] \qquad \text{(C.1)}\end{aligned}$$

where

$$q_{t+i} = \left(\frac{1+x_{t+i}}{1+r_{t+i}}\right)$$

Using the approximation procedure described in Appendix B, the VAR for x_t and r_t is approximated as a finite-state discrete-valued Markov chain. Using grids of N_x and N_r points to approximate the real-valued variables x_t and r_t respectively, the discrete-valued system consists of a state space of $N^* = N_x \times N_r$ points and a $N^* \times N^*$ matrix $\boldsymbol{\Phi}$ of transition probabilities describing the dynamic behaviour of the system. The typical element of $\boldsymbol{\Phi}$ is:

$$\phi_{k,l} = \text{prob}[\text{system will be in state } k \text{ at } t+1|\text{system is in state } l \text{ at } t]$$

The discrete variables $\hat{x}$ and $\hat{r}$ can be used to form the discrete variable $\hat{q}$ which approximates the real-valued variable q. Let $\boldsymbol{Q}$ be the $N_x \times N_r$ matrix of $\hat{q}$'s in every state of the system, and define $\vec{\boldsymbol{Q}}$ as the $N_xN_r \times 1$ vector obtained by stacking the columns of $\boldsymbol{Q}$ one on top of the other. If we index the elements of $\vec{\boldsymbol{Q}}$ by $k = 1, \ldots, N^*$ where $N^* = N_xN_r$, the typical element of $\vec{\boldsymbol{Q}}$ can be written as $\hat{q}(k)$. With this investment in notation, the expected geometric averages of the $\hat{q}_{t+i}$ can be computed as

$$E[\hat{q}_{t+1}|\hat{q}_t = \hat{q}(k)] = \sum_{l=1}^{N^*} \phi_{k,l}\hat{q}(l) \qquad \text{(C.2)}$$

[25] For more on the properties of Markov chains see Grimmett and Stirzaker (1982), or Stokey and Lucas (1989).

$$E[\hat{q}_{t+1}\hat{q}_{t+2}|\hat{q}_t = \hat{q}(k)] = \sum_{l=1}^{N^*}\sum_{m=1}^{N^*} \phi_{k,l}\phi_{l,m}\hat{q}(l)\hat{q}(m) \tag{C.3}$$

$$E[\hat{q}_{t+1}\hat{q}_{t+2}\hat{q}_{t+3}|\hat{q}_t = \hat{q}(k)] = \sum_{l=1}^{N^*}\sum_{m=1}^{N^*}\sum_{n=1}^{N^*} \phi_{k,l}\phi_{l,m}\phi_{m,n}\hat{q}(l)\hat{q}(m)\hat{q}(n) \tag{C.4}$$

Terms further into the future may be constructed as a straightforward extension of (C.2)-(C.4), with each step into the future necessitating an additional summation. The solutions (C.2)-(C.4) bring home the magnitude of the task of computing the cumulative growth factors. The terms (C.2)-(C.4) are only the first three terms in the infinite sum (C.1), *and* this infinite sum must be calculated for each of the N^* states of the system.

To make this task more manageable, it is convenient to write the finite state Markov chain solution to (C.1) in matrix notation. Let $\boldsymbol{\Gamma}$ be the $N_x \times N_r$ matrix of cumulative growth factors $\hat{\Gamma}$. The typical element of $\boldsymbol{\Gamma}$ is $\hat{\Gamma}_{ij}$, which is expected growth factor when the current state is $\hat{x} = \hat{x}_i$, $\hat{r} = \hat{r}_j$. Let $\vec{\boldsymbol{\Gamma}}$ be the $N_xN_r \times 1$ matrix obtained by stacking the columns of $\boldsymbol{\Gamma}$ one on top of the other. Indexing the elements of $\boldsymbol{\Gamma}$ by $k = 1, \ldots, N^*$, the typical element of this vector is $\hat{\Gamma}(k)$. Finally, let $\boldsymbol{\Delta}$ be an $N_xN_r \times N_xN_r$ matrix with its rows all being $\vec{\boldsymbol{Q}}^\tau$, where τ denotes the transpose. The vector of cumulative growth factors in every state of the system then has the following representation

$$\vec{\boldsymbol{\Gamma}} = \sum_{\alpha=1}^{\infty}(\boldsymbol{\Phi} \star \boldsymbol{\Delta})^{\alpha}\boldsymbol{\iota} \tag{C.5}$$

$$= ([\boldsymbol{I} - \boldsymbol{\Phi} \star \boldsymbol{\Delta}]^{-1} - \boldsymbol{I})\boldsymbol{\iota} \tag{C.6}$$

where $\star$ denotes element by element multiplication and $\boldsymbol{\iota}$ is a $N^* \times 1$ vector of ones. Computing the vector of cumulative growth factors in every state of the system therefore amounts largely to computing the inverse of a matrix.

To see why (C.5) is the solution for the cumulative growth factor in every state of the world, consider the case in which $\hat{x}$ and $\hat{r}$ are each drawn from two point grids in which case $\hat{q}$ can take on four values. In this case (C.5) is

$$\begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \\ \Gamma_4 \end{bmatrix} = \sum_{\alpha=1}^{\infty} \begin{bmatrix} \phi_{1,1}\hat{q}(1) & \phi_{1,2}\hat{q}(2) & \phi_{1,3}\hat{q}(3) & \phi_{1,4}\hat{q}(4) \\ \phi_{2,1}\hat{q}(1) & \phi_{2,2}\hat{q}(2) & \phi_{2,3}\hat{q}(3) & \phi_{2,4}\hat{q}(4) \\ \phi_{3,1}\hat{q}(1) & \phi_{3,2}\hat{q}(2) & \phi_{3,3}\hat{q}(3) & \phi_{3,4}\hat{q}(4) \\ \phi_{4,1}\hat{q}(1) & \phi_{4,2}\hat{q}(2) & \phi_{4,3}\hat{q}(3) & \phi_{4,4}\hat{q}(4) \end{bmatrix}^{\alpha} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{C.7}$$

Although somewhat tedious, it is now straightforward to verify that the terms which make up the infinite sum in (C.7) are precisely the solutions for the expected values in (C.1). For example, the generic row of the first term in the sum (C.7) — i.e. the $\alpha = 1$ term — is $\sum_{l=1}^{4} \phi_{k,l}\hat{q}(l)$ which is simply (C.2) for the case of $N^* = 2 \times 2 = 4$.

Appendix D

Wealth and its Components for $\theta = 0.75$

(per capita, constant 1986 dollars)

	total wealth	human component	non-human component
64:1	145321.41	105497.05	39824.36
64:2	148310.27	107764.77	40545.50
64:3	150355.89	108345.13	42010.76
64:4	152100.50	109510.77	42589.73
65:1	155079.69	111832.25	43247.45
65:2	157725.70	114373.30	43352.41
65:3	159718.22	115061.88	44656.35
65:4	162012.16	116441.20	45570.96
66:1	164392.23	118721.75	45670.50
66:2	168910.88	123119.68	45791.20
66:3	170328.72	123710.98	46617.73
66:4	168164.08	121454.30	46709.78
67:1	170384.73	122984.89	47399.84
67:2	170351.91	122615.94	47735.96
67:3	172589.45	123677.23	48912.23
67:4	171217.72	122683.80	48533.92
68:1	166745.34	118973.33	47772.01
68:2	170607.25	122283.09	48324.15
68:3	172471.48	122890.84	49580.64
68:4	174144.53	123985.72	50158.80
69:1	177087.52	126721.52	50366.00
69:2	174920.77	124369.10	50551.66
69:3	176961.61	126641.92	50319.69
69:4	176649.45	126128.69	50520.77
70:1	176902.38	126766.09	50136.29
70:2	174398.23	125938.51	48459.72
70:3	176205.84	126532.67	49673.17
70:4	176002.38	125888.79	50113.59
71:1	180011.25	129147.09	50864.16
71:2	184709.31	133551.47	51157.85
71:3	190543.42	138454.72	52088.70
71:4	190188.14	138860.53	51327.61
72:1	192735.02	139524.75	53210.27

72:2	194584.66	140308.27	54276.39
72:3	199746.47	143699.31	56047.14
72:4	206370.38	150327.48	56042.89
73:1	209677.38	153196.39	56480.98
73:2	211706.97	154795.47	56911.51
73:3	211641.09	151592.88	60048.23
73:4	216791.53	156308.17	60483.36
74:1	221386.20	161297.95	60088.25
74:2	216061.00	156636.25	59424.75
74:3	224197.38	163199.97	60997.41
74:4	226824.50	167242.42	59582.08
75:1	234569.91	174619.05	59950.87
75:2	232298.59	171797.41	60501.18
75:3	229627.59	168226.34	61401.25
75:4	227382.97	167458.92	59924.05
76:1	229570.22	168960.81	60609.41
76:2	229485.70	168482.59	61003.10
76:3	225784.19	163201.78	62582.41
76:4	226736.22	164911.98	61824.24
77:1	227214.80	165886.05	61328.74
77:2	231609.56	170224.14	61385.43
77:3	233960.27	171056.75	62903.51
77:4	228701.09	166489.08	62212.00
78:1	229280.72	167305.97	61974.74
78:2	226033.17	163215.66	62817.51
78:3	227129.91	161999.17	65130.73
78:4	227649.33	161889.17	65760.16
79:1	223729.11	157268.75	66460.36
79:2	229324.14	160933.20	68390.93
79:3	235052.16	163649.94	71402.22
79:4	229915.38	158268.08	71647.30
80:1	232594.36	158552.70	74041.66
80:2	236129.50	162414.31	73715.19
80:3	247662.63	169881.02	77781.62
80:4	242269.58	164577.83	77691.75
81:1	235368.91	159722.86	75646.04
81:2	236175.73	159529.25	76646.47
81:3	227643.98	151663.39	75980.59
81:4	234362.86	161266.97	73095.88
82:1	229322.39	159485.56	69836.84
82:2	218585.06	151540.41	67044.65

82:3	216674.53	149629.98	67044.55
82:4	218357.02	150642.50	67714.52
83:1	218434.20	148851.19	69583.02
83:2	218241.09	146531.22	71709.88
83:3	216552.55	144046.39	72506.16
83:4	218138.72	145997.09	72141.63
84:1	213477.70	142038.83	71438.88
84:2	213279.11	142022.78	71256.33
84:3	211813.80	139794.66	72019.14
84:4	215407.44	143021.19	72386.25
85:1	217358.80	143883.02	73475.78
85:2	223084.70	148974.28	74110.42
85:3	227993.08	153077.17	74915.91
85:4	226753.64	151956.73	74796.91
86:1	224388.33	148989.45	75398.88
86:2	233539.95	156717.58	76822.38
86:3	232788.63	156330.14	76458.48
86:4	233354.86	157057.20	76297.66
87:1	243221.73	163955.39	79266.35
87:2	245252.14	164014.53	81237.63
87:3	244842.97	161500.70	83342.27
87:4	242568.64	165684.58	76884.07
88:1	245502.45	167696.13	77806.33
88:2	245961.59	166789.92	79171.67
88:3	248324.73	168530.25	79794.47
88:4	248256.89	168169.17	80087.72
89:1	249150.80	167186.39	81964.41
89:2	251050.03	168793.11	82256.93
89:3	254762.89	172267.89	82494.98
89:4	253377.19	173003.84	80373.34

References

Barro, R. J. "Are Government Bonds Net Wealth?" *Journal of Political Economy* 82 (1974): 1095-1117.

——— "On the Determinants of the Public Debt." *Journal of Political Economy* 87 (1979): 940-971.

——— "The Ricardian Approach to Budget Deficits." *Journal of Economic Perspectives* 3 (1989): 37-54.

Bernanke, B. S. "Permanent Income, Liquidity and Expenditure on Automobiles: Evidence From Panel Data." *Quarterly Journal of Economics* 99 (1984): 587-614.

Bernheim, D. B. "Ricardian Equivalence: An Evaluation of Theory and Evidence." In *NBER Macroeconomics Annual, 1987*. Cambridge: MIT Press, 1987.

——— "A Neoclassical Perspective on Budget Deficits." *Journal of Economic Perspectives* 3 (1989): 55-72.

Boothe, P.; Reid, B.; and Talen, F. "The Market Value of Canada Saving Bonds." Research Paper no. 89-1. University of Alberta, 1989.

Campbell, J. Y. and Mankiw, N. G. "Consumption, Income and Interest Rates: Reinterpreting the Time Series Evidence." Working Paper no. 2924. National Bureau of Economic Research, 1989.

Christiano, L. J. "Is Consumption Insufficiently Sensitive to Innovations in Income" *American Economic Review* 77 (1986): 337-341.

Cox, W. M. and Hirschhorn, E. "The Market Value of U.S. Government Debt: Monthly 1942-80." *Journal of Monetary Economics* 11 (1983): 261-75.

Cox, W. M. and Hoslag, J. "The Market Value of Canada Debt, Monthly, 1937-84." *Canadian Journal of Economics* XIX (1986): 469-497.

Davies, J. B. "Uncertain Lifetime Consumption and Dissaving in Retirement." *Journal of Political Economy* 89 (1981): 561-577.

Dea, C. and Ng, S. "Sources of Business Cycles in Canada." Working Paper no. 90-4. Ottawa: Bank of Canada, 1990.

Deaton, A. "Life-Cyle Models of Consumption: Is Evidence Consistent With the Data." Working Paper no. 1910. National Bureau of Economic Research, 1986.

Diamond, P. A. and Hausman, J. A. "Individual Retirement and Savings Behaviour." *Journal of Public Economics* 23 (1984): 81-114.

Dickey, D. A. and Fuller, W. A. "Likelihood Ratio Statistics for Autoregressive Time Series

With a Unit Root." *Econometrica* 49 (1981): 1057-1072.

Durlauf, S. N. and Phillips, P. C. B. "Trends and Random Walks in Time Series Analysis." *Econometrica* 56 (1988): 1333-1354.

Friedman, M. *A Theory of the Consumption Function.* Princeton: Princeton University Press, 1957.

Flavin, M. "The Adjustment of Consumption to Changing Expectations About Future Income." *Journal of Political Economy* 89 (1981): 974-1009.

Fuller, W. A. *Introduction to Statistical Time Series.* New York: John Wiley and Sons, 1976.

Greenwood, J.; Hercowitz, Z.; and Huffman, G. "Investment, Capacity Utilization and the Real Business Cycle." *American Economic Review* 78 (1988): 402-417.

Grimmett, G. R. and Stirzaker, D. R. *Probability and Random Processes.* Oxford: Oxford University Press, 1982.

Hall, R. E. and Mishkin, F. S. "The Sensitivity of Consumption to Transitory Income: Estimates From Panel Data on Households." *Econometrica* 50 (1982): 461-481.

Hayashi, F. "The Permanent Income Hypothesis and Consumption Durability: An Analysis Based on Japanese Panel Data." *Quarterly Journal of Economics* 100 (1985): 1083-1115.

Hurd, M. D. "Savings of the Elderly and Desired Bequests." *American Economic Review* 77 (1987): 298-312.

King, M. A. and Dicks-Mireaux, L. "Asset Holdings and the Life Cycle." Working Paper no. 614. National Bureau of Economic Analysis, 1981.

Kotlikoff, L. J. and Summers, L. H. "The Role of Intergenerational Transfers in Aggregate Capital Formation." *Journal of Political Economy* 89 (1981): 706-732.

Lucas, R. E. Jr. "Interest Rates and Currency Prices in a Two-Country World." *Journal of Monetary Economics* 10 (1982): 335-359.

Macklem, R. T. "Forward Exchange Rates and Risk Premiums in Artificial Economies." *Journal of International Money and Finance* , forthcoming, 1991.

Mankiw, N. G. and Shapiro, M. D. "Trends, Random Walks, and Test of the Permanent Income Hypothesis." *Journal of Monetary Economics* 16 (1985): 165-174.

Mehra, R. and Prescott, E. C. "The Equity Premium: A Puzzle." *Journal of Monetary Economics* 15 (1985): 145-161.

Mirer, T. W. "The Wealth-Age Relation Amoung the Aged." *American Economic Review* 69 (1979): 435-443.

Nelson, C. R. and Plosser, C. I. "Trends and Random Walks In Macroeconomic Time Series." *Journal of Monetary Economics* 10 (1982) 139-162.

Perron, P. "The Great Crash, the Oil Price Shock and the Unit Root Hypothesis." Department of Economics, University of Montreal, mimeo, 1987.

Phillips, P. C. B. and Perron, P. "Testing for a Unit Root in Time Series Regression." Discussion Paper no. 2186. C.R.D.E., University of Montreal, 1986.

Poitras, G. "The Market Value of Government of Canada Debt: A Comment on the Importance of Correct Valuation of Non-Marketable Debt." *Canadian Journal of Economics* XXII (1989): 395-405.

Robb, A. L. and Burbidge, J. B. "Consumption, Income and Retirement." *Canadian Journal of Economics* XXII (1989): 522-542.

Rose, D. and Selody, J. "The Structure of the Small Annual Model." Technical Report no. 40. Ottawa: Bank of Canada, 1985.

Seater, J. "The Market Value of Outstanding Government Debt: 1919-75." *Journal of Monetary Economics* 8 (1981): 85-102.

Stokey, N. L. and R. E. Lucas Jr. *Recursive Methods in Economic Dynamics.* Cambridge: Harvard University Press, 1989.

Tauchen, G. "Finite State Markov-Chain Approximation to Univariate and Vector Autoregressions." *Economic Letters* 20 (1986): 177-181.

Wasserfallen, W. "Non-Stationarities in Macro-Economic Time Series — Further Evidence and Implications." *Canadian Journal of Economics* XIX (1986): 498-510.

Watson, M. W. "Univariate Detrending Methods with Stochastic Trends." *Journal of Monetary Economics* 18 (1986): 49-75.

Zeldes, S. P. "Consumption and Liquidity Constraints: An Empirical Investigation." *Journal of Political Economy* 97 (1989): 305-346.

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