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On the Aggregate Implications of Optimal Price Adjustment

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ON THE AGGREGATE IMPLICATIONS OF OPTIMAL PRICE ADJUSTMENT

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ABSTRACT

In macro models, some form of price or wage stickiness is often needed in order to generate real effects from nominal disturbances. This paper examines the implications for macroeconomic relationships of costly price adjustment on the part of firms. In the model, intertemporally optimizing, price-setting firms face quadratic costs of changing prices, an idea first proposed by Rotemberg. A version of the traditional Phillips curve emerges quite naturally from this framework. The Phillips curve can thus be justified as a useful rule-of-thumb which approximately describes optimal price adjustment. The aggregate price adjustment equation also constitutes a framework for addressing credibility issues. This arises because prices are partly forward-looking and the optimal speed of adjustment depends on the degree of policy credibility. The aggregate price adjustment formulas developed in the paper might also be useful in modeling price behaviour in a macroeconomic model.

Résumé

Pour que les chocs nominaux produisent des effets réels dans les modèles macroéconomiques, il faut souvent postuler une certaine inertie des prix ou des salaires. Dans la présente étude, l'auteur analyse les effets qu'exercent sur les relations macroéconomiques les coûts reliés aux ajustements de prix qu'effectuent les entreprises. Dans le modèle retenu par l'auteur, les entreprises sont des leaders en matière de prix et poursuivent des objectifs d'optimisation intertemporelle; les coûts associés aux changements de prix sont formalisés par une fonction quadratique, formulation que Rotemberg a été le premier à proposer. L'auteur constate que ce cadre de raisonnement donne naissance tout naturellement à une version de la courbe de Phillips traditionnelle. Cette dernière peut donc se justifier comme une règle empirique utile décrivant de façon approximative l'ajustement optimal des prix. L'équation d'ajustement global des prix constitue aussi un cadre d'analyse de la crédibilité des politiques. La question de la crédibilité se pose parce que les prix sont en partie de nature prospective et qu'il se trouve que la vitesse d'ajustement optimale est fonction de la confiance qu'inspirent les politiques. Les équations d'ajustement global des prix élaborées dans ce modèle pourraient aussi servir à formaliser le comportement des prix dans un modèle macroéconomique.

I INTRODUCTION

In macro models, some form of price or wage stickiness is often needed in order to generate real effects from nominal disturbances. This paper pursues an idea proposed by Rotemberg (1982), in a model of price setting by intertemporally optimizing firms facing quadratic costs of changing prices. The implications for aggregate macroeconomic relationships of this behaviour on the part of firms are examined. We find the aggregate relationships implied by this idea a useful and effective way of implementing the notion of price stickiness in a macro model.

One class of new Keynesian macro models has tended to focus on nominal wage stickiness -- usually some form of nominal wage contracting. Despite the apparent realism of this assumption, models that incorporate it have been criticized for predicting that real wages should move countercyclically. In fact, the evidence is that real wages tend to be either procyclical or not cyclical at all, a finding that has long been thought to be at variance with sticky-wage Keynesian models. More recently, the new Keynesian research agenda has paid more attention to price-stickiness as a source of demand-induced business cycles. This paper is among them.

The sticky-price new Keynesian models usually start by assuming monopolistic competition, where price-setting firms stand to lose clientele if they raise prices too rapidly. Quadratic costs of changing prices are usually justified in these models as arising from reputational considerations, as well as from menu costs of changing prices. Moreover, a temporary rise in demand is less likely to cause a firm to raise price than is a permanent change in demand. These ideas can be rationalized by the quadratic costs framework.

Much of the research on price stickiness has focused on the assumption of fixed costs of changing prices. This is presumably a more realistic description of how monopolistically competitive firms actually go about setting prices. I use the quadratic costs framework in this paper for two reasons: (a) it is analytically more tractable than the fixed costs assumption, particularly when working out aggregate implications; (b) its implication that price level changes are smooth seems to be consistent with the aggregate price index data, even if its prediction for individual prices might not be. For our purposes, not much is lost by taking an explicit aggregative macroeconomic approach as we do here, and much is gained in terms of tractability.

A version of the Rotemberg model is solved and its aggregate implications worked out. A version of the traditional Phillips curve emerges quite naturally from this framework. The Phillips curve can thus be justified as an optimal price adjustment equation. The aggregate price adjustment equation also constitutes a promising framework for addressing credibility issues. This arises because prices are partly forward-looking, and it turns out that the optimal speed of adjustment depends on the degree of credibility of policy.

The remainder of the paper works out the details of optimal price adjustment, looks at the optimal responses to shocks, derives versions of the Phillips curve and deals with credibility issues. Conclusions are in Section 6.

II OPTIMAL PRICE ADJUSTMENT

The following model, like Rotemberg's, abstracts from trend growth. There are however several differences between our formulation and his. The parameterization here is simpler, to facilitate aggregation. Also, we have exogenous nominal aggregate demand instead of exogenous money. Finally, we have firm-specific as well as aggregate shocks.

Output of a single good is assumed to be produced in a large number, n, of monopolistically competitive firms. Firms face demand and supply shocks of two types: aggregate and firm-specific. In the absence of costs of adjusting prices, demand for the output of firm i at time t, Y^d_{it}, is given by:

$$(1) \qquad Y^d_{it} = e^{a_{it}}\left(\frac{P_{it}}{P_t}\right)^{-b}\cdot\left(\frac{X_t}{P_t}\right), \qquad b > 1, \qquad \sum_{i=1}^{n} a_{it} = 0,$$

where a_{it} is a firm-specific zero-mean demand shock, P_{it} is the price of firm i's output, P_t is an aggregate price index (to be defined later), and X_t is the level of nominal aggregate demand. Both P_t and X_t are assumed to be exogenous to the individual firm, and X_t is further assumed to be exogenous to the economy as a whole. The nominal production costs, $C_i(Y_{it})$, facing firm i are assumed to be given by the quadratic function:

$$(2) \qquad C_i(Y_{it}) = \left(\frac{1}{2f}\right)e^{-(u_{it}+\varepsilon_t)}\cdot P_t\left(Y_{it}\right)^2, \qquad \sum_{i=1}^{n} u_{it} = 0,$$

where u_{it} is a firm-specific, zero-mean supply shock, ε_t is an aggregate supply shock, and f is a positive constant.

In the absence of costs of changing prices, the equilibrium price P^*_{it} is given by,

$$(3) \qquad P^*_{it} = \left(\frac{b}{b-1}\right) f^{-1} e^{-(u_{it}+\varepsilon_t)} \cdot P_t Y_{it}.$$

Equation (3) will prove a useful definition of an equilibrium price, and we shall henceforth refer to it as the long-run equilibrium price.

In the presence of quadratic costs, it turns out that we can write the firm's intertemporal optimization problem as,

$$(4) \qquad \underset{\{p_{it}\}}{\text{Max}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left[\Pi(p^*_{it}) - k_i(p_{it} - p^*_{it})^2 - c_i(p_{it} - p_{it-1})^2\right]$$

$$0 \le \beta \le 1, \qquad k_i, > 0, \; c_i \ge 0,$$

where p_{it} is the log of P_{it}, p^*_{it} is the log of P^*_{it}, $\Pi(p^*_{it})$ is the profit in the absence of costs of adjustment, β is the discount factor, and E is the expectations operator. To simplify aggregation, assume that $c_i/k_i = c \; \forall \; i$. Then the Euler equations for this problem are:

$$(5) \qquad p_{it} = \left(\frac{1}{1+c+\beta c}\right) p^*_{it} + \left(\frac{c}{1+c+\beta c}\right) p_{it-1} + \left(\frac{\beta c}{1+c+\beta c}\right) p^e_{it+1},$$

$$i = 0, 1, 2, \ldots..$$

where p^e_{t+1} is the expectation of p_{t+1} conditional on information available at time t. Equation (5) says that, in logarithmic terms, the optimal price in the face of costs of adjustment will be a weighted average of the past price, the expected future price, and the long-run equilibrium price. The transversality condition is:

$$(6) \qquad \lim_{T \to \infty} \beta^T \left[(p_{iT} - p^*_{iT}) + c(p_{iT} - p_{iT-1}) \right] = 0.$$

This essentially means that, given stable paths for the determinants of the long-run equilibrium price, firms will choose stable paths for actual prices.[1]

Turning now to a characterization of aggregate behaviour in this economy, define aggregate output and the price level as geometrically weighted averages of individual firms' output and price respectively. Working in logarithms, the level of aggregate demand y^d_t is found from (1) to be:

$$(7) \qquad y^d_t = \frac{1}{n} \sum_{i=1}^{n} y^d_{it} = x_t - p_t.$$

The aggregate long-run equilibrium price is found by using equation (1) in (3) to give,

$$(8) \qquad p^*_t = \frac{1}{n} \sum_{i=1}^{n} p^*_{it} = x_t - y^* - \varepsilon_t$$

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1 Sufficient conditions for the transversality condition to hold are that both the equilibrium and the actual price sequences be of exponential order less than $1/\beta$ (see Sargent (1979) for a proof).

where y^* is the long-run equilibrium level or natural rate of output defined by:

(9) $$y^* = \log\left(\frac{b-1}{b}\right) + \log(f).$$

Finally aggregation of equation (5) yields:

(10) $$p_t = \psi p_{t-1} + \beta\psi p^e_{t+1} + (1-\psi-\beta\psi)p^*_t ,$$

$$\psi = \frac{c}{1 + c + \beta c} .$$

Equations (7), (8) and (10) describe aggregate behaviour in this simple economy. Although we shall characterize this behaviour further, for now it is easy to see some interesting implications of the price adjustment equation, (10). For example, if the discount factor is set to zero (which is equivalent to an infinite rate of time preference), the price adjustment formula becomes,

(11) $$p_t = \left(\frac{1}{1+c}\right)p^*_t + \left(\frac{c}{1+c}\right)p_{t-1}$$

which is a standard partial adjustment formulation. Also if expectations are backward looking such that $p^e_{t+1} = p_{t-1}$, then we get,

(12) $$p_t = \left(\frac{1}{1+c+\beta c}\right)p^*_t + \left(\frac{c+\beta c}{1+c+\beta c}\right)p_{t-1}$$

which is also a partial adjustment formulation. Yet another partial adjustment formulation arises if $p^e_{t+1} = p_t$.

These partial adjustment interpretations of the price adjustment formula are not valid in general. Equation (10) is a second-order stochastic difference equation in p_t with exogenous forcing variable p^*_t, which may be solved using the method of Sargent (1979). The roots of (10) are solutions to the homogeneous equation:

$$(13) \qquad 1 - (\beta\psi)^{-1}B + \beta^{-1}B^2 = (1 - \lambda_1 B)(1 - \lambda_2 B) = 0$$

where B is an operator defined by $BE[X_{t+1}|I_t] = E[X_t|I_t]$. The two roots, λ_1 and λ_2, are defined by:

$$(14) \qquad \lambda_1 + \lambda_2 = 1/(\beta\psi)$$

$$(15) \qquad \lambda_1\lambda_2 = 1/\beta$$

While it is not possible to solve exactly for these roots, it is possible to show quite generally that

$$0 \le \lambda_1 < 1 < \lambda_2$$

which means that one root is stable and the other unstable. To satisfy the transversality condition, it is appropriate to solve the unstable root forward to obtain:

$$(16) \qquad (1 - \lambda_1 B)p_t = \frac{(\beta c)^{-1}\lambda_2^{-1}B^{-1}}{(1 - \lambda_2^{-1})B^{-1}} \cdot p^*_{t-1} + D\lambda_2^t$$

where D is a constant of integration. Setting D equal to zero in order to fully satisfy the transversality condition produces the solution:

$$(17) \qquad p_t = \lambda_1 p_{t-1} + \lambda_1 c^{-1} \sum_{i=0}^{\infty} \left(\frac{1}{\lambda_2}\right)^i p^{*e}_{t+i}.$$

Equation (17) says that the current price is determined by the lagged price, with coefficient less than unity, and by a weighted average of current and all expected future values of the equilibrium price.

III OPTIMAL RESPONSES TO SHOCKS

In this section we assess the effects on aggregate output and the price level of temporary and permanent shocks to nominal demand and aggregate supply. Our model now is:

(18) $$y_t = x_t - p_t \ ,$$

(19) $$p_t^* = x_t - y^* - \varepsilon_t \ ,$$

(20) $$p_t = \lambda_1 p_{t-1} + \lambda_1 c^{-1} \sum_{i=0}^{\infty} \left(\frac{1}{\lambda_2}\right)^i p_{t+i}^{*e}, \quad 0 \le \lambda_1 < 1 < \lambda_2,$$

(21) $$x_t = q_t + \omega_t$$

(22) $$q_t = q_{t-1} + v_t$$

(23) $$\varepsilon_t = z_t + \eta_t$$

(24) $$z_t = z_{t-1} + s_t.$$

Equations (18), (19) and (20) are simply equations (7), (8) and (17) once again. Equations (21) and (22) describe the process specified for exogenous nominal demand, which is assumed to consist of a permanent component q_t and a transitory component ω_t. v_t is a shock to the permanent component. We assume that ω_t and v_t are zero-mean, independently distributed random variables. Equations (23) and (24) describe the process specified for the exogenous supply shock, which is assumed to consist of a permanent component z_t and a transitory component η_t. s_t is a shock to the permanent component. We assume that η_t and s_t are zero-mean, independently distributed random variables.

Under these assumptions, and making use of the fact that from equations (14) and (15) $\lambda_1 c^{-1}/(1-\lambda_2^{-1}) = (1-\lambda_1)$, the solutions for the price level and output may be written as:

$$(25) \qquad p_t = \lambda_1 p_{t-1} + (1-\lambda_1)(q_t - y^* - z_t) + \lambda_1 c^{-1}(\omega_t - \eta_t),$$

$$(26) \qquad y_t = \lambda_1 q_t + (1-\lambda_1 c^{-1})\omega_t + \lambda_1 c^{-1}\eta_t + (1-\lambda_1)(y^* + z_t) - \lambda_1 p_{t-1}.$$

Equation (25) shows that in this economy the price level will depend on "fundamentals" such as nominal demand and supply shocks, as well as on its own past history. This of course means that nominal shocks (including monetary shocks) will not be neutral in the short run, as equation (26) shows.

Both temporary and permanent demand shocks have positive effects on real output because of the sluggish adjustment of prices. However, optimal price adjustment dictates different responses to these shocks. The impact effects of a temporary demand shock on the price level and output are:

$$\frac{\delta p_t}{\delta \omega_t} = \lambda_1 c^{-1} \geq 0, \qquad \frac{\delta y_t}{\delta \omega_t} = 1 - \lambda_1 c^{-1} \geq 0.$$

The impact effects of a permanent demand shock are:

$$\frac{\delta p_t}{\delta v_t} = 1 - \lambda_1 \geq 0, \qquad \frac{\delta y_t}{\delta v_t} = \lambda_1 \geq 0.$$

From equations (14) and (15), we can show that $\delta p_t/\delta v_t > \delta p_t/\delta \omega_t$, and also that $\delta y_t/\delta v_t < \delta y_t/\delta \omega_t$. This means that a permanent demand shock has a greater effect on the price level and a smaller effect on output than a temporary demand shock. Forward-looking firms faced with fluctuations in demand will adjust prices more (less) rapidly the more that these fluctuations are perceived to be permanent (transitory). One implication of this for monetary policy is that erratic but transitory fluctuations in the money supply will result in a greater degree of price stickiness and output variability than would occur in a regime in which most shifts in the money supply were permanent.

Optimal price adjustment also has some interesting implications for the response to supply disturbances. Both temporary and permanent supply shocks will produce negatively correlated movements in output and prices, as is usual in a macro model. However, once again there are differences in the magnitudes of the responses to the shocks. The impact effects of temporary and permanent supply shocks on the price level and output are given by:

$$\frac{\delta p_t}{\delta \eta_t} = - \lambda_1 c^{-1} \leq 0, \qquad \frac{\delta y_t}{\delta \eta_t} = \lambda_1 c^{-1} \geq 0.$$

$$\frac{\delta p_t}{\delta s_t} = - (1-\lambda_1) \leq 0 \qquad \frac{\delta y_t}{\delta s_t} = 1 - \lambda_1 \geq 0.$$

We can show that $|\delta p_t/\delta s_t| > |\delta p_t/\delta \eta_t|$, and $|\delta y_t/\delta s_t| > |\delta p_t/\delta \eta_t|$. Thus the optimal response to supply shocks involves slower price and output response to transitory shocks than to permanent shocks.

In the long run, nominal demand is of course neutral. To see this, let $x_t = \bar{x}$ and $\varepsilon_t = 0$ for all t. This means that the price level is given by,

$$p_t = \lambda_1 p_{t-1} + (1 - \lambda_1)(\bar{x} - y^*), \quad (27)$$

which means that nominal demand has a one-to-one effect on the price level (and therefore no effect on output) in the steady state.

IV THE PHILLIPS CURVE AS AN OPTIMAL PRICE ADJUSTMENT EQUATION

A nice, though surprising, feature of this quadratic costs formulation for price adjustment is that an equation resembling the familiar Phillips curve emerges as an alternative description of optimal price adjustment. To see this, we shall use equations (7), (8) and (10).[2] Using (7) and (8) to eliminate p_t^* in equation (10) yields:

$$(28) \qquad p_t = \left(\frac{1}{1+\beta}\right)p_{t-1} + \left(\frac{\beta}{1+\beta}\right)p_{t+1}^e + \frac{1}{c(1+\beta)}(y_t - y_t^*) + u_t$$

where $\quad u_t = -\dfrac{\varepsilon_t}{c(1+\beta)}.$

Equation (28) relates the current price level positively to a weighted average of past and expected future price levels, with coefficient equal to unity, positively to the deviation of output from its natural rate (potential output), and to a supply shock.[3] This equation bears some resemblance to the price adjustment equation in the Bank of Canada's Small Annual Model (SAM) (see Rose and Selody 1985), which is also motivated by considerations of how rational agents would adjust prices from one equilibrium to another. Our Phillips-curve version of the price adjustment

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2 Note that equations (10) and (17) are equivalent representations of the optimal price adjustment equation. We are now about to derive a third representation.

3 An equivalent formulation of the Phillips curve, which some may find more intuitive, was pointed out to me by Bob Ford. It is:

$$p_t - p_{t-1} = \beta(p_{t+1}^e - p_t) + (1/c)(y_t - y^*).$$

equation does differ in some important ways from the traditional Phillips curve, or even more recent formulations based on the Lucas supply function. Firstly, this Phillips curve is partly forward and partly backward-looking, with the degree of *forward-lookingness* depending on the size of the discount factor β. This is a result of the dynamic cost minimization problem facing firms. Secondly, the slope of the Phillips curve depends negatively on both the cost factor, c, and the discount factor, β. A high value of c means that prices are very costly to adjust. A high value of β (maximum value is unity) means that the future is as important as the present, and therefore agents care less about being in equilibrium now than about being in equilibrium on average in the future. Both of these factors will slow down price adjustment and tend to make the Phillips curve flatter.

In the literature on the Phillips curve, prices are often written as a function of present and past values of the deviation of output from potential, the output gap. Let $\hat{y}_t$ denote the output gap, $y_t - y^*$. Then equation (28) may be written as a second-order difference equation with the gap $\hat{y}_t$ as the forcing variable, the solution of which will involve finding the roots θ_1 and θ_2 of the homogeneous equation:

$$(29) \qquad 1 - \left(\frac{1+\beta}{\beta}\right)B + \left(\frac{1}{\beta}\right)B^2 = (1 - \theta_1 B)(1 - \theta_2 B) = 0.$$

In this case, we can find the exact solutions for the roots as:

$$\theta_1 = 1, \qquad \theta_2 = 1/\beta \geq 1.$$

Solving the larger root forward gives us,

$$(30) \qquad \pi_t = c^{-1}\left[\hat{y}_t + \sum_{i=0}^{\infty} \beta^i \hat{y}^e_{t+1+i}\right],$$

where π_t is the rate of inflation $(p_t - p_{t-1})$. Equation (30) says that the rate of inflation is a geometrically weighted average of present and expected future values of the output gap. Moreover, this equation produces stable paths for inflation when there are stable paths for the output gap. Thus a temporary gap produces only a temporary effect on the rate of inflation.[4]

It might be argued that our Phillips curve is derived using a simplistic structure for aggregate demand. Essentially, by assuming that nominal demand is exogenous, we are assuming that the interest elasticity of money demand is zero. We shall now pursue the implications of interest-sensitive money demand for the form of the price adjustment equation. It should be noted that the versions of the price adjustment formula in equations (10) and (17) remain valid under this extension. Only the form of the Phillips curve changes.

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[4] Our derivation of the Phillips curve is inconsistent with the traditional backward-looking approach. To see this, take the backward solution to get:

$$\pi_t = c^{-1} \sum_{i=0}^{\infty} (1/\beta)^i \hat{y}_{t-i},$$

which produces explosive paths for the rate of inflation.

Let the money demand equation be:

$$(31) \qquad m_t - p_t = y_t - \gamma r_t, \qquad \gamma \geq 0,$$

where m_t is the log of the money supply, assumed to be exogenous, and r_t is the nominal interest rate. Equation (31) implies that the long-run equilibrium price level may be written as:

$$(32) \qquad p_t^* = m_t - y^* - \varepsilon_t + \gamma r_t^*,$$

where r_t^* is the equilibrium nominal interest rate in the absence of adjustment costs. In the simplest models of a closed economy, r_t^* is determined by real factors: the rate of time preference, the rate of population growth, and the rate of technical progress, as well as by velocity-adjusted money growth less potential output growth. In an open economy model, it might be more convenient to determine r_t^* by the world rate of interest plus relative money growth rates less relative output growth rates. In either case, r_t^* can be treated as exogenous. Substituting equation (32) into equation (10) and rearranging gives,

$$(33) \qquad p_t = \left(\frac{1}{1+\beta}\right)p_{t-1} + \left(\frac{\beta}{1+\beta}\right)p_{t+1}^e + \frac{1}{c(1+\beta)}(y_t - y_t^*)$$

$$- \frac{\gamma}{c(1+\beta)}(r_t - r_t^*) + u_t.$$

In comparison with equation (28) the only difference is the interest rate gap term. Thus optimal price adjustment involves not only a positive response to the output gap but also a negative response to the interest rate gap.[5]

We now have three equivalent formulas which describe optimal price adjustment. These are equations (10), (17) and (28) or (33). I see the latter two formulas, but particularly (33), as useful and practical ways to implement costly price adjustment in large macro models. While equations (10) and (17) are useful for theoretical analyses of the implications of costly price adjustment, a limitation is that they are written in terms of the long-run equilibrium price p_t^* which will in general be a complex function of the entire structure the model. The advantage of (33) is that it explains deviations of prices from average prices as arising from the output and interest rate gaps.

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[5] The microcomputer version of the Small Annual Model (SAM) has interest rate and output disequilibrium terms in the price adjustment equation, more or less as implied by the optimal price adjustment theory described here (see Selody, Wen and Golob 1986). Theirs is derived, like ours, from the money market equilibrium conditions.

V CREDIBILITY AND THE DEGREE OF PRICE FLEXIBILITY

The price adjustment mechanism can also be used to address the issue of the effects of varying degrees of credibility of monetary policy. Rewrite equation (10) in first-difference form as:

$$(34) \qquad \pi_t = \psi\pi_{t-1} + \beta\psi\pi^e_{t+1} + (1-\psi-\beta\psi)\pi^*_t.$$

Now π^*_t is the long-run equilibrium rate of inflation. In the context of monetary policy, if the monetary authority has an announced target for the rate of inflation, π^T, it is reasonable to assume that this will be equal to π^*_t, which is consistent with some target for the rate of growth of the money supply or nominal income. While the monetary authority may set this target for the rate of inflation, private agents may not believe that the target will be attained, at least not in the short run. This could be interpreted as a consequence of the time-inconsistency of optimal policy. Optimal discretionary policy in this model will involve taking advantage of the short-run Phillips curve trade-off, which will produce a higher rate of inflation than is optimal. Optimal time-consistent policy will, on the other hand, involve setting a target for inflation, such as zero, and sticking to it.[6] Thus, while the monetary authority may announce a target for inflation, private agents know that it always has an incentive to renege

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[6] This description of optimal policy in the model is probably roughly correct. It is, however, based on the work of Barro and Gordon (1983), and not on our specific model. Due to the fact that the our Phillips curve has lags as well as leads of the price level, the precise form of an optimal monetary policy is likely to be somewhat more complicated than in their model. This means, among other things, that monetary policy rules will have to take into account the delayed impact of money on prices. This will be the subject of a future paper.

on its promises and to try to exploit the Phillips curve. This credibility problem means that π^e_{t+1} may not reflect a belief that the actual rate of inflation will adjust towards π^T. Equation (34) shows that this would slow the actual adjustment process.

It is plausible that private agents will base their inflation expectations on a weighted average of past inflation, which reflects the past behaviour of the monetary authority, and the inflation target.[7] That is,

$$(35) \qquad \pi^e_{t+1} = \Phi\pi^T + (1-\Phi)\pi_{t-1}, \qquad 0 \le \Phi \le 1,$$

where Φ is the degree of credibility of monetary policy. Note that this interpretation of Φ as the degree of credibility assumes that the monetary authority wishes to reach its target immediately. If, as is more likely, the announced target is to be attained gradually, then Φ would reflect the speed of attainment of the target in addition to the degree of credibility. Substituting equation (35) into equation (34) yields:

$$(36) \qquad \pi_t = \psi[1 + \beta(1-\Phi)]\pi_{t-1} + \{1 - \psi[1 + \beta(1-\Phi)]\}\pi^T.$$

Equation (36) says that the rate of inflation is a weighted average of past inflation and the inflation target. The coefficient on π^T, $1 - \psi[1+\beta(1-\Phi)]$, has the natural interpretation of the degree of price flexibility. Notice that the degree of price flexibility depends *positively* on the degree of credibility of policy. This occurs despite the price rigidity which is

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7 Rose and Selody (1985) derive an expectations equation that is similar to equation (35). Their formulation models forward-looking expectations on the adjustment path to a long-run equilibrium inflation rate.

present due to the costs of adjustment. This result complements the result of Section 3, that the degree of price flexibility depends positively on the degree of permanence of shocks. Together, these results imply that changes in monetary policies that are perceived to be permanent and credible will increase price flexibility and reduce output variability.

VI CONCLUSION

In this paper, the implications for aggregate behaviour of quadratic costs of changing prices are considered. There are several interesting findings:

1. The Phillips curve emerges as a natural consequence of costly price adjustment. Thus one can interpret the traditional Phillips curve as a useful rule-of-thumb which approximately describes optimal price adjustment.

2. The specific form of the Phillips curve differs from the usual forms in that expectations are forward-looking, though there is a backward-looking aspect to the function.

3. Optimal price adjustment implies different responses to temporary and permanent shocks. In particular, temporary monetary shocks have greater output effects and smaller price effects than permanent monetary shocks. However, temporary supply shocks have smaller output and price effects than permanent ones.

4. Increased credibility is stabilizing in the sense that the higher the degree of policy credibility, the higher the effective degree of price flexibility and the lower the degree of output variability.

We see the formulas developed in this paper as useful not only in describing the implications of costly price adjustment, but also in providing suggestions for the implementation of this form of price adjustment in large-scale macro models.

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