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Monthly Estimates of Canadian GDP and its Price Deflator

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MONTHLY ESTIMATES OF CANADIAN GDP AND ITS PRICE DEFLATOR

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Abstract

Statistics Canada publishes a monthly series for real domestic product at factor cost and quarterly series for real domestic product at market price and the associated implicit deflator. There are no official data for a monthly deflator or for real gross domestic product at market prices. Researchers often rely on proxies or interpolated values in their empirical work. Milbourne, Guay, Otto and Smith (1988) applied the Chow and Lin (1971) technique to produce a monthly seasonally unadjusted series for these variables. This paper uses the same technique to produce seasonally adjusted estimates of monthly real GDP at market price and the corresponding deflator. An important part of the methodology is the imposition of consistency between the monthly and quarterly values. With respect to the deflator, the method chosen by Milbourne et al. to impose this consistency is shown to produce unbelievable erratic movements in the monthly estimates. An alternative method is suggested and the price deflator generated using this approach tends to be much less choppy, and therefore more credible, as a measure of monthly price changes.

Résumé

Statistique Canada publie une série mensuelle du produit intérieur réel au coût des facteurs et des séries trimestrielles du produit intérieur réel aux prix du marché et de l'indice implicite de prix qui s'y rapporte. Il ne publie pas d'indice implicite de prix mensuel ni de données sur le produit intérieur brut en termes réels aux prix du marché. Pour quantifier ces deux variables, les chercheurs doivent souvent procéder dans leurs travaux empiriques par approximation ou par interpolation. Milbourne, Guay, Otto et Smith (1988) ont produit une série mensuelle non désaisonnalisée de ces variables en appliquant la technique de Chow et Lin (1971). L'auteur de la présente étude a fait de même pour construire des estimations mensuelles désaisonnalisées du PIB réel aux prix du marché et de l'indice implicite de prix qui s'y rapporte. L'imposition d'une contrainte de cohérence entre les valeurs mensuelles et trimestrielles fait partie intégrante de la méthode retenue. La technique que Milbourne et ses collègues emploient pour imposer la contrainte de cohérence en ce qui concerne l'indice implicite de prix provoque des variations erratiques incroyables des estimations mensuelles. C'est pourquoi l'auteur adopte une autre méthode, qui tend à produire un indice implicite de prix moins instable et, par conséquent, plus plausible en tant que mesure des variations mensuelles des prix.

1. INTRODUCTION

This paper presents seasonally adjusted estimates of monthly real (i.e. constant 1981 dollars) gross domestic product (GDP) at market price for Canada and the corresponding implicit price deflator. Currently, only quarterly averages of these series are available from Statistics Canada.

Macroeconomic studies that require monthly data for real GDP at market price and its implicit price deflator have typically relied on data from proxy variables or on monthly data interpolated from the observed quarterly data.[1] While Statistics Canada publishes the monthly real GDP at factor cost, a close proxy for real GDP at market price, data on the corresponding implicit price deflator are not available. In the United States, one frequently used proxy is the monthly series on industrial production. However, industrial production (measured at factor cost) currently accounts for only about 27 per cent of total production in Canada. As well, the (quarterly) GDP series and the industrial production series are quite different. As seen in Figure 1, GDP is much smoother over time than is the index of industrial production.

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[1] Milbourne, Guay, Otto and Smith (1988) provide a selection of circumstances in which monthly data would be useful. They also discuss the means by which researchers have attempted to compensate for the lack of data.

Interpolation techniques that use only the quarterly series to generate estimates of monthly data cannot capture month-to-month variations. Since much of the interest in higher frequency models concerns intra-quarter movements, simple interpolation techniques are unlikely to generate adequate estimates.

The interpolation procedure followed in this paper is an application of the Kalman filter techniques derived by Chow and Lin (1971). A summary of their derivation is presented in Section 2. Milbourne, Guay, Otto and Smith (1988)[2] have applied this technique to Canadian data, and some of the issues they raised are outlined in Section 3, along with motivation for introducing changes to their approach.

Section 4 discusses in more detail the Chow-Lin procedure and offers an alternative approach to that followed by MGOS. In particular, when constructing the price series, MGOS impose consistency between their monthly price series estimates and the quarterly observations through the value assigned to the third month within each quarter. This paper adopts an alternative procedure which treats all months within the quarter symmetrically. The constructed price series therefore does not suffer from the spurious variation found in the MGOS estimates.

A few concluding remarks are given in Section 5. The appendix lists the constructed series and the data sources.

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[2] Henceforth referred to as MGOS.

2. CHOW-LIN ESTIMATION TECHNIQUE

In general terms, the problem is to construct a series of monthly values which satisfies the aggregation, or consistency, condition, i.e. that the average of the monthly values must equal the published quarterly values.[3] Friedman (1962), in a survey of the techniques for interpolating time series, concluded that a simple regression approach that takes account of the correlation between the series of interest and related series would improve upon simple linear interpolation.

The Chow-Lin procedure followed in this paper represents a generalization and an extension of Friedman's work. When monthly values of a quarterly series are required, Chow and Lin suggest using a Kalman filter to incorporate information from correlated monthly variables. Monthly values for the series of interest are estimated while ensuring that the estimates consistently aggregate to the known quarterly values.

There are two basic steps in the technique. First, both monthly and quarterly data on variables believed to contain information on the series of interest are collected. The relationship between the series of interest and these other

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[3] In some cases the appropriate quarterly variable could be a sum of the monthly observations. In this paper it is assumed that an average is the appropriate aggregate.

variables is econometrically estimated using observed quarterly data. By assuming that this relationship also holds for monthly data, a first guess at the unobserved monthly values of the series of interest can be made using the monthly data of the correlated series. In this step, the aggregation condition holds only by chance. In the second step, the first "guesses" are aggregated to the quarterly frequency. If these aggregated estimates differ from the known quarterly values, a correction is made. A fraction of the quarterly error is allocated to the initial monthly estimates. This procedure guarantees consistency of the monthly estimates with the quarterly observations. A summary of the formal derivation of Chow-Lin procedure follows.

Assume that the (unobserved) monthly measurements of the series to be estimated, y, satisfy a multiple regression relationship with p related series $x_1, \ldots, x_p$. Over the sample period of 3n months (n quarters) the relation is

$$y = X\beta + u \tag{1}$$

where y is 3nx1, X is 3nxp, β is px1 and u is a random vector. Assume the series u_i, i=1,...,3n has zero mean and is serially uncorrelated with constant variance σ^2. Thus, the covariance matrix of u is $\sigma^2 I$, where I is a 3nx3n identity matrix.

Let C denote the nx3n matrix converting the 3n monthly observations into n quarterly observations. Thus

$$C = 1/3 \begin{bmatrix} 1\ 1\ 1\ 0\ 0\ 0\ 0 & \ldots & 0\ 0\ 0\ 0 \\ 0\ 0\ 0\ 1\ 1\ 1\ 0 & \ldots & 0\ 0\ 0\ 0 \\ 0\ 0\ 0\ 0\ 0\ 0\ 1 & \ldots & 0\ 0\ 0\ 0 \\ \ldots & \ldots & \ldots \\ 0\ 0\ 0\ 0\ 0\ 0\ 0 & \ldots & 0\ 1\ 1\ 1 \end{bmatrix} \quad (2)$$

and from equation (1) it follows that

$$Cy = CX\beta + Cu.$$

Let z represent the nx1 vector of quarterly observations on the dependent variable and W represent the nxp matrix of quarterly observations on the regressors. Thus

$$z = Cy$$

$$\text{and} \qquad W = CX. \quad (3)$$

Given (1), (2) and (3) it can be seen that

$$z = W\beta + Cu. \quad (4)$$

Note that the error term in this regression equation, Cu, has zero mean and covariance matrix

$$E[Cuu'C'] = \sigma^2 CC'. \quad (5)$$

$$= 1/3\sigma^2 I_n$$

The problem is to estimate a vector y^* of 3n monthly observations on the dependent variable. Consistency is imposed by requiring the monthly estimates to correctly aggregate to the known quarterly values z. That is,

$$z = Cy^*. \quad (6)$$

Chow and Lin derive the best (minimum variance) unbiased estimator for this problem. Their estimator takes the form

$$y^* = X\beta^* + C'(CC')^{-1}w^* \tag{7}$$

where

$$\beta^* = (W'(CC')^{-1}W)^{-1}W'(CC')^{-1}z \tag{8}$$

and

$$w^* = z - W\beta^*. \tag{9}$$

Using the specification for C as given in (2) and the assumptions on the random variable u, (7) and (8) can be simplified to

$$y^* = X\beta^* + 3C'w^* \tag{7'}$$

$$\beta^* = (W'W)^{-1}W'z \tag{8'}$$

so that β^* is the least squares estimate of the coefficients in a quarterly regression of z on the quarterly averaged data given by W.[4] w^* is the nx1 vector of residuals from this quarterly regression.

A simple intuitive interpretation can be given to the two components of the estimator given by (7). The first component applies coefficients β^*, estimated with the quarterly model, to the observed high frequency (monthly) series of related variables in X to obtain an estimate of $X\beta$, the first term in the expression for y in (1). The second component is an estimate of the 3nx1 vector u of disturbances. Given our specification of C and the assumptions that u is serially uncorrelated and homoscedastic, this amounts to adding the quarterly residual w^* to each of the monthly fitted values within the quarter. It is this second component that

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[4] Recall that z, which is observed, is a quarterly average of the unobserved y and W is the corresponding matrix of quarterly averages of related monthly series in X.

ensures the consistency of monthly estimates y^* with known quarterly averages z.

Fernandez (1981) provided an alternative means of formulating a solution to the problem of distribution. Using the same notation as above, his approach was to minimize the quadratic loss function

$$(y^* - X\beta)'A(y^* - X\beta) \tag{10}$$

subject to the consistency condition $z = Cy^*$. By specifying the weighting matrix A to be the 3nx3n identity matrix I, Fernandez shows that his approach renders the same results as Chow and Lin (i.e. the same expression for an estimator of y^*) when high frequency residuals are homoscedastic and serially uncorrelated. Under these specifications, the estimator y^* as given by (7) is the best linear unbiased estimator.

3. ESTIMATION OF MONTHLY CANADIAN GDP AT MARKET PRICE

Milbourne et al. used the Chow-Lin technique to create estimates of a monthly real GDP series at market prices for Canada, as well as a consistent price deflator for the period 1962-1985.

The exercise conducted here is conceptually the same as that of MGOS. The only differences are that the equations estimated here use seasonally adjusted data and the sample is extended to 1988M3. Most of the data used for the analysis in this paper are the seasonally adjusted analogs of those used in the study by MGOS.

Seasonally adjusted, monthly estimates of GDP at market prices could be used for a number of purposes. For example, they would enable researchers to model the linkages between movements in total output and movements in other monthly series, such as retail trade and merchandise trade, which are available on a seasonally adjusted basis. This could provide lead information on the seasonally adjusted, quarterly estimates of GDP -- the series most often used as a reliable measure of overall economic activity.

3.1 ESTIMATION OF MONTHLY GDP AT MARKET PRICE: PROCEDURE

In broad terms, the procedure for constructing a monthly real GDP series at market price is as follows. Statistics Canada produces a monthly series of real GDP at factor cost. Real GDP at market prices is equal to real GDP at factor cost plus real net indirect taxes plus the statistical discrepancy. To produce a monthly GDP at market price series we need to add to monthly GDP at factor cost a monthly measure of real net indirect tax revenue, and a monthly measure of the statistical discrepancy. A discussion of the treatment of net indirect taxes and statistical discrepancy follows.

3.2 STATISTICAL DISCREPANCY AND NATIONAL ACCOUNTS ESTIMATES

Statistics Canada calculates quarterly GDP at market prices from both the income and the expenditure sides of the National Income and Expenditure Accounts. To ensure that the totals from both sides are equal, the statistical discrepancy - defined as one half the difference between the sum of the individual income and expenditure items - is added back in to calculate total GDP.[5]

For our purpose, there are two ways of treating the statistical discrepancy. The first is to assume that measurement errors are more likely to occur in the calculation of income at factor cost. This means that the correct monthly values for GDP at factor cost are the published monthly values adjusted by an estimate of the statistical discrepancy. The alternative is to assume that the statistical discrepancy is more appropriately included with the net indirect tax component. This latter approach would require adjusting the monthly real net indirect tax series for the monthly statistical discrepancy. While a monthly indirect tax series can be constructed on the basis of related series such as retail sales and tax receipts using the Chow-Lin technique, it is unclear whether these series would contain information about the monthly statistical discrepancy. Therefore, it seems

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[5] We assume that the correct quarterly data are the ones reported in the National Accounts. In principle, the quarterly data can also be aggregated from the monthly data, but the result need not equal the ones reported in the National Accounts. We do not deal with this type of discrepancy.

reasonable to adopt the MGOS convention of adjusting GDP at factor cost for the statistical discrepancy. Moreover, it is assumed that the statistical discrepancy takes on the same value in each month of the quarter. Monthly GDP at factor cost is adjusted by the same quarterly average value of the statistical discrepancy. In subsequent discussion, reference to GDP at factor cost is to be taken to be GDP at factor cost plus the statistical discrepancy.

3.3. INDIRECT TAXES: ESTIMATION AND RESULTS

Estimates of real net indirect taxes are interpolated from monthly series related to indirect taxes using the Chow-Lin technique, under the maintained hypothesis that the monthly relationship between real net indirect taxes and the related series is the same as at the quarterly frequency.

To implement the Chow-Lin procedure, real quarterly net indirect tax revenue (z) is first regressed on a set of quarterly related series for which monthly observations are available. The estimation period is 1962:Q1 to 1988:Q1.

$$z = W\beta + w \qquad (11)$$

The dependent variable in the regression, z, is real indirect tax revenue. It is the result of subtracting quarterly real GDP at factor cost and the statistical discrepancy from quarterly real GDP at market price. W is an nxp matrix of related variables which includes current and one-quarter lagged observations on new motor vehicle sales deflated by the consumer price index (RMV), retail sales of department stores deflated by the consumer price index (RRS), seasonally adjusted net indirect tax receipts for the federal government deflated by the consumer price index (RNIT), and production disaggregated across the following sectors: agricultural (AP), total manufacturing (MP), construction (CP), wholesale trade (WP), retail trade (RP) and services (SP). Data references are described in Appendix 1.

MGOS split the sample in 1972. Their results suggested that parameter estimates differed between the two periods. A possible explanation is that the monthly GDP series at factor cost had been significantly revised in 1971. Our results also support this split and therefore the convention of estimating two subperiods is maintained. For our sample period the subperiods were 1961Q4 to 1972Q4 and 1973Q1 to 1988Q1.

From these regressions, least squares estimates of β, denoted β^*, and the vector of regression residuals, w^* (as defined in (7) and (8) respectively) are obtained. The estimates of monthly indirect tax revenue, y^*, are computed by postmultiplying the matrix of monthly observations of related series, X, by the vector of least squares coefficients, β^*, and adding the corresponding component of the quarterly residual vector, w^*. The latter ensures that the monthly indirect taxes, when aggregated, equals the reported quarterly values. In other words, the final estimate of monthly real GDP at market price is

$$gdp^{mp*} = gdp^{fc} + X\beta^{*} + 3C'w^{*} \qquad (12)$$

or

$$gdp^{mp}{}_t{}^{*} = gdp^{fc}{}_t + X_t\beta^{*} + w_t^{*}$$

$$t = 1,\ldots,n$$

where C is as defined in (2).

Figure 3 graphs the monthly series at annual rates for real GDP at market prices, for real GDP at factor cost and for industrial production. Appendix 2 lists the estimated monthly series of real GDP at market prices. The values listed are seasonally adjusted at annual rates in 1981 dollars.

4. ESTIMATION OF GDP DEFLATOR

An initial estimate of the monthly implicit deflator for GDP at market prices is constructed by extrapolating information from the monthly related series. This can be viewed as an application of Kalman filtering techniques. The transition equation for the monthly deflator used in this paper is

$$P_{ti} = \pi_0 + PI_{ti}\pi_1 + \pi_2(3t-i) + \pi_3(3t-i)^2 + e_{ti} \qquad (13)$$

$$\text{for} \quad i = 1,2,3 \quad \text{and} \quad t = 1,\ldots,n$$

where P_{ti} is the unobserved monthly deflator for the i^{th} month of the t^{th} quarter, PI is a (1x15) vector which includes current and lagged values of other monthly price indices. These indices include: the consumer price index (CPI), the industry product price index (IPPI), the residential construction price index (RPI) and the input price of non-residential construction (NRI) and an index of the Canada-U.S. exchange rate (PFX). The final three terms in equation (13) are, respectively, linear and quadratic time trends and an error term. Data references are given in Appendix 1.

Parameters π_0, π_1, π_2 and π_3 are estimated by fitting the quarterly regression equation

$$P = \pi_0 + PIND\pi_1 + \pi_2T + \pi_3QT + e \qquad (14)$$

Here, P is the (nx1) vector of observations on the quarterly GDP deflator, PIND is the (nx15) matrix of current, one- and four-quarter lagged data on related variables (CPI, IPPI, RPI, NRI, PFX), and T and QT are (nx1) vectors representing linear and quadratic time trends. PIND, T and QT are generated by starting with the monthly observations and aggregating to quarterly frequency. Although not immediately intuitive, aggregation of the time trend terms is necessary to maintain consistency with the underlying monthly model given by (14). More detailed descriptions of the aggregation methods used are given in the next two sections.

One notable difference between the MGOS specification and ours is that they did not include the exchange rate or time trend terms in the list of regressors. Estimation results corresponding to the MGOS specification were found to be inferior to those using equation (13). Only the latter set of results is included here.

4.1 IMPOSING CONSISTENCY

To satisfy the aggregation condition, the estimated monthly values must add up to the published quarterly data. Consistency of the monthly estimates of the GDP deflator with the quarterly observations cannot be guaranteed by adding the same estimate of the disturbances (the quarterly regression residual) to each of the component months, as was the case with net indirect taxes. This is because the quarterly GDP deflator is not a simple arithmetic average of the monthly deflators, but rather a weighted average where the weights are based on the ratio of monthly GDP to quarterly GDP.

Let y_{ti} (i=1,2,3) denote the real monthly GDP at market prices for the i^{th} month in quarter t. As MGOS showed, the condition which relates the quarterly (P_t) and monthly (P_{ti}, i=1,2,3) price deflators is

$$P_t = \frac{(P_{t1}y_{t1}+P_{t2}y_{t2}+P_{t3}y_{t3})}{(y_{t1}+y_{t2}+y_{t3})} \quad t = 1,\ldots,n. \tag{15}$$

It is this relationship that is used to impose consistency on monthly price deflator estimates.[6]

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[6] The alternative approach is to construct the price deflator as the ratio of monthly nominal GDP to monthly real GDP. In theory, the series of monthly current dollar GDP at market prices could be constructed by using the procedure outlined in Section 4.1 with current dollar analogs of the related data series. Using seasonally unadjusted data, MGOS found the results using this alternative less satisfactory.

There are two straightforward ways of imposing consistency on the monthly price deflator estimates. They differ in how they aggregate the monthly observations on price indices, exchange rate and trend terms to quarterly frequency, and in how they deal with the quarterly regression residual w^*. In the next section, the method used to impose consistency is that presented by MGOS. The subsequent section presents an alternative approach that is closer to the procedure used in Section 3.1 to generate monthly GDP estimates. A comparison of the results of these two different methods suggests a preference for the latter technique.

4.1.1 Consistency imposed using the MGOS method

The first method, as proposed by MGOS, uses arithmetic averages of the monthly price indices and time trends as measures of the corresponding quarterly series. This is equivalent to applying the conversion matrix C as defined in (2) to the (3nx15) matrix PI and to the two (3nx1) time-trend vectors with typical elements k and k^2 (k = 1,...,3n), respectively. Thus, in the regression equation (14), PIND = C•PI, T = C•[k] and QT = C•[k^2].[7] Estimates of the coefficients are obtained by estimating equation (14) over the observation period 1971Q1-1988Q1.

[7] [k] and [k^2] represent the (3nx1) linear and quadratic time trend vectors with k^{th} element k and k^2 respectively (k = 1,...,3n).

For each quarter P_1 and P_2 are constructed by multiplying the coefficient estimates from the quarterly regression by the corresponding monthly data observations. This generates an estimate for P_1 and P_2. P_3 is constructed to satisfy the aggregate consistency condition as implied by (13). Denoting the parameter estimates with an asterisk, we have

$$P_{ti} = \pi_0^* + PI_{ti}\pi_1^* + \pi_2^*(3t-i) + \pi_3^*(3t-i)^2 \qquad (16)$$

$$i = 1,2 \qquad t = 1,\ldots,n$$

and $$P_{t3} = [P_t(Y_{t1}+Y_{t2}+Y_{t3})-P_{t1}Y_{t1}-P_{t2}Y_{t2}]/Y_{t3}. \qquad (17)$$

The created monthly series is given in Appendix 3 under the label PGDPA.

4.1.2 Consistency imposed symmetrically

Another way to impose consistency is to add a value proportional to the quarterly residual to each of the component months. This method treats all months within the quarter symmetrically and follows more closely the approach of Section 2. More precisely, quarterly price indices in PI are calculated using weights based on monthly real GDP at market prices. Appropriate fractions are the ratio of monthly GDP to corresponding quarterly GDP. Define g_{ti} as the weight to be used on the i^{th} month of the t^{th} quarter in calculating the quarter t weighted average. If Y_{ti} is

real monthly GDP at market prices for the i^{th} month in quarter t, then

$$g_{ti} = y_{ti} \ / \ (y_{t1} + y_{t2} + y_{t3}) \\ = y_{ti} \ / \ 3z_t \qquad (18)$$

where z_t is the observed real GDP at market prices for quarter t. A new conversion matrix, D, can be defined as

$$D = \begin{bmatrix} g_{11} & g_{12} & g_{13} & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & g_{21} & g_{22} & g_{23} & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & & & & & \cdots & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & g_{n1} & g_{n2} & g_{n3} \end{bmatrix} \qquad (19)$$

The expression for D is used to aggregate monthly series to a quarterly frequency.[8] Thus PIND = D•PI, T = D•[k] and QT = D•[k²]. For example, the quarterly series for CPI is generated according to

$$CPI_t = g_{t1}CPI_{t1} + g_{t2}CPI_{t2} + g_{t3}CPI_{t3}, \qquad t = 1,\ldots,n. \qquad (20)$$

Using more compact notation, let M be the (3nx18) matrix of

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[8] It is worth noting that the resulting series do not differ substantially from the quarterly series calculated as arithmetic averages. Since the GDP series is a relatively smooth series, the GDP-based weights are all quite close to 1/3.

monthly data and Q be the corresponding (nx18) matrix of quarterly data. Define π to be the (18x1) vector of coefficients. Then[9]

$$M = [[1]_{3n} \quad PI \quad [k] \quad [k^2]]$$

$$Q = D \cdot M$$

$$= [D \cdot [1] \quad D \cdot PI \quad D \cdot [k] \quad D \cdot [k^2]]$$

$$= [[1]_n \quad PIND \quad T \quad QT] \quad \text{and}$$

$$\pi = [\pi_0 \quad \pi_1 \quad \pi_2 \quad \pi_3].$$

Equation (15) can be rewritten as

$$P = D \cdot M\pi + D \cdot e$$

$$= Q\pi + D \cdot e \qquad (21)$$

which has the form of equation (4). Thus the Chow-Lin estimator of the monthly price deflators is

$$[P_{ti}^*] = M\pi^* + D'(DD')^{-1}(P - Q\pi^*)$$

where $$\pi^* = (Q'(DD')^{-1}Q)^{-1}Q'(DD')^{-1}P.$$

The matrix $(DD')^{-1}$ is a diagonal (nxn) matrix with typical element $(y_{t1}+y_{t2}+y_{t3})^2/(y_{t1}^2+y_{t2}^2+y_{t3}^2) = (g_{t1}^2+g_{t2}^2+g_{t3}^2)^{-1}$. Thus π^* can be calculated by estimating (21) using weighted least squares with weights $(g_{t1}^2+g_{t2}^2+g_{t3}^2)$. Coefficient estimates are obtained by estimating (14) over the observation period 1971Q1-1988Q1.

The first component of the estimates of the GDP deflator is formed by multiplying the coefficient estimates, π^*, from the

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9 Using similar notation to that used for the time-trend vectors, $[1]_{3n}$ is a (3nx1) unit column vector and $[1]_n$ is a (nx1) unit column vector.

quarterly regression by the monthly observations on corresponding variables. The second component is a function of the quarterly regression residuals. The matrix of residual multipliers is $D'(DD')^{-1}$. By expanding this expression, the weight for the i^{th} month in the t^{th} quarter is $g_{ti}/(g_{t1}^2+g_{t2}^2+g_{t3}^2)$.

Appendix 3 contains listings of the price deflator, calculated using both the method of section 4.1.1 and the method of this section (labelled PGDPA and PGDPW, respectively).

4.2. A COMPARISON OF RESULTS

The GDP deflator series generated using the methods described in the previous two sections are plotted in Figure 4 along with the consumer price index (CPI). The procedure described in Section 4.1.2, in which months within the quarter are treated symmetrically, produces a smooth series (PGDPW), whereas the procedure of Section 4.1.1 produces a very choppy series (PGDPA). The month-to-month variations seem to increase in magnitude later in the estimation period[10]. CPI is also a relatively smooth series and it is probable that the actual GDP deflator series would experience variations in amplitude similar to those of CPI. The choppiness of PGDPA can be attributed to the consistency

[10] It should be noted that this discussion refers to results generated when specification (14) was used. The variation was even more pronounced when the lagged-dependent-variable specification was used.

requirement that is imposed only on the value assigned to the third month within each quarter and is therefore almost certainly spurious. The benefits of extra information from a higher frequency series seems, in this instance, to be overcome by the noise introduced through the way consistency was imposed.

The procedure by which PGDPW has been created does not introduce the spurious variation found in PGDPA yet maintains consistency. Therefore PGDPW is a better estimate of the unobserved monthly deflator and would be the preferred data for use in future empirical studies.

5. SUMMARY

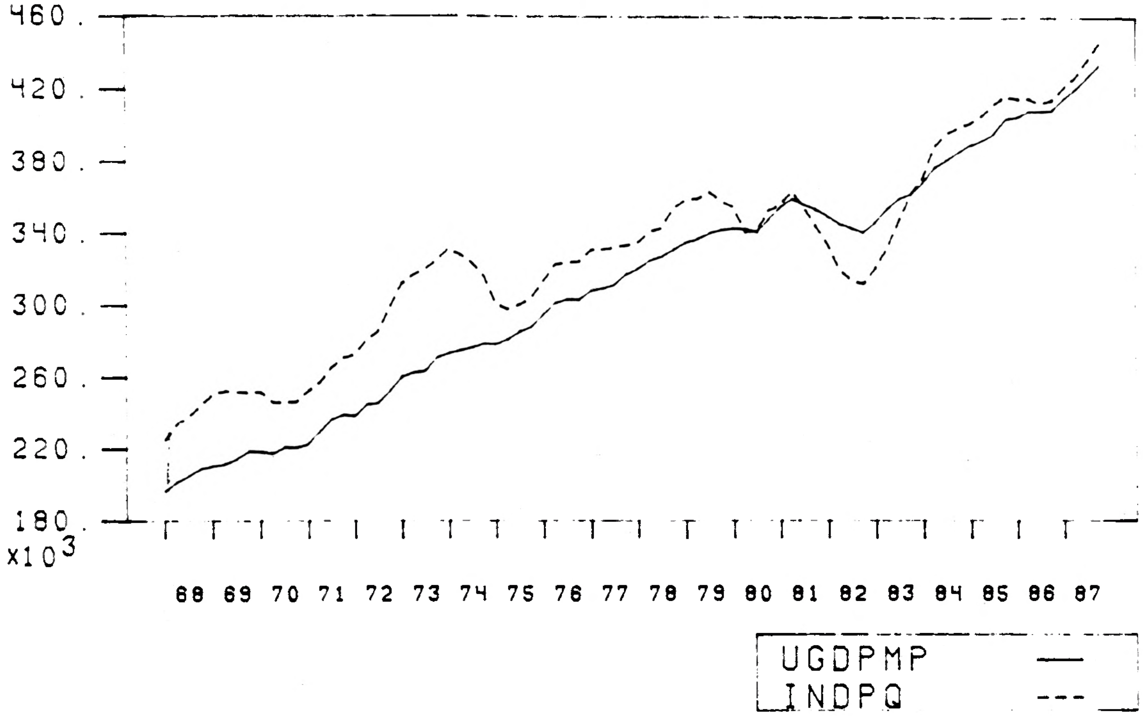
Seasonally adjusted estimates of monthly Canadian real GDP at market prices for Canada and the corresponding deflator are generated using an application of Kalman filtering techniques suggested by Chow and Lin (1971). The procedure preserves consistency between the observed quarterly and constructed monthly series, while taking account of the correlation between the series of interest and other related series.

The issue of consistency must be addressed because monthly values of GDP are aggregated to quarterly frequency in a different fashion than monthly values of the deflator. A choppy series results when consistency on the estimated deflator is imposed through the estimate of the value for the third month within each quarter (as in the study of Milbourne, Gray, Otto and Smith, 1988). An alternative technique is suggested, which treats all months within the quarter symmetrically; it generates a price deflator series that does not suffer from spurious choppiness.

The created series can be used in empirical studies to replace proxy variables or simple interpolations of quarterly data.

FIGURE 1

Quarterly gross domestic product and industrial production

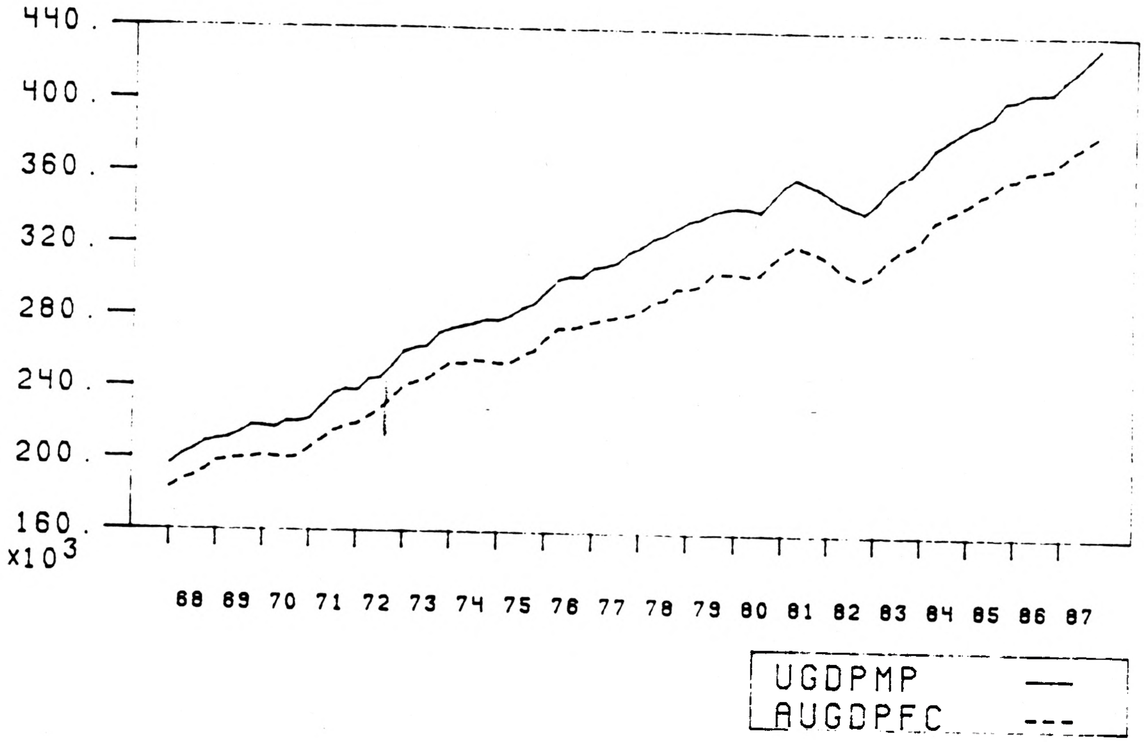


UGDPMP is quarterly GDP at market prices in millions of 1981 dollars, expressed at an annual rate (i.e. times 4).

INDPQ is quarterly industrial production at an annual rate at factor cost in millions of 1981 dollars scaled up by a factor of 4.

FIGURE 2

Quarterly gross domestic product at market price and gross domestic product at factor cost

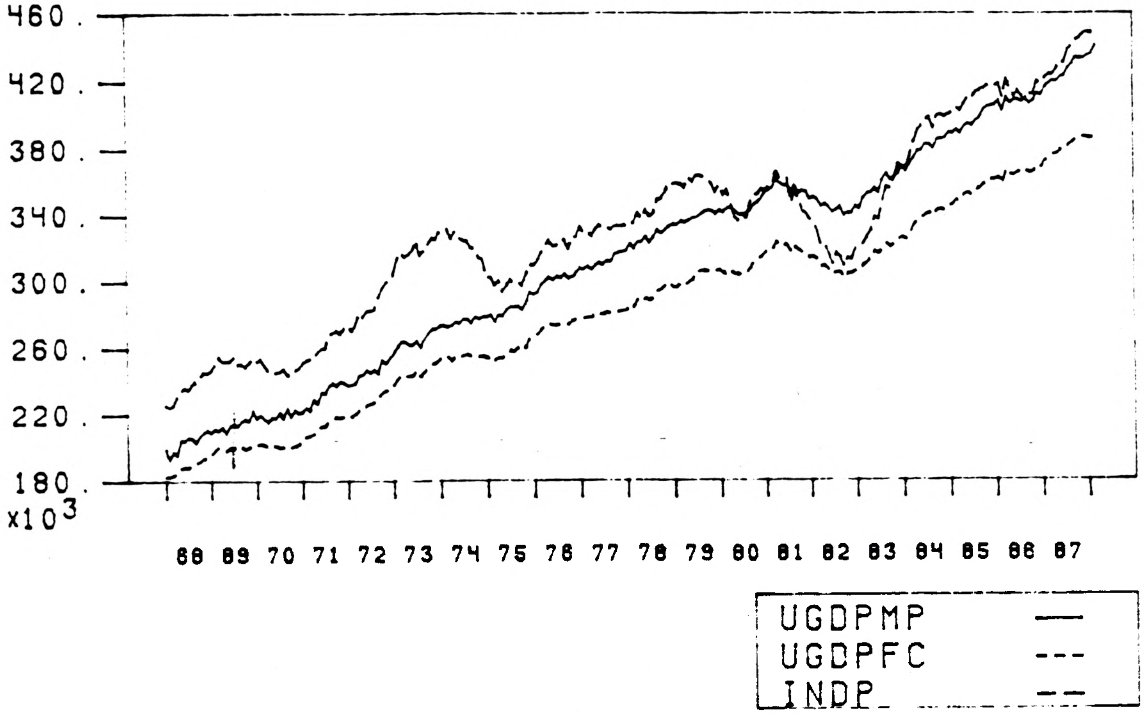


UGDPMP is quarterly GDP at market prices in millions of 1981 dollars.

AUGDPFC is quarterly GDP at factor cost in millions of 1981 dollars adjusted for the statistical discrepancy.

FIGURE 3

Monthly gross domestic product and industrial production: 1968-1988



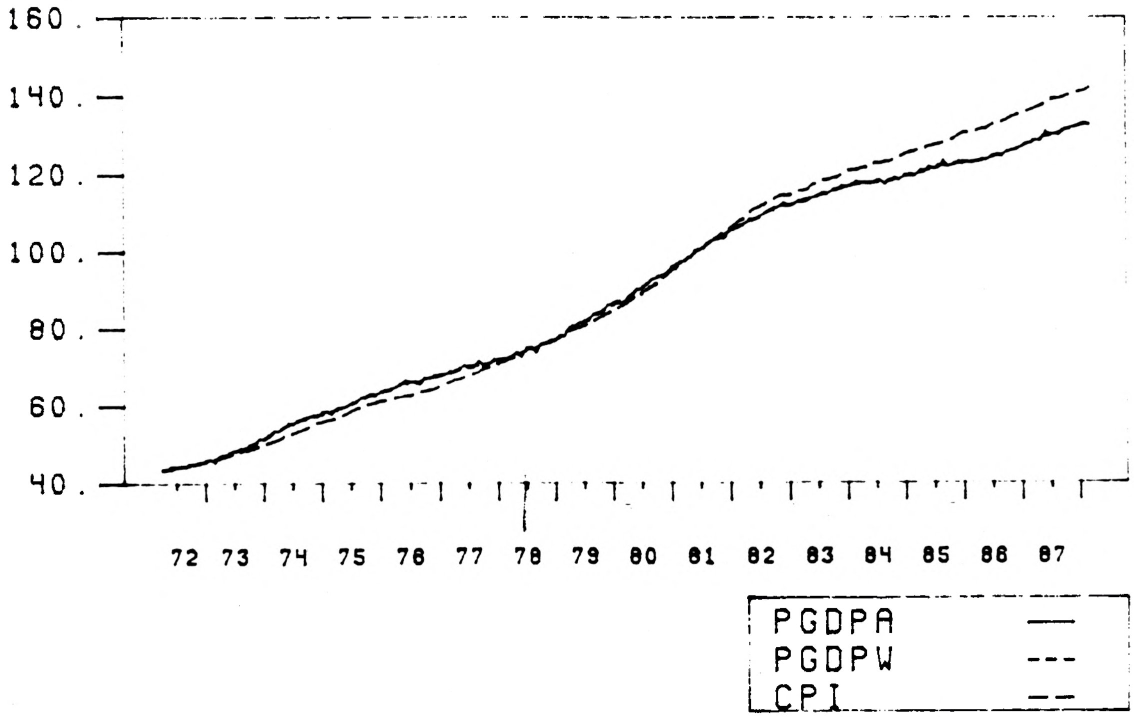
UGDPMP is monthly GDP at market prices in millions of 1981 dollars.

UGDPFC is monthly GDP at factor cost in millions of 1981 dollars.

INDP is monthly industrial production at factor cost in millions of 1981 dollars.

FIGURE 4

Gross domestic product deflators and CPI : 1972-1988



PGDPA is the monthly GDP deflator constructed using the method described in Section 4.1.1.

PGDPW is the month GDP deflator constructed using the method described in Section 4.1.2.

APPENDIX 1

DATA REFERENCES

Quarterly series		**Cansim 1988 reference**
Real GDP at market prices, 1981 ≡ 100		D20031
Real GDP at factor cost, 1981 ≡ 100		I30026
Statistical discrepancy, 1981 ≡ 100		D20052
Real net indirect taxes, 1981 ≡ 100 (z)		D20031-(I30026+D20052)
Monthly series (X)		
Real GDP at factor cost, 1981 ≡ 100		I32026
New motor vehicle sales	(adjusted)	D2363
Retail sales (department stores)	1968 - 1970	D655483
	1971 - 1988	D650091
Monthly output figures:		
Agriculture (AP)		I32001
Total manufacturing (MP)		I32005
Construction (CP)		I32006
Wholesale trade (WP)		I32010
Retail trade (RP)		I32011
Services (SP)		I32034

Indirect tax receipts and fiscal transfer payments by the federal government come from the Statement of Financial Operations (prepared by the Accounting, Banking and Compensation Directorate of the Department of Supply and Services) (NIT)

Monthly series (PI)	
Price indices (1981 ≡ 100)	
Consumer price index (CPI)	B82000
Industry selling price (IPPI)	IPPIA1 (in DATCON*PRICES)
Residential construction (RPI)	D649830,D610002
Input price, non-residential construction (NRI)	D649835,D476602
Canada-U.S. exchange rate: $CDN/$US (PFX)	B3400

APPENDIX 2
GDP AT MARKET PRICES
(Constant 1981 dollars at annual rate)

DATE	MUGDPMP	DATE	MUGDPMP	DATE	MUGDPMP
61: 4	134663	65: 5	172236	69: 6	213693
61: 5	134587	65: 6	173159	69: 7	214143
61: 6	138978	65: 7	174997	69: 8	213493
61: 7	135390	65: 8	176064	69: 9	216116
61: 8	142724	65: 9	176687	69:10	217439
61: 9	137278	65:10	179042	69:11	216199
61:10	140762	65:11	179380	69:12	222905
61:11	139573	65:12	182958	70: 1	217646
61:12	141862	66: 1	180771	70: 2	220367
62: 1	143655	66: 2	185945	70: 3	217500
62: 2	144319	66: 3	187060	70: 4	215709
62: 3	146222	66: 4	187033	70: 5	218334
62: 4	144799	66: 5	185720	70: 6	218157
62: 5	145069	66: 6	188547	70: 7	221478
62: 6	144808	66: 7	187995	70: 8	217587
62: 7	146790	66: 8	188206	70: 9	224463
62: 8	146884	66: 9	186671	70:10	218786
62: 9	147627	66:10	189482	70:11	222857
62:10	149355	66:11	189622	70:12	221094
62:11	150505	66:12	190104	71: 1	221532
62:12	149336	67: 1	190692	71: 2	224649
63: 1	151166	67: 2	187487	71: 3	222471
63: 2	151114	67: 3	189817	71: 4	229457
63: 3	149831	67: 4	189784	71: 5	226424
63: 4	150686	67: 5	194748	71: 6	233507
63: 5	152444	67: 6	194864	71: 7	233128
63: 6	154574	67: 7	194810	71: 8	237444
63: 7	153180	67: 8	193951	71: 9	239481
63: 8	154242	67: 9	193155	71:10	237557
63: 9	154853	67:10	193396	71:11	240462
63:10	157510	67:11	196104	71:12	239533
63:11	157970	67:12	194216	72: 1	237749
63:12	163115	68: 1	199084	72: 2	238365
64: 1	162666	68: 2	192957	72: 3	239146
64: 2	162330	68: 3	197627	72: 4	243515
64: 3	162649	68: 4	195517	72: 5	244378
64: 4	161044	68: 5	20472	72: 6	246891
64: 5	164472	68: 6	204467	72: 7	245389
64: 6	164133	68: 7	206302	72: 8	247169
64: 7	165655	68: 8	206065	72: 9	245035
64: 8	163061	68: 9	202897	72:10	253128
64: 9	168313	68:10	207044	72:11	250694
64:10	166686	68:11	209117	72:12	253834
64:11	165719	68:12	211067	73: 1	256422
64:12	167323	69: 1	209366	73: 2	260835
65: 1	168908	69: 2	211656	73: 3	263823
65: 2	171455	69: 3	210790	73: 4	264033
65: 3	174713	69: 4	212924	73: 5	262247
65: 4	174709	69: 5	208627	73: 6	261868

DATE	MUGDPMP	DATE	MUGDPMP	DATE	MUGDPMP
73: 7	264770	77:11	317274	82: 3	348523
73: 8	260395	77:12	317741	82: 4	344880
73: 9	265647	78: 1	318119	82: 5	346485
73:10	268985	78: 2	322805	82: 6	344487
73:11	271774	78: 3	319592	82: 7	341145
73:12	271630	78: 4	323392	82: 8	342980
74: 1	273597	78: 5	323800	82: 9	344959
74: 2	274104	78: 6	327316	82:10	339298
74: 3	272764	78: 7	323269	82:11	340924
74: 4	273908	78: 8	326601	82:12	340654
74: 5	276197	78: 9	331250	83: 1	345143
74: 6	275111	78:10	328705	83: 2	343170
74: 7	277627	78:11	330970	83: 3	349904
74: 8	277608	78:12	333193	83: 4	352503
74: 9	274973	79: 1	334285	83: 5	353345
74:10	279293	79: 2	333839	83: 6	355732
74:11	277708	79: 3	336275	83: 7	353005
74:12	279184	79: 4	335364	83: 8	360807
75: 1	279000	79: 5	336683	83: 9	364820
75: 2	280666	79: 6	338076	83:10	360626
75: 3	275629	79: 7	338657	83:11	362153
75: 4	280126	79: 8	340033	83:12	364133
75: 5	279686	79: 9	341598	84: 1	369066
75: 6	283332	79:10	342630	84: 2	365743
75: 7	285142	79:11	342673	84: 3	370031
75: 8	285000	79:12	340229	84: 4	373752
75: 9	285542	80: 1	343011	84: 5	377748
75:10	283111	80: 2	341317	84: 6	378804
75:11	287311	80: 3	344001	84: 7	381860
75:12	293698	80: 4	344552	84: 8	382012
76: 1	292258	80: 5	342147	84: 9	379175
76: 2	293461	80: 6	340093	84:10	384939
76: 3	298153	80: 7	339841	84:11	385335
76: 4	300575	80: 8	340395	84:12	385914
76: 5	302633	80: 9	341911	85: 1	389243
76: 6	300824	80:10	346260	85: 2	388344
76: 7	303355	80:11	347272	85: 3	390829
76: 8	301986	80:12	349807	85: 4	388357
76: 9	304908	81: 1	353234	85: 5	392653
76:10	301689	81: 2	355123	85: 6	394547
76:11	302850	81: 3	356151	85: 7	392900
76:12	304965	81: 4	361311	85: 8	396191
77: 1	308045	81: 5	359063	85: 9	397733
77: 2	307214	81: 6	357682	85:10	402240
77: 3	309354	81: 7	357280	85:11	404405
77: 4	307038	81: 8	352934	85:12	405163
77: 5	309508	81: 9	358241	86: 1	405443
77: 6	311534	81:10	352663	86: 2	408330
77: 7	309980	81:11	354463	86: 3	401539
77: 8	312308	81:12	353782	86: 4	410000
77: 9	311745	82: 1	349351	86: 5	406459
77:10	316309	82: 2	350829	86: 6	408490

DATE	MUGDPMP
86: 7	408728
86: 8	407485
86: 9	409528
86:10	406284
86:11	408189
86:12	412359
87: 1	411243
87: 2	416428
87: 3	418445
87: 4	419925
87: 5	419576
87: 6	422203
87: 7	423134
87: 8	427555
87: 9	430000
87:10	434253
87:11	433344
87:12	433527
88: 1	435122
88: 2	435988
88: 3	441054

APPENDIX 3
GDP PRICE DEFLATOR
(1981 =100)

DATE	PGDPA	PGDPW	DATE	PGDPA	PGDPW
72: 4	43.3766	43.5697	76: 7	66.2284	65.7849
72: 5	43.6497	43.8432	76: 8	66.3534	65.9129
72: 6	44.5483	44.1662	76: 9	65.5974	66.4751
72: 7	44.3094	44.1919	76:10	67.1930	67.4397
72: 8	44.5602	44.4419	76:11	67.3358	67.5829
72: 9	44.5010	44.7374	76:12	68.0942	67.6044
72:10	45.1413	45.0158	77: 1	67.9377	68.1471
72:11	45.3292	45.2053	77: 2	68.1427	68.3511
72:12	45.2569	45.5047	77: 3	68.9494	68.5333
73: 1	45.9029	45.5872	77: 4	68.8850	69.2391
73: 2	46.2987	45.9771	77: 5	69.2539	69.6103
73: 3	45.6424	46.2684	77: 6	70.6614	69.9595
73: 4	46.9932	46.9461	77: 7	69.9887	70.3586
73: 5	47.3993	47.3528	77: 8	70.0581	70.4317
73: 6	47.8070	47.8983	77: 9	71.6615	70.9188
73: 7	48.4137	48.4147	77:10	70.7110	70.7877
73: 8	48.8875	48.8880	77:11	70.9659	71.0423
73: 9	49.2495	49.2489	77:12	71.9688	71.8157
73:10	49.5930	49.9529	78: 1	72.0089	71.8091
73:11	50.0181	50.3830	78: 2	72.2115	72.0084
73:12	51.6728	50.9510	78: 3	72.1192	72.5222
74: 1	51.3995	51.6114	78: 4	73.0880	72.4950
74: 2	52.2609	52.4717	78: 5	73.9472	73.3516
74: 3	53.6447	53.2167	78: 6	72.6375	73.8129
74: 4	53.4999	53.7026	78: 7	75.1266	74.6602
74: 5	54.3638	54.5686	78: 8	75.3263	74.8589
74: 6	55.5907	55.1850	78: 9	73.9751	74.8930
74: 7	55.3531	55.4848	78:10	75.8830	76.0524
74: 8	56.1472	56.2801	78:11	76.4275	76.5960
74: 9	56.9057	56.6391	78:12	77.0499	76.7166
74:10	57.0993	56.9152	79: 1	77.3641	77.0968
74:11	57.5516	57.3681	79: 2	78.2200	77.9540
74:12	57.3759	57.7422	79: 3	78.2567	78.7854
75: 1	58.4117	58.0172	79: 4	80.2669	80.4288
75: 2	58.8167	58.4207	79: 5	80.7114	80.8757
75: 3	57.8896	58.6910	79: 6	81.5981	81.2736
75: 4	59.3892	59.3664	79: 7	81.7720	82.0563
75: 5	59.5422	59.5194	79: 8	82.2654	82.5519
75: 6	60.0553	60.1023	79: 9	83.8705	83.3014
75: 7	60.6454	60.7811	79:10	83.7734	84.0619
75: 8	61.1772	61.3113	79:11	84.5522	84.8407
75: 9	62.2098	61.9400	79:12	86.2743	85.6901
75:10	62.5062	62.3519	80: 1	86.5107	86.1250
75:11	63.1551	62.9980	80: 2	87.2922	86.9074
75:12	62.8164	63.1208	80: 3	86.2814	87.0472
76: 1	63.7103	63.6039	80: 4	87.8784	87.9284
76: 2	64.0969	63.9906	80: 5	88.9142	88.9639
76: 3	64.1370	64.3444	80: 6	90.0717	89.9670
76: 4	64.9817	65.2284	80: 7	90.5476	90.6162
76: 5	65.4764	65.7252	80: 8	91.4437	91.5129
76: 6	66.6478	66.1493	80: 9	92.6614	92.5268

DATE	PGDPA	PGDPW	DATE	PGDPA	PGDPW
80:10	93.2628	93.2411	84: 7	118.032	117.621
80:11	93.7636	93.7413	84: 8	118.304	117.891
80:12	94.1985	94.2436	84: 9	117.041	117.873
81: 1	95.8294	95.8670	84:10	118.243	117.936
81: 2	96.5123	96.5516	84:11	118.835	118.527
81: 3	97.2231	97.1474	84:12	118.236	118.850
81: 4	97.8228	98.0003	85: 1	119.534	119.427
81: 5	98.4436	98.6191	85: 2	119.658	119.554
81: 6	100.068	99.7108	85: 3	119.312	119.523
81: 7	100.343	100.467	85: 4	120.254	120.619
81: 8	101.075	101.198	85: 5	120.267	120.636
81: 9	102.519	102.274	85: 6	121.754	121.029
81:10	102.987	102.611	85: 7	121.223	121.795
81:11	103.853	103.474	85: 8	121.324	121.901
81:12	103.388	104.143	85: 9	123.255	122.116
82: 1	105.198	105.100	85:10	121.851	121.973
82: 2	105.818	105.720	85:11	122.192	122.314
82: 3	106.214	106.409	85:12	122.881	122.637
82: 4	107.133	107.071	86: 1	122.824	122.714
82: 5	107.979	107.915	86: 2	122.967	122.856
82: 6	108.130	108.259	86: 3	122.688	122.910
82: 7	108.781	108.799	86: 4	123.214	123.160
82: 8	109.741	109.759	86: 5	123.520	123.466
82: 9	110.466	110.432	86: 6	123.593	123.702
82:10	110.873	111.179	86: 7	124.519	124.342
82:11	111.465	111.773	86: 8	124.884	124.707
82:12	112.581	111.967	86: 9	124.517	124.872
83: 1	111.774	111.846	86:10	125.587	125.580
83: 2	112.123	112.195	86:11	125.982	125.974
83: 3	113.001	112.859	86:12	126.281	126.298
83: 4	113.024	112.899	87: 1	126.933	126.901
83: 5	113.373	113.246	87: 2	127.688	127.655
83: 6	113.784	114.034	87: 3	127.939	128.001
83: 7	114.454	114.228	87: 4	128.574	129.071
83: 8	114.955	114.722	87: 5	128.524	129.023
83: 9	114.748	115.201	87: 6	130.668	129.675
83:10	115.693	115.885	87: 7	129.846	129.631
83:11	115.775	115.968	87: 8	130.174	129.956
83:12	116.706	116.322	87: 9	129.976	130.404
84: 1	116.596	116.878	87:10	131.200	131.286
84: 2	116.891	117.169	87:11	131.402	131.489
84: 3	117.851	117.294	87:12	132.010	131.838
84: 4	117.485	117.296	88: 1	132.504	132.188
84: 5	117.523	117.334	88: 2	132.836	132.518
84: 6	117.212	117.586	88: 3	132.440	133.066

TABLE 1: COEFFICIENT ESTIMATES OF EQUATION (11)

NET INDIRECT TAX REVENUE

Variable	1961:2 - 1972:4 Coefficient	1961:2 - 1972:4 t-stat.	1973:1 - 1988:2 Coefficient	1973:1 - 1988:2 t-stat.
Constant	6986.178	0.65	-18099.570	-2.53
RMV	-696.100	-1.09	551.838	1.44
RMV(-1)	1686.541	2.65	-261.374	-0.65
AP	0.450	1.22	-0.018	-0.03
AP(-1)	-0.093	-0.25	-1.294	-2.24
MP	-0.771	-1.54	0.106	0.52
MP(-1)	-0.269	-0.66	-0.211	-1.15
CP	-0.700	-0.90	-0.881	-2.41
CP(-1)	0.413	0.43	1.678	4.19
WP	-2.350	-1.00	-1.850	-1.67
WP(-1)	5.239	1.99	-1.097	-1.02
RP	8.331	3.32	-0.709	-0.58
RP(-1)	-8.834	-3.77	2.001	1.54
SP	0.211	0.42	1.070	3.54
SP(-1)	-0.185	0.33	-0.646	-2.23
RNIT	1187.655	-1.63	-121.204	-0.47
RNIT(-1)	-1247.186	-2.11	193.320	0.67
RRSQ	-5132.241	-2.98	-2029.204	-1.30
RRSQ(-1)	2515.947	1.43	761.490	0.47
	$\bar{R}^2 = 0.86$	DW = 1.83	$\bar{R}^2 = 0.98$	DW = 1.62

CHOW TEST STATISTIC = 4.64 [F(19,70)]
This easily rejects the null hypothesis that the coefficients are the same in both sample periods at less than 1%.

TABLE 2: COEFFICIENT ESTIMATES OF EQUATION (15)

GDP DEFLATOR

ESTIMATION PROCEDURE A: Quarterly data as arithmetic averages of monthly observations, Estimation period 1972.2-1988.1

Variable	Coefficient	t-stat.
Constant	20.448	7.67
CPI	0.434	2.59
CPI(-1)	0.170	0.96
CPI(-4)	-0.091	-1.39
ISP	0.237	2.78
ISP(-1)	-0.093	-0.87
ISP(-4)	0.217	2.74
RPI	-0.044	-0.83
RPI(-1)	-0.057	1.09
RPI(-4)	0.048	1.09
NRI	0.078	0.90
NRI(-1)	-0.040	-0.50
NRI(-4)	-0.033	-0.57
PFX	-13.485	4.36
PFX(-1)	-2.052	-0.61
PFX(-4)	-1.662	-0.66
T	0.094	5.44
QT	-0.258E-03	-3.90

$\bar{R}^2 = 0.99$ DW = 1.55

TABLE 3: COEFFICIENT ESTIMATES OF EQUATION (21)

GDP DEFLATOR

ESTIMATION PROCEDURE W: Quarterly data as weighted averages of monthly observations; Generalized least squares over the estimation period 1972.2-1988.1

Variable	**Coefficient**	**t-stat.**
Constant	20.455	7.66
CPI	0.433	2.57
CPI(-1)	0.171	0.96
CPI(-4)	-0.918	-1.39
ISP	0.236	2.77
ISP(-1)	-0.092	-0.86
ISP(-4)	0.217	2.37
RPI	-0.044	-0.83
RPI(-1)	0.057	1.09
RPI(-4)	0.486	1.10
NRI	0.078	0.89
NRI(-1)	-0.039	-0.50
NRI(-4)	-0.040	-0.58
PFX	-13.444	-4.34
PFX(-1)	-2.100	-0.62
PFX(-4)	-1.659	-0.65
T	0.093	5.43
QT	-0.257E-03	-3.89

$\bar{R}^2 = 0.99$ DW = 1.55

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