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Analysis of the Variance of the GNE Implicit Price Index and its Components

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ANALYSIS OF THE VARIANCE OF THE GNE IMPLICIT PRICE INDEX AND ITS COMPONENTS

The objective of this short note is to provide some background statistical information on the Implicit Price Index for Gross National Expenditure (GNE) in Canada since 1950. The analysis focuses on the variance of changes in the implicit price index and its subcomponents on the income side of the National Accounts.[1] The note is divided into three sections. The first part presents the component breakdown over the entire sample with the covariances and the correlation coefficients of the changes in the components. The second part breaks the 1950-83 period into two subsamples based on the definition of cyclical upswings and downswings as provided in the paper by Ferley, O'Reilly and Dunnigan (1984). The final part shows results of a statistical test, developed by Anderson (1958), of the equality of the covariance matrices of price components for both subsamples.

Data Preparation

Quarterly seasonally adjusted data on nominal Gross National Product at market prices and its components were drawn from Statistics Canada's National Accounts for the 1950-83 period inclusive. The components were aggregated into the following eight categories:

Gross National Product at market prices (YGNE) =

(1) Wages, salaries, supplementary labour income, military pay and allowances (WI)
(2) Corporate profits before taxes net of dividends paid to non-residents and inventory valuation adjustments (PI)
(3) Interest and miscellaneous investment income (II)
(4) Accrued net income of farm operators from farm production (FI)
(5) Net income of non-farm unincorporated business including rent (BI)
(6) Indirect taxes less subsidies (ITLS)

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1. Jarrett (W9, 1984) has produced similar results in a recent paper but his methodology specifies growth rates in the price deflator and its components. The discussion below will focus on the level differences.

This paper is one of the series of working papers for "Price Flexibility and Business Cycle Fluctuations in Canada - A Survey", a study prepared by the Research Department of the Bank of Canada for the Royal Commission on the Economic Union and Development Prospects for Canada. These research papers were all completed in early 1984.

(7) Capital consumption allowances and miscellaneous valuation adjustment (CCA)[2]

(8) Residual error of estimate (RES)

All nine series were deflated by Gross National Expenditure in constant 1971 dollars (UGNE) to generate the GNE implicit price deflator (PGNE) and price indices of each of its components.

$$PGNE = \frac{YGNE}{UGNE} = \frac{WI}{UGNE} + \frac{PI}{UGNE} + \frac{II}{UGNE} + \frac{FI}{UGNE} + \frac{BI}{UGNE} + \frac{ITLS}{UGNE} + \frac{CCA}{UGNE} + \frac{RES}{UGNE}$$

The first differences of each series were calculated and covariance and correlation matrices were constructed. The actual series are plotted in Figures 1 to 9.[3] The complete decomposition of the covariances of changes in the price deflator for 1950-83 is shown in Table 1. The diagonal gives the variances, the covariances are above the diagonal and the correlation coefficients are below.

The most important component of the change in the deflator is the change in wages and salaries. The two series are highly correlated, represented by a coefficient of 0.821, and the wage and salary component accounts for about 57% of the total variation in the overall price index. This is not a surprising result since wages and salaries comprise the largest portion of Gross National Product. Both farm and unincorporated business income have very little impact on overall price variation. Investment income and capital consumption allowances both have considerably more influence; the capital consumption series is highly correlated with the change in total price index with a coefficient of 0.784. The most striking result is the unimportance of changes in the corporate profits component: it accounts for only about 1% of the variation in the total price index. It is important to note that the correlation coefficient for the corporate profits component is limited in informational content over this large sample given its cyclical volatility. Jarrett (1984) confirms the strongly pro-cyclical nature of the corporate profits share of Gross National Product over this period. One would expect this statistic to be more meaningful in the subsamples to be presented below.

At this point several caveats with respect to methodology and interpretation should be mentioned. Firstly, both the wages and salaries

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2. This is the same series as CCA$ in the RDXF model.

3. From Figure 5 it can be seen that a significant outlier exists for the observation 1976Q1. This was due to a revision by Statistics Canada back to 1976 for both the imputed and paid rents series which are part of unincorporated business income. A check was done on the significance of the outlier for the covariance and correlation coefficients by splicing the series at that point (1976Q1). The variance was reduced as expected but the covariance matrix was only marginally different and it was decided to retain the original series.

Table 1

DECOMPOSITION OF VARIANCE OF CHANGES IN IMPLICIT PRICE DEFLATOR AND COMPONENTS

	Δ PGNE	Δ WI	Δ PI	Δ II	Δ FI	Δ BI	Δ ITLS	Δ CCA	Δ RES
Δ PGNE	4.0625	2.3079	0.06323	0.44769	0.10506	0.15739	0.43343	0.50518	0.04260
Δ WI	0.82057	1.94718	-0.40609	0.12695	0.00696	0.09584	0.27771	0.34105	-0.08169
Δ PI	0.03347	-0.31052	0.87839	-0.08794	-0.01457	-0.01453	-0.17569	-0.07003	-0.04631
Δ II	0.37125	0.15206	-0.15682	0.35796	0.00242	0.01466	0.02127	0.03137	-0.01902
Δ FI	0.13584	0.01299	-0.04051	0.01054	0.14722	-0.00957	-0.02575	-0.00060	0.00104
Δ BI	0.40172	0.35333	-0.07975	0.12609	-0.12832	0.03778	0.00702	0.02344	0.00275
Δ ITLS	0.40583	0.37558	-0.35378	0.06709	-0.12669	0.06814	0.28077	0.07954	-0.03142
Δ CCA	0.78424	0.76474	-0.23378	0.16408	-0.00491	0.37729	0.46969	0.10214	-0.00174
Δ RES	0.04495	-0.12451	-0.10509	-0.06759	-0.00579	0.03003	-0.12613	-0.01155	0.22109

and the corporate profits components are aggregations of standard National Accounts categories. The wages and salaries data include military pay and allowances but these do not represent a significant sum. However, the corporate profits component includes corporate profits before taxes net of both dividends paid to foreigners (DIV) and the inventory valuation adjustment (IVA). The reason for aggregation was to construct an appropriate economic concept of profits. The IVA removes any capital gains or losses resulting from the inventory accounting procedures of business firms (FIFO method rather than LIFO method) and the subtraction of dividends paid abroad produces a series for corporate profits that remain in Canada. Although the aggregate corporate profits series is positively correlated with PGNE at 0.03347 its components do not all have this relationship: the gross corporate profits component is negatively correlated at -0.01561 as is the dividends component at -0.01636. The component for the inventory valuation adjustment is, however, strongly correlated with PGNE at 0.09870.

Secondly, one must not ascribe too much significance to the strong correlation of capital consumption allowances to the total price variation because this is implicit in the method used in the construction of the CCA series. As it is calculated in the National Accounts on a replacement cost basis, the capital consumption charges represent business costs that are implicitly included in the market price of goods and services.

Thirdly, one possible explanation of the strong positive correlation of the changes in the CCA-PGNE and WI-PGNE series in both the full sample and the cycles can be observed in Figures 1, 2 and 8. One can see after 1970 an upward secular trend in all three graphs that would suggest a strong trend correlation. It may be inferred that this trend movement translates into the cyclical correlation observed in the data.

Fourthly, and most importantly, one should not conclude from the above results any directional causality between changes in the overall price index and in its components. For example, given the strong correlation and the large portion of the price variation accounted for by wages and salaries, one cannot infer that wage changes are causing cyclical price movements or vice versa. The data are presented for expositional purposes in order to point out interesting relationships between the GNE implicit deflator and its income components.

Impact of Cyclical Swings

This section consists of an analysis of variance similar to the above but for two subsamples of the original 1950-83 period. The first subsample comprises the periods of cyclical upswing inclusive of the peak and the second subsample includes the periods of cyclical downswing. The actual breakdown is taken directly from Ferley, O'Reilly and Dunnigan (1984) and is displayed in Table 2. The results of the decomposition of variance for the two subsamples are shown in Tables 3 and 4.

Table 2

Two Subsamples

Cyclical downswing	Cyclical upswing
1951 Q2 to 1951 Q4	1950 Q2 to 1951 Q1
1953 Q3 to 1954 Q2	1952 Q1 to 1953 Q2
1957 Q1 to 1957 Q4	1954 Q3 to 1956 Q4
1960 Q2 to 1961 Q1	1958 Q1 to 1960 Q1
1966 Q2 to 1968 Q1	1961 Q2 to 1966 Q1
1970 Q1 to 1970 Q4	1968 Q2 to 1969 Q4
1974 Q2 to 1975 Q1	1971 Q1 to 1974 Q1
1980 Q1 to 1980 Q2	1975 Q2 to 1979 Q4
1981 Q3 to 1982 Q4	1980 Q3 to 1981 Q2
	1983 Q1 to 1983 Q4
Number of observations = 39	Number of observations = 96

The wages and salaries component dominates the explanation of price variation in both cyclical upswings (51%) and downswings (66%) but is slightly stronger in the latter. Indeed, the price movements and wage movements are highly correlated, but the correlation is greater in downswings (0.88 compared with 0.78 in upswings). The cyclical volatility of the corporate profits component described in the previous section is verified below. In the expansion periods the covariance is 0.29192 whereas in the contraction periods it is -0.37992. The swings are also reflected in the correlation coefficients which vary from a positive correlation of 0.19609 in upswings to a negative one of -0.13737 in downswings. Both unincorporated business and farm income are more dominant in the price movements in upswings but are not important factors in either subsample overall. Investment income is more important in periods of contraction in influencing price variation but there is also a stronger correlation between the two series in the positive turns of the cycle. Both indirect taxes less subsidies and capital consumption allowances are more strongly correlated with price changes and more important in price change variations in the periods of contraction.

The corporate profits series was disaggregated as in the previous section for the two subsamples and it was found that the corporate profits data gross of IVA and DIV were even more volatile than the net profits series, with a correlation coefficient of 0.29013 in cyclical upswings and -0.35376 in downswings. Both the DIV and IVA series also jumped from -0.06839 and -0.11514 in upswings to 0.07359 and 0.35943 in downswings, respectively.

Testing the Importance of Cyclical Swings

The above analysis of the variation in the components' price indices with respect to that of the overall implicit price deflator was useful in providing a general overview of the differences in the two subsamples. We can assess the importance of the observed differences by testing the

Table 3

DECOMPOSITION OF VARIANCE - CYCLICAL UPSWINGS

	Δ PGNE	Δ WI	Δ PI	Δ II	Δ FI	Δ BI	Δ ITLS	Δ CCA	Δ RES
Δ PGNE	3.3206	1.6784	0.29192	0.41072	0.09378	0.07358	0.37182	0.34659	0.05389
Δ WI	0.78449	1.3784	-0.21880	0.09377	0.00667	0.04408	0.22614	0.19679	-0.04868
Δ PI	0.19609	-0.22813	0.66738	-0.09732	-0.01574	-0.00311	-0.16941	-0.03080	-0.03491
Δ II	0.49737	0.17625	-0.26289	0.20535	0.01552	0.00626	-0.01188	0.02724	-0.02287
Δ FI	0.14273	0.01576	-0.05341	0.09502	0.13002	-0.00821	-0.00306	-0.00301	-0.02842
Δ BI	0.22929	0.21321	-0.02164	0.07847	-0.12941	0.03101	-0.00246	0.00503	0.0098
Δ ITLS	0.38674	0.36509	-0.39305	-0.04970	-0.01611	-0.02648	0.27836	0.07625	-0.02212
Δ CCA	0.70976	0.62549	-0.14071	0.22434	-0.03111	0.10667	0.53932	0.07181	0.00327
Δ RES	0.06506	-0.09121	-0.09402	-0.11102	-0.17337	0.01224	-0.09222	0.02688	0.20663

Table 4

DECOMPOSITION OF VARIANCE - CYCLICAL DOWNSWINGS

	Δ PGNE	Δ WI	Δ PI	Δ II	Δ FI	Δ BI	Δ ITLS	Δ CCA	Δ RES
Δ PGNE	5.9031	3.7197	-0.37992	0.56592	0.17722	0.33647	0.58905	0.86244	0.03221
Δ WI	0.88246	3.0099	-0.65112	0.23906	0.08383	0.16398	0.39582	0.61389	-0.13561
Δ PI	-0.13737	-0.32969	1.29588	-0.56809	-0.05533	-0.00711	-0.18572	-0.11478	-0.09357
Δ II	0.26944	0.15940	-0.57730	0.74729	-0.03506	0.04006	-0.10586	0.04861	-0.01182
Δ FI	0.17192	0.11389	-0.11455	-0.09557	0.18003	-0.00139	-0.07982	0.02331	0.06166
Δ BI	0.64708	0.44164	-0.02920	0.21656	-0.01541	0.04580	-0.02809	0.05503	0.01201
Δ ITLS	0.44759	0.42121	-0.30119	-0.22607	-0.34732	-0.24231	0.29339	0.08559	-0.00705
Δ CCA	0.89348	0.89066	-0.25381	0.14154	-0.13827	0.64728	0.39774	0.15784	-0.00705
Δ RES	0.02596	-0.15308	-0.16098	-0.02679	-0.28458	0.10992	-0.19583	-0.03474	0.26076

hypothesis of the equality of the covariance matrices of the two subsamples. Anderson (1958) developed an adaptation of a likelihood ratio test to consider such a hypothesis. The methodology is explained briefly and the results for the price variation data are presented below.

The basic assumption of this test is that each subsample is a set of drawings from a multivariate normal distribution.[4] The null hypothesis is

$$H_1: \quad \Sigma_1 = \Sigma_2 \tag{1}$$

where Σ_g is the covariance matrix of the two subsamples so that the value of g is either 1 or 2. X_α^g is defined as an observation from one of the two subsamples where $\alpha = 1 \ldots N_g$, and N_1 and N_2 are the number of observations in subsamples 1 and 2 respectively ($N_1 = 39$; $N_2 = 96$). The following terms are defined

$$N = \sum_{g=1}^{2} N_g \tag{2}$$

$$\bar{X}^g = \sum_{\alpha=1}^{N_g} X_\alpha^g / N_g \tag{3}$$

$$A_g = \sum_{\alpha=1}^{N_g} (X_\alpha^g - \bar{X}^g)(X_\alpha^g - \bar{X}^g)' \tag{4}$$

$$A = \sum_{g=1}^{2} A_g \tag{5}$$

where N is the total number of observations (N=135), A_g is the covariance matrix in each subsample g multiplied by N_g and A is the sum of the two covariance matrices.

The maximum likelihood estimates of the unconstrained covariance matrices (Σ_g) are

$$\hat{\Sigma}_g = \frac{1}{N_g} A_g \tag{6}$$

and that of the constrained matrix (Σ_w) is

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4. The work here cannot be interpreted as a test for structural differences in economic behaviour. It is useful only to the extent that it provides an indication of the importance of differences observed in the data.

$$\hat{\Sigma}_w = \frac{1}{N} A.$$

Anderson constructs the likelihood ratio criterion for the hypothesis as

$$\lambda_1 = \frac{\prod_{g=1}^{2} | \hat{\Sigma}_g | ^{\frac{1}{2}Ng}}{| \hat{\Sigma}_w | ^{\frac{1}{2}N}} \tag{8}$$

and finds that,

$$- 2\ell \, n(\lambda_1) \sim X^2 \text{ with } \frac{nx(n-1)}{2} \text{ degrees of freedom}$$

where n is the number of rows in the covariance matrix. The subsamples were broken down according to Table 2, and so $N_1 = 39$ and $N_2 = 96$ for the above test. The test statistic was found to be 923.855 with 28 degrees of freedom; this rejects the hypothesis H_1 at any reasonable level of significance. We thus conclude that the covariance matrices in the cyclical upswings and downswings are significantly different.

Conclusion

The following summarizes the main conclusions of this note:

(1) Over the 1950-83 period, the wages and salaries component of the implicit GNE deflator accounts for most of the variation in the overall price index while the corporate profits index represents a strikingly small factor.

(2) Each of the components have different effects on the variation in the overall price index in the contraction and expansion periods of the economy. The corporate profits component demonstrated the most volatile shift.

(3) The hypothesis that the covariance matrices of the cyclical upswing and downswing subsamples are equal is rejected.

Figure 1

CHANGE IN GNE IMPLICIT PRICE DEFLATOR

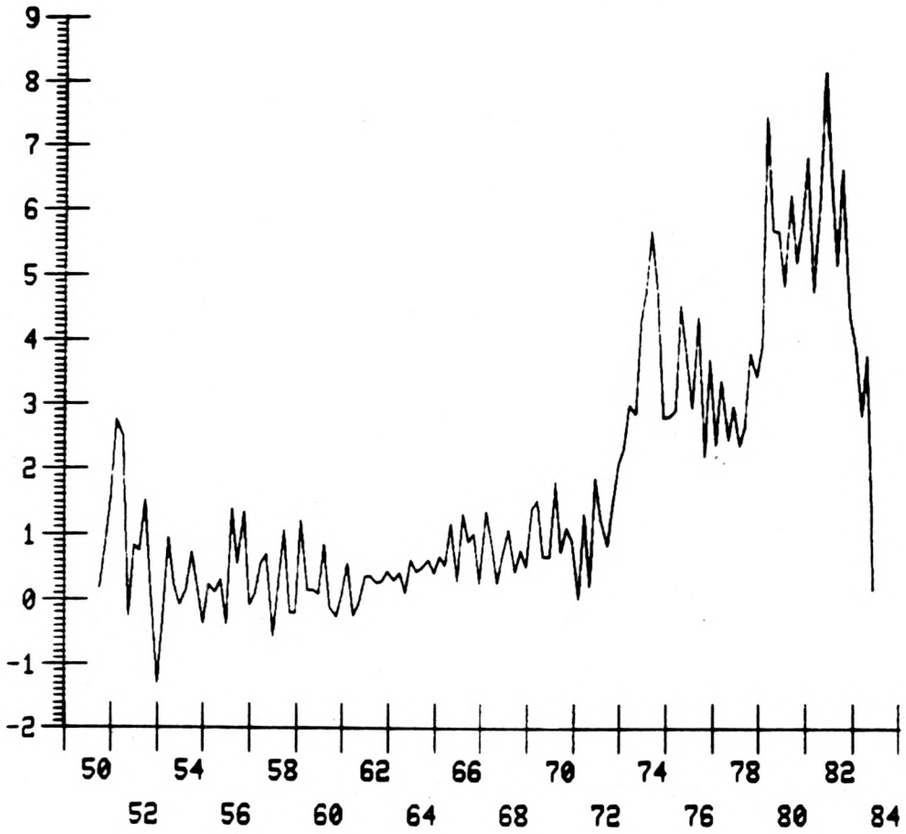


Figure 2

CHANGE IN WAGES AND SALARIES INDEX

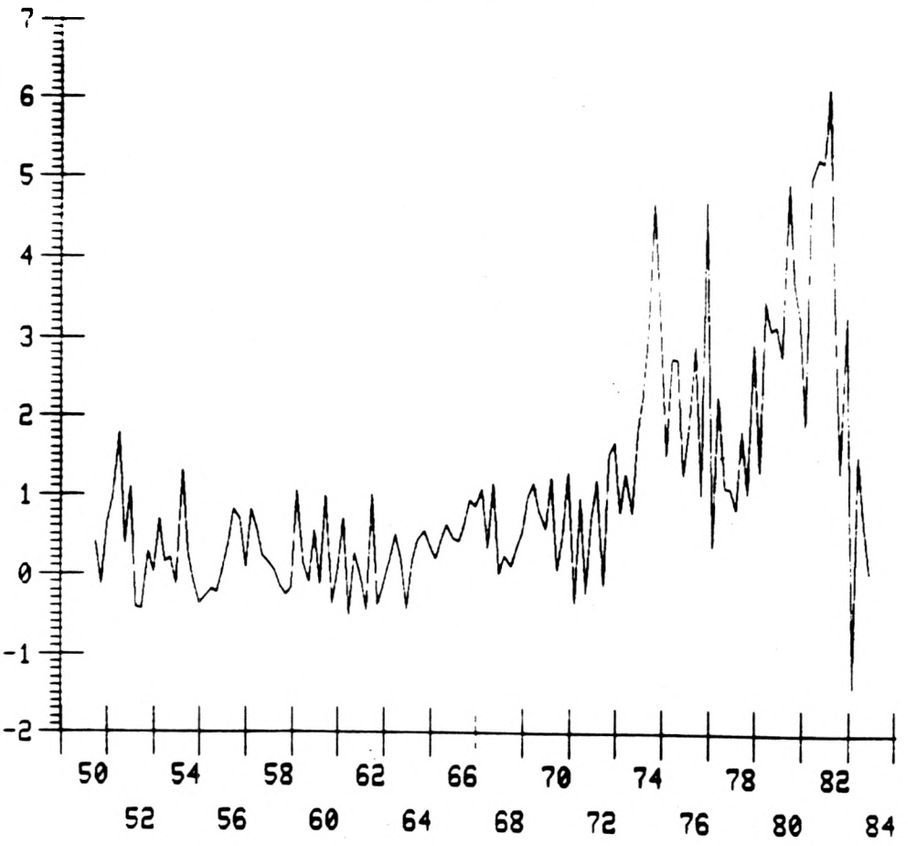


Figure 3

CHANGE IN THE CORPORATE PROFITS INDEX

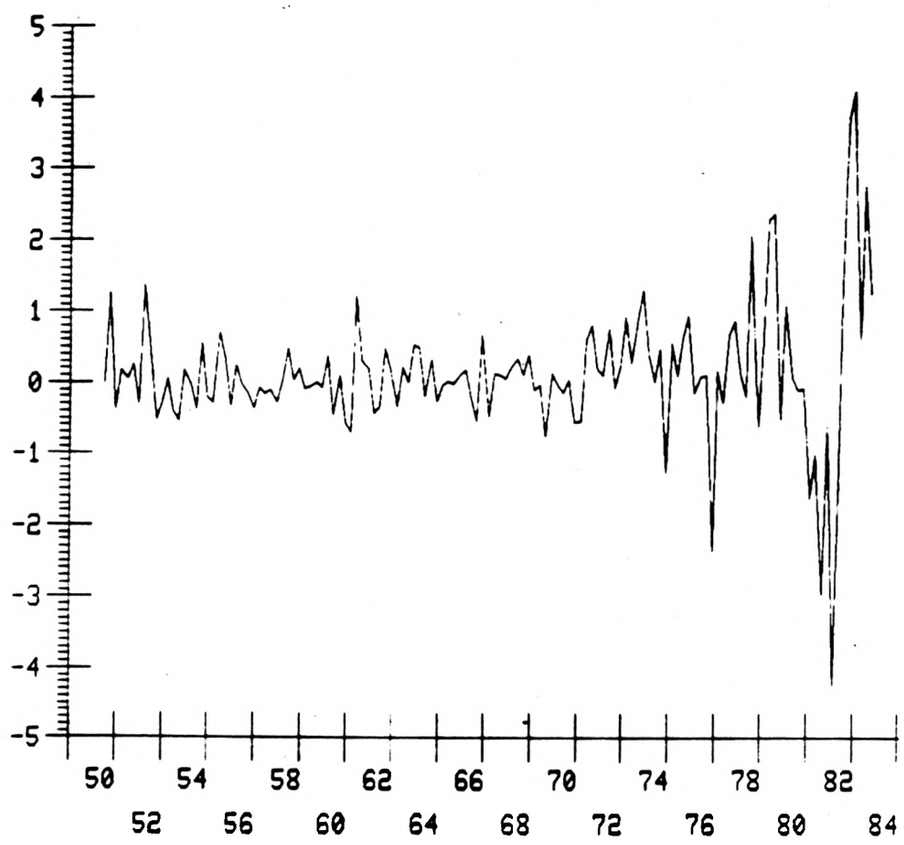


Figure 4

CHANGE IN INVESTMENT INCOME INDEX

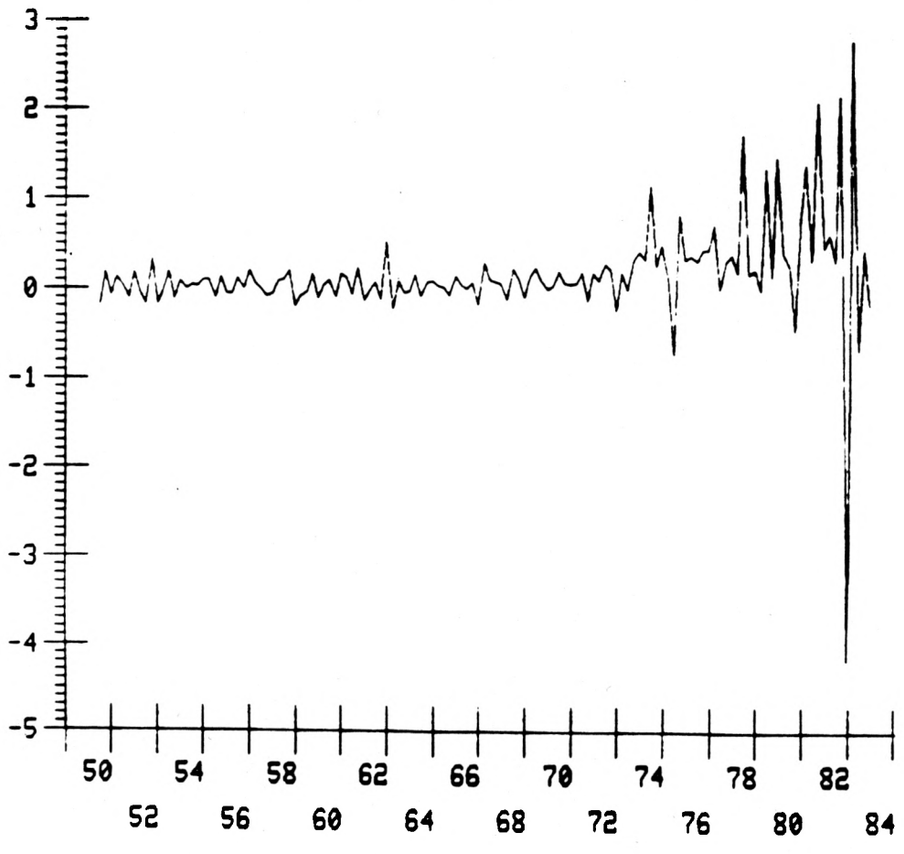


Figure 5

CHANGE IN THE UNINCORPORATED BUSINESS INCOME INDEX

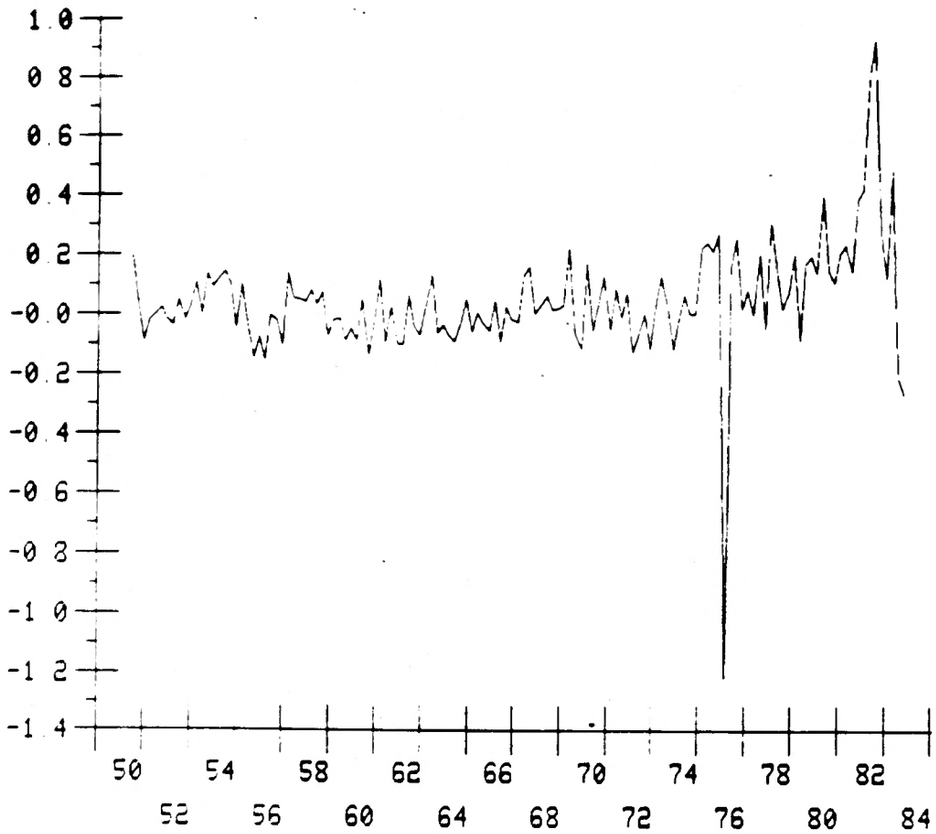


Figure 6

CHANGE IN FARM-OPERATORS' INCOME INDEX

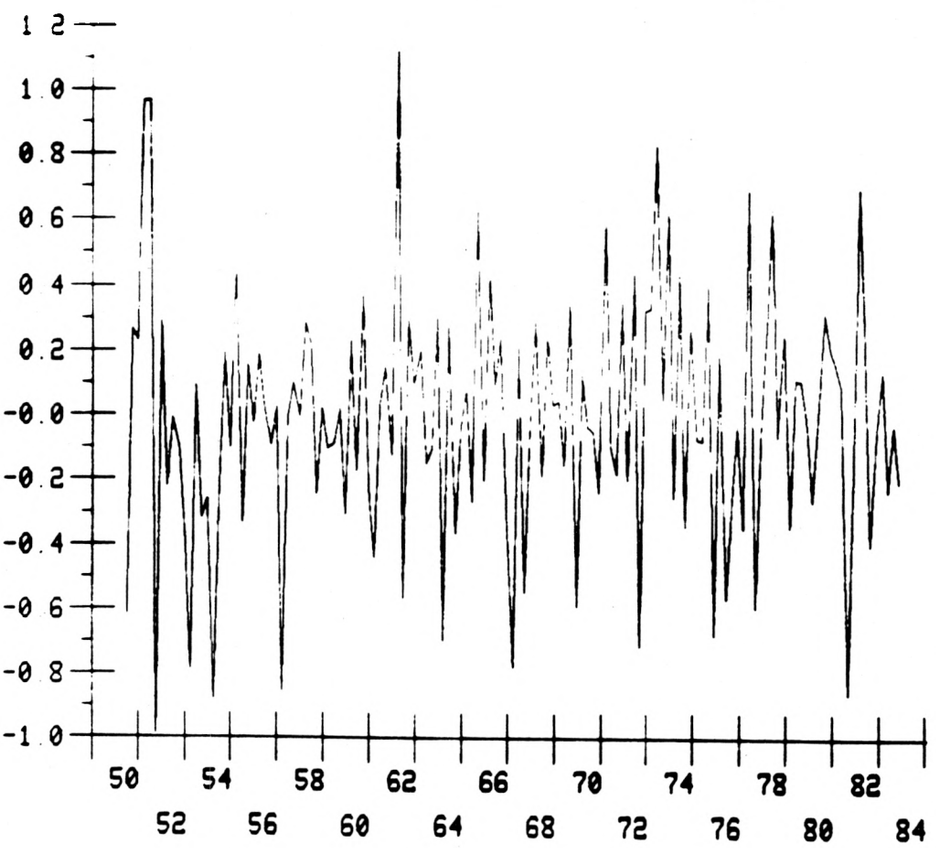


Figure 7

CHANGE IN INDIRECT TAXES LESS SUBSIDIES INDEX

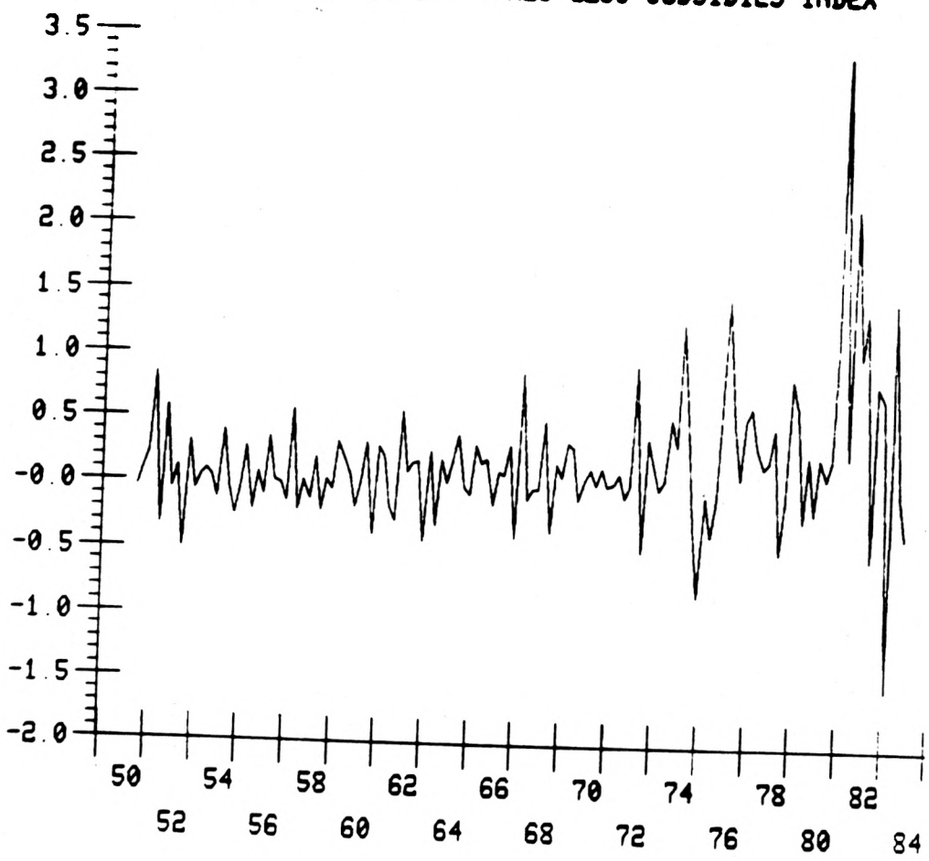


Figure 8

CHANGE IN CAPITAL CONSUMPTION ALLOWANCE INDEX

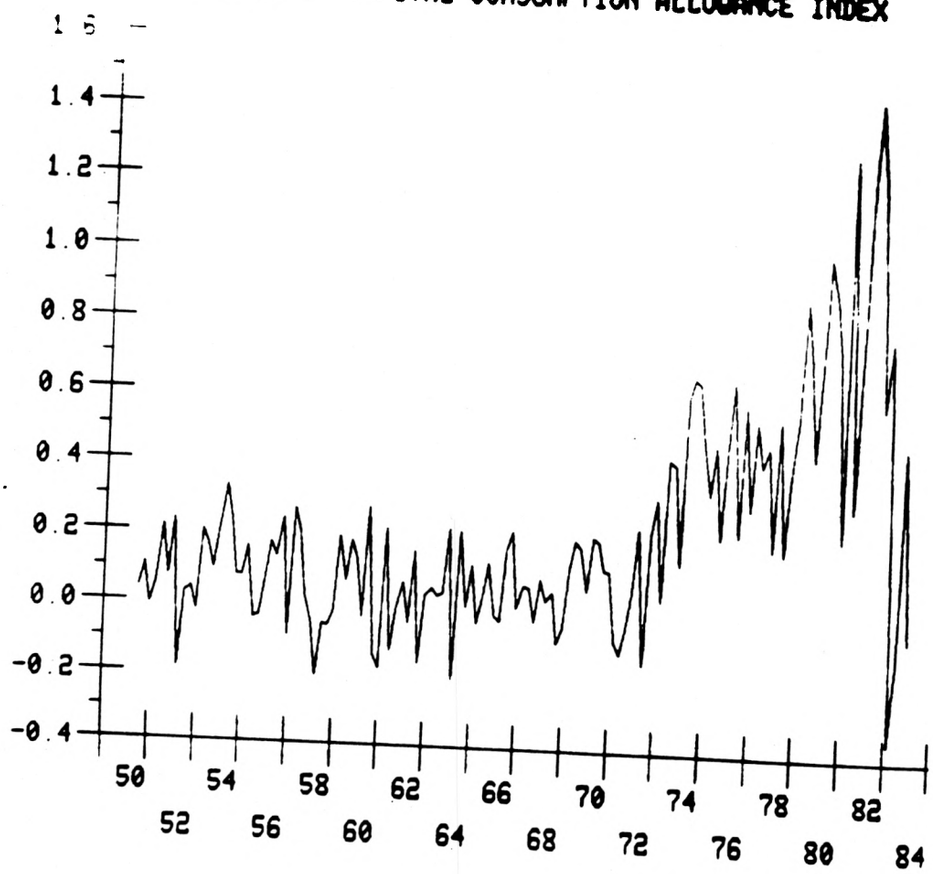
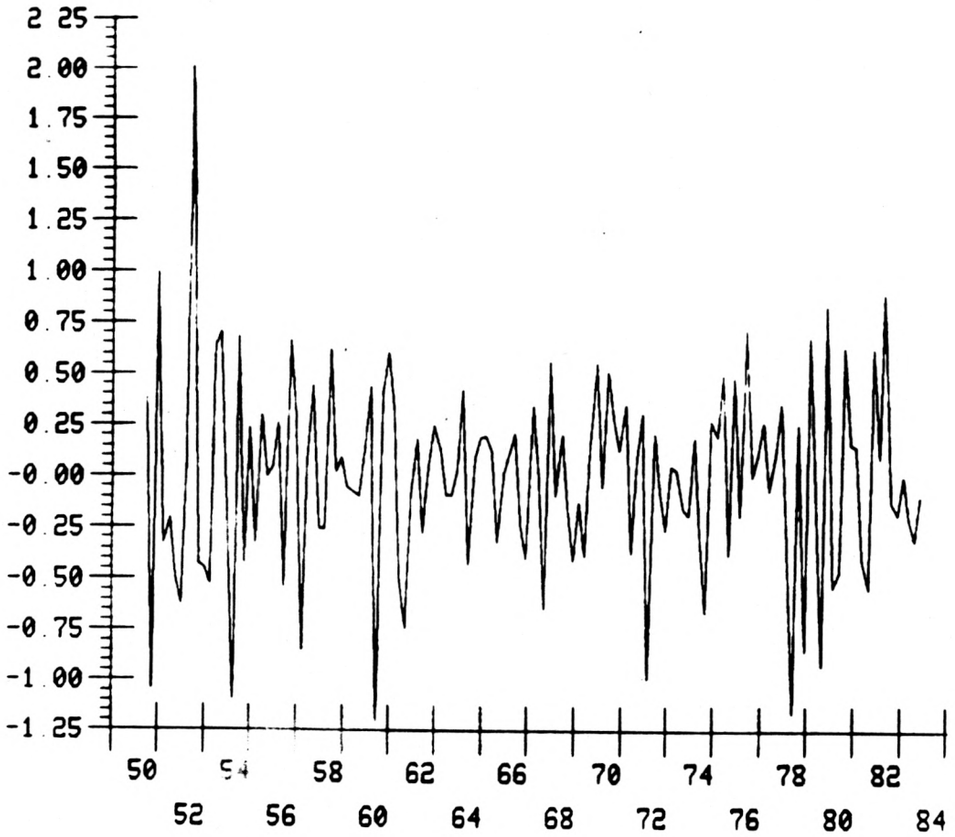


Figure 9

CHANGE IN RESIDUAL ERROR OF ESTIMATE INDEX



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