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MODELS OF INFLATION: A TAXONOMY OF EFFECTS

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ABSTRACT

In this study the author uses three wage equations and three price equations to examine the effect on wages and prices of various shocks. Different combinations of wage and price equations yield very different results when subjected to excess demand shocks and to exogenous price shocks. The results of the shocks are explained in terms of the general characteristics of the models used. The author also examines the effect of increasing the lags in the wage and price equations on the inflationary responses to the shocks.

RÉSUMÉ

Dans ce travail de recherche qui porte sur les réactions des salaires et des prix à certaines modifications de la situation économique, l'auteur utilise trois équations pour étudier le comportement des prix et trois équations pour étudier le comportement des salaires. Des combinaisons de ces équations impliquent des réactions très différentes à une modification de la demande excédentaire ou à une variation de prix due à l'influence de facteurs exogènes. Ces réactions sont expliquées en fonction des caractéristiques générales des modèles utilisés. L'auteur étudie également les phénomènes qui se produisent au niveau des réactions inflationnistes lorsqu'on prolonge les décalages que comportent les équations relatives aux salaires et aux prix.

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Introduction

A wide variety of inflation patterns can be derived by combining different wage and price equations and subjecting them to various shocks. In this report I set up a taxonomy of results based on three wage equations (a Phillips curve equation with the coefficient on expected price inflation equal to 1, a Phillips curve equation with the coefficient on expected price inflation less than 1, an RDX2 equation) and on three price equations (a full immediate mark-up equation, a full mark-up equation with lags, an incomplete mark-up equation). Each combination of a wage and a price equation is subjected to three shocks (a permanent increase in excess demand, a temporary one-period increase in excess demand, a permanent exogenous shock to prices). I then examine the effect of each shock on wages and prices. One can think of the permanent exogenous shock to prices as an oil price increase, an increase in the price of imported goods in general, or an increase in indirect taxes. It is of particular interest to see whether the models under examination respond to a permanent increase in excess demand by an increase in the level of prices, an increase in the rate of inflation to a new constant rate of inflation, or a continuing acceleration in the rate of inflation. The results show that one cannot analyze a wage equation adequately in isolation. An adequate analysis can be made only if the wage and price equations are examined together.

A. The Wage Equations

A.1 The Phillips Curve Equations

The standard augmented Phillips curve equation can be written

$$\Delta \ln(W) = e + f(ED) + g(\text{expected rate of inflation}) \qquad (1)$$

The rate of change in wages, approximated by the first difference in the natural logarithm of wages (W), is a function of current excess demand (ED) and the expected rate of inflation. If the expected rate of inflation is written, as it usually is, as a distributed lag of past rates of inflation, then

$$\Delta \ln(W) = e + f(ED) + g \sum_{1}^{M} v(i) \Delta \ln(P(-i)) \qquad (2)$$

where

ln(P) is the natural logarithm of the price level.

I assume in (2) that current rates of inflation are unknown when expected rates of inflation are being formulated. A special example of the above formulation is that of adaptive expectations in which

$$v(i) = h(1-h)^{i-1}$$

Two important cases are to be distinguished. First, g equals 1. That is, the expected inflation rate is fully incorporated into the wage rate change. Second, g is less than 1. That is, there are elements of price illusion in the determination of wages. I call the first case the augmented Phillips curve equation with no price illusion and the second the augmented Phillips curve equation with price illusion.

Another interpretation of (2) treats the lagged price inflation terms as representing a "catch-up" phenomenon. Here the assumption is that wages are adjusted for price inflation after it occurs and not before. Full catch-up requires that g equals 1. See Stein [6] for the incorporation of a more sophisticated notion of catch-up into a wage equation that also includes expected inflation as an explanatory variable. Note that, in the models under study, the results of shocks are the same for the catch-up interpretation as for the expected inflation interpretation.

A.2 The RDX2 Wage Equation

Ignoring certain terms that have only transitory effects and setting the change in productivity equal to zero one can write the RDX2 [1] wage equation as follows:

$$\Delta \ln(W) = a + c(ED) - d \ln(W(-1)/P(-1)) \qquad (3)$$

The rate of change in wages is a function of current excess demand and the lagged real wage. All coefficients are positive.

Two interpretations can be offered for this kind of equation. In both the Kuh [3] and the Sargan [5] bargaining approach, desired real wages are a function of productivity, represented here by a constant, and of excess demand.

$$\ln(W/P)^* = s + b(ED) \qquad (4)$$

The adjustment of actual wages to desired wages is via the usual partial adjustment model.

$$\Delta \ln(W) = r[\ln(W/P)^* - \ln(W(-1)/P(-1))] \qquad (5)$$

Substituting one gets

$$\Delta \ln(W) = rs + rb(ED) - r \ln(W(-1)/P(-1)) \qquad (6)$$

which is equivalent to (3). Note that there is no anticipation of future inflation in (6) – simply a response of wages to the gap between desired and actual wages.

The second interpretation of (3) follows the McCallum version of the Phillips curve [4]. In [4] the lagged real wage acts as a second proxy for excess demand in the sense that a rise in real wages leads to a decline in the quantity of labour demanded and to an increase in the quantity of labour supplied. Thus an increase in real wages leads to an increase in the excess supply of labour and hence puts downward pressure on the rate of increase in nominal wages. This interpretation also does not allow for anticipated inflation in the wage equation.

Before going on to the other wage equations it is worth analyzing further the RDX2 wage equation to see whether or not it can be made equivalent to the Phillips curve equation with price expectations. I rewrite (3) as

$$\ln(W) = a + c(ED) + (1-d)\ln(W(-1)) + d\ \ln(P(-1))$$

$$\ln(W) = a/d + c\ \sum_{0}^{\infty}(1-d)^{i}ED(-i) + d\ \sum_{1}^{\infty}(1-d)^{i-1}\ln(P(-i))$$

$$\Delta\ln(W) = c\sum_{0}^{\infty}(1-d)^{i}\Delta ED(-i) + d\sum_{1}^{\infty}(1-d)^{i-1}\Delta\ln(P(-i))$$

$$= c\sum_{0}^{\infty}(1-d)^{i}\Delta ED(-i) + \text{the expected rate of inflation}$$

Thus, the RDX2 wage equation can be transformed into a wage equation with the expected rate of inflation based on the usual adaptive expectations process applied to lagged price inflation. In this equation there is a coefficient of unity on the expected inflation term. However, the excess demand term in this reformulation of the RDX2 equation enters in the form of a first difference. In the Phillips curve equation with price expectations the excess demand term enters as a level and this difference between the two wage equations results in very different responses to shocks to excess demand.

B. The Price Equations

The basic price equation in most macro-econometric models can be written

$$\ln(P) = m + \sum_{0}^{I} n(i)\ln(W(-i)) + q(ED) \tag{7}$$

Here I again ignore changes in labour productivity in order to simplify the analysis. Taking first differences one gets

$$\Delta\ln(P) = \sum_{0}^{I} n(i)\Delta\ln(W(-i)) + q\Delta ED \tag{8}$$

Now there are four special cases to consider.

(i) Full Immediate Mark-Up

If n(0) equals 1, n(i) equals zero for each i greater than zero, and q equals zero, then prices will be changed fully and immediately when wages change. This is the simplest case.

(ii) Full Lagged Mark-Up

If the sum of the n(i) equals unity and q equals zero, prices will be marked up fully when wages change. The adjustment will, however, take time.

(iii) Incomplete Mark-Up

If the sum of the n(i) is less than unity and q equals 0, prices will respond less than fully to wage changes even in the long run. This situation will occur if the prices of inputs other than labour do not change when wages change.

(iv) Excess Demand Affects Prices

If q is greater than zero, excess demand changes will have some effect on the time path of wages and prices. But since empirically the effect is not large I ignore it in the remainder of this report.

C. The Results

When the three wage equations are combined with the three price equations, nine models result. (The exact specification of each equation is given in the appendix.) The values chosen for the coefficients in the different equations are such that, at the initial 5 percent unemployment rate and initial levels of prices and wages normalized at unity, all the models yield a zero rate of inflation in both prices and wages. I subject each model to the three shocks (a permanent change in excess demand, a temporary change in excess demand, a permanent exogenous shock to prices) and derive the response of wages and prices. In each case I am interested in the effect of the shock on nominal wages and prices and on real wages. The permanent increase in excess demand is modelled as a permanent decrease in the unemployment rate from 5 percent to 4 percent, whereas the temporary increase in excess demand is modelled as a one-period decrease of unemployment to 4 percent followed by a return to 5 percent. The exogenous price shock is modelled as a once-and-for-all upward shift of 1 percent in the price level (ie, an increase from 0 to 0.01 in the constant of the equation determining the logarithm of price).

The effects of the shocks on the various models are summarized in a series of matrices, Table 1 to 3. For each model and each shock I present the long-run effect on wage inflation (WI), price inflation (PI), and the natural logarithm of the real wage (ln(W/P)). Also, in cases where the rate of inflation returns to zero in the long run, I present the natural logarithm of the nominal wage (ln(W)) as well as a summary description of the path to long-run equilibrium. Details of the time paths to equilibrium are available upon request. The actual numbers presented in the tables are intended mainly to facilitate comparisons across models, although the coefficients of the Phillips curve equations were chosen so as to bear some resemblance to quarterly econometric equations. The effects of the various shocks were found by computer simulations. Analytic solutions of the long-run effects can, however, be determined in all cases.

I now turn to a discussion of the results shown in Tables 1 to 3. Because of the many cases involved, I focus on certain general characteristics of the models. Although all the discussion is in terms of an increase in excess demand and an increase in the rate of inflation, the argument can be reversed for a discussion of decreases in excess demand because the excess demand term enters linearly. If it entered nonlinearly, the qualitative results would be the same but the magnitude of the results would be different as between an increase or a decrease in excess demand.

(i) A permanent increase in excess demand, Table 1, leads to accelerating inflation in the case of a Phillips curve equation with no price illusion and full mark-up pricing, either immediate or lagged. This is the classic monetarist result emphasized by Friedman [2]. In the case of lagged mark-ups the real wage increases .056 percent each period because price inflation always lags behind wage inflation. In the Phillips curve equation with money illusion and in the RDX2 equation such a shock leads to an increase in the rate of inflation as long as full mark-ups are the rule. In the case of an incomplete mark-up the RDX2 equation shows an eventual increase in the real wage that brings the rate of inflation back to zero, whereas both Phillips curve equations display ever-increasing real wages with different rates of inflation for wages and prices.

(ii) A temporary increase in excess demand, Table 2, leads to a permanent change in the rate of inflation only in the case of a Phillips curve equation with no price illusion and full mark-up pricing (either immediate or lagged). Most of the remaining models show sharp initial inflation followed by a period of decelerating rates of inflation until equilibrium is reestablished at higher wage and price levels. The case of the RDX2 equation with incomplete mark-up requires special comment. Clearly a temporary shock leaves desired real wages unchanged in the RDX2 equation. Any amount of inflation leads to an increase in actual real wages in the case of an incomplete mark-up. Hence initial inflation leads to an actual real wage above the desired real wage and thus eventually to deflation since the change in nominal wages is a function of the difference between the desired and the actual real wage in the RDX2 equation. Equilibrium occurs only when the levels of wages and prices return to the pre-shock positions.

(iii) The responses to the price shock, Table 3, comprise an interesting variety of patterns. Both the Phillips curve equation with no price illusion and full mark-up, and the RDX2 equation with full mark-up respond to a once-and-for-all shock to prices with a permanent increase in the rate of inflation. In the Phillips curve equation with no price illusion the shock to the price level feeds into the expected rate of price inflation via the distributed lag on past rates of inflation and consequently leads to an increase in the rate of inflation. In the case of the RDX2 equation the desired real wage is unchanged, the actual real wage falls, and thus there is a continuing attempt to regain the pre-shock position. If mark-ups are incomplete, the response to the price shock in both the Phillips curve equation with no price illusion and the RDX2 equation is zero inflation in the long run. Temporary inflation leads to a restoration of the real wage to its original level at which point inflation ceases. In the case of the Phillips curve equation with price illusion, the price level shock leads to a real wage decline but not to permanent inflation, since price changes feed into wage changes with a coefficient that is less than unity. The initial rate of measured price inflation leads to a lower rate of wage inflation that, in turn, leads to price inflation at the same or lower rate, etc.

(iv) By combining some of the results in the three tables I reach the following conclusions. After a price shock to a system of a Phillips curve equation with no price illusion and a full mark-up price equation a new no-inflation equilibrium and a lowered real wage can be achieved by the imposition of a temporary period of excess supply. With an RDX2 wage equation and a full mark-up price equation, a new zero-inflation equilibrium is possible only when accompanied by a permanent increase in excess supply. In the case of a Phillips curve equation with price illusion and a full mark-up price equation the existence of price illusion allows the real wage to be reduced without generating permanent inflation and without the need for a period of excess supply.

(v) As the results in Tables 1 to 3 make clear, the response of wages and prices to various shocks is very sensitive to the specification of both the wage and price equation. If there is full mark-up of prices, the RDX2 wage equation behaves like a Phillips curve equation with price illusion in the case of shocks to excess demand, and like a Phillips curve equation with no price illusion in the case of price shocks.

(vi) A study of the first row of Tables 1 and 2 reveals that, in the case of a full immediate mark-up price equation, the real wage cannot be affected by changes in excess demand, either temporary or permanent. That is, with full immediate mark-up the real wage is determined in the price equation and nominal inflation is determined in the wage equation.

(vii) When the first two rows in each table are compared, wages and prices are seen to behave qualitatively in very similar fashion in the full immediate mark-up price equation and in the full lagged mark-up equation. That is, if there is constant (zero, accelerating) inflation in one case, there will be constant (zero, accelerating) inflation in the other.

(viii) Further investigation of the case of the Phillips curve equation with no price illusion and the price equation with full mark-up yields the following results. (The results are based on a comparison of simulations using Phillips curve (1) and Phillips curve (4), and price equation (2) and price equation (4); these equations differ only in the length of the lags. The equations are set out in the appendix but the results of the simulations in which Phillips curve (4) or price equation (4) were used are not presented.) Generally, the slower the adjustment in the price equation, the slower will be the rate of acceleration or the rate of inflation in the long run and the higher will be the real wage. The slower the adjustment to past price changes in the Phillips curve equation with no illusion, the slower will be the rate of acceleration or the rate of inflation in the long run but the lower will be the real wage. An exception is the case of full immediate mark-up when the real wage is unaffected by the speed of adjustment in the wage equation, although even here an increase in the length of the lag in the wage equation leads to a lower rate of acceleration or a lower rate of inflation in response to an excess demand or price shock.

Similarly, in the case of the RDX2 wage equation, increasing the lag in the mark-up slows the rate of inflation in response to a given shock. The increased lag results in a higher actual real wage, and the rate of wage inflation in the RDX2 equation derives from the difference between the actual and the desired real wage. Thus, in the case of a permanent increase in excess demand, the desired real wage increases and the actual real wage also increases, although by a smaller amount than the increase in the desired real wage. Therefore the gap between the actual and desired real wage is smaller than it would have been had the actual real wage remained unchanged as occurs in the case of the immediate mark-up.

(ix) More formally, the effect on the real wage of increasing the lag length in the case of a full mark-up price equation with lags can be analyzed as follows: From (7), with q equal to zero and the sum of the n(i) equal to unity, one can obtain

$$\ln(W) - \ln(P) = - m + \ln(W) - \Sigma n(i)\ln(W(-i)) = - m + \Sigma n(i)[\ln(W)-\ln(W(-i))]$$

If the rate of inflation is a constant (x), this equation can be written

$$\ln(W/P) = - m + \Sigma n(i)(ix) = - m + x\, \Sigma i n(i)$$

Since the sum of the n(i) is unity, the average lag is defined as $\Sigma\, in(i)$. Thus the longer the average lag in the mark-up equation the higher will be the real wage for a given rate of inflation. And, for a given average lag, the higher the rate of inflation the higher will be the real wage.

Table 1

THE LONG-RUN RESPONSE TO A PERMANENT INCREASE IN EXCESS DEMAND

Price Equations	Wage Equations: Phillips curve - no illusion	Phillips curve - some illusion	RDX2
Full immediate mark-up	WI accelerates .18% per period	WI up 1.0%	WI up .25%
	PI accelerates .18% per period	PI up 1.0%	PI up .25%
	ln(W/P) unchanged	ln(W/P) unchanged	ln(W/P) unchanged
Full lagged mark-up	WI accelerates .14% per period	WI up 1.0%	WI up .22%
	PI accelerates .14% per period	PI up 1.0%	PI up .22%
	ln(W/P) increases .056% per period	ln(W/P) up .40%	ln(W/P) up .09%
Incomplete mark-up	WI up 1.0%	WI up .57%	WI unchanged
	PI up .75%	PI up .43%	PI unchanged
	ln(W/P) increases .25% per period	ln(W/P) increases .14% per period	ln(W/P) up .71%
			ln(W) up 2.86%

WI is the rate of wage inflation
PI is the rate of price inflation
ln(W/P) is the natural logarithm of the real wage

Table 2

THE LONG-RUN RESPONSE TO A ONE-PERIOD INCREASE IN EXCESS DEMAND

Price Equations	Wage Equations		
	Phillips curve - no illusion	Phillips curve - some illusion	RDX2
Full immediate mark-up	WI up .18%	WI unchanged	WI unchanged
	PI up .18%	PI unchanged	PI unchanged
	ln(W/P) unchanged	ln(W/P) unchanged	ln(W/P) unchanged
		ln(W) up 1.0%	ln(W) up .25%
		Inflation moderating over time	One-period inflation of .25%
Full lagged mark-up	WI up .14%	WI unchanged	WI unchanged
	PI up .14%	PI unchanged	PI unchanged
	ln(W/P) up .06%	ln(W/P) unchanged	ln(W/P) unchanged
		ln(W) up 1.0%	ln(W) up .22%
		Inflation moderating over time	
Incomplete mark-up	WI unchanged	WI unchanged	WI unchanged
	PI unchanged	PI unchanged	PI unchanged
	ln(W/P) up .25%	ln(W/P) up .14%	ln(W/P) unchanged
	ln(W) up 1.0%	ln(W) up .57%	ln(W) unchanged
	Inflation moderating over time	Inflation moderating over time	Inflation followed by deflation

Table 3

THE LONG-RUN RESPONSE TO AN EXOGENOUS PERMANENT SHOCK OF 1 PERCENT TO THE PRICE LEVEL

Price Equations	Wage Equations		
	Phillips curve - no illusion	Phillips curve - some illusion	RDX2
Full immediate mark-up	WI up .71% PI up .71% ln(W/P) down 1.0%	WI unchanged PI unchanged ln(W/P) down 1.0% ln(W) up 3.0% Inflation moderating over time	WI up .35% PI up .35% ln(W/P) down 1.0%
Full lagged mark-up	WI up .56% PI up .56% ln(W/P) down .78%	WI unchanged PI unchanged ln(W/P) down 1.0% ln(W) up 3.0% Inflation moderating over time	WI up .31% PI up .31% ln(W/P) down .88%
Incomplete mark-up	WI unchanged PI unchanged ln(W/P) unchanged ln(W) up 4.0% Inflation moderating over time	WI unchanged PI unchanged ln(W/P) down .57% ln(W) up 1.71% Inflation moderating over time	WI unchanged PI unchanged ln(W/P) unchanged ln(W) up 4.0% Inflation moderating over time

APPENDIX

The notation is the same as that used in the text except for U, which represents the unemployment rate.

A. Wage Equations

Phillips Curve without Money Illusion

$$\Delta \ln(W) = .0125 - .25U + .6\Delta \ln(P(-1)) + .4\Delta \ln(P(-2)) \quad (1)$$

Phillips Curve with Money Illusion

$$\Delta \ln(W) = .0125 - .25U + .45\Delta \ln(P(-1)) + .30\Delta \ln(P(-2)) \quad (2)$$

RDX2

$$\Delta \ln(W) = .0125 - .25U - .35\ln[W(-1)/P(-1)] \quad (3)$$

(1) with Longer Lag

$$\Delta \ln(W) = .0125 - .25U + .4\Delta \ln(P(-1)) + .6\Delta \ln(P(-2)) \quad (4)$$

B. Price Equations

Full Immediate Mark-Up

$$\Delta \ln(P) = \Delta \ln(W) + \Delta K \quad (1)$$

Full Lagged Mark-Up

$$\Delta \ln(P) = .6\Delta \ln(W) + .4\Delta \ln(W(-1)) + \Delta K \quad (2)$$

Incomplete Mark-Up

$$\Delta \ln(P) = .45\Delta \ln(W) + .30\Delta \ln(W(-1)) + \Delta K \quad (3)$$

(2) with Longer Lag

$$\Delta \ln(P) = .4\Delta \ln(W) + .6\Delta \ln(W(-1)) + \Delta K \quad (4)$$

C. Control Solution

$U = .05$ $\quad K = 0$ $\quad \ln(P) = 0$ $\quad \ln(W) = 0$ $\quad \ln(W/P) = 0$

D. Shocks

1. U falls to .04 permanently

2. U falls to .04 for one period and then returns to .05

3. K increases to .01 permanently

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