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SIMULATION ANALYSIS OF A MODEL BASED ON THE LIFE-CYCLE HYPOTHESIS

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CONTENTS

ABSTRACT

This study was undertaken in order to provide better insight into the dynamics of the life-cycle model and to develop a set of quantitative relationships that model the behaviour of macroeconomic aggregates. To begin, a microeconomic model was constructed which could to a certain extent, reproduce behaviour patterns based on the life-cycle hypothesis and, with the aid of observed data on the Canadian population structure, generate a number of macroeconomic variables implied by this behaviour. The dynamics of this model were then examined in a number of simulations in which the effects of variations in population structure, as well as developments in incomes and the interest rate were analyzed. In a third step, we attempted to estimate certain standard consumption functions using the aggregate data generated by the model. While the estimates we obtained met current econometric criteria, these functions, especially where the interest rate was concerned, did not adequately reproduce the model used to generate the data. Finally we tested other formulations which seemed likely to yield a more acceptable representation of the basic model. This exercise, however, proved disappointing.

In conclusion, it was ascertained that the study will have to be further refined in order to integrate into the macroeconomic formulations certain adjustments that would take into account demographic variations as well as the intertemporal substitution effect.

RESUME

La présente étude a été entreprise dans le but de mieux comprendre la dynamique du modèle du cycle de vie et de déduire une série de relations quantitatives qui modélisent le comportement des agrégats économiques. Tout d'abord, nous avons construit un modèle micro-économique capable de reproduire, avec une certaine vraisemblance, les décisions prises selon l'hypothèse du cycle de vie et de créer, à l'aide de statistiques sur la structure de la population canadienne, une série de variables macro-économiques impliquées par ces décisions. Ensuite, nous avons étudié les propriétés dynamiques du modèle en le soumettant, au cours d'une série d'exercices de simulation, à des modifications de la structure démographique et à des variations de revenus et de taux d'intérêt. Puis, à l'aide des agrégats obtenus avec le modèle, nous avons essayé d'estimer quelques fonctions de consommation classiques. Même si nous avons obtenu des estimations satisfaisant les critères économétriques couramment acceptés, ces fonctions, surtout en ce qui a trait au taux d'intérêt, n'ont pas reproduit de façon adéquate le modèle qui a servi à créer les données. Enfin, nous avons testé d'autres équations qui semblaient offrir une meilleure représentation du modèle de base; les résultats de cet exercice ont été décevants.

La conclusion à laquelle nous sommes parvenus, c'est qu'il faudrait pousser plus à fond notre recherche afin d'intégrer dans nos formulations macro-économiques un ensemble d'ajustements qui tiendraient compte à la fois des variations démographiques et de l'effet de substitution dans le temps.

INTRODUCTION

The life-cycle hypothesis is frequently used to explain individual consumption patterns. Its attraction lies in the fact that it forms a coherent whole theory by taking into account such factors as the consumer's age and his expectations concerning income and interest rates, as well as his budgetary constraint. The aggregation of individual behaviour patterns, made possible by the incorporation of a set of simplifying assumptions, facilitates a macroeconomic formulation of the life-cycle theory. However, the empirical testing of this aggregate function does raise a number of problems for econometricians.

This study was undertaken to improve our understanding of the dynamics of the life-cycle model and to develop a set of quantitative relationships that model the behaviour of macroeconomic aggregates. First, a macroeconomic model of consumption was constructed using microeconomic data generated from hypothetical estimates of individual labour income and interest rates as well as from an *a priori* utility function. All parameters were selected to facilitate the construction of a realistic model. Second, several parameters were successively altered in order that the dynamics of the model could be analyzed. We then tested the dynamic response of our model to see if it could be reproduced using standard macroeconomic consumption functions. From this analysis we drew a number of conclusions that we hope will ultimately facilitate the estimation of the consumption function.

1 CONSTRUCTION OF A CLASSICAL LIFE-CYCLE MODEL

To construct a model based on the life-cycle hypothesis, a microeconomic framework was defined subject to the following assumptions:

1) The economy is composed of typical individuals.
2) The typical individual begins working at age 20, retires at 65 and dies at 75.
3) He leaves no estate and receives no inherited wealth.
4) He plans his consumption for the remainder of his lifetime, taking into account a stable utility function, a certain level of accrued savings (real wealth) and his expectations concerning labour income and interest rates (potential wealth).
5) He revises his expectations each year. In the absence of new information, his previous consumption decisions continue to prevail.
6) He invests his wealth in liquid asssets (one-year term deposits) at a single rate of return or he borrows money (one-year term loans) at the same rate.
7) Financial markets have complete information. (The typical individual may borrow on future income).
8) There is no money illusion (real labour income and the real interest rate are used).
9) The economic situation and the behaviour of each

individual are identical within each age group.

10) Labour income has two sources of growth: a fixed rate G arising from technological development and investment in capital, and a fixed rate GA, resulting from increasing productivity due to on-the-job experience.

It should be noted that the model does not explain decisions to purchase durable goods; these decisions are, however, implicit.

Based on the approach of White (1978) and Yaari (1964), the utility function at time t for an individual of age j is defined as

$$U_j = \sum_{i=0}^{74-j} [(\frac{1}{1-d})\, c_{j+i}^{1-d} + b](1+p)^{-i} \quad \text{(for } d > 0 \text{ and } \neq 1) \qquad (1)$$

or

$$U_j = \sum_{i=0}^{74-j} (\ln c_{j+i} + b)(1+p)^{-i} \qquad \text{(for } d = 1) \qquad (2)$$

where

U_j is utility.

c_{j+i} is the flow of consumption to an individual of age $(j + i)$.

p reflects time preference.

d is elasticity of the marginal utility of consumption.

b is a constant.

The budgetary constraint at time t for an individual of age j is

$$\sum_{i=0}^{74-j} c_{j+i}(1+r_{t+i})^{-i} = a_j(1+r_t) + \sum_{i=0}^{74-j} y_{j+i}(1+r_{t+i})^{-i} \qquad (3)$$

where

y_{j+i} is the labour income for an individual of age (j+i).

r_t is the real interest rate at which an individual may lend or borrow funds without limits during period t.

a_j is the net wealth of an individual of age j at the beginning of period t (zero if j = 20).

If the utility function is maximized subject to the budgetary constraint, then

$$c_{j+i} = c_j\left[\frac{1+p}{1+r_{t+i}}\right]^{-i/d} \qquad (4)$$

Substitution of equation (4) into the budgetary constraint yields

$$c_j = \left[a_j(1+r_t) + \sum_{i=0}^{74-j} y_{j+i}(1+r_{t+i})^{-i}\right] / \qquad (5)$$

$$\left[\sum_{i=0}^{74-j} (1+r_{t+i})^{-i} \left(\frac{1+p}{1+r_{t+i}}\right)^{-i/d}\right]$$

If r, p and d do not vary with time, the result is a monotonic function, as can be seen from equation (4). The individual's wealth (a_j), savings (s_j) and total income (yt_j) can be calculated from the following relationships:

$$a_j = a_{j-1} + s_{j-1}, \quad \text{for all } t \tag{6}$$

$$s_j = y_j + a_j r_t - c_j \quad \text{and} \quad yt_j = y_j + a_j r_t \tag{7}$$

Each period, the individual recalculates his lifetime wealth and savings using the same equations. In macroeconomic terms, total saving is calculated by summation of the savings of all individuals at time t:

$$S = \sum_{j=20}^{74} (s_j . P_j)$$

$$P = \sum_{j=20}^{74} P_j$$

where

P_j is the number of persons of age j and

P is the total adult population.

Similarly, total labour income is

$$YW = \sum_{j=20}^{74} (y_j . P_j) \tag{8}$$

Therefore, the rate of saving from labour income can be calculated as follows:

$$SR = S/YW \tag{9}$$

2 CONSTRUCTION OF THE CONTROL SOLUTION

In order to obtain the most realistic model possible, a set of simplifying assumptions was used in the construction of the control solution. Thus, expectations concerning income and the real interest rate are fully realized in the control solution, while consumption levels planned early in the simulation period (100 years), are realized in subsequent years and are valid in that they maximize the utility function in each period.

Further, the anticipated and realized rate of growth of labour income (real disposable income) is set at 2.5 percent annually for each individual. A 2 percent rate of growth is attributed to productivity gains generated by additions to capital stock and technical progress (G), and a 0.5 percent growth rate to productivity generated by one additional year of work experience (GA). At the aggregate level, we obtain a growth rate in labour income of 2 percent per year, similar to the growth rate of real disposable income per worker observed during the last 20 years in

Canada. The real interest rate was set at 2 percent per annum, which is comparable to the average real rate[1] paid by trust companies for one-year term deposits over the period 1953-76.

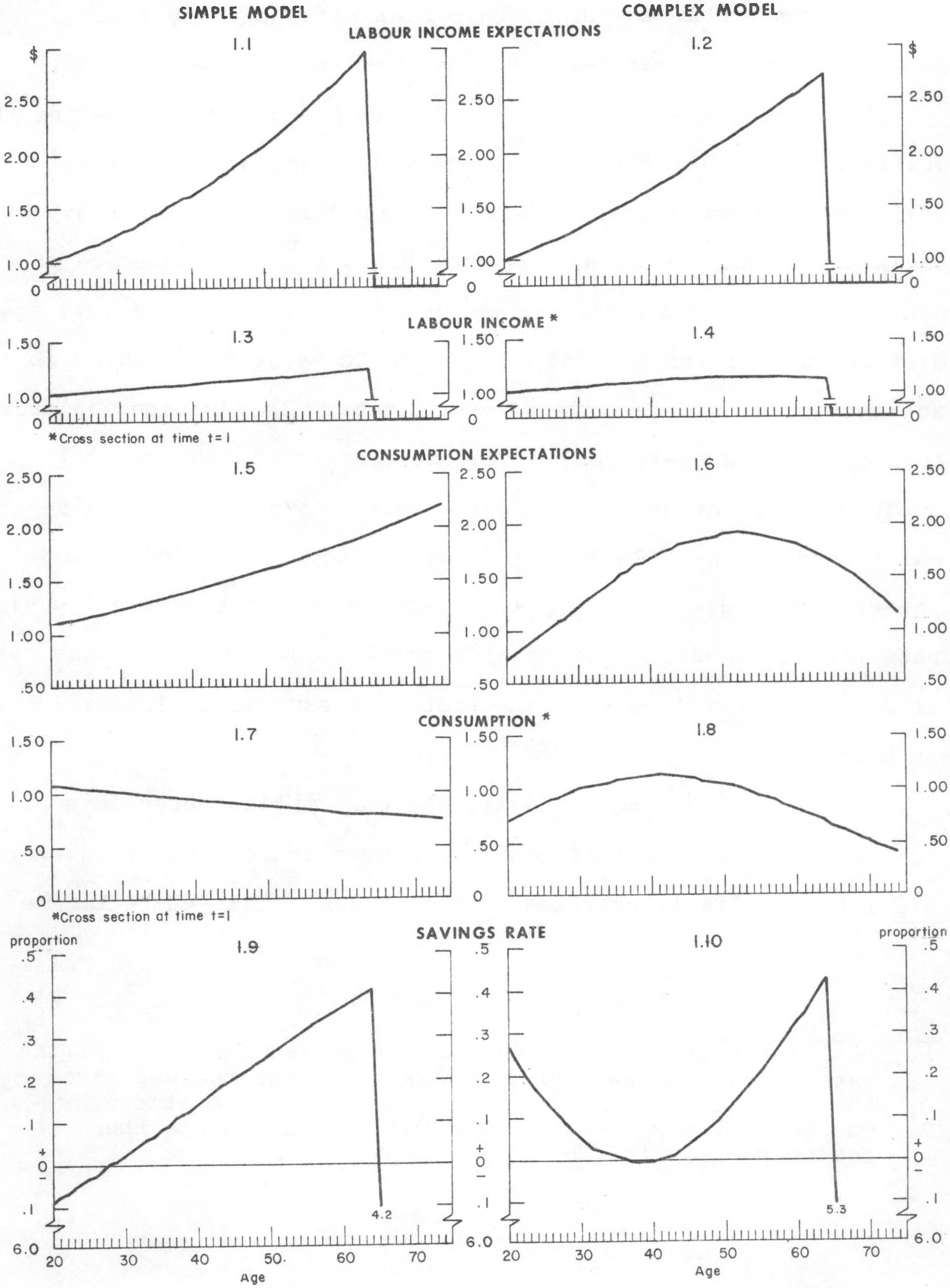
In closing the values of parameters p and d, which define the utility function of the typical individual [equations (1) and (2)], we required a combination that would result in an aggregate savings rate similar to the average 0.075 actually observed in Canada from 1953 to 1977. In the control solution, the 1974 age distribution of the population aged 20 to 74 years was used in the aggregation of microeconomic savings data. In addition, in order to keep the aggregate rate of saving constant in the control solution, population size and distribution were artificially[2] maintained at the 1974 level throughout the simulation period. The choice of d = 1.5 and p = 0 yields a constant over-all savings rate (ratio of savings to total income) of 0.0828, while the rate of saving out of labour income (ratio of savings to labour income) is 0.0854.

At the microeconomic level, the consumption curve as a function of age (the individual's consumption expectations over his lifetime) is illustrated in Figure 1.5. This curve can be

1. Real interest rate was obtained by subtracting the growth rate of the Consumer Price Index (CPI) for the year from the nominal interest rate for the beginning of the year.
2. The resulting inconsistencies will not influence the following results.

Figure 1

LIFE-CYCLE MODELS



derived explicitly, using equation (4) and the selected values of p, d and r:

$$c_{j+i} = c_j(1.02)^{2i/3}$$

The individual will anticipate the savings rate depicted in Figure 1.9. At the macroeconomic level, given that G is 2 percent, consumption in terms of age group (cross section) will follow the path represented in Figure 1.7, while the rate of saving in the economy at any given time will result in the curve shown in Figure 1.9.

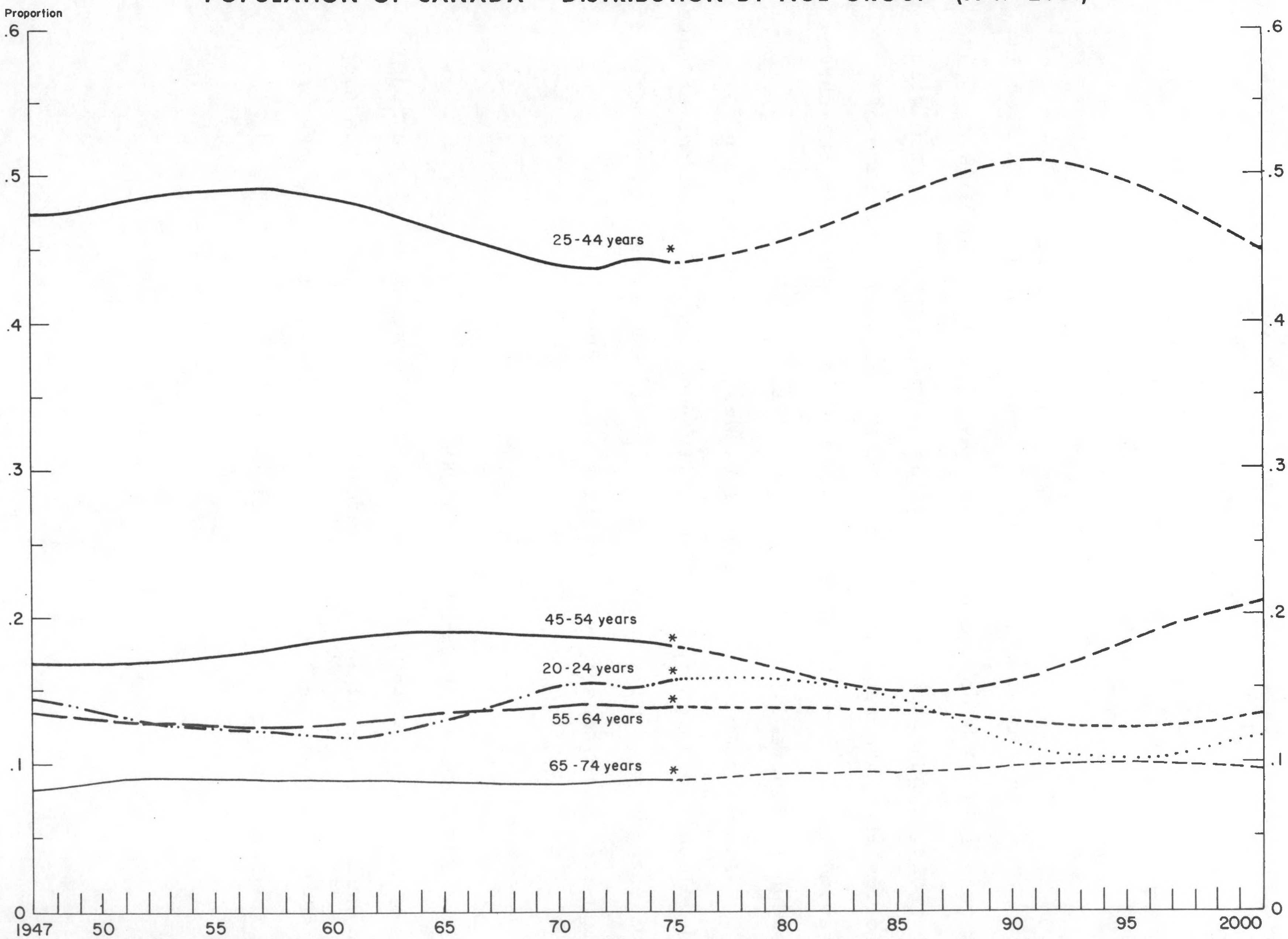
A more complex model that includes a family function is developed in Appendix A. The family function reflects the effects of changes in family size on consumption decisions and produces a non-monotonic consumption function with respect to age.

3 SIMULATIONS OF STANDARD SHOCKS

In order to facilitate a better understanding of the model dynamics, a set of standard shocks was administered to the simple model described in Section 2. The results of these shocks are presented in the following sections.

Figure 2

POPULATION OF CANADA - DISTRIBUTION BY AGE GROUP (1947-2001)



* Population forecasts - Statistics Canada

3.1 Effects of variations in the age composition of the population

As mentioned previously, the actual 1974 population distribution for ages 20 to 74 was used in the control solution.

Age (years)	20-24	25-44	45-54	55-64	65-74
Percentage of adult population	0.1521	0.4423	0.1813	0.1363	0.0878

We solved the model using the age-group distributions from 1947 to 1974 that are presented in Table 1 and Figure 2. This shock was designed to determine how changes in age distribution affect the rate of saving and thus to evaluate the aggregation bias of macroeconomic functions.[3] In effect, the relationships used to estimate the life-cycle theory at the macroeconomic level were obtained through aggregation of microeconomic relationships based on the assumption of fixed age-group distribution over time. (See Ando and Modigliani, 1963).

As can be seen in Figure 3, the rate of saving declined from 0.0851 to 0.0807 between 1947 and 1951. The principal cause of the decline was the decrease in the proportion of persons aged 55 to 64 (who have a high rate of saving) from 0.1335 to 0.1287, in conjunction with the rise in the proportion of those between 65

3. It should be noted that in our model an increase in population will have no influence on the rate of saving if the age structure remains unchanged.

Table 1 POPULATION OF CANADA ACCORDING TO AGE-GROUP DISTRIBUTION 1947-2001

Year	Age group 20-24	25-44	45-54	55-64	65-74
47*	0.1435	0.4732	0.1681	0.1335	0.0816
48	0.1408	0.4743	0.1680	0.1333	0.0837
49	0.1381	0.4770	0.1680	0.1316	0.0854
50	0.1346	0.4796	0.1681	0.1302	0.0873
51	0.1302	0.4834	0.1682	0.1287	0.0895
52	0.1282	0.4858	0.1690	0.1276	0.0895
53	0.1264	0.4873	0.1700	0.1267	0.0895
54	0.1246	0.4888	0.1712	0.1259	0.0896
55	0.1229	0.4898	0.1724	0.1251	0.0898
56	0.1216	0.4904	0.1737	0.1243	0.0899
57	0.1214	0.4908	0.1752	0.1238	0.0888
58	0.1208	0.4899	0.1774	0.1239	0.0880
59	0.1191	0.4874	0.1805	0.1251	0.0879
60	0.1180	0.4847	0.1832	0.1263	0.0879
61	0.1171	0.4818	0.1858	0.1275	0.0880
62	0.1188	0.4768	0.1875	0.1292	0.0878
63	0.1218	0.4712	0.1886	0.1308	0.0875
64	0.1252	0.4663	0.1890	0.1321	0.0873
65	0.1286	0.4614	0.1893	0.1335	0.0873
66	0.1328	0.4567	0.1888	0.1345	0.0871
67	0.1380	0.4526	0.1881	0.1350	0.0863
68	0.1430	0.4482	0.1873	0.1358	0.0857
69	0.1480	0.4434	0.1868	0.1367	0.0850
70	0.1518	0.4402	0.1859	0.1373	0.0847
71	0.1523	0.4366	0.1847	0.1396	0.0869
72**	0.1540	0.4356	0.1836	0.1394	0.0873
73	0.1502	0.4407	0.1833	0.1379	0.0878

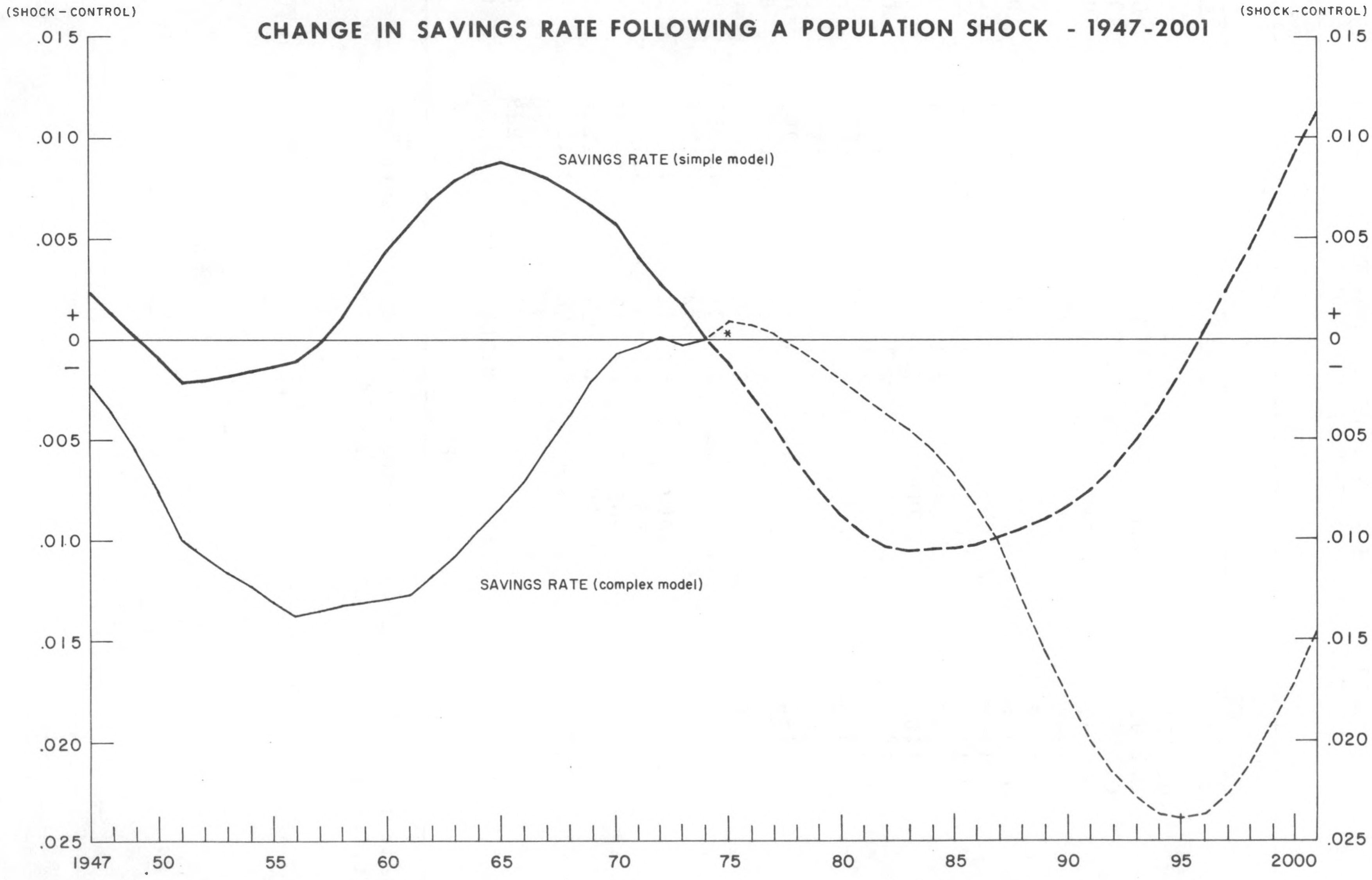
* Data for 1947-1971, Statistics Canada (Cat. no. 91-512).
** Data for 1972-1974, Statistics Canada (Cat. no. 84-201).

Table 1 Cont'd POPULATION OF CANADA ACCORDING TO AGE-GROUP DISTRIBUTION 1947-2001

Year	Age group 20-24	25-44	45-54	55-64	65-74
74	0.1521	0.4423	0.1813	0.1363	0.0879
75*	0.1563	0.4387	0.1790	0.1376	0.0885
76	0.1566	0.4411	0.1762	0.1372	0.0889
77	0.1567	0.4439	0.1730	0.1369	0.0895
78	0.1568	0.4471	0.1692	0.1367	0.0901
79	0.1566	0.4508	0.1652	0.1367	0.0907
80	0.1558	0.4551	0.1614	0.1365	0.0912
81	0.1544	0.4600	0.1578	0.1362	0.0916
82	0.1524	0.4657	0.1544	0.1358	0.0918
83	0.1497	0.4715	0.1513	0.1355	0.0919
84	0.1465	0.4779	0.1488	0.1349	0.0919
85	0.1427	0.4839	0.1471	0.1341	0.0922
86	0.1384	0.4896	0.1464	0.1330	0.0926
87	0.1322	0.4955	0.1469	0.1319	0.0935
88	0.1251	0.5011	0.1484	0.1307	0.0946
89	0.1184	0.5056	0.1509	0.1293	0.0958
90	0.1128	0.5086	0.1540	0.1277	0.0969
91	0.1078	0.5101	0.1580	0.1262	0.0978
92	0.1048	0.5088	0.1628	0.1250	0.0987
93	0.1032	0.5054	0.1682	0.1238	0.0994
94	0.1021	0.5009	0.1741	0.1230	0.0999
95	0.1012	0.4959	0.1801	0.1226	0.1000
96	0.1016	0.4900	0.1857	0.1228	0.0998
97	0.1036	0.4826	0.1911	0.1236	0.0991
98	0.1067	0.4744	0.1958	0.1249	0.0981
99	0.1103	0.4661	0.2000	0.1268	0.0969
00	0.1134	0.4592	0.2032	0.1289	0.0952
01	0.1167	0.4494	0.2074	0.1324	0.0943

* Data for 1975-2001 were calculated from forecasts by Statistics Canada (Cat. no. 91-514).

Figure 3



* Population forecasts - Statistics Canada

and 74 years of age (who have a high rate of dissaving) from 0.0816 to 0.0895. These two trends therefore combined to produce the decline in the savings rate.

In contrast, between 1951 and 1965, the rate of saving increased significantly to 0.0916 from 0.0807 as a result of the following factors. During the period 1951-62 the proportion of those 20 to 24 years of age (youths born during the Depression of the 1930s and before World War II) decreased from 0.1302 to 0.1188. Similarly, between 1951 and 1965 the proportion of 65 to 74 year olds declined from 0.0895 to 0.0873. The individuals that comprise these two age groups are not savers and a decrease in their numbers will contribute to a rise in the over-all rate of saving. Between 1951 and 1957, there was an increase in the proportion of those aged 25 to 44 (individuals who have a higher rate of saving) to 0.4908 from 0.4834. It should be noted that although the relative number of persons in this age group decreased significantly between 1957 and 1965, this was more than offset by an increase to 0.1893 from 0.1752 in the proportion of those aged 45 to 54 (individuals who also have a high rate of saving), and, as a result, the savings rate rose.

Between 1965 and 1974, the rate of saving dropped from 0.0916 to 0.0828, a decrease of 0.0088. Here again there are several causative factors. First, stemming from the baby boom, there was a strong rise to 0.1521 from 0.1286 in the proportion of 20 to 24 year olds, an age group characterized by dissaving. Second, a decline in the proportion of those 25 to 44 years of age (children of the Depression), as well as a decrease in the proportion of those 45 to 54 years of age (falling from 0.1893 in 1965 to 0.1813 in 1974) resulted in a lower proportion of savers in the total population, further contributing to the reduction in the savings rate.

Our model forecasts a rise in the savings rate for the late 1980s, ceteris paribus, because the youths of the baby boom will be in their thirties and will be savers while the decreased birth-rate of the 1960s will significantly reduce the proportion of those aged 20 to 24. Similarly, in the 1990s the rate of saving will remain high owing to a decrease in the proportion of those aged 65 and over (children of the Depression), while the individuals born during the baby boom will still be savers. As can be seen in Figure 3, the incorporation of demographic forecasts by Statistics Canada into our model implies a rise of about 0.0150 in the rate of saving between 1980 and 2001. However, it must be noted that with the addition of a family function, (as described in Appendix A) our model forecasts a large decrease in the savings rate in the 1990s.

Changes in the rate of saving arising from variations in the age composition of the population are important and if disregarded may have significant indirect influences on the estimation of the aggregate consumption function. The largest total change in the rate of saving that can be attributed to age distribution is 1.1

percent[4], three times greater than the value of the standard error reported in V. Rochester's study (1979). Therefore, it follows that an estimation of macroeconomic relationships should take population factors into consideration. It should be noted that a number of demographic factors which could further influence the rate of saving, such as changes in household formation, in the number of workers per household, or in the utility function (over time and between individuals), were not quantified.[5]

3.2 Temporary and unanticipated increase in the real interest rate

In our model, an increase in the real interest rate will raise interest income for lenders and reduce it for borrowers [see equations (5) and (7)]. This will in turn generate a higher (lower) rate of consumption subject to the budgetary constraint. This is the income effect. In the control solution lenders outnumber borrowers, so the income effect is positive. If considered permanent, a rise in the interest rate encourages people to refrain from present consumption in order to consume more advantageously at a later point in time. As we will see, this intertemporal substitution effect is very important in the life-cycle model. The third effect of a rise in the interest

4. For example, a variation of one percentage point represents a variation of more than 10% in the average value of the savings rate.
5. Shocks relating to the age of entry into the labour market, a decrease in the retirement age and an increase in life expectancy, were simulated. See Appendix B.

rate is a revaluation of wealth. In a world where consumers hold long-term assets (or debts) on which the interest rate is fixed for several years in advance, any increase in the interest rate would lower the value of these assets (or debts). However, in this model, a revaluation effect does not exist because, at the beginning of each year, the consumer invests his wealth (or borrows) for one year at the interest rate available at that time (see Section 1, Assumption 5).

In the following simulations, the rise in the real interest rate may be interpreted in two different ways: as an increase in the nominal interest rate with no change in the inflation rate, or as a reduction in the inflation rate with no change in the nominal interest rate. In the latter case, the income effect is not manifested as an increase in nominal interest income, but results instead in a lower price level which allows the acquisition of more consumer goods for the same dollar outlay.

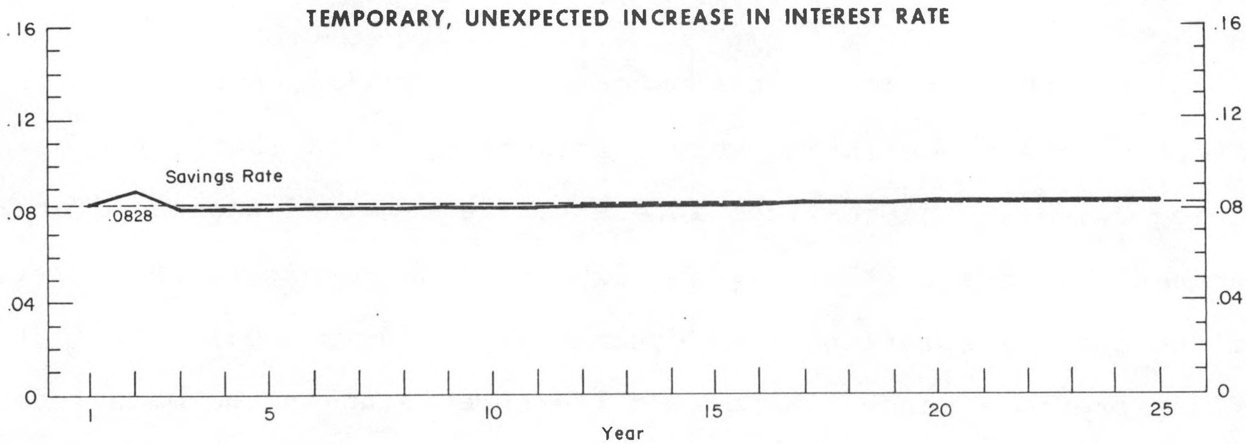
In our control solution, the real interest rate, expected and realized, was set at 2 percent per year. In the shock simulation[6], we increased the real interest rate in the second year by 50 basis points, from 2 to 2.5 percent. Over years 3 to 100, however, the interest rate was maintained at the control solution level of 2 percent. The expected real interest rate was held at 2 percent throughout the entire simulation period.

6. The population remains stable during these simulations.

The increase in unanticipated, realized interest income, in the absence of new consumption decisions, results in an increase of 0.0063 in the rate of saving in the second year. At the beginning of the third year, each worker decides to spread the surplus of savings income from the preceding year over consumption for the remainder of his life, subject to his microeconomic utility function. Thus, during the third year, 6.8 percent of this saved income is spent. This share, as indicated in Figure 4, decreases slowly over the remaining 54 years of the experiment.

Figure 4

TEMPORARY, UNEXPECTED INCREASE IN INTEREST RATE



In our model, the size of the savings rate response to interest rate shocks is a function of the wealth-to-income ratio. In effect, the higher the wealth-to-income ratio, the larger the increase in non-wage income and the greater the rise in the savings rate. In the control solution, the ratio of wealth to income is about 1.6 compared with 3.5 observed in the Canadian household sector.[7] If we had attempted to reproduce this actual situation in our hypothetical model, the temporary increase in the interest rate would have had an impact close to double that depicted in Figure 4. However, it is important to note that in the real world consumers hold only a small part of their wealth in the form of liquid assets and only these are affected by a temporary increase in the interest rate. It should not be inferred, therefore, that our model necessarily underestimates the impact of this shock.

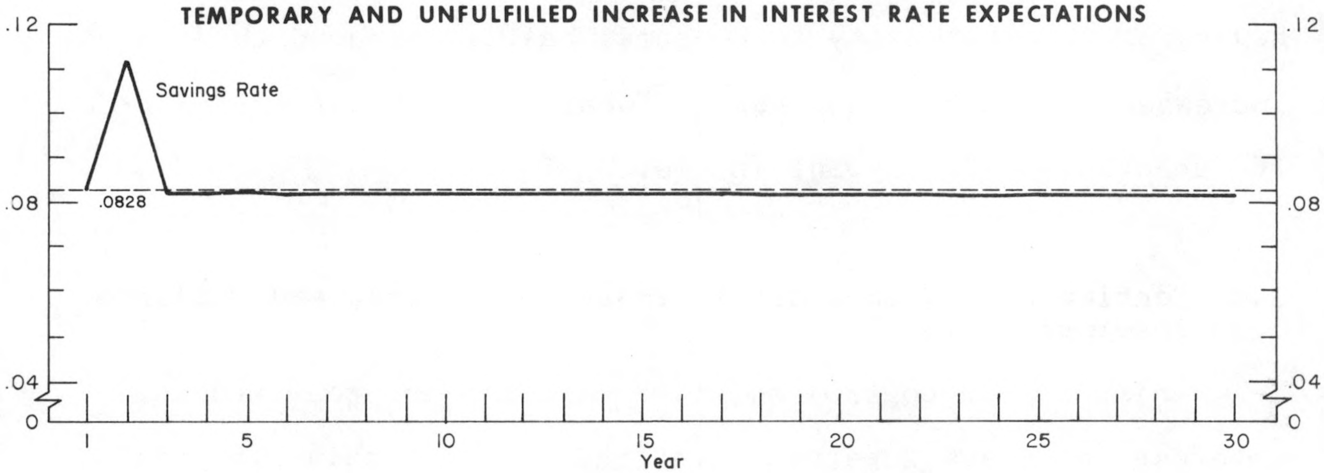
3.3 Temporary but unfulfilled increase in the expected interest rate

Here, in contrast to the shock in Section 3.2, future interest rate expectations were increased by 50 basis points in period 2, while the realized interest rate remained unchanged at 2 percent. In the following year, expectations revert to the level of the control solution. The dynamic response is similar to that of the previous shock, the only difference being in the magnitude of the savings rate response -- an increase of 0.0297 compared to 0.0063. This response is explained by the inherent nature of the life-cycle model, namely that interest rate expectations are more important than current realized interest rates.

7. As calculated from the RDX2 data base: V/(YDP.4).

Figure 5

TEMPORARY AND UNFULFILLED INCREASE IN INTEREST RATE EXPECTATIONS



It is interesting to note that in period 3 the decline in the savings rate relative to the control solution level is smaller than that recorded for shock 3.2. At the outset, one might have expected the opposite to occur, reasoning that since the wealth of the population as a whole is higher - <u>ceteris paribus</u> - consumption should also be higher, causing the rate of saving to decline by a greater amount. Microeconomic analysis of the decision-making process shows that individuals under 50 years of age decide on a higher level of consumption in this situation than in the previous shock. But, as has prevailed throughout, the older segment of the population reacts differently. Since this age group has more wealth than those in the younger age brackets and a shorter time to live, a shock to interest rate expectations has relatively less impact on their spending and saving behaviour.

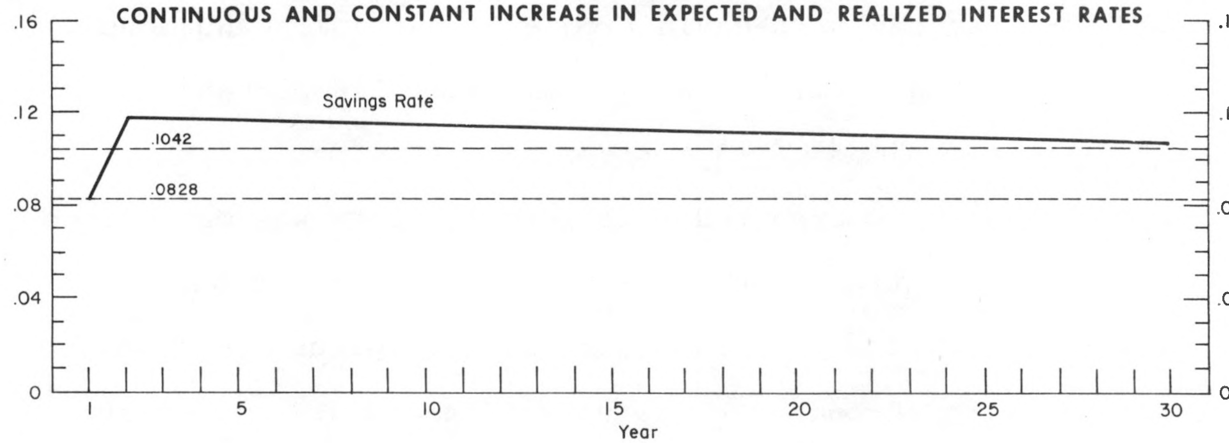
These individuals are more concerned with variations in the current interest rate. For instance, in shock 3.2, older people, having a high propensity to consume, hastily reduced through increased consumption the surplus wealth that had resulted from the unanticipated non-wage income.

3.4 Continuous and constant increase in expected and realized interest rates

While in the control solution expected and realized real interest rates are 2 percent over the entire simulation period, in the current experiment, these rates were set instead at 2.5 percent starting in the second year. At the microeconomic level, an increase in the interest rate encourages some postponement of current consumption so that individuals can benefit from the higher return on savings and consequently enjoy a higher level of consumption in later years. Thus, for this shock we find that the consumption level of those 20 to 40 years of age (50 percent of the population) is lower than in the control solution and their savings rate is higher despite the fact that their over-all income is lower (because of higher interest payments). Older individuals (40 to 64 years of age) have a higher savings rate than in the control solution, because the growth rate of their total income is higher than that of their consumption. Hence, at the aggregate level, the result is an over-all increase in the savings rate.

Figure 6

CONTINUOUS AND CONSTANT INCREASE IN EXPECTED AND REALIZED INTEREST RATES



Relative to the control solution, the equilibrium savings rate depicted in Figure 6 increases by 0.0214 to 0.1042 from 0.0828. This is not a gradual increase. Indeed, during the first year in which the shock was applied, the savings rate increased by 0.0357, which was more than enough to accommodate the necessary long-term adjustment. One explanation for this sudden rise in the savings rate is that consumers over 20 years of age try to reach the desired level of wealth as rapidly as possible in order to maximize their long-term utility. It should be noted that in this simulation the adjustment in period 2 exceeds that in experiment 3.3 because the value of the realized real interest rate is 2.5 percent rather than the 2 percent control level.

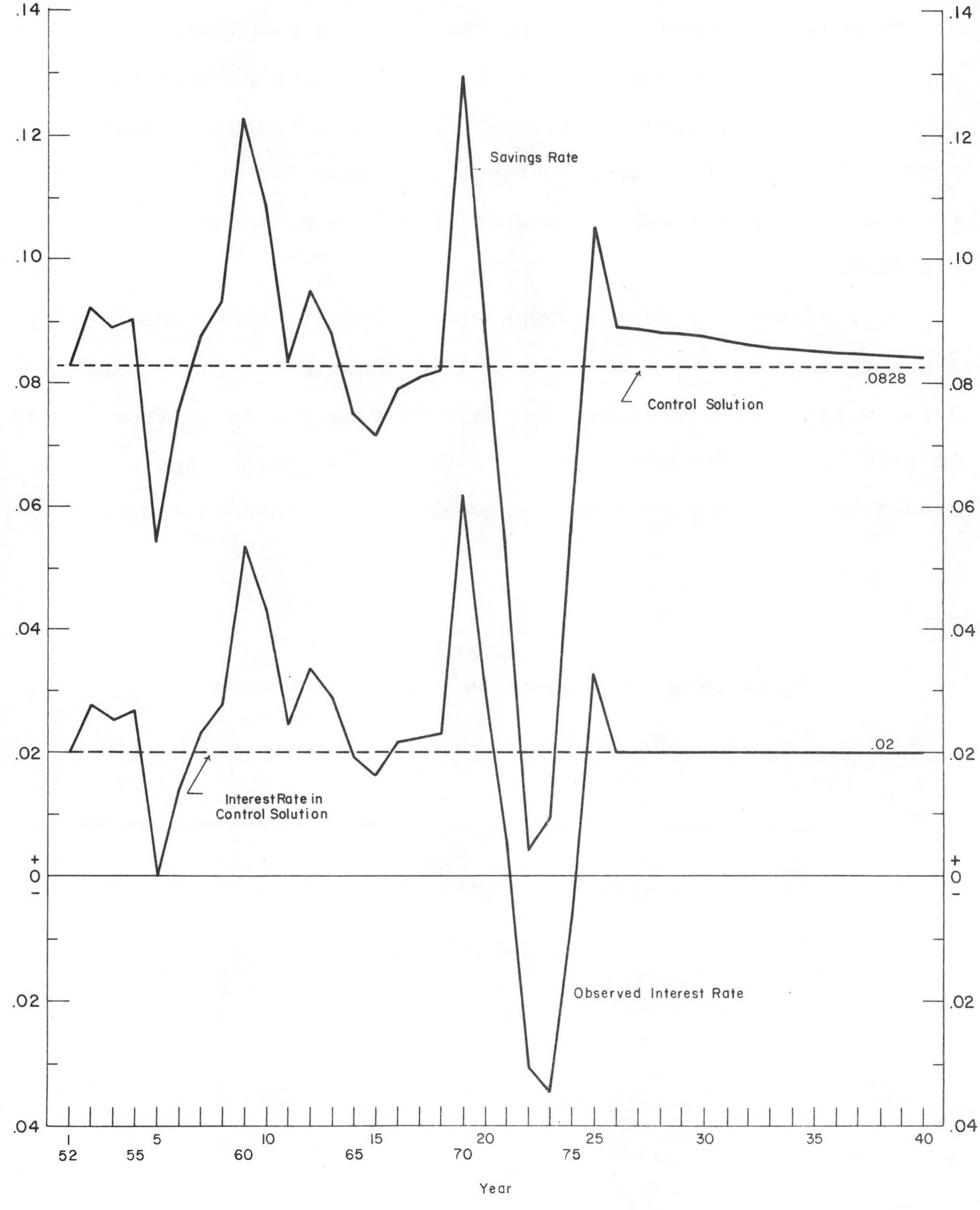
3.5 Observed variations in the real interest rate

The realized real interest rate in this simulation was constrained to follow the same fluctuations as the real interest rate paid on one-year term deposits by trust companies between 1955 and 1976. As can be seen in Figure 7, there are enormous fluctuations in this rate, reaching a maximum of 6 percent in 1971 and falling to a minimum of -3 percent in 1975. During the simulation period, the expected real interest rate was maintained at the 2 percent control level, reflecting the assumption that changes in the realized real interest rate are perceived as being random. Following through the analysis of shock 3.2, it is not surprising to see in Figure 7 that the rate of saving reacts very rapidly and strongly. While the peaks and troughs correspond perfectly, the variations in the savings rate are actually greater than the variations in the realized interest rate.

The shock was then repeated using modified interest rate expectations. A simulation rule was arbitrarily chosen that would reduce by 60 percent and 30 percent, respectively, the gap between the realized rates and the anticipated rates of the control solution (2 percent) for the next two periods. In the remaining simulation periods, the expected rate was held at 2 percent. The results generated from this modification differed only slightly from the previous results (Figure 7) and are therefore not presented.

Figure 7

OBSERVED INTEREST RATE SHOCKS



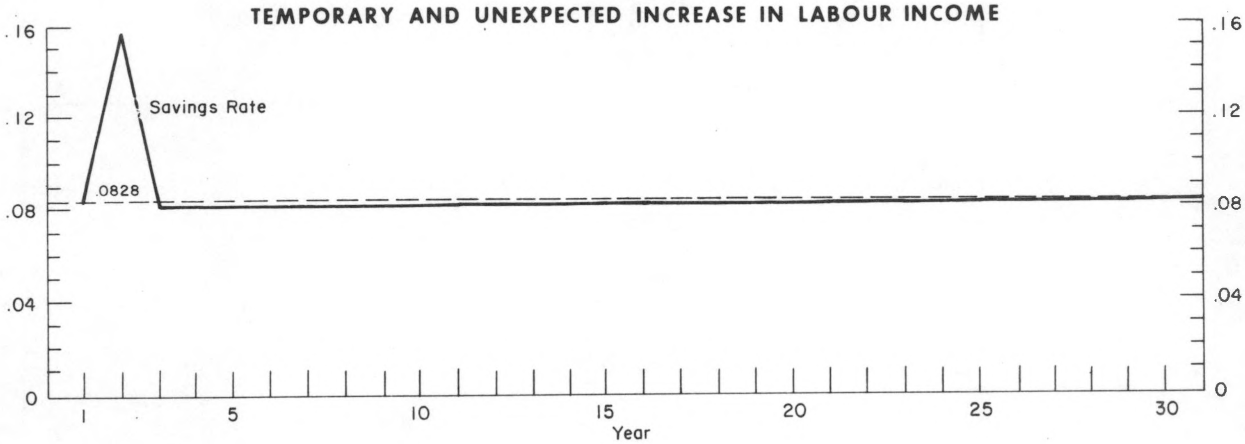
3.6 Temporary and unexpected increases in labour income

In order to investigate the impact of income changes on the savings rate, a set of income shocks was simulated and the effects on the model analyzed. The first shock was a temporary increase of 0.1 (in the second period) in labour income for the total population - including retired persons. This increase, which represents an average of 10 percent of labour income, is assumed to be unanticipated and to have no effect on subsequent expectations.

The effects of such a shock are similar to those produced by the temporary shock in the realized interest rate. The response differs only in magnitude. The savings rate increases from 0.0828 to 0.1577, then drops to 0.0801 in the third period, and climbs gradually back to 0.0828 towards year 50, as shown in Figure 8.

Figure 8

TEMPORARY AND UNEXPECTED INCREASE IN LABOUR INCOME



Initially, aggregate consumption increases by 0.0052 compared with the 0.1 increase in income, resulting in a rise of 0.0948 in savings. Over time however, the savings rate returns to the control level. Marginal propensity to consume in the short run (MPCS = .052) is thus very weak, while in the long run, the total increase in income is consumed (MPCL=1). It is interesting to note that MPCS is strongly influenced by the age distribution of the population and, as can be seen in Table 2, it differs greatly from one age group to another.

Table 2 MARGINAL PROPENSITY TO CONSUME IN THE SHORT RUN
(Temporary shock in labour income)

Age	MPCS in period 2
20	.0222
30	.0256
40	.0319
50	.0433
60	.0698
70	.2027
74	1.0000

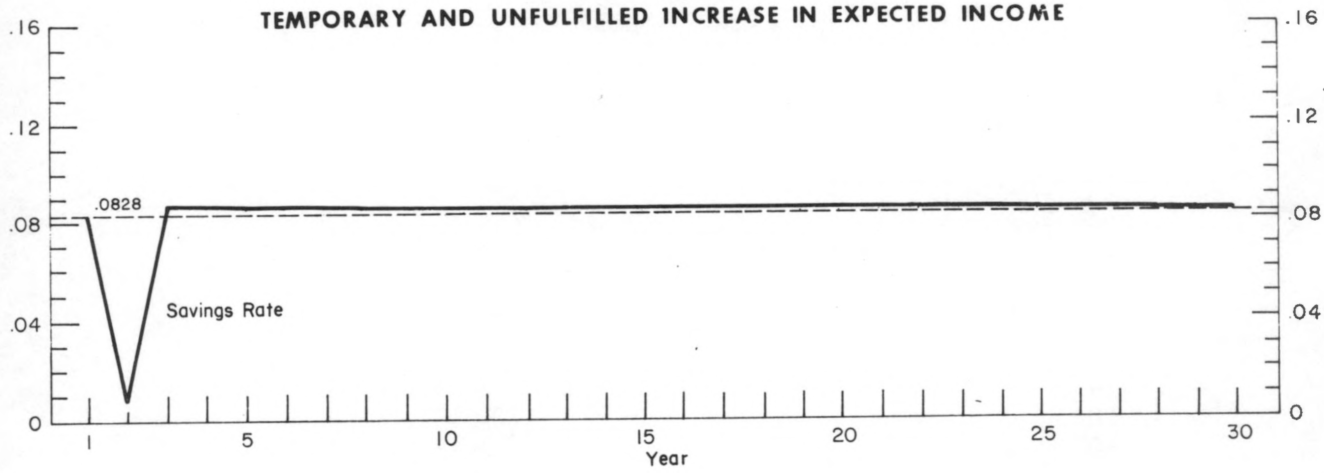
Thus, if there had been a greater proportion of older individuals in our control population, the MPCS would have been much higher because that group is more inclined to dispose of excess income through consumption.

In the second shock, labour income remains at its control level, but anticipated future labour incomes grow by 0.1 in the second period and then return to the control level for all subsequent periods. The results in this case, as seen in

Figure 9, are the opposite of those of the first shock in this section (Figure 8). Individuals in this case immediately increase their consumption to take advantage of the expected rise in income. In the aggregate, the rise in consumption in period 2 reflects a decline of 0.0732 in the savings rate.

Figure 9

TEMPORARY AND UNFULFILLED INCREASE IN EXPECTED INCOME



In Table 3, the ratio (MPC*)[8] of growth in consumption in period 2 to the expected growth in annual income (0.1) is presented as a function of age.

8. MPC* is defined as the difference between consumption under the shock conditions and consumption in the control solution, divided by 0.1.

Table 3 MARGINAL PROPENSITY TO CONSUME IN THE SHORT RUN (Anticipated temporary increase in labour income)

Age	MPC*
20	0.71
30	0.74
40	0.78
50	0.82
60	0.84
70	0.77
74	0.00

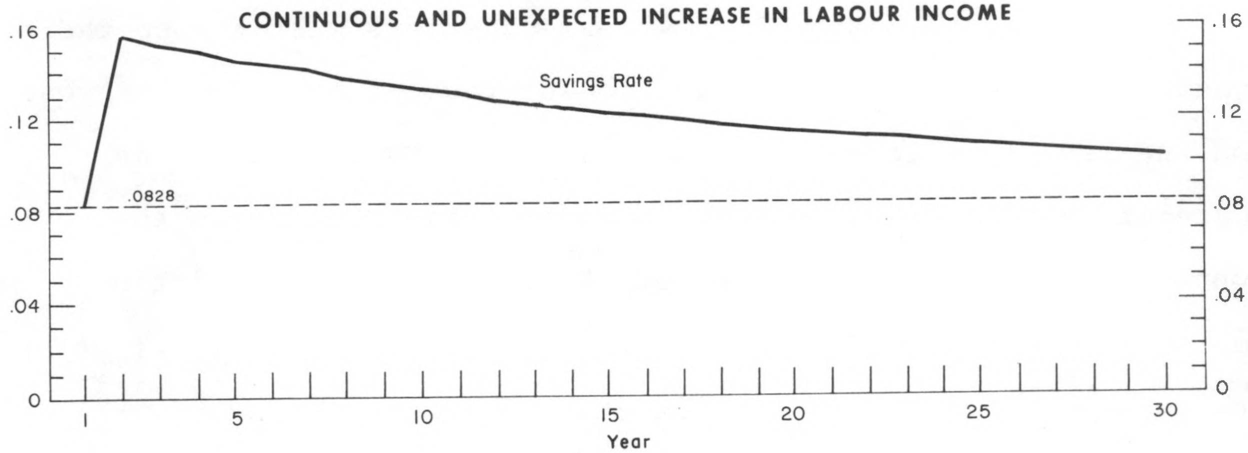
Subsequent to period 2, however, consumers must reduce their spending to offset the excessive consumption of the previous year and the resulting loss in non-wage income. This produces an aggregate rate of saving that is slightly higher than in the control solution. During the next 53 years of the simulation period, consumers gradually return to the equilibrium level of consumption in the control solution.

3.7 Continuous income shocks

In this case, an increase of 0.1 in labour income was maintained over the entire simulation period, while expectations of labour income remained at the control value. The results of this shock are shown in Figure 10. The effect in period 2 is identical to that of the earlier shock (see Figure 8). During subsequent periods, no decline in the savings rate below control is observed because consumers continue to receive unexpected inflows of income. Because consumers do not wish to leave any estate, they are compelled to consume out of wealth and thus the savings rate does decrease slowly over the simulation period. Moreover, the regular increment of 0.1 in labour income represents a steadily decreasing proportion of total income.

Figure 10

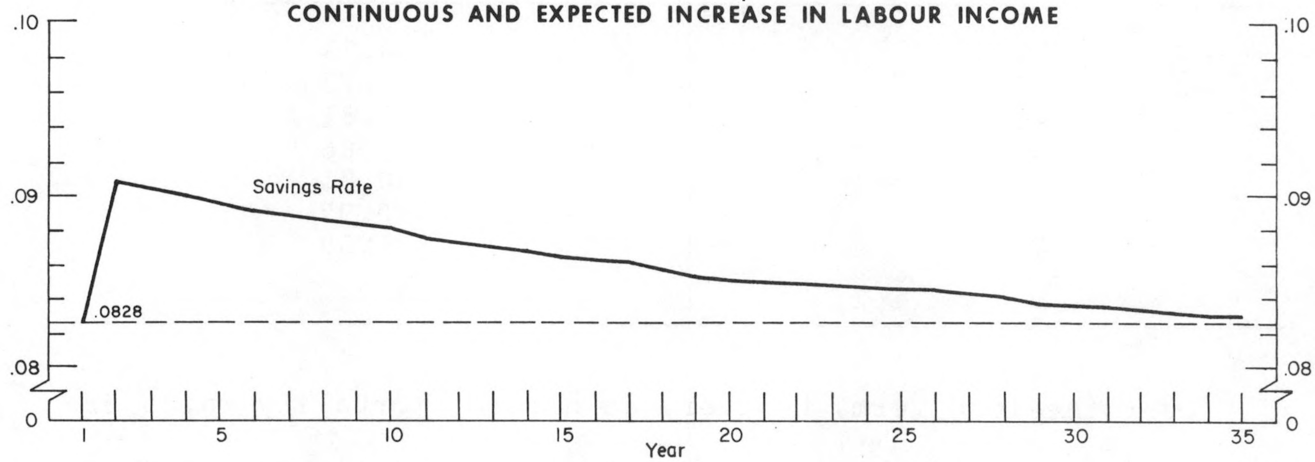
CONTINUOUS AND UNEXPECTED INCREASE IN LABOUR INCOME



In the next simulation, labour income was increased in a continuous and constant manner as before, but expectations were allowed to adjust perfectly to this increase. The result, illustrated in Figure 11, is a rather different dynamic response.

Figure 11

CONTINUOUS AND EXPECTED INCREASE IN LABOUR INCOME



At first one notices that the initial response in the savings rate is much weaker than in the previous case. As was demonstrated in the analysis of an expectations shock (Section 3.6), consumers decide to consume a large part of the expected income increase at once. At the microeconomic level, the MPCS for each age group is much higher in period 2 than with the earlier shock, and at the aggregate level the MPCS is 0.825.

Table 4 MARGINAL PROPENSITY TO CONSUME IN THE SHORT RUN
(Anticipated and actual permanent increase in labour income)

Age	MPCS
20	0.73
30	0.77
40	0.81
50	0.86
60	0.91
70	0.97
74	1.00

Over the long term, however, as natural mortality rates are reflected in the population, fewer and fewer people are affected by the shock and the savings rate returns to the level recorded in the control solution. It should be noted that the same dynamic response would be obtained from a "multiplicative" shock (e.g., a 10 percent increase in labour income).

3.8 Observed trend-income shocks

Finally, we allowed labour income to vary in line with the actual trend of real disposable income per worker in Canada.[9] As in the control solution, individuals expect real income to grow at a rate of approximately 2 percent. However, the level of income to which the growth rate can be applied can be viewed in

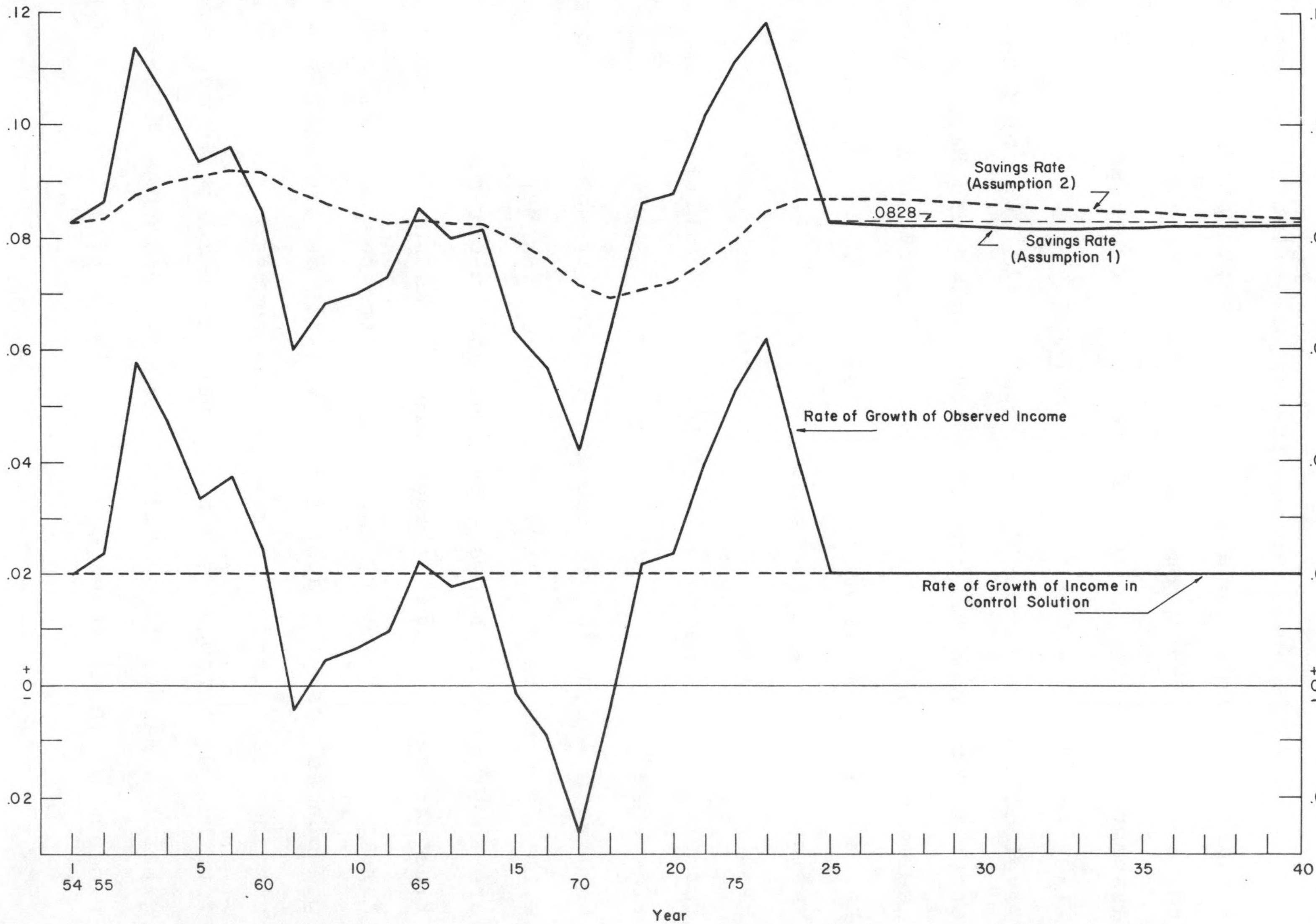
9. As calculated from RDXF data: ln(YDW/PCPI) = A + B QTIME

two ways. First, it can be defined as being that of the control solution. Such a view (which we shall call Assumption 1), implies that people are aware of the transitory effect of income variations and continue to base their expectations of growth on the more standard income level of the control solution. This assumption enables us to compare these income shocks with that illustrated in Figure 8. Alternatively, we could define the base level of income as the actual level in the current period. In other words, expectations of future income are based on current realized income, rather than on the control solution level as in Assumption 1. This hypothesis (Assumption 2) implies much greater variation in expectations and, as will be seen later, is much closer to the views embodied in current macroeconomic formulations. The results of these two simulations are shown in Figure 12.

With Assumption 1, the savings rate response is consistent with the results illustrated in Figure 8. Variations in income are about plus or minus 4 percent and in our model generate fluctuations of similar magnitude in the savings rate. With Assumption 2, however, the savings rate response is much less pronounced (varying approximately 1.3%) and results in a much smoother curve than observed with Assumption I. The explanation for this difference is simply that an increase in current income generates higher expectations for future income which, in turn, produce a higher level of consumption (refer to Figure 9).

Figure 12

OBSERVED INCOME SHOCKS



4 EFFECTS OF POPULATION STRUCTURE ON THE CONTROL SOLUTION

As described in Section 2, the aggregate savings rate is kept constant so that the dynamic response of our model can more easily be analyzed. This implies that the proportions of each age group are fixed for the entire simulation period and, moreover, that total population is constant. The population distribution in the control solution is characterized by a large proportion of youths and a high mortality rate. To measure the effects of such inconsistencies, we redefined the control solution for different population profiles and reran a few income and interest rate shocks. The following paragraphs summarize the results.

First, an increase in total population with a fixed population composition has no effect on the savings rate. In other words, a one percent increase in the population causes a rise of the same magnitude in aggregate consumption, saving and income, so that the aggregate savings rate remains the same.

If, however, the population structure was allowed to follow the pattern of the last 30 years, the changes that have occurred in the age distribution of the population would bring about changes in the savings rate as demonstrated in Figure 3. Let us call this control solution no. 2.

We created another control solution (no. 3) in which population distribution is uniform: total population is constant and each of the 11 age groups contains the same proportion (1/11) of the total. This structure results in a much smaller savings rate, 0.0465 compared with the 0.0828 rate in control solution no. 1 (i.e., the control solution defined in Section 2). This decrease in the savings rate is caused by a reduction in the proportion of those aged 25 to 50 who have a high savings rate, and by an increase in the proportion of older individuals (65 and over), who have a high rate of dissaving.

Finally, a uniform population and a zero rate of growth (G=0) in income are the basic assumptions in control solution no. 4, so that the aggregate savings rate is zero.[10] The advantage of this control solution is that a cross section for any year will illustrate the expected behaviour pattern of an individual over his lifetime.

Three shocks were then applied to each of these four control solutions. The response of the aggregate savings rate to these shocks is presented in Table 5. The results reveal that the use of a historical population profile has little impact on the savings rate response. Slightly different results are obtained in the case of a uniform population where income is growing (G = 0.02). When population is uniform and the income level is not growing (G = 0), quite a different dynamic response to the shocks is observed. Specifically, in Models 1, 2 and 3 a permanent rise in the interest rate causes a permanent rise in the aggregate savings rate in the long run, while Model no. 4 returns to its control solution level (where the savings rate is zero). The

10. The aggregate savings rate is zero because the individual does not wish to leave an estate, no matter what the interest rate may be.

Table 5 VARIATIONS IN SAVINGS RATE
(Shock minus control)

	Model 1	Model 2	Model 3	Model 4
Year	Control solution No. 1	Increasing population No. 2	Uniform population No. 3	Uniform population G = 0, No. 4
	Temporary income shock (Section 3.6)			
2	0.0749	0.0747	0.0815	0.0809
3	-0.0027	-0.0027	-0.0047	-0.0045
4	-0.0025	-0.0025	-0.0041	-0.0040
50	0	0	-0.0001	-0.0003
100	0	0	0	0
	Continuous income shock (Section 3.7)			
2	0.0080	0.0080	0.0094	0.0129
3	0.0076	0.0076	0.0089	0.0126
4	0.0072	0.0073	0.0084	0.0123
50	-0.0004	-0.0004	-0.0017	0.0004
100	-0.0001	-0.0001	-0.0007	0
	Continuous interest-rate shock (Section 3.4)			
2	0.0357	0.0356	0.0324	0.0360
3	0.0351	0.0349	0.0315	0.0346
4	0.0345	0.0342	0.0306	0.0332
50	0.0215	0.0215	0.0108	0.0009
100	0.0214	0.0214	0.0103	0.0002

reason for this difference is that in the first two models, greater weight is given to savers, while in the fourth, all individuals are weighted equally. A scenario with growing income (as in the third model) also gives greater weight to savers.

5 ESTIMATION OF A MACROECONOMIC CONSUMPTION FUNCTION

If we assume that consumers make their decisions as described in the basic life-cycle model, can we aggregate this behaviour in order to obtain a simple macroeconomic function that could be estimated using the usual econometric tools? Ando and Modigliani (1963) studied this problem and, after hypothesizing that population structure is fixed and that current labour income (YW) is a fixed share of actual future income, they obtained the following function:

$$C_t = d + a\ YW_t + b\ A_{t-1} \qquad (10)$$

This relationship can be derived from the aggregation of equation (5).

In order to verify the ability of this function to reproduce the dynamic response of the microeconomic model, equation (10) was estimated from the aggregate data generated by our model, and the dynamics implied by the equation were compared with the dynamic response of the basic model. The model used to generate the data is that of our control solution. Three simultaneous shocks were then administered to the basic model:

1) Structure and size of the population from 1947 to 1974 as in Figure 2.
2) Observed variations in labour income (Figure 12).
3) Observed variations in real interest rate (Figure 7).

The growth of expected labour income and the anticipated real interest rate were fixed at 2 percent per year. Since the second shock was simulated under the two assumptions described in Section 3.8, two data bases were created:

Sample 1 (Assumption 1) - unchanged level of expectations;
Sample 2 (Assumption 2) - level of expectations changes with the income shock.

We analyzed the dynamic properties of the estimated equation using the following model:

$$C_t = d + a\, YW_t + b\, A_t \qquad (11)$$

$$YT_t = YW_t + YNW_t \qquad (12)$$

$$YNW_t = r\, A_t \qquad (13)$$

$$S_t = YT_t - C_t \qquad (14)$$

$$A_t = A_{t-1} + S_{t-1} \qquad (15)$$

$$SR_t = S_t / YW_t \qquad (16)$$

where

C_t	is consumption during year t.
YW_t	is labour income during year t.
YT_t	is total income during year t.
YNW_t	is interest income during year t.
S_t	is saving during year t.
A_t	is wealth at the beginning of year t.
SR_t	is savings rate during year t.
r	is real interest rate ($r = .02$).

The results of estimations with ordinary least squares (OLS) are listed in Table 6. The first run was made using Sample 2, which should help in the reproduction of a dynamic response similar to that of the micro model since the present value of future labour income is a fixed proportion of actual real labour income. If the presence of autocorrelation is disregarded, then the standard aggregate formulation of the life-cycle model seems to pass the empirical tests (see equation 17): the MPC is high, and the wealth coefficient is significantly different from zero. It should be noted, however, that a formulation more akin to the permanent-income hypothesis, but using labour income instead of total income, appears to be more successful, as we observe a large decrease in the standard deviation in equations (20) and (21). The lag structures affecting income and implied by equations (19), (20) and (21) are, however, questionable.

Table 6 CONSUMPTION ESTIMATE WITH DATA GENERATED BY OUR MODEL (Sample 2)

Equation	Explanatory variables							Statistical tests		
	Constant	YW	YW_{-1}	YW_{-2}	C_{-1}	A	YT	see	RB2	dw
17)	.0283	.78920				.0893		.0089	.9995	.57
	(1.6)*	(34)				(9)				
18)	-.0390	.617			.3829			.0128	.9989	.58
	(2.1)	(9)			(5.5)					
19)	-.059	.962	-.3545	.3828				.0155	.9983	.81
	(2.8)	(9.3)	(1.7)	(3)						
20)	.0125	.6822	-.5264	.0044	.815	.0113		.0056	.9998	1.55
	(1.0)	(14.6)	(3.8)	(.07)	(5.3)	(.63)				
21)	.0075	.6815	-.597	.0225	.8942			.0055	.9998	1.45
	(.7)	(15)	(8)	(.4)	(10.4)					
22)	.1051				1.18		-.1734	.028	.9946	1.36
	(2.42)				(12.7)		(1.8)			

* t-test

As shown in Table 7, we obtain appreciably different results using Sample 1. With equation (23), the standard equation of the life cycle, the marginal propensity to consume out of income was slightly lower and the wealth coefficient was sharply higher. Given the results of the income-shock simulations (Sections 3.6 and 3.7) a much larger reduction in MPC had been expected. This expectation was satisfied by the introduction of delayed consumption, which suggests that the formulations that are usually used in the description of the permanent-income hypothesis are preferable.

Two shocks, a continuous increase of 0.1 in labour income and a continuous increase of 0.5 percentage points in the real interest rate were simulated with the following four models:

Model 1: The basic model that generates the data.

Model 2: The standard model of the life-cycle hypothesis (equations 12 to 17).

Model 3: A model that reflects the permanent-income hypothesis, using YW (equations 12-16 and 21).

Model 4: A model that reflects the permanent-income hypothesis, using YT (equations 12-16 and 28).

Since either a continuous or temporary shock, whether expected or not, has the same impact in Models 2, 3 and 4 we chose those that were continuous and expected. From the results obtained, one could say that Models 2, 3 and 4 do not reproduce certain important features of the basic model (Model 1). As can be seen

Table 7 CONSUMPTION ESTIMATE WITH DATA GENERATED BY OUR MODEL (Sample 1)

Equation	Explanatory variables: Constant	YW	YW_{-1}	YW_{-2}	C_{-1}	A	YT	Statistical tests: see	RB2	dw
23)	-.0338	.63				.208		.029	.996	.79
	(1.1)*	(10.6)				(5.6)				
24)	-.01	.0055			1.04			.0103	.9995	1.37
	(.9)	(.11)			(19)					
25)	-.093	.17	.19	.69				.035	.9942	.71
	(2.3)	(.7)	(.5)	(2.4)						
26)	-.018	-.0532	.0824	.026	.999	-.0023		.011	.9994	1.41
	(1.2)	(.6)	(.7)	(.2)	(8.6)	(.1)				
27)	-.018	-.049	.081	.026	.992			.011	.9995	1.41
	(1.3)	(-.6)	(.7)	(.3)	(12.5)					
28)	-.033				.924		.119	.0081	.9997	.93
	(3)				(24)		(3)			

* t-test

in Table 8, the results of an income shock show that the Model 2 response (standard equation of the life-cycle hypothesis) is adequate for estimating the level of consumption and the savings rate. Model 3 is acceptable also. By contrast, Model 4 severely underestimates the consumption response in the short run and overestimates it in the long run.

We observe a much less satisfactory performance when Models 2, 3 and 4 are subjected to an interest rate shock. With these three models, we do not observe any decline in consumption as a result of the postponement over time of consumption decisions. The negative effect is outweighed by the positive impact on consumption of the wealth that results from the higher interest rates (i.e., the income effect). Therefore, the aggregate model is still far from satisfactory.

How can these results be explained? The situation is probably a direct consequence of the simplifying assumptions used in the aggregation to the macroeconomic level. Our aggregate functions, like most others, fail to take account of two important aspects of consumption (or savings) behaviour suggested by life-cycle theories: that changes in the age structure of the population can have a considerable effect on the aggregate outcomes and that the forward-looking nature of the income constraint should be explicitly built into the problem.

Before considering these problems, we wish to present an estimation of the standard life-cycle equation, derived using the RDX2 data base:

Table 8 SIMULATIONS WITH AGGREGATE MODELS
(Shock minus control) x 10,000

Period	Increase in YW = .1								Increase in interest rate = .5%							
	model 1		model 2		model 3		model 4		model 1		model 2		model 3		model 4	
	C	SR	C	SR	C	SR	C	SR	C	SR	C	SR	C	SR	C	SR
1	824	71	789	109	681	201	119	680	-309	351	0	69	0	69	10	60
2	831	66	808	94	694	192	231	587	-312	347	7	65	0	71	19	53
3	837	59	825	81	728	165	336	501	-309	341	14	60	0	73	28	47
4	843	74	842	69	758	142	435	422	-305	329	21	56	0	74	37	41
5	849	57	857	58	784	122	529	350	-302	362	27	52	0	76	45	35
10	880	49	918	18	882	55	911	69	-224	310	56	37	0	85	84	13
20	933	18	991	-20	969	7	1354	-212	- 96	199	102	21	0	103	144	-14
30	950	5	1027	-30	997	0	1510	-289	-154	248	141	15	0	121	189	-26
40	974	-1	1044	-31	1007	4	1496	-276	-110	229	181	12	0	141	227	-29
50	985	-3	1053	-28	1009	9	1388	-230	- 27	218	225	11	0	161	263	-22
100	989	-1	1061	-12	1011	26	650	- 49	-182	214	591	10	0	273	553	-22

OLS, 1956-1974, annual data

$$CON\$_t = \underset{(4.1)}{1546} + \underset{(8.9)}{0.8367}\ YDW_t + \underset{(1.6)}{0.0413}\ V_{-1} \qquad (29)$$

see = 522 RB2 = .9991 dw = 1.28

where, according to RDX2 notation,

CON$ = CNDSD.PCNDSD + CS.PCS + CSMVOD.PCSMVOD

In this model, CON$, YDW and V correspond to C, YW and A respectively. These results are very similar to those of Ando and Modigliani (1963) who obtained coefficients of 0.75 for income and 0.042 for wealth. By using first differences, they succeeded in increasing the wealth coefficient to 0.072, but in our study this transformation had very little impact. A comparison of equations (17) and (29) reveals that the coefficients that affect income are almost equal while those of the wealth variable have a 2 to 1 ratio. The response of the aggregate model constructed from equations (29) and (11) to (16) to an interest rate shock would certainly be weaker than the response of Model 2 as shown in Table 8.

6 DEVELOPMENT OF A MACROECONOMIC CONSUMPTION FUNCTION

In this section four issues related to the aggregate consumption function are addressed. The first concerns the inclusion of the interest rate variable in the consumption function. The second deals with the selection of a variable capable of approximating both permanent and transitory income. The third considers the problem of aggregation, since each age group has a different marginal propensity to consume. Finally, we investigate the problem of temporal substitution between consumption and saving.

The standard consumption functions presented in the preceding section do not permit us to reproduce the responses observed in our basic micro life-cycle model. A review of equation (5) shows that wealth is multiplied by $(1 + r_t)$. This multiplication is necessary if the individual is to maximize his utility function subject to the budgetary constraint without having to liquidate his wealth for immediate consumption. Therefore, in our model a retired person 74 years of age will consume

$$c_{74} = a_{74}(1+r_t)$$

The inclusion of this factor yields the following results with Sample 2 data:

$$C_t = \underset{(-.5)}{-0.0081} + \underset{(38.8)}{0.8338}\, YW_t + \underset{(7.6)}{0.0714}\, A_t(1 + r_t) \qquad (30)$$

see = 0.010 RB2 = 0.9993 dw = 0.13 (OLS)

While this revised formulation does not improve the estimation results (see equation (17)), it was nevertheless simulated using the income and interest rate shocks described in Section 5. Let us call this model, which is formed from the integration of equations (12) to (16) and equation (30), Model 5.

With the income shock, there is only a small increase in consumption which, in turn, results in a high savings rate. These results are comparable to those obtained from Model 2. With an interest rate shock however, consumption increases in the first period rather than decreasing, because of the formulation of equation (30). As expected, the increase in consumption was greater in Model 5 than in Model 2.

Thus Model 5 is not satisfactory, since, as we know, a rise in the interest rate should lead to a reduction in consumption (see Table 8, Model 1).

With Sample I data we obtain the following results for the revised consumption function:

$$C_t = - \underset{(2.68)}{0.0635} + \underset{(21.4)}{0.704}\, YW_t + \underset{(8)}{0.169}\, A_t(1 + r_t) \qquad (31)$$

see = 0.0222 RB2 = .9976 dw = 0.78 (OLS)

Equation (31) gives better results than equation (23), but when it is added to equations (12) to (16) to construct Model 6, the simulation results are equally unsatisfactory.

We next developed a variable to account for the fact that people believe that random variations in labour income are transitory. The following relationship was estimated:

$$\mathrm{Log}(YW_t) = \hat{a}_0 + \hat{a}_1 . Q$$

where

$$Q = 1, 2, 3, \ldots, n$$

Next, estimated (permanent) income was defined as

$$YWEST_t = \exp(\hat{a}_0 + \hat{a}_1 . Q)$$

and transitory income as

$$YWTRA_t = YW_t - YWEST_t$$

Using these variables, we then estimated the consumption function. With Sample 1, the results were as follows:

$$C_t = -\underset{(3.5)}{.074} + \underset{(8)}{1.005}\, YWEST_t + \underset{(.7)}{.15}\, YWTRA_t - \underset{(.15)}{.012}\, A_t(1+r_t)$$

see = 0.01955 RB2 = 0.9982 dw = 0.45 (32) (OLS)

As expected, the propensity to consume out of transitory income is very weak. Unfortunately, wealth is not significant in equation (32); however, if YWTRA is deleted and the equation is re-estimated, the wealth coefficient becomes - 0.0585 with a t-statistic of -2.173.

With Sample 2, the relationship becomes

$$C_t = - \underset{(4.6)}{.043} + \underset{(65)}{.9}\ YWEST_t + \underset{(39)}{.72}\ YWTRA_t + \underset{(6.5)}{.04}\ A_t(1+r_t) \qquad (33)$$

see = 0.005 RB2 = 0.9998 dw = 0.52

(OLS)

The t-tests and standard deviation of this equation are high. Further, we observe that the propensities to consume out of permanent and transitory income are very similar. Several other aggregate consumption functions were estimated but unfortunately none proved to be very desirable.

We must still establish whether there is an aggregation problem arising from the very nature of the life-cycle theory. Specifically, marginal propensities to consume are different for each age group, as are labour income and wealth. Total consumption at time t is simply the sum of the consumption of all age groups:

$$C = CON_1 + CON_2 + CON_3 + \ldots + CON_{11} \qquad (34)$$

Since retired individuals have no labour income, we can specify consumption for each age group as:

$$CON_i = \beta_{0i}A_i(1+r_t) + \beta_{1i}YW_i, \qquad \text{for } i = 1 \text{ to } 9$$

and

$$CON_i = \gamma_{0i}A_i(1+r_t) \qquad \text{for } i = 10 \text{ and } 11$$

Therefore the aggregate consumption function of our life-cycle model actually is

$$C = \sum_{i=1}^{9} (\beta_{0i}A_i(1+r_t) + \beta_{1i}YW_i) + \sum_{i=10}^{11} \gamma_{0i}A_i(1+r_t)$$

There are 20 explanatory variables in the equation. The equation cannot be estimated, however, because observed data on labour income and wealth for all age groups are not available.

Instead, the following variables were chosen to estimate the real aggregate consumption function:

P1 is the proportion of the population aged 20-34 (averaging 0.374);

P2 is the proportion of the population aged 35-64 (averaging 0.539) and

P3 is the proportion of retired persons in the population (averaging 0.0874).

The following relationship was then estimated using Sample 1 data:

$$
\begin{aligned}
C_t = \underset{(5.8)}{.02} - \underset{(.8)}{.25}\, P1_t(A_t)(1+r_t) + \underset{(2.7)}{.64}\, P2_t(A_t)(1+r_t) \\
- \underset{(9.6)}{1.33}\, P3_t(A_t)(1+r_t) - \underset{(.1)}{.14}\, P1_t(YW_t) + \underset{(.5)}{.35}\, P2_t(YW_t) \\
+ \underset{(1.7)}{2.5}\, P1_t(YWEST_t) - \underset{(.6)}{.6}\, P2_t(YWEST_t) \qquad (35)
\end{aligned}
$$

see = 0.0015 RB2 = 0.9999 dw = 2.36 (OLS)

The Durbin-Watson statistic is now acceptable and the population variables P1, P2 and P3 seem to capture the chronological variations which could not be caught by YW_t and A_t.

The marginal propensity to consume out of wealth is calculated as follows:

$$- 0.25 \times 0.374 + 0.64 \times 0.539 - 1.33 \times 0.0874 =$$

$$- 0.0935 + 0.345 - 0.1162 = 0.1353$$

This result is comparable with that of equation (31).

Marginal propensity to consume out of labour income is hardly significant and is calculated to be

$$- 0.14 \times 0.374 + 0.35 \times 0.539 =$$

$$-0.0524 + 0.1886 = 0.1362$$

The result is comparable with that obtained with shock 3.6.

The marginal propensity to consume out of expected labour income is:

$$2.5 \times 0.374 - 0.6 \times 0.539 = 0.935 - 0.323 = 0.612$$

This result approaches that obtained with Model 1 (see Table 8).

The same relationship was then estimated with Sample 2. Since the variable YWEST was not significant, it was not included in the new estimation.

$$C_t = \underset{(21)}{.03} + \underset{(5.4)}{.197}\ P1_t(A_t)(1 + r_t) - \underset{(2.3)}{.06}\ P2_t(A_t)\ (1+r_t)$$

$$+ \underset{(9.7)}{.43}\ P3_t(A_t)\ (1+r_t) + \underset{(13.8)}{.82}\ P1_t(YW_t) + \underset{(20.9)}{.92}\ P2_t(YW_t) \quad (36)$$

see = .00067 RB2 = 1.0 dw = 1.52 (OLS)

The marginal propensity to consume out of labour income is:

$$0.82 \times 0.374 + 0.92 \times 0.539 = 0.3067 + 0.4959 = 0.8026$$

and the marginal propensity to consume out of wealth is:

$$0.197 \times 0.374 - 0.06 \times 0.539 + 0.43 \times 0.0874 =$$

$$0.0737 - .0323 + .0376 = 0.079$$

These results are satisfactory.

The problem of temporal substitution between consumption and saving unfortunately remains unresolved, since aggregate formulations do not reproduce the intertemporal substitution effect. To achieve this, marginal propensities to consume out of income and wealth would have to be functions of variations in the interest rate (past, present and future).

$$MPC = f\ (\text{population structure}, r_{t-n}, r_t, r_{t+m})$$

One possible approach would be to include a lag structure on expected interest rates with a negative short-term effect and a positive long-term effect. Because of lack of time, however, we did not attempt this approach here, but it will be the subject of future research.

Finally, we present the results of an estimation of a general formulation using RDX2 data and based on (equations (35) and (36)) as follows:

OLS, 1956-1974, annual data

$$\begin{aligned} CON\$_t = {} & \underset{(5.1)}{3548.2} + \underset{(.6)}{2.4}\, P1_t (V_{-1_t})(1+r_t) - \underset{(.3)}{1.15}\, P2_t (V_{-1_t})(1+r_t) \\ & - \underset{(.07)}{2.6}\, P3_t (V_{-1_t})(1 + r_t) + \underset{(.9)}{17.8}\, P1_t (YDW_t) + \underset{(1)}{16.9}\, P2_t (YDW_t) \\ & - \underset{(1)}{172.7}\, P3_t (YDW_t) - \underset{(2.7)}{24.5}\, P1_t (YDWEST_t) \\ & - \underset{(1.4)}{10.3}\, P2_t (YDWEST_t) + \underset{(2.1)}{168.7}\, P3_t (YDWEST_t) \end{aligned}$$

see = 306.13 RB2 = 0.99967 dw = 2.186 (37)

where

$$YDWEST = \exp(\hat{a}_0 + \hat{a}_1 \cdot Q).$$

The marginal propensities to consume were calculated as follows:

$$\text{MPC YDW} = 17.86 \times 0.3734 + 16.87 \times 0.5393 - 172.738 \times 0.087322 = 0.6831$$

$$\text{MPC } V_{-1} = 2.4204 \times 0.3734 - 1.1527 \times 0.5393 - 2.6258 \times 0.087322 = 0.0528$$

$$\text{MPC YDWEST} = -24.53 \times 0.3734 - 10.2627 \times 0.5393 + 168.66 \times 0.087322 = 0.0335$$

While the standard deviation in this formulation is much less than in the standard formulation of the life-cycle hypothesis -- (306 versus 522), the signs of certain coefficients as well as their t-statistics are rather perplexing. It is clear, however, that marginal propensities to consume out of income and wealth are comparable with those of equation (29). Once more the empirical results suggest that real labour income has been heavily weighted in the formulation of expected permanent income.

7 CONCLUSIONS

In this study, we attempted to shed some light on the macroeconomic specification of the life-cycle model. First we constructed a microeconomic model which could generate, with a certain degree of verisimilitude, decisions made in terms of the life-cycle hypothesis and, which, with the use of observed data on Canadian population structure, generated a set of macroeconomic variables implied by those decisions. We then analyzed the dynamic response of this system using a series of simulations. In this way the effects of changes in population structure, incomes and interest rates were studied. In a third stage, we tried to estimate the standard consumption functions, using the aggregate data from the model. While we obtained estimations that satisfied the usual econometric criteria, these functions, especially where the interest rate was concerned, did not adequately reproduce the model that generated the data. Finally, tests of different formulations that might have yielded a better representation of the basic model, also proved disappointing.

In general, we can affirm that the problem of aggregation gives rise to serious difficulties when estimating the aggregate consumption functions representing the life-cycle hypothesis. Further research is required in order to integrate into the macroeconomic formulations a group of adjustments that might take into account variations in population structure and intertemporal substitution. Moreover, the microeconomic model will have to be made more complex in order to study the effect of wealth revaluation.

APPENDIX A

COMPLEX MODEL OF THE LIFE-CYCLE THEORY

A.1 Construction of the control solution

The concept of the family was introduced into the budgetary constraint in the belief that a consumption function that is either increasing and monotonic or decreasing and monotonic, is not highly probable (see White, 1978 and Irvine, 1978). The budgetary constraint was altered to reflect the fact that the number of children varies with the ages of the adult members of the population and that financial liabilities are greater during middle age. The new budgetary constraint for an individual of age j at time t is

$$\sum_{i=0}^{74-j} c^0_{j+i} f_{j+i} (1+r_{t+i})^{-i} = \sum_{i=0}^{74-j} y_{j+i} (1+r_{t+i})^{-i} + a_j (1+r_t) \qquad (1a)$$

where

c^0_{j+i} is consumption per family member when the family head is (j+i) years old,

and

f_{j+i} is a function that reflects change in family size (defined later).

The new utility function is

$$U_j = \sum_{i=0}^{74-j} \left[\frac{1}{1-d} c_{j+i}^{0(1-d)} f_{j+i} + b\right](1+p)^{-i}, \quad \text{if } d > 0 \text{ and } d \neq 1 \tag{2a}$$

or

$$U_j = \sum_{i=0}^{74-j} [\ln c_{j+i}^{0} f_{j+i} + b](1+p)^{-i}, \qquad \text{if } d = 1 \tag{3a}$$

Maximizing, we obtain

$$c_{j+i}^{0} = c_j^{0}\left[\frac{1+p}{1+r_{t+i}}\right]^{-i/d} \tag{4a}$$

and

$$c_j^{0} = \left[a_j(1+r_{t+i}) + \sum_{i=0}^{74-j} y_{j+i}(1+r_{t+i})^{-i}\right] /$$

$$\left[\sum_{i=0}^{74-j} \left(\frac{1+p}{1+r_{t+i}}\right)^{-i/d}(1+r_{t+i})^{-i} f_{j+i}\right] \tag{5a}$$

and $c_j = c_j^{0}(f_j)$

where

c_j is total consumption of a household in which the family head is j years old.

In order to define the aggregate savings rate, equations (6) to (9) of the simple model were added to these equations.

Unfortunately, there are no statistics on family consumption behaviour in Canada. However, a study conducted in the United States in 1961 (see Thurow (1969)) showed that consumption expenditures increased steadily from the time of birth, reached a peak at age 40 and then declined until age 75.

Table A-1 gives some population data on families in Canada. Specifically, it shows the average number of children per family broken down by age of the family head. A number of factors, however, complicate a consideration of the family function. First, one must take into account the fact that a large proportion of the population is unmarried (compare Tables A-1 and A-2). Also we can assume that working married women help to support children, and that it costs more to support teenagers than younger children. Because data on families in Canada are very limited, we decided to use total population rather than the total number of families as the aggregation base. As an approximation, therefore, we adjusted the figures for the average number of children in Table A-1. The following family function was finally chosen:

$$Fofa(I) = \sin\,[(I-20)/(54)] + 1, \quad \text{for } I = 20 \text{ to } 74.$$

We are aware that this is a rough approximation, but we were unable to obtain more accurate data.

In our classical life-cycle model, we assumed that income kept growing until retirement. We know that this assumption is not realistic, but there are no accurate figures for recent

Table A-1 CANADIAN FAMILIES, 1971

Age of family head	Number of husband-wife families	Number of persons in families	Average per family	Average number of children per family
-25	294,445	773,350	2.6	0.6
25-34	1,092,440	4,006,405	3.7	1.7
35-44	1,075,205	5,229,215	4.9	2.9
45-54	941,915	4,071,680	4.3	2.3
55-64	686,270	2,118,435	3.1	2.1
65+	515,215	1,200,210	2.3	0.3

Source: "Census of Canada 1971." Statistics Canada, Cat. no. 93-720 vol. 2 Part 2 (Bulletin 2.2-8), July 1975.

Table A-2 POPULATION OF CANADA BY AGE GROUP AND SEX, 1971

Age Group	Total	Men	Women
20-24	1,889,405	941,775	947,630
25-34	2,889,545	1,461,585	1,427,960
35-44	2,526,395	1,285,810	1,240,580
45-54	2,291,575	1,132,310	1,159,270
55-64	1,731,740	854,105	877,635
65+	1,744,410	781,865	962,540

Source: "Census of Canada 1971." Statistics Canada, Cat. 92-715 vol. 1 Part 2 (Bulletin 1.2-3), April 1973.

years on this topic. Therefore, in the complex model we made the assumption that cross-sectional incomes grow at a rate of 0.005 from age 20 to age 50, remain constant from age 50 to age 60 and then decline at a rate of 0.0025 until age 65. This produces the curve shown in Figure 1.4. With a real interest rate of 0.02, a p of 0.0061, and a d of 1.5, we obtain a constant savings rate out of total income of 0.0837 and a constant rate of saving out of labour income of 0.0865.

The individual's expected consumption is illustrated in Figure 1.6 and the aggregate consumption curve at a given point in time is depicted in Figure 1.8. The savings rate for the cross section of groups in the economy will follow the curve in Figure 1.10.

A.2 Shocks to the complex model

Simulations of the more complex model were run for several shocks. As shown in Table A-3, a continuous and expected interest rate shock (as described in Section 3.4), produces an increase in the savings rate, regardless of the model utilized. In the long run an increase in the interest rate from 2 percent to 2.5 percent causes the savings rate to rise from 0.0828 to 0.1042 in the simple model and from 0.0837 to 0.1029 in the complex model.

The dynamic response of the savings rate to a population-growth shock (1947-74) in the model with the family function is different from that of the the simple model. This occurs because savings rate curves for an individual in terms of his age differ significantly from one model to another (see

Table A-3 DEVELOPMENTS IN THE SAVINGS RATE FOLLOWING DIFFERENT SHOCKS

a) Continuous and expected increase in labour income

Year	Simple model	Model with family function
1	0.0828	0.0837
2	0.0908	0.0849
3	0.0904	0.0852
4	0.0900	0.0854
5	0.0897	0.0856
10	0.0880	0.0859
20	0.0854	0.0851
30	0.0837	0.0837
40	0.0828	0.0828
50	0.0824	0.0825
100	0.0827	0.0832

b) Continuous and expected increase in interest rates

Year	Simple model	Model with family function
1	0.0828	0.0837
2	0.1185	0.1189
3	0.1179	0.1183
4	0.1173	0.1178
5	0.1167	0.1172
10	0.1142	0.1144
20	0.1105	0.1096
30	0.1076	0.1059
40	0.1055	0.1038
50	0.1043	0.1030
100	0.1042	0.1029

Figures 1.9 and 1.10). The difference becomes more pronounced from 1975 to 2001 (see Figure 3) because of the sharp reduction in the proportion of youths in the population. The effect on the over-all rate of saving is especially pronounced in the complex model. Thus, the savings rate might decrease from 0.0846 in 1975 to 0.0598 in 1995 and climb to 0.0693 in 2001.

While in the complex model the magnitude of the savings rate response to a continuous income shock is the same as in the simple model (see Section 3.7), it is more gradual. In the complex model individuals aged 20 to 41 years increase their savings rate in the second period, while those 42 to 64 years of age reduce it considerably because of increasing family responsibilities. However in the third period, people aged 43 to 64 years increase their savings rate because they now realize the shock is permanent and this causes the aggregate savings rate to continue to rise. The response is more abrupt in the simple model. In period 2, the savings rate increases significantly for those aged 20 to 42 and declines somewhat for those 43 to 64 years of age. (In this case, individuals begin to consume their increases in income because of their shorter life expectancies rather than because of family obligations.) In the third period, the savings rate decreases for all age groups and therefore the aggregate savings rate begins to decline.

We feel it is unnecessary to reproduce all the simulations that were done since the dynamic responses are similar in both models.

APPENDIX B

EFFECTS OF VARIATIONS IN RETIREMENT AGE, LIFE EXPECTANCY AND LENGTH OF TIME IN THE WORK FORCE ON THE SAVINGS RATE

B.1 Lowering the retirement age

In order to investigate the effects of a reduction in the retirement age, we simulated a simple model using the population structure observed from 1947-74 as the control solution, (see Figure 3). The first shock administered to this model was a five-year reduction in the retirement age to age 60. As a result, the aggregate savings rate was approximately doubled from its level in the control solution to 0.1612 in 1974. On the microeconomic level the savings rate increased by about 0.13 for the entire 20-60 age group. Next, the retirement age was reduced by only one year, from 65 to 64 years of age. In this case, the aggregate savings rate increased, with respect to the control solution, by about 0.03 to 0.1125 in 1974.

Similarly, the retirement age was reduced to 60 in the complex model (model with family function), and the rate of saving also doubled -- from the control level of 0.0837 to 0.1605 in 1974. In contrast to the first simulation, the savings rate registered a considerable increase only for those in the 30-40 year-old age bracket (0.15). It should be noted that the saving potential of individuals in the 40-50 age group is not as strong owing to increasing family expenses. (This is a basic assumption used in the construction of the complex model. See Appendix A.)

B.2 Increase in life expectancy

Two shocks were then simulated in which life expectancy was increased first by one year to 76 years of age and then by 2 years to age 77. The number of years in the labour force (45) remained the same as in the control solution. Life expectancy forecasts were revised for the entire population. As a result, the aggregate savings rate for 1974 increased by about 0.015 when one year was added to the over-all life expectancy of the population, and by about 0.03 with the addition of 2 years.

In Section B.1, where life expectancy remained at age 75, it was shown that if individuals work for 44 years and are retired for 11 years (instead of 10), the aggregate savings rate increases by about 0.03. However, in this next simulation if life expectancy is increased to 76 years of age, then people are in the workforce for 45 years and in retirement for 11 years, with the result that the increase in the aggregate savings rate (with respect to the control solution) is only 0.015. This result seems quite realistic.[11]

B.3 Increase in age of entry into labour market

A situation representing later entry into the labour market

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11. We obtained similar results with the complex model.

was simulated next. In this instance, the individual begins his working life at age 25, works for 40 years and is in retirement for 10 years. As a result, the aggregate savings rate increases by about 0.022. Apparently, it can be said that 2 additional years of retirement will result in a greater increase in the savings rate than if the length of time in the work force is reduced by 5 years. This does not take into account the fact that parents will have to increase their savings rate to support their children until age 25.

B.4 Relevance of the shocks

The simulations described in this Appendix were run in an attempt to reflect certain trends in the Canadian economy. First, the average age at which people enter the labour market, which was 16.2 years in 1931, had risen to 19.2 years by 1971 - a three-year increase. This later entry into the labour market is due to an increase in the average number of years spent in the educational system.[12] Secondly, while the average age of retirement was 65.1 years in 1961, it had decreased to 60.1 years in 1971.[13] Between the first quarter of 1971 and the fourth quarter of 1977, the savings rate in Canada increased by 0.055.[14] A continuation of the trends just noted may have been responsible.

12. Stat. Can. Cat. 71-524E.
13. Ibid.
14. La croissance du taux d'épargne (IT71-4T77): Simulations du secteur de la consommation de RDXF par Jean-Pierre Aubry et Diane Fleurent, RM-79-22, 145-4.

GLOSSARY OF RDX2 MNEMONICS USED IN THIS REPORT

CNDSD	Consumer expenditure on non-durables and semi-durables (millions of 1961 dollars).
CS	Consumer expenditure on services (millions of 1961 dollars).
CSMVOD	Consumer services imputed from the stock of motor vehicles and other consumer durables (millions of 1961 dollars).
PCPI	Consumer Price Index (1961=1).
PCNDSD	Price of consumer non-durables and semi-durables (1961=1).
PCS	Price of consumer services (1961=1).
PCSMVOD	Implicit price deflator for consumer services imputed from the stocks of motor vehicles and other consumer durables (1961=1).
QTIME	Time trend, equals 1.0 in 1Q50, 2.0 in 2Q50, etc.
V	Market value of private sector wealth (millions of current dollars).
YDP	Disposable personal income (millions of current dollars).
YDW	Disposable wage income (millions of current dollars).

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