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THE COMPARATIVE EX POST FORECASTING PROPERTIES OF SEVERAL CANADIAN QUARTERLY ECONOMETRIC MODELS

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AVANT-PROPOS

Dans cette étude, nous comparons le pouvoir prédictif des trois modèles trimestriels de l'économie canadienne accessibles au public: le modèle économétrique trimestriel de prévision à court terme (IRIC) mis au point par le Conference Board au Canada, le modèle économétrique trimestriel de l'économie canadienne (QFM) construit par l'Université de Toronto et le modèle économétrique de l'économie canadienne (RDX2) élaboré par la Banque du Canada. Le pouvoir prédictif de ces modèles a été comparé à celui des modèles ARIMA Box-Jenkins et à celui d'un modèle monétariste de forme réduite choisis comme termes de référence. Nous avons étudié sur différents horizons de prévision seize variables considérées comme importantes par les prévisionnsites, ce dans le but de déterminer les erreurs de prévision contenues dans le niveau des variables et dans leur taux d'accroissement.

Nous n'avons trouvé aucun de ces modèles supérieur aux autres. Toutefois, si les trois modèles économétriques permettent en général d'obtenir des résultats qui se comparent avantageusement à ceux du modèle Box-Jenkins, le modèle monétariste, lui, se révèle constamment supérieur lorsqu'il s'agit de prévoir l'évolution de la dépense nationale brute en termes nominaux. Le pouvoir prédictif des trois modèles économétriques varie considérablement lorsqu'on modifie l'horizon choisi.

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ABSTRACT

In this study we compare the forecasting ability of the three publicly available Canadian quarterly econometric models: The AERIC Short-Term Quarterly Forecasting Model of the Canadian Economy (AERIC) developed in The Conference Board in Canada, the Quarterly Econometric Model of the Canadian Economy (QFM) developed at the University of Toronto, and the Research Department quarterly experimental econometric model of the Canadian economy (RDX2) developed in the Bank of Canada. The standards against which these econometric models are measured are univariate Box-Jenkins models and a monetarist reduced form model. Sixteen variables of general interest to forecasters are examined over various prediction intervals so as to ascertain the forecast errors in the levels of the variables and their percentage changes.

We find that no one model predominates. Although the three econometric models generally perform well in comparison with the Box-Jenkins models, the monetarist model consistently predicts nominal gross national expenditure best. Among the three econometric models there is considerable variation in the ability to predict the variables over different time horizons.

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1. INTRODUCTION

The rapid expansion in the econometric modelling of the Canadian economy noted by Tsurumi [23] has continued into the mid-1970s but with greater emphasis than in the past upon commercial activity and forecasting. For example, The Conference Board in Canada, Data Resources Incorporated in association with the University of Toronto, and Informetrica Incorporated each have an econometric model they use in their services that are available to the public and in the preparation of their forecasts. This recent expansion of Canadian econometric models, as well as their general availability and use in forecasting, makes the present an opportune time[1] to compare some of them. At the Canadian Economics Association meetings in June 1976 a series of papers was presented [8] focussed on the within-sample simulation responses of certain econometric models when subjected to a fairly standard set of shocks. In this report we compare the prediction performance of the three publicly available Canadian[2] quarterly structural econometric models: Quarterly Econometric Model of the Canadian Economy developed at the University of Toronto (QFM), The AERIC Short-Term Quarterly Forecasting Model of the Canadian Economy developed in The Conference Board in Canada, and the quarterly experimental econometric model of the Canadian economy developed in the Research Department of the Bank of Canada (RDX2). The standards against which the performance of each of these models is measured are those of a simple monetarist reduced form model constructed at the Bank of Canada, see Appendix C, and of univariate Box-Jenkins (B-J) models, see Appendix A and [2]. We are concerned with sixteen variables of general interest to forecasters. The forecast period[3] is 1Q73-4Q74. We present results over three time horizons, for both the levels of these variables and the percent changes that take place.

Since we are interested in the ability of the econometric models to forecast and not the ability of the forecasters to forecast, we eliminate the forecaster by a technique called ex post forecasting. This involves providing the econometric model with historical data outside the estimation period for each exogenous variable without allowing the forecaster to manipulate forecasts of the endogenous variables by using intercept adjustments. The result is forecasts of the endogenous variables produced by the econometric model, unaided by the forecaster, with perfect knowledge of variables that the model assumes to be given, ie, the exogenous variables. For excellent discussions of the ex post forecasting procedure from the point of view of model verification,[4] among others, see Maxwell [18], [19], Christ [5], or Haitovsky et al. [11]. Ex post forecasting may also be interpreted as the usual forecasting exercise in the curious situation where the forecaster has perfect knowledge of the future behaviour of all exogenous variables but zero knowledge of the future behaviour of all endogenous variables.

A study as extensive as this one faces several problems.[5] Although each model examined is a quarterly model and basically follows the components of the national income and expenditure accounts, the models are not strictly comparable.[6] The most important difference pertains to the degree of endogeneity. Each group of model builders had somewhat different requirements in mind when the models were designed and these requirements, of course, determined the choice of endogenous variables. As well, the estimation periods are different; seasonally adjusted data are used in some models whereas unadjusted data are used in others; real variables and price indices are expressed in 1961 dollars in some models whereas in others they are expressed in 1971 dollars.[7] Although we do not intend to minimize the problems associated with these differences we believe it is unlikely that any other group of econometric models will ever be as comparable as the versions of the three models we study here. Whenever we think our results may be influenced by our choice of approach we hasten to point this out. Also, the results are set out in as much detail as is practicable so that readers can reach their own conclusions on some of the issues involved.

Our report is not intended to be a critique of any of the econometric models, although we find it difficult to avoid general statements in summarizing our results. Our findings should be taken as being indicative of the potential of the models involved but not as an absolute pronouncement on their relative forecasting abilities. Section 2 is a brief description and comparison of the models studied in this report. In section 3 we discuss the results, and in section 4 we summarize our findings and draw some conclusions.

2. A DESCRIPTION AND COMPARISON OF THE MODELS

The Structural Models

The Bank of Canada has constructed a series of econometric models for use in structural analysis, forecasting, and the simulation of alternative economic policies within a forecasting framework. The best known of these models is RDX2,[8] described in its original form in Helliwell et al. [14]. A useful summary of RDX2 also appears in Tsurumi [23]. We use the latest version of RDX2 here. It is contained in Bank of Canada Technical Report 5 [1].

The Bank of Canada model is characterized by detailed financial, government, personal income tax, and trade sectors but the private sector is not as well developed. All major macro-economic variables are endogenously determined including changes in inventories, the exchange rate, housing starts, long-term capital flows, several components of government expenditure, wealth, mortgage approvals, short-term interest rates, and the money supply. The most recent version of RDX2 is estimated from various starting points and usually to 4Q72. The principal exceptions are the exchange rate, which is estimated to 2Q75, and residential construction, which is estimated to 4Q73. All data are seasonally unadjusted.[9]

The version of the University of Toronto quarterly econometric model of the Canadian economy (QFM) used here is documented in Data Resources, Inc. (DRI) [7] and was the version commercially available in May 1976. Data are seasonally adjusted and expressed in 1961 dollars. The estimated equations have various starting points and generally terminate at 4Q73. It is important to note that, as this is the estimation period for QFM, only half our forecasting interval is outside the estimation period. Thus QFM should have considerable advantage over the other two models. The builders of this version of QFM have, however, relied extensively on the use of intercept adjustments to account for structural change outside the estimation period.

Instead of continually respecifying the equations in the current edition of QFM, the builders have chosen to concentrate upon a complete reestimation[10] of the model. In an ex post forecasting exercise all intercept adjustments are suppressed in order to isolate model errors as opposed to the errors of forecasters. Therefore, the forecasts made by QFM will incorporate known structural deficiencies outside the estimation period.

The original version of the short-term quarterly forecasting model of the Canadian economy constructed by the Applied Economic Research and Information Centre (AERIC) of The Conference Board in Canada is documented in Laurie et al. [16]. However, the model was reestimated after the national income and expenditure accounts were rebased to constant 1971 dollars and some variables are now endogenous that were exogenous in the original version. The version of the AERIC model used for this study is called AERIC753 and is available on the TROLL system [21]. With the exception of the financial sector, AERIC753 is estimated from various starting points to 4Q72. The financial sector is estimated to 4Q73. Seasonally adjusted data are used in AERIC.

Both QFM and AERIC have fewer endogenous variables than has RDX2. For example, the exchange rate and housing starts are exogenous to QFM and AERIC but endogenous to RDX2. In the case of AERIC one can add to the list of exogenous variables inventory investment, which, therefore, means that AERIC has generally fewer endogenous variables than has QFM. In an ex post forecasting exercise the addition of each endogenous variable means that another variable is used in the model with some inaccuracy. This implies greater error in forecasting other endogenous variables apart from errors that are fortuitously offsetting. Indeed, in anticipation of the results of this study, one would obviously expect that aggregate variables such as gross national expenditure would generally be forecast more accurately if components of this variable - such as inventories and investment in residential construction - were exogenous. However, in the usual forecasting exercise when exogenous variables are unknown, use of a model with less endogeneity necessarily requires more judgemental input than would be needed if the model contained more endogenous variables. Thus the extent to which the forecast is a pure model forecast is reduced. In order to determine the importance of the degree of endogeneity for ex post forecasting we also report on the ability of RDX2 to forecast when variables exogenous in AERIC have been exogenized[11] in RDX2.

We noted earlier that the three structural models used in this study were designed with somewhat different requirements in mind. The RDX2 model was designed for policy simulations, AERIC was designed for forecasting, and QFM was designed for both forecasting and policy simulations. Although these distinctions have a direct bearing on the results in this report they raise certain more general but important issues concerning the credibility of model forecasts and policy simulations. Most RDX2 policy simulations are carried out within a forecasting context, that is, the model is tuned to a judgmental forecast and then simulated under alternative policies. Thus the forward-looking policy simulations are directly linked to the ability of the model to simulate the future accurately, conditional upon some value for each exogenous variable. If the model cannot accurately simulate the effect of the levels of the exogenous variables on the levels of the endogenous variables, there may be reason to doubt the credibility of policy simulations that purport to simulate the effect of changes in the levels of exogenous variables on changes in the levels of endogenous variables.

Another difference among the models is the ease with which the data base can be updated. Considerable effort was expended during the construction of RDX2 in defining variables designed to capture economic structure with little regard to ease in updating the data base. For several variables - eg, the market value of private sector wealth, the standard industrial trade classification, and tax variables - the 1974 data used in RDX2 are estimated data and could be considerably revised one or two years hence. Although some forecasting error may be due to these data limitations, the main cause of such error is the inherent structure of RDX2. These problems do not appear to be as pronounced in the smaller AERIC and QFM models as they are in RDX2. The AERIC model, for example, was promptly reestimated with the revised 1971 constant-dollar national accounts and expenditure data.

Mixed, Autoregressive, Moving Average, and Reduced Form Models

In this report we use two naive forecasting models to judge the performance of each of the econometric models. The autoregressive integrated moving average (ARIMA) class economically naive model, described in Box and Jenkins [2] particularly in Part I and Part II, is used together with a monetarist reduced form model. Since the cost of constructing and maintaining an econometric model is many times greater than these costs for a set of naive models, it is desirable that an econometric model intended for forecasting perform at least as well as the less costly models. We emphasize that in ex post forecasting the ARIMA class model is at a considerable disadvantage because it incorporates no information on exogenous variables whereas the econometric models incorporate perfect knowledge. Nevertheless, the ARIMA class model is generally accepted as having a high standard of prediction performance and hence is preferable to a same-change or no-change naive model.

Each of the equations used for forecasting with the Box-Jenkins models is reproduced in Appendix A. We readily acknowledge that several of these equations contain some inadequacies, particularly with respect to seasonality. The kind of seasonality in many of the series is hard to handle with conventional Box-Jenkins procedures, although the technique of Hamilton and Watts [12] is a valuable aid. Also, several series were particularly difficult to model because of known structural change not apparent when the usual B-J tools are used. In some cases we varied the data period. In other cases we ignored structural change completely, preferring a complicated model that attempts to capture sample fluctuations rather than a model based upon a sample as small as five years of data. Consequently the best model was often found to be more complicated than one might otherwise have preferred. Apart from these considerations the criteria determining the choice of each model were consistent with those advccated by Box and Jenkins [2]. The estimation procedure is the nonlinear least squares algorithm of Marquardt [17]. Although this procedure is not equivalent to maximum likelihood estimation in the presence of a moving average error, see Kang [15], it is consistent with the estimation advocated by Box and Jenkins [2], for example, in Section 7.2. All data are seasonally unadjusted from various starting points but always to 4Q72.

The monetarist reduced form model we have used is reproduced in Appendix C. This equation determines the quarterly growth rate of seasonally adjusted nominal gross national expenditure (GNE). The exogenous variables of the equation are the quarterly growth rate of the narrowly defined money supply (M1), the quarterly changes in total exports and quarterly changes in an estimate of the full employment surplus for all levels of government. All explanatory variables enter the equation lagged and contemporaneously. Exports in particular have a large initial impact and, as a majcr component of aggregate demand, could account for part of the success of the equation in predicting nominal GNE.

3. PRINCIPAL RESULTS OF FORECASTS

Each of the four econometric models - RDX2, QFM, AERIC, and monetarist - was used to forecast ex post for the period 1Q73-4Q74 over three convenient time horizons: eight one-quarter forecasts, two four-quarter forecasts, and one eight-quarter forecast. In addition to the four basic econometric models we also consider a version of RDX2 that has been modified to resemble AERIC as closely as possible. The notation we use to describe this version of RDX2 is RDX2 ≡ AERIC. Because RDX2 is the most endogenized and AERIC is the least endogenized of the models, the forecasts are provided so as to give an indication of the importance of the degree of endogeniety of the various models.

Of the summary statistics that may be used to describe forecasting ability we have generally chosen two that to us seem the most relevant. See McNees [20] for an excellent discussion of such summary statistics. The first statistic is the root mean squared error as a percent (RMSE%) of the mean of the variable being predicted. It enjoys favour among model builders as a measure of ability to forecast the levels of a variable. The second statistic is the mean absolute error (MAE) of forecasts of percent changes at annual rates, that is, we calculate the mean absolute error associated with forecasts of the form:

$$[(Y_{t+j}-\hat{Y}_{t+j})/Y_t] * \frac{400}{j} \qquad j = 1, 4, 8 \qquad (1)$$

where $\hat{Y}_{t+j}$ is the forecast of variable Y, j periods ahead. It is crucial to realize that forecasts of this type should not be compared across different time horizons. The reason is simple.[12] If quarterly growth rates fluctuate, year-over-year ($j = 4$) changes, because they are averages of quarter-over-quarter ($j = 1$) changes, will be less variable over time and thus easier to predict. Therefore, when the MAE of forecasts of percent changes at annual rates is used, forecasting precision will improve as the time horizon lengthens. We employ this measure because it captures the probable growth path of some growing economic variable. For the change in inventories we present the MAE in predicting the swing in inventories; for the unemployment rate we present the MAE in predicting the level of the rate.

We show in Tables 1-16 the summary measures of selected variables[13] for AERIC, RDX2, QFM, Box-Jenkins, and for RDX2 $\equiv$ AERIC. The monetarist model is employed only in Table 1. A list of the endogenous variables studied in each model and their definitions is provided in Appendix B. All expenditure variables except nominal GNE are in real terms. The columns in the tables headed 1 step, 4 step, and 8 step represent summary statistics for eight one-quarter and two four-quarter simulations and one eight-quarter simulation of each model, respectively. The columns headed 1Q Ahead, 1 Year Ahead, and 2 Years Ahead contain summary statistics for j = 1, 4, and 8, respectively, in equation 1. In the remainder of this section we discuss the results presented in Tables 1-16 by major category.

Aggregate Demand

For nominal gross national expenditure, Table 1, the monetarist model performs best followed by AERIC, RDX2, B-J, and QFM on 1 step and 4 step simulations for both the levels and percent changes. During the eight-quarter forecasts QFM improves considerably on B-J for the levels and on both B-J and RDX2 for the percent changes. The monetarist, AERIC, and RDX2 models perform in roughly the same way on a one-period-ahead basis, but the RDX2 forecasts degenerate quickly relative to those of the other models as the forecast horizon lengthens. The QFM model performs poorly on the one-step-ahead levels, but the forecasts do not degenerate as the time horizon of the simulations lengthens. Exogenizing RDX2 to resemble AERIC produces a great improvement in the RDX2 forecasts. This improvement is sufficient to rank RDX2 near the monetarist model on either the levels or percent changes criterion for all forecasts. It also indicates that the ability of AERIC to predict nominal GNE may be due to the fact that important components of GNE are exogenous to AERIC - that is, inventories, housing starts, and virtually all government expenditure.

For real gross national expenditure, Table 2, the general ranking of the models is preserved, with B-J providing a stronger performance particularly in the eight-quarter simulation. The QFM model does well on a one-step-ahead basis but not in the longer-run forecasts. The monetarist model is absent in this and all remaining categories since it only predicts nominal GNE. Exogenizing RDX2 to resemble AERIC does not improve the performance of RDX2 in the longer simulations.

Real final domestic demand[14] is forecast better by AERIC than by RDX2 on a one-step-ahead basis. In longer simulations there is little to choose between the forecasting ability of the two models. Use of RDX2 $\equiv$ AERIC worsens the performance of RDX2 for both the levels and percent changes on a one-step-ahead basis but improves it on the levels in longer simulations. The QFM model is strong on a one-step-ahead basis but falls behind both RDX2 and AERIC on four- and eight-quarter predictions.

Table 1

Nominal Gross National Expenditure Forecast Errors

	Levels (Root Mean Squared Error as % of Mean)			Percent Change (at Annual Rates) (Mean Absolute Error)		
	1 Step	4 Step	8 Step	1Q Ahead	1 Year Ahead	2 Years Ahead
AERIC	1.23	1.74	1.68	4.42	1.76	2.01
RDX2	1.67	3.61	5.86	6.50	4.87	5.70
QFM	6.55	6.50	6.58	23.26	7.08	5.39
Box-Jenkins	2.97	4.34	10.01	9.70	5.35	8.25
Monetarist	1.02	.86	1.54	3.78	1.27	.40
RDX2 ≡ AERIC	1.27	1.19	1.02	3.92	1.42	1.04

Table 2

Real Gross National Expenditure Forecast Errors

	Levels (Root Mean Squared Error as % of Mean)			Percent Change (at Annual Rates) (Mean Absolute Error)		
	1 Step	4 Step	8 Step	1Q Ahead	1 Year Ahead	2 Years Ahead
AERIC	1.11	1.61	1.57	3.65	1.89	1.51
RDX2	2.24	2.69	1.69	8.30	2.50	1.21
QFM	1.52	3.80	5.40	5.28	5.55	4.90
Box-Jenkins	1.47	1.97	1.73	4.73	3.29	.23
RDX2 ≡ AERIC	1.94	2.18	1.81	6.00	2.35	1.97

Prices

In Table 3 the implicit price deflator for gross national expenditure is easily forecast most accurately by AERIC, with B-J, RDX2, and QFM following in that order. The forecasts of RDX2 ≡ AERIC improve considerably on those of RDX2, particularly for the longer simulations, but do not approach the performance of AERIC.

In Table 4 the strength of AERIC in predicting the GNE price deflator is not carried over to the consumer price index. The B-J model does well on the levels and the percent changes for one step ahead but its forecasts degenerate quickly as the horizon lengthens. We rank QFM predictions first on both the levels and the percent changes for one step ahead but they slip behind those of RDX2 for the longer simulations. The ranking for performance in estimating the consumer price index is generally RDX2, QFM, AERIC, B-J, with RDX2 ≡ AERIC improving the standing of RDX2.

Consumption

In the case of durables, Table 5, RDX2 does well on the levels over all time horizons but not as well on the percent changes. The QFM predictions miss the first quarter on both the levels and the percent changes but on the levels they improve as the time horizon lengthens. Box-Jenkins does quite the opposite on the levels whereas the AERIC predictions are steady on the levels. The AERIC and QFM estimates are similar on the percent changes, with AERIC estimates usually having a slight edge. The RDX2 model predicts uniformly better than does RDX2 ≡ AERIC. For consumption other than durables, Table 6, there is little to

Table 3

Implicit Price Deflator for Gross National Expenditure

	Levels (Root Mean Squared Error as % of Mean)			Percent Change (at Annual Rates) (Mean Absolute Error)		
	1 Step	4 Step	8 Step	1Q Ahead	1 Year Ahead	2 Years Ahead
AERIC	.43	.48	.61	1.58	.53	.11
RDX2	2.93	5.64	6.58	9.55	6.56	6.51
QFM	7.76	9.68	11.36	27.14	12.29	9.95
Box-Jenkins	2.83	4.92	9.53	8.42	5.94	8.25
RDX2 ≡ AERIC	1.18	1.59	1.45	2.94	1.45	1.23

Table 4

Consumer Price Index

	Levels (Root Mean Squared Error as % of Mean)			Percent Change (at Annual Rates) (Mean Absolute Error)		
	1 Step	4 Step	8 Step	1Q Ahead	1 Year Ahead	2 Years Ahead
AERIC	2.52	2.63	2.85	9.43	3.46	1.93
RDX2	1.55	1.81	1.92	5.20	2.23	2.00
QFM	.35	2.23	3.62	1.25	3.52	3.58
Box-Jenkins	.90	3.40	6.95	3.47	5.50	7.02
RDX2 ≡ AERIC	1.49	1.88	1.61	5.09	2.33	1.61

Table 5

Consumption of Durables

	Levels (Root Mean Squared Error as % of Mean)			Percent Change (at Annual Rates) (Mean Absolute Error)		
	1 Step	4 Step	8 Step	1Q Ahead	1 Year Ahead	2 Years Ahead
AERIC	5.64	5.14	4.87	21.11	4.49	4.53
RDX2	3.72	4.37	3.75	12.08	6.46	4.45
QFM	7.33	6.81	4.17	21.68	3.59	4.73
Box-Jenkins	5.47	6.91	8.10	13.59	9.02	3.85
RDX2 ≡ AERIC	4.05	4.78	4.49	13.68	7.02	5.77

Table 6

Consumption Excluding Durables

	Levels (Root Mean Squared Error as % of Mean)			Percent Change (at Annual Rates) (Mean Absolute Error)		
	1 Step	4 Step	8 Step	1Q Ahead	1 Year Ahead	2 Years Ahead
AERIC	.46	1.09	1.54	1.49	1.51	1.65
RDX2	.79	.66	1.12	2.59	.84	.88
QFM	.42	.81	2.02	1.37	1.08	1.87
Box-Jenkins	.75	.85	.96	2.35	1.28	.09
RDX2 ≡ AERIC	.80	.91	.46	2.62	1.22	.01

chose among the forecasts since the ranking depends on the time horizon chosen and the level or the percent change criterion. Again QFM does well on a one-step-ahead basis but degenerates quickly in longer forecasts.

For total consumption RDX2 performs about as well as AERIC, and both usually do better than QFM. In particular the performance of RDX2 on the levels improves when the time horizon lengthens whereas the performance of AERIC and QFM degenerate. On growth rates for two years ahead the AERIC forecast has a mean absolute error almost twenty times as great as that of RDX2, and the MEA of QFM is even larger than that of AERIC.

Employment

For total employed persons, Table 7, QFM makes a strong showing, the performance of RDX2 and AERIC is about the same, and B-J ranks fourth in all categories. The predictions of RDX2 $\equiv$ AERIC are uniformly poorer than those of RDX2. In Table 8 the unemployment rate is the first variable considered that is not dominated by trend; this is the first time B-J performs well, particularly on a one-step-ahead basis. The QFM forecasts are always poorest; those of RDX2 are better than those of the B-J model over longer time horizons. Again, RDX2 $\equiv$ AERIC predicts uniformly more poorly than does RDX2. The strength of the QFM forecasts of employment clearly does not carry over to the unemployment rate, since the QFM forecasts rank as uniformly the poorest in this category.

Table 7

Total Employed Persons

	Levels (Root Mean Squared Error as % of Mean)			Percent Change (at Annual Rates) (Mean Absolute Error)		
	1 Step	4 Step	8 Step	1Q Ahead	1 Year Ahead	2 Years Ahead
AERIC	.73	1.19	1.53	2.74	1.43	.98
RDX2	.78	1.14	2.39	3.00	1.30	1.46
QFM	.80	.83	.70	2.69	.68	.02
Box-Jenkins	.92	1.64	2.81	3.25	1.45	1.59
RDX2 ≡ AERIC	.91	1.42	2.53	3.32	1.83	1.64

Table 8

Unemployment Rate

	Levels (Root Mean Squared Error (as % of Mean)			Levels (Mean Absolute Error)		
	1 Step	4 Step	8 Step	1 Step	4 Step	8 Step
AERIC	12.60	20.57	26.19	.64	1.05	1.40
RDX2	11.91	9.30	12.07	.55	.40	.57
QFM	21.30	37.11	38.12	1.13	1.98	2.08
Box-Jenkins	5.27	12.74	18.34	.23	.56	.99
RDX2 ≡ AERIC	13.97	11.45	14.09	.70	.56	.68

Investment

A strong performance in forecasting investment in residential construction, Table 9, is given by AERIC, with QFM, B-J, and RDX2 following well behind. The RDX2 ≡ AERIC prediction improves considerably upon that of RDX2 because housing starts are now exogenous. For 1Q Ahead the mean absolute error in forecasting percent changes is three times as large for RDX2 ≡ AERIC as for AERIC. By contrast, the one-year-ahead forecast error for RDX2 ≡ AERIC is 50 percent larger than that for AERIC. For the two-year-ahead forecast RDX2 ≡ AERIC does considerably better than AERIC. The performance of QFM is disappointing, when one considers that housing starts are specified as exogenous.

For business investment in machinery and equipment, Table 10, AERIC again does well, with B-J, and particularly QFM, also forecasting well on a one-step-ahead basis. The RDX2 model turns in a dismal performance. Even RDX2 ≡ AERIC does not improve much on the basic RDX2 performance. The quality of the B-J predictions fades dramatically as the forecast horizon lengthens.

In contrast with business investment in machinery and equipment, RDX2 does well in predicting business investment in non-residential construction, Table 11, but is surpassed by the excellent performance of QFM. The forecasts of the AERIC model rank behind those of RDX2, with B-J forecasts after those of AERIC independently of the choice of time horizon, the level, or the percent change. The RDX2 ≡ AERIC model performs more poorly than does RDX2.

Table 9

Business Investment in Residential Construction

	Levels (Root Mean Squared Error as % of Mean)			Percent Change (at Annual Rates) (Mean Absolute Error)		
	1 Step	4 Step	8 Step	1Q Ahead	1 Year Ahead	2 Years Ahead
AERIC	1.90	1.90	1.95	6.63	2.03	1.41
RDX2	8.36	15.29	20.98	32.29	14.22	4.85
QFM	8.34	8.34	8.34	31.62	7.65	4.36
Box-Jenkins	7.91	12.80	16.09	27.06	17.71	1.29
RDX2 ≡ AERIC	5.54	5.58	5.58	20.09	3.24	.72

Table 10

Business Investment in Machinery and Equipment

	Levels (Root Mean Squared Error as % of Mean)			Percent Change (at Annual Rates) (Mean Absolute Error)		
	1 Step	4 Step	8 Step	1Q Ahead	1 Year Ahead	2 Years Ahead
AERIC	3.81	7.60	7.42	12.53	12.18	1.98
RDX2	13.76	15.13	16.42	56.46	17.34	8.14
QFM	2.70	5.70	8.32	9.99	7.37	3.63
Box-Jenkins	4.71	9.85	16.59	16.88	11.94	11.26
RDX2 ≡ AERIC	14.14	15.54	16.47	58.20	17.42	8.06

Table 11

Business Investment in Non-Residential Construction

	Levels (Root Mean Squared Error as % of Mean)			Percent Change (at Annual Rates) (Mean Absolute Error)		
	1 Step	4 Step	8 Step	1Q Ahead	1 Year Ahead	2 Years Ahead
AERIC	3.00	5.02	8.66	9.74	7.45	7.49
RDX2	2.32	4.14	5.24	7.38	4.98	3.43
QFM	2.59	2.34	2.40	8.16	2.56	.40
Box-Jenkins	3.35	6.69	11.98	11.63	7.51	9.22
RDX2 ≡ AERIC	2.32	4.65	6.54	7.38	6.35	5.35

Table 12

Change in Nonfarm Business Inventories
(Millions of 1961 dollars at quarterly rates)

	Levels (Root Mean Squared Error)			Change Mean Absolute Error		
	1 Step	4 Step	8 Step	1Q Ahead	1 Year Ahead	2 Years Ahead
RDX2	254.75	303.57	359.27	174.35	205.63	218.91
QFM	161.94	183.07	186.96	140.18	139.10	125.66
Box-Jenkins	185.70	206.65	231.18	163.75	127.02	144.18

For the sum of investment in machinery and equipment and non-residential construction, the performance of RDX2 in non-residential construction is not sufficient to offset its dismal showing for machinery and equipment on a one-step-ahead basis. Over the longer simulations there is little to chose between the forecasts of RDX2 or AERIC, although those of QFM are definitely best on the levels. The use of seasonally adjusted data may be a factor in the superior forecasting ability of QFM and AERIC on a one-step-ahead basis, although the AERIC prediction of total investment contains some offsetting errors as well. When residential construction is also considered, the forecasting errors of RDX2 largely offset each other, those of AERIC are mildly offsetting, and those of QFM are reinforcing. As a result, for business investment in residential and non-residential construction and in machinery and equipment, there is little to choose between the performance of AERIC and QFM but that of RDX2 is considerably better in four- and eight-quarter forecasts.

For the change in inventories we present the root mean squared error as an indicator of the ability of the models to predict the levels. On this criterion the models consistently rank QFM, B-J, and RDX2. We also present the mean absolute error for the one-quarter change in inventories, that is, a measure of the ability of the models to predict the one-quarter swing in inventories. This criterion is computed for each of eight one-quarter simulations, two four-quarter simulations, and one eight-quarter simulation. The general ranking of the models is again QFM, B-J, and RDX2. Inventories are exogenous to AERIC.

International Trade

The ranking of the performance of the models in forecasting total exports of goods and services, Table 13, is almost uniformly RDX2, AERIC, and B-J. The RDX2 ≡ AERIC model does even better than RDX2, principally owing to the exogeneity of the exchange rate. Real exports are exogenous in QFM. However, the ranking of the models is exactly opposite when forecasts for imports of goods and services, Table 14, are considered. The B-J model provides its strongest performance in this category, out-predicting the other models on both criteria over all time horizons. Usually the AERIC forecast comes after that of B-J, followed by the RDX2 and then the QFM forecast. The RDX2 ≡ AERIC model strengthens the performance of RDX2 to such an extent that the predictions of the exogenized version of RDX2 rate ahead of the AERIC predictions but behind those of B-J except for the one-quarter-ahead levels and the one-quarter-ahead percent changes.

Financial

The ranking of the forecasting ability of models for both chartered bank business and miscellaneous loans, Table 15, and chartered bank personal loans, Table 16, is virtually uniform: AERIC, B-J, RDX2, and QFM. The RDX2 ≡ AERIC model almost always predicts more poorly than does RDX2. Part of the strong performance of AERIC in these categories may be attributable to the fact that the financial block of AERIC has been estimated through the period 1Q73-4Q73.

Table 13

Exports of Goods and Services

	Levels (Root Mean Squared Error as % of Mean)			Percent Change (at Annual Rates) (Mean Absolute Error)		
	1 Step	4 Step	8 Step	1Q Ahead	1 Year Ahead	2 Years Ahead
AERIC	3.13	4.28	4.92	11.06	5.37	4.21
RDX2	2.77	3.67	4.73	8.40	3.19	3.30
Box-Jenkins	4.46	4.60	7.12	14.87	5.23	6.41
RDX2 ≡ AERIC	2.68	2.59	2.61	8.13	.94	.11

Table 14

Imports of Goods and Services

	Levels (Root Mean Squared Error as % of Mean)			Percent Change (at Annual Rates) (Mean Absolute Error)		
	1 Step	4 Step	8 Step	1Q Ahead	1 Year Ahead	2 Years Ahead
AERIC	4.69	5.11	7.00	15.61	7.67	6.55
RDX2	4.92	7.10	6.94	16.44	7.70	7.65
QFM	5.50	7.26	8.68	18.81	9.68	7.14
Box-Jenkins	4.22	3.04	3.00	15.42	2.93	1.73
RDX2 ≡ AERIC	4.15	4.17	4.29	15.31	3.73	3.15

Table 15

Chartered Bank Business and Miscellaneous General Loans

	Levels (Root Mean Squared Error as % of Mean)			Percent Change (at Annual Rates) (Mean Absolute Error)		
	1 Step	4 Step	8 Step	1Q Ahead	1 Year Ahead	2 Years Ahead
AERIC	2.01	3.83	4.23	7.02	5.30	3.25
RDX2	3.14	6.74	10.81	11.81	10.67	10.31
QFM	12.94	22.66	23.44	48.35	34.30	23.10
Box-Jenkins	1.87	5.58	13.21	6.71	9.61	12.91
RDX2 ≡ AERIC	3.25	6.81	10.77	12.93	10.77	9.53

Table 16

Chartered Bank Personal Loans

	Levels (Root Mean Squared Error as % of Mean)			Percent Change (at Annual Rates) (Mean Absolute Error)		
	1 Step	4 Step	8 Step	1Q Ahead	1 Year Ahead	2 Years Ahead
AERIC	.91	1.35	2.05	3.49	1.06	2.28
RDX2	1.99	6.78	9.95	7.51	9.09	8.55
QFM	6.76	11.26	7.12	26.49	13.75	3.91
Box-Jenkins	1.45	3.26	4.43	5.30	3.91	3.44
RDX2 ≡ AERIC	2.02	6.97	10.17	7.56	9.49	8.47

General

In Tables 17 and 18 we provide a statement summarizing[15] in a particular fashion the results recorded in Tables 1 tc 16. Table 17 is a statement of the number of times each model scored the lowest and highest RMSE% - or RMSE for changes in inventories. In Table 18 we do the same thing for mean absolute error in forecasting the percent change at annual rates - or MAE on the level for the unemployment rate and the swing in inventories. We treat all variables in the tables as being cf equal importance[16] and do not recognize definitional dependence between some of the variables - for example, the ratio of nominal to real GNE equals the GNE price deflator. The monetarist model is not included in these summary tables because its superiority is recognized only in predicting nominal GNE. Neither do we include[17] RDX2 $\equiv$ AERIC, because, although the performance of this model is of scme interest, it is not strategically important in our study.

On the basis of the admittedly arbitrary scoring system in Tables 17 and 18, the performance of AERIC clearly ranks the highest followed by the performance of RDX2, B-J, and QFM more or less in that order. The result is reasonably stable across the various time horizons and the choice of the level or the percent change is the relevant criterion. As one might expect the Box-Jenkins models make their best showing on a one-step-ahead basis, since, in the longer forecasts, known values for the exogenous variables considerably aid the econometric models.

It is possible to conclude from these results that the model incorporating the greatest amount of correct information - ie,

Table 17

Summary of Forecasting Results on Levels

	1 Step Ahead # times RMSE % was;		4 Step Ahead # times RMSE % was;		8 Step Ahead # times RMSE % was;	
	lowest	highest	lowest	highest	lowest	highest
AERIC	6	1	6	1	7	0
RDX2	3	5	5	3	4	3
QFM	4	7	4	7	3	6
B-J	3	3	1	5	2	7

Table 18

Summary of Forecasting Results on Changes

	1 Quarter Ahead # times MAE was;		1 Year Ahead # times MAE was;		2 Years Ahead # times MAE was;	
	lowest	highest	lowest	highest	lowest	highest
AERIC	5	1	6	2	6	0
RDX2	3	5	4	2	2	4
QFM	5	7	4	7	3	6
B-J	3	3	2	5	5	6

AERIC, the most highly exogenized model - will, on average, forecast best. Equivalently, the model that incorporates the least amount of correct information - ie, Box-Jenkins with no exogenous variables - will tend to forecast most poorly. Three points counter this argument. First, the performance of QFM should have ranked well ahead of that of RDX2, and second, the performance of RDX2 ≡ AERIC should have come much closer to matching that of AERIC. Third, though economically naive, the Box-Jenkins models are not statistically naive and do represent a powerful forecasting device, particularly on a one-step-ahead basis.

4. CONCLUSIONS

The ex post forecasting exercise has revealed considerable differences among the three structural econometric models in their ability to predict a selected group of important economic variables. Not surprisingly no one model predominates. The econometric models generally perform reasonably well vis-à-vis the economically naive alternative used here - univariate Box-Jenkins models. The two notable exceptions in favour of the Box-Jenkins models are its forecasts of total imports of goods and services, and, on a one-step-ahead basis, the unemployment rate. As expected the strength of the Box-Jenkins models generally lies in a one-step-ahead forecast.

Without wishing to pass judgement on the ranking of the various models, a simple summary indicating the strengths and weaknesses of each model may be warranted based solely upon ex post forecasting ability during the period 1Q73-4Q74. The results, of course, do not necessarily carry over to subsequent versions of any of the models or to forecasts over different time horizons.

The monetarist model predicts only nominal GNE but does this well. The strength of AERIC lies in forecasting nominal GNE, the price deflator for GNE, and investment in machinery and equipment. Its weaknesses are in forecasting the consumer price index, consumption of durables, and the unemployment rate.

For the consumer price index, consumption of durables, total consumption, investment in non-residential construction, and exports RDX2 predicts well. The weakest RDX2 forecasts are those for nominal GNE, the price deflator for GNE, investment in machinery and equipment, and imports. When RDX2 has several sectors and equations exogenized so that it resembles AERIC as closely as possible, the RDX2 weaknesses in predicting nominal GNE and imports disappear but the forecasts of investment in machinery and equipment remain very weak.

The QFM model predicts well for investment in machinery and equipment and non-residential construction, the change in inventories, total employment, and the consumer price index on a one-step-ahead basis. Its weakest predictions include those for nominal GNE, the unemployment rate, imports, and bank loans.

We hope that this study will provide the users and builders of AERIC, RDX2, and QFM with some idea of the forecasting strengths and weaknesses of these models. We also hope to have given the reader who has only a marginal interest in forecasting or model building some appreciation of the potential of each of the three models. To achieve this potential is the responsibility of the model builders themselves, so that one must look to them for progress based upon any studies that may follow this report.

FOOTNOTES

1 It was possible to produce this paper because of a fortuitous near-coincidence of terminal estimation points for these three publicly available quarterly models. Two of the three have most of the equations estimated to 4Q72 and the third is estimated to 4Q73. Both the naive models used are estimated to 4Q72.

2 In an U.S. context and using actual values for exogenous variables, Nelson [22] considered the intra-sample and extra-sample prediction performance of the FMP model relative to that of univariate Box-Jenkins models. The interested reader who wishes to compare intra-sample to extra-sample statistics for certain variables may, for RDX2 at least, consult the documentation of the last two versions of RDX2 [1] and [13] where these statistics are routinely presented. Cooper and Nelson [6] compare the St. Louis and FMP models with the Box-Jenkins models when a subset of the exogenous variables in each econometric model has to be forecast. Christ [5] compares several U.S. econometric models on various criteria, one of which is post-sample prediction performance with some results for actual exogenous variables and other results for adjusted forecasts. Fair [9] compares three types of forecasts from his model with the forecasts in ASA-NBER Survey of Regular Forecasters.

3 To the reader unfamiliar with forecasting models it may seem that the eight quarters from 1Q73-4Q74 is a very short period over which to draw conclusions regarding the ability of the models to forecast. This is correct. Unfortunately, it is highly unlikely that a longer period will ever be available. Forecasting models are being updated more and more regularly so that it will soon be unusual to find a good forecasting model in current use that has its estimation period more than one year out of date. Indeed, it is the goal of some model builders to update their estimates quarterly.

4 McNees [20] distinguishes three reasons for studying forecasts: "prescriptive, descriptive, and comparative". By studying the forecasting errors made by a particular model, its builders can isolate weakness in the model. The present study falls into the comparative category, although McNees argues convincingly that ex post forecasting is not the best method to use if one seeks to evaluate forecasters - not models.

5 An obvious impediment is cost. It is unlikely that many institutions would be prepared to bear the implicit and explicit cost of subscribing to two or more modelling agencies, the results of composite predictors notwithstanding. See, for example, Cooper and Nelson [6] or Granger and Newbold [10].

6 The lack of standardization among econometric models is McNees's main argument in [20] against ex post forecasting for comparative purposes.

7 The simulation results for AERIC, QFM, and the monetarist model are compared with the national accounts data published by Statistics Canada in [3]. The data for the years 1971 and 1974 were revised in [3] and the constant-dollar data are based on the 1971 dollar. It should be noted that QFM has rebased the constant-dollar data in [3] back to the 1961 dollar. For RDX2 and the Box-Jenkins models we compare the simulation results with the national accounts data available before the revisions and change of base noted above were made. These data are published by Statistics Canada in [4]. The constant-dollar data in this publication are based on the 1961 dollar.

8 Copies of RDX2 are available from the Bank of Canada for the price of the necessary magnetic tapes.

9 We have attempted to measure the importance of using seasonally adjusted data in one model and unadjusted data in another. The forecasts for each variable taken from RDX2 were deflated by the appropriate seasonal factor and compared with seasonally adjusted actuals. The forecast errors for both the levels and the percent changes on these seasonally adjusted forecasts were close to the forecast errors when the raw data were used except in the case of

inventories. Even for inventories the ordering of the models was the same although the differences were less pronounced.

10 In the reestimation to late 1975, with data expressed in 1971 dollars, are incorporated significant revisions to the tax sector, endogenous housing starts, and an equation for the consumption of motor vehicles. Because, in estimating this later version of the model, data for both 1973 and 1974 are included, the results could obviously not be compared with those of any of the other models studied here. Thus rather than exclude QFM from our study, we include QFM with the qualifications noted.

11 Specifically we have exogenized the following RDX2 sectors and equations: housing starts and stock (2.2), (2.3), (2.6); inventories (3.3); government expenditure (Sectors 11, 12, 13, 14); the exchange rate and international capital flows (Sectors 19; 20, 21); and the following price deflators: business investment in machinery and equipment, and non-residential construction (7.5) and (7.7); nonfarm business inventories (7.8); residential and non-residential construction materials (7.9), (7.10); export and import prices (7.11), (7.13), (7.24) to (7.28); current nonwage government expenditure (7.14); government investment in non-residential construction and in machinery and equipment (7.15) and (7.16); gross rent (7.17); gross private business product (7.20); imputed consumer

services from consumer durables (7.21); and the RDX2 expected rate of change in the consumer price index.

12 We are indebted to Kevin Clinton, Pierre Duguay, and Patrick Grady for making it so.

13 The variables are concentrated on the expenditure side of the national income and expenditure accounts because that seems to be the strength of each of the models considered here, residual categories being modelled implicitly or explicitly on the income side. We have, however, some results for corporate profits and personal disposable income. For the former the ordering of forecasts is AERIC, QFM, and RDX2, with RDX2 $\equiv$ AERIC improving on QFM. A typical year-over-year growth-rate forecast error is in the 10-20 percent range. For personal disposable income the ordering of forecasts is RDX2, AERIC, QFM, with RDX2 $\equiv$ AERIC even better than RDX2. The year-over-year growth rate forecast errors are 3-6 percent.

14 No tables are presented for the results asserted for final domestic demand, or for total consumption and total business fixed investment. However, results for these components are given in the text in an attempt to provide information on the importance of the degree of aggregation.

15 Our results in Tables 1-16 do not incorporate the substantial revisions to the national income and expenditure

accounts released by Statistics Canada in June 1976. We have attempted to estimate the effect of these revisions by recomputing the results in Tables 1-16, using the previous predictions of each econometric model but the new national income and expenditure accounts figures as actuals. Generally, the predictions were much worse, the notable exceptions being real GNE, exports, and business fixed investment in machinery and equipment. Each model, except AERIC, predicts revised real GNE and exports better than it did before the revisions. Also, QFM improves when predicting the revised totals of business investment in machinery and equipment. In relative terms the only substantial changes occur for real and nominal GNE and consumption excluding durables. For predicting nominal GNE the AERIC model is now clearly the best. The QFM model is best for real GNE on a one-step-ahead basis but RDX2 is best overall followed by AERIC. For predicting consumption excluding durables AERIC emerges as the best model. The general pattern shown in Tables 17 and 18 is unchanged.

[16] The reader is, of course, free to construct a similar table weighting the variables in any fashion thought to be more suitable.

[17] In every category in Tables 17 and 18 RDX2 ≡ AERIC usually scores at least one more 'lowest' entry than does RDX2, and only once did RDX2 ≡ AERIC score one more 'highest' entry. However, the performance of RDX2 ≡ AERIC still does not generally rival that of AERIC.

APPENDIX A

SIXTEEN BOX-JENKINS MODELS

Real Gross National Expenditure 1950-1972

$$(1-\phi_1 B)(1-B^4)\ UGNE_t = \theta_0 + (1-\theta_1 B)(1-\Theta_4 B^4)a_t$$

$\phi_1 = .951$ (22.98) $\theta_0 = 33.097$ (1.51) $\theta_1 = .351$ (3.01) $\Theta_4 = .610$ (6.10)

$Q = 45.72$ $\hat{\sigma}^2 = 61077.72$

Nominal Gross National Expenditure 1950-1972

$$(1-\Phi_4 B^4-\Phi_8 B^8)(1-B)(1-B^4)YGNE_t^{.75} = (1-\theta_1 B-\theta_2 B^2)a_t$$

$\Phi_4 = -.486$ (4.76) $\Phi_8 = -.409$ (3.87) $\theta_1 = .220$ (2.03) $\theta_2 = -.177$ (1.65)

$Q = 20.75$ $\hat{\sigma}^2 = 272.34$

Implicit GNE Deflator 1956-1972

$$(1-\Phi_4 B^4)(1-B)PGNE_t = \theta_0 + (1-\theta_1 B-\theta_2 B^2)a_t$$

$\Phi_4 = .605$ (5.72) $\theta_0 = .004$ (2.84) $\theta_1 = .332$ (2.78) $\theta_2 = -.335$ (1856.69)

$Q = 50.03$ $\hat{\sigma}^2 = .000061$

Consumer Price Index 1952-1972

$$(1-\phi_1 B-\phi_2 B^2)(1-B^4)\ PCPI_t = \theta_0 + (1-\Theta_4 B^4)a_t$$

$\phi_1 = 1.509$ (44.84) $\phi_2 = -.526$ (14.07) $\phi_0 = .001$ (311.93) $\Theta_4 = .702$ (9.62)

$Q = 47.73$ $\hat{\sigma}^2 = .000039$

Consumption of Non-Durables 1947-1972

$$(1-\phi_1 B-\phi_2 B^2-\phi_3 B^3)(1-B^4)\ \ln CND_t = \theta_0 + a_t$$

$\phi_1 = .521$ (5.29) $\phi_2 = .212$ (1.94) $\phi_3 = .186$ (2.04) $\theta_0 = .020$ (5.48)

$Q = 43.80$ $\hat{\sigma}^2 = .000201$

Consumption of Durables 1952-1972

$$(1-\phi_3 B^3-\phi_4 B^4)(1-B^4)\ \ln CD_t = \theta_0 + (1-\theta_1 B-\theta_2 B^2)a_t$$

$\phi_3 = .406$ (3.78) $\phi_4 = -.339$ (3.73) $\theta_0 = .066$ (4.12) $\theta_1 = -.624$ (5.52) $\theta_2 = -.394$ (3.20)

$Q = 54.99$ $\hat{\sigma}^2 = .003035$

Business Investment in Machinery and Equipment 1947-1972

$$(1-\phi_1 B-\phi_4 B^4-\phi_5 B^5)(1-B^4)\ \ln IME_t = \theta_0 + a_t$$

$\phi_1 = .935$ (14.40) $\phi_4 = -.442$ (4.16) $\phi_5 = .165$ (1.63) $\theta_0 = .014$ (1.97)

$Q = 47.92$ $\hat{\sigma}^2 = .003702$

Business Investment in Non-Residential Construction 1947-1972

$$(1-\phi_1 B-\phi_2 B^2)(1-B^4)\ INRC_t = \theta_0 + a_t$$

$\phi_1 = 1.360$ (15.49) $\phi_2 = -.523$ (5.93) $\theta_0 = 4.546$ (1.62)

$Q = 45.64$ $\hat{\sigma}^2 = 602.18$

Business Investment in Residential Construction 1947-1972

$$(1-\phi_1 B-\phi_2 B^2)(1-B^4)IRC_t^{.5} = \theta_0 + (1-\Theta_4 B^4)a_t$$

$\phi_1 = 1.062$ (10.67) $\phi_2 = -.273$ (2.61) $\theta_0 = .096$ (2.38) $\Theta_4 = .655$ (7.45)

$Q = 45.78$ $\hat{\sigma}^2 = .721$

Business Investment in Non-Farm Inventories 1947-1972

$$(1-\phi_1 B)(1-B)\ IIB_t = \theta_0 + (1-\Theta_4 B^4)a_t$$

$\phi_1 = .260$ (2.64) $\theta_0 = .566$ (.20) $\Theta_4 = .747$ (11.75)

$Q = 36.70$ $\hat{\sigma}^2 = 18406.37$

Total Exports 1952-1972

$$(1-\Phi_4 B^4-\Phi_8 B^8)(1-\phi_1 B)(1-B)(1-B^4)X_t^{.5} = a_t$$

$\Phi_4 = -.540$ (5.09) $\Phi_8 = -.380$ (3.66) $\phi_1 = -.486$ (4.80)

$Q = 53.75$ $\hat{\sigma}^2 = .986$

Total Imports 1952-1972

$$(1-B)(1-B^4)\ \ln M_t = \theta_0 + (1-\Theta_4 B^4)a_t$$

$\theta_0 = .001$ (.31) $\Theta_4 = .694$ (12.46)

$Q = 35.37$ $\hat{\sigma}^2 = .001587$

<u>Unemployment Rate</u> 1953-1972

$$(1-\phi_1 B-\phi_2 B^2)(1-B^4)\ RNU_t = (1-\Theta_3 B^3)a_t$$

$\phi_1 = 1.275$ (13.58) $\phi_2 = -.603$ (6.58) $\Theta_3 = -.450$ (5.08)

$Q = 42.80$ $\hat{\sigma}^2 = .2417$

<u>Total Employed Persons</u> 1953-1972

$$(1-\phi_1 B)(1-B^4)\ \ln NE_t = \theta_0 + (1-\Theta_4 B^4)a_t$$

$\phi_1 = .912$ (25.23) $\theta_0 = .003$ (2.69) $\Theta_4 = .580$ (5.77)

$Q = 24.11$ $\hat{\sigma}^2 = .000054$

<u>Chartered Bank Personal Loans</u> 1950-1972

$$(1-\Phi_4 B^4)(1-\phi_1 B)(1-B)(1-B^4)ABLP_t^{.5} = \theta_0 + (1-\Theta_4)a_t$$

$\Phi_4 = -.353$ (2.80) $\phi_1 = .624$ (884.19) $\theta_0 = .040$ (1.80) $\Theta_4 = .669$ (7.14)

$Q = 43.99$ $\hat{\sigma}^2 = .291$

<u>Chartered Bank Business Loans</u> 1950-1972

$$(1-\Phi_4 B^4)(1-\phi_1 B)(1-B)(1-B^4)ABLB_t^{.5} = \theta_0 + (1-\Theta_4 B^4)a_t$$

$\Phi_4 = -.147$ (1.14) $\phi_1 = .398$ (3.87) $\theta_0 = .040$ (1.38) $\Theta_4 = .796$ (11.33)

$Q = 45.85$ $\hat{\sigma}^2 = .918$

APPENDIX B

DEFINITION OF VARIABLES

	AERIC	RDX2	QFM
Nominal Gross National Expenditure	NGNP	YGNE	GNE$
Real Gross National Expenditure	NGNP7	UGNE	GNE
Gross National Expenditure Price Deflator	NPGNP	PGNE	PGNE
Consumer Price Index	PCPI	PCPI	CPI
Consumption of Durables	NCD7	CDO+CMV	CD
Consumption other than Durables	NC7 -NCD7	CS+CNDSD	C-CD
Total Employed Persons	LEM	NE	E
Unemployment Rate	LUR	RNU	URATE
Business Investment in Residential Construction	NICRB7	IRC	IRC
Business Investment in Machinery and Equipment	NIMEB7	IME	IME
Business Investment in Nonresidential Construction	NINRB7	INRC	INRC
Change in Business Inventories	n/a	IIB	INV
Exports of Goods and Services	NX7	X	n/a
Imports of Goods and Services	NM7	M	M
Chartered Bank Business Loans	FABBGL	ABLB	TBLA9
Chartered Bank Personal Loans	FABPGL	ABLP	CCL9

APPENDIX C

THE MONETARIST REDUCED FORM MODEL

The reduced form equation that we used was estimated by a colleague of ours, Pierre Duguay.

$$J1P(GNE) = \underset{(3.06)}{.76} + \underset{(2.20)}{1.32}\ Z1[J1P(M1)] - \underset{(1.77)}{.97}\ Z2[J1P(M1)]$$

$$+ \underset{(4.03)}{1.55}\ \frac{Z2[J1D(X)]}{.01\ J1L(GNE)} - \underset{(2.93)}{1.24}\ \frac{Z2[J1D(FES)]}{.01\ J1L(GNE)}$$

$\bar{R}^2 = .37$ DW = 2.30 SEE = .99 sample = 1956Q4-1972Q4

where

J1P is the first quarter percentage change operator
J1D is the first difference operator
J1L is the one quarter lag operator
Zi is an Almon variable of degree i (all lags are six quarters long constrained to zero at lag 6)
GNE is nominal gross national expenditure
M1 is currency and demand deposits
X is total export receipts
FES is the full employment surplus for all levels of government.

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