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A COMPARISON OF ALTERNATIVE METHODS OF MONETARY AGGREGATION: SOME PRELIMINARY EVIDENCE

Jon P. Cockerline
and
John D. Murray

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ABSTRACT

The monetary aggregates presently computed have a number of restrictive structural characteristics which could limit their usefulness as economic indicators and policy targets, when these aggregates are used to measure the volume of liquidity or "money" in the economy. Their simple linear structure implicitly treats all the included monetary components as perfect substitutes, and imposes an additive unit-weighted utility function on all asset holders. Additionally, the traditional aggregates are very selective and exclude non-bank monetary assets.

In this paper we construct and test a number of alternative, "superlative", monetary aggregates. These new aggregates have a more flexible functional form and give explicit recognition to the heterogeneous nature of various financial instruments by assigning a unique price weight to each monetary component.

Superlative monetary aggregates are compared with conventional (summation) monetary aggregates in three critical areas: information content, causality, and stability. While superlative aggregates tend to follow more consistent time paths than summation aggregates, their overall performance is very mixed. Superlative monetary aggregates appear to contain less information on contemporaneous and future income levels than their summation counterparts, and fail to provide any new insights on money-income causality. Equations explaining their behaviour do, however, display greater parameter stability at broad levels of aggregation, providing weak evidence in favour of superlative aggregation. This is not the case at narrower levels of aggregation.

All of these findings are generally consistent with the results of similar tests in the United States and, taken together, suggest that narrow conventional aggregates are at least as good as and perhaps better than any existing alternatives. Though superlative aggregation in general has much to recommend it, the superlative monetary aggregates appear to be affected by a number of practical and theoretical problems which may limit their viability as alternatives to the conventional money measures.

RÉSUMÉ

Les agrégats monétaires traditionnels comportent plusieurs caractéristiques structurelles qui pourraient restreindre leur utilité comme indicateurs économiques et comme cibles de la politique monétaire, lorsqu'ils sont utilisés pour mesurer le volume de liquidité ou de "monnaie" dont dispose l'économie. Leur structure linéaire simple fait que toutes leurs composantes sont traitées comme si elles pouvaient se substituer parfaitement les unes aux autres et elle impute aux détenteurs d'actifs une fonction d'utilité linéaire et d'égale pondération. Ils sont de plus très sélectifs, car ils excluent les avoirs monétaires hors banques. Dans le présent rapport, nous construisons et évaluons un certain nombre d'agrégats monétaires dits "superlatifs", susceptibles de remplacer les agrégats traditionnels. Ceux-ci sont dotés de formes plus souples et font ressortir le caractère hétérogène de leurs différentes composantes en donnant à chacune une pondération basée sur les taux d'intérêt.

Les agrégats "superlatifs" sont comparés aux agrégats conventionnels (obtenus par addition) sur trois plans fondamentaux: le plan de l'information, celui de la causalité et celui de la stabilité. Même si les agrégats "superlatifs" ont tendance à suivre des profils de croissance plus cohérents que ne le font les agrégats traditionnels, leur performance globale est très inégale. Les agrégats superlatifs semble fournir moins de renseignements sur les niveaux actuels et futurs du revenu que les agrégats additionnels; de plus, ils ne jettent

aucune lumière nouvelle sur les rapports de causalité qui existent entre la monnaie et le revenu. Toutefois, les équations expliquant leur comportement affichent une plus grande stabilité paramétrique, du moins en ce qui concerne les agrégats au sens large, ce qui atteste en quelque sorte de la validité des agrégats "superlatifs". Ce n'est cependant pas le cas pour les agrégats au sens étroit.

Tous les renseignements que nous avons obtenus avec les agrégats "superlatifs" concordent généralement aux tests analogues effectués aux États-Unis. Ils donnent à penser que les agrégats traditionnels au sens étroit sont au moins d'aussi bons sinon de meilleurs indicateurs que les autres types d'agrégats. Même si, en général, il y a beaucoup d'arguments en faveur des agrégats "superlatifs", ceux-ci semblent présenter plusieurs difficultés d'ordre pratique et théorique, qui en font des solutions de remplacement peu fiables des instruments de mesures traditionnels de la masse monétaire.

INTRODUCTION

Monetary policy is widely regarded as an important instrument for the attainment of various macroeconomic objectives, yet very little attention is paid to the construction of the aggregates used in implementing policy. Researchers and policymakers hold very strong, and often conflicting, views on the appropriate level of monetary aggregation (i.e., which assets should be included), but they seldom question the restrictive manner in which monetary assets are combined in the aggregates. In other words, they focus almost exclusively on the composition of the aggregates and overlook any problems that might be caused by their simple linear structure. It is our intention to address both issues in a series of tests that compare conventional monetary aggregates with new, less restrictive measures of money.

The summation monetary aggregates presently employed by most central banks can be thought of as monetary index numbers with the following peculiarities: 1) they implicitly impose perfect substitutability on all their component assets by assigning each an equal and constant dollar weight; 2) they assume the underlying aggregator functions of all economic agents have the same simple linear structure;[1] and 3) they exclude many liquid assets that are close substitutes for the conventional monetary components. Most financial assets, however, possess a mixture of liquidity and store-of-value characteristics. Although the

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[1] Aggregator function is a general term used to denote both the utility functions of households and the production functions of firms.

proportion varies from asset to asset, few instruments serve strictly as a store of value without in some way augmenting the stock of liquidity or "moneyness" in the financial system. The summation aggregates exclude many assets, such as near-bank deposits, as well as money market instruments, and they ignore the heterogeneous nature of those that are included. It is for these reasons that conventional summation monetary aggregates have been labelled "disreputable" and "inferior".[2]

In their place, several authors have recently recommended a class of alternative indexes termed "superlative" in the index number literature.[3] These indexes assign unique share weights to the component assets, recognizing their limited substitutability and diverse nature. They also provide a functional form which is exact[4] for <u>any</u> flexible aggregator function. While these "superlative" indexes are believed to offer theoretical advantages over the more restrictive summation indexes, it remains to be seen if they provide any practical advantage in terms of superior performance.

In this paper we construct and test a number of "superlative" monetary aggregates, and compare them with summation monetary aggregates in three critical areas: information content, causality, and stability. The results show that the overall performance of the superlative aggregates is very mixed, though they tend to follow more consistent time paths than

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[2] See Barnett, Spindt, and Offenbacher (1981).

[3] Diewert (1976).

[4] There exists a one-to-one relationship between the functional form of an index and its implicit aggregator function. Quantity index $Q(p^0, p^r; x^0, x^r)$ is exact for an aggregator with functional form f if $f(x^r)/f(x^0) = Q(p^0, p^r; x^0, x^r)$, r = 1, 2, ..., N, given the base period normalization $f(x^0) = 1$.

the summation aggregates. It is difficult therefore to reach any strong conclusions as to their relative merits. Superlative monetary aggregates appear to contain less information on contemporaneous and future income levels than their summation counterparts. They do, however, display greater stability at broad levels of aggregation in a series of money demand regressions. Since erratic time series behaviour can be caused by invalid aggregation, the stability tests provide weak evidence that broad superlative aggregates are superior to broad summation aggregates as measures of money in simple demand specifications. Superlative and summation aggregates yield similar results in tests of money-income causality. All of these findings are generally consistent with the findings of similar tests in the United States.[5]

The paper is divided into five sections. The first two sections discuss the concept of a superlative monetary aggregate and describe the data used in its construction. The third section presents five alternative indexes (three superlative and two non-superlative), and documents their disparate behaviour at different levels of aggregation. The empirical tests for information content, causality, and stability are reported in Section 4. The final section summarizes the results and analyzes the reasons for the mixed success of the superlative aggregates.

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[5] See Barnett, Spindt, and Offenbacher (1981).

1 SUPERLATIVE MONETARY AGGREGATES

Our superlative monetary aggregates differ from traditional summation aggregates (M1, M1B, M2, ...) in three important respects. First, they are more inclusive. The existing "M" series contain only currency and certain bank deposit components. The contribution of near-bank deposits and money market instruments to the volume of liquidity or moneyness in the financial system is excluded.[6] Secondly, superlative aggregates attempt to measure the flow of money services in the financial system as distinct from the existing money stock. To this end, each component quantity is weighted by a unique rental price (π_i), reflecting its relative "moneyness". Total expenditure on money services in any period equals the product of each component's price and quantity, summed over all components. Thirdly, superlative indexes are exact for flexible functional forms and thereby avoid the overly restrictive assumptions embedded in the summation aggregates. The latter are exact only for a simple linear aggregator in which all the included components are treated as perfect substitutes and assigned identical unit weights. Since households and firms are unlikely to have identical linear unit-weighted

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[6] Though it is possible to construct alternative superlative aggregates for the same restricted set of components as are presently included in M1, M1B, ..., etc., such an approach is potentially inconsistent with the theory underlying the concept of a superlative index. Ideally, the appropriate level of aggregation is determined by the separability conditions implicit in the related aggregator function (i.e., the substitution relationships in the community utility or production functions). Lacking any evidence to the contrary, our working hypothesis will be that the superlative indexes should include any instrument containing some monetary characteristics.

utility functions, there is reason to believe that the resulting conventional aggregates are imprecise money measures.[7]

One of the more popular superlative indexes is the (Tornquist-Theil) Divisia index,

$$Q_t^D = Q_{t-1}^D \prod_{i=1}^{J} (q_{it}/q_{it-1})^{1/2(S_{it}+S_{it-1})} \qquad (1)$$

where Q_t^D = the superlative (Divisia) quantity index in period t

q_i = the i^{th} quantity component of Q, i = 1, ..., N

S_i = the expenditure share of the i^{th} component

$$(= \pi_i q_i / \sum_{i=1}^{N} \pi_i q_i)$$

π_i = the rental price of one unit of the i^{th} component.

While this equation may appear unnecessarily complicated compared to the simple linear structure of the summation index, one can still give a natural interpretation to the equation once it is translated into logs,

$$\ln Q_t^D - \ln Q_{t-1}^D = \sum_{i=1}^{N} S_{it}^{*}(\ln q_{it} - \ln q_{it-1})$$

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[7] For a more detailed discussion of superlative indexes as applied to monetary aggregates see Cockerline and Murray (1980), or Barnett (1980).

where $S^*_{it} = 1/2(S_{it}+S_{it-1})$.[8]

The growth rate of the Divisia index, measured as first differences in the logged values, is the expenditure-share-weighted average of each component's growth rate.

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[8] Two other superlative indexes include the Fisher ideal,

$$Q^F_t = Q^F_{t-1}\left[\frac{\sum_{i=1}^{J} S_{it-1}(q_{it}/q_{it-1})^{1/2}}{\sum_{i=1}^{J} S_{it}(q_{it}/q_{it-1})^{-1/2}}\right];$$

and the Diewert linear,

$$Q^L_t = Q^L_{t-1}\left[\frac{\sum_{i=1}^{J} S_{it-1}(q_{it}/q_{it-1})}{\sum_{i=1}^{J} S_{it}(q_{it}/q_{it-1})^{-1}}\right]^{1/2}.$$

The summation,

$$Q^S_t = \sum_{i=1}^{N} q_i,$$

and the geometric,

$$Q^G_t = Q^G_{t-1}\prod_{i=1}^{J}(q_{it}/q_{it-1})^{S_{it-1}},$$

are two non-superlative indexes, also examined in later sections.

The expenditure share weights are obtained by multiplying each component quantity by a rental price directly related to its moneyness at the margin. Most deposits and money market instruments offer the investor a variable combination of monetary and non-monetary attributes. Monetary attributes determine an instrument's ability to provide liquidity services, while non-monetary attributes determine its suitability as a store of value. The investor typically expects to receive benefits from both characteristics either through explicit payments or implicit services. At the margin, the value of all these services and returns should just equal the cost of holding each instrument, where the costs include any direct user charges as well as the opportunity costs associated with forgone investment alternatives.

The equalization of marginal benefits and costs in equilibrium can be represented as,

$$M_{it} + R_{it} + O_{it} = R_{Bt} + C_{it} \tag{2}$$

where M_{it} = the implicit return on the monetary portion of the i^{th} component in period t

R_{it} = the explicit own interest and/or expected capital gain (loss) on i

O_{it} = the other non-monetary returns on i

R_{Bt} = forgone interest on the benchmark asset

C_{it} = service charges.

In order to derive a convenient indirect measure of the monetary services embedded in each of the components, the opportunity cost of forgone investments is proxied by the expected return on an asset with little or no moneyness. This return serves as a benchmark allowing us to decompose the returns on all other monetary and quasi-monetary components,

$$M_{it} = (R_{Bt}-R_{it}) + (C_{it}-O_{it}).$$

If one is prepared to assume C_{it} and O_{it} approximately offset one another,[9] variations in M_{it} across instruments can be easily measured as the difference between observed R_{Bt} and R_{it},

$$M_{it} = (R_{Bt}-R_{it}),\ R_{Bt} > R_{it}.$$

Redefining all the variables as nominal rates of return per dollar invested in instruments i and B, the value of monetary services can be expressed as,

$$m_{it} = (r_{Bt}-r_{it})/(1+r_{Bt}). \qquad (3)$$

Monetary services vary inversely with r_{it}, given r_{Bt}, and currency has by definition the most moneyness as its own-interest rate is zero for all t.

Notice that the per unit value of monetary services is equivalent to component i's net opportunity cost or rental price, $\pi_{it} = m_{it}$. Equation (3) can be used therefore to derive a series of rental prices.[10] The important ingredients are a reliable own-interest

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[9] For example, monthly service charges on personal chequing accounts could just offset the value of free safety deposit boxes and other non-monetary services such packages provide. Though this relationship is not likely to hold in all cases the direction of bias introduced by such an assumption is seldom obvious.

[10] More elegant derivations of (3) can be found in Donovan (1978) and Barnett (1978, 1980). A tax adjustment has been added to Barnett's most recent formulation, but it cancels out of the numerator and denominator when the rental price is used to derive Divisia weights for quantity aggregation.

rate and a benchmark rate. Unlike the summation aggregates, the superlative aggregates give explicit recognition to the variable contribution of each component to total money services. In place of the uniform unit weights of the summation index we have unique rental prices and expenditure shares.

To summarize, conventional monetary aggregates are at least potentially very imperfect measures. They assume that all the included components contribute equally to total money services though it is evident they are often imperfect substitutes and vary significantly in their "moneyness". Moreover the summation aggregates make no allowance for changes in the components' monetary characteristics over time except to the extent components are periodically inserted into or deleted from the aggregates so as to reflect institutional changes and innovations. In this regard the unit-weighted summation index used to compute monetary aggregates is at odds with the indexing procedures used to aggregate most other heterogeneous series. It is akin to an output index (e.g., GNE) in which the quantity components are unweighted by either current or base period prices. One simply adds up all the apples and oranges. With this in mind, we have attempted to create and test several different superlative indexes. The following sections of this paper describe their construction and summarize the major empirical results.

2 DATA USED IN CONSTRUCTION

Both price and quantity data are needed to construct superlative monetary aggregates. Whenever possible these series were collected on an average-of-Wednesday basis.[11] When data were not available weekly they were either extracted from month-end sources and averaged, or manufactured from the interest payments information reported annually by chartered banks. Appendix A explains the mnemonics used to denote all the price and quantity components, and Appendix B describes the composition of all our monetary aggregates.

Missing data did pose a problem for certain components. This was not the only problem affecting the aggregate calculations, however. Even when weekly data were readily available, further adjustments were often required to remove biases present in the quoted interest rates and quantities. Two of the major problems we encountered were the distorted interest rates posted on many deposit categories and the biased interest differentials produced by monetary components with widely different maturities.

In many instances the rates posted on savings deposits and other monetary components exaggerate the effective rate individuals actually expect to receive on their investments. Minimum balance

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[11] Monthly data could have been accessed easily from CANSIM, but the series were often incomplete and affected by anomalous month-end behaviour.

requirements, early encashment penalties, and other charges can significantly reduce investors' returns.[12] This is particularly true for personal savings accounts if interest is calculated on the minimum monthly balance. More representative rates can be obtained by multiplying the posted rates by the lowest Wednesday balance in each month and dividing the resulting interest payment by the average monthly balance (see for example RPDEFQ in Table 1).

A second, more serious bias is introduced when the own rates used to calculate rental prices are taken from instruments with markedly different maturities. Most of the disparity we observe between long and short rates is caused by expected inflation over different holding periods. Systematic differences related to an instrument's liquidity and risk characteristics are typically overwhelmed by differences in the inflation premiums associated with different maturities. This is unfortunate, since we are trying to capture liquidity differences with $(R_{Bt}-r_{it})$, not the investor's inflation expectations.

It was necessary therefore to maturity-adjust the data. All the own-interest rates with a maturity greater than one year were normalized to an effective 91-day holding period return. This was accomplished by subtracting an interest spread equal to the difference between the current yield on 91-day treasury bills and longer term

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[12] These can be related to the unobserved C_{it}'s and O_{it}'s in equation (2). The posted r_{it} exaggerates the effective return $(r_{it}-C_{it}+O_{it})$.

government bonds with the same maturity as the unadjusted own-interest rate,[13]

$$r_{it}^{A} = r_{it}^{u} - (r_{Gt} - r_{TBt}) \tag{4}$$

where r_{it}^{A} = the maturity-adjusted own-interest rate on the i^{th} component

r_{it}^{u} = the unadjusted rate

r_{Gt} = the yield on a Canada bond with the same maturity as i

r_{TBt} = the 91-day treasury bill rate.

Without this correction, monetary components with longer terms to maturity would have been assigned larger price weights than components with shorter terms whenever the yield curve was inverted.

The one drawback to this procedure is its implicit acceptance of the pure expectations hypothesis of the term structure. Expected holding period returns are assumed to be equalized across all maturities. Any liquidity or marketability premium on longer term bonds is ignored. As a result, the adjustment introduced in equation (4) could overstate the maturity bias. The existence of positive liquidity premiums produces rental price estimates on long-term monetary components which are too high. A further adjustment was therefore

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[13] Since we do not have Canada bonds outstanding at all maturities, theoretical bond yields were used for the adjustment. These rates were obtained by fitting a curve to the observed yields. The resulting term structure provided a continuous series of yields over the 1- to 25-year range, and removed any distortions caused by extendibility features and coupon effects.

required after the raw interest rate series had been maturity-adjusted. Own-interest rates on monetary components with a maturity greater than one year were augmented with a liquidity premium set equal to the average spread on Canadas over the 1960:01 to 1965:12 period.[14]

2.1 The Benchmark Rate

To ensure non-negative rental prices, the benchmark rate should have a value at least as high as the own-interest rates on assets with some monetary attributes. Negative rental prices would be inconsistent with our theory (and our intuition) which suggests that investors must be prepared to sacrifice some of the returns on a purely non-monetary asset in order to receive monetary services. The benchmark rate was therefore forced to dominate all other own rates by construction. The series was formed by taking the maximum of three rates.

$$R_{Bt} = \max\text{(adjusted 10-year Industrial bond rate, 90-day finance company paper rate, adjusted 3-year Canada rate).}$$

In all but 3 of the 52 quarters (1968Q1 - 1980Q4) the maximum rate was the adjusted 10-year Industrial yield.[15] This is not surprising given the price uncertainty of these long-term bonds, their non-zero

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[14] Inflation expectations were assumed to be flat during this period. Unique liquidity premiums were not assigned to each maturity, however. Instead, only two premiums were created; one for the 1- to 3-year components and one for the 5-year and over components.

[15] The 10-year Industrial yield is a maturity- and liquidity-adjusted McLeod Young Weir index of rates on prime corporate issues, purged of any special features (e.g., low coupons, retractables, convertibles).

default risk, and limited marketability. The three quarters in which the 10-year Industrials failed to dominate the 3-year Canadas and/or 90-day paper rates were periods of turbulence in which unusual interest rate relationships developed.

2.2 Unresolved Problems

Despite our efforts to use the best data available, a number of unresolved and potentially serious problems remain.

The own-interest rates assigned to credit union deposits and some mortgage loan company instruments are actually based on trust company rates. No separate data are collected at present for these two institutional classes. Similarly, the effective rates constructed from annual data in order to fill other data gaps may be very poor proxies. They are based on historic averages and perhaps fail to adequately represent the current marginal rates relevant for portfolio decision-making.

The yield on CSBs is assumed to be equal to the first year's coupon rate. This is an admittedly crude approximation, but more elegant measures such as the application of modern options pricing theories were deemed to be infeasible.[16] Computational difficulties also precluded the use of time-varying liquidity premiums on the own-interest rates and the benchmark rate(s). No recognition is given to movements in the liquidity premium which many researchers believe are inversely related to economic activity.[17]

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[16] See, however, Brennan and Schwartz (1977) for an application of these theories.

[17] There were also theoretical reasons for treating it as a small constant. This is still a controversial area in the literature and several researchers question the existence of a liquidity premium let alone its relationship with the business cycle.

At a more fundamental level, one can question the legitimacy of our assumption concerning the equality of O_{it} and C_{it}. Some deposit categories may offer the deposit holder positive or negative net non-monetary services which serve to distinguish them from other components which carry the same explicit own-interest rate. Demand deposits, for example, often carry zero nominal yields, but could offer some non-monetary services which provide an unobserved return. These features would differentiate them from other zero interest assets such as currency. Here as elsewhere our strategy was to proceed cautiously and not adjust the data unless we were sure of the direction of bias and had a reasonable means by which to quantify it.

It is obviously impossible to know the extent to which the new superlative indexes are biased by inexact rental prices and quantity data. We can only hope that the share weights at least approximate the true expenditure patterns and that the tests on the new indexes are not invalidated by the compromises made in their construction.

3 COMPARISON OF THE AGGREGATES

Alternative non-superlative and superlative quantity aggregates are computed in this section in the hope of avoiding one or all of the problems associated with the summation aggregates. The geometric quantity index (GA), for example, is exact for a Cobb-Douglas aggregator function and assumes a constant and unitary elasticity of substitution between any two goods. It is non-superlative. The Diewert linear (DLA), or quadratic mean of order 1, the Fisher ideal (FIA), or quadratic mean of order 2, and the Divisia (DIV) are all superlative indexes. These four indexes together with the summation index (SUM) are computed for six different levels of bank-only and system-wide monetary aggregates.

The bank-only aggregates correspond to the conventional aggregates published by the Bank of Canada. System-wide aggregates are analogous to the bank-only aggregates but include the deposit liabilities of trust and mortgage loan companies, local credit unions, caisses populaires, and Quebec savings banks, as well as selected money market instruments.[18]

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[18] The composition of these aggregates is illustrated in the table of Appendix B. Mnemonics beginning with a single M or L represent bank-only aggregates. L is a bank-only aggregate consisting of M3 plus bankers' acceptances. Mnemonics beginning with a double M or L represent system-wide aggregates. LL is a system-wide aggregate consisting of the components of M3 plus deposit liabilities of the near-banks plus selected money market instruments. The aggregates are computed for the period 1968Q1 to 1980Q4. Much of the data pertaining to the near-banks are not available prior to that period. Data names and constructions are listed in Appendix A.

Plots of the five quantity indexes at the narrowest and broadest levels (M1 and LL) are shown in Graphs 1 and 2 respectively.[19] As expected, the difference between summation and superlative indexes for M1 is not pronounced (the range between the summation and Divisia measures in 1980Q4 is $0.5 billion). At the broader levels, however, the summation and superlative indexes follow very different growth paths. The superlative indexes have grown at similar rates, unlike the non-superlative indexes. The geometric and summation indexes for LL are separated by $141 billion in 1980Q4 while the Divisia and the Fisher ideal differ by only $2.1 billion. Because all three superlative measures are so similar, only one, the Divisia, is compared with the summation index in subsequent empirical tests.[20]

One attractive feature of the Divisia measures is apparent without econometric testing. The growth rates of summation M1 and the broader bank aggregates often follow divergent time paths. For example, at times when the thrust of monetary policy was to slow the growth of M1 it has often been the case that the growth rates of the broader measures have increased. This is reflected in Graph 3; M1 growth over much of the sample period is inversely related to that of M3. It is possible then to assess monetary policy very differently depending on one's definition of money. This is not the case for Divisia

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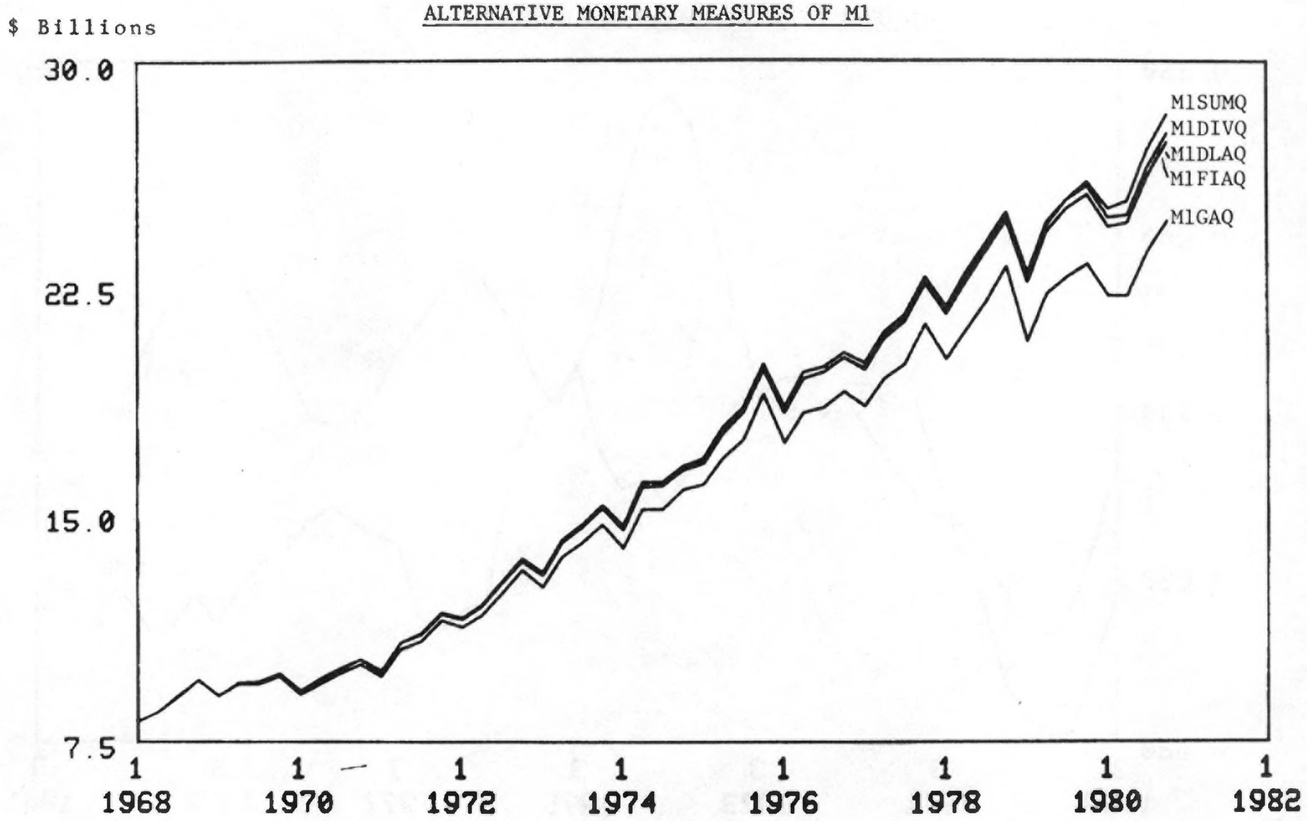
[19] M1SUMQ, M1DIVQ, M1DLAQ, M1FIAQ and M1GAQ correspond to summation, Divisia, Diewert linear, Fisher ideal, and geometric indexes for M1.

[20] Of the two nonsuperlative indexes only the summation index is used in the remaining sections of the paper. Preliminary tests, described in the next section, have shown the geometric index to be the least informative of all five indexes with respect to nominal income.

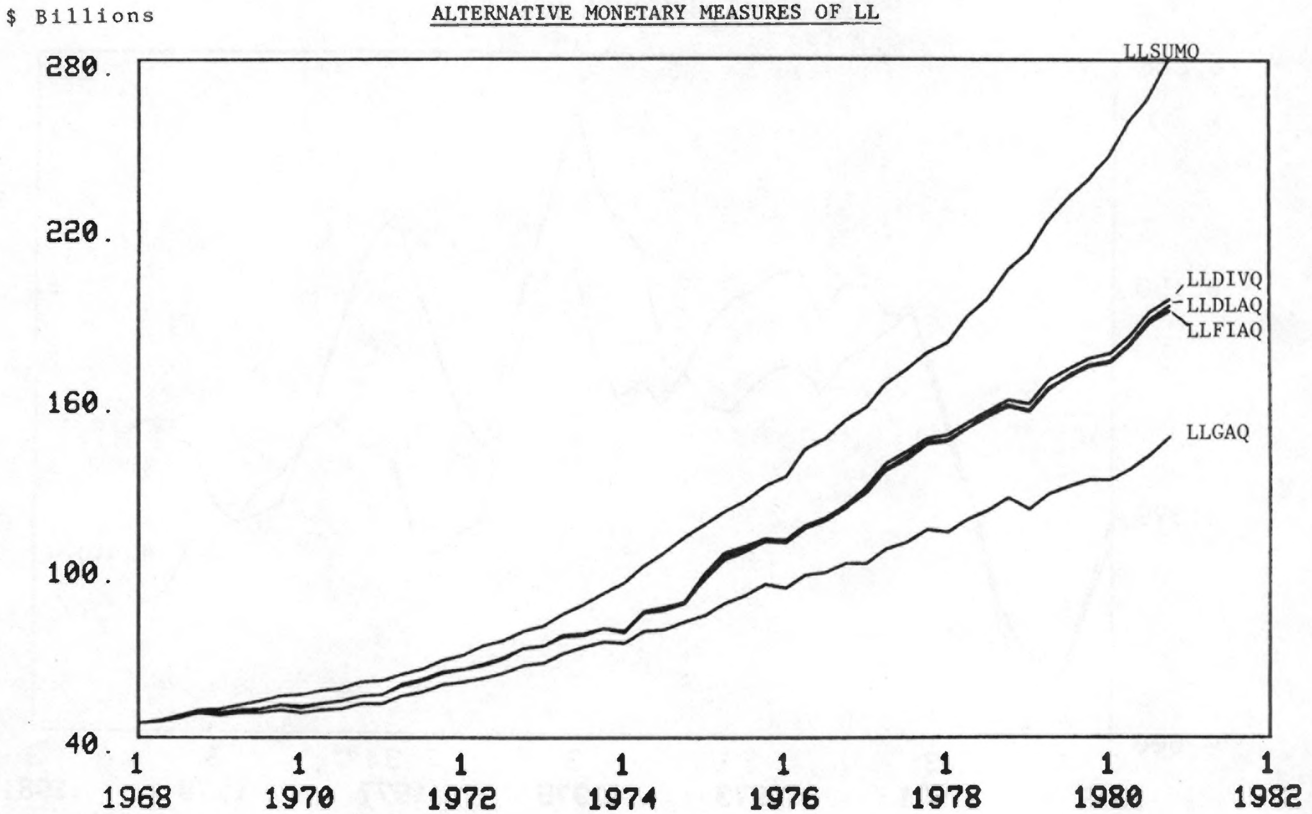
GRAPH 1

ALTERNATIVE MONETARY MEASURES OF M1



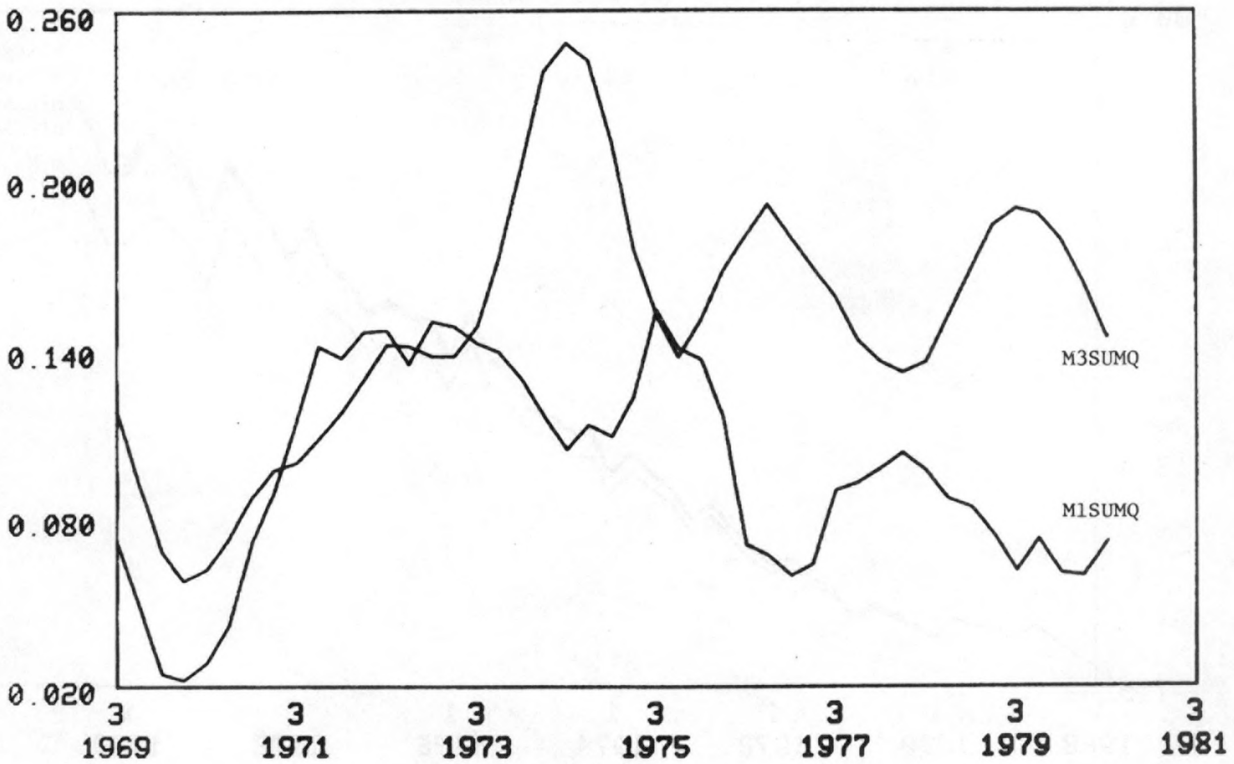
GRAPH 2

ALTERNATIVE MONETARY MEASURES OF LL



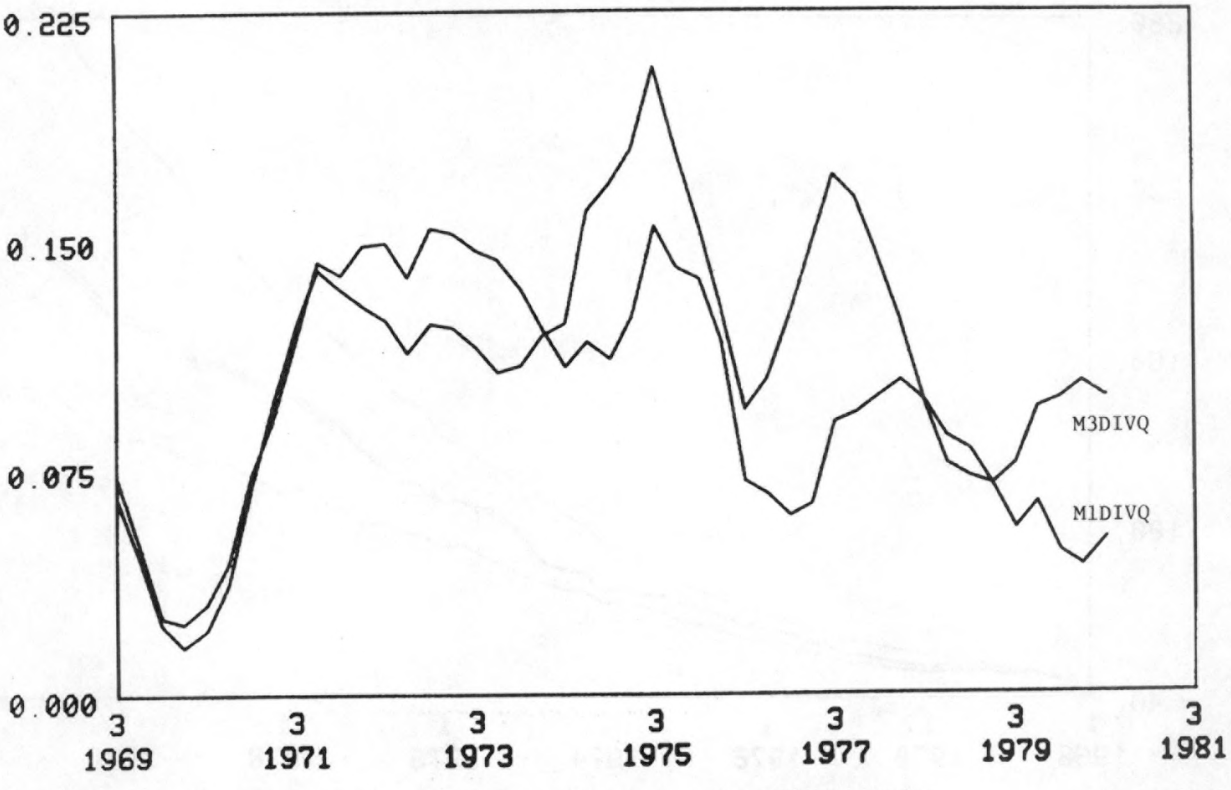
GRAPH 3

GROWTH RATES OF M1SUMQ AND M3SUMQ*



GRAPH 4

GROWTH RATES OF M1DIVQ AND M3DIVQ*



* Growth rates are year-over-year four quarter centered moving averages.

measures of M1 and M3 (see Graph 4). Though Divisia M1 is less inclusive than Divisia M3, they are shown to exhibit similar growth patterns. The same consistency at even broader levels of Divisia aggregation suggests that the growth of liquidity is dominated by the growth of the M1 components.

4 EMPIRICAL RESULTS

This section presents the results of our comparison of the aggregates with respect to information content, causality and stability.

4.1 Information Content

The theoretical superiority of the superlative monetary aggregates over conventional measures is well documented. One question which remains open is whether or not these new aggregates outperform the old in the roles we typically assign to money. It is possible that the practical problems encountered in their construction have nullified any advantages which might have been gained through a more flexible functional form and variable price weights. Given these concerns it is important to examine the usefulness of summation and Divisia monetary aggregates according to criteria relevant for the formation and conduct of monetary policy.

One potentially useful attribute of money is that it contains information relevant to the movement of current nominal income. To improve its performance as an indicator, systematic biases in its measurement should be removed if possible. Furthermore, developments which sharpen our focus on the relationship between money and income should facilitate the setting of appropriate monetary targets and the evaluation of current policy action.[21] Evidence from the U.S. economy

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[21] The usefulness of money as an information variable and as a monetary target are quite separate issues. See White (1979) for a discussion of the choice of a monetary target and questions relating to controllability.

suggests that Divisia monetary aggregates are much higher in information content with respect to inflation and unemployment and marginally higher with respect to nominal income than their summation counterparts.[22] Preliminary results presented in this section suggest that an information loss is apparent in the Canadian Divisia aggregates, at least with respect to nominal income.

Two measures of information content are used here. The first is developed from information theory as proposed by Shannon (1948) and as applied to economic indicators by Tinsley, Spindt and Friar (1980). The information content of one random variable relative to another is defined by Tinsley, Spindt and Friar (TSF) to be the expected uncertainty of the first minus the expected uncertainty of the first conditional upon the second.[23] As applied here, this technique measures the value of using contemporaneous information only. The second technique, based upon Akaike's (1969) final prediction error (FPE), is used to measure the value of using contemporaneous and historical information simultaneously.

The TSF information measure is easily developed for the bivariate model. Let Y (nominal income) and M (money) represent joint normally distributed vectors of T observations each. They are assumed to have a mean vector, μ,

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[22] Barnett and Spindt (1979).

[23] Shannon's proposed expression for expected uncertainty of random vector v with density function f(v) is

$$H_v = - \int f(v) \ln f(v) dv.$$

$$\mu' = (\mu_y', \mu_m'),$$

a variance-covariance matrix, Ω,

$$\Omega = \begin{bmatrix} \Omega_{yy} & \Omega_{ym} \\ \Omega_{my} & \Omega_{mm} \end{bmatrix},$$

and a joint density function. $f(Y,M)$,

$$f(Y,M) = 2\pi^{-T}|\Omega|^{-\frac{1}{2}} \exp -\tfrac{1}{2}\{\binom{y}{m}' \Omega^{-1}\binom{y}{m}\},$$

where $y = Y - \mu_y$ and $m = M - \mu_m$.

The expected information content of M with respect to Y is defined by TSF to be:

$$I_{Y|M} = \frac{1}{2} \ln \left\{\frac{|\Omega_{yy}|}{|\Omega_{yy} - \Omega_{mm}^{-1}\Omega_{my}|}\right\}. \tag{5}$$

Assumptions of constant variance, zero serial correlation and zero noncontemporaneous cross-correlation in Y and M allow us to rewrite (5) as:[24]

$$I_{Y|M} = \frac{1}{2} \ln \left[\frac{1}{1-R^2}\right],$$

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[24] The need for these assumptions is illustrated in Cockerline (1981).

where R^2 is the coefficient of determination of the following bivariate linear model:

$$Y_t = \alpha + \beta M_t + e_t. \tag{6}$$

The usual classical properties of zero mean, constant variance and lack of serial correlation are assumed for disturbance e_t.

Since the requirement for lack of serial correlation in the error is seldom, if ever, met in a regression of income on money, a first order autoregressive transformation has been applied to the data. The procedure has been to regress over $t = 2, \ldots, T$ the equation

$$Y_t = \hat{\rho} Y_{t-1} + \alpha(1-\hat{\rho}) + \beta(M_t - \hat{\rho} M_{t-1}) + u_t, \tag{7}$$

where $\hat{\rho}$ is the GLS estimate of the autocorrelation coefficient ρ from (6) regressed over $t = 1, \ldots, T$.

The R^2 statistics from regressions of nominal GNE on the summation and Divisia quantity indexes at 12 different levels of aggregation are presented in Tables 1 and 2.[25] Table 1 presents statistics for regressions performed on level data and Table 2 presents statistics for first-differenced data. Since the data are seasonally unadjusted, seasonal dummies have been included in all regressions. The first six aggregates correspond to those presently

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25 Since each aggregate is assigned a unique $\hat{\rho}$, the dependent variables differ across regressions and the R^2s are not strictly comparable. Nevertheless, they should serve as reasonable measures of relative information content provided differences in $\hat{\rho}$ are not extreme.

TABLE 1

R^2s OF NOMINAL GNE REGRESSED ON ALTERNATIVE MONEY MEASURES
(Data in levels, 1968Q2 - 1980Q4)

Monetary aggregate	Summation measures			Divisia measure		
	R^2	D.W.	$\hat{\rho}$	R^2	D.W.	$\hat{\rho}$
M1	.9926	2.33	.6802	.9918	2.38	.7844
M1B	.9916	2.20	.7959	.9938	2.15	.9995
M2	.9948	1.95	.7100	.9938	1.81	.5562
M2C	.9952	1.94	.7506	.9941	1.81	.6025
M3	.9954	2.12	.7881	.9946	1.87	.5975
L	.9953	2.16	.8226	.9947	1.86	.5787
MM1	.9930	2.29	.6267	.9922	2.36	.7312
MM1B	.9922	2.18	.7680	.9919	2.17	.8034
MM2	.9940	1.82	.5711	.9938	1.75	.6000
MM2C	.9949	1.98	.6812	.9942	1.78	.6400
MM3	.9950	2.07	.7131	.9945	1.83	.6437
LL	.9943	2.11	.8100	.9945	1.93	.6381

TABLE 2

R^2s OF NOMINAL GNE REGRESSED ON ALTERNATIVE MONEY MEASURES

(Data in first differences, 1968Q3 - 1980Q4)

Monetary aggregate	Summation measure			Divisia measure		
	R^2	D.W.	$\hat{\rho}$	R^2	D.W.	$\hat{\rho}$
M1	.8063	2.06	-.1394	.8027	2.04	-.1087
M1B	.8069	2.05	-.1349	.8047	2.03	-.1087
M2	.8198	2.14	-.1587	.8143	2.11	-.1056
M2C	.8142	2.10	-.1012	.8146	2.11	-.1012
M3	.8396	2.13	-.1700	.8144	2.10	-.1006
L	.8386	2.14	-.1806	.8145	2.10	-.1044
MM1	.8086	2.07	-.1645	.8050	2.05	-.1336
MM1B	.8120	2.04	-.1634	.8089	2.03	-.1371
MM2	.8136	2.11	-.1056	.8141	2.11	-.1044
MM2C	.8139	2.11	-.1037	.8143	2.10	-.1012
MM3	.8138	2.10	-.1025	.8142	2.10	-.1006
LL	.8141	2.15	-.1832	.8142	2.10	-.1000

published by the Bank of Canada. The final six aggregates consolidate the monetary components of both bank and non-bank sectors.

Conclusions drawn from Tables 1 and 2 are that:

a) contemporaneous information obtained from money increases with the level of aggregation. The most informative summation measure is M3. That for Divisia is either L or M2C;

b) information content is almost uniformly higher for summation aggregates than for Divisia aggregates;

c) information content is higher for bank aggregates (M1, ..., L) than for system aggregates (MM1, ..., LL) for both summation and Divisia indexes;

d) the superiority of the summation measures over the Divisia measures is less pronounced for the system aggregates than for the bank aggregates.

The interpretation of R^2 in the simple bivariate case as a measure of information content requires the off-diagonal elements of Ωym to be zero. In the present context, this assumes that past values of M are not informative as to current values of Y. Since this is thought to be an overly stringent requirement, a second measure which incorporates historical as well as contemporaneous information is reported in Table 3.

Table 3 records Akaike's final prediction error (FPE) for the optimum lag structure of Y regressed on M, where Y and M are first differences of logs of nominal GNE and money. The optimum lag structure is defined as that structure which minimizes an estimate of the asymptotic mean square prediction error. For the model:

TABLE 3

FPEs OF Y REGRESSED ON DISTRIBUTED LAG MODELS OF Y AND ALTERNATIVE MONEY MEASURES

(Data in first difference of logs, 1968Q2 - 1980Q4)

Monetary aggregate	Summation measure			Divisia measure		
	a	b	FPE*	a	b	FPE*
M1	1	6	.101319	1	6	.101839
M1B	2	2	.106329	2	2	.106800
M2	2	0	.118203	1	1	.124075
M2C	2	0	.113302	1	1	.124157
M3	1	1	.101466	1	1	.124388
L	2	0	.109845	1	1	.124062
MM1	1	6	.102038	1	6	.102016
MM1B	1	3	.104677	2	2	.107830
MM2	1	3	.120929	1	1	.123512
MM2C	1	1	.121130	1	1	.123730
MM3	1	1	.119625	1	1	.123859
LL	2	0	.120383	1	1	.122809

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* The optimum autoregressive structure of Y is AR(2) with an FPE of .118681. The FPE figures are multiplied by 10^3.

$$Y_t = \alpha + \Psi_{11}{}^{a}(L)\, Y_t + \Psi_{12}{}^{b}(L)\, M_t,$$

where Y and M are stationary time series for $t = 1, \ldots T$ and $\Psi_{11}{}^{a}(L)$ and $\Psi_{12}{}^{b}(L)$ are lag operators of orders a and b respectively; such an estimate is defined by Akaike (1969) to be:

$$\mathrm{FPE}_Y(a,b) = \frac{(T + a + b + 1)}{(T - a - b - 1)} \sum_{t=1}^{T} \frac{(Y_t - \hat{Y}_t)^2}{T},$$

where $\hat{Y}_t$ is the predictor of Y_t. In order to reduce the number of regressions required to minimize FPE for 'a' ranging from 1 to 15 and 'b' ranging from 0 to 15,[26] the Hsiao (1981) computational simplification was used. FPEs for Y regressed upon a constant term, $\Psi_{11}{}^{a}(L)\, Y$ and $\Psi_{12}{}^{b}(L)\, M$ are reported in Table 4 (page 34).

Conclusions drawn from Table 3 are as follows:

a) contemporaneous plus historical information is at a maximum (lowest final prediction error) for the narrowest definitions of money. The only exception to a general reduction in information as the level of aggregation increases appears at summation M3;

b) the summation aggregates dominate the Divisia aggregates in all cases;

c) the bank aggregates appear more informative than the system aggregates for both summation and Divisia indexes;

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[26] It is noted that contemporaneous M is included as an explanatory variable. This is consistent with the usage of M as an indicator of Y.

d) the superiority of the summation measures over the Divisia measures is less pronounced for the system aggregates than for the bank aggregates.

4.2 Causality

The published literature regarding Canadian money-income causality appears to have reached a consensus. Barth and Bennett (1974), Auerbach and Rutner (1978) and Hsiao (1979) all find evidence of bidirectional causality between M1 and GNP for data periods including the 1960s and early 1970s. This is not inconsistent with the view that the Bank of Canada followed an interest rate targeting strategy over much of that period. In general, the results of causality tests performed over periods of mixed policy structures are difficult to interpret. As our data samples become more and more dominated by post-1975 data, however, bivariate causality may be expected to give way to univariate causality from money to income.[27] There is less published evidence regarding broader aggregates and GNP for Canada. Hsiao (1978) reports univariate causality from GNP to M2 for the period 1955Q1 to 1977Q4.

The bivariate causality tests of Hsiao are applied in this study to Canadian income and various definitions of money for the period 1968Q2 to 1980Q4. Hsiao's methodology employs Akaike's FPE

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[27] This is not enough to ensure the short-run exogeneity of money, however. That depends upon the policy reaction function employed by the monetary authority. For anything other than immediate response the causal pattern could still appear bidirectional in the short run.

criterion in choosing the appropriate lag structure to be used in the familiar Granger model. The bivariate model may be written:[28]

$$Y_t = \Psi_{11}{}^{a}(L)Y_t + \Psi_{12}{}^{b}(L)M_t + u_t,$$

$$M_t = \Psi_{21}{}^{c}(L)Y_t + \Psi_{22}{}^{d}(L)M_t + v_t,$$

where Y and M are stationary time series of income and money respectively, and u and v are zero mean, white noise disturbances with constant covariance matrix. It has been shown by Granger (1969) and later by Sims (1972) and Pierce and Haugh (1977) that $\Psi_{12}{}^{b}(L) = 0$ is consistent with unidirectional causation running from Y to M (Y→M), that $\Psi_{21}{}^{c}(L) = 0$ is consistent with unidirectional causation from M to Y (M→Y), and that $\Psi_{21}{}^{c} = \Psi_{12}{}^{b} = 0$ is consistent with independence of Y and M (Y-M). Rejection of all three hypotheses has been taken to imply bidirectional causality ($Y \rightleftarrows M$). Hsiao's innovation allows a, b, c and d, the orders of the four lag operators, to be empirically determined.

Table 4 identifies the optimum bivariate structure of income and eight different definitions of money. Though the lag structures are very similar for summation and Divisia M1, there is a pronounced difference between them at the broader levels. Haugh's residual cross-correlation test rejects independence of the two series at the 95 per cent level in all cases.

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[28] Unlike the bivariate models employed in Section 3.1 contemporaneous values of M and Y are excluded from the right-hand side.

TABLE 4

OPTIMUM BIVARIATE AUTOREGRESSIVE STRUCTURES OF INCOME AND MONEY VARIOUSLY DEFINED*

Money variable	a	b	c	d	Haugh's test statistic**
M1SUM	1	6	5	6	53.96
M1DIV	1	6	6	4	53.66
M2SUM	2	1	3	5	54.87
M2DIV	1	1	7	9	60.85
M3SUM	2	1	1	6	72.05
M3DIV	1	1	6	9	73.21
LLSUM	2	1	1	3	61.43
LLDIV	1	1	9	1	56.31

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* Box-Pierce tests were applied to the residuals of these models and failed to detect autocorrelation of orders 1 through 20 in all cases.

** Haugh's statistic is calculated as the sum of squared cross-correlation coefficients (of the two residual vectors) for lags from 1 to 20 multiplied by the number of observations. It is compared with Chi-square with 20 degrees of freedom.

Results of the causality tests appear in Table 5. In brief, they do not reject unidirectional causality from M1 to GNP whether in summation or Divisia at the 95-per-cent level. At the three broader levels of aggregation, unidirectional causation from GNP to money is not rejected. With respect to causality, then, the Divisia numbers are not markedly different from the summation aggregates. That is, for both measures of M1 the tests suggest the existence of unidirectional causality from money to income. For both measures of M2, M3 and LL the reverse pattern, from income to money, is implied.[29]

4.3 Stability

The existence of an identifiable and stable demand function is an important requirement for any monetary aggregate that serves as an intermediate target. In this section we test the stability of conventional single-equation models using four different monetary aggregates: narrow (M1) and broad (LL) for summation (SUM) and Divisia (DIV) indexes respectively. Though the approach is admittedly simplistic, since it fails to account for institutional developments that are known to have affected the relative stability of narrow versus broad aggregates, it is informative as to the relative stability of summation

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[29] These results were obtained from data transformed as first differences of logarithms. When growth rates are first differenced the similarity in causal patterns of summation and Divisia aggregates remains. However, the finding of univariate causality from M1 to GNP switches to one of bivariate causality. The apparent causal pattern at the broader levels seems more robust as to choice of filter.

TABLE 5

BIVARIATE MONEY-INCOME CAUSALITY
(sample 1968Q2 to 1980Q4)

Likelihood ratio* (d.o.f) / Variables**	Null Hypothesis***		
	$Y \rightarrow M$ (Unidirectional causality)	$M \rightarrow Y$ (Unidirectional causality)	$Y - M$ (Independence)
Y, M1SUM	21.11**** (6)	11.12 (5)	32.12**** (11)
Y, M1DIV	20.81**** (6)	14.60 (5)	35.26**** (7)
Y, M2SUM	1.10 (1)	8.98**** (3)	10.07**** (4)
Y, M2DIV	0.46 (1)	33.65**** (7)	34.11**** (8)
Y, M3SUM	0.00 (1)	8.56**** (1)	8.03**** (2)
Y, M3DIV	0.28 (1)	32.01**** (6)	32.29**** (7)
Y, LLSUM	0.13 (1)	13.69**** (1)	13.83**** (2)
Y, LLDIV	0.13 (1)	23.57**** (9)	23.70**** (10)

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* Obtained as $-2\,[\ln\Theta_{\text{restricted}} - \ln\Theta_{\text{unrestricted}}]$ where Θ is the maximum of the likelihood function. This value is compared with Chi-square at the degrees of freedom indicated in parentheses.

** Variables are expressed as first differences of logarithms. Y corresponds to nominal GNE.

*** The alternative hypothesis in each case is one of bidirectional causality ($Y \rightleftarrows M$).

**** Indicates rejection of the null hypothesis in favour of ($Y \rightleftarrows M$) at the 95 per cent level of significance.

versus Divisia aggregates at two levels of aggregation using simple theoretic models of money demand.[30]

Estimates from conventional lagged dependent variable models for M1 and LL are summarized in Tables 6 and 7. Scale variables are taken to be either real income or real wealth. Opportunity cost variables are: the 90-day finance company paper rate for M1SUMQ; the McLeod Young Weir average of 10 Industrial bonds rate for LLSUMQ; the Divisia price index of the rental prices of currency and current accounts for M1DIVQ: and the rental price of currency for LLDIVQ. LLSUMQ has, in addition, the rate on trust company 90-day GICs as an "own" rate variable. It is noted that, consistent with the theoretical derivation of the Divisia aggregates, opportunity cost measures are in the form of own prices for the M1DIVQ and LLDIVQ. In the case of M1DIVQ, the own price is itself a Divisia price index of the price of currency and the price of interest-bearing current accounts. For LLDIVQ, a Divisia price index for all included components was tried but found to be statistically insignificant. The price of currency was used as a second best measure of the price of LL.

The demand equations are estimated over two different sample periods in Tables 6 and 7. The first is the full sample period from 1968Q2 to 1980Q4. The second is a truncated sample spanning 1968Q2

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[30] The issue of suitability of narrow versus broad money as a monetary target cannot be addressed using these models. The relevant prediction models should be allowed to change as the institutional environment changes. As long as the monetary authority is cognizant of change and of its impact upon the demand for a given aggregate, then the predictability of that aggregate is not affected.

TABLE 6

ESTIMATION RESULTS*: SUMMATION AGGREGATES

Dependent variable equation**	M1SUMQ (1)	(1)	LLSUMQ (3)	(3)
Sample period	1968Q2 - 1980Q4	1968Q2 - 1975Q4	1968Q2 - 1980Q4	1968Q2 - 1975Q4
Constant	0.64 (2.18)	0.57 (1.55)	2.94 (3.89)	3.79 (3.45)
Log (real income)	0.13 (0.68) (3.30)	0.30 (0.81) (3.03)		
Log (real wealth)			0.21 (0.58) (3.93)	0.26 (0.56) (3.34)
Log (R90Q)	-0.06 (-0.28) (-5.59)	-0.05 (-0.14) (-4.01)		
Log (RTR90Q)			0.03 (0.10) (1.52)	0.05 (0.11) (1.78)
Log (RINDQ)			-0.08 (-0.21) (2.77)	-0.11 (-0.23) (-2.88)
Log (lagged dependent)	0.80 (12.83)	0.63 (5.26)	0.64 (6.75)	0.54 (3.94)
SER	0.0229	0.0253	0.0154	0.0176
Auto 1	-0.4073	-0.3608	-0.4168	-0.3758
DW	2.17	2.00	2.01	2.08

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* Estimated steady-state elasticities are presented in parentheses beside the short-run estimates. Large-sample t-statistics appear in parentheses below the short-run estimates.

** Equations are:

$$(1)\quad \text{Log (M1SUMQ/PGNE)} = A_0 + A_1 * \log \text{(YP)} + A_2 * \log \text{(R90Q)} + A_3 * \log \text{(M1SUMQ(-1)/PGNE(-1))} + \text{seasonals}$$

$$(3)\quad \text{Log (LLSUMQ/PGNE)} = A_0 + A_1 * \log \text{(RWQ)} + A_2 * \log \text{(RTR90Q)} + A_3 * \log \text{(RINDQ)}$$

$$+ A_5 * \log \text{(LLSUMQ(-1)/PGNE(-1))} + \text{seasonals}$$

where YP is real GNE and RWQ is real wealth.

TABLE 7

ESTIMATION RESULTS*: DIVISIA AGGREGATES

Dependent variable equation**	M1DIVQ		LLDIVQ	
	(2)	(2)	(4)	(4)
Sample period	1968Q2 - 1980Q4	1968Q2 - 1975Q4	1968Q2 - 1980Q4	1968Q2 - 1975Q4
Constant	0.84 (2.69)	0.89 (2.41)	3.59 (4.46)	4.26 (3.83)
Log (real income)	0.06 (0.40) (1.51)	0.34 (0.74) (3.17)		
Log (real wealth)			0.17 (0.41) (4.68)	0.20 (0.40) (3.93)
Log (OPM1)	-0.70 (-4.63) (-4.38)	-0.68 (-1.48) (-3.71)		
Log (PCURQ)			-0.98 (2.32) (-4.37)	-0.97 (-1.95) (-3.61)
Log (lagged dependent)	0.85 (13.06)	0.54 (4.10)	0.58 (6.21)	0.50 (3.92)
SER	0.0253	0.0256	0.0234	0.0274
Auto 1	-0.3420	-0.3010	-0.0713	-0.1745
DW	2.08	1.91	1.99	2.00

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* Estimated steady-state elasticities are presented in parentheses beside the short-run estimates. Large-sample t-statistics appear in parentheses below the short-run estimates.

** Equations are:

(2) Log (M1DIVQ/PGNE) = $A_0 + A_1$*log (YP) + A_2*log (OPM1) + A_3*log (M1DIVQ(-1)/PGNE(-1)) + seasonals

(4) Log (LLDIVQ/PGNE) = $A_0 + A_1$*log (RWQ) + A_2*log (PCURQ) + A_3*log (LLDIVQ(-1)/PGNE(-1)) + seasonals

where OPM1 is a Divisia Price Index of PCURQ and PRCAQ.

and 1975Q4. A rough assessment of functional stability is to note the relative constancy of the coefficient estimates over different samples. In this regard, LL reveals greater uniformity in long-run elasticities than does M1. The M1 models, when estimated over the period 1968Q2 to 1975Q4 and simulated to 1980Q4, show strong out-of-sample over-predictions. The broader aggregate shows a systematic underprediction, though less pronounced, for the same period. In both broad and narrow aggregation the summation aggregate shows a lower root mean square prediction error than the Divisia.

Table 8 reports the results of Chow tests for structural change applied at four potential breakpoints within the sample. Though all four models show some instability, the statistics of Table 8 suggest that instability is more pronounced, or begins earlier, for models explaining summation aggregates than for models explaining Divisia aggregates. Somewhat in contrast with this evidence, at least at the M1 level, are plots comparing the estimated coefficients of different models over time. Graph 5 plots long-run income elasticities over time for M1SUMQ and M1DIVQ. The leftmost data point corresponds to the value of the coefficient taken from the regression 1968Q2 to 1975Q3 and divided by that taken from the full-sample regression (1968Q1 to 1980Q4). Each successive point corresponds to a regression period augmented by one observation. The M1SUMQ income elasticity exhibits greater constancy over time than that of M1DIVQ. Graph 6 plots long-run wealth elasticities for LLSUMQ and LLDIVQ. This time, the Divisia aggregate shows greater constancy than the summation aggregate with respect to wealth elasticity.

TABLE 8

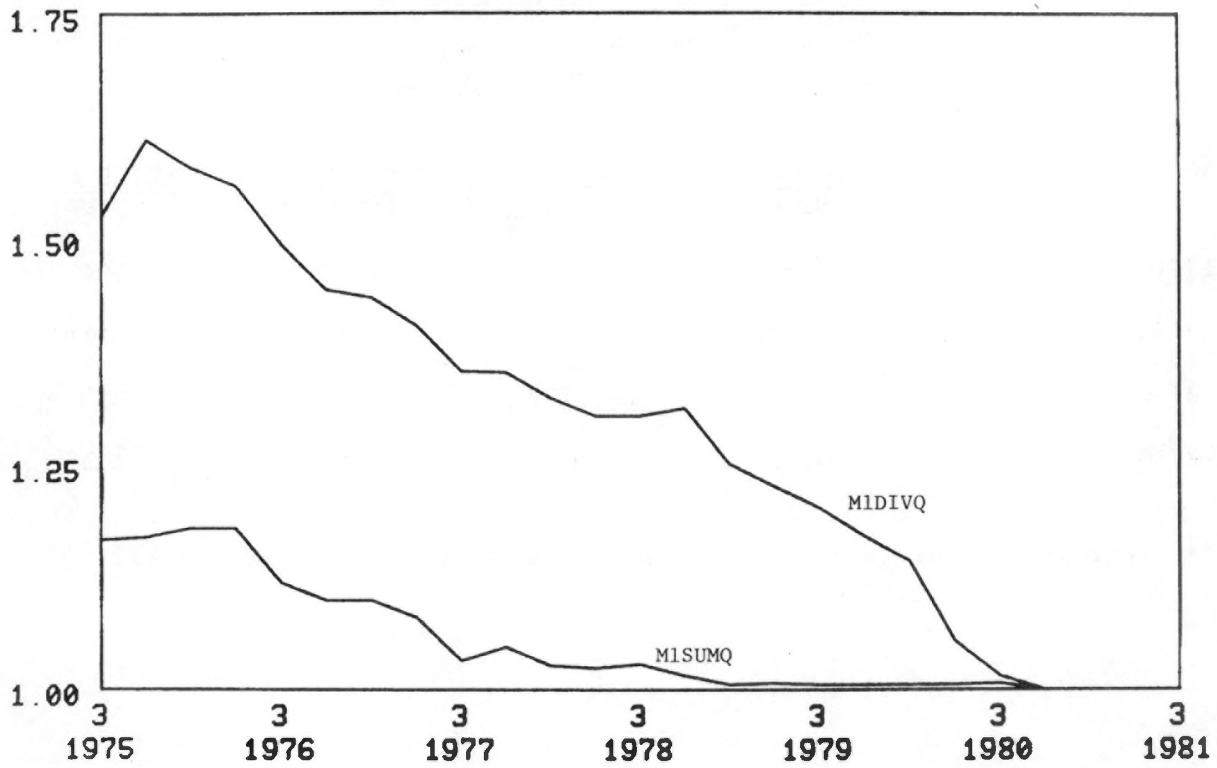
TESTS FOR STRUCTURAL CHANGE AT SELECTED BREAKPOINTS FOR FOUR DEMAND MODELS

BREAKPOINT	MODEL 1 (M1SUMQ)	MODEL 2 (M1DIVQ)	MODEL 3 (LLSUMQ)	MODEL 4 (LLDIVQ)
1971Q4	2.802**	1.927	4.677**	2.104
1973Q4	5.582**	3.129**	4.839**	3.361**
1975Q4	0.996	1.792	0.816	1.083
1977Q4	0.590	0.691	0.990	0.434

** Indicates significant structural change at the 95 per cent level.

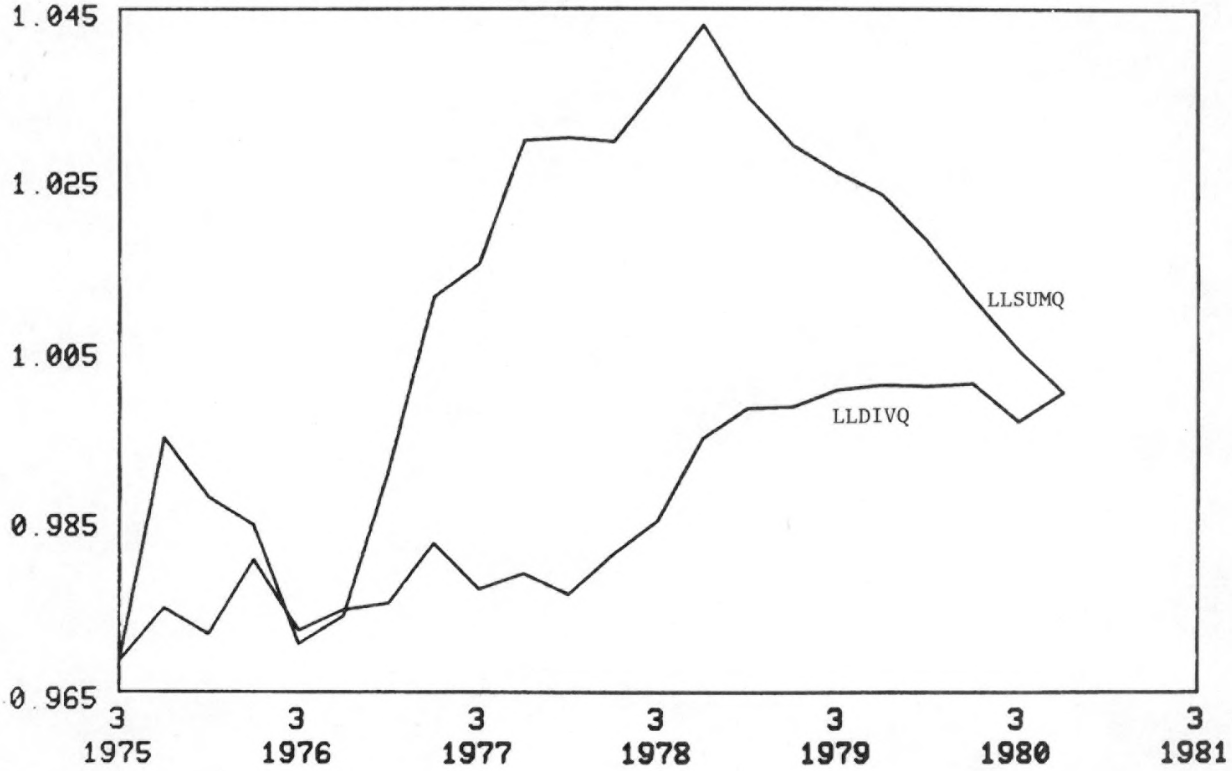
GRAPH 5

LONG-RUN INCOME ELASTICITIES* FOR M1SUMQ AND M1DIVQ



GRAPH 6

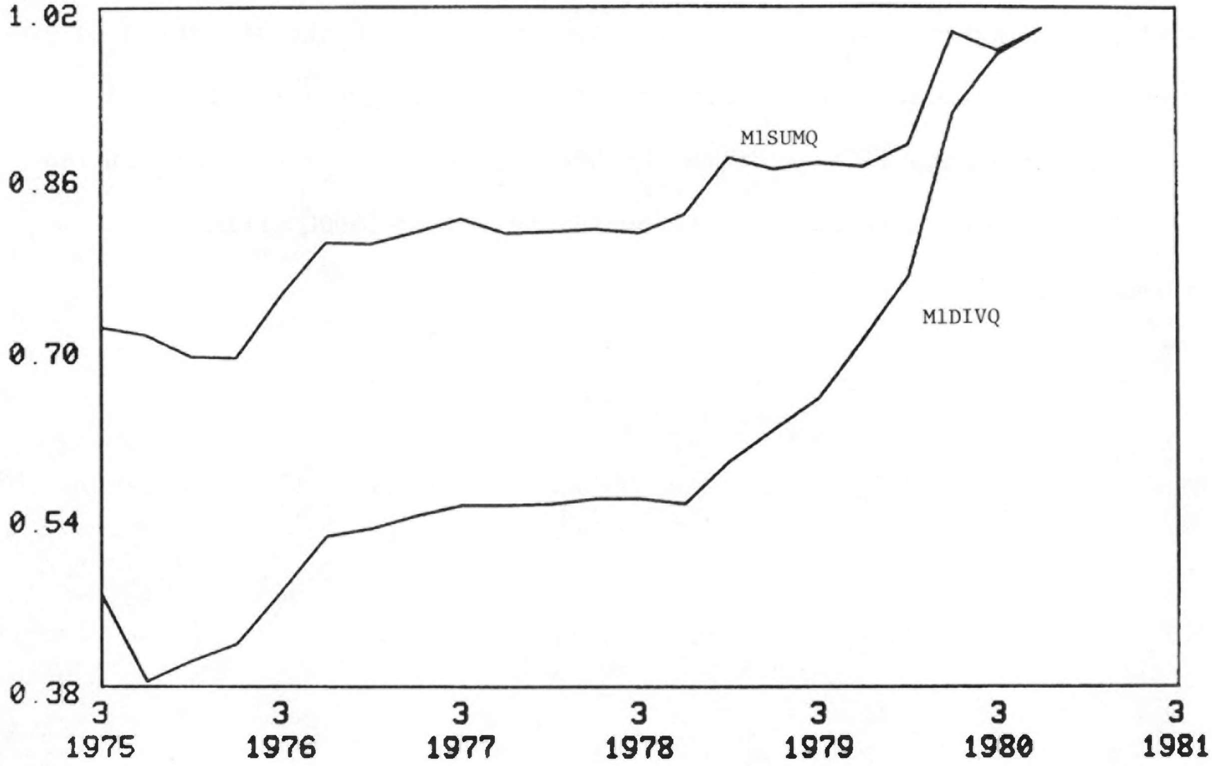
LONG-RUN WEALTH ELASTICITIES* FOR LLSUMQ AND LLDIVQ



* Elasticities are normalized by their 1980Q4 values.

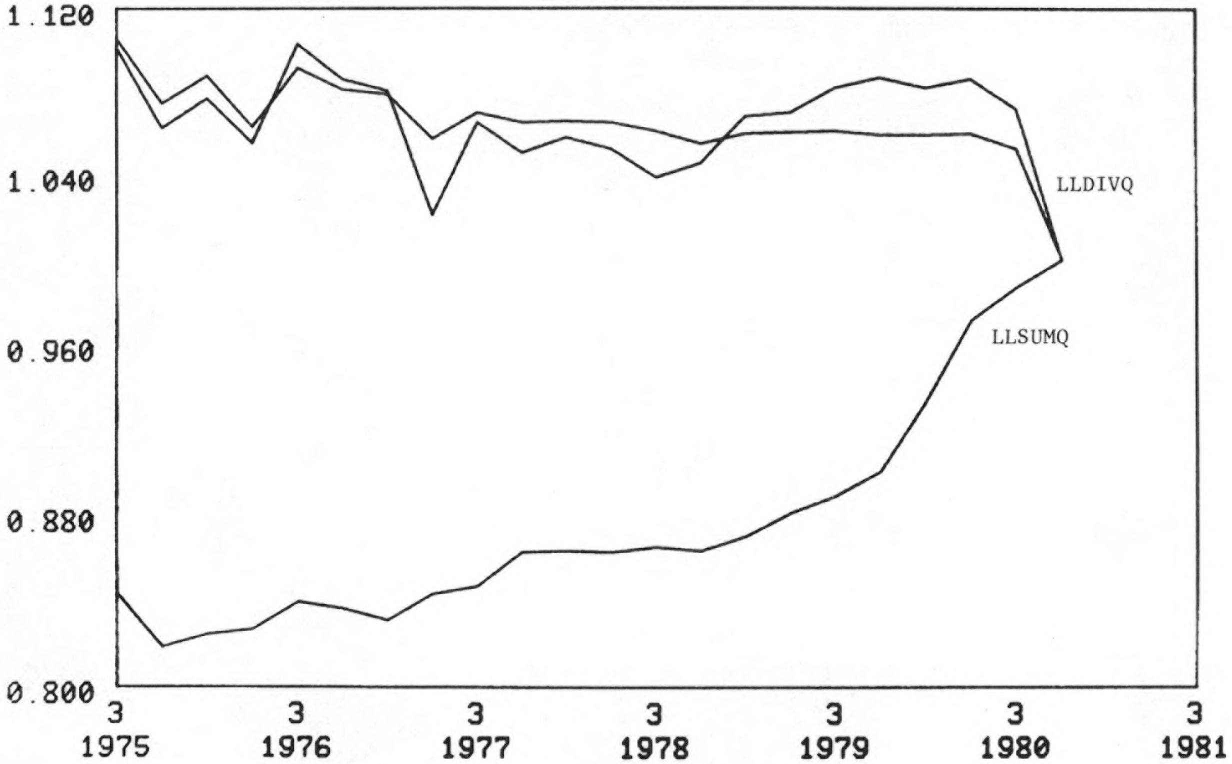
GRAPH 7

LONG-RUN INTEREST AND PRICE ELASTICITIES*
FOR M1SUMQ AND M1DIVQ



GRAPH 8

LONG-RUN INTEREST AND PRICE ELASTICITIES*
FOR LLDIVQ AND LLSUMQ



* Elasticities are normalized by their 1980Q4 values.

Graphs 7 and 8 plot long-run interest and price elasticities for the four aggregates. As observed in Graphs 5 and 6 the summation aggregate shows greater uniformity than the Divisia aggregate at the M1 level. The opposite is true at the broader level. The price elasticities of LLDIVQ are more uniform over the period 1975Q3 to 1980Q4 than the interest elasticity of LLSUMQ.

5 CONCLUSIONS

Though the proper level of aggregation is an oft-debated issue in matters concerning monetary policy, scant attention has been given in the past to the proper method of aggregation. This paper has identified three basic problems associated with the structure of the conventional summation aggregates: 1) they imply perfect substitutability between any two components; 2) they assume a particularly restrictive functional form for the underlying aggregator function; and 3) they focus exclusively on banking system liabilities and currency. Superlative quantity indexes can be constructed in a way that avoids all of these theoretical problems. They may, however, be affected to varying degrees by practical problems not present in the conventional measures.

The empirical portion of this paper compared the performance of Divisia superlative and conventional summation aggregates with respect to their information content, money-income causality, and stability in simple money demand equations. Our purpose was to analyze the aggregates in each of these important areas and to see if the new aggregates were viable alternatives to the inelegant yet serviceable summation approach. The results, as noted above, were very mixed.

The Divisia aggregates were inferior to their summation counterparts as indicators of nominal income in both contemporaneous and distributed lag models. Indeed, the narrower aggregates seemed to be more informative with respect to nominal income than the broader aggregates when historical information was taken into account. On the other hand, the Divisia aggregates displayed greater relative parameter stability at broad levels of aggregation. This could reflect greater theoretical consistency or simply the vagaries of the demand models we employed. All the specifications failed conventional Chow tests over the 1968-1980 period. In the causality tests there were no clear winners, as plausible and very similar bivariate patterns were obtained from both sets of aggregates.

The apparent information loss of the broader superlative aggregates seems inconsistent with the theory underlying their construction. This result suggests that either money properly defined is not a good indicator of nominal income or that the superlative aggregation employed here is in some way deficient. Biased rental prices could be distorting our superlative aggregates, but it would be misleading to place sole responsibility for the mixed results on imprecise data. Similar results have been reported in other tests using U.S. data and different rental price calculations.[31]

It is possible that other problems of a theoretical nature affect the performance of the superlative aggregates. For example, the weak separability assumptions needed to distinguish monetary and non-monetary commodities in the underlying aggregator functions may not be met. In this case it is impossible to meaningfully aggregate the various monetary components currently included in any of our aggregates. Narrow money may dominate the broader aggregates because they are less

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[31] Barnett, Spindt and Offenbacher (1981).

inclusive and therefore less distorted. Furthermore, our approach assumes a common aggregator function for all economic agents. By failing to disaggregate by sector, we ignore the segmented nature of many of our financial markets and the heterogeneous behaviour of households and businesses.

Before concluding, we would like to draw attention to an additional positive feature of the new aggregates which may have been obscured by the mixed statistical results reported in Section 4. This concerns the similar growth paths observed across the spectrum of superlative aggregates. The consistent movement of narrow and broad superlative monetary aggregates may provide policymakers and the public with a less ambiguous barometer of credit conditions than the conflicting pattern occasionally observed among the summation aggregates.

At times the various summation aggregates move in opposite directions, making it difficult to assess monetary policy and monitor economic activity. Much of this anomalous behaviour is believed to be caused by the heterogeneous nature of the components comprising these crude aggregates, as well as their linear unit-weighted structure. Once the components are weighted according to their relative moneyness and embedded in a more flexible non-linear structure the negative correlations disappear. Since M1 summation contains more homogeneous elements it is less affected by these distortions and follows a time path similar to the Divisia aggregates. It might be expected therefore to serve as a better money proxy than the broader summation aggregates.

Finally, we should note that our investigation of superlative monetary aggregates is still at a very preliminary stage. Extensions to it will include an examination of the separability of monetary components in consistently derived asset demand models, as well as the computation and testing of separate indexes for households and firms.

APPENDIX A

MNEMONICS

BACCQ - bankers' acceptances outstanding

BCAQ - current accounts

BCAIQ - interest-bearing current accounts

BCANIQ - non-interest-bearing current accounts
source: BCAQ-BCAIQ

BCSQ - personal chequing savings deposits at banks

BDIAOW - daily interest savings deposits at banks

BDIMMB - daily interest savings deposits at banks
(Monthly minimum of Wednesdays)

BDIQ - daily interest savings deposits at banks

BFTQ - personal fixed-term deposits at banks

BNCSMMBQ - personal non-chequable savings deposits at banks
(Monthly minimum of Wednesdays)

BNCSQ - personal non-chequable savings deposits at banks

BNCSXDIQ - personal non-chequable savings excluding daily interest deposits at banks
source: BNCSQ-BDIQ

BNPBTQ - non-personal bearer-term notes at banks

BNPCQ - non-personal chequable deposits at banks

BNPNCQ - non-personal non-chequable deposits at banks

BNPFTQ - non-personal fixed-term deposits at banks

BOTHFQ - non-swapped foreign currency deposits booked in Canada at banks

BPCAQ - personal chequing accounts at banks

BPROP90Q - proportion of fixed-term deposits at banks by original
BPROP1Q term to maturity in 90-day, 1-2 year, 2-5 year and
BPROP2Q over 5-year maturities
BPROP5Q

BSWAPQ - SWAP deposits at banks

CCDDQ - chequable demand deposits at credit unions

CDDQ - demand deposits at local credit unions and caisses populaires

CNCDDQ - non-chequable demand deposits at credit unions
source: CDDQ-CCDDQ

CSBQ - Canada Savings Bonds outstanding

CSHAREQ - credit union shares

CTDQ - credit union term deposits

CURQ - currency outside banks

LPREM1 - average spread between 1-3 year Canadas and RTB90 over the period 1960:01 to 1965:12 (≈ 60 bp.)

LPREM5 - average spread between 5-10 year Canadas and RTB90 over the period 1960:01 to 1965:12 (≈ 104 bp.)

MMBONDQ - 1-3 year Canada bonds

MMPAPERQ - total corporate short-term paper minus Canadian dollar bankers' acceptances

MMTBQ - treasury bills held by the general public

PCURQ - the rental price of CURQ, BPCAQ, BCANIQ, TMPCAQ
source: RBENCHQ / (1 + RBENCHQ)

PRACCQ - the rental price of BACCQ
source: (RBENCHQ-RACCQ) / (1 + RBENCHQ)

PRB3Q - the rental price of MMBONDQ
source: (RBENCHQ-RB3ADQ) / (1 + RBENCHQ)

PRCAQ - the rental price of BCAIQ
source: (RBENCHQ-RCAQ) / (1 + RBENCHQ)

PRCSBQ - the rental price of SBQ
source: (RBENCHQ-RCSBQ) / (1 + RBENCHQ)

PRCSQ - the rental price of BCSQ
source: (RBENCHQ-RCSBQ) / (1 + RBENCHQ)

PRDSDQ - the rental price of BDIQ
source: (RBENCHQ-RDSDCOM) / (1 + RBENCHQ)

PRFTQ - the rental price of BFTQ
source: (RBENCHQ-RFTQ) / (1 + RBENCHQ)

PRNPCQ - the rental price of BNPCQ
source: (RBENCHQ-RNPCQ) / (1 + RBENCHQ)

PRNPNCQ - the rental price of BNPNCQ
source: (RBENCHQ-RNPNCQ) / (1 + RBENCHQ)

PRNSWAPQ - the rental price of BOTHFQ
source: (RBENCHQ-RNSWAPQ) / (1 + RBENCHQ)

PROVSB - provincial savings bonds outstanding

PRPDQ - the rental price of BNCSXDIQ
source: (RBENCHQ-RPDQ) / (1 + RBENCHQ)

PRSWAPQ - the rental price of BSWAPQ
source: (RBENCHQ-RSWAPQ) / (1 + RBENCHQ)

PRTBQ - the rental price of MMTBQ
source: (RBENCHQ-RTBQ) / (1 + RBENCHQ)

PRTRCDQ - the rental price of TMXPCAQ, CCDDQ
source: (RBENCHQ-RTRCDQ) / (1 + RBENCHQ)

PRTRNCDQ - the rental price of QSBSDQ, TMNCDDQ, CNCDDQ
source: (RBENCHQ-RTRNCDQ) / (1 + RBENCHQ)

PRTR1Q - the rental price of TMTD1Q
source: (RBENCHQ-RTR1ADQ) / (1 + RBENCHQ)

PRTR5Q - the rental price of TMTD5Q
source: (RBENCHQ-RTR5ADQ) / (1 + RBENCHQ)

PRTR90Q - the rental price of CSHAREQ, CTDQ
source: (RBENCHQ-RTR90Q) / (1 + RBENCHQ)

PR90CDQ - the rental price of BNPFTQ, BNPBTQ
source: (RBENCHQ-R90CDQ) / (1 + RBENCHQ)

PR90Q - the rental price of MMPAPERQ
source: (RBENCHQ-R90Q) / (1 + RBENCHQ)

QSBSDQ - deposits at Quebec savings banks other than those of the federal Government

RACC30 - 30-day bankers' acceptance rate

RACCQ - 30-day bankers' acceptance rate

RB3ADQ - maturity-adjusted, liquidity-premium-adjusted rate on 1-3 year Canada bonds
source: RB3Q-SPRD2 + LPREM1

RB3Q - rate on 1-3 year Canada bonds

RBENCHQ - benchmark rate
source: RINDQ, except for 1973:04 and 1974:01 when R90Q is dummied in and 1980:04 when RB3ADQ is dummied in

RBFT90Q - rate on 90-day personal fixed-term deposits at banks

RBFT1Q - rate on 1-year personal fixed deposits at banks

RBFT1ADQ - maturity-adjusted, liquidity-premium-adjusted rate on 1-year personal fixed terms at banks
source: RBFT1Q-SPRD1 + LPREM1

RBFT2Q - rate on 2-year personal fixed terms at banks

RBFT2ADQ - maturity-adjusted, liquidity-premium-adjusted rate on 2-year personal fixed terms at banks
source: RBFT2Q-SPRD2 + LPREM1

RBFT5Q - rate on 5-year personal fixed terms at banks

RBFT5ADQ - maturity-adjusted, liquidity-premium-adjusted rate on 5-year personal fixed terms at banks
source: RBFT5Q-SPRD5 + LPREM5

RBTH1 - theoretical Canada bond rate at 1 year

RBTH2 - theoretical Canada bond rate at 2 years

RBTH5 - theoretical Canada bond rate at 5 years

RCAQ - effective rate on interest-bearing demand deposits other than those of the Receiver General

RCD90AOW - 90-day certificate of deposit rate at banks

RCSBADJQ - maturity-adjusted first year coupon rate on Canada Savings Bonds
source: RCSBQ-SPRD1

RCSBQ - first year coupon rate on Canada Savings Bonds

RCSDB - quoted rate on personal chequable savings deposits at banks

RCSQ - effective rate on personal chequable savings deposits at banks

RDSDCOMQ - quoted rate on daily interest savings deposits

RIND - McLeod Young Weir average of 10 Industrials

RINDAD - maturity-adjusted, liquidity-premium-adjusted McLeod Young Weir average of 10 Industrials
source: RIND-RINDCAN + RTB90

RINDCAN - theoretical Canadian bond rate at the maturity corresponding to the average term to maturity of the McLeod Young Weir 10 Industrial bonds

RFTQ - representative rate on personal fixed-term deposits at banks
source: RBFT90Q * BPROP90Q + RBFT1ADQ * BPROP1Q + RBFT2ADQ * BPROP2Q + RBFT5ADQ * BPROP5Q

RNPCQ - effective rate on non-personal, chequable deposits at banks

RNPNCQ - effective rate on non-personal, non-chequable deposits at banks

RNSWAPQ - 90-day covered Euro-dollar rate

RPDEFQ - effective rate on personal savings deposits at banks (adjusted for minimum monthly balance)
source: RPDQ * (BNCSMMBQ-BDIMMBQ) / BNCSXDIQ

RPDQ - quoted rate on personal savings deposits at banks

RSWAPQ - 90-day swap deposit rate

RTB90 - 90-day treasury bill rate (monthly)

RTBQ - 90-day treasury bill rate (quarterly)

RTRCD - quoted rate on chequable deposits at trust companies

RTRCDQ - effective rate on chequable deposits at trust companies
(adjusted for minimum monthly balance)
source: RTRCD-RCSDB + RCS

RTRNCDQ - rate on non-chequable savings deposits at trust companies

RTR90Q - 90-119 day trust company GIC rate

RTR1ADQ - maturity-adjusted, liquidity-premium-adjusted 1-2 year
GIC rate
source: RTR1Q-SPRD2 + LPREM1

RTR1Q - 1-2 year trust company GIC rate

RTR5ADQ - maturity-adjusted, liquidity-premium-adjusted 5-year
trust company GIC rate
source: RTR5Q-SPRD5 + LPREM5

RTR5Q - 5-year trust company GIC rate

RUS90 - 90-day Euro-dollar deposit rate

R90CDQ - rate on 90-day certificates of deposit

R90Q - 90-day finance company paper rate

SBQ - provincial plus Canada Savings Bonds outstanding
source: PROVSBQ + CSBQ

SPRD1 - theoretical spread between 1-year Canada bonds and
90-day treasury bills
source: RBTH1-RTB90

SPRD2 - theoretical spread between 2-year Canada bonds and
90-day treasury bills
source: RBTH2-RTB90

SPRD5 - theoretical spread between 5-year Canada bonds and
90-day treasury bills
source: RBTH5-RTB90

TMCDDQ - chequable demand deposits at TMLs

TMNCDDQ - non-chequable demand deposits at TMLs

TMPCAQ - PCA-like deposits at TMLs
source: TMCDDQ * TPROPCDQ

TMTD1Q - less-than-1-year term deposits at TMLs

TMTD5Q - greater-than-1-year term deposits at TMLs

TMSPCAQ - chequable savings-type deposits at TMLs
source: TMCDDQ-TMPCAQ

TPROPCDQ - proportion of trust company chequable deposits
that are personal chequing accounts

APPENDIX B

COMPOSITION OF THE MONETARY AGGREGATES

Aggregates	Quantities		Prices
	CURQ	(1)	PCURQ
	BPCAQ	(2)	PCURQ
	BCANIQ	(3)	PCURQ
	BCAIQ	(4)	PRCAQ
M1* = 1, 2, 3, 4			
	TMPCAQ	(16)	PCURQ
MM1** = M1, 16			
	BCSQ	(5)	PRCSQ
	BNPCQ	(6)	PRNPCQ
M1B = M1, 5, 6			
	TMXPCAQ	(17)	PRTRCDQ
	CCDDQ	(18)	PRTRCDQ
MM1B = MM1, 5, 6 17, 18			
	BNCSXDIQ	(7)	PRPDQ
	BDIQ	(8)	PRDSDQ
	BFTQ	(9)	PRFTQ
	BNPNCQ	(10)	PRNPNCQ
M2 = M1B, 7, 8, 9, 10			
	QSBSDQ	(19)	PRTRNCDQ
	TMNCDDQ	(20)	PRTRNCDQ
	TMTD1Q	(21)	PRTR1Q
	TMTD5Q	(22)	PRTR5Q

Aggregates	Quantities		Prices
	CSHAREQ	(23)	PRTR90Q
	CTDQ	(24)	PRTR90Q
	CNCDDQ	(25)	PRTRNCDQ
	SBQ	(26)	PRCSBQ
MM2 = MM1B, 7, 8, 9, 10, 19, 20, 21, 22, 23, 24, 25, 26			
	BNPBTQ	(11)	PR90CDQ
	BNPFTQ	(12)	PR90CDQ
M2C = M2, 11, 12			
MM2C = MM2, 12			
	BSWAPQ	(13)	PRSWAPQ
	BOTHFQ	(14)	PRNSWAPQ
M3 = M2C, 13, 14			
MM3 = MM2C, 13, 14			
	BACCQ	(15)	PRACCQ
L = M3, 15			
	MMPAPERQ	(27)	PR90Q
	MMTBQ	(28)	PRTBQ
	MMBONDQ	(29)	PRB3Q
LL = MM3, 11, 15, 27, 28, 29			

Notes: * M and L = Bank aggregates.
** MM and LL = System aggregates (bank deposits plus near-bank deposits).

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