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INVESTMENT: A SURVEY OF MODELS WITH SOME IMPLICATIONS FOR THE EFFECTS OF MONETARY POLICY

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ABSTRACT

Investment is one of several linkages through which monetary policy may influence the economy. The nature of the presumed linkage has been defined in various theories of investment, each apparently emphasizing a different channel of influence. It is useful, therefore, to understand the differences among the theories by assessing their strengths and weaknesses from both theoretical and empirical points of view, in order to arrive at a clearer understanding of the issues involved in determining the impact of monetary policy on aggregate demand.

The four models of investment surveyed in this report, chosen on the basis of popularity and vintage, are the accelerator, the cash flow, the securities valuation or Tobin's q, and the standard neoclassical models. Briefly, the accelerator model attributes variations in net investment to movements in the demand for output and disregards direct sensitivity to monetary or fiscal parameters. The cash flow model points to corporate profits and depreciation allowances as the major driving force behind investment demand. The securities valuation or Tobin's q model indicates that the driving force of investment is the market value of the real capital stock (as determined in the securities market) relative to its replacement value. In this model, monetary factors affect investment via their effect on the valuation of

capital in the stock market. The <u>standard neoclassical model</u> suggests two possible investment specifications, (a) as a function of output and relative factor prices, and (b) as a function of output and the relative prices of output and capital. Under specific conditions the two specifications are theoretically equivalent. In this model the implicit rental price of capital is defined to be a function of a number of policy levers, including the discount rate, thus providing a direct linkage to monetary policy.

These investment models are surveyed in a fashion similar to U.S. studies by Charles Bischoff and Peter Clark; that is, they are compared on the basis of motivating theory, estimation properties, intra- and extra-sample simulation performance and their implications for the effects of monetary policy.

The empirical findings of the survey indicate that there is not much difference among the models when they are assessed on their ability to track investment. When their theoretical properties are compared, however, the standard neoclassical model is found to be the most appropriate framework for determining the effects of monetary policy on investment as well as for future research.

The survey does not explore at least two important issues: the effect on investment of inflation and uncertainty and the impact of the rise in energy prices. These issues represent, in themselves, major research projects.

RÉSUMÉ

L'investissement est l'un des biais par lesquels la politique monétaire peut influencer l'activité économique. La nature du présumé lien entre l'investissement et la politique monétaire a été mise en lumière dans diverses théories de l'investissement, dont chacune privilégie un aspect différent. En évaluant les avantages et les inconvénients que ces théories présentent, tant au point de vue conceptuel qu'au point de vue empirique, on peut saisir les différences existant entre ces théories et mieux comprendre les problèmes qui se posent lorsqu'on essaie de déterminer l'incidence de la politique monétaire sur la demande globale.

Quatre types de modèles, choisis parmi les plus connus et les plus représentatifs, sont étudiés dans ce rapport : le modèle de l'accélérateur, le modèle de la marge brute d'autofinancement, le modèle de l'évaluation des titres, ou modèle q de Tobin, et le modèle néo-classique type. Le modèle de l'accélérateur explique les variations des investissements nets par les modifications de la demande de biens et services et ne fait aucun cas de l'influence directe des paramètres monétaires et fiscaux sur les investissements. Le modèle de la marge brute d'autofinancement considère les bénéfices des sociétés et les provisions pour amortissement du capital comme la principale force qui sous-tend les investissements. Le modèle de l'évaluation des stocks,

représenté par l'indice q de Tobin, considère comme élément moteur des dépenses d'investissement le rapport entre l'évaluation que le marché des valeurs fait du stock de capital réel et le coût de remplacement de ce stock. Dans ce modèle, les facteurs monétaires influencent l'investissement par le biais de leur incidence sur l'évaluation du stock de capital faite par les marchés boursiers. Le modèle néo-classique propose deux équations possibles de l'investissement. Dans la première, l'investissement est défini comme une fonction du volume de production et des coûts respectifs des facteurs, dans la deuxième, comme une fonction du volume de production, des prix relatifs des biens et services et du capital. Dans certaines conditions, ces deux équations sont théoriquement équivalentes. Dans ce modèle, le loyer implicite du capital est présenté comme une fonction d'un certain nombre de leviers, notamment le taux d'escompte, qui établit un lien direct avec la politique monétaire.

Ces modèles sont examinés selon une méthode comparable à celle qu'ont utilisée Charles Bischoff et Peter Clark dans leurs travaux sur l'économie américaine, c'est-à-dire qu'ils sont comparés en fonction de leur base théorique, de leur aptitude à expliquer le comportement des investissements aussi bien au cours de l'estimation qu'au cours de simulations et aussi en fonction de leurs implications au point de vue de la politique monétaire.

Il ressort de la présente étude que les modèles examinés ne présentent guère de divergence sous le rapport de leur aptitude à expliquer le comportement de l'investissement. Cependant, lorsqu'on compare leurs propriétés théoriques, on se rend compte que le modèle néo-classique permet mieux de déterminer les effets de la politique monétaire sur l'investissement et offre davantage de possibilités sur le plan de la recherche.

Au moins deux points importants ne sont pas abordés dans l'étude : l'effet que l'inflation et les incertitudes produisent sur l'investissement et l'incidence de la hausse des prix de l'énergie sur l'investissement. Ces points peuvent à eux seuls faire l'objet d'importants travaux de recherche.

INTRODUCTION

Investment is one of several linkages through which monetary policy may influence the economy. The nature of the linkage, however, is unclear, and theories of investment differ on which channel of influence is the correct one. To obtain a better understanding of the issues involved in determining the impact of monetary policy on aggregate demand, this study compares four investment models, assessing their strengths and weaknesses from both empirical and theoretical points of view.

The four models, chosen on the basis of their popularity and vintage, are the accelerator model, the cash flow model, the securities valuation or Tobin's q model, and the standard neoclassical model. They are surveyed in a fashion similar to the U.S. studies by Bischoff [7] and Clark [9]. In the first section of the report the motivating theory behind each model is presented and the respective model is derived. This is followed by a discussion of the differences or similarities between models, and the implications for monetary policy in each case. Next, the models are specified as equations, estimated and then simulated within and outside the estimation period. Their empirical properties are compared against each other as well as their respective priors.

All of the models were originally defined in terms of explaining the movement of net investment. Although the literature is notably sparse with respect to replacement investment, a digression is made on this topic since, typically modelled, replacement investment is estimated to be about 50 per cent of gross investment. The report concludes with a summary of the implications of the survey as well as suggestions for future work in this area.

1 THE MODELS

In this section the four main models are described, and the nature of the linkage between monetary policy and investment is pointed out. The problem of accounting for replacement investment is discussed at the end of the section.

A The Accelerator Model

The most stable argument in any investment equation relates to the accelerator principle. In its oldest form the accelerator principle assumes a fixed relationship between the stock of capital and output:

$$K_t = \kappa \cdot Q_{t-1} \tag{A1}$$

where

K_t = stock of capital at the start of the current period or at the end of the last period

Q_t = flow of output during the current period,

and

$$I_t = (1-B)K_{t+1} \tag{A2}$$

where

I_t = net investment

B = backward shift operator.

The assumptions underlying the relation in equation (A2) imply that the accelerator is inoperative when excess capacity exists and that there are lags between ordering and actual delivery.

The accelerator theory has been modified in a number of ways. Koyck modified the notion of timing in the model by introducing a model of adaptive expectations. He assumed that the capital stock was a function of expected future output, which could be modelled by a series of geometrically declining weights on past output:

$$K_t = \kappa(1-\lambda)(1+\lambda B+\lambda^2 B^2+\lambda^3 B^3+ \ldots +\lambda^k B^k+ \ldots)Q_{t-1}. \qquad \text{(A3)}$$

Applying what later came to be known as the Koyck transformation,

$$I_t = \kappa(1-\lambda)Q_t - (1-\lambda)K_t. \qquad \text{(A4)}$$

This specification for investment can also be arrived at using a partial stock adjustment approach. Basically, this view holds that in any given time period net investment reflects an adjustment from the existing stock of capital to some desired position. If we assume that the desired stock of capital is proportional to current output and that in any given time period only a partial adjustment, $\mu < 1$, to the desired position is achieved, then it can be shown that an expression identical to (A4) can be derived for $\mu=1-\lambda$.

If

$$K^d_{t+1} = \kappa Q_t, \tag{A5}$$

where K^d_{t+1} = desired capital stock and

$$I_t = \mu(K^d_{t+1}-K_t), \tag{A6}$$

then

$$I_t = \kappa\mu Q_t - \mu K_t. \tag{A7}$$

It is important to note that although both views, (A4) and (A7), generate the same functional form of the accelerator model, the Koyck transformation implies a moving average process on the error term. This is not the case in the stock adjustment specification.

Equation (A6) is commonly referred to as the flexible accelerator principle. It is flexible in that it permits an increasing rate of investment in response to larger gaps between K^d_{t+1} and K_t.

Why should $\mu < 1$, when $K^d_{t+1} = K_{t+1}$ is the static optimum? Assume a firm pays a penalty for having a capital stock different from K^d and incurs adjustment costs in trying to move to that level. The two costs can be incorporated into an overall loss function, A, where

$$A = f(K^d_{t+1}-K_{t+1}) + h(K_{t+1}-K_t), f'>0, h'>0. \tag{A8}$$

Thus the actual net investment undertaken is one that minimizes costs in the tradeoff where $f(\cdot)$ represents the forgone profits associated with not meeting the static optimum while $h(\cdot)$ represents the adjustment costs. The arguments for

$h(\cdot)$ might include installation costs, rising supply price for capital goods and production lags. Of interest in equation (A8) is the outcome if $g(.)$ is quadratic. This would imply that $\mu < 1$ is optimal in equilibrium and that investment should move the capital stock only part way towards its desired level in any one period.

It is evident that this theory yields no direct monetary linkages since, in the accelerator model, investment is not a function of such factors as the rental price of capital, the discount rate or relative factor prices. As Bischoff points out in [7], the model's pure dependence on output may result from technological rigidities that permit only one capital-output ratio for each product. On the other hand, the model may perform well relative to other models, not because of such technological rigidities, but because the others do not specify the precise way in which different factors determine the optimum capital-output ratio.

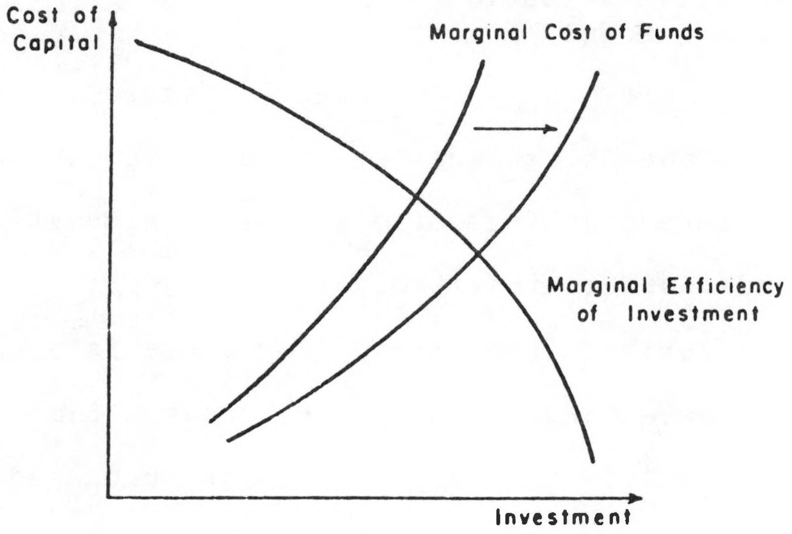
Output could, however, provide an important indirect link between investment and monetary policy insofar as monetary factors affect other expenditures and thus demand variations.

B The Cash Flow Model

Cash flow can affect corporate investment in two ways. First, increased cash flow suggests to the firm greater profitability in the future. In seeking to meet its objective of maximizing its own net worth, the firm increases its demand for capital. Second, increased cash flow permits greater use of internal funds as a financing source compared to equity or debt issues. Since the cost of financing increases as the firm moves successively from internal funds to debt and finally to equity, a marginal cost of funds schedule would slope upward. This concept, originally posited by Duesenberry [11], defines the cost of capital financing to be at the point of intersection of a marginal efficiency of investment schedule with a marginal cost of funds curve. An increase in profits therefore would be depicted as a shift in the marginal cost of funds schedule, and would result in more investment for the individual firm. (See Figure 1.)

Figure 1

COST OF CAPITAL FINANCING



In direct response to this hypothesis Modigliani and Miller [20] derive the conditions under which the cost of capital is independent of the source of financing. They argue that all the relevant information concerning a firm's financial position is incorporated in the share price of the firm; consequently, the relevant variable in determining investment is the cost of capital as determined in the equity market. The source of financing is not an issue.

Different versions of the model have emphasized different flows, i.e., profits versus cash flow. In any case, the monetary link exists only indirectly, to the extent that profits and expected profits are affected by financial shocks. Moreover, the influence of the accelerator is implicitly captured by this theory to the degree that profits are determined by aggregate demand.

C The Securities Valuation or Tobin's q Model

In his General Theory, [19] Keynes wrote: "... daily revaluations of the Stock Exchange, though they are primarily made to facilitate transfers of old investments between one individual and another, inevitably exert a decisive influence on the rate of current investment. For there is no sense in building up a new enterprise at a cost greater than that at which a similar existing enterprise can be purchased; whilst there is an inducement to spend on a new project what may seem an extravagant sum, if it can be floated off on the Stock

Exchange at an immediate profit." Thus the idea that investment is influenced by the value of physical capital, as determined in the securities market, relative to its replacement cost has been around for some time. Tobin, however, is credited as being its principal expounder and defender and has termed the ratio q.

In his essay "Principles of Debt Management" [24], Tobin is very clear on how q affects investment and how it is affected by monetary policy. Simply put, managers, seeking to maximize the market value of their corporations, will add to their fixed capital stock whenever such a marginal addition to the firm's market value, measured by the marginal increase in the stock, exceeds the actual cost or replacement value of the addition. In other words, investment will respond to the marginal productivity of capital stock relative to the rate of return on capital equity. If this ratio is greater than one, the market valuation of capital exceeds its replacement cost and there is an incentive to expand production, hence to invest. The numerator, the marginal productivity of capital, is determined by factor supplies, technology and expectations about the economy, none of which can be directly controlled by policy makers. The denominator is that rate at which households will hold the existing stock of capital, valued at current prices. "It is this rate of return, <u>the supply price of capital</u>, which the monetary and debt authorities may hope to influence through changing the supplies and yields of assets and debts that compete with real capital for place in the portfolios and balance sheets of economic units."

As Bischoff points out, there are at least two inherent problems in applying this theory. First, no information exists on the marginal effects on market valuation of increased spending on fixed capital. Instead, the average ratio of the market value of existing physical capital--as determined in the financial market--to its reproduction cost is used. Secondly, it is difficult to sort out the market valuation of physical capital from that of the rest of a firm's assets.

A more basic question emerges, however. Why won't arbitrage always instantaneously close any incipient gap between the present value of returns from investment projects calculated with market discount rates and their cost? Tobin's answer to this is that investment takes time and that the acquisition and installation of capital goods cost more, both on average and at the margin, for both individual firms and the economy at large, the faster the capital stock is expanded. In other words, the speed of additions to the capital stock and the level of q are correlated.

D The Standard Neoclassical Model

The neoclassical theory of investment reminds us that capital is but one factor in a firm's production process. The theory of the firm tells us that the combination of factor inputs will be dependent on their relative factor prices. While the rental price of capital is not observable since physical capital is generally purchased rather than rented, there is an implicit rental price of capital that arises from the opportunity cost of purchasing physical capital.

The expression for the rental price of capital is as follows:

$$c_t = p_t(r_t - g_t + \delta) \tag{D1}$$

where

p_t = price of investment goods

r_t = nominal discount rate (usually the long-term bond rate)

g_t = $(p_t - p_{t-1})/p_{t-1}$, proportional capital gain

δ = depreciation rate of physical goods.

This expression can be explained intuitively. Suppose a piece of capital equipment is purchased at price p in period t. Three components make up the user cost to its owner in each period of the life of the equipment. The first is the interest cost of the capital good, $p_t.r_t$, representing the opportunity cost of tying funds up in the capital good. Although the long-term bond rate is typically chosen to reflect this interest cost, the true cost is the next best forgone opportunity. Secondly, if the market price of a new piece of capital equipment is rising through time, the price of the old stock will be pulled up above the original purchase price less depreciation. This lowers the user cost of capital in any time period by an amount that corresponds approximately to the capital gain, $p_t \cdot g_t$. The final cost component of the user cost, in each time period related to this piece of equipment, is the amount of depreciation, $p_t \cdot \delta$.

A number of algebraic derivations of the standard neoclassical theory have been developed, all of which reflect one of two approaches: maximizing profits or minimizing costs.

Maximizing Profits. Following Jorgenson's example [17], assume a firm maximizes profits where

R_t = profits
P_t = price of output
Q_t = real output
w_t = nominal wage rate
N_t = labour input
K_t = capital input
I_t^G = gross investment
H_t = cash flow.

We require a production function and a definition of gross investment:

$$Q_t = f(K_t, N_t) \tag{D2}$$

$$I_t^G = K_{t+1} - K_t + \delta K_t \tag{D3}$$

In a static world, assuming constant factor utilization rates, profits are given by

$$\begin{aligned} R_t &= P_t Q_t - c_t K_t - w_t N_t \\ &= P_t f(K_t, N_t) - c_t K_t - w_t N_t. \end{aligned} \tag{D4}$$

The first order conditions for maximizing current profits are

$$\frac{\partial R_t}{\partial N_t} = P_t f^N(K_t^*, N_t^*) - w_t = 0 \tag{D5}$$

$$\frac{\partial R_t}{\partial K_t} = P_t f^K(K_t^*, N_t^*) - c_t = 0. \tag{D6}$$

where f^N and f^K are the partial derivatives with respect to N and K. Maximizing the present value of all future profits does not change these first order conditions. If the production technology is defined to be Cobb-Douglas, then

$$Q_t = AK_t^{\alpha} \cdot N_t^{1-\alpha} \tag{D7}$$

so that

$$f^K(K_t^*, N_t^*) = \alpha Q_t/K_t. \tag{D8}$$

Substituting (D8) into (D6) gives the optimal capital input:

$$K_t = \alpha \frac{P_t Q_t}{c_t} \tag{D9}$$

Jorgenson generates c_t by noting that the current market price for a unit of capital should reflect the discounted present value of its stream of future rental payments. Assuming constant interest rates, inflation and depreciation from time t to ∞, then:

$$p_t = \frac{c_t}{r_t - g_t + \delta} \qquad \text{(D10)}$$

Alternatively, Brechling [8] maximizes the present value of future cash flows:

$$V = \sum_{i=1}^{n} \left(\frac{H_i}{\prod_{k=1}^{i} (1+r_k)} \right) \qquad \text{(D11)}$$

where

$$\begin{aligned} H_t &= P_t Q_t - p_t I_t^G - w_t N_t && \text{(D12)} \\ &= P_t f(K_t, N_t) - p_t(K_{t+1} - K_t + \delta K_t) - w_t N_t . \end{aligned}$$

If we define $\overline{V}$,

$$\overline{V} = V.\left(\prod_{i=1}^{t} (1+r_i) \right),$$

then the first order conditions for maximizing the present value of future cash flows are

$$\frac{\partial \overline{V}}{\partial N_t} = P_t . f^N(K_t^*, N_t^*) - w_t = 0 \qquad \text{(D13)}$$

$$\frac{\partial \overline{V}}{\partial K_t} = -\,(1+r_t)\, p_{t-1} + P_t \cdot f^k(K_t^*, N_t^*) + p_t(1-\delta) = 0. \qquad \text{(D14)}$$

Noting that $g_t = (p_t - p_{t-1})/p_{t-1}$ and that

$$\frac{(1+r_t)}{(1+g_t)} = (1+r_t-g_t) \text{ for small values of } r_t \text{ and } g_t,$$

we obtain

$$P_t f^k(K_t^*, N_t^*) \doteq p_t(r_t - g_t + \delta). \tag{D15}$$

Again, use of the Cobb-Douglas production technology implies

$$\alpha P_t(Q_t/K_t) = p_t(r_t - g_t + \delta). \tag{D16}$$

The right-hand side of this equation is simply equation (D1). Of interest is that the Jorgenson derivation relates to profits and capital stock while the Brechling one uses cash flow and investment; however, both generate the same first order conditions for optimal factor usage. The two are equivalent because adjustment costs are not considered.

Minimizing costs. Rather than maximize profits, one can minimize costs subject to an exogenous level of output. This is, in fact, the usual starting point of interrelated factor demand theory, and the basis upon which the factor specifications are determined in the Bank of Canada's RDX2 quarterly econometric model. (See [1] and [2].)

Minimizing costs subject to an output constraint requires minimizing the following Lagrangean,

$$Z_t = w_t N_t + c_t K_t + \lambda(\overline{Q} - f(K_t, N_t)) \tag{D17}$$

where $\overline{Q}$ = exogenous level of output.

The first order conditions for minimizing costs are

$$\frac{\partial Z_t}{\partial N_t} = w_t - \lambda f^N(K_t^*, N_t^*) = 0 \tag{D18}$$

$$\frac{\partial Z_t}{\partial K_t} = c_t - \lambda f^K(K_t^*, N_t^*) = 0. \tag{D19}$$

Equations (D18) and (D19) imply that the ratio of the marginal products of the two factors must equal relative factor prices. Substituting this back into the production function and solving for capital generates the following equation for demand for capital, provided the production function is Cobb-Douglas,

$$K_t = \frac{\overline{Q}_t}{A} \left[\frac{1-\alpha}{\alpha} \frac{w_t}{c_t} \right]^{1-\alpha} \tag{D20}$$

It can be shown that the cost minimization and profit maximization solutions are equivalent when $P = \lambda$, where $\lambda = \partial Z_t / \partial \overline{Q}_t$ is the long-run marginal cost.

Translating the standard neoclassical model into an investment specification involves both setting the desired capital stock equal to either (D16) or (D20) and assuming that the desired capital stock for next the period, K^d_{t+1}, depends on current output and factor prices. Based on (D16), the stock adjustment formulation would yield

$$I_t = \mu((\alpha P_t Q_t / c_t) - K_t). \tag{D21}$$

Therefore, investment depends not only on planned output but also on the ratio of the output price to the implicit rental price of the services of capital. Based on (D20), however, the stock adjustment formulation would yield

$$I_t = \mu\left[(Q_t/A) \left[((1-\alpha)/\alpha)(w_t/c_t)\right]^{1-\alpha} - K_t \right] \tag{D22}$$

In (D22) investment would depend on planned output and on the ratio of the relative factor prices.

The direct monetary linkage in these specifications is quite obviously the discount factor in the implicit rental price definition. As the interest rate rises, the opportunity cost of holding physical capital rises, implying an increase in the user cost of capital; in response investment will fall.

E Replacement Investment

Most empirical investigations of fixed capital formation have partitioned gross investment into two broad categories, net and replacement. The stock adjustment model, upon which all previously mentioned theories are based, is basically a theory of net investment. From a survey of work done by others both within and outside the Bank of Canada it is clear that there is a widespread assumption that replacement investment is mechanistic and that it may be eliminated from the analysis by the subtraction of some proportion of estimated capital stock from gross investment:

$$I_t^G = I_t + I_t^R \tag{E1}$$

$$I_t = K_{t+1} - K_t \tag{E2}$$

$$I_t^R = \delta K_t \tag{E3}$$

where

I^G = gross investment

I = net investment

I^R = replacement investment.

This practice is not a trivial matter since 50% of gross investment assessed in RDX2 by this method is replacement investment. The scarcity of data on replacement investment, however, and the convenience of the proportional replacement hypothesis (PRH), depicted in equations (E1), (E2) and (E3) are justifications for its use. Since replacement investment is assumed independent of the direct influence of policy instruments, it will respond to public policy with long adjustment lags.

As data become available, tests of this hypothesis are being made. For example, in Feldstein's article [13] a number of tests of the PRH for the United States are made using a survey, conducted by McGraw-Hill, of the proportion of gross investment planned and intended for replacement. The conclusion is a rejection of the PRH for the short run. This is particularly worrisome given the size of replacement investment when modelled in this fashion. Unfortunately, tests of the nature described in [13] cannot be done for Canada since a comparable intentions survey was initiated only a few years ago by the Department of Industry, Trade and Commerce.

Such tests aside, a great deal of concern should be paid to the economics implied by the PRH. Do economic agents simply replace the capital that has depreciated without regard for current or anticipated economic conditions? Are not the factors that determine whether corporations wish to replace worn-out capital the same as those determining net investment? Would not the timing of replacement be related to such factors as financing and the current position in the business cycle? Clearly, as more survey data are collected, these questions must be addressed.

2 MONETARY POLICY CONSIDERATIONS

All of the models surveyed imply some role, direct or indirect, of monetary factors in the determination of investment. Of interest is the relationship of the monetary effects among the models. For at least three of the investment specifications already discussed, it can be demonstrated that, in terms of their implications for the effect of monetary policy, they are conceptually equivalent.

The standard neoclassical model of investment generated two specifications for the desired stock of capital, the first under the assumption of profit maximization and the second under the assumption of cost minimization. The duality theorem can be used to show that if either assumption leads to a finite optimum solution then the other assumption will also lead to a finite optimum solution and the objective functions will have equal values. In other words, the two assumptions lead to the same desired factor stocks, with the same implications for monetary policy. Under conditions of constant or increasing returns to scale this same conclusion holds given an exogenous level of output. Using a partial stock adjustment format the standard neoclassical model, assuming profit maximization, takes on the following form:

$$I_t = \mu(\alpha P_t Q_t / c_t - K_t). \qquad \text{(D21)}$$

If Tobin's q theory of investment is specified in the flexible accelerator format, it can be shown to be conceptually equivalent to (D21). Since q is scaleless, as noted by Bischoff, it is scaled in this paper by the capital stock. In other words,

$$I_t = \mu(q_t-1)K_t$$

where

q = market value of capital/replacement value of capital.

The replacement value of capital at the beginning of the current period is p_tK_t. The market value of capital can be modelled as the appropriate earnings stream divided by the appropriate real supply price of capital. Assuming the Cobb-Douglas production function in (D7), the stream of expected earnings accruing to capital would be αP_tQ_t. If the real supply price of capital is related to the nominal discount rate by $r_t-g_t+\delta$, then substitution of these relations into the above equation generates (D21), the standard neoclassical specification. Therefore, Bischoff's claim that q is a scaleless number is somewhat misleading; q can be interpreted as the ratio of desired to actual capital stock. The main advantage of q is that unobserved expectations of future output as well as the rental price of capital do not have to be approximated; they can be indirectly observed from the stock exchange evaluation. This, however, does not solve the forecaster's problem, since he must in turn explain q or the market value of capital, q_tK_t, as a function of $\alpha P_tQ_t/(r_t-g_t+\delta)$.

It is interesting that these three specifications of the desired capital stock, and by implication investment, are conceptually identical since these are the three specifications which can be interpreted as making specific allowances for monetary effects.

3 ESTIMATION RESULTS

All the preceding models were estimated using quarterly data, seasonally adjusted at annual rates. The results are presented in this section. Investment in machinery and equipment was estimated for the period 1962Q1 to 1977Q4. Investment in non-residential construction was estimated over the sample period 1967Q1 to 1977Q4. A documentation of the variables used in the paper is provided in Appendix A. The output variable, UGPP, which starts in period 1960Q1, proved to be the variable that constrained the starting observation of the estimation.

Both investment series, machinery and equipment and non-residential structures, were defined to exclude energy-related capital formation for three reasons (see [5]). First, energy investment tends to occur in large discrete quantities that can generate problems in identifying a stable lag structure; second, energy investment is characterized by long lags between the time of investment and the time the resulting capital stock is used in the production process, again implying problems for dynamic modelling; finally, many of the factors determining energy-related capital formation are non-market in nature.

It is important to note in the following regression analysis that the time subscripts on the capital stock have a different meaning from the interpretation used in the preceding discussion; that is, K_t now refers to the stock of capital at the end of the current time period. This shift in interpretation was required in order to be consistent with the definition employed in the data base. Using this interpretation, the stock adjustment principle--the formulation used in the specification of all the theories in this survey--would be written as

$$I_t = \mu(K_t^d - K_{t-1}).$$

It is also important to note, however, that the investment series used in the estimations represent gross investment flows. That is,

$$I_t = K_t - K_{t-1}$$

and

$$I_t^G = I_t + \delta K_{t-1}$$

so that in a stock adjustment specification of gross investment,

$$\begin{aligned} I_t^G &= \mu(K_t^d - K_{t-1}) + \delta K_{t-1} \\ &= \mu K_t^d + (\delta - \mu)K_{t-1}. \end{aligned}$$

In other words, the lagged coefficient on the capital stock does not equal $-\mu$, but $(\delta - \mu)$.

In RDXF [3], a more recent econometric model at the Bank of Canada, δ takes on the value of 8.39% per annum for machinery and equipment and 3.54% per annum for non-residential structures. The stock adjustment coefficient, μ, cannot be deduced from K^d since K^d is typically a function. The replacement rate should, therefore, be deducted from the coefficient on the lagged capital stock in order to obtain μ.

Values for K^d were determined by assuming that economic agents form their expectations based on past movements in the respective variables determing K^d for each particular model. The general procedure for identifying specific lag structures on explanatory variables was to subject the functional forms to a wide variety of alternative Almon lag restrictions. Final equation estimates were chosen on an assessment of lag structure significance and equation performance; no rigorous tests were employed to determine the optimal lag structure. References to the nature of the lag structure are consistent with RDX2 and RDXF, and are documented in the TSP[4] manual.

The classic assumption with respect to the disturbance term in an investment equation is that it is distributed identically and independently over time. However, there are reasons to expect the error term to be autocorrelated. In the stock adjustment model there are really two kinds of lagged adjustment responses. First, there is stock adjustment itself: firms will not adjust their actual capital stocks to the desired position immediately but they will aim at removing only a fraction of the discrepancy because of adjustment costs. Moreover, in each successive time period the desired stock position itself will also change. Second, there are lags in investment due simply to technical factors. The nature of fixed capital formation is such that, quite apart from economic considerations, investment takes time. Therefore, it is likely that the unexplained part of current investment is related to errors in explaining investment in previous time periods. In the estimation results that follow, without exception, every equation exhibits a high degree of autocorrelation and was estimated using the TSP [4] iterative non-linear least squares routine allowing for ρ, the autocorrelation coefficient.

A The Accelerator Model

The basic specification used to estimate an investment equation based on the accelerator model, consistent with equation (A7), is as follows:

$$I_t^G = \gamma_0 + \sum_{i=0}^{k} \beta_i \, UGPP_{t-1-i} + \gamma_1 \cdot K_{t-1} + \varepsilon_t \qquad \text{(A9)}$$

where

I^G = gross investment

K = capital stock

ε_t = random disturbance.

Equation (1) in Table 1 was chosen as representative of the accelerator model for investment in machinery and equipment. The numbers in brackets refer to t-values of the estimated coefficients. Extending the lag on UGPP, $k > 8$, did not add any more statistically significant coefficients or increase the statistical significance of the sum of the lags.

Equation (1) suggests a stock adjustment coefficient of about .22 per quarter; the long run desired capital output ratio is about 1.25. The strength of the autocorrelated error term, as in the other equations that follow, probably reflects both the technical dependence of investment from one period to the next and misspecification due to the omission of some other explanatory variable.

As an alternative to an Almon lag on UGPP, a Koyck lag response was estimated by including the lagged dependent variable:

$$IMEXE_t = \underset{(-.55)}{69.2695} + \underset{(3.85)}{.05575UGPP_{t-1}} - \underset{(-2.97)}{.0447KMEXE_{t-1}} \qquad (A10)$$

$$+ \underset{(11.96)}{.80539IMEXE_{t-1}}. \qquad \begin{array}{ll} \overline{R}^2 & = .98332 \\ D.W. & = 2.065 \\ S.E.E. & = 201.3 \end{array}$$

From an econometric point of view there is little difference between (1) in Table 1 and (A10). The overall fit of the two equations is roughly the same; when allowance for the lagged dependent variable in (A2) is made, the coefficient on output is .28 as compared to .27 in (1) and the stock adjustment parameter is .25 compared to .22 in (1).

Estimation of investment in non-residential structures generated equation (6) in Table 2. As is evident, this investment series generates much longer lags on output for given values of the capital stock, a finding consistent with other research. The indicated stock adjustment parameter is, however, quite high at .42; the long-run capital/output ratio is estimated at just below .75. A disturbing element from this estimation is the remaining significant amount of autocorrelation.

Table 1

INVESTMENT IN MACHINERY AND EQUIPMENT
1962Q3 - 1977Q4

Accelerator Model

$$IMEXE = \underset{(-1.04)}{-588.93} + \sum_{i=0}^{8} \beta_i UGPP_{t-1-i} - \underset{(-2.69)}{.202}KMEXE_{t-1} \qquad (1)$$

i	β_i	
i = 0	.032	(1.41)
1	.044	(3.85)
2	.049	(4.28)
3	.046	(3.42)
4	.040	(2.90)
5	.030	(2.61)
6	.020	(2.42)
7	.009	(2.29)
8	.003	(2.20)
	.273	Z2Z3

ρ = .79 (9.48)
D.W. = 1.89
$\bar{R}^2$ = .983
S.E.E. = 201.5

Cash Flow Model

$$IMEXE = \underset{(1.25)}{2127} + \sum_{i=0}^{10} \beta_i((YCR + CCAC\$)/PIME)_{t-1-i} \qquad (2)$$

$$+ \underset{*(1.18)}{.030}KMEXE_{t-1}$$

i	β_i		i	β_i	
i = 0	.066	(1.88)	i = 6	.014	(1.88)
1	.055	(1.88)	7	.009	(1.88)
2	.044	(1.88)	8	.005	(1.88)
3	.035	(1.88)	9	.002	(1.88)
4	.027	(1.88)	10	.001	(1.88)
5	.020	(1.88)		.277	Z2

ρ = .89 (11.75)
D.W. = 1.85
$\bar{R}^2$ = .980
S.E.E. = 219.3

Tobin's q Model

$$IMEXE = (\underset{(10.47)}{.105} + \sum_{i=0}^{8} \beta_i q1_{t-1-i})KMEXE_{t-1} \qquad (3)$$

i	β_i	
i = 0	.011	(3.22)
1	.009	(3.22)
2	.007	(3.22)
3	.005	(3.22)
4	.004	(3.22)
5	.002	(3.22)
6	.001	(3.22)
7	.001	(3.22)
8	.000	(3.22)
	.040	Z2

ρ = .86 (13.51)
D.W. = 1.90
$\bar{R}^2$ = .982
S.E.E. = 209.9

Table 1

INVESTMENT IN MACHINERY AND EQUIPMENT
1962Q3 - 1977Q4
(continued)

Unconstrained Neoclassical Model

Profit Maximization

$$IMEXE_t/KMEXE_{t-1} = \underset{(.62)}{.124} + \sum_{i=0}^{6} \beta_i \ln(UGPP)_{t-1-i} + \sum_{i=0}^{2} \Theta_i \ln(PGPP/RCME)_{t-1-i} - .326 \cdot \ln(KMEXE)_{t-1} \qquad (4)$$

	β_i		Θ_i	
i = 0	.060	(1.71)	.033	(1.89)
1	.073	(4.08)	.015	(1.89)
2	.071	(3.40)	.004	(1.89)
3	.059	(2.68)	.051	Z2
4	.040	(2.31)		
5	.021	(2.10)		
6	.006	(1.96)		
7	.331	Z2Z3		

ρ = .81 (10.23)
D.W. = 1.86
$\bar{R}^2$ = .869
S.E.E. = .005

Cost Minimization

$$IMEXE_t/KMEXE_{t-1} = \underset{(.20)}{.037} + \sum_{i=0}^{6} \beta_i \ln(UGPP)_{t-1-i} + \sum_{i=0}^{2} -\Theta_i \cdot \ln(RCME/(WNIC/(HAWMM.ELEFF_{t-1-i}))) - .329 \cdot \ln(KMEXE)_{t-1} \qquad (5)$$

	β_i		Θ_i	
i = 0	.056	(1.60)	-.032	(-1.86)
1	.074	(4.00)	-.014	(-1.86)
2	.074	(3.49)	-.004	(-1.86)
3	.062	(2.81)	-.050	Z2
4	.043	(2.46)		
5	.023	(2.25)		
6	.066	(2.12)		
	.337	Z2Z3		

ρ = .82 (10.78)
D.W. = 1.86
$\bar{R}^2$ = .869
S.E.E. = .005

Table 2

INVESTMENT IN NON-RESIDENTIAL CONSTRUCTION
1967Q1 - 1977Q4

Accelerator Model

$$INRCXE_t = \underset{(4.64)}{7416.5} + \sum_{i=0}^{18} \beta_i \left[.25 \sum_{j=0}^{3} UGPP_{t-j}\right]_{t-1-i} - \underset{(-4.07)}{.412KNRCXE_{t-1}} \tag{6}$$

	β_i			β_i	
i = 0	.018	(2.57)	i = 10	.017	(3.32)
1	.021	(4.22)	11	.015	(3.21)
2	.024	(5.39)	12	.012	(3.13)
3	.025	(5.38)	13	.009	(3.05)
4	.026	(4.90)	14	.007	(2.98)
5	.026	(4.44)	15	.005	(2.93)
6	.025	(4.08)	16	.003	(2.88)
7	.024	(3.82)	17	.001	(2.84)
8	.022	(3.61)	18	.000	(2.81)
9	.020	(3.44)		.299	Z2Z3

ρ = .70 (7.71)
D.W. = 1.28
$\bar{R}^2$ = .959
S.E.E. = 82.9

Cash Flow Model

$$INRCXE_t = \underset{(2.02)}{1476.6} + \sum_{i=0}^{10} \beta_i \left[.25 * \sum_{j=0}^{3} ((YCR_{t-j} + CCAC\$_{t-j})/PINRC_{t-j})\right]_{t-1-i} \tag{7}$$

$$- \underset{(-.42)}{.012KNRCXE_{t-1}}$$

	β_i			β_i	
i = 0	.017	(.62)	i = 6	.030	(2.71)
1	.032	(1.83)	7	.022	(2.58)
2	.040	(2.99)	8	.014	(2.48)
3	.043	(3.25)	9	.007	(2.35)
4	.042	(3.08)	10	.002	(2.35)
5	.037	(2.87)		.286	Z2Z3

ρ = .78 (8.73)
D.W. = 1.20
$\bar{R}^2$ = .950
S.E.E. = 99.3

Tobin's q Model

$$INRCXE_t = (\underset{(15.36)}{.054} + \sum_{i=0}^{10} \beta_i q1_{t-1-i}) KNRCXE_{t-1} \tag{8}$$

	β_i			β_i	
i = 0	-.001	(-.70)	i = 6	.003	(4.36)
1	.001	(1.16)	7	.003	(4.17)
2	.003	(4.57)	8	.002	(4.04)
3	.004	(5.54)	9	.001	(3.94)
4	.004	(5.04)	10	.000	(3.86)
5	.004	(4.63)		.023	Z2Z3

ρ = .67 (7.80)
D.W. = 1.31
$\bar{R}^2$ = .959
S.E.E. = 88.8

Table 2

INVESTMENT IN NON-RESIDENTIAL CONSTRUCTION
1967Q1 - 1977Q4
(Continued)

Unconstrained Neoclassical

Profit Maximization

$$INRCXE_t/KNRCXE_{t-1} = \underset{(4.44)}{.510} + \sum_{i=0}^{8} \beta_i \ln(UGPP)_{t-1-i} \qquad (9)$$

$$+ \sum_{1=0}^{4} \Theta_i \ln(PGPP/RCNR)_{t-1-i}$$

$$- .115 \cdot \ln(KNRCXE)_{t-1}$$

	β_i		Θ_i	
i = 0	-.020	(-1.69)	.008	(2.87)
1	.002	(.31)	.005	(2.87)
2	.015	(3.21)	.003	(2.87)
3	.021	(3.54)	.001	(2.87)
4	.021	(3.46)	.001	(2.87)
5	.018	(3.39)		
6	.013	(3.34)		
7	.007	(3.30)		
8	.002	(3.27)		
	.078	Z2Z3	.019	Z2

ρ = .64 (7.20)
D.W. = 1.43
$\bar{R}2$ = .901
S.E.E. = .002

Cost Minimization

$$INRCXE_t/KNRCXE_{t-1} = \underset{(4.16)}{.484} + \sum_{i=0}^{8} \beta_i \ln(UGPP)_{t-1-i} \qquad (10)$$

$$+ \sum_{i=0}^{4} \Theta_i \ln(RCNR/(WNIC/(HAWMM.ELEFF_{t-1-i})))$$

$$- \underset{(-3.41)}{.118} \cdot \ln(KNRCXE)_{t-1}$$

	β_i		Θ_i	
i = 0	-.022	(-1.90)	-.009	(-3.03)
1	.001	(.25)	-.006	(-3.03)
2	.015	(3.44)	-.003	(-3.03)
3	.022	(3.90)	-.001	(-3.03)
4	.022	(3.84)	-.001	(-3.03)
5	.019	(3.77)	-.020	(-3.03)
6	.013	(3.72)	-.040	Z2
7	.007	(3.67)		
8	.002	(3.64)		
	.081	Z2Z3		

ρ = .65 (7.78)
D.W. = 1.41
$\bar{R}^2$ = .903
S.E.E. = .002

B The Cash Flow Model

The explanatory variable in equation (2) of Table 1 is retained corporate profits plus capital consumption allowances, YCR + CCAC$, deflated by the investment goods price index. The equation is inconsistent with a stock adjustment format since, even after allowing for the depreciation rate, the coefficient on the lagged capital stock is positive, even though not significantly different from zero.

Equation (7) in Table 2 represents the final specification for non-residential structures. As for the accelerator model, autocorrelation is still quite strong. There is a large difference between the stock adjustment coefficients suggested by equations (6) and (7). In separate regressions, the output variable was added to equations (2) and (7); in both cases output dominated the equations while the profit variable was statistically insignificant.

C The Securities Valuation or Tobin's q Model

Since q is not an observable variable in the economy, estimation results will be dependent on the definition of this variable. In RDX2 a measure of this concept can be obtained by dividing VKB, the market value of capital, by KB$, its replacement value. However, since no such variable, VKB, is defined in the more current RDXF, two measures of q were defined instead. The first, q1, is defined as follows:

$$q1 = \frac{.25 \sum_{i=0}^{3} ((YCR_{t-i} + YDIVII_{t-i} + YDIVF_{t-i})TSEPE_{t-i})}{PIME_t KME_t + PINRC_t KNRC_t + PKIB_t KIB_t} \qquad (C1)$$

TSEPE is the Toronto Stock Exchange price-earnings ratio. Equation (C1), therefore, provides an estimate of the market value of capital by discounting an earnings stream with the earnings-price ratio. Thus this index uses the value of equity as a measure of the market value of capital. A second measure of q is defined in (C2):

$$q2 = \frac{.25 \sum_{i=0}^{3} ((YCR_{t-i} + YDIVII_{t-i} + YDIVF_{t-i})TSEPE_{t-i}) + \frac{ECINT}{RL10P} .01}{PIME_t KME_t + PINRC_t KNRC_t + PKIB_t KIB_t} \tag{C2}$$

ECINT = net interest payments by private corporations, and
RL10P = average rate of interest on ten provincial bonds.

In other words, q2 includes measures of debt-financed capital.

Historically, these two definitions are so close that plots of the series almost lie on top of each other; q1 is plotted in Figure 2.

As the ratio of desired to actual capital stock, q pertains to the total capital stock; it cannot be allocated specifically to machinery and equipment and non-residential structures. The general functional form was

$$I_t^G = (\gamma_0 + \sum_{i=o}^{n} \beta_i q_{t-1-i}) K_{t-1} + \varepsilon_t. \tag{C3}$$

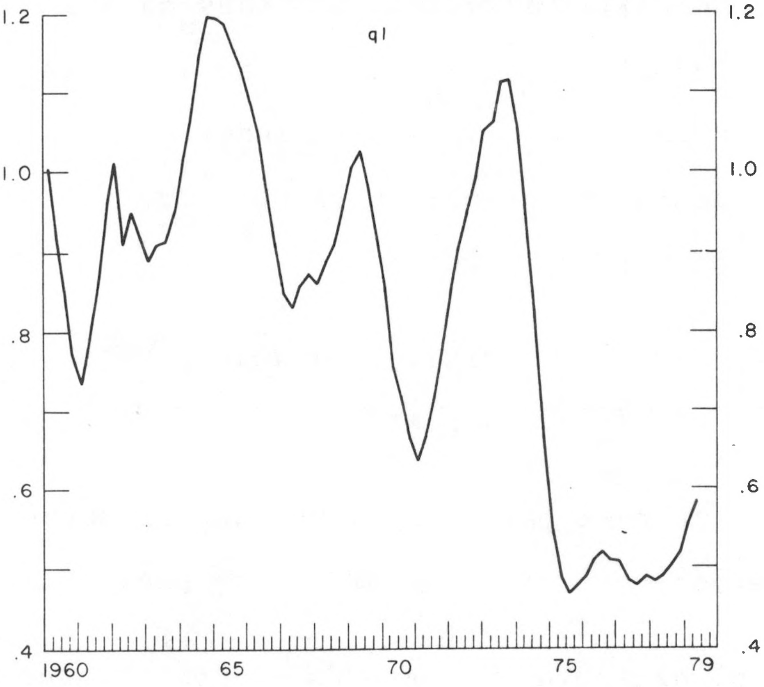
The final specifications chosen are equations (3) and (8) in Tables 1 and 2 respectively.

D The Standard Neoclassical Model

Estimation of this version of investment theory was

Figure 2

A MEASURE OF TOBIN'S q



preceded by the development of measures of the implicit rental price of capital. The definition employed is taken directly from Jorgenson and work done at the Bank of Canada. The basic definition of the rental price is

$$RC_t = p_t(RHOR_t(1-u_t) + \delta)(1-u_t Z_t)/(1-u_t)$$

where

p = price of investment goods

RHOR = real supply price of capital

u = corporate tax rate

δ = depreciation rate of physical good

Z = present value of depreciation allowances.

The general specification used in estimation consistent with (D21) is as follows:

$$I_t^G = \gamma 0 + \sum_{i=0}^{n} \beta_i (\mathrm{PGPP} \cdot \mathrm{UGPP}/\mathrm{RC})_{t-1-i} + \gamma 1 K_{t-1} + \varepsilon_t \qquad \text{(D23)}$$

The lag structure on the ratio of the value of output to the implicit rental price of capital was very difficult to identify econometrically for machinery and equipment. A battery of alternative Almon lag restrictions were imposed on the profile of β_i's; however, in general, the coefficients went negative at i=3. The lags were pushed out from i=0 to i=20 with this same general result prevailing.

In addition to this estimation problem, the output and factor price elasticities were significantly different from their expected values. The anticipated output and factor price elasticities from (D21) is 1.0 but the estimated response was only .7.

Final estimated equations for both machinery and equipment and non-residential structures are shown in equations (D24) and (D25).

$$\mathrm{IMEXE}_t = \underset{(1.13)}{1823.2} + \sum_{i=0}^{5} \beta_i \left(.25 \cdot \sum_{j=0}^{3} (\mathrm{PGPP}_{t-j})\ (\mathrm{UGPP}_{t-j}/\mathrm{RCME}_{t-j})_{t-1-i}\right)$$

$$+ \underset{(.34)}{.015} \cdot \mathrm{KMEXE}_{t-1} \qquad \text{(D24)}$$

	β_i	
i = 0	.163	(2.08)
1	.113	(2.08)
2	.072	(2.08)
3	.047	(2.08)
4	.018	(2.08)
5	.004	(2.08)
	.411	Z2

ρ = .90 (12.57)
D.W. = 1.85
$\bar{R}^2$ = .981
S.E.E. = .217

$$INRCXE_t = \underset{(1.02)}{710.6} + \sum_{i=0}^{4} (.25 \cdot \sum_{j=0}^{3} (PGPP_{t-j})\ (UGPP_{t-j}/RCNR_{t-j}))_{t-1-i}$$

$$+ \underset{(1.64)}{.034} \cdot KNRCXE_{t-1} \qquad (D25)$$

	β_i
i = 0	.103 (2.25)
1	.066 (2.25)
2	.037 (2.25)
3	.016 (2.25)
4	.004 (2.25)
	.225 Z2

ρ = .77 (8.83)
D.W. = 1.09
$\overline{R}^2$ = .946
S.E.E. = 103.3

Following the original suggestion by Bischoff [6], the neoclassical model was respecified with separate lags on the output and factor price terms in a manner consistent with linear stock adjustment. This was done to investigate the possibility that restricting both the relative price and the output variables to the same lag structure might bias the coefficients. Starting with the stock adjustment model for gross investment,

$$I^G_t = \mu(K^d_t - K_{t-1}) + \delta K_{t-1} \qquad (D26)$$

and dividing both sides of (D25) by K_{t-1} gives

$$I^G_t/K_{t-1} = \mu((K^d_t/K_{t-1}) - 1) + \delta. \qquad (D27)$$

Recognizing that for values of (K^d_t/K_{t-1}) close to one,

$$\ln K^d_t - \ln K_{t-1} = K^d_t/K_{t-1} - 1 \qquad (D28)$$

then equation (D26) can be rewritten as

$$I^G_t/K_{t-1} = \mu(\ln K^d_t - \ln K_{t-1}) + \delta. \qquad (D29)$$

Using this functional form and deriving specifications of K^d_t from (D16) and (D20), profit-maximizing and cost-minimizing versions of this unconstrained neoclassical model were estimated for both IMEXE and INCRXE. The estimated equations appear in Table 1 as (4) and (5) for IMEXE and in Table 2 as (9) and (10) for INRCXE. The equations are referred to as the unconstrained neoclassical model to differentiate them from (D24) and (D25). The estimation differences between the profit-maximizing and cost-minimizing versions of this model are virtually zero, indicating the high correlation between output price and the rental price (wage rate) of labour. In addition, the same functional format for INRCXE yields a much slower accelerator than for IMEXE, as evidenced by the coefficient on the lagged stock term.

[illegible]

[illegible]

4 SIMULATION RESULTS

Equations (1) to (10) in Tables 1 and 2 were dynamically simulated through their relevant sample periods. The results are plotted in Figures 3 and 4 on the following pages, and the root mean squared errors are tabulated in Table 3 on page 43. Simulated values for the lagged capital stocks were based on the following equations:

$$KMEXE_t = .97965KMEXE_{t-1} + IMEXE_t/4$$

$$KNRCXE_t = .99125KNRCXE_{t-1} + INRCXE_t/4.$$

The investment variables in these two equations also use their simulated values. In addition the ρ value, estimated for each equation, is applied against each previous period's error.

For both IMEXE and INRCXE, the intra-sample results indicate that superior tracking is obtained using either version of the unconstrained neoclassical specification, followed by the accelerator model, Tobin's q and the cash flow model, in that order. However, in the extra-sample simulations, plotted in Figure 5 on page 42, results show ranking changes for both IMEXE and INRCXE. For IMEXE the preferred ranking of models is the cash flow, Tobin's q, the accelerator and the unconstrained neoclassical model. For INRCXE the ranking changes to the accelerator, Tobin's q, the unconstrained neoclassical and the cash flow model. A striking feature of the extra-sample results is the nature of the simulation error. After 1978Q2 all the equations for IMEXE underpredict actual investment in machinery and

Figure 3

DYNAMIC INTRA-SAMPLE SIMULATIONS: INVESTMENT IN MACHINERY AND EQUIPMENT 1962Q3 - 1977Q4

Millions of Dollars

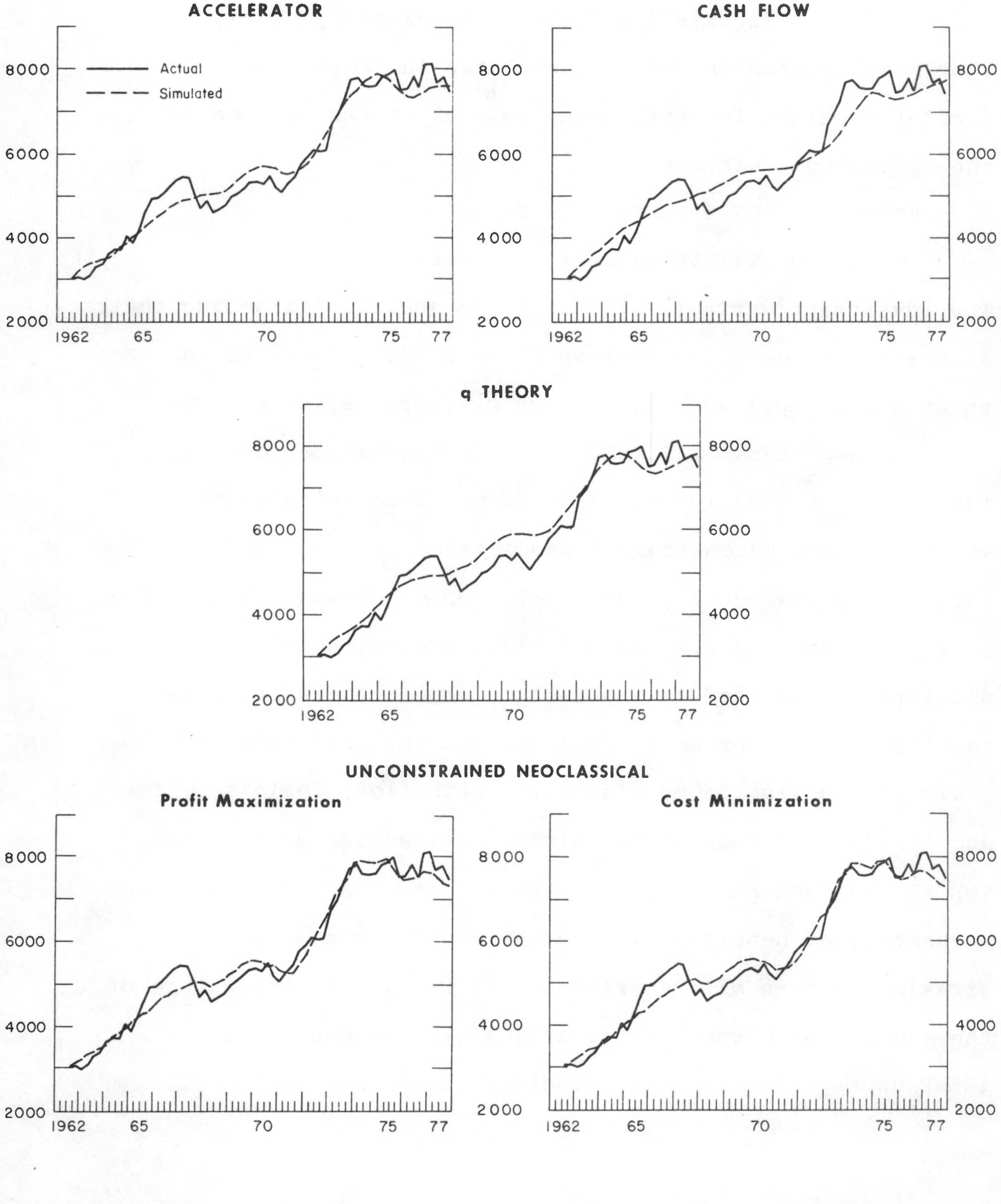


Figure 4

DYNAMIC INTRA-SAMPLE SIMULATIONS:
INVESTMENT IN NON-RESIDENTIAL STRUCTURES
1967Q1 - 1977Q4

Millions of Dollars

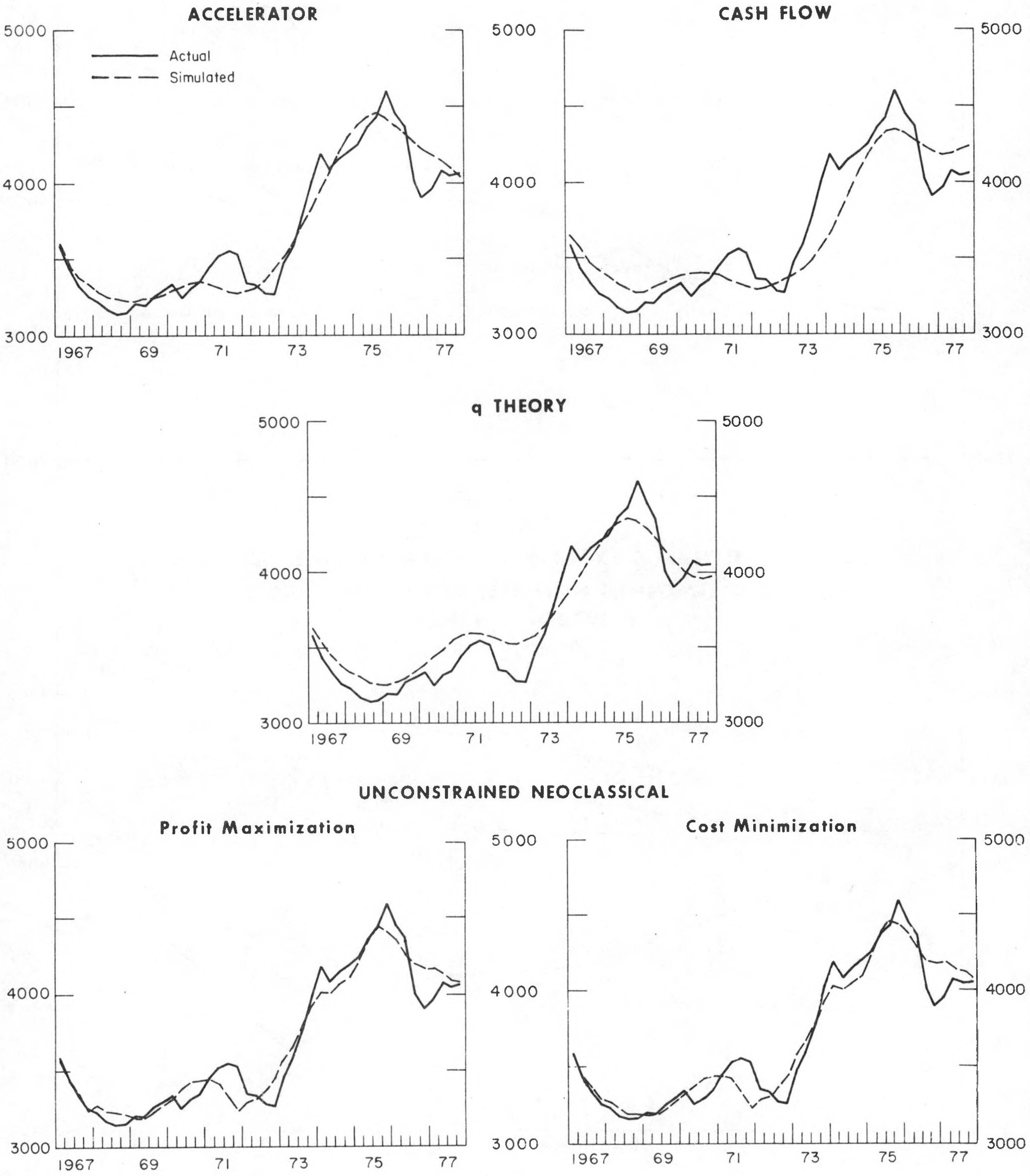


Figure 5

DYNAMIC EXTRA-SAMPLE SIMULATIONS:
INVESTMENT IN MACHINERY AND EQUIPMENT
1978Q1 - 1979Q2

Millions of Dollars

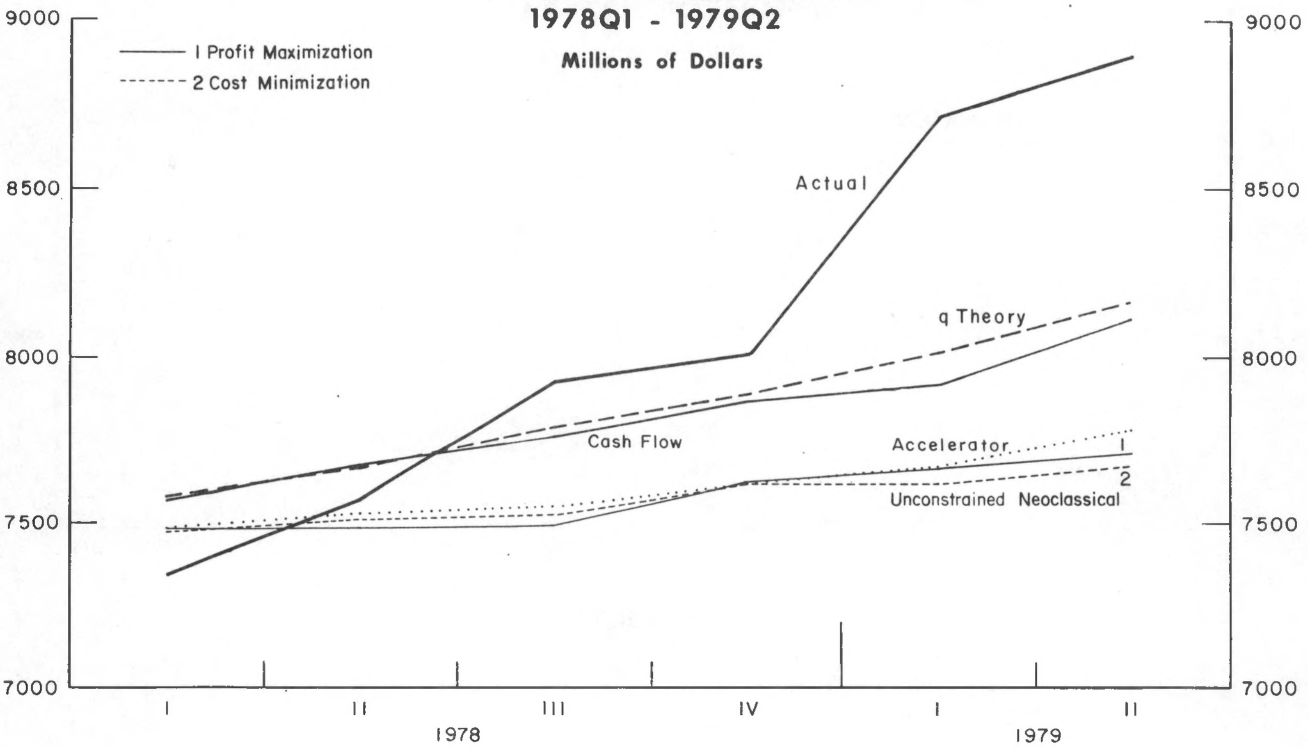


Figure 6

DYNAMIC EXTRA-SAMPLE SIMULATIONS:
INVESTMENT IN NON-RESIDENTIAL STRUCTURES
1978Q1 - 1979Q2

Millions of Dollars

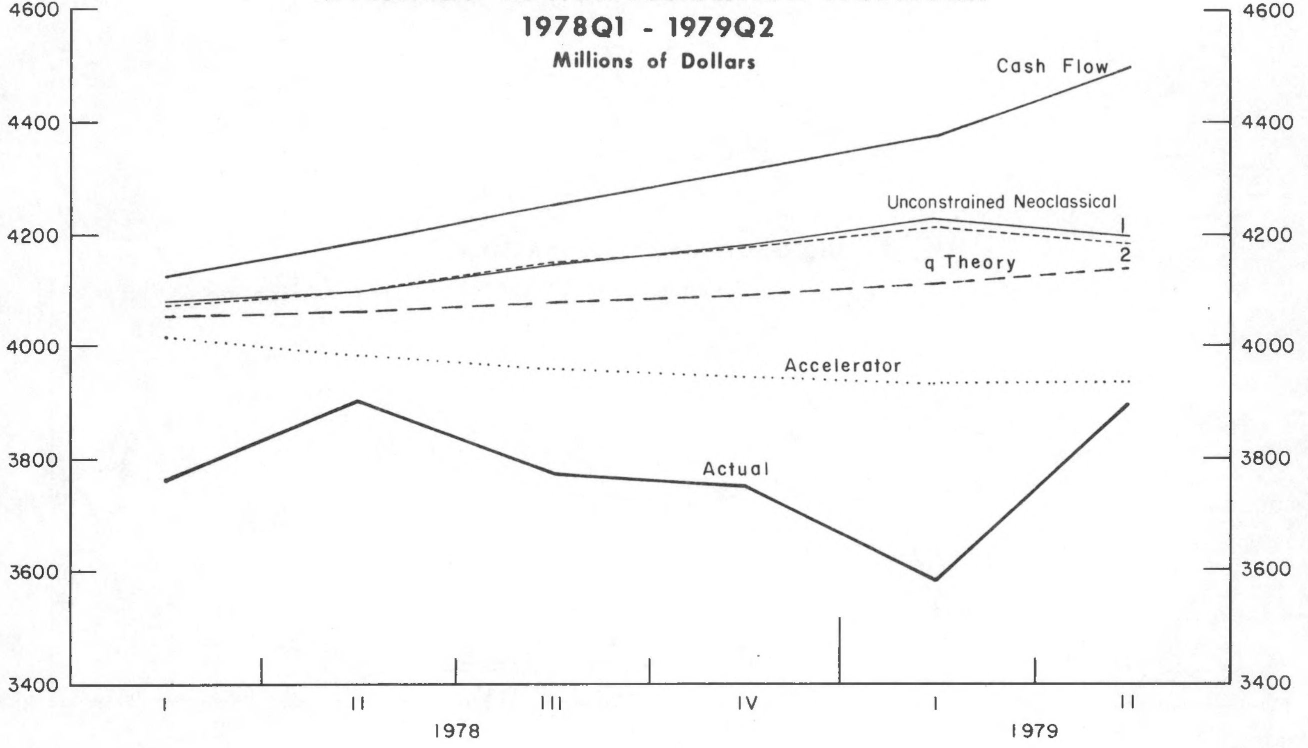


Table 3

ROOT MEAN SQUARED ERRORS FROM INVESTMENT SIMULATIONS*

(Millions of dollars)

Investment in Machinery and Equipment (IMEXE)

Model	1962Q3-1977Q4	1978Q1-1979Q2
Accelerator	308	693
Cash Flow	399	429
Tobin's q	369	450
Unconstrained Neoclassical		
Profit Maximization	276	713
Cost Minimization	279	730

Investment in Non-Residential Structures (INRCXE)

Model	1967Q1-1977Q4	1978Q1-1979Q2
Accelerator	125	210
Cash Flow	181	531
Tobin's q	134	332
Unconstrained Neoclassical		
Profit Maximization	114	379
Cost Minimization	114	374

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* Capital stock endogenous.

equipment, while all the equations for INRCXE overpredict actual investment in non-residential structures.

Additional simulations were done to determine the elasticity of investment with respect to each explanatory variable for each model. The results are tabulated in Table 4.

Table 4

ELASTICITIES* OF GROSS INVESTMENT

IMEXE Variable	Model	Years After Change					Long-Run Elasticity
		1	2	3	4	5	
UGPP	Accelerator	1.67	2.22	1.82	1.56	1.4	1.04
	Unconstrained Neoclassical						
	Profit Maximixation	1.93	1.95	1.62	1.42	1.31	1.02
	Cost Minimization	1.94	2.00	1.65	1.44	1.32	1.03
YCR+CCAC$	Cash Flow	.33	.40	.41	.42	.43	.58
q1	Tobin's q	.25	.34	.42	.44	.44	.66
RCME	Unconstrained Neoclassical						
	Profit Maximization	-.36	-.29	-.24	-.21	-.20	-.16
	Cost Minimization	-.35	-.28	-.23	-.20	-.19	-.15

INRCXE Variable	Model	Years After Change					Long-Run Elasticity
		1	2	3	4	5	
UGPP	Accelerator	.67	1.73	2.28	2.24	1.86	.91
	Unconstrained Neoclassical						
	Profit Maximization	.22	1.00	.99	.95	.98	.83
	Cost Minimization	.22	1.04	1.02	.99	1.02	.85
YCR+CCAC$	Cash Flow	.16	.53	.69	.70	.69	.76
q1	Tobin's q	.02	.19	.30	.33	.32	.35
RCNR	Unconstrained Neoclassical						
	Profit Maximization	-.23	-.24	-.23	-.22	-.23	-.19
	Cost Minimization	-.24	-.25	-.24	-.24	-.24	-.20

---

* Elasticities were determined by shocking explanatory variables 1% above their control value and observing the dynamic simulated response. Long-run elasticities are defined as the elasticity at the end of the simulation period. If a model is not listed opposite a variable, an elasticity of zero is implied.

It is apparent from Table 4 that the estimated long-run elasticities of the three relevant models, the accelerator and both versions of the unconstrained neoclassical, are very close to 1.0. Theoretically the output elasticity should be 1.0 as demonstrated in Appendix B. In terms of dynamics, the accelerator exhibits a strong acceleration and then tapers off; in the unconstrained neoclassical model, investment rises more sharply in the first year and then tapers off more quickly to roughly the same long-run elasticity.

The cash flow equations, although they are not specified as a function of UGPP, contain an implicit elasticity with respect to output. Assuming a long-run profit elasticity of 1.0 with respect to a change in output (obtained from RDXF) the elasticities reported for the cash flow would seem to be quite low; however, the short-run elasticity of profit with respect to output is greater than 1.0, given the residual nature of profits, and this may explain the results obtained.

The elasticities of investment with respect to ql and the rental price of capital are particularly interesting since these variables provide monetary linkages. The elasticity of IMEXE with respect to a change in q moves slowly initially and approaches a value of .66, which is less than the theoretical value of 1.0. That it is may be indicative of the problem with defining a measure of q; nevertheless, the result is particularly interesting since it is very close to the elasticity of investment with respect to the desired capital stock in the standard neoclassical model, equations (D23) and (D24), as suggested it should be in Chapter 2.

Finally, the rental price elasticities in the unconstrained neoclassical model are discouragingly low. This problem, however, could be largely due to the construction of an unknown variable, the rental price, as in the case of the q construct.

5 CONCLUSION

While reasonably extensive, this survey has left unexplored two key issues that should not be ignored in assessing the impact of monetary policy on investment. They were not addressed in the preceding chapters because they are in themselves major research projects.

The first issue deals with the effects of inflation on investment. Since nominal interest payments on debt are tax deductible, inflation generates a downward effect on the real net-of-tax cost of debt, vis-à-vis the before-tax cost, and thus serves to stimulate investment. However, historic cost accounting of depreciation allowances serves to dampen profitability and investment spending. The standard neoclassical theory of the firm offers a framework within which these opposing effects can and should be examined.

The other main issue generally ignored in this survey focuses on the effect that recent large movements in the price of energy have had on capital formation. Again the standard neoclassical theory of investment offers a framework of analysis. Energy can be thought of as a factor of production; however, preliminary work suggests that a production function more sophisticated than the Cobb-Douglas may be required.

What can be concluded from this survey, given these caveats? Clearly, the econometric results point strongly to the importance of output as the underlying determinant of investment, but there is not much difference among the models when they are assessed on their ability to track investment; each captures the trend and broad cyclical variations. Their theoretical properties do, however, provide a clearer choice. It was shown that the models in which some explicit recognition of monetary policy is provided are conceptually equivalent, a result that received some empirical support. The neoclassical model stands out as the most tractable in terms of analysis. Moreover, it includes all the arguments of the other models and in a somewhat more rigorous fashion. While its estimation shortcomings may arise from problems with a variable construct, i.e., the rental cost of capital, as well as from a failure to incorporate the key issues just mentioned, this model can distinguish the price and the income effects associated with monetary policy. Furthermore, its foundation appears to be a useful framework for future research.

Appendix A

SERIES, SOURCES AND NOTES

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CCAC$   (END=I05)

        CAPITAL CONSUMPTION ALLOWANCES,CORPORATIONS

        SOURCE:  NATIONAL INCOME AND EXPENDITURE ACCOUNTS
                 (UNPUBLISHED)--B815500
                 $MILLIONS  (CURRENT)  S.A.A.R.
                 AVAILABLE FROM 1952:1

HAWMM   (END=N02)

        AVERAGE WEEKLY HOURS WORKED IN MINING AND MANUFACTURING

        SOURCE:  GENERATED IN 'COMPLX' AT THE BANK OF CANADA FROM
                 STATSCAN LABOUR DIVISION DATA
                 (PUBLISHED AND UNPUBLISHED)
                 HAWMM=(NMI*HAWMI+NMF*HAWMF)/(NMI*NMF)
                      =((B802503*H802400)+(B802504*D4870))/
                        (B802503*H802504)
                 HOURS PER WEEK  S.A.
                 AVAILABLE FROM 1961:1

IMEXE   (END=I60)

        BUSINESS INVESTMENT IN MACHINERY AND EQUIPMENT
            EXCLUDING ENERGY INVESTMENT

        SOURCE:  GENERATED IN 'COMPLX' AT THE BANK OF CANADA
                 FROM NATIONAL INCOME  EXPENDITURE ACCOUNTS AND
                 AND RESEARCH DEPT. DATA
                 (UNPUBLISHED)
                 IMEXE=IME-IMENRG
                 $MILLIONS  (1971 DOLLARS)  S.A.A.R.
                 AVAILABLE FROM 1952:1

INRCXE  (END=I61)

        BUSINESS INVESTMENT IN NON-RESIDENTIAL CONSTRUCTION
           EXCLUDING ENERGY INVESTMENT

        SOURCE:  GENERATED IN 'COMPLX' AT THE BANK OF CANADA FROM
                 NATIONAL INCOME AND EXPENDITURE ACCOUNTS AND
                 RESEARCH DEPT. DATA
                 (UNPUBLISHED)
                 INRCXR=INRC-INRCNR
                 $MILLIONS  (1971 DOLLARS)  S.A.A.R.
                 AVAILABLE FROM 1952:1

KMEXE   (END=I62)

        STOCK OF NON-FARM MACHINERY AND EQUIPMENT
          EXCLUDING ENERGY STOCKS

        SOURCE:  GENERATED IN 'COMPLX' AT THE BANK OF CANADA FROM
                 NATIONAL INCOME AND EXPENDITURE ACCOUNTS
                 AND RESEARCH DEPT. DATA
                 (UNPUBLISHED)
                 SET KMEXE(52:1) = 17192.49 - 1425.5
                 DEFINED AS BELOW
                 $MILLIONS  (1971 DOLLARS)  S.A.Q.R.
                 AVAILABLE FROM 1952:2
```

KNRCXE (END=I63)

STOCK OF NON-FARM NON-RESIDENTIAL CONSTRUCTION
EXCLUDING ENERGY STOCKS

SOURCE: GENERATED IN 'COMPLX' AT THE BANK OF CANADA FROM
NATIONAL INCOME AND EXPENDITURE ACCOUNTS AND
RESEARCH DEPT. DATA
(UNPUBLISHED)
SET KNRCXE(52:1) = 23236.86 - 5684.1
DEFINED AS BELOW
$MILLIONS (1971 DOLLARS) S.A.C.R.
AVAILABLE FROM 1952:2

PGPP (END=P51)

PRICE DEFLATOR FOR GROSS PRIVATE BUSINESS PRODUCT

SOURCE: GENERATED IN 'COMPLX' AT THE BANK OF CANADA FROM
NATIONAL INCOME AND EXPENDITURE ACCOUNTS
(PUBLISHED AND UNPUBLISHED)
DEFINED AS BELOW
1971=1.0 S.A.
AVAILABLE FROM 1961:1

PIME (END=P07)

PRICE DEFLATOR FOR BUSINESS INVESTMENT IN
MACHINERY AND EQUIPMENT

SOURCE: GENERATED IN 'COMPLX' AT THE BANK OF CANADA FROM
NATIONAL INCOME AND EXPENDITURE ACCOUNTS
(PUBLISHED)
PIME=IME$/IME
=D40264/IME
1971=1.0 S.A.
AVAILABLE FROM 1952:1

RCME (END=I53)

IMPUTED RENTAL PRICE FOR MACHINERY AND EQUIPMENT

SOURCE: GENERATED IN 'COMPLX' AT THE BANK OF CANADA.
SEE BANK OF CANADA TECHNICAL REPORT 6, CH.4
DEFINED AS BELOW
PERCENT PER QUARTER
AVAILABLE FROM 1952:1

RCNR (END=I54)

IMPUTED RENTAL PRICE FOR NON-RESIDENTIAL CONSTRUCTION

SOURCE: GENERATED IN 'COMPLX' AT THE BANK OF CANADA.
SEE BANK OF CANADA TECHNICAL REPORT 6, CH.4
DEFINED AS BELOW
PER CENT PER QUARTER
AVAILABLE FROM 1952:1

UGPP (END=I55)

GROSS PRIVATE BUSINESS PRODUCT (EXCLUDING AGRICULTURE
AND NON-COMMERCIAL BUSINESS)

SOURCE: GENERATED IN 'COMPLX' AT THE BANK OF CANADA
FROM NATIONAL INCOME AND EXPENDITURE ACCOUNTS
DEFINED AS BELOW
$MILLIONS (1971 DOLLARS) S.A.A.R.
AVAILABL FROM 1961:1

```
WNIC     (END=W01)

          WAGES AND SALARIES OF INDUSTRIAL COMPOSITE
             EXCLUDING COMMUNITY SERVICES, GOVERNMENT AND AGRICULTURE

          SOURCE:  GENERATED IN 'COMPLX' AT THE BANK OF CANADA FROM
                   STATISTICS CANADA DATA
                   (PUBLISHED)
                   WNIC=(WSA29-WSA19-WSA27-WSA1)/(NIC*52)
                   DOLLARS PER WEEK  S.A.
                   AVAILABLE FROM 1961:1

YCR       (END=Y51)

         RETAINED CORPORATE PROFITS

         SOURCE:  NATIONAL INCOME AND EXPENDITURE ACCOUNTS--D40414
                  $MILLIONS  (CURRENT)  S.A.A.R.
                   AVAILABLE FROM 1952:1
YDIVF    (END=Y01)

         DIVIDENDS (BEFORE WITHHOLDING TAX) PAID TO FOREIGN
            SHAREHOLDERS BY CANADIAN CORPORATIONS

         SOURCE:  NATIONAL INCOME AND EXPENDITURE ACCOUNTS--D40243
                  $MILLIONS  (CURRENT)  S.A.A.R.
                  AVAILABLE FROM 1952:1

YDIV11  (END=Y04)

        DIVIDENDS PAID TO CANADIAN RESIDENTS BY CANADIAN CORPORATIONS

        SOURCE:  NATIONAL INCOME AND EXPENDITURE ACCOUNTS - D31523
                 (UNPUBLISHED)
                 $MILLIONS  (CURRENT)  S.A.A.R.
                 AVAILABLE FROM 1952:1

PINRC    (END=P09)

        PRICE DEFLATOR FOR BUSINESS INVESTMENT IN NON-RESIDENTIAL
           CONSTRUCTION

        SOURCE:  GENERATED IN 'COMPLX' AT THE BANK OF CANADA FROM
                 NATIONAL INCOME AND EXPENDITURE ACCOUNTS
                 (PUBLISHED)
                 PINRC=INRC$/INRC
                      =D40263/INRC
                 1971=1.0  S.A.
                 AVAILABLE FROM 1952:1
```

Appendix B

THE ELASTICITY OF INVESTMENT WITH RESPECT TO OUTPUT

The basic Cobb-Douglas production function defines

$$Q = A.L^{\alpha}.\ K^{1-\alpha} \qquad (1)$$

where

Q = output

L = labour

K = capital

Assuming profit maximization, the optimal capital input, K , can be derived as

$$K = \alpha\frac{PQ}{c}. \qquad (2)$$

If we define gross investment to be composed of net investment and replacement investment, and model the latter using the constant proportional replacement hypothesis, then:

$$I^G = I + \delta K \qquad (3)$$

In steady state, however,

$I = gK$, where g = steady state growth rate, implying

$$I^G = (g+\delta)\ K. \qquad (4)$$

The elasticity of gross investment with respect to output is, therefore,

$$\frac{dI^G}{dQ}\cdot\frac{Q}{I^G} = \frac{\partial I^G}{\partial K}\cdot\frac{\partial K}{\partial Q}\cdot\frac{Q}{I^G}$$

$$= (g+\delta)\frac{\alpha P}{c}\cdot\frac{Q}{I^G}$$

$$= 1.0.$$

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