Bank of Canada — Banque du Canada

Technical Report 2

Econometric Estimation of Constrained Demand Functions for Assets

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AVANT-PROPOS

Cette étude a pour but d'exposer une méthode d'estimation des fonctions de gestion de portefeuille. Le modèle de base est formé d'un ensemble d'équations déterminant de façon exhaustive la répartition en différentes catégories d'un montant donné d'avoirs. Pour des raisons de cohérence, il faut que, pour tous les postes de l'actif, la somme des coefficients de l'avoir total soit égale à l'unité et que celle des coefficients des autres variables explicatives soit égale à zéro. Ces contraintes sont imposées lors de l'estimation simultanée du modèle. On y parvient en utilisant une version modifiée de la technique d'estimation des régressions en apparence indépendantes, développée par Zellner. On obtient l'estimateur en utilisant d'abord les résidus provenant des régressions des moindres carrés ordinaires pour estimer la matrice de variance-covariance, puis en appliquant la méthode des moindres carrés généralisés. Etant donné qu'il s'agit ici d'une matrice de variance-covariance singulière, on a recours au concept d'inversion généralisée pour calculer l'estimateur des moindres carrés généralisés. En raison de la somme énorme de calculs qu'implique l'estimation simultanée, l'auteur présente une méthode utilisant les formules d'inversion de matrices sectionnées. On peut ainsi réduire considérablement les calculs à effectuer si une proportion significative des variables explicatives est commune à chaque équation.

L'existence de délais d'ajustement est également abordée dans cette étude. Le modèle bien connu des ajustements de stocks ne peut être appliqué ici, parce qu'il ne tient pas compte des interrelations entre les avoirs. L'auteur étudie également un modèle proposé par Brainard et Tobin, dont il présente une version modifiée en imposant à tous les avoirs la même vitesse d'ajustement. Cette méthode est illustrée à l'aide d'un exemple tiré du RDX2, modèle économétrique trimestriel de l'économie canadienne mis au point par le Département des recherches de la Banque du Canada. Les équations sont estimées pour huit catégories d'avoirs, dont la monnaie et plusieurs formes de dépôt dans les banques à charte et dans les sociétés de fiducie ou de prêt hypothécaire.

ABSTRACT

This paper is designed to present a procedure for estimating demand functions for assets subject to portfolio consistency constraints. The basic model consists of a set of equations determining the allocation of a given quantity of wealth among categories that exhaust the total. Consistency requires that the coefficients on total wealth sum to unity across all assets and that the coefficients on other explanatory variables such as rates of return sum to zero. These constraints are imposed on the coefficients by estimating the system of equations simultaneously. This is done with the use of a modified version of the technique introduced by Zellner for estimating "seemingly unrelated regressions". His estimator is obtained by first employing residuals from ordinary least squares regressions to estimate the variance-covariance matrix among the disturbances for each asset and then by applying generalized least squares. Since in the present case the variance-covariance matrix is singular, use is made of the concept of a generalized inverse to obtain the generalized least squares estimator. In view of the considerable computational burden involved in the simultaneous estimation, a method for calculating the coefficients is presented that uses the formulas for the inverse of a partitioned matrix. A considerable saving in computation can be realized by this means if a significant proportion of the explanatory variables appears in the equation for every asset.

An additional problem taken up in the paper is the introduction of lagged adjustments. The familiar stock adjustment model cannot be imposed directly because it does not account consistently for the interactions among assets. A model proposed by Brainard and Tobin is discussed and a modified version of it is presented in which a common speed of adjustment is imposed on all assets. This procedure is illustrated by using an example drawn from RDX2, the quarterly econometric model of the Canadian economy developed in the Research Department of the Bank of Canada. Equations are estimated for eight assets including currency and several categories of deposits in chartered banks and in trust and mortgage loan companies.

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ECONOMETRIC ESTIMATION OF CONSTRAINED DEMAND FUNCTIONS FOR ASSETS

The purpose of this paper is to present a procedure for estimating demand functions for assets subject to portfolio consistency constraints. In Section I of the paper, the basic model is formulated and the nature of the econometric problem is indicated. Section II is a description of an estimation procedure that is a modified version of the technique for estimating "seemingly unrelated regressions" introduced by Zellner [17]. In Section III the problem of introducing lagged adjustments into the model is taken up, and in Section IV the estimation of constrained demand functions is illustrated by an example drawn from RDX2, the quarterly econometric model of the Canadian economy developed in the Research Department of the Bank of Canada.

I The Constrained Portfolio Model

The basic model to be considered is represented by the following system of equations:

$$y_r = \sum_{s=1}^{m} \beta_{rs} x_s + u_r \qquad r = 1,2,\ldots,n \tag{1}$$

where

y_r is the quantity of the r^{th} asset held,

u_r is a disturbance term,

$x_m = \sum_{r=1}^{n} y_r$ is total wealth, and

$x_1, x_2, x_3, \ldots, x_{m-1}$ are other variables influencing asset choice.

The model determines the allocation of a given quantity of wealth among n categories that exhaust the total. The other explanatory variables are typically rates of return but may include other factors influencing portfolio choice as well. The model may be applied to households, firms, or financial institutions and its theoretical interest has been established in many recent contributions to the literature of financial model building - particularly by Tobin [15] and by Brainard and Tobin [3].

Since total wealth is included among the explanatory variables, constraints must be imposed on the coefficients to preserve the identity relating total wealth to the sum of its components. For example, any change in rates of return may cause a shift in asset choices, but the effect must sum to zero over the portfolio. Similarly, a change in total wealth must be distributed over the portfolio so that the effect sums to unity. Thus we have:

$$\sum_{r=1}^{n} \beta_{rm} = 1 \qquad (2)$$

$$\sum_{r=1}^{n} \beta_{rs} = 0 \qquad s = 1,2,3,\ldots,m-1$$

In practice these constraints are frequently not imposed because demand functions are estimated for only a subset of assets that does not exhaust total wealth. However, as emphasized by

Brainard and Tobin [3], failure to consider the constraints may lead to inappropriate implicit specification of the remaining demand functions. Peculiar behaviour may be implied, particularly when lagged adjustment models are used. If the residual assets are simply ignored, all the offsetting effects are automatically assigned to the missing equations.

If the same set of explanatory variables is employed in each of the asset demand functions, the constraints of Equation 2 do not require the use of any special estimation procedure and it is easily shown that they will automatically be satisfied by ordinary least squares estimates of the equations taken one at a time. If, however, some coefficients are set equal to zero (as would be the case, for example, if some asset pairs are not substitutes), the constraints must be incorporated into the estimation procedure. This can be accomplished by combining all the demand functions into a single equation as follows:

$$\begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix} = \sum_{s=1}^{m} \beta_{1s} \begin{bmatrix} x_s \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} + \sum_{s=1}^{m} \beta_{2s} \begin{bmatrix} 0 \\ x_s \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} + \dots + \sum_{s=1}^{m} \beta_{ns} \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ x_s \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \\ \cdot \\ \cdot \\ \cdot \\ u_n \end{bmatrix} \qquad (3)$$

where each element in the partitioned vectors is to be interpreted as a vector of T observations. The equations can then be estimated simultaneously by running one regression on all nT observations and the constraints can be imposed by substitution. However, it is clear that the usual assumption of homoscedasticity cannot be imposed on this combined regression, since, in general, the residual variance will not be constant across the assets. In these circumstances a simple procedure can be used: this is to obtain estimates of the variances from the least squares residuals and then to rerun the equations with the data appropriately weighted so as to eliminate heteroscedasticity. This is the method employed by Sparks [14] and Hendershott [7]. Following the technique used in an earlier paper by Gramlich and Kalchbrenner [5], Hendershott further reduces the number of independent parameters to be estimated by imposing symmetry on the interest rate coefficients. In terms of the notation introduced in Equation 1 above, he assumes that[1]

$$\beta_{rs} = \beta_{sr} \qquad r,s = 1,2,\ldots,m-1$$

II Efficient Estimation of the Portfolio Model

1. Estimation by Generalized Least Squares

The estimation procedure suggested above, which allows for heteroscedasticity across assets, will still be inefficient if there is correlation between residuals for different assets. An efficient method has been suggested by Zellner [17] that involves the estimation of the variance-covariance matrix from the least squares residuals and then the application of generalized least squares. Zellner's method is based on the assumption that, for each asset, the disturbances are homoscedastic and free of autocorrelation, but that there is a different variance for each asset and correlation between contemporaneous disturbances for different assets. In this case the variance-covariance matrix can be written in the following partitioned form:

$$\Sigma = \begin{bmatrix} \sigma_{11}I_T & \sigma_{12}I_T & \cdots & \sigma_{1n}I_T \\ \sigma_{21}I_T & \sigma_{22}I_T & \cdots & \sigma_{2n}I_T \\ \vdots & \vdots & & \vdots \\ \sigma_{n1}I_T & \sigma_{n2}I_T & \cdots & \sigma_{nn}I_T \end{bmatrix} = V \otimes I_T \qquad (4)$$

where

I_T is a T x T identity matrix, and

V is the n x n matrix of distinct variances and covariances.

If it is assumed that zero constraints are imposed on the mn parameters (β_{rs}) (so that p coefficients remain to be estimated) and also that the variables are replaced with vectors of T observations, the model can be written in matrix form as follows:

$$y = X\beta + u$$
$$A\beta = a \qquad (5)$$

where

y is a vector of nT observations on the n assets,

X is a nT x p matrix of observations on the explanatory variables,

β is a vector of p coefficients,

u is a vector of nT disturbances,

A is a m x p matrix with rank m,[2] and

a is a vector of length m with unity in the last position and zeros elsewhere.

Then, the generalized least squares estimates are obtained by minimizing

$$(y-Xb)' \Sigma^{-1} (y-Xb) \tag{6}$$

subject to the constraint

$$Ab = a \tag{7}$$

where

b is a vector of p estimated coefficients.

Under Zellner's assumptions it is not necessary to invert an nT x nT matrix, since Σ^{-1} is given by

$$\Sigma^{-1} = V^{-1} \otimes I_T \tag{8}$$

In the present case, however, this method cannot be applied directly, because the constraints imply that the disturbances (u_r) must sum to zero. This implies that the rows and columns of the matrix V will sum to zero so that V will be singular. Thus the inverse cannot be obtained as required in the application of generalized least squares. A solution to this problem has been provided by Barten [2] in a paper concerned with the analagous problem of estimating a system of demand equations for commodities. He suggests the use of the concept of the generalized inverse, which can be applied to a singular matrix [2] p 82.

Consider the n x n matrix G defined by

$$G = \begin{bmatrix} & & 0 \\ & V_*^{-1} & \vdots \\ 0 & \cdots & 0 \end{bmatrix} \qquad (9)$$

where

V_* is the (n-1) x (n-1) matrix obtained by deleting the last row and column of V. Then G is by definition a generalized inverse of V since it satisfies the condition[3]

$$VGV = V \qquad (10)$$

To see this, we partition V as follows:

$$V = \begin{bmatrix} V_* & v \\ v' & s \end{bmatrix} \qquad (11)$$

where

v is a column vector of length n-1 and

s is a scalar

Carrying out the matrix multiplication, we obtain

$$VGV = V \begin{bmatrix} I_{n-1} & V_*^{-1}v \\ 0 & 0 \dots 0 \end{bmatrix} = \begin{bmatrix} V_* & v \\ v' & v'V_*^{-1}v \end{bmatrix} \tag{12}$$

Since the rows and columns of V sum to zero, we have

$V_*u = -v$

or

$$V_*^{-1}v = -u \tag{13}$$

where

u is a unit vector of length n-1.

Similarly we have

$$s = -v'u \tag{14}$$

Substituting Equation 13 into Equation 14 we obtain

$$s = v'V_*^{-1}v \tag{15}$$

so that the matrix derived in Equation 12 is equal to V. Substituting G in place of the undefined inverse of V, we minimize

$$(y-Xb)'H(y-Xb) \tag{16}$$

subject to Equation 7,

where

$$H = G \otimes I_T \tag{17}$$

Introducing a vector γ of m Lagrangian multipliers, we minimize

$$(y-Xb)'H(y-Xb) + 2b'A'\lambda \tag{18}$$

by solving the following equations:

$$X'HXb + A'\lambda = X'Hy$$
$$Ab = a \tag{19}$$

These equations can be written in the form,

$$\begin{bmatrix} X'HX & A' \\ A & 0 \end{bmatrix} \begin{bmatrix} b \\ \lambda \end{bmatrix} = \begin{bmatrix} X'Hy \\ a \end{bmatrix} \tag{20}$$

The vector of estimated coefficients b can then be obtained from:

$$b = PX'Hy + C'a \tag{21}$$

where

$$\begin{bmatrix} P & C' \\ C & D \end{bmatrix} = \begin{bmatrix} X'HX & A' \\ A & 0 \end{bmatrix}^{-1} \tag{22}$$

2. The Variance-Covariance Matrix of the Constrained Estimator

The variance-covariance matrix of the coefficients can be obtained as follows: Given the model (Equation 5) and the properties of P and C implied by their definition (Equation 22) we can write

$$b = PX'H(X\beta+u) + C'a = (I_p-C'A)\beta + PX'Hu = \beta + PX'Hu \tag{23}$$

Thus we have

$$E[(b-\beta)(b-\beta)'] = E[PX'Huu'HXP']$$

$$= PX'H\Sigma HXP' \qquad (24)$$

$$= PX'HXP'$$

since

$$H\Sigma H = (GVG) \otimes I_T \qquad (25)$$

and[4] $$GVG = G \qquad (26)$$

Simplifying further, we have

$$PXHXP' = (I_p - C'A)P'$$

$$= P'$$

$$= P$$

so that

$$E[(b-\beta)(b-\beta)'] = P \qquad (27)$$

3. Estimation by a Two-Stage Procedure

It can easily be seen from the form of G, defined by Equation 9, that the use of the generalized inverse to deal with the singularity problem has a simple interpretation. The estimator given by Equation 21 could be obtained by simply dropping out one asset and applying Zellner's method to the remaining n-1 assets. The effect of placing zeros in the last row and column of G is to eliminate the terms involving the coefficients in the nth asset from the minimand given in Equation

16. This raises the question of whether different estimates would be obtained for different choices of residual asset. This question has been considered by Powell [12] who has shown that the estimates are invariant with respect to the choice of the generalized inverse and therefore with respect to the choice of the residual asset.

However, it should be noted that if an initial run is made using ordinary least squares in order to estimate V, then to drop out one asset is not appropriate. In this case, V is replaced by an identity matrix, which is of course not singular, and is its own inverse. As can be shown, the ordinary least squares estimates obtained when one equation is dropped are not independent of the asset chosen for exclusion.[5] Thus the two-stage procedure should be carried out in the following way: First obtain ordinary least squares estimates by using Equation 21 with H obtained from Equation 17 and an identity matrix substituted for G. Use the estimated residuals to estimate V with the arbitrary choice of an omitted asset and form G by adopting the definition given in Equation 9. The coefficients can then be reestimated using generalized least squares.

Alternatively, the first stage estimates could be obtained by simply applying unconstrained ordinary least squares to the equations individually. This would lead to an estimated V matrix that need not obey the singularity constraint. Thus the second stage estimates would no longer be invariant with respect to the choice of an omitted asset. Singularity could be insured by using all explanatory variables in all asset equations in the first stage, but this would lead to inefficient estimation of V if there were a significant number of zero constraints. Thus it seems preferable to proceed as suggested above and estimate the equations simultaneously at both stages.[6]

4. Calculation of the Coefficients Using a Partitioned Inverse

Unless the number of assets and explanatory variables is small, the computation of the estimated coefficients in Equation 21 will be onerous, and will require the inversion of a matrix the dimension of which equals the number of coefficients (p) plus the number of constraints (m). For example, in the empirical model discussed below, there are eight assets and sixteen explanatory variables. The number of nonzero coefficients is seventy-five so that the matrix has dimension ninety-one.

As noted above, the estimation procedure collapses to ordinary least squares applied to the equations taken one at a time if the same set of explanatory variables is used for each asset. This fact suggests that the computations in the general case can be simplified by an appropriate partitioning. Let us write the basic model given in Equation 5 as follows:

$$Y = X\beta + u = Z\alpha + Z_*\alpha_* + u \tag{28}$$

where

Z is the nT x nq matrix of observations of the q explanatory variables that appear in the equation for every asset,

α is the corresponding vector of nq coefficients,

Z_* is the nT x (p-nq) matrix of observations on the remaining m-q explanatory variables, and

α_* is the corresponding vector of p-nq coefficients.

Thus the matrix given in Equation 22, which is to be inverted, can be written

$$\begin{bmatrix} X'HX & A' \\ A & 0 \end{bmatrix} = \begin{bmatrix} Z'HZ & Z'HZ_* & \\ & & A' \\ Z_*'HZ & Z_*'HZ_* & \\ & A & 0 \end{bmatrix} \tag{29}$$

If a significant proportion of the explanatory variables appears in every equation, a large saving in computation can be realized by using the fact that

$$Z'HZ = (W'W) \otimes G \tag{30}$$

where

W is the T x q matrix of observations on the q common explanatory variables.

Furthermore, the dimension of the matrix given in Equation 29 can be reduced by recognizing that the set of equations for the coefficients on the common variables combined with the corresponding constraints is redundant. These constraints will automatically be satisfied if the entire set of coefficients is estimated subject to the remaining constraints applicable to variables that do not appear in every equation.

If redundancy is eliminated by dropping the equations for the coefficients on the common variables in the nth asset and replacing them by equations derived from the constraints,[7] the system of equations numbered 20 can be reduced to

$$\begin{bmatrix} M & B \\ R & S \end{bmatrix} \begin{bmatrix} c \\ c_* \\ \lambda_* \end{bmatrix} = \begin{bmatrix} r \\ Z_*'Hy \\ a_* \end{bmatrix} \quad (31)$$

where

$$M = \begin{bmatrix} (W'W) \otimes G_* & 0 \\ I_q \; \ldots \; I_q & I_q \end{bmatrix},$$

$$R = \begin{bmatrix} Z_*'HZ \\ 0 \end{bmatrix},$$

$$S = \begin{bmatrix} Z_*'HZ_* & A_*' \\ A_* & 0 \end{bmatrix},$$

G_* is the $(n-1) \times (n-1)$ matrix obtained by deleting the last row and column of G,

A_*, a_*, λ_* are the parts of the matrix A and of the vectors a and λ that pertain to the m - q constraints on the noncommon variables,

c is the vector of nq coefficients on the common variables,

c_* is the vector of p-nq coefficients on the remaining variables,

B is the matrix composed of the first (n-1)q rows of $Z'HZ_*$ bordered with q rows and m-q columns of zeros, and

r is the vector of length nq obtained by taking the first (n-1)q elements of Z'Hy and the q relevant entries from the vector a.

The inverse of the above partitioned matrix can be obtained as follows:

$$\begin{bmatrix} M & B \\ R & S \end{bmatrix}^{-1} = \begin{bmatrix} N & Q \\ E & F^{-1} \end{bmatrix} \tag{32}$$

where

$N = M^{-1}[I_{nq} - BE]$,

$E = -F^{-1}RM^{-1}$,

$Q = -M^{-1}BF^{-1}$,

$F = S - RM^{-1}B$, and

$$M^{-1} = \begin{bmatrix} (W'W)^{-1} \otimes V_* & 0 \\ (W'W)^{-1} \otimes (-u'V_*) & I_q \end{bmatrix}.$$

Thus the inversion of the q x q matrix W'W and the (p+m-nq-q) x (p+m-nq-q) matrix F is required. In the example discussed below, these dimensions are six and thirty-seven, respectively, so that a considerable saving is achieved by partitioning rather than by direct inversion of a matrix of dimension ninety-one.

III Lagged Adjustments

1. The Brainard-Tobin Model

In this section the problem is considered of incorporating adjustment lags into the demand functions while maintaining the consistency constraints. That old reliable workhorse the stock adjustment model cannot be imposed directly since it does not account consistently for the interactions among assets. If it is assumed that the change in the holdings of each individual asset depends on the gap between the desired and lagged actual stock and if a function of observables is substituted for the desired stocks, a set of relationships is obtained in which the lagged stock of each asset appears in only one equation. Thus the consistency constraints would require the lagged stocks to enter with zero coefficients. To get around this difficulty, Brainard and Tobin [3] postulate a model of the following form:

$$\Delta y_r = \sum_{s=1}^{m} \beta_{rs} x_s - \sum_{t=1}^{n} \gamma_{rt} y_{t_{-1}} + \delta_r \Delta x_m \tag{33}$$

The change in holdings of each asset is assumed to depend on the gaps between desired and actual holdings of all assets and the change in total wealth. The coefficients γ_{rt} represent the response of the r^{th} asset to the gap between the desired and actual stocks of the t^{th} asset. The consistency constraints can be derived by summing over the n equations of the model in Equation 33 as follows:

$$\sum_r \Delta y_r = \sum_{s=1}^{m-1} (\sum_r \beta_{rs}) x_s + \sum_r \beta_{rm} x_m - \sum_t (\sum_r \gamma_{rt}) y_{t_{-1}} + (\sum_r \delta_r) \Delta x_m$$

Rearranging the terms involving total wealth, we obtain

$$x_m = \sum_{s=1}^{m-1} (\sum_r \beta_{rs}) x_s + \sum_r (\beta_{rm} + \delta_r) x_m - \sum_t (\sum_r \gamma_{rt}) y_{t_{-1}} + (1 - \sum_r \delta_r) x_{m-1}$$

Thus we must have

$$\sum_r \beta_{rs} = 0 \qquad s=1,2, \ldots , m-1$$

$$\sum_r (\beta_{rm} + \delta_r) = 1 \tag{34}$$

$$\sum_r \gamma_{rt} = 1 - \sum_r \delta_r$$

Note that the third condition implies that $\sum_r \gamma_{rt}$ is independent of t.[8]

An additional set of n constraints is required because of the exact linear dependence between the explanatory variables X_m, $X_m - X_{m_{-1}}$, and the set of lagged stocks. One option is to eliminate the change in wealth term, but Brainard and Tobin [3] choose to eliminate the lagged stock of one particular asset from all equations. Since Ladenson [8] p 1007 seems to argue to the contrary, it is perhaps worth emphasizing that the choice of constraint to eliminate the redundancy is entirely arbitrary and has no effect on the substance of the model. In terms of the notation used here, Ladenson argues that: if the coefficients β_{rm} (r=1,2,...,n) are set equal to zero, then it is assumed that asset demands adjust instantaneously to a change in wealth, while, if the coefficients δ_r (r=1,2,...,n) are set equal to zero, it is assumed that asset demands adjust to a change in wealth at the same speed as they adjust to changes in other variables.

This interpretation is clearly invalid because the reaction to a change in wealth will not depend on the particular constraint used to eliminate the redundancy. Since it is impossible to identify separately the influence of wealth, the change in wealth, and the set of lagged stocks, Ladenson's distinction is meaningless.

2. The RDX2 Model

In the financial sector of the RDX2 model of the Canadian economy, a modified version of the Brainard-Tobin model is used.[9] In order to reduce the number of parameters to be estimated further restrictions are introduced on the speed of adjustment coefficients, namely

$$\gamma_{rt} = 0 \qquad r \neq t$$
$$\gamma_{rr} = \gamma \tag{35}$$

Thus a model with a common speed of adjustment is considered that takes the form

$$\Delta y_r = \sum_{s=1}^{m} \beta_{rs} x_s - \gamma y_{r_{-1}} + \delta_r \Delta x_m \tag{36}$$

The consistency constraints are

$$\sum_r \beta_{rs} = 0 \qquad s=1, 2, \ldots, m-1 \tag{37}$$

$$\sum_r (\beta_{rm}+\delta_r) = 1$$

$$\gamma = 1 - \sum_r \delta_r$$

In this context, the coefficients on the change in wealth can be interpreted as an inertia effect that accounts for the temporary allocation of an increase in wealth while the stock adjustment process works itself out. The impact effect on the r^{th} asset of an increase in wealth consists of two parts: first there is the stock adjustment component that equals γ times the long run equilibrium effect of wealth on the r^{th} asset, and second the inertia effect represented by δ_r. The constraints ensure that these effects just exhaust the change in wealth as it is distributed among the assets. In subsequent periods the change in wealth is reallocated until the long-run desired portfolio is reached. On the other hand for certain values of the coefficients, the lagged adjustment is short-circuited. In particular, should

$$\beta_{rm} + \delta_r = \frac{1}{\gamma} \beta_{rm}$$

then the impact effect and the equilibrium effect on the r^{th} asset are identical. A general condition for a zero lag wealth effect can be derived to cover Equation 33, although such a condition does not take the form $\beta_{rm} = 0$ as claimed by Ladenson.

IV Empirical Results

In order to illustrate the specification and estimation procedure discussed above, a modified form of the model used to determine liquid asset holdings in RDX2 [6] is estimated. Demand functions for eight assets are considered. These include currency, three types of bank deposits, two types of deposits in trust companies and mortgage loan companies (which are the major nonbank deposit-taking institutions in Canada), Canada Savings Bonds, and liquid marketable securities net of short-term bank loans. The definitions of the variables used are as follows:

ANFN Government of Canada marketable debt held by the general public plus short-term paper outstanding less chartered bank day, call, and short loans

CSB End-of-quarter stock of Canada Savings Bonds

CUR Currency held outside chartered banks

DDB Demand deposits in chartered banks (excluding float, Government of Canada deposits, and personal chequing accounts)

DNPTB Nonpersonal term and notice deposits in chartered banks

DPB Personal savings and personal chequing accounts in chartered banks

DSTL Chequable and nonchequable demand and savings deposits in trust and mortgage loan companies

DTTL Receipts and guaranteed investment certificates deposited in trust and mortgage loan companies

The dependent variables in the regressions are the proportions of total wealth held in each form and therefore the constant terms are constrained to sum to unity.[10] Portfolio scale effects are allowed for by including the reciprocal of total assets among the explanatory variables. Since levels rather than first differences are used for the dependent variables, the coefficients on the lagged dependent variables are interpreted as 1 minus the speed of adjustment.

The explanatory variables used are defined as follows:

A Total liquid assets

QC1, QC2, QC4 Seasonal dummies for the first, second, and fourth quarters, respectively

QDBA Dummy variable for revisions of the Bank Act, which governs the operations of chartered banks (unity from 3Q67 forward)

RCSB Rate of interest on Canada Savings Bonds

RNPT Rate of interest on nonpersonal term and notice deposits

RPD Rate of interest on personal deposits

RST Weighted average of rate of interest on Government of Canada bonds (1-3 years) and 90-day paper

RSTL Rate of interest on trust and mortgage loan savings deposits

RTTL Rate of interest on trust and mortgage loan certificates

Y Gross national expenditure in current dollars

The income variable, which is included to represent the effect of transactions requirements on the choice of assets, is expected to enter positively in the equations for assets that serve as, or are close substitutes for, a medium of exchange.

The regression coefficients obtained by using quarterly data for the period 2Q56-3Q72 are shown in Table 1. Certain aspects of the results should be noted. First, a modification was introduced because of the low speeds of adjustment that were obtained when the model outlined above was fitted. To allow for the possibility of a faster speed of adjustment for demand deposits (DDB), the lagged stock of this asset was included in the equations for demand deposits (DDB) and ANFN. A different reaction lag for currency is similarly permitted by including the lagged stock in the equations for currency (CUR) and for chartered bank personal deposits (DPB). The coefficients imply speeds of adjustment of 46% and 35% per quarter for CUR and DDB, respectively, compared with 14% for the other assets.

In view of the problem of multicollinearity among interest rates, the range of possible substitution effects was narrowed down by dividing the assets into two groups with zero cross-elasticities between assets not in the same group. The first group of assets (Equations 1-4) is held primarily by households that have relatively small portfolios, whereas the second group (Equations 6-8) comprises assets held primarily by businesses or by households that have relatively large financial portfolios. Substitutability was permitted between the marketable securities variable (ANFN) and assets in both groups. Within the first group significant substitution effects were obtained between personal savings and chequing deposits in chartered banks (DPB) and both savings deposits in trust and mortgage loan companies (DSTL) and Canada Savings Bonds (CSB). The marketable securities

Table 1

COEFFICIENTS AND T-RATIOS

	(1) CUR/A	(2) DPB/A	(3) DSTL/A	(4) CSB/A	(5) ANFN/A	(6) DNPTB/A	(7) DTTL/A	(8) DDB/A	Sum Across Equations
CONSTANT	.09415 (5.11)	.19647 (7.24)	.00662 (1.63)	.04755 (3.84)	.11007 (5.35)	.16728 (4.64)	.01363 (2.62)	.36422 (10.31)	1
QC1	.00145 (1.42)	.00782 (3.85)	-.00077 (1.01)	-.00088 (.43)	-.01553 (3.77)	.00639 (2.87)	.00266 (2.28)	-.00113 (.46)	0
QC2	.00384 (5.02)	.00232 (1.43)	-.00036 (.61)	-.00231 (1.68)	-.01134 (3.76)	.00296 (1.75)	.00071 (.82)	.00418 (2.50)	0
QC4	.00544 (3.20)	.00087 (.25)	-.00142 (1.08)	.01344 (4.39)	-.03327 (4.81)	.00631 (1.65)	.00261 (1.31)	.00602 (1.50)	0
Y/A	.04749 (3.49)	.10339 (3.83)	-.01242 (1.25)	-.06060 (2.59)	-.38032 (7.02)	.11211 (3.69)	.02301 (1.44)	.16734 (5.36)	0
1000/A	.26336 (5.19)	-.41656 (4.09)	-.08464 (3.14)	-.10365 (1.18)	.58141 (2.99)	-.18489 (2.03)	-.40711 (6.11)	.35209 (3.73)	0
A_{-1}/A	-.09022 (4.78)	-.19600 (7.28)				-.18447 (5.01)		-.39191 (10.72)	-.86260
RPD		.00630 (5.88)	-.00079 (2.19)	-.00297 (3.50)	-.00254 (2.68)				0
RSTL		-.00227 (3.64)	.00227 (3.64)						0
RCSB		-.00198 (3.27)		.00198 (3.27)					0
RNPT					-.00511 (3.92)	.00511 (3.92)			0
RTTL					-.00220 (3.45)		.00220 (3.45)		0
RST		-.00207 (5.21)			.01329 (8.44)	-.00797 (5.80)	-.00169 (3.29)	-.00156 (3.07)	0
QDBA						.00564 (5.24)		-.00564 (5.24)	0
CUR_{-1}/A	-.32421 (4.04)	.32421 (4.04)							0
DDB_{-1}/A					.21424 (3.26)			-.21424 (3.26)	0
Lagged D.V./A	.86260 (39.44)	.86260 (39.44)	.86260 (39.44)	.86260 (39.44)	.86260 (39.44)	.86260 (39.44)	.86260 (39.44)	.86260 (39.44)	.86260
R^2	.946	.977	.983	.980	.991	.984	.998	.952	
D-W	1.95	1.65	1.44	1.70	1.68	1.90	1.68	2.03	

asset was found to be a substitute for each of the three categories of Canadian dollar deposits in chartered banks, ie, personal deposits (DPB), demand deposits (DDB), nonpersonal term and notice deposits (NDPTB), and term deposits in trust and mortgage loans companies (DTTL).

In the equations for currency and bank deposits the coefficients on income are positive whereas income has a negative effect on ANFN. This indicates that the proportions of these assets shift in response to transactions requirements as hypothesized above. The results for the temporary wealth effect represented by lagged total assets were, however, somewhat puzzling. The only sizeable coefficient appeared in the ANFN equation rather than in the currency and bank deposit equations where one would have expected this effect to be concentrated. In the equations shown, coefficients more consistent with this a priori view were obtained by excluding lagged total assets from the ANFN equation.

In summary, this example illustrates the major features of the model and the estimation procedure proposed. Given the considerable number of zero constraints, joint estimation is necessary to insure consistency across the portfolio. Use of the partitioned inverse was found to reduce substantially the computer time required to obtain the estimated coefficients. The problem of incorporating lagged adjustments in a consistent manner is handled by initially assuming a constant speed of adjustment for all assets so that only one lagged stock appears in each equation. This assumption is then relaxed to allow different speeds of adjustment in two of the eight assets with consistency maintained in each case by including the mirror image of the differential lag in the equation for the closest substitute.

FOOTNOTES

1. This constraint is also used by Parkin [11] who points out that it will hold if the demand functions are derived from a special case of the mean-variance approach to portfolio choice as proposed by Tobin [16] and by Markowitz [9]. Parkin assumes the objective function to be a weighted sum of the mean and the variance.

2. The condition that A has rank m implies that no explanatory variable is omitted from the equation for every asset.

3. For a discussion of this concept, see Searle and Hausman [13], pp 170-173.

4. This result does not follow simply from the fact that G is a generalized inverse of V but can easily be verified by carrying out the multiplication in terms of the partitioned matrices.

5. This proposition of course does not apply in the trivial case where the same set of explanatory variables is used for each asset. For a further discussion of this point see McGuire et al [10] p 1207. The inappropriate

procedure of dropping an asset was followed in the RDX2 model [6] Part 1, pp 171-172, and this error is corrected in the empirical results reported in Section IV below.

6. Of course further iterations can be carried out until convergence of the estimates is obtained. For a discussion of this, see Aigner [1].

7. This procedure for eliminating the redundancy is convenient for calculating the partitioned inverse as it leads to a nonsingular submatrix in the upper left hand corner. Simply dropping the constraints would yield a singular submatrix in the generalized least squares case where G is singular.

8. Brainard and Tobin [3] use a special case of the constraints on the adjustment coefficients, namely, $\sum_r \gamma_{rt} = 0$ and $\sum_r \delta_r = 1$. For a further discussion, see Clinton [4] and Ladenson [8].

9. See Helliwell, et al [6].

10. This constraint follows directly from Equation 36 and 37 given that lagged total wealth is used rather than the change in wealth. The constant terms are thus estimates of $\beta_{rm} + \delta_{r}$.

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