No. 4

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GOVERNMENT SECTOR EQUATIONS FOR MACROECONOMIC MODELS

1969

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CORRECTION

MAY 8 1970

Page 42, equation (1.2) is:

1Q52-4Q65

$$TPS = \underset{(107.21)}{1.178}\left(2/3\ AW + 1/3\ AW_{t-1}\right) - \underset{(6.84)}{.036}\ Q1 \sum_{j=-1}^{-4} AW_{t+j} - \underset{(21.51)}{.116}\ Q2 \sum_{j=-2}^{-5} AW_{t+j}$$

SEE = 21.1 $\quad \bar{R}^2$ = .929 $\quad$ D/W = 1.80

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GOVERNMENT SECTOR EQUATIONS FOR MACROECONOMIC MODELS

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This study was prepared as part of the econometric research program sponsored by the Research Department of the Bank of Canada. The views expressed are the personal views of the authors and no responsibility for them should be attributed to the Bank.

PREFACE

In this study we discuss the various ways of building government sectors of macroeconomic models, present the analysis underlying the government sector equations in the first version of the Bank of Canada's aggregate quarterly model, RDX1, and provide some basis for constructing government sectors for other macro-models being built for various purposes. The government sector designed for RDX1 may be too detailed for macro-models not intended for the analysis of alternative monetary and fiscal policies. However, despite the extensive detail in the government sector of RDX1 and in this paper, we treat numerous items inadequately——particularly expenditures and changes in asset and liability accounts. In view of the many facets of government activity to be modelled, we have had to establish priorities and deal first with the topics that seemed to promise the greatest return for our efforts. We set out in the first section of the paper some of the criteria used in deciding how to allocate our efforts during the first years of research. It is our hope that the range of experiments to date is wide enough to provide some guidance for economists designing government sectors for other macro-models. Since this is one of our main goals, we shall be explicit about the definitions of variables and the sources and details of our data. We have also developed an indexing system, intended to cover the various components of the Canadian federal, provincial, and municipal government sectors. This index is printed just after the data appendix.

As we are still in the midst of our research this paper is essentially a progress report on what we have found so far. Fairly complete sets of equations have been developed for the major revenues and transfer payments of the federal government and for the revenues of the provincial governments. We are gradually working our way down our list of priorities, so that RDX2 ought to contain a reasonably full treatment of provincial and municipal revenues, some expenditure equations for all levels of government, and equations explaining certain major changes in federal government asset and liability accounts.

This study is divided into four parts. Part 1 is an outline of the theoretical and practical problems involved in constructing a government sector model. Part 2 contains federal government tax equations. In Part 3 we present results of work related to the principal federal government transfer payments, and Part 4 contains equations for major provincial revenues. Although this study is very much a joint effort, some division of labour occurred during the course of the research. Helliwell was responsible for Part 1 and for the specification of the revenue equations. Stephenson, Evans, and Jarrett were mainly responsible for Parts 2, 3, and 4, respectively. Gorbet has recalculated some of the models, and extensively redrafted certain parts of the paper to make them dovetail more smoothly with the RDX1 fiscal sector.

PRÉFACE

Cette étude comporte, en plus d'un exposé des différentes façons de procéder à la construction du secteur public dans les modèles macroéconomiques, l'analyse des principes fondamentaux qui ont servi à établir les équations de ce secteur dans la 1ère version du modèle trimestriel élaboré par la Banque du Canada—le RDX1. Elle offre une certaine base susceptible de servir à la construction du secteur public dans d'autres modèles macroéconomiques destinés à des emplois différents. Le secteur public conçu pour le RDX1 est probablement trop détaillé pour des modèles qu'on n'envisagerait par d'utiliser pour analyser les incidences possibles de différentes politiques monétaires et fiscales. Cependant, malgré la variété des détails incorporés dans le secteur public du RDX1 et dont fait état la présente étude, plusieurs composantes n'ont pas été suffisamment analysées, notamment les dépenses et les variations des éléments de l'actif et du passif. C'est qu'en raison des aspects multiples de l'activité gouvernementale dont notre modèle devait tenir compte, nous avons dû établir un ordre de priorité et commencer par les domaines où nos efforts promettaient d'être le plus fructueux.

Dans la première section de la présente étude, nous avons exposé quelques-uns des critères que nous avons adoptés dans la répartition de nos efforts au cours de nos premières années de recherche. Nous espérons que les expériences auxquelles nous avons procédé jusqu'ici ont été suffisamment variées pour servir de guide, dans une certaine mesure, aux économistes qui auront à élaborer les équations du secteur public dans d'autres modèles macroéconomiques. C'est d'ailleurs un de nos principaux objectifs et nous serons très explicites au sujet des définitions de nos variables et de la source et des détails de nos données. Nous avons en outre développé un système d'indexation susceptible d'englober les divers éléments du secteur public, aux niveaux fédéral, provincial et municipal. L'index est reproduit à la suite des données publiées en annexe.

Comme nous n'avons pas encore terminé nos recherches, la présente étude est essentiellement un rapport provisoire sur les résultats obtenus jusqu'à présent. Nous avons mis au point des séries assez complètes d'équations représentant les principales catégories de revenus et de paiements de transfert du gouvernement fédéral et les revenus des gouvernements provinciaux. En suivant graduellement l'ordre de priorité que nous avons établi, nous devrions aboutir, avec le modèle RDX2, à une présentation relativement détaillée des revenus provinciaux et municipaux, à certaines équations des dépenses publiques aux niveaux fédéral, provincial et municipal et à des équations expliquant les plus importants changements enregistrés aux comptes de l'actif et du passif du gouvernement fédéral.

Cette étude est divisée en quatre parties. La 1ère partie est un exposé des problèmes théoriques et pratiques que comporte la construction d'un modèle du secteur public. La 2ième partie présente les équations des impôts du gouvernement fédéral et la 3ième partie, les résultats des travaux relatifs aux principaux paiements de transfert du gouvernement fédéral; enfin, la 4ième partie présente des équations des principales recettes des provinces. Bien que cette étude soit essentiellement le fruit d'un effort collectif, une certaine répartition des tâches s'est imposée au cours des travaux de recherche. M. Helliwell a été chargé de la 1ère partie et de la mise en équations des revenus. MM. Stephenson, Evans et Jarrett ont été respectivement chargés des 2e, 3e et 4e parties. Enfin M. Gorbet a vérifié les calculs d'un certain nombre de modèles et a revisé, dans une large mesure, le texte de certaines parties de l'étude, de façon à les harmoniser davantage avec le secteur fiscal du RDX1.

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PART 1 OUTLINE OF THE GOVERNMENT SECTOR

In this part of the paper we outline the necessary features of the government sector of a policy-oriented model of the economy, and discuss some of the problems involved in specifying the necessary equations and establishing the appropriate links between the government and non-government sectors.

We should presumably be working towards econometric models that specify the expenditures and related patterns of financing of each aggregate sector of the economy. Even if the present flow-of-funds data do not allow the adequate specification of changes in the financial asset and liability accounts of the private sector, we can make the necessary improvements in the specification of the public sector transactions. This ought to be done for the national government at least, since the links between government expenditure, taxation, and the pattern of government financing must be established before a model can take adequate account of the constraints these links place on the choice of monetary, fiscal and debt management policies.

In this part of the paper we will show, in three stages, how appropriate government submodels may be developed and integrated with a variety of macro-models used for different purposes. Section A contains an outline of the structure of a government submodel suitable for policy analysis, section B a discussion of the specification and estimation of the various types of equations required, and section C a consideration of how various descriptions of the government sector can be built up and linked to the relevant macro-models. No single specification of the government sector is right for all policy models, and in section C we indicate how the government submodel should be built up for different sorts of policy analysis.

A. The Structure of the Government Sector

The government sector model must reflect government income and expenditure as well as the pattern of government finance and the necessary relationships between the two must be made explicit. For a policy model this complete explanation of government income, expenditure, and debt management behaviour must be given separately for each autonomous government unit. There should be a distinct government submodel for each level of government, since in Canada each level has different expenditure obligations, income sources and policy instruments within its control.

The simplest way of accounting for the interdependence between income-expenditure flows and changes in government asset and liability accounts is to set up a source- and application-of-funds statement. The analytic distinction between this procedure and the one usually adopted in econometric models should be made clear. This is done by putting the major national accounts income-expenditure items together at the beginning of the source and application statement. The usual econometric model sets out to explain all the important expenditure and income items in the *National Accounts*,[1] so that any item in the source and application statement below the national accounts balance is not included.

If the balance sheet identity is to hold exactly (with income minus expenditure plus changes in asset and liability accounts equal to zero) then all the elements must have the same dimensions. The most obvious numéraire is current dollars. However, some of the expenditure equations might better be expressed in constant rather than current dollars. In the abstract, assuming we could get appropriate parameter estimates for the chosen equations, we can describe all expenditure decisions as relating to current rather than constant dollars.[2] We assume, at this stage, that it is appropriate to express all relevant government income, expenditure, and changes in balance sheet accounts in terms of current

[1]*National Accounts, Income and Expenditure* issued quarterly and annually by the Dominion Bureau of Statistics, catalogue nos. 13-001 and 13-201, respectively.

[2]If G is the (current-dollar) expenditure, X an (constant-dollar) activity measure indicating the requirement for government expenditure, and P the price level, the equation can equivalently be expressed as $G = f_1 (XP)$ or $G/P = f_2 (X)$. Only when it comes to estimation of parameters is there a difference between these two formulations. The choice here is between normalization rules, and theory should tell us which rule is preferable. See pp. 9-10 for further discussion.

dollars. In the next section we discuss the estimation problems that must be dealt with if the use of current-dollar magnitudes is to be strictly appropriate. The general procedure followed in the construction of RDX1 was to express all government revenue and expenditure in current dollars, all non-government expenditure in constant dollars, and all non-government income items in current dollars.

The operations of a government sector are outlined in Table 1. Items chosen for this table are the major elements in the operations of the Canadian federal government, although the statement would have the same form for any other level of government. However, in practice, only federal government securities are purchased by the central bank in the course of its monetary policy transactions. It is therefore convenient for analytical purposes to consolidate the operations of the central bank with those of the federal government, and to regard all cash and deposits at the central bank (net of federal government deposits) as part of the federal government debt. Thus we can show in Table 1 all the basic elements of government income, expenditure and financial management.

The items included in part B of Table 1 are not usually brought into econometric models. Among other things, these items reflect changes in foreign exchange reserves and other federal government purchases and sales of foreign assets (all encompassed in item 2-3-2-1). Item 2-3-2-2 shows the net transfers on loan or investment accounts to crown corporations. The largest elements here are the transfers to Central Mortgage and Housing Corporation and to the Farm Credit Corporation. Item 2-3-2-6 refers only to the corporation income tax since it is the one tax that appears in the *National Accounts* on an accrual basis,[3] all others being recorded in the *National Accounts* at the time the collections are received. The various categories of item 2-3-3 measure changes in the mix and total size of the government debt. The sums of the two columns of Table 1, source and use of funds, must be equal. It is this constraint that is ignored when only

[3]One can also consider the withholding tax to be on an accrual basis because of the way it is recorded in the *National Accounts*. See footnote 10, p. 16.

Table 1

OUTLINE OF THE OPERATIONS OF A GOVERNMENT SECTOR*

A. Income and Expenditure Flows on a National Accounts Basis

Index Code No.**		Source	Use
2-1-1	Direct taxes, persons (approx. 96% income tax)	2,716	
2-1-2	Corporation income tax accruals	1,675	
2-1-3	Non-Resident withholding tax	168	
2-1-4	Indirect taxes	3,252	
2-1-5	Investment income	630	
2-1-6	Insurance and pension accounts	618	
2-2-1	Goods and services, defence		1,559
2-2-2	Goods and services, non-defence		1,734
2-2-3	Transfers to persons		2,312
2-2-4	Interest on the federal public debt		1,052
2-2-5	Transfers to business and agriculture		343
2-2-6	Transfers to other levels of government		1,434
	Total revenues and expenditures	9,059	8,434

B. Net Changes in Asset and Liability Accounts

		Source	Use
2-3-2-1	Federal government claims on non-residents		127
2-3-2-2	Funds advanced to government enterprises and agencies		657
2-3-2-6	Corporation income taxes accrued but not collected	40	
2-3-2-3, 4, 5, 7	Miscellaneous asset and liability accounts	415	
2-3-3	Government debt outstanding, excluding amounts held by government accounts and the Bank of Canada:		
	Canada Savings Bonds		
	Treasury bills		
	Bonds with term to maturity:		
	less than 2 years		
	2 to 5 years		196
	5 to 10 years		
	Over 10 years		
	Demand liabilities of the Bank of Canada (all currency and central bank deposits held outside the federal government)		
2-3-4	Federal government cash balances at chartered banks and Quebec savings banks***		100
	Total sources and uses	9,514	9,514

* This table is based on the 1965 operations of the Canadian federal government; figures are in millions of dollars.

** These code numbers are used throughout our government sector research, and are based on a classification scheme outlined in the government sector index located at the end of this paper.

*** On November 10, 1969 Banque Populaire, formerly Banque d'Économie de Québec (a Quebec savings bank), commenced operations as a chartered bank. Thus The Montreal City and District Savings Bank is now the only Quebec savings bank still in existence.

the national accounts portion of the government sector is included in econometric models. The interdependence of monetary and fiscal policies is here made quite explicit. Government income-expenditure decisions, government lending, and foreign exchange operations jointly determine the total change in government debt outstanding.

In this context it is clear that monetary policy and debt management decisions are logically interdependent, because once the government income-expenditure decisions are taken, the only policy decision left is how to alter the term structure of the federal debt——the monetary base (currency and chartered bank deposits at the central bank) being treated as the non-interest-bearing portion of that debt——so as to achieve the desired credit conditions. Changes in the size of the monetary base are likely to be the key element of the policy decision, since switching government borrowing from cash to interest-bearing debt, or vice versa, is likely to influence behaviour outside the federal government sector more profoundly than changes in the term structure of the interest-bearing debt.

B. Specification of the Equations

The specification of the equations of the government sector must show explicitly each of the instrument variables that might be altered for policy reasons. Ideally, the equations in the other sectors of a macro-model would be specified in such a way that the same policy instruments would be included. Thus estimates of the expenditure effects of all the various policy changes could be provided. These specification problems lie outside the government sector itself, since the government sector equations show only the determination of the various items listed in Table 1. Here we shall consider some of the problems to be faced in specifying equations for tax receipts, government expenditures, transfer payments, and decisions concerning financing.

When determining which government sector equations to include in a macro-model, or indeed when designing a research strategy for building models of the government sector, it is necessary to establish some ground rules. The criteria we have employed when deciding which items to model first were roughly as follows:

(1) Each explained revenue, expenditure, or asset change should have a structure that can be modelled precisely.

(2) Each explained item should have substantial size and variance.

(3) Each explained item should depend directly on the levels of other variables determined endogenously within a macro-model.

(4) Each explained item should depend on rates or other parameters that have been or might be altered for policy purposes.

The first two criteria are concerned with whether or not an item is worth modelling at all, while the last two are concerned with the requirements of the macro-model our research is intended to service.

In the government sector index at the end of the paper we indicate which items have been modelled and included in this paper and which items have been left for further research. The selection reflects our application of the above priorities. We explain in the following pages some of the factors that influenced these choices.

1. Tax Equations

There are two different approaches to tax equations. The most common procedure is to find one variable or several having significant covariation with the tax receipt, and simply regress the tax receipts series thereon. (See Ando, Brown and Adams [1].) A slight modification of this procedure is to choose a single variable representing the tax base, and then use simple division to find the average tax rate. (See Canadian annual models by Brown [2] and May [5].) An alternative approach is to reject a simple equation that forecasts well in favour of one embodying the essential complexities of the tax structure. Although these more complex equations may be no better for the purposes of short-term forecasting assuming an unchanged tax structure, they have obvious advantages if alternative taxation policies are to be assessed.

The specification of the relevant equations is refreshingly different from the specification of the more usual types of equations in an econometric model. In many behavioural equations the lack of a priori knowledge about the formation of expectations and the lag structure of response makes it almost impossible to choose amongst a number of alternative structural equations. What a relief it is to turn occasionally to a tax equation, where the relevant behaviour is laid out explicitly (although not always clearly) in the tax statutes. The tax law and associated regulations specify both the tax base and the tax rates. If appropriate data were available, one would just construct a tax series by multiplying an appropriate tax base series by the relevant tax rates. Econometric testing would only provide a check to ensure that the sums had been done right. As might be expected, the available data are seldom defined in a manner consistent with the tax law, so that a multitude of fine points (and even some basic features) have to be built into the equations with the aid of the economist's (necessarily) imaginative choice of proxy variables. Since the tax law is quite specific about the rates that are supposed to apply and about the time at which payment must be made, there are very clear (and harsh) criteria for judging the success of a specification. If a tax receipts series is regressed on another series selected and scaled to represent the actual tax base, the estimated coefficient must equal the statutory tax rate——or else! The requirement that coefficients should have specified values if an equation is to be satisfactory means that the equation can be used to derive information about the relationship between the actual tax base and other economic variables. Perhaps this relationship will involve a lot of unravelling, because the data may be unsatisfactory in several ways and the tax structure itself may be quite complex.

An example might help to indicate the kind of approach being advocated. The personal income tax has a number of policy parameters used for stabilization purposes, to increase the supply of effort, or to improve the distribution of income. The important features include basic personal exemptions, dependents' allowances, special allowances (such as the dividend tax credit), and the height and shape of the rate schedule. Some people pay taxes through deductions at source, others on an instalment basis. Different types of income fluctuate differently and make up different fractions of the total income in each income group. To make matters even worse, there are fluctuations in the proportion of total income assessed for tax purposes, in the proportion of allowances and exemptions actually offset against income, and, of course, in the number of taxpayers in each income group. If all these features are to be included in the model no shortcut is possible. Once such a model is working satisfactorily, however, it allows the consequences of different tax structures to be assessed very quickly. Each of the important features of a tax structure appears explicitly and one can thus assess the pattern of tax receipts under alternative assumptions about the level of aggregate income, or about the tax structure itself.

A multi-equation model may be necessary to depict adequately the personal income tax, but for many purposes a less detailed model is to be preferred. Alternative versions of the model should be available to provide just that amount of detail required for analysis of the question at hand.

In all cases the explanatory variables used in the tax equations should be policy variables, exogenous aspects of economic or demographic structures, or variables explained elsewhere in the model. Since the expenditure equations in the private sector of most models are in real terms (constant dollars), it is not appropriate to use simply the current-dollar expenditure series as an independent variable in the tax equations. The procedure we recommend is the construction of a new variable——the product of two variables explained in the private sector. The exact two to be used will depend upon the structure of the model, but the principle is simple: obtain a current-dollar approximation by finding the product of a constant-dollar expenditure series and the most relevant price series explained in the model.

Most taxes are less complex than the personal income tax and may be modelled more simply, but the basic modelling procedure followed should be the same. Each equation should be an accurate representation of the tax structure, so that the effects of alternative values of the tax parameters may be quickly assessed.

2. *Expenditure Equations*

People with experience of government budgeting often suggest that economists are too quick to assume that government expenditure can be handily used to adjust the level of aggregate demand. The object of the government expenditure equations is to indicate the extent to which changes in particular government expenditures have been related to economic variables, political events, and economic policy. There is no chance here of duplicating the closely-knit analytic structure of the tax equations. All that the estimated equations can tell us (even assuming that the appropriate economic, political, and policy variables have been included) is how the expenditure has varied in the past in relation to these variables. Suppose, for example, that spending on highways is closely related to the number of registered vehicles and to nothing else. The relationship then would suggest that highway spending has not in the past been varied countercyclically for policy reasons, leaving open the question of how feasible it would be to make such variations in the future. This close relationship may have existed either because the budget makers felt constrained to keep expenditure closely in line with some measure of need, or because they simply did not wish to make countercyclical variations in expenditures. Thus government expenditure equations are largely forecasting equations, useful as a means of indicating what expenditure is likely to be if policy is not changed. Alternative policies can be conveniently assessed only when they are specific about the proposed relationships between future expenditure and other variables appearing in the model.

What estimation problems arise from the decision to have the government expenditure equations expressed in current dollars? Using the procedure suggested in the last section, one can derive current-dollar approximations for any independent variables in the government expenditure equations. A problem would then arise only if the extra weighting attached to observations in the periods of high prices gave rise to inappropriate estimates. If linear regression in the untransformed current-dollar variables is indicated by the structure of the equation, then the usual assumptions require that the variance of the disturbance term be constant over time, when expressed in current dollars. Should the disturbances in the chosen form for the equation have the necessary characteristics for efficient estimation in terms of constant rather than current dollars, one can still save the current-dollar equation by using weighted regression to get efficient parameter estimates.[4]

RDX1 treats all government expenditure on goods and services as exogenous, and to date we have devoted little time to the search for appropriate equations to explain expenditures. For the provincial and municipal governments, which often relate expenditures to the demand for services rather than the requirements of countercyclical fiscal policy, it may be possible to develop expenditure equations with independent variables that include the demand for government services, tax yields, and credit conditions.[5] For a few categories of federal expenditure this approach may also be fruitful.

3. *Transfer Payments*

Although most of the analysis of the characteristics of the federal budget as an 'automatic stabilizer' has been concerned with tax receipts, the cyclical variations in some transfer payments have been more marked than in the case of tax receipts. The transfer payments equations share the potential precision of the tax equations, since the enabling legislation usually defines the basis for the payments and stipulates a schedule of rates.

[4]If G is in current dollars, X in constant dollars, and P is the price level, we have (at least) two possibilities:

(a) $G = a_1 + b_1 PX + u_1$ where $E(u_1^2) = \sigma_1^2$

(b) $G/P = a_2 (1/P) + b_2 X + u_2$ where $E(u_2^2) = \sigma_2^2$

If (a) holds, then (b) does not, and vice versa. We want to get consistent estimates of a_1 and b_1. If (a) holds, there is no problem. If (b) holds, then the equation should be transformed to that form for estimation. a_1 and b_1 are then found easily, as $\hat{a}_1 = \hat{a}_2$ and $\hat{b}_1 = \hat{b}_2$. If the equation is used to provide point estimates only, no further problems arise. If the error properties are to be used for simulation purposes, the appropriate approximation for the standard error of the estimate is $\sigma_{u_2} P$, which varies with P.

[5]This kind of specification has been adopted for some state and local government expenditure equations in the F.R.B.-M.I.T. model, as outlined in Rasche and Shapiro [6].

The specification problems arise in developing the appropriate demographic and economic variables. As with the tax equations, it is important that the transfer payments equations be structurally apt. Thus they can be used easily and accurately to estimate the size and cyclical variability of a change in transfer payments induced by some change in the policy parameters defining the terms on which payments are made. Equations for the most important Canadian federal transfer payments are presented in Part 3 of this paper.

4. Changes in Asset and Liability Accounts

The most ambitious macro-models include a 'monetary sector' with equations showing how the private sector financial institutions adjust their portfolios in response to given changes in the monetary base and the rediscount rate (bank rate).[6] These equations occasionally explain the policy decisions that produce the 'given' changes in the monetary base. If changes in the monetary base are treated as a predetermined policy variable, then simultaneous equations bias will arise in the estimation of parameters unless one can assume that the policy authorities react only to predetermined variables.

The problem is not just that the usual models may have been founded on biased estimates because the exogeneity of the monetary base has been too freely assumed; but that, once the questioning has proceeded this far, one is tempted to wonder if the monetary base is the appropriate policy variable. It will be recalled from section A, that, given a total volume of debt outstanding, monetary policy through open market and debt management operations is free to decide only the term structure of that debt, including the split between money and interest-bearing debt.[7] Each term structure of the debt will, in general, produce a different term structure of interest rates. Do the authorities choose a feasible (attainable) term structure of interest rates for policy reasons, and then buy and sell securities so as to attain this structure?

[6]For example: de Leeuw [3], and Goldfeld [4].

[7]Other aspects of monetary policy——changes in bank rate, changes in required reserve ratios, moral suasion, etc.——are not considered in this paper. If these policy variables have some independent variance and identifiable impacts on behaviour they should be included explicitly in the model.

Or is a desired term structure of the government debt (including the monetary base) chosen and are interest rates then accepted at whatever magnitudes may be required to lodge the debt in private portfolios? The answer depends on what is regarded as the appropriate index of credit conditions. If the authorities make periodic decisions about the state of credit ease or tightness desired for policy reasons and describe that state in terms of interest rates; then those rates, which are the focus of attention, should be regarded as the policy variables. If the desired state of credit conditions is defined in terms of the size of the monetary base and other measures of the term structure of the debt, these measures should be regarded as the policy variables and interest rates should be determined as implicit endogenous variables. When monetary policy decisions are taken frequently, and when the authorities may be assumed to know roughly the extent and nature of open market operations implied by a chosen structure of interest rates and vice versa, then it will be difficult to tell which are the policy variables. If monetary policy has an identifiable 'posture' over fairly long periods, correlations between some index of this posture and the two candidate sets of policy variables might help to decide the issue. Since the financial sector of any comprehensive model contains equations indicating the demand for money and for bonds, including government bonds, the government sector of the model can contain only the equations determining the chosen values of the policy variables. Thus, if interest rates are assumed to be the policy variables, there can be no equations determining the amount of the various maturities of government debt bought or issued. Conversely, if the policy variables are thought to be the quantities of debt bought or issued, then there can be no interest rate equations, since the rates must equilibrate supply and demand for each maturity of government debt.

In RDX1, we have an explicit equation determining the short-term interest rate (R03) as a function of the U.S. treasury bill rate and the supply and demand for credit in Canada. The demand variable is equal to the sum of aggregate capital expenditures and the government national accounts deficit minus corporate retained earnings and capital cost allowances. The supply of credit is represented by the ratio of chartered bank earning liquid assets to the trend value of total chartered bank assets, with higher values of the ratio indicating faster expansion of bank assets and greater willingness of banks to acquire securities. The cash base (BCD) of the banking system is treated as an exogenous policy variable, and the supply of interest-bearing government debt is therefore left to be determined by the flow-of-funds identity. We recognize that treating BCD as an exogenous variable may have led to some simultaneous equations bias in the estimation of the parameters of RDX1. This is possible either if interest rates rather than BCD are the main policy instrument or if the values given to the policy instrument, whichever it may be, are chosen with regard to contemporary disturbances elsewhere in the model.

Thus we see that no explicit equations are required for the debt items in Table 1, and the only asset-management equations needed are for the components of 2-3-2. The change in the "federal government claims on non-residents" item could be obtained, assuming appropriate adaptations, from the foreign sector of RDX1. We must build up the "investments in government enterprises" item from a knowledge of the specific, underlying legislation and estimates of future policy. The "taxes accrued but not collected" item can be estimated by procedures similar to those used for the tax collection equations, and the miscellaneous items must be treated by miscellaneous methods. Item 2-3-2-6 "corporate taxes accrued but not collected" is the only change in asset and liability accounts dealt with explicitly in this paper.

C. Using the Government Sector Equations

The equations for the government sector have a number of different uses. Detailed models of particular taxes, transfers, or expenditures may be used on their own to study, in a partial way, the consequences of alternative structures. For example, the choice between alternative patterns of exemptions and rates of personal income tax may depend, to some extent, on their effects on the efficiency of the tax as an automatic stabilizer. On certain assumptions about the relative spending propensities of those paying the tax when incomes are following various fluctuating growth paths, a model of the personal income tax can be used to assess the behaviour of tax receipts and the stabilizing effects of the tax. Such detailed analysis should reveal which are the key characteristics of the tax structure from the point of view of stabilization, since it is relatively easy to do a sensitivity analysis using a range of values for each of the tax parameters.

Equations for government sector activity may also be included in models of the government sector as a whole or of the entire economy. A model of the government sector may be useful for forecasting the results of changes in interdependent taxes, transfers, and expenditures. However, the primary advantages of a well-specified government sector can only be achieved if it is linked to an appropriate macro-model. Linking the government sector to a model of the entire economy provides an automatic consistency check on the assumptions made about the time paths of various private sector income and expenditure items. If the macro-model can be developed to the point of explaining private sector balance sheet positions, then grafting the government sector on to a macro-model also allows alternative monetary policies to be analyzed.

The amount of extra information to be gained by linking the government sector models to a macro-model depends on the extent to which the two models are compatible. Maximum compatibility is achieved when all the variables (endogenous to the economy as a whole) used but not explained in one sector are explained in another.[8] An efficient specification of government sector equations for a given macro-model takes advantage of all the explanatory power of that macro-model (e.g. choosing the most appropriate series to define a tax base), yet does not have any unexplained

[8]Compatibility also requires that price variables be used consistently throughout a particular macro-model. It was suggested in section B that the entire government sector should be explained in current-dollar terms. The simplest possible price scheme would then be to obtain a single 'private' Gross National Expenditure implicit price deflator as the ratio of current-dollar to constant-dollar non-government expenditure. The consequences of different sorts of price aggregation on the specification of the government sector tax equations can be assessed by experimenting with more and less aggregated price indices when defining the current-dollar private sector expenditure series used as independent variables in the government tax receipts equations.

endogenous variables left over.[9] Particularly, if the values of important target variables are sensitive to alternative specifications of the group of government sector equations, then those equations should be specified with the utmost accuracy and structural detail.

The advantage of estimating government sector equations at various levels of aggregation and degrees of detail is that one can then see with relative ease how sensitive the forecasts of the dependent variables are to changes in the structure of the equations. This insight provides the model assembler with evidence that helps him to decide how much structural detail is worth giving up in order to obtain a smaller and more easily manipulated macro-model. The trade-off a particular model user is willing to make between simplicity and structural detail will depend on the potential gains from greater precision (which will in turn depend on the range of policy alternatives being assessed), and on the costs of estimation and simulation for models of different sizes.

Because considerable research is necessary to develop adequate data and equations for the government sector, much can be said for generating equations at various levels of structural detail and making them freely available to researchers interested in policy models. The wide distribution of suitable data and equations for the government sector can do much to improve the quality of estimated macro-models, and to clarify the issues in discussions of the relative merits of policy alternatives.

[9]For example, the different types of income—wages, salaries, farm income, dividends, etc.—go to people in different income classes in different proportions. Since the various types of income have distinct patterns of cyclical variation, and since the tax rates are progressive, an income tax model is likely to be more accurate if separate forecasts are available for each type of income. But there is no point in using an income tax model of this complexity unless the macro-model in question has separate explanations for each type of income.

PART 2 FEDERAL GOVERNMENT TAX REVENUE

In our index we split federal government revenue into three main categories: taxes, investment income, and insurance and pension accounts. Taxes are the predominant source of federal revenue, producing 86 per cent of the $9,059 million total in 1965. This part of the paper contains an explanation of six taxes (see Table 2), which in aggregate provided 95 per cent of 1965 federal tax revenue, or 82 per cent of total federal government revenue. In Part 3 of the paper we explain, as segments of our models of the Public Service Superannuation Account (2-1-6-1) and of the Unemployment Insurance Fund (2-1-6-2), the revenue components of the federal government insurance and pension accounts. The six tax equations plus these insurance and pension fund revenue equations explain 90 per cent of total 1965 federal revenue. We treat investment income and revenue from a variety of small taxes as exogenous. They can be fairly easily forecast as a group by means of a regression on Gross National Expenditure (GNE).

Because the rates and structural features of certain taxes are thought of as instruments of public policy, we decided to deal with these taxes in detail. Our aim is to make explicit the major elements of the tax structure that might reasonably be altered for policy purposes, and to relate tax revenues as precisely as we can to the appropriate tax base. Where possible, we used variables that match the legal tax base and that are also either predictable exogenous variables or endogenous variables likely to be explained within a macro-model.

All of these taxes, except the corporation income tax, are recorded in the *National Accounts* on a collection basis.[10] The corporation tax is recorded on an accrual basis, which means that there is a discrepancy between the tax revenue as recorded in the *National Accounts* and the actual flow of funds to the federal

[10] Beginning in 1962, the withholding tax series was adjusted by shifting collections back one month to make the series conform more closely with the flow of interest and dividends to non-residents. This, in effect, puts the series on an accrual basis. Since revenue from the withholding tax is relatively insignificant, we have not taken into account the difference between withholding tax accruals and collections when explaining federal revenues on a cash basis.

Table 2

FEDERAL GOVERNMENT TAX REVENUE MODELLED
(Millions of dollars)

Index Code No.		1965 Revenue
2-1-1-1	Personal income tax	2,612
2-1-2	Corporation income tax accruals	1,675
2-1-3	Non-Resident withholding tax	168
2-1-4-1	Customs duties	665
2-1-4-2	Manufacturers' sales tax	1,837
2-1-4-4	Excise duties	431
	Total	7,388

government. To explain federal tax revenues on a cash basis, therefore, we need an additional relationship defining, for each period, the difference between corporation taxes accrued and corporation taxes collected. This relationship is discussed in section 2-3-2-6 below.

The two largest revenue sources, the personal and corporation income taxes, are shared by the provincial and federal governments. In this part of the paper we explain the total yield of these taxes, and suggest (in conjunction with sections 3-1-1-1 and 3-1-3-1 of Part 4) how the federal and provincial portions of that total yield may be separated. Economists preparing macro-models with fairly simple government sectors may prefer to use the models for the combined federal and provincial income taxes; in constructing such models there is no need to split revenues into federal and provincial components.

2-1-1-1 PERSONAL INCOME TAX

We have developed two models of the personal income tax,[11] either of which could be used in an aggregate macro-model. *Model 1* disaggregates personal income both by type of income and by income level, while *Model 2* disaggregates only by income level.[12]

In *Model 1* total personal income is split into wage income (WSSL) and nonwage personal income (NW).[13] This separation allows us to take explicit account of the differing methods of collecting income tax and the payment of refunds on WSSL and NW. Since wages are taxed mainly by deductions at source, and since the tax on nonwage income is usually paid in quarterly instalments, we treated

[11]The complete sets of equations for these models are given on pp. 47-50, followed by definitions of all the variables included.

[12]After trying various income-class sizes, we chose four income classes based on assessed income: Class 1, zero to $3,000; Class 2, $3,000 to $5,000; Class 3, $5,000 to $10,000; Class 4, over $10,000.

[13]Throughout this part of the paper, wage income (WSSL) refers to wages, salaries and supplementary labour income, while all other personal income is referred to as nonwage income (NW). The major components of NW are: net farm income; interest, dividends and rental income; and transfer payments, excluding interest on the public debt.

these two methods of collection separately, allowing each method to have its own lag distribution relating tax payments to income. A different time pattern of refunds is also involved for each type of income. Taxes on wages in most instances are overpaid during the year, therefore year-end adjustments involve refund payments to the taxpayer. On the other hand, quarterly tax instalments on nonwage income tend to be less than the accrued tax liability so that nonwage taxpayers must often pay additional taxes at the year-end. Most refunds are made between March and the end of June while make-up payments on nonwage income are made primarily during the second quarter. In *Model 1* we accounted for both these processes by using variables specifically related to each type of income. Since the primary object in constructing a policy-oriented model is to make explicit as much of the tax structure as is feasible, disaggregation by type of income is to be preferred if it does not yield inferior results.

The problems that arose from disaggregating income by type were sufficient, however, to make a second approach desirable. The main obstacle encountered in constructing *Model 1* concerned data. Although personal income data were available split into wage and nonwage components, many other supporting series could not be correspondingly disaggregated. In *Model 2*, the use of aggregate income allows us to avoid some of the error that was introduced into *Model 1* through the use of variables based on data arbitrarily split into wage and nonwage components.

However, in *Model 2* we cannot take explicit account of the different tax treatment accorded to wage and nonwage income. As well, the reliability of this model's compound payment and refund variables depends crucially upon how stable the relationship is between wage and nonwage income. While over our estimation period this relationship has not been constant, we think it has been sufficiently fixed to make the aggregate approach a reliable alternative to the method used in *Model 1*. Thus *Model 2* is considerably smaller and less complex than *Model 1*. Also the relative simplicity of *Model 2* may be more valuable for some purposes than the additional information retained by disaggregation.

Personal income is disaggregated by income level in both models owing to progressive tax rates. The usual way of dealing with progressivity in the rate structure is to measure the income elasticity of the tax according to past changes in total income, and then assume that the same elasticity will apply to future changes in income. This assumption is inappropriate if the rate structure is not uniformly progressive, or if aggregate income changes are not distributed evenly over time amongst income classes. In any case, a personal income tax model disaggregated by income class is worthwhile because this model can be used to simulate the aggregate effects of a variety of changes in the level and structure of tax rates. A policy-oriented model of the personal income tax should also identify the nature and amount of exemptions and deductions by income class so that the revenue consequences of changes in these allowances may be more accurately assessed.

When building a personal tax model to be included in the government sector of a macro-model, one must not only choose the level of disaggregation but one must also decide which of the required variables should be made endogenous to the macro-model and which should be generated outside. If a variable used exogenously in the tax model depends crucially on the value of some other variable endogenous to the macro-model, the former variable should also be made endogenous to the macro-model. On the other hand, if the future values of a variable can be better forecast by a scheme that does not depend in an important way on the values of other endogenous variables of the macro-model, then this variable can safely be generated outside the model and treated as an exogenous input to both the tax sector and the entire macro-model. If the model is to be used by other economists, some instructions should be provided to aid them in forecasting the future values of the exogenous, as well as the endogenous, variables. Therefore, in the discussion that follows, we shall try to identify any regularities that will make it easier to forecast the exogenous variables.

Our experiments yielded good equations for both models, so, depending upon one's objective and the size of the model with which one wishes to work, either can be used. *Model 2* may be chosen for forecasting purposes, while for policy analysis the more structurally-explicit *Model 1* should be preferred.

The larger model, *Model 1*, has five stochastic equations explaining tax collections at source (TPS), other income tax collections (TPO), total number of tax returns (NT), assessed wage income (WAS), and assessed nonwage income (NWAS). Two non-stochastic equations, based on the tax law, define tax accruals on wages (AW) and accruals on nonwages (ANW). These two equations in turn depend upon five exogenous variables: quarterly average utilized exemptions and deductions for wage and nonwage income ($\overline{WEX}$, $\overline{NEX}$ respectively), the weighted average tax rate (RW), the rate of dividend tax credit (RDC), and dividends paid by Canadian corporations to Canadian residents (DIVC). To define NT by its components, wage tax returns (NTW) and nonwage tax returns (NTNW), and then to separate NT, NTW, NTNW, WAS and NWAS into each of the four income classes, we require another twenty identities. These identities depend upon nine exogenous ratios: N_i, W_i, and NW_i, (i = 1, 2, 3).

Because *Model 2* is based on aggregate income its structure is much simpler than that of *Model 1.* Only three stochastic equations are required: one for total tax collections (TP), a second for total assessed income (YAS), and a third for the total number of tax returns, NT. Notice that TP = TPS + TPO, and YAS = WAS + NWAS. One non-stochastic equation is needed to obtain total tax accruals (AY), and this equation requires four exogenous variables: average quarterly utilized exemptions and deductions ($\overline{YEX}$), RW, RDC, and DIVC. Except for $\overline{YEX}$, these exogenous terms are the same as those used in *Model 1.* Since in *Model 2* it is not necessary to split NT and YAS into income-type components, only eight identities are needed to define these variables for the four income classes. Six exogenous ratios are used in these identities: N_i and Y_i (i = 1, 2, 3).

Finally, total personal income (YP), wages, salaries and supplementary labour income (WSSL), and nonwage income (NW) (where YP = WSSL + NW) are exogenous to both models, although naturally endogenous to almost any macro-model in which either income tax model might be used. WSSL and NW are required in the WAS and NWAS equations of *Model 1,* while YP is used in a similar equation for YAS of *Model 2.*

Thus there are twenty-seven endogenous variables in *Model 1* and twelve in *Model 2*——the number of endogenous variables equals the number of equations in each model. Seventeen exogenous variables are used in *Model 2,* while twenty-four are required for *Model 1.* At the end of this section all the variables are listed and defined. Data sources are also given. A detailed discussion of the methods used in constructing many of the variables is contained in the data appendix, pages 138-148. We also suggest forecasting schemes that might be useful in predicting future values of most of the exogenous variables.

A. Model 1

Relating tax collections to income involves two conceptual steps, and both differ for wage and nonwage income. The first step is to determine tax accruals on the basis of income earned, exemptions, and tax rates. The second step is to estimate the timing of the process by which a given time series of tax accruals gives rise to tax collections and refunds. In the case of the corporation income tax one can estimate separately the parameters applicable to each conceptual step, since data are available for corporate tax accruals as well as corporate tax collections. For the personal income tax, however, we have data only for collections, so that we must estimate all the parameters jointly.

1. *Tax Accruals on Wage Income* (AW)

The links between income and tax accruals on the one hand, and accruals and tax collections on the other, are simpler for the wage than for the nonwage equation. Tax accruals for wages (AW) are obtained by subtracting quarterly utilized exemptions[14] and deductions from assessed wage income in each of the four income classes, then multiplying this taxable income by the weighted average tax rate for each class and summing over the four classes.

$$AW = \sum_{i=1}^{4} RW_i \, [WAS_i - (NTW_i)(\overline{WEX_i})] \qquad (1.1)$$

Each of the accrual equations is based on the assumption that, within any given income class, all taxpayers earn the mean pre-tax income in that class, receive an average level of exemptions (primarily dependent upon the average family structure for that

[14]The term "utilized exemptions" is explained in detail in the data appendix.

class), and incur tax liabilities according to the current tax rate schedule applied to their taxable income.

The WAS_i are obtained from a stochastic equation giving estimated WAS, ($\hat{WAS}$), and four identities described on pages 29 to 34. The NT_i are also obtained from a stochastic equation that estimates NT, ($\hat{NT}$), and four similar identities outlined on page 29. Each NT_i is split into a wage and nonwage component (NTW_i and $NTNW_i$ respectively) by the identities discussed on pages 34 and 35. Quarterly average utilized exemptions for wage income, $\overline{WEX}_i$, are derived by allocating to each quarter average annual exemptions and deductions ($\overline{EX}_i$), (obtained from *Taxation Statistics*)[15] according to a ratio of quarterly $\hat{WAS}$ to annual $\hat{WAS}$. This ratio is given in the data appendix. The ratios for $N\hat{WAS}$ and $Y\hat{AS}$, which are used to derive the similar quarterly exemption series $\overline{NEX}_i$ and $\overline{YEX}_i$, also appear in the data appendix. We tried a constant allocation of .25 of the annual series $\overline{EX}_i$ in our earlier experiments, but the above procedure gave better results. A priori it seems plausible that the quarterly pattern of utilized exemptions would be related to the quarterly movement of assessed income.

2. Tax Collections——Deductions at Source (TPS)

Deductions at source, TPS, are related to estimated AW lagged one month (since deductions are recorded in the *Public Accounts*[16] in the month following that to which they relate), plus first- and second-quarter adjustment terms, to reflect refunds. These refunds are assumed to be proportional to the total tax

[15] *Taxation Statistics, Part One (Individuals)* issued annually by the Department of National Revenue.

[16] *Public Accounts of Canada* issued annually by the Department of Finance.

liability of the relevant year.[17] Thus

$$TPS = \beta_1 \left[2/3\ AW + 1/3\ AW_{t-1}\right] + \beta_2 Q_1 \sum_{j=-1}^{-4} AW_{t+j} + \beta_3 Q_2 \sum_{j=-2}^{-5} AW_{t+j} \qquad (1.2)$$

Equation (1.2) was fitted to quarterly data and the results are discussed below, beginning on page 42.

3. Tax Accruals on Nonwage Income (ANW)

The relationships of income to accruals and of accruals to collections are slightly more complicated for nonwage income than for wage income. In calculating ANW one must consider the dividend tax credit[18] that permits taxpayers to reduce their tax liability by a certain fraction of dividends received from taxable Canadian corporations. Thus

$$ANW = \sum_{i=1}^{4} RW_i \left[NWAS_i - (NTNW_i)(\overline{NEX}_i)\right] - RDC\ (DIVC) \qquad (1.3)$$

The $\overline{NEX}_i$ term is obtained from $\overline{EX}_i$ in the same manner as $\overline{WEX}_i$ — using a ratio of quarterly NWAS to annual $\widehat{NWAS}$. The derivation of $NWAS_i$ and $NTNW_i$ is described below.

4. Tax Collections—Other (TPO)

The collection equation for nonwage income is also more complex than that for wage income, primarily because the nonwage taxpayer has the option of basing quarterly instalment payments on

[17] All subscripts relating to time represent quarters of a year. Hence, the first summation in equation (1.2) refers to the four quarters preceding quarter t. The Q_i are quarterly dummies with a value of 1 in the specified quarter, zero otherwise.

[18] From 1949 until 1953 the rate of dividend tax credit was 10 per cent. Since then it has been 20 per cent.

either actual taxable income in the previous year or on an estimate of income for the current year. Thus the behaviour underlying the equation may shift if incomes follow a pronounced cycle. When incomes are rising fast most nonwage taxpayers would choose to base their instalments on their tax liabilities for the preceding year, while when aggregate income is rising slowly or falling many nonwage taxpayers may choose the other option and pay on the basis of the current year's estimated income. Since most incomes have been rising generally throughout our data period, we have assumed that nonwage taxpayers in general prefer to pay on the basis of the previous year's taxable income. This gives the basic equation:

$$TPO = \beta_1 Q_1 \sum_{j=-1}^{-4} ANW_{t+j} + \beta_2 Q_2 \sum_{j=-2}^{-5} ANW_{t+j} + \beta_3 Q_3 \sum_{j=-3}^{-6} ANW_{t+j} + \beta_4 Q_4 \sum_{j=-4}^{-7} ANW_{t+j} \qquad (1.4)$$

One could add to this equation various cyclical variables intended to capture the changes in payment practices accompanying changes in the proportion of taxpayers with declining incomes. The results of some experiments with a number of these alternative cyclical variables are discussed on pages 45 and 46. Note that this equation requires a separate term for each quarter, not only to take account of refunds and supplementary payments, but also because the quarterly instalments are based on a particular calendar year. This necessitates a different summation expression on ANW_{t+j} for each quarter.

In support of these four main equations of *Model 1* are the following twenty-three equations and identities:

$NT = f_1 (NE, T)$ (1.5)

$WAS/WSSL = f_2 (NU/NL, T)$ (1.6)

$NWAS = f_3 (NW)$ (1.7)

$$NT_i = N_i\,(NT) \quad (i=1,2,3) \qquad (1.8 \text{ to } 1.10)$$

$$NT_4 = NT - \sum_{i=1}^{3} NT_i \qquad (1.11)$$

$$WAS_i = W_i\,(WAS) \quad (i=1,2,3) \qquad (1.12 \text{ to } 1.14)$$

$$WAS_4 = WAS - \sum_{i=1}^{3} WAS_i \qquad (1.15)$$

$$NWAS_i = NW_i\,(NWAS) \quad (i=1,2,3) \qquad (1.16 \text{ to } 1.18)$$

$$NWAS_4 = NWAS - \sum_{i=1}^{3} NWAS_i \qquad (1.19)$$

$$NTW_i = (NT_i)\,[(WAS_i)/(WAS_i + NWAS_i)] \quad (i=1,2,3,4) \qquad (1.20 \text{ to } 1.23)$$

$$NTNW_i = NT_i - NTW_i \quad (i=1,2,3,4) \qquad (1.24 \text{ to } 1.27)$$

5. *Total Number of Tax Returns Filed* (NT)

The total number of tax returns filed each year is closely related to the income earning population. Everyone, other than the dependent of a taxpayer, is required by tax law to submit a return declaring his income whether taxable or not. This law should result in NT almost equalling the total civilian labour force (NL). But NT and NL are not equal as Chart 1 reveals. Part of the difference is accounted for by workers who are unemployed for the entire year (they have no incomes and hence file no returns), and workers who fail to file returns. Much more numerous, however, are taxpayers' dependents who enter the labour force but do not earn sufficient income to lose their dependent status——hence they need not file tax returns. Also, the group of taxable income earners who are over sixty-five and retired is increasing rapidly in importance as a component of the difference between NT and NL.

Chart 1

TOTAL CIVILIAN LABOUR FORCE, EMPLOYMENT AND TAX RETURNS

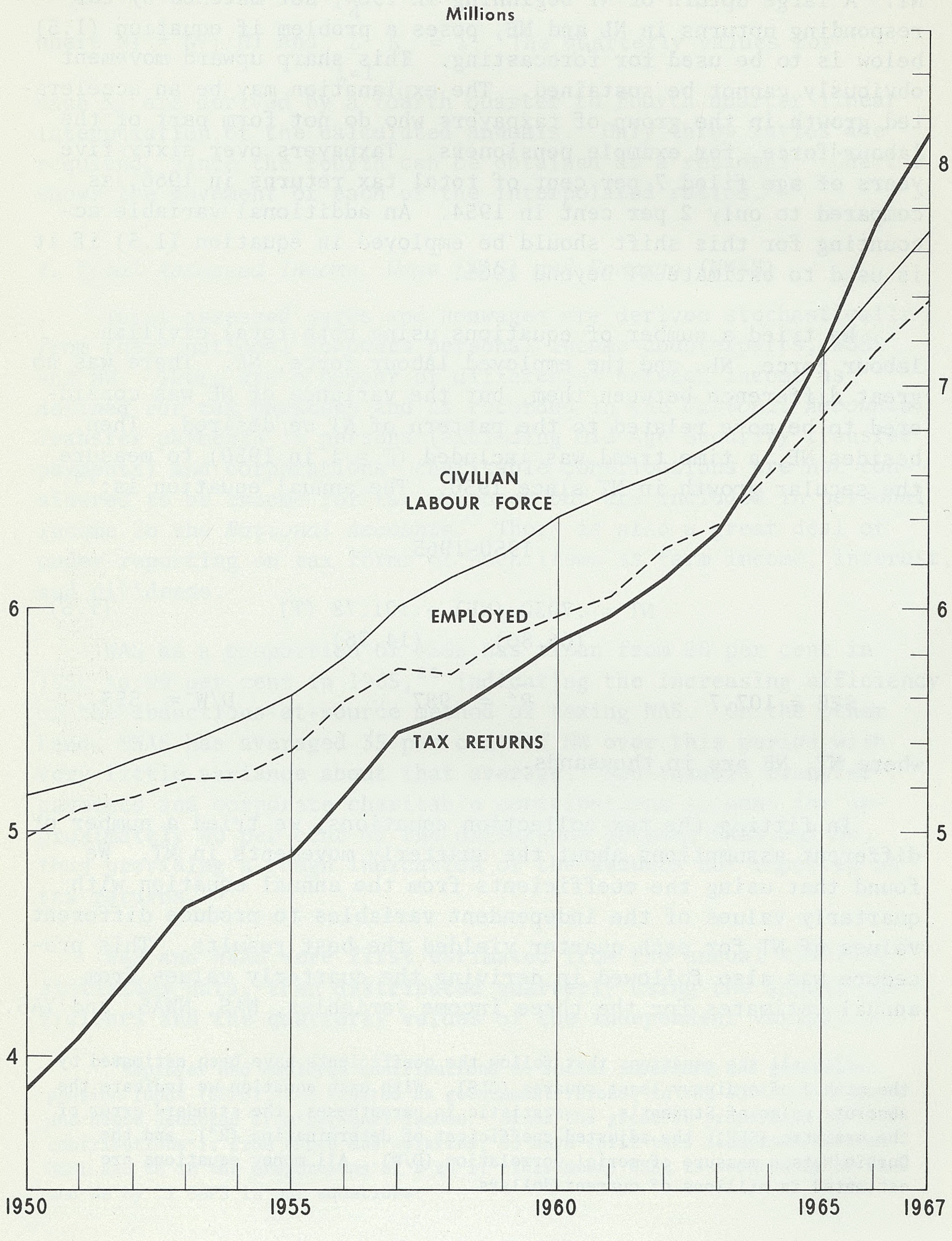


Chart 1 shows the relative movements of the total civilian labour force, NL, employed labour force (NE), and total tax returns, NT. A large upturn of NT beginning in 1964, not matched by corresponding upturns in NL and NE, poses a problem if equation (1.5) below is to be used for forecasting. This sharp upward movement obviously cannot be sustained. The explanation may be an accelerated growth in the group of taxpayers who do not form part of the labour force, for example pensioners. Taxpayers over sixty-five years of age filed 7 per cent of total tax returns in 1966, as compared to only 2 per cent in 1954. An additional variable accounting for this shift should be employed in equation (1.5) if it is used to estimate NT beyond 1965.

We tried a number of equations using both total civilian labour force, NL, and the employed labour force, NE. There was no great difference between them, but the variance of NE was considered to be more related to the pattern of NT we desired. Then, besides NE, a time trend was included (T = 1 in 1950) to measure the secular growth in NT since 1950. The annual equation is:[19]

1950-1965

$$NT = \underset{(66.86)}{.7930\,(NE)} + \underset{(14.36)}{101.78\,(T)} \qquad (1.5)$$

SEE = 107.7 $\bar{R}^2 = .987$ D/W = .553

where NT, NE are in thousands.

In fitting the tax collection equations, we tried a number of different assumptions about the quarterly movements in NT. We found that using the coefficients from the annual equation with quarterly values of the independent variables to produce different values of NT for each quarter yielded the best results. This procedure was also followed in deriving the quarterly values from annual estimates for the three income variables, WAS, NWAS, and YAS.

[19]In all the equations that follow the coefficients have been estimated by the method of ordinary least squares (OLS). With each equation we indicate the absolute value of Student's t statistic in parentheses, the standard error of the estimate (SEE), the adjusted coefficient of determination ($\bar{R}^2$), and the Durbin/Watson measure of serial correlation (D/W). All money equations are estimated in millions of current dollars.

The NT_i, equations (1.8) to (1.11), are obtained from NT using three of four ratios calculated from the *Taxation Statistics*, where $N_i = NT_i/NT$ and $\sum_{i=1}^{4} N_i = 1$. The quarterly values for each N_i are derived by a fourth quarter to fourth quarter linear interpolation of the calculated annuals. Only three ratios are required, since the fourth can be obtained as a residual. Chart 2 shows the movement of each of the interpolated ratios.

6. Total Assessed Income, Wage (WAS) *and Nonwage* (NWAS)

Total assessed wages and nonwages are derived stochastically from their national accounts personal income counterparts, WSSL and NW. There are a number of differences between income as defined for tax purposes and as recorded in the *National Accounts*. Transfer payments to persons (excluding Old Age Security transfer payments) and corporations' charitable contributions are not considered to be income for tax purposes but are included in personal income in the *National Accounts*. There is also a great deal of under-reporting on tax forms of such items as farm income, interest, and dividends.

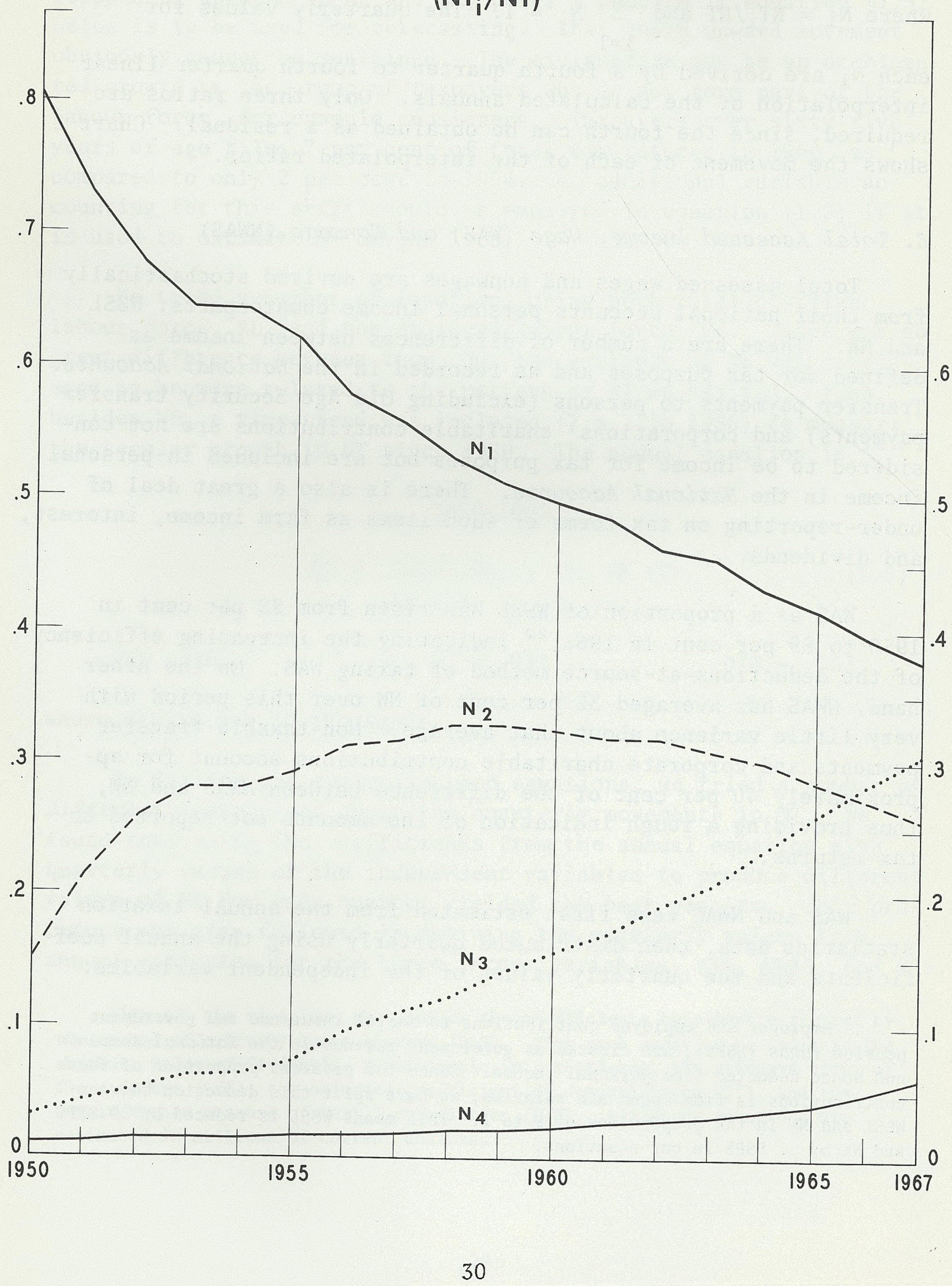
WAS as a proportion of WSSL has risen from 88 per cent in 1950 to 99 per cent in 1965,[20] indicating the increasing efficiency of the deductions-at-source method of taxing WAS. On the other hand, NWAS has averaged 35 per cent of NW over this period with very little variance about that average. Non-taxable transfer payments and corporate charitable contributions account for approximately 40 per cent of the difference between NWAS and NW, thus providing a rough indication of the amounts not reported on tax returns.

WAS and NWAS were first estimated from the annual taxation statistics data, then distributed quarterly using the annual coefficients and the quarterly values of the independent variables.

[20]Employer and employee contributions to social insurance and government pension funds (SSPS), are treated as government revenue in the *National Accounts* and hence deducted from personal income. Since the greatest proportion of these contributions is from wages and salaries, we have split this deduction between WSSL and NW in the proportions of 9 to 1. This means WSSL is reduced by .9 SSPS and NW by .1 SSPS in our equations.

Chart 2

PROPORTION OF TOTAL TAXPAYERS IN EACH INCOME CLASS (NT_i/NT)



In equation (1.6) we estimated the quarterly ratio WAS/WSSL using the above procedure. We then used this ratio with WSSL[21] to derive quarterly $\widehat{WAS}$. WAS/WSSL is made dependent upon the unemployment rate and a time trend equal to 1 in 1950.

1950-1965

$$WAS/WSSL = \underset{(2.60)}{.4339}\,(NU/NL) + \underset{(15.13)}{.0081}\,(T) + \underset{(125.25)}{.8450} \qquad (1.6)$$

SEE = .008 $\bar{R}^2$ = .967 D/W = 1.22

The NWAS equation is much simpler than equation (1.6), making use of the quite stable relationship between NWAS and NW.

1950-1965

$$NWAS = \underset{(99.65)}{.3555}\,(NW) \qquad (1.7)$$

SEE = 130.6 $\bar{R}^2$ = .980 D/W = 1.40

To disaggregate WAS and NWAS by income class we used six exogenous ratios, W_i and NW_i (Charts 3 and 4) where:

$$W_i = WAS_i/WAS$$

$$NW_i = NWAS_i/NWAS \quad (i=1,2,3, \text{ or } 4)$$

These ratios are similar to the N_i, obtained from the *Taxation Statistics*, and their quarterly values are also derived by interpolation of the annual series. Only three ratios are required the ratio for the fourth income class being a residual.

To obtain future values for these ratios (and for the N_i, Chart 2), one would extrapolate. The various ratios follow quite distinct time paths. There is an obvious movement of a modal body

[21]That is, WSSL - .9 SSPS as indicated in the preceding footnote.

Chart 3

PROPORTION OF TOTAL ASSESSED WAGE INCOME IN EACH INCOME CLASS (WAS_i/WAS)

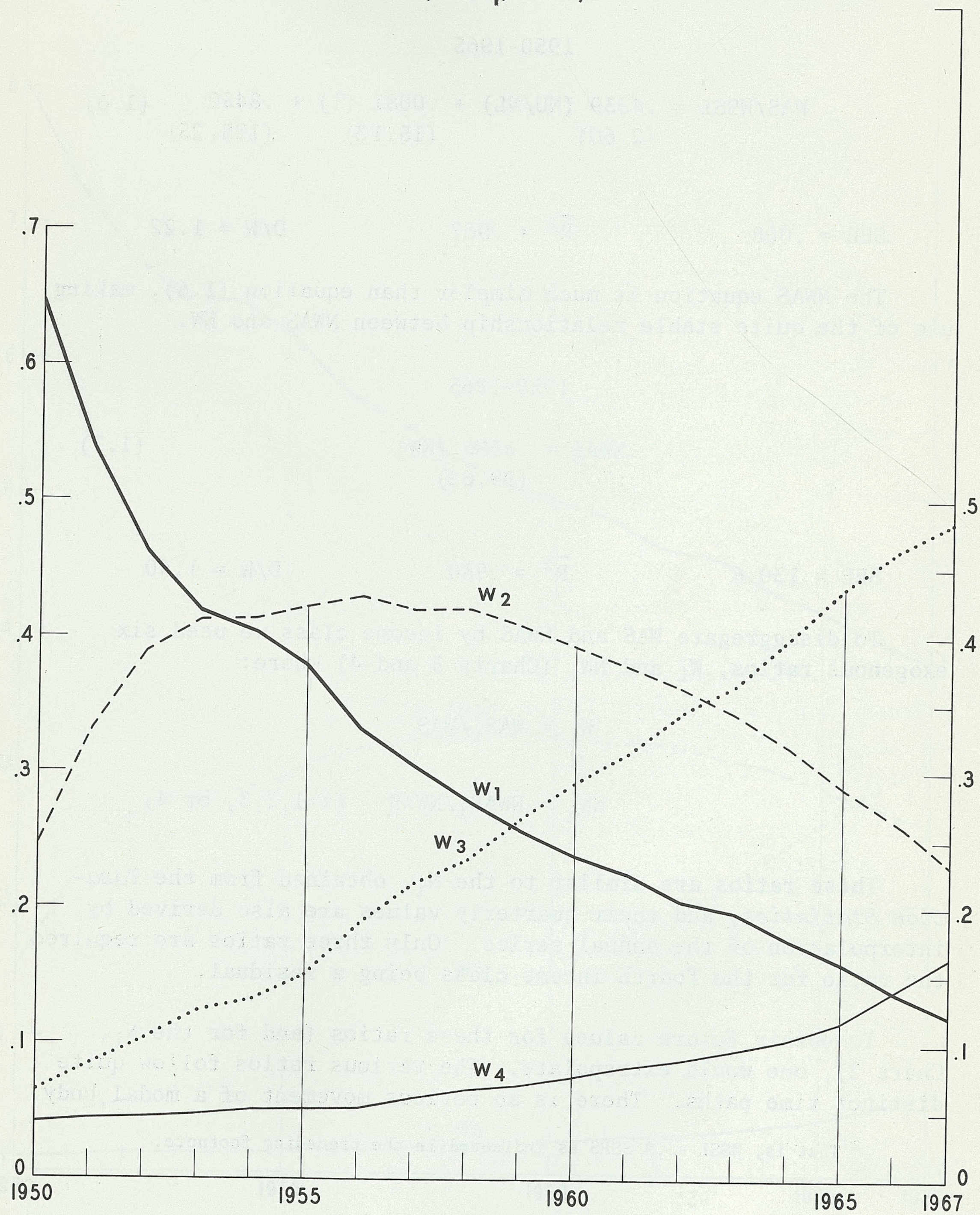
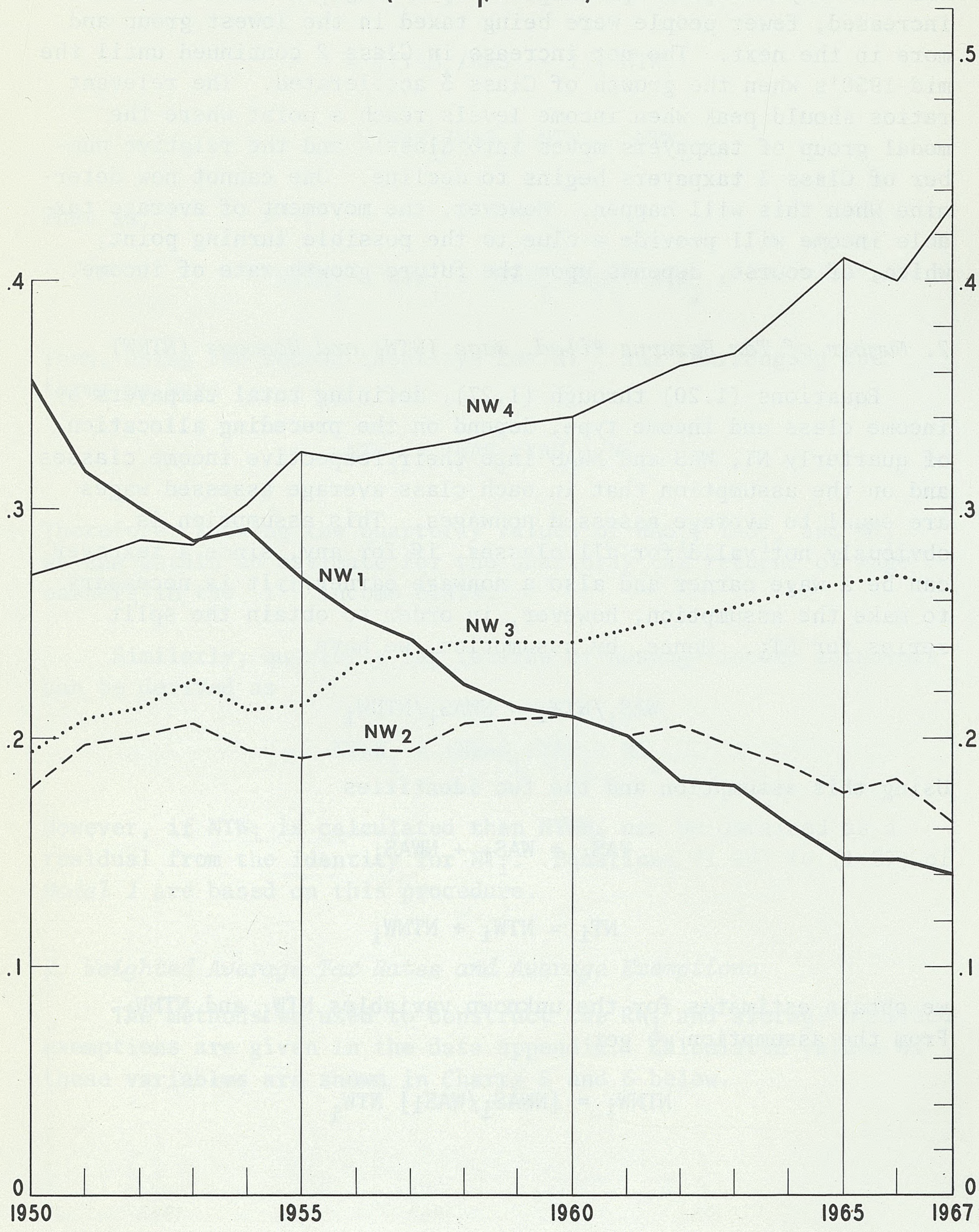


Chart 4

PROPORTION OF TOTAL ASSESSED NONWAGE INCOME IN EACH INCOME CLASS ($NWAS_i/NWAS$)



of income and a related group of taxpayers from lower to higher income classes, beginning in 1950 in Class 1 and now passing through Class 3. As the charts show, the relative size of Class 1 has steadily declined since 1950. As average levels of incomes increased, fewer people were being taxed in the lowest group and more in the next. The net increase in Class 2 continued until the mid-1950's when the growth of Class 3 accelerated. The relevant ratios should peak when income levels reach a point where the modal group of taxpayers moves into Class 4 and the relative number of Class 3 taxpayers begins to decline. One cannot now determine when this will happen. However, the movement of average taxable income will provide a clue to the possible turning point, which, of course, depends upon the future growth rate of income.

7. Number of Tax Returns Filed, Wage (NTW) *and Nonwage* (NTNW)

Equations (1.20) through (1.27), defining total taxpayers by income class and income type, depend on the preceding allocation of quarterly NT, WAS and NWAS into their respective income classes, and on the assumption that in each class average assessed wages are equal to average assessed nonwages. This assumption is obviously not valid for all classes, if for any, since a taxpayer can be a wage earner and also a nonwage earner. It is necessary to make the assumption, however, in order to obtain the split series for NT_i. Hence, by assumption, we have

$$WAS_i/NTW_i = NWAS_i/NTNW_i$$

Using this assumption and the two identities

$$YAS_i = WAS_i + NWAS_i$$

$$NT_i = NTW_i + NTNW_i$$

we obtain estimates for the unknown variables NTW_i and $NTNW_i$. From the assumption we get

$$NTNW_i = [NWAS_i/WAS_i]\ NTW_i$$

Substituting the definition of $NWAS_i$ and rearranging the terms we have

$$NTNW_i = [(YAS_i - WAS_i)/WAS_i]\ NTW_i$$

$$= [(YAS_i/WAS_i - 1)]\ NTW_i$$

$$= (YAS_i/WAS_i)\ NTW_i - NTW_i$$

That is,

$$NTNW_i + NTW_i = (YAS_i/WAS_i)\ NTW_i$$

Then, using the second identity, for NT_i, and rearranging the terms we have

$$NTW_i = (WAS_i/YAS_i)\ NT_i$$

Therefore, knowing the quarterly values of WAS_i, YAS_i, and NT_i we can obtain an estimate for the quarterly tax returns of wage earners in the i^{th} income class.

Similarly, quarterly tax returns of nonwage-income taxpayers can be derived as

$$NTNW_i = (NWAS_i/YAS_i)\ NT_i$$

However, if NTW_i is calculated then $NTNW_i$ can be obtained as a residual from the identity for NT_i. Equations (1.24) to (1.27) of *Model 1* are based on this procedure.

8. Weighted Average Tax Rates and Average Exemptions

The methods we used to construct the RW_i and average utilized exemptions are given in the data appendix. Calculated values of these variables are shown in Charts 5 and 6 below.

Chart 5

WEIGHTED AVERAGE PERSONAL INCOME TAX RATES

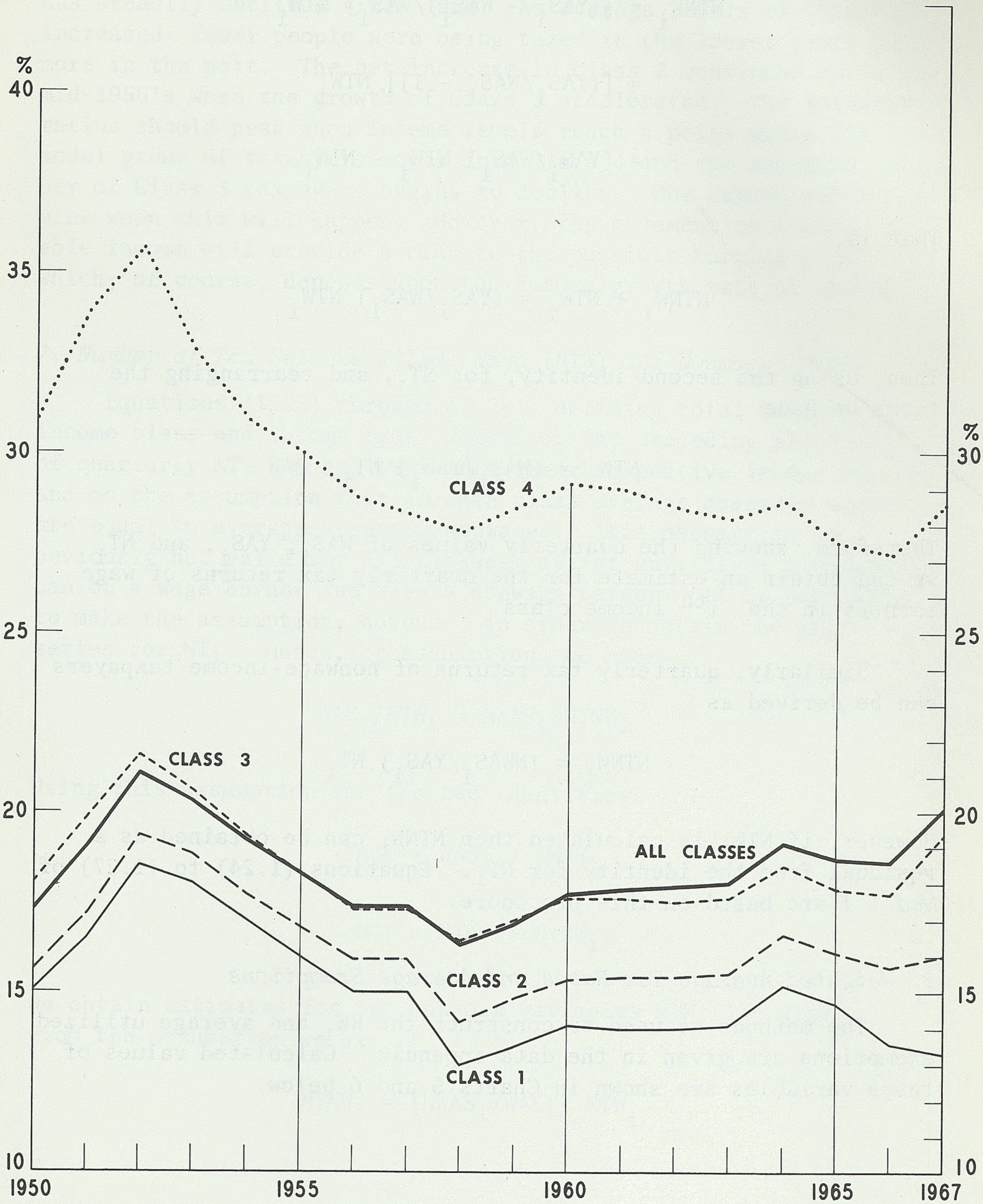
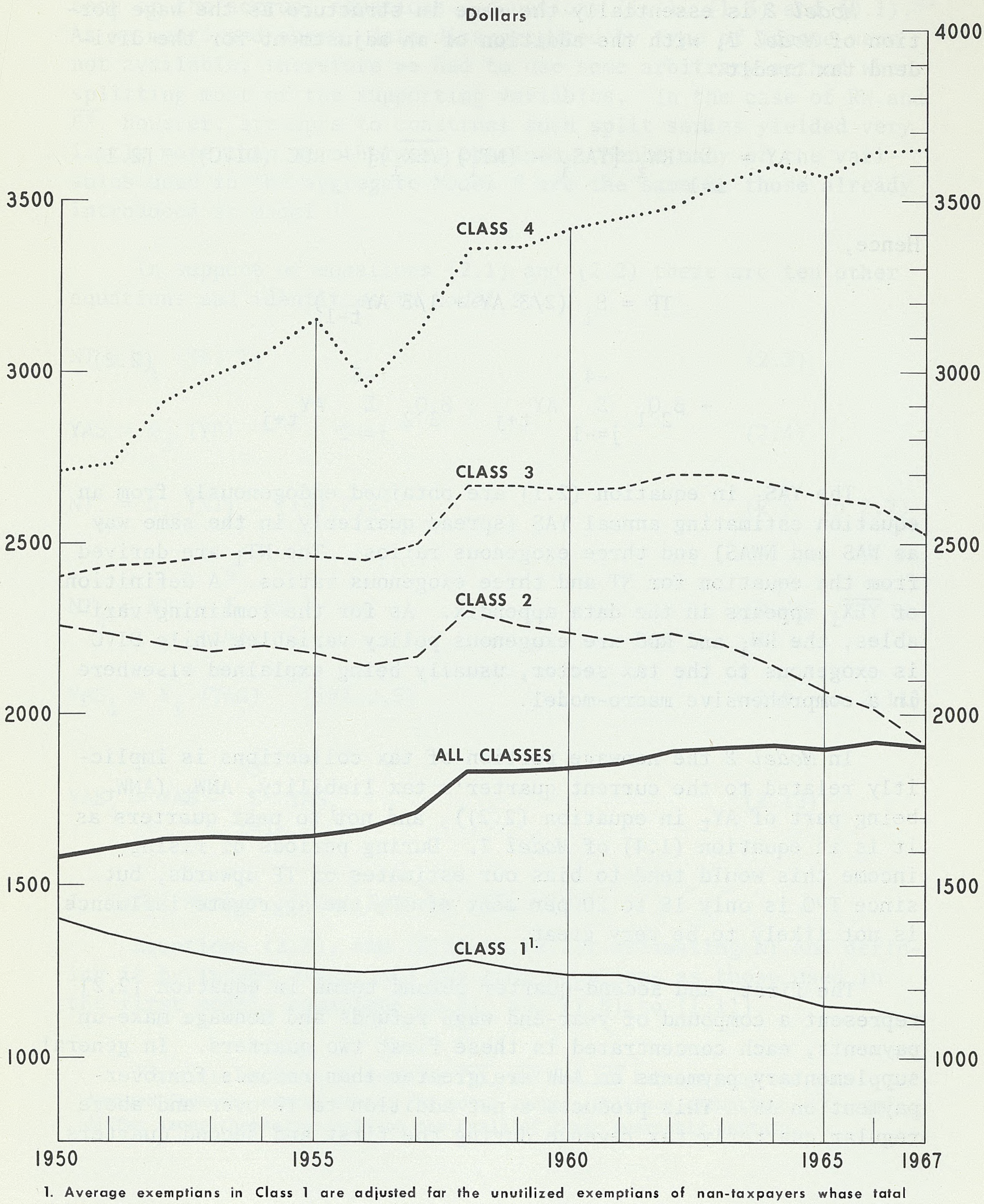


Chart 6

AVERAGE UTILIZED EXEMPTIONS AND DEDUCTIONS



1. Average exemptians in Class 1 are adjusted far the unutilized exemptians of nan-taxpayers whase tatal exemptians are greater than assessed incame. The ratio of utilized to total exemptions is given in the data appendix.

B. Model 2

Model 2 is essentially the same in structure as the wage portion of *Model 1*, with the addition of an adjustment for the dividend tax credit.

$$AY = \sum_{i=1}^{4} RW_i \, [YAS_i - (NT_i)(\overline{YEX_i})] - RDC\ (DIVC) \qquad (2.1)$$

Hence,

$$TP = \beta_1 \, (2/3\ AY + 1/3\ AY_{t-1})$$

$$+ \beta_2 Q_1 \sum_{j=-1}^{-4} AY_{t+j} + \beta_3 Q_2 \sum_{j=-2}^{-5} AY_{t+j} \qquad (2.2)$$

The YAS_i in equation (2.1) are obtained endogenously from an equation estimating annual YAS (spread quarterly in the same way as WAS and NWAS) and three exogenous ratios. The NT_i are derived from the equation for NT and three exogenous ratios. A definition of $\overline{YEX_i}$ appears in the data appendix. As for the remaining variables, the RW_i and RDC are exogenous policy variables while DIVC is exogenous to the tax sector, usually being explained elsewhere in a comprehensive macro-model.

In *Model 2* the nonwage portion of tax collections is implicitly related to the current quarter's tax liability, ANW_t (ANW_t being part of AY_t in equation (2.2)), and not to past quarters as it is in equation (1.4) of *Model 1*. During periods of rising income this would tend to bias our estimates of TP upwards, but since TPO is only 15 to 20 per cent of TP, the aggregate influence is not likely to be very great.

The first- and second-quarter refund terms in equation (2.2) represent a compound of year-end wage refunds and nonwage make-up payments, each concentrated in these first two quarters. In general, supplementary payments on ANW are greater than refunds for overpayment on AW. This produces a net addition to TP over and above regular quarterly tax revenue during the first and second quarters.

Note also that the two main policy variables, the weighted rate, RW, and quarterly average exemptions, $\overline{YEX}$, are the same in each of the accrual equations——equations (1.1), (1.3) and (2.1).[22] As we mentioned above, data disaggregated by type of income were not available; therefore we had to use some arbitrary method when splitting most of the supporting variables. In the case of RW and $\overline{EX}$, however, attempts to construct such split series yielded very little more than we otherwise obtained. Hence many of the variables used in the aggregate *Model 2* are the same as those already introduced in *Model 1*.

In support of equations (2.1) and (2.2) there are ten other equations and identities in *Model 2*:

$$NT = f_1 \ (NE, T) \qquad (2.3)$$

$$YAS = f_2 \ (YP) \qquad (2.4)$$

$$NT_i = N_i \ (NT) \quad (i=1,2,3) \qquad (2.5 \text{ to } 2.7)$$

$$NT_4 = NT - \sum_{i=1}^{3} NT_i \qquad (2.8)$$

$$YAS_i = Y_i \ (YAS) \quad (i=1,2,3) \qquad (2.9 \text{ to } 2.11)$$

$$YAS_4 = YAS - \sum_{i=1}^{3} YAS_i \qquad (2.12)$$

1. Total Number of Tax Returns Filed (NT)

Equations (2.3), and (2.5) to (2.8), estimating NT and defining it by income class, are the same equations as those used in the first model, equations (1.5), and (1.8) to (1.11).

[22]This is essentially true for $\overline{YEX}$, $\overline{WEX}$ and $\overline{NEX}$ because they are each derived from the annual series for total average utilized exemptions ($\overline{EX}$), and differ among themselves only on the basis of their quarterly pattern.

2. *Total Assessed Personal Income* (YAS)

YAS is derived from national accounts personal income, YP. Like its wage and nonwage components, total assessed income differs from YP by the amount of unreported and non-taxable income. YAS as a per cent of YP has increased since 1950 from 67 to 77 per cent, averaging 74 per cent over the fifteen-year period. The growth in this proportion represents primarily the increased coverage of the wage component of income, as described earlier.

The equation for YAS was fitted using annual data.

1950-1965

$$YAS = \underset{(71.99)}{.8153}\ (YP) - \underset{(6.81)}{1974.74} \qquad (2.4)$$

SEE = 324.4 $\bar{R}^2 = .997$ D/W = 1.52

Similarly to WAS and NWAS, the quarterly values are obtained by using the annual coefficients from this equation with quarterly values of YP and the constant term.[23]

The four class variables, YAS_i, are obtained using the exogenous ratios,

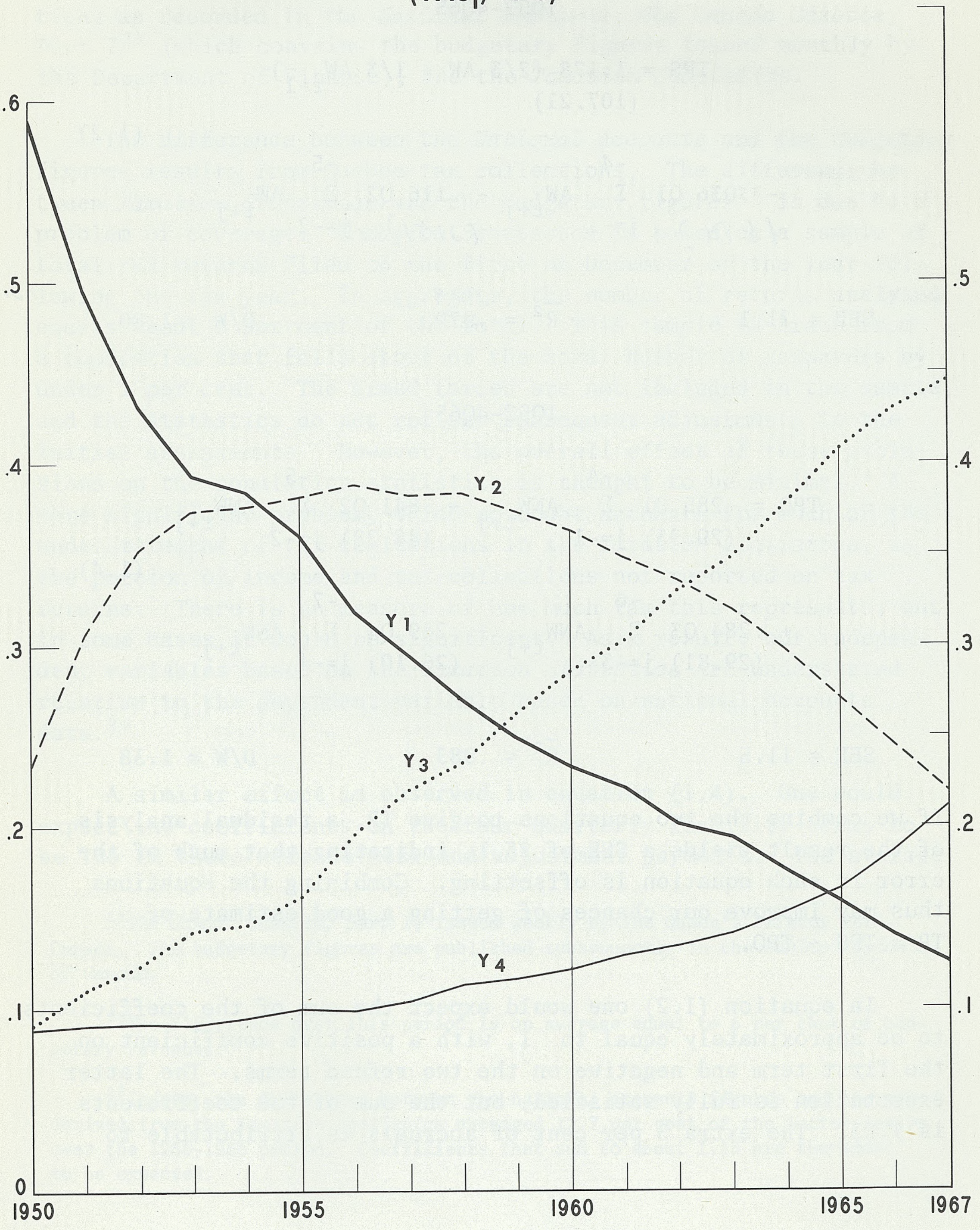
$$Y_i = YAS_i/YAS$$

derived from interpolating the annual series, calculated from the *Taxation Statistics* and shown in Chart 7. The Y_i are in every way similar to the other ratios of this sort discussed above. Classes 1 and 2 display the effect of the movement of taxpayers and income into and out of the respective groups, with Class 3 yet to level off. Three of the four ratios are used to obtain YAS_i according to equations (2.9) to (2.12).

[23]That is, the quarterly value of the annual constant term, which would be 493.69.

Chart 7

PROPORTION OF TOTAL ASSESSED INCOME IN EACH INCOME CLASS (YAS_i/YAS)



C. Econometric Results

1. Model 1

1Q52-4Q65

$$TPS = \underset{(107.21)}{1.178}\,(2/3\ AW + 1/3\ AW_{t-1}) - \underset{(6.84)}{.036}\ Q1 \sum_{j=-1}^{-4} AW_{t+j} - \underset{(21.51)}{.116}\ Q2 \sum_{j=-2}^{-5} AW_{t+j} \qquad (1.2)$$

SEE = 21.1 $\bar{R}^2$ = ~~.979~~ .929 D/W = 1.80

correct BBP

1Q52-4Q65

$$TPO = \underset{(29.93)}{.285}\ Q1 \sum_{j=-1}^{-4} ANW_{t+j} + \underset{(88.28)}{.841}\ Q2 \sum_{j=-2}^{-5} ANW_{t+j} + \underset{(29.81)}{.284}\ Q3 \sum_{j=-3}^{-6} ANW_{t+j} + \underset{(26.10)}{.249}\ Q4 \sum_{j=-4}^{-7} ANW_{t+j} \qquad (1.4)$$

SEE = 11.5 $\bar{R}^2$ = .983 D/W = 1.38

If we combine the two equations to give TP, a residual analysis of the result yields a SEE of 23.1, indicating that much of the error in each equation is offsetting. Combining the equations thus may improve our chances of getting a good estimate of TP = TPS + TPO.

In equation (1.2) one would expect the sum of the coefficients to be approximately equal to 1, with a positive coefficient on the first term and negative on the two refund terms. The latter expectation is fully satisfied, but the sum of the coefficients is 1.03. The extra 3 per cent of accruals is attributable to an underestimate of AW because of discrepancies between the figures in the *Taxation Statistics*, upon which our independent variables are based, and the national accounts figures, which we are attempting to estimate. Chart 8 shows annual personal income tax collections as recorded in the *National Accounts*, *The Canada Gazette, Part I*[24] (which contains the budgetary figures issued monthly by the Department of Finance), and the *Taxation Statistics*.

The difference between the *National Accounts* and the budgetary figures results from Quebec tax collections. The difference between *Taxation Statistics* and the budgetary figures[25] is due to a problem of coverage. *Taxation Statistics* is based on a sample of total tax returns filed to the first of December of the year following the tax year. In aggregate, the number of returns analyzed equals about 6 per cent of the total. This sample is drawn from a population that falls short of the total number of taxpayers by under 1 per cent. The armed forces are not included in the sample, and the statistics do not reflect subsequent adjustments to the initial assessments. However, the overall effect of these exclusions on the population statistics is thought to be minimal. A more significant problem, which no doubt accounts for much of the understatement of tax collections in the *Taxation Statistics*, is the portion of income and tax collections not reported on tax returns. There is no measure of how much tax this represents, but in some cases it could be significant. As a result, our independent variables based on the *Taxation Statistics* are understated relative to the dependent variable based on national accounts data.[26]

A similar effect is observed in equation (1.4). One would expect the coefficients on the four quarterly liability terms to be .25 if there were no year-end adjustment payments. The average

[24]*The Canada Gazette, Part I*, issued weekly by The Queen's Printer for Canada. The budgetary figures are published subsequently in the *Public Accounts of Canada*.

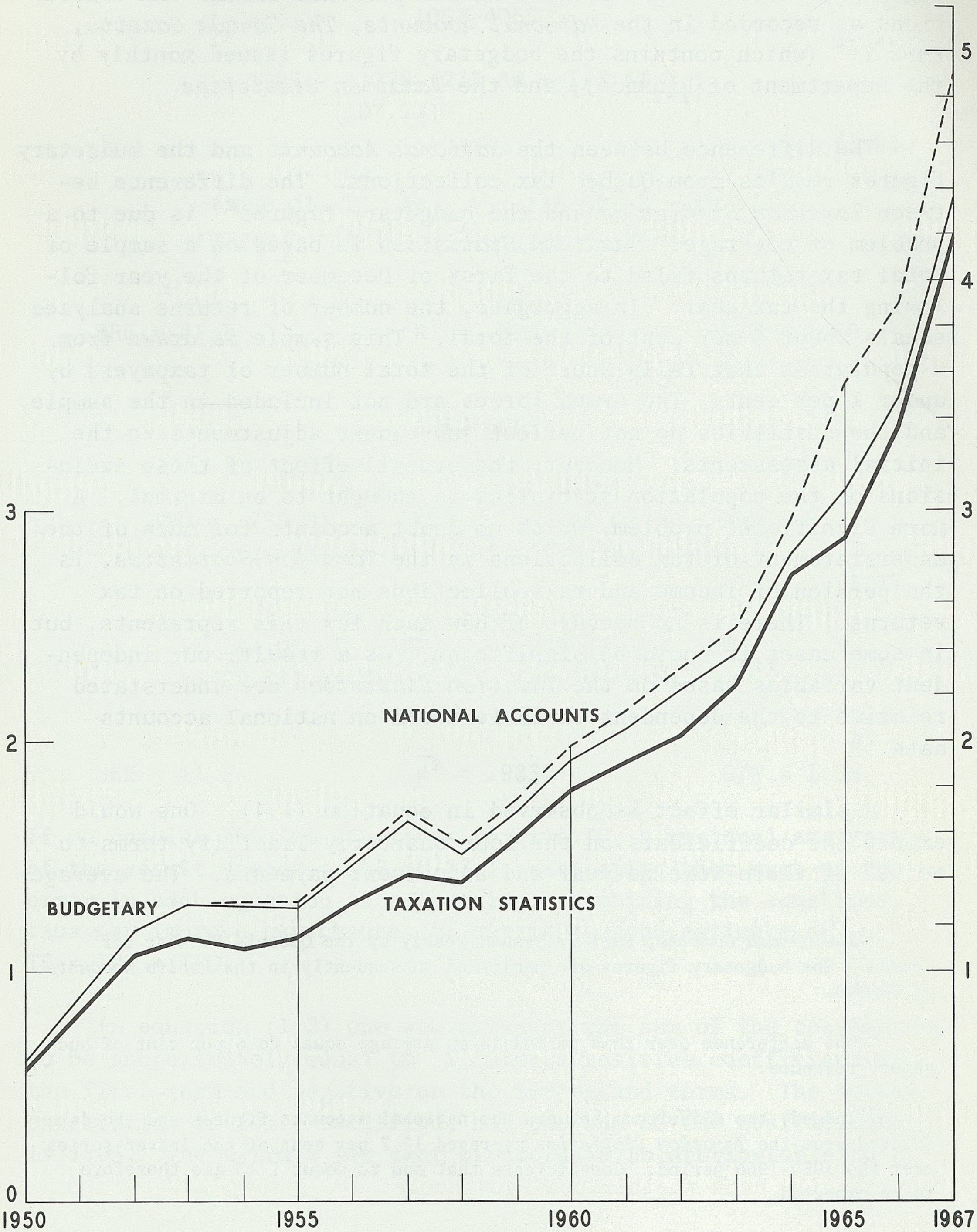
[25]The difference over this period is on average equal to 6 per cent of budgetary revenues.

[26]Indeed, the difference between the national accounts figures and the data derived from the *Taxation Statistics* averaged 12.7 per cent of the latter series over the 1950-1966 period. Coefficients that sum to about 1.13 are therefore to be expected.

Chart 8

TOTAL PERSONAL INCOME TAX COLLECTIONS ON THREE ACCOUNTING BASES

Billions of Dollars



value of the coefficients is substantially higher than .25, supporting our suspicion that accruals, based on the *Taxation Statistics*, are understated.

Two additional variables were tried in equation (1.4) in an attempt to capture the effects of large deviations from our assumption that all nonwage taxpayers base their quarterly instalments on the preceding year's actual liabilities, instead of on an estimate of the liabilities of the current year. The first of these variables, $(ANW_{-4} - ANW)^+$, has non-zero values only when the term is positive. $(ANW_{-4} - ANW)^+$ expresses the fact that when many individual nonwage incomes are declining more nonwage taxpayers will probably choose to base their tax payments on an estimate of the current year's income than when such incomes are rising, not (as we had assumed) on the previous year's accruals. If so equation (1.4), estimating tax payments in terms of the previous year's income, will be an overestimate of the tax actually payable. The coefficient on this variable should be negative and close to 1. As it turned out, the expression has non-zero values in all quarters of 1953 and 1954, in the second quarter of 1957, the fourth quarter of 1962, and the first quarter of 1963. The coefficient was negative whenever the variable was used in the equation, but always had a value much less than 1. This would seem to indicate that not all nonwage taxpayers switched between taxpaying options, either because they were unwilling or unable to make an accurate forecast of the current year's tax liability on which to base their tax payments. Gains would not have been large for many such people and we suspect that most of them regarded the benefits as not worth the effort. The term was never very significant and did little to improve our equation.

The other variable we tried represented an attempt to capture the nonlinear effect that cyclical changes in income have on the expected size of second-quarter nonwage adjustment payments. The variable used was

$$Q_2 \sum_{j=-2}^{-5} ANW_{t+j} \left[\sum_{j=-2}^{-5} (ANW_{t+j}) / \sum_{j=-6}^{-9} (ANW_{t+j}) \right]$$

The bracketed expression will vary around the value of 1 depending primarily upon whether nonwage incomes are rising (>1), falling (<1), or remaining steady (=1). Of course, tax structure changes will also affect this ratio, possibly offsetting income changes. The usefulness of the term is reduced because of this possibility. However, if, with a given tax structure, accruals are rising over the previous year's accruals because of income changes, more quarterly instalments will be based on the preceding year's liabilities. This switch of taxpaying options will cause make-up payments in the following year to be higher than they otherwise would have been. The variable is designed to capture these changes in tax collections due to fluctuating make-up payments, concentrated primarily in the second quarter. Highly collinear with the main second-quarter term, the variable raises the standard error of the coefficient on this latter term quite significantly. We did not use the variable in our final model since it contributed little to the explanatory power of the equation within the data period.

2. Model 2

The estimated equation giving total tax collections, TP, directly is:

1Q52-4Q65

$$TP = \underset{(97.31)}{1.105}\,(2/3\ AY + 1/3\ AY_{t-1}) - \underset{(1.20)}{.007}\, Q_1 \sum_{j=-1}^{-4} AY_{t+j} + \underset{(7.41)}{.041}\, Q_2 \sum_{j=-2}^{-5} AY_{t+j} \qquad (2.2)$$

SEE = 27.9 $\quad \bar{R}^2 = .972 \quad$ D/W = 1.77

Here, as in equations (1.2) and (1.4), the coefficient on the accrual term is greater than the expected 1. The understatement of AY represented by the enlarged coefficient is approximately of the same magnitude as that obtained from AW and ANW, supporting the previous arguments.

The second-quarter adjustment term is positive, reflecting the preponderance of make-up payments on ANW relative to refunds on AW in this quarter. The first-quarter refund term is negative, but the low coefficient and t-value suggest a near balance of refunds and make-up payments during this period.

The standard error of the estimate of TP is $23.1 million from *Model 1* and $27.9 million from *Model 2*, or, 2.8 per cent and 3.3 per cent of the average quarterly TP in 1965. Both models should predict well. Depending upon one's objectives, each model has its uses. Much of the improvement possible in both sets of equations will come from improved data. The basic specification is fairly straightforward as the tax structure is given quite explicitly in tax law. Additional experimentation must be undertaken to improve the specification of a number of the supporting equations, such as those for NT, YAS, WAS and NWAS, and, finally, exemptions could be made endogenous.

D. Models and Variables

1. *Model 1*

1.1 $$AW = \sum_{i=1}^{4} RW_i \left[WAS_i - (NTW_i)(\overline{WEX_i})\right] \quad (i=1,2,3,4)$$

1.2 $$TPS = 1.178\ (2/3\ AW + 1/3\ AW_{t-1}) - .036\ Q_1 \sum_{j=-1}^{-4} AW_{t+j} - .116\ Q_2 \sum_{j=-2}^{-5} AW_{t+j}$$

1.3 $$ANW = \sum_{i=1}^{4} RW_i \left[NWAS_i - (NTNW_i)(\overline{NEX_i})\right] - RDC\ (DIVC) \quad (i=1,2,3,4)$$

1.4 $$TPO = .285\ Q_1 \sum_{j=-1}^{-4} ANW_{t+j} + .841\ Q_2 \sum_{j=-2}^{-5} ANW_{t+j}$$

$$+ .284\, Q_3 \sum_{j=-3}^{-6} ANW_{t+j} + .249\, Q_4 \sum_{j=-4}^{-7} ANW_{t+j}$$

1.5 $NT = .7930\,(NE) + 25.445\,(T_{1950})$

1.6 $WAS/WSSL = .4339\,(NU/NL) + .002\,(T_{1950}) + .8450$

1.7 $NWAS = .3555\,(NW)$

1.8 $NT_1 = N_1\,(NT)$

1.9 $NT_2 = N_2\,(NT)$

1.10 $NT_3 = N_3\,(NT)$

1.11 $NT_4 = NT - \sum_{i=1}^{3} NT_i$

1.12 $WAS_1 = W_1\,(WAS)$

1.13 $WAS_2 = W_2\,(WAS)$

1.14 $WAS_3 = W_3\,(WAS)$

1.15 $WAS_4 = WAS - \sum_{i=1}^{3} WAS_i$

1.16 $NWAS_1 = NW_1\,(NWAS)$

1.17 $NWAS_2 = NW_2\,(NWAS)$

1.18 $NWAS_3 = NW_3\,(NWAS)$

1.19 $NWAS_4 = NWAS - \sum_{i=1}^{3} NWAS_i$

1.20 $NTW_1 = NT_1 \, [WAS_1/(WAS_1 + NWAS_1)]$

1.21 $NTW_2 = NT_2 \, [WAS_2/(WAS_2 + NWAS_2)]$

1.22 $NTW_3 = NT_3 \, [WAS_3/(WAS_3 + NWAS_3)]$

1.23 $NTW_4 = NT_4 \, [WAS_4/(WAS_4 + NWAS_4)]$

1.24 $NTNW_1 = NT_1 - NTW_1$

1.25 $NTNW_2 = NT_2 - NTW_2$

1.26 $NTNW_3 = NT_3 - NTW_3$

1.27 $NTNW_4 = NT_4 - NTW_4$

2. Model 2

2.1 $AY = \sum_{i=1}^{4} RW_i \, [YAS_i - (NT_i)(\overline{YEX_i})] - RDC \, (DIVC)$

2.2 $TP = 1.105 \, (2/3 \, AY + 1/3 \, AY_{t-1}) - .007 \, Q_1 \sum_{j=-1}^{-4} AY_{t+j}$

$+ .041 \, Q_2 \sum_{j=-2}^{-5} AY_{t+j}$

2.3 $NT = .7930 \, (NE) + 25.445 \, (T_{1950})$

2.4 $YAS = .8153 \, (YP) - 493.69$

2.5 $NT_1 = N_1\ (NT)$

2.6 $NT_2 = N_2\ (NT)$

2.7 $NT_3 = N_3\ (NT)$

2.8 $NT_4 = NT - \sum_{i=1}^{3} NT_i$

2.9 $YAS_1 = Y_1\ (YAS)$

2.10 $YAS_2 = Y_2\ (YAS)$

2.11 $YAS_3 = Y_3\ (YAS)$

2.12 $YAS_4 = YAS - \sum_{i=1}^{3} YAS_i$

3. *Variables and Data Sources*

Much of the data is available on a tape created for the Bank of Canada Research Department Experimental Model (RDX1) and maintained in the Research Department of the Bank. Reference to this source will be in the form RDX 12345, where 12345 is the tape location of the particular series being discussed. The Research Department maintains two additional tapes, a master databank tape containing about 6,000 time series and a special tax tape containing some 700 series. Where variables are not entered on the RDX tape reference will be made to the databank tape or the tax tape in the form DB 12345 or TT 12345, respectively. Variables marked with an asterisk are discussed further in the data appendix, pages 138 to 162. All money magnitudes are in millions of current dollars, unless otherwise indicated.

Variables	Definition	Source
Endogenous		
ANW	Accrued tax liabilities on taxable nonwage personal income	Identity (1.3)
AW	Accrued tax liabilities on taxable wage income	Identity (1.1)
AY	Accrued tax liabilities on total taxable personal income	RDX 11600
NT	Total number of tax returns, taxable and non-taxable, annual values, units	(TT 6020 + TT 6030)
NT_i	Number of taxable and non-taxable returns in the i^{th} income class, calculated values, quarterly	RDX 11545-11548
$NTNW_i$	Number of nonwage tax returns in the i^{th} income class, calculated values, quarterly	See page 34.
NTW_i	Number of wage tax returns in the i^{th} income class, calculated values, quarterly	See page 35.
NWAS	Assessed nonwage personal income, annual values, thousands of dollars	(TT 6440 + TT 6450) - (TT 6050 + TT 6060)
$NWAS_i$	Assessed nonwage personal income in the i^{th} income class, calculated values, quarterly	NW_i (NWAS)

Variables	Definition	Source
Endogenous		
*TP	Total personal income tax collections, national accounts basis	RDX 11560
*TPO	Personal income tax collections; other payments, net of refunds	RDX 11024
*TPS	Personal income tax collections; deductions at source, net of refunds	RDX 11023
WAS	Assessed wage income, annual values, thousands of dollars	(TT 6050 + TT 6060)
WAS_i	Assessed wage income in the i^{th} income class, calculated values, quarterly	W_i (WAS)
YAS	Total assessed personal income, annual values, thousands of dollars	(TT 6440 + TT 6450)
YAS_i	Total assessed personal income in the i^{th} income class, calculated values, quarterly	RDX 11551-11554

Exogenous

DIVC	Dividends paid to Canadians by Canadian corporations (exogenous to government sector)	RDX 2406
*$\overline{EX}_i$	Average utilized exemptions and deductions claimed by taxpayers in the i^{th} income class, annual figure repeated in each quarter, thousands of dollars	RDX 11030-11033
*N_i	Proportion of total tax returns in the i^{th} income class	RDX 11302-11305
NE	Total employed (exogenous to government sector), millions of persons	RDX 11065
*$\overline{NEX}_i$	Average utilized exemptions and deductions claimed by taxpayers with nonwage income in the i^{th} income class. $\overline{NEX}_i$ equals average annual utilized exemptions ($\overline{EX}_i$) proportioned quarterly by assessed nonwage income	Data Appendix, pp. 144-148
NL	Total civilian labour force (exogenous to government sector), millions of persons	RDX 11141
NU	Total unemployed (exogenous to government sector), millions of persons	RDX 11063

Variables	Definition	Source
Exogenous		
NW	Total nonwage personal income, as per national accounts, equal to YP less WSSL (exogenous to government sector)	Derived from YP and WSSL
*NW_i	Proportion of assessed nonwage income in the i^{th} income class	RDX 11015-11018
*PE1	An adjusting ratio used to derive utilized exemptions in Class 1	RDX 11029
RDC	Rate of dividend tax credit, per cent/100	RDX 11006
*RW_i	Weighted average income tax rate for the i^{th} income class, per cent/100	RDX 11019-11022
SSPS	Social security and government pension contributions (exogenous to government sector)	(RDX 11285 + RDX 2178)
T_{1950}	Quarterly time trend (T = 1 in first quarter of 1950, 2 in second quarter, etc.)	
*W_i	Proportion of assessed wage income in the i^{th} income class	RDX 11011-11014

*$\overline{WEX}_i$	Average utilized exemptions and deductions claimed by taxpayers with wage income in the i^{th} income class. $\overline{WEX}_i$ equals average annual utilized exemptions ($\overline{EX}_i$) proportioned quarterly by assessed wage income	Data Appendix, pp. 144-148
WSSL	Total wages, salaries and supplementary labour income as per national accounts (exogenous to government sector)	RDX 224
*Y_i	Proportion of total assessed personal income in the i^{th} income class	RDX 11393-11395, 11398
*$\overline{YEX}_i$	Average utilized exemptions and deductions claimed by taxpayers in the i^{th} income class. $\overline{YEX}_i$ equals average annual exemptions ($\overline{EX}_i$) proportioned quarterly by assessed personal (i.e. wage plus nonwage) income, dollars	RDX 11556-11559
YP	Total personal income as per national accounts (exogenous to government sector)	RDX 240

2-1-2 CORPORATION INCOME TAX ACCRUALS

A. Total Corporation Income Tax Accruals

Unlike almost all other government tax revenues, corporation direct taxes are recorded in the *National Accounts* on an accrual basis. In this subsection we present four estimating equations for corporation income tax accruals. We constructed the first two equations, (2) and (2'), to estimate total corporation accruals as they appear in the *National Accounts* (TCA, RDX 1352); and the second two, (3) and (3'), to estimate TCA net of provincial logging and mining taxes (PLMT, RDX 11626). In section 2-3-2-6 we explain the equation that converts our accrual series to a collection basis. From our accrual and collection equations the cash-budget item can be derived. It consists of corporation taxes accrued but not collected.

In modelling corporation income tax accruals we followed the procedure used to explain all of our tax revenue series. We calculated a weighted tax rate and used the product of this rate with an appropriate base as the independent variable in the regression equation.

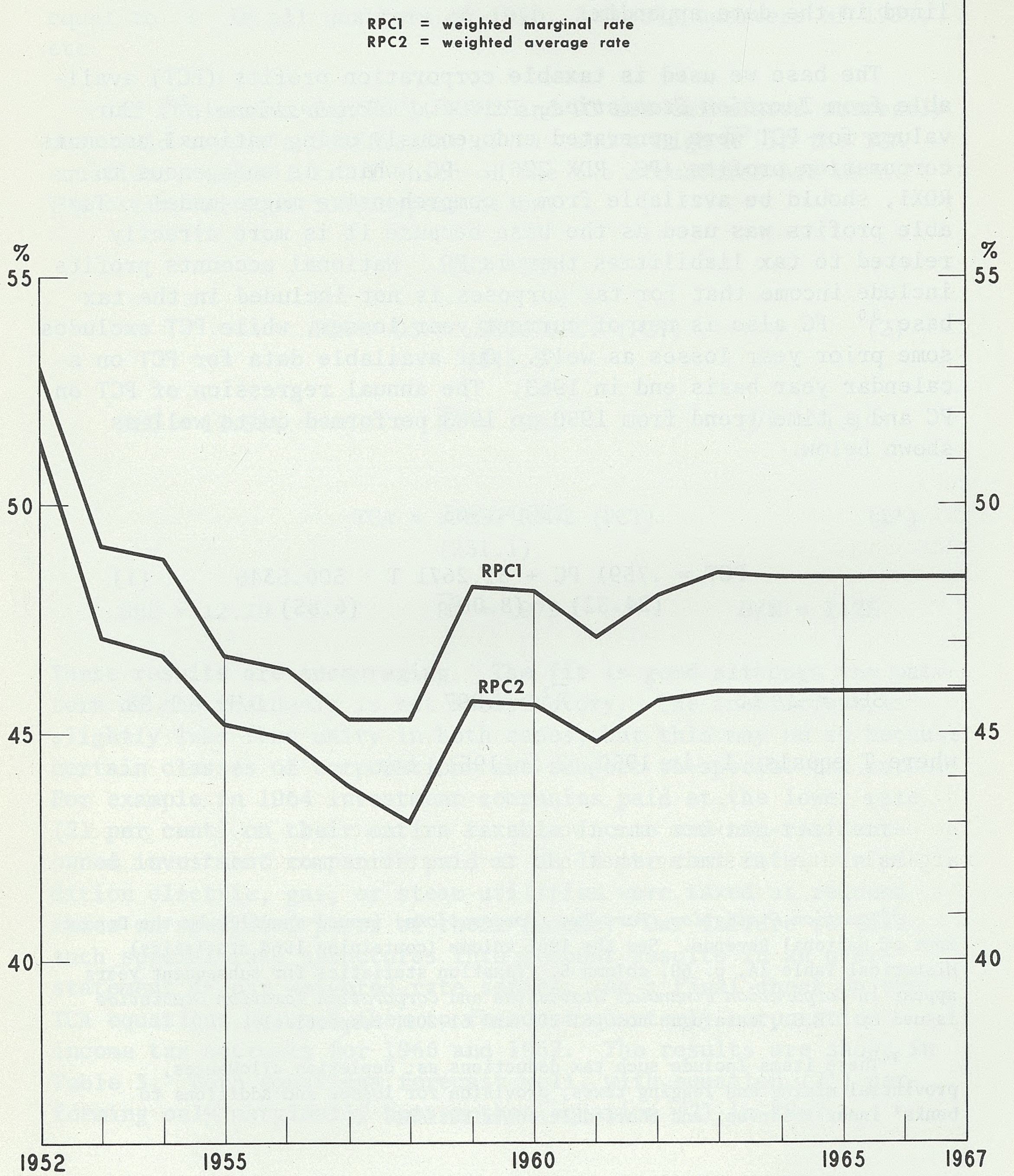
The weighted corporation tax rate is easier to calculate than that for the personal income tax, since there are only two marginal corporation rates at the federal level[27] and a single rate at the provincial level. We calculated two sets of weighted rates, RPC1 and RPC2, which include both federal and provincial levies. Computational methods and values for these rates are given in the data appendix. Chart 9 shows RPC1 and RPC2 from 1952 to 1967.[28] The difference between the two series is that RPC1 is a weighted marginal rate, assuming that all corporations taxed at the high rate pay that rate of tax on their total taxable income, while

[27]Corporations pay 21 per cent on their first $35,000 of taxable income plus 50 per cent on the amount over $35,000. These rates include a 3 per cent levy for the old age security tax. In addition corporations were subject to a 3 per cent surcharge in 1968 and 1969.

[28]1964 is the last year for which the tax data needed to derive our weighted rates are available. Rates shown for 1965, 1966 and 1967 are therefore estimates obtained by extrapolation.

Chart 9

WEIGHTED CORPORATION INCOME TAX RATES



RPC2 is a weighted average rate. One would expect, therefore, that the use of RPC1 would overstate accruals. Allowance was made for this by calculating as a separate variable (D3), the approximate amount by which TCA would be overstated, and by adding D3 to TCA in the regressions that use RPC1. Construction of D3 is outlined in the data appendix.

The base we used is taxable corporation profits (PCT) available from *Taxation Statistics, Part Two (Corporations)*.[29] Our values for PCT were generated endogenously using national accounts corporation profits (PC, RDX 226). PC, which is endogenous to RDX1, should be available from a comprehensive macro-model. Taxable profits was used as the base because it is more directly related to tax liabilities than is PC. National accounts profits include income that for tax purposes is not included in the tax base.[30] PC also is net of current year losses, while PCT excludes some prior year losses as well. Our available data for PCT on a calendar year basis end in 1963. The annual regression of PCT on PC and a time trend from 1950 to 1963 performed quite well as shown below.

1950-1963

$$PCT = \underset{(24.32)}{.7591}\,PC + \underset{(8.00)}{31.2671}\,T + \underset{(6.85)}{500.5346} \qquad (1)$$

SEE = 27.63 $\qquad \bar{R}^2 = .997 \qquad$ D/W = 1.98

where T equals 1 in 1950, 2 in 1951, etc.

[29]*Taxation Statistics, Part Two (Corporations)* issued annually by the Department of National Revenue. See the 1966 volume (containing 1964 statistics) Historical Table 1A, p. 69, column 5. Taxation statistics for subsequent years appear in *Corporation Financial Statistics* and *Corporation Taxation Statistics* issued by D.B.S., catalogue nos. 61-207 and 61-208, respectively.

[30]These items include such tax deductions as: depletion allowances, provincial mining and logging taxes, provision for losses and additions to banks' inner reserves, and charitable contributions.

Quarterly estimates of PCT were obtained from the equation

$$PCT = .7591\ PC + 7.8168\ T + 125.1337 \qquad (1')$$

where PC is the quarterly series and T is now a step function equal to 1 in all quarters of 1950, 2 in all quarters of 1951, etc.

If the series for RPC1, RPC2 and D3 are calculated correctly one would expect a coefficient of 1 and a high $\overline{R}^2$ in the TCA equations, regardless of which of the two structures are used. The estimated equations appear below.

1Q52-4Q65

$$TCA + D3 = \underset{(245.5)}{.9816}\ RPC1\ (PCT) \qquad (2)$$

$SEE = 12.58$ $\qquad \overline{R}^2 = .982$ $\qquad D/W = 1.19$

$$TCA = \underset{(231.1)}{.9855}\ RPC2\ (PCT) \qquad (2')$$

$SEE = 12.79$ $\qquad \overline{R}^2 = .979$ $\qquad D/W = 1.25$

These results are encouraging. The fit is good although the pattern of the residuals is not satisfactory. The coefficient is slightly less than unity in both cases, but this may be so because certain classes of corporations are subject to special tax rates. For example in 1964 investment companies paid at the lower rate (21 per cent) on their entire taxable income and non-resident-owned investment companies paid at an 18 per cent rate. In addition electric, gas, or steam utilities were taxed at reduced rates on specified parts of their income. Our failure to take such special rate structures into account results in an overstatement of our weighted-rate series. As a final check on the TCA equations we used them to forecast quarterly corporation income tax accruals for 1966 and 1967. The results are shown in Table 3. Both equations forecast well, with equation (2') performing only marginally better than equation (2) in 1967.

Table 3

TOTAL CORPORATION INCOME TAX ACCRUALS 1966-1967
(Millions of dollars)

	Actual Values*	Forecast Values Equation (2)	Forecast Error	Forecast Values Equation (2')	Forecast Error
1966 Q1	462	502	+40	505	+43
2	626	614	-12	611	-15
3	559	531	-28	532	-27
4	605	587	-18	586	-19
Year	2,252	2,234	-18	2,234	-18
1967 Q1	441	464	+23	469	+28
2	604	585	-19	585	-19
3	555	555	0	556	+1
4	608	595	-13	594	-14
Year	2,208	2,199	-9	2,204	-4

* *National Accounts* — Quarterly, Table 5, line 4.

Table 4

TOTAL CORPORATION INCOME TAX ACCRUALS
LESS PROVINCIAL LOGGING AND MINING TAXES 1966-1967
(Millions of dollars)

	Actual Values*	Forecast Values Equation (3)	Forecast Error	Forecast Values Equation (3')	Forecast Error
1966 Q1	437	495	+58	497	+60
2	610	605	-5	602	-8
3	552	523	-29	524	-28
4	598	579	-19	577	-21
Year	2,197	2,202	+5	2,200	+3
1967 Q1	414	457	+43	462	+48
2	596	577	-19	577	-19
3	545	547	+2	548	+3
4	602	586	-16	585	-17
Year	2,157	2,167	+10	2,172	+15

* *National Accounts* — Quarterly, Table 5, line 4 minus PLMT (RDX 11626).

To explain corporation tax collections (2-3-2-6) and to disaggregate by level of government our corporation accrual series must be net of provincial logging and mining taxes, PLMT. Subtracting PLMT from TCA and reestimating equations (2) and (2') we get the following results:

1Q52-4Q65

$$TCA - PLMT + D3 = \underset{(195.6)}{.9679}\ RPC1\ (PCT) \qquad (3)$$

SEE = 15.57 $\quad \bar{R}^2 = .973 \quad$ D/W = 1.42

$$TCA - PLMT = \underset{(178.2)}{.9711}\ RPC2\ (PCT) \qquad (3')$$

SEE = 16.34 $\quad \bar{R}^2 = .967 \quad$ D/W = 1.56

The goodness of fit has decreased slightly although there has been some improvement in the pattern of the residuals. Excluding PLMT from TCA does not appear to worsen the forecasting ability of the equations to any large extent. The PLMT series is on a collection basis and most of the annual figure is concentrated in the first two quarters. Some experiments were made with quarterly dummies in an attempt to improve the performance of the equations but the results generally were less satisfactory than those from the equations presented. While more experiments are currently underway, at present the accrual equation used in RDX1 is equation (3') above.[31]

B. Federal Corporation Income Tax Accruals

We attempted to explain federal corporation accruals (FCA) in a manner analogous to our explanation of total corporation accruals. The results are presented in equations (4) and (4').

[31]The rate used in RDX1 is RPC2. However, since no other corporation tax rates appear in the model, RPC2 is entered simply as RPC (RDX 11007).

1Q52-4Q65

$$FCA + D3 = \underset{(203.1)}{.9455} \; FRPC1 \; (PCT) \tag{4}$$

SEE = 12.61 $\quad \bar{R}^2 = .954 \quad$ D/W = 1.50

$$FCA = \underset{(188.6)}{.9478} \; FRPC2 \; (PCT) \tag{4'}$$

SEE = 12.89 $\quad \bar{R}^2 = .946 \quad$ D/W = 1.48

FRPC1 and FRPC2 are the weighted marginal and average federal rates of corporation tax. They are equal to RPC1 and RPC2 less the weighted average of federal abatement rates and any levies in excess of the abatement rate imposed by provincial governments. See data appendix pages 152-153 for values of FRPC1 and FRPC2.

In equations (4) and (4') the fits are good and the patterns of residuals satisfactory although the coefficients are disappointingly low, being about ten standard errors away from their expected values of unity. While the forecasting performance of these equations is satisfactory it is not nearly as good as that of the TCA equations. Equation (4) understates actual FCA by $53 million (3.12 per cent) in 1966 and $46 million (2.82 per cent) in 1967. Forecasts using equation (4') are also too low by $54 million in 1966 and by $41 million in 1967.

The problem involved, in attempting to estimate federal corporation tax accruals by using structural equations such as those outlined above, is that we do not know how accurately the federal accrual figures reported in the *National Accounts* reflect true federal accruals. The only source of quarterly tax accrual data available prior to publication of *Taxation Statistics* is the corporation profits survey.[32] Firms responding to this survey report only the quarter's additions to current liabilities for total corporation income taxes. Therefore, no source of data exists from which to build up a series for federal corporation tax accruals.

[32] *Corporation Profits* issued quarterly by D.B.S., catalogue no. 61-003.

In section 3-1-3-1 a rationale is presented for calculating provincial corporation tax accruals based on our a priori knowledge of the tax structure. If such a procedure is followed, we would obtain federal corporation accruals as a residual, denoted as FCAR. This series is presented in the data appendix on page 160. FCAR does not correspond to the national accounts series, FCA, but there is no reason why these two series should be identical.

We thus have two series for federal corporation tax accruals——FCA, which corresponds to the series in the *National Accounts* and which we estimate by equation (4) or (4'), and FCAR, which is the difference between the TCA series in the *National Accounts* and our theoretically constructed PCA series defined in section 3-1-3-1 below.[33] We present in section 2-3-2-6 two estimated equations for federal corporation tax collections, one based on FCA and the other based on FCAR.

C. Provincial Corporation Income Tax Accruals

Provincial corporation tax accruals are explained in detail in section 3-1-3-1.

2-1-3 NON-RESIDENT WITHHOLDING TAX

In deciding to include the withholding tax in our federal tax sector, we considered several factors. First, this tax is not a major source of revenue to the federal government——in 1966 it yielded $203 million or 2 per cent of total federal revenues, and only 1 per cent prior to major changes in 1960. Second, as a short-run policy instrument, the value of the tax is limited due to its international aspects and the treaty negotiations that must precede any short-run discretionary rate change or base changes. Even if a non-resident withholding tax were more flexible

[33]If we use the FCA series then provincial corporate accruals must be determined residually. We thus have a PCAR series as well as a PCA series and the following identities must hold:

$$TCA = FCA + PCAR$$

$$TCA = PCA + FCAR$$

for policy purposes, the relatively small amount of revenue involved implies a minimal impact on the economy. Over the longer run, however, major changes such as those initiated in 1960 will have some effect, and since a good estimating equation was not too difficult to find we decided to include it among the federal tax equations.

A withholding tax of 15 per cent (with some exceptions noted below) is levied on the gross Canadian income of non-residents received in the form of dividends, interest, estate or trust income, management fees, gross rents, royalties, and alimony. The withholding tax on film payments is 10 per cent. Except for a number of special rates and exemptions, which applied to some dividend and interest payments prior to the 1960 changes, the tax has changed very little during the postwar period. Before 1960 dividend payments by a wholly-owned subsidiary of a U.S. company were taxed at 5 per cent. Interest payments on bonds of, or guaranteed by, the federal government were exempted completely, as well as all interest payable in foreign exchange to non-residents. In 1960 the dividend tax rate applicable to U.S. subsidiaries was increased to 15 per cent and the main interest exemptions were cancelled. Various other exemptions and special rate categories give an effective rate of tax on this income of less than 15 per cent—in 1965 the average effective rate was about 12 per cent.[34]

No data were available with which to construct a weighted average of the various rates applied to each of the income components, so we used a constant 15 per cent rate for the entire estimating period. We obtained from the Department of National Revenue a detailed breakdown of the tax base for 1965 and 1966. This revealed that interest and dividends accounted for 81 per cent of the tax base in these two years, and if royalties were included the remainder amounted to less than 10 per cent. It seemed possible, therefore, that our tax base need be composed only of dividend and interest payments—in any event interest and

[34]In 1963 a 10 per cent rate on dividends paid by a company having a prescribed degree of Canadian ownership was introduced. Also certificates of exemption were issued to certain non-residents for interest on bonds and debentures payable after June 13. These changes were not significant during the period we used, but should be considered if the data period extends beyond 1965.

dividends are the sole published components of the withholding tax base. The time series of dividends paid to non-residents gross of the withholding tax (DIVF, RDX 227) is available from the *National Accounts*, while interest payments net of the tax (INT, DB 3716) can be obtained either from the *National Accounts* or from the balance of payments statistics.[35] To provide our tax base, the interest series was divided by the factor (1 - .15) and then added to DIVF. The use of a constant tax rate and a tax base restricted to interest and dividends means that the coefficient will be unlikely to take on the value of 1 as we would have expected if a weighted rate and the true base had been used. Instead, the coefficient will reflect the net effect of having a rate higher (<1) and a base lower (>1) than the true values.

The dependent variable, quarterly federal withholding tax revenues (TW), is composed of the budgetary collection series (DB 4006) shifted back one month to conform more closely to the flow of interest and dividends. This puts TW essentially on an accrual basis. A dummy variable is included to measure the effect of the 1960 changes. Thus

1Q54-4Q65

$$TW = .8819\ (X1) - .1762\ (D)(X1) \qquad (1)$$
$$(41.95) \qquad (4.95)$$

SEE = 3.71 $\bar{R}^2 = .882$ D/W = 1.72

where:

X1 = .15 [DIVF + INT/(1 - .15)]

D = a dummy variable with a value of 1 in all quarters of 1954-1960, zero elsewhere.

The coefficient of .8819 on X1 shows that the effect of using a constant tax rate of 15 per cent, greater than the average effective rate, is more important than having a base equal to only 80 per cent of the actual base. Predictably the coefficient on the

[35] *Canadian Balance of International Payments and International Investment Position* issued annually by D.B.S., catalogue no. 67-201.

dummy variable is negative and gives some indication of how widespread the effects of the 1960 changes in the tax were.

We also ran the equation with a variable (X1)(T), where T is a time trend, to see if there had been any significant change over this period in the relationship of dividends and interest to the other components of the tax base. The variable was not significant and had very little effect on the equation, suggesting that there had been no major shift in the tax base components relative to the tax liabilities; hence we did not use it.

2-1-4-1 CUSTOMS DUTIES

If rates of import duty influence behaviour, there will be substitution over time in the pattern of imports with untaxed or lightly-taxed items substituted for relatively higher-tariff goods. If trade-flow statistics are not divided into categories corresponding to different tariff rates, it will be difficult to obtain a weighted average tariff rate that remains accurate when one type of good is substituted for another. We are therefore trying to obtain trade statistics that correspond as closely as possible to the duty categories. In the meantime, since the meantime may be a long time, we have developed for use in macro-models approximate relationships, which we offer with misgivings.

In RDX1 all taxes and government revenues are in current dollars, while all private expenditure is in constant dollars. Since current-dollar merchandise imports are the base for import duties, our equation must use constant-dollar merchandise imports (MG, RDX 9147) multiplied by the implicit price deflator for these imports (PMG, RDX 9145). We used only imports of goods because tariffs do not apply to service imports.

Three equations are presented. The first is a simple regression of customs duties on current-dollar imports, a specification that is only appropriate when the weighted average rate of import duty has been constant over the data period. Our first specification also ignores the effects of the 1962-1963 import surcharges.

$$TCUS = .090\ (MG)(PMG) \qquad (1)$$
$$(78.9)$$

1Q50-4Q65

SEE = 12.6 $\bar{R}^2$ = .812 D/W = .486

where:

TCUS = import duties (RDX 2157)

MG = imports of goods, 1957$

PMG = implicit price deflator for current $ MG, 1957=1

The coefficient .090 tells us just what we could have discovered by dividing the mean of TCUS by the mean of (MG)(PMG), that the average rate of duty over the whole period was 9 per cent.

In calculating equation (2) we adjusted for the impact of the 1962-1963 surcharges. This was done by constructing a variable SUR (RDX 11010), the weighted average rate of surcharge with weights based on the structure of imports just before the surcharge was introduced. We used this variable multiplicatively with (MG)(PMG) expecting a coefficient of approximately 1 if no substitution of unsurcharged for surcharged goods occurred during the surcharge period.

1Q50-4Q65

$$TCUS = .089\ (MG)(PMG) + 1.035\ (SUR)(MG)(PMG) \qquad (2)$$
$$(88.3) \qquad\qquad (4.9)$$

SEE = 10.9 $\bar{R}^2$ = .862 D/W = .327

The coefficient of 1.035 on the surcharge variable indicates that no significant substitution against the surcharged goods occurred during the surcharge period. The Durbin/Watson statistic reflects a string of positive residuals from 4Q51 to 4Q60, and primarily negative residuals at the end of the data period. Negative residuals are so predominant at the end that the equation overestimates TCUS by an average of 15 per cent in 1965. Clearly the weighted average rate of duty has been declining throughout the period, either because of statutory rate changes (which were not large) or substitution of lightly-taxed for heavily-taxed items in the import mix.

Since the forecasting properties of the linear equation, equation (2), with an assumed constant duty rate are so unsatisfactory, we developed an alternative that should forecast much better in the short term. This third equation contains a quadratic term in imports. If the quadratic term receives its expected negative sign, then the resulting equation takes account of continuous substitution against the highly-taxed items. The equation also contains a first-quarter term, proportional to imports, intended to capture the apparent first-quarter bulge in lightly-taxed imports.

1Q52-4Q65

$$\begin{aligned} TCUS = \; & \underset{(61.1)}{.1201}\,(MG)(PMG) - \underset{(16.2)}{.000019}\,[(MG)(PMG)]^2 \\ & + \underset{(10.8)}{.9876}\,(SUR)(MG)(PMG) - \underset{(4.1)}{.0046}\,Q1\,(MG)(PMG) \end{aligned} \qquad (3)$$

SEE = 4.69 $\quad \bar{R}^2 = .963 \quad$ D/W = 1.04

Equation (3), which is employed in RDX1, has a markedly better fit and much less autocorrelation of its residuals than has equation (2). The danger with equation (3), of course, is that it will produce ridiculous results if extrapolated far enough beyond the estimation period. Estimated customs duties would become negative by the time constant-dollar imports of goods reached $6,500 million. Either the equation has to be reestimated every year (and even then only used for short-term forecasting) or an adequate, weighted tariff-rate variable must be constructed. In the meantime, equation (3) will have to do.

2-1-4-2 MANUFACTURERS' SALES TAX

Until 1963, the sales tax rate of 11 per cent applied to all non-exempt goods. The primary exemptions were most foods, fuels, construction materials, machinery and equipment used in the production of goods, and materials incorporated into manufactured goods. The June 13, 1963 budget proposed the removal, by stages, of the exemption for construction materials and machinery and equipment. The rate on these items was set at 4 per cent from June 14, 1963 to March 31, 1964, 8 per cent until December 31, 1964, and at the full 11 per cent thereafter. The fall budget of 1966 raised the regular rate from 11 to 12 per cent from January 1, 1967, but left at 11 per cent the rate applicable to construction materials and machinery and equipment. This budget proposed the complete removal, in stages, of the tax on production machinery and equipment. The rate on these items was reduced from 11 to 6 per cent on April 1, 1967, and became zero on June 2, 1967.

Our forecasting equation should therefore have separate variables for each of three major categories of expenditure now treated differently under the manufacturers' sales tax. In making our estimation, however, we have added together the terms for construction and for machinery and equipment, since the investment variables are strongly collinear and during the fitting period (we used data up to 4Q65) both types of investment received the same tax treatment.

If the forecasting equation is reestimated using data from subsequent years, it should be possible to split the two types of investment now treated separately under the sales tax. Our present equation is based on a weighted average of construction expenditures and machinery and equipment expenditures as the investment variable, and the sum of consumer expenditures on durables and non-durables as the general consumer expenditure variable. The relative weighting of construction and machinery and equipment in the investment variable is 42:100, based on a Dominion Bureau of Statistics (D.B.S.) estimate of the proportion of total construction expenditures comprising taxable materials.

Our equation is thus

$$\text{1Q55-4Q65}$$

$$TS = \underset{(74.1)}{.6327}\ (RSC)[(CD + CND)(PGNE)] + \underset{(9.9)}{.5482}\ [(RSIM)(IME)(PGNE)$$

$$+ .42\ (RSIR)(INRC + IRC)(PGNE)] \qquad (1)$$

SEE = 19.29 $\bar{R}^2$ = .943 D/W = 1.66

where:

TS = federal sales tax collections (RDX 11270)

CD = consumer expenditure on durables, 1957$ (RDX 141)

CND = consumer expenditure on non-durables, 1957$ (RDX 140)

IME = investment in machinery and equipment, 1957$ (RDX 11306)

INRC = investment in non-residential construction, 1957$ (RDX 11307)

IRC = investment in residential construction, 1957$ (RDX 145)

PGNE = private GNE deflator, 1957 = 1 (RDX 9153)

RSC = basic sales tax rate, applicable to consumer durable and non-durable expenditure (RDX 11025)

RSIM = tax rate applicable to machinery and equipment (RDX 11620)

RSIR = tax rate applicable to construction materials and building supplies (RDX 11621)

The coefficients on both variables should be less than 1, since the federal sales tax is levied on the manufacturers' sale price, while (CD + CND)(PGNE) is the value of final consumer expenditure (including federal and retail sales taxes plus retailers' margins), and the three investment series measure the total value of work put in place (including manufacturers' sales taxes and installation costs). In addition, a number of tax-exempt items are included in the expenditure series.

2-1-4-4 EXCISE DUTIES

We shall present here two equations, the first intended to picture the structure of the tax with some accuracy, and the second designed for use in a macro-model. Equation (2) does not contain the exact base for the excise duties, but tries to make use of an appropriate proxy series. Equation (1) uses constant-dollar expenditures on tobacco products and alcoholic beverages (both inflated by the consumer non-durable deflator) as independent variables, while equation (2) uses the sum of consumer expenditures on durables and non-durables inflated by their appropriate price indices. The first equation is fitted over a shorter time period than the second since the quarterly expenditure series, disaggregated by commodity, is available only from 1956.

1Q56-4Q65

$$TEX = \underset{(4.9)}{.165}\ BOOZE\ (PND) + \underset{(6.1)}{.283}\ BUTTS\ (PND) \qquad (1)$$

SEE = 5.30 $\bar{R}^2$ = .902 D/W = 2.60

1Q52-4Q65

$$TEX = \underset{(109.2)}{.02299}\ [CD\ (PD) + CND\ (PND)] \qquad (2)$$

SEE = 5.60 $\bar{R}^2$ = .920 D/W = 1.78

where:

TEX = excise duties (RDX 2158)

CD = consumer expenditure on durable goods, 1957$ (RDX 141)

CND = consumer expenditure on non-durable goods, 1957$ (RDX 140)

BOOZE = consumer expenditure on alcoholic beverages, 1957$ (DB 2310)

BUTTS = consumer expenditure on tobacco products, 1957$ (DB 2309)

PD = implicit price index of consumer durable expenditure, 1957 = 1 (RDX 11384)

PND = implicit price index of consumer non-durable expenditure, 1957 = 1 (RDX 11423)

The additional structural detail of the first equation does not achieve a high payoff in terms of goodness of fit, presumably because the detailed expenditure data are not very reliable. Equation (2), based on total consumer expenditure on durables plus non-durables, fits well and apparently has reasonable error properties. It is used in RDX1. If excise duties were increased enough to shift consumption away from the excised goods, the straightforward use of equation (2) would naturally be dangerous.

2-3-2-6 *CORPORATION INCOME TAXES ACCRUED BUT NOT COLLECTED*

Equations for total corporation tax accruals (TCA)[36] and federal corporation tax accruals (FCA) were presented in section 2-1-2. To determine corporation taxes accrued but not collected, we require equations to explain corporation tax collections, both total (TCC)[36] and federal (FCC). Then, by subtracting TCC from TCA and FCC from FCA one can determine the items required for the cash budgets of the federal and provincial governments.

[36] Both of these series are net of provincial logging and mining taxes (PLMT, RDX 11626).

A. Total Corporation Income Tax Collections

In constructing the collection equation several features of the tax law have to be considered. The 'taxation year' of a corporation coincides with its fiscal year. During the taxation year a corporation pays taxes in regular instalments, each instalment being a prescribed amount of either (1) the actual accrued tax liability of the corporation for the preceding taxation year or (2) an estimate of accrued tax liability in the current taxation year. After paying the required number of instalment payments, the corporation must make adjustment payments based on the difference between actual liabilities incurred in the taxation year in question and total tax paid on account for that year.

Although the basic rules for payment of taxes have not changed, since 1962 they have often been amended. These amendments have altered the timing of payments and the fraction of tax liabilities due in each instalment. Table 5 is a simplified exposition of the number, amount and timing of instalment and adjustment payments made by corporations as amendments to the rules of payment have occurred.

Tax payments are usually made on the last business day of the month in which they are due and, as a result, are recorded as tax collections in the following month. Our equation must take account of this recording lag.

The collection equation is constructed on the assumptions that all corporations have fiscal years ending on December 31,[37] that the figures recorded as tax accruals in the *National Accounts* actually represent a corporation's tax liability for the relevant taxation year, and that all firms make their interim payments on the basis of their actual taxes in the preceding taxation year rather than on their estimate of their tax liability for the current year.

[37]Because the fiscal years of the chartered banks end on October 31, we constructed an equation to deal separately with tax collections from banks and other corporations. Since the split did little to improve the explanatory power of the model, we present here only the aggregate model based on the assumption that all fiscal years end on December 31.

Table 5

SCHEDULE OF INCOME TAX INSTALMENT PAYMENTS BY CORPORATIONS

	Months of First Taxation Year												Months of Second Taxation Year					
	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	12th	1st	2nd	3rd	4th	5th	6th
Taxation Year Ending:																		
Before Dec. 1, 1963	-	-	-	-	-	-	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/3*	1/3*	1/3*
Dec. 1, 1963 to Nov. 30, 1964	-	-	-	-	-	-	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/2*	1/2*	-
Dec. 1, 1964 to Nov. 30, 1965	-	-	-	-	-	1/11	1/11	1/11	1/11	1/11	1/11	1/11	1/11	1/11	1/2*	1/2*	-	-
Dec. 1, 1965 to Nov. 30, 1968	-	-	-	-	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/2*	1/2*	-	-
Dec. 1, 1968 to Nov. 30, 1969	-	-	-	-	-	1/5	-	1/5	-	1/5	-	1/5	-	1/5	*	-	-	-
Dec. 1, 1969 to Nov. 30, 1970	-	-	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	-	-	*	-	-	-
After Dec. 1, 1970	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/12	-	-	*	-	-	-

* Adjustment Payment - balance of tax is due with final adjustment payment.

Source: *Canadian Master Tax Guide*, published by Commerce Clearing House Canadian Limited, various issues.

The detailed collection equation from 1953 to 1971 is presented in Table 6. This equation is extremely complex due to changes that have occurred in the legislation since 1963; and an examination of the terms separately, quarter by quarter, is helpful. As an example of the procedure we employed, consider the term for the second quarter of 1965. The assumption is that all corporations have fiscal years ending December 31, 1965. Thus their payments schedule would be that shown in the fourth row of Table 5. In the second quarter of 1965 corporations pay two-twelfths of liabilities accrued in 1964. Because of the recording lag, however, only one-half of their payment is entered as having been made in the second quarter. Thus we get the first term of our sum:

$$1/12 \left[\sum_{-2}^{-5} TCA \right]$$

The second term in the sum takes account of the adjustment payments made in 1965 for taxes paid in 1964. One-half of the difference between accrued liabilities in 1964 and payments in 1964 (which were based on accrued liabilities in 1963) is to be paid in March and the other half in April 1965. Because of the recording lag both of these payments are entered as collections in the second quarter of 1965. Therefore the expression for the adjustment payments is:

$$\sum_{-2}^{-5} TCA - \sum_{-1}^{-3} TCC$$

The summations appearing in the other cells of Table 6 are derived in the same way.

We used our accrual series, appropriate quarterly dummies and the relevant algebraic transformations from Table 6, to construct the variables X1, X2, X3 and X4, from 1953 to 1967. Each of these variables represents the appropriate quarterly term of the collection equation, for example:

Table 6

QUARTERLY TERMS OF TOTAL CORPORATION INCOME TAX COLLECTION EQUATION

Calendar Year*	Q1	Q2	Q3	Q4
1952 to 1963	$1/4[\sum_{-5}^{-8} \text{TCA}]$	$1/12[\sum_{-6}^{-9} \text{TCA}] + 2/3[\sum_{-2}^{-5} \text{TCA} - (\sum_{-1}^{-2} \text{TCC} + 1/4 \sum_{-6}^{-9} \text{TCA})]$	$1/6[\sum_{-3}^{-6} \text{TCA}] + [\sum_{-3}^{-6} \text{TCA} - (\sum_{-1}^{-3} \text{TCC} + 1/6 \sum_{-7}^{-10} \text{TCA})]$	$1/4[\sum_{-4}^{-7} \text{TCA}]$
1964	$1/4[\sum_{-5}^{-8} \text{TCA}]$	$1/12[\sum_{-6}^{-9} \text{TCA}] + [\sum_{-2}^{-5} \text{TCA} - (\sum_{-1}^{-2} \text{TCC} + 1/4 \sum_{-6}^{-9} \text{TCA})]$	$3/11[\sum_{-3}^{-6} \text{TCA}]$	$3/11[\sum_{-4}^{-7} \text{TCA}]$
1965	$3/11[\sum_{-5}^{-8} \text{TCA}]$	$1/12[\sum_{-2}^{-5} \text{TCA}] + [\sum_{-2}^{-5} \text{TCA} - \sum_{-1}^{-3} \text{TCC}]$	$1/4[\sum_{-3}^{-6} \text{TCA}]$	$1/4[\sum_{-4}^{-7} \text{TCA}]$
1966 to 1967	$1/4[\sum_{-5}^{-8} \text{TCA}]$	$1/12[\sum_{-2}^{-5} \text{TCA}] + [\sum_{-2}^{-5} \text{TCA} - (\sum_{-1}^{-3} \text{TCC} + 1/12 \sum_{-6}^{-9} \text{TCA})]$	$1/4[\sum_{-3}^{-6} \text{TCA}]$	$1/4[\sum_{-4}^{-7} \text{TCA}]$
1968	$1/4[\sum_{-5}^{-8} \text{TCA}]$	$[\sum_{-2}^{-5} \text{TCA} - (\sum_{-1}^{-3} \text{TCC} + 1/12 \sum_{-6}^{-9} \text{TCA})]$	$2/5[\sum_{-3}^{-6} \text{TCA}]$	$1/5[\sum_{-4}^{-7} \text{TCA}]$
1969	$2/5[\sum_{-5}^{-8} \text{TCA}]$	$3/10[\sum_{-2}^{-5} \text{TCA}] + [\sum_{-2}^{-5} \text{TCA} - \sum_{-1}^{-3} \text{TCC}]$	$3/10[\sum_{-3}^{-6} \text{TCA}]$	$3/10[\sum_{-4}^{-7} \text{TCA}]$
1970	$1/10[\sum_{-5}^{-8} \text{TCA}] + 1/6[\sum_{-1}^{-4} \text{TCA}]$	$1/4[\sum_{-2}^{-5} \text{TCA}] + [\sum_{-2}^{-5} \text{TCA} - (\sum_{-2}^{-3} \text{TCC} + 2/5 \sum_{-6}^{-9} \text{TCA})]$	$1/4[\sum_{-3}^{-6} \text{TCA}]$	$1/4[\sum_{-4}^{-7} \text{TCA}]$
1971 and beyond	$1/12[\sum_{-5}^{-8} \text{TCA}] + 1/6[\sum_{-1}^{-4} \text{TCA}]$	$1/4[\sum_{-2}^{-5} \text{TCA}] + [\sum_{-2}^{-5} \text{TCA} - (\sum_{-2}^{-3} \text{TCC} + 1/2 \sum_{-6}^{-9} \text{TCA})]$	$1/4[\sum_{-3}^{-6} \text{TCA}]$	$1/4[\sum_{-4}^{-7} \text{TCA}]$

* Coincides with taxation year, by assumption.

$$X4_{1953-1967} = Q4\,[\tfrac{1}{4}\,(\sum_{-4}^{-7} TCA)]\,D1 + Q4\,[\tfrac{1}{4}\,(\sum_{-4}^{-7} TCA)]\,(D1)(D2)$$

where:

D1 = 1 from 1Q53 to 4Q67, zero elsewhere

D2 = 12/11 in 4Q64, zero elsewhere

Fitting the equation to quarterly accrual and collection data for the 1953-1965 period yielded the following result:

1Q53-4Q65

$$TCC = \underset{(50.7)}{1.0064}\,X1 + \underset{(61.2)}{1.0125}\,X2 + \underset{(55.2)}{1.0667}\,X3 + \underset{(51.3)}{.9808}\,X4 \quad (1)$$

SEE = 25.92 $\bar{R}^2 = .916$ D/W = 1.16

Coefficients are all close to their expected value of unity. The equation was used to forecast total corporation tax collections for 1966 and 1967. We present the results in Table 7. The performance of the equation is quite satisfactory except for the second quarter of 1967. If in 1966 corporations had switched the base on which instalment payments are calculated from the prior year's accruals to the current year's estimated liabilities, then we would expect to get an overestimate in the third and fourth quarters of 1966 and the first quarter of 1967, and an underestimate in the second quarter of 1967. This is the pattern we observe in the forecast period. We have no direct evidence that corporations did in fact make this switch in their payment practices, but corporation profits in 1966 declined from their 1965 levels, particularly in the third and fourth quarters.[38] An expected decline in corporation profits would provide a rationale for switching the base on which instalment payments are calculated in the manner described above.

[38]Corporation profits in 1966 were $5,145 million, down from $5,199 million in 1965. In the second half of 1966 profits declined by $160 million from their level in the second half of 1965.

Table 7

TOTAL CORPORATION INCOME TAX COLLECTIONS
LESS PROVINCIAL LOGGING AND MINING TAXES 1966-1967
(Millions of dollars)

	Actual Values*	Forecast Values	Forecast Error
1966 Q1	516	506	-10
2	671	641	-30
3	549	581	+32
4	508	534	+26
Year	2,244	2,262	+18
1967 Q1	522	548	+26
2	721	627	-94
3	575	586	+11
4	548	539	-9
Year	2,366	2,300	-66

* *National Accounts* — Quarterly, Table 5, line 6 plus line 8 minus PLMT (RDX 11626).

Table 8

FEDERAL CORPORATION INCOME TAX COLLECTIONS 1966-1967
(Millions of dollars)

	Actual Values*	Forecast Values	Forecast Error
1966 Q1	398	392	-6
2	528	514	-14
3	409	436	+27
4	392	405	+13
Year	1,727	1,747	+20
1967 Q1	400	425	+25
2	569	513	-56
3	423	442	+19
4	413	410	-3
Year	1,805	1,790	-15

* *National Accounts* — Quarterly, Table 5, line 6.

B. Federal Corporation Income Tax Collections

Federal corporation tax collections (FCC) are explained by an equation exactly analogous to that in Table 6, except that federal corporation tax accruals (FCA, explained in section 2-1-2) are substituted for total accruals. The fitted result is:

1Q53-4Q65

$$FCC = 1.0139\ X1 + 1.0323\ X2 + 1.0417\ X3 + .9667\ X4 \qquad (2)$$
$$(45.4) \qquad (53.1) \qquad (46.8) \qquad (44.3)$$

SEE = 24.98 $\qquad \overline{R}^2 = .827$ $\qquad$ D/W = 1.40

Again, the fit is good (though not as good as that of the equation for total collections) and the coefficients are reasonably close to their expected value of unity. The 1966 and 1967 forecasts are presented in Table 8. The forecast error in 1967 is much lower than the error obtained from the total forecasting equation, equation (1); but the pattern of overestimation, in the third and fourth quarters of 1966 and the first quarter of 1967, followed by underestimation in the second quarter of 1967, is again apparent.

In section 2-1-2 above we pointed out that the federal accrual figures in the *National Accounts* were not based on reports from taxpayers. We developed a separate series for federal accruals, denoted as FCAR and listed in the data appendix on page 160. Our equation for federal tax collections was reestimated, using the FCAR series rather than the national accounts series, FCA.

1Q53-4Q65

$$FCC = .9700\ X1 + .9501\ X2 + .8856\ X3 + .9268\ X4 \qquad (2')$$
$$(32.7) \qquad (38.3) \qquad (33.5) \qquad (31.9)$$

SEE = 34.61 $\qquad \overline{R}^2 = .668$ $\qquad$ D/W = .95

Although the coefficients are still fairly close to unity, the overall fit of the equation is quite poor and the low Durbin/

Watson statistic indicates that the residuals are serially correlated. Assuming that the equation is theoretically correct, one would expect a much better fit. The poor results indicate that the FCAR series may not reflect true federal corporation tax liabilities as accurately as the national accounts series does.

C. Total and Federal Tax Accruals Minus Collections

These items are obtained by subtracting the appropriate collection series from the corresponding accrual series.

PART 3 FEDERAL TRANSFER PAYMENTS

Coverage of the transfer payments sector of the federal government submodel is incomplete. Only transfers to persons are included, and even within this category several small classes of transfers, reported separately in the *National Accounts*, are omitted. In part this lack of coverage is due to a shortage of time and effort input——certain other sectors of RDX1 were given higher priority. As research on the government sector advances and the structure of RDX increases in scope, further classes of transfer payments may be covered. But there are some transfers that we have no intention of including since their analysis would lead neither to structural information nor to predictive power.

In the first place, numerous programs are relatively trivial in scope. This is particularly true of the many transfers to business and agriculture, some of which involve no more than a few million dollars on a quarterly basis. From the viewpoint of recipients or of the overall effectiveness of government operations these amounts are, of course, significant. But, in the context of an aggregate model, analysis of such relatively small flows is unrewarding. Nor can we explain the total of all transfers to business treated as a lump sum because the resulting heterogeneous mix of policy parameters and exogenous variables in any such aggregate would make explanation of its behaviour meaningless. This size criterion also applies to programs currently in their terminal stages, such as the variety of war-associated transfers reported in the *National Accounts*, now, with two notable exceptions, of historical interest only.

Second, a problem arises when the exogenous variables in any analytic explanation of a transfer item are exogenous to the model as a whole and not susceptible to reliable projection. In a tax function the tax base is exogenous to the government sector yet endogenous to the entire model. In the old age pension equation the eligible population is exogenous to the model as a whole yet can be predicted into the middle-run future with reasonable certainty. But in the agricultural sector many transfers depend on the notoriously uncertain yields of the wide variety of crops produced. These exogenous variables may shift substantially from year to year owing to the vagaries of the weather so that a highly variable subsidy, based on a highly variable set of output measures, interacts with a complicated set of parameters such as floor prices, storage costs, premiums, etc. Since the total impact of the program is unpredictable, elaborate dissection of its internal structure seems clearly unprofitable. Admittedly, we cannot then assess directly the impact of a change in agricultural policy directly in our model, but approximate adjustments to an exogenous agricultural transfers factor would probably yield results just as good as those obtainable from a fitted model.

Third, we excluded transfer programs that are currently in a state of flux because of recent establishment or major overhaul. The whole structure of such programs may shift from year to year. This problem occurs in the case of transfers to other levels of government, because many of these payments may be entirely renegotiated at relatively frequent intervals. Econometric analysis of time-series data requires the assumption that at least some aspects of a structural relation remain constant over time. If programs receive major overhauls in structure and parameters every few years, time-series analysis is clearly an inappropriate way of modelling their impact. Thus the behaviour of statutory grants and payments under tax-sharing agreements, health and welfare payments, technical and vocational training financing, all represent programs that do not provide enough historical experience of operation within a constant structure to be amenable to time-series analysis.

The transfers modelled, while representing a minority of all programs, still account for a large portion of the dollar volume of federal transfer payments. As shown in Table 9 federal transfers in 1965 totalled $5,141 million. Of these, our equations cover $2,190 million in transfers to persons and $1,052 million in interest on the federal public debt, for a total of $3,242 million or 63 per cent of all federal transfers. The bulk of the remaining 37 per cent is concentrated in transfers to other levels of government, a sector that will eventually be analyzed in conjunction with a disaggregated model of the provincial and municipal government sectors.

The transfer programs analyzed in this paper may be divided into three categories. In the first category are programs cover-

Table 9

TOTAL FEDERAL GOVERNMENT TRANSFER PAYMENTS
(Millions of dollars)

Index Code No.			1965 Transfer Payments
2-2-3	Transfers to persons		2,312
	Modelled	2,190	
	Excluded	122	
2-2-4	Interest on the federal public debt		1,052
2-2-5	Transfers to business and agriculture		343
2-2-6	Transfers to other levels of government		1,434
	Total		5,141

Source: *National Accounts* — Annual

ing transfers bound to specific revenue sources, and so entering the federal budget on both sides. It is clearly appropriate to treat these revenue and expenditure equations as parts of a single problem to be analyzed in a sub-submodel. In this category we have the Unemployment Insurance Fund and the (public service) Superannuation Account in the Consolidated Revenue Fund. The same treatment was not adopted for old age pensions because revenues of the Old Age Security Fund, from which pensions are formally paid, are raised through supplements to the personal income tax, the corporation income tax and the manufacturers' sales tax, rather than through a separate levy. The fund's revenue base is thus unrelated to its base for pension payments. Since Old Age Security Fund deficits are made up from general revenue, old age pension payments should clearly be treated as charges on the general revenue and Old Age Security Fund revenues as supplements to the relevant federal taxes. The Old Age Security Fund, linking pension payments and revenues, should therefore be disregarded for our purposes. Accordingly, the old age pension is classified in the second category of transfers——the straightforward transfer payment having a rate structure, determined by the federal government, applied to a set of demographic variables that defines the eligible population to yield a volume of federal payments. This second category includes family and youth allowances, old age pensions, war veterans pensions, and war veterans allowances.

We decided that interest on the public debt should constitute our third category of transfers, because these payments are determined by an analytic structure significantly different from those of the other two transfer categories.

2-1-6-1 PUBLIC SERVICE PENSION RECEIPTS
2-2-3-5 PUBLIC SERVICE PENSIONS

The public service pension transfers were handled in an exceptionally rough manner that enabled us to develop crude but usable equations. The model presented provides considerable scope for increased complexity and precision. On the other hand, the behaviour of the (public service) Superannuation Account in the Consolidated Revenue Fund is relatively uninteresting and does not appear to justify the time input we devoted to the highly cyclical Unemployment Insurance Fund. Public service pensions have not to our knowledge ever been identified as an automatic stabilizer, although they undoubtedly represent one channel through which an expanding (or contracting) government sector may influence the long-run performance of the economy.

The revenues of the Superannuation Account (PSPR, DB 2166) are derived from three sources. First, all eligible federal employees pay a proportion of their salaries (males 6 1/2 per cent, females 5 per cent) into the account. Second, the federal government as employer matches this amount. Third, the government as trustee for the account pays interest on the amount in the account at the rate of 1 per cent per quarter. As well, the government makes up any actuarial deficits arising from changes in public service pay schedules, and hence in the expected liabilities of the account.

The flow of payments into the Superannuation Account in current dollars is modelled quarterly from 1955 to 1965. Although the Public Service Superannuation Act took effect in 1954, fitting equations from 1954 to 1965 generated large residuals in 1954, which suggested that there had been a start-up lag in the revenue collections of the account. PSPR is clearly a function of federal wage and salary payments (FGW, DB 2169) with the proportion varying for male and female employees. It would undoubtedly be easy to get a statistical breakdown of the public service showing the ratio of males to females, but for this equation we require a division of total wage payments to males and to females. Thus if the split were

$$FGW = FGWM + FGWF \qquad (1)$$

where FGWM, FGWF are federal government wage payments to males and to females, respectively, the proper explanatory variables would be .065 FGWM and .05 FGWF. One would expect these variables to be highly collinear, however, and for estimation purposes a composite variable (.065 FGWM + .05 FGWF) would be appropriate. But such a composite variable would require generation of these series on a quarterly basis, and time considerations suggested instead the approximation FGWM = 2/3 FGW, assumed to hold for all quarters. This assumption probably does not introduce serious errors, though it should be investigated. It yields the independent variable .06 FGW, using equation (1) above and the known percentage levies.

Furthermore an anomaly observed in 2Q60 suggests that there is an error in the revenue series. Reported revenues are \$54 million in 1Q60, \$43 million in 2Q60 and \$56 million in 3Q60. Thus a dummy variable was included for 2Q60.

The resulting equation is:

$$\text{1Q55-4Q65}$$

$$PSPR = \underset{(6.06)}{9.386} + \underset{(29.89)}{3.156}\,(.06\ FGW) - \underset{(5.14)}{9.933}\ DUM \qquad (2)$$

$$SEE = 1.91 \qquad \bar{R}^2 = .956 \qquad D/W = 1.63$$

This equation is generally satisfactory, although the significant constant term is unfortunate. There are at least two possible sources of specification error: the choice of .06 as the weighted average of .05 and .065 may be wrong, as may be the assumption of a stable average. Moreover the government's contribution, made to preserve the actuarial soundness of the account, is related to the rate of wage change as well as to FGW. Thus the constant term probably represents some form of specification error. The high coefficient on .06 FGW suggests that the government's interest and actuarial payments are roughly equal to the employees' contributions, so that the government pays two-thirds of the account's total revenues at the margin. On the standard test statistics the equation is adequate although not impressive. The proportion of explained variance at .956 is good but not great for this type of equation, the Durbin/Watson statistic suggests but does not establish autocorrelation in the residuals and corresponding systematic misspecification; but the standard error of estimate is very low, and the equation will do. There is not much variance around a trend in the PSPR series in any case.

Pension payments from the account (PSPP, DB 2180) are made to each retired public servant on the basis of 2 per cent of his average salary during the six consecutive years when that salary was highest, multiplied by the number of years worked up to a maximum of thirty-five years. Thus a full pension is 70 per cent of the salary paid in the top six-year earning period.

A theoretically satisfying explanatory variable for this series would require the use of a distributed lag of past retirements, each weighted by some average wage at retirement, with the lag pattern depending on the mortality experience of retired public servants. But the data from which such a series could be constructed do not appear to be available. The annual report of the Superannuation Account[39] contains data on contributors and beneficiaries, and on initiation and termination of pensions, but these data are annual and sparse at best. Moreover the material reported is not always consistent from year to year. Thus, as a quick approximation, we tried regressing pension payments on total FGW, and on FGW multiplied by a time trend (equal to 1 in 1Q54) to allow for a changing relation between the wage bill and pensions. Changing the initial period made very little difference, so we fitted the equations from 1954 to yield:

1Q54-4Q65

$$PSPP = \underset{(11.98)}{-12.074} + \underset{(28.10)}{.1189}\ FGW \qquad (3)$$

SEE = 1.451 $\qquad \bar{R}^2 = .944 \qquad$ D/W = .69

1Q54-4Q65

$$PSPP = \underset{(30.44)}{7.099} + \underset{(44.67)}{.0013}\ (T)(FGW) \qquad (4)$$

SEE = .928 $\qquad \bar{R}^2 = .977 \qquad$ D/W = .90

Autocorrelation is a serious problem in both these equations, as we might expect from the nature of our specifications. Any change in the rate of growth of public service wage payments should certainly lead to shifts in the ratio of pensions to wage bill, since pension liabilities are a function not of the current wage bill but of a distributed lag of past levels. Indeed, a strong argument could be made for testing distributed lags in

[39] *Report on the Administration of the Public Service Superannuation Act* issued annually by the Department of Finance.

this framework, though the caveat must be added that most estimation methods involving lagged endogenous variables would come to grief on the strong autocorrelation in the pension series. As for the test statistics, the trended equation (4) is clearly superior to equation (3). Autocorrelation is less severe, presumably because we have allowed the proportion of pensions to wage bill to vary. Efforts were also made to capture this variation with a linear time trend, but they were not successful. The linear terms were insignificant. For equation (4) the fit is good and the standard error small. The coefficients are strongly significant so that one may place some confidence in them in spite of the underestimate in their standard errors, which results from residual autocorrelation. It is not clear why pensions should be rising as a proportion of wages and salaries when the public service has been increasing so rapidly. Perhaps the greatest rate of increase in wages and salaries took place during World War II and immediately after——current increases being large in absolute magnitudes but proportionately smaller than in that period. This behaviour would produce the observed effect if the relative rate of growth were falling from 1954 on. We expect we could do better with the trended equation, particularly by introducing distributed lags; but for the time being it will serve the purposes of the model adequately. Changes in the Superannuation Account are never likely to be so dramatic that our dynamic misspecification will do serious harm to the actual behaviour of that account.

To attempt a disaggregated analysis of the heterogeneous mix of pensions included in PSPP does not seem to be a particularly profitable exercise. Most of the odd pensions (to pilots, Members of Parliament, and others) are in amounts too small to justify separate treatment, and since our analytic scheme is rough in any case these items may as well be lumped in with the main public service pensions.

2-1-6-2 UNEMPLOYMENT INSURANCE RECEIPTS
2-2-3-3 UNEMPLOYMENT INSURANCE BENEFITS

Total enrolment under the Unemployment Insurance Act depends on the size of the labour force involved and the level of income per capita, since coverage is restricted to workers with annual salaries below a certain ceiling. Annual salary per earner and income per capita are of course not equivalent, and given the many highly variable non-salary components of income it is quite conceivable that the coverage level of the fund would be more closely related to a time trend than to movements in per capita income. The fundamental equation of the Unemployment Insurance Fund model determines the number of workers insured by the fund. Enrolment data used in this model were generated on a quarterly basis by averaging the monthly data in the D.B.S. reports on the operations of the fund.[40]

The proportion of labour force covered is subject to seasonal, cyclical and secular movements in labour force composition. Seasonally, one would expect to find that in the summer the labour force is augmented by agricultural workers, students, and other part-time workers who are not generally insured. The proportion of labour force covered should fall accordingly in the second quarter and more significantly in the third quarter. This pattern was modified by the extension of insurance to agricultural and horticultural workers as of April 1, 1967, but effects of the change in coverage will be hard to determine until more observations are available.

Cyclically, one would expect unemployment to cause casual workers and those in lower income brackets to drop out of the labour force (the 'discouraged worker' effect). This may lessen proportionate coverage because the remaining labour force is more heavily weighted toward workers above the salary ceiling. On the other hand, if such 'drop-out' workers are long-term unemployed whose insurance coverage has expired, they may reduce the uninsured labour force. Moreover, unemployment for the primary earner may also lead to the entry into the work force of other members of a family (the 'additional worker' effect). Such a secondary earner is generally employed at a lower salary level than that of the primary earner and is likely to be insurable, thus raising proportionate insurance coverage. Fluctuations in coverage are therefore subject to a number of factors pulling in different directions and one can hardly say a priori what the net effect will be. In fact there may be little net effect, but this issue may be resolved by statistical investigation.

[40] *Statistical Report on the Operation of the Unemployment Insurance Act* issued monthly by D.B.S., catalogue no. 73-001.

Finally, long-term shifts in the labour force between covered and uncovered occupations will produce corresponding shifts in the proportion of labour force covered. Thus expansion of the health and education industries, in which employees are largely uninsured, lowers the proportion covered, while reduction of the armed forces raises it.

After some experimentation, we found a definite trend in proportionate insurance coverage, but we picked up this trend best with a time-trend variable rather than with per capita income variables, as suggested in the first paragraph. We modelled seasonal shifts by using linear dummies; this produced better results than those obtained by using dummies proportionate to the labour force. Finally we tested an unemployment variable as a cyclical indicator. This variable had a significantly positive sign in early formulations but lost significance and thus was dropped from the final formulation.

The fundamental equation is:

1Q52-4Q65

$$\begin{aligned} INS = &- \underset{(.48)}{.4422}\ Q1 - \underset{(.80)}{.7563}\ Q2 - \underset{(.96)}{.9460}\ Q3 \\ &- \underset{(.79)}{.7588}\ Q4 + \underset{(3.70)}{.9695}\ NEP\ (D5) + \underset{(4.90)}{1.0020}\ NEP\ (D6) \qquad (1) \\ &+ \underset{(1.21)}{.00218}\ T1\ (NEP)(D5) - \underset{(2.08)}{.00346}\ T2\ (NEP)(D6) \end{aligned}$$

$SEE = .098 \qquad \bar{R}^2 = .956 \qquad D/W = 1.12$

This equation determines the average quarterly level of enrolment in the fund (INS, RDX 11257) as a function of quarterly seasonal dummies, the total number of paid workers (NEP, RDX 11064) and time trends. D5 is a quarterly dummy equal to 1 from 1Q52 to 3Q59 and zero thereafter. D6 is similarly defined as 1 in 4Q59 and all following quarters, and as zero in all quarters prior to 4Q59. The reason for this split is that on September 27, 1959 the annual ceiling was changed from a wage and salary ceiling of

$4,800 to a ceiling of $5,460 thus extending coverage to a larger proportion of the labour force. T1 and T2 are time trends: T1 is 6 in 1Q52 and increases to 36 in 3Q59 while T2 is 1 in 4Q59 increasing to 25 in 4Q65.[41]

Equation (1) fulfils some expectations, yet has puzzling features. There appears to be much variation in the seasonal pattern. Although the coefficients on the quarterly dummies match our a priori expectations, the t-test values indicate very high standard errors. The expected downward trend in coverage is clearly a feature of the post-1959 period, as shown by the negative coefficient on T2 (NEP)(D6). This negative coefficient implies that the rate of coverage of additions to the paid labour force has fallen from 100.2 per cent in 4Q59 to 91.5 per cent in 4Q65. Even more puzzling is the positive coefficient on the earlier term T1 (NEP)(D5). A positive coefficient implies a rise in marginal coverage rates throughout the 1950's (could this coefficient reflect shifts out of the uncovered agricultural labour force?) until coverage was 104.8 per cent in 3Q59 indicating a drop in the percentage covered in 4Q59——a most implausible result. Still, much depends on the size of the coefficient of T1 (NEP)(D5), which is only about 1.2 times its own standard error. The low Durbin/Watson statistic implies that the calculated standard errors are underestimates due to autocorrelation of residuals. Thus we may well suggest that the coefficient of T1 (NEP)(D5) is too large due to statistical error. As for general fit, equation (1) is easily the best located, with an $\overline{R}^2$ of .956. The autocorrelation problem, though obviously present, is much less severe than in other tests. We tried to develop a logarithmic format in order to remove some of the autocorrelation, but these efforts led to worse overall fits and low Durbin/Watson statistics.

Given an explanatory equation for the total enrolment of the fund, the next problem is to break down that enrolment into employed persons paying into the fund and unemployed persons receiving benefits. Data on both insured employed and claimants are also available from the monthly report on the operations of the Unemployment Insurance Fund. Since insured population is the sum

[41] T1 starts at 6 because the original estimation period for the equation began in 4Q50. When the equation was reestimated for inclusion in RDX1 the estimation period was changed to correspond more closely with the rest of the model.

of employed contributors plus claimants, and since our first equation determines the total insured, it is sufficient to develop an equation explaining either employed contributors or claimants. The other category follows by subtraction. We chose to focus on the number of claimants (CL, RDX 11247), which clearly depends on the total number of insured, the total unemployed (NU, RDX 11063), and seasonal factors. The fitted equation for claimants on the fund is:

1Q52-4Q65

$$\begin{aligned} CL = &-.2784 - .00328\ T + .1064\ INS + 1.2181\ Q1\ (NU) \\ &\ \ (2.55)\quad\ \ (3.30)\qquad\ (3.00)\qquad\ (22.42) \\ &+ 1.0621\ Q2\ (NU) + .7248\ Q3\ (NU) + 1.0201\ Q4\ (NU) \\ &\ \ (13.78)\qquad\qquad\ \ (6.76)\qquad\qquad\ (11.71) \end{aligned} \tag{2}$$

SEE = .0416 $\quad\bar{R}^2$ = .953 $\quad$ D/W = 1.92

NU enters proportionately to the seasonal dummies Q1 to Q4. T is a trend term equal to 1 in 1Q52 and 56 in 4Q65. Why there should be a downward drift over time in claimants for a given level of insured population and unemployed is not clear. Still, this term is undoubtedly useful. If it is suppressed, the fit of the equation is weakened by all tests and the coefficient on INS becomes insignificant and negative. Seasonal constants proportional to the total unemployed produced better results than either linear dummies or dummies proportional to INS. The low coefficient in the third quarter and the peak in the first quarter suggest that a substantially higher proportion of winter than of summer unemployed are unemployment insurance fund claimants. Statistics for the winter quarters probably pick up the group entitled to seasonal benefits; the unemployed in the summer may include a higher proportion of 'hard-core' unemployed who have exhausted their benefits or who have never built up benefit rights. It is puzzling to find that the coefficients on unemployment are above 1 in three of the four quarters, although the average over four quarters is not significantly different from 1.

Given the determining equations for INS and CL, we can derive a series for employed contributors (EMPS, RDX 11246) by subtraction,

$$EMPS = INS - CL \qquad (3)$$

EMPS then becomes the basic explanatory series for the federal revenue components, employer and employee payments into the Unemployment Insurance Fund (UIR, RDX 2178). Similarly the determining equation for the unemployment insurance benefits paid out by the fund (UIB, RDX 2167) makes use of the series CL indicating the number of claimants on the fund.

Total benefits are determined by the number of claimants and the rate at which each claimant is paid. The rate of payment is determined by the claimant's income, up to a maximum level, and by whether or not the claimant has dependents. Because a consistently high proportion of claimants receive payment at the maximum rate, this rate was chosen to represent the whole structure. Moreover, since the proportion of claimants with dependents to single claimants appears to be quite stable at 53:47, the composite rate of benefit payment was derived by adding .53 times the maximum weekly rate for persons with dependents to .47 times the maximum weekly rate for persons with no dependents. The changes over time in maximum weekly benefit rates are shown in Table 10. Rate changes took place so close to quarter ends that in no case was it necessary to adjust the weighted benefit rate for a quarter to allow for rate changes within that quarter.

This calculated weighted rate variable (WR, RDX 11248) is the policy variable that, when multiplied by the quarterly average of the number of claimants, yields the average weekly payment in millions of dollars for a given quarter. Since there are thirteen weeks in a quarter and since the dependent variable is in millions of dollars the expected coefficient on WR (CL) should be about 13. Moreover the extension of the scope of the fund in 4Q59 creates a seasonal pattern in benefit payments not apparent before that year. This pattern is modelled by introducing short quarterly dummies (SQ1 to SQ4) running only from 1959 to 1965 and multiplicative with the WR (CL) variable.[42] Thus the benefit equation is:

[42]This is the version of the Unemployment Insurance Fund submodel that appears in RDX1. The fits of equations (4) and (5) are significantly improved by starting the seasonal pattern in 1Q59 rather than in 4Q59 when the shift in the structure actually occurred. Data revisions have been such that in later submodels of the fund, including the submodel designed for RDX2, the seasonal pattern does in fact start in 4Q59. Our preliminary results using the new data indicate that the seasonal pattern from 4Q59 does not differ markedly from the seasonal pattern shown in equations (4) and (5).

Table 10

MAXIMUM BENEFIT RATES UNDER THE UNEMPLOYMENT INSURANCE ACT
(Dollars per week)

Effective Date	Person without Dependents	Person with Dependents
July 1/41	12.24	14.40
Oct. 1/46	12.30	14.40
Oct. 4/48	14.40	18.30
July 1/50	16.20	21.00
July 14/52	17.10	24.00
Oct. 2/55	23.00	30.00
Sept. 27/59	27.00	36.00
June 30/68	42.00	53.00

Source: *The National Finances*, Canadian Tax Foundation, various issues.

$$\text{1Q52-4Q65}$$

$$UIB = \underset{(.49)}{-1.6573} + \underset{(22.04)}{8.7352}\ WR\ (CL) - \underset{(1.78)}{.5816}\ SQ1\ (WR)(CL) \qquad (4)$$

$$+ \underset{(2.32)}{.8562}\ SQ2\ (WR)(CL) - \underset{(2.68)}{1.6359}\ SQ3\ (WR)(CL) - \underset{(9.72)}{3.5823}\ SQ4\ (WR)(CL)$$

$$SEE = 9.80 \qquad \bar{R}^2 = .966 \qquad D/W = 1.62$$

Results from this equation are somewhat curious. The constant term is insignificant as desired and the fit is good. On the other hand the quarterly coefficients are not strongly significant, and, in view of the possibility of autocorrelation, their true t-values may be even smaller than those shown. The coefficient on WR (CL) is smaller than one would like, particularly after 1959. One would expect it to be below 13, since obviously the 'average' claimant must receive less than the maximum payment. But it is surprising that he should receive 67 per cent of the maximum before 1959 and even less thereafter. This implies that unemployed claimants are drawn from lower income strata in times of prosperity (i.e. after 1959), though the extent of the overstatement of UIB by WR (CL) is hard to justify on these grounds.

The seasonal pattern suggests that claimants come from relatively high income groups in the second quarter, and from the lowest income groups in the fourth quarter. Does this mean that the seasonally unemployed tend to have relatively low weekly earnings? It is possible, but rationalization on the basis of these coefficients is a pretty shaky business. Basically we simply do not know very much about the composition of the unemployed either on a seasonal basis or over time. If and when we can develop more detailed labour-market equations in the model, we may be able to make less speculative statements about the structure underlying our unemployment insurance subsector.

In the equation explaining UIR, there is no satisfactory way of introducing the rate structure as a policy variable. Payments are made by workers in covered industries receiving less than a set amount per year in wage and salary income; within that class equal weekly payments are made by employer and employee of an amount that depends on the weekly earnings rate of the employee. After the 1959 amendments to the Unemployment Insurance Act, this amount varied from a low of 10 cents per week each when an employee earned less than $9 weekly to a high of 94 cents each when weekly earnings were $69 or more; prior to these amendments the range extended from 8 cents to 60 cents per week. In addition, the government pays one-fifth of the total employer and employee contribution. A model of the rate structure would thus require disaggregation of the contributing population by earning class. Alternately, we could assume that the maximum rate is representative of the rate structure, as we assumed in the case of the benefit equation. This would lead, after 1959, to a weekly payment of $2.256 or a quarterly payment of $29.33 per employed insured worker. If this constant value were multiplied by the total employed population, an exogenous variable for the revenue equation would result.

In fitting the UIR equation a simpler procedure was used, which consisted of regressing UIR on the series for employed contributors, EMPS, from 1952 to 1965. In addition a second series, EMPS (D6), was set equal to zero from 1Q52 to 3Q59 and equal to EMPS thereafter, allowing for the shift in contributions that took place in 1959. A seasonal pattern was also introduced from 1Q59 to 4Q65 to capture the change in seasonality introduced by the 1959 amendments. The resulting revenue equation is:

1Q52-4Q65

$$\begin{aligned} UIR = {} & \underset{(2.82)}{7.9902} + \underset{(12.31)}{10.7738}\ EMPS + \underset{(20.80)}{6.4002}\ EMPS\ (D6) + \underset{(3.83)}{1.3212}\ SQ1\ (EMPS) \\ & - \underset{(4.90)}{1.7033}\ SQ2\ (EMPS) - \underset{(.40)}{.1407}\ SQ3\ (EMPS) + \underset{(.15)}{.0585}\ SQ4\ (EMPS) \end{aligned} \tag{5}$$

SEE = 1.79 $\bar{R}^2$ = .987 D/W = 2.04

This equation looks most impressive by all the standard tests. The only questionable feature is the obviously low significance of the third- and fourth-quarter seasonals. But if the seasonals are eliminated the standard errors on EMPS and EMPS (D6) are increased, and both the proportion of explained variance and the Durbin/Watson statistic are significantly reduced. The last two seasonals are left in for symmetry. As for the coefficients on EMPS and EMPS (D6), the shift means that after 1959 the coefficient on total insured employed is 17.05. If all the employed contributed at the maximum rate, the coefficient should be about 29.33. Thus our equation suggests that the choice of full-rate contribution, as representative of the whole structure, would be less appropriate here than in the benefit equation. This is probably the case because the range of contribution rates for different weekly-earning classes is much greater than the range of benefit rates. The highest benefit rate was 4.5 times the lowest in 1965, while the highest contribution rate was 9.4 times the lowest. But our inability to model the rate structure satisfactorily does not preclude analysis of changes. The roughly 50 per cent rise in contribution rates in 4Q59 is clearly reflected in the coefficient on EMPS (D6). Thus the total coefficient increases from 10.77 to 17.05. Any future increase in the rate structure, if proportionate across the whole structure, could be modelled by an additional EMPS (D7) variable with an appropriate coefficient.

The entire unemployment insurance model therefore consists of four stochastic equations and an identity, explaining one federal revenue series and one transfer series. Included are two policy levers——one explicit in the benefit equation (WR) and one implicit in the total coefficient on EMPS in the revenue equation. Both these equations require certain assumptions about the rate structure: first, that all contribution or benefit rates at different income levels are adjusted equiproportionately when changes are made, and second, that the distribution of contributors and beneficiaries among income classes is unchanged over time. These assumptions cannot be avoided unless a separate analysis is performed for each weekly-earning class, which would require twelve models instead of one as well as analysis of income-class interactions. The aggregation used here seems clearly preferable. Moreover the fit of the whole model is very good. If any area were to be improved, further work should be done on the INS explanatory equation, which is fundamental to the whole system.

2-2-3-1 FAMILY AND YOUTH ALLOWANCES

These payments constitute one of the most straightforward of the transfer programs from a modelling point of view. Eligible recipients are exogenous to the model but very closely related to total population in the relevant age groups. The rate structure is a pure policy variable, and the product of the two is the obvious explanatory variable for the equation.

The family allowance program, initiated on July 1, 1945, provides a monthly payment to the mother or guardian on behalf of every child under 16 fulfilling certain residence requirements. Allowances are currently \$6 per month for each child younger than 10 years of age and \$8 per month for each child 10 to 15 years old. Prior to September 1, 1957, monthly rates were \$5 for each child under 6, \$6 from 6 to 9, \$7 from 10 to 12, and \$8 from 13 to 15. This rate structure is the main policy variable controlled by the federal government. Although it can change the eligibility requirements, very little scope exists for broader eligibility within the present age range, other than through extending the allowance to dependents of Canadians serving abroad.

Payments under the youth allowance program were instituted in September 1964. They are made at the rate of \$10 per month on behalf of all children 16 or 17 years of age attending school or university full time. Quebec opted out of this program in return for an additional income tax abatement of 3 percentage points.

The Dominion Bureau of Statistics provides annual data as of June 1 on the number of persons in the age categories 0-4, 5-9, 10-14, and up by five-year ranges to 90 and over. Quarterly series derived from these annual series by linear interpolation are stored on tape as DB 3033 (population aged 0-4) to DB 3051 (population aged 90 and older) and DB 3032 (total population). From these data, base series for the family allowance were derived by proportional allocation. Thus the population 0-4 added to one-fifth of the population 5-9 was assumed to equal population 0-5 for any given quarter. The remaining four-fifths of the 5-9 cohort is the population 6-9. Similarly, three-fifths of the 10-14 category is 10-12 and the population 13-15 is determined as two-fifths of the 10-14 category plus one-fifth of the 15-19 category. The four series thus created are denoted $KIDS_i$ where i goes from 1 to 4 to denote the population classes 0-5, 6-9, 10-12, and 13-15, respectively. Such a procedure is of course unreliable if sharp fluctuations occur in birth rates, since this would lead to changing proportions of a given five-year cohort being accounted for by any one year. We thought, however, that errors of this kind would not be very serious given the relatively regular behaviour of the Canadian postwar birth rate. The sharp drop in birth rates in the early and middle 1960's may require that more precise numbers be obtained.

The population series required for modelling the youth allowance component of total payments was not so readily attainable. We need the subset of the population 16-17 residing outside Quebec and attending school or university full time. The annual reports of the Department of National Health and Welfare list the number of students receiving youth allowances as of March 31st. As a first approximation we assumed that this number was applicable in each quarter of the appropriate school year, denoting the series so created as STUD. This assumption ignores the seasonal variation attributable to dropouts and seasonal irregularities in births, but since youth allowances affect only six quarters in our data set we decided the approximation would serve. As the model is run forward alternative schemes for forecasting the population receiving youth allowances should be investigated.

The family allowance variable is the product of the quarterly family allowance rate (FAR) and the population in the relevant age group (KIDS). There are four FAR variables, corresponding to the four KIDS variables. FAR_1, which is applied to the 0-5 cohort, is equal to \$15 prior to 3Q57 and \$18 after 3Q57. In 3Q57 FAR_1 is equal to \$16 to allow for the change in monthly rates on September 1. Similarly FAR_3, which applies to the 10-12 cohort, equals \$21 before 3Q57, \$22 in 3Q57 and \$24 in following quarters. Throughout our estimation period, FAR_2 and FAR_4 are equal to \$18 and \$24, respectively. The youth allowance variable is the product of the quarterly youth allowance rate (YAR) and the number of students to whom allowances are payable. YAR had a value of \$10 in 3Q64, since payments were initiated in September of that year, and \$30 thereafter. Prior to 3Q64 YAR equalled zero. We attempted to regress family and youth allowance payments (FAYP, DB 2173) on all the compound variables at once, but it was found that collinearity among the variables made this procedure value-less. Consequently the rate-times-base variables were summed across age groups to yield a single compound variable on which to regress FAYP.

The resulting equation involves several possible sources of error. The population data used are only estimates of the actual quarterly population, while these in turn should be reduced by an unknown amount in order to make estimates of the recipient population. At any given time some small part of the Canadian population may be ineligible for such payments or may simply not apply for allowances.

The fitted equation for the compound variable alone is:

1Q52-4Q65

$$FAYP = \underset{(1277.1)}{.992325} \left[\sum_{i=1}^{4} (FAR_i)(KIDS_i) + (YAR)(STUD) \right] \quad (1)$$

$SEE = .684$ $\quad \bar{R}^2 = .999$ $\quad D/W = .802$

The coefficient is close to unity, as it should be since the population variables are in millions and the rates are quarterly, but its standard error indicates that the coefficient is significantly below 1. Before we conclude that some diehard free enterprise group continues to reject government charity, however, it should be noted that the severe autocorrelation of residuals suggests that the standard error is underestimated.

Further experimentation seemed in order, both to reduce the autocorrelation and to check the test statistics. The constant term was readmitted, and in addition equations were fitted with trend and reciprocal-trend variables running from 1Q52 to 4Q65. The reciprocal trend was no help at all, but the other two specifications gave:

1Q52-4Q65

$$FAYP = .98219 \left[\sum_{i=1}^{4} (FAR_i)(KIDS_i) + (YAR)(STUD) \right] + .03864\, T \qquad (2)$$
$$\qquad (539.8) \qquad\qquad\qquad\qquad\qquad\qquad\qquad (5.92)$$

$$SEE = .538 \qquad \bar{R}^2 = .9994 \qquad D/W = 1.31$$

$$FAYP = 1.01168 \left[\sum_{i=1}^{4} (FAR_i)(KIDS_i) + (YAR)(STUD) \right] - 2.313 \qquad (3)$$
$$\qquad (283.29) \qquad\qquad\qquad\qquad\qquad\qquad\qquad (5.51)$$

$$SEE = .552 \qquad \bar{R}^2 = .9993 \qquad D/W = 1.29$$

Equation (2) seems to be the better of these two equations. Both are superior to the equation with a compound variable alone, and the trended equation seems to suffer somewhat less from autocorrelation. But equations (2) and (3) tell the same story—the coefficient on the compound variable differs significantly from unity and the total payments rise over time as a proportion of the compound variable. Perhaps a small group has been left out of the program, a group shrinking relatively over time. In any case the observed fit makes it clear that extra refinement to isolate this phenomenon will not repay the effort.

2-2-3-2 VETERANS PENSIONS AND ALLOWANCES

The behaviour of war veterans pensions may be modelled by simply applying the rate structure prevailing in any quarter to the existing eligible population. But a complication arises because the eligible population is defined in a rather restrictive manner and cannot be projected into the future by using a subset of existing demographic forecasts. One cannot assume the behaviour of war veterans and their dependents to be similar to that of the whole population. For a complete model of this sector one must try to predict total transfers, given rate and base, as well as the base itself for future years. The rate structure, of course, remains as a policy variable. Although the scope for expansion of eligibility creates a further lever by which the

1Q52-4Q65

$$WWP = -5.0838 + .24768\,(WVR)(VETS) \qquad (1)$$
$$\qquad\qquad\quad (3.77) \qquad (1.81)$$
$$+ 1.9579\,(DR)(DEPS) + 2.0852\,DUM1$$
$$(2.10) \qquad\qquad (3.17)$$

$SEE = .641 \qquad \bar{R}^2 = .985 \qquad D/W = 1.61$

DUM1 is a dummy with a value of 1 in 2Q61 and zero elsewhere. On March 1, 1961 pension rates were increased for both disability and death payments. One would therefore expect an increase in the WWP series in 1Q61 and a larger increase in 2Q61. The national accounts figures for WWP are $34 million in each quarter of 1960 and in 1Q61, $43 million in 2Q61 and $40 million in 3Q61, suggesting that increases made in March 1961 were not entered in the *National Accounts* until the second quarter of that year. To take account of this apparent recording lag we have ignored the within-quarter rate increase in calculating WVR and DR for 1Q61 and added DUM1 to pick up the additional payments, over and above those due to the rate increase, in 2Q61. Our rate variables are in dollars per quarter, the population variables in millions, and WWP is in millions of dollars per quarter. The coefficients on WVR (VETS) and DR (DEPS) should thus reflect the extent to which the rate we assumed to be representative truly reflects the rate structure. The coefficient on DR (DEPS) suggests that our DR series is only about one-half the true weighted rate while that for WVR (VETS) indicates that the average VET is only 25 per cent disabled. High standard errors associated with these terms, however, indicate the presence of severe multicollinearity, and we require more reliable estimates of the impact of changes in WVR and DR on WWP, particularly for use in simulation experiments.

We therefore proceeded by combining our two main independent variables in the form of a weighted sum. The weights were determined by taking fiscal year totals for WVR (VETS) and DR (DEPS) and comparing them with the actual annual liability at fiscal year-end for disability and dependent pensions, respectively. Actual data used for comparison are given in the annual reports of the Department of Veterans Affairs. The ratio 'actual disability pension liabilities/WVR (VETS)' was calculated from the fiscal year ending March 31, 1948 to the fiscal year ending March 31, 1965. A trend in this ratio would indicate that the representativeness of our WVR rate was changing over time. Evidence of such a trend was very slight,[44] and we used the average of the ratio (.3844) to weight the WVR (VETS) term. A similar ratio computed for DR (DEPS) revealed even less evidence of a trend. Again we used the average (.8704) to weight the DR (DEPS) term.

The fitted equation obtained is shown below.

1Q52-4Q65

$$WWP = \underset{(5.63)}{-3.7651} + \underset{(57.55)}{1.0443}\,[.3844\,(WVR)(VETS) + .8704\,(DR)(DEPS)] + \underset{(3.25)}{2.1361}\,DUM1 \qquad (2)$$

SEE = .642 $\bar{R}^2$ = .984 D/W = 1.56

These test statistics are quite satisfactory and almost identical with those for equation (1), as they should be. Implicit coefficients on WVR (VETS) and DR (DEPS) are now quite different from those previously obtained, being .4014 and .9090 respectively. The coefficient on the compound term is significantly larger than the value of unity one would expect. This may be due to the use of only World War I and World War II data to compute our independent variables. A small percentage of pension payments in WWP arises from other wars and special circumstances, and to this extent our independent variable understates the actual liabilities under the pension legislation.

War Veterans Allowances (WVA, DB 2177) were instituted in 1930. From September 1, 1964 to August 31, 1966 the rate of payment was

[44]The first observation was .3756 and the last was .3974. Eleven of the first differences were positive and six were negative.

$94 per month or $282 per quarter to a single individual unable to provide for his own support.[45] Higher rates were payable for married couples or orphaned families, but the single rate was chosen as representative of the rate structure as a whole. In addition, no effort was made in the equation structure to allow for the impact of the income ceiling. The permissible income ceiling for a single person from September 1, 1964 to August 31, 1966 was $133 a month and for married couples $222. These ceilings were raised to $145 and $245 a month, respectively, on September 1, 1966. The quarterly series of recipients (RVA) and quarterly rates (VAR) adjusted for quarters of change are taken from the annual reports of the Department of Veterans Affairs as are the base series for the pension equation. Our estimated equation is:

1Q52-4Q65

$$WVA = \underset{(5.34)}{1.0402} + \underset{(81.04)}{1.0536}\,(VAR)(RVA) + \underset{(4.73)}{2.8157}\,DUM2 \qquad (3)$$

$$SEE = .581 \qquad \bar{R}^2 = .992 \qquad D/W = 2.02$$

DUM2 takes the value 1 in 3Q52, and is zero elsewhere to allow for a retroactive rate increase in that quarter. The equation fits at the 99 per cent level and there is no evidence of autocorrelation. The coefficient on the base variable is, however, significantly greater than 1. This understatement in our base variable is due to offsetting factors, with the presence of married recipients tending to push the coefficient above 1 and the effect of the income ceiling acting to depress the coefficient below 1.

The rate variables in these two equations are clearly government policy parameters requiring no further consideration. But the series VETS, DEPS, and RVA are exogenous to the model and must be projected in some manner if the model is to be run forward. Of the three, DEPS is the easiest to project. Although DEPS consists of two distinct cohorts, World War I (WWI) and World War II (WWII) dependents, graphing the two separately yielded no addi-

[45]On November 30, 1966 the single rate was increased, retroactive to September 1, 1966, to $105 a month and the married rate to $175 (from $161).

tional information. After 1955, the total DEPS series follows a simple autoregressive scheme of the form:

1Q55-4Q65

$$DEPS = .99755\ DEPS_{t-1} \qquad (4)$$
$$(5991.11)$$

SEE = .0000356 $\quad \overline{R}^2 = .9989 \quad$ D/W = .160

or, alternatively

$$DEPS = -.000909 + 1.02575\ DEPS_{t-1} \qquad (4')$$
$$(9.85) \quad (357.94)$$

SEE = .0000198 $\quad \overline{R}^2 = .9997 \quad$ D/W = .523

Either equation seems satisfactory, although the latter is probably preferable because of its better fit. Before 1955 the DEPS series follows no regular pattern. What is extremely surprising is the very slow rate of decline of DEPS, a feature not only of the years after 1955 but of the whole period. The total dependents' pensions resulting from World War I reached a maximum of 20,015 in fiscal 1925, yet had only declined to 14,027 by 1966, forty-one years later. Total dependents' pensions resulting from both world wars peaked at 34,403 in 1950 and were 29,913 in 1966. If the decline is linear, pensions will reach zero in about 2077, while if the post-1955, autoregressive pattern holds DEPS will be with us for generations. Consequently it is doubtful if this structure can be maintained into the middle-run future. Widows and orphans seem to be extraordinarily long-lived.

In the case of disabled veterans we obtain a clear gain in information from separating the World War I and World War II cohorts. There appears to be a definite postwar veteran cycle, which is seen after World War I and may be followed with roughly a twenty-year lag by the World War II cohort. The World War I group rises irregularly to a peak of 80,133 in 1940, then falls exponentially on a smooth curve down to the present. This group is thus easy to project into the future. The World War II group seems to have reached a peak in 1964, and one may therefore assume that it will decline thereafter according to the same pattern as the World War I group. Thus we break down VETS by war into

$$VETS = VETS\ I + VETS\ II$$

Then we subtract VETS I in each post-1940 quarter from the maximum 1940 value to get a series

$$DVETS\ I = .080133 - VETS\ I$$

DVETS I is the divergence between current and peak numbers of disabled World War I veterans, and it increases exponentially from 1940 onward. Then if we fit a log relation the equation is:

1Q47-4Q65

$$\ln DVETS\ I = \underset{(172.17)}{-9.0008} + \underset{(100.39)}{1.26904} \ln T \qquad (5)$$

SEE = .040 $\bar{R}^2$ = .993 D/W = .026

where T is a trend equal to 1 in 2Q40, and 103 in 4Q65.

We have an excellent fit for the equation but the extreme autocorrelation shows that the exponential form is misspecified. As it stands, we have a model saying

$$\ln D = a + b \ln T$$

so

$$D = AT^b$$

where

$$A = e^a$$

Given this form, D = .080133, or VETSI = 0, when ln T = 5.1036 or when T = 165. This will occur in approximately the middle of 1981, and our equation thus implies that veterans disability pensions arising from World War I will be zero by then. The functional form implies

$$\frac{dD}{dT} = bAT^{b-1} = \frac{b}{T} D$$

and thus the rate of increase of D rises as T rises, because the observed b is greater than 1. An alternative form

$$\ln D = a + bT$$

was tried but yielded a weaker fit. This of course implies

$$D = Ae^{bT}$$

and $$\frac{dD}{dT} = bAe^{bT} = bD$$

which indicates a rate of rise of D increasing faster than the previous form, though this rate is initially slower due to smaller values of b. Neither form is fully satisfactory, but the former seems reasonably adequate for short-term prediction. We can then plug into this equation the value T = 1 in 1Q64, if that is the peak quarter, and derive successive values for DVETS II, which, when subtracted from the peak value .106628, will yield a series VETS II. The two series can then be summed to yield projections of VETS.

But for the recipients of War Veterans Allowances (RVA) no projection scheme seems adequate. The series rises steadily at least until 1966 with no sign of a peak. On March 31, 1966 the number of recipients stood at 85,835. This upward trend is a combination of aging veteran population and expanding eligibility, but surely a peak must come soon. We have no way of knowing when the downturn will occur or what form it will take. RVA may begin to drop sharply in the near future, or may just keep rising. One can project RVA at its latest level until a down-trend is asserted; alternately one could apply an autoregressive scheme such as that used for DEPS. But both efforts would be based on guesswork.

2-2-3-6 OLD AGE PENSIONS

The old age pension was initiated on January 1, 1952. Monthly payments at the rate of $40 were made to any Canadian over 70 years of age who met certain residence requirements previous to application. In 1960 coverage was extended to non-residents with twenty-five years of residence in Canada after age 21. The pension was increased to $46 per month on July 1, 1957, and to $55 on November 1, 1957. On February 1, 1962 the pension was again increased to $65, and on October 1, 1963 to $75. In April 1965 provision was made for a cost-of-living adjustment to the pension (with a maximum increase of 2 per cent per year) based on the Canada Pension Plan Pension Index, effective in 1968, but this is outside our data period.[46] The April 1965 legislation provided as well for the payment of old age pensions in 1966 to persons born in 1897 and earlier. Coverage was extended progressively so that in 1970 and thereafter pensions become payable at age 65.

We combined monthly rates to form the quarterly series for the old age pension rate (OAPR). The base series, Canadians 70 years of age and over, is the sum of DB 3047 to DB 3051, and is designated AGED. These population series are derived by quarterly interpolation of annual demographic data supplied by the Dominion Bureau of Statistics. The dependent variable is old age pension payments (OAPP, DB 2182).

1Q52-4Q65

$$OAPP = \underset{(808.78)}{1.0019}\ (OAPR)(AGED) \qquad (1)$$

SEE = 1.352 $\bar{R}^2$ = .9992 D/W = .574

[46]On January 1, 1967 a program of Guaranteed Income Supplements became effective. This program guarantees that old age security recipients will receive a minimum monthly income of $105 in 1967. In 1968, the GIS was also made subject to a cost-of-living adjustment with a maximum increase of 2 per cent per year.

or, alternatively

$$OAPP = 1.0030\ (OAPR)(AGED) - 5.7488\ (1/T) \qquad (2)$$
$$(1105.79) \qquad\qquad (7.18)$$

$$SEE = .976 \qquad \bar{R}^2 = .9996 \qquad D/W = 1.038$$

T in equation (2) is a time trend from 1Q52 to 4Q53 allowing for an apparent initial registration lag. Without this time trend there were substantial negative residuals in the first few years. Because the independent variable is quarterly payments in millions of dollars, the basic coefficient is 1. The inverse trend term is clearly helpful in dealing with autocorrelation, although further examination of residuals suggests that the term might well have been extended to 4Q54. On balance this is a very simple and straightforward equation, and further tinkering would be unproductive.

2-2-4 INTEREST ON THE FEDERAL PUBLIC DEBT

Interest on the federal public debt (IFD, DB 2181) is the largest category of transfers to persons. It now runs at over a billion dollars each year. The amount of the transfer obviously depends on the total amount of debt outstanding and the average coupon rate on this debt. Within the block of federal debt it is useful to distinguish three classes of instruments usually sold in different markets and subject to different forms of rate behaviour. These are treasury bills (TB, DB 636), other direct market securities (DMS), and Canada Savings Bonds (CSB). Treasury bills are short-term, highly liquid assets with a rate structure assumed to be based on the key rate, the three-month treasury bill rate (RTB, DB 601), average of Thursdays. DMS comprises all the regularly traded federal debt with varying maturities and yields and semiannual coupons. DMS also includes a small block of perpetuals, sold at a $55 million face value and a 3 per cent coupon rate, carrying a constant annual interest charge of $1.65 million. The third category, CSB, consists of fixed-yield securities, redeemable on demand at par plus accrued interest. The coupons are redeemable annually on November 1st.[47]

Prior to 1960 the patterns of coupon redemption described above created a seasonal pattern in the interest liabilities series, because all interest was reported on a paid basis in the *National Accounts*. Thus CSB interest payments were concentrated in the fourth quarter, while interest payments on direct market securities shifted about depending upon which pair of quarters had the highest proportion of semiannual payments. Starting in 1960 interest liabilities have been reported in the *National Accounts* on an accrual basis, which should have eliminated the seasonal pattern. However, this pattern has not been entirely eliminated. Consequently our final equation embodies two seasonal patterns: AQ1 to AQ4 running from 1Q55 to 4Q59, and BQ1 to BQ4 running from 1Q60 to 4Q65. The need for the second set of quarterly dummies suggests that some problems remain in recording interest liabilities on an accrual basis.

The independent variables needed to construct the equation were generated as described below. To arrive at the treasury bill interest liabilities variable, we multiplied the treasury bill rate (RTB) by the value of bills outstanding at the end of the quarter (TB). Since RTB is expressed in per cent per annum, the expected coefficient on the compound variable is .0025. In dealing with the other two classes of debt, a more complicated procedure was required due to the wide variety of rates on the debt outstanding at any one time. We attempted, with little success, to find a single rate or a simple moving average of rates that would serve as a representative for the whole structure. Instead we broke down the DMS variable into individual issues for each quarter in our data period,[48] and derived an interest liabilities series by multiplying the amount of each issue outstanding at the end of the quarter by the coupon rate for that issue. An issue outstanding for only part of a quarter was assumed to be outstanding for the whole quarter but to be reduced proportionately. Thus a $100 million issue retired

[47]Although coupons of the 1956 CSB issue were redeemable annually on May 1, we ignored this irregularity in calculating our interest liabilities series.

[48]See the table "Details of Unmatured Outstanding Issues" in *Bank of Canada Statistical Summary* issued monthly by the Bank of Canada.

at mid-quarter would be counted as $50 million for the whole quarter. Summing the resulting quarterly liabilities series across all issues gave us total quarterly interest liabilities for direct market securities (DMSL).

A similar procedure was followed to obtain a series for Canada Savings Bond interest liabilities (CSBL), although the calculations in this case were more complex. These instruments are issued in the fourth quarter of the year and dated November 1st. Sales of an issue are generally continued through the first three quarters of the following year and are then discontinued. The amount of CSB's outstanding, disaggregated by issue, is not readily available on a quarterly basis. However, estimates are available for the amount of each issue outstanding as of January 1st of each year.[49] They are taken as the stock outstanding of each CSB issue at the end of the fourth quarter of the previous year. For all issues except the current one the change from fourth quarter to fourth quarter is a measure of redemptions. For the current issue, however, the change in the calendar year following the year of issue is a combination of both sales and redemptions. This net change has been negative for all issues, indicating that redemptions typically exceed sales (in most cases by relatively large amounts) in the year after a new CSB issue. The belief that gross sales of CSB's in the calendar year following issue are small, and probably insignificant compared with the total sales in the quarter of issue, led us to adopt the convenient assumption that these sales are not significantly different from zero. This implies that we treated the first year change in current CSB's outstanding as entirely due to redemptions, and, in effect, ignored minor CSB sales occurring in three quarters.

Given an annual series of redemptions for each CSB issue we had to decide how to spread these on a quarterly basis. We have no information on the quarterly pattern of redemptions, but it seems reasonable to expect that a large proportion of annual redemptions are due to 'roll-overs' when a new issue is sold. If so, annual redemptions would be heavily concentrated in the fourth quarter. We experimented with three spreading ratios: the first was a simple linear interpolation; the latter two involved a non-

[49]These estimates are published in *Loans of Government of Canada and Loans Guaranteed by the Government of Canada* issued annually by the Bank of Canada.

linear interpolation with quarterly weights of .2, .2, .2, .4, and .1, .1, .1, .7, respectively. Each of these three approximative procedures yielded a different series for quarterly CSB's outstanding by issue. These three series were used to generate three CSBL series. The relative performance of these alternative CSBL series in the estimated GINT equation, both within the estimation period and in the forecast period, was our criterion for selecting the 'most reasonable' method of interpolation. Both of the nonlinear procedures were superior to the simple linear interpolation (as one might expect), and the CSBL series based on the assumption that 70 per cent of the redemptions occurred in the fourth quarter proved to be the best of the three.

Using this quarterly series for CSB's outstanding, CSBL was calculated in the following way. We first generated two liabilities series, one on a paid basis (CSBLP) and one on an accrued basis (CSBLA). We then have

$$CSBL = D1\ (CSBLP) + D2\ (CSBLA) \qquad (1)$$

where D1 = 1 from 1Q55 to 4Q59 and zero thereafter, and D2 = 1 from 1Q60 forward. Interest on an accrued basis, CSBLA, was calculated as follows:

$$CSBLA = 1/4 \sum_{i=1}^{m} \left[\frac{X_{it-1} + X_{it}}{2}\right]\left[\frac{CR_{it-1} + CR_{it}}{2}\right] \qquad (2)$$

where X_{it} is the amount of the CSB of issue i outstanding at the end of quarter t, and CR_{it} is the annual coupon rate (expressed as a decimal) applicable to X_{it} in quarter t. The amount of a CSB issue outstanding fluctuates daily. Therefore quarterly CSB's outstanding for each issue were found by averaging the beginning- and end-of-quarter values. Such a procedure gave us an approximation of CSB's outstanding at mid-quarter.[50]

[50]The coupon rate on a CSB may be changed during the life of the issue. For this reason we averaged successive quarterly coupon rates to get the rate that most accurately reflects the mid-quarter rate. Changes that have occurred in the past have always taken place in the fourth quarter.

The CSBLP series is more complex. What we required for the first three quarters is the amount of redemptions by issue multiplied by the annual coupon rate for that issue (expressed as a decimal) and scaled by some fraction representing the actual proportion of the annual interest received at the time of redemption (remembering that the 'year' for purposes of interest payments runs from November 1 to October 31). For the fourth quarter, we needed a similar expression, but we also needed a measure of the interest paid on the stock of CSB's outstanding on November 1. We assumed that redemptions are spread evenly throughout the quarter, and that all monthly redemptions are made on the last day of the month. This gave us the expression:

$$\begin{aligned} CSBLP = \sum_{i=1}^{m} [&1/3\ Q1\ (\Delta X_{it})(CR_{it}) + 7/12\ Q2\ (\Delta X_{it})(CR_{it}) \\ &+ 10/12\ Q3\ (\Delta X_{it})(CR_{it}) + 1/12\ Q4\ (\Delta X_{it})(CR_{it}) \\ &+ Q4\ (X_{it-1})(CR_{it-1})] \end{aligned} \qquad (3)$$

The CSBL and DMSL series calculated in the manner described above are reproduced in the data appendix.

The estimated IFD equation is shown below.

1Q55-4Q65

$$\begin{aligned} IFD = {} & \underset{(3.14)}{.00227}\ (TB)(RTB) + \underset{(6.91)}{1.50581}\ CSBL + \underset{(17.97)}{.28029}\ AQ1\ (DMSL) \\ & + \underset{(26.25)}{.42515}\ AQ2\ (DMSL) + \underset{(20.92)}{.35666}\ AQ3\ (DMSL) + \underset{(2.29)}{.11272}\ AQ4\ (DMSL) \\ & + \underset{(10.55)}{.27656}\ BQ1\ (DMSL) + \underset{(12.78)}{.32316}\ BQ2\ (DMSL) + \underset{(12.71)}{.31890}\ BQ3\ (DMSL) \\ & + \underset{(11.88)}{.29831}\ BQ4\ (DMSL) \end{aligned} \qquad (4)$$

SEE = 8.373 $\bar{R}^2 = .975$ D/W = 2.15

On the whole this equation is satisfactory, although it has some disturbing features. The fit is good, the pattern of residuals reassuring and the coefficient on (TB)(RTB) is not significantly different from its expected value of .0025. As for the CSBL variable, it should have a coefficient of 1 and its estimated value is clearly too large. Similarly the sums of AQ_i (DMSL) and BQ_i (DMSL) variables are 1.175 and 1.217 respectively, while one would expect them to be unity. Another puzzling feature of the equation is the distribution of the coefficients on the AQ_i (DMSL) variables. The indication is that interest payments on direct market issues are highest in the second and third quarters, but such a pattern is unreasonable if coupon payments are made semi-annually. Prior to 1960 the CSBL variable was relatively large in the fourth quarter. This, coupled with the high coefficient on CSBL, must be responsible to some extent for the low coefficient and relatively high standard error on AQ4 (DMSL). Despite these difficulties, however, equation (4) forecasts well. Forecasts of the interest on the debt for 1966 and 1967 were too low by $55.6 million (4.83 per cent) and $58.3 million (4.68 per cent), respectively.

A criticism of this procedure might be levelled at the complexity of the independent variables CSBL and DMSL. These will be a nuisance to forecast, as their future values must take account of the whole past debt structure. On the basis of our present approach, we can calculate the interest liabilities that equation (4) will generate into the future until such time as the last series currently outstanding has been redeemed. To this calculation must be added in each quarter the interest liability generated by gross new issues at the going market rate. Thus, if in the last observed quarter DMSL has been calculated, we can derive $DMSL_{t+1}$ by subtracting the interest liability for any issue redeemed in period t+1 and then adding the interest liability on gross new issues in period t+1. We can assume that gross new issues will be equal to the net government deficit in t+1 plus the value of the securities that will be redeemed in period t+1. Assumptions will then have to be made about the term to maturity of these gross new issues, so that we will know in what future period (t+n) they will be redeemed. Thus the liabilities series used can be projected forward logically given the net deficit, the government borrowing rate, and the desired maturity structure from the rest of the model.

PART 4 PROVINCIAL GOVERNMENT REVENUE

Total revenue of the provincial governments was $6,361 million in 1965, of which tax revenue made up 62 per cent and transfers from other levels of government (primarily the federal government) 22 per cent. In this part of the paper we present eight equations, which explain $3,598 million (or 72 per cent of total revenue excluding transfers from other levels), comprising 81 per cent of tax revenue and 41 per cent of the remaining non-tax revenue. These equations are listed in Table 12.

Our three personal direct tax equations explain 88 per cent of total provincial personal direct taxes, while our corporation income tax equation explains 92 per cent of corporate direct tax revenue, and our three indirect tax equations explain 74 per cent of total indirect tax revenue. The equation in section 3-1-6-3 covers profits of provincial government business enterprises, and explains 64 per cent of provincial investment income.

In deciding which provincial revenue sources to model, we were influenced by the relative sizes of various components, the extent to which explanatory variables were likely to be produced elsewhere in a macro-model, the availability of necessary data, and the amount of variance to be explained. Most of the items excluded are either quite small (e.g. amusement taxes) or are the product of such a jumble of rates applied to such a wide variety of tax bases (e.g. natural resources taxes and real property taxes) that little can be learned from detailed modelling. Treated as a group, however, the excluded items are easier to handle. The sum of related revenues moves fairly steadily over time, since it results from a large number of different tax rates (with substantial intertemporal and interprovincial differences) applied to a considerable range of tax bases. This cancellation of random variations is even more obvious if certain small non-tax revenues are included. For example, if all revenue items not modelled separately, except transfers from other levels of government, are

Table 12

LIST OF PROVINCIAL GOVERNMENT REVENUE ITEMS MODELLED
(Millions of dollars)

Index Code No.		1965 Revenue
	Personal Direct Taxes	
3-1-1-1	Personal income taxes	743
3-1-1-2	Motor vehicle licences and permits, persons	96
3-1-1-3	Hospital insurance premiums	187
	Corporate Direct Taxes	
3-1-3-1	Corporation income tax accruals	503
	Indirect Taxes	
3-1-4-2	Gasoline taxes	659
3-1-4-4	Motor vehicle licences and permits, businesses	139
3-1-4-7	Retail sales taxes	839
	Investment Income	
3-1-6-3	Profits of government enterprises	432
	Total	3,598

added together[51] and regressed on time and a constant term we can account for 90 per cent of the variance. The regression of that sum on GNE (RDX 223) multiplied by quarterly dummies and a time trend accounts for over 98 per cent of the variance, as shown below.

1Q48-4Q66

$$\begin{aligned} SUM = \underset{(33.03)}{.01748}\,(GNE) + \underset{(4.75)}{.00218}\,Q1\,(GNE) - \underset{(4.44)}{.00192}\,Q2\,(GNE) \\ - \underset{(9.15)}{.00370}\,Q3\,(GNE) + \underset{(18.16)}{.00015}\,(GNE)(T) \end{aligned} \qquad (1)$$

SEE = 11.51 $\bar{R}^2$ = .985 D/W = 1.33

However, such a regression is an unsatisfactory means of explaining major revenue sources regardless of the fit obtained, since there is no way of using the resulting equation to indicate how and when revenues would change in response to specific changes in government policy. The method is appropriate for dealing with a heterogeneous collection of leftovers in which there is little policy interest or for which there are no suitable sources of explanatory data. But as long as we deal with the provinces as an aggregate group a number of revenue sources are best treated by an essentially defeatist regression on GNE or a trend. From time to time it will be desirable to break out specific items from the aggregate group, and attempt to develop particular models. If individual provinces wish to forecast their own revenues, then specific treatment for separate revenue items now lumped together would become at once necessary and more feasible. For the time being, we shall deal only with the larger items for which there is some homogeneity of experience across provinces.

[51] Code numbers of the items are: 3-1-1-4, 3-1-1-5, 3-1-3-2, 3-1-4-1, 3-1-4-3, 3-1-4-5, 3-1-4-6, 3-1-4-8, 3-1-6-1, 3-1-6-2, 3-1-6-4, 3-1-8-1, 3-1-8-2, 3-1-8-3, and 3-1-8-4. The last four items (which represent employer and employee contributions to pension, vacation, and compensation funds) could be modelled in the same way as the income side of the similar fund examined in section 2-1-6-1. Construction of a single model for all provinces would involve additional problems caused by aggregation of data from provinces with differing contribution rates.

One major provincial item we shall bypass is "transfers from other levels of government", because forecasting this item would not be easily improved by using the kind of econometric models that we employ in forecasting other revenue sources. Few economic or demographic variables help much to explain the outcome of negotiations on the size of transfer payments from one level of government to another.

Our approach to the explanation of provincial revenue sources has been similar to that used in the federal tax section, that is to duplicate as nearly as possible typical tax calculations underlying payments recorded in the *National Accounts*. In most cases we selected an appropriate proxy for the tax base, and then applied a weighted average tax rate. The weighting across provinces was usually done according to the distribution of the tax base among provinces. Since these weighting patterns do not usually change dramatically in the short run, the weighted rates can be regarded as predetermined variables derived from policy-determined tax rates and stable weighting patterns. In some instances, one can assume that there is a simultaneous feedback from the expenditure requirements to the tax rates. In our initial model, which treats government expenditure as being predetermined, this feedback does not prevent us from dealing with tax rates as predetermined variables. When a number of expenditure items are made endogenous to the model, some of the more volatile tax rates may have to be treated as simultaneously determined endogenous variables.

In the revenue models discussed here, we employ a number of weighted average tax rates. We describe in the data appendix the calculation procedure used in constructing each of these rates, and list the resulting series.

3-1-1-1 PERSONAL INCOME TAXES

Under the federal-provincial tax-sharing agreements of 1962 the federal government partially withdrew from the personal income tax field. The provinces were offered an abatement equal to a fixed percentage of the 'basic tax' collected.[52] Formally, the

[52]'Basic tax' is defined as total tax accruals (AY, RDX 11600) net of the Old Age Security Tax, the 1965 tax cut, the temporary surcharge imposed in 1968 and the new Social Development Tax, which took effect at the beginning of 1969.

provinces levy their own taxes at rates equal to, above, or below those of the federal abatements. The provincial tax is collected by the federal government without cost to the province——as long as the provincial tax base is identical to that of the federal government. Quebec is the one province now collecting its own personal income tax. In Table 13 we summarize the federal abatements and the provincial rates of personal income tax since 1962. The provincial levies shown for Quebec are somewhat misleading since, as already noted, Quebec has its own collection system with a separate tax base and a progressive rate structure. We assume in our analysis, however, that total Quebec tax collections do not differ significantly from the dollar value of the abatement received.[53] Manitoba and Saskatchewan each levy a surtax of 5 per cent (formerly 6 per cent) over and above the federal abatement rate, while for the other seven provinces the rates of income taxation are equal to the federal abatement rate.[54]

Two models explaining total personal tax collections were presented in section 2-1-1-1. Here we needed some method of breaking out provincial collections from the total. Our procedure was essentially the same as that used in constructing *Model 2* of section 2-1-1-1, but modified to take account of the fact that the relevant base for calculating provincial tax collections is federal basic tax, not assessed taxable income.

We proceeded by defining a new series of weighted average tax rates (BRW_i) analogous to the RW_i of section 2-1-1-1 except that we substituted basic tax payable for total tax payable in our calculations.[55] The identity

[53]In 1967 marginal tax rates in Quebec were exactly one-half of the federal marginal (basic tax) rates in each income category, although the Quebec tax base and exemption levels were not analogous to those of the federal government. *Principal Taxes and Rates 1967* issued annually by D.B.S., catalogue no. 68-201, pp. 8 and 14.

[54]Effective July 1, 1969 a 5 per cent surcharge was imposed in Alberta and in Newfoundland. New Brunswick adopted a 10 per cent surcharge effective April 1, 1969.

[55]See data appendix, pp. 155-156.

Table 13

PERSONAL INCOME TAX: RATES OF FEDERAL ABATEMENT AND PROVINCIAL TAX
(Expressed as a percentage of basic tax)

	Taxation Year						
	1962	1963	1964	1965	1966	1967	1968
Federal Abatement							
To all provinces except Quebec	16	17	18	21	24	28	28
To Quebec*	16	17	18	44	47	50	50
Provincial Levies							
Manitoba	22	23	24	26	29	33	33
Saskatchewan	22	23	24	27	29	33	33
Quebec	16	17	18	44	47	50	56
All Others	16	17	18	21	24	28	28

* In 1965 an 'opting-out' arrangement was instituted whereby a province could elect not to participate in various shared-cost programs with the federal government. Quebec opted out of several programs in return for additional abatements.

Source: *Provincial Finances*, Canadian Tax Foundation, various issues.

$$ABY = \sum_{i=1}^{4} BRW_i \, [YAS_i - NT_i \, (\overline{YEX_i})] - RDC \, (DIVC) \qquad (1)$$

was then used to define personal basic tax accruals (ABY). The YAS, NT, YEX, RDC, and DIVC variables, as well as the four income classes employed, are defined in section 2-1-1-1.

The provincial rates of personal income tax shown in Table 13 are weighted by the proportion of total tax payable originating in each province and summed to form the weighted provincial tax rate variables TPER1 for all provinces except Quebec and TPER2 for Quebec. (See the data appendix.) Total provincial accruals are then defined as

$$TPA = (TPER1 + TPER2)(ABY) \qquad (2)$$

Taxes collected by the federal government are passed on to the provinces with a two-month lag. This lag, coupled with the one-month lag between tax accruals and collections by the federal government, results in a delay of one quarter between the time provincial taxes are accrued and the time they are actually received and recorded by the provinces. Since Quebec collects its own tax we expect the lag between accruals and collections to be shorter in this instance; indeed we suspect it would be close to the one-month lag between federal accruals and collections. We thus construct the collection variable

$$TPC = [(TPER1)(ABY)]_{t-1} \qquad (3)$$

$$+ [1/3[(TPER2)(ABY)]_{t-1} + 2/3 \, [(TPER2)(ABY)]]$$

Regressing actual provincial personal tax collections (TPP, DB 4051) on the calculated collection variable, we get the following equation:

2Q62-4Q66

$$TPP = 1.1368 \ TPC \qquad (4)$$
$$(51.98)$$

SEE = 14.13 $\qquad \bar{R}^2 = .959 \qquad$ D/W = 1.98

This equation fits fairly well and the Durbin/Watson statistic is reassuring. Ideally we would expect the coefficient on TPC to be 1. However, our experience with the TP equation in section 2-1-1-1 indicates that a coefficient of 1.13 is not unreasonable since the independent variable, based on data in the *Taxation Statistics*, is consistently lower than the dependent variable, based on the *National Accounts*.[56]

Equation (4) allows us to assess directly the revenue effects of changes in the federal basic tax rate schedule, in the levels of abatements to the provinces, in provincial surtaxes, or in any combination of these three policy instruments. Federal personal income tax collections can be determined by deducting TPP from TP.

3-1-1-2 MOTOR VEHICLE LICENCES AND PERMITS, PERSONS
3-1-4-4 MOTOR VEHICLE LICENCES AND PERMITS, BUSINESSES

Since motor vehicles of any size can be owned either for personal or for business use, it makes sense to explain revenue from motor vehicle licences and drivers' permits, issued to persons and to businesses, as a single item (MVLP, DB 4052 + DB 4064). For this purpose we need a weighted average licence fee per registered motor vehicle, and a series representing the number of licenced vehicles. Neither is easy to develop.

Licence fees vary by province and also by type of vehicle. Since no data exist on the numbers of each type of vehicle registered in a given province, we calculated an average rate per vehicle for each province. This was done by dividing provincial revenue derived from licences and permits by the number of regis-

[56]See the discussion preceding Chart 8, pp. 43-44.

tered motor vehicles in a province. When the equation is used for forecasting, a problem arises in translating any licence fee change for a particular sort of vehicle into the appropriate change in the average rate for the province concerned. Weighting across provinces was established by using provincial net (i.e. taxable) gasoline sales. The resulting pattern of weights is stable enough to permit the equation to provide useful forecasts. Sources for all series used to compute the weighted average rate of licence fee per registered motor vehicle (MVPR), as well as values of MVPR from 1955 to 1967, are given in the data appendix.

The base we employed in constructing the MVLP equation is the total number of registered motor vehicles (RMV) as recorded in *The Motor Vehicle, Part III Registrations*.[57] A separate independent variable is required for each quarter because most licence fees are paid annually rather than quarterly. These quarterly variables were obtained by interpolating the annual RMV series (from fourth quarter to fourth quarter) and using proportional quarterly dummies. The estimated relationship is shown below.

1Q55-4Q65

$$\begin{aligned} MVLP = \; & \underset{(7.4)}{.0469} \, (MVPR)(RMV) + \underset{(66.7)}{.6126} \, Q1 \, (MVPR)(RMV) \\ & + \underset{(24.0)}{.2186} \, Q2 \, (MVPR)(RMV) + \underset{(2.5)}{.0226} \, Q3 \, (MVPR)(RMV) \end{aligned} \qquad (1)$$

$SEE = 3.546 \qquad \overline{R}^2 = .993 \qquad D/W = 2.00$

Equation (1) appears to be satisfactory in all respects. The sum of the quarterly coefficients is 1.041 and their pattern indicates that the bulk of licence revenue is received in the first quarter with a steady decline throughout the year. We used the equation to forecast MVLP for 1966 and 1967 with encouraging results. The 1966 forecast value was $5 million (2.03 per cent) less than the actual value and the 1967 forecast was only $2 million (.769 per cent) too low.

[57] *The Motor Vehicle, Part III Registrations* issued by D.B.S., catalogue no. 53-219.

When we attempt to use equation (1) in a macro-model, registered motor vehicles are unlikely to be endogenously determined. Sales of new passenger cars are more likely to be explained in such a model. To utilize this latter variable we created a series for the stock of passenger cars in existence (CARS). CARS is defined as a fifteen-year weighted sum of annual new car sales, with weights corresponding to U.S. scrappage rates.[58]

We reestimated the MVLP equation using the interpolated CARS series in place of RMV. The test statistics were not significantly different from those obtained for equation (1), although they were slightly inferior. The residuals were quite large in 1965, however, and when the equation was used to forecast 1966 and 1967 the forecast values of MVLP were higher than the actual values by 4.9 and 6.9 per cent respectively. This relatively poor forecasting performance is caused by a change over time in the proportion of passenger cars to the total stock of registered motor vehicles. There was a nonlinear decline in the ratio RMV/CARS from 1.58 in 1955 to 1.38 in 1965.

Employing an alternative procedure we estimated directly the relationship between RMV and CARS, and used the forecast values of RMV in equation (1) to forecast MVLP. An annual regression of RMV on CARS and CARS/T (where T is an annual time trend equal to 1 in 1955) provided the best results and is shown below.

1955-1965

$$RMV = \underset{(7.3)}{.9597} + \underset{(40.9)}{1.1906}\ CARS + \underset{(1.4)}{.0532}\ (CARS/T) \qquad (2)$$

$SEE = .0418$; $\bar{R}^2 = .998$; $D/W = 1.21$

The equation indicates that the RMV/CARS ratio declines to an asymptotic value of 1.19. RMV was forecast for 1966 and 1967 from equation (2). The interpolated values, used in equation (1)

[58]New car sales are available from *New Motor Vehicle Sales* issued monthly by D.B.S., catalogue no. 63-007. Scrappage rates are given in Friedman, Charles S.: "Stocks of Passenger Cars: Postwar Growth and Distribution." In: *Survey of Current Business*, U.S. Department of Commerce, September 1963, p. 20.

produced revenue forecasts for MVLP that were only $3 million too low in 1966 and $1 million too high in 1967.

3-1-1-3 HOSPITAL INSURANCE PREMIUMS

Under the federal Hospital Insurance and Diagnostic Services Act of 1957, the costs of hospitalization are shared approximately equally by the federal and provincial governments. Since the provinces are free to devise their own methods for financing the provincial share of these costs, a variety of financing methods are in use. Three provinces——Ontario, Manitoba and Saskatchewan——require the payment of hospital insurance premiums. These revenues appear under "Direct Taxes" in the *National Accounts*.

To estimate the aggregate premium receipts, we needed a weighted average rate of premium (with zero weights for provinces other than Ontario, Manitoba and Saskatchewan), and a measure to represent the insured population. We prepared our weighted average premium rate (RWH) by weighting according to the provincial share of the national civilian labour force (NL, RDX 11141) and therefore the appropriate base figure is NL. We chose NL as our base because most of the insured population is insured through payroll deductions——coverage is a condition of employment in many firms. If we regress total hospital insurance premiums (HIP, DB 4053) on RWH (NL) the coefficient could differ from 1 because the number of insured persons not in the civilian labour force may be either more or less than the number of uninsured persons in the civilian labour force.

To calculate a weighted average premium rate, an average premium for each province was first obtained by multiplying the premium for married contributors by the proportion of married males in the provincial labour force and adding the product of the premium for single contributors multiplied by the proportion of single persons in the provincial labour force.[59] The average premium for a particular province was then weighted by that prov-

[59]We assume that all married females in the labour force have husbands who are also in the labour force. The category 'single persons in the labour force' includes those who are widowed or divorced. The relevant provincial proportions are derived from 1961 census data and assumed to be constant throughout the estimation period.

ince's share in the national civilian labour force. Summing over the three provinces requiring premium payments gave a weighted average hospital insurance premium rate for Canada.

The use of quarterly dummy variables was considered to be necessary because of the method of making premium payments in Saskatchewan. Payments in Ontario and Manitoba are made on a quarterly or monthly basis, generally through payroll deductions. In Saskatchewan however, contributors assessed at the single rate must pay at least one-half of the premium on November 30 and the balance on the following May 31; contributors assessed at the married rate may make bulk payments in November or may pay in quarterly instalments. One would therefore expect the total fourth-quarter premium receipts to be consistently larger than those received in the previous three quarters. While the anticipated results were obtained, the relative magnitudes involved are not great owing to the small number of single contributors in Saskatchewan compared with total contributors in all provinces. Another reason for expecting the coefficient to be lower in the second and third quarters is that any seasonal increase in the labour force would not necessarily be matched by an equivalent increase in the insured population, since some seasonal workers would remain uninsured, while others would maintain their insurance even when out of the labour force.

We present below our estimated equation.

1Q59-4Q66

$$\begin{aligned} HIP = {} & \underset{(55.3)}{1.1627}\ Q1\ (RWH)(NL) + \underset{(54.9)}{1.1243}\ Q2\ (RWH)(NL) \\ & + \underset{(55.1)}{1.0447}\ Q3\ (RWH)(NL) + \underset{(59.9)}{1.1644}\ Q4\ (RWH)(NL) \end{aligned} \qquad (1)$$

SEE = 1.795 $\bar{R}^2$ = .949 D/W = 1.69

The quarterly pattern of coefficients is the pattern we expected. The average of the quarterly coefficients is 1.124 indicating that there are more insured people outside the labour force than uninsured people in the labour force. Since about 98 per cent of the population in Ontario, Manitoba and Saskatchewan is insured, the high coefficients were expected. Because of differing family sizes the proportion of the labour force insured can, of course, be higher or lower than 98 per cent. The forecast value of HIP for 1967 was $2 million (1.010 per cent) higher than the actual value. For 1968 the equation produced a value $2 million (.667 per cent) lower than the actual value of HIP.

3-1-3-1 CORPORATION INCOME TAX ACCRUALS

In the federal tax sector we presented equations for total and federal corporation income tax accruals (section 2-1-2) and for the corresponding corporation tax collections (section 2-3-2-6). In this section we explain the provincial share of the corporation income tax on an accrual basis.

For total corporation tax accruals (TCA) it was possible to estimate a structural equation, since we have some knowledge of the structure of the tax as well as direct survey data on both total corporation profits and total corporation tax accruals. For provincial tax accruals, however, no suitable data are available.[60] This leaves us with a choice of two procedures. Since we have equations for total accruals and federal accruals as they appear in the *National Accounts*, we can obtain a series for provincial accruals as a residual. The series obtained in this way (PCAR) would correspond to the national accounts series on provincial accruals. Alternatively, we can create a series for provincial corporation accruals (PCA) based on our a priori information about the structure of the tax. Since in any one province there is only a single rate at which corporations pay provincial income tax and since this rate is applied directly to taxable corporate profits, it is fairly easy to calculate a series for provincial accruals according to the identity

$$PCA = PCTR\ (PCT) \qquad (1)$$

where PCT is taxable corporation profits generated from national accounts profits (PC) according to the relationship

[60]See p. 62.

$$PCT = .7591\ PC + 7.8168\ T + 125.1337 \qquad (2)$$

discussed fully in section 2-1-2.

PCTR is a weighted average of provincial corporation income tax rates. Between 1947 and 1951 eight of the provinces received a transfer from the federal government equal to 5 per cent of taxable corporation profits collected in their jurisdictions; Ontario and Quebec collected their own corporation profits tax at a rate of 7 per cent. In 1952 Ontario rented its corporation tax field to the federal government; Quebec continued to levy the 7 per cent rate until 1957. Under the terms of the Federal-Provincial Tax-Sharing Arrangements Act of 1957 the provinces received either a transfer of 9 per cent of taxable corporation profits accruing in a province, or an equal abatement of federal tax. Quebec and Ontario took the abatement. In addition Ontario levied a tax of 11 per cent on corporation profits. The remaining provinces took the transfer. From 1962 to 1967 all provinces except Quebec received a federal abatement of 9 per cent; Quebec received an extra 1 per cent in lieu of federal grants toward the support of universities in that province. During these abatement agreements both Ontario and Quebec had rates that were 2 per cent greater than the abatement, while Manitoba and Saskatchewan levied an extra 1 per cent. All the other provinces re-entered the corporation tax field under the 1962 agreements with rates equal to the 9 per cent abatement. In 1967 this abatement was increased to 10 per cent for all provinces. We weighted these provincial rates by the proportion of taxable corporation profits in each province to get PCTR. Values for PCTR are given in the data appendix.

The use of equation (1) to calculate PCA implies that federal accruals are given as a residual (FCAR).[61] Values for PCA and FCAR obtained in this way appear in the data appendix. It should be noted that these series differ from their national accounts counterparts. Since no direct observations underlie either set

[61]See footnote 33, p. 63.

of accrual series one's choice should depend on the plausibility of the derivation procedure and on how they perform in experiments that can discriminate between the two series.[62]

3-1-4-2 *GASOLINE TAXES*

In generating a gasoline tax equation we attempted to duplicate as closely as possible the tax calculations involved in arriving at the national accounts totals. Our basic equation is of the form

$$GTAX = RWG\ [(GAS/CARS)\ CARS] + RWD\ [(DO/CV)\ CV] \quad (1)$$

where:

GTAX = gasoline tax receipts, millions of dollars (DB 4062)

RWG = weighted average gasoline tax rate, dollars per gallon

RWD = weighted average diesel oil tax rate, dollars per gallon

GAS = taxable sales of gasoline, millions of gallons

DO = taxable sales of diesel oil, millions of gallons

CARS = stock of existing passenger automobiles, millions

CV = commercial vehicle registrations, millions

We constructed a gasoline tax model that leaves only CARS, RWG and RWD exogenous to the government sector. CARS is a Bank of Canada series generated by applying U.S. scrappage rates to Canadian new car sales. (See section 3-1-4-4, p. 126.) To calculate RWG we weighted the gasoline tax rate in a particular province by that province's share in taxable sales of gasoline. Similarly we calculated RWD by weighting the diesel oil tax rate by taxable

[62] One such experiment is suggested and reported in section 2-3-2-6, pp. 79-80.

sales of diesel oil. Sources for the data used to compile RWG and RWD, as well as the values of these weighted rates from 1959 to 1967, are given in the data appendix.

Our initial step in constructing the gasoline tax model was to generate a series that approximates CV and that can be readily forecast. The ratio CV/CARS declines nonlinearly from .40 in 1955 to .27 in 1967, and an annual regression similar to that used to forecast RMV in section 3-1-4-4 produced the following results:

1955-1966

$$CV = \underset{(11.69)}{.4494} + \underset{(23.00)}{.1861}\ CARS + \underset{(2.26)}{.0276}\ (CARS/T) \qquad (2)$$

SEE = .0145 $\qquad \bar{R}^2 = .990 \qquad$ D/W = 1.96

Quarterly values of CV used in equation (1) (denoted $\hat{CV}$) were generated by using the coefficients of equation (2) with the CARS series linearly interpolated from fourth quarter to fourth quarter and with T a step trend, equal to 1 in all quarters of 1955, 2 in all quarters of 1956, etc.

Next, we needed equations with which to forecast the ratios GAS/CARS and DO/CV. As one would expect the ratio GAS/CARS is almost constant over the time period, a factor that led to the following equation:

1Q59-4Q66

$$GAS/CARS = \underset{(85.15)}{169.1217}\ Q1 + \underset{(110.21)}{218.9053}\ Q2 + \underset{(125.93)}{250.1134}\ Q3 + \underset{(105.65)}{209.8438}\ Q4 \qquad (3)$$

SEE = 5.618 $\qquad \bar{R}^2 = .965 \qquad$ D/W = 1.77

These coefficients indicate the number of gallons of gasoline used per existing passenger car in each quarter. The quarterly pattern is predictable, with the highest average gasoline consumption occurring in the third quarter and the lowest in the first quarter. Although the coefficients seem high some upward bias is to be expected, since the denominator does not include all gasoline-burning vehicles.

The following equation, used to forecast the DO/CV ratio, proved to be slightly more difficult to construct than equation (3) because of the more intensive use of capital (CV), reflected in the rapid increase of the ratio over the 1962-1966 period.

1Q59-4Q66

$$DO/CV = \underset{(14.71)}{25.5043} + \underset{(1.97)}{.4305}\ (T1)(Q1) - \underset{(.74)}{.1582}\ (T1)(Q2)$$

$$+ \underset{(3.28)}{.6842}\ (T1)(Q3) + \underset{(3.61)}{.7271}\ (T1)(Q4) + \underset{(1.45)}{.0201}\ (T1)(T2)(Q1) \quad (4)$$

$$+ \underset{(4.47)}{.0572}\ (T1)(T2)(Q2) + \underset{(1.57)}{.0185}\ (T1)(T2)(Q3) + \underset{(.85)}{.0092}\ (T1)(T2)(Q4)$$

SEE = 3.419 $\bar{R}^2 = .887$ D/W = 1.66

T1 is a time trend equal to 1 in 1Q59 and continuing to 4Q66. T2 is a second time trend inserted to capture the increase in the ratio referred to above. T2 equals 1 in 1Q62 and continues to 4Q66. One would not expect this rate of increase to be maintained indefinitely. A close watch should therefore be kept on the forecast values of equation (4) to try to ascertain when T2 should stop rising.

Using the calculated values from equations (2), (3) and (4) along with the exogenous series for RWG, RWD and CARS, we estimated equation (1) with the following results:

1Q59-4Q66

$$GTAX = \underset{(105.11)}{1.0061} \left[RWG \left(\frac{\widehat{GAS}}{CARS}\right) CARS + RWD \left(\frac{\widehat{DO}}{CV}\right) \widehat{CV}\right] \quad (5)$$

SEE = 7.283 $\bar{R}^2$ = .958 D/W = 2.35

The fit of equation (5) is good with the coefficient not significantly different from 1. We used the model to forecast GTAX for 1967 and the forecast value exceeded the actual value by $5 million (.644 per cent).

The gasoline tax model presented above should be useful both for straight forecasting and for policy analysis using a macro-model, since the only exogenous variables are weighted tax rates and the number of cars existing. It appears that the weights are stable enough to allow RWG and RWD to be easily forecast. Since scrappage rates are assumed to be predetermined, one can derive the CARS series from any model that has a demand equation for automobiles.

3-1-4-7 RETAIL SALES TAXES

The retail sales tax (TRS, DB 4067) is simple to model, as a variable much like the actual tax base is explained within most macro-models. Our base is the sum of current-dollar consumer expenditures on durables (CD, DB 243) and nondurables (CND, DB 242). The weighted sales tax rate (RWS) is obtained by weighting each provincial rate by that province's proportion of retail trade. Since receipts from the retail sales tax are remitted by merchants and recorded by the governments as tax receipts in the month following the sales of the relevant goods,[63] both the weighted rate and the base series must be lagged one month. This can be done easily for the rate, because the weights are available from a monthly

[63]In Saskatchewan the taxes are remitted and recorded during the month following the end of the quarter in which goods are sold. In our estimation procedure, apart from the calculation of the weighted rate variable, no special allowance was made for this situation.

series.[64] For the base series, however, only quarterly data are available. Therefore, to approximate this lag we defined a new expenditure variable equal to two-thirds of actual expenditure in the current quarter plus one-third of expenditure in the preceding quarter. By making these adjustments to capture the lags in the collection and recording procedures, we obtained a closely-fitting equation without the aid of dummy seasonals. Since the national accounts expenditure series are adjusted to include the amount of retail sales tax collected, they must be divided by (1 + RWS) to obtain an estimate of the tax base. Thus our equation is:

1Q52-4Q66

$$TRS = \underset{(102.01)}{1.13952}\ RWS \left[\frac{1/3\ (CD + CND)_{t-1} + 2/3\ (CD + CND)}{1 + RWS}\right] \quad (1)$$

SEE = 8.942 $\qquad \bar{R}^2 = .985 \qquad$ D/W = 1.80

Many taxable sales of secondhand goods are only partially included[65] in CD + CND, and a special sales tax on tobacco and tobacco products is not covered in our weighted rate variable. Offsetting these items are a number of expenditures included in the base series, but not subject to tax. These comprise small purchases (those under 10, 15 or 20 cents depending on the province concerned), foodstuffs, and in some provinces, meals, drugs, some fuels and a variety of miscellaneous items. Our coefficient indicates that the receipts from the tobacco tax and the tax on sales of secondhand goods more than outweigh the effect of exempt items included in the expenditure series.

[64]Total retail trade by province was obtained from *Retail Trade* issued monthly by D.B.S., catalogue no. 63-005. Prior to 1961 our trade figures are based on the 1949 Standard Industrial Classification; from 1961 on they are based on the 1960 SIC. The weights do not appear to be affected by these revisions and, indeed, are remarkably stable over the 1952-1968 period.

[65]Sales of used consumer durables from the business sector to the personal sector are reduced by the estimated value of trade-in allowances received by consumers. See *National Accounts, Income and Expenditure, 1926-1956* issued by D.B.S., catalogue no. 13-502, p. 158.

Forecast values for TRS generated with equation (1) were \$5 million (.394 per cent) too high in 1967 and \$67 million (4.400 per cent) too low in 1968.

3-1-6-3 PROFITS OF GOVERNMENT ENTERPRISES

Provincial governments engage in a wide range of enterprises, from the sale of liquor to the operation of near-banks, with a heavy emphasis on utilities. Given the monopolistic nature of many of these enterprises and the relative stability of demand for the goods and services involved, one would expect trading profits (PGTP, DB 4072) to be less volatile than the profits of private corporations (PC, RDX 226) though more volatile than GNE (RDX 223). If this were so both variables should help to determine provincial government trading profits. Since there is no reason to expect these relationships to be independent of the seasons, one should allow some of the coefficients to take different values in each quarter.

We experimented with two basic specifications—the first using GNE and quarterly dummies proportional to PC, and the second using PC with seasonal differences proportional to GNE. The latter specification was marginally superior but in general the coefficients on PC were not significant. We present the two equations below, as well as a third one using GNE alone to explain PGTP.

1Q51-4Q65

$$PGTP = \underset{(13.30)}{.00780}\ GNE + \underset{(1.01)}{.00640}\ Q1\ (PC) - \underset{(.24)}{.00128}\ Q2\ (PC) \qquad (1)$$

$$- \underset{(1.36)}{.00863}\ Q3\ (PC) + \underset{(3.11)}{.01826}\ Q4\ (PC)$$

SEE = 3.666 $\qquad \bar{R}^2 = .969 \qquad$ D/W = 2.00

1Q51-4Q65

$$\begin{aligned} PGTP = {} & .00538\ PC + .00791\ Q1\ (GNE) + .00704\ Q2\ (GNE) \\ & \quad (.93) \qquad\qquad (14.51) \qquad\qquad\qquad (10.85) \\ & + .00649\ Q3\ (GNE) + .00911\ Q4\ (GNE) \\ & \quad (11.91) \qquad\qquad\quad (15.44) \end{aligned} \qquad (2)$$

SEE = 3.563 $\bar{R}^2$ = .970 D/W = 2.02

1Q51-4Q65

$$\begin{aligned} PGTP = {} & .00965\ GNE - .00124\ Q1\ (GNE) \\ & \quad (94.69) \qquad\quad (7.95) \\ & - .00201\ Q2\ (GNE) - .00266\ Q3\ (GNE) \\ & \quad (13.59) \qquad\qquad\quad (19.19) \end{aligned} \qquad (3)$$

SEE = 3.559 $\bar{R}^2$ = .970 D/W = 1.96

DATA APPENDIX

2-1-1-1 PERSONAL INCOME TAX

A. Total Personal Income Tax Collections (TP): Deductions at Source (TPS) and Other Collections (TPO)

Using *Model 1* and *Model 2* we attempted to estimate national accounts income tax collections, net of refunds (TP, DB 1368). Unfortunately this series is not divided into its constituent series, deductions at source (TPS) and other collections (TPO), which we require for *Model 1*. To obtain values for these variables we used the two budgetary series "Total Personal Income Taxes Collected, Gross Excluding Refunds, Deductions at Source" and "Total Personal Income Taxes Collected, Gross Excluding Refunds, Other Collections". These monthly series are available in the Research Department of the Bank of Canada. The aggregate of the two series differs from the national accounts series in two respects. The first difference is that Quebec's share of the personal income tax is not included in budgetary revenues since it is collected directly by the province. The second, but minor, difference arises from the treatment of supplementary period revenues, received in April but recorded as March budgetary revenues. In the *National Accounts* these revenues are recorded in April. This latter difference does not affect annual revenues, but it makes the first- and second-quarter amounts different under the two accounting procedures.

To reconcile the budgetary series with the national accounts series we calculated the difference between the two [TP - (TPS + TPO)], and allocated this difference to TPS and TPO in the proportions

$$TPS/(TPS + TPO)$$

$$\text{and } TPO/(TPS + TPO)$$

respectively. After spreading the difference in this way the series used in equations (1.2) and (1.4) are:

DEDUCTIONS AT SOURCE, NET OF REFUNDS (TPS)
(Millions of dollars)

	1Q	2Q	3Q	4Q
1950	80	42	85	96
1951	108	91	152	178
1952	189	143	208	237
1953	226	161	226	240
1954	223	148	247	261
1955	235	152	258	267
1956	256	175	309	328
1957	310	226	347	364
1958	304	150	319	336
1959	295	204	377	401
1960	342	278	412	426
1961	360	291	442	462
1962	399	334	486	513
1963	451	325	513	567
1964	504	427	625	664
1965	611	477	687	733
1966	736	521	818	919
1967	1,027	606	1,045	1,157
1968	1,227	752	1,232	1,418

OTHER COLLECTIONS, NET OF REFUNDS (TPO)
(Millions of dollars)

	1Q	2Q	3Q	4Q
1950	64	142	52	51
1951	71	177	57	56
1952	77	189	71	63
1953	86	220	65	63
1954	75	211	67	64
1955	75	184	64	62
1956	75	216	73	64
1957	83	219	79	65
1958	76	225	75	70
1959	80	240	77	70
1960	87	273	88	73
1961	94	295	98	83
1962	104	293	101	86
1963	105	321	115	90
1964	120	374	131	112
1965	135	437	143	132
1966	165	447	163	134
1967	187	561	179	143
1968	214	664	220	195

B. Exogenous Spreading Ratios (N_i, W_i, NW_i, Y_i)

Data used to calculate the N_i, W_i, NW_i and Y_i ratios are obtained from tables in the Department of National Revenue's annual publication, *Taxation Statistics, Part One (Individuals)*. The values of the ratios presented in the following tables are annual values. When these ratios are used in the tax models a quarterly pattern is imparted to them by means of simple linear interpolation from fourth quarter to fourth quarter.

PROPORTION OF TOTAL TAX RETURNS FILED IN EACH INCOME CLASS (N_i)

($N_i = NT_i/NT$)

	N1	N2	N3	N4
1950	.806	.150	.032	.011
1951	.731	.213	.043	.012
1952	.677	.258	.052	.014
1953	.644	.280	.062	.014
1954	.642	.280	.064	.014
1955	.620	.292	.073	.016
1956	.577	.312	.095	.017
1957	.557	.315	.110	.018
1958	.529	.327	.122	.021
1959	.511	.327	.140	.022
1960	.496	.324	.156	.024
1961	.485	.318	.170	.027
1962	.461	.318	.191	.029
1963	.454	.308	.206	.032
1964	.431	.300	.233	.036
1965	.414	.285	.260	.041
1966	.394	.272	.284	.050
1967	.377	.256	.306	.061

PROPORTION OF ASSESSED WAGE INCOME IN EACH INCOME CLASS (W_i)
($W_i = WAS_i/WAS$)

	W1	W2	W3	W4
1950	.647	.246	.065	.042
1951	.538	.331	.085	.045
1952	.462	.388	.103	.046
1953	.418	.412	.124	.047
1954	.404	.413	.132	.051
1955	.378	.421	.149	.052
1956	.332	.428	.188	.053
1957	.305	.419	.217	.060
1958	.279	.420	.234	.067
1959	.257	.407	.266	.070
1960	.238	.393	.293	.076
1961	.225	.378	.314	.083
1962	.205	.362	.345	.088
1963	.197	.343	.365	.096
1964	.177	.319	.400	.104
1965	.160	.289	.435	.116
1966	.138	.259	.464	.139
1967	.121	.231	.484	.164

PROPORTION OF ASSESSED NONWAGE PERSONAL INCOME IN EACH INCOME CLASS (NW_i)
($NW_i = NWAS_i/NWAS$)

	NW1	NW2	NW3	NW4
1950	.357	.178	.194	.271
1951	.317	.197	.208	.278
1952	.300	.201	.213	.286
1953	.286	.206	.225	.284
1954	.291	.195	.212	.302
1955	.270	.191	.214	.325
1956	.253	.195	.232	.321
1957	.243	.194	.237	.326
1958	.223	.206	.242	.330
1959	.213	.208	.242	.337
1960	.209	.209	.242	.340
1961	.201	.201	.247	.352
1962	.181	.205	.252	.363
1963	.179	.196	.257	.368
1964	.162	.188	.263	.388
1965	.147	.176	.267	.410
1966	.147	.182	.271	.400
1967	.140	.163	.264	.433

PROPORTION OF ASSESSED INCOME IN EACH INCOME CLASS (Y_i)
($Y_i = YAS_i/YAS$)

	Y1	Y2	Y3	Y4
1950	.588	.232	.091	.089
1951	.495	.305	.110	.091
1952	.431	.352	.124	.092
1953	.393	.373	.143	.091
1954	.384	.375	.146	.095
1955	.359	.381	.161	.100
1956	.318	.388	.196	.099
1957	.294	.382	.220	.104
1958	.269	.383	.235	.113
1959	.249	.373	.262	.116
1960	.233	.362	.285	.121
1961	.221	.347	.302	.129
1962	.201	.335	.329	.134
1963	.194	.319	.347	.141
1964	.175	.297	.376	.152
1965	.158	.270	.407	.165
1966	.140	.245	.429	.186
1967	.125	.219	.445	.211

C. Weighted Average Personal Income Tax Rates

The average tax rates for the i^{th} income class are found by calculating the tax payable (using the current rate schedule) on the mean taxable income in j groups of the i^{th} class. The average group tax rate is then the ratio of tax payable to mean taxable income in that group. To obtain a class weighted average (RW_i), group averages are weighted by the ratio of assessed income taxable in the group (that is, total assessed income of those filing returns who did pay some tax) to assessed income taxable in the class concerned and summed over all groups. Thus

$$RW_{it} = \sum_{j=1}^{n} [(T_{jit}/\overline{YT}_{jit})(YAST_{jit}/YAST_{it})] \quad (i=1,2,3,4)$$

where:

RW_{it} = weighted average personal income tax rate, class i, year t.

T_{jit} = tax on mean taxable income in group j, class i, year t, calculated by using the rate schedule applicable in year t.

$\overline{YT}_{jit}$ = mean taxable income in group j, class i, year t.

$YAST_{jit}$ = total assessed income taxable in group j, class i, year t.

$$YAST_{it} = \sum_{j=1}^{n} YAST_{jit}$$; total assessed income taxable, class i, year t.

Calculating T_{jit} using income groups within the four income classes enabled us to capture most of the progression in the rate schedule. The larger is n (that is, the more groups employed in any one class) the more precise the estimate of RW_i. In this calculation we used $1,000 groups up to the $10,000 level of assessed income,[66] and $5,000 groups up to the $25,000 level. The remaining incomes we divided into two groups, $25,000-$50,000 and over $50,000. Hence n = 2 in Class 1 and Class 2, and n = 5 in Class 3 and Class 4.

The rates do not include provincial levies in excess of the standard federal rates. Amounts would not be great, however, since only Quebec had higher rates between 1954 and 1961, and following the 1962 taxation agreements only Manitoba and Saskatchewan levied an extra tax on personal income.[67]

Annual values for RW_i are given in the following table. The annual value is used in each quarter of the year.

[66]Except for assessed income between zero and $2,000, which we treated as one group.

[67]Saskatchewan levied a 6 per cent surcharge from 1962 to 1965, then a 5 per cent surcharge from 1966 to 1968. In Manitoba the surcharge was 6 per cent from 1962 to 1964, and 5 per cent from 1965 to 1968. Quebec introduced a 6 per cent surcharge in 1968.

WEIGHTED AVERAGE PERSONAL INCOME TAX RATES (RW_i)
(Percentages)

	RW1	RW2	RW3	RW4	Overall Weighted Average
1950	15.0	15.6	18.0	30.8	17.3
1951	16.5	17.2	19.8	33.9	19.0
1952	18.5	19.3	21.6	35.7	21.1
1953	18.0	18.8	20.6	32.8	20.3
1954	17.0	17.7	19.3	30.9	19.2
1955	16.0	16.8	18.3	30.0	18.3
1956	15.0	15.9	17.3	28.8	17.4
1957	15.0	15.9	17.3	28.3	17.4
1958	13.0	14.2	16.4	27.8	16.3
1959	13.5	14.8	17.0	28.5	16.9
1960	14.1	15.3	17.6	29.2	17.7
1961	14.0	15.4	17.6	29.0	17.8
1962	14.0	15.4	17.5	28.5	17.9
1963	14.1	15.5	17.6	28.2	18.0
1964	15.2	16.6	18.5	28.7	19.1
1965	14.7	16.1	17.8	27.5	18.7
1966	13.6	15.7	17.7	27.2	18.6
1967	13.3	16.0	19.3	28.6	20.1
1968(E)*	13.3	16.1	19.6	29.3	20.6
1969(E)	15.3	18.1	21.6	30.3	22.5

*(E) = a forecast estimate

D. Average Utilized Exemptions and Deductions for Each Income Class

The average level of exemptions and deductions per tax return in each income class ($\overline{EX_i}$) is exogenous to both income tax models. It is possible to estimate these series within the models, but a number of experiments with various types of equations suggested this could not be done simply enough to justify inclusion in our initial specification. The series is merely EX_i/NT_i—total class exemptions and deductions per taxpayer in the class. Annual values are as follows:

AVERAGE UTILIZED EXEMPTIONS AND DEDUCTIONS ($\overline{EX}_i$)
(Dollars)

	$\overline{UEX1}$*	$\overline{EX2}$	$\overline{EX3}$	$\overline{EX4}$	Overall Weighted Average
1950	1,404	2,256	2,397	2,712	1,579
1951	1,361	2,232	2,429	2,730	1,609
1952	1,325	2,203	2,438	2,915	1,631
1953	1,294	2,180	2,461	2,986	1,638
1954	1,272	2,200	2,476	3,050	1,634
1955	1,260	2,177	2,463	3,154	1,645
1956	1,250	2,140	2,450	2,962	1,669
1957	1,262	2,170	2,499	3,118	1,718
1958	1,282	2,301	2,668	3,360	1,828
1959	1,259	2,262	2,666	3,366	1,831
1960	1,243	2,240	2,654	3,421	1,839
1961	1,243	2,217	2,659	3,450	1,854
1962	1,218	2,237	2,698	3,480	1,892
1963	1,213	2,207	2,698	3,553	1,900
1964	1,191	2,143	2,670	3,605	1,908
1965	1,152	2,068	2,628	3,570	1,896
1966	1,142	2,017	2,614	3,643	1,922
1967	1,111	1,913	2,524	3,649	1,904

*Average exemptions in Class 1 are adjusted for the unutilized part of nontaxpayers' total exemptions and deductions.

In Class 1 ($0-$3,000) we must adjust $\overline{EX1}$ to take into account the nontaxable returns (that is, returns on which the amount of eligible exemptions and deductions exceeds assessed income). Here the only exemptions actually utilized are the amounts necessary to offset reported income; any excess over these amounts lapses. Average exemptions in Class 1 therefore must be adjusted downward. To arrive at the adjustment factor we calculated PE1, the proportion of utilized exemptions in Class 1. PE1 can be derived from data in the *Taxation Statistics*. It is equal to the ratio of utilized exemptions——total exemptions claimed by taxpayers plus total assessed income that is nontaxable——to total exemptions, taxable plus nontaxable. Thus

$$PE1 = \frac{EX1\ (\text{taxable}) + YAS1\ (\text{nontaxable})}{EX1\ (\text{taxable}) + EX1\ \ (\text{nontaxable})}$$

The average utilized exemption series, $\overline{UEX1}$, is then equal to

PE1 (EX1/NT1)

No utilization adjustments are needed in the other classes as most nontaxable returns fall in Class 1. Therefore $PE_i = 1$, $i = 2, 3, 4$.

CALCULATED VALUES FOR PE1

1950	.852	1959	.807
1951	.855	1960	.799
1952	.851	1961	.800
1953	.844	1962	.795
1954	.825	1963	.800
1955	.824	1964	.803
1956	.840	1965	.794
1957	.826	1966	.788
1958	.810	1967	.779

To obtain $\overline{WEX_i}$, $\overline{NEX_i}$, and $\overline{YEX_i}$, which are the quarterly values of the annual $\overline{EX_i}$ series, and which, in turn, are used in the accrual equations of *Model 1* and *Model 2*, $\overline{EX_i}$ is spread by a ratio of estimated quarterly assessed income to annual assessed income. For the wage equation (1.1) $\hat{WAS}$ would be used to derive the spreading ratio, while in equations (1.3) and (2.1) $N\hat{WAS}$ and $\hat{YAS}$ would be used, respectively. The ratios are, in effect, quarterly proportions giving a quarterly pattern to an annual value. $\overline{EX_i}$, therefore, is made to vary quarterly in proportion to the relevant income variations. These spreading ratios are given below.

RATIO OF QUARTERLY $\hat{WAS}$ TO ANNUAL $\hat{WAS}$

	1Q	2Q	3Q	4Q
1950	.233	.243	.256	.268
1951	.231	.245	.258	.266
1952	.238	.243	.255	.264
1953	.240	.248	.256	.255
1954	.240	.247	.256	.257
1955	.236	.246	.258	.260
1956	.230	.247	.261	.262
1957	.236	.249	.260	.255
1958	.238	.251	.257	.254
1959	.239	.250	.255	.256
1960	.239	.251	.257	.253
1961	.237	.250	.258	.256
1962	.237	.250	.257	.255
1963	.237	.249	.257	.258
1964	.235	.248	.258	.259
1965	.232	.247	.258	.263
1966	.234	.247	.258	.261
1967	.236	.248	.257	.259
1968	.233	.247	.258	.262

RATIO OF QUARTERLY $\hat{NWAS}$ TO ANNUAL $\hat{NWAS}$

	1Q	2Q	3Q	4Q
1950	.185	.208	.376	.232
1951	.171	.235	.374	.221
1952	.183	.195	.391	.231
1953	.183	.213	.386	.218
1954	.206	.229	.319	.246
1955	.200	.229	.347	.224
1956	.193	.221	.356	.229
1957	.210	.228	.317	.245
1958	.216	.233	.310	.241
1959	.217	.238	.311	.234
1960	.224	.228	.308	.240
1961	.232	.235	.283	.250
1962	.219	.237	.307	.238
1963	.210	.234	.317	.239
1964	.230	.229	.297	.243
1965	.231	.220	.305	.244
1966	.227	.217	.313	.243
1967	.236	.227	.289	.248
1968	.220	.234	.298	.248

RATIO OF QUARTERLY $\hat{YAS}$ TO ANNUAL $\hat{YAS}$

	1Q	2Q	3Q	4Q
1950	.205	.226	.314	.255
1951	.199	.241	.312	.248
1952	.211	.222	.316	.251
1953	.214	.234	.312	.240
1954	.224	.240	.283	.253
1955	.217	.239	.296	.248
1956	.212	.237	.300	.251
1957	.224	.242	.283	.250
1958	.227	.244	.280	.249
1959	.227	.246	.279	.248
1960	.231	.242	.279	.248
1961	.232	.244	.270	.255
1962	.227	.245	.279	.249
1963	.223	.243	.282	.251
1964	.231	.241	.274	.254
1965	.230	.237	.277	.257
1966	.230	.236	.279	.255
1967	.235	.240	.270	.255
1968	.227	.242	.274	.257

2-1-2 CORPORATION INCOME TAX ACCRUALS

Data used for the calculation of non-databank variables in this section are obtained from *Taxation Statistics, Part Two (Corporations),*[68] where statistics are presented on a fiscal year basis. However recent issues of this publication include an historical table showing most of the data on a calendar year basis. Wherever possible we have used calendar year data.

A. Weighted Marginal (Federal and Provincial) Corporation Tax Rate (RPC1)

There are two rates of tax at the federal level. The low rate (currently 21 per cent) is paid on taxable profits up to a low-rate maximum ($35,000 from 1961 to the present), and the high rate (currently 50 per cent) is paid on the remaining profits. To calculate RPC1 we assumed that all firms paying tax at the high rate (HR) on part of their taxable income pay this rate on

[68]See footnote 29, p. 58.

all their taxable income. We then have

$$RPC1U = \frac{(HR)\ PCTH + (LR)\ PCTL}{PCT}$$

where:

RPC1U = weighted marginal rate, unadjusted for provincial levies

HR = high rate of taxation

LR = low rate of taxation

PCTH = taxable corporation profits of firms that pay at the high rate

PCTL = taxable corporation profits of firms that pay at the low rate

PCT = PCTH + PCTL——total taxable corporation profits

No data are available on the disaggregation of taxable corporation profits between high-rate and low-rate firms, but we do have observations on current-year profits (CYP) for these two categories of firms from Table 6 of *Taxation Statistics, Part Two*. We also know that

$$PCT = CYP - PYL$$

where PYL is prior-year losses. Analysis of Table 2 in *Taxation Statistics, Part Two* reveals that the ratio of prior-year losses of high-rate to low-rate firms has been fairly constant at .25. We thus calculate PCTH and PCTL as

$$PCTH = CYPH - .20\ PYL$$

$$PCTL = CYPL - .80\ PYL$$

where CYP = CYPH + CYPL.

Finally, to arrive at RPC1 we must adjust RPC1U to reflect any provincial levies that exceed allowable provincial tax credits.

From 1957 to 1967 Ontario levied corporation taxes at a rate 2 per cent greater than the allowed abatement. Manitoba and Saskatchewan levied an extra 1 per cent from 1962 to 1967 while Quebec levied an additional 2 per cent. In 1967 Newfoundland levied an additional 1 per cent. These extra levies are weighted by the proportion of total taxable corporation profits in each of the respective provinces (Table 7, *Taxation Statistics, Part Two*), and the weighted provincial excess (WPE) is added to RPC1U to give RPC1. Data values from 1952 to 1967 appear below. The annual rate is assumed to hold in each quarter.

WEIGHTED MARGINAL CORPORATION INCOME TAX RATES

	RPC1U	WPE	RPC1
1952	.515	.015	.530
1953	.476	.015	.491
1954	.472	.016	.488
1955	.451	.016	.467
1956	.448	.016	.464
1957	.444	.009	.453
1958	.444	.009	.453
1959	.473	.009	.482
1960	.472	.009	.481
1961	.462	.009	.471
1962	.465	.015	.480
1963	.470	.015	.485
1964	.469	.015	.484
1965(E)*	.469	.015	.484
1966(E)	.469	.015	.484
1967(E)	.469	.015	.484

*(E) = a forecast estimate

As pointed out in the text, the use of RPC1 overstates total accruals by assuming that firms with taxable profits in excess of the low-rate maximum (LRM) pay the high rate on all their profits. This overestimate would be equal to

$$D3 = (HR - LR)(LRM)(NHRF)$$

That is, RPC1 would overestimate by an amount equal to the difference between the low rate actually applied to the low-rate portion of a high-rate firm's profits (LR) and the rate we apply to it (HR), times the allowable profit per firm that is taxable at the low rate (LRM), times the number of firms that pay tax at both rates (NHRF). This amount (D3) is added to the dependent variable (TCA) to offset the bias in RPC1. Values for D3 are given below. It is assumed that the annual value is spread evenly over the four quarters of the year.

APPROXIMATE OVERESTIMATE OF TCA DUE TO USE OF RPC1 (D3)
(Millions of dollars)

1952	37.2	1960	77.3
1953	49.8	1961	74.9
1954	48.1	1962	81.7
1955	41.5	1963	97.6
1956	46.5	1964	101.8
1957	45.3	1965(E)*	106.6
1958	61.3	1966(E)	111.7
1959	81.1	1967(E)	116.7

*(E) = a forecast estimate

B. Weighted Average (Federal and Provincial) Corporation Tax Rate (RPC2)

To avoid having to use D3 in the accrual equation we created a weighted average high rate (WHR) applicable to firms that pay at the high rate (HR) on some portion of their profits. We have

$$WHR = \frac{LR\ (LRM)(NHRF) + HR\ [PCTH - (LRM)(NHRF)]}{PCTH}$$

where all symbols have been previously explained. Given WHR we proceed as before, with

$$RPC2U = \frac{WHR\ (PCTH) + LR\ (PCTL)}{PCT}$$

and RPC2 = RPC2U + WPE.

Data values for RPC2U, RPC2 and WPE are shown below. The annual value is assumed to hold in each quarter.

WEIGHTED AVERAGE CORPORATION INCOME TAX RATES

	RPC2U	WPE	RPC2
1952	.500	.015	.515
1953	.456	.015	.471
1954	.451	.016	.467
1955	.436	.016	.452
1956	.433	.016	.449
1957	.429	.009	.438
1958	.421	.009	.430
1959	.448	.009	.457
1960	.447	.009	.456
1961	.439	.009	.448
1962	.442	.015	.457
1963	.444	.015	.459
1964	.444	.015	.459
1965(E)*	.444	.015	.459
1966(E)	.444	.015	.459
1967(E)	.444	.015	.459

*(E) = a forecast estimate

C. Weighted Marginal and Average Federal Corporation Income Tax Rates (FRPC1 and FRPC2)

These may be derived from RPC1, RPC2, and PCTR, the weighted average provincial corporation income tax rate explained in section 3-1-3-1 of this appendix. We have

$$FRPC1 = RPC1 - PCTR$$

and

$$FRPC2 = RPC2 - PCTR$$

Data values are presented below. Once again the annual value is assumed to hold in each quarter.

WEIGHTED FEDERAL CORPORATION INCOME TAX RATES

	FRPC1	FRPC2
1952	.515	.500
1953	.476	.456
1954	.472	.451
1955	.451	.436
1956	.448	.433
1957	.403	.388
1958	.403	.380
1959	.432	.407
1960	.431	.406
1961	.421	.398
1962	.372	.349
1963	.377	.351
1964	.376	.351
1965(E)*	.376	.351
1966(E)	.376	.351
1967(E)	.369	.344

*(E) = a forecast estimate

2-1-4-2 MANUFACTURERS' SALES TAX

The manufacturers' sales tax equation utilizes three tax rates: the basic sales tax rate applicable to durable and nondurable consumption expenditure (RSC), the tax rate applicable to machinery and equipment (RSIM), and the tax rate applicable to construction materials and building supplies (RSIR). Values for these rates, adjusted for within-quarter changes, are shown below expressed in decimal form.

BASIC SALES TAX RATE (RSC)

	1Q	2Q	3Q	4Q
1955	.10	.10	.10	.10
1956	.10	.10	.10	.10
1957	.10	.10	.10	.10
1958	.10	.10	.10	.10
1959	.10	.11	.11	.11
1960	.11	.11	.11	.11
1961	.11	.11	.11	.11
1962	.11	.11	.11	.11
1963	.11	.11	.11	.11
1964	.11	.11	.11	.11
1965	.11	.11	.11	.11
1966	.11	.11	.11	.11
1967	.12	.12	.12	.12
1968	.12	.12	.12	.12

SALES TAX RATE ON MACHINERY AND EQUIPMENT (RS1M)

	1Q	2Q	3Q	4Q
1955	-	-	-	-
1956	-	-	-	-
1957	-	-	-	-
1958	-	-	-	-
1959	-	-	-	-
1960	-	-	-	-
1961	-	-	-	-
1962	-	-	-	-
1963	-	.006	.04	.04
1964	.04	.08	.08	.08
1965	.11	.11	.11	.11
1966	.11	.11	.11	.11
1967	.11	.04	-	-
1968	-	-	-	-

SALES TAX RATE ON CONSTRUCTION MATERIALS AND BUILDING SUPPLIES (RS1R)

	1Q	2Q	3Q	4Q
1955	-	-	-	-
1956	-	-	-	-
1957	-	-	-	-
1958	-	-	-	-
1959	-	-	-	-
1960	-	-	-	-
1961	-	-	-	-
1962	-	-	-	-
1963	-	.006	.04	.04
1964	.04	.08	.08	.08
1965	.11	.11	.11	.11
1966	.11	.11	.11	.11
1967	.11	.11	.11	.11
1968	.11	.11	.11	.11

2-2-4 INTEREST ON THE FEDERAL PUBLIC DEBT

In the equation explaining interest on the federal public debt (IFD) we make use of interest liabilities series for direct market securities (DMSL) and Canada Savings Bonds (CSBL). The methods used to construct these variables, as well as the data sources, are indicated in the text. Below, we present the quarterly time series for DMSL and CSBL, calculated from 1955 to 1967.

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INTEREST LIABILITIES FOR DIRECT MARKET SECURITIES (DMSL)
(Millions of dollars)

	1Q	2Q	3Q	4Q
1955	309.45	309.45	309.95	309.95
1956	308.55	307.65	291.55	298.76
1957	295.45	294.33	292.07	299.64
1958	303.15	313.38	334.65	383.40
1959	392.51	390.21	389.17	396.04
1960	391.34	410.24	412.96	406.63
1961	406.37	416.91	416.01	426.34
1962	426.69	421.89	425.63	437.67
1963	444.92	458.77	460.37	471.94
1964	474.09	475.19	479.72	487.43
1965	478.90	478.62	488.65	483.15
1966	488.77	492.99	495.11	508.89
1967	521.11	536.64	545.91	544.76

INTEREST LIABILITIES FOR CANADA SAVINGS BONDS (CSBL)
(Millions of dollars)

	1Q	2Q	3Q	4Q
1955	.34	.60	.86	65.44
1956	.75	1.31	1.88	73.22
1957	1.17	2.04	2.92	73.47
1958	.59	1.03	1.48	80.49
1959	1.22	2.13	3.05	88.47
1960	34.44	33.99	33.54	33.41
1961	37.40	36.96	36.52	37.81
1962	43.60	42.48	41.36	42.40
1963	52.54	51.95	51.36	52.04
1964	58.64	58.10	57.56	58.23
1965	64.23	63.56	62.89	63.84
1966	68.73	66.39	64.05	61.96
1967	74.00	72.54	71.08	70.13

3-1-1-1 PERSONAL INCOME TAXES

A. Weighted Average Basic Tax Rates (BRW_i)

These rates are analogous to the RW_i of section 2-1-1-1 in all respects except that basic tax payable is substituted for total tax payable in our calculations. Thus

$$BRW_{it} = \sum_{j=1}^{n} [(BT_{jit}/\overline{YT}_{jit})(YAST_{jit}/YAST_{it})] \quad (i=1,2,3,4)$$

where BT_{jit} is the basic tax payable on mean taxable income in group j, class i, year t calculated by using the actual rate schedule applicable in year t. All other variables and subscripts have been previously discussed.[69] Values for BRW_i expressed in decimal form are given below. The annual value is assumed to hold in each quarter.

WEIGHTED AVERAGE BASIC TAX RATES

	BRW1	BRW2	BRW3	BRW4	Overall Weighted Average
1962	.110	.124	.151	.278	.154
1963	.112	.125	.152	.275	.156
1964	.112	.126	.153	.277	.160
1965	.112	.127	.154	.276	.163
1966	.112	.128	.156	.271	.167

B. Total Provincial Effective Rates (TPER1 and TPER2)

TPER1 is the total provincial effective rate for all provinces except Quebec. It is calculated by weighting abatements and provincial surtaxes by the proportion of total tax payable originating in each province. The rates are given in Table 13 on page 122 and the weights are derived from Table 1 of *Taxation Statistics, Part One (Individuals)*. But since Quebec collects its own tax, only that portion of Quebec residents' tax liabilities that is payable to the federal government is shown in Table 1. Thus, before we calculate the weights, we must inflate this number by

$$100/[100 - ARQ]$$

where ARQ is the abatement to Quebec. TPER2 is the total provincial effective rate for Quebec. It is calculated in the same way as TPER1. Values for TPER1 and TPER2 are given below. Once again the annual value is assumed to hold in each quarter.

[69]See this appendix, section 2-1-1-1, pp. 142-144.

TOTAL PROVINCIAL EFFECTIVE RATES,
QUEBEC (TPER2) AND ALL OTHER PROVINCES (TPER1)

	TPER1	TPER2
1962	.126	.039
1963	.133	.042
1964	.140	.045
1965	.159	.116
1966	.181	.123

3-1-1-2 MOTOR VEHICLE LICENCES AND PERMITS, PERSONS
3-1-4-4 MOTOR VEHICLE LICENCES AND PERMITS, BUSINESSES

The data we used to calculate the weighted average rate of licence fee per registered motor vehicle (MVPR) were taken from *The Motor Vehicle, Parts II, III and IV.*[70] The appropriate revenue series for each province was divided by the total number of motor vehicles registered in that province to get a provincial average rate. These rates were weighted by the proportion of total net (i.e. taxable) gasoline sales in each province and the weighted rates were summed over the ten provinces to get MVPR. Values of MVPR from 1950 to 1967 are presented in the following table. The annual rate is assumed to hold in each quarter.

WEIGHTED AVERAGE RATE OF LICENCE FEE (MVPR)
(Dollars per registered motor vehicle)

1950	25.3	1959	30.6
1951	24.9	1960	31.0
1952	24.7	1961	31.2
1953	24.3	1962	30.9
1954	24.7	1963	32.6
1955	28.2	1964	32.9
1956	28.2	1965	34.4
1957	29.1	1966	34.0
1958	29.1	1967	34.5

[70]*The Motor Vehicle, Part II Motive Fuel Sales, The Motor Vehicle, Part III Registrations* and *The Motor Vehicle, Part IV Revenues* issued by D.B.S., catalogue nos. 53-218, 53-219 and 53-220, respectively.

3-1-1-3 HOSPITAL INSURANCE PREMIUMS

To calculate the weighted average hospital insurance premium rate (RWH) we initially obtained a weighted average rate for each of the three provinces concerned (Ontario, Manitoba and Saskatchewan). This calculation was made by weighting the family rate in each province by the proportion of married males in the provincial labour force and weighting the single rate by the proportion of single persons in the provincial labour force. The relevant rates are available from *Principal Taxes and Rates; Federal, Provincial and Selected Municipal Governments.*[71] Appropriate proportions for each province were obtained from 1961 census data[72] and assumed to be constant over the period.

The weighted rate for each province was then weighted by that province's share of the total labour force, available from the *Canadian Statistical Review,*[73] Table 20. Summing the results across the provinces yielded the following series:

WEIGHTED AVERAGE HOSPITAL INSURANCE PREMIUM RATE (RWH)
(Dollars per member of the labour force)

	1Q	2Q	3Q	4Q
1959	3.94	3.92	3.92	3.94
1960	3.98	3.96	3.92	3.94
1961	4.26	4.24	4.01	4.03
1962	4.03	4.02	4.00	4.01
1963	4.03	4.03	4.00	4.03
1964	3.97	3.97	5.71	5.72
1965	5.79	5.75	5.74	5.74
1966	5.77	5.77	5.74	5.75
1967	5.80	5.81	5.78	5.77
1968	5.82	9.26	9.23	9.28

[71]*Principal Taxes and Rates; Federal, Provincial and Selected Municipal Governments* issued by D.B.S., catalogue no. 68-201.

[72]*The Canadian Labour Force* special series issued by D.B.S., catalogue no. 99-522.

[73]*Canadian Statistical Review* issued monthly by D.B.S., catalogue no. 11-003.

3-1-3-1 CORPORATION INCOME TAX ACCRUALS

A. Weighted Average Provincial Corporation Tax Rate (PCTR)

PCTR is equal to the weighted average federal abatement to the provinces (WAAR) plus the weighted average of any additional levies imposed by provincial governments (WPE). The relevant rates are set out in the text and these were weighted by the share of taxable corporation profits in each province, obtained from *Taxation Statistics, Part Two (Corporations).* The relevant rates, expressed as decimals, are:

WEIGHTED AVERAGE PROVINCIAL CORPORATION INCOME TAX RATES

	WAAR	WPE	PCTR
1952	-	.015	.015
1953	-	.015	.015
1954	-	.016	.016
1955	-	.016	.016
1956	-	.016	.016
1957	.041	.009	.050
1958	.041	.009	.050
1959	.041	.009	.050
1960	.041	.009	.050
1961	.041	.009	.050
1962	.093	.015	.108
1963	.093	.015	.108
1964	.093	.015	.108
1965(E)*	.093	.015	.108
1966(E)	.093	.015	.108
1967(E)	.100	.015	.115

*(E) = a forecast estimate

Provincial corporate accruals generated as described in the text (PCA) are shown below.

PROVINCIAL CORPORATION INCOME TAX ACCRUALS (PCA)
(Millions of dollars)

	1Q	2Q	3Q	4Q
1952	9	10	10	10
1953	9	11	10	9
1954	8	10	10	10
1955	9	12	13	12
1956	11	14	14	13
1957	35	43	40	36
1958	33	40	41	42
1959	37	47	45	45
1960	39	45	44	41
1961	34	46	47	47
1962	87	110	106	108
1963	93	120	113	118
1964	106	134	125	135
1965	114	143	136	142
1966	121	146	127	140
1967	119	149	141	151

As explained in the text (section 2-1-2, part B), if this series is used for PCA the corresponding federal accrual series is FCAR, given below.

FEDERAL CORPORATION INCOME TAX ACCRUALS (FCAR)
(Millions of dollars)

	1Q	2Q	3Q	4Q
1952	288	368	361	328
1953	280	342	307	252
1954	228	278	279	259
1955	226	327	350	323
1956	280	376	371	334
1957	283	339	307	254
1958	241	298	325	295
1959	286	390	369	362
1960	310	380	357	328
1961	268	376	395	399
1962	275	346	335	343
1963	294	370	347	372
1964	333	417	392	411
1965	346	452	426	466
1966	342	480	432	465
1967	322	455	414	457

3-1-4-2 *GASOLINE TAXES*

A. Weighted Average Gasoline Tax Rate (RWG)

Gasoline tax rates are also obtained from *Principal Taxes and Rates; Federal, Provincial and Selected Municipal Governments.* These rates were weighted by the proportion of taxable gasoline sales in each province, available on a monthly basis in *The Motor Vehicle, Part II Motive Fuel Sales.* Summing across the provinces we obtained the following series:

WEIGHTED AVERAGE GASOLINE TAX RATE (RWG)
(Dollars per gallon)

	1Q	2Q	3Q	4Q
1959	.125	.125	.126	.126
1960	.125	.126	.126	.126
1961	.127	.133	.134	.134
1962	.134	.134	.134	.134
1963	.134	.139	.139	.139
1964	.143	.147	.147	.147
1965	.147	.151	.151	.151
1966	.151	.156	.155	.155
1967	.155	.155	.156	.156

B. Weighted Average Diesel Oil Tax Rate (RWD)

The calculation of this rate involved weighting each province's diesel oil tax rate by that province's share in net (i.e. taxable) diesel oil sales, and summing across the ten provinces. Sources used for the rates and weights were the same as those used to obtain the corresponding data for RWG. Statistics on sales of taxable diesel oil by province are available monthly only from 1962 on, and no figures are available for Nova Scotia prior to 1964. In order to obtain the data needed we first estimated an annual total for Nova Scotia for each year from 1959 to 1963, assuming that Nova Scotia's annual share of total diesel oil sales in each of those years was equal to its share in aggregate sales in 1964 and 1965 (that is, 1.4 per cent). The 1959-1961 annual totals for each province were then spread quarterly according to the 1962 quarterly distribution. The resulting RWD series is given below.

WEIGHTED AVERAGE DIESEL OIL TAX RATE (RWD)
(Dollars per gallon)

	1Q	2Q	3Q	4Q
1959	.141	.148	.142	.142
1960	.139	.145	.141	.141
1961	.139	.163	.155	.154
1962	.178	.178	.180	.180
1963	.180	.189	.189	.187
1964	.190	.194	.194	.195
1965	.195	.200	.201	.199
1966	.198	.204	.203	.201
1967	.201	.202	.203	.203

3-1-4-7 RETAIL SALES TAXES

The weighted sales tax rate (RWS) was obtained by weighting the provincial rates by each province's proportion of monthly retail trade. The value of total retail trade by province is available monthly in the *Canadian Statistical Review*, Table 85. Since tax receipts are received and recorded by the provinces with a lag of one month (one quarter in the case of Saskatchewan), the monthly retail trade figures used in the weighting have been lagged accordingly. The following series was calculated by using this procedure.

WEIGHTED AVERAGE RETAIL SALES TAX RATE (RWS)
(Percentages)

	1Q	2Q	3Q	4Q
1952	1.124	1.091	1.136	1.130
1953	1.129	1.093	1.144	1.138
1954	1.131	1.171	1.328	1.317
1955	1.308	1.269	1.313	1.311
1956	1.327	1.296	1.336	1.325
1957	1.337	1.315	1.332	1.318
1958	1.325	1.296	1.313	1.307
1959	1.415	1.397	1.425	1.409
1960	1.416	1.391	1.467	1.457
1961	1.463	1.458	2.399	3.189
1962	3.304	3.267	3.292	3.279
1963	3.302	3.284	3.308	3.294
1964	3.317	3.676	3.835	3.848
1965	3.835	3.810	3.798	3.803
1966	3.777	4.555	4.551	4.543
1967	4.606	5.238	5.403	5.402
1968	5.350	5.465	5.413	5.407

INDEX TO THE GOVERNMENT SECTOR OF THE CANADIAN ECONOMY

The first section of this index is reserved for quantitative forecasts, while in subsequent sections reference is made to the analytic framework and fitted equations from which the forecasts are derived.

Each item in the index is preceded by a code number. From section 2 on, the first digit in each code number refers to the level of government. Thus 2 refers to the federal government and 3 to the provinces and municipalities. The second digit indicates the nature of the item: 1 refers to a revenue item, 2 to an expenditure item or a transfer payment, and 3 to a change in an asset or liability account. The next digits indicate subclassifications.

[74]These will be disaggregated by department and by type of expenditure.

[75]The federal government announced on August 30, 1968 that no incentives would be provided for winter works projects in the winter of 1968-1969.

[76]On October 1, 1969 ECIC was succeeded by the Export Development Corporation which has wider powers than ECIC had and greater financial resources.

[77]On November 10, 1969 Banque Populaire, formerly Banque d'Économie de Québec (a Quebec savings bank), commenced operations as a chartered bank. Thus The Montreal City and District Savings Bank is now the only Quebec savings bank still in existence.

3-1-8 Insurance and pension accounts——provincial

3-1-8-1 Public service pension receipts
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3-2-3-5 Public service pensions
3-2-3-6 Grants to private non-commercial institutions
3-2-3-7 QPP payments
3-2-3-8 Miscellaneous transfers (including industrial employees vacations benefits)

3-2-4 Transfers to persons——municipal

3-2-4-1 Direct relief payments
3-2-4-2 Public service pensions
3-2-4-3 Grants to private non-commercial institutions
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3-3-2-2 Municipal non-cash asset accounts

3-3-2-2-1 Loans and advances
3-3-2-2-2 Holdings of investment accounts
3-3-2-2-3 All other financial assets and liabilities

3-3-3 Net changes in debt accounts—provincial and municipal

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3-3-3-1-1-1 Marketable bonds and treasury bills
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