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CANADIAN INVENTORY INVESTMENT

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by

R.G. Evans

See p. 37 Equation (2.9)

t values omitted in error on the coefficients of variables

Y_t^P, ΔY_t^P, UI_t and H_{t-1}

Equation should read:

1Q56 - 4Q65 (SOIV)

$$\Delta H_t = \underset{(3.83)}{603.846}\ Q_1 + \underset{(3.37)}{518.098}\ Q_2 + \underset{(0.46)}{53.381}\ Q_3 - \underset{(1.46)}{86.778}\ Q_4$$

$$+ \underset{(2.02)}{.135}\ Y_t^P - \underset{(0.22)}{.030}\ \Delta Y_t^P - \underset{(5.16)}{260.046}\ UI_t - \underset{(2.64)}{.120}\ H_{t-1} \qquad (2.9)$$

SEE = 76.7 $\bar{R}^2$ = .786 D/W = 2.01

CANADIAN INVENTORY INVESTMENT

R.G. Evans

This paper is a report on the research underlying the business inventory investment equations used in RDX1, the experimental aggregate model of the economy being developed at the Research Department of the Bank of Canada. The views expressed are the personal views of the author and no responsibility for them should be attributed to the Bank.

PREFACE

The experimental econometric model of the Canadian economy developed at the Bank of Canada Research Department, RDX1, has been built up from partial studies of the major sectors of the economy. Results of investigations of quarterly business inventory investment carried out as part of the study of business investment modelled in RDX1 are reported in this paper. Later results, primarily based on single-equation analysis completed in September of 1967, are also included. They were obtained from simultaneous reestimation of the basic equations with the whole-economy model. These consistent parameter estimates are largely in accordance with the conclusions derived from the earlier work.

In this study I attempt to model the behaviour of quarterly business inventory investment within a single equation linking sales proxy variables, expectational variables, and cost-of-finance variables. These finance variables are of primary interest, as they represent the channels through which inventory investment may be influenced by the policy instruments available to the various levels of government. Unfortunately the results of the study in this respect are negative, since no useful evidence of inventory responsiveness to cost-of-finance variables could be found. It is possible that these negative results are due to my focus on aggregate inventories; but the requirements of the simultaneous model, for which the equation was designed, largely dictated this focus. Some analysis of inventories broken down by industry division was attempted in the summer of 1966. However this line of approach was not very successful and was broken off before completion.

The research recorded in this paper was all carried out at the Bank of Canada, and was aided greatly by the environment of continuous consultation which characterizes the econometric model project. Assistance was received with each phase of the study from so many members of the Research Department that it now seems impossible to acknowledge all this help. Where would one start, or stop? In particular, however, John Helliwell of the University of British Columbia, a Bank of Canada Research Consultant, was a constant source of inspiration and advice while Ian Stewart, as production manager for the whole model, was tirelessly patient with the endless tinkering that went into the equation. Moreover during my absence from the Bank they conducted part of the research here described. To them, and to the others who were involved, should go much of the credit for whatever this study has contributed.

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CANADIAN INVENTORY INVESTMENT

1. The Framework of the Study

a) The General Problem

In attempting to specify and estimate a single equation determining Canadian inventory investment on a quarterly basis, I am reminded of those intrepid investigators of an earlier day who set out to hunt the snark. They also were engaged in a venture whose importance was unquestioned, with a quarry whose characteristics were but dimly descried. And like them, I run the risk of eventually finding something looking very like a snark yet being in fact disastrously different in its structural properties. In that case I can only "softly and suddenly vanish away". It is not recorded what happens in the sequel.

The importance of the search is indeed unquestioned. The investment sector of the economy lies at the heart of any econometric model, being both the main source of variance in the behaviour of the private sector and the focus for much of traditional public policy. From the policy point of view, the behaviour of the investment sector is fundamental to the attainment both of short-run stability in the levels of income and employment and of long-run growth through the expansion and technological improvement of productive capacity. In the long-run context, of course, inventory investment is of little consequence compared to investment in plant, equipment, and housing stock. Adding nothing to productive capacity, the swings of inventory investment tend to cancel out over time. But for short-run predictions over several quarters or for the analysis of the short-run impact of policy changes, the behaviour of inventory investment becomes crucial because of its volatility and the substantial size of inventory movements. During the postwar period there have been several quarters in which inventory investment has run at 5 per cent or more of total Gross National Expenditure (GNE), and others in which disinvestment has been between 2 per cent and 3 per cent of GNE. Such sharp inventory movements can account, or more than account, for total GNE movements between quarters. In recent years inventory movements relative to GNE have been somewhat more moderate, but the absolute swings have often run between a quarter- and a half-billion dollars in constant 1957 dollars per quarter.

In addition to the speed and size of inventory movements, inventory investment has generated a particular concern in its potential as a focus for macroeconomic policy. The hypothesized importance of inventory investment has diminished since the days when R.G. Hawtrey based a whole theory of business cycles on the reaction of trade inventories to changes in the cost of credit. (See Haberler [11] pp. 14-28.) But it is still true that holding inventory ties up capital, and capital costs money whether borrowed or owned. If the inventory-holder borrows to finance his holdings, monetary policy may operate through the cost of funds to make this borrowing more expensive, or through the availability of funds to make further accumulation impossible. If the inventory-holder merely ties up his own capital, rising interest rates will increase the opportunity cost of such holdings, and presumably will discourage them. The mechanism can also be expected to work in reverse, subject to the usual question of whether cheap finance actually encourages borrowing as much as dear finance discourages it. If monetary policy can thus manipulate the optimum level of each holder's inventory, and if his decisions are sensitive to this optimum level, then inventory investment provides a particularly direct means by which monetary policy can influence the levels of investment, income and employment almost independently of the longer-run rates of capacity accumulation and growth, and without the long lags characteristic of the reactions of fixed investment. The appeal of such a rapid and direct channel of policy control is obvious — if it works.

Thus the ideal inventory investment equation, which we should like to build into an aggregate macroeconomic model of the Canadian economy, would provide a good fit to the sharp inventory swings observed over the postwar period, plus a plausible and temporally stable structural form isolating behavioural rather than associative relationships. In addition, an ideal equation should contain in its structure a set of financial variables indicating the extent to which policy makers can influence the level of investment. Unfortunately no one has yet found such an equation — indeed it may very well not exist. Certainly previous attempts to fit a single equation to Canadian data, usually in the context of an aggregate econometric model, have not met with great success.[1] The fits have been weak and the implied structures at best plausible but not immediately convincing. In general the investigators have been dissatisfied, but, since the primary interest and focus of their work was in other sectors of the economy, they have simply shored up the inventory investment section of the model and concentrated their efforts elsewhere. In the United States considerable work has been done usually with quite elaborate theoretical structures.[2] The reduced forms from such models have tended to be similar, and have yielded good fits but unstable implied structures. Different investigators, working with roughly similar estimating equations, have produced widely divergent estimates of the structural parameters. (See Mack [17] pp. 224-231.) Much of the problem seems to stem from efforts to combine in one equation hypothesized processes of expectations formation and of investment based on expectations. Results may improve with the use of the new data on business expectations now becoming available. (See Foss [9].) But such Canadian data are not presently available nor will they be in the forseeable future, so for us this improvement is cold comfort.

Another line of attack, which may have considerable promise in dealing with this problem, has recently been opened by T.J. Courchene. (See [3] and [4].) He challenges the established assumption that aggregation in inventory analysis does no harm and may be positively beneficial by producing some very interesting results from the analysis of disaggregated data. He examines subsectors of the Canadian manufacturing industry, emphasizing the distinction between those that produce largely to meet orders for specific commodities (Production-to-Order) and those that produce

[1]Inventory equations embodied in simultaneous econometric models of the Canadian economy include Bakony [1], Officer [18], Rhomberg [19], Shapiro [20] and Stewart [21]. Johnson and Winder [12] also attempt to specify a single-equation model.

[2]A review of the earlier literature is included in the survey of investment studies for the Commission on Money and Credit by Eisner and Strotz [8]. Further discussion of the literature and additional results are presented in Lovell [15]. The Brookings Quarterly Econometric Model of the United States contains an inventory investment sector within a simultaneous model framework, Darling and Lovell [6]. More recent is the preliminary work of Lovell using anticipations surveys, Lovell [16].

and accumulate output in anticipation of future sales (Production-to-Stock). In addition, he breaks down inventories in each manufacturing industry by stage of fabrication, and analyzes separately the raw material, goods-in-process, and finished goods components of inventory holdings. Thus one can segregate the different categories of inventory according to the motives for holding each, and so derive a much more stable causal structure.

For a number of reasons this procedure could not be used here. In the first place I had to try to explain all inventory holdings and not simply holdings in the manufacturing sector. But, outside of manufacturing, data by stage of fabrication are not available. This fact limits the application of the Courchene approach in an aggregate model. A more serious difficulty involved in adapting his procedure is that the disaggregated data for manufacturing are available only in current dollars while, to conform to RDX1, my equation had to be fitted in constant dollars. (A check on the equations, described below, showed that at the aggregate level the constant-dollar equations gave a much better fit than equations using current dollars, in spite of the elimination of parallel price movements in dependent and independent variables.) There is a partial breakdown of the manufacturing deflator into durables and non-durables, from 1959 to the present, but this is clearly not enough to work with. In addition, because the data on non-manufacturing inventories are notoriously shaky, it seemed perhaps wiser to pool the errors in variables and hope for some cancellation rather than to try to handle each series separately. Certain experiments were made with the inventories held by non-manufacturing industries, but the results were in general not encouraging. More time and effort spent on these experiments might have led to some sort of breakthrough; however this further endeavour will have to wait for another occasion.

Finally there are sheer size constraints imposed by the need to fit the inventory sector into a simultaneous econometric model. Had I surmounted in one way or another the problem outlined above and arrived at reliable disaggregated inventory equations I should have had an inventory sub-sector of some twenty or so equations. This would represent one-third of the capacity of a very respectable model, and ten per cent of a model even more ambitious than the currently published form of The Brookings Quarterly Econometric Model of the United States. Moreover, the virtue of a disaggregated approach is that it allows different causal variables for each category of inventory. But in a complete model, each new variable requires its own explanatory equation and so expands the model even further. Unfilled orders, for instance, may play a powerful role in explaining some categories of inventory; but to explain unfilled orders may be no easier than to explain inventory. And one can hardly leave unfilled orders as an exogenous variable. Thus the limitations of data and constraints imposed by the simultaneous model framework led me to stay with the aggregated approach.

The results fall somewhat short of the ideal inventory equations described above, which is not really surprising. I have not succeeded in fitting the data period as well as I should like, although the fits compare quite favourably with previous Canadian experiments. Nor have I been able to match the Americans' ability to generate an $\overline{R}^2$ of .95 on almost anything. The final structure, like the goodness of fit, is satisfactory without being exciting; but I was particularly disappointed in my efforts to introduce financial variables subject to policy control. This disappointing result may have several explanations. Inventory-holders may not be fully 'rational' — through ignorance or inertia they may not take account of the costs of carrying inventory. Alternatively, given the relatively small changes historically observed in such costs, the savings to be derived from optimal inventory management may not justify the effort and expense involved. Where information is costly, 'irrationality' may be rational. And finally the uncertainties surrounding sales and supply considerations may be so great that they swamp any cost-of-funds effects. If businesses could forecast future sales, or even some frequency distribution of future sales, with perfect accuracy they might take account of the impact of financial variables; but in the full uncertainty of the real world such considerations are of decidedly secondary importance. There is some survey evidence to support this view. (See Young and Helliwell [23].)

As for the goodness of fit, it is possible that given the highly expectational nature of inventory investment decisions, there may be a substantial segment of variance that cannot be explained. Such an 'animal spirits' component cannot be fitted into a deterministic structure, or at least not into a structure that relies on economic causal variables. If we want better-fitting equations, we must wait for operations research techniques of inventory control to spread through the economy. In addition, of course, our equations are subject to all the usual open-economy-type problems: firms' suppliers and/or markets may be outside the country and more or less independent of Canadian economic developments, foreign parents or affiliates may make the costs of financing inventory in Canada irrelevant, and so on. There is no shortage of explanations of why the ideal equation has not yet been found.

b) The Analytic Background

A search for the relevant causal variables bearing upon inventory investment can begin simply by making choices among the many plausible candidates which suggest themselves. It is preferable, however, to start with some hypothesis on the internal structure of the inventory investment decision in order to organize the search and provide a criterion by which to interpret the results. Since the flexible accelerator mechanism is now solidly established in the literature[3] and provides a very logical and satisfying way of organizing the equation, this structure was chosen as a starting point.

The flexible accelerator mechanism is based on the hypothesis that there exists at any time some equilibrium or desired level of inventories H_t^* that, if achieved, would tend to be maintained. Zero inventory investment is thus implied. This level of inventories would be a function of current and expected future sales, the cost of carrying inventory, and a wide variety of other factors. The discrepancy between the desired level and the actual level at the end of period t-1 would be eliminated by investment or disinvestment in this period. For several possible reasons, however, the discrepancy is only partially eliminated in the current period, and the change in inventories is proportionate to the size of this gap. Thus we get the basic formulation:

$$\Delta H_t = b(H_t^* - H_{t-1}) \qquad (1.1)$$

[3]The mechanism is derived from Goodwin, [10] and was used in inventory analysis by Darling [5] and subsequently by Lovell, Courchene, and others.

Now H_t^* is of course not directly observable, so it may be represented as a function of various sales and cost variables:

$$H_t^* = f(X_1,\ldots,X_n) \qquad (1.2)$$

These variables in turn include such magnitudes as expected sales, so they are a combination of observable and non-observable variables. For full generality:

$$X_i = g_i(Y_1,\ldots,Y_m) \qquad (1.3)$$

In the event that X_i is an observable variable, $X_i = Y_i$; otherwise X_i must be represented by some combination of observed Y_j whose form is subject to one's hypotheses about expectations formation. Also the distinction may shift depending on the availability of new data. Thus next quarter's expected sales may be measurable in the United States from survey data, but in Canada they may be assumed to be a function of present and past sales — in turn represented by a proxy such as GNE, shipments, or some other observed variable. If, through a heroic (or naive) process of simplification, all functions are expressed linearly the equation is:

$$H_t^* = a_0 + a_1X_1,\ldots,a_nX_n$$

$$X_i = c_{0i} + c_{1i}Y_1,\ldots,c_{mi}Y_m$$

$$\Delta H_t = bd_0 + bd_1Y_1 + bd_2Y_2,\ldots, + bd_mY_m - bH_{t-1} \qquad (1.4)$$

an expression in observable variables susceptible to estimation.

There is, of course, a limit to the number of possible Y_j that can be used for estimation purposes — a limit imposed by available data sources.

But there are few limits on the possible determinants of H_t^* or on the possible functional forms relating these determinants to the available observed variables. Thus most of the efforts to estimate an inventory equation have used roughly similar sets of

Y_j but have derived them from differing definitions of X_i and forms of f and of the g_i. As a simple example, I can define H_t^* as a function of expected sales and the interest rate, and let expected sales be a linear combination of current sales and those of the previous quarter.

$$H_t^* = a_0 + a_1 S_t^e + a_2 r_t \qquad (1.5)$$

$$S_t^e = (1 - \rho) S_t + \rho S_{t-1} \qquad (1.6)^4$$

The estimating equation becomes:

$$\Delta H_t = ba_0 + ba_1 (1 - \rho) S_t$$

$$+ ba_1 \rho S_{t-1} + ba_2 r_t - bH_{t-1} \qquad (1.7)$$

On the other hand if I assume that inventory investment includes a 'passive' term equal to the deviation between actual and expected sales, the equation is:

$$\Delta H_t = b(H_t^* - H_{t-1}) + (S_t^e - S_t) \qquad (1.8)$$

The estimating equation is:

$$\Delta H_t = ba_0 + [ba_1 (1 - \rho) - \rho] S_t$$

$$+ [ba_1 + 1] \rho S_{t-1} + ba_2 r_t - bH_{t-1} \qquad (1.7')$$

[4]This form derived from the projection of past levels with a partial prediction of the change,

$$S^e = S_{t-1} + \delta \Delta S_t$$

$$= \delta S_t + (1 - \delta) S_{t-1}$$

$$\text{or } (1 - \rho) S_t + \rho S_{t-1}$$

A discussion of this forecasting form can be found in Theil [22] pp. 154-161.

The equation is unchanged but the interpretation of the coefficients differs. In fact it is not difficult to build models in which the reduced-form estimates will not yield unique values of the structural parameters and the model is underidentified.

The estimated values from the final equation could, of course, be used as a way of distinguishing between different model specifications, given a priori notions of plausible ranges for the model parameters. But this eliminates one of the criteria for choosing a good final form of the estimated equation. Thus a chicken-and-egg problem results in that the model ultimately chosen and the best possible estimating equation have to be jointly selected in a way that weakens most of the statistical tests of significance. The methodological implications of this procedure are at best unclear; but, given the extensive degree of our ignorance about the state of the world, this procedure is undoubtedly superior to marrying oneself to a specific a priori model. Since not enough is known on theoretical grounds to do this, I have fitted equations of form (1.4) with an interpretation that is explicitly as simple as possible within the framework of the flexible accelerator model, and I have interpreted the parameters as well as I could afterwards.

Since the model as outlined above is virtually unrestricted in form and content, I should comment on my interpretation of it and on the considerations which governed the choice of variables for experimentation. To begin with, this type of model rests on the assumption that H_t^*, the desired level of inventories, exists for the whole economy. This assumption can be based on an extension of the well known theory of inventory management developed as a branch of operations research. Given enough information on the actual values or the probability distributions of the relevant variables affecting each inventory-holder, an individual H^* can be derived for him in each time period. Aggregating these H^* yields an optimum level for the whole economy. It is not necessarily true, however, that the optimal inventory strategy located by operations research techniques will define a desired inventory level. In some cases this problem is not serious, as in the two-bin or (S,s) strategy that calls for the firm to reorder up to some level S whenever stocks decline to level s. Here S may be defined as the optimal level and the adjustment coefficient b may be relied on to embody the delayed adaptation of actual inventories to the desired level. But for some inventory models this strategy is not optimal, and at any given time no 'best' level can be defined. A simple example of such a situation is given by Dorfman [7] pp. 45-47.

This problem can be avoided by saying that whatever the 'best' level may be, every inventory-holder has some idea about what his inventories should be. His idea might be no more precise than 'higher', 'lower', or 'unchanged', but some level will satisfy each inventory-holder. In such a case the individual H_t^* is likely to be a band rather than a single value. But from this assumption it does not immediately follow that we can aggregate these individual levels to yield an economy-wide value of equilibrium inventories. It is quite reasonable to assume that the desired inventory levels of all firms are interdependent, quite apart from their dependence on the same exogenous factors. If a firm's suppliers have lower desired inventory levels, then its own desired levels should rise. One could argue that suppliers' actual inventories are more relevant to the firm, as these influence reorder lags; but suppliers' desired levels of inventories will have more significance for the future, and in its inventory policy a firm should take account of this fact.

If the justification of H_t^* for the whole economy on the basis of aggregating micro-values is unacceptable, some more general grounds for its assumption are needed. I can argue that relatively high inventories prompt holders to cut back, and relatively low inventories prompt them to accumulate. This implies that, ceteris paribus, the change in aggregate H_t is a declining function of H_t, taking on positive and negative values. If the function is assumed to be single-valued and continuous, this implies that for some level of H_t the change in H_t is zero. An equilibrium level is established that will, in the absence of changes elsewhere in the economy, tend to perpetuate itself. If the function is also monotonic, there will be a unique and stable equilibrium; otherwise there may be multiple equilibria some of which are unstable. If monotonicity is plausible, I can assume a unique and stable equilibrium level (not necessarily desired) of H_t yielding net inventory investment of zero.[5]

[5]If on the other hand the mapping of $H \rightarrow \Delta H_t$ is point-to-set, equilibrium solutions would seem to require restrictions on the nature of the mapping equivalent to the extension from Brouwer to Kakutani fixed-point theorems. (See Lancaster [13] pp. 336-8 and pp. 342-352.)

If the unique macro-level H_t^* is assumed to exist, we may then consider the nature of the reaction coefficient b. Inventories tend to be very volatile and subject to rapid adjustment; so why do not inventory-holders try to eliminate all the gap within the quarter, rather than just a fraction of it? Lovell suggests several reasons [14] pp. 295-296. There may be ordering costs involved in changing the level of inventories, ordering intervals may be infrequent, liquidation of recently acquired stocks may be gradual. These reasons can be summarized in the (S,s) strategy, implying that at any given time most holders are somewhere between S and s stock levels, and will not act to eliminate the gap until the stocks fall to s. The batch cost makes it uneconomic to order at frequent intervals so adjustment will be slow. But this framework involves two types of problems. First of all, the implied reaction time is unlikely to be much longer than a quarter; lags of two or three quarters are not consistent with what is known about the rapid responsiveness of inventory. This implies that b for quarterly data should be quite close to 1.0, certainly above 0.5. Yet 0.5 is not the estimated value in most empirical work, including mine. Secondly, the identification of the upper bound S with H_t^* is also pretty shaky because this rationale implies that each holder is almost always below S and that the economy in total is always below. A desired reduction in inventories does not fit the apparatus. If the (S,s) model is discarded, the delayed reaction may be justified by saying inventory-holders are cautious and do not move to the equilibrium level all at once in case it should shift by the time they get there. Another possible rationale is that total inventories adjust slowly, because one firm's disinvestment is another firm's investment, and if all firms try to change their inventories at once in the same direction all will be more or less unsuccessful. Again, however, long lags are hard to justify by this explanation. So given the very long implied adjustment lags derived from most empirical work, there may be suspicion about whether the model is measuring the right thing.

If misgivings about the definitions of H_t^* and b are ignored, attention must be turned to the determinants of H_t^*. These can be subdivided into positive and negative categories — the reasons for holding inventory and the costs of doing so. In the positive category, clearly the desired variable is expected sales. Whatever the class of inventory and whatever the industrial division, all inventory is held for contribution to future sales. In a complete model the desired level of inventory should also depend on the probability distribution of future sales, or at least on the standard error; but at the level of macro-aggregates these concepts are hard to define. Expected sales are not available in the data set, therefore some hypothesis is required about the process of expectations formation by which expected sales are generated from presently observable variables such as: current and past sales, new orders, and unfilled-order backlogs. Data on the last two variables are available for the manufacturing sector only; no explicit sales data exist for any sector. Sales data may be approximated quite well in the manufacturing sector by shipments, but for the whole economy some estimate must be developed from GNE or Gross Domestic Product, or elsewhere. The new-orders variables and unfilled-orders variables have the advantage of being explicitly related to future sales but these variables are incomplete in coverage and involve either the use of a partially disaggregated model, with manufacturing and non-manufacturing sectors, or else the assumption that such variables from the manufacturing sector have an equivalent impact on non-manufacturing inventories. Moreover, new-orders and unfilled-orders variables would require separate determining equations and would create extra difficulties for a complete model. Unfilled orders have the additional difficulty of being causally ambiguous because any delay in production will lead to a simultaneous increase in work-in-progress inventory and to unfilled orders in the case of industries producing to order. The resulting relation is associative rather than causal. GNE-based proxies for current sales, while convenient in a simultaneous model, necessitate explicit hypotheses concerning expectations formation. The parameters of these hypotheses must be calculated, along with those of the accelerator relation, from the estimated coefficients of whatever equation is finally derived.

The costs of carrying inventory (apart from the physical elements of storage cost, wear, depreciation, and so on) are the costs of the financing necessary to pay for the inventory. Thus some type of interest or discount rate must be introduced to represent the marginal cost of funds to the inventory-holder — a cost variable subject to all the usual problems of discount rates in an imperfect capital market. Is the relevant rate short-term, such as the treasury bill rate, measuring the opportunity cost of funds tied up in inventory? Should the rate be the industrial bond yield, the average (rather than marginal) cost of borrowed funds to the firm? The bank loan rate may be the average cost of short-term funds, but it is far from being the marginal rate and has little variance anyway. Does the firm have some concept of the internal short-term discount rate, less volatile than the bill rate, incorporating various risk elements as well? For that matter, as suggested above, the historical fluctuations in borrowing costs may be altogether too small relative to the great uncertainties surrounding other determinants of desired inventory. In this case, measures of credit availability or credit rationing may indicate the impact of infinite marginal borrowing costs that are not represented in market interest data. In the face of such uncertainties one can only try various combinations of cost and availability measures to see how they perform.

Further influential variables are limited only by the imagination of the researcher, but one commonly suggested variable is price change as represented by combinations of past price movements. This rests on the hypothesis that firms will build up inventory if prices are expected to rise and cut back if they are expected to fall — a hypothesis usually combined with an extrapolative expectations mechanism. But results with this variable are generally unsatisfactory, and mine are no exception. This could be due to the weakness of the price data, or because the expectations mechanism is inadequate. It could also be because firms consider that they are in business to produce and sell goods, not to speculate on price movements. If they buy in fluctuating markets, they are more likely to cover with forward contracts and 'dis-speculate' than to play the market. Another variable used with some success in the American studies is the size of defence expenditures, recognizing the long lags and large work-in-progress elements in defence procurement. Given the relative size of the Canadian military establishment, however, this variable does not seem worth importing.

Yet another category of variables might be introduced by admitting explicitly that inventory investment depends largely upon how businessmen 'feel' about the state of the economy, and by searching for variables such as unemployment rates and share prices, that are likely to be widely observed and to condition the psychological climate in which inventory decisions are made.

Some success was achieved in this study with transformations of such variables.

One might also question whether the model outlined in (1.1) to (1.4) is complete because this implies that all inventory investment is in some sense deliberate. Yet clearly misforecasting of sales does occur, and may lead to unplanned accumulation or reduction of inventories unrelated to equilibrium or past stock levels. This can be handled by including all such unplanned elements in an error term — (1.4) can hardly be expected to fit exactly! A more involved theoretical formulation, which boils down to the same estimating relation, assumes an unplanned element in accumulation which is proportionate to the forecasting error:

$$\Delta H_t = b(H_t^* - H_{t-1}) + c(S_t^e - S_t) \qquad (1.8')$$

The parameter c is the production inflexibility coefficient. It is zero if production can be fully adjusted within the quarter so that the 'unexpected' sales changes can be met without affecting inventory levels, and 1.0 if production cannot be adjusted within the quarter and all the error is reflected in unplanned inventory change.

If this model is combined with (1.5) and (1.6) it reduces to:

$$\Delta H_t = ba_0 + [ba_1 - \rho(ba_1 + c)]\ S_t$$

$$+ (ba_1 + c)\ \rho S_{t-1} + ba_2 r_t - bH_{t-1} \qquad (1.7'')$$

Again, the equation to be estimated is unchanged but now it is impossible to distinguish between c and ρ, so the model is underidentified. I have some a priori information about each, but hardly enough to be confident in my estimates. And the more parameters to be derived from a given reduced form, the more sensitive is the model to errors of specification and estimation. There is also the theoretical question of whether it is appropriate to 'tack on' the forecast error in this way. The discussion of the formation of H_t^* and of the adjustment mechanism suggested that these might embody some form of optimal inventory strategy. Without being too precise, it can be said that end-of-period target inventory may be dependent on the relation between actual and expected sales. One of the motives for carrying inventory is to provide a buffer stock so that unexpectedly large sales can be met without disrupting the production process. Since inventories have this production-smoothing function, the inventory target is in fact a range, with each value assuming a different level of actual sales. Thus, the 'unplanned' component is already included implicitly in the equilibrium level — not an additional factor independent of the target inventory. This implies that (1.5) is an inadequate specification, something I certainly do not deny. But the estimation problem is unchanged, and there does not seem to be any theoretical justification for grafting an 'unplanned' investment term onto the model.

2. Estimation Results

In presenting these results I have adopted the philosophy that the cataloguing of failure is almost as important as the description of final success. ('Success', of course, is a relative term.) From the point of view of economic science it is important to present this work in as complete and reproducible a fashion as possible, however galling that record may be. One of the most valuable products of the exercise may then become the information it yields on unfruitful approaches — other researchers can thus avoid these blind alleys. Better still, they may see light where I saw only darkness and discover that it was the researcher who was blind. (Better from the point of view of economic knowledge, not from my point of view.) Therefore I have recorded the results in considerable detail (both good and bad), marking as clearly as possible the pattern followed.

The dependent variable throughout the research was the quarterly change in Canadian non-farm inventories, in constant 1957 dollars seasonally unadjusted (Databank (DB) 150, see [24]). The basic data period was first quarter 1947 to fourth quarter 1965, but most equations were fitted over shorter periods both to avoid what appeared to be structural shifts in the earlier years and to examine potential explanatory variables not available earlier in this period. The model used required a series for inventory stocks as well, which were constructed by cumulating inventory change onto a base-period value for end of fourth quarter 1955 supplied by the Dominion Bureau of Statistics. Since seasonally unadjusted data were used, (1.1) was reformulated:

$$\Delta H_t = Q_1 + Q_2 + Q_3 + Q_4 + b(H_t^* - H_{t-1}) \qquad (2.1)$$

where the Q_i are quarterly seasonal dummies. Since the specification of H_t^* can lead to a constant term, one of the seasonals must be dropped in estimation.

a) First Phase: Estimation with Gross National Expenditure

As outlined above, the experiments began with the idea that the production-smoothing role of inventory should lead to a direct impact of sales-forecast errors on the equilibrium inventory level. Consequently I hypothesized that businessmen have some idea of the 'normal' level of sales, and deviations from this level are partially reflected in H_t^*. This leads to the formulation:

$$H_t^* = c + dS_t^e + e(S_t^* - S_t) + fr_t \qquad (2.2)$$

Here S_t^e is expected sales, S_t^* is 'normal' sales, and r_t is some measure of the cost of funds. Clearly the linear specification of r is rather unsatisfactory, but given the highly uncertain nature of the structure and the difficulties of a non-linear approach it seems a reasonable compromise. The 'normal' level of sales was derived by fitting quarterly GNE (DB 157) in logs to a time trend from 1947 to 1965. The calculated value of GNE was selected as the 'normal' level. Although the use of GNE for a sales proxy is dubious, it has the advantage of ready availability in a small simultaneous model and so seemed to be a point where experimentation could reasonably begin. More refined variables than those used here should lead to better results but not to qualitatively different ones.

Businessmen's sales expectations were modelled in two ways, first by the form outlined in (1.6) and second with a Koyck-type distributed lag of the form:

$$S_t^e = (1 - \lambda)\ [S_t + \lambda S_{t-1} + ,\ldots,\ \lambda^n S_{t-n}] \qquad (2.3)$$

These forms differ very little for λ and ρ with both close to zero but (2.3) implies a much greater importance for sales in the more distant past if λ is close to 1.0. Smaller parameters imply better forecasting by inventory-holders.

Each of the expectations structures implies a different estimating model. Bringing together (2.1), (2.2), and (1.6), the equation is:

$$\Delta H_t = (Q_1 + Q_2 + Q_3 + bc) + [bd(1 - \rho) - be] S_t$$

$$+ bd\rho S_{t-1} + beS^*_t + bfr_t - bH_{t-1}$$

Substituting GNE (Y_t) for sales, and noting that $Y_t = \Delta Y_t + Y_{t-1}$:

$$\Delta H_t = (Q_1 + Q_2 + Q_3 + Q_4 + bc) + b[d(1 - \rho) - e] \Delta Y_t$$

$$+ b(d - e) Y_{t-1} + beY^*_t + bfr_t - bH_{t-1} \qquad (2.4)$$

This is an estimating equation in observable variables whose coefficients are fully identified.

Substituting (2.3) for (1.6), going through the familiar Koyck transformation, and letting $\lambda\Delta H_{t-1} = \lambda H_{t-1} - \lambda H_{t-2}$, we obtain:

$$\Delta H_t = (1 - \lambda)(Q_1 + Q_2 + Q_3 + bc) + [bd(1 - \lambda) - be] Y_t$$

$$+ be\lambda Y_{t-1} + (be - be\lambda k) Y^*_t + bfr_t - bf\lambda r_{t-1}$$

$$- (b - \lambda) H_{t-1} - \lambda(1 - b) H_{t-2} \qquad (2.5)$$

Since Y^*_t is a log trend, $Y^*_{t-1} = kY^*_t$ where $0<k<1$. Equations (2.4) and (2.5) are the initial estimating equations.

Equations (2.4) and (2.5) were fitted over varying time periods using several alternatives for r_t. I tried the threemonth treasury bill rate (DB 601), the average rate on Government of Canada bonds due in less than three years R03 (DB 1365), and the average rate on Government of Canada bonds due or callable in ten years or more RLC (DB 2764) to cover a range of possible terms to maturity; but failed to find significant differences between these alternatives. The results are shown in Tables 1 and 2.

Table 1 indicates clearly that the specification used is inadequate. The 'normal' sales level variable is uniformly insignificant and varies in sign. Worse, the interest variables are all moderately significant but have the wrong signs. While inventories may well perform a 'buffer stock' function the equation does not show this. Weaknesses in the overall equation structure may prevent interest rates from playing their proper theoretical role; certainly they have no place in this formulation.

On the other hand, the coefficients of ΔY_t, Y_{t-1}, and H_{t-1} are significant and relatively stable. This stability extends to H_{t-2}, which was accidentally included in certain of the regressions in place of H_{t-1}. Since in general $H_{t-2} < H_{t-1}$ one might expect H_{t-2} to have a larger coefficient, but the accelerator mechanism suggests that the coefficient should be smaller and this is in fact the case. Implied values of b can be derived from the coefficient of H_{t-2}. These are shown in Table 1 and tend to be somewhat larger than the measured coefficients of H_{t-1}. Only in one case is b undefined.

In deriving the values of the model parameter from these coefficients it can be seen that d, the target marginal stock/sales ratio, lies between 0.5 and 1.0, with the H_{t-2} equations yielding the lower values. The value of ρ is small, suggesting that most of the change in sales is correctly forecast. In the short-period equations beginning in 1955, however, the implied forecast is an overestimate. Examination of residuals from the long-period equations suggests that sharp inventory movements in 1956 and 1957 may be distorting the regression plane of the short-period equation, leading to better fits but less reliable structures. Such a conclusion can hardly be based on the Table 1 results alone, but these results do suggest that a long-period equation may be better. This is confirmed by the general similarity between the coefficients of the 1947-1965 and 1952-1965 equations, a similarity borne out in later experiments. The markedly

TABLE 1

Lovell-type Expected Sales

(Equation 2.4)

	Q1	Q2	Q3	Q4	ΔY_t	Y_{t-1}	Y^*_t	RLC	RTB	R03	H_{t-1}	H_{t-2}	d	ρ	b	SEE	$\overline{R}^2$	D/W
3Q47	501.66	236.45	-.85	131.68	.165	.220	-.018	46.29				-.226	.586	.272	.345	103.4	.517	2.24
4Q65	(5.85)	(2.72)	(0.01)	(1.40)	(3.64)	(4.24)	(0.29)	(1.27)				(4.54)						
1Q55	587.41	107.63	-135.39	189.85	.254	.242	-.028	67.17				-.245	.500	-.056	.429	76.6	.765	1.82
4Q62	(2.36)	(0.42)	(0.44)	(0.63)	(3.51)	(3.13)	(0.32)	(1.58)				(3.43)						
1Q55	624.81	162.60	-58.68	195.29	.228	.244	-.057	74.55				-.228	.531	.086	.354	76.5	.773	2.05
4Q65	(4.04)	(1.03)	(0.32)	(1.17)	(3.67)	(4.58)	(0.72)	(2.18)				(3.47)						
1Q52	577.33	316.82	44.55	166.18	.160	.217	.027			32.45	-.260		.938	.234	.260	89.1	.658	1.66
4Q65	(4.30)	(1.83)	(0.26)	(1.05)	(3.79)	(4.30)	(0.42)			(1.84)	(3.47)							
1Q52	607.38	269.06	105.81	257.95	.143	.163	.062			26.36		-.245	.524	.089	.429	88.9	.659	2.20
4Q65	(3.80)	(1.69)	(0.57)	(1.45)	(3.38)	(3.39)	(0.85)			(1.51)		(3.51)						
1Q55	543.65	75.68	-120.91	180.88	.222	.200	.055		18.44			-.254	1.004	-.086	?	78.5	.753	1.79
4Q62	(2.12)	(0.29)	(0.36)	(0.56)	(2.97)	(2.21)	(0.66)		(1.12)			(3.49)						
1Q55	643.11	193.32	44.97	278.07	.182	.175	.040		22.89			-.232	.927	-.033	.366	78.7	.760	1.97
4Q65	(4.03)	(1.20)	(0.25)	(1.62)	(3.13)	(3.59)	(0.59)		(1.60)			(3.44)						
3Q47	614.30	409.77	105.51	210.20	.172	.249	-.010		22.92		-.255		.937	.322	.255	104.1	.511	1.69
4Q65	(5.48)	(3.37)	(0.89)	(1.78)	(3.81)	(4.43)	(0.18)		(1.38)		(4.47)							

TABLE 2

Koyck-type Expected Sales

(Equation 2.5)

	Q1	Q2	Q3	Q4	Y_t	Y_{t-1}	Y^*_t	R_t	R_{t-1}	H_{t-1}	H_{t-2}	SEE	$\bar{R}^2$	D/W
								(R03)						
2Q52	623.64	300.40	76.90	245.67	.160	.022	.044	29.70	.679	-.149	-.098	88.7	.666	1.83
4Q55	(3.68)	(1.63)	(0.40)	(1.36)	(3.62)	(0.42)	(0.60)	(1.04)	(0.02)	(1.01)	(0.72)			
								(RLC)						
3Q47	538.94	305.82	16.21	130.26	.179	.079	-.030	37.28	17.42	-.130	-.126	104.2	.510	1.98
4Q65	(5.62)	(2.74)	(0.16)	(1.37)	(3.63)	(1.46)	(0.45)	(0.42)	(0.18)	(1.02)	(1.14)			
								(RLC)						
1Q55	589.80	39.63	-66.85	311.12	.212	-.081	.021	141.01	-97.65	.214	-.395	78.2	.755	2.07
4Q62	(2.24)	(0.15)	(0.20)	(0.91)	(2.50)	(0.83)	(0.21)	(1.43)	(0.88)	(0.91)	(2.06)			
								(RLC)						
1Q55	592.54	117.80	-67.45	194.71	.212	.0002	-.035	110.79	-49.53	.050	-.260	78.5	.761	2.11
4Q65	(3.47)	(0.61)	(0.35)	(1.10)	(2.98)	(0.002)	(0.39)	(1.33)	(0.50)	(0.28)	(1.67)			
								(RTB)						
1Q55	564.12	46.68	-79.90	255.44	.210	-.058	.061	22.62	-7.37	.137	-.355	81.3	.735	2.04
4Q62	(2.02)	(0.17)	(0.22)	(0.70)	(2.63)	(0.62)	(0.68)	(0.91)	(0.31)	(0.65)	(1.96)			
								(RTB)						
1Q55	617.15	142.90	23.50	270.23	.179	-.020	.039	27.31	-8.25	.069	-.281	80.8	.747	2.11
4Q65	(3.53)	(0.74)	(0.12)	(1.46)	(2.95)	(0.29)	(0.56)	(1.17)	(0.35)	(0.42)	(1.86)			

better fit of the 1952-1965 equation is probably due to its avoidance of the Korean War inventory boom in 1950 and 1951.

Thus some information can be gleaned from Table 1: the accelerator structure may be useful, the cost variables in their present form are no use, and the 1952-1965 period may be optimal. But no further information can be derived from Table 2. Here again Y^* is insignificant, and all the interest rate variables are wrongly signed. According to the structure of (2.5) the lagged rates should have positive coefficients. The structural parameters in this form are overidentified and different estimates of the same parameter do not converge. Multicollinearity reduces the significance of almost all parameters. The implied values of the reaction coefficient b are quite unstable — one case yields a large negative result. The values of d, the target stock/sales ratio, are even worse. It appears that the Koyck lag structure introduces too many collinear variables and contributes nothing to the usefulness of the model. Further examination of the parameter values implied by these equations does not change this conclusion.

b) Second Phase: The New Sales Variable

It seemed fairly clear that substantial improvements were required in the specifications either of structure or of estimating variables. And since most changes in theoretical structure had little impact on the estimating equation, priority was given to the latter. First an improved sales proxy was developed from the *National Accounts*[6] by taking the sum of: expenditures on consumer durable and non-durable goods (DB 141, DB 140), business gross fixed capital formation (DB 144), total government non-wage expenditure on national accounts basis (DB 2171, DB 4079, DB 4104), and exports less imports (DB 153, DB 154). The resulting variable was thought to be a much closer approximation to actual sales of goods by Canadian inventory-holders than was the GNE used initially and as such to be a more relevant measure of expected pressures on inventories. This variable was denoted as 'purged' GNE or Y^P.

[6]*National Accounts Income and Expenditure* issued quarterly and annually by the Dominion Bureau of Statistics.

The variable for cost of funds was also changed, because it appeared that the strong positive trend in interest rates in the postwar period together with the rising level of inventories might be creating part of our problem. If the interest rate is influential, not as an absolute magnitude but as an index of the relative cost of funds, the trend component should be removed. Thus the interest index is defined:

$$i_t = [\sum_{j=1}^{12} r_{t-j}]/12r_t \quad (2.6)$$

This index can be interpreted not only as the relevant magnitude, if interest rates are important only relative to their recent levels; but also as a measure of credit stringency, on the hypothesis that inventory-holders do not react to cost levels but do react when their sources of funds dry up entirely. We are assuming that the 'fringe of unsatisfied borrowers' moves with relative, not with absolute, interest rates — a fairly safe assumption.

The equation fitted under these hypotheses can be derived from (2.1), (1.5) and (1.6) with Y_t^P substituted for S_t.

$$\Delta H_t = (Q_1 + Q_2 + Q_3 + bc) + bd(1 - \rho)\, Y_t^P$$

$$+ bd\rho Y_{t-1}^P + bei_t - bH_{t-1}$$

or grouping with $Y_{t-1}^P = Y_t^P - \Delta Y_t^P$,

$$\Delta H_{t-1} = (Q_1 + Q_2 + Q_3 + bc) + bdY_t^P$$

$$- bd\rho\Delta Y_t^P + bei_t - bH_{t-1} \quad (2.7)$$

This equation was estimated from 1953 to 1965 in three forms, one with no cost-of-funds term, one with R03, and one with RLC, both interest rates in index form. The results are shown in Table 3. The first obvious point is that the interest index is no help here either. It should be positively signed, but it is negative in both equations. The RLC index is insignificant and weakens the equation fit; but R03, which on a priori grounds might be more

TABLE 3

<u>'Purged' GNE and Interest Index</u>

(Sample Period: 1Q53 to 4Q65)

$$\Delta H_t = \underset{(1.80)}{-208.26} + \underset{(0.85)}{105.91}\, Q_1 + \underset{(2.86)}{271.67}\, Q_2 + \underset{(1.71)}{93.22}\, Q_3$$

$$+ \underset{(3.04)}{.225}\, Y^P_t - \underset{(3.14)}{.295}\, \Delta Y^P_t - \underset{(2.42)}{.119}\, H_{t-1}$$

SEE = 99.4 $\bar{R}^2$ = .596 D/W = 1.23

$$\Delta H_t = \underset{(0.63)}{-82.58} + \underset{(0.78)}{94.28}\, Q_1 + \underset{(2.53)}{237.92}\, Q_2 + \underset{(1.55)}{82.60}\, Q_3$$

$$+ \underset{(2.55)}{.189}\, Y^P_t - \underset{(2.95)}{.272}\, \Delta Y^P_t - \underset{(2.00)}{.098}\, H_{t-1} - \underset{(1.90)}{105.10}\, R03$$

SEE = 96.7 $\bar{R}^2$ = .618 D/W = 1.38

$$\Delta H_t = \underset{(0.40)}{-75.26} + \underset{(0.92)}{115.86}\, Q_1 + \underset{(2.77)}{264.48}\, Q_2 + \underset{(1.62)}{88.63}\, Q_3$$

$$+ \underset{(2.97)}{.221}\, Y^P_t - \underset{(2.99)}{.284}\, \Delta Y^P_t - \underset{(2.35)}{.116}\, H_{t-1} - \underset{(0.91)}{141.91}\, RLC$$

SEE = 99.6 $\bar{R}^2$ = .594 D/W = 1.24

relevant, is moderately significant and improves the equation slightly. Unfortunately its causal implications cannot be accepted.

These equations provide little support for the basic structural model. The implied value of b is so low that it can be rejected on a priori grounds — it yields implausibly slow reaction times. This coefficient may measure the generally depressing effect of past inventory levels on this year's rate of accumulation, but it does not look much like a reaction coefficient. Moreover the implied value of d is about two, and that of ρ between one and two; neither of which goes down very well with our basic model. The value of ρ implies systematic error in the estimates of the change in the direction of sales; that of d implies that for every unit increase in the number of items they expect to sell, firms try to expand inventory by two items. Neither value is very plausible. The trouble may stem in part from decisions made in the first quarter, when inventories are built up in expectation of sales in the second quarter rather than based on actual sales in the first and fourth quarters of the previous year. It is possible that the strong seasonality in the data is too much for the theoretical model to encompass.

I did, however, find substantial support for the procedure of fitting in constant dollars at this point. A set of equations similar to those of Table 3 was run on current-dollar data, and resulted in a markedly lower fit with almost all coefficients insignificant including the quarterly dummies. It seemed fairly well established that the constant-dollar approach was appropriate.

Starting with the basic structure in the Table 3 equation, various forms of cost-of-funds and expectations variables were tested in an effort to capture some of the strong swings that the basic model bypassed. The results of some of these efforts are shown in Table 4. To get at the impact of credit conditions, I introduced the variable $(L/A)_t$, which is the ratio in period t of total business loans outstanding (DB 687) to the level of loan authorizations provided by the banking system (DB 608). The hypothesis is that when businesses start to run up against the limits of their lines of credit, they tend to cut back on inventories in an effort to conserve borrowing capacity. If inventory-holders behave in this way, (L/A) should come in with a negative

TABLE 4

'Purged' GNE with Loans/Authorizations (L/A) Ratio, Stock Index and Applications/Vacancies (APP/VAC) Ratio

	Constant	Q1	Q2	Q3	Y^P_t	ΔY^P_t	$(L/A)_t$	$(L/A)_{t-1}$	H_{t-1}	$STOCK_t$	$(APP/VAC)_t$	$(APP/VAC)_{t-1}$	EQR_t	SEE	$\bar{R}^2$	D/W
3Q56	-561.11	65.22	136.57	25.22	.137	-.234		3.48	-.047					96.7	.629	1.21
4Q65	(1.32)	(0.36)	(1.01)	(0.38)	(1.43)	(1.91)		(0.73)	(0.69)							
3Q56	-1,159.51	69.91	151.66	11.22	.183	-.261	11.61		-.072					87.7	.694	1.38
4Q65	(3.15)	(0.43)	(1.32)	(0.19)	(2.15)	(2.38)	(2.66)		(1.19)							
2Q47	-5.48	-48.96	169.50	69.31	.104	-.280			-.057	2.82				108.9	.461	1.36
4Q65	(0.08)	(0.56)	(2.61)	(1.63)	(1.62)	(4.49)			(1.36)	(2.20)						
1Q52	-256.56	2.20	258.59	94.97	.184	-.326			-.085	2.58				93.6	.622	1.48
4Q65	(2.57)	(0.02)	(3.05)	(2.00)	(2.55)	(4.18)			(1.76)	(2.02)						
3Q56	-350.25	-18.54	-23.43	-31.01	.008	-.160			.044	4.08				86.5	.703	1.72
4Q65	(2.12)	(0.11)	(0.20)	(0.51)	(0.09)	(1.47)			(0.68)	(2.85)						
3Q56	143.97	-50.10	-54.47	-104.79	.026	-.203			-.005		-5.44			85.2	.712	1.64
4Q65	(0.68)	(0.31)	(0.45)	(1.51)	(0.31)	(1.94)			(0.08)		(3.05)					
3Q56	-11.96	66.91	175.49	15.29	.049	-.227			-.014			-5.92		83.2	.725	1.52
4Q65	(0.07)	(0.43)	(1.60)	(0.27)	(0.63)	(2.21)			(0.25)			(3.35)				
2Q52	-22.43	-5.93	159.08	11.03	.141	-.303			-.067		-4.21			90.9	.649	1.57
4Q65	(0.20)	(0.05)	(1.68)	(0.19)	(1.92)	(3.68)			(1.40)		(2.69)					
2Q52	-127.47	82.80	304.30	86.83	.116	-.298			-.046			-5.89		85.2	.691	1.59
4Q65	(1.33)	(0.79)	(3.99)	(1.99)	(1.70)	(3.86)			(1.03)			(3.83)				
2Q52	-352.50	45.59	269.79	76.08	.088	-.292			-.024			-5.47	183.37	83.7	.702	1.79
4Q65	(2.12)	(0.43)	(3.47)	(1.75)	(1.27)	(3.84)			(0.53)			(3.56)	(1.64)			

sign and preferably with a lag. It would form a very clear channel for policy even if businesses were insensitive to interest rates.

Unfortunately, as can be seen in Table 4, (L/A) comes in strongest currently and with a positive sign. Clearly I have identified a demand for loans, not a supply relation, and shown that when business inventories are rising, firms tend to borrow more. If this is true, then credit constraints may inhibit inventory growth; but not through the structure suggested in the equation. This equation was only tested from 1956 to 1965 because the data on authorizations are not available before that date.

Efforts were also made to include as expectational variables the difference between the Toronto Stock Exchange common stock index (DB 2597) and its log trend 1946 to 1965 $STOCK_t$; the ratio of the index to its trend EQR_t; and the ratio of unplaced applicants to unfilled vacancies in all industry divisions $(APP/VAC)_t$ (DB 3979, DB 3965). As can be seen in Table 4 these variables all enter with the correct sign but tend to weaken the other variables in the equation and to help the fit only marginally. Moreover, none of the expectational variables included have a particularly good theoretical justification, unless they are indeed relevant to the formation of businessmen's expectations about the state of the economy.

One expectational variable which is widely watched, however, is the unemployment level (DB 1202, DB 1203). Although it has less variance than $(APP/VAC)_t$, the unemployment level is, with Gross National Product, probably the best known of the major economic aggregates. Rapidly and easily available to the public, the level of unemployment is generally accepted as an indicator of business conditions. But it is not clear in what form the variable should be introduced into these equations. Do inventory-holders look at the level of unemployment, recent changes in that level, the level relative to recent changes, or what? With these questions in mind, four new variables were defined to enter the inventory equation: U_t is the average level of unemployment over the quarter, ΔU_t is the current first difference in the level, U_t/U_{t-4} is the ratio of this quarter's level to the same quarter last year, and UI_t is a ratio of the current level divided by a twelve-quarter moving average with equal weights given to the levels from t-1 to t-12.

The results of computations using these variables are shown in Table 5, and they are distinctly interesting. Neither U_t nor ΔU_t contributes much to the equation fit — both weaken the coefficients of other variables. But U_t/U_{t-4} and particularly UI_t produce substantial improvements in fit while the other variables in the equation hold up well. On all the standard tests of equation quality, Durbin/Watson (D/W) statistic, Standard Error of Estimate (SEE) and so on, the UI_t equation for both 1952-1965 and 1956-1965 is a considerable improvement over any of its predecessors. Whereas before, the presence of autocorrelation had to be accepted with a 5 per cent confidence level, now for the 1952-1965 equation the hypothesis can be rejected at a 5 per cent level. Moreover, the standard errors of all coefficients are reduced. It would appear that the level of unemployment relative to its average over the last twelve quarters (roughly a full business cycle) captures part of the process of expectations formulation about the general state of the economy to which businesses react in setting their inventory targets.

The equation with a UI_t term fitted from 1956 to 1965 also may contain some confirmation of the flexible accelerator structure. It implies quite a low b. But the near equality of the Y^P_t and ΔY^P_t terms might imply that both are only part of a Y^P_{t-1} term with a marginal stock/sales ratio somewhat greater than one. If expected current sales are some ratio to last quarter sales, a ratio greater than one, this yields a marginal stock/sales ratio near one. Then businesses project current sales as a per cent rise on last quarter; and target end-of-quarter inventories are, on the average, equal to next quarter's expected sales. The structure is quite plausible except for the very slow implied rate of adjustment. Still, this equation allows for a more sensible interpretation of all coefficients than any other located so far. Consequently I accept the last two equations of Table 5 as a new 'basic model'.

c) Extensions of the Basic Model

If the new equation is better specified as to structure than were the previous ones, it is possible that a priori significant

TABLE 5

'Purged' GNE with Unemployment-based Expectations Variables

	Constant	Q1	Q2	Q3	Y^P	ΔY^P	U_t	ΔU_t	U_t/U_{t-4}	UI_t	H_{t-1}	SEE	$\bar{R}^2$	D/W
1Q52	-254.32	-0.38	236.27	64.02	.120	-.331	-.404				-.029	93.2	.625	1.48
4Q65	(2.56)	(0.00)	(2.69)	(1.23)	(1.38)	(4.27)	(2.12)				(0.47)			
1Q56	97.58	171.92	-15.58	-89.51	-.028	-.121	-.755				.045	87.9	.718	1.39
4Q65	(0.62)	(1.25)	(0.14)	(1.39)	(0.29)	(1.22)	(3.52)				(0.66)			
1Q52	-243.89	56.57	334.51	130.42	.241	-.331		.135			-.126	97.3	.592	1.21
4Q65	(2.24)	(0.52)	(3.17)	(2.02)	(3.41)	(3.91)		(0.41)			(2.74)			
1Q56	-13.77	294.56	15.34	-72.20	.174	-.169		-.540			-.101	101.0	.628	1.31
4Q65	(0.08)	(1.84)	(0.10)	(0.77)	(2.00)	(1.49)		(1.28)			(1.69)			
1Q52	59.42	-0.87	262.28	98.12	.175	-.322			-142.97		-.097	86.7	.676	1.86
4Q65	(0.49)	(0.01)	(3.44)	(2.26)	(2.71)	(4.46)			(3.56)		(2.34)			
1Q56	335.26	154.70	100.81	5.25	.133	-.167			-196.51		-.091	81.6	.757	1.73
4Q65	(2.00)	(1.21)	(1.03)	(0.10)	(0.10)	(1.89)			(4.42)		(1.89)			
1Q52	185.84	143.91	250.96	38.13	.155	-.300				-211.46	-.092	79.6	.726	1.85
4Q65	(1.55)	(1.58)	(3.58)	(0.90)	(2.60)	(4.50)				(4.89)	(2.43)			
1Q56	546.63	342.47	160.45	-37.33	.158	-.179				-269.90	-.126	73.1	.805	1.70
4Q65	(3.29)	(3.00)	(1.83)	(0.77)	(2.53)	(2.18)				(5.67)	(2.91)			

variables formerly of no use might now enter the equation in an appropriate manner. With this in mind I attempted to reintroduce the R03 bond yield in index form. It had the wrong sign and was insignificant, weakening the equation. But at least the new specification no longer draws assistance from wrongly signed cost variables. I also tried the treasury bill yield previously used in the first testing phase RTB (DB 601), the industrial bond yield (McLeod, Young, Weir IBY (DB 268)), and the yield on a sample of equities (Moss, Lawson EY (DB 2765)). Each of these was tested currently, with a one-quarter lag; and as a four-quarter moving average, first with equal weights then with declining weights (t-1 = .30, t-2 = .35, t-3 = .25, t-4 = .10). In addition, I tried allowing the unemployment index to come in multiplicatively with a four-quarter seasonal; I tried using a price difference term, the first difference of the GNE deflator both current and lagged once; and I tried the cash flow ratio variable, which had been used successfully in other experiments, to see if I could measure the response of inventories to pressures on working capital. The last formulation consists of cash flow, which is the sum of corporate retained earnings (DB 1393) and corporate capital consumption allowances (DB 3711), fitted to a linear trend from 1950 to 1965 and then divided by the series of trend values.

Results from these experiments are comparatively meagre. The financial variables are all wrongly signed except for the equations noted in Table 6. In general the interest rate variables seem to conform more closely to theoretical expectation when the shorter period is used; this agrees with Courchene's results on manufacturing inventory for the period 1955 to 1962. But the interest rates also perform better in conjunction with the equity yield, which is probably bringing in an expectations effect. On the whole, the few equations with correct a priori signs do not inspire much confidence, and they improve the fit of the equation very little. The experiments with a price term and with a seasonally-spread UI_t term produce insignificant results and weaker equations.

Thus efforts to extend the basic model led to few conclusive results. There are several new equations that cannot be rejected on the ground of a wrong a priori sign or on the ground of the weakening of the basic equation. Not one of the six has any clear-cut superiority over the others; although the equations in-

TABLE 6

Testing Financial Variables with 'Purged' GNE and Unemployment Index

	Constant	Q1	Q2	Q3	Y^P_t	ΔY^P_t	UI_t	H_{t-1}	CFR	RTB 4Q	CFR 4Q	EY 4Q	SEE	$\bar{R}^2$	D/W
Basic 1952-65	185.84	143.91	250.96	38.13	.155	-.300	-211.46	-.092					79.6	.726	1.85
	(1.55)	(1.58)	(3.58)	(0.90)	(2.60)	(4.50)	(4.89)	(2.43)							
1952-65	-0.15	113.50	197.29	11.95	.114	-.297	-200.79	-.065	188.28				78.9	.732	1.95
	(0.00)	(1.22)	(2.48)	(0.26)	(1.73)	(4.50)	(4.61)	(1.53)	(1.38)						
1956-65	515.60	311.08	146.25	-38.83	.142	-.180	-258.57	-.107		-15.34			73.7	.802	1.71
	(2.98)	(2.53)	(1.62)	(0.79)	(2.12)	(2.18)	(5.12)	(2.12)		(0.72)					
1952-65	645.86	82.36	254.29	57.92	.154	-.313	-144.82	-.106		-10.09	55.33	-89.62	77.8	.739	1.98
	(1.48)	(0.81)	(3.41)	(1.30)	(2.27)	(4.69)	(2.58)	(1.91)		(0.48)	(0.33)	(1.83)			
1956-65	1,026.08	203.67	138.34	-14.00	.135	-.198	-142.87	-.108		-24.26	34.92	-127.56	70.9	.817	1.89
	(2.23)	(1.51)	(1.37)	(0.27)	(1.67)	(2.41)	(1.94)	(1.60)		(1.09)	(0.18)	(2.09)			
Basic 1956-65	546.63	342.47	160.45	-37.33	.158	-.179	-269.90	-.126					73.1	.805	1.70
	(3.29)	(3.00)	(1.83)	(0.77)	(2.53)	(2.18)	(2.91)	(5.67)							
Omitting 1Q56 1952-65	140.12	50.57	251.25	45.55	.133	-.331	-198.81	-.073					73.2	.744	1.93
	(1.26)	(0.57)	(3.91)	(1.16)	(2.41)	(5.34)	(4.98)	(2.06)							
Omitting 1Q56 1956-65	431.12	210.27	171.28	-22.18	.132	-.232	-255.33	-.097					71.0	.795	1.75
	(2.46)	(1.56)	(2.01)	(0.46)	(2.12)	(2.71)	(5.44)	(2.14)							

cluding financial variables imply such trivial impacts that they are perhaps better ignored on grounds of simplicity.

More information has now become available from that most reliable of sources, the passage of time. Since the equations in Table 6 were developed, observations for 1966 and the first quarter of 1967 have been generated to provide a check on the validity of the final equations. This testing was carried out in several ways. First, the equations fitted from 1952 to 1965 were projected forward for five quarters. Then they were refitted to the end of 1966 and their performance checked over the same five quarters. These equations are given in Table 7, and as can be seen the coefficients differ very little indicating a stable structure into 1966. Examination of the residuals suggested that a major movement in the first quarter of 1956 might be twisting the regression plane, so this quarter was dropped and the basic equations refitted over 1952-65, 1952-66, 1956-65, and 1956-66. In all cases the residuals, the sums of squared residuals for 1966 and 1Q67, and the absolute sums of residuals for these periods were calculated. The results are shown in Table 8. Finally a naive test was performed by projecting forward the quarterly means of inventory investment over the various time periods and calculating the same residual test statistics for them. The projection test is questionable on one count; in that, for simplicity, H_{t-1} is taken as the actual H_{t-1} in each projected quarter rather than what H_{t-1} would be if predicted inventory investment were being cumulated. This procedure simplifies computations greatly without having much impact on the actual result.

Comparison of equation performance shows that the equations fitted to the end of 1966, though little different in structure, are better 'predictors' for 1966 and 1Q67. This is hardly surprising. Perhaps more interesting is the fact that dropping 1Q56 from the calculations has so little effect. As can be seen from Tables 6 and 7, the 'gap' tends to draw the 1956 equation closer to the 1952 structure. But the 'predictive' capacity of the gap equation is generally weaker. On almost all parallel tests, as shown in Table 8, the gap equations are inferior. In no case are they significantly superior; consequently this approach does not seem worth pursuing. As for our other financial variables, their insignificance in the equations is confirmed in the prediction results. In the 1952-65 and 1952-66 equations these variables

TABLE 7

Equations in Table 6 Fitted to 1966

	Constant	Q1	Q2	Q3	Y^P_t	ΔY^P_t	UI_t	H_{t-1}	CFR	RTB 4Q	CFR 4Q	EY 4Q	SEE	$\bar{R}^2$	D/W
Basic 1952-66	170.83	175.37	247.11	27.66	.153	-.268	-206.05	-.090					82.7	.713	2.04
	(1.59)	(1.87)	(3.56)	(0.65)	(2.64)	(4.03)	(4.74)	(2.35)							
1952-66	0.06	143.18	185.23	-0.25	.104	-.260	-196.38	-.060	191.08				81.9	.718	2.13
	(0.00)	(1.50)	(2.27)	(0.01)	(1.56)	(3.93)	(4.51)	(1.37)	(1.42)						
1952-66	474.70	124.75	251.73	46.22	.148	-.276	-152.91	-.099		-6.24	105.59	-71.58	81.3	.723	2.16
	(1.31)	(1.22)	(3.40)	(1.06)	(2.13)	(4.08)	(2.83)	(1.81)		(0.32)	(1.65)	(0.67)			
Basic 1956-66	417.54	380.64	180.99	-36.65	.177	-.159	-249.84	-.128					79.5	.771	1.93
	(2.83)	(3.10)	(1.99)	(0.73)	(2.73)	(1.85)	(5.05)	(2.74)							
1956-66	82.97	303.95	58.21	-80.41	.078	-.136	-223.87	-.057	271.62				78.9	.774	2.05
	(0.27)	(2.23)	(0.44)	(1.32)	(0.76)	(1.56)	(4.21)	(0.77)	(1.26)						
1956-66	712.07	223.99	128.44	-24.07	.115	-.158	-131.17	-.083		-21.11	174.23	-121.51	77.0	.785	2.21
	(1.85)	(1.59)	(1.25)	(0.47)	(1.33)	(1.81)	(1.85)	(1.22)		(0.97)	(0.95)	(2.07)			
Omitting 1Q56 1952-66	128.49	82.53	247.86	34.72	.130	-.299	-194.76	-.071					77.1	.727	2.15
	(1.27)	(0.89)	(3.83)	(0.88)	(2.39)	(4.77)	(4.79)	(1.95)							
Omitting 1Q56 1956-66	329.53	243.79	188.17	-23.88	.146	-.211	-239.14	-.098					77.6	.761	2.02
	(2.15)	(1.68)	(2.12)	(0.48)	(2.20)	(2.36)	(4.92)	(1.99)							

bring about some slight improvements, but in the 1956 equations they do more harm than good. Moreover it appears that what little they do contribute comes in through the equity yield and the cash flow ratio rather than the treasury bill rate. Since the equity yield reflects stock prices and the cash flow is highly correlated with profits, the 1956 equations appear to be picking up a weak expectational factor rather than a cost-of-finance impact — therefore there seems to be no gain in maintaining these forms.

The best equation on the basis of Table 8 is clearly the basic model fitted from 1956 to 1966. This equation has the lowest sums of absolute and of squared residuals over all five quarters, and is superior by a substantial margin in most cases. That it should dominate those equations fitted only to the end of 1965 is not particularly surprising — one is surprised to find that it is also the best of the 1966 group. Of equations fitted to the end of 1965, the 1956-65 equation is clearly better than the 1952-65, and is even slightly better than the 1956-66 equation in 1967. However the difference is hardly large enough to outweigh the dominance of the 1956-66 equation in 1966. Thus our equation of choice is:

(1956-66)

$$\Delta H_t = \underset{(2.83)}{417.54} + \underset{(3.10)}{380.64}\, Q_1 + \underset{(1.99)}{180.99}\, Q_2 - \underset{(0.73)}{36.65}\, Q_3$$

$$+ \underset{(2.73)}{.177}\, Y_t^P - \underset{(1.85)}{.159}\, \Delta Y_t^P - \underset{(5.05)}{249.84}\, (UI)_t - \underset{(2.74)}{.128}\, H_{t-1} \qquad (2.8)$$

$$SEE = 79.5 \qquad \bar{R}^2 = .771 \qquad D/W = 1.93$$

Comparing this equation to the naive quarterly mean projections shows that it comes out a long way ahead in 1966 and trails in 1Q67 in all cases. 1Q67 was a bad quarter for all our equations. But its superiority in 1966 outweighs this disadvantage — over these five quarters the 1952-66 quarterly means do best with $\Sigma u^2 = 81,477$ $\Sigma|u| = 555$. As against this, our equation fitted 1956-66 gives $\Sigma u^2 = 71,965$ $\Sigma|u| = 442$ despite its relatively much weaker performance in 1Q67.

TABLE 8 Part 1

Prediction Capacity of Sample Equations

		Basic Equation 1952-65	Current CFR 1952-65	MOV.AVG. CFR,RTB,EY 1952-65	Basic Equation 1956-65	MOV.AVG. RTB 1956-65	MOV.AVG. CFR,RTB,EY 1956-65	Basic Equation 1952-66	Current CFR 1952-66	MOV.AVG. CFR,RTB,EY 1952-66	Basic Equation 1956-66	Current CFR 1956-66	MOV.AVG. CFR,RTB,EY 1956-66
Predicted Value													
1966	1	494	515	498	444	448	443	478	491	464	457	477	446
	2	69	91	61	15	20	28	89	104	85	69	94	79
	3	46	45	38	-38	-30	-27	47	42	49	0	10	33
	4	149	180	117	95	107	85	149	170	129	129	171	124
1967	1	451	468	431	342	344	355	437	441	427	359	387	383
Residual													
1966	1	-114	-135	-118	-64	-68	-63	-98	-111	-84	-77	-97	-66
	2	-203	181	211	257	252	244	183	168	187	203	176	193
	3	-50	-49	-42	34	26	23	-51	-46	-53	-4	-14	-37
	4	-20	-51	12	34	22	44	-20	-41	0	0	-42	5
1967	1	-250	-267	-230	-141	-143	-154	-236	-240	-226	-158	-186	-182
Σu^2	66	57,105	55,988	60,353	72,457	69,288	65,970	46,094	44,030	44,834	47,001	43,053	42,999
u^2	67	62,500	71,289	52,900	19,881	20,449	23,716	55,696	57,600	51,076	24,964	34,596	33,124
$\Sigma \|u\|$	66	367	416	383	389	368	374	352	366	324	284	329	301
$\|u\|$	67	250	267	230	141	143	154	236	240	226	158	186	182

TABLE 8 Part 2

Prediction Capacity of Sample Equations

Predicted Value		Basic 1Q56OUT 1952-65	Basic 1Q56OUT 1956-65	Basic 1Q56OUT 1952-66	Basic 1Q56OUT 1956-66	Quarterly Means 1952-65	Quarterly Means 1952-65	Quarterly Means 1952-66	Quarterly Means 1952-66
1966	1	483	448	471	457	258	314	266	318
	2	63	21	86	72	15	-8	32	11
	3	49	-17	53	14	13	7	12	6
	4	152	114	154	143	43	86	49	89
1967	1	458	373	439	383	258	314	266	318
Residual									
1966	1	-103	-68	-91	-77	122	66	114	62
	2	209	251	186	200	257	280	240	261
	3	-53	13	-57	-18	-17	-11	-16	-10
	4	-23	15	-25	-14	86	43	80	40
1967	1	-257	-172	-238	-182	-57	-113	-65	-117
Σu^2	66	56,628	68,519	46,751	46,449	88,618	84,683	77,252	73,665
u^2	67	66,049	29,584	56,644	33,124	3,249	12,656	4,225	13,689
$\Sigma \lvert u \rvert$	66	388	347	359	309	482	400	490	373
$\lvert u \rvert$	67	257	172	238	182	57	113	65	117

Returning from the empirical results to the theory on which the experiments were based, (2.8) may be interpreted in the light of the model (2.7). Then $\rho = .159/.177 = .898$. This implies $S_t^e = .102\ S_t + .898\ S_{t-1}$ or relatively weak forecasting by inventory-holders. According to the model only 10 per cent of sales change is forecast, an implausibly low result much below that of Lovell for the U.S. [16] pp. 542-550. 'Rational' business forecasting as defined by Bossons and Modigliani [2] implies a $(1 - \rho)$ of less than one if fully rational forecasts have an expected value of zero for the difference between forecast and actual, or $E(u_t) = 0$ where $S_t^e - S_t = u_t$. Similarly Theil finds that the conditions under which $(1 - \rho) < 1$ are quite broad [22] pp. 154-161. But a forecast of only 10 per cent of sales change is implausibly low.

On the other hand, if (2.8) is regarded as fitting the model of (1.7') with a passive inventory term, equivalent to (1.7") with $c = 1$, then the estimate of ρ drops to .135, implying relatively good business forecasting or $S_t^e = .865\ S_t + .135\ S_{t-1}$. This makes the passive or production inflexibility model more plausible. In general, for (1.7"), $\rho = (.159)/(c + .177)$ where $0 \leq c \leq 1$ and the larger is c, the better is business forecasting implied to be by the equation.

The other parameters of the model are b, the reaction coefficient, and d, the marginal stock/sales ratio. b is rather low, implying a long adjustment period,[7] while the marginal stock/sales ratio is 1.38. This is rather high, but below the average stock/sales ratio of about 1.6. It is worth noting that the flow variable approximating sales, Y^P, is flow per quarter; this explains the size of the stock/sales ratio. The low value of b is disturbing, given that inventory is believed to be rapidly adjustable by the holder. Perhaps the very large and apparently random error in inventory movements (reflected in the Coefficient of Variation of 73.5 per cent and the Durbin/Watson statistic of 1.93 in the final equation) leads to great caution on the part of inventory-holders in accepting and moving toward equilibrium levels. The fact that inventories are volatile need not imply that they are easy to adjust swiftly; large random movements may make the adjustment process a slow and cautious one.

[7]If $b = .128$, and H^* is constant, 42.2% of any gap is closed in four quarters.

Perhaps the most disappointing feature of this equation from the point of view of a policy-oriented model is the consistent failure of all financial variables to yield any measurable impact on inventory investment that could be justified in theory. Despite the range of variables tested and introduced with a variety of lag patterns and index forms (from the treasury bill rate to the over ten-year government bond yield and the industrial bond yield, including credit availability and price variables) the most assiduous data mining yielded only the rocks presented in Tables 6 and 7. Tests of predictive capacity soon reduced these to powder. Financial variables may have the hypothesized impact on inventory investment, but if so it does not show up in any form I have yet tested.

There is, however, another possibility. The whole purpose of building a simultaneous model is to derive consistent estimates of the individual equation parameters by using estimation techniques which avoid simultaneous equations bias. The inventory equation might be expected to be particularly subject to this form of bias since multiplier effects can be expected to lead from inventory behaviour to the activity variables which are treated as independent in the Ordinary Least Squares (OLS) estimation. In fitting the simultaneous model, (2.8) was reestimated using a type of two-stage least squares procedure in which a subset of the exogenous variables is chosen as instruments according to a causal ordering hierarchy developed by F. Fisher. The procedure (called Structurally Ordered Instrumental Variables, or SOIV) is discussed in the overall RDX1 model paper. Estimating the equation in the form yields:

1Q56 - 4Q65 (SOIV)

$$\Delta H_t = \underset{(3.83)}{603.846}\, Q_1 + \underset{(3.37)}{518.098}\, Q_2 + \underset{(0.46)}{53.381}\, Q_3 - \underset{(1.46)}{86.778}\, Q_4$$

$$+ .135\, Y_t^P - .030\, \Delta Y_t^P - 260.046\, UI_t - .120\, H_{t-1} \qquad (2.9)$$

$$SEE = 76.7 \qquad \overline{R}^2 = .786 \qquad D/W = 2.01$$

The same equation estimated by OLS yields:

1Q56 - 4Q65 (OLS)

$$\Delta H_t = \underset{(3.81)}{627.323}\, Q_1 + \underset{(2.83)}{326.314}\, Q_2 + \underset{(1.92)}{160.728}\, Q_3 - \underset{(0.85)}{39.285}\, Q_4$$

$$+ \underset{(2.66)}{.160}\, Y^p_t - \underset{(2.26)}{.190}\, \Delta Y^p_t - \underset{(5.74)}{271.921}\, UI_t - \underset{(3.03)}{.135}\, H_{t-1} \quad (2.10)$$

$$SEE = 72.4 \qquad \bar{R}^2 = .809 \qquad D/W = 1.70$$

(The OLS equation differs slightly from that reported in Table 6 due to minor changes in the input data made for consistency with the simultaneous model.) Fitted by OLS, the 1956-1965 equation looks quite similar to the 1956-1966 equation. This implies $\rho = 1.188$ if $c = 0$, suggesting that some degree of production inflexibility must be present. Also d and b are little changed. The significant feature, however, is that in the SOIV results, the ΔY^p_t variable loses almost all significance. Its coefficient implies a value of $\rho = .222$, or business forecasting of 77.8 per cent of quarter by quarter change. The result would be very interesting, except that ρ is not significantly different from zero ($t = .22$). This could imply very high accuracy in business sales forecasts; it could also cast some doubt on the model structure. Efforts to model the formation of sales expectations may simply be inadequate, and this inadequacy may be masked by simultaneous equations bias in the OLS estimates.

The SOIV results do, however, provide strong confirmation for the financial variables findings. After fitting (2.9) a range of cost-of-capital variables was retested to see if the SOIV form might enhance their role. I used the 90-day finance company paper rate RCP (DB 1129), the average rate on Government of Canada bonds due in less than three years R03 (DB 1365), and the rate on Government of Canada bonds due or callable in ten years or more RLC (DB 2764) both current and lagged one quarter. In addition the same variables were used in the index form current and lagged. Some of the results are presented in Table 9 where the SOIV results confirm the OLS. Whatever the theoretical model implied by

TABLE 9

Final Equation Form Tested with Assorted Financial Variables, Single and Simultaneous Equation Estimates

(All equations include the eight variables of the chosen equation plus, in most cases, an interest rate variable.)

Financial Variable*	Coefficient on Financial Variable	t Value of Coefficient	Estimation Method	SEE	$\overline{R}^2$
NONE			OLS	72.4	.809
NONE			SOIV	76.6	.786
RCP	9.47	.65	OLS	73.0	.809
RCPR	.006	.00	OLS	73.5	.803
R03	13.08	.69	OLS	73.0	.806
R03R	-1.61	.11	OLS	73.5	.803
RLC	52.63	1.47	OLS	71.1	.815
RLCR	-1.33	.09	OLS	73.5	.803
RCP_{-1}	1.24	.09	OLS	73.5	.803
$RCPR_{-1}$	-7.52	.65	OLS	73.0	.805
$R03_{-1}$	.04	.00	OLS	73.6	.803
$R03R_{-1}$	-12.02	.87	OLS	72.7	.807
RLC_{-1}	33.99	.85	OLS	72.7	.807
$RLCR_{-1}$	-12.17	.72	OLS	72.9	.806
$RCPI_{-1}$	25.27	.49	OLS	73.2	.804
$RCPIR_{-1}$	-.01	.00	OLS	73.6	.803
$R03I_{-1}$	39.03	.62	OLS	73.1	.805
$R03IR_{-1}$	.038	.24	OLS	73.5	.803
$RLCI_{-1}$	-87.22	.44	OLS	73.3	.804
$RLCIR_{-1}$	.93	.33	OLS	73.4	.803
RCP	10.99	.66	SOIV	78.6	.774
RCPR	21.83	1.03	SOIV	81.8	.756
R03	15.11	.71	SOIV	78.4	.775
R03R	-1.50	.08	SOIV	77.9	.779
RLC	62.23	1.48	SOIV	76.8	.784
RLCR	7.04	.36	SOIV	77.9	.779
RCP_{-1}	-4.17	.26	SOIV	77.4	.781
$R03_{-1}$	-2.75	.12	SOIV	77.7	.779
$RCPI_{-1}$	22.60	.39	SOIV	77.0	.783
$R03I_{-1}$	.67	.00	SOIV	77.8	.778

* R03, RLC, R03I and RLCI are as used previously. RCP is the rate on 90 day prime finance paper (DB 1129). RCPI is the same in index form. The real interest rates R03R, RLCR, etc. are defined as:

$$RLCR = RLC - 100 \left[\frac{PGNE - PCNE_{t-4}}{PCNE_{t-4}}\right]$$

where pgne (DB 9153) is the implicit private CNE deflator used in the aggregate model RDX1.

the results may be, it seems fairly conclusive that no role is left for cost-of-capital variables. Table 9 also shows the results of using real rather than money rates of interest as financial variables. In almost all instances, the adjustment of interest rates for expected price changes (using the price change over the preceding four quarters as a proxy for expected price changes) improves the performance of the financial variables. However, it remains the case that none of them enter significantly, with the right sign, into either the OLS or the SOIV parameter estimates. A radically different structural specification might show some impact of financial variables, but in the various specifications tested in this study I was unsuccessful in isolating a significant influence of interest rates on inventory investment.

Postscript:

The conclusions stated above cannot be asserted too confidently since evidence has recently become available that casts suspicion on the whole structure of the aggregate flexible accelerator inventory model. This is an inevitable risk when a dynamic specification process is combined with long production lags. The basic work of this study was done in the summer of 1966 and the simultaneous model RDX1, in which the inventory equation belongs, has been developed considerably since then. In particular, a revision of the structurally ordered instrumental variables technique which changes the 'instrument package' by reducing the number of quasi trends, leads to an estimate of our basic model as follows:

1Q56 - 4Q65 (SOIV-B)

$$\Delta H_t = \underset{(5.68)}{610.6} + \underset{(3.05)}{366.8}\, Q_1 + \underset{(0.55)}{58.80}\, Q_2 - \underset{(1.54)}{83.63}\, Q_3 - \underset{(1.63)}{0.0830}\, H_{t-1}$$

$$+ \underset{(1.21)}{0.0855}\, Y_t^P - \underset{(0.95)}{0.1020}\, \Delta Y_t^P - \underset{(5.64)}{287.9}\, UI_t \qquad (2.11)$$

SEE = 74.62 $\qquad \bar{R}^2 = 0.797 \qquad$ D/W = 1.80

The extreme weakness of the coefficients on Y^P and ΔY^P, and the implausibly small value of b make it very difficult to interpret these coefficients as representing the structural parameters of the flexible accelerator model. No effort was made to refit equation (2.11) with financial or other cost variables, since their weakness in earlier experiments did not seem to be due to any relationship with the activity variables. When we compare (2.11) with (2.9) and (2.10), it appears that 1966 equations do not fit well into a 1969 model — thus the need for further research is amply demonstrated.

VARIABLES

DB — Numbers in brackets with the prefix DB refer to the index numbers of these series on the Databank Master Tape at the Bank of Canada. A master tape containing all series referred to in the Bank of Canada Staff Research Studies is available to the public.

Dependent Variable

ΔH_t — Change in Canadian non-farm inventories in 1957 dollars, not seasonally adjusted, in quarter t (DB 150).

Activity Variables

All are in 1957 dollars, not seasonally adjusted, unless otherwise specified.

H_t — Canadian non-farm inventory stock (DB 11636) at the end of period t, constructed by cumulating ΔH_t from the *National Accounts Income and Expenditure* (issued quarterly and annually by the Dominion Bureau of Statistics) on a base value of $5,185 million in 4Q46.

H_t^* — Equilibrium or desired level of non-farm inventories in period t (unobservable) defined as a function of sales, cost, and expectations variables to be determined in the study.

Y_t — Quarterly Gross National Expenditure (DB 157) used as a proxy for sales (S_t) hypothesized to influence expected sales (S_t^e), desired inventories, and actual inventories.

Y_t^* — The estimated value of Y_t found by fitting Y_t to a time trend quarterly from 1947 to 1965 in the form

$$\log Y_t^* = a_0 + a_1 t.$$

Y_t^P A 'purged' sales variable differing from Y_t in being built up from those components of Gross National Expenditure believed to be most closely related to sales of goods. Y_t^P is the sum of consumer durable and non-durable goods expenditures (DB 141, DB 140), business gross fixed capital formation (DB 144), total government non-wage expenditure on national accounts basis (DB 2171, DB 4079, DB 4104) and exports less imports (DB 153, DB 154).

Expectational Variables

$STOCK_t$ The difference between the Toronto Stock Exchange index of common stocks (DB 2597) and its log trend fitted from 1946 to 1965 in the form log stock index = $a_0 + a_1 t$.

EQR_t The ratio of the Toronto Stock Exchange index to its trend value, the same variables whose difference yields $STOCK_t$.

(APP/VAC) The ratio of unplaced applicants for employment (DB 3979) to reported unfilled job vacancies (DB 3965) across all industry divisions.

U_t The quarterly average unemployment level (DB 1202, DB 1203).

ΔU_t $(U_t - U_{t-1})$

UI_t $12\, U_t / \sum_{i=1}^{12} U_{t-i}$

Financial Variables

r Any general measure of the cost of finance.

RTB The average interest rate in quarter t on Canadian treasury bills of three months to maturity (DB 601).

R03 Average interest rate on Government of Canada bonds maturing in less than three years (DB 1365).

RLC Average interest rate on Government of Canada bonds due or callable in ten years or more (DB 2764).

i Any general financial variable in index form,

$$i_t = (\sum_{j=1}^{12} r_{t-j})/12\ r_t$$

$(L/A)_t$ The ratio of total business loans outstanding (DB 687) to the level of loan authorizations provided by the banking system (DB 608) both in quarter t.

IBY (Not reported in tables) The yield on a sample of industrial bonds prepared by McLeod, Young, Weir (DB 268).

EY The yield on a sample of selected equities prepared by Moss, Lawson (DB 2765).

CFR_t The cash flow ratio, calculated by summing corporate retained earnings (DB 1393) and corporate capital consumption allowances (DB 3711) and fitting these to a linear trend from 1950 to 1965. CFR_t is then the ratio of actual to fitted trend value both in quarter t.

CFR 4Q A four-quarter moving average of the variable with weights t-1 = .30, t-2 = .35, t-3 = .25, t-4 = .10.

RCP The average interest rate on 90-day finance company paper (DB 1129).

RCPR, RLCR, etc. Real interest rates, found by subtracting the rate of change of the implicit private GNE deflator from the nominal interest rate, e.g.

$$RLCR_t = RLC_t - 100\ [\frac{PGNE_t - PGNE_{t-4}}{PGNE_{t-4}}]$$

RCPI, RLCI, etc. Interest rates in the index form defined above for i_t. Note that they should have positive signs a priori.

Parameters

b The speed of adjustment in the flexible accelerator,

$$\Delta H_t = b(H^*_t - H_{t-1}) + \text{other variables.}$$

d The marginal quarterly stock/sales ratio; if expected quarterly sales increase by 1, desired inventory stock increases by d.

ρ, λ Parameters associated with alternative mechanisms of sales expectations formation. If expectations for period t are formed by forecasting the change ΔS_t and adding this to the level of last period, then

$$S^e_t = S_{t-1} + (1 - \rho)\,\Delta S_t = (1 - \rho)\,S_t + \rho S_{t-1}.$$

If expectations are formed from a moving average of past values with geometrically declining weights, then

$$S^e_t = (1 - \lambda)\,[S_t + \lambda S_{t-1} + \;\ldots\;, \lambda^n S_{t-n}]$$

In equation forms (1.8') and (1.7") only

c A production inflexibility or passive accumulation term which represents the proportion of the error in sales forecast $(S^e_t - S_t)$ which is added to or subtracted from inventories e.g. if c = 1.0, then production is not adjusted at all in period t to allow for errors in sales forecast and the whole forecast error is reflected in inventory change. (c is also used as a general constant in other equation forms, with and without subscript.)

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TRAVAUX DE RECHERCHE À LA BANQUE DU CANADA

ÉTUDE N° 2 - 1969

CANADIAN INVENTORY INVESTMENT*

par

R. G. EVANS

RÉSUMÉ

Cette étude représente une tentative d'intégrer en une seule équation les divers facteurs de causalité qui déterminent le taux d'investissement en stocks des entreprises non agricoles sur une base trimestrielle. Cette équation a été mise au point comme élément du modèle économétrique simultané de l'économie canadienne, RDX 1, élaboré au Département des recherches de la Banque du Canada. Les restrictions imposées par les données et le cadre du modèle RDX 1 ont conduit à l'adoption d'une formulation en une seule équation.

La formation des stocks est représentée par un mécanisme d'accélération flexible; notre discussion porte donc d'abord sur les fondements théoriques de ce concept et sa relation avec la notion de contrôle optimal des stocks. Dans le choix, pour le modèle, d'une structure spécifique qui tienne compte des retards et des attentes, on a accordé une attention spéciale au problème d'identification qui se pose lorsque différentes structures théoriques exigent que l'on estime à partir des mêmes données divers ensembles de paramètres. Même lorsque les estimations obtenues sont identifiables, elles ont tendance à être très instables par suite de légères différences dans l'échantillon. Comme les équations ajustées empiriquement aux mêmes données tendent de toute façon à être presque semblables, on a décidé de recourir surtout aux ajustements d'ordre empirique et d'analyser ensuite les coefficients ainsi obtenus afin de déterminer la gamme des structures hypothétiques susceptibles d'être raisonnablement soutenues par l'équation finale.

Notre travail empirique a d'abord porté sur une première spécification visant à expliquer l'investissement en stocks. Les variables explicatives utilisées dans cette relation sont: le niveau des stocks - obtenu par addition cumulative des flux trimestriels à une valeur de base - le niveau trimestriel ainsi que la variation trimestrielle de la Dépense Nationale Brute pour représenter les valeurs actuelle et anticipée des ventes et les taux d'intérêt sur des avoirs à échéances diverses pour mesurer le coût des capitaux immobilisés dans les stocks. L'ajustement de cette formulation s'est avéré peu satisfaisant et les coefficients des variables de taux d'intérêt ont été contraires à nos attentes. On a alors apporté

(Suite au verso)

un raffinement à la variable servant à représenter les ventes, en déduisant de la Dépense Nationale Brute toutes les composantes qui ne représentent pas une dépense en biens; en outre, les variables de taux d'intérêt furent divisées par la valeur retardée de leurs moyennes mobiles - calculées sur douze trimestres à pondération uniforme - ce qui donnait, pour représenter le coût de financement des stocks, un indice dépourvu de toute tendance à long terme. L'équation s'est alors améliorée, mais les coefficients des variables de taux d'intérêt sont demeurés non significatifs ou affectés de signes erronés a priori, ou même les deux. D'autres expériences ont été faites en utilisant le rapport entre les prêts en cours accordés aux entreprises et les montants autorisés; cette méthode nous a cependant donné une fonction de demande plutôt que d'offre. Un accroissement des stocks avait tendance à se traduire par une augmentation des autorisations, tandis qu'une réduction de celles-ci n'entraînait pas une compression des stocks.

Comme, de toute évidence, l'évolution des ventes dans le passé ne permettait pas de prévoir de manière satisfaisante le volume des ventes dans l'avenir, on a procédé à de nombreuses expériences avec des variables prévisionnelles basées sur les cours des actions, les bénéfices des sociétés et les taux de chômage. L'utilisation d'un indice de chômage - calculé en divisant le taux courant par la valeur retardée de sa moyenne mobile sur quatre trimestres avec pondération uniforme - a permis d'obtenir les meilleurs résultats. L'utilisation de cet indice a permis d'améliorer sensiblement l'ajustement de l'équation. Cependant, tous les efforts visant à introduire des variables de taux d'intérêt, sous forme réelle ou d'indice, ont été caractérisés par l'insuccès, quelle que soit la méthode d'estimation utilisée. On a, en outre, effectué plusieurs expériences avec des variables de prix, mais les résultats n'étaient pas significatifs. L'équation, sous sa forme finale, présente l'investissement trimestriel en stocks comme étant fonction du niveau courant et de la première différence de la variable servant à représenter les ventes, de l'indice de chômage et de la valeur retardée du stock.

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*Cette étude présente les travaux de recherche à la base de l'équation relative à l'investissement en stocks des entreprises utilisée dans le RDX 1 - modèle économétrique trimestriel de l'économie canadienne construit au Département des recherches de la Banque du Canada. Les opinions exprimées sont celles de l'auteur et n'engagent en rien la responsabilité de la Banque.

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BANK OF CANADA STAFF RESEARCH STUDIES

1969

No. 1 Quarterly Business Capital Expenditures

R. G. Evans John Helliwell