Bank of Canada
Technical Reports

Rapports techniques
Banque du Canada

June 1978

Technical Report 13

AN ANALYSIS OF THE MAJOR DYNAMIC PROPERTIES OF RDX2

L. de Bever, U. Kohli
and
T. Maxwell

Research Department

CONTENTS

ABSTRACT

In this report the authors study the dynamic response of the latest version of the RDX2 econometric model. The response of the model to a fiscal shock - a permanent increase in the level of federal government nonwage expenditure - is analyzed in detail under different monetary and exchange-rate conditions. The primary aim is to isolate those features of RDX2 that contribute most to the magnitude and timing of the model's response. This is accomplished by starting with a highly simplified version of the model and then successively reintroducing sectors to obtain a measure of the marginal impact of the last component added.

An extended IS-LM model is used as a theoretical framework for the exposition of the results.

RÉSUMÉ

Dans ce rapport, les auteurs étudient la dynamique de la plus récente version du modèle économétrique RDX2. Ils analysent en détail la réaction du modèle à un choc budgétaire - une augmentation permanente des dépenses du gouvernement fédéral autres que celles imputables aux traitements et salaires - selon diverses hypothèses relatives à la politique monétaire et au régime des changes. Le principal objectif consistait à isoler les éléments de RDX2 qui contribuent le plus à déterminer l'ampleur et le profil temporel de la réaction du modèle. Les auteurs sont parvenus à déterminer l'impact marginal des principaux éléments du modèle en les ajoutant successivement à une version extrêmement simplifiée du RDX2.

Aux fins de présentation des résultats, les auteurs utilisent comme cadre théorique une version élargie du modèle IS-LM.

INTRODUCTION

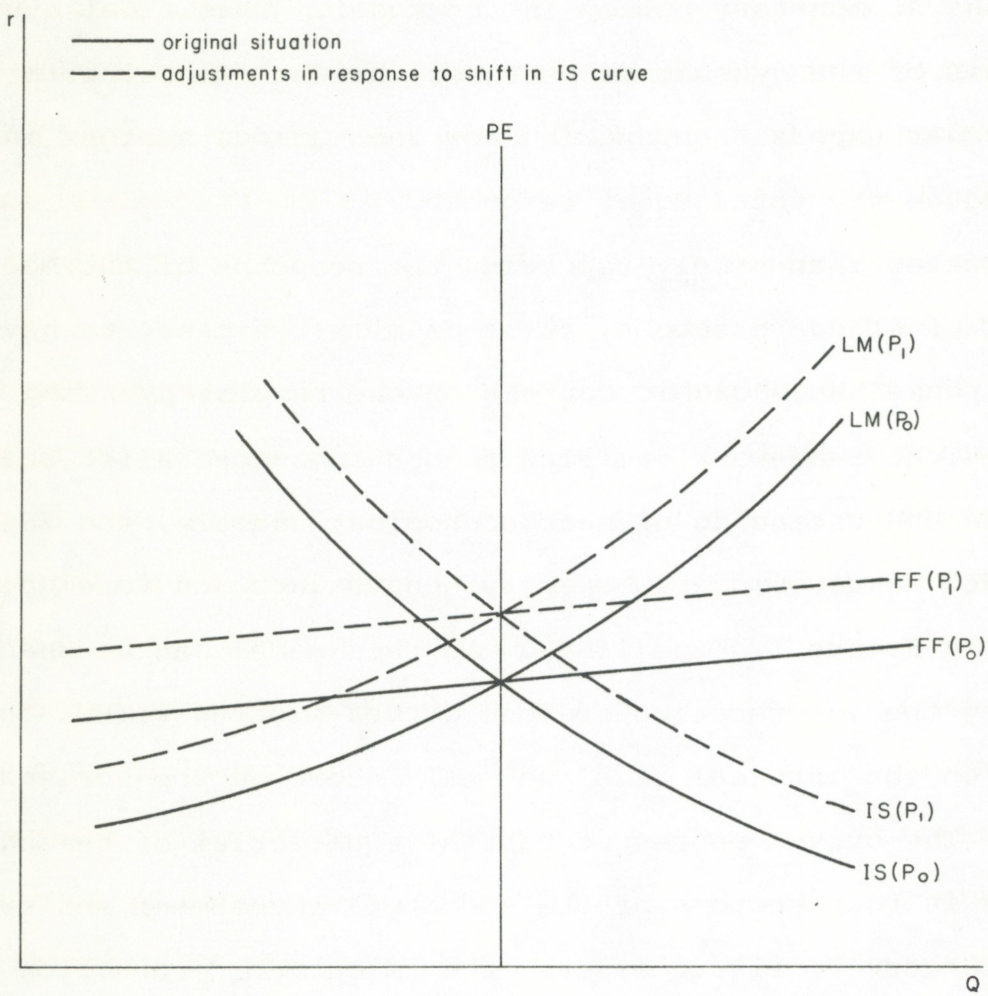
This is the fourth and final report in a series of studies dealing with the latest version of the RDX2 econometric model. The first report (Bank of Canada, 1976) deals with the equations themselves, the second (Bank of Canada, 1977) is a discussion of the theory and the partial dynamics of the sectors of the model, and the third (Freedman and Longworth, 1975) deals with the workings of monetary policy in the model. This final study is an analysis of the dynamic response of the whole model with particular emphasis on showing how the various sectors interact to produce the total model response.

Rather than simply reporting the response of the model to a number of standard shocks, as is usually the case, we have tried to increase the economic content of the results by using a theoretical framework familiar to most non-specialist readers with an understanding of macroeconomics; namely, the IS-LM model extended to accommodate capacity constraints and foreign trade as shown in Figure 1. Equilibrium in this model can be analyzed by finding the intersection of four curves plotted with interest rates on the vertical axis and real income on the horizontal axis. The curves represent partial equilibrium of demand and supply in four sectors of the economy: investment and savings (the IS curve), liquid assets (the LM curve), production and employment (the PE curve), and foreign trade (the FF curve).

Although RDX2 is much more complex than this simple model, for purposes of exposition a shock to RDX2 is represented by a

Figure 1

EQUILIBRIUM IN AN EXTENDED IS-LM MODEL WITH A VERTICAL PE CURVE



shift of one or more of the curves. In other words, we regard RDX2 as an empirical realization of the IS-LM-FF model, thereby creating a useful shorthand for explaining the dynamic behaviour of the model. In this study we concentrate on the effect of an initial movement in either the IS or the FF curve in terms of induced shifts in the remaining curves. We do not consider the case of a simultaneous initial movement of two or more curves. Since the response of RDX2 to a monetary shock, i.e., an initial movement of the LM curve, has been dealt with elsewhere (Freedman and Longworth, 1975) we keep the LM curve fixed in this study (apart from movements induced by price changes) by keeping M1A at its control values. Finally, we do not consider exogenous shifts in the PE curve.

The report is divided into four sections. In the first, by providing a simplified overview of RDX2, we attempt to put some flesh onto the bare bones of the IS-LM-FF model and to point out those structural features of RDX2 that have a bearing on its dynamic properties. Because RDX2 is a large model those parts of the model (in particular the government sector) that do not play an important part in the experiments described in this report are not treated in detail. The second section is an analysis of the effects of a shift of the IS curve (occasioned by an increase in government expenditure), initially with the FF curve fixed (i.e., with all international links suppressed), and then with the FF curve endogenous. Because of the importance of the FF curve in determining the response of the model, the third section of the report is an elaboration on the role of the FF curve, examining the impact of shifts of this curve. Since in the previous sections we concentrate on the response of the model as sectors are added one by one, in the last section we round out the analysis by providing a detailed examination of the response of the complete model to a shift in the IS curve.

1 A SIMPLIFIED OVERVIEW OF RDX2

The kernel of RDX2 is a relatively simple equilibrium growth model, which takes into account stock and flow equilibrium as well as equilibrium between demand and supply. However, it is not assumed that all markets clear or that the system is in equilibrium in every period. The model therefore represents an empirical realization of the adjustment process towards a new position of equilibrium. In this section we consider the model to be a set of equilibrium relationships, leaving the empirical representation of these adjustment processes (except those central to an understanding of the model) to be discussed in subsequent sections. To do this we have made liberal (but we hope judicious) use of Occam's razor.

The central variable in RDX2 is real business product (UGPP),[1] which is defined as the sum of personal consumption (CON), business fixed investment (IBUS), residential construction

---

1. Where possible, we have used RDX2 mnemonic notation. However, when these were not available, we created mnemonics following the schema outlined in Bank of Canada Technical Report 5, pp. 9-11.

(IRES), government nonwage expenditure (GNW) and net exports (X-M):

UGPP = CON + IBUS + IRES + GNW + X - M.

The supply side of RDX2 is characterized by a Cobb-Douglas production function with constant returns to scale, which explains UGPP in terms of the factor inputs, capital (K), man-hours (NH), and technical progress. A distinction is made between long-run technical progress (ELEFF), which is assumed to be exogenous and labour-augmenting, and short-run productivity which is endogenous.

This explicit, albeit aggregate, treatment of the production process enables us to integrate more fully the elements of demand and supply into the market for real goods. The production function serves two purposes in the model: first to define a series of output measures; second, in conjunction with an assumption of cost minimization, to derive desired factor inputs.

If UGPPS is defined as the output from the production function at existing rates of factor utilization - i.e., UGPPS = F(K, NH*ELEFF) - the production function can be used to split out the contribution of short-run productivity from UGPP. The change in inventories is explained in terms of the change in business output broken down into the major expenditure categories. In RDX2 any change in inventories is divided into an intended and an unintended component (neither of which are directly observable) by assuming that the part of the change in inventories that is induced by the portion of the change in business output directly attributable to short-run productivity, corresponds to the unanticipated component.

Under the assumption that over a long period the economy has, on average, been operating at its desired level of capacity utilization, the series UGPPD, defined as F(K, $\overline{NH}$*ELEFF) where $\overline{NH}$ is the average labour input over the sample period, provides a measure of the output that would be produced if all factors were being used at their desired rates. Thus the ratio of UGPPA (UGPP purged of unintended inventory change) to UGPPD provides a measure of capacity utilization, CAP.

An interrelated factor demand network is explicitly introduced into RDX2 by assuming cost-minimizing behaviour on the part of entrepreneurs. The costs minimized in determining the desired capital/output ratio (KYD^d) associated with a decision to increase output are the expected labour cost (WH^e) and the expected rental price of capital (RCK^e). The latter is the price the entrepreneur would have to pay in the market to obtain the services of physical capital and is a function of the real supply price of capital (RHOR) and the corporate tax rate (RTC). Investment is then determined (with the appropriate realization lag) by a forward-looking, flexible-accelerator vintage model (i.e., putty-clay) on expected UGPPA. If $\overline{KYD}$ is a weighted average of past capital/output ratios (to capture the vintage aspect), we can write

$$IBUS = f(KYD^d, (UGPPA^e - K/\overline{KYD})).$$

The same cost-minimizing behaviour should be applicable to the decision to hire new labour. However, it was found that for the economy as a whole, the technological constraints in the production process were dominant. Thus, once the desired amount of new investment has been determined, the associated labour input (NHD) is determined by the magnitude of the increase in output and the technology of the production process, i.e., by an inversion of the production function,

$$NHD = F^{-1} (K, UGPP, ELEFF).$$

Actual labour input adjusts with a lag to this desired level. The model distinguishes between the number of workers and the hours worked, principally in terms of their relative speeds of adjustment, a point that is discussed in Section 2.

Prices are determined for each main category of aggregate domestic demand and for imports and exports. These prices are then weighted by expenditure weights to obtain the gross national expenditure deflator (PGNE), the business output deflator (PGPP), and by the appropriate weights to obtain the consumer price index (PCPI). Domestic prices are based on a markup on unit labour costs and capital costs, which have been normalized to exclude the effects of short-run productivity. In addition, import prices, the state of capacity utilization, and a cumulant of unintended inventory changes, which represents stock disequilibrium, play an important role. Thus for a typical price "P":

P = f(NH*WH/UGPPS, K*RCK/UGPPS, PM, CAP, UIC)

where

NH*WH/UGPPS is normal unit labour costs

K*RCK/UGPPS is normal unit capital costs

PM is import prices

CAP is the ratio of UGPPA to UGPPD

UIC is a cumulant of unintended inventory changes.

Import prices are simply the Canadian equivalent of foreign prices, i.e., PM = PFX*PF where PF is the foreign price and PFX is the exchange rate in Canadian dollars per U.S. dollar. Export prices (PX) are a function of the price of domestic output and the Canadian dollar equivalent of foreign prices:

PX = f(PGPP, PFX*PF).

Still on the supply side of the model, the supply of labour (NHS) is explained by demographic factors (DEM) and the state of capacity utilization (CAP), which is regarded as explaining movements in the participation rate, thus

NHS = f(DEM, CAP).

Immigration depends on the Canadian unemployment rate and the relationship between real wages in Canada and those in the rest of the world. Emigration depends on the relationship between the Canadian and the U.S. unemployment rates.

The theory underlying the determination of private-sector wages is that in the long run workers bargain for and obtain a real wage that rises *pari passu* with the trend rate of productivity. However, in the short run they may be prevented from attaining their target by labour-market conditions and unanticipated inflation. Conversely, an unexpected decrease in the rate of inflation or tight labour-market conditions may result in workers exceeding their real wage target. Thus,

WH = f(WH/PCPI, ELEFF, NHD, NHS).

Consumption is explained in RDX2 by a permanent-income hypothesis and to a limited extent by the market value of private-sector wealth (V), which consists of the sum of equities and corporate bonds, high-powered money, government bonds held by the public, and net claims on foreigners. A distinction is made between two major sources of permanent income: that derived from wages (YW) and that derived from nonwage sources (YNW). The latter has its theoretical basis in components of V and provides the major link between wealth and consumption. Government transfers to persons (GT) also enter the consumption function. Since all these variables are allowed to take on different marginal propensities to consume, then if RTP is the personal tax rate,

CON = f[(1-RTP)(YW, YNW, GT), V].

Investment in housing is linked to mortgage approvals, which in turn are explained by the change in the assets of financial intermediaries and by the differential between the mortgage rate and the bond rate. The change in the assets of banks and of trust and mortgage loan companies is explained basically by the change in the demand for loans, which in turn depends on interest rates and activity variables. The change in the assets of life insurance companies is explained by changes in permanent income. Thus, for our purposes, it is sufficient to regard mortgage approvals as determined by the size of the financial sector, which in turn depends on income (both permanent and current) and on the interest rate.

The import equations are cast in the framework of consumer-demand theory. By solving the profit-maximizing conditions in terms of the final demands from which they are derived, this approach can be extended to cover the case of intermediate goods which comprise a large proportion of imports. Thus imports are determined by the various categories of expenditure (proxied here by UGPP), the ratio of domestic prices to foreign prices, and the state of capacity utilization. The latter term recognizes the "safety valve" role of imports when demand is strong relative to productive capacity. Hence,

M = f(UGPP, P/PFX*PF, CAP).

Exports are driven by foreign activity (FA) and relative prices:

X = f(FA, PX/PFX*PF).

Although the major categories of government expenditure (both federal and provincial-municipal) are explained in RDX2, their diverse nature does not permit a simple unified explanation; hence, we make no attempt to expand on this sector in this paper.

On the income side of the accounts, the three main components are wage income (YW), corporate income (YC), and other income (YOTHER) which encompasses all remaining income components. The variable YW is the sum of private wages and government wages (GW), i.e., YW = NH*WH + GW ; YOTHER is represented by a collection of equations, befitting its varied nature. Corporate income is determined residually.

In the government sector the most important concept is the government balance (GBALF) defined as

GBALF = RTP*YP + RTC*YC - GT - GW - GNW

where personal income (YP) is defined as the sum of YW and the personal components of YOTHER. The financing requirements of the government enter directly into the financial sector.

The main function of the financial sector is to determine the balance sheets of the major financial institutions. The liabilities of these institutions are specified as part of a constrained-portfolio model covering all major liquid assets held by the nonfinancial public (ANFLIQ), an important component of which is the debt associated with the financing of the government deficit. Because of the diversity of the determinants of items on the asset side of the balance sheet, they are explained by a series of independent equations and therefore do not admit to a unified explanation.

In the experiments reported in the subsequent sections, where M1A (a component of ANFLIQ) is constrained to follow its control solution values, the short-term interest rate is adjusted by a simulation rule to the level necessary to ensure that the monetary target will be met (see the Appendix for details). Since this is conceptually equivalent to renormalizing the demand-for-money function, then

RS = f(ANFLIQ, YGNE, PGNE, M*)

where

PGNE is the GNE deflator, and

M* is the control solution value of M1A.

Long-term interest rates (RL) are explained by long-term U.S. interest rates (RLUS) and by a term-structure relationship on Canadian short-term rates,

RL = f(RLUS, RS).

Another major function of the financial sector is to determine the real supply price of capital (RHOR). This variable presents a special problem since it is not directly observable. It is obtained by assuming that the market value of the capital stock (v) is the present discounted value of the expected future earnings of capital (which in the absence of any information to the contrary is assumed to grow at a constant rate g from its present value x). If g is assumed to equal the expected rate of change in the price level then

$v = x/(r-g)$

where

r represents the theoretical measure of the supply price (compared with the empirical representation (RHO) in RDX2).

The above formulation is underdetermined as there are two unkowns and only one equation. To obtain an empirical measure of RHO, we assume equilibrium in the markets for equities and government bonds. Then, assuming that the risk premium remains constant, the difference between RHO and RL can be expressed as a function of the relative supplies of the respective instruments, measured by the ratio of after-tax corporate profits and the flow of interest payments (h) from the government debt. By subtracting the expected rate of inflation (g) from RHO, RHOR can be obtained.

RHOR = f[(1-RTC)*YC, h, RL, g].

A portfolio model provides the underlying theory for international capital flows in RDX2. Since, with the exception of foreign pay issues, Canadian securities do not represent a distinct commodity with separate demand and supply equations, there are two relevant portfolio magnitudes: the stock of borrowers' debts and the stock of lenders' assets. The resulting equations encompass both borrower and lender behaviour because they contain elements of the relevant supply and demand variables. If LSCAN is the scale variable in the domestic liability portfolio and LSF the scale variable in the foreign asset portfolio, and if the appropriate rates of return are RL and RLUS, then (ignoring first differences) a typical equation for long-term capital flows (FIL) would be

FIL = f(LSCAN, LSF, RL, RLUS).

The exchange rate (PFX) is regarded as the price that clears the balance of payments once short-term capital flows induced by exchange-rate expectations have stabilized. However, instead of explicitly estimating the individual components of the balance of payments and then solving for the implicit exchange rate, the exchange rate has been explicitly estimated with the remaining balance-of-payments component (short-term capital flows) implicitly determined. Hence,

PFX = f[(X*PX - M*PM + FIL), RS, RSF, LSCAN, LSF]

where

RSF is the short-term foreign interest rate.

2 THE FISCAL MULTIPLIER IN RDX2

In this section we analyze the dynamic response of RDX2 to a fiscal shock - a permanent increase ($400 million per annum in constant 1961 dollars) in the level of federal government nonwage expenditure distributed over current expenditures, investment in machinery and equipment, and non-residential construction in the same proportion as the total 1961 outlays on these items. (Details of the shock are contained in the Appendix). Our primary aim is to isolate those features of RDX2 that contribute most to the magnitude and timing of the model's response. Our starting point is a simplified version of RDX2, exogenized to the point at which it resembles a textbook macromodel of the type described by Solow (1956) or Tobin (1965). We then successively reintroduce the RDX2 equations (or sectors) to obtain a crude measure of the marginal impact of the last component added. In principle, this procedure is not order independent because of the high degree of simultaneity in the model and the resulting possibility of interaction effects. Fortunately, the quantitative results do not appear to be affected significantly by a reversal in the order in which the components are introduced.

The RDX2 model, with more than 300 endogenous variables, gives rise to significant problems of presentation of results. We adopted the GNE multiplier, measured as the ratio of induced constant-dollar GNP to the magnitude of the shock (also in constant dollars), as the best single summary statistic for reporting the results of our analysis of the partial models in this section. We use the IS-LM model, developed by Hicks (1937) and extended to the case of an open economy by Mundell (1960), as a simple framework for understanding the results of this exercise. The elements of the extended IS-LM model are shown in Figure 1, and since a detailed description of the IS-LM model can be found in most intermediate macroeconomic textbooks (e.g., Branson and Litvak, 1976) the following summary is brief.

The extended IS-LM model describes equilibrium in the economy as a whole in terms of mutual consistency of partial equilibrium in the markets for goods, money, labour, and foreign exchange. Because of Walras' law, the fifth market (for bonds) will be in balance if the other four are. Partial equilibria are depicted as curves in a diagram with "the" interest rate (r) on the vertical axis, and output (Q) on the horizontal axis. If all relevant variables other than interest rate and output are constant, one can trace a locus of combinations of Q and r that satisfies equilibrium conditions in each market. If variables other than Q and r are allowed to vary, the curves describing equilibrium in each market in the IS-LM model will shift.

In the IS-LM diagram (Figure 1), the IS curve shows the interest rate at which consumers and investors can be induced to absorb a given level of real output. The IS curve is downward sloping, indicating that interest rates must be lowered to induce consumers and investors to increase spending.

The LM curve describes equilibrium in the market for liquid assets. If the price level and the supply of money are fixed, and the demand for money is assumed to be positively related to income and negatively related to interest rates, the LM curve will be upward sloping. An increase in the price level, not accompanied by an increase in the money supply, will reduce the supply of real cash balances, which will cause the LM curve to shift upwards. A decrease in M1 has the same effect, assuming that the price level remains unchanged.

In the simplest IS-LM model, M1 and P are fixed, and equilibrium is determined at the intersection of the IS and LM curves. An exogenous upward shift of the IS curve will (ignoring the supply constraints discussed below) result in a new equilibrium where both output and interest rates are higher.

The PE curve describes equilibrium between production and employment. Assume that the country's technology can be represented by an aggregate production function, with man-hours (NH) and capital (K) as primary inputs. Firms hire labour until the physical marginal product of labour equals the real wage. Increases in long-run or short-run factor productivity shift the labour-demand curve upwards. The supply of labour is assumed to be a positive function of the real wage; however, if workers are subject to money illusion, the price level should enter the supply function as a separate variable. In the absence of money illusion, the production function and the labour-supply curve determine a unique level of Q, resulting in a vertical PE curve.

Simultaneous equilibrium in the three markets discussed thus far is brought about by price adjustment. For instance, if the PE curve is vertical, an exogenous shift in the IS curve will result in intersection with the PE curve at the same level of Q but at a higher level of r. To re-establish over-all equilibrium, the price level must rise, shifting the LM curve inwards. If the PE curve is upward sloping, the new equilibrium will be characterized by a higher level of output, as well as a higher price level and interest rate.

In the case of an open economy, the foreign exchange market must be included in the analysis. The balance of payments consists of the balance of trade (net exports, a negative function of the domestic price level and income, and a positive function of the exchange rate) and of the capital account (net capital inflow, an increasing function of domestic interest rates). Equilibrium in the balance of payments can be represented by a positively sloped FF curve that may be steeper than the LM curve. An increase in the price level or an appreciation of the exchange rate shifts the FF curve upwards. If the exchange rate is flexible, equilibrium requires that the FF curve pass through the intersection of the IS and the LM curves. If the exchange rate is fixed, however, this requirement (that all three curves pass through the same point) does not necessarily hold as official interventions will have to match any disequilibrium in the balance of payments. An FF curve to the right of the intersection of the IS and LM curves would indicate a balance-of-payments surplus.

Because the extended IS-LM model is essentially a tool of comparative statics, it might seem paradoxical that it is used here to analyze the dynamic properties of RDX2. We feel, however, that although the use of this model may sometimes be cumbersome, it is sufficiently well anchored in economists' minds to be a useful reference point.

If we look ahead to the multiplier results for the full RDX2 model, presented in Figure 13, we see that RDX2 exhibits a noticeably cyclical response to a fiscal shock. Samuelson (1939) has shown that as few as two lags in a simple multiplier-accelerator model may produce cycles in response to a sustained government shock. Since most stocks, flows, and prices require considerable time to reach new equilibrium levels following a shock such as that under discussion, RDX2 contains equations estimated with lags and consequently many potential sources of cyclical behaviour. However, the realization that lags may interact to produce cycles is of little value in identifying the model components that contribute most to this phenomenon. Therefore, this section of the study is an attempt to identify and explain the reasons underlying this cyclicity.

We start with model 0, a version of RDX2 in which factor prices and output prices adjust instantly to a change in demand. The economy is closed by fixing all trade variables at their historical levels. In the same manner, we exogenize the stock of money, the supply of inputs, inventories, factor productivity, and desired outputs and inputs. In this system, supply is insensitive to the interest rate. Equilibrium in the IS-LM model occurs at the point where the vertical production-employment curve intersects with the IS curve.

We did not simulate model 0 because we know that its GNE multiplier in response to a fiscal shock is zero. The IS curve shifts outwards, but real income will remain unchanged and rising prices will cause the LM curve to shift inwards, resulting in a new equilibrium characterized by a higher interest rate and price level. At this new equilibrium, new public expenditures are accommodated through reductions in other demand components.

In each subsequent model (described in detail in the Appendix) we substitute RDX2 formulations for the hypothesized instantaneous adjustment of prices and wages, as well as for the immobility of the factor stocks and trade flows assumed above.

The order of introduction is:

(1) Prices and interest rates are determined by RDX2 equations. Labour earnings increase pari passu with productivity and prices.

(2) Lags in the response of wages to price increases are activated.

(3) Labour inputs respond to changes in activity.

(4) Investment equations determine the augmentation of capital stocks.

(5) Capacity adjusts to previous activity levels.

(6) Inventory equations are reintroduced.

(7) Trade becomes endogenous.

(8) The equations of RDX2 determine capital flows.

(9) The exchange rate is allowed to float.

Since constant returns to scale are assumed in the production function for business output discussed in Section 1, it must be that the long-run impact of a government shock is only positive to the extent that it induces a long-term upward shift in productivity or growth in factor inputs.

In keeping with our choice of the extended IS-LM model as an expository tool, we concentrate initially on the partial dynamics of a closed economy (i.e., models 1-6, where the elements of the FF curve are assumed to be exogenous) and then on the dynamics of an open economy (i.e., models 7-9, where the elements of the FF curve are successively introduced). The results of the first set of experiments are presented in Figure 2; those of the second set in Figure 3.

In model 0 prices and interest rates are assumed to respond instantly: in model 1 RDX2 equations determine prices and interest rates. In the face of a constant nominal money supply, the short-term interest rate (RS) moves upward to choke off increased demand for liquid assets. Other interest rates react to RS with a lag, and this in turn has a weak secondary effect on prices via a change in normalized unit capital costs. As in model 0, and contrary to the full RDX2 model, we assume that any increase in productivity is immediately reflected in normalized unit labour costs. The other variables in the "typical" RDX2 price equation (P) described above are still exogenous. Since adjustments are no longer intantaneous, equilibrium of demand and supply is no longer assured. In RDX2, the resulting excess demand is assumed to be satisfied by a rise in short-run productivity, which allows the production-employment curve to shift outwards, at least in the short run. Total demand in model 1 initially increases by almost the amount of the government shock. Induced demand from increases in real disposable income raise the multiplier above one until price increases bring it down asymptotically to zero because of unchanged productive capacity. Hence, model 1 is asymptotically equivalent to model 0, the neoclassical polar case of the IS-LM model.

In model 1 nominal wages adjust instantly to the value of

Figure 2

REAL GNE MULTIPLIERS GENERATED BY AN INCREASE IN GOVERNMENT EXPENDITURE, OBTAINED BY SEQUENTIAL INTRODUCTION OF THE MAJOR COMPONENTS OF THE IS CURVE

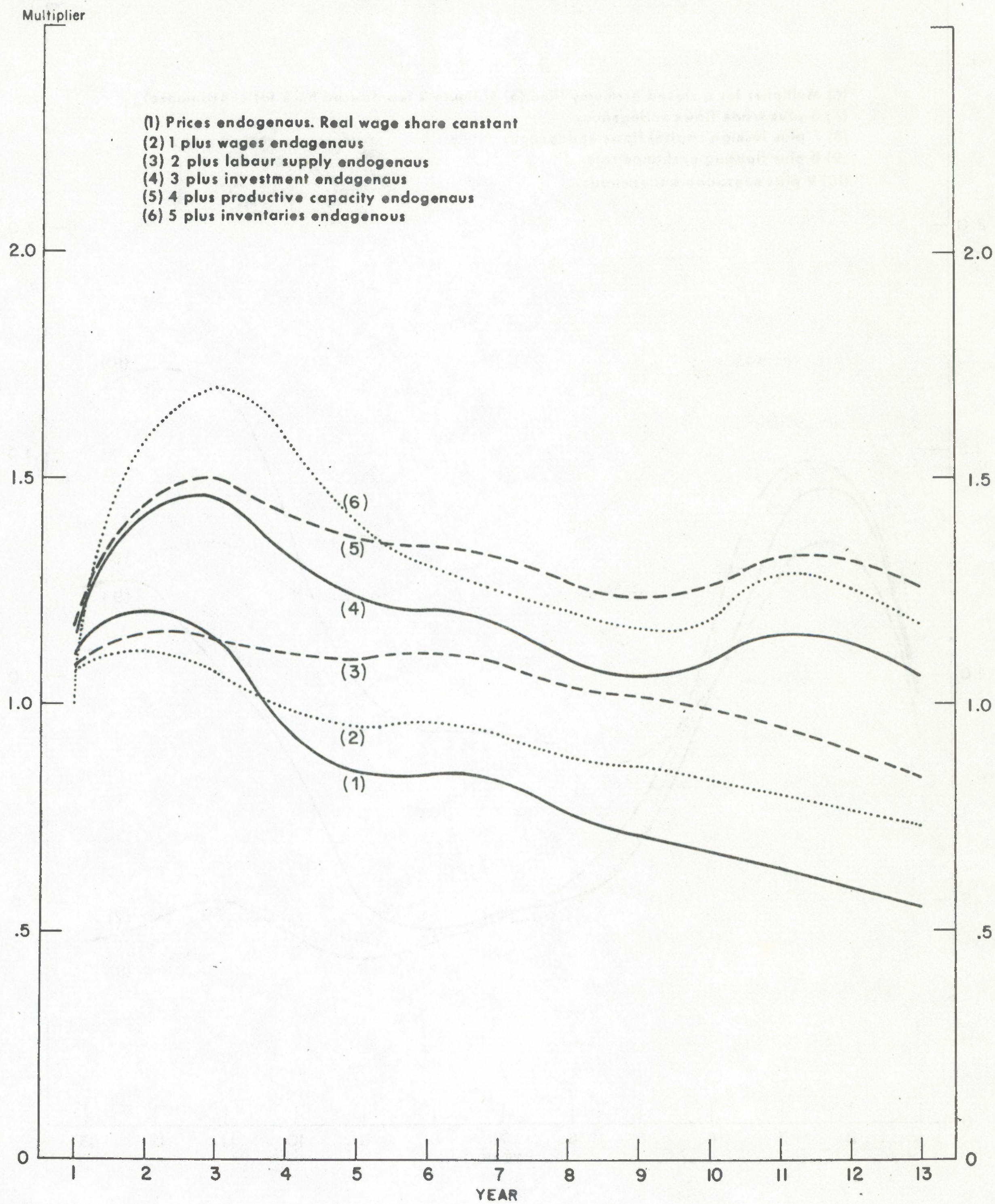
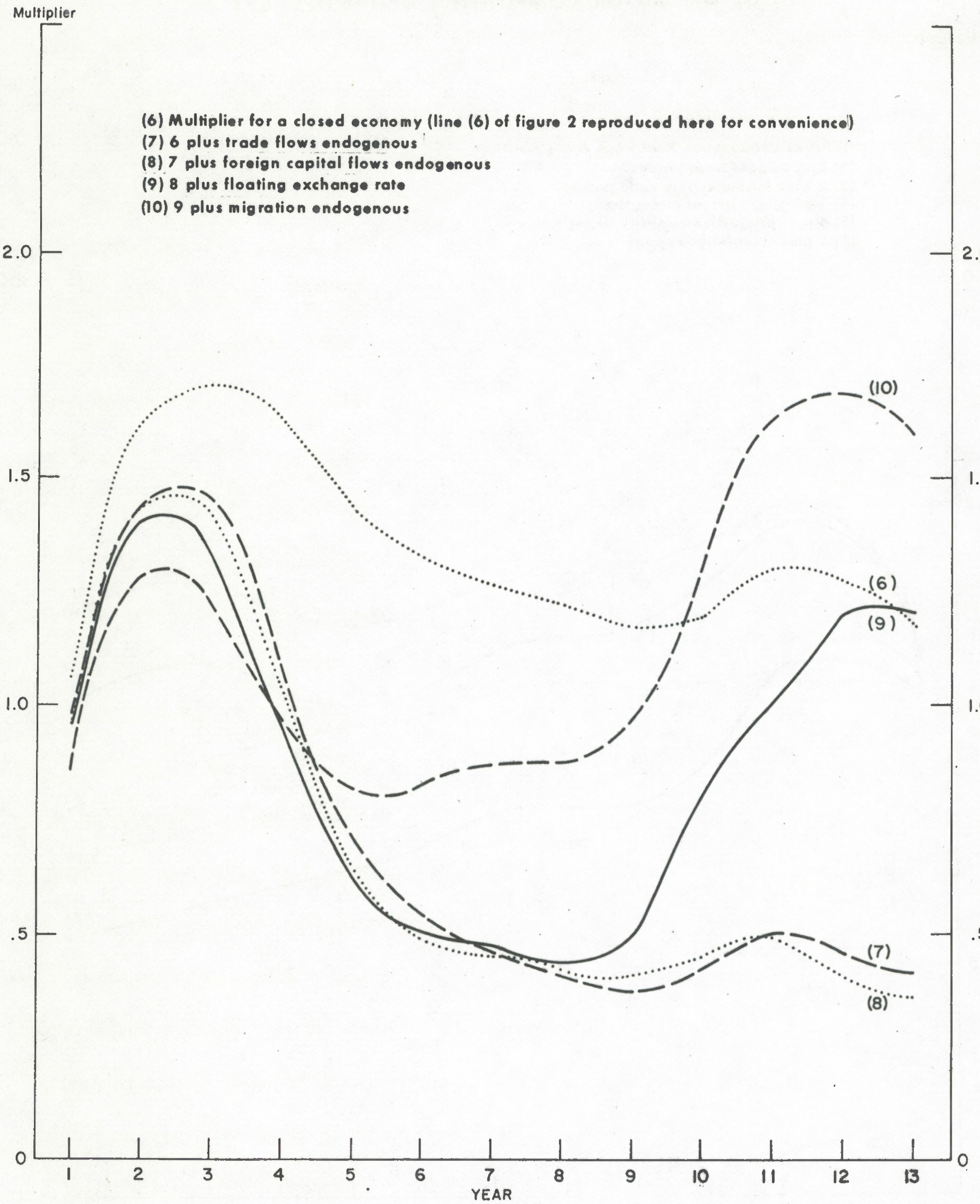


Figure 3

REAL GNE MULTIPLIERS GENERATED BY AN INCREASE IN GOVERNMENT EXPENDITURE, OBTAINED BY SEQUENTIAL INTRODUCTION OF THE MAJOR COMPONENTS OF THE FF CURVE



efficiency wage implied by the production function. In model 2, wages are determined by RDX2 equations that arrive at this same result with a lag. Labour inputs are still fixed at this stage. Endogenization of wages lengthens the total adjustment time. Wages respond with a lag to consumer prices and real output, and prices respond with a lag to changes in normalized unit labour costs. The slow response of wages shifts the distribution of income in favour of capital. Since the propensity to spend out of returns from capital is below the propensity to spend out of disposable wage income in RDX2, the shock induces less consumption on impact in model 2 than in model 1. In all other respects, model 2 is equivalent to models 0 and 1.

In model 3, labour inputs expand in response to the increased demand for labour. This implies some money illusion on the part of workers, which causes the PE curve to slope upward, at least in the short run. This condition requires a smaller (price-induced) shift in the LM curve in response to the shock than is required in model 2. The interrelated factor demand network in RDX2 implies a two-stage labour response. First, hours worked increase. Next, hirings respond with a lag to the number of workers desired on the basis of long-run labour productivity and trended hours worked. As indicated in Figure 2, the multiplier initially rises faster in model 3 than in model 2. The output produced by newly hired workers relieves the upward pressure on prices and interest rates transmitted via normalized unit capital and labour cost. Because of lags in real-wage adjustment and the consequent shift in income distribution towards capital, induced demand is lower than in model 2. When prices and wages start to catch up, real demand and desired hirings begin their downward trend, causing the production-employment curve to shift inwards.

So far, we have seen little evidence of a cycle in the GNE multiplier. Lag structures of prices, wages, and the labour force tended only to modify a clear pattern of demand-induced increases in the multiplier, followed by price-induced decreases. This situation changes with the introduction of RDX2 private investment equations in model 4. For the time being, we allow only induced investment to affect demand and ignore the effects of induced changes in capital stock on UGPPD. Both non-residential construction and machinery and equipment are driven by accelerators. Thus, a shift of the IS curve originating in the government sector will be amplified by the demand for investment goods induced via the accelerator. Higher prices and interest rates eventually affect normalized unit capital cost, decreasing investment. Hence, cyclical behaviour would be expected to originate in this area. Investment in machinery and equipment does, indeed, show a mild cyclical tendency. The first peak of investment in machinery and equipment follows the same pattern as the real GNE multiplier. The second peak appears to represent a substitution effect away from non-residential construction, because the rate of inflation in the cost of that factor is higher than the rate of price inflation for machinery and equipment. This in turn can be explained by the comparatively large amount of non-residential construction, induced by the government shock, which causes wages in the construction sector to increase faster than in the business sector as a whole. The shock-minus-control increase in investment in machinery and equipment and in non-residential construction is virtually flat after the initial activity-induced boom. Thus, it appears that the second peak in total investment is mainly due to the remaining item - residential construction. The mortgage approval equations in RDX2 reflect the empirical fact that mortgage money is tight during periods of economic expansion. As a result, the housing stock remains below its historical value throughout the simulation. However, an increae in imputed housing returns and a subsidence of the effect of the government shock on other investment cause a strong recovery of housing investment in the last four or five years. The total effect is a moderate cycle.

Until now, we have not allowed demand-induced changes in capital stock and employment to affect the level of output that producers desire based on long-run average utilization rates of factors of production (UGPPD). In model 5, UGPPD (the denominator of the term measuring the state of capacity utilization) is endogenous and hence induced increases in capital stock and new employment are incorporated into the measure of capacity utilization. This causes a reduction in capacity-induced pressure on prices (compared with the previous models), which is the main cause of the upward displacement of the multiplier.

In model 6 inventories are endogenous. Initially, this reduces the value of the GNE multiplier, since part of the demand resulting from the government shock can be met out of existing stocks of goods. In the face of increased activity added demand for inventories eventually raises the multiplier above the value observed in model 5.

We have now exhausted the major domestic mechanisms which can influence the multiplier in RDX2 or, alternatively, we have finished building up the IS curve from its components.

In Figure 3, we examine links between the domestic economy and the rest of the world, that is we start introducing the elements of the FF curve into the analysis. These include

i) trade flows

ii) capital flows

iii) the exchange rate (via its impact on domestic prices and trade flows)

iv) migration.

Since financial markets in the United States and Canada are so closely integrated, foreign interest rates enter directly into the reduced-form interest rate equations in RDX2, providing an important link between domestic and foreign capital markets. However, since in these experiments foreign variables remain unchanged, this link is not relevant. In the subsequent analysis we treat the first three elements as the components of the FF curve and regard migration as an additional factor, which tends to reduce induced pressures on the supply side.

In the previous models, we assumed that foreign demand remained unchanged as domestic supply prices rose. In model 7 trade flows are endogenous, allowing rising domestic prices to eventually reduce foreign demand for Canadian products. At the same time, increased domestic demand can be met by imports. As a result, the expansionary effect of the fiscal shock is drastically reduced, once the trade sector adjusts to the initial shock. Because of our assumption of a fixed exchange rate the balance-of-payments deficit gives rise to substantial reserve losses.

In model 8 capital flows are allowed to change in response to the shock. Higher interest rates attract foreign capital, thereby reducing reserve losses to some extent, and the multiplier pattern is essentially the same as in model 7.

In models 7 and 8 we assume that reserve losses impose no constraint on monetary and fiscal policy. Considering the magnitude of these losses, this may not be a tenable position. In model 9, we allow the exchange rate to be determined by RDX2 equations, completing our buildup of the components of the FF curve.

For the first nine years of the simulation, the multiplier responses of the fixed-rate and floating-rate versions are similar, but from this point on their paths diverge. Table 1 is a good starting point in understanding why this occurs. In this

Table I THE CONTRIBUTION OF THE MAJOR COMPONENTS OF GNE TO THE PERIOD-TO-PERIOD CHANGE IN THE REAL GNE MULTIPLIER IN RDX2 WITH BOTH A FIXED AND A FLOATING EXCHANGE RATE

i) RDX2 with fixed exchange rate and migration exogenous (Model 8)

	C_t-C_{t-1}	I_t-I_{t-1}	G_t-G_{t-1}	X_t-X_{t-1}	$-M_t-(-M_{t-1})$	GNE_t-GNE_{t-1}
1961	.18	.00	.98	.00	-.20	.96
1962	.14	.44	.11	-.01	-.21	.47
1963	-.08	.12	-.00	-.02	-.05	-.03
1964	-.12	-.20	-.07	-.04	.06	-.37
1965	-.06	-.22	-.09	-.06	.04	-.39
1966	.04	-.14	-.04	-.06	.03	-.17
1967	.02	.02	.01	-.04	-.03	-.04
1968	.00	.03	.01	-.03	-.04	-.03
1969	-.01	.01	.03	-.01	-.03	-.01
1970	.02	.09	-.01	-.01	-.05	.04
1971	.02	.09	-.01	.00	-.06	.04
1972	-.03	-.04	.00	-.01	-.01	-.09
1973	-.04	-.08	.00	-.01	.08	-.04

ii) RDX2 with floating exchange rate and migration exogenous (Model 9)

	C_t-C_{t-1}	I_t-I_{t-1}	G_t-G_{t-1}	X_t-X_{t-1}	$-M_t-(-M_{t-1})$	GNE_t-GNE_{t-1}
1961	.18	.02	1.01	-.01	-.23	.97
1962	.12	.44	.07	-.02	-.17	.45
1963	-.12	.00	-.01	.01	.03	-.08
1964	-.18	-.24	-.07	.01	.10	-.38
1965	-.08	-.17	-.08	-.01	.03	-.31
1966	.05	-.08	-.05	-.03	-.01	-.12
1967	.02	.01	.00	-.05	-.01	-.06
1968	-.02	-.04	.00	-.05	.05	-.06
1969	-.06	-.06	.02	.01	.15	.06
1970	-.06	.13	-.02	.09	.13	.27
1971	-.09	.28	-.03	.14	.09	.40
1972	-.19	.10	-.04	.16	.12	.15
1973	-.22	-.02	-.07	.14	.15	-.02

table we show the year-to-year change in the constant-dollar contribution of the major aggregates to the change in the real GNE multiplier. The last column presents the time derivatives of GNE with respect to an exogenous government expenditure shock, G*.

In the latter part of the simulation period, when the divergence starts, one can see that the major differences are in the behaviour of exports and imports. Consumption and investment also show different responses but these are induced primarily by changes in other sectors. The behaviour of consumption can be attributed largely to the effects of higher import prices following the change in the exchange rate and to a lesser extent to the lagged response of income to any change originating on the expenditure side of the accounts. (This facet of the model is discussed further in Section 4). Investment behaviour is,in turn, principally induced by the accelerator mechanism.

During the whole period, the Canadian dollar is weaker than in the control solution. This is mainly caused by deterioration in the trade balance brought on by increased levels of economic activity. The higher interest rates associated with increased activity during the first part of the period cause capital inflows that almost offset the trade balance and keep the depreciation in the region of 1 cent. The nearly constant exchange rate explains the similarity between the fixed and floating exchange rate models during the first part of the simulation. However, by the midpoint of the simulation, interest rates have fallen as a consequence of relatively lower levels of activity and therefore capital flows are not sufficient to offset the continuing trade deficit. Thus, after the eighth year, the exchange rate starts to depreciate and towards the end of the period has depreciated by as much as 4 cents. This depreciation initially causes a drop in imports and then a more gradual increase in exports, the net result of which is to cause the trade balance to swing into surplus and capital inflows (from the higher interest rates associated with the increased levels of activity) cause the exchange rate to start appreciating in the final year.

In model 10 we consider the effects of endogenizing migratory flows. The result is not so much an alteration of the cyclical pattern as a change in the level of the pattern. Migration has the effect of easing the supply constraint imposed on the labour market by a fixed population. When the second peak is reached, there has been a net addition to the labour force of 70,000 workers through migration, which is sufficient to produce the additional $290 million (constant dollars) of GNE implied in the difference between the second peaks of lines 9 and 10 in Figure 3.

The fact that the second peak exceeds the first should not lead to the conclusion that the model is exhibiting explosive tendencies. Rather it reflects the fact that migration has increased the growth rate of the economy.[2] The role of

---

2. The importance of migratory flows in the international links between economies has been noted by Helliwell (1974).

migration in determining the supply-side response of the model is discussed further in Section 5 where the contributions to output of the factors of production are analyzed.

3 THE EFFECT OF A CHANGE IN THE EXCHANGE RATE

One striking feature of the above analysis is that the introduction of the FF curve is responsible for a major change in the dynamic response of the model. In view of this observed difference between models of open and closed economies we felt that a more thorough analysis of the FF curve and its interaction with the rest of the model was warranted. Therefore, in this section we examine the dynamic response of RDX2 to an exogenous change in the exchange rate (keeping M1A at historical values).

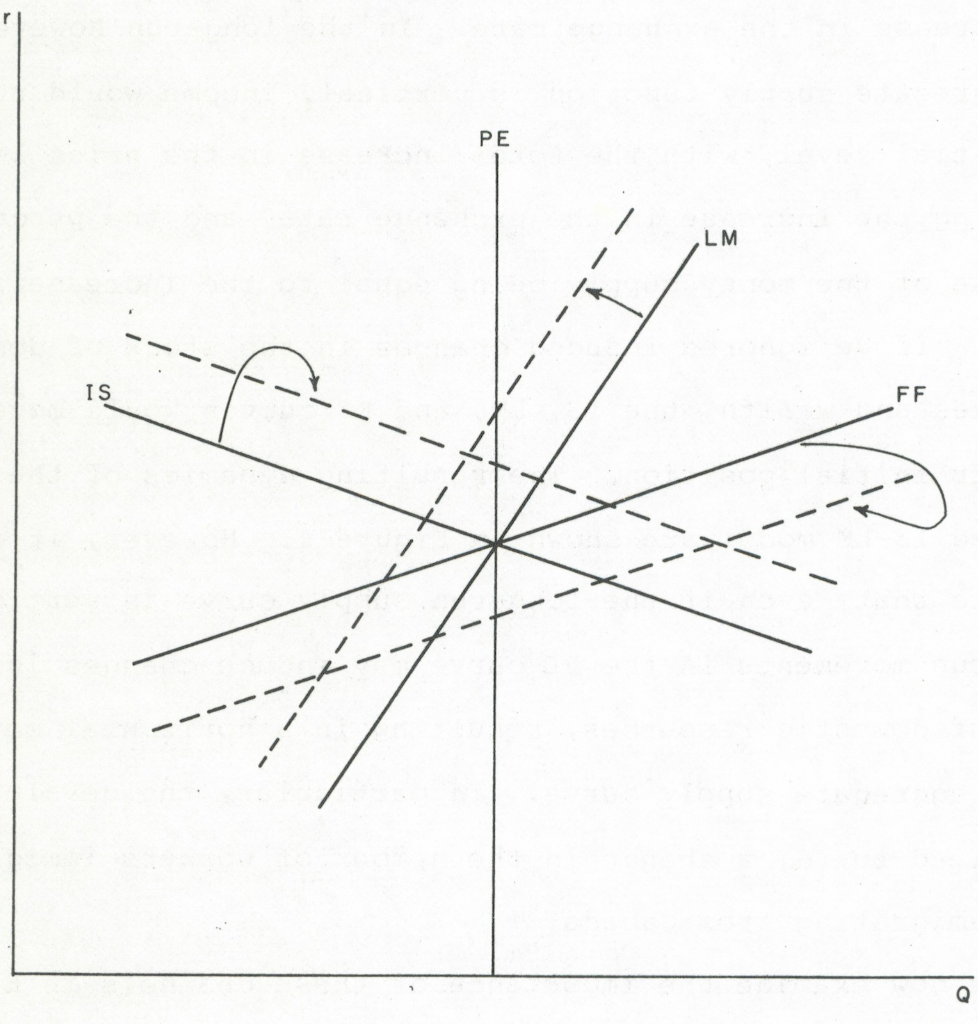
In the framework of the expanded Hicksian IS-LM model, an exogenous change in the exchange rate leads to a shift of the FF curve accompanied by a shift of the IS curve. If we assume that the Marshall-Lerner condition holds, a devaluation of the national currency would shift both curves to the right. However, the final outcome, depends crucially on the sterilization policy with respect to the balance-of-payments surplus and on the functioning of the labour market. The assumption of whether the balance-of-payments surplus is sterilized determines whether equilibrium requires a zero private excess demand for foreign exchange. The assumptions about the functioning of the labour market, particularly the prevailing degree of money illusion, determine the slope of the long-run supply curve.

We can identify a number of channels whereby a change in the exchange rate would alter the equilibrium position in the IS-LM-FF model. A devaluation of the national currency would (1) shift the aggregate demand curve to the right as a result of increased net exports following the change in foreign prices relative to domestic prices; (2) shift the aggregate supply curve upwards because of the rising cost of imports, thus raising the price level; (3) shift the aggregate supply curve even further upwards as money wages increase as a result of higher prices. Since the aggregate supply curve is not flat, a shift in the aggregate demand curve will (4) cause an increase in the price level though, since imports are a function of aggregate income, the aggregate demand curve will be flatter and its rightward shift will be reduced by import leakages. The price increases observed in (2), (3), and (4) will tend to reduce the extent of the rightward shifts of the IS and the FF curves. If sterilization takes place, the real money supply will tend to fall, resulting in (5) a leftward shift of the LM curve. In the short-run, equilibrium should be reached at a higher level of income and at a higher price level. In the longer run, if the long-run supply curve is vertical, income may fall back to its initial level, as further price increases offset the effect of the devaluation. The increase in the price level will be less than the increase in the exchange rate because of the reduction in the real money supply. This and the resulting increase in the interest rate imply that the balance of payments would remain in a surplus position. In the absence of sterilization, equilibrium requires private excess demand for foreign exchange to be zero. The initial surplus in the balance of payments would lead to an increase in the nominal money supply, further contributing to the increase in the price level. The real money supply, however, must increase if the aggregate supply curve is upward sloping. In this case the increase in domestic prices would be less than the increase in the exchange rate. In the long-run however, if the aggregate supply function is vertical, income would return to its initial level, with the total increase in the price level equalling the increase in the exchange rate, and the percentage increase of the money supply being equal to the increase in prices. If we ignored induced changes in the stock of domestic resources and wealth, the IS, LM, and FF curves would move back to their initial position. The resulting dynamics of the extended IS-LM model are shown in Figure 4. However, it is possible that, even if the long-run supply curve is vertical, short-run movements in the PE curve may induce changes in the stock of domestic resources, resulting in a horizontal movement of the aggregate supply curve. In particular, the devaluation might lead to (6) a change in the number of workers immigrating to or emigrating from Canada.

We now examine the importance of these channels in RDX2, using the nested partial-model approach developed in Section 2 (i.e., we start from a simple model and add sectors until the full RDX2 model is simulated). However, since the role of

Figure 4

DYNAMICS OF AN EXTENDED IS-LM MODEL WITH A VERTICAL PE CURVE



migratory flows has been discussed in Sections 2 and 4 and is not essential to an understanding of the economic processes following a devaluation, we concentrate on the first five channels. Each of the partial models described (see the Appendix for details) corresponds to the addition of one more RDX2 representation of these channels, assuming that sterilization takes place. Thus, the marginal impact of the introduction of another channel is represented by the difference between the results for that version of the model and the version preceeding it.

In the case of a change in the exchange rate, there is no single summary statistic (such as the multiplier used in Section 2) which can adequately describe the processes involved. We have therefore chosen to report, in Figures 5-9, the effect of the devaluation on five major economic aggregates: real GNE (UGNE), real exports (X), real imports (M), the current account balance in current dollars (XBAL$), and the consumer price index (PCPI). For the sake of symmetry, we present these variables for all the models simulated (with the exception of the change in PCPI for model 1 which is zero).

The first model considered, model 1, has only the trade sector and import prices endogenous with all other prices and activity variables kept exogenous. The results obtained are therefore solely and directly implied by the price elasticities and lags of the aggregate demand for imports and the aggregate foreign demand for Canadian exports. As expected, the 10 percent

Figure 5

BEHAVIOUR OF REAL GNE (SHOCK MINUS CONTROL EXPRESSED AS A PERCENTAGE OF CONTROL) FOR FIVE VARIANTS OF RDX2 FOLLOWING A 10% DEVALUATION

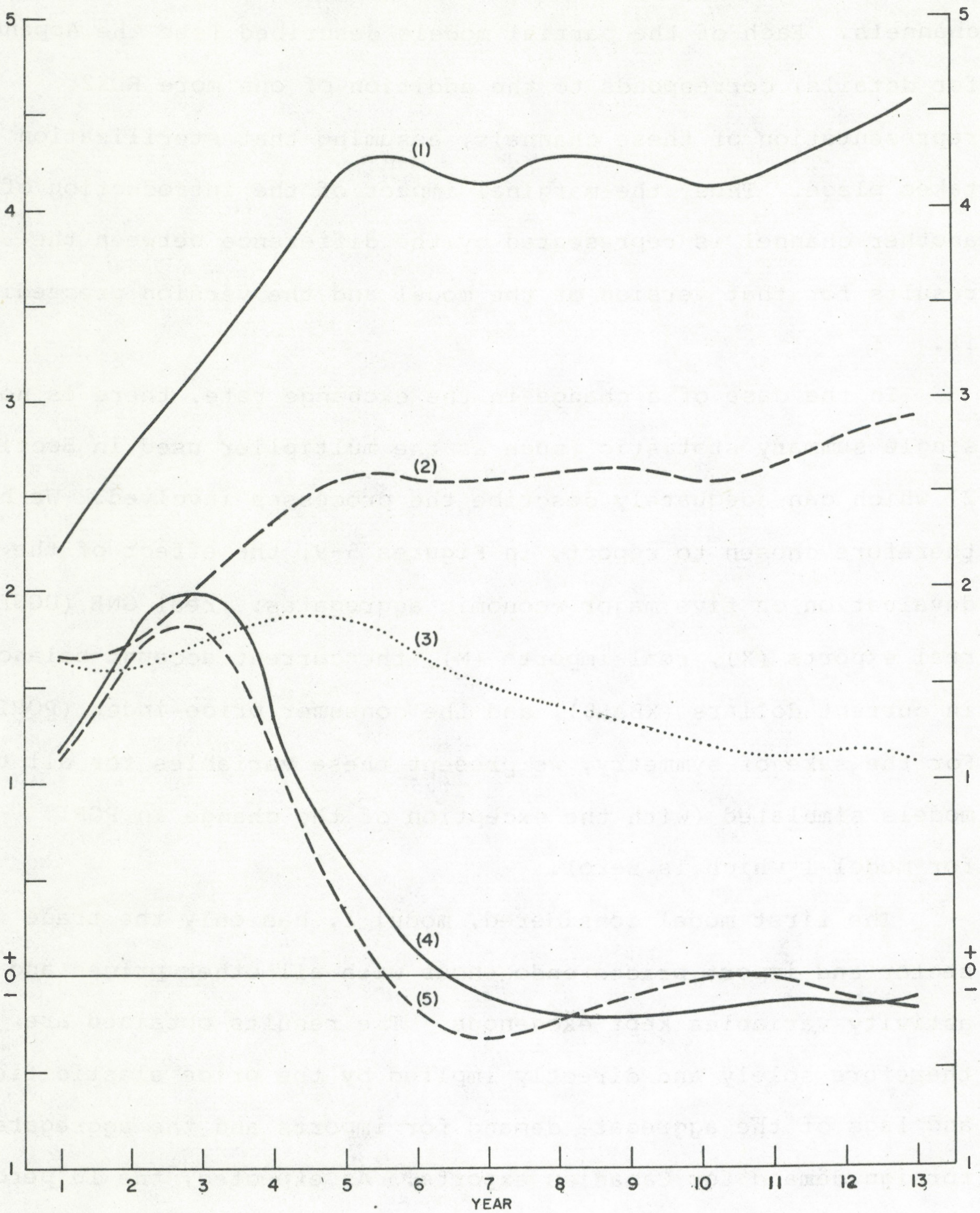


Figure 6

BEHAVIOUR OF REAL EXPORTS (SHOCK MINUS CONTROL EXPRESSED AS A PERCENTAGE OF CONTROL) FOR FIVE VARIANTS OF RDX2 FOLLOWING A 10% DEVALUATION

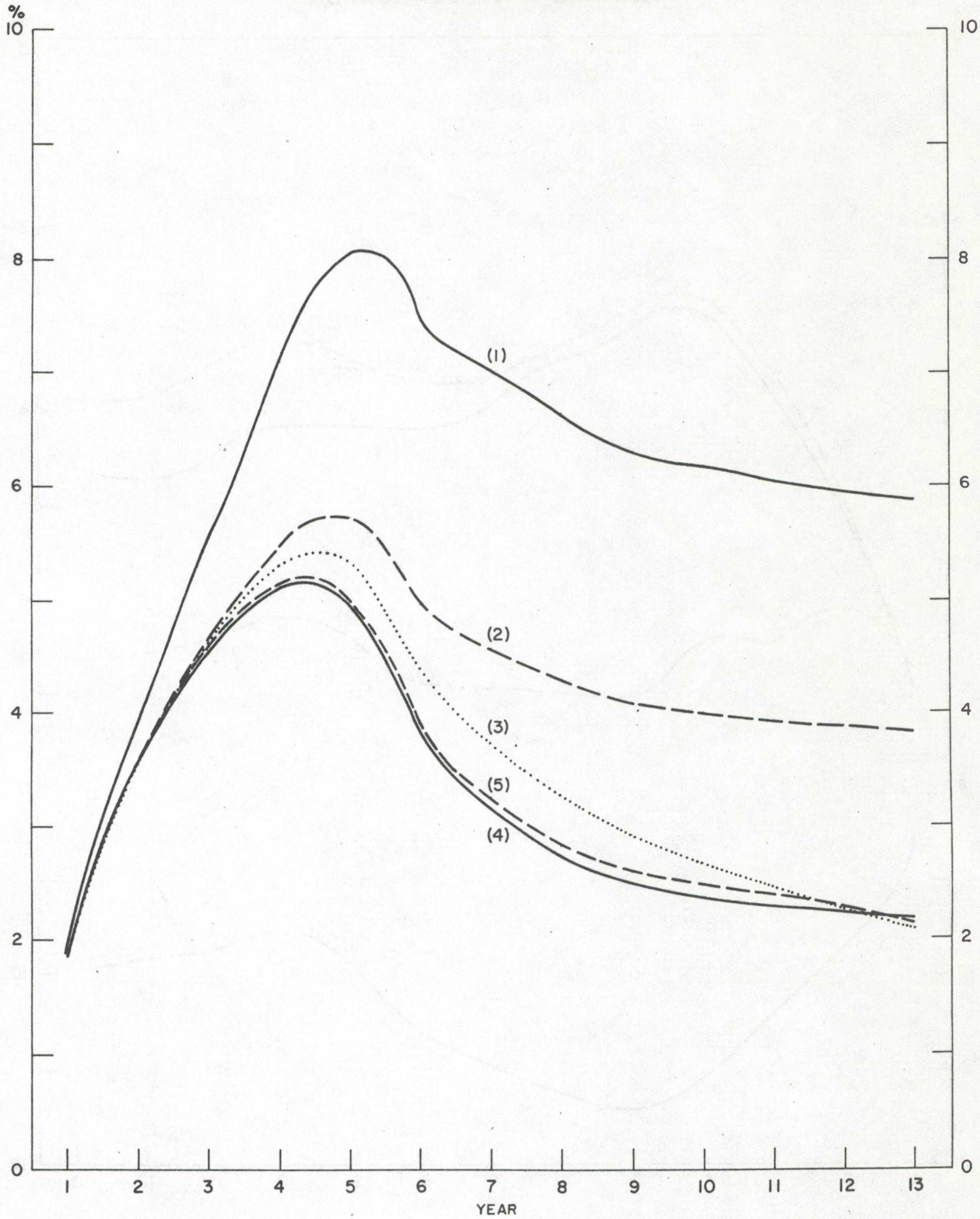


Figure 7

BEHAVIOUR OF REAL IMPORTS (SHOCK MINUS CONTROL EXPRESSED AS A PERCENTAGE OF CONTROL) FOR FIVE VARIANTS OF RDX2 FOLLOWING A 10% DEVALUATION

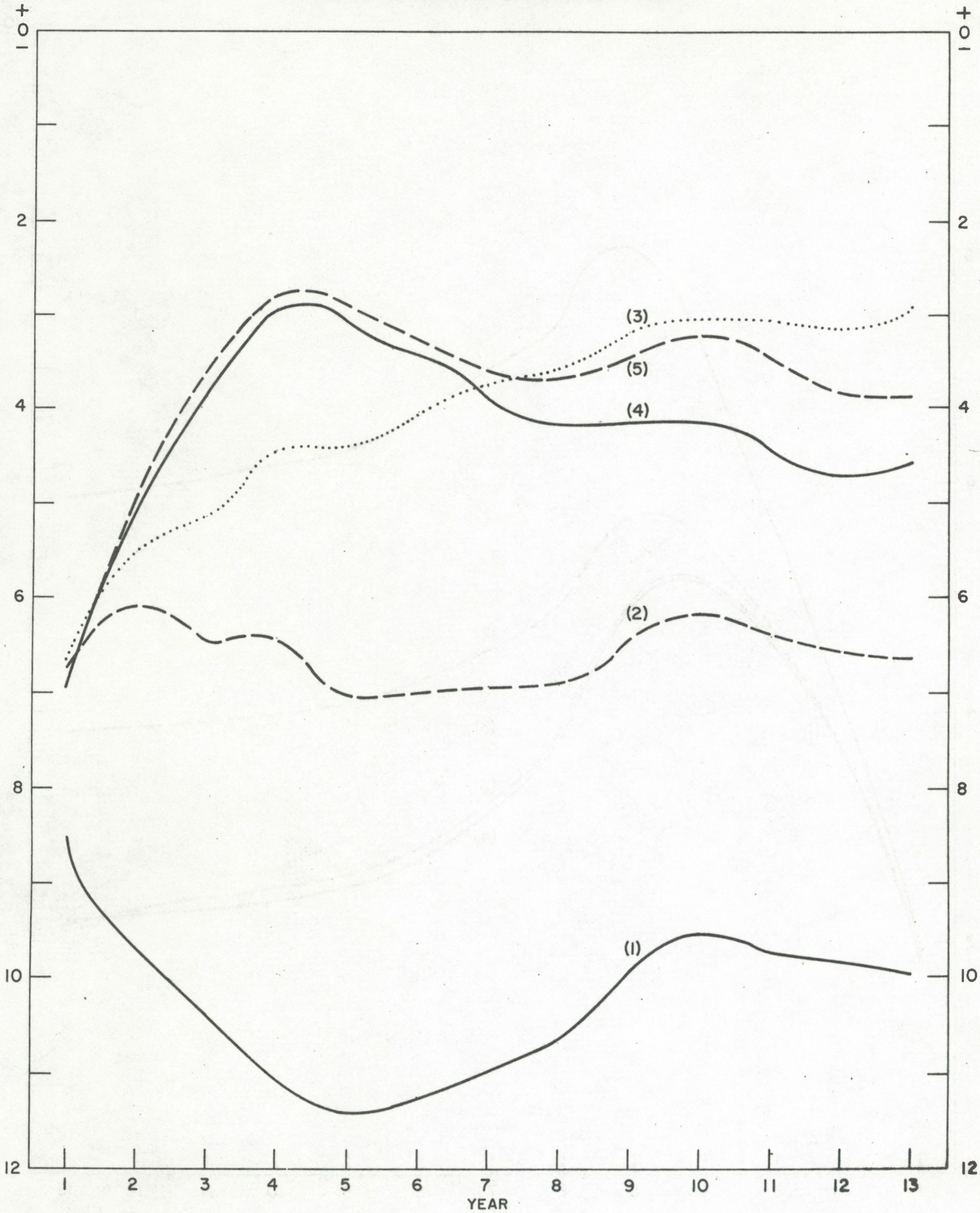


Figure 8

BEHAVIOUR OF THE CURRENT ACCOUNT BALANCE (LEVEL, IN CURRENT DOLLARS) FOR FIVE VARIANTS OF RDX2 FOLLOWING A 10% DEVALUATION

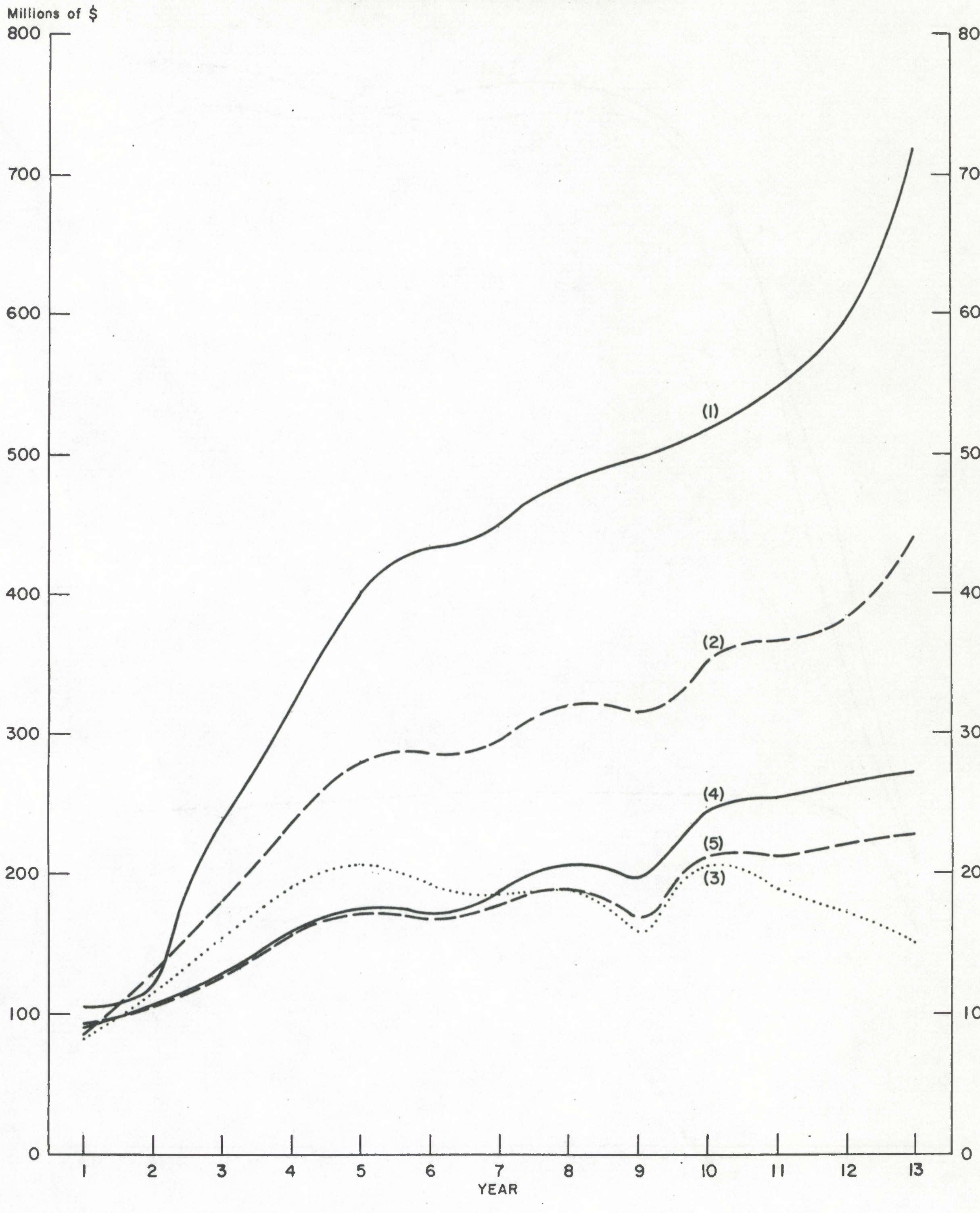
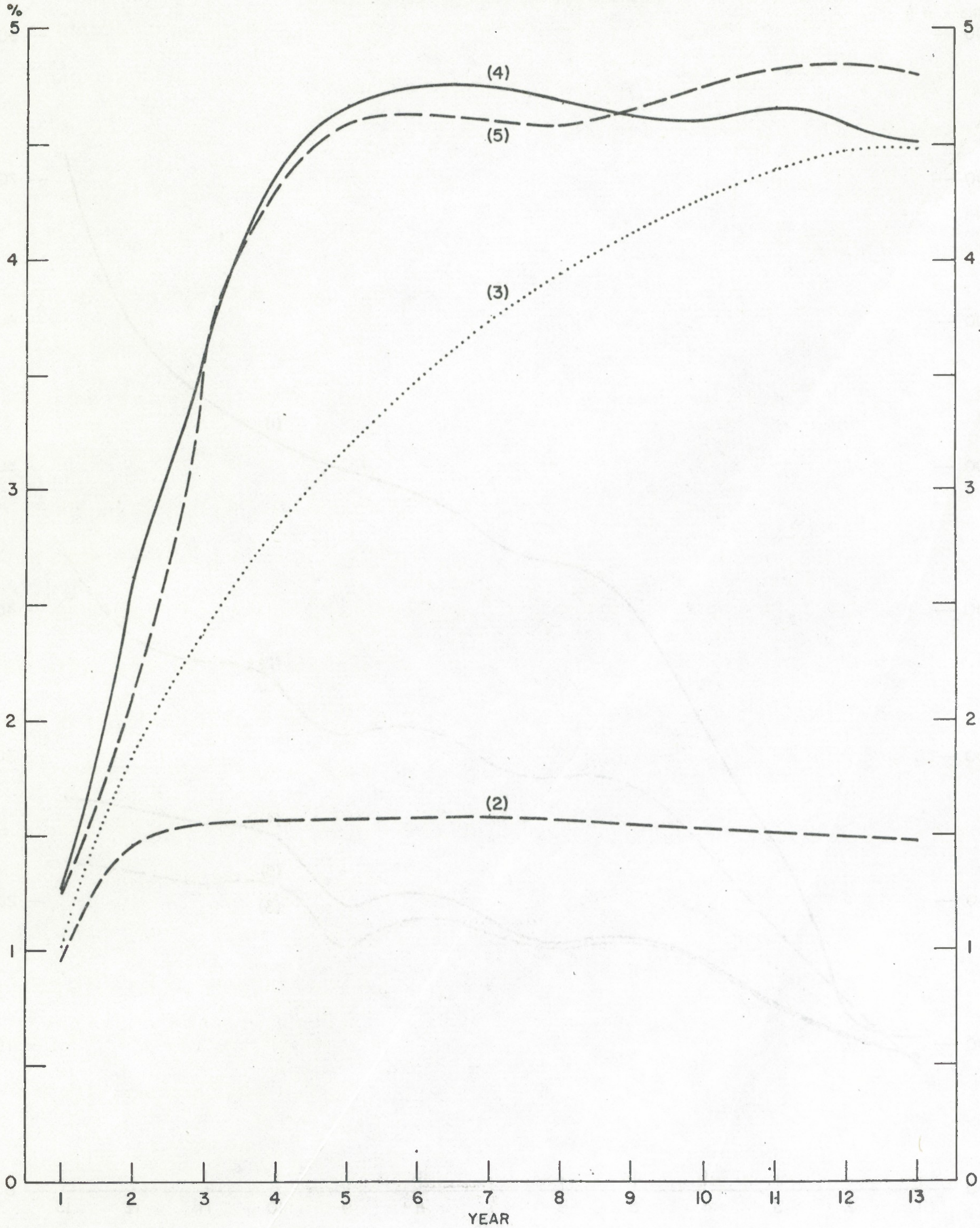


Figure 9

BEHAVIOR OF THE CONSUMER PRICE INDEX (SHOCK MINUS CONTROL EXPRESSED AS A PERCENTAGE OF CONTROL) FOR FIVE VARIANTS OF RDX2 FOLLOWING A 10% DEVALUATION



devaluation has a particularly strong effect on the flow of trade as the domestic price level remains constant. The quantity of exports (line 1, Figure 6) rises gradually (reflecting the long lags in the export equations) to about 6.6 percent above control and the quantity of imports (line 1, Figure 7) falls by an average of 10.7 percent below control values.[3] These translate directly into real GNE (line 1, Figure 5), which in turn shows a substantial increase.

In model 2 domestic prices (including the price of exports) are allowed to increase in response to the exchange-rate change. Since the relative difference between domestic and import prices has been reduced, we would expect a similar though muted response to the shock, with the difference in patterns attributable to the lags in the price equations (Bank of Canada, Technical Report 6 pp. 103-131). As a consequence of the higher cost of imports feeding into domestic prices and then into export prices, and as a consequence of higher world prices, the volume of exports (line 2, Figure 3) shows a muted response compared with that in model 1. Imports (line 2, Figure 7) now fall by only 6.5 percent on average. The effect on the current account balance (line 2, Figure 8) is also considerably weaker.

The higher import prices increase the consumer price index (line 2, Figure 9) by 1.5 percent (through increases in PCMV and PCNDSD) and it takes nearly two years for the full impact to be

3. The variation in real exports and imports once the shock has worked its way through the lags is a consequence of non-homogeneities in the model.

felt. The GNE deflator is not greatly affected by the devaluation because the small increases in the prices of most final demand components are largely offset by the increase in the price of exports. Thus the behaviour of real GNE (line 2, Figure 5) still largely mirrors the behaviour of real exports and imports.

In model 3, domestic prices are allowed to increase not only because of higher import costs, but also because labour costs rise as a result of increases in the general price level. This builds a price-wage inflation spiral into the model (Bank of Canada, Technical Report 6 pp. 103-121) and consequently prices increase over a much longer period. The consumer price index (line 3, Figure 9) increases steadily to 4.5 percent above control at the end of the simulation period, and wages follow essentially the same pattern. As a result, the effect of the devaluation on the quantity of imports weakens as the relative gap between domestic and import prices narrows. Real imports (line 3, Figure 7) are 7.4 percent below control in the first quarter, but only 2.7 percent below control after 13 years. The import bill is now increasing over time relative to its control value: it is 4.5 percent higher than control after 13 years (3.5 percent above control on average).

On the export side, we find that the price deflator is steadily increasing. The quantity of real exports (line 3, Figure 6) peaks at 6.2 percent above control in the 15th quarter, but the gain is rapidly reduced as domestic prices rise, leaving exports only 1.9 percent higher than control at the end of the simulation. In terms of the current account (line 3, Figure 8), we observe that the steady increase in prices now weakens the effect of the devaluation even more. We find that the increase is diminishing in the second half of the simulation (despite the increasing volume of foreign trade) when the current account is only 87 million dollars above control after 13 years: on average, XBAL$ is $168 million above control. The behaviour of real GNE (line 3, Figure 5) now reflects not only the behaviour of real exports but also the increase in the GNE deflator.

We next allow the level of activity to change as a result of the devaluation (model 4). In the short-run at least, the activity level will increase and could be expected to lead to increased imports both because of the direct demand effect and because of increased domestic prices. This would further reduce the effect of the devaluation on the balance of trade. It is apparent from Figure 8 that this is not the case in the first five quarters of the simulation and then only holds for approximately half the simulation period. On average, the current account is actually $189 million above control (vs. $168 million in model 3). This somewhat unexpected result is due to the different import propensities of the various GNE components. For the first half of the simulation UGNE is above control but IME remains below control after 15 quarters and consumption is below control for the entire simulation. These are the components with the largest import propensity. The decrease in consumption, which results from the fall in disposable income brought about by the devaluation, is particularly severe for motor vehicles (CMV). Imports of motor vehicles are in turn severely affected by this drop, particularly during the early part of the simulation.

The fall in imports is fairly sharp over the first part of the simulation as aggregate demand is significantly above control during the first years following the devaluation but then weakens while domestic prices keep increasing. The effect of the devaluation on exports is slightly reduced as the higher level of activity further increases domestic prices and export prices. The devaluation has its largest impact on exports after four years (X is then 6.0 percent above control), but this gain is gradually eroded as export supply prices continue to increase. At the end of the simulation period, exports are only 2.0 percent above control. The GNE price deflator has increased by 3.5 percent on average and the additional increase in the PCPI over that of model 3 can be attributed to additional activity-induced price pressures. If the results of simulations with models 3 and 4 are compared with those of model 2 we see that the direct impact of an increase in import prices on the consumer price index is less than the indirect effect due to the accompanying increase in labour costs and the increase in activity. We also note that, as expected (because of sterilization and non-homogeneities in the price equations), the increase in price level is less than the increase in the exchange rate.

Not surprisingly, we find that the increased level of domestic activity has an essentially favourable effect on the rate of unemployment: RNU falls by 102 points within the first 11 quarters, but increases somewhat above control after 7 years as the rate of growth of UGNE falls below control.

In model 5, the financial sector of RDX2 is endogenized; only immigration and emigration are exogenous. As expected, the devaluation, through larger income and a higher price level, tends to increase interest rates. In the seventh quarter RS increases very rapidly to about 55 points above control and then gradually falls back to its control value. Consequently all long-term and medium-term interest rates are above control for most of the simulation. The inclusion of the financial sector does not have as crucial an effect as the monetary approach to the balance of payments would suggest, since the balance-of-payments surplus is sterilized. The higher interest rates nevertheless contribute somewhat to the reduction in aggregate demand. All investment components fall relative to model 4. The same holds for all consumption components, except consumption of other durable goods (CDO). Because private sector wealth is larger than control, CDO is actually above control for almost the entire simulation period. The increase in wealth, which takes place in spite of higher interest rates, is largely due to the short-run increase in profits that follows the devaluation.

We can now also examine the impact of the devaluation on the

Canadian balance of payments. The devaluation not only has a positive effect on the current account, but long-term capital inflows are also above control (especially FIDI12, FIDI13, FITOBB12 and FITOGB12) as a result of the higher level of activity. The initial increase in interest rates also attracts short-term capital inflows; however, increasing international indebtedness and a persisting surplus in the basic balance (UBAL) rapidly lead to an outflow of short-term capital. Nevertheless, the balance of payments is in a strong surplus position which requires massive official interventions in the foreign exchange market. Official exchange reserves increase dramatically during the entire simulation period, with URES 179 percent above control in the seventh year.

In these simulations we have examined the importance of

(1) the effect of higher import costs on domestic prices;
(2) the additional cost-multiplier effect due to wage increases;
(3) the direct effect on imports of changes in the level of activity and the indirect effect, through further increases in domestic prices, in both on both imports and exports, and
(4) the restrictive influence of higher interest rates.

One important point that emerges is the significance of price and wage behaviour and of the activity variables in determining the outcome of the simulations. In particular these simulations reveal that a partial equilibrium analysis of a devaluation that ignores these four effects (such as model 1) would grossly overstate the impact of a devaluation on the current account. Finally, we should emphasize that the increase in the current account occurs mainly at the cost of real consumption and residential construction.

4 THE DYNAMICS OF THE COMPLETE MODEL

Sections 2 and 3 were devoted to the presentation and discussion (in rather condensed form) of the partial dynamics of RDX2. This approach provides a useful set of building blocks with which to develop an understanding of the dynamic response of the model as a whole, the topic to which this section is devoted.

Once again we were faced with the problem of presentation: how to report succinctly the simulation behaviour of all the endogenous variables in RDX2. The presentation scheme that we adopted centres around three aspects of model behaviour:

i) The behaviour of the components of total demand (i.e., from domestic and foreign sources) for Canadian goods and the total supply (from both domestic and foreign sources) of goods in Canadian markets.

ii) Movements in business output and the contributions of individual factor inputs to business output induced by the shock.

iii) The marginal changes in income distribution for the major classes of income recipients induced by the shock.

However, before examining the dynamics of RDX2 in detail, it is worthwhile to set the stage for the subsequent analysis by presenting the nominal and real multipliers for the complete model. These are shown in Figure 10. The real GNE multiplier exhibits cyclical behaviour (the reasons for this have been discussed in Sections 2 and 3) around a rising trend. This trend is discussed later in this section and is attributable to the inflow of migrants increasing the labour force and so diminishing the effects of supply constraints. The nominal multiplier follows the same pattern as the real multiplier (apart from an upward displacement) except in the final years when the gap is widening due to the impact of the depreciating exchange rate on domestic prices.

Figure 11 records the constant-dollar change in the major components of real GNE expressed as a ratio of the constant-dollar shock in government expenditure. The demand components (representing total demand for Canadian goods) are plotted in the upper panel, and the components of supply (treating imports as a source of supply), plotted with a negative sign, appear in the lower panel. Since the decomposition is additive, the figure admits to an easy explanation. Line 1 is the response of total government expenditure. Line 2 represents the sum of total government expenditure plus exports; thus the contribution of exports is the difference between lines 1 and 2.

Figure 10

REAL AND NOMINAL GNE MULTIPLIERS GENERATED BY AN INCREASE IN GOVERNMENT EXPENDITURE WITH (a) MONEY SUPPLY HELD AT CONTROL LEVEL AND (b) INTEREST RATES HELD AT CONTROL LEVEL

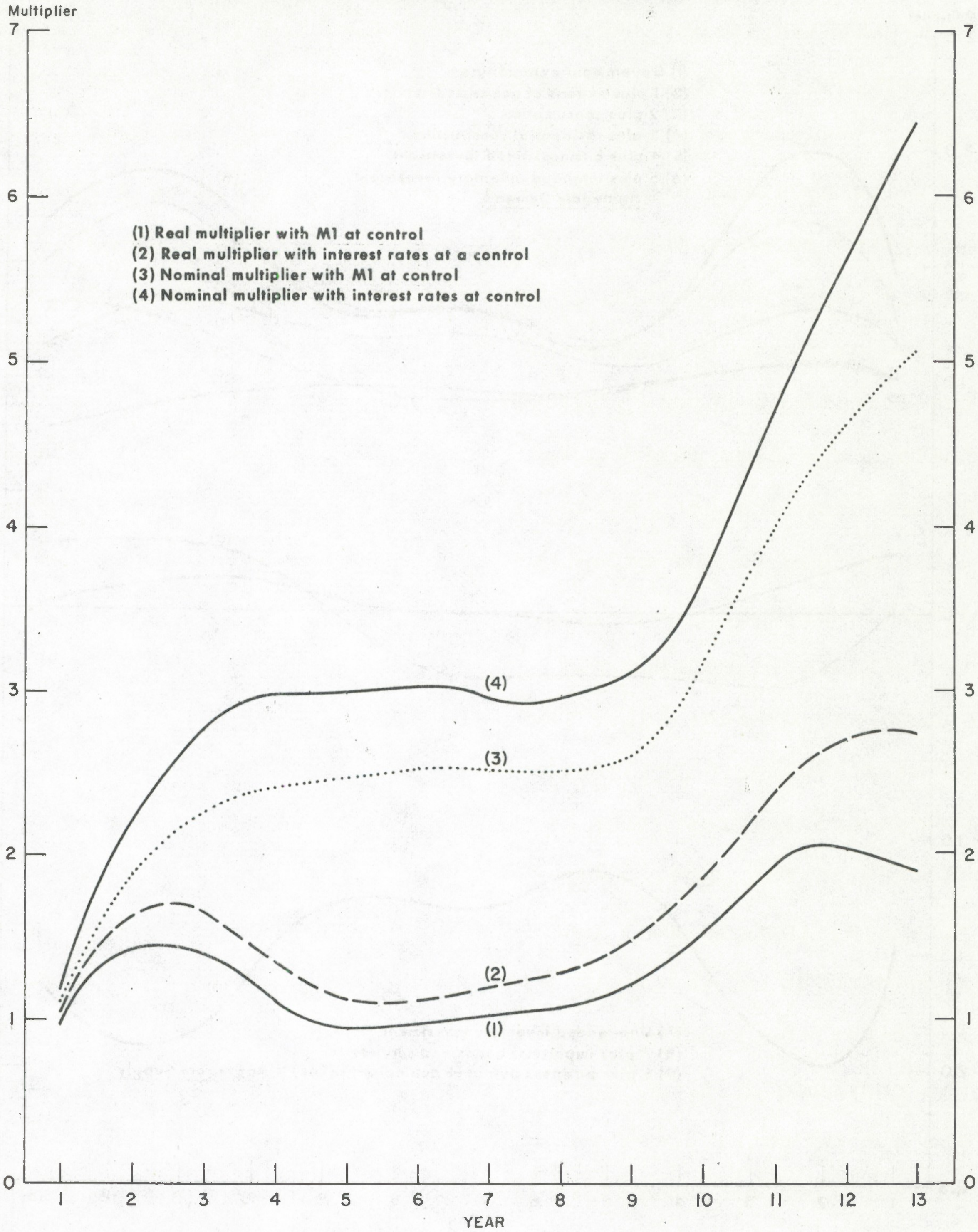
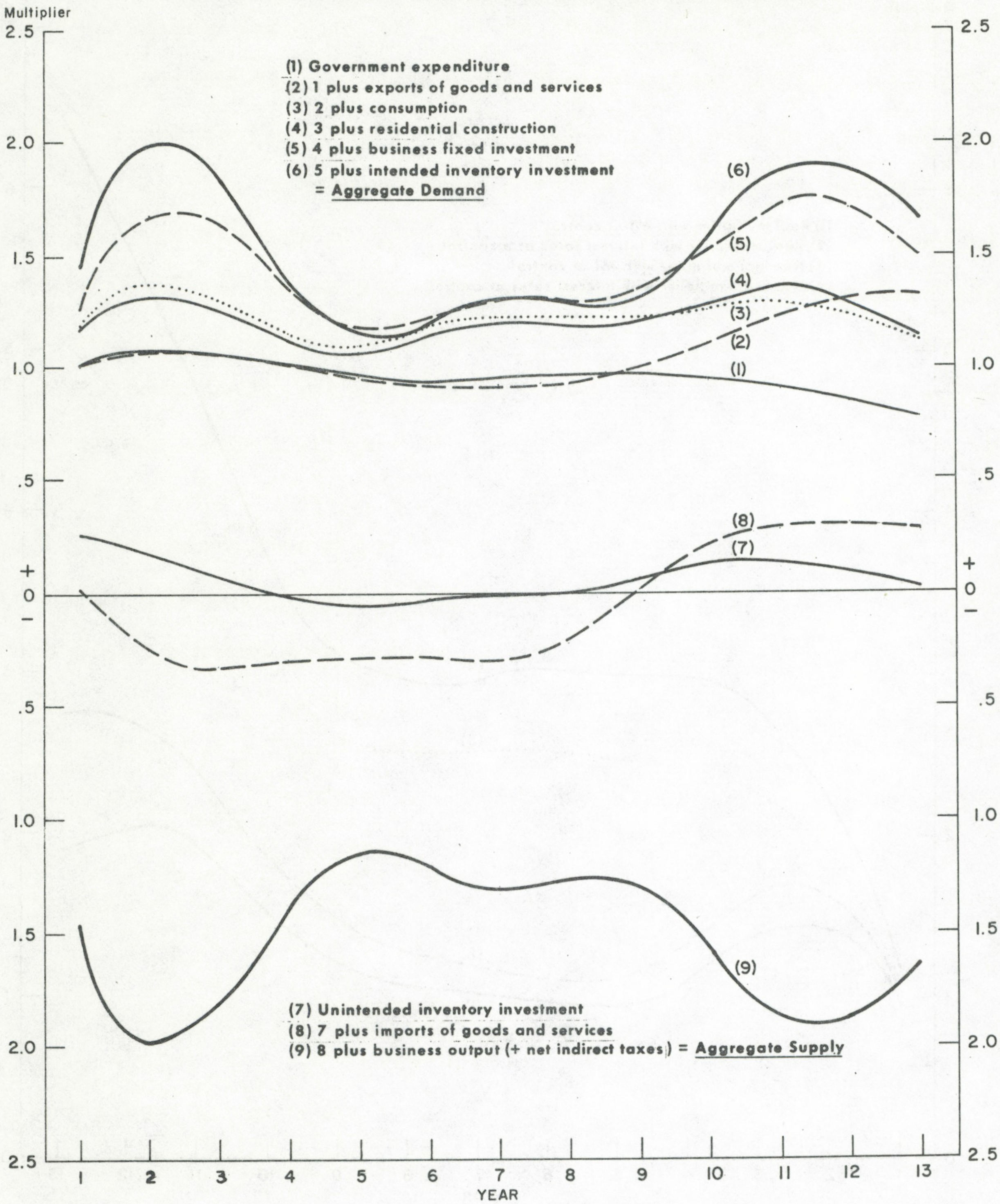


Figure 11

DECOMPOSITION OF REAL DEMAND AND SUPPLY RESPONSE GENERATED BY AN INCREASE IN GOVERNMENT EXPENDITURE



This process is continued until all the major demand components have been considered. A similar approach is adopted for the supply components.

An examination of Figure 11 reveals the same story as was told piecemeal in the previous two sections. If we look first at the demand components, we see that during the first cycle, while consumption adds somewhat to the initial upswing, it is investment (initially intended inventory investment and then business fixed investment as the accelerator mechanism comes into play) that provides the greatest contribution.[4] During the first cycle, since the exchange rate differs by less than one percent from its control values, imports and exports tend to mirror activity levels in the economy.

The roles played by the different components change during the second cycle. In this case, the exchange rate starts to depreciate in the eighth year. Consequently, exports start to

---

4. This ordering is largely a consequence of the nature of the shock being examined. The demand shock enters almost, immediately on an one-for-one basis into real expenditure since prices react with a lag. This activates the accelerator mechanism in the investment equations. Since it takes time for the effects to appear in labour income and then in consumption via the distributed lag (representing permanent income), the response of consumption is understandably muted. If the shock had been on the income side, (i.e., a cut in personal income taxes) the relative importance would have been reversed: real demand would have increased more slowly with the accompanying investment response muted. This arises since the increase in current disposable income is included in permanent income and so affects consumption and then investment via the accelerator. These differences can be seen by comparing Tables 5A and 5C on pp. 272 and 274 of Technical Report 5.

rise and imports start to fall; i.e., it is the foreign sector of the model that initiates the second cycle. As a result of this increase in activity, the accelerator (both business fixed investment and inventories) is brought into play and augments the upswing. Consumption rises in sympathy with this increase in activity.

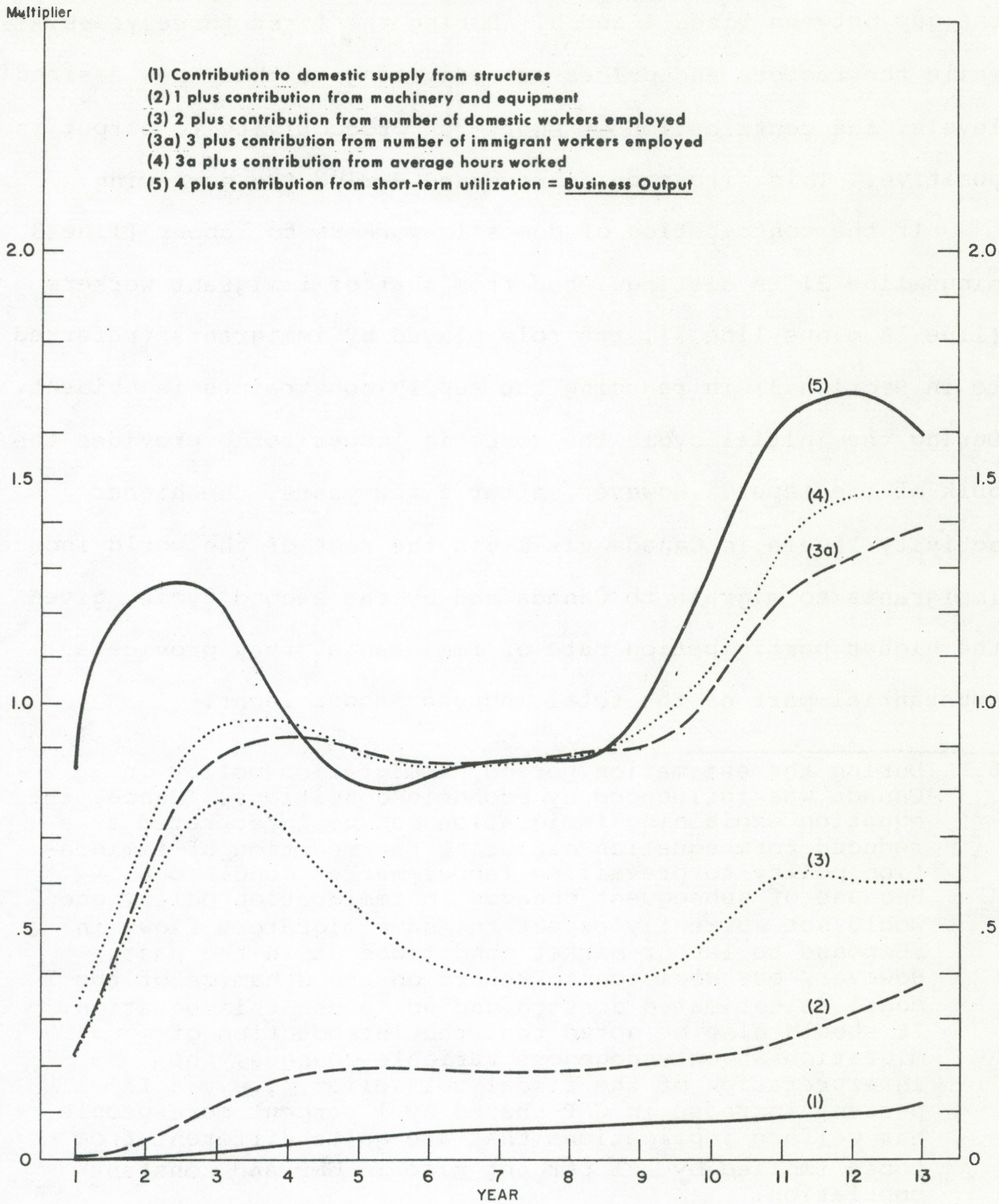
In the above discussion, there is no consideration of the behaviour of the major component of supply, business output, to which we now turn our attention. Business output is regarded as the output variable of the aggregate Cobb-Douglas production function representing the domestic-supply side of the economy. Figure 12 depicts the contribution of the factor inputs (weighted by their shares in the production function) to business output. Since the contribution of the factors is cumulative, the graph admits to the same interpretation as Figure 11.

One distinguishing feature of RDX2 is the use of an interrelated factor demand network with a hierarchy of adjustment speeds. If we ignore the role of short-run productivity for a moment, Figure 12 reveals that, of all the factor inputs, labour makes the fastest adjustment to the new desired levels, with hours worked adjusting faster than the number of workers.[5] Investment in machinery and equipment is the next input to respond and non-residential construction is the last. Initially,

5. The faster response of hours worked is somewhat masked by the use of annual data in Figure 12. The quarterly data obtained directly from the RDX2 simulation reveals a much sharper initial response of hours worked.

Figure 12

CONTRIBUTION OF FACTOR INPUTS TO BUSINESS OUTPUT GENERATED BY AN INCREASE IN GOVERNMENT EXPENDITURE



before factor inputs and prices can fully adjust to the new desired levels, the gap between demand and supply induced by the shock must be met by short-run productivity, as indicated by the gap between lines 4 and 5. During the first three years, while the factors and prices are adjusting to their new desired levels, the contribution of short-run productivity to output is positive. This situation is reversed during the downswing.

If the contribution of domestic workers to labour (line 3 minus line 2) is distinguished from that of immigrant workers (line 3A minus line 3), the role played by immigrants (referred to in Section 3) in reducing the supply constraints is evident. During the initial cycle the domestic labour force provides the bulk of the input. However, after a few years, the higher activity levels in Canada vis-à-vis the rest of the world induce immigrants to migrate to Canada and by the second cycle, given the higher participation rate of immigrants, they provide a substantial part of the total induced labour input.[6]

---

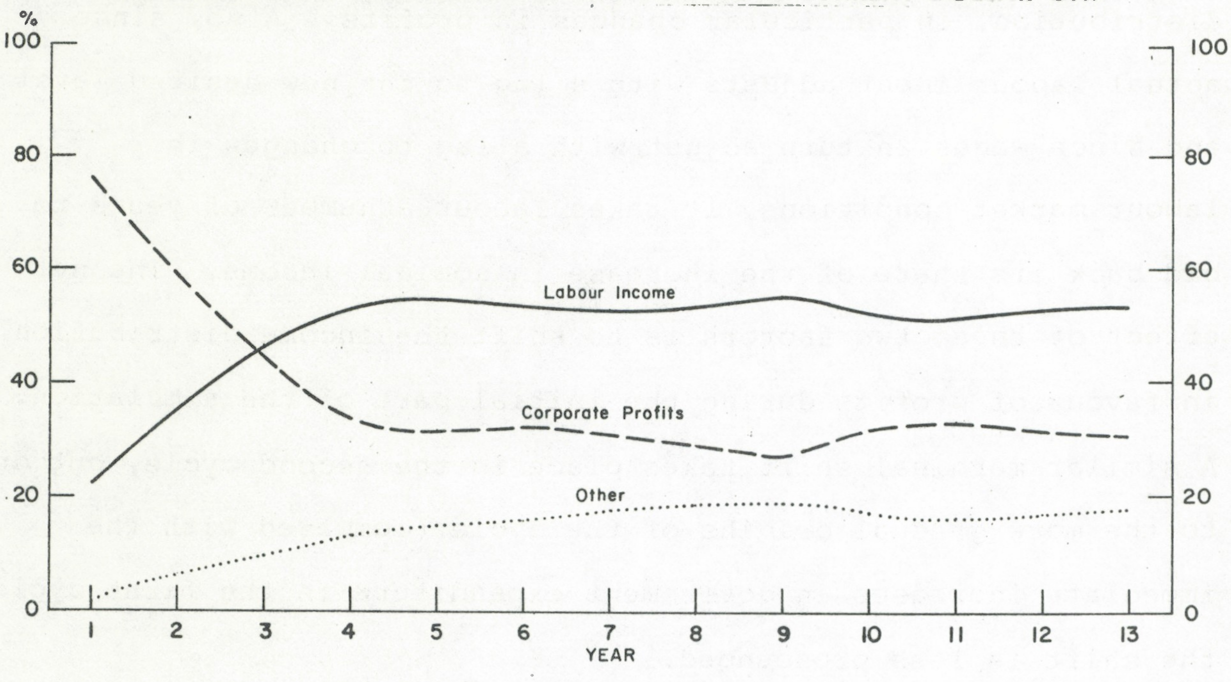
6. During the estimation period, immigration policy in Canada was influenced by economic conditions. Hence, the equation explaining immigration can be interpreted as a reduced-form equation capturing the reaction of immigration policy to prevailing labour-market conditions. Because of subsequent changes in immigration policy one would not currently expect the same migratory flows in response to labour-market conditions as in the past. However, our decision to report on the dynamics of the model as estimated constrained us to use this equation. It should also be noted that the introduction of migration as an endogenous variable changes, the interpretation of the fiscal multiplier; i.e., a 1 percent increase in GNP shared by 1 percent more people has welfare implications that are quite different from those implied by a 1 percent rise in GNP and constant population.

The final aspect of the model's response is illustrated in Figure 13, which shows the marginal changes in income distribution (expressed as a percentage of the induced change of current-dollar GNP). Since unit labour costs in the price equations have been normalized to exclude the effect of short-run productivity, the productivity gains referred to above are not reflected in prices. They appear instead as changes in income distribution, in particular changes in profits. Also, since actual labour input adjusts with a lag to the new desired level and since wages in turn adjust with a lag to changes in labour-market conditions, it takes labour a number of years to bid back its share of the increase in nominal income. The net effect of these two factors is to shift the income distribution in favour of profits during the initial part of the simulation. A similar marginal shift takes place in the second cycle, but due to the more gradual origins of the cycle (compared with the immediate increases in government expenditure in the first cycle) the shift is less pronounced.

Even though the subject has been treated in detail elsewhere (Freedman and Longworth, 1975), no analysis of model dynamics would complete without some description of how a shift of the LM curve modifies the model's response to a shift of the IS curve. To do this we ran the same fiscal shock as previously, but instead of keeping M1A at its historical values, we kept the key short-term interest rate RS (the average yield on Government of Canada bonds with maturities of 1-3 years) at historical levels.

Figure 13

INCOMES INDUCED BY THE INCREASE IN GOVERNMENT EXPENDITURE EXPRESSED AS A PERCENTAGE OF THE CHANGE IN CURRENT DOLLAR GNP



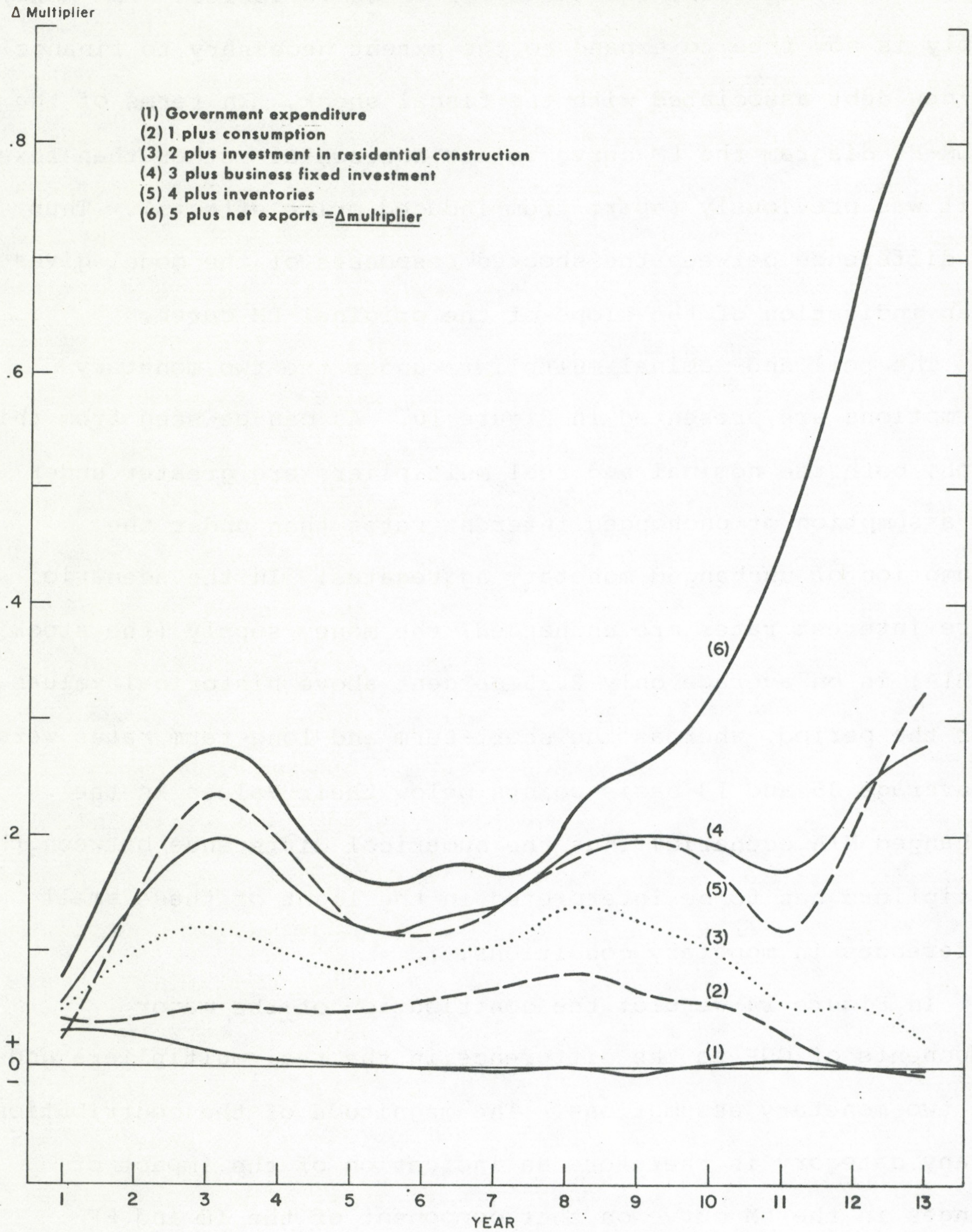
Since the yields on longer term government bonds (RMS), (RML) and (RL) are explained by a term-structure relationship based on RS, we also exogenized the equations for these variables. The money supply is now free to expand to the extent necessary to finance the new debt associated with the fiscal shock. In terms of the IS-LM-FF diagram the LM curve is now horizontal rather than fixed as it was previously (apart from induced price effects). Thus the difference between the shocked responses of the model gives us an indication of the slope of the original LM curve.

The real and nominal multpliers under the two monetary assumptions are presented in Figure 10. As can be seen from this graph, both the nominal and real multipliers are greater under the assumption of unchanged interest rates than under the assumption of unchanged monetary aggregates. In the scenario where interest rates are unchanged, the money supply (the stock of M1A) is on average only 2.35 percent above historical values over the period, whereas the short-term and long-term rates were on average 35 and 18 basis points below their values in the unchanged M1A scenario. Thus the numerical difference between the multipliers has to be interpreted in the light of these small differences in monetary conditions.

In Figure 14 we plot the contribution of the major components of GNP to the difference in the real multipliers under the two monetary assumptions. The magnitude of the contribution of any category is therefore an indication of the impact of changes in the LM curve on that component of the IS and FF

Figure 14

CONTRIBUTION OF THE MAJOR COMPONENTS OF GNE TO THE DIFFERENCE IN REAL MULTIPLIERS WITH (a) MONEY SUPPLY HELD AT CONTROL LEVEL AND (b) INTEREST RATES HELD AT CONTROL LEVEL



curves.

Since these results are a combination of two effects, namely, the direct impact of the monetary sector on the IS and FF curves and the secondary effects induced by these shifts, they should not be interpreted as partial elasticities (the approach followed in Freedman and Longworth, 1975) but rather as total elasticities. It is impossible to distinguish between direct and induced effects without detailed knowledge of the links between the financial and real sectors, therefore these results complement the partial results of Freedman and Longworth, rather than substituting for them. Since the contributions of the major components of the IS curve to the difference in multipliers are cumulative, Figure 14 admits to the same interpretation as Figures 11 and 12.

The first component of GNP plotted, government expenditure (line 1), makes a very small contribution to the difference in multipliers: in other words, the difference in GNP cannot be attributed to the impact of monetary factors on government expenditure (i.e., the links between the monetary sector and government expenditure are not very strong). The next component is consumption (line 2); differences in consumption behaviour contribute somewhat to the differences in the multipliers, but given our knowledge of the direct links between the monetary sector and consumption, this difference is induced by the behaviour of the other components of GNE. This is especially true for the "bulge" in consumption at about year eight which represents the impact on consumption of the income changes (see Figure 12 for an idea of the lags involved) following the cyclical peak in year three.

Investment in residential construction (line 3) shows a continuous and significant contribution to the difference in multipliers. Given our knowledge of the partial elasticities of the links between the monetary and real sectors, most of the contribution can be directly attributed to links with the monetary sector.

Business fixed investment (line 4) is directly affected by monetary conditions (the cost of capital), but since the direct impact of monetary factors on investment is small, the major contribution of investment to the GNE multiplier has been induced by the accelerator. Since there are no direct links between the monetary sector and inventory investment (line 5), the total contribution of inventories is induced by other components.

The final component of GNE, net exports (line 6), makes a major contribution to the difference in the multipliers. Since we know both from theory and from this study that one of the major linkages between the monetary sector and real activity in an open economy is via the effect of monetary factors on the exchange rate, most of the contribution of net exports to the GNE multiplier is caused by monetary factors. The relatively weaker exchange rate under the constant-interest-rate scenario initially encourages import substitution followed by an increase in exports, with the dynamic pattern essentially that discussed in

Section 3.

5 CONCLUSION

We have now finished our analysis of the major determinants of the dynamic response of RDX2 to an exogenous shock to the IS curve by means of an increase in federal government expenditure. We also analyzed the effect a shock to the FF curve by means of an imposed devaluation. A number of salient features of the models' response are worthy of repetition.

i) The main cause of the cyclical response of the domestic sector is the accelerator mechanism in the investment equations.
ii) Trade leakages (assuming a fixed exchange rate) are a very important element in determining the magnitude of the fiscal multiplier.
iii) There is a noticeable difference in the dynamics of the model (in particular the existence of the second cycle in the GNE multiplier) depending on whether the exchange rate is fixed or floating.

These results are similar to conclusions obtained from simple theoretical models. It is reassuring to be able to demonstrate that the introduction of detail in a large econometric model like RDX2 has not been at the expense of consistency with economic theory.

REFERENCES

Bank of Canada. The Equations of RDX2 Revised and Estimated to 4Q72, Ottawa, Bank of Canada, 1976, 279 p. (Bank of Canada Technical Report 5).

Bank of Canada. A Sectoral Analysis of RDX2 Estimated to 4Q72, Ottawa, Bank of Canada, 1977, 287 p. (Bank of Canada Technical Report 6).

Branson, W.H. and J.M. Litvack. Macroeconomics, New York, Harper and Row, c 1976, 433 p.

Freedman, C. and D. Longworth. "Channels of Monetary Policy in RDX2", Paper Presented to the Queen's Conference on Canadian Monetary Issues, Aug. 17-20th, 1975. (Revised)

Helliwell, J.F. "Trade, Capital Flows and Migration as Channels for the International Transmission of Stabilization Policies", in Ando, A., R. Herring and R. Marston (eds.), International aspects of Stabilization Policies, Boston, Federal Reserve Bank of Boston, 1974. (Conference series no. 12) pp. 241-278.

Hicks, J.R. "Mr. Keynes and the 'Classics'; A Suggested Interpretation", Econometrica, 5, April 1937, pp. 147-159.

Mundell, R.A. "The Monetary Dynamics of International Adjustment Under Fixed and Flexible Exchange Rates", The Quarterly Journal of Economics, 74, May 1960, pp. 227-250.

Samuelson, P.A. "Interactions Between the Multiplier Analysis and the Principle of Acceleration", The Review of Economic Statistics, 21, May 1939, pp. 75-78.

Solow, R.M. "A Contribution to the Theory of Economic Growth", The Quarterly Journal of Economics, 60, Feb. 1956, pp. 65-94.

Tobin, J. "Money and Economic Growth", Econometrica, 33, Oct. 1965, pp. 671-684.

APPENDIX

In this appendix we detail the model changes and the accounting concepts employed in this study. In the first section we discuss various details relevant to the simulations performed and some of the simulation techniques used. The second section is a description of the nested sequence of models simulated in Section 2 of the report and the third a description of the models used in Section 3. The final section details the accounting underlying Figures 11 and 12.

1. Simulation Techniques

In all the experiments reported in this paper, RDX2 was forced to track history exactly by adding back in to the simultaneous solution the single equation errors. Thus the value of any variable in the control solution is its actual value. This was done principally to facilitate the mechanics of keeping a variable at control in the RDX2 simulator.

The quarterly output of RDX2 was converted to annual values for presentation: quarterly data stocks were converted to annual values by taking the end-of-year values; flows were summed over four quarters; rates and prices were averaged. Consistency problems arise when a flow variable is defined as a function of stock variables. For this reason, all statistics were initially computed on a quarterly basis and converted to annual equivalents in the last step of the analysis.

As mentioned in the text, we are concerned with the response of RDX2 to a sustained $100 million (1961 dollars) increase per quarter in federal government expenditure distributed over all categories in proportion to their share in federal outlays. Specifically, we increased the following variables by the amounts indicated in millions of 1961 dollars.

Variable	Description	Constant-dollar shock
IMEGF	Federal government investment in machinery and equipment	5
INRCGF	Federal government investment in non-residential construction	19
GCNWF/PGCNWG	Federal current nonwage expenses	76
		100

Even though it was our intention to examine the response of the model exactly as published, we found it necessary to make a number of modifications in order to perform the experiments satisfactorily.

In RDX2 the equations for federal government expenditure exhibit strong countercyclical tendencies. When we added (by means of add-factors) the shock to the respective equations in the model, we found that the countercyclical tendencies were such that, after a time, the level of government expenditure (including the shock) for these components was returning to its pre-shock value. Since we felt that this would not produce a true picture of the multiplier in the longer run, we decided to exogenize real federal investment in non-residential construction (INRCGF) and real federal investment in machinery and equipment (IMEGF). The respective price indices (PINRCG) and (PIMEG) remained endogenous. The equation for federal current nonwage expenditure (GCNWF), which is estimated in current dollars in RDX2, was suppressed. It was replaced by an equation which kept real federal current nonwage expenditures at their historical levels and converted them (and the real shock) to current dollars by the solution values of the appropriate price index (PGCNWG).

In RDX2 the exchange rate is regarded as the price that clears the balance of payments. Thus if the exchange rate is to remain unchanged, the private excess demand for foreign exchange must be exactly offset by official transactions. Hence we designed the changes made in the fixed-exchange-rate model of the foreign exchange market in RDX2 primarily to keep the spot exchange rate (PFX) at historical values. The equation explaining official intervention under the Bretton Woods system (FXO) was suppressed and FXO was defined as the inverse of the private-excess-demand equation (FXP). This ensures that the market-clearing condition is satisfied. In addition, the dummy variables used to capture the effect of historical changes during the fixed and flexible exchange-rate regime (QFIX) and (QFLEX2) were set at 1 and 0, respectively.

When we simulated the floating-exchange-rate regime, the settings of QFIX and QFLEX2 were reversed. The equations for the spot and the forward exchange rates (PFX) and (PFXF) were left unchanged and tuned to track the actual values (even when the exchange rate was fixed) by means of constant-form adjustments.

One danger of performing such simulation experiments over a long historical period is that the results may be "contaminated" by any structural changes explicitly incorporated into the model. We explicitly made the decision to take account of only those changes that caused a major change in the simulation results and to simulate the post-change structure over the whole period. With the exception of the change in the exchange-rate regime the only quantitatively important structural change during the simulation period was the introduction of the autopact.

In RDX2 there are two equations explaining the imports of motor vehicles and parts, including internal combustion engines, from the United States (MMVP12). The first, estimated from 1Q58 to 4Q64, represents the pre-autopact environment; the second, estimated from 1Q65 to 4Q72, represents the post-autopact environment. We assume in this report that the second equation holds over the entire simulation period. The equation tracks history (including the pre-autopact period) by means of constant-term adjustments.

In the financial sector of RDX2 public holdings of liquid assets are explained at a fairly disaggregated level. This enabled us to split out demand functions for such quantities as high-powered money, M1, and M2. We chose, in these experiments, to concentrate on M1A when simulating the adherence of the monetary authority to an unchanged monetary target. We define

M1A as the sum of currency outside the chartered banks held by the non-financial public, business demand deposits in chartered banks (excluding float), and personal chequing accounts in chartered banks. In RDX2 nomenclature, M1A = ANFCUR + DDB + ERBPCA*DPB.

In RDX2, the public's demand for liquid assets is subject to portfolio constraints. While it is theoretically possible to invert the demand-for-money function to obtain the interest rates associated with a desired path of money holdings, it involves considerable algebraic manipulation. Rather than perform this manipulation we decided to implicitly renormalize the demand function using a simulation rule in which RS was changed (during iteration) to offset any deviation of the simulated demand for money from its (historical) target values.

Thus the RS equation was replaced by the rule:

$$RS_i = RS_{i-1} + a(M^* - M1A_i)$$

where

i is the iteration number

M* is the historical value of M1A

$0 < a < 1$

When performing the simulations with interest rates unchanged, we exogenized the equation for RS. Since the yields on long-term government bonds (RMS), (RML), and (RL) are explained by a term-structure relationship based on RS, we exogenized the equations explaining these variables in order to highlight the links between the financial and real sectors.

2. Variants of RDX2 Simulated in Section 2

Model 0 corresponds to RDX2 with the following sectors or equations exogenized.

(i) Prices (sector 7) and interest rates (sector 17)

(ii) Private sector wages (WQMMOB and WQC)

(iii) Labour markets (NL, NE, NMMOB, NMMOBD, NMMOBS, NIS, NGPAF, NGPAPM, NC)

(iv) Non-inventory investment (IRC, INRC, IME, IMEGPM, INRCSM)

(v) Capacity (UGPPD)

(vi) Inventory investment (IIB). In addition UGPPA = UGPP

(vii) Trade flows (sector 4)

(viii) International capital flows (sectors 19, 20 and part of 21)

(ix) The market for foreign exchange (PFX and PFXF)

(x) Migration (NIMS and NEMS).

Models 1 - 10 were obtained from model 0 by the sequential endogenization of the above:

Model 1 Endogenize the RDX2 price equations (sector 7), the interest rate equations (sector 17) including the simulation rule for RS, and endogenize WQMMOB and WQC by expressions of the form simulated wage = ((simulated

PCPI)/(historical PCPI))*(historical real wage).

Model 2 Replace the wage equations of Model 1 by the RDX2 formulation.

Model 3 Model 2 plus endogenous labour markets (i.e., NL, NE, NMMOB, NMMOBD, NMMOBS, NIS, NGPAF, NGPAPM, NC).

Model 4 Model 3 plus engodenous non-inventory investment (i.e. IRC, INRC, IME, IMEGPM, INRCSM).

Model 5 Model 4 plus UGPPD endogenous.

Model 6 Model 5 plus endogenous inventory investment (IIB). In addition UGPPA is allowed to differ from UGPP.

Model 7 Model 6 plus endogenous trade flows (sector 4).

Model 8 Model 7 plus endogenous international capital flows (sectors 19, 20 and part of 21).

Model 9 Model 8 plus endogenous foreign exchange market (i.e., PFX and PFXF).

Model 10 Model 9 plus endogenous migration (i.e., NIMS and NEMS).

This is the full RDX2 model.

3. Nested Sequence of Models Used in Section 3

In Model 1 the only endogenous variables are the trade equations (sector 4) and import prices (equations 7.24 to 7.40).

Model 2 is model 1 with all other prices endogenous (i.e., the rest of sector 7 plus the identities necessary to calculate PGNE and PGPP).

Model 3 is model 2 with private sector wages (WQMMOB and WQC) endogenous.

Model 4 is model 3 with all domestic GNP components endogenous (i.e., sectors 1-14 with the exception of the equations for NIMS and NEMS).

Model 5 is model 3 with all the remaining sectors endogenous (with the exception of the equation for PFX).

4. Accounting Definitions Underlying Figures 11 and 12.

The decomposition of the demand and supply response reported in Figure 11 corresponds to the shock-minus-control values (scaled by the real value of the shock) of the following quantities.

A. DEMAND COMPONENTS

(1) Government expenditure

G = (GCNWF + GCNWPM + GCGSM + GCGSCPP + GCGSQPP)/PGCNWG + IMEGF + IMEGPM + 621*(GWIF+GWIPM) (WQIOS) + 1333*NIS + 1261*NGPAF + 916*NGPPAPM

(2) Exports of goods and services

X

(3) Private consumption expenditure

CP = CNDSD + CS + CMV + CDO

(4) Investment in residential construction

IRC

(5) Business fixed investment

IME + INRC

(6) Intended inventory investment

III = IIB + IIF + IIG - UGPPA + UGPP

B. SUPPLY COMPONENTS

(7) Unintended inventory investment

IIU = UGPP + UGPPA

(8) Imports of goods and services

M

(9) Business output, government wages, and net indirect taxes

UGPIT = UGPP + TILGS/PGNE + 1333*NIS + 1261*NGPAF

+ 916*NGPAPM + 621*(GWIF+GWIPM)/WQIOS)

Actual business output (UGPP) in RDX2 is implicitly defined as a multiplicative function of short-term productivity (STP) and a synthetic measure of business output (UGPPS) based on the contribution of the factors of production in an explicit Cobb-Douglas production function. These factors are:

KNRC	Stock of nonfarm non-residential structures
KME	Stock of nonfarm machinery and equipment
NMMOB	Paid employees in mining, manufacturing, and other business
HAWMM	Average hours worked in mining and manufacturing

The production function has weights of .16, .12, .72 for non-residential construction, machinery and equipment and man-hours respectively. If the function is differentiated with respect to time, then:

$$\frac{U\dot{G}PP}{UGPP} = .16\,\frac{K\dot{N}RC}{KNRC} + .12\,\frac{K\dot{M}E}{KME} + .72\,\frac{NM\dot{M}OB}{NMMOB} + .72\,\frac{HA\dot{W}MM}{HAWMM} + 1\,\frac{S\dot{T}P}{STP}$$

where

$\dot{X}/X$ denotes the rate of change of X at time t.

We then define $a(\dot{X}/X)/(U\dot{G}PP/UGPP)$ (where a is the coefficient of the factor in the production function) to be the proportional contribution of factor X to the total shock-minus-control change in UGPP divided by the real government shock. These quantities are illustrated in Figure 12.

Printed in Canada Imprimé au Canada