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Abstract

Bank liability guarantee schemes have traditionally been viewed as costless measures to shore up investor confidence and prevent bank runs. However, as the experiences of some European countries, most notably Ireland, have demonstrated, the credibility and effectiveness of these guarantees are crucially intertwined with the sovereign's funding risks. Employing methods from the literature on global games, we develop a simple model to explore the systemic linkage between the rollover risks of a bank and a government, which are connected through the government's guarantee of bank liabilities. We show the existence and uniqueness of the joint equilibrium and derive its comparative static properties. In solving for the optimal guarantee numerically, we show how its credibility can be improved through policies that promote balance-sheet transparency. We explain the asymmetry in risk transfer between the sovereign and the banking sector, following the introduction of a guarantee as being attributed to the resolution of strategic uncertainties held by bank depositors and the opacity of the banks' balance sheets.

JEL classification: G01, G28, D89
Bank classification: Financial stability; Financial system regulation and policies

Résumé

Les programmes de garantie des passifs bancaires ont traditionnellement été perçus comme des moyens, à coût nul, de soutenir la confiance des investisseurs et de prévenir les retraits massifs de dépôts. Toutefois, comme en témoigne l'expérience de certains pays européens, surtout l'Irlande, la crédibilité et l'efficacité de ces programmes et le risque de financement souverain sont imbriqués de façon cruciale. À l'aide de méthodes inspirées de la littérature sur les jeux globaux, les auteurs élaborent un modèle simple qui permet d'explorer le lien systémique que la garantie de l'État sur les passifs bancaires crée entre le risque de refinancement d'une banque et le risque de financement souverain. Les auteurs démontrent l'existence et l'unicité de l'équilibre conjoint et en établissent les propriétés de statique comparative. En déterminant numériquement le niveau de garantie optimal, ils montrent que l'adoption de politiques favorisant la transparence des bilans peut accroître la crédibilité de la garantie offerte. Enfin, ils expliquent l'asymétrie du transfert du risque entre le secteur bancaire et l'État, après la mise en place d'un programme de garantie, par la levée de l'incertitude stratégique à laquelle sont confrontés les déposants et par l'opacité des bilans bancaires.

Classification JEL : G01, G28, D89
Classification de la Banque : Stabilité financière; Réglementation et politiques relatives au système financier

1. Introduction

In the aftermath of the collapse of Lehman Brothers in September 2008, a great many, particularly European, countries issued sizable bank debt guarantee programs. In this paper we analyze the conditions conducive to the success of such schemes. We address this issue by answering several smaller, but more tractable, questions. Firstly, how does a government's issuance of a banking sector liability guarantee scheme influence the lending behavior of sovereign and bank creditors? Secondly, what is the impact of the guarantee on the *ex ante* probabilities of banking and sovereign default, as well as on the likelihood of a systemic crisis? Thirdly, is there a guarantee that optimally trades off the risk of sovereign and bank default? And, finally, how does the effectiveness of the (optimal) guarantee depend on balance-sheet transparency and on the liquidity of banks and sovereigns alike?

The global financial crisis was marked by a severe loss of confidence by investors in financial markets around the world. Triggered by losses on U.S. subprime mortgages and other toxic financial assets, interbank markets froze as banks ceased lending to each other.[1] Figure 1(a) illustrates this development. It shows the EURIBOR-OIS spread, a measure of interbank market tensions in the euro area, sharply and abruptly increasing three-fold following the collapse of Lehman Brothers in September 2008. Figure 1(b) shows the changes in the spreads for banking sector and sovereign credit default swaps (CDSs), between January 2007 and late September 2008 (shortly after the default of Lehman Brothers). Viewed as proxies for the probabilities of bank default, we note a marked increase in the fragility of the banking sector in several countries.

In light of such deteriorating conditions, many governments introduced contingent guarantee schemes for retail and wholesale bank deposits. These schemes were viewed as cost-effective measures to stave off bank runs, whereby governments would lend their own creditworthiness to the financial sector.[2] Figure 1(c) compares the sizes of schemes introduced in several countries, relative to their GDP. The schemes in Italy and Spain amounted to about 3 percent and 9 percent of GDP, respectively, while in Austria and the Netherlands they totalled roughly 30 percent of GDP. Albeit sizable, all these programs were dwarfed by the measures introduced in Ireland, wherein the state guaranteed *all* bank liabilities for a period of two years with no monetary cap. The broad mandate of the Irish scheme, which amounted to roughly 244 percent of GDP, followed from the consensus that, as Patrick Honohan (2010), governor of the Central Bank of Ireland, noted, "No Irish bank should be allowed to fail."

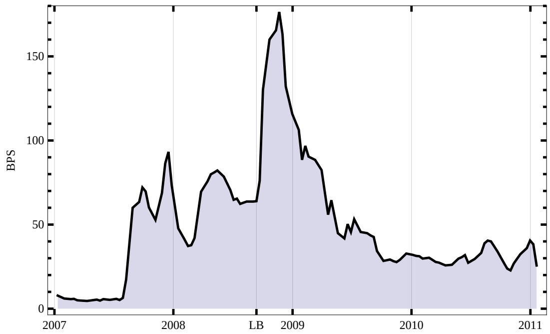
In general, the guarantee schemes were successful in alleviating banking sector default risk. Yet, they led to a simultaneous increase in sovereign default risk. This can be seen in Figure 1(d), which compares the change in sovereign CDS spreads with the change in banking sector CDSs. Based on this measure, it appears that the increase in the sovereigns' default probabilities was of much smaller magnitude than the reduction in the respective banking sector default probability. This phenomenon indicates that the guarantees not only led to a reallocation of risks between banks and governments, but they may have also reduced economy-wide risks.

The case of Ireland requires particular attention, since it can be considered exemplary for the dramatic and systemic consequences that may follow from tying the government's

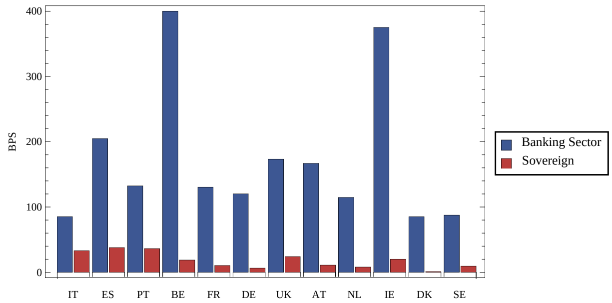
[1] See Taylor and Williams (2008) or Holthausen and Pill (2010) for a detailed investigation of interbank money markets during the 2007–08 financial crisis.

[2] Table A1 in the appendix summarizes schemes introduced in several countries. See also Schich and Kim (2011) for an overview of different banking sector safety nets.

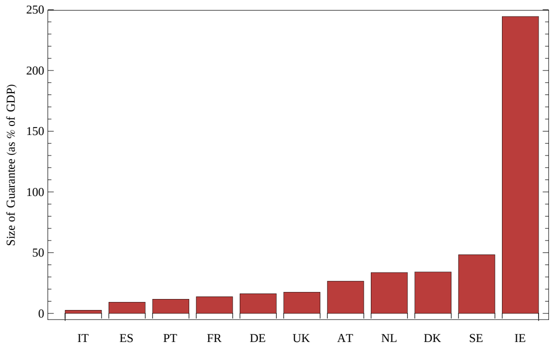
Figure 1: Stylized Facts



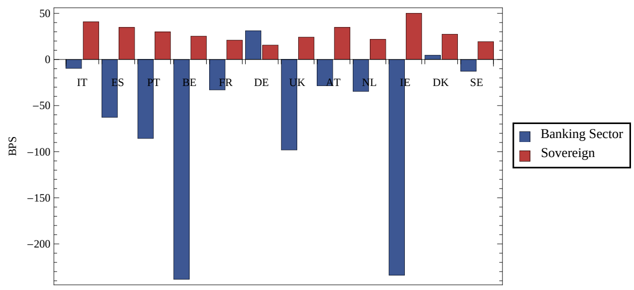
Panel (a): Time series of the 3-month Euribor-OIS spreads in basis points. The marker 'LB' indicates the date that Lehman Brothers filed for bankruptcy (15 September 2008). Data from Bloomberg.



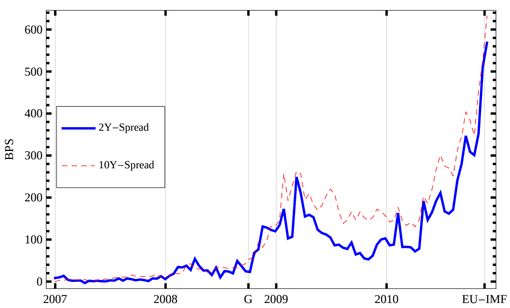
Panel (b): Change in CDS spreads for banks and sovereigns between 1 January 2007 and 25 September 2008. Bank CDSs are unweighted averages of banks with headquarters in respective countries. Data from Bloomberg.



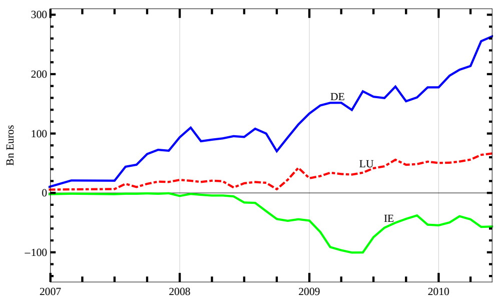
Panel (c): Guarantee sizes in % of GDP. Data from OECD.



Panel (d): Change in CDS spreads for banks and sovereigns between 26 September 2008 and 21 October 2008. Bank CDSs are unweighted averages of banks with headquarters in respective countries. Data from Bloomberg.



Panel (e): Irish spreads over German bund, in basis points. The marker 'G' indicates the date that the Irish government introduced the guarantee scheme, while 'EU-IMF' indicates the date that the joint European Union and International Monetary Fund bailout for Ireland was announced. Data from Bloomberg.



Panel (f): Net TARGET2 Liabilities of selected euro-area national central banks against the Eurosystem in millions of euro. Negative values reflect a Target2 liability, positive numbers a Target2 asset. Data from University of Osnabrück's Euro Crisis Monitor.

funding situation to that of its banking sector by means of debt guarantees. Prior to the crisis, the Irish economy was considered 'sound,' with a low government debt and deficit, prospects for growth, and as Figure 1(e) depicts, low sovereign funding costs. Against this background, Ireland issued its first bank liability guarantee program in October 2008. The guarantee had the immediate effect of driving down CDS spreads for the banking sector. However, as concerns rose among sovereign creditors as to whether the Irish government would be able to pay out the guarantee, Ireland's funding costs skyrocketed. Moreover, the guarantee failed to prevent large withdrawals away from Irish banks to the perceived safe havens like Germany or Luxembourg. Figure 1(f) illustrates this trend by showing net TARGET2 liabilities of the Irish Central Bank, which serve as a proxy for the cumulative net outflows of euro-denominated liquidity.[3] The systemic events culminated in the nationalization of Anglo-Irish Bank in January 2009, and in the Irish government seeking a bail-out on 21 November 2010, jointly from the European Union's European Financial Stability Facility and the International Monetary Fund.

The resulting 'Celtic crisis' differs dramatically from the actual goal of governments issuing bank debt guarantee schemes. The systemic crisis was a direct consequence of the false belief that a guarantee would shore up investor confidence, without placing any strain on a government's own funding needs, and hence, on the credibility in keeping its guarantee promises. Or, as one financial market participant bluntly put it to the Wall Street Journal (2011) when asked to comment on the on-going banking sector problems in the euro area, "How useful would bank guarantees from member states be if these member states are themselves shut out of financial markets?"

In this paper, we model a systemic liquidity crisis. The model consists of a government, one bank and a large pool of bank and sovereign creditors. Bank creditors must decide whether to roll over their loans to the bank or to withdraw. Their decisions depend on the bank's recourse to liquidity and the contingent guarantee provided by the government. Sovereign creditors, in turn, decide whether to continue lending to the government or to withdraw. The decisions of sovereign creditors depend on the government's available resources and the possible payment of the bank guarantee. Using standard techniques from the literature on global games, we embed our model in an incomplete information setting, where creditors face strategic uncertainty concerning the actions of other creditors, as well as fundamental uncertainty over the bank's and the government's recourse to liquidity. Following well-established lines of reasoning, we show that our model exhibits a unique equilibrium in threshold strategies, and that there are no other equilibria in non-threshold strategies.

Our model displays *strategic complementarities* within each group of creditors. That is, the incentives of individual bank (sovereign) creditors to roll over are increasing in the mass of bank (sovereign) creditors who also roll over. Furthermore, bank creditors' incentives to roll over are also increasing in the mass of sovereign creditors who lend to the government. Hence, sovereign creditors' actions are strategic complements for bank creditors. But the converse does not hold. The incentives of a sovereign creditor to lend are decreasing in the mass of bank creditors who roll over. The actions of bank creditors are therefore *strategic substitutes* for sovereign creditors. To better understand the latter

[3] While the Irish guarantee scheme was introduced in October 2008, the outflows continued until May 2009, when they peaked at approximately €100 billion. While there was a reversal of trends between May and September 2009, the pace of withdrawals accelerated shortly thereafter and continued throughout 2010, peaking in January 2011. See Bindseil and König (2012) for details on the role and mechanics of the TARGET2 system during the financial crisis.

property, suppose that, following the introduction of a guarantee, a large fraction of bank creditors roll over their loans. However, were the bank still to fail, a large guarantee payout would come due, adding to the government's liabilities. Anticipating such an outcome, sovereign creditors would become doubtful about the government's liquidity and thus more reluctant to roll over their own claims. This property of our model must be interpreted with caution and against the background of the questions that we address. Although the government in the model wishes to avoid a bank default, we abstract away from direct payments being made by the bank to the government. If, for example, the government could collect taxes from the bank, its liquidity situation would be directly intertwined with the bank and the strategic substitutes effect would be less pronounced. However, since such taxes may distort the incentives of the bank to act with prudence and remain solvent, we abstract from their inclusion in order to derive the 'pure' strategic interactions between the different groups of creditors.

Finally, using numerical methods, we investigate how the optimal guarantee size, and the welfare properties it induces, relates to the underlying model parameters. The optimal guarantee is obtained by minimizing a cost-of-crisis function, which is a weighted sum of the output losses attributed to individual bank and government defaults, and the systemic crisis. Increases in the *ex ante* expected recourse to liquidity for both the bank and government lead to larger guarantees. We also find that policies that promote the bank's balance-sheet transparency are welfare enhancing. These gains are further improved by the added balance-sheet transparency of the government. We also explain why the reduction in banking sector CDS spreads that followed the introduction of guarantee schemes was often larger in absolute magnitude than the accompanying increase in sovereign CDS spreads. We argue that the strong reduction in banking sector CDSs may have been due to the guarantee's effect of removing strategic uncertainty among bank creditors, while the higher sovereign CDSs are attributed to the opacity of the bank's balance sheets.

The paper is structured as follows. We introduce the canonical bank debt rollover model in section 3. In section 4 we introduce the government that issues a guarantee but is itself subject to rollover risk. The comparative statics properties of this extended model are provided in section 5. In section 6 we report numerical results for the effects of transparency in a calibrated exercise. Section 7 concludes. Most of the mathematics and all proofs are deferred to the appendix.

2. Relation to the Literature

The modern theoretical perspective on banks' maturity and liquidity mismatches and deposit guarantees is based on the seminal model of Diamond and Dybvig (1983). They show the existence of multiple, self-fulfilling equilibria for a bank with short-term financed illiquid assets. In one equilibrium, the bank is run by all depositors and fails, since its liquid reserves are not sufficient to cover depositors' aggregate claims. In the second equilibrium, only a small amount of withdrawals occurs and the bank's liquidity is sufficient to avoid default. The two equilibria are brought about by a mis-coordination of beliefs. Deposit insurance financed by taxes helps to overcome this multiplicity by increasing depositors' expected payoffs from rolling over. The mere existence of such deposit insurance is sufficient to coordinate creditors on the efficient equilibrium and to avoid a bank run. In equilibrium, the insurance is never paid out.

Morris and Shin (2000) and Goldstein and Pauzner (2005) solve the multiple equilibria problem by extending the set-up of Diamond and Dybvig to an incomplete information setting where information on the liquidity of the bank is not common knowledge. By employing the global games approach of Morris and Shin (1998, 2003), they solve for the unique equilibrium in threshold strategies. If the information received by depositors is sufficiently precise and a bank's fundamentals are below a critical threshold, most depositors withdraw, thus causing the bank's failure. If liquidity is sufficiently high, then depositors stay. Importantly, in equilibrium, the amount actually paid out due to the deposit guarantee is low, since there are only a few depositors who roll over despite the bank's default. This logic has recently been translated to government guarantee schemes by Kasahara (2009) and Bebchuk and Goldstein (2010). Kasahara considers a standard global game model, where creditors to a firm enjoy the benefit of a government-financed debt guarantee. He shows that the guarantee removes inefficient coordination failures only if the government combines this policy with an information policy where it provides a sufficiently precise public signal about the firm's fundamental. Although the guarantee in Kasahara's model is exogenously financed, he also considers potential costs that may arise when the guarantee creates adverse incentives and leads to a moral hazard problem on the side of the firm.

Bebchuk and Goldstein (2010) consider a stylized global game model where the coordination failure occurs among banks that can decide whether to lend to the real economy or not. Among other policy measures, they consider how a guarantee of banks' loans could overcome the no-lending or 'credit-freeze equilibrium.' Similar to the effect of a deposit insurance in a bank-run model, they find that when the guarantee is sufficiently high, the risk of coordination failure may be reduced to zero. Bebchuk and Goldstein focus especially on the 'global game solution' of vanishing fundamental uncertainty, concluding that "government's guarantees ... do not lead to any capital being spent ... this mechanism leads to an improvement in the threshold below which a credit freeze occurs without any actual cost" (p. 25). The authors nevertheless acknowledge that the validity of a guarantee mechanism crucially "depends on the credibility of the government in providing the guarantee" (p. 26). Our model contributes to this recent literature by explicitly considering the credibility of the guarantee by introducing a refinancing problem for the sovereign guarantor. As will be explained in greater detail below, Bebchuk and Goldstein's conclusion still holds in our model whenever fundamental uncertainty vanishes. Yet, whenever bank creditors face some fundamental uncertainty, the guarantee leads to a higher default risk of the sovereign.

Cooper (2012) reports a similar result in a multiple equilibrium model of sovereign debt pricing. He studies how a guarantee by a sound country shifts strategic uncertainty toward the guarantor. In the absence of fundamental uncertainty, beliefs of creditors are not affected and the guarantee simply acts as a device that selects the good equilibrium. Yet, when fundamental uncertainty is present, the guarantee may influence the price of the sound country's debt. The guarantee thereby creates a contagion channel between the countries that was not previously present.

Acharya et al. (2011) consider the related problem of financial sector bailouts and their impact on sovereign credit risk. Bank bailouts are financed by taxing the non-financial sector of the economy. While the bailout is successful in alleviating problems of the banks, the higher tax burden of the non-financial sector reduces the economy's growth rate. Thus, the government's task is to set the optimal tax rate in order to maximize the economy's welfare. In this paper, we abstract from taxation and finely focus on the coordination problem between bank creditors and sovereign creditors. This emphasis on joint coordination failures allows us to address more adequately the issues of the governments' "ability to pay" and the credibility of the guarantee. The government in our model then sets the optimal guarantee in order to minimize the expected costs of crises and coordination failures.

Closely related to our model is the 'twin crises' global game of Goldstein (2005) that also includes two groups of agents: currency speculators and bank creditors. The former attack a pegged exchange rate, while the latter hold foreign currency-denominated claims against a domestic bank. The (exogenous) political decision by a government to peg the exchange rate connects the actions of the two groups of agents. The greater the fraction of speculators who attack the currency, the more likely a devaluation of the currency becomes, and hence the more likely is the bank to default due to the currency mismatch on its balance sheet. Conversely, the greater the fraction of bank creditors who withdraw their funds, the larger is the outflow of foreign reserves, and it becomes more likely that the currency peg will break down. The actions of bank creditors and speculators are strategic complements. They reinforce each other, giving rise to a vicious circle. In our model, the actions of sovereign and bank creditors are also connected through an exogenous political decision (guaranteeing bank debt). But, in contrast to Goldstein's twin crises theory, only the actions of sovereign creditors are strategic complements for bank creditors, while bank creditors' actions are strategic substitutes for sovereign creditors. Moreover, in Goldstein's model, the bank's and the sovereign's financial strength is determined by the same fundamental, whilst the financial strength of the respective institutions in our model is driven by different, independently distributed fundamentals.

Global games with different fundamentals have not been extensively studied. Two examples related to our model are Dasgupta (2004) and Manz (2010). Dasgupta models financial contagion in a global game between two banks in different regions that are exposed to independent regional shocks. Linkages between banks are created by cross-holdings of deposits in the interbank market, and regional shocks may therefore trigger contagious bank failures in equilibrium. Manz considers a global game with two independently distributed fundamentals to study information-based contagion between distinct sets of creditors of two firms. Creditors have imperfect information about both their debtor firm's fundamental and a common hurdle function that a fundamental must pass for the respective firm to become solvent. In contrast to Dasgupta, Manz's model has a sequential structure where creditors to the second firm can observe whether the first firm failed. This observation functions as a common signal and provides creditors to the second firm with information about the hurdle, which in turn influences their decision to liquidate their own claim. While we also resort to the assumption of independently distributed fundamentals, creditor decisions are taken simultaneously, which implies that informational contagion, based on the observation of a particular outcome in one refinancing game, cannot occur. Rather, the spillovers between the bank's and the sovereign's refinancing problem are determined by the guarantee.

3. Canonical Bank Debt Rollover Game

In this section, we describe the canonical rollover game that serves as the workhorse for the remainder of the paper. We introduce an exogenously financed guarantee and discuss the relationship between balance-sheet transparency and the costliness of the guarantee.

3.1. Model description

A bank, indexed b, is indebted to risk-neutral creditors $n_b \in [0,N_b]$, where $N_b \in \mathbb{R}_+$ measures the bank's exposure to funding illiquidity. Creditors hold identical claims against the bank, each with a face value of one monetary unit. The bank's recourse to liquidity is summarized by the random variable $\theta_b \sim U[-\eta_b, \eta_b + \theta_b^0]$, with the *ex ante* mean recourse to liquidity being $\theta_b^0/2$. θ_b comprises two parts. First, there are the liquid assets on the bank's balance sheet, which directly contribute to increasing θ_b. Second, the bank can raise cash by entering into secured finance arrangements – for example, repurchase agreements and covered bonds – where it pledges illiquid assets to investors in exchange for cash. These investors, who are not explicitly modelled, include other commercial banks, hedge funds and also the central bank.

Creditors simultaneously decide whether to roll over their loans to the bank, or to withdraw. We express the set of actions for a typical bank creditor by $\{0,1\}$, where 0 denotes rolling over, while 1 denotes withdrawing. Defining $\lambda_b \in [0,1]$ as the fraction of bank creditors who withdraw, the bank defaults whenever aggregate withdrawals exceed the available liquid resources; i.e.,

$$\lambda_b N_b \geq \theta_b. \tag{1}$$

We assume that all bank creditors have common payoffs, which are summarized in Table 1. Withdrawal by a creditor may entail additional transaction costs, which are subtracted

		Bank	
		Default	Survive
Bank Creditor	Withdraw	C_b	C_b
	Roll over	ℓ	D_b

Table 1: A typical bank creditor's payoffs

from the unit claim held against the bank. Thus, the net payoff from withdrawing is $C_b \leq 1$, independent of whether the bank defaults or survives.[4] If, however, the creditor rolls over the loan and the bank survives, the creditor is paid back $D_b > 1$, which includes both the original amount lent and additional interest payments. Finally, if the bank defaults, creditors who rolled over their loans receive a fraction ℓ of their unit claim. We interpret ℓ as the payment stemming from a bank liability guarantee scheme. In what follows, we assume that ℓ is exogenously financed and that creditors receive the amount

[4]The fact that creditors *always* receive C_b when they choose to withdraw deserves comment. The interpretation of θ_b as available liquid resources implies that the bank is unable to pay one unit per claimant for $\theta_b < \hat{\theta}_b$. A more plausible set-up would then be to impose a 'sequential service constraint' and assume that creditors receive only a fraction of the available resources in the case of bank default, which may be determined by θ_b, the fraction λ_b and possible transaction costs. The resulting payoff from withdrawing would inherit a negative dependency on λ_b. However, the realism added by modelling the problem in this way has to be traded off against technical difficulties that arise due to the resulting *partial strategic complementarities*. The proof of equilibrium employed above relies on the existence of *global strategic complementarities*; i.e., creditors' actions strictly decrease in λ_b. But with the more realistic assumption of a 'sequential service constraint,' the expected payoff differential (rolling over vs. withdrawing) becomes increasing in λ_b over a certain range. However, as Goldstein and Pauzner (2005) show, under the alternative assumption of the payoff differential obeying a single-crossing property, the nature of the equilibrium remains unaltered. There is still a unique symmetric threshold equilibrium. Under the additional restriction to uniform distributions, there are also no other non-threshold equilibria. Yet, their proof is more involved, leading to more complicated comparative statics calculations that continue to remain qualitatively identical. Thus, to keep the model tractable, we stick to the less-realistic assumption that the payoff from withdrawing is fully safe which guarantees the global strategic complementarity property. This is in line with standard practice in the literature; e.g., Chui et al. (2002) or Morris and Shin (2006). Rochet and Vives (2004) further motivate this approach by appealing to institutional managers who seek to make the right decision, while their payoffs do not depend directly on the face value of their claims.

whenever it comes due. We further assume that $D_b > C_b \geq \ell \geq 0$, which entails that creditors face a coordination problem.[5]

3.2. *Tripartite classification of the fundamental*

The bank debt rollover game exhibits a *tripartite classification* of the fundamental θ_b, which is a characteristic of such coordination games.[6] For $\theta_b < 0$, the bank always defaults, irrespective of the fraction λ_b of creditors who withdraw. We refer to this as the *fundamental insolvency* case or the efficient default. It is a dominant action for creditors to withdraw in this case. For $\theta_b > N_b$, the bank always survives, even if all creditors were to withdraw their funds. Here it is dominant for all creditors to roll over.

If $\theta_b < 0$, there exists a unique Nash equilibrium where all creditors withdraw and the bank defaults. For $\theta_b > N_b$, there is a unique Nash equilibrium where all creditors roll over their loans and the bank survives. However, under the assumptions of *common knowledge* of θ_b, the game exhibits multiple equilibria – in pure strategies – for intermediate values $\theta_b \in [0, N_b]$. The equilibria in this interval are sustained by common self-fulfilling expectations about the behavior of other creditors. In one equilibrium, each creditor expects that all other creditors will withdraw, and hence withdrawing is the best response to this belief. Creditors' aggregate behavior leads to the bank's default, validating the initial beliefs. In the second equilibrium, each creditor expects all other creditors to roll over their loans. This implies that each creditor chooses to roll over as the best response to this belief. The resulting outcome is one where the bank survives, which once again vindicates the beliefs of creditors.

3.3. *Information structure and strategies*

To eliminate the multiplicity of equilibria, we use the global games approach and relax the assumption of common knowledge about θ_b. Instead, we make the weaker assumption that creditors have heterogeneous and imperfect information concerning the bank's fundamental. Specifically, creditors receive private signals about the fundamental before choosing their action. The signals are modelled as $x_{n_b} = \theta_b + \varepsilon_{n_b}$, where ε_{n_b} is an idiosyncratic i.i.d. noise term uniformly distributed over the support $[-\varepsilon_b, \varepsilon_b]$. Following the literature on transparency (i.e., Heinemann and Illing (2002); Bannier and Heinemann (2005); Lindner (2006)), we interpret ε_b as the *degree of balance-sheet transparency* in the banking sector. When ε_b is small there is a high degree of transparency, since the signals that bank creditors receive enable them to better infer the true fundamental from their observed signals. Creditors use their private signals and the commonly known prior to form individual posteriors $\theta_b|_{x_{n_b}}$ by means of Bayesian updating. Furthermore, to apply global game methods, we need to ensure that the support of the fundamental distribution is sufficiently large to include an upper and a lower dominance region.[7]

[5]For simplicity, we deliberately ignore the possibility of default due to insolvency at some later date, which may occur even though the rollover has been successfully managed.

[6]See, e.g., Diamond and Dybvig (1983), in the context of bank runs, and Obstfeld (1996) in the context of currency crises.

[7]Given the support of the signal error, after receiving a signal $x_{n_b} < -\varepsilon_b$, a creditor is certain that the bank defaults (even if all other creditors roll over). And similarly, after receiving a signal $x_{n_b} > N_b + \varepsilon_b$, the creditor is certain that the bank survives (even if all other creditors withdraw). We assume that the support of θ_b is sufficiently large to include states where all creditors find either rolling over or withdrawing dominant; i.e.,

$$[-2\varepsilon_b, N_b + 2\varepsilon_b] \subset [-\eta_b, \theta_b^0 + \eta_b].$$

A strategy for a typical creditor is a complete plan of action that determines for each realization of the signal whether the creditor rolls over or withdraws. Formally, a strategy is a mapping $s_{n_b} : x_{n_b} \mapsto \{0,1\}$. Strategies are *symmetric* if $s_{n_b}(\cdot) = s_b(\cdot)$ for all n_b. A strategy is called a *threshold strategy* if a creditor chooses to withdraw for all x_{n_b} below some critical $\hat{x}_{n_b}$ and rolls over otherwise. Finally, a *symmetric threshold strategy* is a threshold strategy where *all* creditors use the same critical $\hat{x}_b$.

3.4. Equilibrium

A symmetric equilibrium of the bank debt rollover game with heterogeneous information is given by the strategy $s_b(\cdot)$ and the aggregate choice $\lambda(\theta_b)$ such that creditors maximize their expected payoffs and

$$\lambda_b(\theta_b) = \frac{1}{2\varepsilon_b}\int_{\theta_b-\varepsilon_b}^{\theta_b+\varepsilon_b} s_b(x_{n_b})dx_{n_b}.$$

It is a well-established result that coordination games such as our bank debt rollover game exhibit a unique equilibrium in symmetric threshold strategies.[8] The following proposition restates this result in terms of our model.

Proposition 1. *The bank debt rollover game has a unique equilibrium summarized by the tuple* $(\hat{x}_b, \hat{\theta}_b)$ *where*

$$\hat{x}_b = \hat{\theta}_b + 2\varepsilon_b\left(\frac{\hat{\theta}_b}{N_b} - \frac{1}{2}\right) \tag{2}$$

and

$$\hat{\theta}_b = \frac{N_b(C_b-\ell)}{D_b-\ell}. \tag{3}$$

Creditors with signals x_{n_b} *withdraw if* $x_{n_b} < \hat{x}_b$ *and roll over if* $x_{n_b} > \hat{x}_b$. *The bank defaults if and only if* $\theta_b < \hat{\theta}_b$.

Proof. See Morris and Shin (2003) for the proof of existence and uniqueness, and the appendix for the calculations of equations (2) and (3). □

3.5. Changes to the guarantee size

Albeit stylized, we interpret ℓ as the payment from a bank liability guarantee scheme provided by the government. Creditors receive ℓ in case they roll over their loans and the bank defaults. If creditors choose to withdraw, they always receive C_b. In the absence of a guarantee (i.e., $\ell = 0$), bank creditors will choose to roll over their loans as long as the probability attached to the bank's survival is sufficiently high. In terms of the payoffs, they will roll over as long as the spread between D_b and C_b is large enough to compensate for incurring the risk of ending up with a zero payoff in case of bank default. A positive guarantee $\ell > 0$ reduces the opportunity cost of rolling over (given by $C_b - \ell$), and therefore increases creditors' incentives to roll over. All other things equal, a larger guarantee lowers the critical thresholds $\hat{\theta}_b$ and $\hat{x}_b$, thereby leading to a higher *ex ante* survival probability; i.e.,

$$\frac{\partial\hat{\theta}_b}{\partial\ell} = \frac{N_b(C_b-D_b)}{(D_b-\ell)^2} < 0.$$

[8] See Morris and Shin (2003). For a general class of distributions of the fundamental other than the uniform distribution, uniqueness requires that the private signals of creditors are sufficiently precise; i.e., they require ε_b to be sufficiently small.

3.6. Transparency and expected costs of a guarantee

The comparative static result above and its implications may have contributed to the widely held perception that bank liability guarantee schemes are a costless measure to shore up confidence in financial institutions. While it is true that the guarantee serves as a device to change the incentives of creditors to coordinate on the efficient equilibrium, the question remains whether this is indeed a costless policy. To better appreciate the conditions under which this holds true, consider the case where creditors face only *strategic uncertainty* about the behavior of other creditors and no *fundamental uncertainty* about the true realization of θ_b. This corresponds to a high degree of balance-sheet transparency with $\varepsilon_b \to 0$, which implies that $\hat{x}_b \to \hat{\theta}_b$. All creditors now receive almost identical signals. Since they all use the same threshold strategy around $\hat{x}_b$, in equilibrium, either everyone rolls over and the bank survives or everyone forecloses and the bank defaults. The payoffs to the creditors are D_b if everyone rolls over their loans, or C_b if they all withdraw. While the guarantee payment ℓ raises the creditors' incentives to roll over, it is never paid out. A policy-maker could therefore issue an arbitrarily large guarantee and effectively control the likelihood of default without ever having to follow up on its promises. In particular, by setting $\ell = C_b$, the bank's failure threshold converges to $\hat{\theta}_b = 0$ such that only a fundamentally insolvent bank defaults. By making such a choice, a policy-maker can prevent inefficient bank runs due to coordination failures.

The result that guarantees are costless changes, however, with a lower degree of balance-sheet transparency and creditors facing fundamental uncertainty; i.e., $\varepsilon_b > 0$. In this case, some creditors may decide to roll over their loans due to 'misleading' signals $x_{n_b} > \hat{x}_b$, even though $\theta_b < \hat{\theta}_b$ and the bank defaults. These creditors become benefactors of the guarantee scheme and receive ℓ. Denote by γ_b the fraction of agents who receive the guarantee payment. By the law of large numbers, γ_b equals the probability that a single signal x_{n_b} is above $\hat{x}_b$ conditional on the realized θ_b; i.e.,

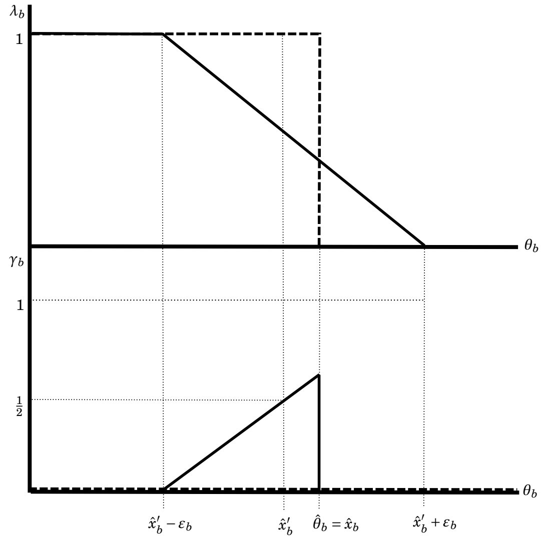
$$\gamma(\theta_b, \hat{x}_b, \hat{\theta}_b) = \begin{cases} 0 & \text{if } \theta_b > \hat{\theta}_b \\ \frac{\theta_b - \hat{x}_b + \varepsilon_b}{2\varepsilon_b} & \text{if } \hat{x}_b - \varepsilon_b < \theta_b < \hat{\theta}_b \\ 0 & \text{if } \theta_b < \hat{x}_b - \varepsilon_b. \end{cases} \tag{4}$$

Figure 2 plots λ_b and γ_b against the fundamental θ_b for the cases of full balance-sheet transparency, $\epsilon_b = 0$ (dashed lines), and with lower transparency, $\epsilon_b > 0$ (solid lines).

In the case of full transparency, λ_b is a step function with a jump discontinuity at $\hat{\theta}_b$, while γ_b is always equal to 0. With lower transparency, however, λ_b decreases linearly from 1 to 0 over the range $[\hat{x}_b - \varepsilon_b, \hat{x}_b + \varepsilon_b]$, with γ_b increasing linearly in θ_b from 0 to $(\hat{\theta}_b - \hat{x}_b + \varepsilon_b)/2\varepsilon_b$ over the range $[\hat{x}_b - \varepsilon_b, \hat{\theta}_b]$. The increase in γ_b illustrates the potential costs stemming from the guarantee scheme. The *ex ante* expected fraction of agents who benefit from the guarantee, and hence expected costs, rise when the bank becomes less transparent. When balance-sheet transparency is rather low, creditors' information is widely dispersed and many creditors may erroneously believe that the bank will not default even if, in fact, it does. These creditors, in turn, become eligible for the guarantee payment.

Several vital questions arise from these considerations. To what extent do the costs stemming from the guarantee pose a threat to the guarantor's own solvency or liquidity position? Are guarantees still effective in reducing the likelihood of bank default whenever one takes the funding risk of the guarantor into account? What are the effects of variations in bank and guarantor liquidity parameters on the behavior of creditors? In what follows, we answer these questions by explicitly modelling the guarantor's (i.e., the government's) funding risks.

Figure 2: Upper diagram: Fraction of bank creditors who withdraw, λ_b. Lower diagram: Fraction of bank creditors who receive guarantee payment, γ_b. The case $\varepsilon_b = 0$ is represented by the dotted lines, whereas the case $\varepsilon_b > 0$ is represented by solid lines. An increase in ε_b does not affect $\hat{\theta}_b$, but it changes $\hat{x}_b$ to $\hat{x}'_b$. The diagram is drawn under the assumption that $\frac{C_b-\ell}{D_b-\ell} < \frac{1}{2}$ so that $\hat{x}'_b < \hat{\theta}_b$ if $\varepsilon_b > 0$.



4. Bank Debt Rollover Game with Endogenous Sovereign Funding Risk

4.1. Model description

Building on the canonical bank debt rollover model outlined in section 3, we now explicitly introduce the refinancing problem of the government that issued the guarantee. In case of bank default, the government pays out ℓ to those bank creditors who rolled over their loans. However, the government is itself facing a rollover game involving a set of sovereign creditors $n_g \in [0, N_g]$ who are all different from the bank's creditors. We normalize the mass of sovereign creditors to unity, $N_g \equiv 1$. Each sovereign creditor holds a claim with a face value of one monetary unit against the government. Sovereign creditors decide simultaneously whether to continue lending to the government or to withdraw. The government defaults whenever its liquid resources are insufficient to service debt withdrawals and guarantee payments. We represent the government's liquidity by the random variable θ_g, which is uniformly distributed over $[-\eta_g, \theta_g^0 + \eta_g]$, where $\theta_g^0/2$ is the *ex ante* mean recourse to liquidity. Moreover, with respect to the relation between θ_b and θ_g, we impose the following assumption.

Assumption: *The government's liquidity, θ_g, and the bank's liquidity, θ_b, are independently distributed.*

Sovereign creditors receive noisy signals $x_{n_g} = \theta_g + \varepsilon_{n_g}$ concerning the government's liquidity θ_g, where ε_{n_g} is a uniform i.i.d. random variable with support $[-\varepsilon_g, \varepsilon_g]$. As in the banking game, reduced information dispersion (i.e., a lower ε_g) is associated with a higher degree of transparency of the government's financial situation. By assumption, the signals of bank and sovereign creditors are completely uninformative about the fundamental of the respective other entity.

Table 2 gives the payoffs in the sovereign rollover game. A sovereign creditor who withdraws early receives $C_g < 1$, which is the unit claim less potential transaction costs. If the government survives, creditors who rolled over their loans receive D_g. If the government defaults, the sovereign creditors who rolled over get a zero payoff, since there is

		Government	
		Default	Survive
Sovereign Creditor	Withdraw	C_g	C_g
	Roll over	0	D_g

Table 2: A typical sovereign creditor's payoffs.

no guarantee in place for them.

The bank's creditors, however, continue to enjoy the benefit of a guarantee in case the bank defaults and the government survives. The payoffs for a typical bank creditor are shown in Table 3, where we have normalized $C_b = 1$ in order to reflect the relatively small transaction costs in bank funding markets.

		Bank Default		Bank Survive
		Govt Survive	Govt Default	
Bank Creditor	Withdraw	$C_b = 1$	$C_b = 1$	$C_b = 1$
	Roll over	ℓ	0	D_b

Table 3: Updated bank creditor's payoffs.

Since our assumption of independence between sovereign and banking sector liquidity appears restrictive, some comments are in order.

- Firstly, the assumption should be judged against the clear but narrow objective of our paper, namely that we want to demonstrate how, and to what extent, the introduction of a guarantee induces a dependency between the refinancing situation of the sovereign and the banking sector. For example, it is by now widely known that some euro-area members got stuck in a 'diabolic loop,' where problems in the banking sector and problems of the sovereign tend to amplify each other. One casual explanation usually given for the high exposure of sovereigns vis-à-vis their banking sectors is that governments, through the issuance of guarantees, linked their own funding situation with that of the bank.[9] Yet, this implies that the observed correlation has been *caused*, among other things, by governments issuing guarantees. It was not necessarily present *before* the introduction of guarantees. From this perspective, our objective in this paper is to provide analytical underpinnings to this side of the diabolic loop. The simplest setting for such an analysis is one where, absent the guarantee, no dependency between the two coordination games exists.

- Secondly, since our focus is on liquidity crises, it can be argued that the correlation between the banking sector's liquidity and that of its government is rather low. Indeed, the liquidity of the government is essentially determined by its revenues from taxes, public dues and tariffs. In contrast, as Shin (2012) notes, internationally active banks may tap domestic as well as international markets, and can issue a greater variety of financial instruments. Moreover, if banks have branches in other countries, there may be intra-banking group liquidity transactions, so that a bank's

[9]See, e.g., DIW (2012).

liquidity may depend on the economic fundamentals in those countries as well. Consequentially, the liquidity situation of banks need not be strongly correlated with the liquidity situation of their resident government. Figure A1 illustrates this for the case of Ireland. The top panel of Figure A1 shows claims of banks in foreign countries on Irish banks against the Irish government's revenue, expressed relative to Irish GDP. As can be readily gleaned, the linear correlation between the time-series is low. The bottom panel of Figure A1 plots foreign claims of Irish banks on banks in other countries against the Irish government's revenue, both as fractions of Irish GDP. Once again, the linear correlation between the series is close to zero. The claims of and on Irish banks serve as a proxy for θ_b, while government revenue is captured θ_g. As such, Figure A1 provides some evidence in favor of our independence assumption.

- Finally, on purely technical grounds, the assumption of independence allows us to devise a simple proof for the existence of a unique equilibrium in threshold strategies, and for the non-existence of equilibria in other strategies. The intuition behind this result is straightforward. The independence assumption implies that a bank (sovereign) creditor's signal is informative only about the liquidity situation of the bank (sovereign), and completely uninformative about the liquidity of the sovereign (bank). We can therefore treat the behavior of sovereign creditors in the bank rollover game, and, respectively, the bank creditors in the sovereign game, as exogenously given. Hence, given any arbitrary strategy used by creditors in the other group, each rollover game has a unique equilibrium in threshold strategies. The following proposition summarizes this result.

Proposition 2. *There exists a unique equilibrium where sovereign and bank creditors use threshold strategies. There are no other equilibria in non-threshold strategies.*

Proof. See the appendix. □

As a consequence of Proposition 2, we restrict our attention to threshold strategies for sovereign and bank creditors. Absent a guarantee ($\ell = 0$), the two rollover problems are independent of each other and the critical thresholds for the government and the bank can be calculated from the respective formulae in Proposition 1. However, once the government issues a guarantee ($\ell > 0$), its refinancing problem becomes tied to the bank's rollover problem. For states of the world where the bank defaults, the government faces additional costs due to the guarantee payout. This alters the critical threshold for sovereign creditors, which in turn changes the government's default point *in all states of the world*, even in those where the bank survives. Moreover, the possibility that the government may default changes the critical threshold of bank creditors and thus the bank's default point.

We next turn to an explicit derivation of the threshold equilibrium. Firstly, we solve for the bank's and the government's default conditions. Secondly, we exploit the indifference of agents at the threshold signal to characterize the equilibrium.

4.2. Bank and sovereign default conditions

The possibility of government default does not alter the bank's failure condition, which remains $\lambda_b N_b > \theta_b$. Suppose that bank creditors use a threshold strategy around $\hat{x}_b$. From equation (2) we obtain that the bank's default point $\hat{\theta}_b$ can be written as a function of the critical threshold signal $\hat{x}_b$ as

$$\hat{\theta}_b(\hat{x}_b) = \frac{\hat{x}_b + \varepsilon_b}{1 + 2\varepsilon_b N_b^{-1}}. \tag{5}$$

Thus, the bank fails if and only if $\theta_b < \hat{\theta}_b(\hat{x}_b)$.

In calculating the government's failure point we must distinguish between two cases. Firstly, if $\theta_b > \hat{\theta}_b$, the bank survives and the government does not pay out the guarantee. Assuming that government creditors use a symmetric threshold strategy around $\hat{x}_g$, the government defaults whenever $\lambda_g > \theta_g$, where λ_g is the fraction of sovereign creditors whose signals are below $\hat{x}_g$. The government's failure point is calculated as the solution to $\hat{\theta}_g = \lambda_g(\hat{\theta}_g)$, yielding

$$\hat{\theta}_g = \frac{\hat{x}_g + \varepsilon_g}{1 + 2\varepsilon_g}.$$

Secondly, suppose that $\theta_b < \hat{\theta}_b$ and the bank defaults. The government is obliged to pay ℓ to each bank creditor who rolled over their loan. Since bank creditors use the threshold strategy around $\hat{x}_b$, we can use equation (4) to calculate total guarantee payments conditional on the realized θ_b, as

$$N_b \, \ell \, \gamma\left(\theta_b, \hat{x}_b, \hat{\theta}_b \middle| \theta_b < \hat{\theta}_b\right) = \frac{\ell N_b}{2\varepsilon_b} \int_{\hat{x}_b}^{\theta_b + \varepsilon_b} \mathrm{d}u.$$

The government's failure point in case of a bank default then follows by solving

$$\hat{\theta}_g - \frac{\ell N_b}{2\varepsilon_b} \int_{\hat{x}_b}^{\theta_b + \varepsilon_b} \mathrm{d}u = \lambda_g(\hat{\theta}_g)$$

yielding

$$\hat{\theta}_g = \frac{\hat{x}_g + \varepsilon_g}{1 + 2\varepsilon_g} + \frac{\varepsilon_g}{\varepsilon_b} \frac{\ell N_b (\theta_b + \varepsilon_b - \hat{x}_b)}{1 + 2\varepsilon_g}.$$

Taken together, the government's failure point is

$$\hat{\theta}_g(\hat{x}_g, \hat{x}_b, \theta_b) = \begin{cases} \frac{\hat{x}_g + \varepsilon_g}{1 + 2\varepsilon_g} & \text{if } \theta_b \geq \hat{\theta}_b(\hat{x}_b) \\ \frac{\hat{x}_g + \varepsilon_g}{1 + 2\varepsilon_g} + \frac{\ell N_b \varepsilon_g}{\varepsilon_b (1 + 2\varepsilon_g)} (\theta_b + \varepsilon_b - \hat{x}_b) & \text{if } \theta_b < \hat{\theta}_b(\hat{x}_b). \end{cases} \tag{6}$$

The government defaults if and only if $\theta_g < \hat{\theta}_g(\hat{x}_g, \hat{x}_b, \theta_b)$.

4.3. Creditors' expected payoffs

Given the default points of the bank and government, we next examine the differences in expected payoffs for typical bank and sovereign creditors who observe signals x_{n_b} and x_{n_g}, respectively, and believe that all other bank and sovereign creditors are using the threshold strategy around $\hat{x}_b$ and $\hat{x}_g$.

For the typical bank creditor with signal x_{n_b}, the expected payoff difference between rolling over and withdrawing is given by

$$\pi^b\left(\hat{x}_b, \hat{x}_g, x_{n_b}\right) \equiv \frac{D_b}{2\varepsilon_b} \int_{\hat{\theta}_b(\hat{x}_b)}^{x_{n_b} + \varepsilon_b} \mathrm{d}u + \frac{\ell}{2\varepsilon_b} \int_{x_{n_b} - \varepsilon_b}^{\hat{\theta}_b(\hat{x}_b)} \left(\frac{1}{\sigma_g} \int_{\hat{\theta}_g(\hat{x}_g, \hat{x}_b, u)}^{\tilde{\sigma}_g} \mathrm{d}v \right) \mathrm{d}u - 1, \tag{7}$$

where

$$\sigma_g = (\theta_g^0 + 2\eta_g), \quad \text{and} \quad \tilde{\sigma}_g = \theta_g^0 + \eta_g,$$

are the width of the support for the θ_g and the upper bound of the support, respectively. The second summand is the payment from the guarantee ℓ multiplied by the probability attached by the bank to the survival of the government.

The difference in expected payoffs from rolling over and withdrawing for a typical sovereign creditor with signal x_{n_g} is

$$\pi^g\left(\hat{x}_g, \hat{x}_b, x_{n_g}\right) \equiv \frac{D_g}{\sigma_b} \int_{-\eta_b}^{\tilde{\sigma}_b} \left(\frac{1}{2\varepsilon_g} \int_{\hat{\theta}_g(\hat{x}_g, \hat{x}_b, u)}^{x_{n_g}+\varepsilon_g} \mathrm{d}v \right) \mathrm{d}u - C_g, \tag{8}$$

where

$$\sigma_b = (\theta_b^0 + 2\eta_b), \quad \text{and} \quad \tilde{\sigma}_b = \theta_b^0 + \eta_b\,,$$

are the width of the support for θ_b and the upper bound, respectively. Using the piecewise definition of $\hat{\theta}_g$ from equation (6), we can rewrite the double integral in equation (8) as

$$\frac{D_g}{\sigma_b} \left(\frac{\sigma_b}{2\epsilon_g} \left(x_{n_g} + \epsilon_g - \frac{\hat{x}_g + \epsilon_g}{1+2\epsilon_g} \right) - \frac{\ell N_b}{(1+2\varepsilon_g)} \int_{-\eta_b}^{\hat{\theta}_b} \int_{\hat{x}_b}^{u+\varepsilon_b} \frac{\mathrm{d}u}{2\varepsilon_b} \right).$$

Note further that no guarantee payments come due in the case that all bank creditors receive signals $x_{n_b} < \hat{x}_b$ and withdraw. By virtue of the uniform distribution assumption, the signals lie on the interval $[\theta_b - \varepsilon_b, \theta_b + \varepsilon_b]$. If the upper bound $\theta_b + \varepsilon_b$ is less than the threshold $\hat{x}_b$, all creditors will withdraw. Thus, for realizations of the fundamental $\theta_b < \hat{x}_b - \epsilon_b$ the bank fails, but because all bank creditors withdrew, no guarantee payout has to be made by the government. Utilizing this fact, we can finally write the payoff difference between rolling over and withdrawing for a sovereign creditor as

$$\pi^g\left(\hat{x}_g, \hat{x}_b, x_{n_g}\right) = \frac{D_g}{2\epsilon_g} \left(x_{n_g} + \epsilon_g - \frac{\hat{x}_g + \epsilon_g}{1+2\epsilon_g} \right) - \frac{D_g \ell N_b}{(1+2\varepsilon_g)\sigma_b} \int_{\hat{x}_b-\varepsilon_b}^{\hat{\theta}_b} \frac{u+\varepsilon_b - \hat{x}_b}{2\varepsilon_b} \mathrm{d}u - C_g. \tag{9}$$

4.4. Equilibrium

From Proposition 2, we know that there exists a unique equilibrium in threshold strategies. Creditors who receive the critical signals $(\hat{x}_b, \hat{x}_b)$ must be indifferent between rolling over and withdrawing. Hence,

$$\pi^b\left(\hat{x}_b, \hat{x}_g, \hat{x}_b\right) = 0, \tag{10}$$

and $\pi^b(\hat{x}_b, \hat{x}_g, x_b) \gtrless 0$ if and only if $x_{n_b} \gtrless \hat{x}_b$, and

$$\pi^g\left(\hat{x}_g, \hat{x}_b, \hat{x}_g\right) = 0, \tag{11}$$

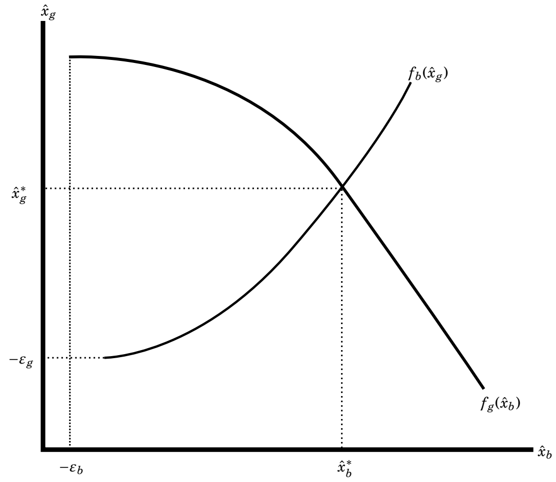
and $\pi^g(\hat{x}_g, \hat{x}_b, x_g) \gtrless 0$ if and only if $x_{n_g} \gtrless \hat{x}_g$.

An equilibrium is a combination of critical signals simultaneously solving equations (10) and (11). We explore the properties of the equilibrium using graphical techniques.

Proposition 3. *The solutions to creditors' indifference conditions, equations* (10) *and* (11)*, can be characterized by functions f_b and f_g, where $\hat{x}_b = f_b(\hat{x}_g)$ and $\hat{x}_g = f_g(\hat{x}_b)$. Moreover, f_b is strictly increasing, whereas f_g is strictly decreasing.*

Proof. See the appendix. □

Figure 3: Best reply curves f_b and f_g. The joint equilibrium in the rollover games occurs at the intersection point $(\hat{x}_b^*, \hat{x}_g^*)$.



The functions f_b and f_g can be interpreted as aggregate best response functions between bank and sovereign creditors. The equilibrium of the model is then given by the intersection of the two curves.

Figure 3 illustrates the equilibrium. The best response curve for bank creditors, f_b, is strictly increasing over the entire range of $\hat{x}_g$, implying that the actions of sovereign creditors are *strategic complements* for bank creditors. As sovereign creditors increase their critical signal, the risk of a government default increases and the likelihood that the guarantee will be paid out decreases. In response, bank creditors increase their critical signal as well. In contrast, f_g is strictly decreasing over the entire range of $\hat{x}_b$, implying that the actions of bank creditors are *strategic substitutes* for sovereign creditors. This deserves some comment. We show in the proof of Proposition 3 that

$$f_g'(\hat{x}_b) \propto \frac{\partial}{\partial \hat{x}_b}\left(\int_{\hat{x}_b-\varepsilon_b}^{\hat{\theta}_b(\hat{x}_b)} (u+\varepsilon_b-\hat{x}_b)\,\mathrm{d}u\right).$$

Suppose that bank creditors increase their critical signal $\hat{x}_b$. This exerts two opposing effects on sovereign creditors' payoffs and thus on their critical signal $\hat{x}_g$. Firstly, a higher $\hat{x}_b$ increases $\hat{\theta}_b$ and enlarges the range of θ_b realizations where the bank may default and the guarantee comes due. This, in turn, decreases sovereign creditors' expected payoffs from rolling over and induces them to increase their critical signal as well. From the expression above, this effect is up to a constant given by

$$(\hat{\theta}_b+\varepsilon_b-\hat{x}_b)\frac{\partial \hat{\theta}_b}{\partial \hat{x}_b}.$$

There is, however, a second, opposing effect. As $\hat{x}_b$ increases, fewer bank creditors *mistakenly* roll over their debt whenever the bank fails. Consequently, the guarantee payout for the government is lowered. This is true for all states $\theta_b < \hat{\theta}_b$. In turn, the likelihood that the government survives rises and a typical sovereign creditor's expected payoff from rolling over increases. Formally, this effect is, up to the same constant, given by

$$-(\hat{\theta}_b+\varepsilon_b-\hat{x}_b).$$

The second effect outweighs the first one as long as $\varepsilon_b > 0$, since

$$\frac{\partial \hat{\theta}_b}{\partial \hat{x}_b} = \frac{N_b}{N_b+2\varepsilon_b} < 1,$$

leading to the downward-sloping aggregate best response curve for the sovereign creditors.

4.5. Comparative statics

We next analyze the comparative statics properties of the critical signals with respect to the guarantee size ℓ, the degree of the bank's funding illiquidity N_b, and the *ex ante* expected liquidities θ_b^0 and θ_g^0 for the bank and the government, respectively.

Figure 4(a) depicts the effects of a marginal increase in ℓ. The increase shifts the f_b curve to the left. For any given $\hat{x}_g$, a higher guarantee increases bank creditors' expected payoff from rolling over and lowers their critical signal. The f_g curve is shifted to the right. For any given $\hat{x}_b$, a higher guarantee lowers the probability that the government survives and, in response, sovereign creditors raise their critical signal. The increase in the guarantee thereby exerts a direct effect on the payoffs for both bank and sovereign creditors. In addition, it exerts an indirect effect through the change in the critical signal of the respective other type of creditors. For sovereign creditors, both effects work in the same direction and produce a clear-cut total effect. For bank creditors, the two effects work in opposite directions. An increase in the critical signal of sovereign creditors lowers bank creditors' expected payoffs from rolling over and thereby countervails the positive effect of the higher guarantee. If, however, the rightward shift in the f_g curve is sufficiently small, then the latter effect outweighs the former and bank creditors' critical signal is lowered. The following proposition provides a necessary and sufficient condition for this to occur.

Proposition 4. *A marginal increase in the guarantee lowers bank creditors' critical signals, i.e.* $\partial \hat{x}_b / \partial \ell < 0$, *if and only if*

$$\frac{\ell N_b}{\sigma_b} \int_{\hat{x}_b - \varepsilon_b}^{\hat{\theta}_b} \frac{u + \varepsilon_b - \hat{x}_b}{2\varepsilon_b} \mathrm{d}u < \widetilde{\sigma}_g - \hat{\theta}_g(\hat{\theta}_b). \tag{12}$$

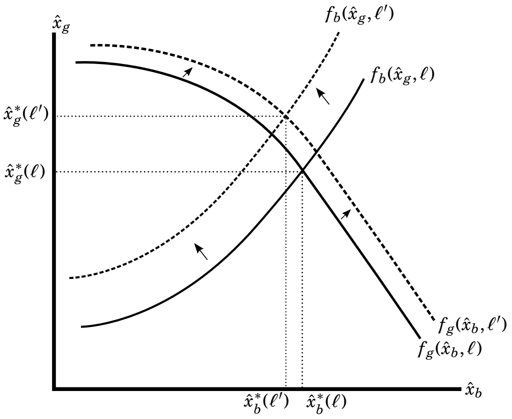
Proof. See the proof of Lemma A6 in the appendix. □

The left-hand side of condition (12) is the *ex ante* expected guarantee payout, conditional on the government surviving. The right-hand side is the difference between the government's maximal cash flow (i.e., the upper bound $\widetilde{\sigma}_g$ of the support for θ_g) and the minimal cash flow it needs to survive. We may interpret the right-hand side as the 'slack' in available liquidity for the government.

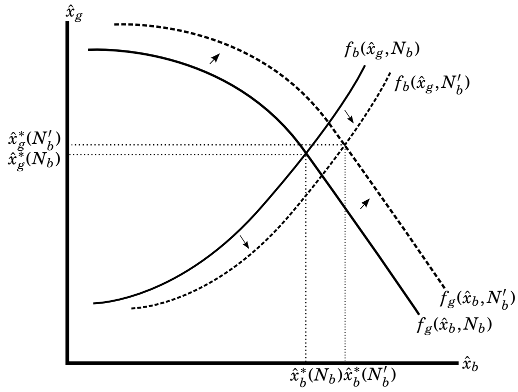
A marginal increase in the guarantee induces bank creditors to decrease their critical signal if and only if the *ex ante* expected guarantee payout is less than the government's slack in liquidity. Condition (12) can thus be interpreted as a 'credibility condition.' We say that a guarantee $\ell = \tilde{\ell}$ is *credible* if condition (12) is satisfied when evaluated at $\ell = \tilde{\ell}$. If condition (12) fails to hold, bank creditors may *ex ante* judge the government's resources to be insufficient to cover the guarantee promise and respond by raising their critical signal. It is straightforward to show that the condition always holds for $\ell = 0$, implying that the introduction of a small guarantee is always credible and lowers bank creditors' critical signal. However, as the following corollary states, if a guarantee is credible, then further increases in the guarantee can lead to a reversal of the condition; i.e., by increasing the expected burden on the government's budget, the guarantee erodes its own credibility.

Corollary 1. *Suppose the condition* (12) *is satisfied for a given guarantee* $\tilde{\ell}$. *A further marginal increase in the guarantee increases the* ex ante *expected guarantee payout and simultaneously diminishes the government's slack in liquidity.*

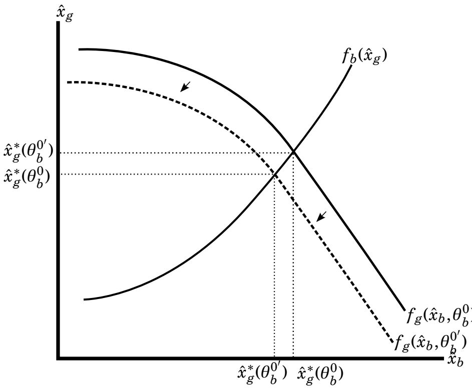
Figure 4: Comparative statics of $(\hat{x}_b^*, \hat{x}_g^*)$



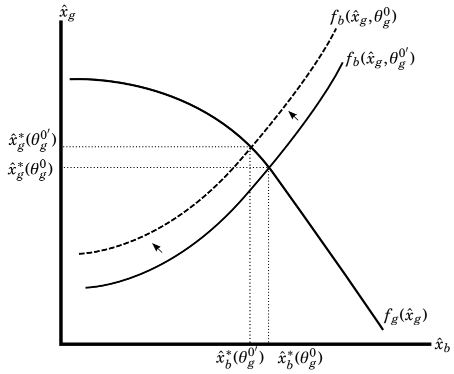
Panel (a): Change in $(\hat{x}_b^*, \hat{x}_g^*)$ due to increase in guarantee from ℓ to ℓ', given that (12) holds.



Panel (b): Change in $(\hat{x}_b^*, \hat{x}_g^*)$ due to increase in funding illiquidity from N_b to N_b'.



Panel (c): Change in $(\hat{x}_b^*, \hat{x}_g^*)$ due to increase in bank's expected liquidity from $\theta_b^0/2$ to $\theta_b^{0'}/2$.



Panel (d): Change in $(\hat{x}_b^*, \hat{x}_g^*)$ due to increase in government's expected liquidity from $\theta_g^0/2$ to $\theta_g^{0'}/2$.

Figure 4(b) depicts the effect of an increase in the bank's exposure to funding illiquidity, N_b. A higher degree of funding illiquidity is associated with a higher probability of bank failure and consequently with larger expected guarantee payments. Thus, increases in N_b shift both the f_b and f_g curves to the right. This leads to a higher critical signal for bank creditors. From the graphical analysis alone, the sign of the effect on the sovereign creditors' critical signal is not clear-cut. On the one hand, a larger N_b increases the *ex ante* guarantee payments, which diminishes the government's liquidity and increases the critical signal for sovereign creditors (given ℓ and $\hat{x}_b$). However, as a consequence of strategic substitutability, a higher critical signal for bank creditors makes sovereign creditors more willing to roll over, thereby mitigating the effect on the sovereign creditors' critical signal. As shown in Lemma A6 in the appendix, the latter 'substitutability effect' is smaller in magnitude than the former 'complementarities effect,' implying that a larger N_b always leads to an increase in the sovereign creditors' critical signal.

Figures 4(c) and 4(d) show the effects of increases in the *ex ante* expected liquidity for the bank θ_b^0 and government θ_g^0, respectively. An increase in θ_b^0 leaves the f_b curve unaffected and shifts f_g to the left, thereby lowering the critical signals for both bank and sovereign creditors. The decisions of bank creditors are based on updated information on θ_b that is obtained from the signals x_{n_b}, which do not depend on θ_b^0. Sovereign creditors, on the other hand, do not receive updated information about θ_b, and must instead rely on θ_b^0. A higher *ex ante* liquidity for the bank raises the probability that the bank survives and lowers the government's expected payments due to the guarantee promise. This, in turn, increases the sovereign creditors' expected payoffs from rolling over and lowers their critical signal. By virtue of the strategic complementarities, the lowering of $\hat{x}_g$ leads to a lowering of the critical signal $\hat{x}_b$ for the bank's creditors.

An increase in θ_g^0, on the other hand, has a significantly different effect. Following similar lines of reasoning as above, θ_g^0 affects only bank creditors' expected payoffs and leaves sovereign creditors' expected payoffs unaffected. An increase in θ_g^0, then, increases the likelihood that the government manages to roll over its debt and therefore it increases the probability that the guarantee can be paid out. This leads bank creditors to lower their critical signal. However, since the actions of bank creditors are strategic substitutes for sovereign creditors, the critical signal for sovereign creditors is increased.

These results suggest that whenever the bank and the sovereign are connected through the guarantee promise, a positive spillover effect exists from the bank's liquid resources to the likelihood that the government manages its debt rollover and survives. Similarly, an improvement in the government's *ex ante* liquidity also spills over to the likelihood that the bank survives. Yet, this comes at the cost of a higher critical signal of sovereign creditors that, in turn, may jeopardize the beneficial effect of the improved θ_g^0 on the government's likelihood of managing the debt rollover.

5. The Optimal Guarantee and Its Properties

In this section, we determine the optimal guarantee based on a stylized measure of the expected costs of crises. Moreover, we discuss how the guarantee affects the probabilities of sovereign default, bank default and dual default (a systemic crisis).

5.1. A measure of expected costs of crises

In determining the appropriate guarantee to provide the bank's creditors, the government faces a trade-off between lowering the expected costs stemming from a bank default and placing additional strains on its own budget, thereby raising the likelihood that it enters into default itself. We formalize this trade-off by defining a measure of the expected costs of crises, which the government minimizes by setting ℓ optimally.

We denote by ϕ_b the costs incurred when the bank defaults and the government survives. Similarly, ϕ_g denotes the costs of a sovereign default, where the bank survives. Finally, the costs of a systemic crisis (i.e., a crisis where both government and bank default) are denoted ϕ_s. We normalize all costs by setting $\phi_s \equiv 1$. We interpret the costs as the loss in the economy's output that materializes following a default event. In particular, ϕ_b results from a disruption in financial intermediation and the reduction in available bank credit in the aftermath of default. Banks typically make sizable investments into screening and monitoring technologies, while building long-term relationships with borrowers. Following a bank default, the soft information accrued is lost and has to be acquired anew, which involves costs for the economy as a whole. Moreover, due to the specificity of this information, some of the bank's borrowers cannot easily find a new bank and may become credit constrained. Such constraints may become binding for households and small businesses which, faced with high costs when attempting to borrow on financial markets directly, are highly dependent on financial intermediation via the banking sector.[10]

[10]See, for example, Leland and Pyle (1977) and Allen and Gale (2001).

Equivalently, ϕ_g is the foregone output due to a sovereign default. The default may impose reputation costs on the government, implying higher borrowing costs in the future or even a full exclusion from financial markets. A government default may also exert a negative effect on trade through either sanctions and retaliations, or reduced access to trade credit. Moreover, empirically, sovereign default is also associated with an immediate effect on economic growth in the default period.[11]

Denoting by $K(\ell)$ the expected default costs stemming from the different scenarios, the government's objective is to

$$\min_{\{\ell\in[0,1]\}} K(\ell) \equiv \phi_g\left(P_g(\ell)-q(\ell)\right)+\phi_b\left(P_b(\ell)-q(\ell)\right)+q(\ell), \tag{13}$$

where $P_g(\ell)$ is the probability of government default, $P_b(\ell)$ the probability of bank default and $q(\ell)$ the probability of a systemic crisis.

We compare the expected costs under the optimally chosen guarantee denoted by $K^{opt} \equiv K(\ell^{opt})$ to two benchmarks, (1) the first-best outcome K^{FB} that occurs in the absence of coordination risks for both sovereign and bank creditors, and (2) the costs $K^0 \equiv K(0)$ incurred in the absence of a guarantee.

Without coordination failures, the government and the bank default if and only if θ_b and θ_g are less than zero. Following the uniform distribution assumption, the first-best benchmark can be calculated as

$$K^{FB} = \phi_g\frac{\eta_g}{\sigma_g}+\phi_b\frac{\eta_b}{\sigma_b}+(1-\phi_g-\phi_b)\frac{\eta_b}{\sigma_b}\frac{\eta_g}{\sigma_g}. \tag{14}$$

While K^{FB} provides a floor to the expected costs, the ceiling is given by the costs incurred in absence of a guarantee; i.e.,

$$K^0 = K^{FB}+\phi_g\frac{C_g/D_g}{\sigma_g}+\phi_b\frac{1/D_b}{\sigma_b}+(1-\phi_g-\phi_b)\frac{(C_g/D_g+\eta_g)(1/D_b+\eta_b)-\eta_g\eta_b}{\sigma_g\sigma_b}. \tag{15}$$

5.2. Probabilities of crises

In what follows, we write the equilibrium critical signals as $\hat{x}_b^*(\ell)$ and $\hat{x}_g^*(\ell)$ to emphasize their dependency on the guarantee ℓ.[12] The probabilities of bank, government and systemic crises, as expressed in the cost function $K(\ell)$, are

$$P_b(\ell) \equiv \mathbf{Pr}\left(\theta_b < \hat{\theta}_b^*(\ell)\right) \qquad \text{and} \qquad P_g(\ell) \equiv \mathbf{Pr}\left(\theta_g < \hat{\theta}_g^*(\ell)\right),$$

and

$$q(\ell) \equiv \mathbf{Pr}\left(\{\theta_b < \hat{\theta}_b^*(\ell)\} \cap \{\theta_g < \hat{\theta}_g^*(\ell)\}\right),$$

respectively. Moreover, the probability that there is at least one crisis is

$$Q(\ell) \equiv \mathbf{Pr}\left(\{\theta_b < \hat{\theta}_b^*(\ell)\} \cup \{\theta_g < \hat{\theta}_g^*(\ell)\}\right).$$

With respect to the probability of a bank default, the guarantee influences $\hat{\theta}_b^*$ via the critical signal $\hat{x}_b^*$. This can be seen by writing explicitly

$$P_b(\ell) = \frac{1}{\sigma_b}\int_{-\eta_b}^{\hat{\theta}_b^*(\ell)} \mathrm{d}u = \frac{\frac{N_b(\hat{x}_b^*(\ell)+\varepsilon_b)}{N_b+2\varepsilon_b}+\eta_b}{\sigma_b}. \tag{16}$$

[11]See, for example, Eaton and Gersovitz (1981) and Borensztein and Panizza (2009).

[12]The default points of the government and the bank are written as $\hat{\theta}_b^*(\ell) \equiv \hat{\theta}_b(\hat{x}_b^*(\ell))$ and $\hat{\theta}_g^*(\ell,\theta_b) \equiv \hat{\theta}_g(\hat{x}_g^*(\ell),\hat{x}_b^*(\ell),\theta_b)$.

The guarantee's influence on the probability of a government crisis works through two channels. Firstly, there is an effect on the critical signal $\hat{x}_g^*(\ell)$ that induces a level-shift in the default point $\hat{\theta}_g^*(\ell,\theta_b)$. This effect is similar to that induced by the guarantee on the bank's default point $\hat{\theta}_b^*(\ell)$. Secondly, the government's default point depends directly on the bank's liquidity θ_b. This induces a functional interdependence between the likelihood of a government default and the bank's liquidity. Calculating the government's probability of default therefore requires integration over both θ_b and θ_g. Formally,

$$
\begin{aligned}
P_g(\ell) &= \frac{1}{\sigma_b}\int_{-\eta_b}^{\hat{\theta}_b^*(\ell)}\left(\frac{1}{\sigma_g}\int_{-\eta_g}^{\hat{\theta}_g^*(\ell,u)}\mathrm{d}v\right)\mathrm{d}u \quad + \quad \frac{1}{\sigma_b}\int_{\hat{\theta}_b^*(\ell)}^{\tilde{\sigma}_b}\mathrm{d}u\,\frac{1}{\sigma_g}\int_{-\eta_g}^{\hat{\theta}_g^*(\ell)}\mathrm{d}v \\
&= \frac{\frac{\hat{x}_g^*(\ell)+\varepsilon_g}{1+2\varepsilon_g}+\eta_g}{\sigma_g} \quad + \quad \frac{1}{\sigma_b\sigma_g}\frac{\ell N_b 2\varepsilon_g}{(1+2\varepsilon_g)}\int_{\hat{x}_b^*-\varepsilon_b}^{\hat{\theta}_b^*(\ell)}\frac{u+\varepsilon_b-\hat{x}_b^*(\ell)}{2\varepsilon_b}\mathrm{d}u,
\end{aligned}
\tag{17}
$$

where the final term illustrates the functional dependency between the government's default probability and the bank's fundamental. This clearly shows how the government's fate does not exclusively lie in the hands of its own creditors but, through the guarantee, becomes closely tied to that of the bank, even though the liquidity resources that otherwise govern individual default probabilities are fully independent.

In much the same way, the probability of a systemic crisis can be calculated as

$$
\begin{aligned}
q(\ell) &= \frac{1}{\sigma_b}\int_{-\eta_b}^{\hat{\theta}_b^*(\ell)}\left(\frac{1}{\sigma_g}\int_{-\eta_g}^{\hat{\theta}_g^*(\ell,u)}\mathrm{d}v\right)\mathrm{d}u \\
&= \frac{\frac{\hat{x}_g^*(\ell)+\varepsilon_g}{1+2\varepsilon_g}+\eta_g}{\sigma_g}\times\frac{\frac{N_b(\hat{x}_b^*(\ell)+\varepsilon_b)}{N_b+2\varepsilon_b}+\eta_b}{\sigma_b} \quad + \quad \frac{1}{\sigma_b\sigma_g}\frac{\ell N_b 2\varepsilon_g}{(1+2\varepsilon_g)}\int_{\hat{x}_b^*-\varepsilon_b}^{\hat{\theta}_b^*(\ell)}\frac{u+\varepsilon_b-\hat{x}_b^*(\ell)}{2\varepsilon_b}\mathrm{d}u.
\end{aligned}
\tag{18}
$$

Figure 5 shows the impact of the guarantee on the default points $\hat{\theta}_g^*(\ell,\theta_b)$ and $\hat{\theta}_b^*(\ell)$. The guarantee decreases $\hat{x}_b^*(\ell)$ and increases $\hat{x}_g^*(\ell)$. The dotted lines separate the regions of default and survival in absence of the guarantee. The introduction of a guarantee ℓ shifts the bank's default point to the left (dashed line) and enlarges the region where the bank survives. Moreover, as the guarantee increases the sovereign creditors' critical signal, the dotted horizontal line moves to the solid line, increasing the region where the government defaults. In the region where the bank defaults (to the left of the dashed line), the government's default point is a function of θ_b and therefore the solid line slopes upwards.

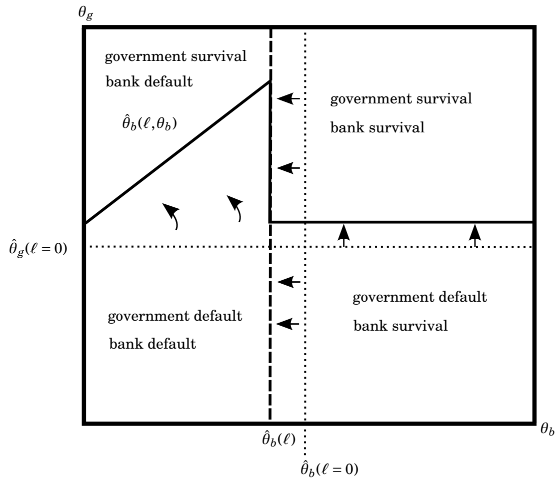
5.3. The influence of transparency on the optimal guarantee

The influence of the guarantee in reducing the likelihood of bank default depends on its 'credibility,' which in turn is determined by the risk of sovereign default. The pertinent question is, then, whether, and to what degree, a particular guarantee promise undermines the government's credibility to pay by placing undue strains on its refinancing needs. As discussed in section 3.6, the costs associated with a guarantee promise are crucially dependent on the degree of balance-sheet transparency. To better understand the effects of changes in the degrees of balance-sheet transparency, ε_b and ε_g, on the optimal policy, we explore two extreme cases.

5.3.1. Transparent bank

With a high degree of balance-sheet transparency for the bank (i.e., ε_b becoming negligibly small) bank creditors face only strategic uncertainty about the behavior of other

Figure 5: Regions of bank and/or sovereign default in θ_b–θ_g–space.



bank creditors. The coordination failure of bank creditors can be avoided, at zero cost to the government, by issuing a sufficiently large guarantee promise.[13]

Lemma 1. *In the limit when the bank is fully transparent ($\varepsilon_b \to 0$) and for any degree of transparency of the government ($\epsilon_g \geq 0$), the default points for the bank and the government are given by*

$$\hat{\theta}_b^*(\ell) = \frac{N_b\left(1-\ell(1-P_g)\right)}{D_b-\ell(1-P_g)} \quad \text{and} \quad \hat{\theta}_g^* = \frac{C_g}{D_g},$$

where $P_g = \left(C_g/D_g+\eta_g\right)/\sigma_g$.

Proof. See the appendix. □

While the sovereign default risk influences the critical threshold $\theta_b^*(\ell)$, the guarantee does not put any additional strains on the government, and its threshold converges to the one in the canonical model. This implies a clear-cut negative effect of a higher guarantee on the costs of crises $K(\ell)$. The government's program has a corner solution.

Lemma 2. *If the bank is fully transparent, the first-order necessary condition for the government's program is given by*

$$K'(\ell) = -\frac{N_b}{\sigma_b}\frac{(1-P_g)(D_b-C_b)}{(D_b-\ell(1-P_g))^2}\left(\left(1-P_g\right)\phi_b+P_g\left(1-\phi_g\right)\right) < 0. \tag{19}$$

Proof. See the appendix. □

The optimal guarantee for a fully transparent bank is provided in the following proposition.

Proposition 5. *If the bank is fully transparent, the optimal guarantee becomes $\ell^{opt} = 1$, and it provides full coverage of bank creditors' claims.*

Proof. See the appendix. □

[13] This is the result obtained by Bebchuk and Goldstein (2010).

Although the full guarantee diminishes the range of fundamentals where inefficient bank runs occur, it does not completely remove the possibility of inefficient bank failures. Since the government itself defaults with probability P_g, even a full guarantee is not enough to achieve $\hat{\theta}_b(1) = 0$. To remove all inefficient bank failures, the government would have to set

$$\ell = \frac{1}{1 - P_g} > 1, \tag{20}$$

which is tantamount to rewarding bank creditors for a bank failure.

5.3.2. Opaque bank and transparent government

The result provided in Proposition 5 depends only on the transparency of the bank and is independent of the government's transparency. Indeed, transparency of the government plays an entirely different role than transparency of the bank. ε_g has no decisive influence on whether the guarantee creates an actual cost. Equation (17) suggests that higher government transparency can reduce the guarantee's effect on the government's critical threshold in cases where the bank's balance sheet is rather opaque. But even when the government is fully transparent, the optimal policy set out in Proposition 5 may change if the bank is opaque. For example, for $\varepsilon_b > 0$ and $\varepsilon_g \to 0$, the default points of the bank and the government are given by

$$\hat{\theta}^*_b(\ell) = \frac{N_b(\hat{x}^*_b(\ell) + \varepsilon_b)}{N_b + 2\varepsilon_b} \quad \text{and} \quad \hat{\theta}^*_g(\ell) = \hat{x}^*_g(\ell),$$

and the derivative of the cost-of-crisis function becomes

$$\begin{aligned} K'(\ell) = & \frac{1}{\sigma_b} \left(\phi_b(1 - P_g(\ell)) + (1 - \phi_g)P_g(\ell)\right) \frac{N_b}{N_b + 2\varepsilon_b} \frac{\partial \hat{x}^*_b(\ell)}{\partial \ell} \\ & + \frac{1}{\sigma_g} \left(\phi_g(1 - P_b(\ell)) + (1 - \phi_b)P_b(\ell)\right) \frac{\partial \hat{x}^*_g(\ell)}{\partial \ell}, \end{aligned} \tag{21}$$

with $P_g(\ell) := \frac{\hat{x}^*_g(\ell) + \eta_g}{\sigma_g}$ and $P_b(\ell) := \frac{N_b(\hat{x}^*_b(\ell) + \varepsilon_b + \eta_b) + 2\varepsilon_b\eta_b}{\sigma_b(N_b + 2\varepsilon_b)}$.

The sign of $K'(\ell)$, and hence the optimal guarantee policy, are no longer parameter-independent. In particular, they crucially depend on the costs of crises ϕ_b and ϕ_g, and on the remaining parameters governing the model. While conceptually simple, the government's program does not yield tractable analytical solutions. We therefore resort to a numerical analysis in order to determine the optimal guarantee, and examine its dependency on the degrees of transparency and on the parameters governing the liquidity situations of the government and the bank.

6. Numerical Analysis

In this section, we explore the consequences of changes in the degrees of transparency in the banking sector and the government through a set of numerical exercises, where we fix the cost parameters ϕ_b and ϕ_g, associated with bank and sovereign defaults, respectively, at some empirically plausible values and where we calibrate, in broad strokes, the model to the Irish economy.

6.1. Calibrating the Celtic crisis

According to Table A1, the first guarantee scheme introduced by the Irish government covered banking sector liabilities that amounted to 244 percent of Irish GDP. According to the International Monetary Fund (2011), the refinancing needs of the Irish banks amounted to around 25 percent of their total liabilities. This roughly equates to refinancing needs in the order of 61 percent of GDP. In contrast, the Irish government faced financing needs of only 19.5 percent of GDP in 2011. This implies that the amount of maturing claims of Irish banks was approximately three times that of the Irish government, resulting in a value of $N_b = 3$, where we maintain $N_g = 1$. Moreover, in line with the experience prior to the crisis, we assume that the risk premia of Irish banks were higher than the risk premium of the Irish government, and thus set $D_b = 1.75$ and $D_g = 1.5$. We also set $C_b = C_g = 1$.

To ensure that the dominance regions of the two rollover games are well-defined, we take $\eta_b = 4.01$, $\eta_g = 1.01$, $\theta_b^0 = 3$ and $\theta_g^0 = 4$. Consequently, the banking sector is exposed to a large rollover risk with expected liquidity $\theta_b^0/2$ covering only 50 percent of total maturing claims. For the government, in contrast, expected liquidity is double the amount of maturing claims.[14]

We normalize the cost of a systemic crisis to $\phi_s = 1$. Cost parameters ϕ_b and ϕ_g are thus interpreted as the output losses due to individual bank and sovereign crises, respectively, relative to the loss due to a systemic crisis. Table 4 provides a brief overview of the empirical estimates of such losses. The cumulative output losses associated with a systemic crisis amount to 54 percent of the pre-crisis GDP. The output loss of a sovereign default-only event is around 10 percent of GDP. Estimated losses due to a solo banking crisis range from 6.3 percent to 28 percent of GDP. For the first exercise in this section we set $\phi_g = 0.2$ (which approximates $\frac{10\%}{54\%} = 0.185 \approx 0.2$) and $\phi_b = 0.1$ (approximating $\frac{6.3\%}{54\%} = 0.116 \approx 0.1$). In the second exercise, we maintain the value of ϕ_g, but we change ϕ_b to 0.5, thus approximating $\frac{28\%}{54\%}$.

Source	Type of crisis	Duration	Average annual output loss
Hoggarth et al. (2002)	Banking	3.2	1.9%
Honohan and Klingebiel (2000)	Banking	3.5	3.6%
Hutchison and Noy (2005)	Banking	3.3	3.0%
De Paoli et al. (2009)	Sovereign	4	2.5%
De Paoli et al. (2009)	Twin (Sovereign and Banking)	11	4.9%
Boyd et al. (2005)	Banking	5.1	5.4%

Table 4: Costs of different types of crises. Output loss in percent of annual GDP. Reported values are the average losses reported in the respective studies.

In what follows, we measure the welfare gain from introducing the optimal guarantee as

$$\text{welfare} = K^0 - K^{opt}.$$

[14] The choice of η_b allows for variations of ε_b up to 2, whereas the choice of η_g allows for variations of ε_g up to 0.5. As the preceding sections illustrated, the choice of ε_g is of less importance for the outcome of the model, which is why we restrict ourselves to only a limited range of variations.

Moreover, in order to assess the impact of the optimal guarantee on the likelihood of crises, we consider the differences in the probabilities of different crises between having the optimal guarantee and having no guarantee; i.e., we write

$$\Delta P_b \equiv P_b(\ell^{opt}) - P_b(0) \quad \text{and} \quad \Delta P_g \equiv P_g(\ell^{opt}) - P_g(0),$$

as well as

$$\Delta Q \equiv Q(\ell^{opt}) - Q(0) \quad \text{and} \quad \Delta q \equiv q(\ell^{opt}) - q(0).$$

6.2. Results

Figure 6 shows the comparative statics exercises with respect to ε_b and ε_g, where we have set $\phi_g = 0.2$ and calibrate the costs of a banking crisis to $\phi_b = 0.1$. As can be seen from Panel (a), a lower degree of transparency in the banking sector (higher ε_b) may decrease the optimal guarantee. Moreover, as the difference between the black, the gray and the dashed gray line in Panel (a) indicates, this effect is more pronounced when the degree of government transparency is also lower (i.e., ε_g is higher). As shown in Panel (b), the expected welfare gain is highest when the transparency of banks and the government is maximal, amounting to roughly 1.2 percent of GDP ($\approx 0.022 \times 54$ percent). Reductions in the government's transparency are associated with an expected welfare loss of at most 0.27 percent of GDP. Panels (c)–(f) in Figure 6 show how the probability differences ΔQ, Δq, ΔP_b and ΔP_g vary with changes in ε_b and ε_g. As one would expect, the probability of a sovereign crisis rises with the introduction of the optimal guarantee. However, it rises by less than the reduction in the probability of a banking crisis, which in turn explains why the probabilities q and Q are decreasing. Higher bank balance-sheet transparency is clearly enhancing the effect of the guarantee on probabilities P_b, q and Q, while it mitigates the adverse effect on P_g. When the bank becomes fully transparent ($\varepsilon_b \to 0$), the introduction of a guarantee comes at no cost for the government and therefore exerts no effect on the probability P_g. Moreover, a less-transparent government significantly dampens the effect of the guarantee on all probabilities.

Figure 7 shows the numerical results when ϕ_g is kept at 0.2 and when $\phi_b = 0.5$, thereby approximating the highest output loss of a solo banking crisis in Table 4. Several important differences emerge compared to the previous exercise. Firstly, as can be seen from Panel (a), given the high costs of a bank default, the government finds it now optimal to provide a full guarantee ($\ell^{opt} = 1$) independent of its own degree of transparency and the degree of bank transparency. Secondly, from Panel (f), ΔP_g increases linearly with lower transparency of the banks, yet it is unaffected by changes in the government's transparency.[15] Thirdly, Panel (c) shows that a combination of low degree of bank and government transparency (high ε_b and ε_g) may now increase the probability of experiencing a systemic crisis above the level that it reaches in the absence of a guarantee. This effect is essentially driven by the increase in ΔP_g, since, from Panel (e), the change in the probability of a banking crisis is rather flat. Quantitatively, this effect seems to be small, yet it constitutes a marked qualitative difference to the previous exercise, where the costs of a banking crisis were smaller than the costs of a sovereign crisis. Finally, the maximum welfare gain (when the government and the banking sector are quite transparent) amounts to roughly 2.38 percent of GDP, which is larger than previously.

A robust finding throughout these numerical exercises is that the increase in the government's default probability is, in absolute magnitude, significantly smaller than the

[15] This result is robust to other numerical specifications whenever $\phi_b > \phi_g$.

reduction in the bank's default probability. This replicates the empirical behavior of CDS spreads that we alluded to in the introduction (see Figure 1(d)), and allows us to put forward an interpretation of this stylized fact. Recall that in our model, under a regime of full bank transparency ($\varepsilon_b \to 0$), no guarantee payout will ever come due. This implies, as can be seen from the corresponding panels in Figures 6 and 7, that for a relatively high degree of bank transparency, the sovereign's default probability remains almost unchanged when the guarantee is introduced, whereas the impact on the bank's default probability is large. The guarantee removes strategic uncertainty, thereby serving as a device to coordinate bank creditors on the efficient equilibrium. When the degree of bank transparency becomes smaller, the mass of bank creditors who may eventually claim the guarantee increases and, in case the bank defaults, the guarantee creates an actual cost burden for the government. As a result, the government's default probability begins to increase. The large decrease in CDS spreads across countries (and especially in Ireland) that was observed right after the issuance of bank debt guarantees may therefore mirror the removal of strategic uncertainty among bank creditors. However, sovereign CDS spreads increased at the same time, suggesting that the corresponding banking sectors may not have operated under a regime of full transparency. Market participants in sovereign funding markets may have conjectured that the guarantees would create an actual cost for the sovereign and therefore withdrawn funding.

Moreover, while it is tempting to criticize the Irish government for having provided an enormous guarantee, at least our numerical exercises suggest that even such a guarantee may have been the optimal one. In particular, as Ireland's financial industry constituted an important sector of its economy, the output costs of an economy-wide banking crisis may have been quite so large that for any degree of transparency, the government would have considered 100 percent coverage optimal (see the exercise in Figure 7). Yet, if transparency was rather low, such a policy may have contributed to heighten *ex ante* the likelihood of the systemic crisis that Ireland eventually experienced. Figure 7 suggests that, given the strong reduction in the probability of a relatively costly banking crisis, the government may optimally drive up the likelihood of its own default, which is less costly, to avert the cost burden of a banking crisis, even if this also means raising the probability of a systemic crisis above the level in absence of a guarantee.

7. Conclusion

In this paper, we analyze the effects of a bank debt guarantee provided by the government and the role of the degree of balance-sheet transparency in making the guarantee costly. To examine this phenomenon, we use a stylized global games framework to address the following questions: (i) How does the introduction of a bank liability guarantee by a government affect the behavior of banking and sovereign creditors? (ii) How does the guarantee affect the likelihood of crises? (iii) What is the optimal guarantee that trades off the expected costs associated with the different types of crises? and (iv) How do changes in the parameters governing fundamental uncertainty/transparency and liquidity impact on the optimal guarantee?

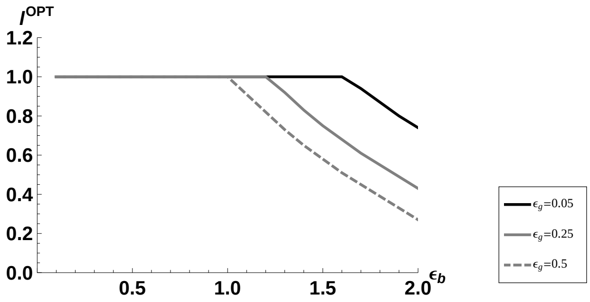
Since the guarantee promise increases the sovereign's expected liabilities, sovereign creditors may lend to the government less often, thereby increasing the government's own likelihood of default. This in turn can jeopardize the effectiveness of the guarantee as bank creditors become less eager to rely on the guarantee when they expect that the government is becoming unable to fund its promises.

Proposition 4 provides a necessary and sufficient condition for the guarantee to be effective in raising the incentives of bank creditors to roll over their loans. Moreover, our model provides a theoretical foundation for the empirically observed behavior of credit default spreads during the recent crisis across the different countries that issued bank debt guarantees.

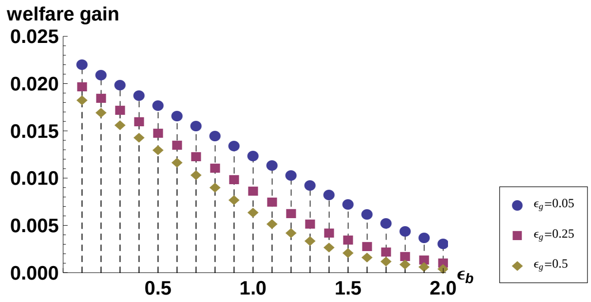
Our results show clear-cut welfare improvements with greater transparency lowering fundamental uncertainty. This would suggest that, in designing guarantee schemes, authorities can improve on their credibility by mandating greater disclosure on the part of the banks. These findings are in line with the new approaches being sought by several countries, as discussed by the Basel Committee for Banking Supervision (2011). Moreover, by improving on the government's own transparency, these gains can be further enhanced.

While reduced-form, the model captures key strategic interactions across sovereign and bank creditors in the design of optimal guarantee schemes that are often assumed to be exogenous. Such cautionary tales apply equally to the design of new regulations, where authorities focus on effects in partial- rather than general-equilibrium models.

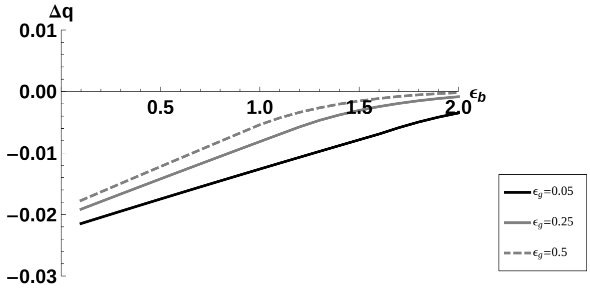
Figure 6: Comparative statics of ε_b and ε_g with $\phi_b = 0.1$ and $\phi_g = 0.2$



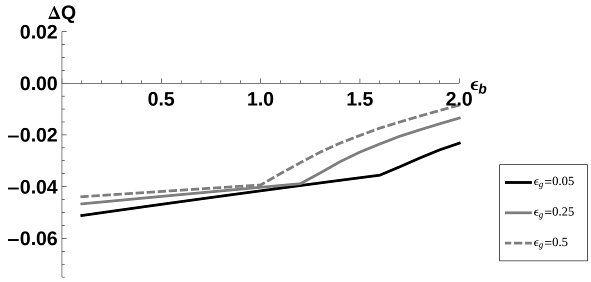
Panel (a): Optimal guarantee: ℓ^{opt}



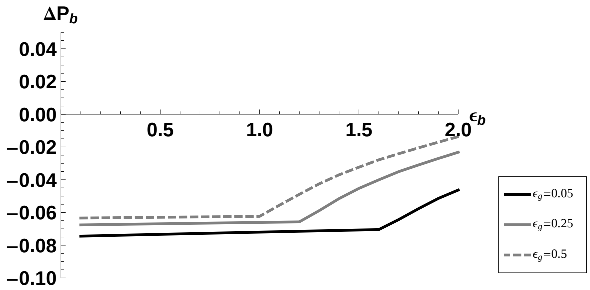
Panel (b): Welfare gain: $K^0 - K^{opt}$



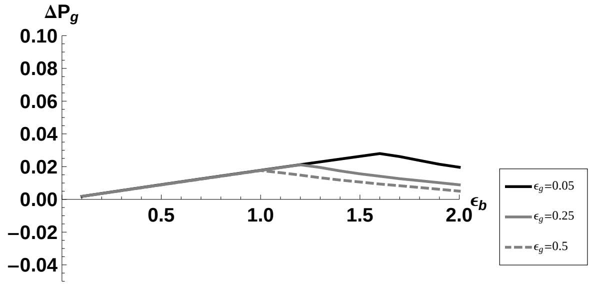
Panel (c): Change in probability of systemic crisis: Δq



Panel (d): Change in probability of any crisis: ΔQ

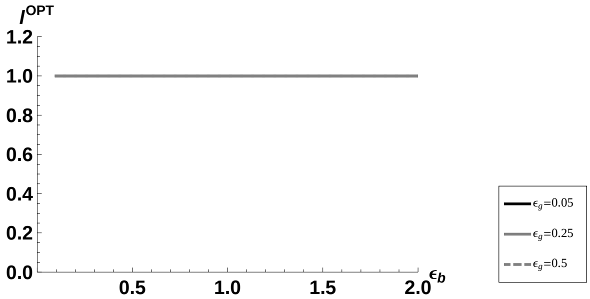


Panel (e): Change in probability of banking crisis: ΔP_b

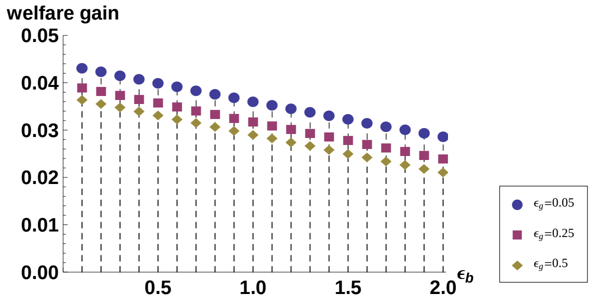


Panel (f): Change in probability of sovereign crisis: ΔP_g

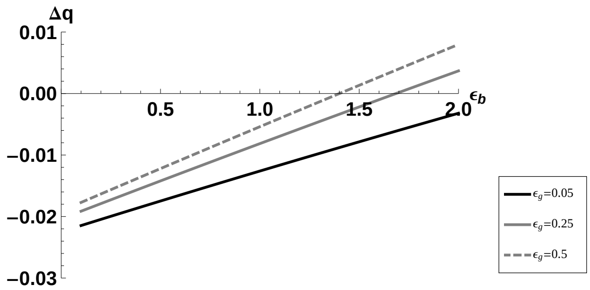
Figure 7: Comparative statics of ε_b and ε_g with $\phi_b = 0.5$ and $\phi_g = 0.2$



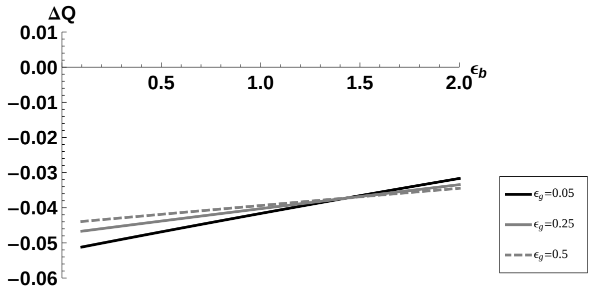
Panel (a): Optimal guarantee: ℓ^{opt}



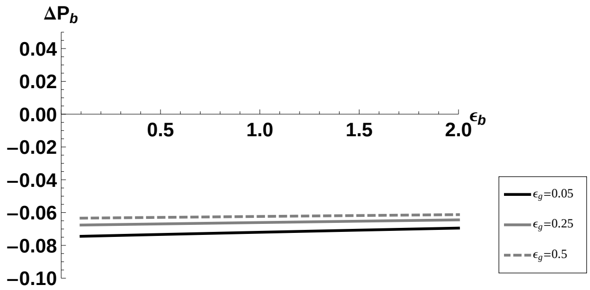
Panel (b): Welfare gain: $K^0 - K^{opt}$



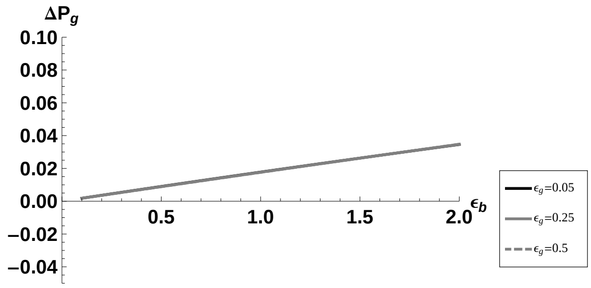
Panel (c): Change in probability of systemic crisis: Δq



Panel (d): Change in probability of any crisis: ΔQ



Panel (e): Change in probability of banking crisis: ΔP_b



Panel (f): Change in probability of sovereign crisis: ΔP_g

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Appendix

Proof of Proposition 1. Morris and Shin (2003) show that the model has a unique symmetric threshold equilibrium where creditors use the strategy around $\hat{x}_b$ and the bank defaults whenever $\theta_b < \hat{\theta}_b$. The creditor who observes $x_{n_b} = \hat{x}_b$ must therefore be indifferent between rolling over and withdrawing. Thus, the expected payoff difference between rolling over and withdrawing is given by

$$D_b \mathbf{Pr}\left(\theta_b > \hat{\theta}_b \middle| \hat{x}_b\right) + \ell \mathbf{Pr}\left(\theta_b \leq \hat{\theta}_b \middle| \hat{x}_b\right) - C_b = 0, \tag{A22}$$

which, by using the assumed uniform distributions, can be written as

$$\frac{D_b - C_b}{D_b - \ell} = \frac{1}{2\varepsilon_b} \int_{\hat{x}_b - \varepsilon_b}^{\hat{\theta}_b} \mathrm{d}u. \tag{A23}$$

Due to the law of large numbers, $\lambda_b(\theta_b) = \mathbf{Pr}\left(x_{n_b} \leq \hat{x}_b \middle| \theta_b\right) = \frac{\int_{\theta_b-\varepsilon_b}^{\hat{x}_b} \mathrm{d}u}{2\varepsilon_b}$. Combining the latter with failure condition (1) yields

$$\frac{1}{2\varepsilon_b} \int_{\hat{\theta}_b - \varepsilon_b}^{\hat{x}_b} \mathrm{d}u = \frac{\hat{\theta}_b}{N_b}. \tag{A24}$$

From equation (A23),

$$1 - \frac{D_b - C_b}{D_b - \ell} = \frac{C_b - \ell}{D_b - \ell} = 1 - \frac{1}{2\varepsilon_b} \int_{\hat{x}_b - \varepsilon_b}^{\hat{\theta}_b} \mathrm{d}u = \frac{1}{2\varepsilon_b} \int_{\hat{\theta}_b - \varepsilon_b}^{\hat{x}_b} \mathrm{d}u,$$

and combining the latter with equation (A24) gives equation (3) in the text,

$$\frac{N_b(C_b - \ell)}{D_b - \ell} = \hat{\theta}_b.$$

Moreover, solving equation (A24) for $\hat{x}_b$, gives equation (2) in the text,

$$\frac{1}{2\varepsilon_b} \int_{\hat{\theta}_b - \varepsilon_b}^{\hat{x}_b} \mathrm{d}u = \frac{\hat{x}_b - \hat{\theta}_b + \varepsilon_b}{2\varepsilon_b} = \frac{\hat{\theta}_b}{N_b} \quad \Rightarrow \hat{x}_b = \hat{\theta}_b\left(1 + \frac{2\varepsilon_b}{N_b}\right) - \varepsilon_b.$$

□

Proof of Proposition 2. By our assumption on the independence between random variables θ_b and θ_g, we can consider each game separately and treat the fundamental and the strategy in the respective other game as exogenously given. Thus, as shown in the following Lemmas A3 and A4, bank creditors respond to any strategy played by sovereign creditors by using a unique threshold strategy. Moreover, as shown in Lemma A5, government creditors respond to any strategy played by bank creditors by using a unique threshold strategy. As a direct consequence, the unique equilibrium in the model is a threshold equilibrium.

□

To prove Lemmas A3 - A5, the following Claims A1 and A2 provide some properties of the payoff differentials of bank and sovereign creditors respectively.

Denote the fraction of bank creditors who withdraw by λ_b and suppose that government creditors play any symmetric strategy $s_g(x_{n_g})$. Given the government's liquidity θ_g, we can write the fraction of government creditors who withdraw as $\int_{\theta_g-\varepsilon_g}^{\theta_g+\varepsilon_g} s(x_{n_g})\mathrm{d}x_{n_g}$. The payoff differential between rolling over and withdrawing for a typical bank creditor can then be written as

$$\pi^b(\theta_b, \lambda_b, \theta_g, s_g(\cdot)) = \begin{cases} D_b - C_b & \text{if } \lambda_b < \theta_b,\ \forall \theta_g \\ \ell - C_b & \text{if } \lambda_b > \theta_b,\ \int_{\theta_g-\varepsilon_g}^{\theta_g+\varepsilon_g} s_g(x_{n_g})\mathrm{d}x_{n_g} < \theta_g - (1-\lambda_b)\ell \\ -C_b & \text{if } \lambda_b > \theta_b,\ \int_{\theta_g-\varepsilon_g}^{\theta_g+\varepsilon_g} s_g(x_{n_g})\mathrm{d}x_{n_g} > \theta_g - (1-\lambda_b)\ell. \end{cases} \tag{A25}$$

Claim A1. *The bank creditors' payoff differential* (A25) *has the following properties.*

1. *Action single-crossing in λ_b: For any θ_b and θ_g, there exists λ_b^* such that $\pi^b > 0$ for any $\lambda_b < \lambda^*$ and $\pi^b < 0$ for any $\lambda_b > \lambda_b^*$.*
2. *State monotonicity in θ_b: π^b is non-decreasing in θ_b.*

3. *Laplacian State Monotonicity: There exists a unique θ_b^* such that*

$$\int_0^1 \pi(\theta_b^*, \lambda_b, \theta_g, s_g(\cdot))\mathrm{d}\lambda_b = 0.$$

4. *Uniform Limit Dominance: There exist $\underline{\theta}_b$ and $\overline{\theta}_b$ such that $\pi^b < -\delta$ for $\theta_b < \underline{\theta}_b$ and $\pi^b > \delta$ for $\theta_b > \overline{\theta}_b$ for some $\delta > 0$.*

Moreover, the noise distribution satisfies

5. *Monotone Likelihood Property.*
6. *Finite expectations of signals.*

Proof of Claim A1. 1. Note that $D_b - C_b > 0 > \ell - C_b > -C_b$. Action single-crossing then follows by setting $\lambda_b^* = \theta_b$.

2. Can be inferred immediately from equation (A25).
3. We can write the integral $\int_0^1 \pi(\theta_b, \lambda_b, \theta_g, s_g(\cdot))\mathrm{d}\lambda_b$ as follows

$$\begin{aligned}&(D_b - C_b)\int_0^{\theta_b} \mathrm{d}\lambda_b - C_b \int_{\theta_b}^{\min\left\{1, 1-\ell^{-1}(\theta_g - \int_{\theta_g-\varepsilon_g}^{\theta_g+\varepsilon_g} s_g(x_{n_g})\mathrm{d}x_{n_g})\right\}} \mathrm{d}\lambda_b \\ &+ (\ell - C_b)\int_{\min\left\{1, 1-\ell^{-1}(\theta_g - \int_{\theta_g-\varepsilon_g}^{\theta_g+\varepsilon_g} s_g(x_{n_g})\mathrm{d}x_{n_g})\right\}}^1 \mathrm{d}\lambda_b = 0.\end{aligned}$$

   As the left hand side of the equality sign is negative for $\theta_b = 0$, positive for $\theta_b = 1$ and otherwise strictly increasing in θ_b, there exists a unique θ_b^* such that $\int_0^1 \pi(\theta_b^*, \lambda_b, \theta_g, s_g(\cdot))\mathrm{d}\lambda_b = 0$.

4. The claim follows by setting $\underline{\theta}_b = 0$, $\overline{\theta}_b = 1$ and $\delta = \min\{C_b - \ell, D_b - C_b\}$.
5. Uniform noise satisfies MLRP, see (Shao, 2003, p. 399).
6. This follows immediately from the assumption of a uniform distribution with bounded support.

□

Suppose that bank creditors play any strategy $s_b(x_{n_b})$. Given any θ_b, we can then write the fraction of bank creditors who withdraw as $\int_{\theta_b-\varepsilon_b}^{\theta_b+\varepsilon_b} s_b(x_{n_b})\mathrm{d}x_{n_b}$. The payoff differential between rolling over and withdrawing for a typical government creditor is then given by

$$\pi^g(\theta_g, \lambda_g, \theta_b, s_b(\cdot)) = \begin{cases} D_g - C_g & \text{if } \lambda_g < \theta_g,\ \int_{\theta_b-\varepsilon_b}^{\theta_b+\varepsilon_b} s_b(x_{n_b})\mathrm{d}x_{n_b} < \theta_b \\ D_g - C_g & \text{if } \lambda_g < \theta_g - (1 - \int_{\theta_b-\varepsilon_b}^{\theta_b+\varepsilon_b} s_b(x_{n_b})\mathrm{d}x_{n_b})\ell,\ \int_{\theta_b-\varepsilon_b}^{\theta_b+\varepsilon_b} s_b(x_{n_b})\mathrm{d}x_{n_b} > \theta_b \\ -C_g & \text{if } \lambda_g > \theta_g - (1 - \int_{\theta_b-\varepsilon_b}^{\theta_b+\varepsilon_b} s_b(x_{n_b})\mathrm{d}x_{n_b})\ell,\ \int_{\theta_b-\varepsilon_b}^{\theta_b+\varepsilon_b} s_b(x_{n_b})\mathrm{d}x_{n_b} > \theta_b \\ -C_g & \text{if } \lambda_g > \theta_g,\ \forall \theta_b. \end{cases} \tag{A26}$$

Claim A2. *Government creditors' payoff differential* (A26) *has the following properties.*

1. *Action monotonicity in λ_g: π^g is non-increasing in λ_g.*
2. *State monotonicity in θ_g: π^g is non-decreasing in θ_g.*
3. *Laplacian State Monotonicity: There exists a unique θ_g^* such that*

$$\int_0^1 \pi(\theta_g^*, \lambda_g, \theta_b, s_b(\cdot))\mathrm{d}\lambda_g = 0.$$

4. *Uniform Limit Dominance: There exist $\underline{\theta}_g$ and $\overline{\theta}_g$ such that $\pi_g < -\delta$ for $\theta_g < \underline{\theta}_g$ and $\pi_g > \delta$ for $\theta_g > \overline{\theta}_g$ for some $\delta > 0$.*

Moreover, the noise distribution satisfies

5. *Monotone Likelihood Property.*
6. *Finite expectations of signals.*

Proof of Claim A2. 1. Suppose $\theta_b > \lambda_b$, then, since $D_g - C_g > -C_g$, π^g is clearly non-increasing in λ_g for any θ_g. Similarly for the case where $\theta_b < \lambda_b$.

2. Suppose $\theta_b > \lambda_b$, then π^g is increasing in θ_g for any λ_g. Similarly for $\theta_b < \lambda_b$.
3. If $\theta_b > \lambda_b$, then $\theta_g^* = C_g/D_g$. If $\theta_b < \lambda_b$, then $\theta_g^* = C_g/D_g + (1-\lambda_b)\ell$.
4. This follows by setting $\overline{\theta}_g = 1+\ell$ and $\underline{\theta}_g = 0$ and $\delta = D_g - C_g$.
5. Uniform noise satisfies MLRP, see (Shao, 2003, p. 399).
6. This follows immediately from the assumption of a uniform distribution with bounded support.

□

Lemma A3. *For any strategy $s_g(\cdot)$ played by government creditors, the roll over game between bank creditors has a unique threshold equilibrium.*

Proof of Lemma A3. Since the payoff differential satisfies properties (1) to (6) in Claim A1, the Lemma follows from Morris and Shin (2003, Lemma 2.3). □

Lemma A4. *There are no other equilibria in non-threshold strategies.*

Proof of Lemma A4. Since noise terms are uniformly distributed and the payoff differential satisfies action single-crossing, the Lemma follows immediately from the proof to Goldstein and Pauzner (2005, Theorem 1). □

Lemma A5. *For any strategy $s_b(\cdot)$ played by bank creditors, the roll over game between government creditors has a unique equilibrium in threshold strategies. Moreover, there are no equilibria in non-threshold strategies.*

Proof of Lemma A5. Since the payoff differential satisfies properties (1) to (6) in Claim A2, the Lemma follows immediately from Morris and Shin (2003, Proposition 2.2). □

Proof of Proposition 3. From the proof of Proposition 2 follows that each game has a unique equilibrium in threshold strategies. That is, for given $\hat{x}_g$, there exists a unique $\hat{x}_b$ that satisfies equation (10) and for given $\hat{x}_b$, there exists a unique $\hat{x}_g$ that satisfies (11). To see this directly, fix $\hat{x}_g$. Due to the existence of dominance regions there exist $\underline{\hat{x}}_b$ and $\overline{\hat{x}}_b$ such that $\pi^b(\hat{x}_b, \hat{x}_g, \hat{x}_b) < 0$ for any $\hat{x}_b < \underline{\hat{x}}_b$, and $\pi^b(\hat{x}_b, \hat{x}_g, \hat{x}_b) > 0$ for any $\hat{x}_b > \overline{\hat{x}}_b$. Similarly, for $\pi^g(\hat{x}_g, \hat{x}_b, \hat{x}_g)$. Since $\pi^b(\cdot)$ and $\pi^g(\cdot)$ are continuous they both cross the x-axis at least once. To show that there exists exactly one threshold signal, it suffices to show that $\pi^b(\hat{x}_b, \hat{x}_g, \hat{x}_b)$ is strictly increasing in $\hat{x}_b$ and $\pi^g(\hat{x}_g, \hat{x}_b, \hat{x}_g)$ is strictly increasing in $\hat{x}_g$.

The derivative of $\pi^g(\cdot)$ with respect to $\hat{x}_g$ is given by

$$\frac{\partial \pi^g(\hat{x}_g, \hat{x}_b, \hat{x}_g)}{\partial \hat{x}_g} = \frac{D_g}{1+2\varepsilon_g} > 0, \tag{A27}$$

where we have used

$$\frac{\partial \hat{\theta}_g}{\partial \hat{x}_g} = \frac{1}{1+2\varepsilon_g} \quad \forall \theta_b.$$

Next, consider the derivative of π^b with respect to $\hat{x}_b$. Observe first that $\hat{\theta}_b'(\hat{x}_b) = N_b(N_b+2\varepsilon_b)^{-1}$ and $(1-\hat{\theta}_b'(\hat{x}_b)) = 2\varepsilon_b(N_b+2\varepsilon_b)^{-1}$. Moreover, if $\theta_b < \hat{\theta}_b$, then $\partial\hat{\theta}_g/\partial\hat{x}_b = -\ell N_b \varepsilon_g(\varepsilon_b(1+2\varepsilon_g))^{-1}$. Let $\hat{\theta}_g^T := (\hat{x}_g + \varepsilon_g)(1+2\varepsilon_g)^{-1}$, so that we can write $\hat{\theta}_g(\hat{x}_g, \hat{x}_b, \theta_b) = \hat{\theta}_g^T + \frac{\ell N_b \varepsilon_g}{1+2\varepsilon_g} \frac{\theta_b - \hat{x}_b + \varepsilon_b}{N_b + 2\varepsilon_b}$, while $\hat{\theta}_g(\hat{x}_g, \hat{x}_b, \hat{x}_b - \varepsilon_b) = \hat{\theta}_g^T$. Using these facts and definitions, the derivative of $\pi^b(\cdot)$ with respect to $\hat{x}_b$ is given by

$$\begin{aligned}
\frac{\partial \pi^b\left(\hat{x}_b, \hat{x}_g, \hat{x}_b\right)}{\partial \hat{x}_b} &= \frac{D_b}{2\varepsilon_b}\left(1-\hat{\theta}_b'(\hat{x}_b)\right) + \frac{\ell}{2\varepsilon_b}\left(\frac{\hat{\theta}_b'(\hat{x}_b)}{\sigma_g}\int_{\hat{\theta}_g(\hat{x}_g,\hat{x}_b,\hat{\theta}_b)}^{\tilde{\sigma}_g} \mathrm{d}v - \frac{1}{\sigma_g}\int_{\hat{\theta}_g(\hat{x}_g,\hat{x}_b,\hat{x}_b-\varepsilon_b)}^{\tilde{\sigma}_g} \mathrm{d}v - \int_{\hat{x}_b-\varepsilon_b}^{\hat{\theta}_b} \frac{\frac{\partial\hat{\theta}_g(\cdot)}{\partial\hat{x}_b}}{\sigma_g}\mathrm{d}u\right) \\
&= \frac{D_b}{N_b+2\varepsilon_b} + \frac{\ell}{2\varepsilon_b\sigma_g}\left(\frac{N_b}{N_b+2\varepsilon_b}\left(\theta_g^0 + \eta_g - \hat{\theta}_g(\hat{\theta}_b)\right) - \left(\theta_g^0 + \eta_g - \hat{\theta}_g^T\right) + \left(\frac{\ell N_b 2\varepsilon_g}{1+2\varepsilon_g}\frac{(N_b - \hat{x}_b + \varepsilon_b)}{N_b+2\varepsilon_b}\right)\right) \\
&= ((N_b+2\varepsilon_b)\sigma_g)^{-1}\left[\sigma_g D_b - \ell(\tilde{\sigma}_g - \hat{\theta}_g^T) + \frac{\ell}{2\varepsilon_b}\left(\frac{\ell N_b 2\varepsilon_g}{1+2\varepsilon_g}\left(1-\frac{N_b}{N_b+2\varepsilon_b}\right)(N_b - \hat{x}_b + \varepsilon_b)\right)\right].
\end{aligned} \tag{A28}$$

Now observe that $\sigma_g D_b - \ell(\tilde{\sigma}_g - \hat{\theta}_g^T) = \sigma_g\left(D_b - \ell\frac{(\tilde{\sigma}_g - \hat{\theta}_g^T)}{\sigma_g}\right) > 0$ since $D_b > \ell$ and $\frac{(\tilde{\sigma}_g - \hat{\theta}_g^T)}{\sigma_g} \leq 1$ because it is a probability. Furthermore $N_b + \varepsilon_b - \hat{x}_b \geq 0$ because the existence of an upper dominance region implies that $\hat{x}_b$ is bounded above by $N_b + \varepsilon_b$. Thus, $\frac{\partial\pi^b(\hat{x}_b, \hat{x}_g, \hat{x}_b)}{\partial\hat{x}_b} > 0$.

The existence of functions f_b and f_g follows by applying the implicit function theorem to equations (10) and (11). The slopes of the two functions are given by

$$f_b'(\hat{x}_g) = -\frac{\partial\pi^b/\partial\hat{x}_g}{\partial\pi^b/\partial\hat{x}_b} \quad \text{and} \quad f_g'(\hat{x}_b) = -\frac{\partial\pi^g/\partial\hat{x}_b}{\partial\pi^g/\partial\hat{x}_g}.$$

We have $f_b' > 0$, since

$$\frac{\partial\pi^b}{\partial\hat{x}_g} = -\frac{\ell}{2\varepsilon_b}\int_{\hat{x}_b-\varepsilon_b}^{\hat{\theta}_b}\frac{1}{\sigma_g}\mathrm{d}u < 0, \tag{A29}$$

and $f_g' < 0$, because

$$\begin{aligned}
\frac{\partial\pi^g}{\partial\hat{x}_b} &= -\frac{D_g\ell N_b}{2\varepsilon_b\sigma_b(1+2\varepsilon_g)}\frac{\partial}{\partial\hat{x}_b}\left(\int_{\hat{x}_b-\varepsilon_b}^{\hat{\theta}_b(\hat{x}_b)}(u+\varepsilon_b-\hat{x}_b)\mathrm{d}u\right)\\
&\propto -\frac{\partial}{\partial\hat{x}_b}\left(\int_{\hat{x}_b-\varepsilon_b}^{\hat{\theta}_b(\hat{x}_b)}(u+\varepsilon_b-\hat{x}_b)\mathrm{d}u\right)\\
&= (\hat{\theta}_b+\varepsilon_b-\hat{x}_b)\left(1-\frac{\partial\hat{\theta}_b}{\partial\hat{x}_b}\right) > 0,
\end{aligned} \tag{A30}$$

since $\frac{\partial\hat{\theta}_b}{\partial\hat{x}_b} = \frac{N_b}{N_b+2\varepsilon_b} < 1$. □

Lemma A6. *The signs of the derivatives of the critical signals $\hat{x}_b$ and $\hat{x}_g$ with respect to parameters $\{\ell, N_b, \theta_b^0, \theta_g^0\}$ are given by*

$$\begin{aligned}
&\frac{\mathrm{d}\hat{x}_g}{\mathrm{d}\ell} > 0 \quad \textit{and} \quad \frac{\mathrm{d}\hat{x}_b}{\mathrm{d}\ell} \lesseqgtr 0\\
&\frac{\mathrm{d}\hat{x}_b}{\mathrm{d}N_b} > 0, \quad \textit{and} \quad \frac{\mathrm{d}\hat{x}_g}{\mathrm{d}N_b} > 0\\
&\frac{\mathrm{d}\hat{x}_b}{\mathrm{d}\theta_b^0} < 0 \quad \textit{and} \quad \frac{\mathrm{d}\hat{x}_g}{\mathrm{d}\theta_b^0} < 0\\
&\frac{\mathrm{d}\hat{x}_b}{\mathrm{d}\theta_g^0} < 0 \quad \textit{and} \quad \frac{\mathrm{d}\hat{x}_g}{\mathrm{d}\theta_g^0} > 0.
\end{aligned}$$

Proof of Lemma A6. Let $\xi = (\ell, N_b, \theta_b^0, \theta_g^0)$ with typical element ξ_k. The total effects $\frac{\mathrm{d}\hat{x}_b}{\mathrm{d}\xi_k}$ and $\frac{\mathrm{d}\hat{x}_g}{\mathrm{d}\xi_k}$ can be found by applying the implicit function theorem to the set of equations

$$\begin{aligned}
\pi^g(\hat{x}_g, \hat{x}_b, \xi) &= 0\\
\pi^b(\hat{x}_b, \hat{x}_g, \xi) &= 0.
\end{aligned}$$

The Jacobian of this system is given by

$$\mathbf{J} = \begin{pmatrix} \frac{\partial\pi^b}{\partial\hat{x}_b} & \frac{\partial\pi^b}{\partial\hat{x}_g} \\ \frac{\partial\pi^g}{\partial\hat{x}_b} & \frac{\partial\pi^g}{\partial\hat{x}_g} \end{pmatrix} = \begin{pmatrix} (+) & (-) \\ (+) & (+) \end{pmatrix},$$

and thus its determinant is positive, $|\mathbf{J}| > 0$.

The total effects can be computed as

$$\frac{\mathrm{d}\hat{x}_b}{\mathrm{d}\xi_k} = \frac{\begin{vmatrix} -\frac{\partial\pi^b}{\partial\xi_k} & \frac{\partial\pi^b}{\partial\hat{x}_g} \\ -\frac{\partial\pi^g}{\partial\xi_k} & \frac{\partial\pi^g}{\partial\hat{x}_g} \end{vmatrix}}{|\mathbf{J}|} = \frac{-\frac{\partial\pi^b}{\partial\xi_k}\frac{\partial\pi^g}{\partial\hat{x}_g} + \frac{\partial\pi^b}{\partial\hat{x}_g}\frac{\partial\pi^g}{\partial\xi_k}}{|\mathbf{J}|}. \tag{A31}$$

and

$$\frac{\mathrm{d}\hat{x}_g}{\mathrm{d}\xi_k} = \frac{\begin{vmatrix} \frac{\partial\pi^b}{\partial\hat{x}_b} & -\frac{\partial\pi^b}{\partial\xi_k} \\ \frac{\partial\pi^b}{\partial\hat{x}_g} & -\frac{\partial\pi^g}{\partial\xi_k} \end{vmatrix}}{|\mathbf{J}|} = \frac{-\frac{\partial\pi^g}{\partial\xi_k}\frac{\partial\pi^b}{\partial\hat{x}_b} + \frac{\partial\pi^g}{\partial\hat{x}_g}\frac{\partial\pi^b}{\partial\xi_k}}{|\mathbf{J}|}. \tag{A32}$$

The partial derivatives with respect to ℓ are given by

$$
\begin{aligned}
\frac{\partial \pi^b}{\partial \ell} &= \frac{1}{2\varepsilon_b}\int_{\hat{x}_b-\varepsilon_b}^{\hat{\theta}_b}\frac{1}{\sigma_g}\int_{\hat{\theta}_g(u)}^{\widetilde{\sigma}_g}\mathrm{d}v\,\mathrm{d}u - \frac{\ell}{2\varepsilon_b}\int_{\hat{x}_b-\varepsilon_b}^{\hat{\theta}_b}\frac{\varepsilon_g N_b}{\varepsilon_b(1+2\varepsilon_g)}\frac{(u+\varepsilon_b-\hat{x}_b)}{\sigma_g}\mathrm{d}u \\
&= \frac{1}{2\varepsilon_b\sigma_g}\int_{\hat{x}_b-\varepsilon_b}^{\hat{\theta}_b}\left[\int_{\hat{\theta}_g(u)}^{\widetilde{\sigma}_g}\mathrm{d}v - \frac{\varepsilon_g \ell N_b}{\varepsilon_b(1+2\varepsilon_g)}(u+\varepsilon_b-\hat{x}_b)\right]\mathrm{d}u \\
&= \frac{1}{2\varepsilon_b\sigma_g}\int_{\hat{x}_b-\varepsilon_b}^{\hat{\theta}_b}\left[\widetilde{\sigma}_g - \hat{\theta}_g^T - \frac{2\varepsilon_g \ell N_b}{\varepsilon_b(1+2\varepsilon_g)}(u+\varepsilon_b-\hat{x}_b)\right]\mathrm{d}u \\
&= \frac{\hat{\theta}_b-\hat{x}_b+\varepsilon_b}{2\varepsilon_b}\left\{\left(\frac{\widetilde{\sigma}_g-\hat{\theta}_g^T}{\sigma_g}\right) + \frac{2\ell\varepsilon_g N_b(\hat{x}_b-\varepsilon_b)}{\sigma_g\varepsilon_b(1+2\varepsilon_g)} - \frac{2\ell\varepsilon_g N_b(\hat{\theta}_b+\hat{x}_b-\varepsilon_b)}{2\sigma_g\varepsilon_b(1+2\varepsilon_g)}\right\} \\
&= \frac{\hat{\theta}_b-\hat{x}_b+\varepsilon_b}{2\varepsilon_b}\left\{\frac{\widetilde{\sigma}_g-\hat{\theta}_g^T}{\sigma_g} + \frac{\ell\varepsilon_g N_b(\hat{x}_b-\varepsilon_b-\hat{\theta}_b)}{\sigma_g\varepsilon_b(1+2\varepsilon_g)}\right\} \\
&= \frac{\hat{\theta}_b-\hat{x}_b+\varepsilon_b}{2\varepsilon_b}\left\{\frac{\widetilde{\sigma}_g-\hat{\theta}_g(\hat{\theta}_b)}{\sigma_g}\right\} \geq 0, \qquad \text{(A33)}
\end{aligned}
$$

where we have used the abbreviation $\hat{\theta}_g(u) := \hat{\theta}_g(\hat{x}_g, \hat{x}_b, u)$.

Furthermore,

$$
\begin{aligned}
\frac{\partial \pi^g}{\partial \ell} &= \frac{-D_g}{2\varepsilon_g\sigma_b}\int_{-\eta_b}^{\hat{\theta}_b}\frac{\varepsilon_g N_b(u+\varepsilon_b-\hat{x}_b)}{\varepsilon_b(1+2\varepsilon_g)}\mathrm{d}u \\
&= \frac{-D_g N_b}{\sigma_b(1+2\varepsilon_g)}\int_{\hat{x}_b-\varepsilon_b}^{\hat{\theta}_b}\frac{u+\varepsilon_g-\hat{x}_b}{2\varepsilon_b}\mathrm{d}u < 0. \qquad \text{(A34)}
\end{aligned}
$$

Given the signs of equations (A33) and (A34), it follows from equations (A31) and (A32) that

$$
\frac{\mathrm{d}\hat{x}_g}{\mathrm{d}\ell} > 0 \quad \text{and} \quad \frac{\mathrm{d}\hat{x}_b}{\mathrm{d}\ell} \lesseqgtr 0.
$$

Condition (12) in the text can be derived by explicitly calculating

$$
-\frac{\partial \bar{\pi}^b}{\partial \ell}\frac{\partial \bar{\pi}^g}{\partial \hat{x}_g} + \frac{\partial \bar{\pi}^b}{\partial \hat{x}_g}\frac{\partial \bar{\pi}^g}{\partial \ell}.
$$

Using equations (A27), (A29), (A33) and (A34), we obtain

$$
-\frac{D_g}{1+2\varepsilon_g}\left(\frac{\hat{\theta}_b-\hat{x}_b+\varepsilon_b}{2\varepsilon_b}\right)\left(\frac{\widetilde{\sigma}_g-\hat{\theta}_g(\hat{\theta}_b)}{\sigma_g}\right) + \left(\frac{\hat{\theta}_b-\hat{x}_b+\varepsilon_b}{2\varepsilon_b}\right)\frac{D_g \ell N_b}{\sigma_g\sigma_b(1+2\varepsilon_g)}\int_{\hat{x}_b-\varepsilon_b}^{\hat{\theta}_b}\frac{u+\varepsilon_b-\hat{x}_b}{2\varepsilon_b}\mathrm{d}u,
$$

which is negative if and only if

$$
\frac{\ell N_b}{\sigma_b}\int_{\hat{x}_b-\varepsilon_b}^{\hat{\theta}_b}\frac{u+\varepsilon_b-\hat{x}_b}{2\varepsilon_b}\mathrm{d}u < \widetilde{\sigma}_g - \hat{\theta}_g^*(\hat{\theta}_b^*),
$$

which is condition (12).

The derivatives with respect to N_b are given by

$$
\frac{\partial \pi^b}{\partial N_b} = \frac{1}{(N_b+2\varepsilon_b)^2}\left[\left(-D_b + \ell\frac{\widetilde{\sigma}_g-\hat{\theta}_g(\hat{\theta}_b)}{\sigma_g}\right) - \frac{\ell^2}{\sigma_g(1+2\varepsilon_g)}\frac{(N_b+\varepsilon_b-\hat{x}_b)}{(N_b+2\varepsilon_b)}\left((N_b+2\varepsilon_b)^2 + N_b(N_b+4\varepsilon_b)\right)\right] < 0 \qquad \text{(A35)}
$$

and

$$
\frac{\partial \pi^g}{\partial N_b} = -\frac{D_g \ell}{2\varepsilon_b(1+2\varepsilon_g)\sigma_b}\left[\int_{-\eta_b}^{\hat{\theta}_b}(u+\varepsilon_b-\hat{x}_b)\mathrm{d}u + \frac{2\varepsilon_b(\hat{x}_b+\varepsilon_b)}{(N_b+2\varepsilon_b)^2}\left(\hat{\theta}_b-\hat{x}_b+\varepsilon_b\right)\right] < 0. \qquad \text{(A36)}
$$

Given the signs of equations (A35) and (A36), it follows from equations (A31) and (A32) that

$$
\frac{\mathrm{d}\hat{x}_b}{\mathrm{d}N_b} > 0, \quad \text{and} \quad \frac{\mathrm{d}\hat{x}_g}{\mathrm{d}N_b} \gtrless 0.
$$

To show that $\frac{\mathrm{d}\hat{x}_g}{\mathrm{d}N_b} > 0$, we calculate

$$
-\frac{\partial \pi^g}{\partial N_b}\frac{\partial \pi^b}{\partial \hat{x}_b} + \frac{\partial \pi^g}{\partial \hat{x}_b}\frac{\partial \pi^b}{\partial N_b}.
$$

Using equations (A28), (A30), (A35) and (A36), we obtain

$$\Omega\left(\frac{\hat{\theta}_b+\varepsilon_b-\hat{x}_b}{4\varepsilon_b}+N_b(\hat{x}_b+\varepsilon_b)\right)-\frac{\Omega N_b}{N_b+2\varepsilon_b}\frac{\hat{x}_b+\varepsilon_b}{N_b+2\varepsilon_b}-\frac{\ell^2 N_b\varepsilon_g(\hat{\theta}_b+\varepsilon_b-\hat{x}_b)^2}{4\varepsilon_b^2\sigma_g(N_b+2\varepsilon_b)(1+2\varepsilon_g)},$$

where $\Omega := \frac{D_b-\ell\frac{\tilde{\sigma}-\hat{\theta}_g(\hat{\theta}_b)}{\sigma_g}}{N_b+2\varepsilon_b}$. Since $N_b \geq 1$, we have

$$\Omega N_b(\hat{x}_b+\varepsilon_b) > \frac{\Omega N_b}{N_b+2\varepsilon_b}\frac{\hat{x}_b+\varepsilon_b}{N_b+2\varepsilon_b}.$$

Moreover,

$$\begin{aligned}
&\Omega\left(\frac{\hat{\theta}_b+\varepsilon_b-\hat{x}_b}{4\varepsilon_b}\right)-\frac{\ell^2 N_b\varepsilon_g(\hat{\theta}_b+\varepsilon_b-\hat{x}_b)^2}{4\varepsilon_b^2\sigma_g(N_b+2\varepsilon_b)(1+2\varepsilon_g)}>0\\
\Leftrightarrow\ &\Omega > \frac{\ell^2 N_b\varepsilon_g(\hat{\theta}_b+\varepsilon_b-\hat{x}_b)}{\varepsilon_b\sigma_g(N_b+2\varepsilon_b)(1+2\varepsilon_g)}\\
\Leftrightarrow\ &D_b-\ell\frac{\tilde{\sigma}-\hat{\theta}_g^T}{\sigma_g}+\frac{\ell^2 N_b\varepsilon_g(\hat{\theta}_b+\varepsilon_b-\hat{x}_b)}{\varepsilon_b(1+2\varepsilon_g)\sigma_g} > \frac{\ell^2 N_b\varepsilon_g(\hat{\theta}_b+\varepsilon_b-\hat{x}_b)}{\varepsilon_b\sigma_g(1+2\varepsilon_g)}\\
\Leftrightarrow\ &D_b-\ell\frac{\tilde{\sigma}-\hat{\theta}_g^T}{\sigma_g}>0.
\end{aligned}$$

We thus have $-\frac{\partial\pi^g}{\partial N_b}\frac{\partial\pi^b}{\partial\hat{x}_b}+\frac{\partial\pi^g}{\partial\hat{x}_b}\frac{\partial\pi^b}{\partial N_b}>0$, which implies $\frac{\mathrm{d}\hat{x}_g}{\mathrm{d}N_b}>0$.

Finally, the derivatives with respect to θ_b^0 and θ_g^0 are given by

$$\frac{\partial\pi^b}{\partial\theta_g^0}=\frac{\ell}{2\varepsilon_b}\int_{x_{n_b}-\varepsilon_b}^{\hat{\theta}_b(\hat{x}_b)}\left(\frac{1}{\sigma_g^2}\int_{\hat{\theta}_g(\hat{x}_g,\hat{x}_b,u)}^{\tilde{\sigma}_g}\mathrm{d}v\right)\mathrm{d}u>0,$$

$$\frac{\partial\pi^b}{\partial\theta_b^0}=0,$$

$$\frac{\partial\pi^g}{\partial\theta_g^0}=0,$$

$$\frac{\partial\pi^g}{\partial\theta_b^0}=\frac{D_g\ell N_b}{(1+2\varepsilon_g)\sigma_b^2}\int_{-\eta_b}^{\hat{\theta}_b}\frac{u+\varepsilon_b-\hat{x}_b}{2\varepsilon_b}>0.$$

Combining these with equations (A31) and (A32), we obtain

$$\frac{\mathrm{d}\hat{x}_b}{\mathrm{d}\theta_b^0}<0,\ \frac{\mathrm{d}\hat{x}_b}{\mathrm{d}\theta_g^0}<0,\ \frac{\mathrm{d}\hat{x}_g}{\mathrm{d}\theta_b^0}<0,\ \frac{\mathrm{d}\hat{x}_g}{\mathrm{d}\theta_g^0}>0.$$

□

Proof of Corollary 1. Suppose $\ell=\tilde{\ell}$ and condition (12) holds when evaluated at $\tilde{\ell}$. This implies that $\mathrm{d}\hat{x}_b(\tilde{\ell})/\mathrm{d}\ell<0$.

The derivative of the left–hand side of condition (12) is given by

$$\frac{N_b}{\sigma_b}\int_{\hat{x}_b-\varepsilon_b}^{\hat{\theta}_b}\frac{u+\varepsilon_b-\hat{x}_b}{2\varepsilon_b}\mathrm{d}u\ -\ \frac{\ell N_b}{\sigma_b}\cdot\frac{\hat{\theta}_b+\varepsilon_b-\hat{x}_b}{2\varepsilon_b}\frac{2\varepsilon_b}{N_b+2\varepsilon_b}\frac{\mathrm{d}\hat{x}_b}{\mathrm{d}\ell},$$

which is positive by the supposition that (12) holds.

Consider the derivative of the right–hand side with respect to ℓ. It is given by

$$-\frac{\frac{\mathrm{d}\hat{x}_g}{\mathrm{d}\ell}}{1+2\varepsilon_g}-\frac{\varepsilon_g N_b(\hat{\theta}_b+\varepsilon_b-\hat{x}_b)}{\varepsilon_b(1+2\varepsilon_g)}+\frac{\varepsilon_g\ell N_b}{\varepsilon_b(1+2\varepsilon_g)}\frac{2\varepsilon_b}{N_b+2\varepsilon_b}\frac{\mathrm{d}\hat{x}_b}{\mathrm{d}\ell},$$

which is negative by the supposition that (12) holds. □

Proof of Lemma 1. Observe that for given θ_b, total guarantee payments are given by

$$\begin{cases} \frac{N_b \ell}{2\varepsilon_b} \int_{\hat{x}_b}^{\theta_n+\varepsilon_b} \mathrm{d}u & \text{if } \theta_b < \hat{\theta}_b^* \\ 0 & \text{else} \end{cases}.$$

Hence, whenever $\varepsilon_b \to 0$, $\hat{x}_b^* \to \hat{\theta}_b^*$ and the integral collapses to zero. But then, the guarantee does not appear anymore in the government's default condition and the threshold for government default converges to $\hat{\theta}_g^* = C_g/D_g$, as in the canonical model. The probability of a government default can then be calculated as $P_g \equiv \mathbf{Pr}\left(\theta_g < \hat{\theta}_g^*\right) = \frac{C_g/D_g+\eta_g}{\sigma_g}$.

The critical bank creditor's indifference condition can be explicitly written as

$$\bar{\pi}^b(\hat{x}_b, \hat{x}_g) = \frac{D_b(\hat{x}_b + 2\varepsilon_b)}{N_b + 2\varepsilon_b} + \frac{\ell(\widetilde{\sigma}_g - \widehat{\theta}_g^T)(\widehat{\theta}_b - \widehat{x}_b + \varepsilon_b)}{\sigma_g 2\varepsilon_b} - \frac{\ell\varepsilon_g N_b(\widehat{\theta}_b - \widehat{x}_b + \varepsilon_b)^2}{4\varepsilon_b\sigma_g(1+2\varepsilon_g)} - 1 = 0.$$

Observe that $\widehat{\theta}_b - \widehat{x}_b + \varepsilon_b = 2\varepsilon_b(N_b - \widehat{x}_b + \varepsilon_b)/(N_b + 2\varepsilon_b)$. Substituting this into the indifference condition and taking the limit $\varepsilon_b \to 0$ leads to

$$\bar{\pi}^b(\widehat{x}_b) = D_b\widehat{x}_b + (1-P_g)\ell(N_b - \widehat{x}_b) - N_b = 0,$$

which can be solved for the critical signal,

$$\widehat{x}_b = \widehat{\theta}_b = \frac{N_b(1-\ell(1-P_g))}{D_b - \ell(1-P_g)}. \tag{A37}$$

□

Proof of Lemma 2. We obtain from equation (A37)

$$\frac{\partial \widehat{\theta}_b^*}{\partial \ell} = \frac{N_b(1-P_g)(1-D_b)}{(1-\ell(1-P_g))^2} < 0.$$

The probability of a systemic crisis can be computed as

$$q(\ell) = P_g \times P_b(\ell),$$

with $P_b(\ell) = \frac{\widehat{\theta}_b^* + \eta_b}{\sigma_b}$. Since P_g does not depend on ℓ, the derivative of the cost-of-crisis function with respect to ℓ can then be computed as

$$K'(\ell) = (1-P_g)\phi_b \frac{\partial P_b}{\partial \ell} + P_g(1-\phi_g)\frac{\partial P_b}{\partial \ell}.$$

Substituting

$$\frac{\partial P_b}{\partial \ell} = \frac{1}{\sigma_b}\left(\frac{N_b(1-P_g)(1-D_b)}{(1-\ell(1-P_g))^2}\right)$$

gives the expression in the text. □

Country	Name of program	Issued	Tenor	Size (bn of domestic currency)	GDP (bn of domestic currency)	Size (% of GDP)	GSD (bn of domestic currency)	Size (% of GSD)
Austria	Interbank Market Support Act (Finanzmarktstabilisierungsgesetz)	Oct 2008	3-5 years	75	283	27%	181	42%
France	Société de Refinancement des Activités des Etablissements de Crédit	Oct 2008	5 years	265	1,931	14%	1,319	20%
Ireland	Financial Support Act 2008	Oct 2008	2 years	440	180	244%	80	553%
Ireland	Eligible Liabilities Guarantee Scheme 2009	Dec 2009	5 years	150	161	93%	105	143%
Italy	Italian Guarantee Scheme	Nov 2008	5 years	40	1,575	3%	1,667	2%
Netherlands	2008 Credit Guarantee Scheme	Oct 2008	5 years	200	594	34%	348	58%
Portugal	Portuguese State Guarantee Scheme 2008	Oct 2008	3 years	20	172	12%	123	16%
Spain	Spanish Guarantee Scheme	Dec 2008	5 years	100	1,088	9%	437	23%
United Kingdom	2008 Credit Guarantee Scheme	Oct 2008	3 years	250	1,434	17%	752	33%

Table A1: Summary of guarantee schemes introduced in several developed economies following the collapse of Lehman Brothers. All monetary figures are provided in the country of origin's local currency. The column labelled 'Size' refers to the size of the guarantee; GDP is gross domestic product; GSD is gross sovereign debt.

Figure A1: Claims on and of Irish banks vs. the Irish government's revenue as fractions of Irish GDP