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Abstract

We model the behavior of dealers in Over-the-Counter (OTC) derivatives markets where a small number of dealers trade with a continuum of heterogeneous clients (hedgers). Imperfect competition and (endogenous) default induce a familiar trade-off between competition and risk. Increasing the number of dealers servicing the market decreases the price paid by hedgers but lowers revenue for dealers, increasing the probability of a default. Restricting entry maximizes welfare when dealers' efficiency is high relative to their market power. A Central Counter-Party (CCP) offering novation tilts the trade-off toward more competition. Free-entry is optimal for all level of dealers' efficiency if they can constrain risk-taking by its members. In this model, dealers can choose CCP rules to restrict entry and increase their benefits. Moreover, dealers impose binding risk constraints to increase revenues at the expense of the hedgers. In other words, dealers can use risk controls to commit to a lower degree of competition. These theoretical results provide one rationalization of ongoing efforts by regulators globally to promote fair and risk-based access to CCPs.

JEL classification: G10, G18
Bank classification: Financial markets; Financial system regulation and policies; Financial stability

Résumé

Les auteurs modélisent le comportement des courtiers sur des marchés de dérivés de gré à gré dans lesquels un petit nombre de courtiers traitent avec un éventail de clients hétérogènes. La concurrence imparfaite et le risque de défaut (endogène) donnent lieu à un arbitrage bien connu entre concurrence et risque. Si l'on accroît le nombre de courtiers desservant le marché, on observe une baisse des prix pour les clients, mais aussi une diminution des revenus des courtiers, ce qui augmente la probabilité d'une défaillance. Des restrictions à l'entrée optimisent le bien-être tant que l'efficience des courtiers est élevée relativement à leur pouvoir de marché. La présence d'une contrepartie centrale assurant la novation fait pencher la balance en faveur de la concurrence. La libre entrée offre une situation optimale quel que soit le niveau d'efficience des courtiers si la contrepartie centrale est en mesure de limiter les risques que prennent ses membres. Dans le modèle, les courtiers peuvent influer sur les règles qu'applique la contrepartie centrale de manière à restreindre l'entrée et à augmenter leurs avantages. De plus, ils peuvent imposer des mécanismes contraignants de limitation des risques afin de grossir leurs revenus aux dépens des clients. Autrement dit, les dispositifs de contrôle des risques permettent aux courtiers de s'engager à réduire la concurrence entre eux. Ces résultats théoriques fournissent une justification aux efforts déployés actuellement dans le monde par les autorités de réglementation pour favoriser un accès aux contreparties centrales qui soit équitable et fondé sur les risques.

Classification JEL : G10, G18
Classification de la Banque : Marchés financiers; Réglementation et politiques relatives au système financier; Stabilité financière

1 Introduction

Over-the-counter (OTC) markets are decentralized markets where dealers offer intermediation services to investors. In practice, a small number of dealers provide customized intermediation services to a large number of diverse investors raising questions about the extent to which OTC dealers can exercise market power and increase profits. Could new entrants and greater competition among dealers improve welfare? On the other hand, OTC markets concentrate couterparty default risk in the hand of dealers and intensifying competition among dealers may increase dealers default risk, reducing welfare. Indeed, counterparty default risk played a key role in the propagation of the 2007-2009 crisis into OTC markets (Duffie, 2010a). In response, regulators are pursuing the increased use of Central Counter-parties (CCPs) to control default risk and mitigate the costs of default.[1]

We model an OTC market where a small number of dealers meet a continuum of risk-averse agents with an uncertain endowment (i.e., hedgers). Dealers are differentiated and have market power. For instance, hedgers' utility from trading can vary across dealers because of preferences, differentiated ancillary services, or because the costs of establishing a relationship vary across dealers. There is no information asymmetry. Dealers offer hedgers an homogenous swap contract that promises a fixed payment in exchange for their random endowment. Dealers have limited liability, face an idiosyncratic shock, and may default. They can transfer the endowment risk, but dealers' hedging strategies, and their exposures from other markets or business lines, introduce introduces idiosyncratic default risk. Hence a swap contract transfers the endowment risk to the dealer but does not eliminate risk for hedgers. Markets are incomplete since hedgers cannot insure against dealers' default. The reduction in risk for hedgers depends on the scale of dealers' idiosyncratic shocks. We interpret the reduction of risk as a measure of dealers' efficiency to transfer risk. Our analysis leads to the following four theoretical contributions.

Our first contribution is to characterize the trade-off between competition and default risk in the context of an OTC market.[2] Increasing the number of dealers improves welfare via two channels and reduces welfare via a third channel. A higher number of dealers intensifies competition, lowering the price paid by hedgers. More dealers also implies that the pool of dealers is more diverse, lowering transaction costs for hedgers. On the other hand, a lower

[1] The leaders of the G-20 group of nations stated in September 2009 that all standardized OTC derivatives contracts should be cleared through CCPs.

[2] A vast literature assesses the interaction between competition and stability in the context of banking. Competition may reduce or promote stability. See Keeley (1990), Boyd and De Nicolò (2005), Martinez Miera and Repullo (2010) and the review in Vives (2010). However, our results show that market infrastructure, such as CCPs, introduces an important difference between market-based intermediation and banking.

price reduces revenue and increases the probability of a dealer's default. This reduces welfare. The socially optimal level of entry involves trading-off the benefits of a lower price and of lower transaction costs against the consequences of a higher default probability. Depending on model parameters, restrictions to entry may be optimal to balance the conflicting effects of competition. The scale of dealers' default risk plays a key role. Hedgers prefer to restrict entry if dealers reduce risk efficiently. In this case, restricting entry at a relatively high level reaps much of the gain from competition but with a relatively small increase in default risk. In this case, and all else constant, raising dealers' market power increases the gains from competition and raises the restricted level of entry.

Idiosyncratic default justifies the introduction of novation to reap the gains from diversification (Koeppl and Monnet, 2010). Novation is introduced via a CCP that stands between hedgers and dealers, becomes a party to every trade, and absorbs counterparty risk in the OTC market.[3] As in Koeppl and Monnet (2010) novation is not a guarantee. Instead, a CCP can only pool the resources available from its members to fullfill its obligations. We analyze the effect of novation on the competitive environment, the endogenous (joint) distribution of dealers' default, and the optimal level of entry. We also consider the effect of a CCP that offers novation, set membership requirements, and set limit on the probability of losses due to dealers' default.[4]

In a second contribution, we find that the introduction of novation tilts the social optimum toward a greater level of competition. This arises because the diversification benefits affect the trade-off associated with increasing the number of dealers. Novation reduces the welfare (default) costs associated with higher competition. In fact, free-entry becomes optimal for a range of dealers' efficiency where restrictions to entry were previously optimal. Novation raises the efficiency threshold needed to justify restrictions to entry. This argument can be extended and applied to other aspects of CCPs that reduce the (marginal) welfare costs of default associated with new entrants (e.g., such as the collateralizing of dealers' obligations).

In a third contribution, we show that free-entry is socially optimal if the CCP has the ability to directly limit dealers' risk. The CCP chooses a low level of risk when dealers are more efficient, trading-off the costs and benefits of lower default probability. In equilibrium,

[3] CCPs have long played an important role in markets for exchange-traded securities but have been absent from most OTC derivative markets. Counterparty credit risk is more salient in derivative markets since contractual relationships between counterparties are long-lasting and, therefore, the interaction between controlling default risk and competition is especially relevant.

[4] CCPs constrain and mitigate counterparty risk via different means, including margins, haircuts, default funds, mutualisation, multilateral netting and others. Rules and institutions designed to mitigate the effect of a dealer's default on the market are the focus of this paper but are not the only barriers to entry. For instance, the required technology to offer intermediation services requires substantial fixed investments.

dealers' revenues must be consistent with the default probability imposed by the CCP. Hence, risk controls constrain the dealers' ability to reduce the price excessively. Still, free-entry eliminates any price mark-ups due to market power. In addition, free-entry increases the diversity of the dealers' pool reducing transaction costs for hedgers. Hedgers reap the benefits from both greater competition and lower default risk.

Finally, we show that a CCP that maximizes its members' profits uses strict membership rules to achieve a level of entry lower than what is socially optimal. Membership requirements can act as a barrier to entry. Realistically, the number of dealers servicing a given market may be determined from history, whereas incumbents may not be forced to withdraw, and the CCP cannot reduce the number of operating dealers. In this case, the CCP chooses to limit new entry and to implement stringent risk controls on its members' trading activities. This is perhaps surprising since, individually, dealers with limited liability prefer to avoid restrictions on their ability to take risk. But CCP members as a group prefer binding risk-controls. In effect, these binding risk-controls act as a commitment device that reduces the effect of competition and leads to more favorable price in equilibrium.[5]

Barriers to entry into a CCP have practical relevance and the potential conflict between welfare and the incentives of the clearinghouse's members provide one explanation as to why regulators have made open membership to CCPs a central feature of the new regulatory architecture.[6] National regulators have been implementing these principles and some CCPs have eased some of their membership requirements in response. For instance, the US Commodity Futures Trading Commission has imposed rules capping minimum capital requirements at $50 million. London Clearing House's LCH.Clearnet's SwapClear facility, which until 2012 required that members have $5 billion in equity capital and a $1 trillion derivatives book, now require $50 million in capital and has no requirement on book size.[7]

To our knowledge, this paper is the first theoretical analysis of the interactions between CCP rules, competition and counterparty risk in an OTC market. The relevance of this

[5]Analog results exist in the literature on the industrial organization of firms. Dixit (1980) and Kreps and Scheinkman (1983) find that firms can strategically choose ex-ante small levels of productive capacity in order to decrease competition ex-post.

[6]See also the discussion in Fontaine, Perez Saiz, and Slive (2012). The Financial Stability Board (FSB) identified fair and open access to CCPs as one of four safeguards needed to establish a safe environment for clearing OTC derivatives. The new Principles for Financial Markets Infrastructures (FMI) states that: "...participation requirements should be justified in terms of the safety and efficiency of the FMI and the markets it serves,[...] an FMI should endeavour to set requirements that have the least-restrictive impact on access that circumstances permit." The Committee on Payment and Settlement Systems (CPSS) together with the International Organization of Securities Commissions (IOSCO) made fair, open and risk-based access to CCPs one of its Principles for Financial Markets Infrastructures (CPSS-IOSCO, 2012).

[7]See Lazarow (2011) for details on the governance structure of some large CCPs.

issue, however, has been highlighted by Duffie (2010b) and Pirrong (2011).[8] The industrial organization of OTC markets has been studied in the microstructure literature (Stoll 1978, Kyle 1985 and Kyle 1989) but dealers' potential default are typically neglected. A recent literature justifies the introduction of a CCP. Consistent with more general results in Koeppl and Monnet (2010), we find that novation improves welfare. They study the benefits of novation and mutualization with a continuum of dealers. We study the effect of novation on the trade-off underlying the optimal level of entry by dealers. Acharya and Bisin (2011) emphasize that with asymmetric information, two parties to a trade cannot commit to limit risky trading and maintain their default risk in the future – the counter-party risk externality. Thompson (2010) also analyzes the moral hazard problem associated with the re-investment of premium by insurance sellers. Carapella and Mills (2012) show that clearing transactions through a CCP reduces the extent to which debt or equity securities are information-sensitive. However, Thomson and Stephens (2012) show that when transaction costs are low, asymmetric information on the risk being transferred and the quality of the counterparties can exist in equilibrium.

Another strand of the nascent literature discusses the effect of different CCP configurations on counterparty risk. Haene and Sturm (2009) model the optimal balance between default fund and margin contributions in capitalizing a CCP. Rausser, Balson, and Stevens (2010) suggest that a CCP is not capable of internalizing all the benefits it creates and therefore should be run as a public-private partnership. Duffie and Zhu (2011) consider the effect of inefficient netting by CCPs on the counterparty risk of members. Renault (2010) models the optimal number of CCPs for a market. Finally, there is some empirical evidence that membership restrictions may not address the most salient sources of risk. Jones and Perignon (2012) argue that a substantial share of the systemic risk faced by the CME clearinghouse is due to proprietary trading on the part of its members. Still, the empirical literature remains thin, in large part due to limited access to data.

2 Model

We consider a trading environment similar in spirit to that of Duffie, Gârleanu, and Pedersen (2005) and Lagos and Rocheteau (2009) where hedgers and dealers (described below) meet

[8] Duffie (2010b), comment 4: "Large [clearing members], which tend to be dealers, may have a conflict of interest in claiming that smaller CMs are unsafe merely because of being small. [...] Allowing smaller CMs introduces more competition between large dealers and smaller market participants." Pirrong (2011), page 28: "One way to exercise this market power for the benefit of members is to limit membership to an inefficiently small number through the imposition of unduly restrictive membership requirements. Therefore, it cannot be ruled out that CCPs will utilize membership requirements for strategic, competitive purposes."

to trade an OTC contract. Of course, our focus and the market environment differ. First, we study how risk and competition change as the number of dealers changes. Hence, we abstract from search frictions, where results rely on a law of large numbers applied to a continuum of homogenous dealers. One implication is that dealers' market power does not arise from the costs that hedgers face to search for an alternate dealer. Instead, we rely on horizontal differentiation across dealers where dealers provide different levels of ancillary services which are reflected by heterogenous trading costs borne by hedgers. Second, dealers carry an inventory of residual risks and, eventually, may find themselves unable to fulfill their side of the contract. Markets are incomplete since hedgers cannot insure against dealers' default. This risk plays a key role in our analysis. We interpret a low level of residual risk (low variance) as a proxy for the ability of dealers to transfer risk efficiently. Dealers' default is endogenous and we derive the joint distribution of defaults. Finally, we introduce a CCP that novates all trades, restricts entry via membership rules, and controls the risk arising from its members' trading activities. Clearly, real-world CCPs also control risk via, for instance, margin calls, a default fund and the mutualization of losses. All of these additional features could add to the trade-off between competition and default risk but would not change the thrust of our results.

2.1 Over-the-Counter Market

There are two types of agents : a unit measure of non-atomistic risk-averse agents, called hedgers, and a small number, $n \in \mathbb{N}$, of specialized agents, called dealers. There is one asset and one numéraire good. Each hedger owns m unit of the numéraire and one unit of the asset. The asset produces a random endowment of the consumption good, $\tilde{e}$, with

$$\widetilde{e} = \begin{cases} e > 0 & \text{with probability} \quad q \\ 0 & \text{with probability} \quad 1 - q. \end{cases} \tag{1}$$

Following, e.g., Lagos and Rocheteau (2009) and Koeppl and Monnet (2010), hedgers have quasi-linear preferences over the consumption good and the numéraire,[9]

$$u_h(\widetilde{e}, m) = \log(1 + \widetilde{e}) + m.$$

[9] As in the extant literature, studying an economy with log-linear preferences yields greater insight in the underlying mechanism due to its analytical tractability. However, it implies that wealth effect from hedging the risk are absent.

The asset is not tradable – hedgers are exposed to a common endowment shock.[10] Dealers are specialized agents that offer a contingent contract s with payoffs $\widetilde{s}$ correlated with the endowment shock, $\widetilde{e}$. Dealers have linear preferences over the numéraire but with limited liability. The exchange between dealers and hedgers is an over-the-counter (OTC) financial market. All dealers offer the same contract but they also offer differentiated ancillary services to their clients, for example, cash accounts, accounting services and other customized services. Alternatively, establishing a relationship between a hedger and a dealer may be costly or dealers may have varying knowledge about different hedgers. This can also be seen as an approximation to an environment with search frictions.

Specifically, hedger i incurs a costs of $c_{i,j}$, in terms of the numéraire, to trade with dealer j. These costs can be seen as a transaction costs, or costs arising from participating in the market, in which case, they are incurred directly by the hedgers. They can also be seen as paid upfront to cover for the costs incurred by dealers to offer customized services. We use the circular road model of Salop (1979) to model price competition with horizontal differentiation in the OTC market. The Salop circle is an analytical device where heterogeneity is captured by the location of hedgers and dealers on the circular road (See Figure 1). In this representation of the market, hedgers are uniformly dispersed around a circle with length $H = 1$ and the n dealers are equally spaced along the circle. Horizontal differentiation is captured by varying trading costs with the distance between hedgers and dealers. Hedger i's costs of trading with dealer j located at a distance of $d_{i,j}$ on the road is given by $c_{i,j} = d_{i,j} \cdot t$ with $t > 0$.[11]

2.2 Dealers' Intermediation

The timing of events is the following. First, hedgers trade a swap contract with dealers at a given price, p_j, and pay the transaction cost $c_{i,j}$. Second, all shocks are realized, the contract's payoff is determined and some dealers may default.[12] The contingent contract is a fixed-for-floating swap. Both legs of the transaction occur in the second period. The terms of the contract specify the exchange of a fixed quantity, known today, against an uncertain quantity. It is natural to set the variable payment to be $\widetilde{e}$ and the fixed payment to the mean

[10] Hedgers have the same exposure and there is no gain from trading between themselves. We abstract from idiosyncratic shocks to hedgers. The common shock is the hedgers' systemic exposure remaining after diversification benefits have been exhausted.

[11] We exclude the case where hedgers can trade with each others. Dealers are truly specialist of offer services that hedgers cannot replicate easily. We will consider entry of additional, competing, dealers below.

[12] Hedgers do not default in equilibrium since they only promise to deliver their random endowment. Moreover, we assume that m is large enough, so that $m > p$ and there is no wealth effect, and that hedgers can commit to pay the contract's price. Any monitoring costs by dealers can be included in the trading costs t without loss of generality.

of $\widetilde{e}$, $\overline{e} = qe$. The net payoff to hedgers from this contract is $\widetilde{s} = \overline{e} - \widetilde{e}$. The distribution of $\widetilde{s}$ is given by:

$$\widetilde{s} = \begin{cases} (q-1)e & \text{with probability} \quad q \\ qe & \text{with probability} \quad 1-q. \end{cases} \tag{2}$$

Each Hedger makes an additional transfer of the numéraire to the dealer which is the price, p, of the contract. The price is paid upon settlement of the contract in the second period.[13] In equilibrium, hedgers are willing to pay a non-negative price to buy this contract from dealers, to receive a fixed payment and eliminate uncertainty. Risk-neutral investors would be indifferent to take either side of this swap with $p = 0$.

Dealers incur losses from the swap contract whenever $\widetilde{s} > p$. For simplicity, we do not endow dealers with capital to cover for these losses. Instead, dealer j has access to different trading strategies in other markets where he can trade to hedge this risk. In our model, a dealer is not a bank or an insurance company but a trading specialist in financial markets.[14] In particular, dealer j can establish different trading strategies, $\widetilde{h}_j$ to hedge the risk. The net payoff from the combination of a swap contract and the hedge position is $\widetilde{\Delta_j} = \widetilde{s} - \widetilde{h}_j$ and is idiosyncratic to each dealer. Its distribution is given by:

$$\widetilde{\Delta_j} = \begin{cases} (q-1)\sigma e & \text{with probability} \quad q \\ q\sigma e & \text{with probability} \quad 1-q, \end{cases}$$

and $\sigma \geq 0$. The shocks $\widetilde{e}$ and $\widetilde{\Delta_j}$ are uncorrelated and we fix $P(\widetilde{\Delta_j} = (q-1)\sigma e) = P(\widetilde{s} = (q-1)e) = q$ for parsimony. This parametrization implies that $E[\widetilde{\Delta_j}] = 0$ and, more importantly, that the relationship between the variance of the swap and of the hedged positions is given by:

$$Var[\widetilde{\Delta_j}] = \sigma^2 Var[\widetilde{s}].$$

The residual risk introduces the possibility that a dealer may default on its obligations. The hedge contract mirrors the payoff from the swap contract but with a gap $\widetilde{\Delta}_j$ that represents residual risk, or basis risk. This differs from the common assumption in the

[13] The fixed payment in a typical swap or futures transaction is set such that there is no initial exchange of money. This is purely a matter of notional convention. We could define the fixed payment as $\overline{e} - p$ with no loss of generality. Our approach highlights the role of risk on the exchanges between hedgers and dealers.

[14] We refrain from including capital for two reasons. First, our focus is on varying the number of competing dealers in the market. This raises important general equilibrium questions about how much capital is available in the economy to cover default risk in the OTC market and to allocate it among dealers (incumbents and new entrants) as we vary the number of dealers n. Second, although capital could be modeled by adding a constant payoff to the hedging strategies, this addition would not alter the main results below.

literature on OTC markets where a dealer has access to a frictionless markets to trade its risk away entirely (see e.g., Duffie, Gârleanu, and Pedersen 2005). The gap can represent risk that simply cannot be traded away by dealers, for instance, if the best hedging strategy is not perfectly correlated with the payoff from the swap contract. The deviations may also correspond to gains or losses incurred by dealer j from trading strategies in other markets or business lines. The parameter σ controls the variance reduction that results from hedging activities. It is a deep parameter that represents how efficiently dealers can transfer the risk borne by hedgers. In particular, the hedge is perfect if $\sigma = 0$. In this case, dealers never default, since $\widetilde{\Delta}_j = 0$ with probability one, and hedgers face no risk in equilibrium. On the other end, $\sigma = 1$ corresponds to the case where dealers do not reduce variance. Importantly, the shocks $\widetilde{\Delta}_j$ are uncorrelated with hedgers' endowment shocks. Therefore, trading away the endowment risk may be beneficial to hedgers even if σ is close to one, and the reduction of risk is low, since the hedging strategy offer diversification benefits.[15]

2.3 Dealers' Default Probability

Dealer j defaults whenever the price obtained from entering the swap contract, p_j, is less than the loss incurred from its partially hedged position.[16] The probability that dealer j will default as a function of price, $D_j(p) \equiv \Pr\left(p_j < \widetilde{\Delta}_j\right)$, is given by:

$$\begin{aligned}
\text{Low-Risk Region:} \quad & D_j(p) = 0 && \text{if} \quad \overline{p} \leq p_j \\
\text{High-Risk Region:} \quad & D_j(p) = 1 - q && \text{if} \quad \underline{p} \leq p_j < \overline{p} \\
\text{Default Region:} \quad & D_j(p) = 1 && \text{if} \quad p_j < \underline{p},
\end{aligned} \tag{3}$$

where, the thresholds $\overline{p} \equiv \sigma q e$ and $\underline{p} \equiv \sigma(q-1)e$ define three distinct regions. Nonetheless, the low-risk and high-risk regions are the only economically relevant cases. The default region, where dealers always default is irrelevant in equilibrium since it is never optimal for dealers. The hedgers's demand is zero in this case (see below). Using a simple distribution for the shock $\widetilde{\Delta}$ allows us to derive clear analytical results related to the joint probability distribution of events where some, many or all dealers default. This is crucial when studying losses within the CCP.[17]

[15] The distribution of $\widetilde{\Delta}_j$ is exogenous and homogenous across dealers for parsimony. Thompson (2010) analyzes the moral hazard that arises when the hedging or re-investment strategies of the dealers are endogenous in the related context of financial insurance contracts for the purpose of credit risk transfer.

[16] One important consideration is whether hedging by dealers is consistent with their preferences. We show in Appendix B that, for any price p_j, the hedgers' demand is higher when dealers hedge their risk, even if only partially.

[17] It may appear simplistic that the probability of default is zero in the low risk region. Adding one point or more to the support of $\widetilde{\Delta}_j$ would lead to one or more risk regions but the extra cases only complicate

2.4 Novation by a CCP

Dealer's default is an idiosyncratic risk and, therefore, re-allocating the losses across hedgers can improve welfare (Koeppl and Monnet, 2010). Novation by a CCP is one mechanism to implement this re-allocation and reach an efficient equilibrium when $n \geq 2$. With novation, every contract between a hedger and a dealer is superseded by two contracts: one between the hedger and the CCP, and one between the CCP and the dealer.

In case of a default, the CCP fulfills its obligations as follows. Consider the event where $k < n$ of the n dealers face a low realization of their hedge strategy and default on their contracts with the CCP. Then, the CCP can use the proceeds from contracts with the $n-k$ surviving dealers to fulfill a pro-rata share $(n-k)/k$ of its obligations toward hedgers. In return, hedgers pay the same pro-rata share of the agreed price. Then, the net quantity of the consumption good obtained by one hedger when k of the n dealers default, $F_{\tilde{e}}(k)$, is given by:

$$F_{\tilde{e}}(\widetilde{k}) = 1 + \widetilde{e} + \frac{n - \widetilde{k}}{n} (qe - \widetilde{e}) \, . \tag{4}$$

The hedger entering a swap contract has utility given by:

$$\log\left(F_{\tilde{e}}(\widetilde{k})\right) - \frac{n - \widetilde{k}}{n} p_j - d_j t, \tag{5}$$

if a CCP operates in the market and the expected utility is given by:

$$E[u_h^{CCP}(p_j; n)] = qE\left[\log(F_e(k)\right] + (1-q)E\left[\log(F_0(k)\right] - E\left[\frac{n-k}{n} p_j\right] - d_j t. \tag{6}$$

For instance, if all dealers default then $F_{\tilde{e}}(n) = 1 + \tilde{e}$ and if no dealer defaults then $F_{\tilde{e}}(0) = 1 + \bar{e}$. These cases correspond to the hedger's payoffs in the absence of a CCP. We can derive the the joint distribution of dealers' default. Dealer j defaults when $p_j < \widetilde{\Delta}_j$. The payoffs $\widetilde{\Delta}_j$ are independent of the endowment $\widetilde{e}$, and they have independent bernoulli distributions across dealers. Therefore, the random variable $\widetilde{k}$, the number of default among n dealers, follows a binomial distribution with parameters $D_j(p)$ and n.[18]

Since it improves welfare, the CCP's emergence can be seen as endogenous or following

the analysis and lengthen the propositions. Similarly, extending the length of the support, e, without bound would eliminate regions where $D_j(p) = 0$ and where $D_j(p) = 1$. However, much of the tractability would be lost without changing the thrust of the paper.

[18] A random variable $\widetilde{x}$ with a Binomial distribution with parameters p and n has a mean np and variance $np(1-p)$.

a mandate by the regulatory authority to clear all trades.[19] Moreover, as in Koeppl and Monnet (2010), novation is not a guarantee. The CCP can only re-distribute the resources from dealers that did not fail and uses them to fulfil its obligations to hedgers. In returns, hedgers only pay a pro-rata share of the price. This guarantees that the CCP can fulfill its obligation to surviving dealers but, yet, that no resources remains idle within the CCP and all the benefits of diversification are re-distributed outside of the CCP. Introducing a fixed default fund, or other exogenous resources, does change the nature of the trade-off between risk and competition.

2.5 Risk Controls and Membership Rules

The CCP can implement membership rules to restrict entry and select the number of dealers that become members. Hedgers are not clearing members but, instead, hedgers use indirect clearing and the CCP still novates all the hedgers' trades. A CCP can also imposes ex-ante rules on the level of risk that each member can carry. Specifically, it can set the number of members n freely, as well as the default constraint parameter α such that,

$$\Pr\left(p_j < \widetilde{\Delta}_j\right) \leq \alpha. \tag{7}$$

Note that real-world CCPs do not impose a probability constraint directly. However, several of the practices that CCPs put in place are used to minimize the probability of a large loss following a member's default. For instance, initial margins, variation margins and haircuts on collateral secure the CCP's claim on each dealers and directly reduces the risk of a loss following default. Members' contribution to a default fund, mutualization of losses, and other institutional features of a CCP also reduce the probability of losses. Similarly, real-world CCPs do not fix the number of dealers directly but, for instance, set capital or fixed investment requirements that have the effect of limiting further entry.

3 Equilibrium in the OTC Market

This section describes the equilibrium when the OTC market is unregulated and there is no CCP. We first consider the case where n is low, and dealers are monopolist. In this case, the equilibrium does not "cover" the market: some hedgers will not trade since the sum of the price and the trading costs is too high. We label this case the *uncovered equilibrium.* With more dealers in the market, the equilibrium eventually covers the market and there is

[19]The G20 have agreed that markets for OTC derivatives should be centrally cleared. We analyze the case with only one CCP operation in a given market. Economies of scale and network effects present in clearing – but not modeled in this paper – push towards a single CCP for each market.

imperfect competition in the market. We label this case the *covered equilibrium*. The latter is the economically relevant case for many real-world OTC markets. In this case, a low level of competition yields a high price and the equilibrium lies in the low-risk region while more competition leads to lower price and the equilibrium eventually reaches the high-risk region.

3.1 Hedgers' Demand

3.1.1 Demand in a Covered Equilibrium

The covered equilibrium corresponds to the area of the parameter space in Salop (1979) where dealers are competing with each other. As in Duffie et al. (2005), dealers are ex-ante homogenous and it is natural to consider an equilibrium where a unique price prevails across the OTC market. Hence, we follow Salop (1979) and focus on the symmetric equilibrium. To derive hedgers' demand, compare the hedger's utility obtained from trading with different dealers. The utility of a hedger from trading with the nearest dealer j located at a distance d_j is $\log(1+\bar{e})+m-p_j-d_jt$ if the dealer does not default, and $\log(1+\widetilde{e})+m-d_jt$ if the dealers default. Then, the expected utility is given by:

$$E[u_h(p_j)] = \begin{cases} \Pi + q\log(1+e) - p_j - d_jt & \text{if} \quad \overline{p} \leq p_j \\ q(\Pi + \log(1+e) - p_j) - d_jt & \text{if} \quad \underline{p} \leq p_j < \overline{p} \\ q\log(1+e) - d_jt & \text{if} \quad p_j < \underline{p} \end{cases}, \tag{8}$$

where Π is the surplus derived from trading,

$$\Pi = \log(1+\bar{e}) - q\log(1+e) > 0, \tag{9}$$

since hedgers are risk-averse with respect to the consumption good. In a symmetric equilibrium, all dealers but dealer j set their price to $\widehat{p}$. Consider the second nearest dealer, $j-1$ say, located at distance $\frac{1}{n}-d_j$ on the other side of the hedger. The utility from trading with dealer $j-1$ is given by:

$$E[u_h(\hat{p})] = \begin{cases} \Pi + q\log(1+e) - \hat{p} - (\frac{1}{n}-d_j)t & \text{if} \quad \overline{p} \leq \hat{p} \\ q(\Pi + \log(1+e) - \hat{p}) - (\frac{1}{n}-d_j)t & \text{if} \quad \underline{p} \leq \hat{p} < \overline{p} \\ q\log(1+e) - (\frac{1}{n}-d_j)t & \text{if} \quad \hat{p} < \underline{p} \end{cases}, \tag{10}$$

and, therefore, the hedger will prefer to trade with dealer j if

$$E[u_h(p_j)] \geq E[u_h(\hat{p})] \Leftrightarrow \begin{cases} \frac{-p_j+\hat{p}+\frac{1}{n}t}{2t} \geq d_j & \text{if} \quad \overline{p} \leq p_j, \hat{p} \\ \frac{-qp_j+q\hat{p}+\frac{1}{n}t}{2t} \geq d_j & \text{if} \quad p_j, \hat{p} \in [\underline{p}, \overline{p}) \\ \frac{1}{2n} \geq d_j & \text{if} \quad p_j, \hat{p} < \underline{p} \end{cases} . \tag{11}$$

We obtain a similar condition for those hedgers between dealer j and $j+1$. Then the demand schedule for dealer j is given by,

$$y_c(p_j, \widehat{p}) = \begin{cases} H\frac{-p_j+\widehat{p}+\frac{1}{n}t}{t} & \text{if} \quad \overline{p} \leq p_j, \hat{p} \\ H\frac{-qp_j+q\widehat{p}+\frac{1}{n}t}{t} & \text{if} \quad p_j, \hat{p} \in [\underline{p}, \overline{p}) \\ 0 & \text{if} \quad p_j, \hat{p} < \underline{p}, \end{cases} \tag{12}$$

where $y_c(p_j, \widehat{p})$ is the quantity of contracts sold by dealer j as function of its own price, p_j, and the price of other dealers, $\widehat{p}$.[20] This results confirms the irrelevance of the case $\hat{p} < \underline{p}$, which corresponds to the Default region above, since dealers face zero demand in this case.

3.1.2 Demand in the Uncovered Equilibrium

A hedger does not trade with any dealer if its expected utility is less than its reservation value,

$$R = \log(1+e)q + \log(1)(1-q) + m = q\log(1+e) + m. \tag{13}$$

Comparing Equation 8 and Equations 13, shows that some hedgers choose not to trade if the prices offered by dealers or the trading costs, t, are large enough. In this case, each dealer is a local monopoly, corresponding to the monopolistic area of the parameter space in Salop (1979). The demand schedule in the uncovered case is

$$y_u(p_j) = \begin{cases} \frac{2H}{t}\max(\Pi - p_j, 0) & \text{if} \quad \overline{p} \leq p_j \\ q\frac{2H}{t}\max(\Pi - p_j, 0) & \text{if} \quad p_j \in [\underline{p}, \overline{p}) \\ 0 & \text{if} \quad p_j < \underline{p}, \end{cases} \tag{14}$$

[20] Strictly speaking, the demand schedule also exists and is well-defined for cases where the prices of two neighboring dealers are located on each side of $\overline{p}$. We do not present the results here to economize on space since we focus on symmetric equilibrium prices are identical across dealers and any two prices always lie in the same region.

3.1.3 Social Welfare

The total expected utility of hedgers trading a swap contract is given by:

$$E[U_h] = 2n \int_0^{d^*} E[u_{h_i}(p^*)]di \tag{15}$$

where the integral is over all hedgers on one side of a given dealer, up to distance, d^*, where the marginal hedger is located. This distance in given in equilibrium. Multiplying by two includes hedgers on both sides of this dealer, and multiplying by n homogenous dealers gives the total expected utility of hedgers in equilibrium.

3.2 Equilibrium without a CCP

All dealers choose a price simultaneously taking into account the hedgers' demand. The expected utility of individual dealers, is defined by:

$$E[u_d] = E[\max(y(p_j - \widetilde{\Delta}_j), 0)], \tag{16}$$

where y is the quantity of contract sold to hedgers.[21] The following Proposition characterizes the equilibrium price, as well as the equilibrium expected utility of hedgers and dealers. We normalize the mass of hedgers to one, $H = 1$, and consider cases where dealers are sufficiently efficient to reduce risk for hedgers, $0 \leq \sigma \leq \bar{\sigma}$. In this case, the sequence of equilibrium type as n increases is the following: *(i)* monopoly in the low-risk region, *(ii)* imperfect competition in the low-risk region and, *(iii)* imperfect competition in the high-risk region.[22] If the scale of the dealers' risk is too high, $\sigma > \bar{\sigma}$, then the equilibrium lies in the high-risk region for all n because the hedgers' surplus from trading is not sufficiently high. All proofs can be found in the Appendix for the more general case where $H > 0$ and $\sigma \geq 0$.

Proposition 1 ***Equilibrium in the OTC Market without CCP***
Fix $H = 1$ and consider cases where dealers' efficiency ranges in $0 \leq \sigma \leq \bar{\sigma} = \frac{\Pi}{2qe}$. The subscripts u and c designate the uncovered equilibrium and the covered equilibrium, respectively. The superscripts lr and hr designate the low risk and high risk regions, respectively.

Low-Risk Uncovered Equilibrium :

For any $1 \leq n \leq n_u^{lr}$, where $n_u^{lr} = \frac{t}{\Pi}$, a symmetric equilibrium exists, is unique, and

[21] Dealers' utility is linear in the numéraire. Implicitly, dealers have access to a spot market where they can exchange any surplus $p_j - \widetilde{\Delta}_j > 0$ in the form of the numéraire. Alternatively, dealers' utility is linear in the commodity and the numéraire.

[22] When $\sigma < \bar{\sigma}$ the efficiency of dealers and the price that hedgers are willing to pay is to low so that the equilibrium never lies in the low-risk region.

lies in the low-risk uncovered region (dealers are monopolies). This equilibrium is such that,

$$p_u^{lr} = \frac{\Pi}{2}$$
$$E[U_{h,u}^{lr}] = n\frac{\Pi}{t}\left(\frac{\Pi}{4} + q\log(1+e)\right).$$

Low-Risk Covered Equilibrium :

For any $n_c^{lr} \leq n \leq \bar{n}_c^{lr}$, where $n_c^{lr} = \frac{2t}{\Pi}$, $\bar{n}_c^{lr} = \frac{t}{\bar{p}}$ and $n_c^{lr} \leq \bar{n}_c^{lr}$, a symmetric equilibrium exists, is unique, and lies in the low-risk covered region (dealers compete). This equilibrium is such that,

$$p_c^{lr} = \frac{t}{n}$$
$$E[U_{h,c}^{lr}] = \left(\log(1+qe) - \frac{5t}{4n}\right),$$

and dealers's expected profits decreases with n (see Appendix).

High-Risk Covered Equilibrium :

For any $n > n_c^{hr}$, where

$$n_c^{hr} = \max\left(\bar{n}_c^{lr}, \frac{t(1+H)}{q(\Pi - (q-1)\sigma e)}\right),$$

a symmetric equilibrium exists, is unique, and lies in the high-risk covered region (dealers compete). This equilibrium is such that,

$$p_c^{hr} = \frac{t}{qn} + (q-1)\sigma e$$
$$E[U_{h,c}^{hr}] = \left(q(\Pi - (q-1)\sigma e) + q\log(1+e) - \frac{5t}{4n}\right),$$

and dealers's expected profits decreases with n (see Appendix).

The monopolistic equilibrium exists only if $n_u^{lr} = t/\Pi \geq 1$. That is, if the costs of differentiation, measured by t, is larger than the surplus from a trade, Π. Then, some hedgers do not to trade and the number of dealers n does not affect the equilibrium. Eventually, as the number of dealers increase, or if differentiation is low enough, the equilibrium is covered and there is imperfect competition between adjacent dealers.[23] Then, each dealer trades with all hedgers located on either of the dealer side but within a maximum distance of $1/2n$. The quantity traded by every dealer is $y^* = H/n$.

The equilibrium price balances market power and default risk. When the number of dealers is relatively low, and the equilibrium still lies in the low-risk region, the contract's price exceeds the expected payoff ($p_u^{lr} \geq p_c^{lr} > E[\tilde{s}] = 0$). However, as n rises beyond n_c^{hr}, and we reach the high-risk region, the equilibrium price reflects the probability that a dealer may default. Eventually, as competition increases, default risk dominates market power and the contract price declines below the expected payoff to compensate hedgers for losses due to default. The threshold, n_c^{hr}, between the low-risk and the high-risk regions compares the level of differentiation t and the dealers' risk $\sigma q e$. If dealers are efficient (σ is small) or if differentiation is high, then a larger increase in the number of dealers competing in the market is required before the equilibrium reaches the high-risk region ($n_c^{hr} \gg n_c^{lr}$).

3.3 Equilibrium with Novation

One way to see the benefits of novation is to assess the diversification benefits for the (risk-averse) hedgers directly. Using the binomial distribution of dealers' default, a hedger's expected utility in the high-risk region is given by:

$$E[u_h^{CCP}(p_j; n)] = v_h(n) - qp_j - d_j t, \tag{17}$$

where $qp_j = E[\frac{n-k}{n} p_j]$. The first term, $v_h(n)$, corresponds to the expected utility from the consumption goods, given by:

$$\begin{aligned} v_h(n) &\equiv E[\log(F_{\tilde{e}}(k))] \\ &= qE\left[\log(F_e(k))\right] + (1-q)E\left[\log(F_0(k))\right]. \end{aligned} \tag{18}$$

It follows that for a given price, the expected utility, increases with n due to the benefits of diversification. The effect of n works through its effect on $F_{\tilde{e}}(\widetilde{k})$. The means of $F_e(k)$ and

[23] The demand schedule for all values of p_j has a kink at the point between the uncovered and the covered cases. This kink reflects the effect of competition on the price elasticity of hedgers (compare the slope of the hedgers' demand between the uncovered case and the covered case). The transition between an uncovered equilibrium toward a covered equilibrium in the low-risk region, as we vary n from n_u^{lr} to n_c^{lr} can be made smooth by proper choice of the parameter values. For instance $n_c^{lr} - n_u^{lr} \leq 1 \Leftrightarrow H \leq \Pi$. Similarly, parameters values can be chosen such that the transition between the covered equilibrium is smooth.

$F_0(k)$ do not change with n,

$$E(F_e(k)) = 1 + qe + e(1-q)^2$$
$$E(F_0(k)) = 1 + qe - eq(1-q),$$

but their variances decrease strictly with n,

$$Var(F_e(k)) = e^2 \frac{(1-q)q}{n}(1-q)^2$$
$$Var(F_0(k)) = e^2 \frac{(1-q)q}{n}q^2,$$

and reach zero in the limit. Therefore, the distribution of consumption when the number of dealers is n and the distribution when the number of dealers is $n+1$ differ by a mean-preserving spread where the dispersion of the distribution decreases with n. Then, from standard results, $v_h(n)$ and is increasing in n because the log function is concave (Rothschild and Stiglitz 1970 and, e.g., the review in Levy 1992). Next, define Π^{CCP} as the surplus from trading when the CCP provides diversification benefits,

$$\Pi^{CCP}(n) \equiv \frac{v_h(n) - q\log(1+e)}{q}.$$

This definition is an analog to the notation previously introduced for the hedger's surplus. The difference between $\Pi^{CCP}(n)$ and Π measures the benefits from diversification. Indeed, there is no diversification benefits when $n = 1$ and we have that, $\Pi^{CCP}(1) = \Pi$. Moreover, the benefits from diversification increases with the number of members since $\Pi^{CCP}(n) > \Pi$ for all $n > 1$ ($v_h(n)$ increases with n). In turns, hedgers, expected utility increases with n,

$$E[u_h^{CCP}(p_j;n)] = \begin{cases} \Pi + \log(1+e) - p_j - d_j t & if \quad p_j \geq \overline{p} \\ q(\Pi^{CCP}(n) + \log(1+e) - p_j) - d_j t & if \quad p_j \in [\underline{p}, \overline{p}) \end{cases}, \tag{19}$$

where, again, the case $p_j < \underline{p}$ is irrelevant. For a given price, the hedger's expected utility increases with n in the high-risk region but there is no effect in the low-risk region since default is absent. Of course, the net effect also depends on the equilibrium price. The following proposition builds on Proposition 1 and verifies that expected utility also increases in Equilibrium. we focus on the effect of novation. The effect of entry restrictions and that of risk controls will be analyzed in the following section.

Proposition 2 ***Equilibrium in the OTC Market with Novation***

Fix $H = 1$ and consider cases where $n > 1$ and $0 \leq \sigma \leq \bar{\sigma} = \frac{\Pi}{2qe}$. The subscripts u *and c designate the uncovered equilibrium and the covered equilibrium, respectively. The superscripts lr and hr designate the low risk and high risk regions, respectively.*

The introduction of novation,

- *does not affect the equilibrium price given in Proposition 1 (Equilibrium without a CCP),*
- *does not affect dealers' expected profit,*
- *but novation improves welfare. Hedgers' total utility is given by*

$$E[U^{hr}_{h,c}] = \left(q(\Pi^{CCP}(n) - (q-1)\sigma e) + q \log(1+e) - \frac{5t}{4n} \right)$$
$$\text{for} \quad n^{hr}_{c,CCP} < n < +\infty.$$

where $n^{hr}_{c,CCP}$ is given in the Appendix.

Novation does not affect the price in a covered equilibrium. Due to competition, dealers cannot raise their price beyond a level consistent with the extent of differentiation. Novation does not affect the price in the uncovered equilibrium either since diversification has no effect in the low-risk region.[24] Therefore, dealers' profit is unchanged. On the other hand, hedgers' expected utility increases due to novation ($\Pi^{CCP}(n) \geq \Pi$) and hedgers capture all of the welfare gains due to novation. Therefore, the introduction of novation improves social welfare.[25] For our purpose, this framework captures the key effect of novation. As the number of dealers increases, and competition intensifies, novation reduces the welfare declines associated with the increase in individual dealer's default risk. This trade-off determines the optimal level of entry on the OCT market.

4 Novation, Restrictions to Entry and Risk Controls

4.1 Socially Optimal Level of Entry

We first study the optimal level of entry in a CCP that only offers novation but does not control risk. Consider the free-entry equilibrium where new dealers enter the market as long as expected profits are positive. Free-entry leads to an equilibrium where dealers have

[24]This is an artefact of our specification where the dealers' hedging shocks only take two, bounded, values. Otherwise, monopolistic dealers would capture part of the diversification benefits from novation.

[25]The CCP only affects the price in the high-risk region, i.e., $\Pi^{CCP}(n) = \Pi$ in the low-risk region.

no market power but which lies in the high-risk region. Therefore, free-entry may not be socially optimal since the lower price associated with a higher degree of competition may not warrant the higher default probability in the high-risk region.[26] The following proposition characterizes the free-entry equilibrium with and without novation. In each case, there is a role for a CCP to restrict entry. The analysis yields two results. First, hedgers prefer free-entry when dealers' market power is high relative to their ability to transfer risk efficiently. Otherwise, hedgers would prefer a situation where entry is restricted and competition is inhibited. Second, novation tilts the optimal level of entry toward more competition.

Proposition 3 ***Free-Entry Equilibrium***

Define the social welfare as the sum of all hedgers' utility. Define σ^*_{OTC} *and* σ^*_{CCP} *as,*

$$\sigma^*_{OTC} = \frac{4(1-q)\Pi}{9qe - 4q^2 e})$$
$$\sigma^*_{CCP} = \frac{4\left(\Pi - q\Pi^{CCP}(+\infty)\right)}{9qe - 4q^2 e},$$

with $0 < \sigma^*_{CCP} < \sigma^*_{OTC} < \bar{\sigma}$.

- *In the free entry equilibrium, i.e., as* $n \to +\infty$,

$$p_c^{hr} \to (q-1)\sigma e, \quad E[u_{d,c}^{hr}] \to 0, \quad \textit{and} \quad D \to 1-q.$$

- *Hedgers' utility without and with novation is given by*

$$E[U_{h,c}^{hr}] \to q(\Pi - (q-1)\sigma e) + q\log(1+e)$$
$$E[U_{h,c}^{hr}] \to \left(q(\Pi^{CCP}(+\infty) - (q-1)\sigma e) + q\log(1+e)\right),$$

  respectively.

- *In the OTC market without novation, social welfare is maximized by restricting entry to* $n^*_{OTC} \leq n_c^{hr}$ *if* $\sigma < \sigma^*_{OTC} \leq \bar{\sigma}$. *Then, the equilibrium lies in the low-risk region with imperfect competition. Otherwise, if* $\sigma > \sigma^*_{OTC}$, *then the social welfare in the OTC market without CCP is maximized in the free-entry equilibrium.*

- *In the OTC market with novation, social welfare is maximized by restricting entry to* $n^*_{CCP} \leq n_{c,CCP}^{hr}$ *if* $\sigma < \sigma^*_{CCP}$. *Otherwise, if* $\sigma > \sigma^*_{CCP}$ *the social welfare in the OTC market with a CCP offering novation is maximized in the free-entry equilibrium.*

[26] The same trade-off arises if we consider the sum of dealers and hedgers' expected utility as a criteria for social welfare. For instance, if hedgers hold all the share of the dealer's firm.

The free-entry equilibrium reaches the high-risk region and dealers default with probability $1 - q$.[27] Whether free entry is optimal depends on the following trade-off. Increasing the number of dealers improves welfare via two channels. A higher number of dealers intensifies competition, lowering the price paid by hedgers. More dealers also implies that the pool of dealers is more diverse, lowering transaction costs for hedgers. On the other hand, a lower price reduces revenue and increases the probability of a dealer's default. This reduces welfare. Whether the trade-off favors more or less entry depends in a significant way on the ability of dealers to reduce risk for hedgers. When dealers are sufficiently efficient, $\sigma < \sigma^*_{OTC}$, a relatively large number of new entrants is required to reach the high-risk region. At this point, just before crossing into the high-risk region, competition is sufficiently intense and the potential benefit of further entry, via a lower price, does not compensate for the increased default risk. Hedgers would prefer that entry be restricted. Higher efficiency implies that the optimal level of entry is higher.

Novation increases the hedgers' expected utility in the free-entry equilibrium. But free-entry man still not be optimal. New entrants are beneficial as long as their effect on the equilibrium default risk is less than the benefits due to lower price. Novation pushes this threshold lower, $\sigma^*_{CCP} < \sigma^*_{OTC}$ and tilts the optimum toward greater competition in two ways. If restrictions to entry are still optimal, novation increases the number of dealers needed before reaching a point where the benefits of higher competition do not compensate for the higher default risk, raising the optimal level of entry. Moreover, if $\Pi - q\Pi^{CCP}(+\infty)$ is sufficiently large, then the introduction of novation can switch the social optimum to the free-entry equilibrium.

4.2 Social Optimal Level of Entry with Risk Controls

Next, we consider how the introduction of risk controls within the the CCP, via the constraint in Equation 7, affects the optimal level of entry.

Proposition 4 ***Optimal CCP Rules***

Define social welfare as the sum of all hedgers' utility and σ^+_{CCP},

$$\sigma^*_{CCP} < \sigma^+_{CCP} = \frac{\Pi - q\Pi^{CCP}(+\infty)}{q(2 - qe)}.$$

- *Free-entry to a CCP that controls risk is socially optimal.*

[27]The implication that $n \to +\infty$ should not be interpreted literally. The number of new entrants is bounded if they must incur a fixed cost. Then, the Proposition still holds unless the level of fixed cost is so high that competition cannot materially reduce risk.

- *If $\sigma < \sigma^+_{CCP}$, then the optimal risk control is $\alpha^+ < 1 - q$ and the equilibrium is in the low-risk region. Otherwise, the optimal risk control is $\alpha^+ = 1 - q$ and the economy is in the high-risk region.*

Free-entry is optimal in the presence of risk controls. As above, the free-entry entry equilibrium in the high-risk region is socially optimal for low level of dealers' efficiency. Reaching the low-risk regions via entry restrictions implies increases of dealers' revenues that are too high relative to the gain in terms of default risk. But free-entry is also optimal for high levels of efficiency. The ability to control risk directly allows the CCP to maintain the equilibrium in the low-risk regions for higher levels of dealers' efficiency. The binding constraint implies that dealers must seek a price that is consistent with their risk limit. But free-entry eliminates any mark-up of prices due to market power. Hence, hedgers reap the benefits of lower-risk and greater competition. The level of dealers' efficiency dictates the level of the CCP's risk controls. The CCP selects a lower level of default risk when dealers are relatively efficient.

4.3 Dealers' Optimal CCP Rules

This Section derives the choice of CCP's rule by dealers that can coordinate their actions to maximize their total expected profits,

$$\frac{1}{n}\sum_{i=1}^{n} E[u_d^*]. \tag{20}$$

By symmetry, this is equivalent to maximizing each dealer's utility ED_i^*.[28] We consider two cases. In the first case, dealers can set both n and α. Alternatively, dealers must take the initial number incumbents as given from history and they cannot exclude any incumbent member (but they could allow for new entrants). The following Proposition summarizes the results.

Proposition 5 ***Dealers' Optimal CCP Rules***

- *If members can determine n and α, then they choose either n_u^{lr} or the minimum n where a covered equilibrium exists, the equilibrium is in the low-risk region and the risk constraint is not binding. (Otherwise, they would also choose $\alpha < 1 - q$).*

- *If n is exogenous and cannot be reduced then members choose $\alpha < 1 - q$ and exclude new entrants.*

[28] There is a large literature that studies stability of cartels and other coalition structures. See, e.g., Donsimoni et al. (1986), Bloch (1996) or Thoron (1998).

Dealers' choices of n and α differ from the social optimum. If the dealers control the CCP structure, they can maximize expected profit by choosing a low number of members n, in which case the risk constraint becomes irrelevant. Dealers choose to nearly cover the market but nonetheless remain local monopolist. Otherwise, if n is given exogenously, dealers can maximize profit by setting a binding risk constraint while restricting further entry. Incumbent members set binding risk constraints ex-ante to dampen the effect of competition, leading to higher profits.[29]

Several comments are in order. First, CCP rules and regulation that reduce access excessively may induce the emergence of competing CCP. This is outside the scope of this paper. Note that there are strong network effects that favors concentration of clearing services in OTC markets within a single entity (Duffie and Zhu, 2011). In any case, potential entry by an alternative CCP involves large fixed costs that should limit but not eliminate members' rents. But the exogenous trading costs parameters t controls the level of rents. If entry by a competing CCP is viable at lower level of rents, this corresponds to lowering t. Second, while the CCP can monitor dealers' activity and serve as a commitment device, we did not discuss how dealers agree to a set of CCP rule ex-ante. A formal game-theoretic analysis of this problem is beyond the scope of this paper. But note that the threat to break discussion and leave the CCP is not credible. Hedgers value trading with CCP members more. Moreover, with homogenous dealers, a coalition grouping all members that agree to the a common set of rules described above is likely to be stable against deviations by subset of dealers.

5 Conclusion and Extensions

Clearing houses and CCPs have for long played a key role in securities markets and their importance will increase in OTC derivatives markets over the next few years. In order to manage risk, a CCP typically imposes stringent membership requirements and other risk controls. Our analysis emphasizes that these restrictions affect the industrial organization of the financial markets that the CCP serves. We have shown conditions for which allowing free-entry to the CCP maximizes welfare implies. This minimizes rent and maximizes the diversity of dealers servicing the market, reducing trading costs. We also find that the optimal risk controls trades-off the costs to hedgers arising from a default against the costs from increasing stability. A very different outcome arises when members determine the CCP

[29] Analog results exist in the literature on the industrial organization of firms. Dixit (1980) and Kreps and Scheinkman (1983) find that firms can strategically choose ex-ante small levels of productive capacity in order to decrease competition ex-post.

rules and regulations. Members use risk controls to commit to a lower degree of competition and increase their profit. This has positive implications for the stability of the market but at a greater costs to hedgers relative to the social optimum.

Other aspects of a CCP's structure could be brought within the model. Margins, default funds, and mutualization may also affect the trade-off between competition and default risk. But the thrust of the message would likely remain. In this more general model, a CCP could use these risk-controls to achieve the benefits from competition with lower increase in default risk while dealers could use these risk-controls to achieve high profits and limit the effect of competition. Similarly, considering fixed costs to entry would not affect the results.

Several extensions would be of interest. First, the social planner's problem ignored potential externalities that different choices of competition and default risk in one market may have in other markets. For instance, a negative externality from default of too-big-to-fail dealers or positive externality from having a robust pool of diverse dealers might tilt the social optimum away from or close to competition, respectively. Second, the potential coordination between dealers, or the lack thereof, could be the focus of a formal game-theoretic analysis. More interesting, is the possibility that incumbent dealers may allow access to new entrants but at a price. Third, though in this model we considered the case where the level of dealers' hedging efficiency is homogenous and exogenous, a broader perspective would ask what would be the dealers' optimal choice of hedging strategy. The interaction between dealers behavior and the CCP's structure would lead us to ask how can the planners' implement free-entry equilibrium in low-risk regions. In other words, under what mechanism would dealers chooses strategies that reduces risk optimally from the point of view of the hedgers.

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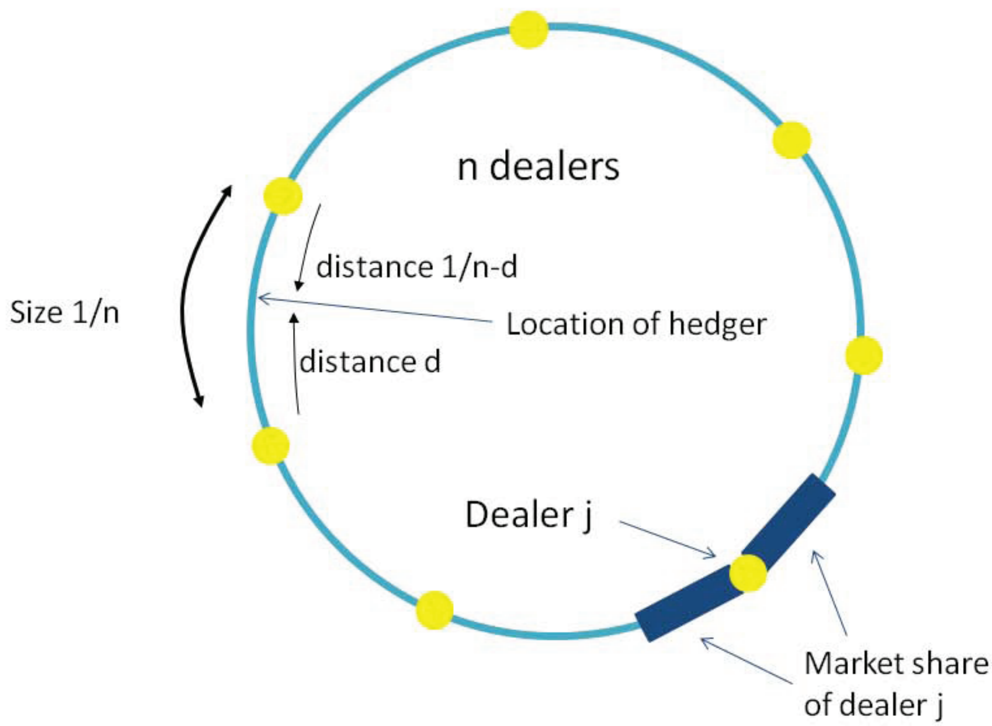

Figure 1: **Salop's Circular Road**

A Proofs of Propositions

A.1 Proof of proposition 1

We probe a general case of the proposition for any value of H.

A.1.1 Uncovered equilibrium case

For the dealers, the expected utility is

$$\begin{aligned} E[u_{d,u}] &= E[y_u(p_j)(p_j - \widetilde{\Delta}_j)/y_u(p_j)(p_j - \widetilde{\Delta}_j) \geq 0] \cdot \Pr(y_u(p_j)(p_j - \widetilde{\Delta}_j) \geq 0) = \\ &= y_u(p_j)E[p_j - \widetilde{\Delta}_j/p_j - \widetilde{\Delta}_j \geq 0] \cdot \Pr(p - \widetilde{\Delta}_j \geq 0), \end{aligned} \tag{21}$$

which can be rewritten as

$$E[u_{d,u}] = \begin{cases} y_u(p_j)p_j & if \quad p_j \geq \overline{p} \\ y_u(p_j)(p_j - \underline{p})q & if \quad p_j \in [\underline{p}, \overline{p}) \\ 0 & if \quad p_j < \underline{p} \end{cases} .$$

If we assume that the equilibrium price is in the low risk region ($\overline{p} \leq p_j$), and also that $p_j \leq \Pi$, then the expected utility is

$$E[u_{d,u}(p_j = p_u^{lr})] = \frac{2H}{t} (\Pi - p_j) \, p_j.$$

which is a concave quadratic function. To derive this expression, we have used the function y_u derived in the main text. Therefore, first order conditions are necessary and sufficient to find an optimum price which is equal to

$$p_u^{lr} = \frac{\Pi}{2} < \Pi. \tag{22}$$

In order to satisfy the constraint $\overline{p} = q\sigma e \leq p_u^{lr}$ the following condition on σ must be satisfied:

$$\begin{aligned} q\sigma e &\leq \frac{\Pi}{2} \Leftrightarrow \\ \sigma &\leq \frac{\Pi}{2qe}. \end{aligned} \tag{23}$$

In equilibrium, the utility of the dealer in the low risk region is

$$E[u_{d,u}^{lr}] = y_u(p_u^{lr})p_u^{lr} = \frac{2H}{t} \left(\frac{\Pi}{2} \right)^2 . \tag{24}$$

Now, we assume that the uncovered equilibrium price is in the high risk region. In that case, the equilibrium price p_u^{hr} must satisfy the following first order condition:

$$\frac{\partial E[u_{d,u}^{hr}(p_j = p_u^{hr})]}{\partial p_j} = \frac{\partial y_u}{\partial p_j}(p_u^{hr} - (q-1)\sigma e)q + y_u q$$
$$= -\frac{2qH}{t}(p_u^{hr} - (q-1)\sigma e)q + \frac{2qH}{t}(\Pi - p_u^{hr})q = 0.$$

Therefore, p_u^{hr} is equal to

$$p_u^{hr} = \frac{\Pi + (q-1)\sigma e}{2}, \tag{25}$$

where $p_u^{hr} < p_u^{lr}$. Since $y_u(p_j)(p_j - (q-1)\sigma e)q$ is a concave quadratic function, first order conditions are necessary and sufficient to find an optimum.

In order to satisfy the constraint $(q-1)\sigma e \leq p_u^{hr} < q\sigma e$ the following conditions on σ must be satisfied:

$$p_u^{hr} = \frac{\Pi + (q-1)\sigma e}{2} < q\sigma e \Leftrightarrow$$
$$\sigma > \frac{\Pi}{(1+q)e}, \tag{26}$$

and

$$p_u^{hr} = \frac{\Pi + (q-1)\sigma e}{2} \geq (q-1)\sigma e \Leftrightarrow \sigma \geq -\frac{\Pi}{(1-q)e},$$

which is always satisfied because by assumption $\sigma \geq 0$.

In equilibrium, the utility of the dealer in the high risk region is

$$E[u_{d,u}^{hr}] = y_u(p_u^{hr})(p_u^{hr} - (q-1)\sigma e)q = \frac{2q^2 H}{t}\left(\frac{\Pi - (q-1)\sigma e}{2}\right)^2. \tag{27}$$

In order to have an equilibrium in the low risk region, the expected utility for every dealer in the low risk region must be greater than in the high risk region. This condition is satisfied as long as σ satisfies the condition $\sigma \leq \frac{\Pi}{qe}$ which is obtained as follows:

$$E[u_{d,u}^{hr}] = \frac{2q^2 H}{t}\left(\frac{\Pi - (q-1)\sigma e}{2}\right)^2 \leq E[u_{d,u}^{lr}] = \frac{2H}{t}\left(\frac{\Pi}{2}\right)^2 \Leftrightarrow q\frac{\Pi - (q-1)\sigma e}{2} \leq \frac{\Pi}{2} \Leftrightarrow$$
$$q\Pi - q(q-1)\sigma e \leq \Pi \Leftrightarrow (q-1)\Pi \leq q(q-1)\sigma e \Leftrightarrow \Pi \geq q\sigma e \Leftrightarrow$$
$$\sigma \leq \frac{\Pi}{qe}. \tag{28}$$

Since we can easily show that $\frac{\Pi}{(1+q)e} \leq \frac{\Pi}{2qe} \leq \frac{\Pi}{qe}$, then the inequalities (23), (26) and (28) can be summarized as follows:

- if $\sigma \leq \overline{\sigma} = \frac{\Pi}{2qe}$, then $p_u^{lr} = \frac{\Pi}{2} > \overline{p}$ is the uncovered equilibrium price, and
- if $\sigma > \overline{\sigma} = \frac{\Pi}{2qe}$, then $p_u^{hr} = \frac{\Pi + (q-1)\sigma e}{2} < \overline{p}$ is the uncovered equilibrium price

Now we need to verify that this equilibrium is uncovered, that is, we need to verify that every dealer sells in equilibrium a number of contracts lower than H/n. In equilibrium in the low risk region, every dealer sells $y_u(p_u^{lr}) = \frac{2H}{t}\frac{\Pi}{2} = \frac{H\Pi}{t} > 0$ which must be lower than $\frac{H}{n}$ by definition of the uncovered equilibrium. Since H/n is decreasing with n and tends to 0 as n tends to the infinity, there exists n_u^{lr} such that when $n \geq n_u^{lr}$, we have $y_u(p_u^{lr}) > H/n$ and there will not be uncovered equilibrium. The expression of n_u^{lr} is obtained as follows:

$$\frac{\Pi H}{t} = \frac{H}{n_u^{lr}} \Leftrightarrow n_u^{lr} = \frac{t}{\Pi}.$$

For the high risk region, we can also find n_u^{hr} such that when $n \geq n_u^{hr}$, we have $y_u(p_u^{hr}) > H/n$ and there will not be uncovered equilibrium. The expression of n_u^{lr} is obtained as follows:

$$\frac{2qH}{t}\frac{\Pi - (q-1)\sigma e}{2} = \frac{H}{n_u^{hr}} \Leftrightarrow n_u^{hr} = \frac{t}{q(\Pi - (q-1)\sigma e)}.$$

The total utility of all hedgers that trade with one dealer j is equal to the integral of the utility of all hedgers where we integrate over the distance d_j between every hedger and the dealer. Every hedger trades with a dealer as long as the utility is greater than the reservation value. From this condition, we obtain the maximum distance of hedgers that trade with dealer j. For the low risk region, this distance is given by the following inequality:

$$\Pi + q\log(1+e) - p_j - d_j t \geq q\log(1+e) \Leftrightarrow$$
$$\frac{\Pi - p_j}{t} \geq d_j.$$

From this inequality, we obtain the total utility of all hedgers for the low risk region, which is equal to

$$2H \int_0^{\frac{\Pi - p_u^{lr}}{t}} \left[\Pi + q\log(1+e) - p_u^{lr} - zt\right] dz.$$

This expression can be simplified as follows:

$$2H \int_0^{\frac{\Pi - p_u^{lr}}{t}} \left[\Pi + q\log(1+e) - p_u^{lr} - zt\right] dz = 2H\left((\Pi + q\log(1+e) - p_u^{lr})\frac{\Pi - p_u^{lr}}{t} - \left[\frac{z^2 t}{2}\right]_{z=0}^{z=\frac{\Pi - p_u^{lr}}{t}}\right)$$
$$= 2H\left((\Pi + q\log(1+e) - p_u^{lr})\frac{\Pi - p_u^{lr}}{t} - \frac{(\Pi - p_u^{lr})^2}{2t}\right) =$$
$$= 2H\frac{\Pi - p_u^{lr}}{t}\left(\Pi + q\log(1+e) - p_u^{lr} - \frac{\Pi - p_u^{lr}}{2}\right)$$
$$= 2H\frac{\Pi - p_u^{lr}}{t}\left(\frac{\Pi - p_u^{lr}}{2} + q\log(1+e)\right).$$

By using the equilibrium price p_u^{lr} from (22), we can simplify furthermore the expression in order to obtain

$$2H \int_0^{\frac{\Pi - p_u^{lr}}{t}} \Big[\Pi + q\log(1+e) - p_u^{lr} - zt\Big]\, dz = \frac{2H}{t}\frac{\Pi}{2}\left(\frac{\Pi}{4} + q\log(1+e)\right). \tag{29}$$

Finally, the total utility of hedgers $E[U_{h,u}^{lr}]$ is given by the sum of utilities of hedgers that trade with every dealer. Therefore, this is given by n times (29):

$$E[U_{h,u}^{lr}] = n\frac{\Pi H}{t}\left(\frac{\Pi}{4} + q\log(1+e)\right). \tag{30}$$

Since prices do not depend on n in the uncovered case, this expression is strictly increasing in n.

In the high risk region we can obtain the utility of the hedgers that trade with a dealer using identical derivations used in the low risk region:

$$\begin{aligned}
&2H \int_0^{q\frac{\Pi - p_u^{hr}}{t}} \Big[q(\Pi + \log(1+e) - p_u^{hr}) - zt\Big]\, dz \\
&= 2H\left(\Big(q(\Pi - p_u^{hr}) + q\log(1+e)\Big)\, q\frac{\Pi - p_u^{hr}}{t} - \left[\frac{z^2 t}{2}\right]_{z=0}^{z=q\frac{\Pi - p_u^{hr}}{t}}\right) \\
&= 2H\left(\Big(q(\Pi - p_u^{hr}) + q\log(1+e)\Big)\, q\frac{\Pi - p_u^{hr}}{t} - \frac{q^2(\Pi - p_u^{hr})^2}{2t}\right) \\
&= 2Hq\frac{\Pi - p_u^{hr}}{t}\left(q(\Pi - p_u^{hr}) + q\log(1+e) - q\frac{\Pi - p_u^{hr}}{2}\right) \\
&= 2H\frac{\Pi - p_u^{hr}}{t}\left(q\frac{\Pi - p_u^{hr}}{2} + q\log(1+e)\right).
\end{aligned}$$

By using the equilibrium price p_u^{hr} from (25), we can simplify furthermore the expression in order to obtain

$$2H \int_0^{q\frac{\Pi - p_u^{hr}}{t}} \Big[q(\Pi + \log(1+e) - p_u^{hr}) - zt\Big]\, dz = \frac{2H}{t}\frac{\Pi - (q-1)\sigma e}{2}\left(q\frac{\Pi - (q-1)\sigma e}{4} + q\log(1+e)\right).$$

The total utility of hedgers in the high risk region would be n times this quantity:

$$E[U_{h,u}^{hr}] = n\frac{H}{t}(\Pi - (q-1)\sigma e)\left(q\frac{\Pi - (q-1)\sigma e}{4} + q\log(1+e)\right). \tag{31}$$

Since prices do not depend on n in the uncovered case, this expression is strictly increasing in n.

A.1.2 Covered equilibrium case

In the covered equilibrium case, we can derivate the expected utility of every dealer as in equation (21) which can be rewritten as

$$E[u_{d,c}] = \begin{cases} y_c(p_j, \widehat{p})p_j & if \quad \overline{p} \leq p_j, \hat{p} \\ y_c(p_j, \widehat{p})(p_j - (q-1)\sigma e)q & if \quad p_j, \hat{p} \in [\underline{p}, \overline{p}) \\ 0 & if \quad p_j, \hat{p} < \underline{p} \end{cases} . \tag{32}$$

This expression assumes that all dealers set a price in the same region.

Demand when dealers set prices in different regions:

In the main text we have shown how to derive the demand function y_c where we assume that the prices of all dealers are either in the high risk region or in the low risk region. However, in order to obtain the symmetric equilibrium, we need to calculate the demand function for the case where, given that the rest of the dealers set the price in a given region, a dealer deviates and sets a price in a different region. For the case where a dealer j deviates to set a price p_j in the low risk region given that the rest of the dealers set a price $\widehat{p}$ in the high risk region, the hedgers that trade with dealer j are given by the following inequality:

$$\Pi + q\log(1+e) - p_j - d_j t \geq q(\Pi + \log(1+e) - \widehat{p}) - (\frac{1}{n} - d_j)t \iff$$
$$\frac{(1-q)\Pi - p_j + q\widehat{p} + \frac{1}{n}t}{2t} \geq d_j.$$

From this inequality, we can express the demand for dealer j in this case (denoted as $y_c^{lr}(p_j, \widehat{p})$) as

$$y_c^{lr}(p_j, \widehat{p}) = H\frac{(1-q)\Pi - p_j + q\widehat{p} + \frac{1}{n}t}{t}. \tag{33}$$

For the case where a dealer j deviates to set a price p_j in the high risk region given that the rest of the dealers set a price $\widehat{p}$ in the low risk region, the hedgers that trade with dealer j are given by the following inequality:

$$q(\Pi + \log(1+e) - p_j) - d_j t \geq \Pi + q\log(1+e) - \widehat{p} - (\frac{1}{n} - d_j)t \iff$$
$$\frac{(q-1)\Pi - qp_j + \widehat{p} + \frac{1}{n}t}{2t} \geq d_j.$$

From this inequality, we can express the demand for dealer j in this case (denoted as $y_c^{hr}(p_j, \widehat{p})$) as

$$y_c^{hr}(p_j, \widehat{p}) = H\frac{(q-1)\Pi - qp_j + \widehat{p} + \frac{1}{n}t}{t}. \tag{34}$$

Calculation of the symmetric equilibrium in the low risk region:

If we assume that the equilibrium price is in the low risk region, the symmetric equilibrium price p_c^{lr} must be given by the first order conditions

$$\frac{\partial E[u_{d,c}(p_j = p_c^{lr}, \widehat{p} = p_c^{lr})]}{\partial p_j} = \frac{\partial y_c}{\partial p_j}p_c^{lr} + y_c = 0,$$

where we have differentiated the expected utility expression from (32).

Since in the symmetric covered equilibrium all firms charge the same price it must be the case that $y_c = H/n$. Therefore, we have

$$\frac{\partial E[u_{d,c}(p_j = p_c^{lr}, \widehat{p} = p_c^{lr})]}{\partial p_j} = -\frac{H}{t} p_c^{lr} + \frac{H}{n} = 0 \Leftrightarrow$$
$$p_c^{lr} = \frac{t}{n}. \tag{35}$$

In order for p_c^{lr} to be an equilibrium, the second order conditions must be verified. They are given by

$$\frac{\partial^2 E[u_{d,c}(p_j = p_c^{lr}, \widehat{p} = p_c^{lr})]}{\partial p_j^2} = \frac{\partial^2 y_c}{\partial p_j^2} p_c^{lr} + \frac{\partial y_c}{\partial p_j} + \frac{\partial y_c}{\partial p_j}.$$

Since the second derivative of $y_c(p_j, \widehat{p})$ is zero, second order conditions are satisfied at the optimum:

$$\frac{\partial^2 E[u_{d,c}(p_j = p_c^{lr}, \widehat{p} = p_c^{lr})]}{\partial p_j^2} = 2\frac{\partial y_c}{\partial p_j} = -2\frac{H}{t} \leq 0.$$

Also, in order to have $\overline{p} \leq p_c^{lr}$ we need to satisfy the following inequality:

$$q\sigma e \leq \frac{t}{n} \Leftrightarrow$$
$$n \leq n_c^* = \frac{t}{q\sigma e} = \frac{t}{\overline{p}}. \tag{36}$$

The utility of every dealer is

$$E[u_{d,c}^{lr}] = y_c(p_c^{lr}) p_c^{lr} = \frac{Ht}{n^2}. \tag{37}$$

If there is a symmetric equilibrium, the price must be given by (35). In order for this price to be an equilibrium, we need to verify that a dealer does not want to deviate by setting a price in the high risk region given that the rest of dealers set a price $p_c^{lr} = \frac{t}{n}$ in the low risk region. By using the demand function obtained in (34), the best deviating price chosen by dealer j (denoted by $p_j^{hr\_l}$) is given by first order conditions:

$$\begin{aligned}
\frac{\partial E[u_{d,c}(p_j, \widehat{p} = \frac{t}{n})]}{\partial p_j} &= \frac{\partial y_c^{hr}}{\partial p_j}(p_j - \underline{p})q + y_c^{hr} q = 0 \\
&= -\frac{qH}{t}(p_j - \underline{p})q + H\frac{(q-1)\Pi - qp_j + \frac{t}{n} + \frac{t}{n}}{t} q = 0 \\
&\Leftrightarrow \\
p_j^{hr_l} &= \frac{\underline{p} + \frac{q-1}{q}\Pi + \frac{2t}{qn}}{2}.
\end{aligned}$$

Using this price, we can obtain the expected utility of the dealer deviating:

$$
\begin{aligned}
E[u_{d,c}] &= y_c(p_j^{hr\_l}, \widehat{p})(p_j^{hr\_l} - \underline{p})q \\
&= H\frac{\frac{q-1}{2}\Pi - \frac{q\underline{p}}{2} + \frac{t}{n}}{t} q \frac{-\underline{p} + \frac{q-1}{q}\Pi + \frac{2t}{qn}}{2} \\
&= \frac{H}{t}\left(\frac{q-1}{2}\Pi - \frac{q\underline{p}}{2} + \frac{t}{n}\right)\left(\frac{q-1}{2}\Pi - \frac{q\underline{p}}{2} + \frac{t}{n}\right) \\
&= \frac{H}{t}\left(\frac{(q-1)\Pi - q\underline{p}}{2} + \frac{t}{n}\right)^2 .
\end{aligned} \tag{38}
$$

In order to verify that this is not a profitable deviation, we need to compare the utility from (38) with the utility from not deviating. If we assume that $\sigma \leq \frac{\Pi}{2qe}$, then the following inequality is satisfied:

$$
(q-1)\Pi - q\underline{p} = (q-1)\left(\Pi - q\sigma e\right) \leq 0
$$

Therefore, since the term $(q-1)\Pi - q\underline{p}$ in (38) is negative, it must be the case that the utility in (38) is smaller than the utility $E[u_{d,c}^{lr}]$ in (37). Therefore, it is not a profitable deviation for a dealer to set a low price in order to be in the high risk region given that the rest of the dealers set a price in the low risk region equal to $p_c^{lr} = \frac{t}{n}$.

Also, for this equilibrium to exist it must be that the marginal hedger in $d = H/n$ is having an utility that is greater than the outside value. For the low risk region we have

$$
\begin{aligned}
\log(1+qe) - p_c^{lr} - dt \geq q\log(1+e) \Leftrightarrow \Pi - \frac{t}{n} - \frac{H}{n}t \geq 0 \Leftrightarrow \\
n \geq n_c^{lr} = \frac{t(1+H)}{\Pi}
\end{aligned} \tag{39}
$$

Therefore, it must be that $n \geq n_c^{lr}$ for the covered equilibrium to exist in the low risk region.

Calculation of the symmetric equilibrium in the high risk region:

When n increases, p_c^{lr} decreases, and eventually the equilibrium price will be in the high risk region. The equilibrium price p_c^{hr} in the high risk region is given by

$$
\begin{aligned}
\frac{\partial E[u_{d,c}(p_j = p_c^{hr}, \widehat{p} = p_c^{hr})]}{\partial p_j} &= \frac{\partial y_c}{\partial p_j}(p_c^{hr} - (q-1)\sigma e)q + y_c q = 0 \\
&= -\frac{qH}{t}(p_c^{hr} - (q-1)\sigma e)q + \frac{H}{n}q = 0 \\
&\Leftrightarrow \\
p_c^{hr} &= (q-1)\sigma e + \frac{t}{qn}.
\end{aligned} \tag{40}
$$

Second order conditions are also satisfied:

$$\frac{\partial^2 E[u_{d,c}(p_j = p_c^{hr}, p_k = p_c^{hr})]}{\partial p_j^2} = \frac{\partial^2 y_c}{\partial p_j^2}(p_c^{hr} - (q-1)\sigma e)q + \frac{\partial y_c}{\partial p_j} q + \frac{\partial y_c}{\partial p_j} q =$$

$$= 2\frac{\partial y_c}{\partial p_j} q = 2\frac{-Hq}{t} q \leq 0.$$

In order for price p_c^{hr} in (40) to be an equilibrium in the high risk region, it must be that $p_c^{hr} < q\sigma e$. From this condition, we obtain the following inequality on n:

$$p_c^{hr} = (q-1)\sigma e + \frac{t}{qn} < q\sigma e \Leftrightarrow \frac{t}{q\sigma e} = n_c^* < n. \tag{41}$$

The utility of every dealer is

$$E[u_{d,c}^{hr}] = y_c(p_c^{hr})(p_c^{hr} - (q-1)\sigma e)q = \frac{Ht}{n^2}. \tag{42}$$

Again, in order for this to be an equilibrium, we need to check that a dealer does not want to deviate to set a price in the low risk region given that the rest of dealers set a price $p_c^{hr} = (q-1)\sigma e + \frac{t}{qn}$ in the high risk region. The expression of the demand for a dealer deviating in (33) increases with the deviating price of the dealer (as long as the deviating price is greater than $\overline{p}$). When $p_j = \overline{p}$ demand is the highest possible for the dealer which is equal to

$$y_c^{lr}(p_j = \overline{p}, \widehat{p} = \underline{p} + \frac{t}{qn}) = H\frac{(1-q)\Pi - \overline{p} + q\widehat{p} + \frac{t}{n}}{t} =$$

$$= H\frac{(1-q)\Pi - \overline{p} + q\underline{p} + \frac{2t}{n}}{t}.$$

If we assume that q is high enough and t low enough, the term $-\overline{p} + q\underline{p} < 0$ is greater in absolute value than $(1-q)\Pi + \frac{t}{n} > 0$ and therefore the demand of the deviating dealer is zero because $y_c^{lr} < 0$. Therefore, deviating is not profitable for the dealer when q is high enough and t low enough[30].

Also, for this equilibrium to exist it must be that the marginal hedger in $d = H/n$ is having an utility that is greater than the outside value. For the high risk region we have the following condition:

$$q(\Pi - p_c^{hr}) + q\log(1+e) - dt \geq q\log(1+e) \Leftrightarrow q(\Pi - (q-1)\sigma e - \frac{t}{qn}) - \frac{H}{n}t \geq 0 \Leftrightarrow$$

$$n \geq n_c^{hr} = \frac{t(1+H)}{q(\Pi - (q-1)\sigma e)}.$$

[30] Given the assumed condition on σ, $\sigma \leq \frac{\Pi}{2qe}$, and $H = 1$, the rest of the parameters e and Π can be adjusted to satisfy all these conditions that ensure the existence and unicity of equilibrium.

We can compare n_c^{hr} and n_c^{lr}:

$$\begin{aligned}
&n_c^{hr} \le n_c^{lr} \Leftrightarrow \frac{t(1+H)}{q(\Pi-(q-1)\sigma e)} \le \frac{t(1+H)}{\Pi} \Leftrightarrow \\
&\Pi \le q(\Pi-(q-1)\sigma e) \Leftrightarrow q(q-1)\sigma e \le (q-1)\Pi \Leftrightarrow \\
&q\sigma e \ge \Pi \Leftrightarrow \sigma \ge \frac{\Pi}{qe}.
\end{aligned}$$

Given these inequalities, the covered equilibrium in the low risk region will exist as long as $n_c^{lr} < n_c^*$ therefore, it exists when $n \in [\min(n_c^{lr}, n_c^*), n_c^*]$. Similarly, the covered equilibrium in the high risk region will exist when $n > \max(n_c^{hr}, n_c^*)$.
If $H = 1$ and $\sigma \le \overline{\sigma} = \frac{\Pi}{2qe} < \frac{\Pi}{qe}$ then $n_c^{lr} = \frac{2t}{\Pi}$ and since $n_c^* = \frac{t}{q\sigma e}$ then we have that $n_c^{lr} < n_c^*$ because we have assumed that $\sigma \le \overline{\sigma} = \frac{\Pi}{2qe}$. Therefore, in this case there exists a low risk covered equilibrium in the interval $[n_c^{lr}, n_c^*]$.

Utility of hedgers in the covered equilibrium:

Since in the covered equilibrium, the distance from the hedger indifferent between two dealers and the dealer he is trading with is equal to $1/2n$, then utility for hedgers covered by one dealer for the low risk region is equal to

$$\begin{aligned}
&2H\int_0^{1/(2n)} [\log(1+qe) - zt - p_c]\, dz = 2H\left(\log(1+qe)\frac{1}{2n} - p_c\frac{1}{2n} - \frac{t}{8n^2}\right) && (43)\\
&= 2H\frac{1}{2n}\left(\log(1+qe) - \frac{t}{n} - \frac{t}{4n}\right)\\
&= \frac{H}{n}\left(\log(1+qe) - \frac{5t}{4n}\right). && (44)
\end{aligned}$$

And for the high risk region is equal to

$$\begin{aligned}
2H\int_0^{1/(2n)} [q(\Pi+\log(1+e)-p_c) - zt]\, dz &= 2H\left((q(\Pi-p_c) + q\log(1+e))\frac{1}{2n} - \frac{t}{8n^2}\right)\\
&= 2H\frac{1}{2n}\left(q(\Pi-p_c) + q\log(1+e) - \frac{t}{4n}\right)\\
&= \frac{H}{n}\left(q(\Pi-(q-1)\sigma e - \frac{t}{qn}) + q\log(1+e) - \frac{t}{4n}\right)\\
&= \frac{H}{n}\left(q(\Pi-(q-1)\sigma e) + q\log(1+e) - \frac{5t}{4n}\right). \qquad (45)
\end{aligned}$$

Finally, the total utility of hedgers would be n times the values obtained in (44) and (45):

$$E[U_{h,c}^{lr}] = H\left(\log(1+qe) - \frac{5t}{4n}\right). \qquad (46)$$

$$E[U_{h,c}^{hr}] = H\left(q(\Pi-(q-1)\sigma e) + q\log(1+e) - \frac{5t}{4n}\right). \qquad (47)$$

Note that total utility of hedgers are strictly increasing in n.

A.2 Proof of proposition 2

We probe a general case of the proposition for any value of H.

A.2.1 Uncovered equilibrium case

In the low risk region there is no default, therefore same results as in proposition 1 apply.Using identical analysis to the one used in proposition 1 in the uncovered equilibrium case, we find that the uncovered equilibrium price in the high risk region is

$$p_{u,CCP}^{hr} = \frac{\Pi^{CCP}(n) + (q-1)\sigma e}{2} > p_u^{hr}.$$

In order to satisfy the constraint $p_u^{hr} < \underline{p}$ the following condition on σ must be satisfied:

$$\frac{\Pi^{CCP}(n) + (q-1)\sigma e}{2} < q\sigma e \Leftrightarrow \Pi^{CCP}(n) + (q-1)\sigma e < 2q\sigma e$$
$$\Leftrightarrow \sigma > \frac{\Pi^{CCP}(n)}{(1+q)e} > \frac{\Pi}{(1+q)e}.$$

The utility of the dealer in the high risk region is

$$E[u_{d,u}^{hr}] = \frac{2q^2H}{t}\left(\frac{\Pi^{CCP}(n) - (q-1)\sigma e}{2}\right)^2,$$

which is greater than the case without CCP in (27).
Also, the uncovered equilibrium price, threshold n_u^{lr}, and dealer's utility in the low risk region are:

$$p_{u,CCP}^{lr} = \frac{\Pi}{2}, \quad n_u^{lr} = \frac{t}{\Pi}, \quad E[u_{d,u}^{lr}] = \frac{2H}{t}\left(\frac{\Pi}{2}\right)^2,$$

where as in proposition 1 it is necessary $\sigma \leq \frac{\Pi}{2qe}$ in order for this price to be in the low risk region. The expected utility for dealers in the high risk region is lower than in the low risk region if the following condition on σ is satisfied:

$$E[u_{d,u}^{hr}] = \frac{2q^2H}{t}\left(\frac{\Pi^{CCP}(n) - (q-1)\sigma e}{2}\right)^2 \leq E[u_{d,u}^{lr}] = \frac{2H}{t}\left(\frac{\Pi}{2}\right)^2$$
$$\Leftrightarrow q\frac{\Pi^{CCP}(n) - (q-1)\sigma e}{2} \leq \frac{\Pi}{2} \Leftrightarrow$$
$$q\Pi^{CCP}(n) - q(q-1)\sigma e \leq \Pi \Leftrightarrow q(1-q)\sigma e \leq \Pi - q\Pi^{CCP}(n)$$
$$\Leftrightarrow \sigma \leq \frac{\Pi - q\Pi^{CCP}(n)}{q(1-q)e} < \frac{\Pi}{qe} < \frac{\Pi}{2qe}.$$

Finally, for the high risk region, the value n_u^{hr} is obtained as the solution to the following fixed point equation

$$\frac{2qH}{t}\frac{\Pi^{CCP}(n_{u,CCP}^{hr}) - (q-1)\sigma e}{2} = \frac{H}{n_{u,CCP}^{hr}},$$

where we have that $n_{u,CCP}^{hr} < n_u^{hr}$.

A.2.2 Covered equilibrium case

Using identical operations from proposition 1 we can find

$$p_c^{lr} = \frac{t}{n}$$
$$p_c^{hr} = (q-1)\sigma e + \frac{t}{qn}. \tag{48}$$

Also, as shown in proposition 1 in (36) and (41), if $n \leq n_c^* = \frac{t}{q\sigma e}$ then $p_c^{lr} \geq \overline{p}$, and if $\frac{t}{q\sigma e} = n_c^* < n$ then $p_c^{hr} < \overline{p}$.

As we have done in proposition 1 in (39), it must be that the marginal hedger in $d = H/n$ is having an utility that is greater than the outside value. Therefore, we can find the value of the coefficient $n_{c,CCP}^{hr}$ that solves the following condition:

$$q\Pi^{CCP}(n_{c,CCP}^{hr}) - q(q-1)\sigma e = \frac{t(1+H)}{n_{c,CCP}^{hr}}.$$

Note that n_c^{hr} does not have a closed form solution. Because $\Pi^{CCP}(n)$ is increasing with n, it must be the case that $n_{c,CCP}^{hr} < n_c^{hr}$.

Similarly, the term $n_c^{lr} = \frac{t(1+H)}{\Pi}$ is identical to the case of proposition 1. The utility of every dealer in every region is

$$E[u_{d,c}^{lr}] = y_c(p_c^{lr})p_c^{lr} = \frac{Ht}{n^2}$$
$$E[u_{d,c}^{hr}] = y_c(p_c^{hr})(p_c^{hr} - (q-1)\sigma e)q = \frac{Ht}{n^2}.$$

Finally, identical operations from proposition 1 give the utility of the hedgers for the low and high risk regions and conditions for existence of the equilibrium.

A.3 Proof of proposition 3

A.3.1 Free entry in the OTC market without novation

When $n \to +\infty$, we are in the high risk region and $y_c = H/n \to 0$. From (40) in proposition 1, we also have that

$$p_c^{hr} = (q-1)\sigma e + \frac{t}{qn} \underset{n\to+\infty}{\to} (q-1)\sigma e.$$

Therefore, in the free entry equilibrium the price is the lowest possible in the high risk region. Also, with free entry, dealers will enter in the market as long as $E[u_{d,c}^{hr}] \geq 0$, therefore, in the limit we have $E[u_{d,c}^{hr}] \to 0$.

A.3.2 Social optimum in the OTC market without novation

By summarizing the results from proposition 1 in (30), (31), (46) and (47), we obtain the utility of hedgers for every case equal to

$$
\begin{aligned}
E[U^{lr}_{h,u}] &= n\frac{\Pi H}{t}\left(\frac{\Pi}{4} + q\log(1+e)\right) \text{ for } n \leq \frac{t}{\Pi}\\
E[U^{hr}_{h,u}] &= n\frac{H}{t}(\Pi-(q-1)\sigma e)\left(q\frac{\Pi-(q-1)\sigma e}{4} + q\log(1+e)\right) \text{ for } n \leq \frac{t}{q(\Pi-(q-1)\sigma e)}\\
E[U^{lr}_{h,c}] &= H\left(\log(1+qe) - \frac{5t}{4n}\right) \text{ for } n \in [\min(\frac{t(1+H)}{\Pi}, \frac{t}{q\sigma e}), \frac{t}{q\sigma e}]\\
E[U^{hr}_{h,c}] &= H\left(q(\Pi-(q-1)\sigma e) + q\log(1+e) - \frac{5t}{4n}\right) \text{ for } n \in (\max(\frac{t(1+H)}{q(\Pi-(q-1)\sigma e)}, \frac{t}{q\sigma e}), +\infty).
\end{aligned}
$$

Since every case gives an utility that is increasing with n, we need to compare them at the highest possible n. To simplify the analysis, we treat n here as a real number. This gives the following expected utility values

$$
\begin{aligned}
E[U^{lr}_{h,u}] &= \frac{t}{\Pi}\frac{\Pi H}{t}\left(\frac{\Pi}{4} + q\log(1+e)\right)\\
E[U^{hr}_{h,u}] &= H\left(\frac{\Pi-(q-1)\sigma e}{4} + \log(1+e)\right)\\
E[U^{lr}_{h,c}] &= H\left(\log(1+qe) - \frac{5t}{4\frac{t}{q\sigma e}}\right)\\
E[U^{hr}_{h,c}] &= H\left(q(\Pi-(q-1)\sigma e) + q\log(1+e)\right).
\end{aligned}
$$

First we need to obtain conditions for the inequality $E[U^{lr}_{h,c}] \leq E[U^{hr}_{h,c}]$ to be satisfied:

$$
\begin{gathered}
E[U^{lr}_{h,c}] \leq E[U^{hr}_{h,c}] \Leftrightarrow \log(1+qe) - \frac{5q\sigma e}{4} \leq q(\Pi-(q-1)\sigma e) + q\log(1+e)\\
\Leftrightarrow \Pi - \frac{5q\sigma e}{4} \leq q\Pi - q(q-1)\sigma e \Leftrightarrow (1-q)\Pi - \frac{5q\sigma e}{4} \leq q(1-q)\sigma e \Leftrightarrow\\
(1-q)\Pi \leq \left(\frac{5qe}{4} + q(1-q)e\right)\sigma \Leftrightarrow (1-q)\Pi \leq \left(\frac{5qe+4qe-4q^2e}{4}\right)\sigma \Leftrightarrow\\
(1-q)\Pi \leq \left(\frac{9qe-4q^2e}{4}\right)\sigma \Leftrightarrow\\
0 < \frac{4(1-q)\Pi}{9qe-4q^2e} \leq \sigma. \qquad (49)
\end{gathered}
$$

As we probed in proposition 1, the covered equilibrium in the low risk region will exist as long as $n^{lr}_c < n^*_c$. Therefore, from (36) and (39), the covered case in the low risk region exists if

$$\frac{t(1+H)}{\Pi} \leq \frac{t}{q\sigma e} \Leftrightarrow \sigma \leq \frac{\Pi}{(1+H)qe}. \qquad (50)$$

We can compare the values of (49) and (50) to obtain the following condition on H:

$$\frac{4(1-q)\Pi}{9qe-4q^2e} \le \frac{\Pi}{(1+H)qe} \Leftrightarrow \frac{4(1-q)}{9-4q} \le \frac{1}{(1+H)} \Leftrightarrow 1+H \le \frac{9-4q}{4(1-q)}$$
$$\Leftrightarrow H \le \frac{9-4q}{4(1-q)} - 1 = \frac{5}{4(1-q)} \equiv H^*.$$

Therefore, the comparison between $E[U^{lr}_{h,c}]$ and $E[U^{hr}_{h,c}]$ is only valid when $H \le H^* = \frac{5}{4(1-q)}$ where H^* is a number strictly greater than one.

Also, (49) is always lower than the threshold $\frac{\Pi}{2qe}$ as we can show as follows:

$$\frac{4(1-q)\Pi}{9qe-4q^2e} < \frac{\Pi}{2qe} \Leftrightarrow \frac{4(1-q)}{9-4q} < \frac{1}{2} \Leftrightarrow$$
$$8-8q < 9-4q \Leftrightarrow -1 < 4q \Leftrightarrow -\frac{1}{4} < q. \tag{51}$$

By comparing $E[U^{lr}_{h,u}]$ and $E[U^{hr}_{h,c}]$ we find the following condition on σ:

$$E[U^{lr}_{h,u}] \le E[U^{hr}_{h,c}] \Leftrightarrow \frac{1}{4}\log(1+qe) + \frac{3}{4}q\log(1+e) \le q(\Pi-(q-1)\sigma e) + q\log(1+e) \Leftrightarrow$$
$$\frac{1}{4}\log(1+qe) + q\log(1+e) - \frac{1}{4}q\log(1+e) \le q(\Pi-(q-1)\sigma e) + q\log(1+e) \Leftrightarrow$$
$$\frac{1}{4}\Pi \le q\Pi - q(q-1)\sigma e \Leftrightarrow (\frac{1}{4}-q)\Pi \le q(1-q)\sigma e$$
$$\Leftrightarrow \frac{(\frac{1}{4}-q)\Pi}{q(1-q)e} \le \sigma. \tag{52}$$

The threshold in (52) is always lower than the threshold $\frac{\Pi}{2qe}$ as we can show as follows:

$$\frac{(\frac{1}{4}-q)\Pi}{q(1-q)e} < \frac{\Pi}{2qe} \Leftrightarrow \frac{(1-4q)}{4(1-q)} < \frac{1}{2} \Leftrightarrow$$
$$2-8q < 4-4q \Leftrightarrow -2 < 4q, \tag{53}$$

which is always satisfied because q is a probability. We compare the thresholds in (49) and (52). It is relatively easy to show mathematically that

$$\frac{4(1-q)\Pi}{9qe-4q^2e} \ge \frac{(\frac{1}{4}-q)\Pi}{q(1-q)e} \Leftrightarrow \frac{1-q}{\frac{9}{4}-q} \ge \frac{\frac{1}{4}-q}{1-q} \text{ for any } q \in [0,1].$$

Also, if we assume that $\sigma > \frac{\Pi}{2qe}$, then it must be true that $E[U^{hr}_{h,u}] \le E[U^{hr}_{h,c}]$. This is because when $n = +\infty$, we are in the covered high risk region, the price is the minimum possible and equal to $(q-1)\sigma e$ and the distance from the hedger to the dealer is 0. However, in the uncovered region the price is higher and the distance is not zero.

Therefore, we can summarize these conditions obtained as follows:

- If $H \leq H^*$, then if $\sigma \in [\frac{4(1-q)\Pi}{9qe-4q^2e}, +\infty)$ the free entry equilibrium is socially optimum. This is the relevant case to consider when we assume $H = 1$ because $H^* > 1$.
- If $H > H^*$, then if $\sigma \in [\frac{(\frac{1}{4}-q)\Pi}{q(1-q)e}, +\infty)$ the free entry equilibrium is socially optimum.

Depending on the value of H, we define $\sigma^*_{OTC} = \frac{4(1-q)\Pi}{9qe-4q^2e}$ or $\sigma^*_{OTC} = \frac{(\frac{1}{4}-q)\Pi}{q(1-q)e}$. If $H = 1$, then the first definition applies.

Finally, we want to show in what region there is the optimum for hedgers when the free entry equilibrium is not socially optimum. If $\sigma \leq \frac{\Pi}{2qe}$ we compare $E[U^{lr}_{h,c}]$ and $E[U^{lr}_{h,u}]$ we obtain the following condition on σ:

$$\begin{aligned}E[U^{lr}_{h,c}] \geq E[U^{lr}_{h,u}] &\Leftrightarrow \log(1+qe) - \frac{5q\sigma e}{4} \geq \frac{\Pi}{4} + q\log(1+e) \Leftrightarrow \\ \Pi - \frac{5q\sigma e}{4} &\geq \frac{\Pi}{4} \Leftrightarrow \frac{3\Pi}{4} \geq \frac{5q\sigma e}{4} \\ &\Leftrightarrow \frac{3\Pi}{5qe} \geq \sigma. \end{aligned} \tag{54}$$

Because q is a probability, it must be true that the threshold from (54) is greater than the threshold obtained in 49, as shown as follows:

$$\begin{aligned}&\frac{3\Pi}{5qe} \geq \frac{4(1-q)\Pi}{9qe-4q^2e} \Leftrightarrow \frac{3}{5} \geq \frac{4(1-q)}{9-4q} \Leftrightarrow 27-12q \geq 20-20q \Leftrightarrow \\ &8q \geq -7 \Leftrightarrow q \geq -7/8.\end{aligned}$$

Finally, as we have shown before, if $\sigma > \frac{\Pi}{2qe}$, then it must be true that $E[U^{hr}_{h,u}] \leq E[U^{hr}_{h,c}]$. Therefore, $E[U^{hr}_{h,u}]$ can never be an optimum. Therefore, these results can be summarized as follows:

- If $H \leq H^*$, then if $\sigma \in [0, \frac{4(1-q)\Pi}{9qe-4q^2e})$ the social optimum is in the low risk covered region. This is the relevant case to consider when we assume $H = 1$ because $H^* > 1$.
- If $H > H^*$, then if $\sigma \in [0, \min(\frac{\Pi}{(1+H)qe}, \frac{4(1-q)\Pi}{9qe-4q^2e})$ the social optimum is in the low risk covered region. and if $\sigma \in [\min(\frac{\Pi}{(1+H)qe}, \frac{4(1-q)\Pi}{9qe-4q^2e}), \frac{(\frac{1}{4}-q)\Pi}{q(1-q)e})$ the social optimum is in the low risk uncovered region. Note that depending of the value of some parameters, the low risk uncovered region could not exist.

A.3.3 Free entry in the OTC market with novation

As n increases, we are in the covered case. Therefore from (48) in proposition 2, we have that

$$p^{hr}_{c,CCP} = (q-1)\sigma e + \frac{t}{qn} \underset{n\to+\infty}{\to} \underline{p} = (q-1)\sigma e.$$

With free entry by dealers and no risk controls, dealers will enter in the market as long as $E[u^{hr}_{d,c}] \geq 0$, therefore, in the limit we have $E[u^{hr}_{d,c}] \to 0$ and dealers trade $y_c = \frac{H}{n} \to 0$.

Therefore, in the free entry equilibrium, the price is the lowest for all n. Since all hedgers pay the lowest possible price and they are infinitely close to a dealer, then their utility is the highest possible.

A.3.4 Social optimum in the OTC market with novation

We proceed using identical calculations as in the case without novation, but due to the diversification from the CCP, the surplus for hedgers is $\Pi^{CCP}(n)$. First we need to find conditions for the inequality $E[U_{h,c}^{lr}] \leq E[U_{h,c}^{hr}]$ to be satisfied. As we did in (49), we can find the following condition on σ:

$$\frac{4\left(\Pi - q\Pi^{CCP}(+\infty)\right)}{9qe - 4q^2e} \leq \sigma. \tag{55}$$

The covered case in the low risk region exists if

$$\frac{t(1+H)}{\Pi} \leq \frac{t}{q\sigma e} \Leftrightarrow \sigma \leq \frac{\Pi}{(1+H)qe}. \tag{56}$$

We compare the thresholds in (55) and (56) as follows:

$$\frac{4\left(\Pi - q\Pi^{CCP}(+\infty)\right)}{9qe - 4q^2e} \leq \frac{\Pi}{(1+H)qe} \Leftrightarrow \frac{4\left(\Pi - q\Pi^{CCP}(+\infty)\right)}{9 - 4q} \leq \frac{\Pi}{(1+H)} \Leftrightarrow$$

$$1 + H \leq \frac{(9-4q)\Pi}{4\left(\Pi - q\Pi^{CCP}(+\infty)\right)} \Leftrightarrow$$

$$H \leq \frac{(9-4q)\Pi}{4\left(\Pi - q\Pi^{CCP}(+\infty)\right)} - 1 \equiv H^*_{CCP}.$$

Therefore, the comparison between $E[U_{h,c}^{lr}]$ and $E[U_{h,c}^{hr}]$ is only valid when $H \leq H^*_{CCP}$. Note that because $\Pi^{CCP}(+\infty) \geq \Pi$ then $\Pi - q\Pi^{CCP}(+\infty) \leq (1-q)\Pi$ and

$$H^*_{CCP} = \frac{(9-4q)\Pi}{4\left(\Pi - q\Pi^{CCP}(+\infty)\right)} - 1 \geq \frac{(9-4q)\Pi}{4\left(\Pi - q\Pi\right)} - 1 = \frac{5}{4\left(1-q\right)} > 1.$$

Because $\Pi^{CCP}(+\infty) > \Pi$ it must be the case that the following inequality is satisfied:

$$\frac{4\left(\Pi - q\Pi^{CCP}(+\infty)\right)}{9qe - 4q^2e} < \frac{4(1-q)\Pi}{9qe - 4q^2e}.$$

Since we probed in (51) that $\frac{4(1-q)\Pi}{9qe-4q^2e} < \frac{\Pi}{2qe}$, then it is also true that $\frac{4\left(\Pi-q\Pi^{CCP}(+\infty)\right)}{9qe-4q^2e} < \frac{\Pi}{2qe}$.

Since we assume that $\sigma \leq \frac{\Pi}{2qe}$, then for $E[U_{h,u}^{lr}] \leq E[U_{h,c}^{hr}]$ to be satisfied, we need the following condition as we probed in (50):

$$\frac{\frac{1}{4}\Pi - q\Pi^{CCP}(+\infty)}{q(1-q)e} \leq \sigma.$$

Also, because $\Pi^{CCP}(+\infty) > \Pi$ the following inequality is satisfied:

$$\frac{\frac{1}{4}\Pi - q\Pi^{CCP}(+\infty)}{q(1-q)e} < \frac{(\frac{1}{4} - q)\Pi}{q(1-q)e}.$$

Since we probed in (53) that $\frac{(\frac{1}{4}-q)\Pi}{q(1-q)e} < \frac{\Pi}{2qe}$, then it is also true that $\frac{\frac{1}{4}\Pi-q\Pi^{CCP}(+\infty)}{q(1-q)e} < \frac{\Pi}{2qe}$.

Also it is easy to check numerically that

$$\frac{4\left(\Pi - q\Pi^{CCP}(+\infty)\right)}{9qe - 4q^2e} \geq \frac{\frac{1}{4}\Pi - q\Pi^{CCP}(+\infty)}{q(1-q)e} \quad \text{for any } q \in [0,1].$$

Therefore, we can summarize these conditions as follows:

- If $H \leq H^*_{CCP}$, then if $\sigma \in [\frac{4\left(\Pi - q\Pi^{CCP}(+\infty)\right)}{9qe-4q^2e}, +\infty)$ the free entry equilibrium is socially optimum. This is the relevant case to consider when we assume $H = 1$ because $H^*_{CCP} > 1$.
- If $H > H^*_{CCP}$, then if $\sigma \in [\frac{\frac{1}{4}\Pi - q\Pi^{CCP}(+\infty)}{q(1-q)e}, +\infty)$ the free entry equilibrium is socially optimum.

We define $\sigma^*_{CCP} = \frac{4\left(\Pi - q\Pi^{CCP}(+\infty)\right)}{9qe-4q^2e}$ or $\sigma^*_{CCP} = \frac{\frac{1}{4}\Pi - q\Pi^{CCP}(+\infty)}{q(1-q)e}$ depending on the value of H. For the case $H = 1$, the first definition applies.

Note that because $\Pi^{CCP}(+\infty) > \Pi$ then we have that $\sigma^*_{CCP} < \sigma^*_{OTC}$.

To show where is the optimum when the free entry equilibrium is not socially optimum, we proceed like in the no novation case and we arrive to the following result:

- If $H \leq H^*_{CCP}$, then if $\sigma \in [0, \frac{4\left(\Pi - q\Pi^{CCP}(+\infty)\right)}{9qe-4q^2e})$ the social optimum is in the low risk covered region. This is the relevant case to consider when we assume $H = 1$ because $H^*_{CCP} > 1$.
- If $H > H^*_{CCP}$, then if $\sigma \in [0, \min(\frac{\Pi}{(1+H)qe}, \frac{4\left(\Pi - q\Pi^{CCP}(+\infty)\right)}{9qe-4q^2e})$ the social optimum is in the low risk covered region. and if $\sigma \in [\min(\frac{\Pi}{(1+H)qe}, \frac{4\left(\Pi - q\Pi^{CCP}(+\infty)\right)}{9qe-4q^2e}), \frac{\frac{1}{4}\Pi - q\Pi^{CCP}(+\infty)}{q(1-q)e})$ the social optimum is in the low risk uncovered region. Note that depending of the value of some parameters, the low risk uncovered region could not exist.

A.4 Proof of proposition 4

To probe proposition 4, we first show the existence and unicity of a default constrained equilibrium where the default probability constraint $\Pr(p_j < \widetilde{\Delta}_j) \leq \alpha$ must be satisfied.

A.4.1 ***Existence and unicity of default-constrained equilibrium***

For any $\alpha < 1-q$ and assuming that $\sigma \leq \overline{\sigma} = \frac{\Pi}{2qe}$, we know from proposition 2 that the equilibrium for any $n < \overline{n} = \max(n^{hr}_{c,CCP}, n^*_c)$ will satisfy the default constraint with strict inequality because we are in the low risk region (and therefore default probability is zero). Therefore, for any $n < \overline{n}$ the default constrained equilibrium is just the uncovered/covered case shown before.

If $n \geq \overline{n}$ the unconstrained equilibrium does not satisfy the default probability constraint because we are in the high risk region and $\alpha < 1-q$.

Now we probe existence and unicity of this default-constrained equilibrium. For any $\alpha < 1-q$, price must be in the low risk region, therefore the constraint $\Pr\left(p_j < \widetilde{\Delta}_j\right) \leq \alpha$ is satisfied if and only if $p_j \geq \overline{p}$.

First we check if this equilibrium is uncovered. The uncovered equilibrium problem subject to the constraint is

$$\max_{p_j} y_u(p_j)p_j = 2H\frac{\Pi - p_j}{t}p_j$$
$$st$$
$$p_j \geq \overline{p}.$$

The unique optimum of this problem when the constraint does not bind is $\frac{\Pi}{2}$. Since $\overline{p} \leq \frac{\Pi}{2}$ by assumption and the objective function is concave and quadratic, then $\frac{\Pi}{2}$ is the optimum. However, since $n \geq \overline{n}$ this equilibrium cannot exist (because the amount traded by every agent is $> \frac{H}{n}$). Therefore, the default-constrained equilibrium cannot be uncovered.

Now we show existence of constrained covered equilibrium equal to $\overline{p}$. If all other dealers are charging $p = \overline{p}$, then any dealer j has to maximize the low-risk region profits subject to the constraint. If the equilibrium is covered, the optimization problem is

$$\max_{p_j} y_c(p_j, \widehat{p} = \overline{p})p_j = H\frac{-p_j + \overline{p} + \frac{1}{n}t}{t}p_j$$
$$st$$
$$p_j \geq \overline{p}.$$

The objective function is a concave quadratic function with a maximum at $p_j^* = \frac{1}{2}(\overline{p} + \frac{t}{n})$. If $\frac{1}{2}(\overline{p} + \frac{t}{n}) < \overline{p}$ then the unconstrained optimum for dealer j is lower than $\overline{p}$. Therefore, the optimum for dealer j must be $\overline{p}$. This inequality is satisfied when $\overline{p} > \frac{t}{n}$ or $n > n_c^* = \frac{t}{\overline{p}} = \frac{t}{q\sigma e}$, which is satisfied by assumption (because $n \geq \overline{n} = \max(n_c^{hr}, n_c^*)$). At this equilibrium every dealer has utility

$$\frac{H}{n}\overline{p}. \tag{57}$$

Finally, we show this equilibrium is unique. Let assume that there is an equilibrium where all firms set $\widehat{p} > \overline{p}$. This equilibrium would be covered where the constraint is satisfied with strict inequality (low risk region). Therefore, we can apply results from proposition 2. But from proposition 2, for $n \geq \overline{n}$ the (unconstrained) equilibrium is unique and it is in the high risk region. Therefore, $\widehat{p}$ cannot be an equilibrium.

Therefore, if $n \geq \overline{n}$ there exists a unique default-constrained equilibrium $\overline{p}$.

A.4.2 ***Social Optimum with a CCP***

Let assume that $\sigma \leq \frac{\Pi}{2qe}$. For any $\alpha < 1 - q$, we know from proposition 2 that the equilibrium for any $n < \overline{n} = \max(n_c^{hr}, n_c^*)$ will satisfy the default constraint with strict inequality and will be in the low risk region. Also, for $n \geq \overline{n} = \max(n_{c,CCP}^{hr}, n_c^*)$ the equilibrium is default constrained with price equal to $\overline{p} = q\sigma e$. The choice of $\alpha < 1 - q$ and $n = +\infty$ gives a higher utility for dealers than any $n < \overline{n}$ because hedgers are in the low risk region in any case, they are infinitely close to a dealer, and they pay the lowest minimum price $\overline{p}$ to be in the low risk region. In this case, hedgers utility is

$$H\left(\log(1 + qe) - \overline{p}\right). \tag{58}$$

We need to compare this amount with the utility of hedgers when $\alpha \geq 1-q$. If $n < \overline{n}$, we are in the low risk region and as discussed above, the utility is lower than the case $\alpha < 1-q$ and $n = +\infty$. If $n \geq \overline{n}$ then the utility is maximized at $n = +\infty$ and we are in the high risk region, where the hedgers obtain

$$H\left(q(\Pi^{CCP}(+\infty) - (q-1)\sigma e) + q\log(1+e)\right). \tag{59}$$

This is also the utility in the free entry equilibrium. By concavity of the log function, the following inequality must be satisfied:

$$\begin{aligned}\Pi^{CCP}(+\infty) &\equiv \frac{q\log\left(1+qe+e(1-q)^2\right) + (1-q)\log\left(1+qe-eq(1-q)\right) - q\log(1+e)}{q} \\ &< \frac{\log(1+qe) - q\log(1+e)}{q} = \frac{\Pi}{q}.\end{aligned} \tag{60}$$

We need to compare the two values in (58) and (59):

$$\begin{aligned}H\left(\log(1+qe) - \overline{p}\right) \geq H\left(q(\Pi^{CCP}(+\infty) - (q-1)\sigma e) + q\log(1+e)\right) &\Leftrightarrow \\ \log(1+qe) - \overline{p} \geq q(\Pi^{CCP}(+\infty) - (q-1)\sigma e) + q\log(1+e) &\Leftrightarrow \\ \Pi - \overline{p} \geq q\Pi^{CCP}(+\infty) + q(1-q)\sigma e &\Leftrightarrow \\ \Pi - q\Pi^{CCP}(+\infty) \geq +q(2-q)\sigma e &\Leftrightarrow \\ \sigma^{+}_{CCP} \equiv \frac{\Pi - q\Pi^{CCP}(+\infty)}{q(2-q)e} &\geq \sigma.\end{aligned} \tag{61}$$

And because $q\Pi^{CCP}(+\infty) < \Pi$ as we showed in (60), we have $\frac{\Pi - q\Pi^{CCP}(+\infty)}{q(2-q)e} > 0$.

Therefore, when $\sigma \leq \sigma^{+}_{CCP} \equiv \frac{\Pi - q\Pi^{CCP}(+\infty)}{q(2-q)e}$ we have that a CCP that limits trade through setting $\alpha < 1-q$ and $n = +\infty$ maximizes total utility of hedgers. And when $\sigma > \sigma^{+}_{CCP}$ then $n = +\infty$ without ris controls (so any $\alpha \geq 1-q$) maximizes total utility of hedgers.

Note that when $\sigma = 0$ there is never default in any case and the free entry (unconstrained) equilibrium maximizes the utility of hedgers.

A.5 Proof of proposition 5

A.5.1 Case n exogenous, choice of α

By proposition 4, if $n \geq \bar{n}$ and $\alpha^* \in [0, 1-q)$, then the equilibrium price is $\overline{p}$. We want to show that if $n \geq \bar{n}$, then by choosing any value $\alpha^* \in [0, 1-q)$ the dealers are better off. For that, we need to compare the dealers expected utility when they are unconstrained obtained in (42), $\frac{Ht}{n^2}$, with the default-constrained utility obtained in proposition 4 in (57), $\frac{H}{n}\overline{p}$. The condition to be satisfied is

$$\frac{Ht}{n^2} < \frac{H}{n}\overline{p} \Leftrightarrow \frac{t}{n} < \overline{p} \Leftrightarrow n^*_c = \frac{t}{q\sigma e} < n.$$

This condition is satisfied by the assumption made $n \geq \bar{n} = \max(n^{hr}_c, n^*_c)$.

Now we check if the optimum for the dealers is also the best for the hedgers. When $\alpha^* \in [0, 1-q)$, then the equilibrium price is $\overline{p}$. Therefore, the utility of the hedgers is n times the quantity in (43), equal to

$$H\left(\log(1+qe)-\overline{p}-\frac{t}{4n}\right). \tag{62}$$

If $\alpha^* \geq 1-q$ then the utility of the hedgers is obtained from (47) where we have substituted Π by $\Pi^{CCP}(n)$:

$$H\left(q(\Pi^{CCP}(n)-(q-1)\sigma e)+q\log(1+e)-\frac{5t}{4n}\right). \tag{63}$$

By comparing (62) and (63), we obtain the following condition:

$$\begin{aligned}\log(1+qe)-\overline{p}-\frac{t}{4n} \geq q(\Pi^{CCP}(n)-(q-1)\sigma e)+q\log(1+e)-\frac{5t}{4n} \Leftrightarrow \\ \Pi-q\Pi^{CCP}(n)+\frac{t}{n}-\overline{p}+(q-1)\overline{p}\geq 0 \Leftrightarrow \Pi-q\Pi^{CCP}(n)+\frac{t}{n}\geq(2-q)\overline{p} \\ \Leftrightarrow \frac{\Pi-q\Pi^{CCP}(n)+\frac{t}{n}}{(2-q)qe}\equiv\sigma_{CCP}(n)\geq\sigma.\end{aligned}$$

Note that $\sigma_{CCP}(n)>0$, $\sigma_{CCP}(n)$ is decreasing with n, and $\sigma_{CCP}(n=+\infty)=\sigma^+_{CCP}$ from (61) in the previous proposition.

Therefore, when $n \geq \bar{n}$ is exogenous and $\sigma \leq \sigma_{CCP}(n)$ then hedgers are better off with any $\alpha^* \in [0,1-q)$.

A.5.2 Choice of n and α

From the previous section, we can find the optimum α for a given n. If $n \geq \overline{n}$, it is optimum to choose any $\alpha^* \in [0,1-q)$ and from (57) every dealer obtains $\frac{H}{n}\overline{p}$. For $n<\overline{n}$, α^* is irrelevant because we are in the low risk region (we are assuming that $\sigma \leq \frac{\Pi}{2qe}$). In order to obtain the optimum n^* that maximizes every dealer utility, we need to compare every case. From (24), in the uncovered equilibrium every dealer has utility $E[u^{lr}_{d,u}]=\frac{2H}{t}\left(\frac{\Pi}{2}\right)^2$, independent of n. From (37), in the low risk covered case every dealer has utility $E[u^{lr}_{d,c}]=\frac{Ht}{n^2}$, decreasing with n. Therefore the minimum n for the covered equilibrium to exist will maximize the utility. This is $n=\min(n^{lr}_{c,CCP},n^*_c)$. Finally, in the low risk covered case $\frac{H}{\overline{\overline{n}}}\overline{p}$ gives the highest utility.

Now we compare these utilities of the dealers. If $n^*_c \geq n^{lr}_{c,CCP}$ then there is low risk region in the covered equilibrium and this implies

$$n^*_c \geq n^{lr}_{c,CCP} \Leftrightarrow \frac{t}{q\sigma e}\geq\frac{t(1+H)}{\Pi}\Leftrightarrow\sigma\leq\frac{\Pi}{(1+H)qe}.$$

Since by assumption we have $\sigma\leq\frac{\Pi}{2qe}$, then if $H\leq 1$ we have $\frac{\Pi}{2qe}\leq\frac{\Pi}{(1+H)qe}$ and there exists covered equilibrium in the low risk region. Therefore, the minimum n for the covered equilibrium to exist is $n^{lr}_{c,CCP}=\frac{t(1+H)}{\Pi}$. If $E[u^{lr}_{d,c}]$ exists, then it must be that $E[u^{lr}_{d,c}]\geq\frac{H}{\overline{\overline{n}}}\overline{p}$. This is because $E[u^{lr}_{d,c}]$ exists at smaller n and $p^{lr}_c\geq\overline{p}$. Using similar arguments, we have that $E[u^{lr}_{d,u}]\geq\frac{H}{\overline{\overline{n}}}\overline{p}$.

Therefore, we just need to compare $E[u^{lr}_{d,c}]$ at $n^{lr}_{c,CCP}$ with $E[u^{lr}_{d,u}]$:

$$E[u^{lr}_{d,c}] \geq E[u^{lr}_{d,u}] \Leftrightarrow \frac{\Pi^2 H}{t(1+H)^2} \geq \frac{2H}{t}\left(\frac{\Pi}{2}\right)^2 \Leftrightarrow \frac{1}{(1+H)^2} \geq \frac{1}{2}$$
$$\Leftrightarrow \sqrt{2} \geq 1 + H \Leftrightarrow H \leq \sqrt{2} - 1 \approx 0.41. \quad (64)$$

Therefore, if $H \in (0, \sqrt{2}-1]$ then, $n^{lr}_{c,CCP}$ maximizes the utility of every dealer, and if $H \in (\sqrt{2}-1, 1]$, n^{lr}_u maximizes the utility of every dealer. Also, if $H > 1$ and $\sigma \leq \frac{\Pi}{(1+H)qe}$, then from the inequality (64), we have $E[u^{lr}_{d,c}] < E[u^{lr}_{d,u}]$ so the uncovered low risk equilibrium is optimal.

Finally, if $H > 1$ and $\frac{\Pi}{(1+H)qe} \leq \sigma \leq \frac{\Pi}{2qe}$ we have $n^*_c < n^{lr}_{c,CCP}$ and therefore there is not low risk equilibrium in the covered case. However, since we have previously shown that, $E[u^{lr}_{d,u}] \geq \frac{H}{\overline{n}}\overline{p}$, then n^{lr}_u gives the highest utility for dealers.

Since we have shown that the optimum is in either n^{lr}_u or $n^{lr}_{c,CCP}$, the choice of α is irrelevant because α is only relevant in the high risk covered region.

B Demand of hedgers when dealers cannot hedge the hedgers' risk

Let assume in this section that dealers cannot trade the swap contract $\widetilde{s}$ in the outside market. The utility of hedger trading with a dealer j that does not default is

$$\log(1 + \widetilde{e} + \widetilde{s}) - p_j - d_j t = \log(1 + qe) - p_j - d_j t,$$

where $\widetilde{s} = qe - \widetilde{e}$. Dealer defaults when $p_j < \widetilde{s}$. If $p_j \geq qe$ the dealer never defaults. If $p_j \in [(q-1)e, qe)$ the dealer defaults when $\widetilde{e} = 0$ (and therefore $\widetilde{s} = qe$) which occurs with probability $1 - q$. In that case, the utility of the hedger is

$$\log(1) - d_j t = -d_j t.$$

This is the key difference with the case where dealers can hedge the hedgers' risk. Because the variable $\widetilde{\Delta}_j$ is incorrelated with $\widetilde{e}$, then dealer j defaults with exogenous probability and when there is a default,, the hedger obtains $q \log(1+e) + (1-q)\log(1) = q\log(1+e) > 0$. However, in the case of no hedging, dealer defaults when $\widetilde{e} = 0$ (when the outcome is negative for the dealer) and the hedger obtains $\log(1) = 0$. Therefore, the expected utility of the hedger when $p_j \in [(q-1)e, qe)$ is

$$\begin{aligned} E[u_h(p_j)] &= (\log(1+qe) - p_j) \cdot N_j - d_j t \\ &= q\left(\Pi + q\log(1+e) - p_j\right) - d_j t. \end{aligned}$$

Therefore, the utility of the hedger in every region can be written as

$$E[u_h(p_j)] = \begin{cases} \Pi + q\log(1+e) - p_j - d_j t & if \quad p_j \geq qe \\ q\left(\Pi + q\log(1+e) - p_j\right) - d_j t & if \quad p_j \in [(q-1)e, qe) \\ q\log(1+e) - d_j t & if \quad p_j < (q-1)e \end{cases}$$

We can compare the utility in the high risk region with the case where dealers can hedge the risk from hedgers:

$$q\left(\Pi + q\log(1+e) - p_j\right) - d_j t \leq q\left(\Pi + \log(1+e) - p_j\right) - d_j t \Leftrightarrow$$
$$q\log(1+e) \leq \log(1+e) \Leftrightarrow q \leq 1.$$

Since it is always true that $q \leq 1$, then in the high risk region, hedgers are worse off when dealers do not trade the risk from hedgers.

The number of trades is given by the number of hedgers that get a higher value than the outside value of no trading, $q\log(1+e)$. Note that in the region $p_j \in [(q-1)e, qe)$, we have

$$q\left(\log(1+qe) - p_j\right) - d_j t \geq q\log(1+e) \Leftrightarrow$$
$$q\frac{\log(1+qe) - \log(1+e) - p_j}{t} \geq d_j.$$

Since $\log(1+qe) - \log(1+e) \leq 0$ for any $q \leq 1$, it is necessary that $p_j < 0$ and negative enough in order to have a strictly positive number of hedgers trading with dealer j ($d_j > 0$). Hedgers need to be paid to buy the contract in order to have a positive number of hedgers willing to trade with a dealer. Therefore, the number of swap contracts bought by the hedgers to a dealer for a given price p_j in the uncovered equilibrium when dealers do not hedge the risk is

$$y_{u,nh}(p_j) = \begin{cases} \frac{2H}{t}\max\left(\Pi - p_j, 0\right) & if \quad p_j \geq qe \\ 0 & if \quad p_j \in [\log(1+qe) - \log(1+e), qe) \\ \frac{2qH}{t}\max\left(\log(1+qe) - \log(1+e) - p_j, 0\right) & if \quad p_j \in [(q-1)e, \log(1+qe) - \log(1+e)) \\ 0 & if \quad p_j < (q-1)e \end{cases}$$

Since $\log(1+qe) - \log(1+e) < 0 < \Pi$, we have $y_{u,nh}(p_j) < y_u(p_j)$ for any price p_j. This explain why dealers need to hedge the risk of hedgers. Hedging helps increasing the demand of contrats bought by hedgers.