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by Nellie Zhang

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Abstract

This paper applies a static model of an interest rate corridor to the Canadian data, and estimates the aggregate demand for central-bank settlement balances in the Large Value Transfer System (LVTS). The empirical specification controls for various calendar effects that have been shown to cause fluctuations in LVTS payment flows. The analysis takes into account the downward divergence of the overnight interest rate from the target rate, which has been persistent since 2005. The results suggest that a target of $3 billion for LVTS settlement balances does not seem excessive during the time period when Canadian monetary policy was operating at the effective lower bound (ELB). Specifically, the model projects that, if the consistent downward divergence of overnight interest rate is taken into account, then on average $2.405 billion of LVTS settlement balances would probably have been sufficient to achieve the goal of keeping the overnight interest rate at or very close to the lower bound of the corridor. However, by targeting a slightly higher level, the Bank of Canada could be 95% certain that the overnight interest rate would on average not exceed its policy rate at the lower bound of the corridor. In addition, the estimation shows that the point elasticity of overnight interest rate is around 0.17 when the daily level of settlement balances is targeted at $3 billion under the ELB framework.

JEL classification: G01, E40, E50, C36
Bank classification: Interest rates; Monetary policy implementation; Payment, clearing, and settlement systems

Résumé

L’auteure applique aux données canadiennes un modèle statique formalisant un corridor de taux d’intérêt en vue d’estimer la demande totale de soldes de règlement au sein du Système de transfert de paiements de grande valeur (STPGV). La spécification empirique retenue neutralise différents effets de calendrier reconnus à l’origine de fluctuations dans les flux de paiement du STPGV. L’analyse tient compte de la divergence à la baisse du taux du financement à un jour qu’on observe depuis 2005 par rapport au taux visé. D’après les résultats obtenus, un niveau cible de 3 milliards de dollars pour les soldes de règlement durant la période où le taux directeur au Canada se situait à sa valeur plancher ne paraît pas excessif. Plus précisément, si l’on considère la divergence à la baisse persistante du taux à un jour, des soldes moyens de 2,405 milliards de dollars auraient probablement été suffisants, selon le modèle, pour que le taux à un jour se maintienne à la limite inférieure du corridor ou très près de celle-ci. Toutefois, en visant un montant légèrement supérieur, la Banque du Canada pouvait être sûre à 95 % que le taux à un jour ne dépasserait pas, en moyenne, le taux directeur (la limite inférieure du corridor). L’estimation montre en outre que l’élasticité du taux à un jour s’établit à environ 0,17 quand le niveau quotidien visé pour les soldes de règlement est de 3 milliards de dollars et que le taux directeur se trouve à sa valeur plancher.

Classification JEL : G01, E40, E50, C36
Classification de la Banque : Taux d’intérêt; Mise en œuvre de la politique monétaire; Systèmes de paiement, de compensation et de règlement

1 Introduction

Nowadays, most central banks implement monetary policy through control of overnight interest rates. It is usually the case that the framework for the implementation of monetary policy is tightly connected with the payment system operated and/or overseen by the central bank, because the financial institutions who directly transact in the payment system ultimately settle their final funds positions on the books of the central bank. Being the ultimate supplier of settlement balances (central bank money), the central bank is in a unique position to exert its influence on the overnight interest rate through its influence on the market for settlement balances. This paper estimates the aggregate demand for settlement balances in the Canadian Large Value Transfer System (LVTS) that has been a central setting for the conduct of monetary policy in Canada for more than a decade.[1] The focus of interest is to answer the question of how big the supply of settlement balances should be when the central bank wants to target the overnight interest rate at a very low level, for instance, during economic downturns; and how much is too much?

The question becomes particularly interesting in the context of interest-rate corridors, which many central banks around the world adopt for the implementation of monetary policy, including the Bank of Canada.[2] A pure interest-rate corridor (also known as "tunnel" or "channel") system, in contrast to a period-average reserve regime that relies on traditional reserves requirements to help achieve a target interest rate, utilizes standing facilities to control the overnight interest rate. Specifically, the central bank's standing facilities establish an operating band that contains the movements of the overnight rate; the interest rate on overnight overdraft loans from the central bank provides a ceiling for the overnight rate, while a floor is provided by the interest rate paid on the deposits held overnight at the central bank. With the overnight interest rate contained within the corridor, the downward-sloping aggregate demand curve (for settlement balances) takes an inverted S-shape that becomes increasingly flatter near both the top and bottom boundaries.

Hence, if a central bank wants to exploit the elastic region of the demand curve (where changes in quantities of settlement balances have little impact on the overnight rate), by setting its target for the overnight rate at the bottom

[1]For details about LVTS operations and risk controls, please see Arjani and McVanel (2006).

[2]Countries who are currently using a rate corridor regime include Australia, Mexico, New Zealand and Sweden.

of the corridor, how can it determine the proper level of settlement balances needed to achieve the target rate?

The Canadian experience with a rate corridor regime provides a fitting context and necessary data for examining this question. In spring 2009, in order to preserve the effective functioning of financial markets in a low interest rate environment, the Bank of Canada made several adjustments to its framework for the implementation of monetary policy, which were to remain in effect for a designated period of time. The key changes include: (i) setting the target for the overnight interest rate at the bottom of the operating band as opposed to the midpoint, as at normal times; (ii) narrowing the operating band from 50 basis points previously to 25 basis points; (iii) targeting a daily level of settlement balances in the LVTS at $3 billion rather than a small positive amount (e.g. $25 million). With the first two adjustments, the Bank of Canada was setting an effective lower bound (ELB) on the overnight interest rate; and by providing a significantly more-than-required amount of settlement balances, the Bank of Canada created competition among LVTS participants in loaning out surplus funds at the end of each day, which successfully drove the overnight interest rate quite close to the lower bound of the rate corridor.

Using a generalized linear model regression, this paper aims to estimate the aggregate demand for settlement balances in LVTS and uncover whether or not a large supply of settlement balances such as $3 billion is more than is needed for the Canadian overnight rate to trade at the bottom of the corridor.

The main results of this study suggest that a target level of $3 billion for LVTS settlement balances does not seem excessive in the ELB framework. Taking into account the fact that the average interest rate in Canadian overnight market has been consistently drifting below the Bank of Canada's target rate for the past few years, the regression model projects that $2.405 billion of LVTS settlement balances would probably have been sufficient to achieve the goal of keeping the overnight interest rate at or very close to the lower bound of the corridor. However, by targeting a slightly higher level, the Bank could be 95% certain that the overnight rate would on average not exceed its policy rate at the lower bound of the corridor. In addition, the estimation shows the point elasticity of the overnight rate is around 0.17, when the daily level of LVTS settlement balances is targeted at $3 billion under the ELB framework.

There is plenty of previous work in the literature on both theoretical modelling and empirical estimation of the demand for settlement balances. Empirical analyses are represented by the *normal mixture* time-series models first explored by Hamilton (1996). These models capture observed features of the overnight market for central-bank reserves. Hamilton (1997) followed by studying the liquidity effect of central-bank open market operations on overnight interest rate, which is measured by the slope of the demand curve for central bank funds. One of the theoretical models is proposed by Berentsen and Monnet (2008). The authors developed a dynamic general equilibrium framework for an interest-rate corridor system, and found that such systems require characterization of any monetary policy to be highly consistent with the system properties. This paper, however, draws heavily on the theoretic model proposed by Whitesell (2006), which is elaborated in Section 3 hereinafter.

The rest of this paper proceeds as follows. The next section provides a brief illustration of an interest-rate corridor system and institutional background for Canadian monetary policy implementation framework. Section 3 reviews the interest-rate corridor model proposed by Whitesell (2006). A description of data to which the model is applied and the empirical estimation can be found in Section 4.1. Regression details are provided in Section 4.2 and estimation results are presented in Section 4.3. Conclusions follow in Section 5.

2 Institutional Background

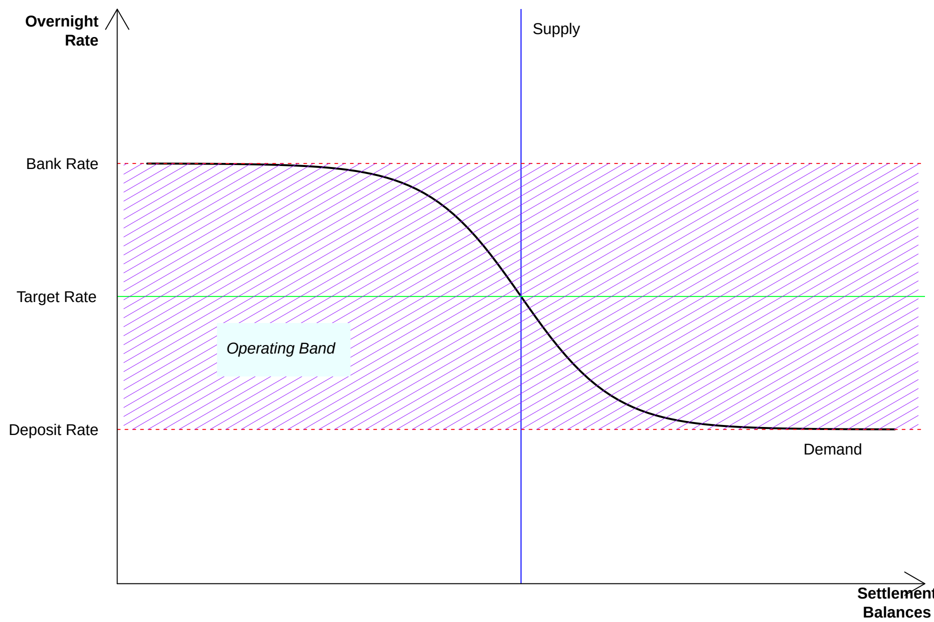
2.1 A Rate Corridor System

In general, there are three main elements in a rate corridor system: the central bank's target for the overnight interest rate and the two boundaries of the corridor. The upper limit of the corridor is usually the interest rate offered by the central bank's standing lending facility (SLF), known as the *Bank Rate* in Canada; the lower limit is the rate of interest that financial institutions earn upon their deposits of excess settlement balances at the central bank, i.e. the *Deposit Rate*.[3] The spread between the two boundaries is often known as the *operating band*. The prices the central bank sets on deposits and overdraft loans create strong incentives for the movements of the overnight interest rate to stay within the corridor. *Figure 1* provides an illustration of an interest-rate corridor. Typically, the central bank sets its target for overnight rate at the midpoint of the operating band, to create symmetric opportunity costs around the target rate. In Canada, based on current market conditions and its outlook

[3] In this paper, a "financial institution" is more specifically referring to a financial organization that has access to central bank liquidity facilities.

for inflation, the Bank of Canada determines its target for the overnight rate, the positioning and the width of the operating band, and announces its decision on fixed dates each year.

Figure 1: Interest Rate Corridor Regime



Consider the interest-rate corridor structure from a theoretical perspective of the dynamics between supply and demand of settlement balances. As illustrated in *Figure 1*, one can picture a vertical supply curve at the level of settlement balances provided by the central bank; and a generally downward-sloping demand curve for aggregate settlement balances being bounded within the two limits of the corridor. The demand curve is steepest at the midpoint of the corridor and becomes increasingly flatter near the two boundaries. Specifically, as the overnight interest rate falls, the quantity of settlement balances demanded rises; when it is approaching the floor of the corridor, the quantity of settlement balances demanded can be anywhere between a very large number and infinity (because of the close-to-perfect interest elasticity of demand near the lower bound.)

In principle, a central bank may prefer an interest-elastic demand function, because the interest rate would not be significantly affected by a relatively large error in the supply of settlement balances, if it occurred. Given an inverted S-shaped demand curve, if a central bank wants to benefit from the interest-elastic region of the demand, it can lower its target rate from the midpoint of the operating band to the floor, and simultaneously supply excess settlement balances that is large enough to help drive the overnight rate to the bottom of the corridor.

2.2 Implementation of Monetary Policy in the LVTS

The Bank of Canada typically sets a target rate (i.e. its policy rate) at the midpoint of a 50-basis-point operating band. Changes in this target for the overnight rate, through a transmission mechanism, affect other longer-term market rates, the foreign exchange rate, the aggregate demand and ultimately inflation.[4]

The Canadian experience shows that without imposing administrative burdens on financial institutions related to reserve maintenance, a pure rate corridor regime can be highly effective in keeping the overnight interest rate quite close to the central bank's policy rate, as well as in providing liquidity to the money market. The difference between the reference overnight interest rate and the target rate is considerably lower in Canada than similar measures in other countries who adopt a rate corridor with reserve requirement.[5]

In response to the substantial stress in money markets during the recent global financial turmoil, the Bank of Canada lowered its target for the overnight interest rate several times, while keeping the spread of the operating band at 50 basis points and the target rate at the midpoint. In addition, given the increased demand for overnight liquidity during this period, the Bank of Canada also periodically increased the level of settlement balances. On April 21^{st}, 2009, overall conditions in money markets improved notably as the financial turbulence began to slowly subside. To preserve the effective functioning of markets in a low interest rate environment, the Bank of Canada adjusted its framework for the implementation of monetary policy by setting an effective lower bound of 25 basis points on the overnight interest rate. As part of this conditional commitment (conditional on inflation outlook and scheduled to be withdrawn in June 2010), the Bank of Canada determined to provide excess

[4]For a more detailed account of the operational framework for the implementation of monetary policy in Canada, please see Engert, Gravelle, and Howard (2008).

[5]Both Whitesell (2006) and Engert, Gravelle, and Howard (2008) show a comparison of the volatility of overnight interest rates between Canada and other countries.

settlement balances to reinforce the target rate at the bottom of the operating band, by setting a daily target level of settlement balances in the LVTS at $3 billion, which is unprecedentedly high compared to a small amount of $25 to $50 million at normal times.[6]

3 The Model

3.1 The Typical Setting

Whitesell (2006) developed a static model for interest-rate corridor systems, and this analysis is based mainly on this theoretical representation.

The basic setup of the model is that a financial institution usually has an approximate idea about its target end-of-day funds position, T, however, not with perfect certainty. Presumably, there is a random liquidity shock ε that occurs during the latter part of the day and can be attributed to events such as unanticipated payment requests from its clients, or accounting-induced delay. The stochastic estimation error is assumed to have a zero mean, i.e. $E(\varepsilon) = 0$. Financial institutions trade with each other at the market rate i in the overnight money market.

Under normal economic conditions where the central bank target rate i^* is typically set at the midpoint between the central bank lending rate and deposit rate, then a representative financial institution, with no prior knowledge of ε, faces a problem of minimizing two types of costs: (i) the opportunity cost of depositing a positive settlement balance at its central bank account versus earing a higher rate i in the market; that is $(T+\varepsilon)\times[i-(i^*-s)]$, where $T+\varepsilon > 0$; s is half of the spread between the central bank lending rate and deposit rate. (ii) the additional cost of taking overnight advances from the central bank to fulfil its negative settlement obligations as opposed to borrowing from the market; that is $(T + \varepsilon) \times (i^* + s - i)$, where $T + \varepsilon < 0$.[7]

[6]Other actions taken by the Bank of Canada include: narrowing the width of the operating band from 50 basis points to 25 basis points, temporarily expanding the list of eligible collateral for *Special Purchase and Resale Agreements* transactions, engaging in multiple rounds of repos to inject extra overnight liquidity, and launching *term Purchase and Resale Agreements* operations, etc.

[7]There are two other main assumptions in support of the model. First, a typical financial institution is indifferent at the margin between dealing with the central bank directly and trading with other financial institutions in the market. Second, the lending and deposit facilities offered by the central bank are perfect substitutes for the corresponding inter-bank transactions in the overnight market. For more details about the model specification, please see Whitesell (2006).

The first-order condition for this minimization problem is derived as follows:

$$F(-T^*) = \frac{1}{2} + \frac{i - i^*}{2s} \tag{1}$$

where $F(\cdot)$ is the cumulative distribution function of the later-day liquidity shock ε; T^* is the optimal amount of settlement balance for a typical financial institution.

Therefore, a financial institution's demand for central bank settlement balances is simply given by the inverse of *equation* (1). And the aggregate demand in the system, denoted by D, is thus obtained by summing the individual demand over all the participants who have access to the central bank liquidity facilities, indexed by j.

$$D = \sum_j T_j^* = -\sum_j F_j^{-1}\left(\frac{1}{2} + \frac{i - i^*}{2s}\right) \tag{2}$$

If the liquidity shock ε is assumed to be normally distributed, with zero mean and non-zero variance σ^2, i.e. $\varepsilon \sim \Phi(0, \sigma^2)$ and $F(-T^*) = \Phi(-T^*/\sigma)$, then the aggregate demand for central bank settlement balances can be written as:[8]

$$D = -\Phi^{-1}\left(\frac{1}{2} + \frac{i - i^*}{2s}\right)\sum_j \sigma_j \tag{3}$$

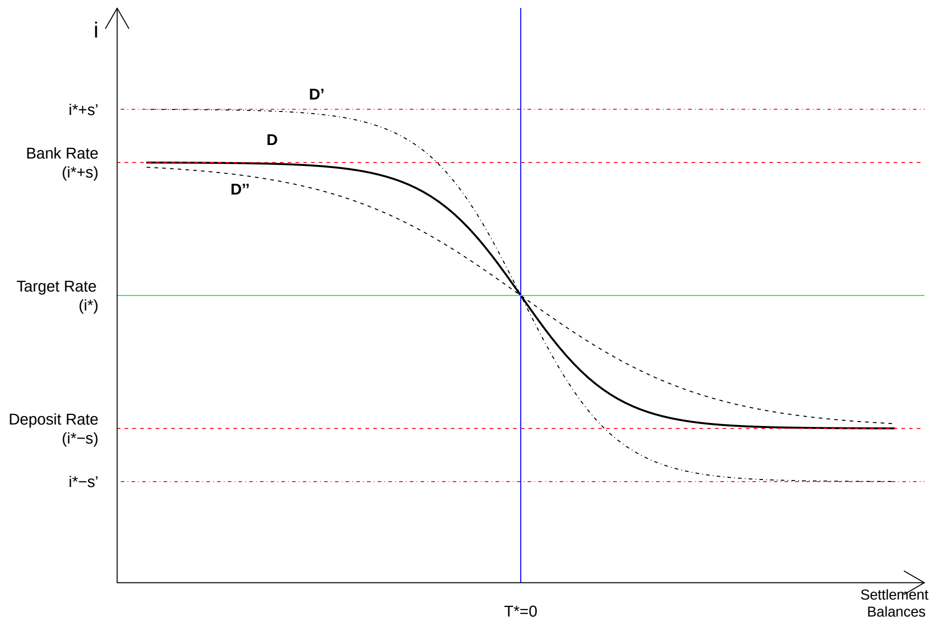
where σ_j^2 represents the uncertainly of end-of-day liquidity shock faced by financial institution j; Φ is the cumulative distribution function (CDF) for the normal distribution.

In light of *equation* (3), the relationship between the overnight interest rate and the quantity of aggregate demand for central bank settlement balances can be illustrated by *Figure 2*. The generally downward-sloping demand curve becomes increasingly flatter near both boundaries of the corridor. This suggests that the demand for settlement balances can be positive or negative infinity when the overnight interest rate asymptotically approaches the central bank's lending and deposit rates. The vertical line represents the supply of settlement balances, solely controlled by the central bank.

Also shown in *Figure 2*, as the spread between the central bank's lending

[8] It is recognized that the normality assumption may not be well justified by the central limit theorem in the cases where there are only a small number of financial institutions who settle their transactions in the books of the central bank. However, it is a simple natural start in the absence of a better assumption. Future work can explore alternative choices for the distribution of the liquidity shock.

Figure 2: Interest Rate Corridor - Financial Institution's Demand for Central Bank Settlement Balances



Note: curve D': increase in spread of the corridor; curve D'': increase in uncertainties about end-of-day liquidity shocks.

and deposit rates widens, e.g. $s' > s$, the demand curve D expands vertically to position D', and the interest-rate elasticity of demand decreases near the target interest rate. Likewise, if the uncertainties about liquidity shocks rise in the system, the demand curve will be stretched horizontally to D'' and become relatively more elastic in the middle.

The Whitesell (2006) model proves one advantage of adopting a pure interest-rate corridor system; that is, as long as the central bank targets the overnight interest rate at the midpoint of the operating band, in theory, the demand for central bank settlement balances should always be close to zero. In addition, the implementation of this corridor framework turns out to be remarkably simple, because several variables become inconsequential and can be eliminated from consideration. The premise of this advantage is the assumption of liquidity shocks having a symmetric distribution.

To see this, if $F(\cdot)$ in *equation* (1) is a symmetric distribution with zero mean, then T^* must equal zero when $i = i^*$. In other words, the central bank can freely move its target interest rate and/or adjust the spread of the operating band, without worrying about any potential effects of these changes on the quantity of demand for settlement balances. Furthermore, by establishing symmetric opportunity costs around its target interest rate, the central bank can spare the effort of frequently re-estimating the demand curve that will be shifted by the constantly changing liquidity uncertainties in the system.

However, one main drawback of an interest-rate corridor system is the intricacy of guaranteeing symmetric opportunity costs around the central bank's target rate, which can be attributed to the imperfect substitutability between central-bank settlement balances and the private supply of broad liquidity, transaction costs and frictions in the overnight market, etc. Central bank overnight advances are not completely equal alternatives to market transactions, because interbank loans are in general uncollateralized whereas borrowing from the central bank usually requires good security. Therefore, the actual market rate may carry a premium reflecting collateral cost and/or credit risk adjustments. Transaction frictions and costs also weaken the effectiveness of an interest-rate corridor system. For example, in Canada, central bank liquidity facilities are only accessible to a small number of direct participants in the LVTS. The majority of financial institutions do not have this option of choosing whether to deal with the central bank in overnight funding or not; hence, they are more likely to borrow overnight funds from the LVTS participants at a higher interest rate than the higher-tier interbank lending rate.

One solution for the central bank to overcome this difficulty is to supply a small positive amount of settlement balances to the financial system, rather than zero quantity. In fact, the Bank of Canada has been adopting this approach for many years. Every day, the Bank intentionally leaves a small amount of excess settlement balances in the LVTS, so that any participant who is in a net negative funds position at the end of day knows with certainty that there is at least one other system member holding an offsetting surplus position. It effectively lowers the possibility of LVTS participants constantly taking small quantities of overnight advances from the Bank of Canada, when they would rather absorb the interest-rate differential and other costs than search for trading partners in the market.

Another limitation of a rate corridor system is the relatively inelastic demand near the central bank target rate, which suggests that a small error in central bank's supply of settlement balances can have a great impact on the overnight interest rate. This is usually the case, but not always. For example, the target rate can be set at the lower bound of the corridor where the demand curve is almost perfectly elastic.

3.2 Targeting at the Lower Bound

An upsurge in demand for liquidity usually occurs during economic downturns due to higher investor risk aversion in financial markets. Financial institutions hoard more liquidity as a precaution and show increasing reluctance to lend. Individual depositors also tend to hold more central bank bank notes and less interest-bearing instruments.

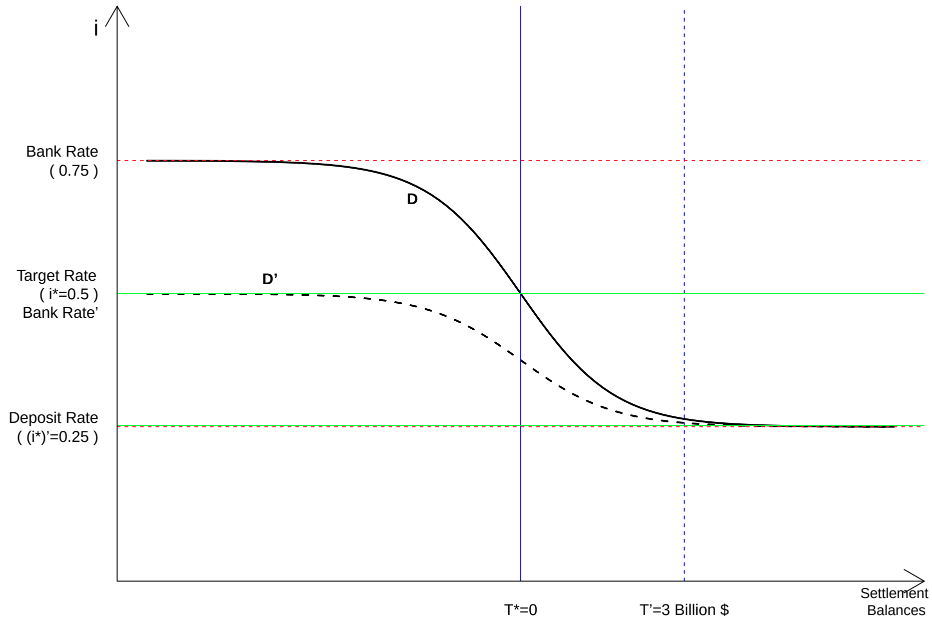
High demand drives up the market prices for money. In order to keep the economy effectively functioning in a low interest rate environment during difficult times, a central bank could target the overnight interest rate at the lower bound of the corridor, by supplying a substantially large amount of excess settlement balances to major financial market participants (also known as implementation of a "floor system"). In addition to a downward pressure on the overnight rate, significant holdings of excess central bank balances in the banking system can help improve market conditions in some other ways. First, they provide additional stimulus to aggregate demand by encouraging major financial market participants to purchase financial assets and/or to increase lending to businesses and households; second, with the overnight funds market trading near the corridor lower bound, the central bank will be able to take advantage of the interest-elastic part of the demand curve.

During the most recent global economic recession, The Bank of Canada adopted such a floor system to help relieve pressures in short-term funding markets. It narrowed the width of the corridor from 50 to 25 basis points, lowered the overnight target rate to the deposit rate of 25 basis points, and tremendously expanded the target supply of LVTS settlement balances from $25 million to $3 billion dollars. As illustrated in *Figure 3*, these adjustments together increased the interest-elasticity of demand for settlement balances.[9]

According to the Whitesell (2006) model, if the central bank targets the overnight interest rate at the lower bound of the operating band, a typical financial institution will face a problem of minimizing two types of costs that are slightly different from those in the normal setup: (i) $(T+\varepsilon) \times [i - i^*]$, when $T + \varepsilon > 0$; (ii) $(T + \varepsilon) \times (i^* + 2s - i)$, when $T + \varepsilon < 0$. And the aggregate

[9]During the financial crisis, the Bank of Canada also took many other actions to mitigate the substantial stress in the money market, such as conducting multiple rounds of repo operations, temporarily expanding the list of eligible collateral for SPRA transactions, launching term PRA operations, and temporarily accepting non-mortgage loan portfolios and US treasury bills as eligible collateral for SLF advances, etc.

Figure 3: Change of Demand for LVTS Settlement Balances in a "Floor System" of Canada during the 2007 – 2008 Global Economic Recession



Note: In a *floor system*, the interest rate paid on deposits at the central bank is pegged at the target overnight rate.

demand function becomes:

$$D = \sum_j T_j^* = -\sum_j F_j^{-1}\left(\frac{i - i^*}{2s}\right) \quad (4)$$

Note that in *equation* (2), the demand for settlement balances in a typical corridor system can be rewritten as:

$$D = \sum_j T_j^* = -\sum_j F_j^{-1}\left(\frac{i - (i^* - s)}{2s}\right)$$

Recall that s is half of the width of the operating band; therefore, if the central bank's target rate i^* is situated at the mid-point of the band, then $i^* - s$ is equal to the deposit rate at the lower bound. In the case of a floor system, by definition, i^* itself is the deposit rate.

Let r^d denote the interest rate on deposits at the central bank, and r^b be the cost of borrowing from the central bank (e.g. the Bank Rate in Canada).

equation (2) and *equation* (4) can thus be generalized to the following form.

$$D = \sum_j T_j^* = -\sum_j F_j^{-1}\left(\frac{i - r^d}{r^b - r^d}\right)$$

And, if random liquidity shocks are assumed be normally distributed, then

$$D = -\Phi^{-1}\left(\frac{i - r^d}{r^b - r^d}\right)\sum_j \sigma_j \tag{5}$$

3.3 Downward Movements of the Overnight Interest Rate

The incentive mechanism of an interest-rate corridor should in theory tremendously reduce the likelihood of overnight interest rate permeating the policy floor formed by the interest rate paid on deposits at the central bank. However, during the recent global financial turmoil, deviations from the theoretical expectation are observed by several central banks around the world that essentially implemented a floor system (e.g. the Federal Reserve, Bank of England, Bank of Canada, and Bank of Japan, etc.). The negative spread between the average overnight interest rate and the central bank deposit rate is generally small in most cases, but a few countries show higher frequencies of occurrence and more significant magnitude of the breach.[10]

Many articles in the literature have looked into this puzzle and found three main possible explanations. First, not all overnight market participants have access to the central bank deposit facility; this limited eligibility leads to a segmentation in the overnight funds market and gives the financial institutions who are eligible to lend to the central bank market power to extract borrowing rates that are lower than the policy deposit rate, from the funds lenders without dual choices. And yet, measures of the overall average cost of overnight funding are often broadly computed from overnight transactions that involve both types of market participants.

Second, there is a lack of arbitrage of the spread between the overnight interest rate and the central-bank deposit rate, which foregoes the chances of bringing the overnight rate back within the rate corridor. Bech and Klee (2009) argue that the unwillingness and/or incapability of financial institutions to conduct arbitrage is possibly related to balance-sheet management and/or constraints.

[10] Detailed cases studies of these central banks' experiences can be found in the discussion paper by Bowman, Gagnon, and Leahy (2010).

Third, due to increases in perceived credit risk and concerns about other market developments during the financial crisis, the frequent lenders in the overnight funds market may have narrowed down their lists of potential business partners. This contraction of credit can weaken these lenders' bargaining power with the remaining funds borrowers, which results in a lower lending rate than otherwise.

In a floor system, the interest rate paid on deposits at the central bank is pegged at the target policy rate. In fact, as the Canadian experience shows, the downward movements in the overnight interest rate are not exclusive to the implementation of a floor system. Since the beginning of 2005, the interest rate in the Canadian overnight market has been consistently drifting below the Bank of Canada's target rate, which was always set at the midpoint of the operating band prior to April 2009. Reid (2007) suggests that the main factor that drives this persistent downward divergence might be the growing demand for collateral and the consequent behavioral changes in market participants' collateral management.

In Canada, two measures of the collateralized overnight interest rate are used as official estimates of overall conditions in the overnight funds market; and the Government of Canada marketable debt (collectively referred to as "general collateral" (GC)) constitutes a major component in the collateral used in the Canadian overnight market. High demand for the collateral that underlies a repo transaction generally leads to a lower repo rate. In particular, the market participants who provide GC (that is in higher demand than other types of securities) as collateral for an overnight loan of funds are able to obtain lower borrowing rates. Furthermore, lending excess balances to the central bank is risk free and thus never *collateralized*. In the presence of high demand for collateral, financial institutions are likely willing to loan out their surplus funds in the overnight market at a lower interest rate (than the central bank deposit rate), in exchange for holdings of collateral that are much needed in other financial transactions.[11]

Repeated incidents of downward movements of the overnight interest rate suggest that the actual overnight rate i (i.e. not influenced by any downward

[11]Broadly speaking, the increase in demand for collateral results from more and more extensive use of it in the past ten years. For instance, collateral management has become an increasingly important part of risk management; collateral is required to control the counterparty risks in the emerging financial derivatives markets; the amount of collateral pledged has been fast growing in major settlement and payments systems, e.g. the LVTS, the Canadian Depository for Securities Limited (CDS) and the CLS Bank. For detailed discussions, please see Reid (2007).

pressure from the factors discussed above) should have been equal to the observed overnight rate $\tilde{i}$ plus the deviations δ between the two, $\delta = i - \tilde{i}$. Rewrite the function of aggregate demand for settlement balances (under the assumption of normal liquidity shocks, i.e. *equation* (5)) in terms of the observed overnight rate.

$$D = -\Phi^{-1}\left(\frac{\tilde{i} + \delta - r^d}{r^b - r^d}\right)\sum_j \sigma_j \tag{6}$$

An alternative hypothesis leads to the same re-illustration of the model. That is, due to the consistent downward pressure on the overnight interest rate, the entire operating band is shifted down by an amount δ, with the *effective* cost of borrowing from the central bank being $r^b - \delta$ and the actual deposit rate being $r^d - \delta$. Thus, the aggregate demand for central bank balances, bounded by this shifted operating band, is also given by *equation* (6).

$$\begin{aligned} D &= -\Phi^{-1}\left(\frac{\tilde{i} - (r^d - \delta)}{(r^b - \delta) - (r^d - \delta)}\right)\sum_j \sigma_j \\ &= -\Phi^{-1}\left(\frac{\tilde{i} + \delta - r^d}{r^b - r^d}\right)\sum_j \sigma_j \end{aligned}$$

This slightly adjusted demand function provides a feasible solution to overcoming the difficulty of applying all of the data to the model later in the regression analysis.

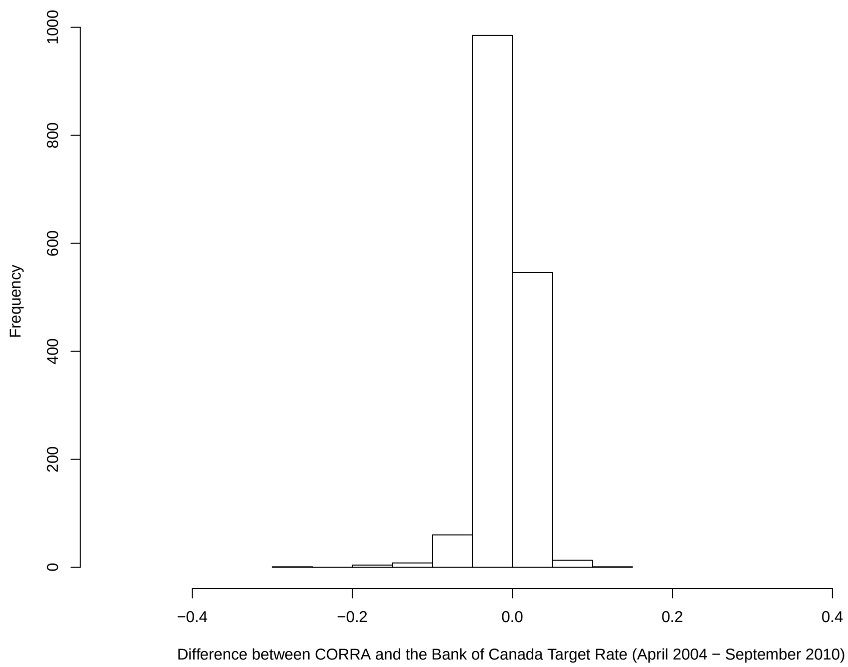
4 Empirical Estimation

4.1 Data Description

The empirical analysis is based on the Canadian experiences of implementing an interest-rate corridor framework during the period of April 1, 2004 to September 16, 2010. The beginning of the sample is determined by the earliest date from which the transactional-level LVTS data is available. The payment flows are needed to compute the actual level of settlement balances in the LVTS. The end of the sample is the nearest possible date to the time when the analysis was conducted.

Canada has adopted a pure interest-rate corridor system since the introduction of the LVTS in 1999. Historical data shows that the Canadian framework for implementing monetary policy is remarkably effective, compared to other relatively more complex models, e.g. the reserve-averaging approach. *Figure 4* shows that the overnight interest rate has been hitting the Bank's target rate exactly almost every day, and financial institutions' demand for settlement balances is always met by the Bank's supply.

Figure 4: Histogram of Deviations of the CORRA from the Bank of Canada Target Rate



The effectiveness of the Canadian framework for monetary-policy implementation is the result of interplay of several elements. Most importantly is the transparency in the LVTS. At the end of every business day, every direct participant is aware, with perfect certainty, of every other system member's funds position as well as the Bank of Canada's. The certainty about the quantity of settlement balances demanded in the system greatly reduces potential errors in supplying the required amount (Recall that the demand curve is mostly inelastic at the midpoint of the operating band).[12]

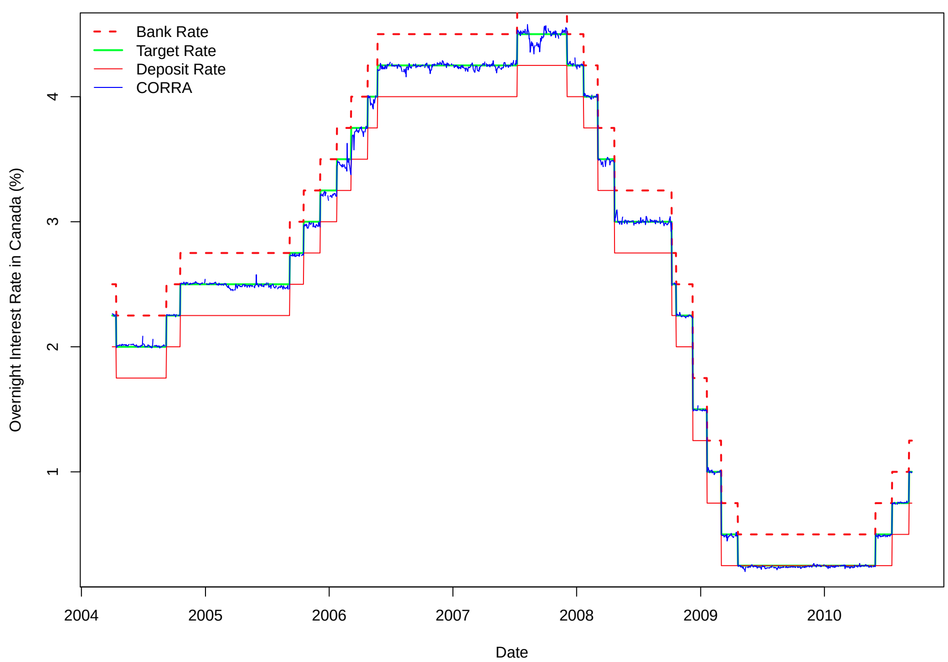
The Canadian Overnight Repo Rate Average (CORRA) is one of the two measures used by the Bank of Canada as proxies for the overall average cost of overnight funding, and therefore is used in this analysis as the reference

[12]To a certain extent, the effective management of settlement balances in LVTS is attributed to the fact that there are only a small number of direct participants in the system.

overnight interest rate.[13] The CORRA is computed as a weighted average of interest rates on overnight repo transactions involving non-specific Government of Canada securities, within the time period of 6:00 and 16:00.

Figure 5 presents a picture of movements of the CORRA over the sample period. It shows that over the course of 2008, the Bank of Canada lowered its target rate numerous times in response to the tremendous stress in money markets, until it reaches the lowest possible level in spring 2009. Moreover, except for the period from April 2009 to June 2010 (the ELB period), the Bank target interest rate is always set at the midpoint of the operating band. The graph also shows that, despite the fact that the operating band has been repeatedly shifted up and down over the course of six years in the sample, the two limits of the interest-rate corridor move in tandem most of the time; the spread remains at 50 basis points. The only exception occurs during the time period when monetary policy was implemented at the effective lower bound.

Figure 5: Bank of Canada Target Rate, Bank Rate, Deposit Rate and the CORRA



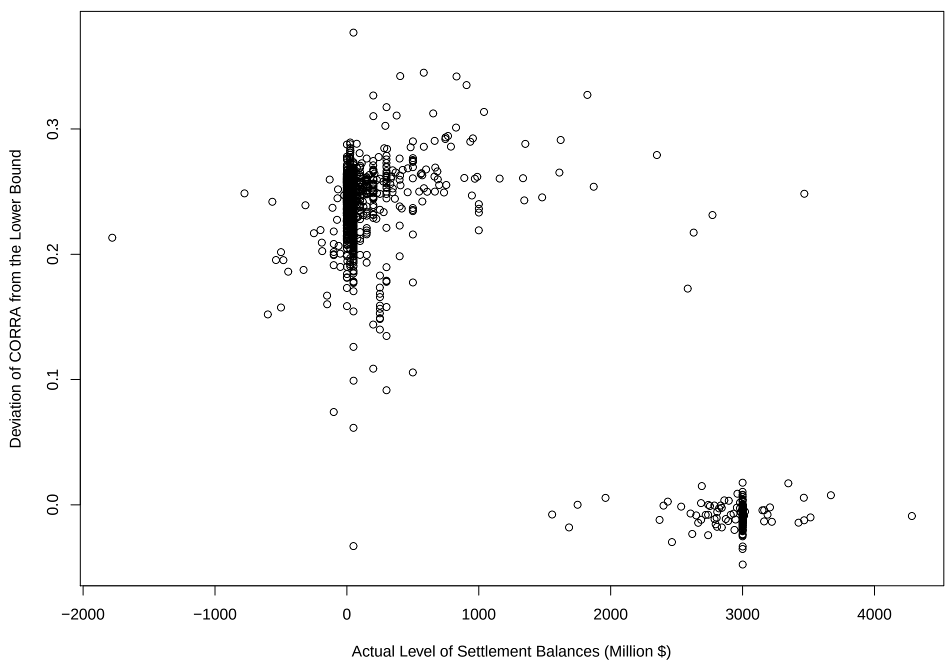
Note: Between April 2009 and June 2010, the Bank of Canada adopted the Effective Lower Bound framework for monetary policy implementation.

Figure 6 presents a scatter plot of the CORRA (in relation to the lower

[13]The other measure is the so-called overnight money-market financing rate.

bound of the interest-rate corridor) versus the actual level of settlement balances in the LVTS. It shows two clusters of data points in the sample period: one is gathered in the intersection area of small amounts of LVTS settlement balances i.e. between zero and \$50 million) and relatively high levels of the overnight interest rate; the other is located in the region where large quantities of settlement balances (mostly around \$3 billion) crosses very low levels of interest rate. The plot confirms the negative relationship between the quantity of settlement balances and the overnight interest rate, and an inverted S-shape demand curve, as proposed by Whitesell (2006), may fit the two clouds of data.

Figure 6: Differences between the CORRA and the Deposit Rate



Note: overall negative relationship between the quantity of settlement balances and the overnight interest rate.

4.2 Methodology

4.2.1 Generalized Linear Model

Generalized Linear Models (GLMs) are commonly used for modelling binary outcome variables. In this analysis, GLM provides a simple and convenient method for capturing the non-linearity of the demand for central bank balances in an interest-rate corridor system. Specifically, the explanatory variables ($\mathbf{x_i}\boldsymbol{\beta}$)

are mapped to a real number by a linear regression, and this linear predictor is then used to fit the outcome variable ($E[y_i]$) through a link function. What makes GLM "generalized" is that the link function can take various forms and does not have to be linear.[14] In the Whitesell (2006) model, the assumption of normally distributed liquidity shocks gives rise to the choice of *probit* as the link function in estimation.

Recall *equation* (5), the aggregate demand function for settlement balances under the assumption of normality. And the inverse of this demand function is given by:

$$\left(\frac{i - r^d}{r^b - r^d}\right) = \Phi\left(\frac{-D}{\sum_j \sigma_j}\right) \tag{7}$$

Apply GLM to *equation* (7). The left-hand side corresponds to the response variable y, i.e. $y = \frac{i-r^d}{r^b-r^d}$ (subscripts i for individual observations being dropped for simplicity). The dependent variable in this case can be interpreted as standardized deviations of overnight interest rate from the lower bound of the rate corridor.

On the right-hand side, the quantity of settlement balances demanded (adjusted by the aggregate liquidity uncertainties in the system) will be explained in a linear regression model by a vector of predictor variables, $\mathbf{x_i}$ i.e. $\frac{-D_i}{\sum_j \sigma_j} = \Phi^{-1}(\frac{i_i-r_i^d}{r_i^b-r_i^d}) = -(\mathbf{x_i}\boldsymbol{\beta} + \epsilon_i) = -\hat{\eta}_i$. $\hat{\eta}_i$ is the so-called *linear predictor*, and the *probit* link function describes the relationship between $E[y_i]$ and the linear predictor.

The persistent downward movements of Canadian overnight interest rate (discussed in Section 3.3) present a challenge when conducting the GLM estimation on the full set of data. During the period from April 2009 to June 2010, nearly 84% of the data shows $i - r^d < 0$, whereas the *probit* function is only defined within the domain $[0, 1]$.[15] However, it does not seem sensible to simply disregard such a large portion of data, especially when these data of the ELB framework play a crucial part in identifying the negative relationship between the overnight interest rate and the quantity of settlement balances (shown in *Figure 6*).

[14] In traditional linear regression models, the link between between $E[y_i]$ and $\mathbf{x_i}\boldsymbol{\beta}$ is invariably *identity*. The other extension featured by GLMs is that the outcome variable y_i is allowed to follow many probability distributions other than the normal distribution, such as the beta, binomial, multinomial, gamma, Poisson and Dirichlet distributions, etc.

[15] As previously mentioned, the overnight rate falling below the effective lower bound is a special case of their downward divergence from the central bank target rate, because in an ELB framework, the target rate is set at the bottom of the operating band. Data shows that the total number of such divergence account for 61% of the entire sample.

One feasible solution is to base the estimation on the aggregate demand (for settlement balances) expressed in the function of the observed overnight rate, i.e. *equation* (6). Hence, in this analysis the generalized linear model is specified as the following.

$$\begin{cases} \hat{\eta}_i = \mathbf{x_i}\boldsymbol{\beta} + \epsilon_i \\ y_i = \Phi(-\hat{\eta}_i) - \Delta_i \end{cases}$$

where

$$y_i = \frac{\tilde{i}_i - r_i^d}{r_i^b - r_i^d} \qquad \text{and} \qquad \Delta_i = \frac{\delta_i}{r_i^b - r_i^d}$$

The width of the operating band, $r^b - r^d$, has only taken two values over the whole sample period: 0.25 under the ELB framework and 0.5 otherwise. Since it mainly provides a scaling effect, $r^b - r^d$ is set equal to 0.25 in the estimation. It is less straightforward to determine the magnitude of δ, which depends on the assumptions about the levels at which the overnight rate would have been in the absence of various sources of downward pressure.

One simple way is to make δ sufficiently large so that all the data points in the sample will be included in the regression. Or, alternatively, δ can be an amount such that the majority of observed data will be qualified for estimation, only with a small fraction excluded and regarded as outliers (even though the downward pressure on the overnight interest rate has been taken into consideration). *Table 1* provides statistics of the negative differences (expressed in absolute values) between the CORRA and the Bank of Canada target rate, over the sub-sample period in which an effective lower bound of 25 basis points was set for the overnight rate. In addition to the average and maximum downward deviations, listed in the table are the percentiles at 50%, 90%, 95%, and 99%. In the GLM estimation, a separate regression is conducted for each of the percentile values for δ, and the results are then compared based on the log-likelihood of every estimated model.

In addition, the *Gaussian* family is specified as the class of GLM, under the assumption of the error distribution being approximately normal.

4.2.2 Two-stage Estimation with Instrumental Variable

In the linear component of the GLM specification, $\hat{\eta}_i = \mathbf{x_i}\boldsymbol{\beta} + \epsilon_i$, the vector of explanatory variables $\mathbf{x_i}$ includes the observed quantities of settlement balances SB_i and a number of covariates controlling for calendar effects ($\mathbf{C_i}$), open market buyback operations ($\mathbf{M_i}$), and various stages of the recent $2007 - 2008$

Table 1: Statistics of the Absolute Values of Negative Differences between the CORRA and the Target Rate (in the ELB framework)

	50%	90%	95%	99%	Mean	Maximum
$\lvert\tilde{i} - r^d\rvert$	0.0101	0.0179	0.0207	0.0318	0.0103	0.0476

[a] The interest rate paid on deposits at the Bank r^d is pegged at the target rate under the ELB framework.
[b] Note: These percentiles are computed over the ELB period (from April 2009 to June 2010), and only for the cases where $\tilde{i} - r^d < 0$. $\tilde{i}$ denotes the CORRA.

financial crisis ($\mathbf{F_i}$), i.e. $\mathbf{x_i} = \{SB_i, \mathbf{C_i}, \mathbf{M_i}, \mathbf{F_i}\}$.

However, SB is considered to be endogenous, because the overnight interest rate is jointly determined by the interaction of supply and demand of settlement balances. Specifically, the Bank of Canada adjusts the supply of settlement balances to accommodate expected changes in demand, and to prevent the overnight interest rate from moving too far away from the its target rate. The observed pairs of the overnight interest rate and the level of settlement balances represent the equilibria over time in the overnight funds market.

To address the concern of endogeneity, the instrumental variable (IV) method is applied in the regression analysis which results in a two-stage estimation. Since demand elasticities are the subjects of interest in this study, a variable that alters the supply of settlement balances is needed to trace out the demand curve. A valid supply shifter is required to have no correlation with movements in demand, but only affecting the changes in supply.

The transaction amount at the daily auctions of the Federal Government of Canada's Receiver General (RG) account balances appears to be a natural candidate for the supply shifter. Because the Bank of Canada neutralizes the net impact of public-sector payment flows by transferring the government's deposits from/to the government's account at the Bank of Canada to/from those at the LVTS participants; and the transfer is made through twice-a-day RG auctions, at 9 : 15 and 16 : 15 respectively. The degree of neutralization determines the Bank's target level of settlement balances in the LVTS. For example, a full neutralization of public-sector transactions leaves zero settlement balances in the LVTS. And a positive excess of settlement balances can be generated by the Bank transferring more government deposits to the participants than the amount required by a complete neutralization.[16]

[16] Public-sector payment flows affect the LVTS settlement balances because when the gov-

Specifically, the PM RG auction amount is used in this study as an instrument for the supply of LVTS settlement balances, since all transactions with regards to government disbursements and receipts are cut off at $15:00$ each day; and by the time of PM RG auction, the Bank of Canada knows with certainly the exact net impact of public-sector payment flows and thus can finalize its decision about the desired level of settlement balances for that day.[17]

The two-stage estimation is conducted as follows. In the first stage, using the ordinary least squares method, regress the actual level of settlement balances on the instrumental variable and a set of covariates.

$$\begin{aligned} SB_i = \; & \beta_0 + \beta_1 \cdot RG_i + \textstyle\sum_{s=1}^{11} \beta_2^s \cdot C_i^s + \sum_{s=1}^{3} \beta_3^s \cdot M_i^s \\ & + \textstyle\sum_{s=1}^{5} \beta_4^s \cdot F_i^s + \epsilon_i \end{aligned} \qquad (1^{st} \text{ stage})$$

where i denotes an individual data observation; β_0 is the intercept to capture fixed effects. RG is the issue amount at PM RG auctions; $\mathbf{C}$ is a set of 12 calendar dummies that captures fluctuations in LVTS payment flows at different times of a year; it includes days of the week (Mondays, Wednesdays, every second Thursdays and Fridays), US holidays, the beginning and end of each month, corporate and fiscal year end, etc. $\mathbf{F}$ is a vector of dummy variables for 5 stages of the recent financial crisis, including the ELB period; the duration of each stage prior to the ELB period is listed in *Table 5* in the appendix. $\mathbf{M}$ is a set of 3 indicator variables for open market operations conducted by the Bank of Canada: SPRAs, SRAs and term PRAs. Definitions of these control variables and the rationale for including them in the estimation are elaborated in the appendix.

In the second stage, using GLM, regress the transformed overnight interest rate (measured by the CORRA) on the fitted values from the first-stage regression ($\widehat{SB_i}$), as well as on the same set of covariates as in the first-stage

ernment makes disbursements from its account at the Bank of Canada, e.g. to pay for their expenditures, it increases deposits at financial institutions and adds to the settlement balances; conversely, a net government receipt of payments will withdraw liquidity from the financial system and reduce the settlement balances.

[17]In addition, there are two major differences between AM and PM RG auctions. First, AM auctions are accessible to a broader group of financial institutions, whereas afternoon auctions are restricted to LVTS participants. Second, since 2002, part of the morning auctions (beyond a certain threshold) require collateral, while PM auctions remain uncollateralized; however, no statistics have suggested any link between the repo-like nature of AM auctions and the movements of the CORRA.

equation.

$$\begin{cases} \hat{\eta}_i = & \gamma_0 + \gamma_1 \cdot \widehat{SB}_i + \sum_{s=1}^{11} \gamma_2^s \cdot C_i^s + \sum_{s=1}^{3} \gamma_3^s \cdot M_i^s \\ & + \sum_{s=1}^{5} \gamma_4^s \cdot F_i^s + \xi_i \\ y_i = & \Phi(-\hat{\eta}_i) - \Delta_i \end{cases} \qquad (2^{nd} \text{ stage})$$

4.3 Regression Results

Regression results of both stages are presented in *Table 2*. The issue amount at daily PM RG auctions is shown negatively related to the actual level of settlement balances in the LVTS, which is not readily inferable from an operational point of view. Precisely speaking, the net amount auctioned (i.e. the difference between the gross amount issued at both AM and PM RG auctions and the total value maturing on that day) consists of two components: the neutralization of public-sector transactions in the LVTS and the Bank of Canada's desired change in the level of settlement balances for that day.

However, the results show a significantly strong correlation between PM RG auction issue amount and the actual level of settlement balances. As proposed by Stock and Yogo (2005), one simple way to test for instrument relevance is to make use of the first-stage F statistic. Implementations of the F-statistic approach depend on users' definitions of weak (or strong) instruments. For example, an instrumental variable can be deemed as non-weak, if the first-stage F statistic is sufficiently large such that no more than a certain percentage (e.g. 10%) of the time, a conventional Wald test of the null hypothesis that $\theta = \theta_0$ (e.g. at significance level of 5%) will be rejected, where θ is the estimate of the parameter of interest (i.e. the coefficient of the endogenous regressor in the structural equation) and θ_0 is the proposed value under the null hypothesis.[18]

Stock, Wright, and Yogo (2002) provide critical values of the first-stage F statistic (in a two-stage least squares (TSLS) regression) for testing the null hypothesis that instruments are weak. In the case of a single endogenous regressor treated with a single instrumental variable, the weak-instruments critical value for the F statistic is 16.38, if the desired maximal rate of rejection (of a 5% Wald test that $\theta = \theta_0$) is 10%. Specifically, it means that if the first-stage F statistic exceeds this critical value, then one can reject the null hypothesis of weak instruments. The first-stage estimation in this study shows that

[18] Alternatively, weak instruments can be detected and tested based on the *relative bias* between the ordinary least squares (OLS) and the TSLS estimators. For more details on various methods of testing for instrument relevance, please see Stock and Yogo (2005) and Stock, Wright, and Yogo (2002).

$F = 32.49$, suggesting no evidence of PM RG auction amount being a weak instrument.[19]

The Pregibon goodness-of-link diagnostic test is conducted to examine the suitability of the *probit* link function that is used to relate the mean of the overnight interest rate to the chosen set of explanatory variables. The test result shows that the p-value is 0.6477, suggesting no evidence against the null hypothesis that it is a reasonable fit of the regression model with *probit* link specification.[20]

Estimated effects of most control variables in the first-stage regression show expected signs and many are statistically significant. The beginning and end of each month, the fiscal year end for financial institutions, the four stages of the $2007 - 2008$ financial crisis, and the ELB time period all show positive correlation with the amount of settlement balances in LVTS, as they indicate the days on which there is a higher-than-usual demand for intraday liquidity.

Correlations between several variables and LVTS settlement balances turn out opposite to the expectations. For instance, the demand for LVTS settlement balances seems to shrink a little on Wednesdays, despite the anecdotal evidence that larger transaction volume usually occurs on the third Wednesday of each quarter-end month. In addition, rather than a catchup effect, the results show a possible continuation of slowdown in business activity on the first business day following a Canadian statuary holiday. The corporate year-end effect, which is expected to be positive, might be offset to a certain degree by the fact that every year end is also a big holiday season during which LVTS transaction activities may slow down.

SPRA and SRA operations are the reverse of each other, and as expected, they show opposite relationship with the level of LVTS settlement balances in the regression. The operation and announcement dates of term PRA are both found statistically insignificant in explaining the level of LVTS settlement balances.

In the second-stage GLM regression, $\delta = 0.0476$, allowing all the data points to be included in the estimation. The level of LVTS settlement balances is

[19] In comparison between two linear models where the difference is only in one variable, the test statistic has an F-distribution with only 1 degree of freedom in the numerator; and hence, the value of the F-test statistic is simply the square of the *Student's t* statistic for the variable being tested. In this study, the results of first-stage estimation show that the t statistic for PM RG auction is -5.70; therefore, $F = 32.49$ for the F-test of the null hypothesis that $\theta = \theta_0$.

[20] An interested reader can refer to Pregibon (1980) for further details of goodness-of-link tests for GLMs.

found to be statistically significant only at the 1% level; however, the weak relationship is not entirely unexpected, because the supply of LVTS settlement balances does not actually respond to the demand on an intraday basis. The Bank of Canada typically sets a target level of LVTS settlement balances for a long period of time and only makes temporary seasonal adjustments. Many control variables such as the beginning and end of each month, Government of Canada Treasury Bill (T-Bill) auction settlement dates, different stages of the financial crisis period and the Bank of Canada intervention operations, etc. also show significant correlations with the overnight interest rate.

The coefficients in the second-stage results (shown in *Table 2*) are estimated effects of right-hand-side (RHS) variables on the scale of the linear predictor $\hat{\eta}$. Given the *probit* link used in the GLM, the marginal effect of a continuous explanatory variable, x_k, on the response variable is given by:

$$\frac{\partial y}{\partial x_k} = -\phi\Big(\sum_{k=1}^{K} \beta_k x_k\Big)\beta_k$$

where K is the total number of independent variables included in estimation.

Recall that the response variable y in the GLM regression is $\frac{\tilde{i}-r^d}{r^b-r^d}$; hence, the marginal effect of x_k on the observed overnight interest rate $\tilde{i}$ becomes:

$$\frac{\partial \tilde{i}}{\partial x_k} = -\Big(r^b - r^d\Big)\phi\Big(\sum_{k=1}^{K} \beta_k x_k\Big)\beta_k \tag{8}$$

As opposed to partial derivatives, the marginal effect of an indicator variable (i.e. a binary explanatory variable), x_j, is derived from the partial difference; i.e. the change in the response variable when the inverse link function is evaluated at $x_{ij} = 1$ and $x_{ij} = 0$, respectively, with all other RHS variables held constant at the same values. Therefore, in this study, the marginal effect of a dummy control variable on the observed overnight interest rate $\tilde{i}$ is given by:

$$\big(r^b - r^d\big)\Big[\Phi\big(-\mathbf{x_{1j}}\boldsymbol{\beta}\big) - \Phi\big(-\mathbf{x_{0j}}\boldsymbol{\beta}\big)\Big] \tag{9}$$

where $\mathbf{x_{1j}}\boldsymbol{\beta}$ is the vector of regressor values with $x_{ij} = 1$, and $\mathbf{x_{0j}}\boldsymbol{\beta}$ is the same vector of regressor values with $x_{ij} = 0$; all other regressors take the same fixed values. Subscripts for observations are dropped for simplicity.

Both *Equation* (8) and (9) show that marginal effects in *probit*-link linear models depend on the values of RHS variables. *Table 3* compares the marginal

Table 2: Two-stage Regression Results

This table reports coefficient estimates of two-stage regressions. The first stage is the ordinary least squared (OLS) regression of the actual level of settlement balances (SB) on the instrumental variable (issue amount at PM RG auctions) and various calendar effects. The second stage is a GLM regression of the standardized deviations of the overnight interest rate from the lower bound, $(\frac{\tilde{i}-r^d}{r^b-r^d})$ on the fitted value from the first stage ($\widehat{SB}$), as well as on the same set of calendar dummies. All control variables are defined in the Appendix. Data of daily frequency is used and robust standard errors are reported in parentheses.

	Stage (I)[b] OLS (SB)	Stage (II)[c] GLM $(\frac{\tilde{i}-r^d}{r^b-r^d})$		Stage (I) OLS (SB)	Stage (II) GLM $(\frac{\tilde{i}-r^d}{r^b-r^d})$
Constant	101.7477***	-0.4630***	corpYearEnd	-8.2293	0.0105
	(15.7860)	(0.0074)		(56.1589)	(0.0291)
pmAuction	-0.0820***	x	fcPeriod0	196.2050***	0.0658**
	(0.0144)			(26.0663)	(0.0212)
$\widehat{SB}$	x	0.00023*	fcPeriod1	130.0574***	-0.0488**
		(0.0001)[d]		(29.1223)	(0.0185)
Monday	-6.6691	0.0044	fcPeriod2	48.7297*	-0.0341**
	(17.0008)	(0.0088)		(21.5191)	(0.0113)
Wednesday	-22.3763	0.0146 ·	fcPeriod3	32.5250	0.0074
	(16.1688)	(0.0085)		(28.9998)	(0.0145)
Thursday[e]	6.0926	-0.0288**	spra	276.3013***	-0.2350***
	(20.6163)	(0.0107)		(22.6861)	(0.0320)
Friday	-1.9144	-0.0081	sra	-29.3169	0.2500***
	(16.4554)	(0.0085)		(23.8949)	(0.0123)
usHoliday	10.8039	-0.0119	fad	43.0984	-0.0308 ·
	(38.8492)	(0.0209)		(33.1906)	(0.0177)
canHolPlusOne	-61.3611*	0.0158	termPra	-17.8439	0.0338**
	(30.8493)	(0.0171)		(22.7967)	(0.0131)
monthBegin	55.7435*	-0.0395**	termPraAnn	-33.8998	-0.0150
	(27.8875)	(0.0148)		(45.5183)	(0.0249)
monthEnd	12.2823	-0.0277**	elbPeriod	2914.9337***	0.7860**
	(20.1837)	(0.0106)		(16.0265)	(0.2700)
fiYearEnd	117.2013*	-0.0442			
	(47.8561)	(0.0274)			

[a] Note: $(\cdot)$, $(*)$, $(**)$ and $(***)$ represent statistical significance at the 5, 1, 0.1 and less than 0.1 per cent level, respectively.

[b] For the first-stage OLS regression, residual standard error is 223 on 1596 degrees of freedom (67 observations deleted due to missing values); the adjusted R-squared is 0.96; F-statistic is 1840 on 21 and 1596 degree of freedom; p-value is < 0.0000.

[c] The GLM regression is based on *equation* (6), where $\delta = 0.0476$. Dispersion parameter for Gaussian family is taken to be 0.0016; Null deviance is 65.2259 on 1613 degrees of freedom; Residual deviance is 2.4677 on 1592 degrees of freedom; 71 observations deleted due to missing values and $AIC = -5838$; the number of Fisher Scoring iterations is 4.

[d] The standard error of the coefficient estimate for LVTS settlement balances, 0.0001, has been corrected to account for the fact that predicted (instead of observed) values of settlement balances are used in the second-stage estimation. It is obtained from scaling the original unadjusted standard-error estimate (0.00009) by the ratio of two residual standard deviations: one is from the GLM prediction equation that is based on the actual level of settlement balances (not the fitted value from the first-stage regression), and the other is the residual standard deviation of the second-stage regression itself (0.0394).

[e] The binary variable *Thursday* indicates the settlement dates for bi-weekly auctions of Government of Canada Treasury Bills. It takes value of 1 every second Thursday, starting on April 8^{th}, 2004.

effects of certain explanatory variables between two circumstances: (i) normal economic conditions where the Bank of Canada typically targets its supply of

LVTS settlement balances at a small positive quantity of 25 million dollars, and the spread of the operating band is 50 basis point. (ii) the ELB framework under which the target level of settlement balances is increased to $3 billion, and the band width is narrowed down to 25 basis points. The indicator variables shown in the table are selected based on two criteria: first, they are relevant in both settings; second, they are statistically significant at the second stage of estimation.

Table 3: Marginal Effects of Selected Variables on the Observed Overnight Interest rate: normal times versus the ELB framework

	Normal Conditions $SB = \$25$ million	ELB Framework $SB = \$3$ billion
SB	-0.00004	-0.000014
Wednesday	-0.0026	-0.0009
Thursday	0.0051	0.0017
Month begin	0.0070	0.0024
Month End	0.0049	0.0017
FAD	0.0055	0.0019
SPRA	0.0397	0.0157
SRA	-0.0471	-0.0130
Term PRA	-0.0061	-0.0020

[a] Note: The marginal effect of LVTS settlement balances, under both circumstances, is calculated when none of the control variables (for calendar effects and open market operations) takes value of 1.
[b] However, under the ELB framework, the variable controlling for this time period ("elbPeriod") is set to 1.
[c] $r^b - r^d = 0.5\%$ under normal economic conditions; $r^b - r^d = 0.25\%$ in the ELB framework.

The results suggest that on a typical business day without any calendar events and intervention operations (i.e. none of the indicator variables takes value of 1), when the daily level of LVTS settlement balances is targeted at $25 million, a one-million-dollar increase in the amount of settlement balances would cause a decrease of overnight interest rate by 0.004 basis points, ceteris paribus. Likewise, at the beginning (end) of each month, with all other variables held constant, the overnight interest rate would on average go up by 0.7 (0.49%) basis points, relative to other times of year. Similar effects show up on every second Thursday in each calendar year; i.e. the settlement date for T-Bill auctions tends to place an upward pressure on the overnight rate, an increase of 0.51 basis points on average, while other factors remain fixed.

As expected, Term PRA operations show a statistically significant and negative impact on the overnight interest rate. However, the control variables for SPRA and SRA operations seem to vary systemically with the overnight rate. In these cases, the coefficent estimates do not represent the causal effects of the two open market buyback operations on the overnight interest rate. This endogeneity issue is undoubtedly unsatisfactory and future work on improving the results should look into different estimation methods or finding additional instruments to eliminate it.

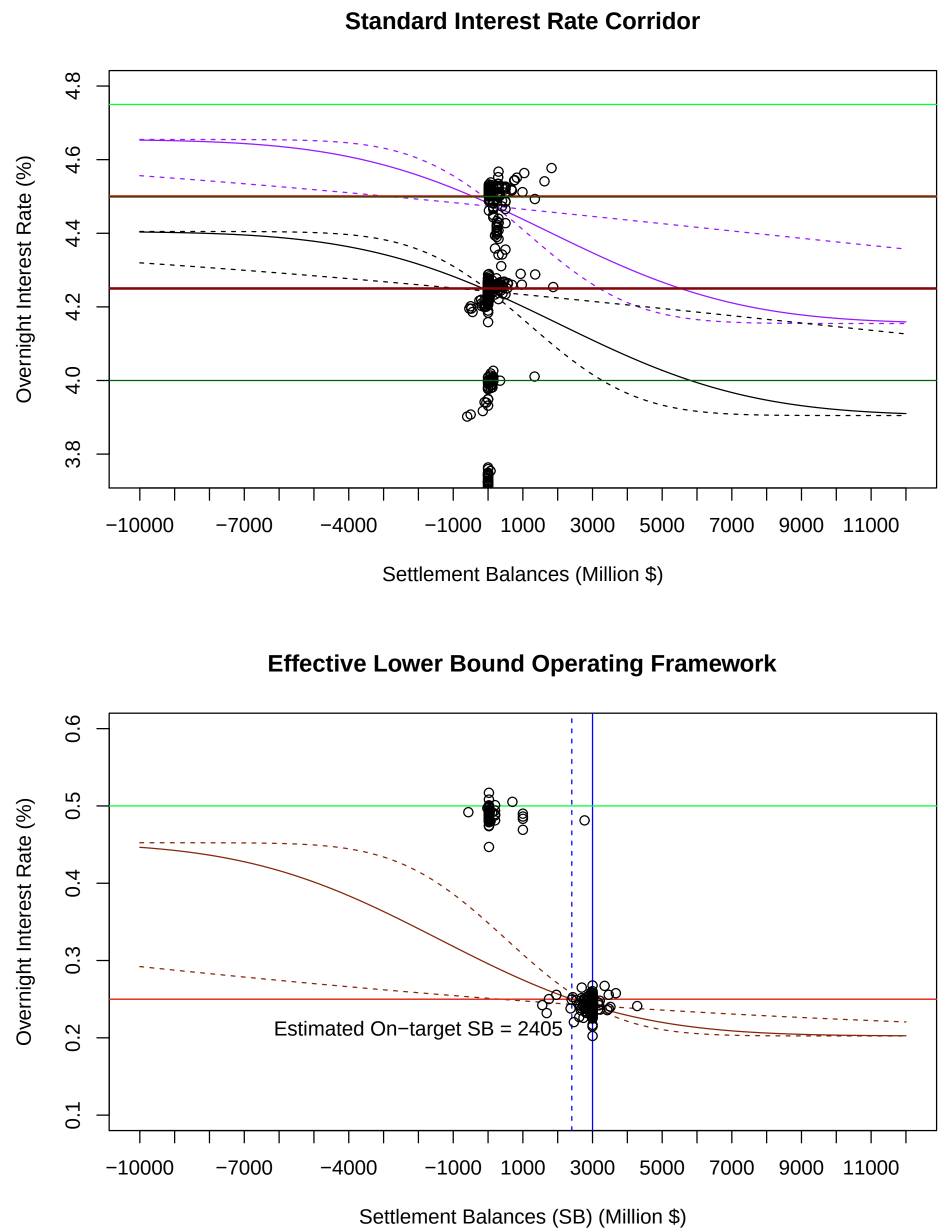
In addition, the second-stage regression results do not show evidence of mounting pressures in the overnight funding markets during the first stage of the financial crisis (from the onset of the crisis to mid-December of 2007). This could imply a transition period for behavioural changes, meaning that it possibly takes time for liquidity hoarding and precautions in inter-bank lending to intensify and drive up the cost of overnight funding.

The marginal effects of the variables shown in *Table 3* are all smaller in the ELB framework than those under normal conditions; for example, the overnight interest rate would drop by only 0.0014 basis points in response to a one-million-dollar increase of LVTS settlement balances. This is both because the demand curve becomes flatter when it is close to the lower bound of the corridor, and because the corridor is narrower in the ELB setting. Hence, the *settlement-balance-elasticity of overnight interest rate* might be a more useful measure to gauge the responsiveness of the overnight rate to a change in the level of LVTS settlement balances. It is defined as follows:

$$E_{\tilde{i}} = \left(\frac{\partial \tilde{i}}{\partial SB}\right)\left(\frac{\overline{SB}}{\bar{\tilde{i}}}\right)$$

Data shows that, during the ELB period, the average overnight interest rate is 0.2416%, and the average actual level of settlement balances in LVTS is \$2967.11 million. Hence, the elasticity of overnight interest rate is estimated to be -0.1689. It is a common practice to ignore the negative sign when analyzing elasticities between price and demand. The result suggests that during the time period when Canadian monetary policy was implemented in the ELB framework, a one percent change in the amount of LVTS settlement balances would prompt an approximately 0.17 percent change in the overnight interest rate. The result supports the theory that when the overnight rate is targeted at the lower bound of the corridor, it is generally *settlement-balance inelastic*, i.e. not sensitive to changes in LVTS settlement balances.

Figure 7: Estimated Demand for LVTS Settlement Balances



Note: the black curve portrays the estimated demand for LVTS settlement balances from May 24^{th} 2006 to July 9^{th} 2007, a period prior to the outbreak of the recent global financial crisis; the purple curve describes the estimated demand during the first stage of financial crisis and the brown curve (in the bottom panel) is the estimated demand for the time period when the ELB framework was implemented. The solid vertical blue line marks the Bank of Canada's daily target supply of $3 billion for LVTS settlement balances during the ELB period, and the dashed vertical blue line indicates the level of LVTS settlement balances at 2.405 billion, the estimated amount that is required to exactly achieve the overnight target rate at 25 basis points. The black circles represent the CORRA (daily). The dashed curves are 95% confidence intervals for the predictions from the GLM regression model.

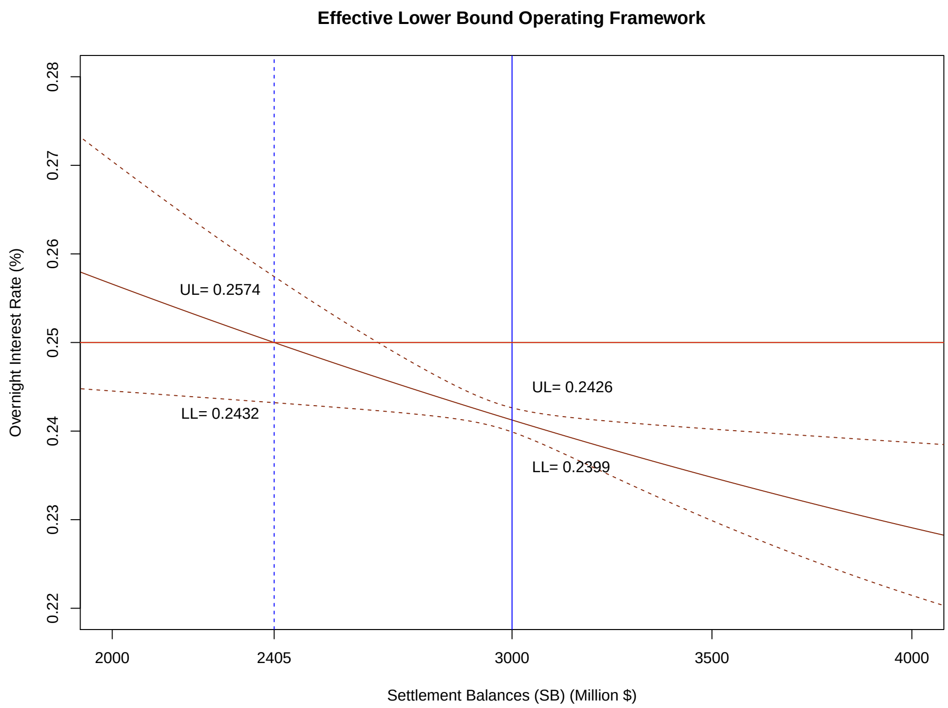
Figure 7 plots the estimated demand for LVTS settlement balances during three different time periods (excluding all calendar effects): a representative pre-financial crisis period (e.g. from May 24^{th}, 2006 to July 9^{th}, 2007), the first stage of the financial crisis and the time period during which Canadian monetary policy was implemented in an ELB framework. The black circles in the figure are a scatter graph of the CORRA at each actual level of LVTS settlement balances.

The estimated demand curves for the first two periods are shown in the upper panel of *Figure 7*, representing the standard corridor system with the target rate set at the mid-point of the operating band. The black curve portrays the estimated demand for LVTS settlement balances immediately prior to the outbreak of the recent global financial crisis, when the Bank Rate was at 4.5% and the deposit rate equal to 4%. The purple line describes the estimated demand during the first stage of the financial crisis (as defined in the Appendix), with the operating band placed between 4.25% and 4.75%.

The lower panel illustrates the estimated demand for LVTS settlement balances over the period from April 21^{st}, 2009 to May 31^{th}, 2010. In this ELB framework, the Bank of Canada set its target rate at the lower bound of the corridor: 0.25%. It shows that, if the consistent downward divergence of the CORRA from the target rate is taken into account, then $2.405 billion of LVTS settlement balances would probably have been sufficient to achieve the goal of keeping the overnight interest rate at or very close to the lower bound of the corridor.

The dashed curves in the figure are 95% confidence intervals for the estimates of overnight rate from the GLM regression model. In all three cases, the confidence intervals are most narrow in the areas where observed data is concentrated, indicating that the closer the given amount of settlement balances is to its sample mean, the smaller the estimation error becomes. *Figure 8* provides a close-up view of the lower panel in *Figure 7*. The plot suggests that if the Bank of Canada had supplied $2.405 billion daily for LVTS settlement balances over a fairly large number of days in the ELB framework, then on average 95% of the overnight interest rates would have fallen within the range between 0.2432% and 0.2574%. This might not be very desirable, because half of the time the overnight rate would have been above the Bank's policy rate 0.25%. However, if the level of LVTS settlement balances is targeted at a slightly higher level, e.g. $3 billion as actually implemented, then with a 95% level of confidence, the Bank could expect that the overnight rate would not exceed its target but be located within the prediction bounds of 0.2399% to 0.2426%.

Figure 8: A Close-up Look at 95% Confidence Intervals for Predictions of Overnight Interest Rates in the ELB Framework



Parameters in a GLM are estimated with maximum likelihood techniques. The residual deviance and the AIC (a version of Akaike's *An Information Criterion*), both computed based on the model's log-likelihood, are measures of goodness of fit of the overall model. In a GLM, the difference in deviance between any two models is the likelihood ratio test statistic and approximately follows a chi-squared distribution, with degrees of freedom equal to the difference in degrees of freedom between the two models.[21]

Table 4 provides a comparison of competing models estimated with different values for δ (i.e. the percentiles of downward deviations of the CORRA from the target rate, listed in *Table 1*). The model specified with $\delta = 0.0476$ (including every data point in the sample) shows the highest log-likelihood score. The column of *p-value* (the probability of obtaining the χ^2 test statistic) indicates

[21]Under usual circumstances, model comparison is conducted between two models with different number of parameters; hence, the null hypothesis in the likelihood ratio test states that the model with fewer parameters, relative to the other model, is true. In this paper, model specifications are the same across all the models in comparison, in terms of the number of parameters; and the only difference lies in the various values taken by δ.

whether a model fits data better than the model immediately above. It shows that model $No.1$ and $No.2$ are statistically equivalent, and the same is true of the last two models; and there is a significant improvement in goodness of fit when 99%, versus 95%, of data is included in the estimation.

Table 4: Comparison of Regression Models Estimated with Different δ Values

Model	Percentiles	δ	Log-likelihood	Deviance	$p - value$
1	50%	0.0101	2709.93	2.3247	NA
2	Mean	0.0103	2712.04	2.3253	15.7249
3	90%	0.0179	2902.83	2.4163	0.0000
4	95%	0.0207	2922.67	2.4409	0.0000
5	99%	0.0318	2935.39	2.4730	0.0000
6	Max	0.0476	**2938.13**	2.4788	0.4985

5 Concluding Remarks

This paper aims to estimate the aggregate demand for LVTS settlement balances, based on the static model proposed by Whitesell (2006). The regression analysis controls for a group of calendar events that affect LVTS intraday payment flows and several time periods of unusually high funding pressures in the overnight market.

The results suggest that a target of $3 billion for LVTS settlement balances does not seem excessive during the period when Canadian monetary policy is operated at the effective lower bound. Specifically, the model projects that, if the consistent downward divergence of the CORRA from the target rate is taken into account, then $2.405 billion of LVTS settlement balances would probably have been sufficient to achieve the goal of keeping the overnight interest rate at or very close to the lower bound of the corridor. However, by raising the target to a slightly higher level, e.g. $3 billion, then with a 95% level of confidence, the Bank could expect that the overnight rate on average would not exceed its policy rate. Furthermore, when an effective lower bound of 25 basis points was set on the overnight interest rate and excess settlement balances (targeted at a daily level of $3 billion) were provided in the LVTS, the point elasticity of the overnight rate is estimated to be 0.1689.

One main concern in this analysis is the potential problem that two of the control variables (for SPRA and SRA operations) are likely influenced by the outcome variable. The results shall be interpreted with caution, since the estimation is conditioned on the endogenous variation of the two control variables. Search for additional instrumental variables is often difficult; other methods such as nonparametric matching could be possible solutions.

Other steps in future work might include first, expanding on the scope of this paper by estimating two separate equations for normal economic conditions and the state of financial stress, respectively. The estimation conducted in this study currently relies on dummy variables for various stages of the recent financial crisis; however, in reality, it is often unclear during the course of events, which phase of new market developments the economy currently resides in. Therefore, a refined estimation can improve the robustness of the results and possibly make the demand model for settlement balances a more effective policy tool;[22] second, finding an alternative instrument variable for LVTS settlement balances, in response to the argument that the issue amount at PM RG auctions may not be absolutely exogenous to the demand.[23]

[22] At present, in the context of LVTS, separate modelling might be challenged by the lack of data support. The negative relationship between the overnight interest rate and the quantity of settlement balances may not be easily identified, due to highly limited variation in the quantity of settlement balances.

[23] Attempts have been made using the forecast error of Government of Canada payment receipts and disbursements (i.e. the Bank of Canada internal data on the differences between the actual and predicted government payment flows) as an instrument for LVTS settlement balances, however, the regression results are substantially identical to what is presented in this paper.

Appendix: Control Variables

In addition to the explanatory variable of interest: the actual level of settlement balances in LVTS, a group of dummy variables are included in both stages of the regression as covariates, to control for (i) calendar effects of numerous events throughout a year on LVTS transaction flows. A larger demand for settlement balances is usually prompted by a higher level of payment activity in LVTS;[24] (ii) the liquidity impact of the Bank of Canada intervention actions (aimed at reinforcing the target rate and stabilizing the financial markets); and (iii) the time periods of extraordinarily high funding pressures in the overnight market.

Mondays, Wednesdays and Fridays These week-day dummy variables are included to capture certain weekly pattern in LVTS transactions. Payment flows are in general believed to be more concentrated on Mondays than on Fridays, for example. Historical data also shows larger transaction volume in the LVTS on the third Wednesday of each quarter-end month, though no sensible explanation has been identified yet. The dummy variable for Wednesday in this analysis, however, indicates the middle of *every* week in the sample.

Thursdays A binary variable that indicates the settlement dates for auctions of Government of Canada Treasury Bills (T-Bills). In the sample period used in this analysis, the variable takes value of 1 every second Thursday, starting on April 8^{th}, 2004. Since the fall of year 1997, auctions of T-Bills take place on Tuesdays and settle on Thursdays, on a bi-weekly basis. On these auction settlement days, payment flows from LVTS direct participants to the Bank of Canada are expectedly larger than usual, which hence increases the demand for intraday liquidity and puts an upward pressure on the overnight interest rate.

US holidays and the next day after Canadian statuary holidays An increase in LVTS payment flows is usually expected on the first business day following a Canadian federal statuary holiday, due to a carryover of transactions from the break. The regression in this analysis takes into account the following Canadian statuary holidays: *New Year's Day*, *Family Day*, *Good Friday*, *Victoria Day*, *Canada Day*, *Civic Holiday*, *Labour*

[24] A Bank of Canada internal study by Arjani (2004) looked into various calendar events in establishing a benchmark for LVTS payment activity. The estimates of these calendar effects on intraday payment flows are found to be statistically significant in general.

Day, *Thanksgiving Day*, *Remembrance Day* and *Boxing day*. Likewise, the close trading relationship between Canada and the United States leads to an expected decrease in LVTS payment activity on a U.S. national holiday. The indicator variable *usHoliday* in this paper identifies eight U.S. national holidays during which LVTS is operational: *Martin Luther King, Jr. Day*, *President's Day*, *Memorial Day*, *Independence Day*, *Columbus Day*, *Veteran's Day* and *Thanksgiving Day*.

Month begin and month end The beginning and end of each month indicate the first and last two business days of the month. Many bill payments become due at the end of each month, e.g. payments for mortgages, rent and government taxes on income and sales, etc. Hence, the LVTS participants in general face an increase of account uncertainty at each month end, and the Bank of Canada usually temporarily expands the supply of settlement balances above its normal level. Higher transaction volume and value also occur at the beginning of each month, largely owing to the carryover from the previous month end. In addition, the redemption of the Government of Canada marketable bonds at maturity mostly takes place on the first business day of March, June, September and December, and noticeably increase the payment flows in LVTS on those days.

Corporate and financial year end October 31^{st} each year is the fiscal year-end for Canadian financial institutions. On several days surrounding this book-closing date, the intense year-end accounting activities tend to increase transaction volume in LVTS and drive up the participants' demand for liquidity. Hence, on these days, the Bank of Canada usually raises its target level for LVTS settlement balances to alleviate the mounting pressures on the overnight interest rate. The dummy variable *fiYearEnd* in this analysis indicates four days of each year: October 29^{th}, 30^{th}, 31^{st} and November 1^{st}. For the same reason as above, the regression also controls for the fiscal year end for most Canadian corporations (*corpYearEnd*), which is defined in this study as the four business days surrounding the end of each calendar year, i.e. the last two business days in December and the first two business days in January of the following year.

Financial crisis period{0-3} Liquidity conditions started improving slowly in early 2009. The period of $2007-2008$ global financial crisis is divided into four stages (inclusive on both ends) as listed in *Table 5*. The breakpoints correspond to the moments of peaking pressures in the short-term funding market, which were marked by several significant incidents such as the first joint effort by central banks around the world to inject additional liquidity into the money markets, the collapse of Bear Stearns, the bankruptcy of Lehman Brothers and the government bailout of American International Group, Inc. (AIG), etc.

Table 5: Stages of the 2007 – 2008 Global Financial Crisis

period 0	August 9^{th}, 2007 ∼ December 12^{th}, 2007
period 1	December 13^{th}, 2007 ∼ March 16^{th}, 2008
period 2	March 17^{th}, 2008 ∼ September 16^{th}, 2008
period 3	September 17^{th}, 2008 ∼ December 31^{st}, 2008

SPRA and SRA Two dummy variables are used to indicate the days in the sample on which the fine-tuning market operations are conducted to reinforce the target overnight rate during the course of a day. The Bank of Canada's ability to intervene in the overnight funds market with buyback operations at its target rate is one of the key features of its monetary-policy implementation framework. Specifically, if the overnight market is consistently trading above the target rate, then the Bank will inject liquidity with Special Purchase and Resale Agreements (SPRAs); conversely, Sale and Repurchase Agreements (SRAs) will be invoked to withdraw liquidity, if the overnight rate is constantly trading below the target. During the recent financial crisis, these fine-tuning facilities were more extensively used than under normal circumstances, to stabilize the financial system.

Term PRA It indicates the days in the sample on which there are term Purchase and Resale Agreements (term PRA) operations.[25] Term PRA is an important liquidity facility frequently used during the recent global financial crisis. On December 12^{th} 2007, the Bank of Canada joined in an international collaboration among many other central banks around the world (e.g. the Bank of England, the European Central Bank and the Federal Reserve, etc.), to launch term PRA operations that were aimed to

[25] Three main characteristics of term PRA operations, in contrast to the Bank's regular SPRA operations are: (i) funds are lent out for a period longer than overnight; (ii) a wider range of securities are accepted as collateral than those eligible for SPRA; and (iii) the rates on transactions are not fixed at the target overnight rate but determined through a competitive auction process.

alleviate pressures in short-term funding markets.[26] It was re-introduced in March 2008 as part of another international joint program by G10 central banks, and again in September 2008 when the financial market turmoil reached another peak. More importantly, during the ELB period when the Bank made a conditional commitment to keeping a low interest rate environment for the economy, term PRA (with the minimum and maximum bid rates matched to the Bank rate and the deposit rate, respectively) played a double role, not only as a liquidity facility, but also as a supplementary tool for monetary policy implementation.

Fixed announcement dates (FAD) It indicates eight "fixed" or pre-specified dates in the course of a year, on which the Bank of Canada announces its decisions on any changes to the target for the overnight interest rate and/or to the operating band. These announcement are made at 9 : 00 in press releases, and the change in the target rate, if there is any, takes effect on the same day as announced. FAD was first introduced in November 2002; it replaced the old regime under which financial markets essentially needed to be prepared for potential changes to the key interest rate on any business day throughout a year. The fixed-date approach improved the effectiveness of monetary policy in Canada, by reducing uncertainties in financial markets, better informing the Canadian public of recent economic developments, and by enhancing the transparancy of monetary policy.

Term PRA announcement date Enenajor, Sebastian, and Witmer (2010) show robust evidence of *announcement* effect of term PRA operations on reducing inter-bank funding costs in short-term money markets. The term PRA announcement dates are obtained from the *Market Notices* page on the Bank of Canada's website.[27] In this analysis, term PRA announcements are defined as releases of information regarding new and/or revised schedules, extensions of the facility and changes to the terms of the operations.

ELB Period The variable takes 1 over the period from April 21^{st}, 2009 to May 30^{th}, 2010 (inclusive), 0 otherwise, indicating the time period during

[26] In this collaboration, the Bank of Canada conducted two operations over the calendar year, with transaction amount of $2 billion each.

[27] Specifically, they are available at http://www.bankofcanada.ca/publications-research/market-notices/.

which Canadian monetary policy was implemented in an ELB framework. During this time interval, besides the changes to the operating framework, the Bank of Canada also increased the target for daily settlement balances in LVTS from $25 million to $3 billion, to provide additional incentive for creating a low interest rate environment for economic recovery.

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