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Fooled by Search: Housing Prices, Turnover and Bubbles

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Abstract

This paper develops and estimates a model to explain the behaviour of house prices in the United States. The main finding is that over 70% of the increase in house prices relative to trend during the increase of house prices in the United States from 1995 to 2006 can be explained by a pricing mechanism where market participants are 'Fooled by Search.' Trading frictions, also known as *search* frictions, have been argued to affect asset prices, so that asset markets are constrained efficient, with shocks to liquidity causing prices to temporarily deviate from long run fundamentals. In this paper a model is proposed and estimated that combines search frictions with a behavioural assumption where market participants incorrectly believe that the efficient market theory holds. In other words, households are 'Fooled by Search.' Such a model is potentially fruitful because it can replicate the observation that real price growth and turnover are highly correlated at an annual frequency in the United States housing market. A linearized version of the model is estimated using standard OLS and annual data. In addition to explaining over 70% of the housing bubble in the United States, the model also predicts and estimation confirms that in regions with a low elasticity of supply, price growth should be more sensitive to turnover. Using the lens of turnover, a supply shock is identified and estimated that has been responsible for over 80% of the fall in real house prices from the peak in 2006 to 2010.

JEL classification: E3, R2, R21
Bank classification: Asset pricing; Business fluctuations and cycles

Résumé

L'auteur construit et estime un modèle en vue d'expliquer l'évolution des prix des maisons aux États-Unis. Principal constat de l'étude : la hausse des prix enregistrée par rapport au niveau tendanciel durant la bulle immobilière qu'a connue ce pays entre 1995 et 2006 peut s'expliquer à hauteur de plus de 70 % par le fait que les agents évaluent les prix des maisons sur la base de prémisses erronées. Certains soutiennent que les frictions qui accompagnent les échanges – les frictions liées à la prospection – influent sur les prix des actifs, si bien que le marché des actifs affiche une efficience limitée, les chocs de liquidité poussant les prix à s'écarter temporairement de la trajectoire induite par les facteurs fondamentaux de long terme. Le modèle proposé par l'auteur conjugue les frictions liées à la prospection avec un postulat voulant que les agents croient à tort en l'efficience du marché. Ce modèle promet d'être fécond, car il parvient à reproduire la corrélation très étroite entre la croissance annuelle des prix réels et le taux de rotation sur le marché américain du logement. Une version linéarisée du modèle est estimée par la méthode des moindres carrés ordinaires à l'aide de données annuelles. Outre sa capacité à expliquer à plus de 70 % l'envolée des prix durant la bulle immobilière aux États-Unis, le modèle peut également prédire, comme le confirme l'estimation, que la progression des prix devrait être plus sensible à la variation du taux de rotation dans les régions du pays où l'offre est peu élastique. En partant du taux de rotation, l'auteur identifie un choc d'offre auquel peut être imputée au-delà de 80 % de la chute des prix réels des maisons observée depuis leur sommet de 2006 jusqu'en 2010.

Classification JEL : E3, R2, R21
Classification de la Banque : Évaluation des actifs; Cycles et fluctuations économiques

1 Introduction

> *In the housing market, people ......just do not know how to judge the overall level of prices. Much more salient in their minds is the rate of increase of prices.*
>
> Robert J. Shiller *Irrational Exuberance*, 2nd Ed., 2005. p. 208

> *Housing markets are less liquid, but people are very careful when they buy houses. Its typically the biggest investment they're going to make, so they look around very carefully and they compare prices. The bidding process is very detailed.*
>
> Eugene F. Fama "Interview with Eugene Fama", *The Region*, December 2007, Minneapolis Federal Reserve Bank.

The behaviour of house prices is notorious. The bubble of 2000-2007 in the United States and subsequent bust in many of the countries in the developed world being the most recent examples. House prices are volatile, with movements that are much larger than the movements in fundamentals such as income, rents, and interest rates. House prices are generally thought to be sticky, slowly adjusting to their long-run equilibrium level. House prices are predictable. High growth in one year implies high growth in the following year. House prices exhibit short to medium run deviations from fundamentals but do exhibit reversion to the fundamentals. All of these observations have led researchers to believe that the housing market is not efficient, for instance Case and Shiller (1989).

At the same time, turnover, defined as sales to the stock of housing, has generally moved in lock-step with house price growth. Therefore, the same behaviour that shows up in house prices also shows up in housing quantities.

This paper provides one possible avenue to understand the inefficient behaviour of house prices. The idea is that buyers and sellers are affected by search frictions while setting house prices, so that when there are a large number of buyers prices rise relative to fundamentals. However, buyers and sellers ignore that search frictions could have affected past house prices. Instead, buyers and sellers interpret past house prices as reflecting the true, or fundamental, value of a house. In this way, when bargaining over house prices, buyers and sellers bargain relative to recent house prices. In other words, they bargain over house price growth. Such a mechanism, which I call 'Fooled by Search' can generate the very tight correlation between house price *growth* and turnover while at the same time can help us understand the large inefficiences noted in the behavior of house prices.

Two competing models are constructed and estimated. The first model is the constrained efficient or 'rational' model while the other model is the 'irrational' or 'Fooled' model. In both models an exogenous stochastic process determines the measures of buyers and sellers which then affect prices via search frictions and market tightness. All buyers and sellers are forward looking and know the exogenous persistent process for buyers and sellers. However, no entry decision of whether to be a buyer or seller is modelled. In this way the models are like typical asset pricing models with exogenous processes for endowments and rational forward looking agents. Here, the exogenous endowments are the measures of buyers and sellers.

The difference between the models is how buyers and sellers interpret past prices. In both models agents do not know the fundamental value of a house, instead they use past data. In the rational model, agents realize that past prices were set in a constrained efficient market where past market conditions affected prices via search frictions. Therefore, agents use both quantity (turnover) and price data to calculate the fundamental value of a house from comparable transactions. However, in the irrational, or 'Fooled', model, agents instead assume that past prices were set in a frictionless efficient asset market and therefore only use price data from comparable properties when calculating the fundamental value of a house.

The two models make two very different predictions regarding the relationship between turnover and prices. In the rational model, real price growth should be related to *changes* in turnover, whereas in the irrational model, real house price growth should be related to the *level* of turnover. To see this, suppose house prices start off at their fundamental value and that there is a temporary one period shock to the number of buyers. This should raise turnover and raise house prices relative to fundamentals in both models. The difference is in what happens in the subsequent period when the number of buyers goes back down to steady-state levels.

In the rational model, when market participants use comparables to figure out the fundamental value of a house, by examining quantities, they understand that prices in the previous period were affected by demand, and therefore adjust for this when calculating the fundamental value of a house. Therefore, when the number of buyers returns to its original level in the second period, turnover and house prices fall back down to their original level. Thus, turnover is related to the level of house prices, alternatively, changes in turnover are related to the growth in real house prices.[1]

In the irrational model, market participants ignore that past prices were affected by search frictions and instead assume that fundamentals are fully captured by prices. Thus, when the numbers of buyers returns to its original level in the second period, turnover falls, but prices do not. Instead prices remain at their level from the previous period. Thus, in the irrational model real price growth is related to the level of turnover.

Put differently, a temporary increase in turnover is related to a temporary increase in real prices in the rational model, but a permanent increase in the 'Fooled' model. Alternatively, a permanent increase in turnover is associated with a permanent increase in the price level in the rational model but a permanent increase in the *growth rate* of real prices in the 'Fooled' model. Therefore, under the same size of a temporary but persistent

[1] Note that this story requires that demand drive turnover. If there are also supply shocks then market pariticipants will also have to adjust for this.

demand shock, potentially caused by a relaxation of credit standards, under the 'Fooled' model prices will deviate much more from fundamentals.

Both models are estimated using annual data from 1975 to 2010, at both the national level and four census regions. An identifying assumption is made that turnover is driven by shocks to the measure of buyers, with the reaction to the measure of sellers depending upon the elasticty of supply. In all estimations the rational model does a significantly worse job at explaining the data than the irrational model. Therefore the data supports that agents are 'Fooled by Search.'

At the regional level, the model predicts that in regions with a higher elasticity of supply the sensitivity of real price growth to turnover should be lower. This result is found in the estimates, with the Midwest and South having price growth being roughly 37% less sensitive to turnover than in the Northeast and West. More specifically, a 20% increase in turnover that lasts for five years leads to a 14% increase in prices relative to trend in the Midwest versus a 24% increase in prices relative to trend in the West. In the estimation turnover has been demeaned by region, so that turnover has no effect on the relative long-run growth rates across regions. Therefore, the estimates suggest that 'Fooled' can lead to larger deviations from fundamentals in regions where the elasticity of supply is lower. In addition, the rational model fits the data much better in the Midwest and South. This is consistent with the idea that the construction of new homes can be used as a comparable to keep prices from rising too much, with this effect being strongest in the Midwest and South.

Given the estimated model, a counterfactual is constructed where the effects of 'Fooled' can be removed under the (false) assumption that there would have been no reaction by turnover. In the national data, 'Fooled' is found to have caused real prices to increase an extra 36% from 1995 until the peak in 2006, which is three-fifths of the total increase of 60%. When trend growth is removed, 'Fooled' explains over two-thirds. Across regions, 'Fooled' explains 57% of the total 107% increase in the West, but only 21% of the total 32% increase in the Midwest. These results suggest that, while not being a direct cause of the driver of housing demand, 'Fooled' can be a strong contributing factor allowing for such large increases in real house prices.

The analysis so far assumes that shocks to demand drive the housing market while, anecdotally, recent falling prices in the United States seem related to a glut of vacant and/or bank owned properties. Therefore, the last section of the paper introduces supply shocks into the identification scheme. A supply shock is identified by deviations between existing home sales and new home sales. Such deviations are not modelled, and so should be interpreted with caution. The idea is that if existing home sales rise relatively more (or fall relatively less as has happened in the recent bust) then this is evidence of an increase in the supply of homes for sale, controlling for changes in demand. The rational and 'Fooled' models are both re-estimated using the supply shock identification scheme (note that demand shocks are still identified using turnover). Once again the 'Fooled' model outperforms the rational model in terms of fit. The supply shock is found to have a significant and negative impact in all regions and in the national estimate. A counterfactual is then performed where the supply shock is set to zero to answer by how much would have prices fallen since the peak in 2006 if the supply shock had not happened. The estimates suggest that in the United States real prices should only have fallen 3.0%, suggesting that without the supply shock nominal prices would not have fallen.

The strong relationship between turnover and prices was first illustrated in Stein (1995). Subsequently, papers by Berkovec and Goodman (1996), Hort (2000)[2], Ortalo-Magné and Rady (2004), Andrew and Meen (2003), Leung, Lau and Leong (2002) and Wheaton and Lee (2009) have confirmed the results. None of these papers focuses on the difference between the price level versus price growth.[3] Andrew and Meen (2003) estimate a non-structural model of the United Kingdom housing market. Using VAR analysis they find that temporary shocks to fundamentals such as interest rates lead to a temporary change in turnover and a permanent change in house prices.

One line of thought to explain the relationship between turnover and sales is the search context used in this paper. The first search model of housing is Wheaton (1990). The model in Williams (1995) is very close to the model in this paper, and is the first paper to consider aggregate uncertainty. Since then several papers have developed search models to explain the general relationship, notably, Berkovec and Goodman (1996), Krainer (2001), Novy-Marx (2009), Leung and Zhang (2007), Ngai and Tenreyo (2010) and Díaz and Jerez (2010). With the exception of Ngai and Tenreyo (2010) and Díaz and Jerez (2010), none of these papers confront the empirical predictions of the model with the data. None of these papers examines the implications for the model regarding the relationship between turnover and price growth versus price levels. The paper by Berkovec and Goodman (1996) is very interesting, sharing similar features to the irrational model in this paper, such as a backward looking element for price setting similar to this paper. However, they assume an equation for the slow adjustment of prices, not placing it in a microfounded theory of why prices are backward looking. Furthermore, they assume that sellers do not know the current level of market tightness. Last, the paper by Albrecht, Anderson, Smith and Vromer (2007) shares several modelling features with the model of this paper, but they look at the implications of impatient buyers and sellers on price dispersion and time-on-the-market in a stationary environment.

Another idea besides search frictions to explain the high correlation between turnover and prices,[4] first put forth by Stein (1995), is that households face a down payment constraint to buy a home. When prices fall, current homeowners have less equity that they can then apply to buy a bigger house, so that turnover also falls. This idea has been

[2]Hort (2000) is VAR analysis estimated in levels for both turnover and prices. Hort's estimated impulse response functions indicate that turnover is more highly related to the change of prices rather than the level of prices.

[3]Wheaton and Lee (2009) have mentioned the difference, but their inclusion of lags in their estimation of levels makes a model in levels equivalent to a model in differences.

[4]Also put forth by Genesove and Mayer (2001) and Engelhardt (2003) is nominal loss aversion. In this case, in a downturn, sellers are reluctant to sell and face nominall losses, so turnover falls.

quite nicely put into a dynamic OLG setting by Ortalo-Magné and Rady (2006).[5] Several authors have been taking the search and the credit constraint theories and trying to see which is more important.[6] I do not see these theories as competing, rather, they reinforce each other: prices drive turnover and turnover drives prices. In my opinion, a model that combines both of these frictions is much closer to reality than a model with only one such friction.

The rest of the paper is laid out as follows. The next section goes over the behaviour of house prices that suggest that the housing market is an inefficient market. The following section covers the tight relationship between turnover and real price growth. In section 4 the 'Fooled by Search' mechanism is introduced. Section 5 covers the complete structural model. The model is estimated in section 6. The subsequent section uses counterfactuals to address by how much did 'Fooled' contribute to the housing bubble. The penultimate section covers the supply shock. The last section concludes.

2 Behaviour of House Prices

House prices are notorious for deviating from fundamentals. Two of the most cited fundamentals for house prices are real personal income and real rents. Figure 1 plots real rents (from the BLS), real personal income (from the BEA) and real house prices (from the FHFA). All series are deviations from a log-linear trend and have been deflated by the GDP deflator (from BEA). Pictures like figure 1 have led various researchers to make several observations about the behaviour of house prices:

- House prices are *volatile relative to fundamentals*, such as income and rents. As can been seen by figure 1 the movements in house prices are much larger than fundamentals such as income or rents. Such an observation has been made by Lamont and Stein (1999), Malpezzi (1999), Ortalo-Magne and Rady (2006), Verbrugge (2008) among others.

- House prices are *predictable*. This is the famous result of Case and Shiller (1989), where they found predictable house prices at the metropolitan level. From figure 1 house prices are clearly not white noise–they have a predictable component.

- House prices are *sticky*. In other words, they adjust slowly to fundamentals. We can see this in figure 1 by the general lagging behavior. DiPasquale and Wheaton (1994) argue strongly that to understand the behavior of house prices we need to allow for stickiness. The idea that house prices are sticky seems to be generally accepted by most real-estate economists and the general public.

[5]Further work by Sanchez -Marcos and Ríos-Rull (2008) has tried to extend the environment of Ortalo-Magné and Rady (2006).

[6]See Leung, Lau and Leong (2002), Wheaton and Lee (2009) and Clayton, Miller and Peng (2010).

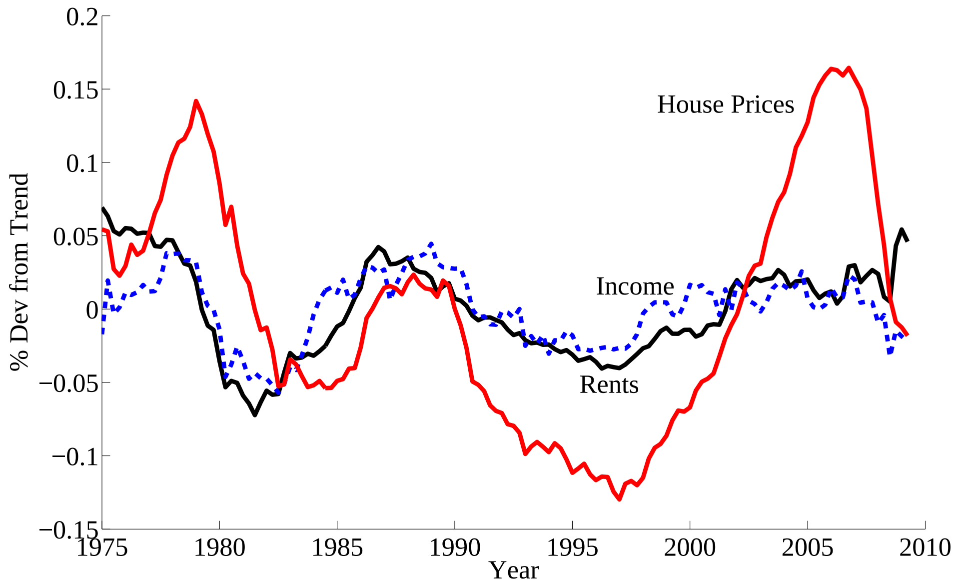

Figure 1: Real house prices, personal income, and rents. Sources: Rents: BLS; Personal Income: BEA; House Prices: FHFA. Made real by GDP Deflator (BEA).

- Last, house prices exhibit *short-run deviations from fundamentals, but long-run reversion*. This is an idea of overshooting or overreaction to fundamentals. Such behaviour of house prices has been found by Meese and Wallace (1994) and Glaeser and Gyourko (2007)

Most of these observations have been found in aggregate data and more disaggregated data such as at the metropolitan or regional levels, and have been found in many different countries. The goal of this paper is to give us a way to provide one possible avenue to understand the divergent behavior between house prices and fundamentals.

3 House Prices, Turnover and Efficient Markets

Figure 2 plots the annual real house price growth rate alongside turnover since 1975. The series for house price growth is the price index put forth by the Federal Housing Finance Agency (FHFA) made real by the GDP Deflator from the Bureau of Economic Analysis

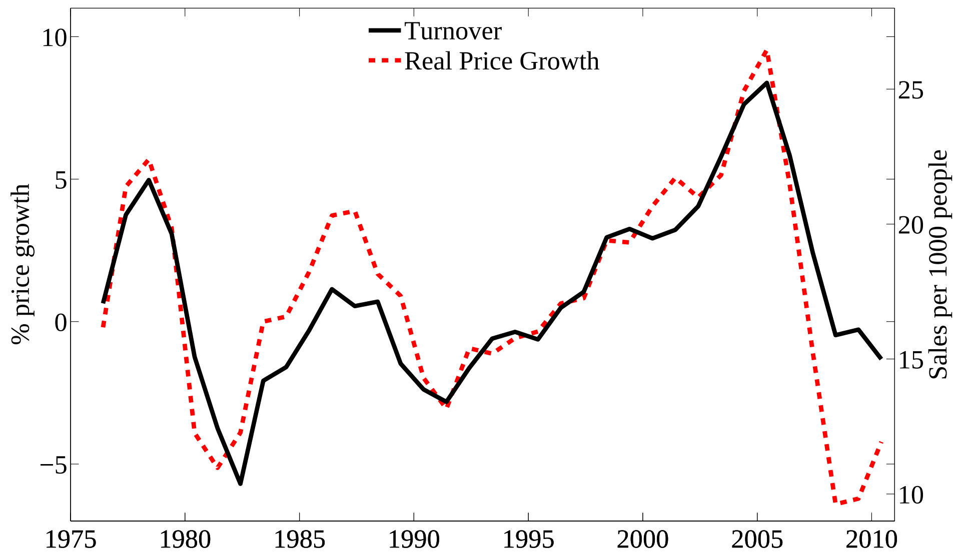

Figure 2: Turnover and Annual Real House Price Appreciation. Sources: Turnover: NAR and Census; Real Price growth: FHFA and GDP Deflator.

(BEA).[7] The price appreciaton series appears stationary with a mean of 1.77%.[8] Turnover is measured as the ratio of total sales relative to the population.[9] Population comes from Census, while the total sales is the sum of existing single family sales reported by the National Association of Realtors (NAR) and new single family sales from Census as part of the Survey of Construction. The turnover series appears stationary over this period with a mean of 17.5 sales per 1000 people.

Annual real house price growth and turnover are highly related. Over the entire sample, the correlation is 0.84. A simple linear regression of house price growth, $\%\Delta p_t$, on a constant, γ, and turnover, ϕ_t, given by

$$\%\Delta p_t = \gamma + \psi \phi_t + \varepsilon_t \tag{1}$$

results in an R^2 of 0.71. Empirically, at an annual frequency, real house price growth and turnover are highly related.

[7] The FHFA produces their price index from Fannie Mae and Freddie Mac conforming mortgages. There are two indices. One consists of purchase only mortgages, the other consists of all transactioncs including refinances. The all transactions index goes back to 1975 while the purchase only index goes back to 1991. The series used in this paper is a combination. From 1975 to the first quarter of 1991 the all transactions index is used. From the second quarter of 1991 on the purchase only index is used. The purchase only index is slightly smoother in quarterly data.

[8] This stationarity was not so obvious before the current downturn, a nice positive externality of the current problems.

[9] Typically turnover is measured at the ratio of sales to stock. Here, for stock I use population instead of the stock of homes since population measurement is a bit more precise, especially at more disaggregated data.

We can write the house price level, p_t, as the product of house price growth and an initial house price level, p_0, by using

$$p_t = \prod_{s=1}^{s=t} (1 + \%\Delta p_s)\, p_0. \tag{2}$$

Taking logs we get

$$\log p_t - \log p_0 = \sum_{s=1}^{s=t} \%\Delta p_s. \tag{3}$$

Inserting the linear relationship from equation (1) we arrive at

$$\log p_t - \log p_0 = t\gamma + \psi \sum_{s=1}^{s=t} \phi_s + \sum_{s=1}^{s=t} \varepsilon_s. \tag{4}$$

Equation 4 suggests that the current price level for housing (relative to an initial price level) is composed of three parts:

1. a trend component, given by $t\gamma$,
2. shocks, given by the summation of the ε_s, and
3. the summation of past turnover.

More importantly, equation (4) implies that, *ceteris paribus*, a shock to turnover at time s is a *permanent* shock to the real house price level. Alternativaly, one can think that there is a shock at time s that causes a temporary change in turnover but a permanent change to the house price level.

The idea that shocks to turnover cause a permanent change to the house price level has the flavor of efficient markets. One interpretation of the Efficient Market Hypothesis of Fama (1970) is that prices fully reflect all of the relevant information, or fundamentals. In order for the market to be efficient, the shocks to these fundamentals should follow a random walk, being (a) *independent and identically distributed* (iid) and (b) *permanent*. In the case of the housing market, the shock to the fundamentals that are driving turnover are related to a permanent shock to prices, so that (b) is satisfied. However, inspection of figure 2 shows this shock fails on the iid criterium, turnover is predictable. Of course, given the strong relationship between turnover and real house price growth, this is simply a restatement of the well known result of Case and Shiller (1989) that real house price growth is predictable. Here, the predictability of real house price growth is shown to also manifest itself in turnover.[10]

To summarize: (1) real house price growth and turnover are highly related in aggregate United States data; (2) this is consistent with the idea that, *ceteris paribus*, a shock causes

[10] Of course the idea that turnover is also predictable could be a sign that the market is trying to grab the predictable returns to housing due to the predictable component of prices.

a temporary change to turnover and a permanent change to the house price level; (3) turnover is predictable, so that the efficient markets hypothesis is potentially violated: housing returns are predictable.

4 Fooled by Search: The Mechanism

What can explain the tight relationship between real house price growth and turnover that is indicative of an inefficiency in the housing market? One possibility is 'Fooled by Search.' The idea is twofold.

Search Inefficiency The first part is that there is an inefficiency in the housing market due to search frictions. These frictions are the difficulties that buyers and sellers face in finding each other in the market, frictions that are clearly present in the housing market. When demand is high, sellers can more easily find buyers. Due to search frictions and bargaining, high demand can raise prices since both buyers and sellers know that sellers can find another buyer more easily than in periods with low demand. For instance, there were many stories during the peak of the housing boom of sellers receiveing multiple offers above their asking price on the day that a house was listed. Such frictions can cause prices to deviate from fundamentals due to demand being temporarily high, even though there has been no change to the underlying fundamentals. In this sense search can cause the housing market to be constrained efficient. Note that this effect should be a *level* effect, raising house prices above the fundamental price, not a growth rate effect.

Belief in Efficient Markets The second part is that households believe that the housing market is efficient. In particular, households believe that the prices of recent transactions reflect the true underlying fundamental value of a house. Households ignore that the housing market is constrained efficient.

Combination: Fooled by Search The combination of the two creates an inefficiency in the housing market where turnover and real house price growth move together in response to demand shocks.[11] A positive shock to demand raises the number of buyers, raising the number of transactions, therefore raising turnover. Furthermore, the number of buyers relative to sellers increases, making it easier for a seller to sell, improving the bargaining position of the seller, resulting in higher prices. When buyers and sellers believe that markets are efficient, they use recent comparable transactions (comparables) as an approximation of the fundamental value of the house being sold.[12] Since buyers and sellers believe that the prices of recent transactions reflect the fundamentals, they bargain relative to recent prices instead of the true underlying fundamental. In other words, buyers and sellers bargain over house price *growth*, not the level. Thus, a positive shock to demand raises turnover and puts upward pressure on prices due to search frictions. Since

[11] A shock to supply would cause turnover and price growth to move in opposite directions. There is some evidence that this has been happening in 2008 and 2009.

[12] Comparables serve as an anchor in the sense of Tversky and Kahneman (1974).

buyers and sellers believe in efficient markets, they bargain over price growth, not the level. Therefore, shocks to demand move both turnover and price growth together.

Because future market participants[13] believe in efficient markets, they ignore that the combination of search frictions and high demand may have raised prices of comparables above fundamentals. In this way, the shock to turnover arising from the shock to demand results in a *permanent* shock to the price level.

4.1 Reduced Form Model

Later sections of the paper cover the model in detail. Here the main ideas of 'Fooled by Search' are covered using the reduced form equations that result from the deeper structural model.

Efficient Markets First consider an efficient markets model, where prices fully reflect fundamentals and there are no search frictions. Letting p_t denote the price at t and z_t the fundamental at t, we get

$$\log p_t = z_t. \tag{5}$$

Shocks to the fundamental should be iid, besides a predictable growth component, γ, so that

$$z_t = z_{t-1} + \gamma + \varepsilon_t, \tag{6}$$

where ε_t is distributed iid. This implies that

$$\log p_t = z_{t-1} + \gamma + \varepsilon_t.$$

Subtracting $\log p_{t-1}$ from $\log p_t$ and using the approximation $\%\Delta p_t \approx \log p_t - \log p_{t-1}$ we get the familiar result, that besides γ, price growth should be unpredictable, or

$$\%\Delta p_t = \gamma + \varepsilon_t. \tag{7}$$

Search Inefficiency Now suppose that there is a search inefficiency. In the presence of search frictions, high demand raises prices above fundamentals. Further, assume that high demand also implies higher turnover. Therefore, high demand in period t results in higher turnover, ϕ_t, and causes prices to deviate from fundamentals, so that

$$\log p_t = \psi \hat{\phi}_t + z_t \tag{8}$$

where $\hat{\phi}$ is the percent deviation of turnover from average. Equation (8) is the constrained efficient price. The parameter ψ measures the strength of the deviation of prices from fundamentals due to search frictions, where $\psi = 0$ means no search frictions, resulting in efficient markets pricing, given by equation (5). Assuming that z_t still follows the process in equation (6), the price at t is given by

$$\log p_t = \psi \hat{\phi}_t + z_{t-1} + \gamma + \varepsilon_t. \tag{9}$$

[13] Note that non-participants may be non-participants because they believe that prices do not reflect fundamentals when prices are high.

Subtracting $\log p_{t-1}$ from $\log p_t$, price growth in the presence of search frictions is given by

$$\%\Delta p_t = \gamma + \psi\left(\hat{\phi}_t - \hat{\phi}_{t-1}\right) + \varepsilon_t. \tag{10}$$

Search frictions provide a possible explanation to the predictability of prices. However, the predictable component should be related to the predictable *change* in turnover, but we saw in figure 2 that the predictable component of price growth is related to the predictable component of the *level* of turnover. Therefore, search frictions alone cannot explain the inefficiencies we observe in house prices.

Fooled by Search: Search Inefficiencies and belief in Efficient Markets Finally, assume that buyers and sellers believe that recent transaction prices satisfy efficient markets, but at the same time allow for search frictions to affect their price setting. In particular, buyers and sellers at time t believe that past prices (the comparables) satisfy the efficient markets hypothesis, or

$$\log p_{t-1} = z_{t-1}.$$

Therefore, when bargaining over prices at t, buyers and sellers set $z_{t-1} = \log p_{t-1}$ in equation (9), so that the price setting equation at t becomes

$$\log p_t = \psi\hat{\phi}_t + \log p_{t-1} + \gamma + \varepsilon_t. \tag{11}$$

Once again, subtracting $\log p_{t-1}$ from $\log p_t$, the combination of search frictions and a belief that past prices reflect fundamentals results in price growth being given by

$$\%\Delta p_t = \gamma + \psi\hat{\phi}_t + \varepsilon_t. \tag{12}$$

We thus arrive our result: real house price growth is related to the level of turnover. In other words, the predictable component of real house price growth stems from the predictable component of the level of turnover.

5 Structural Model

5.1 Full Information Model

5.1.1 Environment: Matching and Turnover

The model presented here is a modified version of the classic Diamond (1982) model, similar to the model presented in Rocheteau and Weill (2011). First consider a full information, fully rational stochastic forward looking model. There are two types of agents: *buyers* and *sellers*. Buyers and sellers meet in a decentralized market to trade an homogenous asset at an endogenous price of $\tilde{p}_t$. Time is continuous and all agents discount the future at rate r. No attempt is made to model the entry decision of buyers and sellers.

Instead, the measures of both evolve stochastically.[14] To keep the model consistent with an exogenous level of buyers and sellers, assume that successful buyers and sellers are both immediately replaced. Let the measure of buyers at time t be given by

$$\mu_t^B = e^{b_t} \kappa_t \mu_O \tag{13}$$

and the measure of sellers at time t be given by

$$\mu_t^S = e^{s_t} \kappa_t \mu_O. \tag{14}$$

The variables b_t and s_t denote the current shock to buyers and sellers, respectively. The measure of the total stock of the asset is given by μ_O, which is assumed to be constant for simplicity. The variable κ_t denotes churn and is given by

$$\kappa_t = e^{k_t} \overline{\kappa}, \tag{15}$$

where k_t is a shock to churn which increases the measures of both buyers and sellers by an equal proportion.

Assume that b_t follows a first-order markov process in continuous time,[15] with an arrival rate of λ_b and let the distribution of the new shock be given by $F_b(b'|b)$. Furthermore, assume that

$$E\left[b' \middle| b\right] = \rho_b b \qquad \text{and} \qquad E[b] = 0$$

where $0 < \rho_b < 1$. This shock is similar to liquidity shocks in the model of Grossman and Miller (1988) or Duffie, Garleanu and Pedersen (2007).

While the model can incoporate separate shocks to sellers and to churn, for expositional reasons make the following assumption regarding shocks to sellers and churn:

Assumption 1. Assume that supply and churn shocks are proportional to shocks to buyers:

$$s_t = \xi b_t \qquad 0 \leq \xi \leq 1, \qquad \text{and} \qquad k_t = \xi_k b_t \qquad \xi_k > 0.$$

Therefore, shocks to the economy are completely demand driven. This assumption is important for identification. First for market participants to identify past shocks in the incomplete information model to follow, and second, for the later empirical section.

The parameter ξ can be interpreted as a measure of the elasticity of supply, or the ability for capital to enter into this asset market. In this way $\xi < 1$ captures the *limits of arbitrage* discussed in Gromb and Vayanos (2010) and Duffie (200x). As expanded on

[14] The methodology of the model is similar to endowment models of asset pricing: quantities evolve stochastically and prices are determined endogenously. The "endowments" in this paper are the exogenous quantities of buyers and sellers that determine turnover.

[15] This model assumes that only one side of the market, buyers, is hit with shocks. Later on in the empirical part of the paper, the model will be modified to allow for independent shocks to both buyers and sellers.

below, shocks to churn have no effect on pricing, but do affect the total number of sales so are important in mapping the model to observables. Consistent with the theories of churn in Stein (1995) or Ortalo-Magné and Rady (2006) in assumption 1 churn is a reaction to demand.[16] In this way the theory presented in this paper incorporates and is consistent with theories of churn linking turnover and prices.

Buyers and sellers meet and trade in a decentralized market. Let the rate of flow[17] of matches, m_t, be determined by a Cobb-Douglas matching function:

$$m_t\left(\mu_t^B, \mu_t^S\right) = A\left(\mu_t^B\right)^{\alpha}\left(\mu_t^S\right)^{1-\alpha}. \tag{16}$$

The parameter α determines the weight that buyers have in creating matches, whereas A determines the productivity of the matching technology. The Cobb-Douglas assumption is standard in the labor search literature. Empirical work in labor search has failed to reject a constant returns to scale matching function,[18] while little justification has been given for the Cobb-Douglas functional form.[19] To the author's knowledge this issue has not been addressed in the housing search literature. For financial assets, Duffie, Garleanu and Pederson (2005, 2007) assume that the number of matches is determined by the short-side of the market, which is also a constant returns to scale matching technology. For an excellent overview on the matching function in models of asset trading see Rocheteau and Weill (2011).

Buyers and sellers are concerned with the likelihood that they find each other. The arrival rate of a buyer for a seller, or the selling rate, is

$$q_t = \frac{m_t}{\mu_t^S},$$

which is simply given by the ratio of the flow rate of matches to the total number of sellers. Likewise, denote the finding rate of a seller for a buyer as

$$f_t = \frac{m_t}{\mu_t^B}.$$

Let θ_t denote the ratio of buyer to sellers, or market tightness. From equations (13) and (14) market tightness is given by

$$\theta_t = e^{b_t - s_t}.$$

[16] In these models, because housing is typically purchased with a down payment, an increase in demand that raises house prices raises the equity of existing homeowners, enabling existing homeowner to be able to purchase larger homes. The combination of an existing homeowner selling their current house and buying another is churn. As noted by Ortalo-Magné and Rady (2006) their theory is complementary to the search based theory presented in this paper.

[17] For those unfamiliar with continuous time search models, the number of matches (or sales in this case) over a time interval Δ would be given by the product of the flow and the length of the time interval, or Δm_t, holding m_t fixed over the interval.

[18] See the survey by Petrongolo and Pissarides (2001)

[19] See Shimer (2005).

Under the Cobb-Douglas assumption, the selling rate and finding rate are given by

$$q(\theta_t) = A\theta_t^{\alpha} \text{ and } f(\theta_t) = A\theta_t^{\alpha-1}.$$

By construction, at the steady-state, market tightness is unity and the finding rate and selling rate are both equal to A. Away from the steady-state, the deviation of market tightness from unity is approximately given by

$$\theta_t - 1 \approx (1-\xi)\, b_t.$$

Therefore, if $\xi < 1$, then shocks to the measure of buyers impact the relative probability that buyers and sellers find each other. Note that the constant returns to scale matching technology rules out that finding and selling rates are increasing in the overall size of the market.[20] As noted below, what matters for pricing in this model is the *relative* probability with which buyers and sellers find each other.

Given the Cobb-Douglas assumption, the rate of turnover, ϕ, is given by

$$\phi_t = \frac{m_t}{\mu_O}.$$

Using equations (13) to (16), turnover can be written as

$$\phi_t = A\overline{\kappa}e^{k_t+\alpha b_t+(1-\alpha)s_t}.$$

At the steady state, where $b_t = s_t = k_t = 0$, denote the level of turnover as

$$\overline{\phi} = A\overline{\kappa},$$

and let $\hat{\phi}_t = \left(\phi_t - \overline{\phi}\right)/\overline{\phi}$ denote the percentage deviation of turnover at time t from the steady-state. Define

$$\hat{\phi}_t^b = \alpha b_t, \qquad \hat{\phi}_t^s = (1-\alpha)\, s_t \qquad \text{and} \qquad \hat{\phi}_t^k = k_t. \tag{17}$$

Refer to $\hat{\phi}_t^x$ as the contribution of variable x the deviation of turnover from the steady-state at time t. By construction

$$\hat{\phi}_t \approx \hat{\phi}_t^b + \hat{\phi}_t^s + \hat{\phi}_t^k. \tag{18}$$

Given the structure of shocks to sellers and churn in assumption 1 shocks to buyers map into turnover via

$$\hat{\phi}_t = (\alpha + (1-\alpha)\,\xi + \xi_k)\, b_t. \tag{19}$$

Equation (19) is crucial for the latter model of incomplete information and for mapping the model into data.

Further, note that the churn variable, k_t, affects turnover but not market tightness. In other words, the finding and selling rates are only functions of b_t and s_t. Buyers and sellers only care about the finding and selling rates, so in the analysis to follow, k_t is not a state variable. The presence of churn will matter for confronting the model with the data.

[20] Ngai and Tenreyo (2010) allow for a thick market externality in housing markets, where households can better match to a house as the size of the market increases.

5.1.2 Environment: Trade, Pricing and Equilibrium

Let y_t denote the expected present discounted value to a buyer from buying the asset at time t. Refer to y_t as the *long run fundamental* value to the asset. In a world of perfect capital markets, if we let d_t denote the dividend flow to the asset at time t, then the present discounted value of the asset would be given by

$$y_t = \int_0^\infty e^{-rs} d_s \mathrm{d}s.$$

For a house, the dividend flow could consist of the value of housing services adjusted for tax benefits, maintenance costs, expected capital gains, etc. Instead of modeling the dividend process, write y_t as

$$y_t = e^{z_t + \gamma t}\overline{y}.$$

Under this formulation, γ represents exogenous constant growth in the fundamental, while z_t represents a shock in the long run fundamental. Assume that new values to z_t follow a poisson process with an arrival rate of λ_z and that the distribution of the new variable, z', follows a martingale, so that

$$E\left[z' \middle| z \right] = z.$$

To motivate trade, let sellers value the asset less than buyers by δ percent so that a seller's valuation of the asset is $(1-\delta) y_t$. The flow utility to the seller from possessing the asset is therefore $r(1-\delta) y_t$.

Turning to the value functions, the aggregate state variables are the two relevant shocks: b_t and z_t.[21] Denote the value function for a buyer at time t as $\tilde{V}_t^B(b_t, z_t)$ and that of the seller as $\tilde{V}_t^S(b_t, z_t)$. Let the pricing function be given by $\tilde{p}_t(b_t, z_t)$. To ease on notation, divide the value functions and prices through by $e^{\gamma t}$ and define

$$V^B(b,z) = \tilde{V}_t^B(b,z)/e^{\gamma t}, \qquad V^S(b,z) = \tilde{V}_t^S(b,z)/e^{\gamma t} \qquad \text{and}$$

$$p(b,z) = \tilde{p}_t(b,z)/e^{\gamma t},$$

where the t subscript has been removed.[22] The value function for a buyer is

$$rV^B(b,z) = f(\theta)\left[e^z\overline{y} - p(b,z) - V^B(b,z)\right] + \lambda_b E_{b'|b}\left[V^B(b',z) - V^B(b,z)\right] + \lambda_z E_{z'|z}\left[V^B(b,z') - V^B(b,z)\right], \quad (20)$$

while that of a seller is

$$rV^S(b,z) = r(1-\delta)e^z\overline{y} + q(\theta)\left[p(b,z) - V^S(b,z)\right] + \lambda_b E_{b'|b}\left[V^S(b',z) - V^S(b,z)\right] + \lambda_z E_{z'|z}\left[V^S(b,z') - V^S(b,z)\right]. \quad (21)$$

[21] Remember that the churn shock k_t does not affect the finding and selling rates.

[22] The growth variable γ and time t will once again be relevant in the incomplete information model and in the empirical work.

These value functions imply that when a buyer and seller make a trade, they never trade again. Further, note that this formulation excludes the possibilities of bubbles, where the value to asset can rise just because it is expected to rise.

When a buyer and seller meet they bargain over the price P. Here we denote P as the price buyers and sellers bargain over, taking the pricing function in potential future matches, $p(b,z)$, as fixed. Bargaining and trades are assumed to happen sufficiently fast that the state variables b and z are fixed from the viewpoint of the bargaining buyer and seller. Assume that the bargaining process can be represented as the solution to a Nash bargaining problem. The threat point for a buyer is to continue being a buyer, implying a surplus to the buyer of

$$X^B(b,z) = e^z\overline{y} - P - V^B(b,z).$$

The threat point for a seller is to continue being a seller, giving a seller a surplus of

$$X^S(b,z) = P - V^S(b,z).$$

The total surplus to the trade, $X(b,z) = X^B(b,z) + X^S(b,z)$, is given by

$$X(b,z) = e^z\overline{y} - V^B(b,z) - V^S(b,z).$$

Let ω be the bargaining power for a buyer. The price setting equation from the Nash solution satisfies

$$X^B(b,z) = \omega X(b,z),$$

which is given by

$$e^z\overline{y} - P - V^B(b,z) = \omega\left[e^z\overline{y} - V^B(b,z) - V^S(b,z)\right].$$

Doing some re-arranging, we arrive at the following price setting equation

$$P(b,z) = (1-\omega)\left(e^z\overline{y} - V^B(b,z)\right) + \omega V^S(b,z). \tag{22}$$

An *equilibrium* is thus $p(b,z)$, $P(b,z)$, $V^B(b,z)$ and $V^S(b,z)$ that satisfy equations (20), (21), (22) and

$$P(b,z) = p(b,z) \qquad \forall b,z. \tag{23}$$

5.1.3 Stationary Solution

In solving the model, first consider the stationary solution where the shocks, b and z, are all set to zero for all t. With such a restriction imposed, the stationary value to being a buyer can be solved from equation (20) to be

$$\overline{V}^B = \frac{f(\overline{y} - p)}{r+f},$$

where the overline denotes a stationary variable. Likewise, from equation (21) the stationary value to being a seller is

$$\overline{V}^S = \frac{r\left(1-\delta\right)e^z\overline{y} + qp}{r+q}.$$

Inserting these into equation (22) and setting $P = p$ the equilibrium stationary price is

$$\overline{p} = \left(1-\omega\delta\right)\overline{y}. \tag{24}$$

This is the long-run price of the asset when the short-run liquidity shocks, b_t, and fundamental shocks, z_t, are shut down. Note that search frictions do not enter into the steady-state price. This is due to the implicit assumption that steady-state market tightness is unity. Search frictions enter into the pricing of the asset due to *relative* search costs, and search costs are equal when there are equal measures of buyers and sellers. The factors that do affect the steady-state price are: the long run fundamental value of the asset, $\overline{y}$; a seller's valuation relative to a buyer, δ; and the bargaining frictions, ω. As the buyer gets more bargaining power, a buyer is able to lower the price down to the seller's valuation of $\left(1-\delta\right)\overline{y}$. On the other hand, as the buyer's bargaining power diminishes to zero, the price approaches the buyer's valuation of $\overline{y}$.

5.1.4 Approximate Linear Solution to Stochastic Model

As shown in Appendix A, it is straight forward to derive the approximate linear solution to the pricing equation for the stochastic model. Defining $\hat{p}$ as the percent deviation of the prices from $\overline{p}$, the approximate linear solution is

$$\hat{p}\left(b,z\right) \approx z + \left(1-\beta\right)\left(1-\xi\right)\sigma b, \tag{25}$$

where

$$\beta = 1 - \left[\frac{r}{r+\lambda_b\left(1-\rho_b\right)}\right]$$

and

$$\sigma = \left[\frac{A}{r+A}\right]\left[\left(1-\omega\right)\omega\right]\left[\frac{\delta\overline{y}}{\left(1-\omega\delta\right)\overline{y}}\right].$$

5.1.5 Turnover and Prices

From equation (19) turnover relates to prices via

$$\hat{p}\left(b,z\right) \approx z + \psi\hat{\phi} \tag{26}$$

where

$$\psi = \frac{\left(1-\beta\right)\left(1-\xi\right)\sigma}{\left(\alpha+\left(1-\alpha\right)\xi+\xi_k\right)}. \tag{27}$$

5.1.6 Price Growth and Turnover

To bring price growth into the model, remember that the actual price at time t is given by

$$\tilde{p}(b_t, z_t) = e^{\gamma t} p(b_t, z_t).$$

Taking logs and using equation (25) we get

$$\log \tilde{p}(b_t, z_t) \approx \tilde{z}_t + (1-\beta)(1-\xi)\sigma b \tag{28}$$

where

$$\tilde{z}_t = \gamma t + \log \overline{p} + z_t,$$

and the approximation $\log p \approx \log \overline{p} + \hat{p}$ has been used. The percentage price growth from $t-\Delta$ to t is approximately given by $\log \tilde{p}_t - \log \tilde{p}_{t-\Delta}$, or

$$\%\Delta p_t \approx (1-\beta)(1-\xi)\sigma(b_t - b_{t-\Delta}) + \Delta\gamma + \Delta\varepsilon_t, \tag{29}$$

where $\Delta\varepsilon_t = z_t - z_{t-\Delta}$ denotes the shock to the fundamental from $t-\Delta$ to t.[23] Using equation (26) price growth relates to turnover via

$$\%\Delta p_t \approx \Delta\gamma + \psi\left(\hat{\phi}_t - \hat{\phi}_{t-1}\right) + \Delta\varepsilon_t. \tag{30}$$

5.2 Incomplete Information Model

In the model of complete information, market participants know the fundamental value of the house, given by $\tilde{z}$. In many asset markets this may be difficult to determine. For instance, in housing markets the actual market value of a house with a pool, a large deck, in a certain neighborhood and a ten minute walk from a nice park may be hard to determine. However, market participants may have some idea that such a house may be rising in value. To capture this concept precisely make the following assumption:

Assumption 2. (*Incompete Information*) Market participants do not observe $\tilde{z}_t$. Instead they observe ε_t and assume

$$\tilde{z}_t = \tilde{z}_{t-\Delta} + \Delta\gamma + \Delta\varepsilon_t. \tag{31}$$

This assumption is similar to the quote by Shiller that starts this paper. People may not have a good understanding of the *levels* of asset values, but they have an understanding of the *growth rates* of asset values.

In order for market participants to bargain over asset prices, they need to know the value of $\tilde{z}_{t-\Delta}$. However, the value of $\tilde{z}_{t-\Delta}$ can be inferred from past market data. Two different assumptions are modeled. In the first, market participants *rationally* take into account the effects of past shocks to market liquidity when inferring $\tilde{z}_{t-\Delta}$. In the second, market participants *irrationally* ignore that market liquidity affect prices in the past, so that market participants can be 'Fooled by Search.'

[23] To be consistent with notation, the shock is multiplied by the length of the time period in question since the size of the shock is proportional to the length of time.

5.2.1 Rational Model

In the rational model, market participants are aware that prices are determined by the combination of long run fundamentals, $\tilde{z}$, and temporary market liquidity conditions, b. Therefore, make the following assumption regarding how market participants infer the value for $\tilde{z}_{t-\Delta}$ from past data:

Assumption 3. (*Rationality*) In the rational model, market participants infer the past long run fundamental, $\tilde{z}_{t-\Delta}$ by

$$\tilde{z}_{t-\Delta} = \log \tilde{p}_{t-\Delta} - \psi \hat{\phi}_{t-\Delta}. \tag{32}$$

Note that market participants may not know the shock b at time $t - \Delta$, however, they can infer it from data on turnover. Combining assumptions 2 and 3, when market participants recognize that past prices can be affected by market liquidity, the inferred current long run fundamental is

$$\tilde{z}_t = \log \tilde{p}_{t-\Delta} - \psi \hat{\phi}_{t-\Delta} + \Delta\gamma + \Delta\varepsilon_t.$$

Therefore, through the bargaining process, prices at time t are

$$\log \tilde{p}_t = \log \tilde{p}_{t-\Delta} + (1-\beta)(1-\xi)\,\sigma b_t - \psi \hat{\phi}_{t-\Delta} + \Delta\gamma + \Delta\varepsilon_t. \tag{33}$$

The information structure in equation 33 is that market participants know current market conditions, b_t, and can observe past market data, $\tilde{p}_{t-\Delta}$ and $\hat{\phi}_{t-\Delta}$. Using the relation between current market conditions and current turnover, the growth rate of prices in the rational model is related to turnover via

$$\%\Delta\tilde{p}_t = \psi\left(\hat{\phi}_t - \hat{\phi}_{t-\Delta}\right) + \Delta\gamma + \Delta\varepsilon_t. \tag{34}$$

Therefore price growth in the rational model with incomplete information, given by equation (34), is the same as in the model with complete information, given by equation (30).

5.2.2 Irrational Model

In the irrational model, market participants ignore that market liquidity affects past prices. Instead market participants assume that past house prices reflect past long run fundamentals, $\tilde{z}$. To be specific, make the following assumption:

Assumption 4. (*Irrationality*) In the irrational model, market participants infer the past long run fundamental, $\tilde{z}_{t-\Delta}$ by

$$\tilde{z}_{t-\Delta} = \log \tilde{p}_{t-\Delta}. \tag{35}$$

Combining assumptions 2 and 4, when market participants ignore that past prices can be affected by market liquidity, the inferred current long run fundamental is

$$\tilde{z}_t = \log \tilde{p}_{t-\Delta} + \Delta\gamma + \Delta\varepsilon_t.$$

Therefore, through the bargaining process, prices at time t are

$$\log \tilde{p}_t = \log \tilde{p}_{t-\Delta} + (1-\beta)(1-\xi)\,\sigma b_t + \Delta\gamma + \Delta\varepsilon_t. \tag{36}$$

Incorporating the relationship between b_t and $\hat{\phi}_t$, the growth rate of prices in the irrational model is given by

$$\%\Delta\tilde{p}_t = \psi\hat{\phi}_t + \Delta\gamma + \Delta\varepsilon_t. \tag{37}$$

Therefore, price growth in the irrational model with incomplete information, given by equation (34), is related to the *level* of the shock to market liquidity, rather than the *growth rate* in the shock to market liquidity as in the other models.

5.3 Comments on the Irrational Model

5.3.1 Growth Rate of House Prices

In the rational model, the growth rate of house prices in a stationary equilibrium (where turnover is constant) is exogenous, given by γ. Relative supply and demand only affects the *level* of house prices. In contrast, in the irrational model, the growth rate of house prices in a stationary equilibrium depends upon γ and the relative levels of supply and demand. To see this, suppose that there is a permanent one time change to the measure of buyers. In the rational model, such an increase would cause a one-time in the price level, but have no effect on future price growth. This is clear in equation (34), since, in future periods, the increase in buyers simply cancels out, leaving price growth at $\gamma\Delta$. However, from equation (37), such a permanent increase in the measure of buyers would permanently increase price growth in the irrational model. Of course, if price growth was to permanently increase, we might expect demand to fall, this lowering the measure of buyers and lowering price growth.[24]

5.3.2 Alternative Interpretation

As I alluded to earlier, there is an alternative interpretation of the irrationality built into assumption 4. Instead households could believe that the fundamental follows the process:

$$\tilde{z}_t = \tilde{z}_{t-\Delta} + \Delta\gamma + \Delta\tilde{\varepsilon}_t.$$

In addition to not knowing $\tilde{z}_t$, buyers and sellers could also not know $\tilde{\varepsilon}_t$ and instead use current market conditions to estimate it. In other words, they interpret the bargaining pressures as the shock $\tilde{\varepsilon}_{t+1}$ so that

$$\Delta\tilde{\varepsilon}_t = \Delta\varepsilon_t + (1-\beta)(1-\xi)\,\sigma b_t. \tag{38}$$

Of course, as we will see below, b_t is likely predictable since turnover is predictable, implying that this interpretation of the fundamental is predictable, a violation of the

[24] This very likely what happened in the boom and bust in the United States.

efficient markets hypothesis. The potential mistake being made is along the lines of the quote by Shiller at the start of the paper. Households seems to think more about growth rates then levels. Under the alternative interpretation, buyers and sellers are mistaking the shock to relative demand and supply as a growth effect when theoretically it should be a level effect. They may be thinking that there is a permanent change to demand, and that such shocks lead to a permanent change to the growth rate of prices. When in reality, there was most likely just a temporary shock to market tightness, and even if demand stays high, supply will most likely respond.

Also note the flavor of self-fulfilling expectations in equation (38): if households believe it is a good time to buy a house, then demand will rise, creating pressure on prices to rise via the search frictions, resulting in rising prices and a perceived positive shock to the fundamental value of houses.

5.3.3 Rate of Information Flow and Price Growth

In equation (37) Δ denotes the time lag between the prices that buyers use as comparables and when they are actually bargaining over their transaction. Let Δ^a denote the length of a year and n satisfy

$$\Delta^a = n\Delta$$

so that $1/n$ denotes the length of the time delay in reporting prices in annual units. Holding $\hat{\phi}_t$ fixed, to get the growth in prices over a year, simply sum equation (37) n times to arrive at a general equation for annual price growth:

$$\%\Delta^a p_t \approx \gamma^a + n\psi\hat{\phi}_t + \varepsilon_t^a, \tag{39}$$

where γ^a and ε_t^a are in annual units. Therefore, the larger is n, the larger will be the fluctuations in price growth. Of course, if prices are released instantaneously, as they are in financial markets, then it might be easier for market participants to notice the pricing errors and correct for them.[25] In other words, a slow release of prices could allow for the mispricing from 'Fooled by Search' to prolong itself. We now turn to estimation and bring turnover back into the model.

6 Estimation of Models

The structural model provides two competing theories to explain the behavior of house prices: the rational model and the irrational model. Using the equations for the annual model (see subsection 5.3.3), the equation for the rational model is

$$\%\Delta p_t \approx \gamma + \psi\left(\hat{\phi}_t - \hat{\phi}_{t-1}\right) + \varepsilon_t, \tag{40}$$

[25] However, it is interesting that such a mechanism could help explain the high volatility of prices in financial markets.

while the equation for the irrational model is

$$\%\Delta p_t \approx \gamma + n\psi\hat{\phi}_t + \varepsilon_t. \tag{41}$$

6.1 Data

In order to estimate the model we have to map observables into the unobservable shocks b_t and s_t. The observables used are turnover. To compute turnover three sources of data are used:

1. Existing home sales: single-family existing home sales taken from the National Association of Realtors (NAR), annual from 1975 to 2010.
2. New home sales: single-family new homes sales, taken from the Census Bureau, annual 1975 to 2010.
3. Population: annual population, 1975 to 2010[26], from the Census Bureau.

The definition of turnover, denoted as ϕ_t, used is

$$\phi_t = \frac{New\_Sales_t + Existing\_Sales_t}{Population_t}.$$

This measure of turnover incorporates both new home sales and existing home sales. Unless otherwise specified, 'turnover' denotes 'total turnover.'

For house prices, annual data from 1975 to 2010 from the Federal Housing Finance Agency (FHFA) is used. The FHFA data is used because it provides the longest time horizon, back to 1975, and it also can be broken down to the state level which is used in estimations at the regional level. Note that the Case-Shiller data only starts in 1989 at the national level. All price series are made real via the core CPI provided by the Bureau of Labor Statistics.

The data can be broken down into the four census regions: Northeast, Midwest, South and West.[27] The price data at the census region level is constructed using the state-level FHFA data.[28]

6.2 Identification

From equations (17) and (18) the percent deviations of turnover from average can be broken down into contributions from the three shocks: b_t, s_t and k_t, resulting in

$$\hat{\phi}_t = \alpha b_t + (1 - \alpha)\, s_t + k_t. \tag{42}$$

[26] The 2010 estimate is from the 2010 census.

[27] Both new home sales and existing sales are only avaiable at the Census region level back to 1975.

[28] Each regional index is constructed by weighting the growth rate from each state for each year by population for that year.

Two different identifying approaches are used. In this section, the approach used assumes that the housing market is completely demand driven. A second approach will be used later in the paper that allows for supply shocks.

6.2.1 Demand Driven Approach

Identifying Assumption 1. *Demand Driven* Both supply and churn are driven by buyers, so that

$$s_t = \xi_s b_t \qquad \text{and} \qquad k_t = \xi_k b_t. \tag{43}$$

This assumption means that all changes in supply and churn are responses to changes in the number of buyers, *i.e.* demand. For supply, this simply means that the supply curve is fixed, but changes in buyers creates an increase in the quantity supplied that is analogous to a movement along a supply curve. The magnitude of the increase in quantity supplied is determined by ξ_s which can be thought of as a measure of the elasticity of supply, with a higher ξ_s implying a higher elasticity of supply. In addition, identifying assumption 1 also means that churn is driven by demand. This is consistent with the theories of churn such as Stein (1995) or Ortalo-Magné and Rady (2006). In this way the theory presented in this paper incorporates and is consistent with theories of churn linking turnover and prices.

6.2.2 Elasticity of Supply

Inspection of equation (27) shows that ψ is decreasing in the elasticity of supply, ξ_s. Therefore, under this identification scheme there is another testable implication of the model: *the sensitivity of price growth to turnover should be decreasing in the elasticity of supply*. This result is later tested by comparing estimates of ψ acrosss regional data. We would expect regions with more elastic supply of house, such as the Midwest, to have a lower estimate of ψ.

6.3 Estimation Results

Equations (40) and (41) are estimated. Results are shown in table 1. For each model, six estimations are done. The first estimation, denoted USA, is the aggregate estimation using standard OLS. The next four, Northeast, Midwest, South, and West, are OLS performed on each census region individually, while the last estimation, denoted Panel, is an OLS panel data estimation using all four census regions together controlling for fixed effects and correlation across regions.

Overall, the estimations strongly support the 'Fooled' model over the rational model. The irrational model does a much better job of fitting the data than the rational model, delivering a significantly higher R-squared in all the estimations. The fit of the two models for the aggregate data (USA) is shown in figure 3. The irrational model does a strikingly good job of explaining the data, capturing the timing of all of the booms and busts while

	USA	NORTHEAST	MIDWEST	SOUTH	WEST	PANEL
Rational Model	$\%\Delta p_t = \gamma + \psi\left(\hat{\phi}_t - \hat{\phi}_{t-1}\right) + \varepsilon_t$					
$\hat{\phi}_t - \hat{\phi}_{t-1}$	15.50**	18.46	18.69***	13.28**	5.840	12.43*
	(5.387)	(9.926)	(4.471)	(4.399)	(7.919)	(5.173)
$E\left(\%\Delta p_t\right)$	0.97	1.65	0.22	0.29	2.29	1.69
	(0.60)	(0.94)	(0.51)	(0.50)	(1.18)	(0.91)
Observations	35	35	35	35	35	140
R^2	0.201	0.095	0.346	0.216	0.016	0.113
Irrational Model, "Fooled"	$\%\Delta p_t = \gamma + n\psi\hat{\phi}_t + \varepsilon_t$					
$\hat{\phi}_t$	17.03***	20.45***	14.03***	13.54***	24.14***	18.22***
	(1.886)	(4.547)	(2.078)	(1.335)	(3.153)	(1.708)
$E\left(\%\Delta p_t\right)$	0.90*	1.58*	0.18	0.23	2.23**	1.60*
	(0.36)	(0.78)	(0.41)	(0.28)	(0.71)	(0.76)
Observations	35	35	35	35	35	140
R^2	0.712	0.380	0.580	0.757	0.640	0.542

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 1: Estimation for rational and irrational models.

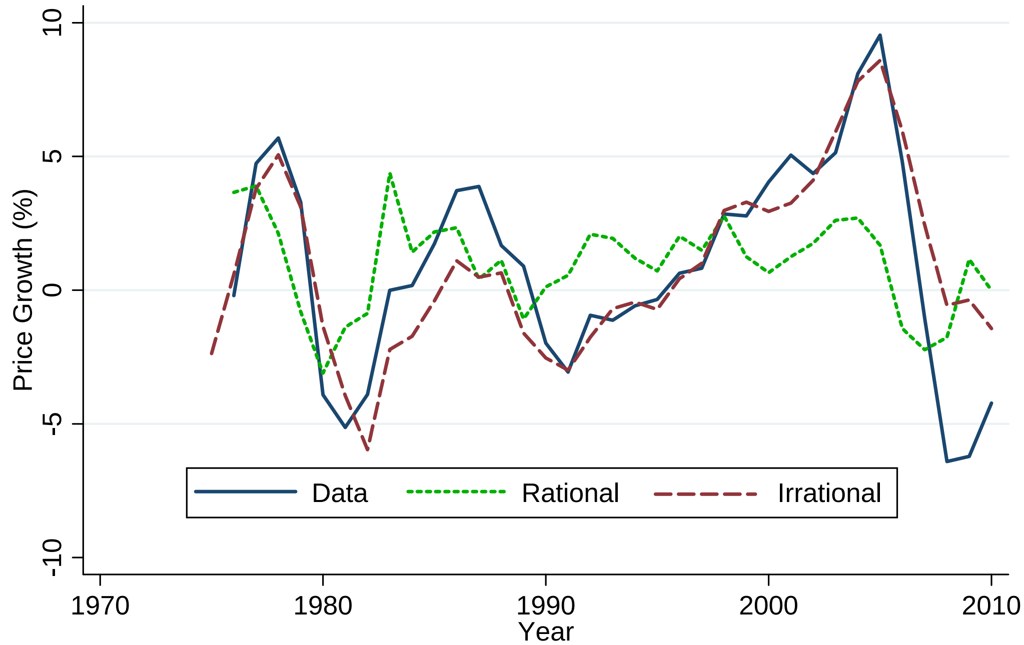

Figure 3: Fit of estimated models for National (USA) data.

also being able to match the magnitudes of the booms and busts. The main deviations are the size of the boom in the late 1980s and the depth of the fall in the current crisis. Later in the paper, a supply-shock will be incorporated into the identification scheme that will be able to explain a large part of the fall in house prices in the current crisis.

6.3.1 Regional Results

The model does an excellent job of explaining the aggregate data. This is expected since the model was constructed with the aggregate data in mind. To better test the model, we can look at the performance of the model at a disaggregated level. As argued earlier (see subsubsection 6.2.2) the model predicts that in regions with a higher elasticity of supply, price growth should be less sensitive to turnover. From previous research (see Saiz 2010), the Midwest and South are thought to be regions with a higher elasticity of supply than the Northeast and West[29].

Examining table 1 this is in fact exactly what is estimated across regions, captured by the higher estimated coefficient on ϕ_t for the Northeast and West in the 20-24 ranges versus about 14 for the Midwest and South in the 'Fooled' estimates. To understand

[29]The West may seem to have abundant land, but a large part of the housing stock for the West is California, Washington, Oregon and Hawaii where there are strong limitations of easily developed land.

the numbers, consider a 20% increase in turnover.[30] In the Midwest, that would imply that real house prices would increase 2.8% over one year, while in the West, real house prices would increase 4.8%. Spread out over 5 years, that would imply that real house prices would increase 14.0% in the Midwest and 24.0% in the West. However, we need to be careful here. This is not to say that this effect is explaining the differences in the long-run growth of real house prices across regions. In the estimation, turnover has been demeaned and entered into the estimation as a % deviation from average over the time period 1976-2010 for each region, so that the net contribution of turnover to each region's long-run growth of real house prices is nil. Therefore, the 12.0% versus 7.0% difference between the West and the Midwest is only a temporary effect.

The long-run estimates of real price growth are shown in table 1, see ($E(\%\Delta p_t)$). Once again we see a divergence across regions. The Northeast and West have much higher estimated growth rates than the Midwest and South, 1.58 and 2.23 versus 0.18 and 0.23. The model estimates imply a positive relationship between the long-run growth rate of real house prices and the sensitivity of real house price growth to turnover. This positive relationship is graphed in figure 4, given by the red (lighter) diamonds. Figure 4 reconfirms the results found in Saiz (2010) that areas with a lower long-run growth rate of real house prices have a higher elasticity of supply. Here, this result is captured through the lens of the relationship between turnover and price growth.

A central feature of the model is that the price of a house diverges from fundamentals due to market participants being 'Fooled' by search frictions into thinking that fundamentals have changed more than is othwerwise justified. Conceptually, when new supply is elastic, this effect should be weaker since market participants can use the cost of a new house to help anchor the value of a house. In addition, theory predicts that in areas where there is an abundance of easily developed land, then the long-run growth of real house prices should be lower. Therefore, we might expect that the ability of the *rational* model to explain the data may be higher the lower is the long-run rate of real price growth. Put in a different way, we would expect that the *fit of the rational model is decreasing in the long-run growth rate of real house prices.*

The relationship between the fit of the rational model and the long-run growth rate of real house prices is plotted in figure 4, given by the blue (dark) squares. Although there are only four independent observations (note that USA is simply a linear combination of the regions) we see a negative relationship. This suggests, that as the long-run growth rate of real house prices increases, it becomes more difficult for market participants to rationally figure out what the correct level of house prices should be.

Summarizing the regional results, the model estimates tell a story about house supply elasticity and real house prices.

> As house supply elasticity falls, the long-run growth rate of house prices increase, the sensitivity of real house price growth to turnover increases, and the

[30] the average standard deviation of turnover across regions is 20%

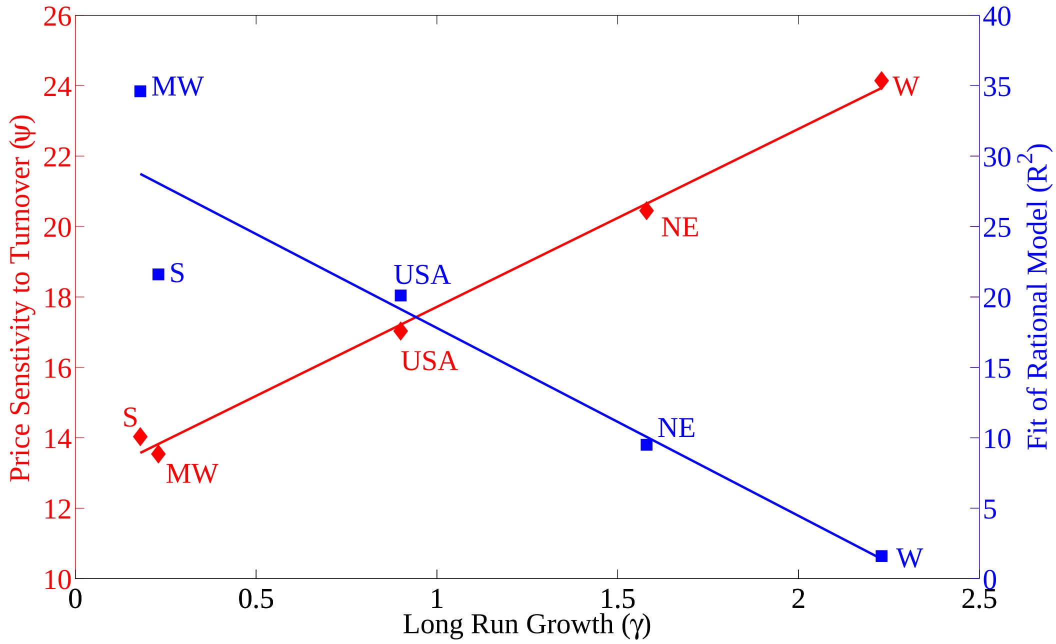

Figure 4: Fit of estimated models. Sources: Real Price growth: FHFA and GDP Deflator.

> ability of the rational model of house prices and turnover to explain real house price growth dimishes.

In other words, the less elastic is the supply of housing, the less anchored is the level of house prices, and the stronger will be the effect of 'Fooled by Search' on real house price growth. On the other hand, the more elastic is the supply of housing, the more grounded is the level of house prices (likely due to construction costs), the weaker will be the effect of 'Foooled by Search' and the better will a rational model of search costs explain the data.

7 Counterfactual: Contribution to Bubbles

The Irrational model is inefficient because current buyers and sellers interpret past price changes as a permanent change in the fundamental value of a house instead of temporary changes induced by temporary demand shocks and search frictions. In this section counterfactuals are constructed where buyers and sellers correctly take into account search frictions on past prices. The counterfactuals are then used to answer the question: *how much of the bubble in house prices leading up to the crisis can be explained by 'Fooled by Search'?*.

To construct the counterfactual, subtract $\psi\hat{\phi}_{t-1}$ from the annual price growth in the data, or

$$\%\Delta p_{i,t}^{CF} = \%\Delta p_{i,t} - \psi\hat{\phi}_{i,t-1}, \tag{44}$$

where 'CF' denotes counterfactual. In constructing this counterfactual several assumptions have been made:

1. the turnover series is unchanged,
2. the parameter for the lag in price reporting (n) is one year, see subsubsection 5.3.3,
3. average turnover is equal to that observed in the data from 1976 to 2000 to construct deviations of turnover from mean, and
4. the series for the fundamental value of a house is unchanged.

Under the above assumptions, the counterfactual then constructs the reduced form rational model equation, given by equation (40) from the estimate for equation (41). The counterfactual is constructed for each region i and for the aggregate data.

Given the counterfactual series for real price growth, a counterfactual series for the price level is constructed to capture the effects of 'Fooled' on the bubble. To do this, prices are normalized to 1 in 1995 and the real price growth series from the data and the counterfactual series are used to construct two different series for the price level. The results for the aggregate data are plotted in figure 5. The 'Data' line is the data while the 'Counterfactual' is the counterfactual where the effects of 'Fooled' have been removed.

From figure 5 we see that the counterfactual and the data were roughly equal until about 2000. There is a sense in which the presence of 'Fooled' lets momentum in the housing market lead to prices to increase more and at a much faster rate than they would otherwise. After 2000, the data starts to grow faster than the counterfactual, with an acceleration around 2003. The counterfactual peaks in 2005, having grown 23.42% percent since. The data peaks later, in 2006, having grown 59.59% since 1995. The results are reported in table 2 under the column 'USA.' The extra increase is 36.17%. Therefore, *'Fooled' explains 60.7% of the house price increase in aggregate data from 1995 until the peak of the bubble in 2006.* If trend growth is removed, then 'Fooled' explains 71.2% of the extra increase of real house prices.

The contribution of 'Fooled' to house price growth is broken down at the regional level in table 2. The effects are the strongest in the West, causing prices to increase an extra 56.59% while the effects are the smallest in the Midwest, causing prices to only increase an extra 21.47%. Relative to trend, the presence of 'Fooled' explains well over half of the growth in real house prices leading up to the collapse.

To summarize, 'Fooled by Search' is able to explain over two-thirds of the above trend real house price growth in aggregate data from 1995 until the peak of the housing bubble in 2006, and is able to explain the much larger price increase in the West and Northeast relative to the Midwest and South.

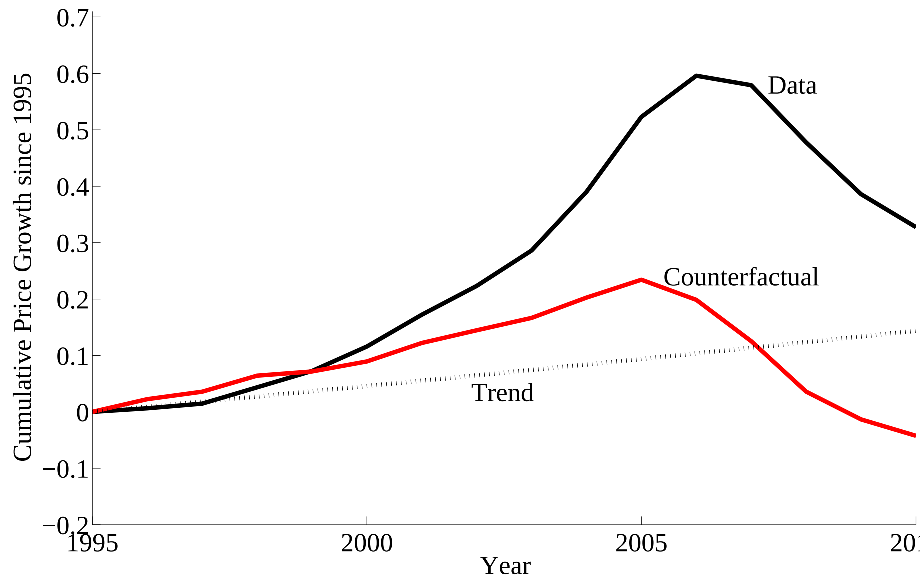

Figure 5: Data and counterfactual price path since 1995 for aggregate. Data Source: OFHEO.

As can be seen in figure 5 a common feature of the data and the counterfactual is a fall in prices. To address this issue, the next section of the paper re-estimates the model usign an identification scheme to identify supply shocks.

8 Supply Shocks

The housing bubble that crashed starting in 2006 has lead to a large fall in house prices and a sharp rise in foreclosures, taking center stage in the financial crisis beginning in 2007. As can be seen from figure 3 the model using the demand approach for identification cannot capture the sharp fall in prices. To address this issue, the model is re-estimated in this section using an identification approach that incorporates supply.

Identifying Assumption 2. *Demand Driven with Supply Shocks* Assume that churn is driven by buyers, so that

$$k_t = \xi_k b_t. \tag{45}$$

and that sellers are driven by buyers and a shock to sellers:

$$s_t = \xi_s b_t + \tilde{s}_t. \tag{46}$$

Furthermore, assume

$$\tilde{s}_t = \tau \left(\hat{\phi}_t^E - \hat{\phi}_t^N \right). \tag{47}$$

	USA	NORTHEAST	MIDWEST	SOUTH	WEST
Peak Growth since 1995					
Data	59.59	72.38	32.47	49.94	107.03
Counterfactual	23.42	23.86	11.00	21.17	50.44
extra increase	36.17	48.52	21.47	28.77	56.59
% of total increase	60.7	67.0	66.1	57.6	52.9
Peak Growth Relative to Trend					
Data	44.61	45.08	30.00	45.86	62.43
Counterfactual	12.84	5.89	9.61	18.42	20.67
extra increase	31.77	39.19	20.39	27.44	41.76
% of total increase	71.2	86.9	68.0	59.8	66.9

Table 2: Counterfactuals: Contribution of 'Fooled by Search' to real house price growth from 1995 until housing market peak.

This second approach to identification allows for shocks to supply, $\tilde{s}_t$, that are not simply responses to demand. These shocks can be thought of as a movement in the supply curve. The shocks to supply are identified by the difference in the deviation of existing turnover from new turnover. Therefore, a positive shock to supply comes from an increase of existing homes relative to what construction is providing in new homes, possibly driven by an increase in vacant homes for sale.[31] A negative shock to supply can be seen as existing homes not responding to an increase in demand that shows up in new home sales. This could be due to the supply of vacant homes not being sufficient to respond to the increase in demand. We should bear in mind that the supply of new and existing homes are not modelled, so the results in this section are a bit ad hoc. With this in mind τ is used in the identifying assumption in equation (47) as a scaling parameter to capture differences between the identifying assumption and the structure of turnover of the deeper model. Given the ad hoc approach used here, the results should be taken lightly. Nonetheless, the results do provide insight.

Under this approach to identification, the equation for deviations to turnover becomes:

$$\hat{\phi}_t = \alpha b_t + (1-\alpha)\left(\xi_s b_t + \tilde{s}_t\right) + \xi_k b_t.$$

Solving for b_t we get

$$b_t = \frac{1}{\alpha + (1-\alpha)\,\xi_s + \xi_k}\left[\hat{\phi}_t - (1-\alpha)\,\tilde{s}_t\right]. \tag{48}$$

[31] An increase in occupied homes for sale is likely indicative of churn.

Letting the stochastic process for $\tilde{s}_t$ be determined by λ_s and ρ_s, the equilibrium pricing equation in the rational model is now

$$\%\Delta p_t \approx \gamma + \left[\frac{r\sigma\left(1-\xi_s\right)}{r+\lambda_b\left(1-\rho_b\right)}\right]\left(b_t - b_{t-1}\right) - \left[\frac{r\sigma}{r+\lambda_s\left(1-\rho_s\right)}\right]\left(\tilde{s}_t - \tilde{s}_{t-1}\right) + \varepsilon_t,$$

while that of the irrational model is now

$$\%\Delta p_t \approx \gamma + n\left[\frac{r\sigma\left(1-\xi_s\right)}{r+\lambda_b\left(1-\rho_b\right)}\right] b_t - n\left[\frac{r\sigma}{r+\lambda_s\left(1-\rho_s\right)}\right]\tilde{s}_t + \varepsilon_t.$$

Demand Driven with Supply Shocks: Reduced Form Equations Using equations (48) and (47) we get the following reduced form equation for the rational model,

$$\%\Delta p_t \approx \gamma + \psi\left(\hat{\phi}_t - \hat{\phi}_{t-1}\right) - \eta\left(\left(\hat{\phi}_t^E - \hat{\phi}_t^N\right) - \left(\hat{\phi}_{t-1}^E - \hat{\phi}_{t-1}^N\right)\right) + \varepsilon_t, \tag{49}$$

and for the irrational model,

$$\%\Delta p_t \approx \gamma + n\psi\hat{\phi}_t - n\eta\left(\hat{\phi}_t^E - \hat{\phi}_t^N\right) + \varepsilon_t, \tag{50}$$

where ψ is given by equation (27) and

$$\eta = \tau\psi\left(1-\alpha\right) + \frac{\tau r\sigma}{r+\lambda_s\left(1-\rho_s\right)}. \tag{51}$$

8.1 Estimated Results

The results of the estimation are shown in table 3. The fit of the aggregate data is shown in figure 6. Overall, incorporating supply shocks into the model provides a much better fit, especially after 2006. At the regional level the supply shocks greatly improve the fit of the model in the Northeast. While not shown, this is due to a much better fit in the housing cycle in the late 1980s. Examining table 3 the supply shock is significant and has a negative impact as predicted by theory in the irrational model. However, for the rational modle the supply shock is not significant.

8.2 Counterfactuals with Supply

The results for the counterfactuals with the supply shock removed are shown in figures 7 and 8. The contributions of the supplhy shock to the fall in house prices after 2006 for all the regions is shown in table 4.

	USA	NORTHEAST	MIDWEST	SOUTH	WEST	PANEL
Rational Model						
$\hat{\phi}_t - \hat{\phi}_{t-1}$	14.74*	15.58	20.94***	13.51**	0.751	9.918
	(6.173)	(12.16)	(4.885)	(4.620)	(7.642)	(5.340)
Δ supply shock	-2.282	-3.785	6.305	1.101	-25.14*	-7.690
	(8.560)	(8.996)	(5.635)	(5.580)	(10.11)	(4.934)
$E\,(\%\Delta p_t)$	1.018	1.765	0.0821	0.272	2.858*	1.896*
	(0.635)	(0.986)	(0.520)	(0.519)	(1.116)	(0.920)
Observations	35	35	35	35	35	140
R^2	0.202	0.100	0.371	0.217	0.175	0.137
Irrational Model, "Fooled"						
$\hat{\phi}_t$	15.18***	23.00***	10.32***	13.28***	20.89***	16.76***
	(1.127)	(3.717)	(2.320)	(0.989)	(2.695)	(1.062)
supply shock	-9.800***	-8.184***	-5.675**	-6.326***	-11.13***	-5.025**
	(1.221)	(1.895)	(2.051)	(1.191)	(2.675)	(1.826)
$E\,(\%\Delta p_t)$	0.949***	1.643*	0.232	0.246	2.280***	1.950**
	(0.211)	(0.626)	(0.371)	(0.207)	(0.582)	(0.735)
Observations	35	35	35	35	35	120
R^2	0.904	0.608	0.661	0.871	0.766	0.648

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 3: Estimation for rational and irrational models.

	USA	NORTHEAST	MIDWEST	SOUTH	WEST
Data	16.8	14.0	13.5	13.3	29.8
Counterfactual	3.0	0.5	4.0	5.0	12.6
extra fall	13.8	13.5	9.5	8.3	17.2
% of total fall	82.2	96.2	70.1	62.5	57.8

Table 4: Counterfactual Fall due to Supply shock.

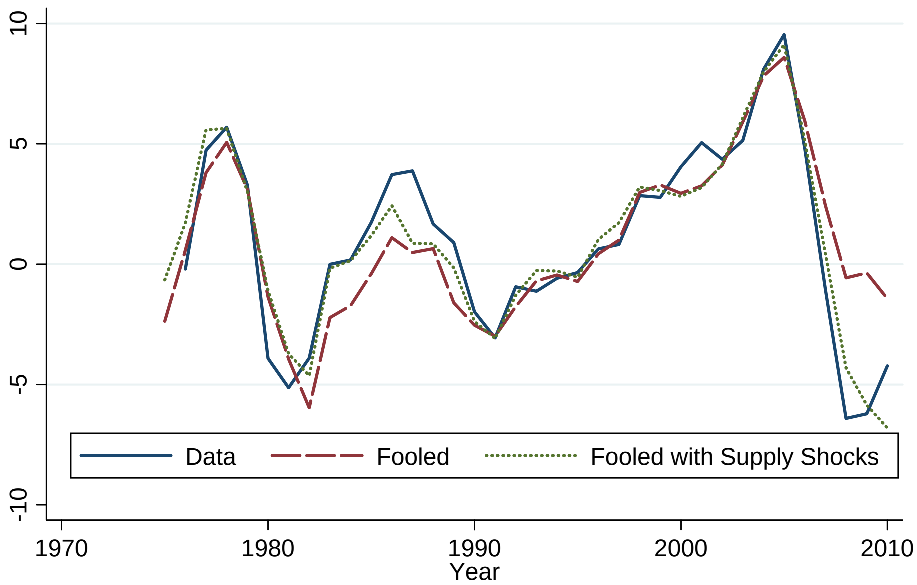

Figure 6: Fit of estimated models. Sources: Real Price growth: FHFA and GDP Deflator.

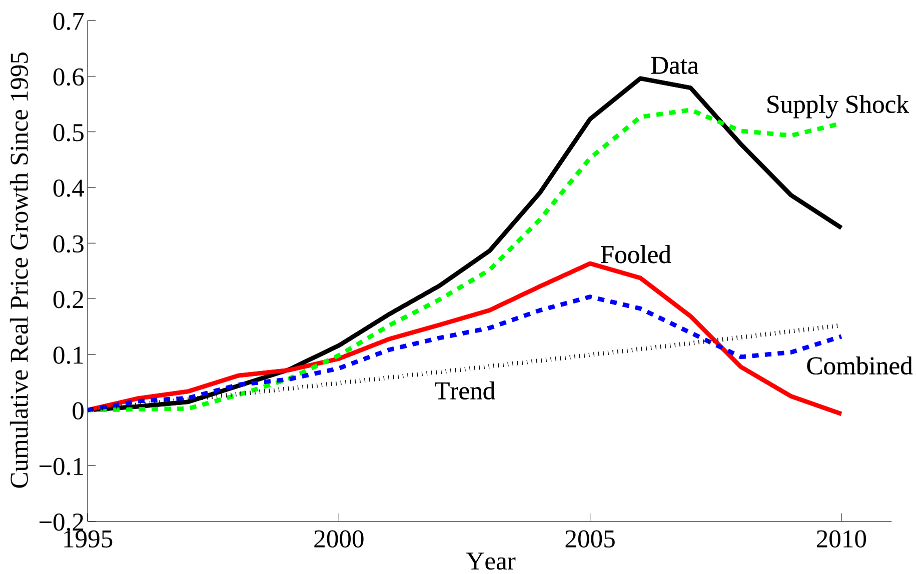

Figure 7: National Counterfactuals. 'Fooled' has the effect of Fooled by Search removed; 'Supply Shock' has the effect of the estimated supply shock removed; and 'Combined' removes both effects. Data Source: OFHEO.

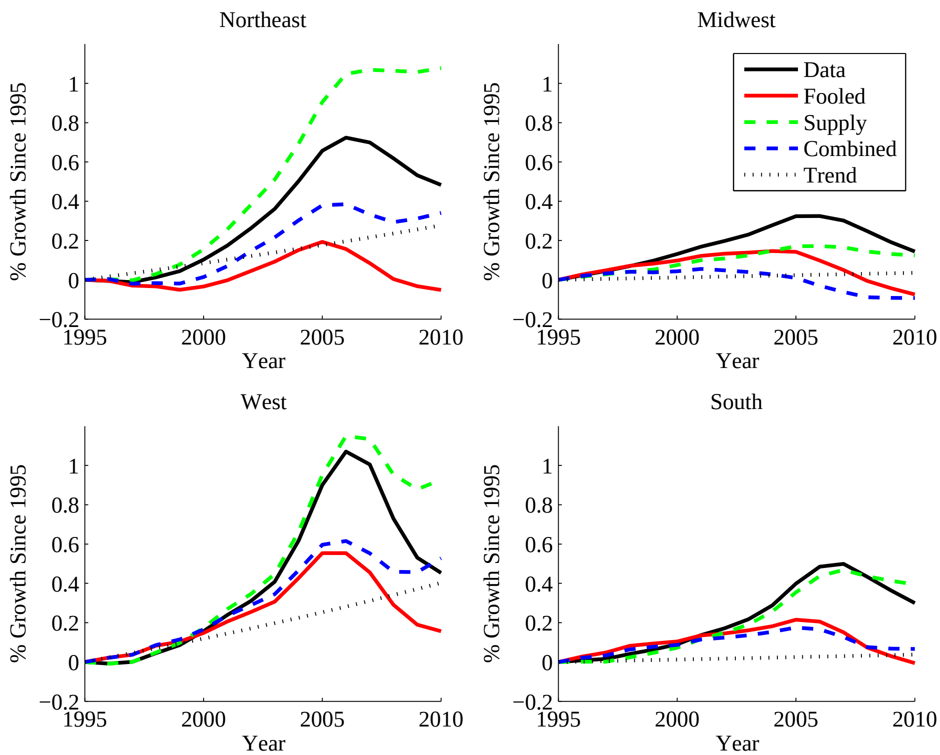

Figure 8: Regional Counterfactuals. 'Fooled' has the effect of Fooled by Search removed; 'Supply Shock' has the effect of the estimated supply shock removed; and 'Combined' removes both effects. Data Source: OFHEO.

9 Conclusion

This paper has highlighted an observation from the United States housing market: there is a very tight relationship between turnover and price growth rather than the price level. This distinction between growth and levels is important since it determines whether a temporary movement in demand causing a permanent price change versus a temporary price change. One way to explain the data is to assume that movements in demand are related to permanent changes to the fundamental value of a house. However, if we believe that search frictions affect house prices, then we face a contradiction between theory and data: *theory predicts that the effects of search frictions on house prices from temporary movements in demand should be temporary, while the data suggests it is permanent.*

This paper offers one possible resolution to the contradiction: *buyers and sellers ignore the effects of search frictions on past prices, instead interpreting all past prices as the best signal of the fundamental value of a house.* This behavioral assumption implies that when bargaining over house prices, current buyers and sellers (who are affected by search frictions) end up bargaining over price growth rather than the price level since they treat past prices as a type of anchor. The change in prices becomes permanent when future buyers and sellers interpret the change in prices as a change to the fundamental value of a house. In this way buyers and sellers are 'fooled by search.'

Such a behavioral assumption seems related to Shiller's observation that households have a very poor understanding of levels, dealing much better with the rate of change of prices. The assumption implies that a shock that raises demand causes an increase in the growth rate of prices, rather than an increase in the level of prices. If we accept Shiller's hypothesis that there are positive feedbacks from growth rates, then shocks to demand (positive or negative) can cause further shocks to demand reinforcing the growth rate.

The behavioral assumption was inserted into a textbook search model, linearized, and estimated using standard OLS. The counterfactuals of the estimated model imply that three-fifths of the increase of house prices during the housing bubble starting in 1995 can be explained by the amplification mechanism of search frictions and the ignorance of search frictions on past prices. When trend price growth is removed, the model explains over two-thirds of the bubble. In addition, a supply shock is identified and estimated that has been responsible for over 80% of the fall in real house prices from the peak in 2006 to 2010.

The model of this paper takes turnover as exogenous and explains prices. Ultimately research should have both endogenous prices and turnover. However, the paper does provide guidance on how to explain turnover. There is evidence that buyers who are choosing to not enter the market do respond to the level. In the counterfactual of the estimated model, whenever houses are overvalued, turnover falls, while when houses are undervalued turnover is increasing. This suggests that renters who are thinking about entering the housing market are affected by levels, consistent with down payment stories. A model that combines the amplification mechanisms in Stein (1995) and Ortalo-Magné and Rady (2006), search, and renters that face down payment constraints seems quite intuitive. The tight correlation between price growth and turnover would then serve as a benchmark for a model to hit.

Finally, if we accept the assumptions of this paper, one way to get around the inefficiency from the behavioral assumption would be to create price indices that *remove* the change in house prices that is due to a combination of changes in market tightness and search frictions.[32] Such an index would be model specific, but could help the public to better price illiquid real assets that trade in decentralized markets, such as houses. This could help to mitigate 'Irrational Exuberance.'

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[32] Note that such an index is different from the proposals of Fisher, Gatzlaff, Geltner and Haurin (2003) and Goetzmann and Peng (2006). Their proposals are to add the increase in liquidity to the price level. The underlying model in their framework is one where bargaining between buyers and sellers, and hence prices, does not respond to changes in liquidity. This would create larger swings in prices. My proposal would be to take out the swings in prices that are due to prices responding to liquidity.

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A Linearization of Rational Model

This sections covers the derivation of the linearization of the stochastic model. For generality with the complete model, both shocks to buyers and sellers are included. To solve the model, insert the pricing equation given by equation (22) into the value functions given by equations (20) and (21). The result is

$$\begin{aligned} rV^B(b,s,z) = {} & \omega f(\theta) X(b,s,z) + \\ & \lambda_z E_{z'|z}\left[V^B(b,s,z') - V^B(b,s,z)\right] + \lambda_b E_{b'|b}\left[V^B(b',s,z) - V^B(b,s,z)\right] + \\ & \lambda_s E_{s'|s}\left[V^B(b,s',z) - V^B(b,s,z)\right] \end{aligned} \tag{52}$$

and

$$\begin{aligned} rV^S(b,s,z) = {} & r(1-\delta)e^z\overline{y} + (1-\omega) q(\theta) X(b,s,z) + \\ & \lambda_b E_{b'|b}\left[V^S(b',s,z) - V^S(b,s,z)\right] + \lambda_s E_{s'|s}\left[V^S(b,s',z) - V^S(b,s,z)\right] + \\ & \lambda_z E_{z'|z}\left[V^S(b,s,z') - V^S(b,s,z)\right]. \end{aligned} \tag{53}$$

where

$$X(b,s,z) = e^z\overline{y} - V^B(b,s,z) - V^S(b,s,z).$$

Rewrite the pricing equation as

$$p(b,s,z) = e^z\overline{y} - V^B(b,s,z) - \omega X(b,s,z). \tag{54}$$

The goal is a linearized approximation to the pricing equation at $b = s = z = 0$. The strategy is to solve for $X(b,s,z)$ and then use implicit differentiation to form a first-order Taylor expansion of the pricing equation.

A.1 X : Surplus

Use equations (52) and (53) to eliminate V^B and V^S to get an implicit solution for $X(b,s,z)$:

$$\begin{aligned} \left[r + \omega f(\theta) + (1-\omega) q(\theta)\right] X(b,s,z) = {} & r\delta e^z\overline{y} + \\ & \lambda_z E_{z'|z}\left[e^z\overline{y} - e^{z'}\overline{y}\right] - \lambda_z E_{z'|z}\left[X(b,s,z) - X(b,s,z')\right] - \\ & \lambda_b E_{b'|b}\left[X(b,s,z) - X(b',s,z)\right] - \lambda_s E_{s'|s}\left[X(b,s,z) - X(b,s',z)\right] \end{aligned} \tag{55}$$

A.1.1 dX/db

Implicitly differentiate equation (55) to solve for dX/db:

$$\begin{aligned} \left[\omega f'(\theta) + (1-\omega) q'(\theta)\right] X(b,s,z) + \left[r + \omega f(\theta) + (1-\omega) q(\theta)\right] \frac{dX}{db} = {} & \\ - \lambda_b E_{b'|b}\left[\frac{dX}{db} - \frac{dX'}{db'}\frac{db'}{db}\right] & \end{aligned} \tag{56}$$

Since we are making a linear approximation, we can use the following approximation:

$$E_{b'|b}\left[\frac{dX}{db}-\frac{dX'}{db'}\frac{db'}{db}\right]\approx\frac{dX}{db}\left(1-\rho_b\right). \tag{57}$$

Plugging this into equation (56), we can solve for dX/db at the steady-state:

$$\frac{dX}{db}\approx\frac{\left[\omega-\alpha\right]A}{\left[r+\lambda_b\left(1-\rho_b\right)+A\right]}\overline{X} \tag{58}$$

where the Cobb-Douglass assumption on the matching function has been used to evaluate f and q at $\theta=1$ and

$$\overline{X}=\frac{r\delta\overline{y}}{r+A} \tag{59}$$

denotes surplus at the steady-state.

A.1.2 dX/ds

Turning to the effects of changes in the number of sellers, since $\theta=e^{b-s}$ the effect of a change in s is simply negative of the change in b but with the persistence parameters coming from the process for s. Therefore

$$\frac{dX}{ds}\approx\frac{\left[\alpha-\omega\right]A}{\left[r+\lambda_s\left(1-\rho_s\right)+A\right]}\overline{X}$$

A.1.3 dX/dz

Implicitly differentiate equation (55) to solve for dX/dz:

$$\left[r+\omega f\left(\theta\right)+\left(1-\omega\right)q\left(\theta\right)\right]\frac{dX}{dz}=r\delta\overline{y}+\lambda_z\overline{y}-\lambda_zE_{z'|z}\left[e^{z'}\overline{y}\frac{dz'}{dz}\right] \\ -\lambda_zE_{z'|z}\left[\frac{dX}{dz}-\frac{dX'}{dz'}\frac{dz'}{dz}\right].$$

Using the assumption that conditional upon a new shock for z, that z follows a martingale, and evaluating at the steady state we arrive at

$$\frac{dX}{dz}\approx\overline{X}. \tag{60}$$

A.2 V^B : Buyer's Value Function

A.2.1 dV^B/db

Implicitly differentiate equation (52) to solve for dV^B/db:

$$r\frac{dV^B}{db}=\left[\omega f'\left(\theta\right)X\left(b,s,z\right)+\omega f\left(\theta\right)\frac{dX}{db}\right]+\lambda_bE_{b'|b}\left[\frac{d\left(V^B\right)'}{db'}\frac{db'}{db}-\frac{dV^B}{db}\right].$$

Once again applying the linearity assumption from equation (57) and evaluating at $b = s = z = 0$, we can solve for dV^B/db to get

$$\frac{dV^B}{db} \approx \frac{\omega A}{r + \lambda_b (1 - \rho_b)} \left[(\alpha - 1) \overline{X} + \frac{dX}{db} \right]. \tag{61}$$

A.2.2 dV^B/ds

As before, a shock to suppliers is the negative of a shock to buyers with the caveat that the parameters for the shock process are different. Therefore

$$\frac{dV^B}{ds} \approx \frac{\omega A}{r + \lambda_s (1 - \rho_s)} \left[(1 - \alpha) \overline{X} + \frac{dX}{ds} \right].$$

A.2.3 dV^B/dz

Since z follows a martingale process, we can solve for dV^B/dz directly from equation (52) to get

$$\frac{dV^B}{dz} \approx \frac{\omega A}{r} \frac{dX}{dz}. \tag{62}$$

A.3 Pricing Equation

We are looking for the approximation to equation (54).

A.3.1 dp/db

From equation (54),

$$\frac{dp}{db} = -\frac{dV^B}{db} - \omega \frac{dX}{db}.$$

Inserting equation (61) we get

$$\frac{dp}{db} \approx -\frac{\omega}{r + \lambda_b (1 - \rho_b)} \left\{ (\alpha - 1) A\overline{X} + [r + \lambda_b (1 - \rho_b) + A] \frac{dX}{db} \right\}.$$

Inserting equation (58) in for dX/db we get

$$\frac{dp}{db} \approx \frac{(1 - \omega) \omega}{r + \lambda_b (1 - \rho_b)} A\overline{X}$$

Finally, using the definition of $\overline{X}$ from equation (59) we arrive at

$$\frac{dp}{db} \approx \left[\frac{r}{r + \lambda_b (1 - \rho_b)} \right] \left[\frac{A}{r + A} \right] [(1 - \omega) \omega] \delta \overline{y}. \tag{63}$$

A.3.2 dp/ds

Once again, the effect of a shock to suppliers is negative of the shock to buyers with the appropriate relabeling of the parameters for the shock process, so that

$$\frac{dp}{ds} \approx -\left[\frac{r}{r+\lambda_s\left(1-\rho_s\right)}\right]\left[\frac{A}{r+A}\right]\left[\left(1-\omega\right)\omega\right]\delta\overline{y}. \tag{64}$$

A.3.3 dp/dz

From equation (54)

$$\frac{dp}{dz} = \overline{y} - \frac{dV^B}{dz} - \omega\frac{dX}{dz}$$

Inserting equation (61) we get

$$\frac{dp}{dz} \approx \overline{y} - \omega\left[\frac{A+r}{r}\right]\frac{dX}{dz}$$

Inserting equation (60) we arrive at

$$\frac{dp}{dz} \approx \left(1-\omega\delta\right)\overline{y} \tag{65}$$

A.3.4 Linearized Price Equation

We now arrive at the final objective of the linearized pricing equation. The first-order Taylor approximation of equation (54) is

$$p\left(b,s,z\right) \approx \overline{p} + \frac{dp}{db}b + \frac{dp}{ds}s + \frac{dz}{ds}z$$

Inserting equations (63) to (65) and using the definition of $\overline{p}$ from equation (24) the linearized pricing equation becomes

$$p\left(b,s,z\right) \approx \overline{p} + \left[\frac{r}{r+\lambda_b\left(1-\rho_b\right)}\right]\sigma\overline{p}b - \left[\frac{r}{r+\lambda_s\left(1-\rho_s\right)}\right]\sigma\overline{p}s + \overline{p}z$$

where

$$\sigma = \left[\frac{A}{r+A}\right]\left[\left(1-\omega\right)\omega\right]\left[\frac{\delta\overline{y}}{\left(1-\omega\delta\right)\overline{y}}\right].$$

Which becomes

$$\hat{p}\left(b,s,z\right) \approx \left[\frac{r}{r+\lambda_b\left(1-\rho_b\right)}\right]\sigma b - \left[\frac{r}{r+\lambda_s\left(1-\rho_s\right)}\right]\sigma s + z, \tag{66}$$

where $\hat{p}$ denotes the percent deviation of p from $\overline{p}$.