Working Paper/Document de travail
2010-29

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Bank of Canada Working Paper 2010-29

November 2010

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ISSN 1701-9397

Acknowledgements

We thank Jean-Sébastien Fontaine, Prasanna Gai, Toni Gravelle, Scott Hendry, and Jun Yang for helpful comments.

Abstract

We offer a multi-period systemic risk assessment framework with which to assess recent liquidity and capital regulatory requirement proposals in a holistic way. Following Morris and Shin (2009), we introduce funding liquidity risk as an endogenous outcome of the interaction between market liquidity risk, solvency risk, and the funding structure of banks. To assess the overall impact of different mix of capital and liquidity, we simulate the framework under a severe but plausible macro scenario for different balance-sheet structures. Of particular interest, we find that (1) capital has a decreasing marginal effect on systemic risk, (2) increasing capital alone is much less effective in reducing liquidity risk than solvency risk, (3) high liquid asset holdings reduce the marginal effect of increasing short term liability on systemic risk, and (4) changing liquid asset holdings has little effect on systemic risk when short term liability is sufficiently low.

JEL classification: G01, G21, C15, C81, E44
Bank classification: Financial stability; Financial system regulation and policies

Résumé

Les auteurs offrent un cadre d'évaluation multipériodique du risque systémique afin d'apprécier globalement l'incidence de récentes propositions relatives aux exigences réglementaires en matière de liquidités et de fonds propres. À l'instar de Morris et Shin (2009), ils font du risque associé à la liquidité de financement une conséquence endogène de l'interaction entre le risque de liquidité du marché, le risque de solvabilité et la structure de financement des banques. Pour mesurer l'impact général d'une combinaison différente de liquidités et de fonds propres, ils simulent la réaction de structures de bilan distinctes à une conjoncture macroéconomique difficile mais plausible. Plusieurs résultats intéressants ressortent des simulations : 1) les fonds propres ont un effet marginal décroissant sur le risque systémique; 2) leur augmentation à elle seule atténue beaucoup moins efficacement le risque de liquidité qu'elle ne réduit le risque de solvabilité; 3) la détention d'un niveau élevé d'actifs liquides restreint l'effet marginal d'une hausse des passifs à court terme sur le risque systémique; 4) la variation du niveau des actifs liquides détenus influe peu sur le risque systémique quand le montant des passifs à court terme est suffisamment faible.

Classification JEL : G01, G21, C15, C81, E44
Classification de la Banque : Stabilité financière; Réglementation et politiques relatives au système financier

Over the years [...], funding liquidity came to replace asset liquidity. The idea was that, so long as bank capital sufficiency was assured, which adherence to Basel II was supposed to achieve, banks could always rely on access to these large, efficient, wholesale markets. [...] The Basel Committee on Banking Supervision (BCBS) had attempted, in the mid 1980s, to put together an Accord on banking liquidity as a supplement to the Capital Accord of 1988. But when that initiative failed, [...] no individual country regulator felt able to halt, let alone to reverse, the developing trend away from asset liquidity. When short-term wholesale markets did collapse after August 9, 2007, banks were left with little internal asset liquidity with which to ride out the storm. (Goodhart, 2010)

1. INTRODUCTION

The collective reactions by market participants during the recent financial crisis led to mutually reinforcing solvency and liquidity problems. As funding liquidity evaporated, strongly capitalised financial institutions had to write-off illiquid assets, pushing the financial system into the tail of the loss distribution (Van den end [2008]). The crisis revealed that regulating solvency at individual institutions was not sufficient for the financial system to be stable.

System stability takes on the attributes of a public good and since, as with any public good, private markets will fail to provide enough of it, many regulatory initiatives to help market participants, banks in particular, internalise the externalities they impose on others are currently under consideration. This is a challenging task as one needs a tractable framework that incorporates those externalities and can be used to assess the impact of different regulatory proposals in a holistic way rather than in a piecemeal fashion. Gauthier, Lehar and Souissi (2010) (GLS thereafter) present the first step toward the development of such a framework for the Canadian banking system,[1] by incorporating some contagion externalities present in the financial system: banks' response in terms of asset sales, which impact other market participants through fair-value accounting, as well as spillover effects through a network of interbank exposures.[2] One of their main findings is that ignoring the interactions between solvency and market liquidity seriously underestimates the importance of risks in the whole financial system.

GLS focus on banks' asset liquidity and do not consider the risks associated with an increase in a bank's funding risk related to asset-liability maturity mismatch or its reliance on volatile wholesale funding markets. Aikman et al (2009) do integrate funding liquidity risk into a framework similar to GLS for large UK banks. They assume that banks whose balance sheets deteriorate beyond certain thresholds, face constraints to their access to funding markets. Van den End's (2008) approach is similar, but his starting point is the liquidity profile of a group of

[1] Other studies focussing on the measurement of systemic risk include Mistrulli (2007), Toivanen (2009), Acharya (2009), Huang, Zhou and Zhu (2010) and Tarashev, Borio and Tsatsaronis (2010).
[2] Upper (2006) present a survey of interbank contagion models.

Dutch banks as reported in regulatory reform. Again the impact of a shock on banks is modelled, and their responses to this shock incorporated in a second-round calculation of the effect on their liquidity profiles. Contagion effects are larger if more banks react or if reactions are more similar. In Van den End, funding difficulties are randomly drawn in a Monte-Carlo simulation while in Aikman et al. they are triggered by exogenously fixed thresholds. Both papers show that ignoring funding liquidity risk seriously underestimates systemic risk.

In this paper, we propose a multi-period framework with which to assess the potential mitigating impact of combined capital and liquidity requirements on systemic risk. Our main contribution is to build on recent theoretical literature to measure funding liquidity risk as an endogenous outcome of the interaction between market liquidity risk, solvency risk, and the structure of banks' funding.

Ahead of the crisis, the theoretical literature was almost silent on how to link funding liquidity and bank solvency risks, concentrating for the most part on one type of risk or the other. In the aftermath of the crisis, a new strand of the literature modelises the interactions between those different types of risks. Examples of recent studies include He and Xiong (2009, 2010), which show that the fear of a firm's future rollover risk can lead creditors to preemptively run ahead of others, and Acharya, Gale and Yorulmazer (2009), which show that high rollover frequency can lead to diminishing debt capacity of risky assets. Brunnermeier and Pedersen (2009) presents a model in which market liquidity risk and traders' funding liquidity mutually reinforce, giving rise to a liquidity spiral. Morris and Shin (2009) (MS thereafter) study rollover risk through a coordination problem between short term creditors. A feature of their model is that illiquidity risk interacts with *future* insolvency risk. In their model, solvency risk is defined as the likelihood of default conditional on there being no run on the bank and liquidity risk as the probability of a run. Total credit risk is therefore the unconditional likelihood of default (or the sum of solvency and liquidity risks). Their main contribution is to provide a tractable theoretical framework to assess how different types of risks vary with balance sheet composition. In particular, they show how the likelihood of a market run on banks is jointly determined by maturity mismatch, liquid asset holdings, and capital level. Their analysis does not, however, address the issue of wide-spread contagion at the system level.

We build on MS and GLS to introduce funding liquidity risk in a systemic risk assessment framework. Although our framework does not include contagion effects through funding markets, we consider spillover effects due to the network effects among banks. We provide a method to decompose the total credit risk of a financial institution according to different underlying economic causes: the solvency risk due to credit losses in the absence of liquidity run, the liquidity risk due to a run by short term creditors and the spillover risks.

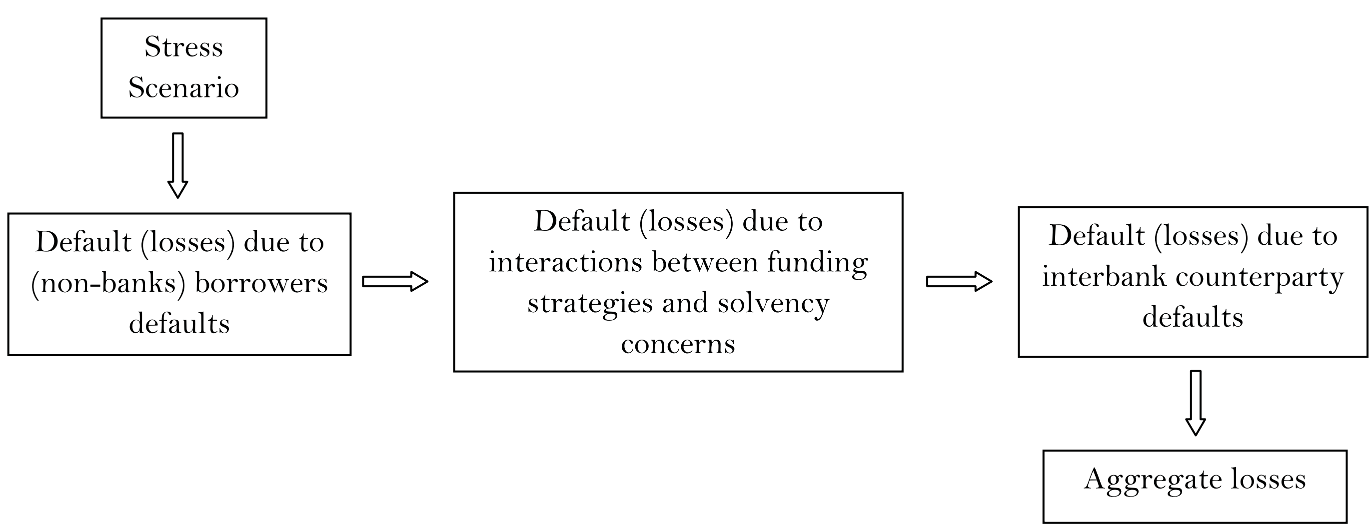
To evaluate the overall impact of capital and liquidity regulatory proposals, we simulate the framework under a severe but plausible macro scenario for different combinations of liquid asset holdings, capital and short-term funding. We show the general pattern that higher capital ratios, more liquid assets or less short term liabilities induces lower systemic risk in the financial system. However, the individual effect of any of the three variables on systemic risk depends in a non linear way on the levels of the other two variables. Of particular interest, we find that (1) capital has a decreasing marginal effect on systemic risk, (2) increasing capital alone is much less effective in reducing liquidity risk than solvency risk, (3) high liquid asset holdings reduce the marginal effect of increasing short term liability on systemic risk, and (4) changing liquid asset holdings has little effect on systemic risk when short term liability is sufficiently low.

The rest of the paper is organised as follows. Section 2 shows how to introduce funding liquidity risk into a systemic risk assessment framework. Section 3 presents the results of simulations under different combinations of liquid asset holdings, capital levels and short-term funding. We conclude in Section 4 and discuss future work[3].

2. A MODEL OF THE BANKING SYSTEM

In this section, we introduce endogenous funding liquidity risk into a systemic risk assessment framework whose basic structure can be best described at an intuitive level by Figure 1.

Figure 1. Basic structure of the model



[3] Appendix B illustrates the use of the model to assess systemic risk at a point in time (2008Q2).

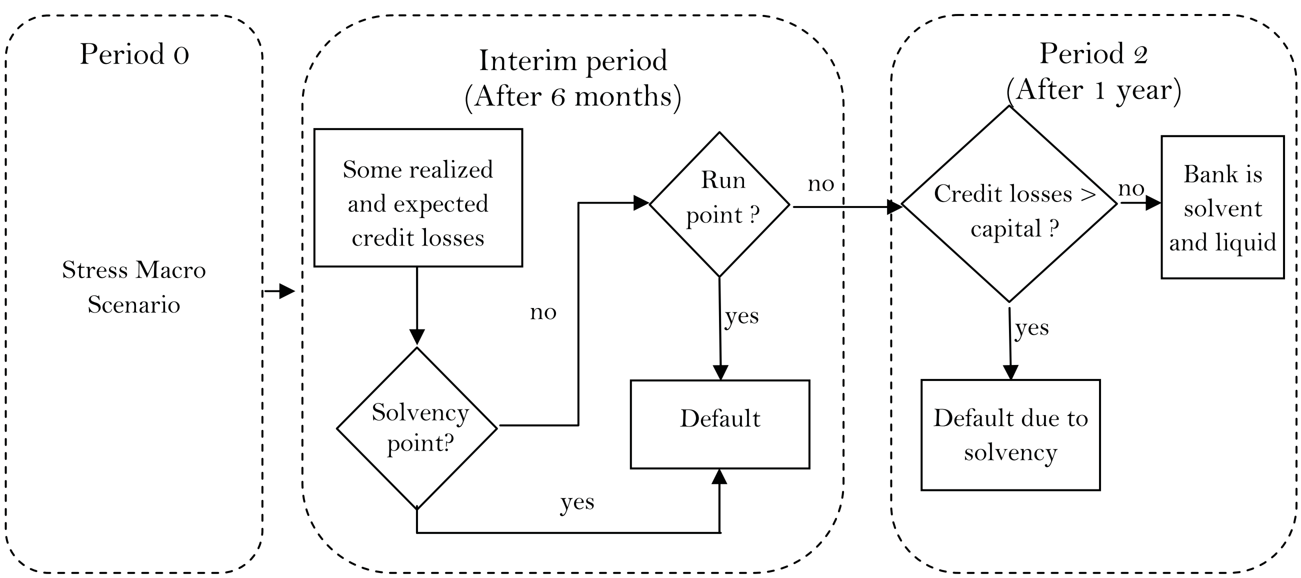
2.1 Stress scenario and credit losses at individual banks

The first step is to simulate credit losses at individual banks under adverse but plausible macroeconomic conditions. The underlying assumption is that loan quality is sensitive to the economic cycle. We therefore estimate the sensitivity of the risk of default in different economic sectors to changes in macro fundamentals. The estimation strategy consists of combining results from two models. The first model captures systematic factors affecting all banks' loans simultaneously. It relates sectoral default rates to a set of selected macroeconomic and financial variables which, according to theory and empirical evidence, affect credit risk. Estimated coefficients are then used to simulate distributions of sectoral default rates under a coherent macro stress scenario, which features a recession that is about one-third larger than experienced in the early 1990s. The second model is an extended CreditRisk+ model as in Elsinger, Lehar and Summer (2006a) and GLS. It captures some idiosyncratic risk factors and is used to generate, for a given default rate, a distribution of expected losses that reflects the size distribution of banks' loans. More details on these models, the macro scenario, data used, and the generated loss distribution are provided in Appendix A.

2.2 Solvency and funding liquidity risks

The second step in our framework is to build on the models developed in MS and GLS to assess credit and funding liquidity risks at individual institutions. Figure 2 gives an overview of our approach. The assessment horizon is divided into three periods: period 0, at the beginning of the year, where only the distribution of credit losses at the end of the year is known; the interim period, after 6 months, at which time some loan book losses are observed, potentially leading to interim illiquidity (of solvent banks) or insolvency; and period 2, the end of the year, at which time total credit losses are observed and liquid banks could be insolvent.

Figure 2. Approach to assessing liquidity and solvency risks



To analyse the interactions between these types of risks, we consider the following balance sheet of a bank in period 2:[4]

Assets	Liabilities
M	S
$Y - p_1 - p_2$	L
	$E - p_1 - p_2$

where M is the amount of liquid assets on the institution's balance sheet in period 0, p_1 and p_2 are the credit losses on the risky asset Y in the semi-annual period 1 and 2 respectively[5], S and L are the short-term (less than one year at the beginning of period 0) and long-term (longer than one year at the beginning of period 0) liabilities respectively, and $E - p_1 - p_2$ is the remaining capital after the total write-downs. We assume that the only available capital to absorb losses is Tier 1 capital.

The total cash available to the bank at the interim date is

$$C^* = M + \Psi(Y - p_1) \tag{1}$$

where $\Psi < 1$ can be seen as either the fire sale price of the illiquid assets or the collateral value of the assets (one minus the haircut) in stressed funding markets.[6] The bank fails from a run if the proportion of short-term debt holders not rolling over is more than

$$\lambda = \frac{C^*}{S} \tag{2}$$

i.e. if the total cash available on the balance sheet is not enough to cover for the creditors not rolling over (λS). The parameter λ is dubbed *liquidity ratio* at the interim date. Similarly, the liquidity ratio at the initial period is defined as

$$\lambda_0 = \frac{M + \Psi Y}{S} \tag{3}$$

[4] This is the balance-sheet before consideration of bankruptcy costs and liquidity risk as defined below.
[5] The distributions of credit losses at each bank are generated over a one-year horizon. To obtain the distributions of credit losses during the 2 semi-annual periods, we assume, for simplicity, that the semi-annual distributions are one half of the annual credit losses distributions and that the credit losses are uniformly distributed over the one-year horizon and independently distributed.
[6] To incorporate another channel of contagion between banks, the parameter Ψ could rather be a function of the number of runs in the system or the clearing price in a market for illiquid assets as in GLS. This is left for future work.

Solvency risk is defined as the likelihood of default conditional on there being no run on the bank and liquidity risk as the probability of a run. Total credit risk is therefore the unconditional likelihood of default (or the sum of solvency and liquidity risks). To characterize the ex ante liquidity and solvency risks, we need to solve the problem backwards and first characterize those risks in the interim period, once p_1 is observed.

2.2.1 Interim solvency risk

An institution is insolvent in period 2 if

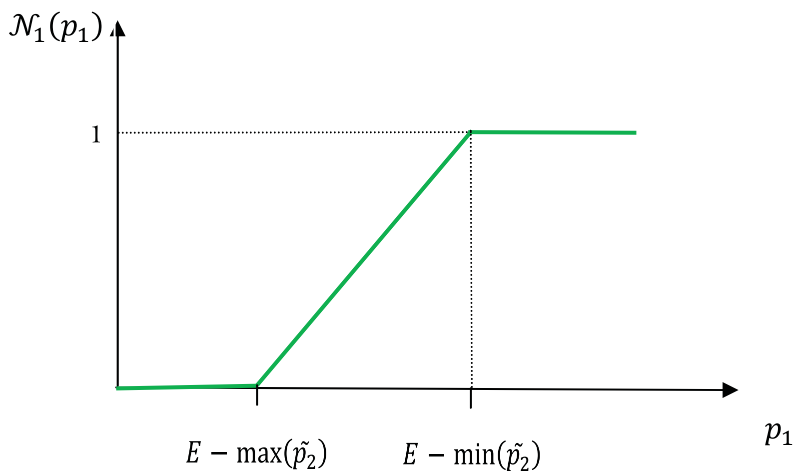
$$E - p_1 - p_2 < 0 \tag{4}$$

The probability of this happening, conditional on having observed losses of p_1 after the first six months, and conditional on not having a run by short-term creditors, is

$$\mathcal{N}_1(p_1) = P(\tilde{p}_2 > E - p_1) = \begin{cases} 1, & \text{if} \quad p_1 > E - \min(\tilde{p}_2) \\ 1 - F_2(E - p_1), & \text{if } E - \max(\tilde{p}_2) < p_1 \le E - \min(\tilde{p}_2) \\ 0, & \text{if} \quad p_1 \le E - \max(\tilde{p}_2) \end{cases} \tag{5}$$

where $\tilde{p}_2$ is the ex-ante credit loss in period 2, in contrast to the realized value p_2, and follows the cumulative distribution $F_2(.)$ on the support $[\min(\tilde{p}_2), \max(\tilde{p}_2)]$. When $p_1 > E - \min(\tilde{p}_2)$, the realized credit losses is such that even the smallest possible loss in period 2 would wipe out all the bank's capital. In the remaining, we refer to $p_1 > E - \min(\tilde{p}_2)$ as the "*insolvency point*". If, on the other hand, bank's capital can cover for the largest possible loss in period 2, given p_1, then the bank will be solvent for sure at the end of the year. In between these two extremes, the likelihood of the bank being insolvent in period 2 is $1 - F_2(E - p_1)$. Interim solvency risk is illustrated in Figure 3.

Figure 3. Interim solvency risk



2.2.2 Interim liquidity risk and the rollover decision of short-term creditors

In the interim period, short-term creditors must decide whether or not to roll over their funds after observing the credit losses in the first period. Let r_S be the notional return of short-term debt from period 1 to period 2. The total interim return of rolling over, conditional on there not being a run is therefore:

$$r_S\big(1-\mathcal{N}_1(p_1)\big) = \begin{cases} 0, & \text{if} \quad p_1 > E - \min(\tilde{p_2}) \\ r_S F_2(E-p_1), & \text{if } E - \max(\tilde{p_2}) < p_1 \leq E - \min(\tilde{p_2}) \\ r_S, & \text{if} \quad p_1 \leq E - \max(\tilde{p_2}) \end{cases} \tag{6}$$

There is a coordination problem among short-term debt holders in the interim period and a key variable in their decision to roll over will be their beliefs about the proportion of other short-term creditors they expect to roll over. We follow MS and use some simple and natural assumptions about these beliefs.[7] Assume each short-term creditor believes that the proportion of short-term creditors not rolling over is uniformly distributed on the interval $[0,1]$.[8] A successful run will not occur if this proportion is smaller than the liquidity ratio λ, and each short-term creditor will assume this to happen with probability λ.[9]

Given those beliefs, the expected return to rolling over is $\lambda r_S F_2(E-p_1)$. Assume that short term creditors have an alternative investment opportunity in which they can earn gross return r^*. Since both the likelihood of not having a successful run, (λ), and the probability of being solvent, $F_2(E-p_1)$, are decreasing with p_1.a successful run will occur for values of p_1 above a "*run point*", p_1^*, at which the expected return to rolling over is just equal to the creditor's opportunity cost, r^* :

$$\lambda(p_1^*) r_S F_2(E-p_1^*) = r^* \tag{7}$$

Define the "*outside option ratio*" $\mu = r^*/r_S$. The run point is solved by

$$p_1^* = E - F_2^{-1}(\mu/\lambda) = E - F_2^{-1}\left(\frac{r^* * \mathrm{S}}{r_S[\mathrm{M} + \psi(\mathrm{Y} - p_1^*)]}\right) \tag{8}$$

Since both $\lambda(.)$ and $F_2(.)$ are decreasing functions of p_1 , Equation (8) shows that the "*run point"* is an increasing function of bank's capital (E), liquid asset holdings (M), and the return on

[7] See MS for a sketch of how results from the global games literature can provide a foundation for the assumptions made about the short-term creditor's beliefs. See also Morris and shin (2003) for a fully rational foundation and Chui, Gai and Haldane (2002) for a very early global games application.

[8] Assuming a uniform distribution is equivalent to assuming total uncertainty about the proportion that will roll over, which sounds like a reasonable assumption in a distress period.

[9] Note that for $\lambda \geq 1$, there will always be enough liquidity to cover for short-term funding withdrawal, and a successful run will not occur with probability 1. We concentrate in the following on cases where a run is possible and relegate singular cases to footnotes.

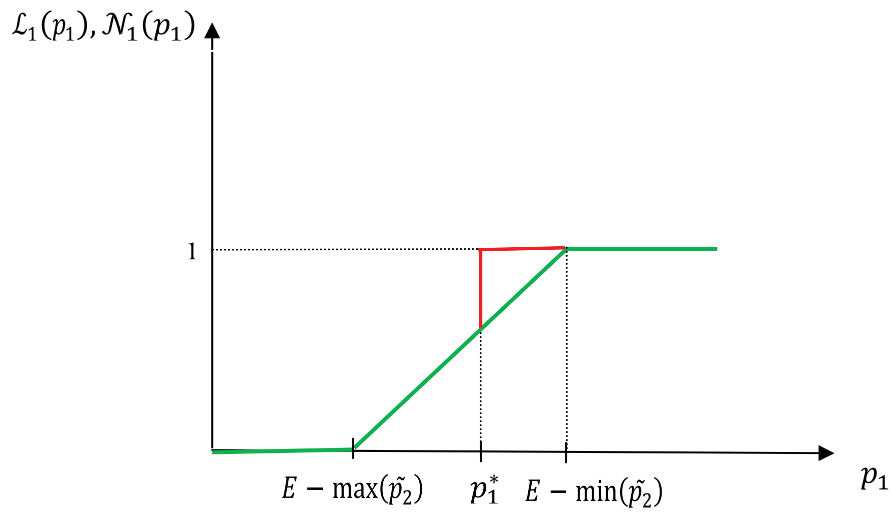
short-term debt (r_s), and a decreasing function of the amount of short-term funding (S) and the opportunity cost of short-term creditors (r^*).

Interim liquidity risk is

$$\mathcal{L}_1(p_1) = \begin{cases} 0, & \text{if} \quad p_1 < p_1^* \\ F_2(E - p_1), & \text{if} \quad p_1^* \leq p_1 \leq E - \min(\tilde{p}_2) \\ 0, & \text{if} \quad p_1 > E - \min(\tilde{p}_2) \end{cases} \tag{9}$$

For values of p_1 smaller than the "run point", the expected return to rolling over is higher than the opportunity cost and liquidity risk is 0. When $p_1 > E - \min(\tilde{p}_2)$, i.e. the observed credit losses in the interim period is sufficiently large, the institution will be bankrupt for sure even without any run by the creditors. Hence, solvency risk is 1 and, by definition, liquidity risk is zero.[10] Finally, for values of p_1 sufficiently close to the "*insolvency point*", the bank defaults because of a run by its short-term creditors whereas it would have been solvent in the absence of a run. Liquidity risk is illustrated by the red triangle in figure 4.

Figure 4. Interim liquidity risk



Summing the solvency and liquidity risk gives the interim (total) credit risk:[11]

$$\mathcal{C}_1(p_1) = \begin{cases} 0, & \text{if} \quad p_1 \leq E - \max(\tilde{p}_2) \\ 1 - F_2(E - p_1), & \text{if} \quad E - \max(\tilde{p}_2) < p_1 < p_1^* \\ 1, & \text{if} \quad p_1 \geq p_1^* \end{cases} \tag{10}$$

[10] Note that it is also possible that $p_1^* > E - \min(\tilde{p}_2)$, in which case $\mathcal{L}_1(p_1) = 0$ for the same reason.

[11] If $\lambda > 1$, or $p_1^* > E - \min(\tilde{p}_2)$, the liquidity risk is 0 and hence $\mathcal{C}_1(p_1) = \mathcal{N}_1(p_1)$. If $\mu/\lambda > 1$, we have $L_1(p_1) = 100 - \mathcal{N}_1(p_1)$.

2.2.3 Ex ante risks

Given the distribution of credit losses in period 1, $f_1(.)$, is known at period 0, the ex ante solvency risk, i.e. the expectation at $t = 0$ of being insolvent at $t = 1$:

$$\mathcal{N}_0 = \int \mathcal{N}_1(p_1) f_1(p_1) dp_1 \tag{11}$$

that is,

$$\mathcal{N}_0 = \int_{E-\max(\tilde{p}_2)}^{E-\min(\tilde{p}_2)} (1 - F_2(E - p_1)) f_1(p_1) dp_1 \ + \int_{E-\min(\tilde{p}_2)}^{\max(\tilde{p}_1)} f_1(p_1) dp_1 \tag{12}$$

Similarly, the ex ante liquidity risk is:

$$\mathcal{L}_0 = \int \mathcal{L}_1(p_1) f_1(p_1) dp_1 = \int_{p_1^*}^{E-\min(\tilde{p}_2)} F_2(E - p_1) f_1(p_1) dp_1 \tag{13}$$

For given $F_2(.)$, $f_1(.)$ and E, the ex ante liquidity risk $\mathcal{L}_0$ is a decreasing function of the run point p_1^* and is maximized when $\mu/\lambda > 1$.[12]

The ex ante total credit risk $\mathcal{C}_0$ is simply the sum of $\mathcal{N}_0$ and $\mathcal{L}_0$:

$$\mathcal{C}_0 = \int_{E-\max(\tilde{p}_2)}^{p_1^*} (1 - F_2(E - p_1)) f_1(p_1) dp_1 + \int_{p_1^*}^{\max(\tilde{p}_1)} f_1(p_1) dp_1 \tag{14}$$

2.3 Network of exposures between banks

Our model is designed to include network externalities caused by counterparties' default. Defaulting banks, either because of solvency or illiquidity, are unable to fully honour their interbank liabilities, potentially causing, in turn, the default of other banks. We model these network externalities explicitly by clearing the interbank market and identifying banks that are in spillover default.[13] The clearing is done by extending the model of Eisenberg and Noe (2001) to include bankruptcy costs and uncertainty as in Elsinger, Lehar and Summer (2006) and GLS.

[12] If $\lambda > 1$, or $p_1^* > E - \min(\tilde{p}_2)$, the interim liquidity risk is always 0 and hence $\mathcal{L}_0 = 0$ and $\mathcal{C}_0 = \mathcal{N}_0$; if $\mu/\lambda > 1$, we have $\mathcal{L}_0 = 100 - \mathcal{N}_0$ and $\mathcal{C}_0 = 1$.

[13] Jorion and Zhang (2009) provide empirical evidence of contagion from counterparty risk for a sample of US banks and corporations.

To model the network of interbank obligations, we distinguish between claims outside the banking system and those to counterparties inside the banking system. Consider a set $\Lambda = \{1, \dots, \mathrm{N}\}$ of banks. Each bank $i \in \Lambda$ has a claim on specific assets A_i outside of the banking system, which we can interpret as the bank's portfolio of non-bank loans and securities. Each bank is partially funded by issuing senior debt or deposits, D_i, to outside investors. Bank *i*'s obligations against other banks $j \in \Lambda$ are characterized by the nominal liabilities x_{ij}.[14]

The total value of a bank after credit losses is the value of its net assets minus the outside liabilities, $A_i - p_{1i} - p_{2i} - D_i$, plus the value of all net payments to and from counterparties in the banking system. We denote by $d \in \mathbb{R}_+^N$ the vector of total obligations of banks toward the rest of the system, i.e. $d_i = \sum_{j \in \Lambda} x_{ij}$. We define a new matrix $\Pi \in [0,1]^{N \times N}$ which is derived by normalizing x_{ij} by total obligations.

$$\pi_{\mathrm{ij}} = \begin{cases} \dfrac{x_{ij}}{d_i}, & if \;\; d_i > 0 \\ 0, & otherwise \end{cases} \tag{15}$$

Thus the net value of bank *i* assuming all interbank claims are paid in full is

$$A_i - p_{1i} - p_{2i} + \sum_{j=1}^{N} \pi_{\mathrm{ji}}\, d_j - D_i \; - d_i \tag{16}$$

where the summation term represents the total interbank assets of bank *i*. We define a *clearing payment vector* X^* which has to respect limited liability of banks and proportional sharing in case of default. It denotes the total payments made by the banks under the clearing mechanism.[15] Each component of X^* is defined by

$$x_i^* = min\left[d_i, max\left(\left(A_i - p_{1i} - p_{2i} - \phi_i + \sum_{j=1}^{N} x_j^* \pi_{\mathrm{ji}} - D_i, 0\right)\right)\right] \tag{17}$$

where the proportional bankruptcy costs is similarly applied to all types of bankruptcies and is defined as

[14] We do not need to distinguish between short-term and long-term liabilities and between liquid and illiquid assets since the "run point" of banks is not reestimated once the network has been cleared. Ideally, the solution of the problem would require an iterative process to solve for a fixed point such that the "run point" is consistent with the level of capital coming out of the network. This is not done to limit the computational burden.

[15] Appendix C provides a numerical illustration of spillover effects through the clearing mechanism with three hypothetical banks.

$$\phi_i = \begin{cases} \psi_i y_i, \ if \ \ p_{1\mathrm{i}} + p_{2\mathrm{i}} > \mathrm{E_i} \ \ or \ \ \mathcal{L}_1(p_{1\mathrm{i}}) > 0 \ \ or \ x_i^* < d_i \\ \\ 0, \ otherwise \end{cases} \tag{18}$$

3. THE IMPACT OF CAPITAL AND LIQUIDITY ON SYSTEMIC RISK

In this section, we study the effect of capital and liquidity on both risks at individual banks, and systemic risk, where systemic risk refers to the probability of at least one bank defaulting.[16] We also run a calibrated exercise for the Big 6 Canadian banks based on observed data in 2008Q2 (see Appendix B for the details of this exercise).

We run simulations for 1000 credit loss scenarios using different combinations of banks' Tier 1 capital ratio (ranging from 4 to 10 percent of risk-weighted assets), short term liabilities (ranging from 25 to 75 percent of total liabilities) and liquid asset holdings (ranging from 5 to 25 percent of total assets).[17] These ranges well contain both the values for the Big 6 Canadian banks used in the calibrated exercise and those relevant from a policy perspective. In each simulation, all banks are assigned the same combination. Finally, for all combinations, the parameters r_s and ψ are set at their average values from the calibrated exercise, and r^* is fixed at its 2008Q4 level. Hence, the only factors that distinguish individual banks are their sectoral loan exposures (which affect their credit losses under the stress scenario) and their bilateral exposures to other banks.

3.1 Impact of capital

Table 1 presents the effect of capital on banks' default risks and its various components. Measures at a given capital level are the averages of PDs obtained with the different combinations of liquid holdings and funding structures.[18]

At a Tier 1 capital level of 4%, the minimum required under Basel II, all individual banks have very high conditional probability of defaults with total PDs of about 95%. This provides further support for either a higher regulatory minimum Tier 1 capital ratio or a countercyclical capital buffer.[19] Solvency risks are the dominant components for all banks but are dramatically reduced as the capital level increases to 6%. Increasing the capital level to 8% (1 percent more

[16] The 6 major domestic Canadian banks hold more than 90% of banking assets in Canada. We consider that the default of even the smallest of these banks constitutes a systemic event.
[17] The number of scenarios is fixed at 1000 allowing for reasonable flexibility and manageable computation burden.
[18] Since the results shown in Table 4 are average PDs over the different combinations of S and M, liquidity and contagion PDs are obviously lower than they would be for the highest values of μ/λ.
[19] We cannot evaluate the relative merits of these alternatives within this framework.

than the regulatory minimum in Canada) reduces the solvency PD of all banks to be less than 0.8%. Solvency risks are virtually eliminated with a capital level of 10% (not shown).

Table 1. Banks' PDs for different levels of capital. A bank is insolvent when it defaults because of credit losses given that all interbank claims are paid in full. A bank is illiquid when there is a succefull run by short-term creditors. Spillover defaults are recorded when a bank defaults because of other banks not honouring their interbank commitments. Total PD is the sum of columns 2, 3 and 4. Results are the outcome of 1000 credit loss scenarios for different combinations of short term liabilities (ranging from 25 to 75 percent of total liabilities) and liquid asset holding (ranging from 5 to 25 percent of total assets). Measures at a given capital level are the averages of PDs obtained with the different combinations.

Bank	Solvency PD ($\mathcal{N}_0$) (%)	Liquidity PD ($\mathcal{L}_0$) (%)	Spillover PD (%)	Total PD (%)
		Panel A: k=4%		
1	84.92	10.46	0	95.38
2	81.69	12.42	1.13	95.24
3	70.57	18.44	6.32	95.33
4	79.11	13.75	2.43	95.29
5	63.25	21.45	10.53	95.23
6	59.86	23.02	12.28	95.16
		Panel B: k=6%		
1	17.9	26.45	0.49	44.84
2	18.39	26.56	0.09	45.04
3	4.78	22.24	17.83	44.85
4	8.66	23.82	12.5	44.98
5	3.26	21.35	20.13	44.74
6	9.32	23.98	11.08	44.38
		Panel C: k=8%		
1	0.55	20.15	0.25	20.95
2	0.77	20.37	0.03	21.17
3	0.01	19.98	0.95	20.94
4	0.06	20	1.02	21.08
5	0.01	19.98	0.92	20.91
6	0.34	20.09	0.48	20.91

Liquidity risk is an important, component of total risk, particularly at higher capital levels. The liquidity PD of most banks increases as the capital rises from 4% to 6%, and then begins to decrease as the capital level becomes higher than 6%. The increase in liquidity risk when capital goes from 4% to 6% is due to those loss scenarios that lead to insolvency when capital is set at

4% but to illiquidity instead when capital is set at 6%.[20] At a capital level of 8%, total PDs remain elevated and are almost totally due to liquidity risks.[21] Relative to solvency risk, increasing capital alone is much less effective in reducing liquidity risk. This result is not surprising, since in the MS model, the risk of a creditor run on banks is jointly determined by capital level, maturity mismatch, and liquidity holding.

Considering only solvency risk, the traditional focus under Basel II, we find that the likelihood of default differs significantly among banks. For example, with a capital ratio of 6%, bank 2's solvency PD is almost 6 times larger than for Bank 5. However, when all types of risk are taken into account, particularly spillover effects, all banks have very similar total PDs. This is explained by the relatively high likelihood of many banks defaulting simultaneously due to common exposure to macro shock and homogenous calibration of banks (same E, M, S, r_s, r^* and ψ).[22] Hence, although only one or two banks might default in some scenarios, the solvent banks' capital is often sufficiently low after the write-downs for spillover to occur. Our results show that this spillover effect is a significant component of total risk, particularly at low levels of capital. This suggests that models that do not account for the network effect among banks would significantly underestimate the extent of systemic risk at low levels of capital. This speaks to the importance of getting timely information about exposures between financial institutions (GLS reached the same conclusion).

We next turn to the measurement of systemic risk. Figure 5 plots the distributions of the number of bank defaults at different capital levels. The distribution of bank defaults is calculated by integrating over short term liabilities and liquid asset holding, which are still assumed to follow uniform distributions on [0.25 0.75] x [0.05 0.25].

For all capital levels, the distributions of bank defaults exhibit a dichotomous pattern. Most of the probability mass is at either the no-default or all-default outcome. The average probabilities of other outcomes are typically less than 0.1%. This dichotomy is again due to the homogenous calibration of banks and their common exposures to the macro shock. In simulations with more heterogeneity, the dichotomous pattern weakens.[23]

As expected, the probability of all-defaults decreases while the probability of no-default goes up as the capital level increases. Increasing capital level from 4% to 8% increases the probability of no-default from about 5% to 80%. Note that there is a decreasing marginal effect of capital on systemic risk. The increase in no-default probability is markedly larger when the capital level

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[20] Assume figure 4 represents the case with a capital ratio set at 6%. The case with 4% would correspond to a shift of the green curve and the run point, p_1^*, to the left, increasing insolvency risk, but such that the red triangle becomes smaller, decreasing liquidity risk.

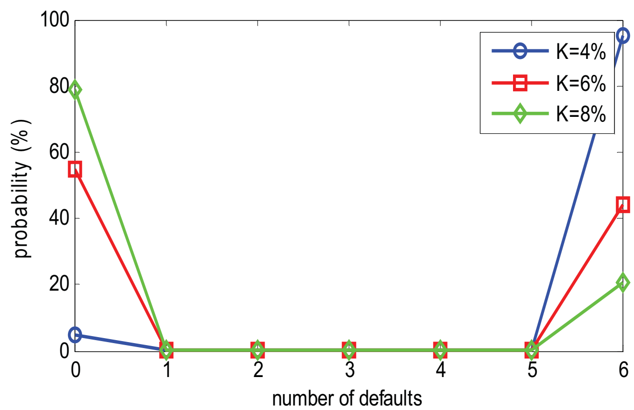
[21] The relatively high level of liquidity risk when capital is set at 8 is due the combinations of high S and low M. When we let S range from 15 to 55 percent and M from 10 to 30 percent instead, liquidity risk averages are below 0.1 percent for all banks.

[22] Remember that the only factors distinguishing individual banks are their sectoral loan exposures (which affect their credit losses under the stress scenario) and their bilateral exposures to other banks.

[23] Results from such simulations are available upon request.

goes from 4 to 6% than from 6 to 8%. This suggests that increasing capital is most effective in reducing systemic risk when the existing capital level is relatively low.

Figure 5. Distribution of Bank Defaults: Capital Level



3.2 *Impact of liquidity*

We now study the effects of liquidity on systemic risk with capital levels ranging from 4 to 8%. In the model of this paper, there are two dimensions of liquidity: the amount of short term liability and the liquid asset holding. We first examine the individual effect of each of these two dimensions of liquidity on systemic risk and then study their joint effect. Figure 6 plots the distributions of bank defaults for different levels of short term liability at capital levels of 4, 6 and 8% respectively. We assume that banks' liquid asset holding, the other dimension of liquidity, follows a uniform distribution on [0.05 0.25]. The distribution of bank defaults integrates over banks' liquid asset holding.

At a given level of capital, the distribution of bank defaults concentrates again on either no default or all-6-defaults. The probability of the events in between is typically less than 1%. As the short term liability goes up, the probability of no-default is reduced while the probability of all-defaults increases correspondingly. For example, at the capital level of 6%, increasing short term liability from 25% to 65% would raise systemic risk from about 20% to 68% while lowering the probability of no-default from about 80% to about 30%. As the capital level goes up from 4 to 8%, the average probability of defaults is significantly reduced. Interestingly, there is an increasing marginal effect of short term liability on systemic risk. The changes in the distribution of bank defaults are much smaller when the short term liability goes up from 25% to 45% than from 45% to 65%. Hence, the effect of changing short term liability level on systemic risk is larger when the existing short term liability level is relatively high.

Figure 6(1): Distribution of Bank Defaults: Short Term Liability, k=4%

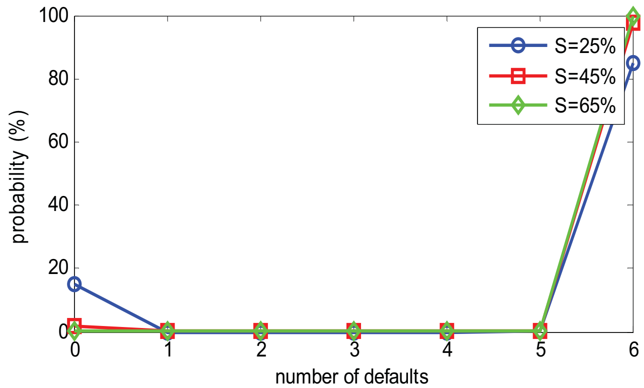


Figure 6(2): Distribution of Bank Defaults: Short Term Liability, k=6%

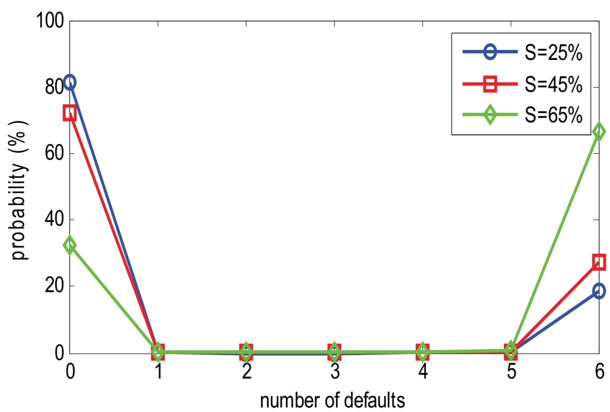


Figure 6(3): Distribution of Bank Defaults: Short Term Liability, k=8%

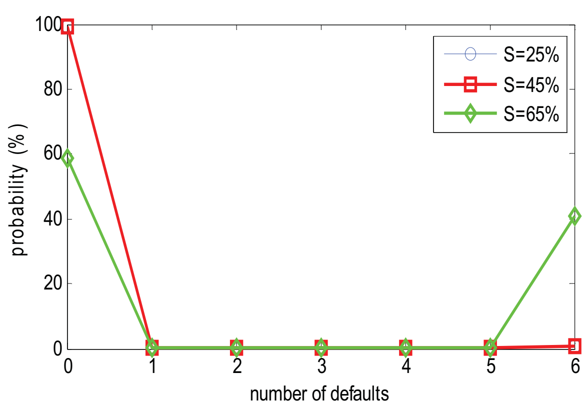


Figure 7 plots the distributions of bank defaults for different levels of liquid asset holdings at capital levels of 4, 6 and 8% respectively. To account for the other dimension of liquidity, short term liability, the distribution of bank defaults integrates over banks' short term liability levels, which is assumed to follow a uniform distribution on [0.25 0.75].

Figure 7(1): Distribution of Bank Defaults: Liquid Asset Holding, k=4%

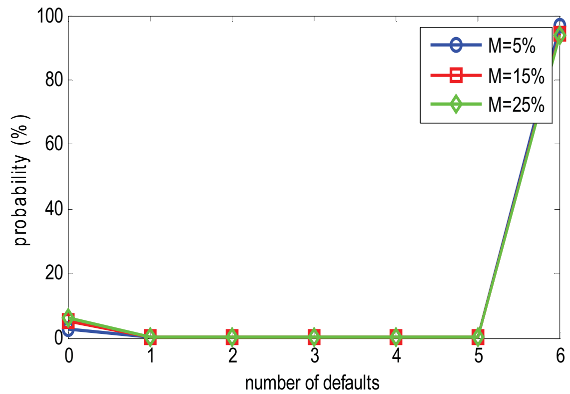


Figure 7(2): Distribution of Bank Defaults: Liquid Asset Holding, k=6%

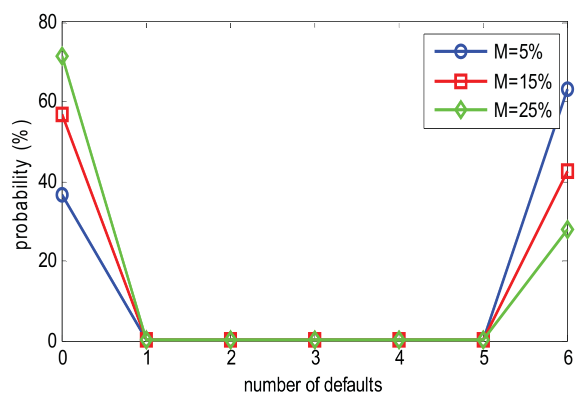
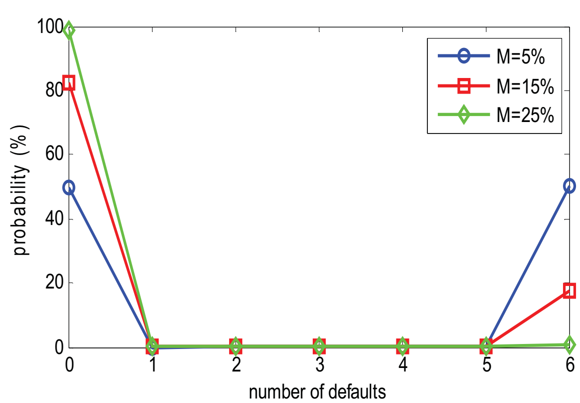


Figure 7(3): Distribution of Bank Defaults: Liquid Asset Holding, k=8%



We see a similar dichotomous pattern of default distribution as in the study of short term liability. At the capital ratio of 4%, the average systemic risk is more than 90% and is essentially independent of liquid asset holding. As the capital level rises from 4% to 8%, systemic risk is significantly reduced. We also find that the changes in the distribution of bank defaults are larger when the liquid asset holding goes up from 5% to 15% than from 15% to 25%. Hence, the effect of changing liquid asset holding on systemic risk is larger when the existing liquid asset holding is relatively low.

Next we study the joint effect of short term liability and liquid asset on systemic risk. Figure 8 plots the probability of at least 1 default as functions of liquid asset and short term liability for capital levels 4, 6 and 8% respectively.

In general, systemic risk decreases as capital level increases from 4 to 8%, consistent with what we find above. For a given capital level, the systemic risk is minimized when the liquid asset holding is at its upper end of 25% and short term liability at its lower end of 25%. As liquid asset holding increases and short term liability decreases, the systemic risk rises as expected.

As opposed to the (average) marginal distributions of Figures 6 and 7, the joint distributions of Figure 8 clearly shows how the relation between systemic risk and one dimension of liquidity varies with the other dimension of liquidity. In particular, the positive relation between systemic risk and short term liability is much steeper at lower levels of liquid assets, for all levels of capital. This suggests that high liquid asset holding will reduce the marginal effect of increasing short term liability on systemic risk. Similarly, the negative relation between systemic risk and liquid asset holding is most significant at higher levels of short term liability. At the relatively low levels of short term liability, e.g. 25%, the relation between systemic risk and liquid asset holding is almost flat, implying that changing liquid asset holdings has little effect on systemic risk when short term liability is sufficiently low.

The three dimensions figures are helpful to better identify the relative impacts of liquid holdings and short-term funding for a given level of systemic risk. Figure 8(2) shows that, at 5 percent of liquid asset holding and 6 percent of capital level, systemic risk could be reduced by 200% by reducing short-term debt from 75 to 25 percent. The trade-off between short-term debt (S) and liquid holdings (M) is illustrated with isoquants. For example, combinations of (15, 25) percent or (25, 25) percent of M and S, respectively, would induce the same level of systemic risk when the capital ratio is set at 4 percent (see Figure 8(1)).

Figure 8(1): Systemic Risk: Liquid Asset Holding and Short Term Liability, k=4%

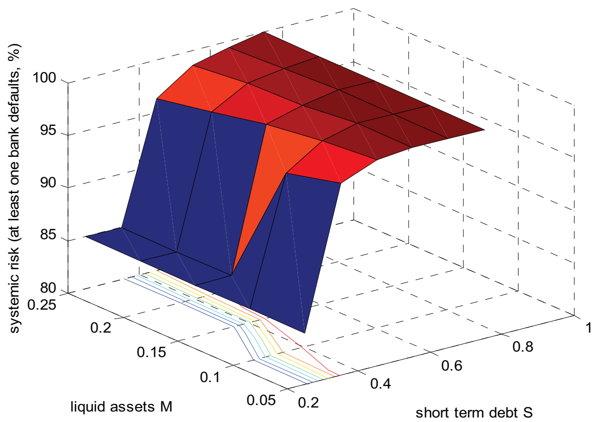


Figure 8(2): Systemic Risk: Liquid Asset Holding and Short Term Liability, k=6%

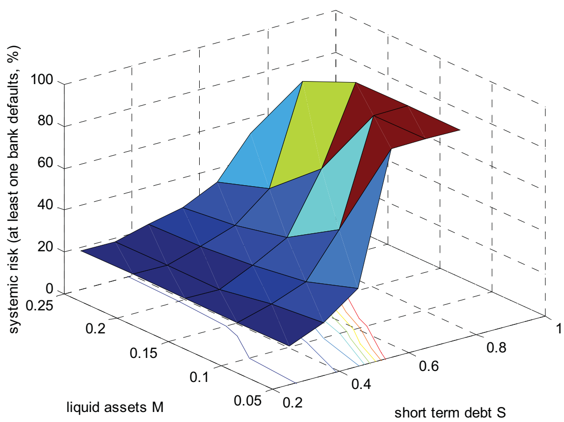
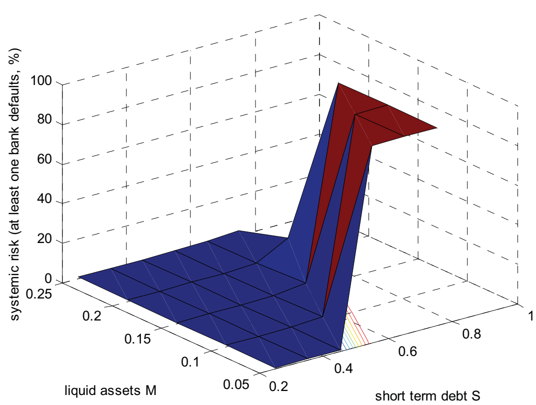


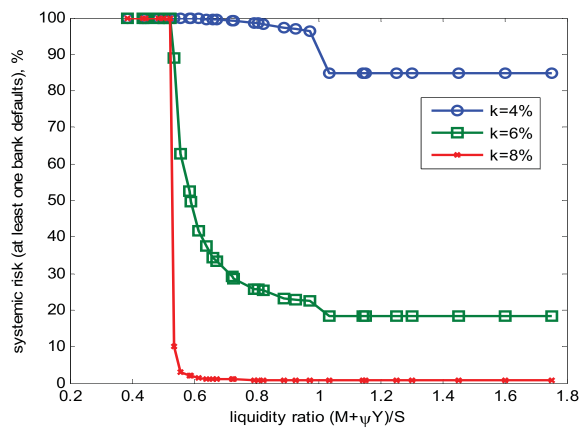
Figure 8(3): Systemic Risk: Liquid Asset Holding and Short Term Liability, k=8%



Finally we examine the effect of liquidity ratio, defined in equation (3), on systemic risk at different capital levels. The liquidity ratio integrates the effects of both liquid asset holding and short-term liability, and also takes into account the fire-sale price of illiquid asset. Hence it provides a holistic measure of the joint effect of liquid asset and short-term liability.

Figure (9) plots systemic risk, defined as the probability of at least 1 default, as function of liquidity ratio at capital levels 4, 6, 8% levels respectively.[24]

Figure 9: Systemic Risk: Liquid Ratio and Capital Level



As liquidity ratio increases, more short-term liability is covered by cash available at short notice, and systemic risk decreases at each given capital level as we found before. Note that systemic risk is flat at high levels of liquidity ratio in the plot. As we have seen in Section 2, liquidity risk of an individual financial institution becomes 0 when its liquidity ratio is higher than one. Hence the systemic risk at high liquidity ratios reflects only the solvency risk and spillover effect of financial institutions.

On the other hand, holding liquidity ratio fixed, systemic risk decreases with higher capital ratio. Similar to what we found before, the marginal effect of capital on systemic risk is decreasing. At a given level of liquidity ratio, the reduction in systemic risk when capital level goes from 4% to 6% is generally markedly larger than that when capital level goes from 6% to 8%.

4. CONCLUSION AND FUTURE WORK

In this paper we propose a multi-period framework for assessing systemic risk and the mitigating impact of currently considered policy measures. Our main contribution is to build on recent theoretical literature to integrate funding liquidity risk as an endogenous outcome of the

[24] We also experimented with 3 dimensional plot of systemic risk against liquidity ratio and capital level, and find it less visually effective than the 2 dimensional plot presented.

interaction between market liquidity risk, solvency risk, and the structure of banks' balance-sheets. Using this framework, we analyse the impact on systemic risk of different combinations of banks' liquid asset holdings, capital position and short-term funding.

In summary, we find that the network effect and liquidity risk are important components of banks' overall risk. Failure to account for them would significantly underestimate the actual default risks facing banks. We show the general pattern that higher capital, more liquid assets or less short term liabilities induces lower systemic risk in the financial system. However, the individual effect of any of the 3 variables on systemic risk depends in a non linear way on the levels of the other 2 variables. Of particular interest, we find that (1) capital has a decreasing marginal effect on systemic risk, (2) increasing capital alone is much less effective in reducing liquidity risk than solvency risk, (3) high liquid asset holdings reduce the marginal effect of increasing short term liability on systemic risk, and (4) changing liquid asset holdings has little effect on systemic risk when short term liability is sufficiently low. Consequently, a regulatory framework that properly controls for systemic risk should consider capital, liquid asset holdings and short term liability in a holistic way. Treating any of them in isolation would produce a misleading assessment of systemic risk and hence impair the effectiveness of the regulatory policies.

The framework presented in this paper can be used to answer different policy questions. For example, work is currently under way to estimate the capital surcharge needed for systemic financial institutions. Our model could also be extended in different directions. A priority is to integrate a mechanism by which funding difficulties at one bank could translate into funding difficulties at other banks.

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APPENDIX A

A.1 Generation of credit losses

Estimating probabilities of default is the first step in assessing potential credit losses faced by Canadian banks under a coherent stress-test macro scenario. As in GLS, we use the macroeconomic scenario designed for the macro stress-testing exercise conducted as a part of Canada's Financial Sector Assessment Program (FSAP) update in 2007. This scenario features a recession that is about one-third larger than experienced in the early 1990s.[25] For the modeling of probabilities of default, we use a macroeconomic-based model which captures systematic factors affecting all banks' loans simultaneously.[26] This type of model is motivated by the observation that default rates in the economic sectors increase during recessions. It analyzes the relationship between a logit transformation of Canadian default rates in different sectors[27] and the overall performance of the economy as captured by a selected set of macroeconomic variables. To address nonlinearities, the specification of the credit-risk model includes higher-order terms as explanatory variables.[28] Using this model, we simulate sectoral distributions of 10,000 default rates for 2009Q2 under stress. The sectoral distributions of default rates are centered on fitted values from sectoral regressions, and are generated using the correlation structure of historical default rates.[29] Descriptive statistics of these distributions as well as historic peaks over the 1988-2006 period are presented in Table A1. Consistent with the severity of the macro scenario, mean default rates are much higher than historic peaks. Default rates in the tail of the distributions are still higher.

Table A1. Summary statistics of simulated default rate distributions for 2009Q2. Columns two to four show the minimum, maximum and average default rates generated for each sector. Column five gives the historic peak over the 1988-2006 period.

	Minimum	Average	Maximum	Historic Peaks
Accommodation	3.0	11.7	21.0	7.6
Agriculture	1.0	1.7	2.0	0.8
Construction	2.0	6.4	10.0	3.3
Manufacturing	5.0	12.2	20.0	8.3
Retail	0.0	4.3	8.0	5.3
Wholesale	2.0	7.0	12.0	4.6
Mortgage	0.0	0.6	1.0	0.6

Source: Gauthier, Lehar and Souissi (2010)

[25] See Lalonde, Misina, Muir, St-Amant, and Tessier (2008) for a detailed description of the scenario.
[26] Chan-Lau (2006) lists advantages of macroeconomic models in forecasting default rates under stress.
[27] For more details on the construction of historical default rates, see Misina and Tessier (2007).
[28] In stressful periods, when the default rate reaches its historical peak; without nonlinearities, even the extreme shocks would have had a very limited impact on default rates.
[29] More specifically, for each sector, the mean of the distribution is the expected default rate under the scenario, while the dispersion is obtained by adding random draws from the variance/covariance matrix of the historical sectoral default rates. See Misina, Tessier, and Dey (2006) for more details on the simulation of default rates.

A bank's expected loss will not depend only on the default rate but also on the structure of its loan portfolio. For a given default rate, a portfolio with a higher number of large exposures will experience higher expected losses. In order to capture idiosyncratic risk factors arising from the size distribution of banks' exposures, we use an extended CreditRisk+ model as in Elsinger, Lehar, and Summer (2006). A key input in this model is banks' loan portfolios compositions by sector, which are estimated using the Bank of Canada Banking and Financial Statistics, and private data on banks' largest exposures towards non-banks obtained from the Office of the Superintendent of Financial Institutions (OSFI). For each bank, we draw 100 independent loan size for each of the 10 000 sectoral default rates simulated previously, yielding a total of 1 000 000 sectoral loan loss scenarios. The individual distributions of total expected losses are derived by adding expected sectoral loan losses. Other key inputs include: the Loss-given-Default reported by banks.

Table A2 shows the importance of considering both sources of uncertainty. When considering systematic factors only, expected losses to the 6 big banks average $45.7 Billion or 47.7 percent of total Tier 1 Capital, with a standard deviation of $7.9 Billion. Taking both systematic and idiosyncratic factors into account, the expected losses are approximately the same ($46.4 Billion on average), and, not surprisingly, are larger in the tail of the distribution (the 99 percent Valueat-Risk (VaR) is $68.7 Billion as compared to $63.7 Billion in the first distribution).

Table A2. Aggregate losses and probabilities of default conditional on extreme stress scenario. It provides descriptive statistics of aggregate losses considering systematic factors only (Column 1) and both systematic and idiosyncratic factors (Column 2).

	Systematic factors		Systematic and idiosyncratic factors	
	$Billion	%of Tier1 capital	$Billion	%of Tier1capital
Mean	-45.7	47.7	-46.4	48.5
Standard Deviation	7.9	8.4	9.5	9.9
Quantiles				
- 99%	-27.3	28.5	-25.7	26.9
- 10%	-55.8	58.4	-58.8	61.4
- 1%	-63.7	66.6	-68.7	71.8

Source: Gauthier, Lehar and Souissi (2010)

In the paper, we use subsamples of the simulated losses to reduce the computational burden. The statistical properties of these subsamples are close to the properties of the full sample shown in Table A2.

A.2 Data on exposures between the Big six Canadian banks

We use the same dataset on interbank exposures as in GLS. As in previous studies of systemic risk in foreign banking systems[30], we collected data on exposures between banks that arise from traditional lending (unsecured loans and deposits). We also cover another on-balance sheet item, cross-shareholdings, and off-balance sheet instruments such as exchange traded and OTC derivatives.[31]

Our dataset excludes zero-risk exposures such as repo style transactions. Also, Owing to data limitations, we have not covered exposures coming from intraday payment and settlement, from bank holdings of preferred banks' shares (and other forms of capital), and from holdings of debt instruments issued by banks like debentures and subordinated debt.

Data on the exposures are collected on a consolidated basis and come from different sources as described below. Available data are collected for May 2008 with the exception of exposures related to derivatives which are recorded as of April 2008. We present descriptive statistics in Table A3.

Data on deposits and unsecured loans come from the banks' monthly balance-sheet reports to OSFI. These monthly reports reflect the aggregate asset and liability exposures of a bank for deposits, and only aggregate asset exposures for unsecured loans. Data on exposures related to derivatives come from a survey initiated by OSFI at the end of 2007. In that survey, banks are asked to report their 100 largest mark-to-market counterparty exposures that were larger than $25 million. These exposures were related to both OTC and exchange traded derivatives. They are reported after netting and before collateral and guarantees.[32]

The reported data are used to construct a matrix of Big 6 banks' bilateral exposures. Data on cross-shareholdings exposures were collected from Bank of Canada's quarterly securities returns.[33]

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[30] See among others Sheldon and Maurer (1998), Wells (2002) and Upper and Worms (2004).

[31] While derivatives are often blamed for creating systemic risk, the lack of data in many countries (including the U.S.) makes it hard to verify. Our expanded dataset enables us to better capture linkages among banks and contagious bank defaults.

[32] The derivatives exposures reported may be biased upward, since they were reported before collateral and guarantees. In particular, anecdotal evidence suggests that the major Canadian banks often rely on high-quality collateral to mitigate their exposures to OTC derivatives.

[33] These returns provide for each bank aggregate holdings of all domestic financial institutions' shares. Due to data limitations, cross-shareholdings among the Big Six banks were estimated by (i) distributing the aggregate holdings of a given bank according to the ratio of its assets to total assets of domestic financial institutions, and (ii) excluding shares that were held for trading (assuming that they are perfectly hedged).

Table A.3. Summary statistics on exposures between Canadian banks. Panel A gives the aggregate size of interbank exposures related to traditional lending, derivatives and cross-shareholdings (reported in $billion and as percentage of banks' tier 1 capital). Panel B gives banks' bilateral exposures as percentage of tier 1 capital under two assumptions: entropy maximization and relationship banking.

Panel A: Aggregate exposures between Canadian banks				
	Aggregate exposure	As percentage of Tier1 capital		
	($Billion)	Minimum	Average	Maximum
Traditional lending	12.7	5.25	16.3	38.6
Derivatives exposures	5.4	0.0	5.9	21.1
Cross-shareholdings	3.5	0.3	4.1	8.8
Total exposures	21.6	5.5	26.4	51.2
Panel B: Banks' bilateral exposures as percentage of Tier 1 capital				
Assumption:		Minimum	Average	Maximum
- Entropy maximization		0.6	4.4	15.6
- Relationship banking		0.5	4.4	16.2

Source: Gauthier, Lehar and Souissi (2010)

The aggregate size of interbank exposures was approximately $21.6 billion for the Six major Canadian banks. As summarized in Table A3, total exposures between banks accounted for around 25 per cent of bank capital on average. The available data suggest that exposures related to traditional lending (deposits and unsecured loans) were the largest ones compared with mark-to-market derivatives and cross-shareholdings exposures. Indeed, in May 2008, exposures related to traditional lending represented around $12.7 billion on aggregate, and 16.3 percent of banks' Tier 1 capital on average. Together, mark-to-market derivatives and cross-shareholdings represented 10 per cent of banks' Tier 1 capital on average.

A complete description of linkages between Canadian banks requires a complete matrix of the bilateral exposures. Such a complete matrix was available only for exposures related to derivatives. Unavailable bilateral exposures were estimated under the assumption that banks spread their lending and borrowing as widely as possible across all other banks using an entropy maximization algorithm (see e.g. Blien and Graef (1997)). A difficulty with this solution is that it assumes that all lending and borrowing activities between banks are completely diversified. This rules out the possibility of relationship banking i.e. a bank preferring some counterparties to others (as reflected in the structure of banks' exposures related to derivatives). As a benchmark, banks' bilateral exposures were also estimated under the assumption that concentrations of exposures between banks are broadly consistent with their asset sizes. As shown in Table A3, banks' bilateral exposures are comparable under these two assumptions. Indeed, they are consistent with the concentration of bilateral exposures related to derivatives.

APPENDIX B. ASSESSMENT OF SYSTEMIC RISK

The framework can be used to assess systemic risk over time. As an illustration, we estimate solvency, liquidity and contagion risks at the Big six Canadian banks in 2008Q2 (period 0 in Figure 2). Remember that no government or central bank interventions are allowed in the model. As a result, the PDs of banks are higher in our model than in a more realistic world where central bank's liquidity facilities and/or mitigating government measures would be put in place.

B.1 Calibration

We need to calibrate M, S, r_S, r^* and ψ to bank balance sheet and market data. Liquid asset holding, M, include cash and government securities as reported on banks' monthly balance sheet reports to the Office of the Superintendent of Financial Institutions (OSFI) as of 2008Q2. On average, liquid holdings of the Big 6 Canadian banks were around13 percent of total assets and as much as 23% for one bank.

With respect to the amount of banks' short term funding, S, we need an estimate of how much funding liabilities are maturing at the interim date which is set at six months after the beginning of the exercise, 2008Q4. [34] We consider both short and long-term maturing liabilities regardless of their original contract maturity. The only source of data that cover all types of funding liabilities is banks' annual reports. We group the data reported at the end of 2007 in (i) funding liabilities with a maturity less than three months, such as bankers' acceptances, repurchase agreements (Repos), and personal and non-personal on-demand and notice deposits, and (ii) other fixed-term liabilities maturing in less than twelve months. For liabilities in (i), we assume that the amount to roll-over in 2008Q4 is the same as the one reported at the end of 2007. For other liabilities in (ii), we assume that half of it matures in 2008Q4.[35]

We present descriptive statistics in Table B1. Under the assumptions outlined above, total funding of the Big six Canadian banks maturing at the interim date was significant, representing close to 47 per cent of total liabilities or 64 per cent of total banks' funding on average. Note that it remains quite large even if personal source of funding is excluded, in which case it represents more than 30 per cent of total liabilities or 42 per cent of total funding on average. Also, funding liabilities with a maturity less than three months that are assumed to roll-over at the interim date is the most important source of major Canadian banks' funding at

[34] We assume for simplicity that M, Y, S, L, and E stay constant over the simulation horizon. Since, as time goes by, some portion of S is retired while some portion of L becomes short term liabilities, the assumption of fixed S and L implies that the banks constantly manage short or long term debt to keep their mix constant.
[35] We collected all data at the end of 2007 and assumed that the ratio of maturing liabilities to total liabilities was constant over time.

around 43 per cent of banks' total funding on average.[36] With respect to the latter type of funding, note that banks rely on comparable mix of funding sources, repos being the only exception.

Table B1: Summary statistics on funding maturing at the interim date for the Big six Canadian banks (as of 2007Q4). Column 2 reports the amounts as percentage of total funding on average, where total funding represents the sum of both short and long-term liabilities. Columns 3 to 5 report minimum, average and maximum amounts of funding as percentage of total liabilities. Panel A gives the amount of funding with a maturity less than three months. Panels B and C provide the fixed-term funding maturing in less than twelve months and total funding, respectively, assumed to roll-over at the interim date.

	As percentage of total funding (Average)	As percentage of total liabilities		
		Minimum	Average	Maximum
Panel A: Funding liabilities with a maturity less than 3 months				
- Repurchase Agreements	8.0	1.8	5.9	8.5
- Bankers' Acceptance	3.7	2.0	2.7	3.6
- On demand and notice deposits				
- Personal	15.8	8.4	11.5	12.7
- Wholesale	15.3	9.6	11.1	14.4
- Total	42.8	24.0	31.2	37.3
Panel B: Fixed-term funding liabilities maturing in less than twelve months				
- Personal deposits	5.6	2.5	4.1	5.7
- Wholesale deposits	15.1	7.9	11.1	13.2
Panel C: Total funding assumed to roll-over at the interim period				
- Total	63.5	37.9	46.4	50.4
- Excluding personal	42.1	25.3	30.8	35.9

For illustrative purposes, we include personal deposits and repos as potential sources of run even though they may be considered as stable sources of funding. This could be seen as a worst case scenario.[37] We relax this assumption in the main text and simulate the model for different levels of stable short-term funding.

For each bank, the cost of short term funding, r_S, is calibrated as the average cost of these short term liabilities weighted by their respective sizes in 2008Q4. The calibrated values range

[36] This ratio is estimated assuming a 6-month roll-over period. Under a shorter roll-over horizon, the amount of short-term funding would be smaller.

[37] Nevertheless, we could argue that retail depositors might run because of concerns about the immediate availability of their funds in the advent of a bank default, even with government deposit insurance (up to $100,000 in the case of Canada). As for Canadian repo markets, which ceased to function at one point during the crisis except for overnight maturities, we could foresee a total breakdown in the absence of central bank liquidity support.

between 2.7 and 3.3 percent.[38] The opportunity cost r^* is calculated as the average 6-month Government of Canada Treasury Bill rates in 2008Q4. The calibrated value is 1.57%.

Finally, the fire-sale price of illiquid assets, ψ, is derived as:

$$\psi = \sum_i \psi_i * y_i \tag{19}$$

where ψ_i is the fire-sale of illiquid asset y_i. Three types of illiquid assets are considered: securities, loans, and others (including derivatives). Individual banks' security holdings come from 2008 banks' annual reports and fire-sale prices are obtained from Table 1 in CGFS (2010). Loans and derivatives are assumed totally illiquid (zero fire-sale price).[39] The calibrated ψ is 25 per cent for the average bank.

B.2 Results

We decompose the bank's credit risk into the three components described in section 2: the solvency risk due to deterioration of asset quality, the liquidity risk due to a run by short term creditors and the contagion risk due to the network effect among banks. Table B2 reports the results obtained from simulations for 10000 generated credit losses. It shows that the Canadian banking system was very stable at that time with total PDs below .01 percent for all banks. These results suggest that as of 2008Q2, the combination of high capital levels (more than 10% Tier 1 ratio on average), large liquidity holdings (more than 13% of total assets on average), together with the funding structures shown in Table B1 did not pose risk to the stability of the financial system.

Table B2. Bank defaults in the 2008Q2 calibrated case. Column 2 shows the individual probabilities of default due to solvency risk. Column **3** shows the PDs due to liquidity risk, and column 4 the PDs due to interbank contagion. Total PD is the sum of columns 2, 3 and 4.

Bank	Solvency PD ($\mathcal{N}_0$) (%)	Liquidity PD ($\mathcal{L}_0$) (%)	Contagious PD (%)	Total PD (%)
1	0	0	0.0010	0.0010
2	0.0023	0	0	0.0023
3	0	0	0.0023	0.0023
4	0	0	0.0023	0.0023
5	0	0	0.0018	0.0018
6	0.0010	0	0.0013	0.0023

[38] The cost of short-term deposits is calculated from the banks' financial statements. The BA rates are based on data from the trading room of the Bank of Canada, which are collected from market dealers. The Repo rates are calculated as the Canadian Overnight Repo Rate Average (CORRA) from the Bank of Canada database.
[39] Different assumptions on banks' liquid holdings are considered below.

Of interest, no liquidity risk was found even under the very severe scenario we consider[40]. Allen, Hortacsu and Kastl (2010) studies the Canadian banks' bidding behaviour at the liquidity auctions of the Bank of Canada during the financial crisis. They reach a similar finding that the participating Canadian banks do not seem to put persistently high value on liquidity before mid September 2008. These results suggest that the perceived risk of default due to a funding liquidity run was generally low for Canadian banks prior to Lehman's collapse in September 2008. Remember however from equation (7) that the probability of a run is a decreasing function of bank's capital and liquid asset holdings, and an increasing function of the amount of short-term funding.[41]

The case $\mu/\lambda > 1$ is not an interesting case to simulate because $\mathcal{L}_0$ = 100 percent. Table B3 reports the liquidity PDs of individual banks when their liquidity structures are such that $\mu/\lambda = 1$.[42] Results are shown for different levels of capital, which is set at the same level for all banks. The other parameters are kept at the calibrated values described above. Our results suggest that liquidity risk is very high for all banks when capital levels are at 6 percent and remains so for most banks even when capital is raised to 8 percent. Risks are muted when capital reaches 10 percent but are nevertheless significantly more important than in the calibrated case for banks 1, 2 and 6.[43] These results points to the importance of monitoring closely the mix of liquid holdings and short-term funding of banks.[44]

Table B3. Liquidity PDs of individual banks when their liquidity structures are such that $\mu/\lambda = 1$. Results are shown for different levels of capital. Capital ratios are set at the same level for all banks. The other parameters are kept at the calibrated values described above.

Bank	Liquidity PD (%) k=6%	Liquidity PD (%) k=8%	Liquidity PD (%) k=10%
1	82.12	44.95	0.7
2	81.52	50.93	1.89
3	79.22	1.49	0
4	90.95	17.84	0.1
5	74.54	1.49	0
6	90.10	35.36	1.00

40 Both the fire sale discount of illiquid assets, ψ_i, and the short-term funding cost, r_s and r^*, are exogenous parameters in the model. Brunnermeier and Pedersen (2009) shows that the market liquidity risk and funding liquidity could be positively correlated and hence mutually reinforce during crisis. Modeling the fire sale discount of illiquid asset and the short-term funding cost as functions of the likelihood of liquidity run is likely to increase the funding liquidity risk of banks and is left for future work.

41 We analyse the relative impact of these factors on the different types of risk for the Canadian banking system in the following section.

42 There are many combinations of S and M such that $\mu/\lambda \geq 1$ when M and S are allowed to vary in plausible ranges.

43 This result is obtained with identical capital ratio (10%) for all banks. It still holds when the simulation is done with the 2008Q2 capital ratios.

44 An interesting exercise would be to assess systemic risk in the Canadian banking system over history. This is left for future work.

APPENDIX C. THE MECHANICS OF THE NETWORK CLEARING ALGORITHM

To illustrate the key concepts of the network model, consider a simple banking system composed of three hypothetical banks in which the structure of interbank claims and liabilities is given by the following matrix:

$$L = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

The columns of L refer to the claims each bank has on each of the remaining two banks. For example, bank 3 holds a claim of 1 with bank 1 and a claim of 1 with bank 2. The rows of L represent the liabilities of each bank vis-à-vis the other banks in the system. In this example, bank 1 has liabilities of 1 against bank 2 and against bank 3. All banks incur no liabilities against themselves (which explains the zeros in the diagonal). Total interbank liabilities of banks toward the rest of the system are therefore given by the vector $d = (2,1,1)$, where the three components correspond to total liabilities of bank 1, 2 and 3 respectively.

Assume further that the outside net wealth of banks 1, 2 and 3, which derives from their non-interbank operations, can be represented by the vector e$= (\frac{1}{2}, 0,0)$. In this example, we can show that all three banks cannot meet their interbank liabilities simultaneously. To understand this, let us first normalize the individual entries of L by total liabilities to get the relative liability matrix:

$$\pi = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

Note that $\pi' d$ provides each bank's total interbank assets. To determine the clearing vector associated with the above structure on the interbank market, let us first assume that all banks fulfill all their interbank liabilities. Under this assumption, the net value of a given bank can be derived as the sum of its full interbank income (i.e. total payments received from other banks) plus its outside net wealth, minus its total promised interbank payment to other banks:

$$\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}' * \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} \frac{1}{2} \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ 0 \\ 1 \end{pmatrix}$$

We arrive at a negative net value for bank 1 which is therefore insolvent. We refer to this as a *fundamental default*.

Lets assume that other banks fulfill their obligations towards bank 1 which implies that bank 1's net value before any interbank payment is $\frac{3}{2}$). This amount, given the assumed proportional sharing rule, is allocated to banks 2 and 3 on a pro rata basisproviding the following banks' net wealth:

$$\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}' * \begin{pmatrix} \frac{3}{2} \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} \frac{1}{2} \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} \frac{3}{2} \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{1}{4} \\ \frac{3}{4} \end{pmatrix}$$

The resulting net value of bank 2 is negative. The insolvency of bank 1 reduces the interbank claim of bank 2 to such an extent that it fails to keep its interbank promises (2 is only able to pay $\frac{3}{4}$ rather than 1), and becomes insolvent as well. This is what we call in the paper a *contagious default*. Applying again the rule of proportional sharing, we can show that bank 3 remains solvent:

$$\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}' * \begin{pmatrix} \frac{3}{2} \\ \frac{3}{4} \\ 1 \end{pmatrix} + \begin{pmatrix} \frac{1}{2} \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} \frac{3}{2} \\ \frac{3}{4} \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \frac{2}{4} \end{pmatrix}$$

The outcome of the network model is a clearing vector which makes all interbank claims consistent. In our example, this vector is $p^* = \left(\frac{3}{2}, \frac{3}{4}, 1\right)$. Eisenberg and Noe (2001) prove the existence and uniqueness of such clearing payment vector in a more general context.