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New Evidence from a Panel of U.S. Dollar Bilateral Exchange Rates

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by

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Abstract

We use a novel approach to identify economic developments that drive exchange rates in the long run. Using a panel of six quarterly U.S. bilateral real exchange rates – Australia, Canada, the euro, Japan, New Zealand and the United Kingdom – over the 1980-2007 period, a dynamic factor model points to two common factors. The first factor is driven by U.S. shocks, and cointegration analysis points to a long-run statistical relationship with the U.S. debt-to-GDP ratio, relative to all other countries in our sample. The second common factor is driven by commodity prices. Incorporating these relationships directly into a state-space model, we find highly significant coefficients. Then, we decompose the historical variation of each exchange rate into U.S. shocks, commodities, and a domestic component. We find a strong role for economic fundamentals: Changes in the two common factors, which are driven by the (relative) U.S. debt-to-GDP ratio and commodity prices, can explain between 36 and 96 per cent of individual countries' exchange rates in our panel.

JEL classification: J31
Bank classification: Exchange rates; Econometric and statistical methods

Résumé

Les auteurs empruntent une approche inédite pour identifier les déterminants économiques de l'évolution des taux de change à long terme. À partir d'un modèle à facteurs dynamiques qu'ils estiment pour la période 1980-2007 au moyen de données de panel trimestrielles concernant le taux de change bilatéral réel du dollar É.-U. par rapport à six autres monnaies (dollars australien, canadien et néo-zélandais, euro, livre sterling et yen), ils dégagent deux facteurs communs. Le premier est influencé par les chocs qui surviennent aux États-Unis. Une analyse de cointégration révèle en effet une relation statistique de long terme entre ce facteur et le ratio de la dette américaine au PIB mesuré comparativement à celui observé dans chacun des autres pays considérés. Le second facteur commun est influencé par l'évolution des prix des produits de base. Si ces relations sont directement intégrées dans un modèle espace d'états, les coefficients obtenus sont très significatifs. Les variations passées de chaque taux de change sont ensuite décomposées en trois éléments : chocs enregistrés aux États-Unis, évolution des prix des produits de base et une composante intérieure. Les variables économiques fondamentales tiennent un rôle explicatif important : les modifications des deux facteurs communs associées au ratio (relatif) de la dette américaine au PIB et à l'évolution des prix des produits de base peuvent rendre compte de 36 à 96 % des fluctuations des six taux de change examinés.

Classification JEL : J31
Classification de la Banque : Taux de change; Méthodes économétriques et statistiques

1 Introduction

International economists and policymakers have struggled to explain the connection between macroeconomic fundamentals and exchange rates. It has become widely accepted that empirical models have rather limited success in explaining exchange rate movements (Meese and Rogoff 1983; Obstfeld and Rogoff 2000; Cheung et al. 2005). Models exploiting the relationship between commodity prices and exchange rates have performed somewhat better. Empirical studies found the link between fluctuations in commodity prices and the real exchange rates to be relatively stable (Gruen and Kortian 1996, Issa et al. 2008, Djoudad et al. 2001, Bayoumi et al. 2007).[1] However, not all currency fluctuations can be explained by movements in commodity prices. Consider the period 2000-2007, during which the rise in commodity prices can likely help explain the rapid appreciation of currencies of commodity-exporters against the U.S. dollar. However, over the same time, currencies of commodity-importing countries or currency areas, like the euro area and the United Kingdom, have also appreciated against the U.S. dollar. If currencies of commodity exporters appreciate, because their economies benefit from rising commodity prices, then currencies of commodity importers should depreciate in response to rising commodity prices, as they face a negative terms-of-trade shock. This suggests that models explaining exchange rates *only* with commodity prices may be missing important other drivers.

In this paper we use a novel empirical approach to identify macroeconomic developments behind long-term exchange rate movements. We combine different methodologies found in the literature: we use a panel of exchange rates and start with the latent factor approach used in Diebold and Nerlove (1989) and Mahieu and Schotman (1994).[2] This purely statistical technique helps uncover patterns or 'factors' in the data (for instance, Dungey, 1999, decomposes movements in six Pacific Rim currencies into a domestic component, a foreign currency component and a world component). However, a shortcoming is that these factors need not have a meaningful economic interpretation. We alleviate this shortcoming by employing regression techniques to link the factors to observable macroeconomic developments. Hence, we can establish statistical links between macroeconomic fundamentals and exchange rates.

Our analysis proceeds in three steps. In our first step, we estimate a dynamic, orthogonal factor model that identifies two common factors in a panel of U.S. dollar bilateral exchange rates. The first factor moves all six bilateral U.S. dollar exchange rates in the

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[1] Amano and van Norden (1998) also find a robust empirical relationship between the real oil price and the real exchange rates of Germany, Japan and the United States.

[2] Carriero et al. (2009) also use a panel of exchange rates, but these authors focus on forecasting exchange rates, while we aim at explaining past behaviour.

same direction, suggesting that it is driven by U.S. shocks (and not shocks to individual countries). The second factor is positively related to the exchange rates of Australia, Canada, New Zealand and negatively related for Europe, Japan and the United Kingdom. This points to a link with commodity prices. Our second step is to explain the macroeconomic drivers behind the factors. Based on economic theory, we establish an empirical, long-run relationship between the first common factor ('U.S. shocks') and the ratio of U.S. debt-to-GDP, relative to all other countries in our panel. The second common factor is cointegrated with world commodity prices. Having determined the economic interpretation of our factors, our third step is to develop a state-space model of real exchange rates, based on these long-run determinants, and estimate all relationships jointly. We conclude that shocks to the U.S. debt position, relative to all other countries, explain a substantial share of low frequency movements in our panel of U.S. dollar exchange rates. Commodity prices also matter, especially for Australia and Canada. Domestic factors – essentially the residual in our model – explain a relatively high portion of the variability in the Japanese yen, the British pound and the New Zealand dollar. Overall, we can explain between (roughly) 35 and 95 per cent of quarterly exchange rate movements for each of the countries in our sample.

In the next section, we estimate a dynamic, orthogonal factor model, and identify two common factors in our panel of exchange rates. In section 3, we link the two factors to macroeconomic developments (relative debt positions and commodity movements), and estimate all empirical relationships between exchange rates, relative debt stock and commodity prices jointly in a state-space model. In section 4 we provide historical decompositions of observed exchange rate movements for these countries. The final section summarizes our key insights.

2 Extracting Common Movements

We use a panel of six real U.S. dollar bilateral exchange rates, covering Australia, Canada, the euro area, Japan, the United Kingdom, and New Zealand. We use quarterly data over the period 1981Q1 to 2007Q4. Almost over the entire period, these currencies floated freely against the U.S. dollar.[3] Figure 1 graphs the real exchange rates of all countries in our sample.

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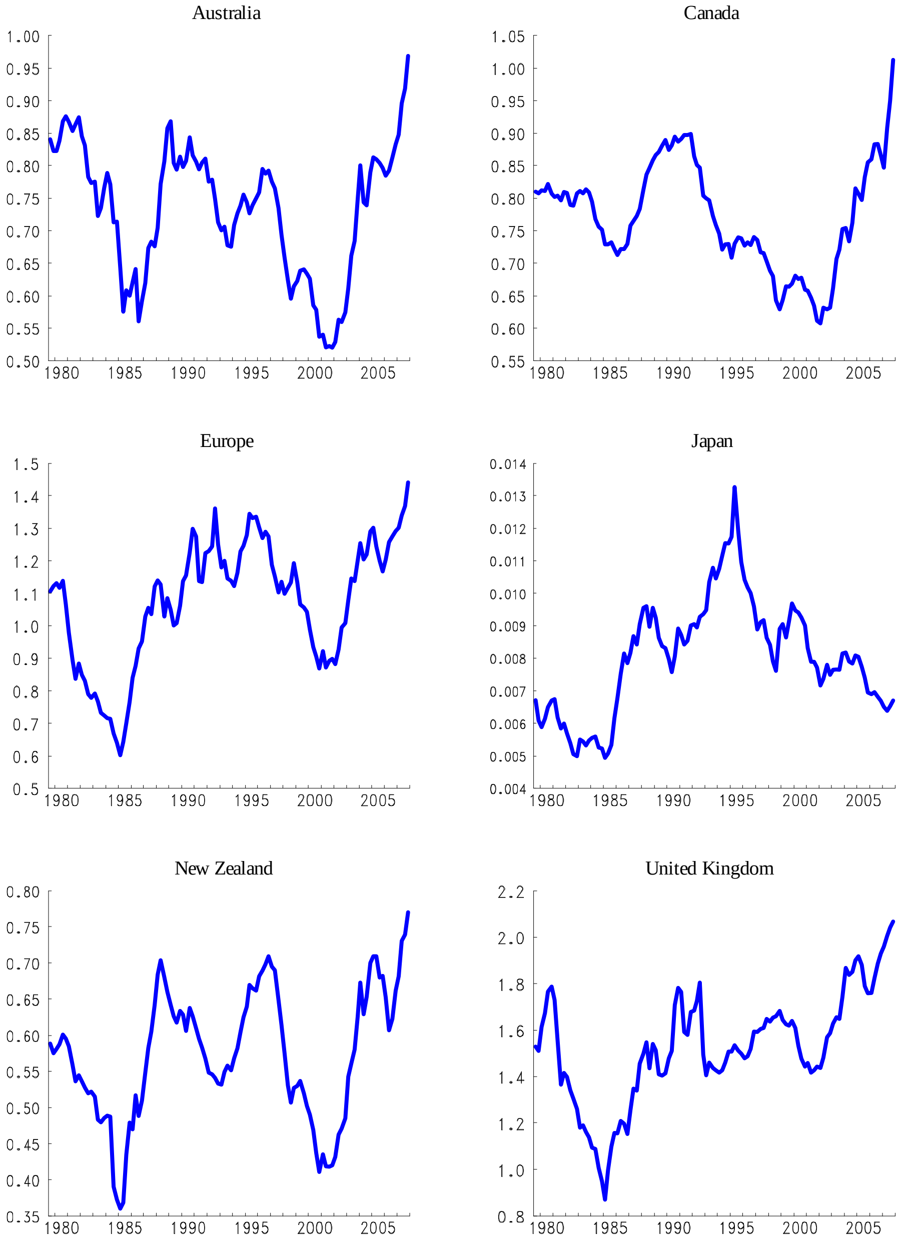
[3] Availability of the quarterly IMF non-energy commodity prices index limits our sample. Prior to the euro's introduction in 1999, we construct a 'synthetic' euro, based on the ECU-U.S. dollar exchange rate. The U.K. pound was briefly pegged within the European Monetary System between 1990 and 1992, but we do not believe that this short period poses a severe constraint for the real U.K bilateral exchange rate against the U.S. dollar. See appendix A for a detailed description of the data.

Figure 1: Real U.S. Dollar Bilateral Exchange Rates



Our empirical strategy is to identify key developments behind exchange rate movements, based on the notion that the covariance (or correlation) structure of the data contains important information to explain currency movements. Multivariate data can exhibit patterns, suggesting the existence of a common structure in the data. Previous studies found that in many cases, a small number of factors account for the bulk of the observed variation of major economic aggregates (Sargent and Sims, 1977; Stock and Watson, 1989; Sargent, 1989). There are essentially two ways of estimating these patterns. First, factor analysis is a simplifying method to identify patterns that can account for most of the variations in the covariance or correlation matrix of the data (Tsay, 2002). Factor analysis is purely statistical and relies on a minimum set of restrictions and assumptions (e.g. Geweke 1977; Sargent and Sims 1977; Forni and Lippi 2001; Bai and Ng 2008). Underlying this technique is the premise that unobservable characteristics – commonly referred to as 'latent factors' – account for the variation and covariation across exchange rates. Factor analysis attempts to reduce the dimensionality of our data set by seeking underlying, unobservable variables, which are reflected in observed variables, by analyzing the correlation matrix.

An alternative approach is state-space modelling, which can also be used to extract common movements among a set of aggregate time-series. State-space models are a generalization of the linear regression model, and have, for instance, been used by Stock and Watson (1991) to build an index of indicators to provide information about the overall state of the economy.[4] Each approach has its advantages and disadvantages: The dynamic factor model is a purely statistical technique, which has the advantage of relying on a minimum of restrictions and assumptions. Results are driven by data, and are not very likely to be subject to misspecification. The state-space model is more flexible, as restrictions can be applied to impose additional structure on the data. This helps explain the driving forces behind the model, and facilitates statistical inference. However, state-space models can be less robust, because they could be sensitive to assumptions regarding the stochastic processes of the unobservable common components.

Given that correct identification of the factors is key to our approach, we estimate both types of models. As shown below, regardless of the approach taken, the results are virtually identical. Given the differences in methodologies, this finding provides confidence that the basic insights in terms of factors are very robust, and that the risk of misspecification is very small.

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[4] Fernandez Macho et al. (1987) discuss state-space modelling to extract common trends among cointegrated variables.

2.1 A dynamic factor model

The orthogonal factor model stipulates that p random variables X ($X = (x_1, ...x_p)'$) can be expressed as linear functions of m ($m < p$) hypothetical common factors F ($F = (f_1, ...f_m)'$), plus an error term. We use a panel of p real U.S. dollar bilateral exchange rates X_t with mean μ, each normalized by its standard deviation (to ensure that each bilateral exchange rate contributes equally to the total variance of the model). We also allow for the possibility of co-movements across time by employing a dynamic model, which stipulates that the factors can be expressed as autoregressive processes. The specification of the dynamic factor model is as follows:

$$\begin{aligned} X_t - \mu &= LF_t + \varepsilon_t && (1) \\ f_t &= l_1 f_{t-1} + l_2 f_{t-2} + ... l_p f_{t-p} + \nu_t && (2) \\ \varepsilon_t &= \alpha_1 \varepsilon_{t-1} + \alpha_2 \varepsilon_{t-2} + ... \alpha_q \varepsilon_{t-q} + \xi_t && (3) \end{aligned}$$

where X_t are the normalized exchange rates, $L = [l_{ij}]_{p \times m}$ is the matrix of factor loadings and l_{ij} is the loading of the ith variable on the jth factor. ε_i is the specific error of X_i, and ν_t and ξ_t are white noise processes.

The orthogonal factor model aims to find a linear combination of the factors f_t by maximizing the variance of $f_{i,t}$, and identifying them by imposing that all factors are orthogonal (technically, $f_{i,t}$ and $f_{j,t}$ are uncorrelated for $i \neq j$, as shown in Tsay, 2002). The number of factors retained should be large enough to account for the bulk of common variation in the sample, but small enough to discard factors that basically represent idiosyncratic movements in the data. A common criterion for the number of factors to retain is the Kaiser-Guttman criterion, which says that only factors with an eigenvalue larger than 1 should be retained.

A preliminary analysis of the data suggests that most real exchange rates in our panel are integrated of order one (see appendix B). We find a statistically significant, but very small, first-order autoregressive term of about 0.3 for all of the real exchange rate series in first differences. Consequently, we estimate our dynamic factor model in first differences.

The factor models suggests retaining two factors, as the eigenvalues for the first and second factor are 3.6 and 1, respectively (the eigenvalue for the third identified factor is 0.3, and thus clearly below the criterion of an eigenvalue larger than 1). The first two dynamic factors are plotted in Figure 2. An interesting pattern emerges with regard to the loading factors: Table 1 shows the loading factors for the contemporaneous factors,

Figure 2: The two factors from the dynamic factor model (cumulated from first differences)

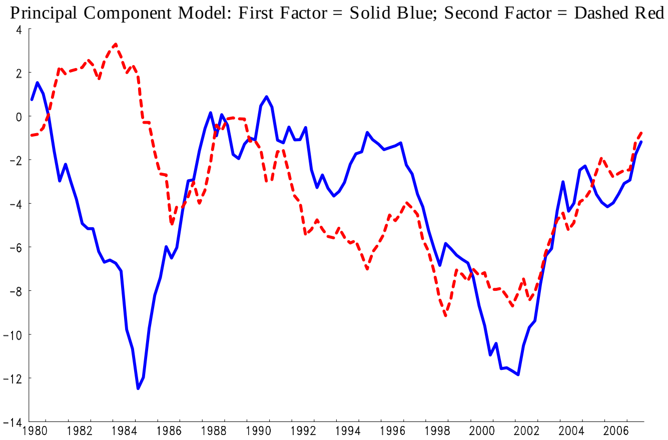

Table 1: Loading factors of the contemporaneous factors

Country	Factor 1	Factor 2
Australia	0.14	0.29
Canada	0.07	0.21
Euro area	0.18	-0.29
Japan	0.09	-0.17
New Zealand	0.17	0.04
United Kingdom	0.14	-0.10

and as can be seen, the loading factors on the first common factor are positive for all countries. This implies that all currencies move in the same direction in response to movements in the first common factor. The sign of the loading factor for the second common component is positive for the Australian, Canadian and New Zealand dollar, but negative for the euro, the yen and the U.K. pound. Given that the first three countries are commodity exporters and the second set of countries are commodity importers, commodity prices might be driving the second factor.

2.2 A simple state-space model to identify common movements in exchange rates

In this section, we check whether we can reproduce the identification of two common factors in our panel of exchange rates with principal factor analysis with a state-space model (Stock and Watson 1991). The state-space representation is given by the following equations:

$$
\begin{aligned}
X_t^i - \mu^i &= \gamma^i C_t + v_t^i && (4)\\
\phi(L) C_t &= \eta_t && (5)\\
D(L) v_t &= \varepsilon_t && (6)
\end{aligned}
$$

where X_t^i is a n x 1 vector representing our panel of i real U.S. dollar bilateral exchange rates, C_t is a set of m common factors, and v_t^i is a n-dimensional component that represents idiosyncratic movements in the series. The common components C_t enter each n equations, but with different weights γ^i.[5]

To make the vector of C_t components the unique source of comovements across the exchange rate series, we assume that the v_t and the C_t are mutually uncorrelated for all leads and lags. This is possible if we assume that the matrix $D(L)$ is diagonal and that the errors terms η_t and ε_t are uncorrelated.[6] We estimate the state-space model by a combination of a Kalman filter and maximum likelihood. As figures 3 and 4 show, the factors identified with the state-space model are practically identical to the factors from the dynamic factor model. Also, we find that the loading factors exhibit the same pattern: The first factor is positively related to all exchange rates, representing a common movement of all currencies against the U.S. dollar; the sign of the second component is not identical for all countries, indicating that the second factor captures developments that do not affect all countries symmetrically (more specifically, the sign of the loading factor of second factor is positive for commodity exporters and

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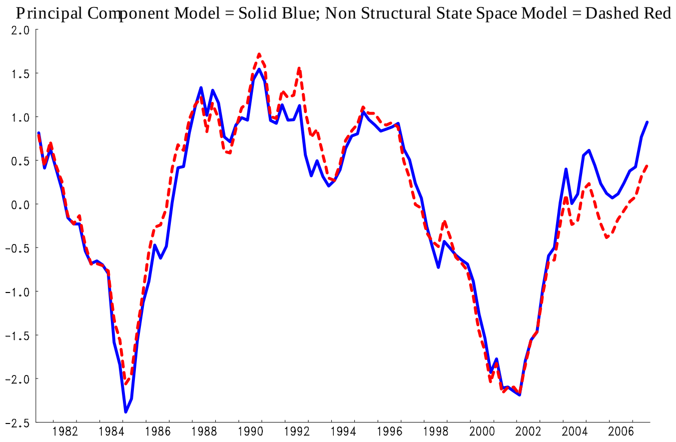
[5] As with the principal factor model, a key issue is the stationarity of the data. There are two possibilities: first, we can assume that the v_t and the C_t contain (some) unit roots. Alternatively, we can assume that these stochastic trends enter through C_t. This is equivalent to saying that the members of X_t are cointegrated, and in this case, the idiosyncratic shocks are stationary by construction. Given that we do not find cointegration for all countries, we allow for the possibility that the idiosyncratic components are non-stationary.

[6] To identify C_t, restrictions on the variance-covariance matrix of η_t (Σ_η) and on γ are required. Fernandez Macho et al. (1987) impose three restrictions: they set Σ_η equal to a diagonal matrix, restrict $\gamma_{ij} = 0$ for $j > i$, and set $\gamma_{ii} = 1$ for $i = 1, ...m$. We set Σ_η as a diagonal matrix, but rather than restricting γ_{ii} to be equal to 1 for $i = 1, 2$, we set the two diagonal elements of Σ_η equal to 1 ($\sigma_{\eta 1} = \sigma_{\eta 2} = 1$). Instead of the restrictions for γ_{ij}, we differentiate the two common factors by the following ARIMA processes: First, for one common factor, we assume an ARIMA(1,1,0) process to capture the autocorrelation found in the first difference of the real exchange rate series; second, for the other factor, we assume an ARIMA(0,1,0) process.

Figure 3: First factor from the simple state-space model and the dynamic factor model



negative for commodity-importing countries). Overall, the factors are very robust to the methodology chosen to extract them.

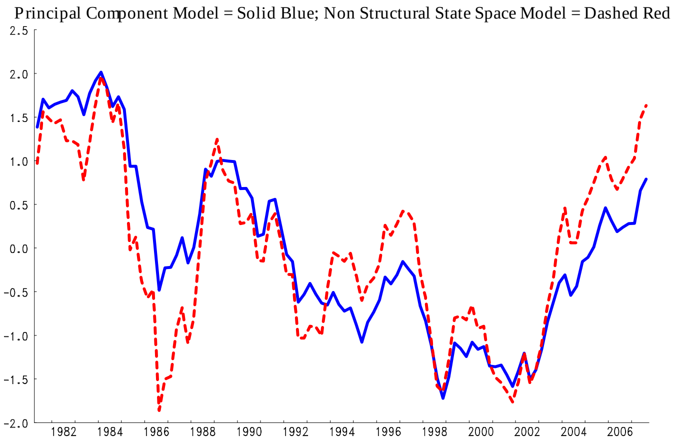
3 What Is Driving the Common Factors?

While latent factor models are useful to detect patterns in the data, they lack economic interpretation. Using the theoretical literature as a guide to motivate candidate explanations, we provide evidence that suggests a long-run empirical relationship between real exchange rates, commodity prices and shock driven by the evolution of the ratio of U.S. debt-to-GDP, relative to the other countries in our sample. We establish this link using two different empirical approaches: i) cointegration analysis of the latent factors identified by the dynamic factor model, and ii) an estimated state-space model.

3.1 What does theory have to say?

The first factor identifies a common movement in the direction of all bilateral U.S. dollar exchange rates, i.e. the U.S. dollar appreciates or depreciates against all currencies simultaneously. Since our panel comprises bilateral exchange rates against the U.S. dollar, the first factor is not driven by idiosyncratic shocks in any of the individual countries in our panel – except for the United States. Hence, we tentatively label the

Figure 4: Second factor from the simple state-space model and the dynamic factor model



symmetric exchange rate factor the 'U.S. common factor', as it is likely driven by U.S. shocks.

The literature suggests (at least) two possible shocks that are likely to affect a panel of bilateral exchange rates in the same direction: productivity differentials or differences in fiscal policy. Considerable emphasis has been placed on the role of productivity shocks, going back to the early work of Balassa (1964) and Samuelson (1964). Their work emphasizes a positive correlation between productivity in the traded goods sector and a country's real exchange rate.[7] The second possible interpretation for the first factor is that it is related to fiscal policy. A rise in a country's government debt-to-GDP ratio should lead to a worsening of its current account and an depreciation of the real exchange rate over the medium to long-run (Ganelli 2005; Kumhoff and Laxton 2007). In overlapping generations models in the spirit of Blanchard (1985) or Weil (1989), agents with finite economic lifetimes discount future tax liabilities at a higher rate than the market real interest rate. As a result, consumers do not increase their saving to sufficiently account for the additional future tax burden. Instead, their investment in government debt crowds out investment in physical capital and foreign

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[7] As an example, if productivity in the U.S. tradables sector, relative to the U.S. non-tradables sector, would rise faster than in other countries, the Balassa-Samuelson hypothesis predicts an appreciation of the U.S. dollar against these currencies (see Corsetti et al., 2007, for a recent empirical study of the impact of productivity shocks on the U.S. dollar).

assets, and the reduction in savings leads to a fall in the long-run net foreign asset-to-GDP ratio. Financing the increased level of foreign indebtedness is facilitated through a depreciation of the currency, which boosts net exports.

For the second factor, the signs of the loading factors suggest a relationship with commodity prices. Commodity prices are typically the most volatile component of the terms of trade. Amano and van Norden (1995) argue that major movements in the price of oil are thus reasonable proxies for the exogenous changes in the terms of trade even for large economies like the United States, Germany and Japan. Similarly, Chen and Rogoff (2003) suggest that commodity prices capture exogenous terms-of-trade shocks, which can play an important role in the determination of real exchange rates (particularly for small economies with a high share of commodity exports).

3.2 Linking factors to macroeconomic variables: A preliminary empirical investigation

We take these candidate explanations from the theoretical literature as the starting point of our empirical investigation of economic developments that could be driving the common factors. To test for the influence of productivity differentials or differences in fiscal policy, we need measures for U.S. shocks. They should be based on the evolution of the ratio of productivity in the manufacturing sector, relative to the economy as a whole, in the United States, and the ratio of the stock of U.S. government debt, relative to GDP.

We proceed as follows: For all countries, we compute the evolution of their stock of debt, relative to GDP, divided by the U.S. debt-to-GDP ratio (and similarly the evolution of productivity differentials, relative to U.S. productivity differentials). We plot those two time series in Figure 5.[8] Next, given our interest in U.S. shocks, we extract common factors from these series to obtain how the United States debt-to-GDP ratio or productivity differential has evolved, *relative to all other countries in our panel*.[9] In both cases, the loading factor of the first factor for all countries has a positive sign, suggesting that we identify U.S. shocks.

Figure 6 plots the first factor and the factors extracted from debt-to-GDP ratio and productivity differentials. As can be seen, both candidate explanations seem to be positively correlated with the first factor over the latter part of our sample. However, over

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[8] Unfortunately, data on productivity differentials for New Zealand were not available, so we estimated the productivity factor excluding New Zealand.

[9] Note that we use the stock of government debt in the debt-to-GDP ratio, not budget deficits. Our variable is slow-moving, which is in line with our attempt to explain long-term currency movements. A preliminary statistical analysis of the data suggests that the government debt and productivity variables are I(1), and we find statistically significant AR(1) terms in both variables. As a result, we estimate dynamic factor models in first differences for both the debt and productivity differentials.

Figure 5: Debt-to-GDP ratios and productivity, relative to the United States

Note: Debt - solid blue line; productivity - dotted red line; no comparable productivity data available for New Zealand. All data have been indexed to 1980Q1.

the first part of our sample – roughly until the early 1990s – relative productivity differentials seem to be negatively correlated with the first factors. The factor extracted from the U.S. relative debt position, however, shows a positive relationship with the first factor from our exchange rate panel over the full sample. To check formally whether the first factor in our exchange rate panel – the U.S. factor – is related to relative differences in the stock of debt, we employ cointegration tests. As Table 2 confirms, the U.S. factor has a long-run empirical relationship with the first factor found in the evolution of the debt-to-GDP ratios, relative to the United States. The negative sign of the coefficient suggests that a deterioration in the U.S. fiscal position, relative to all of the other countries in our sample, leads to a long-run multilateral U.S. dollar depreciation. This relationship is in line with the theoretical link suggested by overlapping generations models.[10] Also, given that fiscal deficits and current account deficits often occur simultaneously (this has certainly been true in the past for the United States), the U.S. factor in the model could represent multilateral adjustment to the U.S. current account deficit.[11] In contrast, we do not find an empirical link between relative productivity differentials and the U.S. factor.[12] Figure 7 plots the U.S. dollar common factor, and the cointegration relationship based on the U.S. debt common factor.

The loading factors for the second common exchange rate factor suggest a link with commodity prices, as commodity-exporting currencies can be expected to appreciate in response to rising commodity prices, while countries that are net importers of commodities are likely to experience a depreciation of their currency (reflecting a negative terms-of-trade shock). Figure 8 shows a scatter plot of each country's net commodity imports, relative to the United States, and the loading factors estimated in section 2.1.[13] The scatter plot points to a relationship between the commodity net export position of each economy, relative to the United States, and the sign of the loading factors.

Cointegration tests confirm the statistical relationship between the second component and commodity prices. As energy and non-energy commodity prices can behave

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[10] A potential caveat could be that fiscal policy – notably budget deficits – contemporaneously affect the exchange rate and vice versa. We avoid this endogeneity issue by considering the relative stock of debt-to-GDP: while the causality between the current budget deficit and the exchange rate might be ambiguous, the relative *stock* of debt-to-GDP is a slow-moving variable, and not likely to be contemporaneously affected by the exchange rate.

[11] See Bailliu et al. (2007) for an approach to model multilateral adjustment empirically.

[12] We considered the relative difference in the evolution of tradable vs. nontradable productivity, as well as the difference in the evolution of tradable productivity, relative to total productivity. In both cases, we failed to establish an empirical link between productivity and the U.S. factor (results available upon request). Ricci et al. (2008) use a very detailed sectoral breakdown, yet still only find a very small relationship between productivity differentials and real exchange rates for a panel of 48 industrial countries.

[13] Net commodity imports, relative to the United States, are defined as the nominal commodity net export position of each country, expressed as a share of nominal GDP, divided by U.S. net exports of commodities, expressed as a share of U.S. GDP.

Figure 6: First factors extracted from U.S. debt-to-GDP ratios and productivity differentials, relative to all other countries (for expositional clarity, we inverted the debt factor; all series cumulated from first differences)

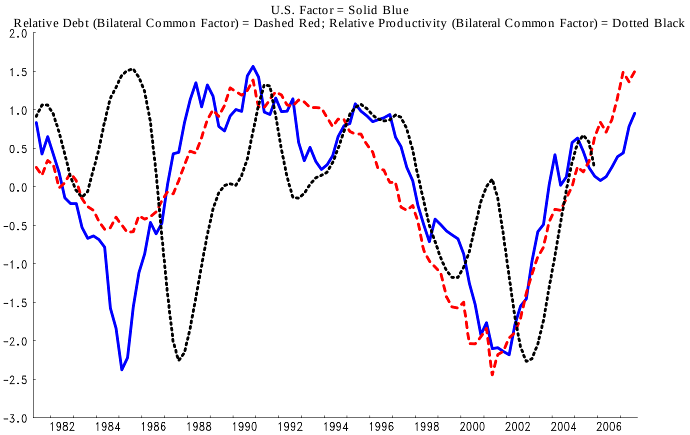


Figure 7: U.S. factor cointegrated with U.S. debt-to-GDP ratio, relative to all other countries (cumulated from first differences)

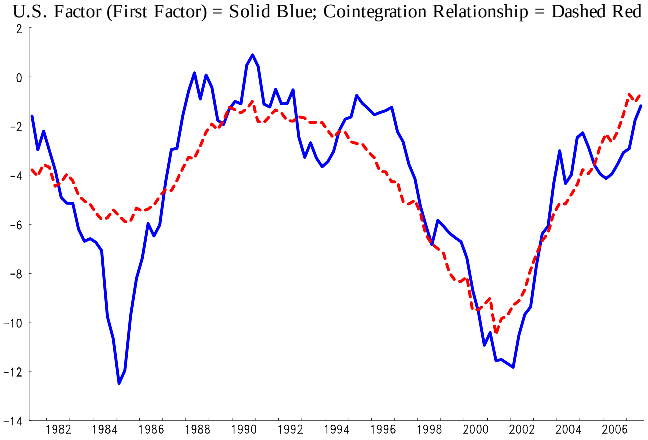


Figure 8: Negative relationship between net commodity imports (relative to the United States) and the loading factor of the second factor

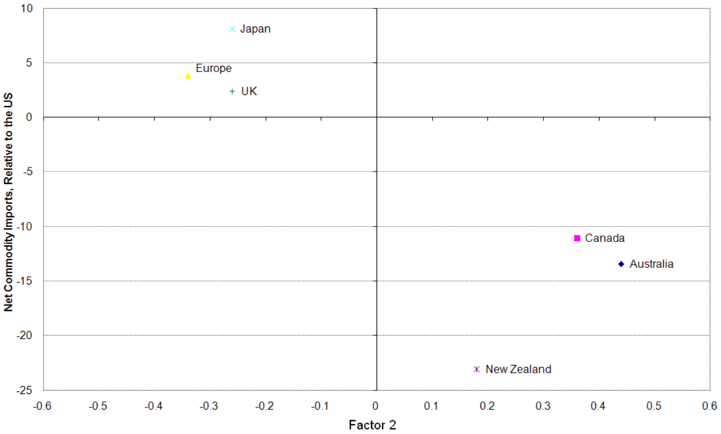


Table 2: Saikonnen cointegration tests (null hypothesis: no cointegration)

		US factor	Commodity factor
Test Statistic (Saikkonen)		-3.56	-4.62
Test Statistic (Engle-Granger)		-3.36	-4.29
Critical values:	1 per cent	-4.00	-4.44
	5 per cent	-3.40	-3.83
	10 per cent	-3.09	-3.52

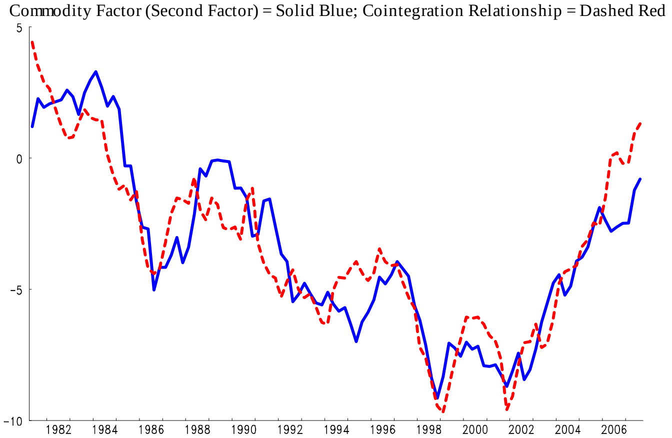
quite differently, we test for a relationship between the second factor and the real price of oil, as well as the real non-energy commodity price index. As can be seen in Table 2, we find evidence for cointegration, and Figure 9 shows the cointegrating relationship for the second factor.

3.3 The short-term dynamics of exchange rates

So far, we have established statistical links between our factors and differences in the stock of debt and to commodities. These relationships comprise the core of our empirical investigation. Note, however, that we would expect the economic relationships represented by the factors – fiscal policy and commodities – to hold over the medium- to long-term, not to explain short-term movements. To improve the model's properties in the shorter terms, we add one last element: differences in interest rates.

Interest rate differentials are used in many theoretical and empirical models as key drivers for exchange rates, as uncovered interest rate parity conditions suggest that

Figure 9: Commodity factor cointegrated with commodity prices



arbitrage occurs if interest rate differentials widen.[14] In terms of our panel of bilateral exchange rates against the U.S. dollar, we are primarily concerned with explaining common movements of the entire panel. The panel as a whole is likely driven by changes in U.S. interest rates, relative to all other countries, or more specifically, by the first common factor in a set of short-term interest rate differentials against the United States. Hence, we identify the U.S. interest rate common factor by estimating a common factor model (in levels) on a panel of short-term real interest rate differentials, and taking the factor that triggers symmetric responses in each of the bilateral exchange rates. In what follows, we add this element to the empirical relationship driving our first factor (the factor representing U.S. shocks).

3.4 A state-space model with commodity prices, relative debt/GDP ratios, and interest rates

Having linked the factor to economic developments and having extracted a common factor from short-term real interest rate differentials, we can proceed to estimate all relevant economic relationships jointly in an extended state-space model. We estimate

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[14] In a recent paper, Murchison (2009) argues that reduced-form estimating of the importance of interest rate differentials might be biased. Estimated using simulated data from a DSGE model – in which interest rate differentials play an important role in driving the exchange rate – it is shown that, by not accounting for the source of the interest rate movement, reduced-form equations tend to find coefficients on interest rate differentials close to zero.

the following model:

$$\begin{aligned} X_t^i - \mu^i &= \gamma_1^i C_t^1 + (\gamma_2^i + D^{93}\gamma^{93}) C_t^2 + \nu_t^i & (7) \\ \Delta C_t^1 &= \phi_1 \Delta C_{t-1}^1 + \beta_1 F_{t-1}^{rdiff} - \lambda_1 (C_{t-1}^1 - \beta_2 F_{t-1}^{Debt}) + \eta_t^1 & (8) \\ \Delta C_t^2 &= \phi_2 \Delta C_{t-1}^2 - \lambda_2 (C_{t-1}^2 - \beta_3 p_{t-1}^{NE} - \beta_4 p_{t-1}^{oil}) + \eta_t^2 & (9) \\ D(L)\nu_t^i &= \varepsilon_t^i \text{ for all } i & (10) \end{aligned}$$

where X_t is a 6 x 1 vector representing our panel of real U.S. dollar bilateral exchange rates. Equation (7) expresses the bilateral exchange rates as a function of the two common factors C_t. The factors have different weights γ^i for each country i, such that each exchange rate will be affected by shocks to each of the factors differently. Note that the equation for Canada also contains the variable D^{93}. This is a 0/1 dummy, which captures a break in the relationship between the Canadian dollar and energy commodities due to liberalization of the energy market in the early 1990s.[15] Hence, the impact of commodities on the Canadian exchange rate after 1993 is given by $(\gamma_2^{CA} + D^{93}\gamma^{93})$. The next two equations specify the cointegration relationships we identified in section 3.2: equation (8) expresses the U.S. factor as a long-run relation with the common factor found in the U.S. debt-to-GDP ratio (F^{Debt}), relative to all other countries in our sample, while the short-term dynamics are driven by the common factor extracted from short-term interest rate differentials F^{rdiff}. The second cointegrating relationship is given by equation (9), which expresses the commodity factor as a function of p^{oil} and p^{NE}, that is, the real prices of energy- and non-energy commodities. The n-dimensional component ν_t represents idiosyncratic movements in the series.

We allow for the possibility that the ν_t and the C_t contain (some) unit roots. As before, we assume that the ν_t and the C_t are uncorrelated at all leads and lags. Given the rich empirical specification, we no longer need to impose any additional restrictions on γ_j^i. As before, we estimate the state-space model by a combination of a Kalman filter and maximum likelihood.

The estimation results are given in Table 3. The results confirm the findings obtained before: First, the loading factors have the same signs as those from the dynamic

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[15] The free trade agreement between Canada and the United States, as well as the liberalization of Canadian energy policy in the 1990s – including deregulation of the North American natural gas market – affected the relationship between the Canadian dollar and energy commodity prices. While rising energy commodity prices used to cause a depreciation of the Canadian dollar, the effect has changed in 1993. Since 1993, rising energy commodity prices have resulted in an appreciating Canadian currency (see Issa et al. 2006 for details).

factor model, and are statistically significant. Also note that while all countries in our sample are affected by the two factors, the impact of the factors on individual exchange rates is not equal. The loading factor for commodities for New Zealand, for instance, is 0.25, which is considerably smaller than the loading factor for commodities on Australia of 0.57. Hence, the model suggests that both the New Zealand and the Australian dollar appreciate when commodity prices rise, but the effect on the Australian dollar is much stronger. Second, as regards the first factor, the coefficient on the U.S. debt common factor $\beta_2 = -0.65$ is highly significant. The parameter for the speed of adjustment of the cointegration relationship between the first factor and the fiscal position variable is relatively high ($\lambda_1 = 0.16$, $t_{\lambda_1} = 3.93$).[16] Thus, the empirical link uncovered earlier with cointegration analysis for the U.S. factor continues to hold. Third, as expected, interest rate differentials play a role in explaining the short-run dynamics of the model ($\beta_1 = 0.25$, $t_{\beta_1} = 2.51$). Fourth, the second common component is linked to both energy- and non-energy commodities, as both variables are highly significant with very similar coefficient estimates ($\beta_3 = 0.65$; $\beta_4 = 0.51$). Also, the speed of adjustment parameter between the second common factor and commodity prices is fairly high ($\lambda_2 = 0.23$, $t_{\lambda_2} = 4.48$). Taken together, the results from the cointegration analysis are confirmed in the richer state-space model.

In line with Issa et al. (2008), our results also confirm that the importance of commodities for Canada has increased over time, as γ^{93} is positive and statistically significant. The loading factor of the commodity factor for Canada is thus increasing after 1993 (the impact of commodities on the Canadian dollar is given by Table 4). We also have two additional pieces of evidence supporting our empirical specification: first, a likelihood ratio test rejects the hypothesis that the coefficients for relative debt/GDP ratios, interest rates and commodity prices (and all the other coefficients that are added to the richer version of the model) are all equal to 0. This supports the model specification we chose.[17] Second, we find that the two factors estimated within the structural state-space model are very similar to the factors obtained from the dynamic factor analysis and the simple state-space model. We interpret this as further evidence confirming the robustness of our results.

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[16] We are not aware of a test for cointegration using a state-space model. However, the Boswijk (1994) test for the presence of cointegration in an error-correction model confirms the cointegration relationship between the U.S. factor and the relative debt-to-GDP ratio.

[17] Stock and Watson (1991) test their specification by examining whether the disturbances of the specific component disturbances are predictable by regressing the residuals against the lags of the residuals and the lags of the observable variables. Judged by that metric, we did not detect substantive evidence of misspecification (results available upon request).

Table 3: Estimation of the structural state-space model

Loading factors	Coefficient	t-stat
γ_1^{AU}	0.46	9.02
γ_1^{CAN}	0.33	4.78
γ_1^{EU}	0.78	17.55
γ_1^{JA}	0.50	7.02
γ_1^{NZ}	0.58	9.68
γ_1^{UK}	0.65	11.14
γ_2^{AU}	0.57	10.73
γ_2^{CAN}	0.21	2.83
γ_2^{EU}	-0.28	-5.67
γ_2^{JA}	-0.18	-2.30
γ_2^{NZ}	0.25	3.86
γ_2^{UK}	-0.16	-2.53
γ^{93}	0.43	3.25
Parameter estimates for the factors		
λ_1	0.16	3.94
β_1	0.25	2.54
β_2	-0.65	-5.98
λ_2	0.23	4.43
β_3	0.65	4.88
β_4	0.51	3.31
AR coefficients		
ϕ_1	0.31	3.50
ϕ_2	0.31	3.10
ν^{CAN}	0.27	2.68
ν^{JAP}	0.28	3.16
ν^{UK}	0.19	1.77
ν^{AU}	0.78	6.84
ν^{EU}	0.63	2.77

Table 4: Loading of the commodity factor for Canada

Period	Loading factor
1980-1993	0.21
1993-2007	0.64

Table 5: Variance of the first difference of the exchange rates explained by the two factors in the structural state-space model

Country	Explained by the two factors
Australia	88 per cent
Canada	46 per cent
Euro area	96 per cent
Japan	36 per cent
New Zealand	53 per cent
United Kingdom	56 per cent

3.5 Robustness checks

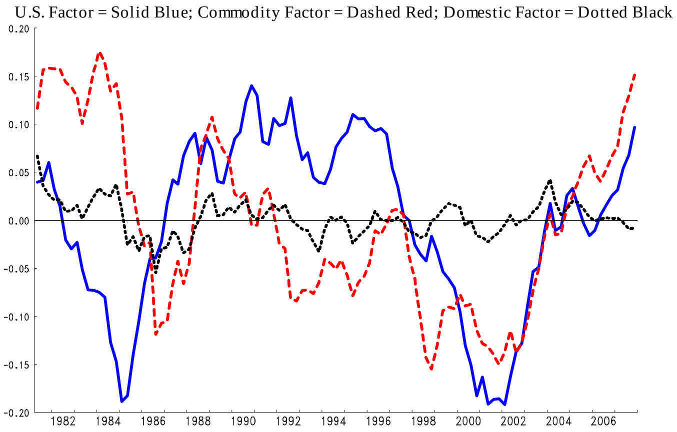
We have carried out a number of robustness checks, none of which changed our basic insights (detailed results available upon request). We estimated the dynamic factor model in levels and first differences and neither the number of significant factors, nor the factors, changed substantially (we also estimated a static factor model with very similar results). Also, we estimated the model using bilateral real exchange rates against the euro, rather than against the U.S. dollar. Our first factor changed from being a ‘U.S. factor’ to a ‘euro factor’, but the key results of our empirical model did not change (the link with commodity prices was virtually identical).

4 Historical Decompositions

Having established these relationships, we can decompose movements in each of the real bilateral U.S. dollar exchange rates into three stochastic components: one explained by U.S. shocks (whose long-run determinant is the relative debt-to-GDP ratio), one driven by commodity prices, and a country-specific component (essentially those movements that are not accounted for by the two factors or the common movements of interest rate differentials). As Table 5 shows, our two factors explain up to 96 per cent of the variation in individual countries’ exchange rates.

Figures 10-14 show the three stochastic components for all countries. The vertical axis is in percentage points and the different components are centred on zero. To keep our discussion focused, in what follows we refer to the 2002 to 2007 period.

Figure 10: Historical decomposition for Australia highlights importance of commodity prices



4.1 Australia

Figure 10 shows that over the entire sample, movements of the real Australian exchange rate against the U.S. dollar are clearly dominated by the two common components.[18] While Australia does not export oil, it is among the world's largest exporters of coal, whose price is highly correlated with energy prices. Our results underline the importance of commodites for the Australian dollar: world commodity prices, in combination with the U.S. factor (whose trend is driven by the relative U.S. debt-to-GDP ratio), account for almost all of the swing in the Australian dollar-U.S. dollar exchange rate over the 2002 to 2007 period, with each factor accounting for roughly 50 per cent of the appreciation.

4.2 Canada

The commodity price and U.S. factor together explain about 46 per cent of the long-run variance in the Canada-U.S. exchange rate (with the country-specific factor accounting for the remaining 54 per cent). As regards the most recent period of appreciation of the Canadian dollar between 2002 to 2007, the two common factors together explain more than 60 per cent of the appreciation in the Canada-U.S. exchange rate. Of this rise, the

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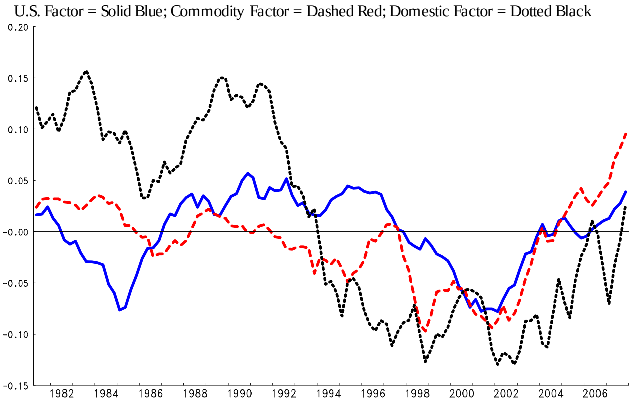
[18] Formally, we can reject the hypothesis of no cointegration between the real Australia-U.S. exchange rate and the two principal factors.

Figure 11: Historical decomposition for Canada shows recent appreciation driven by commodity prices and U.S. weakness



U.S. factor, driven by the U.S. debt common factor, explains about 50 per cent, while commodities explain about the other half (see Figure 11).

4.3 The euro area

Like in the case of Australia, the two common factors explain almost all of the variation in the real euro-U.S. dollar exchange rate (in fact, we are able to reject the hypothesis of no cointegration between the real euro-U.S. dollar exchange rate and the two common factors). Unlike for Australia, however, U.S. shocks can explain almost all of the appreciation in the euro-U.S. dollar real exchange rate between 2002 and 2007 (see Figure 12). This result is consistent with Fratzscher (2007), who argues that the euro and its predecessor currencies have contributed to the bulk of the adjustment of the U.S. dollar effective exchange rate over the past 25 years. World commodity prices also contributed slight downward pressure on the euro. In contrast, the domestic factor is relatively small for the euro area, likely reflecting that over the entire sample period, the currency was less prone to idiosyncratic shocks than other exchange rates.[19]

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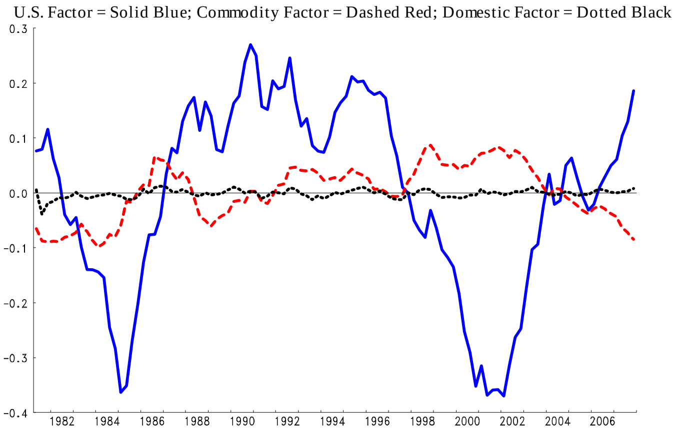
[19] One possible interpretation is that over the bulk of the sample, the euro area experiences less of a common business cycle than other countries. In the past, it was often the case that country-specific shocks tended to 'average out' for the euro area as a whole. This could imply that the exchange rate has also been less prone to area-wide shocks.

Figure 12: Historical decomposition for the euro area shows that the U.S. factor plays a key role



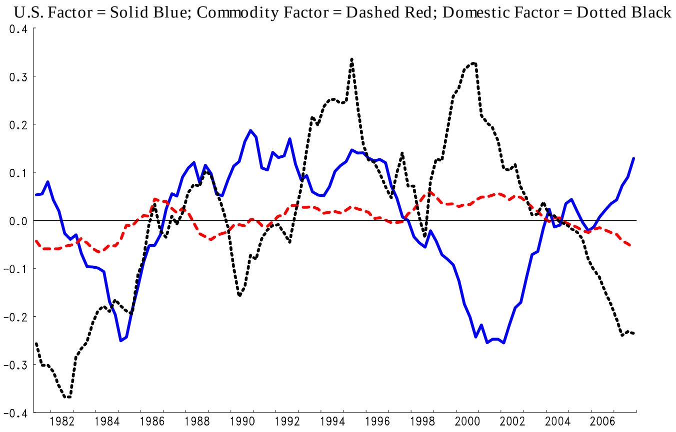
4.4 Japan

The country-specific factor for Japan shows a very distinct pattern (Figure 13). Favourable domestic developments have resulted in upward pressure on the Yen until the mid-1995. Since the start of the Japanese deflation and the associated run-up in budget deficits, the yen has experienced a substantial depreciation. Consequently, between 2002 and the end of our sample, the main factor influencing the bilateral exchange rate for the yen and the U.S. dollar is country-specific. In addition, commodity prices also exert some downward pressure on the yen-U.S. dollar exchange rate, reflecting Japan's dependency on imports for both energy- and non-energy commodities. Lastly, there is upward pressure on the yen due to the worsening of the U.S. budgetary situation over the 2002 to 2007 period (as evidenced by the U.S. factor), which offsets part of the downward pressure from the other two components.

4.5 New Zealand

The case of New Zealand is intriguing. New Zealand is a large commodity exporter, but its exports are largely concentrated in food and agricultural products. As a result, the rise in global commodity prices – which is mostly focused on energy and industrial materials – plays a less important role in explaining the appreciation of the New Zealand-U.S. dollar exchange rate than for other commodity-exporting countries in

Figure 13: Historical decomposition for Japan suggests that depreciation of the yen since 2000 is driven by domestic developments



our sample (econometrically, this is reflected in the relatively low loading factor for commodities). Between 2002 and 2007, the U.S. factor accounts for the bulk of the appreciation of the currency.

4.6 United Kingdom

For the United Kingdom, Figure 15 shows that U.S. factors play a large role in explaining the volatility of the pound-U.S. dollar exchange rate. Domestic factors also seem to have played an important role. In particular, note the upward pressure on the pound-U.S. dollar exchange rate in the late 1990s. Over the 2002 to 2007 period, our decomposition suggests that the U.S. factor played the predominant role in explaining the appreciation of the pound vis-a-vis the U.S. dollar, while commodity prices had only a modest downward impact.[20]

5 Conclusion

Understanding movements in exchange rates is a notoriously difficult task. Using a purely empirical approach, we combine the literature on factor models and exchange rates. Two very different empirical methodologies – a dynamic factor model and a

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[20] Note that the United Kingdom was a net exporter of oil during our sample, but still a net importer of commodities as a whole.

Figure 14: Historical decomposition for New Zealand shows that commodity prices account for most of the variation during the sample

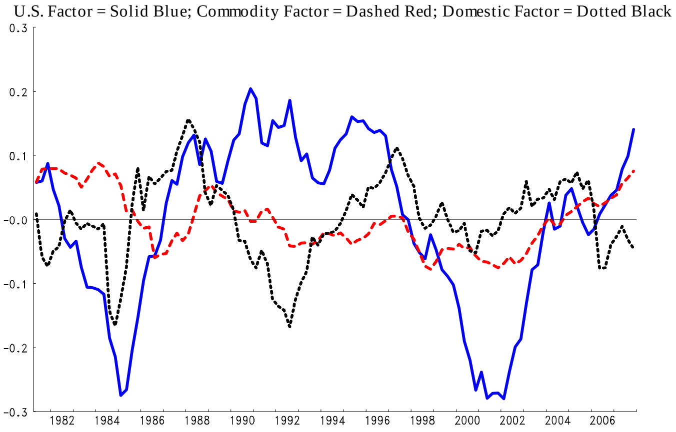
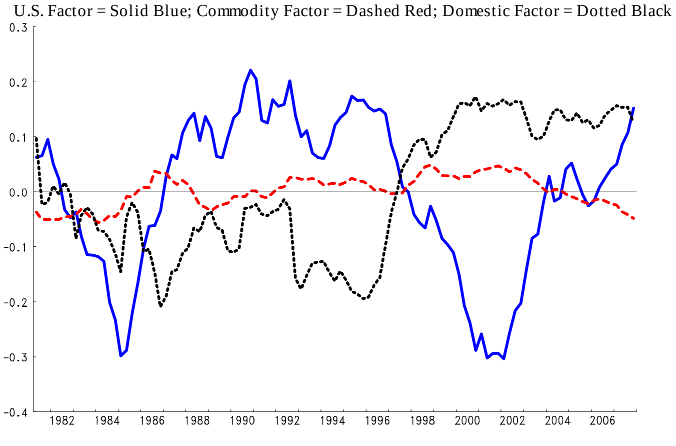


Figure 15: Historical decomposition for the United Kingdom shows that U.S. developments have played an important role since 2002



state-space model – provide very similar findings, namely that the bulk of variation in our panel is explained by two common factors. The pattern of the loading factors suggests that the first factor is driven by U.S. shocks, as all currencies move in the same direction. We subsequently link this factor to changes in the U.S. stock of debt, divided by GDP, relative to other countries' evolution of their debt-to-GDP ratios. The pattern of the second factor suggests a relationship with commodity prices, as commodity exporters and commodity importers move in opposite directions. Indeed, tests show that the second factor and commodity prices are cointegrated.

Estimating all relationships jointly, we find that U.S. fiscal policy and commodity prices have played very important roles in determining the value of a number of U.S. dollar exchange rates. Decomposing exchange rates, we conclude that both factors explain between 36 and 96 per cent of quarterly exchange rate behaviour between 1980 and 2007, depending on the country. In future work, we would like to better understand the 'domestic' factors that may be driving a number of the exchange rates in our panel (basically the residual in our state-space model). In addition, given recent developments in the relative deterioration of the U.S. fiscal position, it would be interesting to project the future evolution of bilateral exchange rates, based on alternative assumptions for the future path of the debt and commodity price variables.

A Data Description

Real exchange rates The nominal exchange rate data are from the IMF's IFS. We construct the euro exchange rate data, over the period 1981 to 1998, by using data on the ECU-U.S. dollar exchange rate (excluding the U.K. pound and Danish krona) and applying GDP weights per country. The real exchange rates are calculated using GDP price deflators.

Commodity prices Our commodity price measures are the IMF non-energy commodity price index and the West Texas Intermediate (WTI) crude oil price. Both are expressed in U.S. dollar terms and are deflated by the U.S. GDP price deflator.

Manufacturing labour productivity Annual productivity for Australia, the euro area, Japan, the United Kingdom and the United States were collected from EU KLEMS. Quarterly productivity data for Canada was collected from Statistics Canada. The relative manufacturing labour productivity of each country is calculated by:

$$\frac{\text{(Real Manufacturing Gross Output) / (Manufacturing Number of Workers)}}{\text{(Real Total Gross Output) / (Total Number of Workers)}}.$$

The resulting relative manufacturing labour productivity is indexed (2000=100) and interpolated to a quarterly frequency.

Debt-to-GDP ratio We use general government net financial liabilities, which include federal, state and local debt, as percentage of GDP. The data for Canada, U.K., Japan, U.S. and Australia are from the OECD Economic Outlook; the debt-to-GDP ratio for New Zealand from the New Zealand Treasury; and the euro area debt-to-GDP ratio was aggregated based on national statistics.

Real interest rates The real interest rate is calculated as $(1+i)/(1+\tau) = (1+r)$, where i denotes the nominal interest rate; r denotes the real interest rate; τ is the year-over-year inflation rate. The nominal interest rate is the 3-month treasury bill rate, collected from various sources (including Bloomberg, DataStream and the International Financial Statistics from the IMF). The inflation rate is calculated using the national CPIs and was collected from Global Insight and the BLS.

Table 6: Augmented Dickey-Fuller unit-root tests show that most series contain a unit root (sample period 1981Q1 to 2007Q4 (Null hypothesis: Series contain a unit root))

Log level of the U.S. real bilateral exchange rate	No Trend	Trend
Australia (AU)	-2.22	-1.97
Canada (CA)	-0.99	-0.42
Euro (EU)	-1.62	-2.31
Japan (JA)	-2.03	-1.75
United Kingdom (UK)	-1.74	-2.04
New Zealand (NZ)	-1.28	-3.12

B Empirical Strategy

In this section, we provide additional details and results of our estimation strategy. First, Table 6 provides unit root tests for all real exchange rates in our panel. As can be seen, most of them are integrated of order one. As outlined in the main text, we therefore estimate the dynamic factor model in first differences.

Second, Figure 16 graphs the gap between the observed and the cointegration relationship for the first factor. The pattern of the loading factors for the second factor suggests a relationship with commodity prices. Visual inspection confirms this notion: Figures 17 and 18 plot the second common factor from the dynamic factor model and the real price of oil and the IMF's real non-energy index, respectively. As can be seen, the second principal factor shares a common trend with world commodity prices. Figure 19 shows the residual from the cointegration relationship.

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Figure 16: Residual from the cointegration relationship: U.S. factor

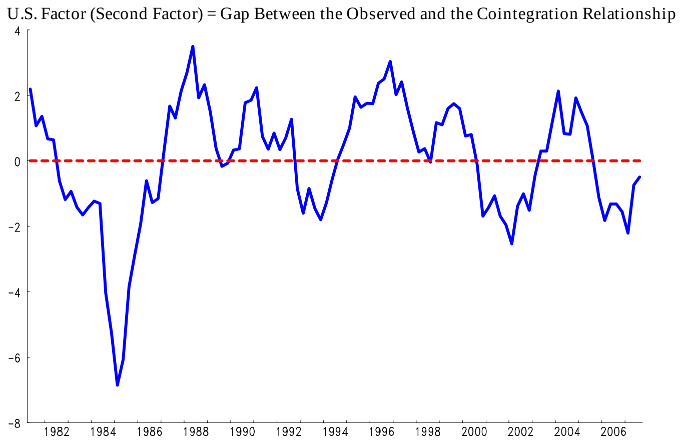


Figure 17: The second factor shows a similar pattern as oil prices (both shown in log-levels)

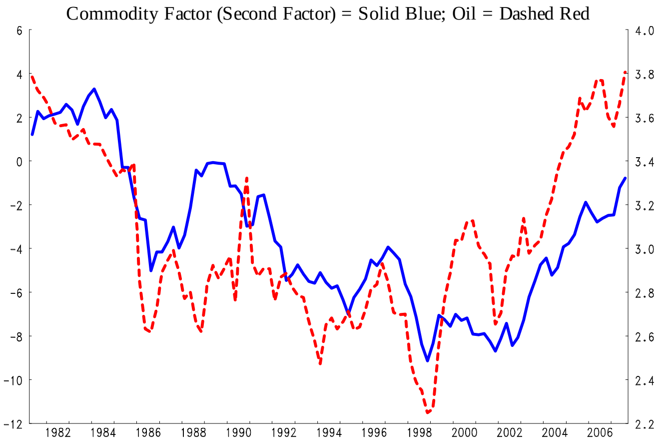


Figure 18: The second factor shows a similar pattern as non-energy commodity prices (both shown in log-levels)

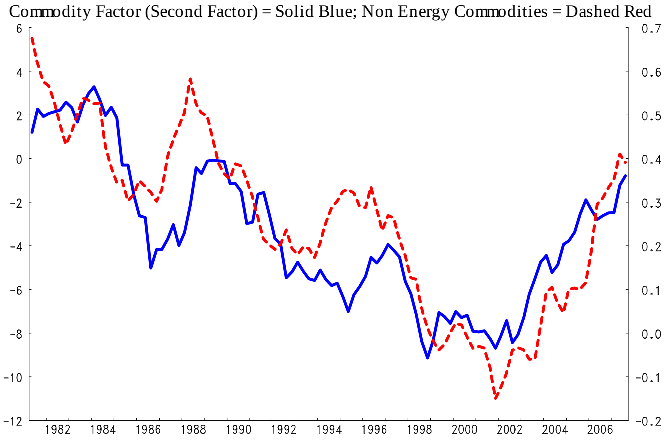
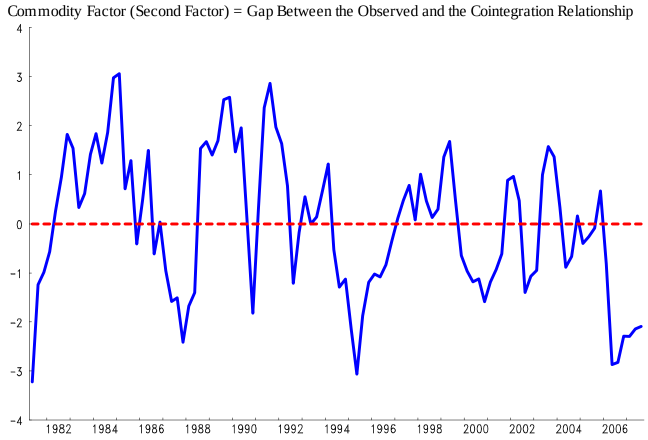


Figure 19: Residual from the Cointegration Relationship: Factor 2



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