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by Hajime Tomura

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Abstract

This paper uses a small-open economy model for the Canadian economy to examine the optimal Taylor-type monetary policy rule that stabilizes output and inflation in an environment where endogenous boom-bust cycles in house prices can occur. The model shows that boom-bust cycles in house prices emerge when credit-constrained mortgage borrowers expect that future house prices will rise and this expectation is neither shared by savers nor realized ex-post. These boom-bust cycles replicate the stylized features of housing-market boom-bust cycles in industrialized countries. In an environment where mortgage borrowers are occasionally over-optimistic, the central bank should be less responsive to inflation, more responsive to output, and slower to adjust the nominal policy interest rate. This optimal monetary policy rule dampens endogenous boom-bust cycles in house prices, but prolongs inflation target horizons due to weak policy reactions to inflation fluctuations after fundamental shocks.

JEL classification: E44, E52
Bank classification: Credit and credit aggregates; Financial stability; Inflation targets

Résumé

Dans le cadre d’un modèle de petite économie ouverte se prêtant à l’étude de l’économie canadienne, l’auteur examine la règle de Taylor optimale qui permet de stabiliser la production et l’inflation en présence de cycles endogènes d’envolée et d’effondrement des prix de l’immobilier résidentiel. Le modèle montre que de tels cycles peuvent émerger si les emprunteurs hypothécaires ayant un accès limité au crédit s’attendent à une hausse des prix des maisons mais que ces attentes ne sont pas partagées par les épargnants et sont déçues par la suite. Ces cycles d’envolée et d’effondrement sont conformes, dans les grandes lignes, aux cycles d’essor et de contraction du marché du logement dans les pays industrialisés. Lorsque les emprunteurs hypothécaires sont à l’occasion exagérément optimistes, la banque centrale devrait réagir moins aux variations de l’inflation et davantage à celles de la production, et modifier moins rapidement le taux d’intérêt directeur nominal. Cette règle de politique monétaire optimale atténue les cycles endogènes d’envolée et d’effondrement des prix des maisons, mais elle a pour effet d’allonger l’horizon nécessaire pour ramener l’inflation au taux visé, puisque la politique monétaire réagit faiblement aux fluctuations de l’inflation après un choc fondamental.

Classification JEL : E44, E52
Classification de la Banque : Crédit et agrégats du crédit; Stabilité financière; Cibles en matière d’inflation

1 Introduction

Strong asset-price booms have tended to end with significant drops in asset prices, leading to severe economic contractions that call for monetary policy responses. Thus, monetary policy reactions to boom-bust cycles in asset prices have become an important policy question for central banks.

In the literature, Bernanke and Gertler (1999) examine the performance of the Taylor-type monetary policy rule during boom-bust cycles in the price of capital, using the Bernanke, Gertler and Gilchrist (1999) model. They assume that boom-bust cycles are caused by exogenous deviations of the market price of capital from the 'fundamental' price implied by the capital market's competitive equilibrium. They find that a strong commitment to stabilizing expected inflation is effective in stabilizing current inflation and output in this environment. Following their finding, Basant Roi and Mendes (2007) examine the optimal Taylor rule that stabilizes the volatility of output and inflation during a similar type of exogenous boom-bust cycle in house prices in a small-open economy model for the Canadian economy. They particularly focus on optimal target horizons for inflation under the optimal Taylor rule (the expected numbers of periods which the inflation rate takes to return to the target after shocks under the optimal Taylor rule).[1] They find that exogenous house price shocks that cause boom-bust cycles lead to persistent inflation dynamics, prolonging the optimal target horizons.

This paper contributes to this literature by conducting optimal monetary policy analysis during boom-bust cycles in house prices, including evaluation of optimal target horizons, using a small-open economy model for the Canadian economy developed by Tomura (2009a). This model improves the exogenous boom-bust models in the literature described above in two dimensions. First, boom-bust cycles in house prices occur endogenously due to over-optimistic expectations of households. This innovation is important for policy analysis, since policy evaluation using exogenous boom-bust models must ignore the feedback effect from monetary policy to boom-bust cycles in asset prices and the consequent spillover effect to the rest of the economy. Second, the model endogenously replicates the stylized features of

[1] The definition of optimal target horizons is adopted from Batini and Nelson (2000).

macroeconomic dynamics during boom-bust cycles in house prices observed in industrialized countries. Thus, optimal monetary policy analysis in this paper is built upon one of the possible mechanisms for boom-bust cycles in house prices in reality.

In the model, there are two types of households: borrowers, who take mortgage loans, and savers, who provide mortgage loans. There exist credit constraints such that households can borrow only up to the collateral value of their housing.[2] Households receive noisy public signals of future fundamentals, which may not be realized ex-post. This assumption follows the so-called "news-shock" literature, which analyzes expectation-driven boom-bust cycles in business cycle models.[3] While the preceding models in the news-shock literature assume that households share identical expectations, the model in this paper relaxes this assumption, allowing households to have heterogeneous prior beliefs on the precision of public signals, which generate heterogeneous expectations among households observing the same public signal.

The model shows that endogenous boom-bust cycles in house prices emerge when credit-constrained borrowers expect that future house prices will rise and this expectation is not shared by savers or realized ex-post. This result is consistent with suggestive evidence on the relationship between heterogeneous expectations and boom-bust cycles in house prices. Borrowers and savers in the model can be interpreted as young and old households, respectively, since mortgage borrowers tend to be young and the holders of positive net financial assets tend to be old. As will be described in Section 3, Tomura (2009a) finds that real house price growth has tended to be higher when young households showed stronger consumer confidence on future economic conditions than old households in household survey data in U.S. and Canada.

As summarized in Section 2, cross-country data for industrialized countries indicate that

[2]Except for heterogeneous beliefs, the features of the model follow Iacoviello (2005).

[3]In the literature, it has been found difficult to generate expectation-driven boom-bust cycles in output and asset prices by overly optimistic household expectations in standard business cycle models. The literature has been focusing on finding a set-up with which overly optimistic household expectations cause boom-bust cycles. For example, Beaudry and Portier (2004) and Jaimovich and Rebelo (2006) modify the production function and the household-utility function, respectively. Christiano, et al. (2007) introduce nominal wage rigidity and an inflation-targeting central bank represented by a Taylor rule.

the nominal policy interest rate and the CPI inflation rate have tended to decline during housing booms and rise after the peaks of housing booms. The model explains this observation as follows. When borrowers expect that future house prices will rise, they increase housing investments, which causes a housing boom. Since borrowers are credit-constrained, they work more to finance their housing investments during the boom. At the same time, when savers do not share the optimistic expectations of borrowers, they instead expect the boom to be temporary and increase savings for a future recession. The increases in labour supply and savings reduce real wages and the real interest rate, respectively. Given sticky prices, a resulting fall in the marginal cost of production lowers the inflation rate, and, in response to this, the central bank cuts the policy rate. When the optimistic expectations of borrowers are not realized ex-post, a housing bust occurs, and savings and labour supply decline. As a consequence, the inflation rate rises, inducing a monetary policy tightening.

This paper finds that taking into account the stylized facts of housing-market boom-bust cycles is very important for optimal monetary policy analysis. Since inflation is counter-cyclical during boom-bust cycles in house prices, strong monetary policy reactions to inflation fluctuations would amplify boom-bust cycles by enhancing the counter-cyclical movement of the nominal policy interest rate, which destabilizes aggregate economic activity, including output. Thus, the optimal Taylor rule in the model implies that the central bank should be less responsive to inflation, more responsive to output, and slower to adjust the nominal policy interest rate. This monetary policy rule prolongs optimal inflation target horizons due to weak policy reactions to inflation fluctuations after fundamental shocks.

The optimal Taylor rule in this model contrasts with the implication of exogenous boom-bust models. As demonstrated by Bernanke and Gertler (1999), exogenous asset-price shocks that cause boom-bust cycles in exogenous boom-bust models generate positive co-movement between output and inflation, which do not replicate the stylized fact of the negative correlation between output and inflation during boom-bust cycles. This feature of exogenous boom-bust models lead to the conclusion that a monetary policy commitment to stabilizing inflation also stabilizes output during boom-bust cycles in asset prices.

On the other hand, the results of this paper and Basant Roi and Mendes (2007) jointly imply that optimal inflation target horizons are longer during boom-bust cycles in house prices, whether they are exogenous or endogenous. The difference between the two papers is in the cause of the longer target horizons. In Basant Roi and Mendes' model, exogenous house-price shocks cause persistent inflation dynamics directly. In this paper, the adjustment of the optimal monetary policy rule to boom-bust cycles in house prices prolongs inflation dynamics in response to fundamental shocks.

The rest of the paper is organized as follows. Section 2 summarizes the stylized features of housing-market boom-bust cycles in industrialized countries. Section 3 shows suggestive evidence for the relationship between heterogeneous expectations and house prices. Section 4 describes the model. Section 5 shows that endogenous boom-bust cycles in house prices in the model replicate the stylized features of housing-market boom-bust cycles in industrialized countries. Section 6 examines the optimal Taylor rule. Section 7 shows optimal target horizons implied by the optimal Taylor rule. Section 8 conducts sensitivity analysis. Section 9 concludes.

2 The stylized features of housing-market boom-bust cycles

Ahearne, et al. (2005) summarize the stylized features of boom-bust cycles in house prices by pooling the time-series of macroeconomic indicators over a 5-year window around the peaks of housing booms in industrialized countries. Taking the median of each indicator in each period in the time window, they find that output, consumption and investment have tended to positively co-move with house prices during boom-bust cycles, while the nominal policy interest rate and the CPI inflation rate have tended to decline during housing booms and rise as house prices fell.

Figure 1 reproduces the charts 3.1-3 reported by Ahearne, et al. for the real GDP growth rate, the CPI inflation rate, and the nominal policy interest rate, following their data appendix. The panels in the figure are not completely identical with their charts, since for the nominal policy interest rate and the CPI inflation rate, I take the medians of changes in these variables from the beginning of the time window (5 years before the peaks of housing booms), while Ahearne, et al. show the medians of the levels of these variables.[4] Also, for the CPI inflation rate, Ahearne, et al. use the rates targeted by central banks if exist, while I use total CPI inflation rates consistently. Moreover, I add the rate of growth of total hours worked to Figure 1, which shows that total hours worked have tended to grow strongly during housing booms and decline significantly during housing busts. See the data appendix of Tomura (2009a) for the construction of Figure 1. Tomura (2009a) also shows that these stylized facts held largely during the housing booms in Canada around 1981 and 1989.

3 Suggestive evidence on the effects of heterogeneous expectations on house prices

The model in this paper will indicate that house prices rise with over-optimistic expectations of mortgage borrowers compared to savers and that the labour supply of mortgage borrowers is pro-cyclical during housing-market boom-bust cycles. This section describes two empirical observations consistent with the model: the average hours worked of young workers co-move with house prices more closely than those of old workers do in Canada; and the real house price growth rate tends to be higher when young households show stronger consumer confidence on future economic conditions than old households in U.S. and Canada. This section compares behaviour of different age groups, since mortgage borrowers tend to be young and the holders of positive net financial assets tend to be old.

3.1 Positive correlations between house prices and the average hours worked of young households

Figure 2 compares the real house price index and the average hours worked of young and old households in Canada. It shows that the average hours worked of young households (less than 45 years old) rose and fell with housing booms and busts, respectively, around 1980 and

[4] Note that the levels of nominal variables tend to be non-stationary and vary across countries. Considering changes in the nominal variables rather than the levels alleviates this problem when pooling data across countries and time.

1990, while the average hours worked of old households (45-65 years old, and 45 years old and over) did not show such a clear correlation with house prices in those periods. The average hours worked in the figure are hours worked per population per week for each age group, which include both intensive and extensive margins. Tomura (2009a) show the correlation coefficient between detrended real house prices and detrended average hours worked is higher for the young than the old.

3.2 Positive response of the real house price growth rate to over-optimistic expectations of young households

This section examines the responses of house prices to the differences in expectations between young and old households. For a proxy to household expectations, I construct an index of household expectations of future economic conditions from a subset of the survey data from the Conference Board of Canada that are used for constructing the Index of Consumer Confidence. More specifically, there are two questions about future economic conditions among the four overall survey questions:

- considering everything, do you think that your family will be better off, the same or worse off financially six months from now?
- how do you feel the job situation and overall employment will be in this community six months from now?

Following the methodology for constructing the Index of Consumer Confidence, I derive an index of household expectations by adding the percentage of positive responses and subtracting the percentage of negative responses for each question. Thus, higher values of the index indicate more optimistic views of households for the future. I measure the over-optimistic expectations of young households compared to old households by the difference of the index for young households (less than 45 years old) from the index for old households (45 years old and over), and regress the real house price growth rate on this difference as well as lagged dependent variables, the average index of household expectations for all ages, the real GDP growth rate and the real interest rate. I estimate the coefficients by OLS, assuming that unobserved house price shocks are orthogonal to the difference in the index of household expectations. Due to availability of the survey data for age groups, the sample period is for

1990:4-2007:1.

Table 1 shows the regression results. There are three regressions with different sets of regressors. In each regression, the lag-2 regressor of the difference in the index of household expectations between the young and the old shows a statistically significant positive effect on the real house price growth rate. Even though the contemporaneous difference in the index of household expectations has negative coefficients in Regressions 1 and 2, the coefficient becomes insignificant when I consider the full set of regressors in Regression 3, adding the average index of household expectations for all ages.

Note that the household expectation survey data for age groups in Canada are available only for the recent period after 1990, so that the sample period for the regressions has to be short. Also the horizon of the questions is 6 month, which is short, too. These properties of the regressions may affect the regression results. To circumvent these problems, I look at U.S. data in addition. In U.S., the University of Michigan Surveys of Consumers provide the Index of Consumer Expectations for age groups from 1978. The horizons of the questions that the Index is based on are 1 and 5 years, depending on each question. Figure 3 compares the real house price growth rate and the difference of the Index of Consumer Expectations for young households (less than 45 years old) from that for old households (45 years old or more) in U.S. The figure shows that the two series have co-moved very closely, especially until the late 1990s. Tomura (2009b) reports that in fact the contemporaneous difference in the Index of Consumer Expectations between the young and the old has robustly significant positive effects on the real house price growth rate in regressions similar to those considered in Table 1.

4 Model

This section describes a small open economy model for the Canadian economy developed by Tomura (2009a). The model includes two types of households who take and provide mortgage loans as well as collateral constraints on residential mortgages as in Iacoviello (2005). The model also incorporates monopolistic firms that produce intermediate inputs, a representative firm that produces final goods competitively, and a monetary authority.

4.1 Production

Final good production. There is a representative firm that acts in a perfectly competitive market and uses composite domestic and imported inputs to produce final goods, y_t, according to the following CES technology:

$$y_t = \left\{(1-\omega)^{\frac{1}{\theta}} (y_{D,t})^{\frac{\theta-1}{\theta}} + \omega^{\frac{1}{\theta}} (y_{M,t})^{\frac{\theta-1}{\theta}}\right\}^{\frac{\theta}{\theta-1}}, \tag{1}$$

where

$$y_{i,t} = \left[\int_0^1 y_{i,t}(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} \qquad \text{for } i = D, M. \tag{2}$$

Domestic inputs are denoted by D, and imported inputs are denoted by M. The parameter $\omega > 0$ denotes the share for imported inputs in the production of final goods, and $\theta > 0$ is the elasticity of substitution between different intermediate inputs.

The resulting demand function for the intermediate inputs is:

$$y_{i,t}(j) = \left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\theta} y_{i,t}, \tag{3}$$

for $i = D, M$. The cost minimization for the final-good firm entails the following demand curves for $y_{D,t}$ and $y_{M,t}$:

$$y_{D,t} = \left(\frac{P_{D,t}}{P_t}\right)^{-\theta} (1-\omega) y_t, \tag{4}$$

$$y_{M,t} = \left(\frac{P_{M,t}}{P_t}\right)^{-\theta} \omega y_t, \tag{5}$$

where the price indices for domestic and imported intermediate inputs are defined by:

$$P_{i,t} = \left(\int_0^1 P_{i,t}(j)^{1-\theta} dj\right)^{\frac{1}{1-\theta}}, \tag{6}$$

for $i = D, M$. The domestic aggregate price level, P_t, is defined by:

$$P_t = \left[(1-\omega) P_{D,t}^{1-\theta} + \omega P_{M,t}^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{7}$$

Final goods can be consumed, invested into capital or exported abroad.

Intermediate inputs. There is a continuum of firms indexed by $j \in [0,1]$ that monopolistically produce $y_{D,t}(j)$ units of each variety of domestic intermediate input according to a standard Cobb-Douglas function:

$$y_{D,t}(j) = (k_t(j))^{\alpha} (A_t l_t(j))^{1-\alpha}, \tag{8}$$

where $k_t(j)$ is the amount of capital, $l_t(j)$ is the units of labour and $\alpha \in (0,1)$ is the constant share for capital in production. A_t denotes labour augmenting technology. The monopolistic firms can only infrequently adjust the prices of their products with probability $1-\chi$ every period. When adjusting the price, each firm maximizes the present discounted value of profits while the price remains fixed:

$$\max_{P_{D,t}(j)} E_t' \left[\sum_{s=t}^{\infty} \chi^{s-t} \Lambda_{t,s} \left(\frac{\Pi_{D,s}(j)}{P_s} \right) \right], \tag{9}$$

subject to the demand function (3). E_t' is the subjective conditional expectation operator for firms and $\Lambda_{t,s}$ is the firms' discount factor between periods t and s. The expectation operator and the discount factor are identical to those of the share holders of firms that will be described below. Firms' profits in real terms are given by:

$$\frac{\Pi_{D,s}(j)}{P_s} = \left[\frac{P_{D,s}(j)}{P_s} - f_s \right] y_{D,s}(j), \tag{10}$$

where

$$f_s \equiv \left(\frac{r_{K,s}}{\alpha} \right)^{\alpha} \left[\frac{w_s}{(1-\alpha)A_s} \right]^{1-\alpha}, \tag{11}$$

which is the marginal cost of production for domestic inputs implied by competitive factor markets and the production function (8).

Each variety of imported inputs is supplied to the domestic market by a monopolistic importing firm. Importers buy homogeneous foreign goods at a unit cost of $e_t P_t^*$ for a given nominal exchange rate, e_t, and foreign price level, P_t^*. Thus the real exchange rate, s_t, becomes the real acquisition price of imported goods. Importers produce each variety of imported intermediate inputs, $y_{M,t}(j)$, from homogeneous foreign goods via one-to-one transformation. Each monopolistic importer sets the price $P_{M,t}(j)$ of each variety of imported input for $j \in [0,1]$. As in the domestic intermediate inputs sector, each importer faces a constant probability $1-\chi$ of being allowed to change her price, solving a similar problem to (9):

$$\max_{P_{M,t}(j)} E_t'\left[\sum_{s=t}^{\infty}\chi^{s-t}\Lambda_{t,s}\left(\frac{P_{M,s}(j)}{P_s}-s_s\right)y_{M,s}(j)\right], \tag{12}$$

subject to the demand function (3).

4.2 Households

Consider two types of households that differ in terms of the subjective discount factor: one type of household has a higher time-discount rate than the other. Following Iacoviello (2005), characterize the former type as 'patient', and the latter type as 'impatient'. The two types of households are of mass $\mu \in (0,1)$ and $1-\mu$, respectively. As described below, the heterogeneity in time discount rates implies that patient households provide mortgage loans to impatient households in the neighbourhood of the deterministic steady state.

Patient Households. Each patient household, denoted by ($'$), derives utility from consumption, c_t', and housing services provided by the housing stock, h_t', and disutility from supplying labour, l_t'. This is specified by the following utility function:

$$E_t'\left\{\sum_{s=t}^{\infty}\beta'^{s-t}\left[\ln(c_s')+\gamma\ln(h_s')-\frac{\eta(l_s')^{1+\xi'}}{1+\xi'}\right]\right\}, \tag{13}$$

where E_t' is the subjective conditional expectation operator for patient households, β' is the time-discount rate, and γ, η, $\xi' > 0$. Patient households control the domestic-input firms and the importers as share holders. Hence assume $\Lambda_{t,s} = (\beta'^{s-t}c_t'/c_s')$ and that the domestic-input firms, the importers and patient households share the identical subjective conditional expectation operator, E_t'. These assumptions ensure that the domestic-input firms and the importers behave as if they maximize the utility function of patient households.

The patient household's budget constraint is given by:

$$c_t' + q_t\Delta h_t' + s_t\left(b_{F,t}'-(1+r_{F,t-1})b_{F,t-1}'\right)+\frac{\zeta_B}{2}(b_{F,t}')^2+\frac{\zeta_K}{2}\left(\frac{i_t'}{k_{t-1}'}\right)^2 k_{t-1}'+b_{D,t}'$$
$$= w_t l_t' + r_{k,t}k_{t-1}' + (1+r_{D,t-1})b_{D,t-1}' + \Gamma_t, \tag{14}$$

where i_t' is investment in capital stock, k_t' is the end-of-period value of capital stock, $\Delta h_t' = h_t' - h_{t-1}'$ is the change in housing stock, $b_{F,t}'$ is foreign bonds denominated in foreign currency,

$b'_{D,t}$ is the supply of mortgage loans to impatient households, q_t is the real price of housing stock, s_t is the real exchange rate, $r_{F,t}$ is the real world interest rate, w_t is the real wage, $r_{k,t}$ is the real rental price of capital stock, $r_{D,t}$ is the domestic real interest rate, and Γ_t is the sum of the profits from the monopolistic domestic-input producers and importers.[5]

The real interest rate is determined by the ratio between the gross nominal interest rate, R_t, controlled by the monetary authority and patient households' expected gross inflation rate of final goods:

$$1 + r_{D,t} \equiv \frac{R_t}{E'_t \Pi_{t+1}}, \tag{15}$$

where Π_t is the gross inflation rate of final goods, that is, P_t/P_{t-1}. While bonds are indexed in the baseline model, Section 8 will introduce nominal bonds to confirm the robustness of the results of the model.

The fourth term on the left-hand side of (14) is an adjustment cost on foreign bond holdings, $(\zeta_B/2)(b'_{F,t})^2$, where $\zeta_B > 0$, which ensures that dynamics around the steady state are stationary in the equilibrium analysis presented below.[6] The fifth term on the left-hand side is an adjustment cost on the installation of capital. In the equilibrium analysis below, the existence of capital adjustment cost will generate co-movement between consumption and investment.

Impatient Households. Impatient households, denoted by ($''$), maximize the utility function:

$$E''_t \left\{ \sum_{s=t}^{\infty} \beta''^{s-t} \left[\ln(c''_s) + \gamma \ln(h''_s) - \frac{\eta (l''_s)^{1+\xi''}}{1+\xi''} \right] \right\}, \tag{16}$$

subject to the following budget and collateral constraints:[7]

$$c''_t + q_t(h''_t - h''_{t-1}) + b''_{D,t} = w_t l''_t + (1 + r_{D,t-1}) b''_{D,t-1}, \tag{17}$$

$$(1 + r_{D,t}) b''_{D,t} \geq -m E'_t \left[q_{t+1} h''_t \right]. \tag{18}$$

[5] Note that money balance does not appear either in the utility function or in the budget constraint. This is equivalent to considering a 'cashless' economy where real money balance enters the utility function in an additive term every period but the real and nominal money balances are so infinitesimal that they do not affect the budget constraint.

[6] See, e.g., Schmitt-Grohè and Uribe (2003) for further details.

[7] The budget constraint (17) implies that impatient households do not invest in capital. It can be shown that their impatience implies no capital holding in the neighbourhood of the deterministic steady state.

As in Iacoviello (2005), the collateral constraint (18) implies that impatient households can only borrow up to the collateral value of their housing.[8] Since impatient households value current consumption more than patient households, it is possible to show that impatient households borrow up to the limit in the neighbourhood of the deterministic steady state.[9] The collateral value of housing is determined by the expectations of lenders, who are patient households in the neighbourhood of the deterministic steady state, and the parameter m representing the maximum loan-to-value ratio for residential mortgages.

Note that the elasticity of labour supply can be different between the patient and the impatient households. This assumption is consistent with the fact that mortgage borrowers tend to be young and the holders of positive net financial assets tend to be old and that the volatility of labour supply is higher for young workers than old workers.

4.3 Monetary policy

Assume that the monetary authority follows a Taylor rule in the following form:

$$\widehat{R}_t = \phi_R \widehat{R}_{t-1} + \phi_\pi \widehat{dCPI}_t + \phi_Y \widehat{GDP}_t + \psi_t, \tag{19}$$

where ϕ_R is a smoothing-term parameter, ϕ_π and ϕ_Y determine the responses of the nominal policy interest rate to CPI inflation and real GDP, respectively, and ψ_t is an i.i.d. monetary policy shock. The variables denoted by the hat symbol "$\widehat{\ }$" are the log deviations from the steady state values. This type of monetary policy rule is standard in the literature. For sensitivity analysis, Section 8 will consider a case where the nominal policy interest rate responds to the expected inflation rate instead of the current inflation rate.

On the right-hand side of (19), GDP_t denotes real GDP and $dCPI_t$ denotes the CPI inflation rate. Real GDP is the value of domestic production, which equals $P_{D,t} y_{D,t} / P_t$. The definition of the CPI inflation rate reflects the treatment of housing rental cost in the

[8] See Kiyotaki and Moore (1997) for the bargaining environment behind the collateral constraint. I assume that borrowers can renegotiate debt contracts only before the realization of aggregate shocks in the next period, so that lenders can seize borrowers' labour income in period $t+1$ if their debts exceed the values of the collateral after the realization of shocks.

[9] See Iacoviello (2005) for more details.

Canadian statistics, such that:

$$\begin{aligned}
dCPI_t =& \frac{(1-\lambda)\frac{P_t}{P_{SS}} + \lambda \frac{P_t r_{h,t}}{P_{SS} r_{h,SS}}}{(1-\lambda)\frac{P_{t-1}}{P_{SS}} + \lambda \frac{P_{t-1} r_{h,t-1}}{P_{SS} r_{h,SS}}} \\
=& \pi_t \cdot \frac{(1-\lambda) + \lambda \frac{r_{h,t}}{r_{h,SS}}}{(1-\lambda) + \lambda \frac{r_{h,t-1}}{r_{h,SS}}},
\end{aligned} \tag{20}$$

where P_t is the nominal price of final goods, λ is the fixed weight on the housing-rent components of the CPI, and $r_{h,t}$ is the real value of the housing-rent components of the CPI. The subscript SS denotes steady state values. Use the steady-state values for the base-year values of the price indices for housing rent and final goods. The treatment of rental costs in the CPI has the potential to be important for optimal policy since they allow house prices to affect the measure of inflation directly.

The model does not incorporate renters formally. Assume the following reduced-form specifications for $r_{h,t}$:

$$\widehat{r}_{h,t} = \kappa \widehat{q}_t. \tag{21}$$

Tomura (2009a) conducts sensitivity analysis and finds that the main feature of the model does not change even with an alternative specification that the housing-rent components are correlated with the user cost of housing for patient households (i.e., $\widehat{r}_{h,t} = \kappa \widehat{u}_t$ where $u_t = q_t - E'_t \Pi_{t+1} q_{t+1} / R_t$).

4.4 Market-clearing conditions

In each period, the following market clearing conditions are satisfied for labour, capital stock, housing stock and mortgage loans, respectively:

$$\mu l'_t + (1-\mu) l''_t = \int_0^1 l_t(j) dj, \tag{22}$$

$$\mu k'_{t-1} = \int_0^1 k_t(j) dj, \tag{23}$$

$$\mu h'_t + (1-\mu) h''_t = 1, \tag{24}$$

$$\mu b'_t + (1-\mu) b''_t = 0. \tag{25}$$

The fixed supply of housing stock (i.e. land) is normalized to 1. Note that $l_t(j)$ and $k_t(j)$ are labour and capital demand, respectively, by the domestic-input firm of variety j. The second equation implies that the capital stock available for production in the current period must be formed in the previous period. Factor demand, $\{k_t(j), l_t(j)\}_{j\in[0,1]}$, is determined by the first-order conditions implied by the cost minimization of domestic-input firms:

$$k_t(j) = \frac{\alpha f_t\, y_t(j)}{r_{K,t}}, \tag{26}$$

$$l_t(j) = \frac{(1-\alpha) f_t\, y_t(j)}{w_t}. \tag{27}$$

4.5 Balance of payments

The trade balance must equal the economy-wide net saving, so that:

$$y_{X,t} - s_t y_{M,t} = \mu s_t \left(b'_{F,t} - (1 + r_{F,t-1}) b'_{F,t-1} \right). \tag{28}$$

In order to close the model, the export demand must be specified. Assume the following simple reduced-form function:

$$y_{X,t} = (s_t)^{\tau} Y_{F,t}, \tag{29}$$

where $\tau > 0$ is the elasticity of the home country's aggregate exports and $Y_{F,t}$ is an export demand shock summarizing business conditions in the rest of the world. As a rise in s_t implies depreciation, the positive value of τ ensures that export demand rises as the home currency depreciates.

4.6 Shock processes, public signals and heterogeneous expectations

Assume that labour augmenting technology, A_t, the world interest rate, $r_{F,t}$, the export demand shock, $Y_{F,t}$, and the monetary policy shock, ψ_t, follow AR(1) processes. I denote the deterministic steady-state values of $r_{F,t}$ and $Y_{F,t}$ by $\overline{r_F}$ and $\overline{Y_F}$, respectively. Each shock process is defined by:

$$x_t = \rho_x x_{t-1} + \varepsilon_{x,t}, \quad \varepsilon_{x,t} \backsim \text{ i.i.d. } N(0, \sigma^2_{\varepsilon_x}),\ 0 < \rho_x < 1, \tag{30}$$

for $x_t \in \{\ln(A_t),\ \ln((1 + r_{F,t})/(1 + \overline{r_F})),\ \ln(Y_{F,t}/\overline{Y_F}),\ \psi_t\}$.

Households receive a public signal $s_{A,t}$ of a future technological shock $\varepsilon_{A,t+n}$. The signal of the shock is generated by the following process:

$$s_{A,t} = \varepsilon_{A,t+n} + \omega_{A,t}, \tag{31}$$

where $\omega_{A,t}$ is an uncorrelated and normally distributed innovation with zero mean and standard deviation ν_A^2.

Assume that each type of household holds a time-invariant belief of the value of ν_A, regardless of the realization of shocks, $\varepsilon_{A,t}$. Denote the beliefs of patient and impatient households by $\nu_A^{'}$ and ν_A'', respectively.[10] It can be shown that the subjective conditional expectations of future shocks are given by:

$$E'[\varepsilon_{A,t+n}|s_{A,t}] = \frac{\sigma_{\varepsilon_A}^2 s_{A,t}}{\sigma_{\varepsilon_A}^2 + (\nu_A')^2}, \tag{32}$$

$$E''[\varepsilon_{A,t+n}|s_{A,t}] = \frac{\sigma_{\varepsilon_A}^2 s_{A,t}}{\sigma_{\varepsilon_A}^2 + (\nu_A'')^2}. \tag{33}$$

Thus, public signals generate heterogeneous expectations of future economic conditions among households.

This assumption of time-invariant heterogeneous beliefs is related to a vast strand of the behavioural finance literature that analyzes the joint behaviour of stock prices and trading volume in the stock market. See Hong and Stein (2007) for a survey. This literature indicates that heterogeneous expectations among investors generated by their heterogeneous beliefs explain non-fundamental fluctuations in stock prices accompanied by an increase in trading volume. This paper investigates the effects of heterogeneous beliefs among households on house prices.

4.7 Equilibrium conditions

Equilibrium conditions are defined as follows. Every period, $\{c_s', h_s', l_s', b_s', i_s'\}_{s=t}^{\infty}$ solves the maximization problem for patient households, $\{c_s'', h_s'', l_s'', b_s''\}_{s=t}^{\infty}$ solves the maximization problem for impatient households, and $P_{i,t}(j)$ for $i = D, M$ and $j \in [0, 1]$ solves the maximization problem for the domestic-input firm or the importer if the price can be adjusted

[10] As described above, the domestic-input firms and the importers share the same beliefs with patient households.

in period t. Otherwise, $P_{i,t}(j)$ equals $P_{i,t-1}(j)$. $\{w_t, r_{K,t}, q_t, f_t, y_t, P_t, R_t, \Gamma_t\}$ is determined to satisfy the market clearing conditions (22)-(25) and the balance of payments (28), given the definition of each variable specified above. Households hold rational expectations of the determination of $\{w_s, r_{K,s}, q_s, f_s, y_s, P_s, R_s, \Gamma_s\}_{s=t}^{\infty}$ conditional on each realization of shocks and public signals. However, households form the subjective likelihood of the realization of future shocks on the basis of their time-invariant beliefs on the precision of public signals.

4.8 Solution method and parameter specification

Equilibrium dynamics are solved by applying the undetermined coefficient method to log-linearized equilibrium conditions described above around the deterministic steady state. See Tomura (2009b) for more details on the solution method.

Table 2 shows the baseline parameter values. The parameter values are calibrated with aggregate Canadian data or adopted from standard values in the business cycle literature. See Tomura (2009a) for details on parameter specification and data sources.[11] An important feature of the parameter specification is that the elasticity of labour supply of impatient households is higher than that of patient households. I set this assumption by considering that mortgage borrowers represented by impatient households tend to be young while the owners of financial assets represented by patient households tend to be old.[12] Given this interpretation, I calibrate the elasticities of labour supplies in the model to replicate the volatility of average hours worked of young households and those of old households in Canadian data, in which the average hours worked of young households show higher volatility than those of old households. Table 3 compares the moments between the model and Canadian data used in calibration.[13] Also, I use an estimated Taylor rule coefficients for the Canadian economy in the monetary policy rule (19).

[11]Following Dib (2008), I assume the elasticity of substitution between varieties, θ, is identical between domestic and imported intermediate inputs.

[12]See Meh and Terajima (2008) for the wealth distribution over age groups in Canada.

[13]Note that the model is calibrated to the moments of key variables for optimal monetary policy analysis, i.e., the inflation rate and output, as well as average hours worked of different age groups. Among the moments of other variables, the model closely replicates the variances of the real exchange rate and the current account, while aggregate consumption and investment in the model are more volatile than data.

5 Replication of the stylized features of housing-market boom-bust cycles

Suppose that, in period 0, agents receive a public signal of technological progress that will occur in 4 periods. The signal turns out to be wrong ex-post (i.e. $s_{A,0} > 0$ and $\epsilon_{A,4} = 0$). I consider the case in which impatient households believe the public signal to be true (i.e. $\nu_A'' = 0$). In contrast, patient households do not believe the public signal to be true and expect no future technological progress (i.e. $\nu_A' = \infty$).

Figure 4 shows that the equilibrium dynamics replicate the stylized pattern of housing-market boom-bust cycles: real GDP and aggregate consumption, investment and labour supply positively co-move with the house price; and the nominal policy interest rate and the CPI inflation rate fall during housing booms and rise as the house price falls. The model explains these stylized facts as follows. When impatient households expect that future house prices will rise, they increase housing investments, which causes a housing boom. Since impatient households are credit-constrained, they work more to finance their housing investments during the boom. At the same time, when patient households do not share the optimistic expectations of impatient households, they instead expect the boom to be temporary and increase savings for a future recession. The increases in labour supply and savings reduce real wages and the real interest rate, respectively. Given sticky prices, a resulting fall in the marginal cost of production lowers the inflation rate, and, in response to this, the central bank cuts the policy rate.[14] When the impatient households' expectations are not realized in period 4, however, impatient households start dissaving the over-accumulated housing stock. This weakens housing demand, which causes a housing bust. Impatient households also reduce labour supply as they no longer have to raise funds for housing investments. At the same time, patient households withdraw savings to support their consumption. This development raises real wages and the real interest rate. As a consequence, the inflation rate rises, inducing a monetary policy tightening.

Tomura (2009a) discusses the features of the model in more details. The paper finds

[14] This relationship between the real marginal cost of production and the inflation rate can be shown by the new-Keynesian phillips curves implied by the Calvo-pricing.

that the stylized features of housing-market boom-bust cycles arises in the model only if impatient households become over-optimistic and the patient do not. For example, when patient households believe an ex-post wrong signal of future technological progress, then they increase consumption and reduce saving, which leads to a rise in the domestic real interest rate. Having a rise in the cost of financing, the impatient households reduce their housing investments. Thus, they do not increase labour supply to raise funds for housing investments, either. As a consequence, both the patient and the impatient households reduce labour supply, and output falls in response to over-optimistic expectations of households. The dynamics of the model becomes closer to this dynamics as the patient households put more trust on signals, believing the signals are less noisy.

6 Evaluation of the optimal Taylor rule during endogenous housing-market boom-bust cycles

This section evaluates the optimal Taylor rule during endogenous boom-bust cycles in house prices. To introduce occasional endogenous boom-bust cycles in house prices into the model, consider the following set-up. In addition to the set of the shocks to fundamentals described above, households observe public signals of future technological progress. Assume that impatient households perceive that $s_{A,t} = \epsilon_{A,t+4}$ (i.e., $\nu''_A = 0$), and that patient households believe that the signals are not informative (i.e., $\nu'_A = \infty$). Assume that the true signal process is a white noise, i.e., $s_{A,t} \sim$ i.i.d.$N(0, \nu_A^2)$. Thus, positive signals always generate over-optimistic expectations of impatient households, and the impulse responses to positive signals endogenously generate boom-bust cycles in house prices. This shocks can be interpreted as 'housing-bubble' shocks that make house prices deviate from their 'fundamental' prices endogenously.

Set the variance of the signal, ν_A, by considering the following scenario: once in 10 years on average, borrowers become over-optimistic to the extent that house prices are pushed up by more than 10% of the steady state value. More specifically, consider $\underline{s}$ such that $\hat{q}_t = 0.1$ at the peak of the impulse response to $s_{A,0} = \underline{s}$, given the baseline parameter values in Table

2. Then set the value of ν_A with which the probability that $s_{A,0} > \underline{s}$ is 0.025.[15]

Here, I do not attempt to pin down the value of ν_A with Canadian data, since it is difficult to identify the fraction of house price movements contributed by over-optimistic expectations of households. This question is left for future research. Instead, I consider a scenario of realistic frequency and magnitude of boom-bust cycles, which is common among exogenous boom-bust models in the literature.[16] Later, Section 8 will show sensitivity analysis by considering a smaller value of ν_A with which the probability that $s_{A,0} > \underline{s}$ is 0.025, where $\hat{q}_t = 0.05$ at the peak of the impulse response to $s_{A,0} = \underline{s}$. It will be shown that the results of the analysis do not change significantly.

The optimal Taylor rule is the coefficients of the monetary policy rule:

$$\widehat{R}_t = \phi_R \widehat{R}_{t-1} + \phi_\pi \widehat{dCPI}_t + \phi_Y \widehat{GDP}_t + \phi_q \hat{q}_t \tag{34}$$

that solve the stabilization problem of output-gap and inflation variances:

$$\min \quad E(\widehat{dCPI}_t^2) + E(\widehat{GDP}_t^2) + zE[(\widehat{R}_t - \widehat{R}_{t-1})^2], \tag{35}$$

where $z \geq 0$. Note that the monetary policy rule (34) does not contain monetary policy shocks. Thus, the optimal monetary policy analysis does not take into account any possibility of policy error. Also, the monetary policy rule allows the central banks to directly respond to house prices. The loss function (35) reflects the mandate for central banks to achieve output and inflation stability. The last term of the loss function (35) is the averseness of the central bank to fluctuating the nominal policy interest rate. This type of loss function is common in the literature on optimal monetary policy analysis.[17] I consider $z = 0$ and $z = 0.5$ to show that the results of the model do not depend on the inclusion of the third term in the loss function. Section 8 will report that alternative weights in the loss function do not change the results of the model.

Table 4 reports the optimal Taylor rules in two scenarios for each type of loss function. In the first scenario, "fundamental shocks only", there is no signal or over-optimistic ex-

[15] $\nu_A = 2.436$. Given the normal distribution of the signals, this number implies that the value of $\hat{q}_t$ at the peak of the impulse response to $s_{A,0}$ is less than 0.15 with the probability of 99.9%. So the deviation of house prices from the 'fundamental value' is in the range between 10% and 15% once in 10 years.

[16] For example, see Bernanke and Gertler (1999) and Basant Roi and Mendes (2007).

[17] For example, see Rudebusch and Svensson (1999).

pectations of households. In the second scenario, "fundamental shocks and over-optimism", ν_A is set to the value specified above, so that impatient households occasionally become over-optimistic, causing boom-bust cycles in house prices. In the second scenario, patient households do not believe public signals and only the impatient households believe them, as outlined above.

Table 4 indicates that, regardless of the type of loss function, the central bank should respond less to inflation and more to output and should be slower to adjust the policy rate when over-optimistic expectations of impatient households can occur. The reason for this result is that since the inflation rate is counter-cyclical during boom-bust cycles in house prices, as shown in Section 5, strong policy reactions to inflation would amplify the counter-cyclicality of the nominal policy interest rate, which would enhance boom-bust cycles in house prices and destabilize aggregate economic activity.

Comparison between the two types of loss functions under each scenario indicates that the averseness to fluctuating the policy rate implied by the optimal Taylor rule is not due to the exogenous cost incorporated by the loss function, but due to the concern that active policy responses to the inflation rate would enhance boom-bust cycles in house prices. This is an interesting result since estimated Taylor-type monetary policy rules for various countries imply that adjustments of the nominal policy interest rates are sluggish and an exogenous cost in the loss function has been necessary to reproduce this observation in the optimal monetary policy analysis in the literature.

The optimal Taylor rule in Table 4 indicates that the central bank does not have to target house prices directly to dampen boom-bust cycles. This is because house prices and output co-move during boom-bust cycles in house prices, as shown in the previous section, so that responding to output is equivalent to responding to house prices.

Figure 5 compares the impulse responses to a positive public signal of future technological progress under the baseline Taylor rule in Table 2 and under the optimal Taylor rule in the "fundamental shocks and over-optimism" scenario. The figure shows that the optimal Taylor rule significantly dampens endogenous boom-bust cycles in house prices and thus their destabilizing spillover effects to aggregate economic activity, including inflation and output. The significantly dampened boom-bust cycle suggests that monetary policy plays an important role in causing boom-bust cycles endogenously in this model.

7 Evaluation of optimal target horizons

In this section, I evaluate optimal target horizons implied by the optimal Taylor rules in the two scenarios considered above. Optimal target horizons are defined as the expected numbers of periods which the inflation rate takes to return to the target after shocks under the optimal Taylor rule.[18]

I adopt the simulation procedure of Basant Roi and Mendes (2007):

- Fundamental shocks and signals are jointly drawn from the probability distributions at period 0.
- For each draw, compute the impulse response and the number of periods that inflation takes to return to the steady state value. No shocks are drawn after period 0. The economy is at the deterministic steady state before the realizations of shocks and signals at date 0.
- I consider ± 0.025% band of the steady state quarterly inflation rate as the convergence criterion.
- Iterate the draw by 100,000 times and obtain the distribution of optimal target horizons.

Table 5 shows the mean as well as the range of optimal target horizons in each scenario under the optimal Taylor rule shown in Table 4. Table 5 indicates that without the possibility of endogenous boom-bust cycles in house prices (i.e., "fundamental shocks only"), the average target horizon is within the horizons adopted by the Bank of Canada (6-8 quarters). With the possibility of endogenous boom-bust cycles in house prices (i.e., "fundamental shocks and over-optimism"), optimal target horizons become longer.

While the average target horizon with boom-bust cycles in house prices (17.7 quarters) might seem to be quite long, note that this is the time that the inflation rate takes to return

[18]This definition of optimal target horizons is proposed by Batini and Nelson (2000).

to the very narrow band of the target (within 0.1% error in terms of the annualized inflation rate). If the width of the band is increased to ± 0.3% in terms of the annualized inflation rate, then the average target horizon declines to 7.56 quarters.

The result of longer optimal inflation target horizons is similar to Basant Roi and Mendes (2007). The difference between this paper and Basant Roi and Mendes is in the reason behind the result. In Basant Roi and Mendes' model, exogenous house-price shocks cause persistent inflation dynamics directly. In this paper, the adjustment of the optimal monetary policy rule to boom-bust cycles in house prices prolongs inflation dynamics in response to fundamental shocks.

8 Sensitivity analysis

This section describes sensitivity analysis. In general, the results reported above are found to be robust.

8.1 Loss function in optimal monetary policy analysis

I consider the following variations of the loss function:

$$
\begin{array}{lllllllll}
\cdot & E(\widehat{GDP}_t^{\,2}) & + & & E(\hat{\pi}_t^{*2}) & & & \\
\cdot & E(\widehat{GDP}_t^{\,2}) & + & & E(\hat{\pi}_t^{*2}) & + & 0.5 & E[(\widehat{R}_t - \widehat{R}_{t-1})^2] \\
\cdot & E(\widehat{GDP}_t^{\,2}) & + & 2 & E(\hat{\pi}_t^{*2}) & + & 0.5 & E[(\widehat{R}_t - \widehat{R}_{t-1})^2] \\
\cdot & E(\widehat{GDP}_t^{\,2}) & + & 0.5 & E(\hat{\pi}_t^{*2}) & + & 0.5 & E[(\widehat{R}_t - \widehat{R}_{t-1})^2] \\
\cdot & E(\widehat{GDP}_t^{\,2}) & + & & E(\hat{\pi}_t^{*2}) & + & & E[(\widehat{R}_t - \widehat{R}_{t-1})^2]
\end{array}
$$

The optimal Taylor rule coefficients reported in Table 4 are not very sensitive to these variations of the loss function.

8.2 The duration of over-optimism of households

Basant Roi and Mendes (2007) set the average duration of housing booms to 3 years in their model. In the model in this paper, the duration of housing booms is determined by the lead time of signals, n. Following Basant Roi and Mendes, I change the value of n

from 4 quarters to 12 quarters. The optimal Taylor rule is not sensitive to this change, so that optimal target horizons are not affected, either. The intuition for this insensitivity is that, since the optimal Taylor rule is set to dampen endogenous boom-bust cycles in house prices, the persistence of shocks behind boom-bust cycles in house prices (i.e., over-optimistic expectations of impatient households) does not affect the duration of inflation under the optimal Taylor rule. In contrast, Basant Roi and Mendes report that optimal target horizons are sensitive to the persistence of exogenous housing booms, since exogenous shocks to house prices cause persistent inflation dynamics in their model.

8.3 The frequency of over-optimism of households

For the value of ν_A, I consider a smaller value of ν_A with which the probability that $s_{A,0} > \underline{s}$ is 0.025, where $\hat{q}_t = 0.05$ at the peak of the impulse response to $s_{A,0} = \underline{s}$. The optimal Taylor rule is not sensitive to this change, so that optimal target horizons are not affected much, either. As explained above, the optimal Taylor rule dampens endogenous boom-bust cycles in house prices. Thus the frequency of shocks behind boom-bust cycles in house prices does not much affect optimal target horizons.

8.4 Nominal bonds and expected inflation rate in the monetary policy rule

The baseline model outlined above assume that bonds are indexed and that the nominal policy interest rate responds to the current inflation rate. To confirm the robustness of the results presented in this paper, this section repeats the main analysis, using a model with nominal bonds and an expected inflation rate in the Taylor rule.

Replace the flow-of-funds constraints for patient and impatient households, Equations (14) and (17), respectively, with the following constraints:

$$c'_t + q_t \Delta h'_t + s_t \left(b'_{F,t} - (1 + r_{F,t-1}) b'_{F,t-1}\right) + \frac{\zeta_B}{2}(b'_{F,t})^2 + \frac{\zeta_K}{2}\left(\frac{i'_t}{k'_{t-1}}\right)^2 k'_{t-1} + b'_{D,t}$$
$$= w_t l'_t + r_{k,t} k'_{t-1} + \frac{R_{t-1} b'_{D,t-1}}{\Pi_t} + \Gamma_t, \quad (36)$$

$$c''_t + q_t(h''_t - h''_{t-1}) + b''_{D,t} = w_t l''_t + \frac{R_{t-1} b''_{D,t-1}}{\Pi_t}. \tag{37}$$

Also, replace the collateral constraint (18) and the monetary policy rule (19) with the following equations, respectively:

$$R_t b''_{D,t} \geq -m E'_t \left[\Pi_{t+1} q_{t+1} h''_t \right], \tag{38}$$

$$\widehat{R}_t = \phi_R \widehat{R}_{t-1} + \phi_\pi E'_t \widehat{dCPI}_{t+1} + \phi_Y \widehat{GDP}_t + \psi_t. \tag{39}$$

Assume that the central bank shares expectations with patient households, so that the central bank forms correct expectations when impatient households have ex-post wrong over-optimistic expectations, causing boom-bust cycles in house prices. The calibrated parameters listed in Section 4.8 are re-calibrated with these equations in the same way as the baseline model.[19]

It is found that the results of the baseline model are robust to the introduction of nominal bonds and the expected inflation rate in the Taylor rule. Table 6 shows optimal Taylor rules under the two scenarios considered above, i.e., the case with fundamental shocks and the case with both fundamental shocks and over-optimism of impatient households. As in the baseline model, the optimal Taylor rule for the second scenario has less weight on the expected inflation rate and more weight on output and the previous nominal interest rate. Figure 6 compares the impulse responses under the optimal Taylor rule and the baseline Taylor rule, showing that the optimal Taylor rule significantly dampens boom-bust cycles in house prices by reducing the counter-cyclicality of the nominal policy interest rate.[20] Table 7 indicates that, since the optimal monetary policy rule responds to the inflation rate less, the inflation dynamics after fundamental shocks become longer, prolonging the optimal target horizons.

9 Conclusions

This paper evaluates the optimal Taylor-type monetary policy rule with boom-bust cycles in house prices by using a small open economy model that can replicate the stylized facts of

[19] $(\xi',\ \xi'',\ \zeta_K,\ \rho_{Y_F},\ \sigma_{\epsilon_{Y_F}}) = (28,\ 1e-15,\ 8,\ 0,\ 0.09)$. Other parameter values, including the frequency of over-optimism in the optimal target horizon analysis, are the same as before.

[20] Since the nominal policy interest rate responds to the expected inflation rate, it rises at the peak of the housing boom.

housing-market boom-bust cycles in industrialized countries. The model is also consistent with suggestive evidence on the relationship between heterogeneous household expectations and house prices.

The optimal monetary policy analysis in the model indicates that the central bank should be less responsive to inflation, more responsive to output, and slower to adjust the nominal policy interest rate during boom-bust cycles in house prices. The reason for this result is that since inflation is counter-cyclical during boom-bust cycles in house prices, strong policy reactions to inflation fluctuations would amplify the counter-cyclicality of the nominal policy interest rate, which would enhance boom-bust cycles in house prices and destabilize aggregate economic activity. It is found that the weaker optimal monetary policy responses to inflation fluctuations prolong the optimal inflation target horizons after fundamental shocks.

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Table 1: Regressions of the real house price growth rate on the difference in consumer expectations between young and old households in Canada

Dependent Variable: Δ ln(Real house price).

Sample period: From 1990:04 To 2007:01.

Regressor	Regression 1		Regression 2		Regression 3	
EXP(< 45) - EXP(≥ 45)	-0.000527*	(0.000299)	-0.000538**	(0.000246)	-0.000303	(0.000241)
EXP(< 45) - EXP(≥ 45) (-1)	0.000133	(0.000299)	-0.0000144	(0.000252)	0.0000849	(0.000240)
EXP(< 45) - EXP(≥ 45) (-2)	0.000522*	(0.000299)	0.000614**	(0.000244)	0.000597**	(0.000236)
EXP(< 45) - EXP(≥ 45) (-3)	0.000283	(0.000304)	0.000102	(0.000270)	-0.00000616	(0.000259)
Constant	-0.00779	(0.0126)	0.00143	(0.0162)	-0.00799	(0.0157)
EXP					0.000168	(0.000140)
EXP (-1)					0.000359**	(0.000163)
EXP (-2)					-0.000355**	(0.000168)
EXP (-3)					-0.0000408	(0.000144)
Δ ln(Real house price) (-1)			0.223*	(0.131)	0.169	(0.136)
Δ ln(Real house price) (-2)			-0.0415	(0.123)	0.0657	(0.134)
Δ ln(Real house price) (-3)			0.0190	(0.122)	0.0679	(0.122)
Δ ln(Real GDP)			1.07***	(0.373)	0.821**	(0.378)
Δ ln(Real GDP) (-1)			-0.654	(0.472)	-0.526	(0.443)
Δ ln(Real GDP) (-2)			0.167	(0.469)	0.140	(0.438)
Δ ln(Real GDP) (-3)			0.0535	(0.401)	0.0333	(0.382)
Real interest rate			-0.00407	(0.00318)	-0.00212	(0.00302)
Real interest rate (-1)			0.00745**	(0.00316)	0.00916***	(0.00303)
Real interest rate (-2)			-0.00905***	(0.00315)	-0.00519	(0.00327)
Real interest rate (-3)			-0.00470	(0.00320)	-0.00629*	(0.00330)
R^2	0.175		0.636		0.704	

Notes: The coefficients are estimated by OLS. The standard errors are in parentheses beside the coefficient values. *, ** and *** indicate that the coefficient in question is significant at the 10%, 5%, and 1% levels, respectively. The minus values in parentheses in the first column indicate lagged regressors. 'EXP (< 45)' is the index of household expectations for less than 45 years old and 'EXP (≥ 45)' for 45 years old and over. 'EXP' is the index of household expectations for all ages. The real house price is the Royal LePage House Price Index denominated by the total CPI. The real GDP and the real interest rate are also denominated by the total CPI. The real interest rate is the ex-post interest rate. Δ ln(·) indicates first-order log difference.

Table 2: Baseline parameter values

$(\beta', \beta'') = (0.99, 0.95)$	Time discount rates
$\gamma = 0.0166$	Housing weight in preference
$(\xi', \xi'') = (19.4,\ 1e-14)$	Elasticity of labour supply
$\mu = 0.8$	Patient's fraction of population
$\alpha = 0.26$	Capital share in production
$\zeta_K = 19.8$	Capital adjustment cost
$\delta = 0.025$	Depreciation rate of capital stock
$\theta = 6$	Elasticity of substitution
$\omega = 0.3$	Import share in final domestic demand
$m = 0.75$	Loan-to-value ratio
$\chi = 0.5$	Probability of price-adjustment
$\zeta_B = 1e-06$	Access to international credit markets
$\tau = 0.8$	Elasticity of export demand
$\lambda = 0.218$	Weight on the housing-rent components of CPI
$\kappa = 0.292$	Elasticity of the housing-rent components of CPI
$(\phi_R, \phi_\pi, \phi_Y) = (0.733,\ 0.496,\ 0.014)$	Monetary policy rule
$(\rho_A, \rho_\psi, \rho_{r_F}, \rho_{Y_F}) = (0.93,\ 0,\ 0.43,\ 0)$	AR(1) coefficients for shocks
$(\sigma_{\varepsilon_A}, \sigma_{\varepsilon_\psi}, \sigma_{\varepsilon_{r_F}}, \sigma_{\varepsilon_{Y_F}}) = (0.008,\ 0.0037,\ 0.011,\ 0.11)$	Standard deviations of shocks

Table 3: Second moments targeted by calibration of the capital adjustment cost (ζ_K), the elasticities of labour supply (ξ', ξ''), and the moments of export-demand shocks (ρ_{Y_F}, σ_{Y_F})

a. Standard deviations of the CPI inflation rate and labour supply (relative to de-trended GDP)

	Model	Data
CPI inflation rate	0.14	0.13
Average hours worked for old households (Age≥45)	0.97	0.69
Average hours worked for young households (Age<45)	1.14	1.25
Aggregate hours worked	0.82	0.91

b. Standard deviation and autocorrelation of detrended GDP

	Model	Data
Standard deviation	0.05	0.03
Lag-1 autocorrelation	0.94	0.96

Table 4: Optimal Taylor rules

Loss function	Scenario	Taylor rule coefficients			
		$\widehat{\pi}_t^*$	$\widehat{GDP}_t$	$\hat{R}_{t-1}$	$\hat{q}_t$
$z = 0.5$	fundamental shocks only	0.96	0	0.2	0
$z = 0.5$	fundamental shocks and over-optimism	0.3	0.06	0.9	0
$z = 0$	fundamental shocks only	1.2	0	0	0
$z = 0$	fundamental shocks and over-optimism	0.3	0.06	0.9	0

Notes: The optimal coefficients of the Taylor rule are found by grid search. The intervals of the grid points are 0.02 for ϕ_Y, 0.1 for ϕ_R, 0.1 for ϕ_q and 0.2(1-ϕ_R) for ϕ_π.

Table 5: Optimal target horizons (OTHs)

Scenario	Mean of OTHs	99 percentile of OTHs
fundamental shocks only	5.06	16
fundamental shocks and over-optimism	17.74	37

Notes: The optimal monetary policy rule for each scenario is in Table 4 ($z = 0.5$).

Table 6: Optimal Taylor rules: with nominal bonds and an expected inflation rate in the Taylor rule

Loss function	Scenario	Taylor rule coefficients			
		$\widehat{\pi}_t^*$	$\widehat{GDP}_t$	$\hat{R}_{t-1}$	$\hat{q}_t$
$z = 0.5$	fundamental shocks only	1.28	0	0.2	0
$z = 0.5$	fundamental shocks and over-optimism	0.8	0.1	0.8	0
$z = 0$	fundamental shocks only	1.6	0	0	0
$z = 0$	fundamental shocks and over-optimism	0.8	0.1	0.8	0

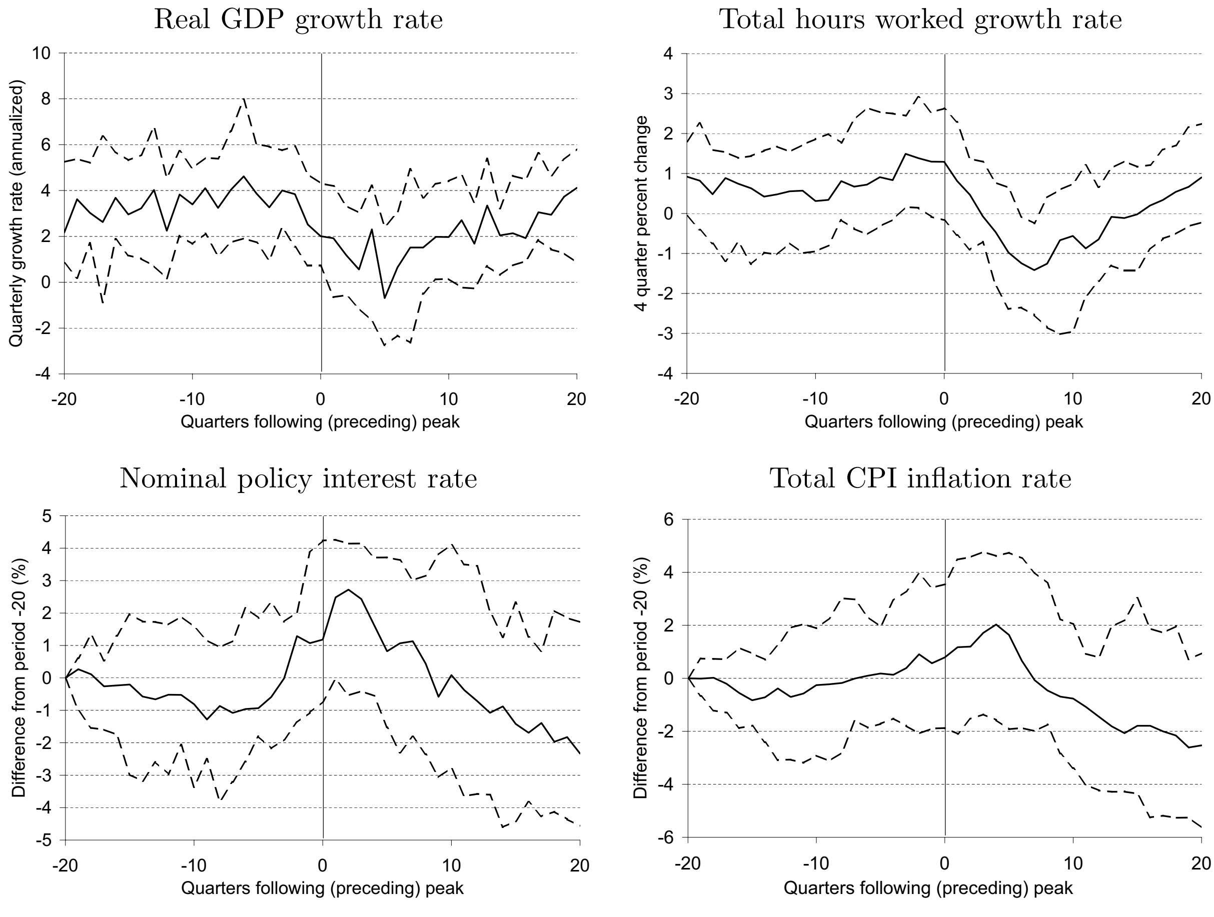
Notes: The optimal coefficients of the Taylor rule are found by grid search. The intervals of the grid points are 0.02 for ϕ_Y, 0.1 for ϕ_R, 0.1 for ϕ_q and 0.2(1-ϕ_R) for ϕ_π.

Table 7: Optimal target horizons (OTHs): with nominal bonds and an expected inflation rate in the Taylor rule

Scenario	Mean of OTHs	99 percentile of OTHs
fundamental shocks only	4.26	7
fundamental shocks and over-optimism	14.57	33

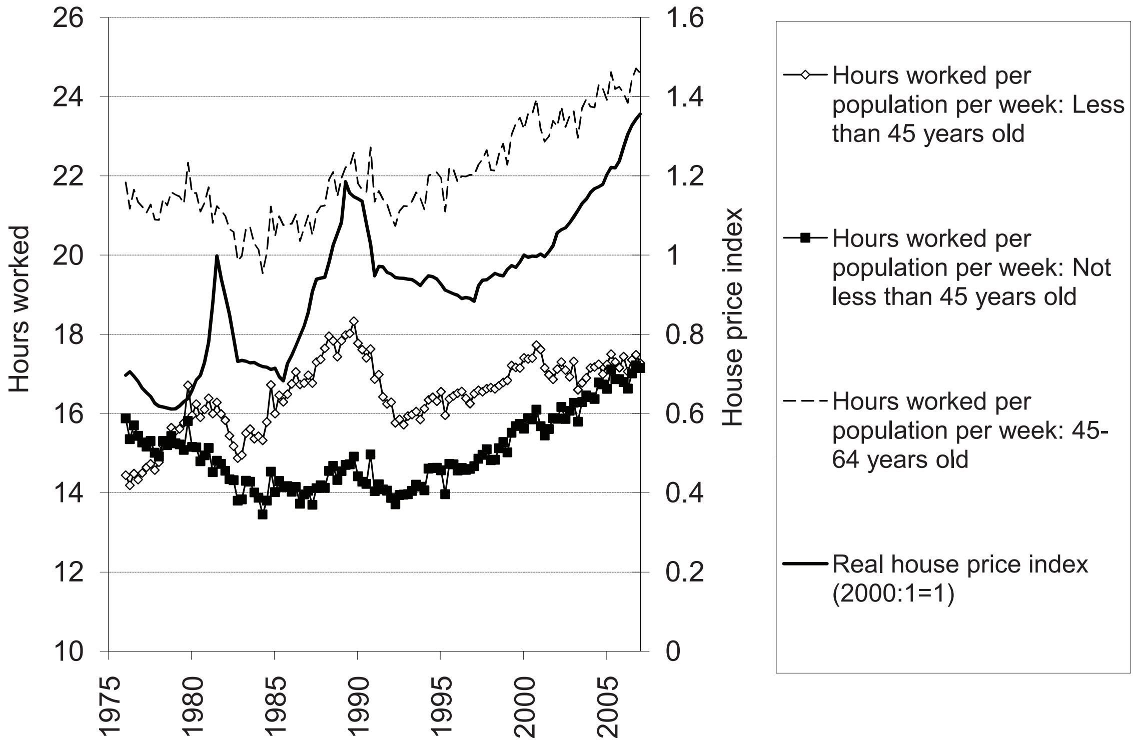
Notes: The optimal monetary policy rule for each scenario is in Table 6 ($z = 0.5$).

Figure 1: Median dynamics around the peaks of housing-market boom-bust cycles in industrialized countries



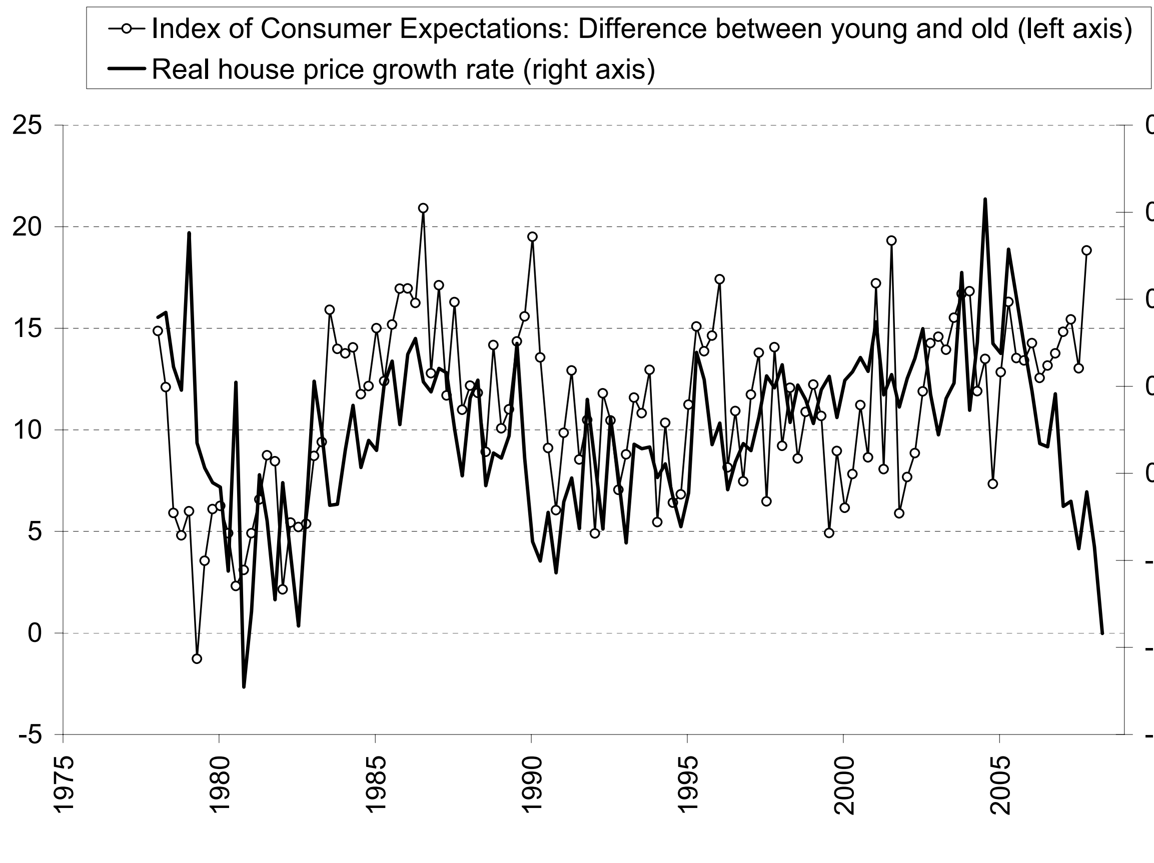
Notes: The solid line is the median of the variable in question in each quarter around the peaks of past housing booms in industrialized countries for 1973 to 2000. The dashed lines below and above the solid line are the first and the third quartiles in each quarter, respectively. Period 0 corresponds to the peaks of housing booms.

Figure 2: House prices and hours worked of different age groups in Canada



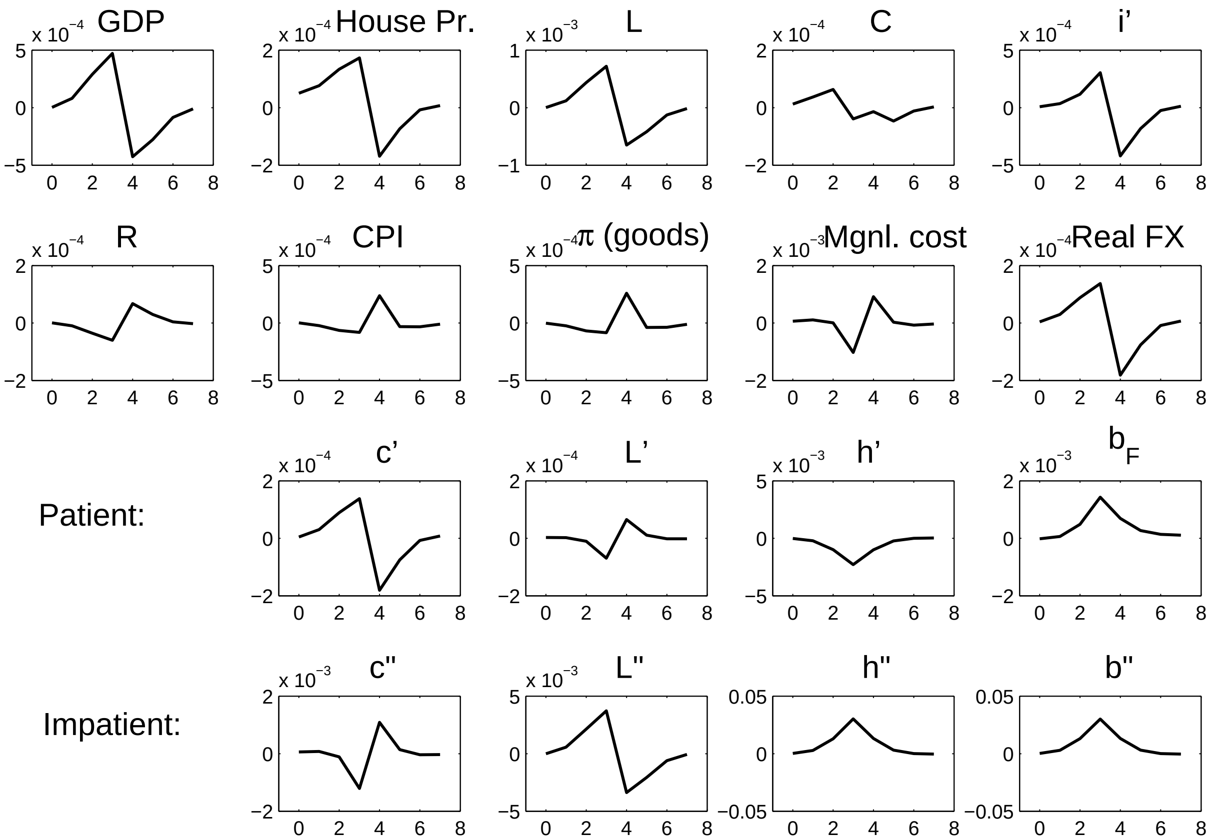
Notes: Labour supply data are from the Statistics Canada. Real house price index is the Royal LePage House Price Index deflated by the total CPI.

Figure 3: Real house price growth rates and differences in the Index of Consumer Expectations between young and old households in U.S.



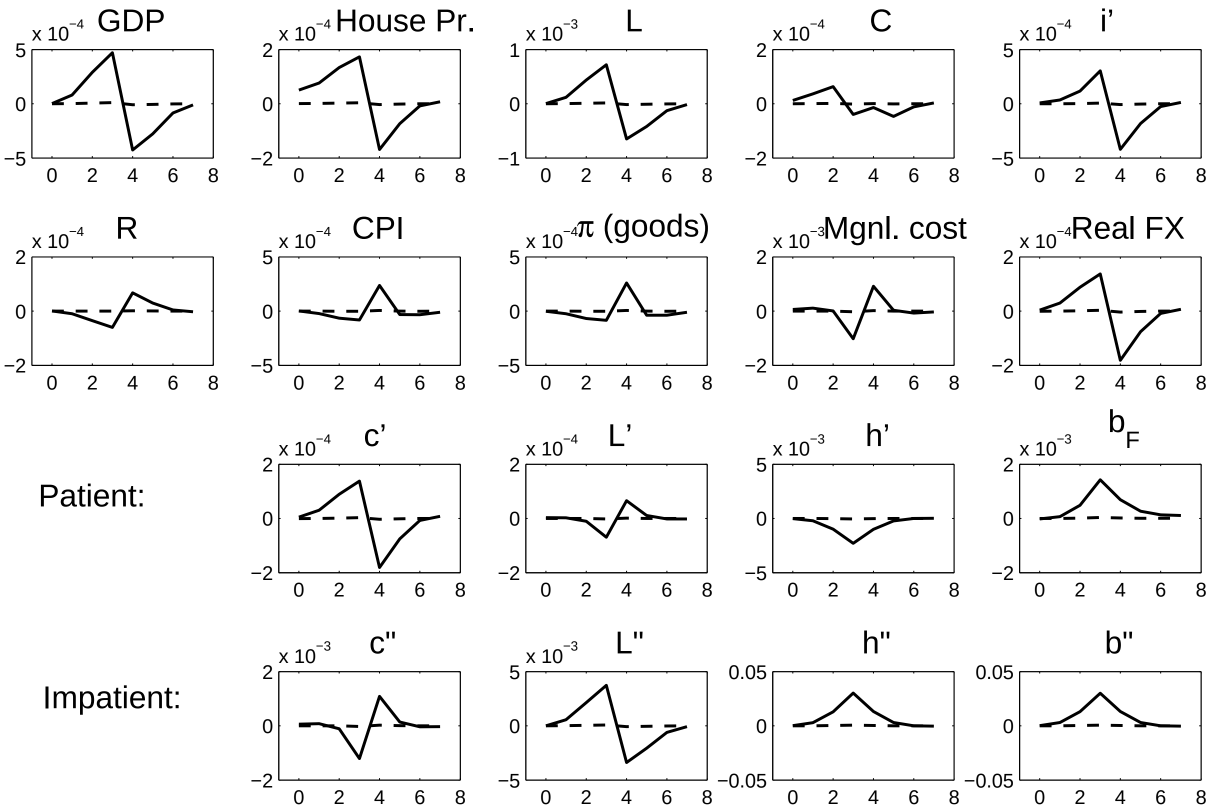
Notes: Positive values of "Difference between young and old" in the figure indicate that young households (44 years old or less) have stronger expectations for future economic conditions than old households (45 years old or more).

Figure 4: Response to over-optimism of impatient households



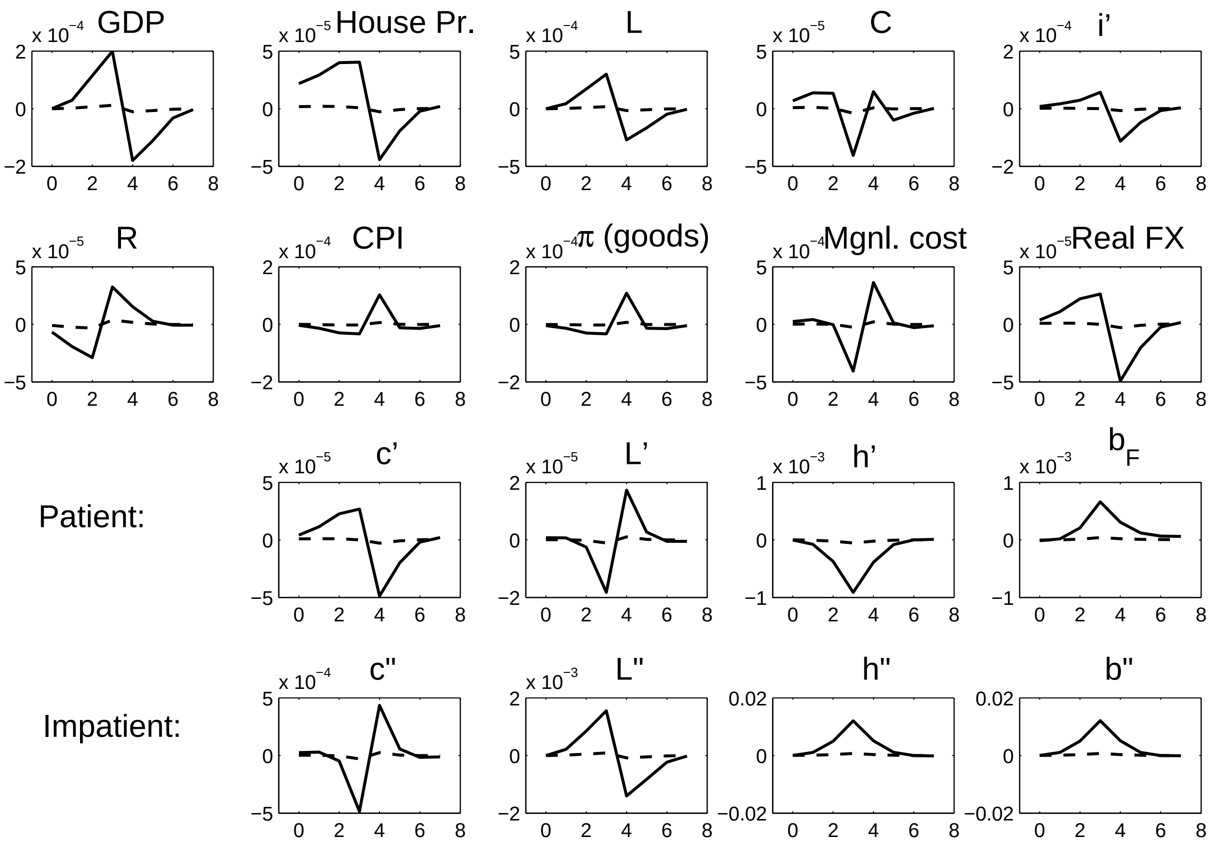
Notes: Figures are % deviations from the deterministic steady-state values except $b_{F,t}$, which is a difference from the steady-state value. The signal is received in period 0, but is not realized in period 4, i.e. $s_{A,0} = \sigma_{\varepsilon A}$ and $\varepsilon_{A,4} = 0$. The economy is at the steady state before period 0. Only the impatient households expect the gain, i.e. $\nu'_A = \infty$ and $\nu''_A = 0$. In the first and second rows, C and L are aggregate consumption and labour supply, respectively. "House Pr." is the house price, q_t, "CPI" is the CPI inflation rate, $dCPI_t$, "Π (goods)" is the inflation rate of final goods, Π_t, "Real FX" is the real exchange rate, s_t, and "Mgnl. cost" is the marginal cost of production, f_t. The third and the forth rows respectively show the actions of the patient and the impatient households.

Figure 5: Responses to over-optimism of impatient households under the baseline Taylor rule and the optimal Taylor rule



Notes: The solid line: the baseline Taylor rule in Table 2. The dashed line: the optimal Taylor rule in the 'fundamental shocks and over-optimism' scenario shown in Table 4. Figures are % deviations from the deterministic steady-state values except $b_{F,t}$, which is a difference from the steady-state value. The signal is received in period 0, but is not realized in period 4, i.e. $s_{A,0} = \sigma_{\varepsilon_A}$ and $\varepsilon_{A,4} = 0$. The economy is at the steady state before period 0. Only the impatient households expect the gain, i.e. $\nu'_A = \infty$ and $\nu''_A = 0$. In the first and second rows, C and L are aggregate consumption and labour supply, respectively. "House Pr." is the house price, q_t, "CPI" is the CPI inflation rate, $dCPI_t$, "Π (goods)" is the inflation rate of final goods, Π_t, "Real FX" is the real exchange rate, s_t, and "Mgnl. cost" is the marginal cost of production, f_t. The third and the forth rows respectively show the actions of the patient and the impatient households.

Figure 6: Responses to over-optimism of impatient households under the baseline Taylor rule and the optimal Taylor rule: with nominal bonds and the expected inflation rate in the Taylor rule



Notes: The solid line: the baseline Taylor rule in Table 2. The dashed line: the optimal Taylor rule in the 'fundamental shocks and over-optimism' scenario shown in Table 6. Figures are % deviations from the deterministic steady-state values except $b_{F,t}$, which is a difference from the steady-state value. The signal is received in period 0, but is not realized in period 4, i.e. $s_{A,0} = \sigma_{\varepsilon_A}$ and $\varepsilon_{A,4} = 0$. The economy is at the steady state before period 0. Only the impatient households expect the gain, i.e. $\nu'_A = \infty$ and $\nu''_A = 0$. In the first and second rows, C and L are aggregate consumption and labour supply, respectively. "House Pr." is the house price, q_t, "CPI" is the CPI inflation rate, $dCPI_t$, "Π (goods)" is the inflation rate of final goods, Π_t, "Real FX" is the real exchange rate, s_t, and "Mgnl. cost" is the marginal cost of production, f_t. The third and the forth rows respectively show the actions of the patient and the impatient households.