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Abstract

This paper studies the steady-state costs of inflation in a general-equilibrium model with real per capita output growth and staggered nominal price and wage contracts.

Our analysis shows that trend inflation has important effects on the economy when combined with nominal contracts and real output growth. Steady-state output and welfare losses are quantitatively important even for low values of trend inflation. Further, we show that nominal wage contracting is quantitatively more important than nominal price contracting in generating these losses. This important result does not arise from price dispersion *per se* but from an effect of nominal output growth on the optimal markup of monopolistically competitive labour suppliers. We also demonstrate that accounting for productivity growth is important for calculating the welfare costs of inflation. Indeed, the presence of two percent productivity growth increases the welfare costs of inflation in our benchmark specification by a factor of four relative to the no-growth case.

JEL classification: E0, E5
Bank classification: Inflation: costs and benefits

Résumé

Les auteurs étudient les coûts de l'inflation en régime permanent au moyen d'un modèle d'équilibre général dans lequel le taux de croissance réel de la production par habitant est positif et qui intègre des contrats échelonnés de prix et de salaires.

L'analyse montre qu'une inflation tendancielle positive a des effets considérables sur l'économie lorsqu'elle est jumelée à des contrats rédigés en termes nominaux et à une croissance réelle de la production. Même faible, l'inflation tendancielle entraîne une baisse substantielle du niveau de la production et du bien-être en régime permanent. Les auteurs démontrent également que cette diminution tient davantage à l'existence de contrats salariaux qu'à celle de contrats de prix. Cet important résultat ne découle pas de la dispersion des prix en soi, mais de l'incidence de la croissance nominale sur le choix de la marge bénéficiaire exigée par les offreurs de travail en situation de concurrence monopolistique. Par ailleurs, les auteurs établissent que la prise en compte de la croissance de la productivité importe pour le calcul des retombées négatives de l'inflation sur le bien-être; selon leurs calculs, un taux de croissance de 2 % de la productivité multiplie par quatre les coûts de l'inflation dans la spécification type du modèle, par rapport au cas où la croissance de la production est nulle.

Classification JEL : E0, E5
Classification de la Banque : Inflation : coûts et avantages

1 Introduction

This paper studies the welfare implications of trend inflation in the presence of nominal contracts and productivity growth. To this end, we formulate a general-equilibrium model that incorporates staggered wage and price contracts and has two key features.

First, we account for steady-state real per capita output growth since most, if not all, industrialized economies exhibit positive trend real output growth. To the best of our knowledge, we are the first to examine the welfare consequences of inflation in an environment where productivity growth and nominal wage and price contracts are present.

Second, our model embeds Taylor (1979) style contracts for both prices *and* wages.[1] Fixed-duration nominal contracts have been found to be better suited for analyzing the welfare costs of inflation than the fixed-hazard rate specifications (such as Calvo (1983)) often used in the literature (see Ascari 2004). Moreover, the inclusion of two sources of nominal rigidity allows us to compare the relative importance of each for the calculation of welfare. As well, an emerging body of research has documented the importance of including rigidities in wage decisions for the analysis of monetary policy and for building data-congruent monetary models. Erceg, Henderson and Levin (2000) examine the welfare costs of different monetary policy rules within a dynamic general-equilibrium environment with staggered price and wage contracts. They report that the presence of staggered nominal wage contracts have important implications for optimal monetary policy. Christiano, Eichenbaum and Evans (2005) have examined the ability of a dynamic general-equilibrium model to match dynamic responses of inflation and real variables to a monetary policy shock. They find, *inter alias*, that sticky wages as opposed to prices is the critical nominal friction for replicating key moments in the data.[2] Huang and Liu (2002) arrive at similar conclusions when they examine the ability of price and wage contracts to generate persistent real effects after a monetary shock.

Despite the possible importance of nominal output growth for our understanding of the steady-state welfare costs of nominal wage and price contracting, there is no work (to the best of our knowledge) that examines these features in concert. Ascari (1998) and Graham and Snower (2004), for instance, study the interaction of money growth rates and nominal wage rigidity in a model that abstracts from sticky prices and real per capita output growth. King and Wolman (1996) and Ascari (2004) examine the steady-state costs of trend inflation in general-equilibrium models that include only nominal goods price rigidity. Finally, Wolman (2006) studies the determinants of optimal inflation in a two-good sticky-price model that encompasses relative productivity growth in the two goods sectors.

We compute output and welfare costs of trend inflation by comparing the

[1] Taylor (1999) emphasizes the importance of including both wage and price rigidities in economic models.

[2] Indeed, a version of the model with only sticky wages performs almost as well as the estimated model with frictions in both nominal wages and prices.

steady state of our model under a given rate of inflation to the one that would occur under the optimal rate. To preview our results, we report (consumption-equivalent) welfare costs of inflation that are sizable. We find that trend inflation and productivity growth together induce output and welfare losses. Notably, the presence of productivity growth leads to a prescription for deflation at a rate very close to the growth rate of the economy as the optimal rate of inflation. Further, the welfare costs of inflation in a growing economy are increased by a factor of four, relative to the case of no growth. The quantitative importance of wage rigidity outweighs price rigidity by a wide margin in the computations of these results. Intuitively, this arises not from relative price dispersion *per se* but from a relatively little discussed effect of trend inflation on the optimal markup of monopolistically competitive labour suppliers which we discuss more fully in the paper. Our results about the quantitative importance of nominal wage rigidity relative to price stickiness for computing optimal inflation echo those obtained in dynamic analyses such as Erceg et al. where a similar type of staggered wage rigidity dominates the structure of optimal (dynamic) monetary policy.[3]

It is important to note that our work, unlike much of the work on optimal monetary policy, does not consider a role for "shoe-leather" costs of inflation. In other words, our model does not generate welfare losses associated with the area under the money demand curve. We make this assumption as it allows us to focus on the novel issue of trend inflation in the presence of nominal wage and price contracts and real productivity growth.

Our paper is organized as follows. Section 2 develops the model used in our simulations while the model's steady state is described in Section 3. Section 4 presents the calibration of the model. Our results and discussion are presented in Section 5. Section 6 then offers concluding remarks.

2 The Model

In this section we describe our model economy and the optimization problems solved by firms and households. The underlying framework is an extension of the model developed by Erceg et al. (2000) to study welfare and optimal business-cycle monetary policy. In particular, we consider a steady-state version of their model extended to account for non-zero trend inflation and steady-state real growth. We then use the modified model to study welfare and optimal long-run monetary policy.

2.1 Firms and Price Setting

Final Good Production

The final good, Y_t, is produced by assembling a continuum of intermediate goods Y_{jt} for $j \in [0,1]$ that are imperfect substitutes with a constant elasticity

[3] Similarly, Wolman (2006) develops a two-good sticky-price model and finds that the more rigid price sector drives the results regarding optimal inflation.

of substitution ε. The production function is constant-returns to scale and is given by

$$Y_t \equiv \left[\int_0^1 Y_{jt}^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}j \right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{1}$$

Aggregate output Y_t is allocated to consumption and investment by households, so we have:

$$C_t + I_t = Y_t, \tag{2}$$

where choices of C_t and I_t are discussed below. The final-good sector is competitive; profit maximization leads to the following input-demand function for the intermediate good j:

$$Y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\varepsilon} Y_t, \tag{3}$$

which represents the economy-wide demand for good j as a function of its relative price P_{jt}/P_t and of aggregate output Y_t. Imposing the zero-profit condition in the sector provides the final-good price index P_t:

$$P_t = \left(\int_0^1 P_{jt}{}^{1-\varepsilon} \mathrm{d}j \right)^{\frac{1}{1-\varepsilon}}. \tag{4}$$

Intermediate Good Production

Intermediate-good producing firm j uses capital K_{jt} and labour L_{jt} to produce Y_{jt} units of good, following a Cobb-Douglas production function *viz.*,

$$Y_{jt} = (A_t L_{jt})^{\alpha} K_{jt}{}^{(1-\alpha)}, \tag{5}$$

where A_t is an economy-wide level of labour-augmenting technology and the parameter α is the share of labour in production.

We assume that the aggregate level of technology is deterministic and grows at the (gross) rate $g \geq 1$ every period, so we have

$$A_t = g^t A_0, \; A_0 = 1. \tag{6}$$

Capital is specific to the firm; each firm's capital input at time t, K_{jt}, is predetermined as a result of past investment decisions. The firm's current-period investment I_{jt} increases its capital stock, following the standard accumulation equation:

$$K_{j,t+1} = (1-\delta)K_{jt} + I_{jt}, \tag{7}$$

with δ the rate of depreciation on capital. In contrast, firms hire labour from a spot market in which they are price takers; in that market, one unit of composite labour is priced at W_t.

To introduce nominal price stickiness into the model, we assume that producers of the differentiated goods set prices according to Taylor-style staggered nominal contracts of fixed duration. Specifically, firms set the price of their good for J quarters and price setting is staggered so that every period, a fraction $1/J$ of firms is resetting prices. Further, the cohorts are fixed, so the same fraction $1/J$ of firms reset prices every J quarters together. Whether they are changing their nominal price or not, however, all firms decide in every period their labour input demand L_{jt} and investment I_{jt}.

Current-period (nominal) profits are paid as dividends D_{jt} to the firm's owners:

$$D_{jt} = P_{jt}Y_{jt} - P_t I_{jt} - W_t L_{jt}. \tag{8}$$

Each intermediate-good producing firm maximizes the discounted, expected sum of future (real) dividends D_{jt}/P_t. The relevant discount factor for dividends is $\beta^k \lambda_{t+k}/\lambda_t$ (the intertemporal rate of substitution of households) because households own all firms.[4] The maximization problem thus consists of choosing prices (every J quarters), labour input, and investment purchases in order to solve the following problem:

$$\max E_t \sum_{k=0}^{\infty} (\beta^k \lambda_{t+k}/\lambda_t) \left[\frac{P_{j,t+k}}{P_{t+k}} Y_{j,t+k} - I_{j,t+k} - \frac{W_{t+k}}{P_{t+k}} L_{j,t+k} \right],$$

with respect to the economy-wide demand for product j (3), the production function (5), and the capital accumulation equation (7).

The first-order condition for labour input is:

$$\frac{W_t}{P_t}\frac{1}{A_t} = s_{jt}\left(\frac{\alpha Y_{jt}}{A_t L_{jt}}\right), \tag{9}$$

where s_{jt} is the real marginal (labour) cost of increasing production, conditional on the stock of installed capital.[5] The first-order condition for investment purchases leads to:

$$1 = \mathbf{E}_t \left\{ (\beta^k \lambda_{t+k}/\lambda_t) \left[s_{j,t+1} \left(\frac{(1-\alpha) Y_{j,t+1}}{K_{j,t+1}} \right) + 1 - \delta \right] \right\}. \tag{10}$$

Anticipating the equilibrium result that the expected intertemporal rate of substitution $\mathbf{E}_t \left\{ \beta^k \lambda_{t+k}/\lambda_t \right\}$ equals the inverse of the real interest rate $1/(1+r_t)$, (10) can be further reduced to

$$r_t + \delta = \mathbf{E}_t \left[s_{j,t+1} \left(\frac{(1-\alpha) Y_{j,t+1}}{K_{j,t+1}} \right) \right], \tag{11}$$

which states that firms invest up to the point where the opportunity cost of the funds engaged in investment purchases equals the expected return from installed capital next period.

[4] As λ_t represents households' marginal utility of income, $\beta^k \lambda_{t+k}/\lambda_t$ measures their valuation of dividends received in period $t+k$.

[5] s_{jt} is the Lagrange multiplier for equation (5).

Finally, the first-order condition for P_{jt}^*, the price chosen by firms that are resetting in period t, is

$$\mathbf{E}_t \sum_{\tau=0}^{J-1} \beta^\tau \lambda_{t+\tau}/\lambda_t \left((1-\epsilon) \left(\frac{P_{jt}^*}{P_{t+\tau}} \right)^{-\varepsilon} P_{t+\tau}^{-1} Y_{t+\tau} + \epsilon s_{j,t+\tau} \left(\frac{P_{jt}^*}{P_{t+\tau}} \right)^{-\varepsilon-1} P_{t+\tau}^{-1} Y_{t+\tau} \right) = 0 \tag{12}$$

After some algebra, this condition yields the following expression:

$$P_{jt}^* = \frac{\varepsilon}{\varepsilon-1} \frac{\mathbf{E}_t \sum_{\tau=0}^{J-1} \beta^\tau \lambda_{t+\tau} P_{t+\tau}^{\varepsilon} Y_{t+\tau} s_{j,t+\tau}}{\mathbf{E}_t \sum_{\tau=0}^{J-1} \beta^\tau \lambda_{t+\tau} P_{t+\tau}^{\varepsilon-1} Y_{t+\tau}}. \tag{13}$$

Since the Taylor pricing structure allocates firms within fixed cohorts through time, firms resetting prices all behave identically. We can therefore omit the j subscript from the optimal price and write P_t^*. In equilibrium, there are now only J different prices in the economy and, following the definition in (4), the aggregate price index P_t becomes:

$$P_t = \left(\frac{1}{J} \sum_{\tau=0}^{J-1} {P_{t-\tau}^*}^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}, \tag{14}$$

where $P_{t-\tau}^*$ is the optimal price of the $1/J$ portion of firms who reset their price τ periods ago.

2.2 Households and Wage Setting

Composite Labour

We assume the presence of a multi-agent, infinitely-lived representative household. Each member i ($i \in [0,1]$) of this extended household supplies L_{it} units of differentiated labour to labour aggregators. These firms assemble composite labour from differentiated, individual-specific labour according to the following aggregation function:

$$L_t \equiv \left[\int_0^1 L_{it}^{\frac{\theta-1}{\theta}} \mathrm{d}i \right]^{\frac{\theta}{\theta-1}}. \tag{15}$$

where θ represents a constant elasticity of substitution. Labour aggregators in turn sell this composite labour to firms, at the economy-wide price (the aggregate wage) W_t. Each unit of differentiated labour L_{it} costs these aggregators W_{it}, which is determined as part of the household's optimization problem described below. Labour aggregators are price takers in both their output and input markets. Cost minimization, thus, leads to the following input demand for type-i labour:

$$L_{it} = \left(\frac{W_{it}}{W_t} \right)^{-\theta} L_t, \tag{16}$$

which is the economy-wide demand for type-i labour, as a function of its relative wage W_{it}/W_t and of aggregate composite labour L_t. This sector is competitive; imposing the resulting zero-profit condition for labour aggregators yields a definition of the aggregate wage index W_t:

$$W_t = \left(\int_0^1 {W_{it}}^{1-\theta} \mathrm{d}i \right)^{\frac{1}{1-\theta}}. \tag{17}$$

Household Optimization

The representative household receives utility from shared consumption, C_t, and experiences disutility from supplying the differentiated labour, L_{it} ($i \in [0,1]$). Expected lifetime utility is

$$\mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \gamma \int_0^1 \frac{{L_{it}}^{1+\upsilon}}{1+\upsilon} \mathrm{d}i \right), \tag{18}$$

where $0 < \beta < 1$ is the discount factor and the parameters γ, and $\upsilon > 0$.

Household revenues consist of labour earnings $W_{it}L_{it}$ from each household member, dividends D_{jt} ($j \in [0,1]$), derived from ownership of intermediate-good producing firms (see below) and bond holdings B_{t-1}. These revenues must be sufficient to cover consumption and new bond purchases, so that the budget constraint is obeyed:

$$P_t C_t + \frac{B_t}{R_t} = \int_0^1 W_{it} L_{it} \mathrm{d}i + \int_0^1 D_{jt} \mathrm{d}j + B_{t-1}, \tag{19}$$

where B_t represents nominal bond holdings and R_t is the (gross) nominal interest rate ($1/R_t$ thus is the price of one-period, nominal discount bonds).

The household is assumed to set the wage for each of its members' differentiated labour input in a staggered fashion, under assumptions similar to those described above for price contracts. In particular, the duration of each wage contract is fixed for I periods and every period, a fraction $1/I$ of wages are reset. The household's intertemporal optimization problem is thus to choose consumption, bond holdings, and wages (when resetting) in order to maximize (18) with respect to aggregate demand for each type of labour (16) and the budget constraint (19).

The first-order conditions for C_t and B_t are standard and yield

$$\lambda_t = C_t^{-1}; \tag{20}$$

$$1/R_t = \beta \mathbf{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t \pi_{t+1}} \right]; \tag{21}$$

where π_t represents the gross rate of growth in the aggregate price index ($\pi_t \equiv P_t/P_{t-1}$) and λ_t is the household's marginal utility of (real) income, that is, the

Lagrange multiplier for equation (19). The first-order condition for wage, W_{it}, when resetting, is

$$\mathbf{E}_t \sum_{s=0}^{I-1} \beta^s \left[\theta\gamma \left(\frac{W_{it}^*}{W_{t+s}} \right)^{-\theta(1+\upsilon)} (W_{it}^*)^{-1} L_{t+s}^{1+\upsilon} + (1-\theta)\lambda_{t+s} \left(\frac{W_{it}^*}{W_{t+s}} \right)^{-\theta} P_{t+s}^{-1} L_{t+s} \right] = 0; \tag{22}$$

where W_{jt}^* is the optimal choice for the wage of type-i labour when resetting. After some algebra, this expression yields

$$W_{it}^* = \left(\frac{\theta\gamma}{\theta-1} \frac{\mathbf{E}_t \sum_{s=0}^{I-1} \beta^s \left(W_{t+s}^{\theta} L_{t+s} \right)^{\upsilon+1}}{\mathbf{E}_t \sum_{s=0}^{I-1} \beta^s \lambda_{t+s} \left(W_{t+s}^{\theta} L_{t+s}/P_{t+s} \right)} \right)^{\frac{1}{1+\theta\upsilon}}. \tag{23}$$

Since wage-setting cohorts are fixed through time, all wages being reset are fixed equally and the i subscript on the optimal choice can be eliminated; we therefore denote it W_t^*. There are now only I different wages in the economy and, following the definition in (17), the aggregate wage index W_t becomes:

$$W_t = \left(\frac{1}{I} \sum_{s=0}^{I-1} \left(W_{t-s}^* \right)^{1-\theta} \right)^{\frac{1}{1-\theta}} \tag{24}$$

where W_{t-s}^* is the period-t wage applying to the $1/I$ portion of differentiated labour that was reset s periods ago.

2.3 Monetary Policy

Monetary policy consists of a constant, targeted rate of (steady-state) inflation π. This rate is to be consistent with equilibrium decisions of firms, so that in the (balanced growth) steady state of the economy, we have

$$\pi_t \equiv \frac{P_t}{P_{t-1}} = \pi, \ \forall t. \tag{25}$$

This monetary policy can be interpreted as implemented by a interest rate targeting rule or a money growth rule. Since we focus our analysis on the balanced-growth steady-state of the economy, either interpretation is valid.[6] There is no government taxation or spending in the model.

2.4 Price and Wage Dispersion

The extent of dispersion in price and wage present in the economy can potentially be an important factor in understanding our results. Therefore, it is useful to define dispersion in prices of intermediate-goods as

[6] To interpret monetary policy through a money-growth rule, we need to reserve a role for money in the model. This can be done, without changing any of our behavioral results, by reserving a role for money balances in utility (in a separable form relative to consumption and labour).

$$\Delta_{pt} = \frac{1}{J}\sum_{\tau=0}^{J-1}\left(\frac{P_{\tau t}}{P_t}\right)^{-\varepsilon}, \tag{26}$$

and the dispersion in wages of differentiated labour as

$$\Delta_{wt} = \frac{1}{I}\sum_{s=0}^{I-1}\left(\frac{W_{st}}{W_t}\right)^{-\theta}. \tag{27}$$

2.5 Equilibrium

The equilibrium to this economy consists of allocations and prices such that households, labour aggregators, final-good producing firms and intermediate-good producing firms optimize, the monetary policy rule (25) is satisfied, and all markets clear.

We focus on cohort-symmetric equilibria in which all resetting, intermediate-good producing firms choose the same price P^*_{jt} for the good they produce. As described above, this implies that only J different prices coexist in equilibrium at any time. It also implies that the firms within each price-setting cohort are characterized by identical demand for their product (so we can write $Y_{\tau t},\ \tau = 0, ..., J-1$) and also identical input mix to satisfy demand ($L_{\tau t}, K_{\tau t},\ \tau = 0, ..., J-1$).

This symmetry extends to wage choices. All wages W^*_{it} reset in a given period are equal and in equilibrium, the economy is characterized by I different wages ($W_{st},\ s = 0, ..., I-1$) and corresponding, differentiated labour demand ($L_{st},\ s = 0, ..., I-1$).

Market Clearing

For the labour market to clear, total supply of the composite labour produced by labour aggregators must equal total demand arising from intermediate-good producing firms. Further, total investment purchases by these firms must cover aggregate investment. We thus have the following market-clearing conditions:

$$L_t \equiv \int_0^1 L_{jt}\mathrm{d}j;\quad I_t \equiv \int_0^1 I_{jt}\mathrm{d}j. \tag{28}$$

Data Transformations

Allowing for a deterministic trend in the level of aggregate technology – recall (6) – implies that a balanced-growth path exists where aggregate output Y_t, aggregate consumption C_t, aggregate investment I_t and the real wage W_t/P_t all grow at the same rate g as technology. We transform these variables to induce stationarity; this allows for a solution to the steady-state values of these transformed variables:

$$y_t \equiv \frac{Y_t}{A_t};\quad c_t \equiv \frac{C_t}{A_t};\quad i_t \equiv \frac{I_t}{A_t};\quad w_t \equiv \frac{W_t}{P_t}\frac{1}{A_t}. \tag{29}$$

Steady-state growth is also present at the level of intermediate-good producing firms. That is, their production $Y_{\tau t}$, $\tau = 0, ..., J-1$ also grows at the rate of technology. Correspondingly, individual investment purchases, as well as their installed capital stock, also share this growth rate. Again, stationarity is induced by dividing all these variables by the level of technology, so we have:

$$y_{\tau t} \equiv \frac{Y_{\tau t}}{A_t};\ \ i_{\tau t} \equiv \frac{I_{\tau t}}{A_t};\ \ k_{\tau t} \equiv \frac{K_{\tau t}}{A_t};\ \ \tau = 0, ..., J-1. \tag{30}$$

Finally, trend inflation means that newly-set prices P_t^* and newly-set wages W_t^* are growing along the balanced-growth path. To induce stationarity and facilitate the computation of the deterministic steady state of the economy, we define the choice of resetting agents, relative to economy-wide counterparts, as follows:

$$p_t^* \equiv \frac{P_t^*}{P_t}\ \ w_t^* \equiv \frac{W_t^*}{W_t}, \tag{31}$$

and modify equations (13), (14), (23) and (24) accordingly. Details on variable transformations and their impact on the model's equations are available from the authors.

3 The steady state of the economy

Intermediate-good Production

Computing the deterministic steady-state of the economy first involves rewriting production of each intermediate-good producing firm as follows:

$$y_\tau = {L_\tau}^\alpha {k_\tau}^{(1-\alpha)},\ \ \tau = 0, ..., J-1. \tag{32}$$

whereas the first-order conditions for labour choice (9) and for capital accumulation (11), evaluated at steady state, yield

$$L_\tau = \alpha\left(\frac{s_\tau}{w}\right) y_\tau,\ \ \tau = 0, ..., J-1; \tag{33}$$

$$k_\tau = (1-\alpha)\left(\frac{s_\tau}{r+\delta}\right) y_\tau,\ \ \tau = 0, ..., J-1. \tag{34}$$

Combining the last three expressions yields the following determination for the marginal cost s_τ:

$$s_\tau = s = \left(\frac{w}{\alpha}\right)^\alpha \left(\frac{r+\delta}{1-\alpha}\right)^{1-\alpha}; \tag{35}$$

note that in our steady state, marginal cost is equated across all firms, irrespective of the last time they reset their price.

Next we aggregate (32) to (34) by defining the following, simple-sum, production and input aggregates:

$$y^* \equiv \sum_{\tau=0}^{J-1} y_\tau; \quad k^* \equiv \sum_{\tau=0}^{J-1} k_\tau; \quad L^* \equiv \sum_{\tau=0}^{J-1} L_\tau. \tag{36}$$

Since each firm uses the same capital-labour ratio and marginal cost is constant across firms, simple algebra shows that (32) to (34) map over to the aggregate level:

$$y^* = L^{*\alpha} k^{*(1-\alpha)}; \tag{37}$$

$$L^* = \alpha \left(\frac{s}{w}\right) y^*; \tag{38}$$

$$k^* = (1-\alpha)\left(\frac{s}{r+\delta}\right) y^*. \tag{39}$$

Prices

The definition of the aggregate price index in (14), evaluated at steady state, allows us to solve for the newly-set price p^* as a function of steady-state inflation π and model parameters ϵ and J:

$$p^* = \left(\frac{1}{J}\sum_{\tau=0}^{J-1} \pi^{\tau(\varepsilon-1)}\right)^{\frac{1}{\epsilon-1}}. \tag{40}$$

In turn, inserting this value for p^* into the optimal pricing equation (13) at steady state leads to a solution for marginal cost s as a function of model parameters:

$$s = \frac{\varepsilon}{\varepsilon-1} \frac{\left(\frac{1}{J}\sum_{\tau=0}^{J-1} \pi^{\tau(\varepsilon-1)}\right)^{\frac{1}{\epsilon-1}} \sum_{\tau=0}^{J-1} \left(\beta\pi^{\varepsilon-1} g\right)^\tau}{\sum_{\tau=0}^{J-1} \left(\beta\pi^{\varepsilon} g\right)^\tau}. \tag{41}$$

The expression in (26) defining price dispersion across intermediate goods becomes the following in steady state:

$$\Delta_p = \frac{\frac{1}{J}\sum_{\tau=0}^{J-1} \pi^{\tau\varepsilon}}{\left(\frac{1}{J}\sum_{\tau=0}^{J-1} \pi^{\tau(\varepsilon-1)}\right)^{\frac{1}{\epsilon-1}}}. \tag{42}$$

Using this expression, the market demand for each intermediate-good (3), and the definition of y^* in (36) above establishes the following relationship between y and y^*:

$$y^* = \Delta_p y; \tag{43}$$

Note from this last expression that a high level y^* of production at the firm level is not necessarily translated into high levels of GDP (y), but could be instead dissipated into price dispersion Δ_p. This arises because efficient production of the final good requires equal amounts of each intermediate good –recall (1)– whereas in practice these goods will be used in production according to their relative price as in (3). Section 5 examines the quantitative importance of this 'dissipating' effect.

Wages

On the labour market side, the steady-state wage index, (24), implies that we can solve for the optimal relative wage, w^*, as a function of model parameter only, that is,

$$w^* = \left(\frac{1}{I} \sum_{s=0}^{I-1} (\pi g)^{s(\theta-1)} \right)^{\frac{1}{\theta-1}}. \tag{44}$$

Using similar logic as above, inserting this expression into (23), the condition for optimal choice of wage, leads to the following expression, which links the real wage w, aggregate consumption c, and aggregate composite labour L:

$$w = \frac{\theta}{\theta-1} \gamma c L^{\upsilon} \frac{\sum_{s=0}^{I-1} \beta^s (\pi g)^{s\theta(1+\upsilon)}}{\sum_{s=0}^{I-1} \beta^s (\pi g)^{s(\theta-1)} \left(\frac{1}{I} \sum_{s=0}^{I-1} (\pi g)^{s(\theta-1)} \right)^{\frac{1+\theta\upsilon}{\theta-1}}}. \tag{45}$$

Further, the expression in (27), defining wage dispersion, becomes the following in steady state:

$$\Delta_w = \frac{\frac{1}{I} \sum_{s=0}^{I-1} (\pi g)^{s\theta}}{\left(\frac{1}{I} \sum_{s=0}^{I-1} (\pi g)^{s(\theta-1)} \right)^{\frac{1}{\theta-1}}}. \tag{46}$$

Finally, using this last expression, the market demand (16) for each labour variety, and definition of L^* in (36) above establishes the following relationship between L and L^*:

$$L^* = \Delta_w L. \tag{47}$$

Again, note from this expression that high levels of labour effort at the individual levels, in L^*, are not necessarily translated to high levels of composite labour L but could instead be dissipated through dispersion in wage prices Δ_w.

In the following sections, the model parameters are calibrated, its steady state computed and the effects of trend inflation on the economy are analyzed.

4 Calibration

In order to compute the steady state of the economy, numerical values are assigned to parameters. On the production side, the labour share parameter, α, is set to 0.64 and the parameter δ, representing capital depreciation, is set to 0.035, implying an annual rate of depreciation around 13 percent. The per capita growth rate of the economy, g, is set to 1.005 or an annualized rate of output growth of 2 percent in our benchmark case. The elasticity of substitution parameters for intermediate goods (ε) and differentiated labour (θ) are both set to 11. These values imply steady-state markups of 10 percent in both goods and labour markets in economies with zero inflation and no growth, and are consistent with the finding reported in Basu (1996) and Basu and Fernald (1997). The disutility of labour parameter, ν, is initially assigned a value of one, as in Hornstein and Wolman (2005). The length of price contracts J is set to 2, based on results reported in Bils and Klenow (2004). Finally, the length of wage contracts I is set equal to 4, as in Erceg et al. (2000) and Huang and Liu (2002). Further support is found by Taylor (1999) who conducts a review of the empirical literature and concludes that the average frequency of wage changes is about one year, and Christiano et al. (2005) who estimate a dynamic general-equilibrium model and find average wage changes to be about 3.3 quarters.

Given the uncertainty around some of these parameter values, we will also examine the sensitivity of our results to alternative calibrations. We view the current calibration as capturing the midpoint of the ranges for the parameters under consideration.

5 Results

This section presents the main findings of the paper and is structured as follows. The next subsection reports the optimal rate of long-run inflation. Subsection 5.2 then compares the effect of different levels of positive inflation on the economy while intuition for the results is offered in subsection 5.3. Subsection 5.4 presents results from a set of parameter perturbation experiments.

5.1 Levels of Optimal Inflation

A natural question that arises for our analysis is: What is the optimal level of inflation admitted by the model described above? We define the optimal level of inflation as the rate of inflation that maximizes the (steady-state) welfare of the household. We consider two versions of our model: (i) A no growth version ($g = 1$); and (ii) a version with annual real per capita output growth to equal 2 percent (or $g = 1.005$). The former allows for comparison with previous studies that do not incorporate steady-state growth in their analysis whereas the latter will serve to emphasize the importance of output growth for determining the optimal level of inflation.

The no-growth version of the model admits an optimal inflation rate of 0.04 percent (on an annualized basis). This result is similar in spirit to that reported in Wolman (2001). Wolman studies optimal inflation in a sticky-price model and argues that a small degree of inflation is necessary to offset some of the monopoly distortion within his model. The result is also akin to that in King and Wolman (1996), who report optimal inflation to be greater than that prescribed by the Friedman rule.

The optimal inflation rate is significantly lower in the model version with growth, approximately -1.9 percent per annum. This result is explained largely by the effect of trend inflation offsetting the wage markup distortion. Under the model specification with growth, the wage markup distortion is eliminated when $\pi/g = 1$ or a deflation rate of 1.96 percent per annum. This deflation, however, does not eliminate the price markup distortion which requires near-zero inflation. Since the price markup distortion is not as important quantitatively, the optimal annual inflation rate settles at -1.9 percent, indicating that offsetting the wage markup distortion is the most welfare-enhancing policy. This conclusion is the steady-state analog to the dynamic monetary policy result reported in Erceg et al. (2000), in which the objective of eliminating the effects of wage rigidities is the prominent task for optimal monetary policy. Interestingly, we arrive at a Friedman-rule-like result for optimal inflation in a model without a transactions role for money. Consistent with the standard Friedman-rule prescription, our results call for deflation to offset another (non-monetary) feature – the inability of nominal wages to freely adjust – of the economy.

5.2 Output and Welfare Costs of Inflation

The analysis focuses on the steady state of the economy using the calibration described above. Throughout, we report results by pairing the steady state computed with a given annual net inflation rate to the one arising under the optimal rate discussed above. We examine rates of inflation up to the 4 percent mark, which covers the empirical range of trend inflation in many industrialized economies over the last decade.

Figure 1 depicts the output costs of trend inflation. The numbers along the horizontal axis correspond to the rate of steady-state inflation measured on a net, annualized basis while the value on the vertical axis represents the loss in output arising from the corresponding rate of inflation, relative to the GDP level occurring under optimal inflation.[7] The graphs can thus be interpreted as presenting the output that would be gained by an economy moving from a currently observed average inflation rate of, say, 2 percent to the optimal.[8] It is apparent that the effects of inflation on the economy are sizeable. Although the effects are relatively modest at very low rates of positive trend inflation, an economy with 2 percent inflation nonetheless admits output that is 0.77 percent lower than the optimal-inflation economy. At 4 percent inflation, distortionary effects have increased to a point where output is reduced by 1.71 percent relative to its optimal counterpart.

Figure 2 reveals that similar, sizeable effects of trend inflation are present when welfare measures are presented. The figure presents the (consumption-

[7] Specifically, the quantity on the vertical axis is $100(1 - GDP_{\pi}/GDP_{\pi=\pi^*})$, where π^* represents the optimal rate of inflation.

[8] This interpretation of the graph lessens the severity that Lucas-critique arguments can have on our results. For example, the fact that we keep the length of wage contracts fixed throughout is not important if the experiment involves comparing the current, observed situation (with, say, 2 percent average inflation) to the optimal (where the length of wage contracts is largely irrelevant).

equivalent) welfare loss of trend inflation, measured by comparing steady states.[9] Again, the effects start out being relatively modest, although a 2 percent rate of trend inflation already causes welfare losses amounting to around 0.8 percent of consumption. At 4 percent annual inflation, the welfare losses have increased to represent around 1.8 percent of consumption.

In order to identify the source of these sizeable effects of trend inflation, Figure 3 and 4 report markup and relative price dispersion as a function of π in the market for intermediate goods (Figure 3) and in the market for labour (Figure 4). Figure 3 shows that distortions in the market for intermediate goods are minimized at (essentially) zero inflation. Inflation rates higher, but also lower, than this figure exacerbate the distortions in the goods market, and both markups and price dispersion increase. However, the effects are quantitatively very small. The top panel shows the markup rising with positive trend inflation, but even at 4 percent inflation this upward movement is extremely small and the markup distortion is barely distinguishable (around 10.014 percent) from its level in the zero inflation benchmark (10.0 percent). Similarly, the price dispersion measure (Δ_p), which is exactly one in the zero inflation benchmark, increases smoothly in trend inflation but the magnitude of the increase is again very small. Even under 4 percent trend inflation, Δ_p increases by less than 0.015 *percent* relative to its optimal-inflation benchmark.

Figure 4, analyzing the labour market, displays markedly different quantitative patterns. The top panel of the figure shows that the markup in the market for labour increases significantly as inflation increases. From its benchmark level of 10 percent at the optimal rate, the labour wage markup quickly climbs to 12 percent for 2 percent annual inflation and to 14 percent when annual inflation is 4 percent. Inflation, therefore, significantly affects the decision of household wage setters. As inflation increases, households choose increasingly greater markups of wage over (utility) marginal costs, pricing themselves out of the market (to avoid fluctuating demand for their labour services) and, in doing so, exacerbating the distortion that arises from the monopolistic nature of labour supply. We discuss this effect in more detail in the next section. As a result of these responses of wage setters to inflation, relative wage distortions remains a quantitatively modest phenomenon, as evidenced by the bottom panel of Figure 4. Overall, these results accord well with the conclusions reported in Christiano et al. (2005) and Huang and Liu (2002) regarding the crucial role of nominal wage rigidity for understanding the effects of monetary policy. The results also support the conclusion in Erceg et al. (2000) regarding the importance of staggered nominal wage contracts for economic welfare as well as understanding optimal monetary policy.

5.3 Discussion

Firms and households in this economy make constrained choices: Although they would like to re-optimize every period, the price they choose today will

[9] Appendix A details how the measure for welfare loss is constructed.

prevail for the length of the contract and they are thus unable to achieve their desired markups every period. In an environment with positive trend inflation, the relative price chosen by a price (wage) setter will decrease as the contract progresses. In the early periods of the contract, relative prices will tend to be higher, resulting in lower demand but higher markup. In the later stages of a contract, the relative price will be lower, delivering greater demand but lower per unit markup.

To develop intuition for our results, Figure 5 plots profit for a given intermediate-good producer (full line), and utility for a given wage setter (dashed line) from our model.[10] The figure highlights an important difference in the shape of the profit and utility functions around the optimum, *viz.*, the utility function of wage setters is strongly asymmetrical around the optimum while the profit function of intermediate-good producing firms is not. This asymmetry around optimal choices is the key to understanding the relationship between markups and positive trend inflation as well as differences in the magnitude of effects arising from nominal price and wage contracts. The asymmetry arises from our assumption regarding the labour and goods aggregator. In particular, we use an aggregator developed by Dixit and Stiglitz (1977) that admits non-linear demand curves for labour and goods and, as an artifact, asymmetry in the profit and utility functions. The degree of asymmetry is then controlled by our calibration.[11] Owing to their inability to reoptimize, firms and households find themselves at different points along these profit or utility functions over the course of their contracts; the shape of these functions is thus a key factor determining pricing behaviour. Consider first the household's problem. Lower relative wages lead to high levels of labour demand and rapidly decreasing utility (recall (18)). To avoid these periods of low relative wages and high demand for their labour, households increase their markup, so that the range of relative prices over the course of their contract shifts to the right of the optimum. In contrast, this mechanism has a quantitatively negligible effect on the pricing behaviour of firms as firm-specific marginal cost is constant even in periods of relatively low price and high demand for their goods. Hence, the region of rapidly decreasing profits, at the left of the optimum in Figure 5, is considerably more pronounced for wage setters than for price setters.

To summarize, the asymmetry implies that firms (households) choose a higher markup of price over marginal cost (wage over the marginal rate of substitution) when inflation is positive, because profits (utility) decline faster with a markup that is below the optimum than with a markup above the optimum. This effect is quantitatively sizeable for households and wages, but very modest for firms and prices. Reducing trend inflation reduces the impact of this asymmetry on markups. Said otherwise, higher trend inflation, by increasing markups, increases the monopoly distortion that is already present in the model

[10] More specifically, the figure depicts the objective functions in a one-period, unconstrained maximization problem for a monopolist whose demand and cost structures are similar to those facing firms (full lines) and households (dashed lines) in our economy.

[11] In the senstivity analysis below, we consider the case where the degree of asymmetry in the goods and labour markets is identical (that is, $\nu = 0$).

owing to imperfect competition. Moreover, in the presence of trend inflation and nominal wage contracts, each non-adjusting wage cohort sees its relative real wage become lower as the price level rises in each period. This cohort receives relatively higher demand for its labour than the other higher-priced cohorts and average real wage across cohorts falls. This substitution towards the lower cost labour (and away from higher wage labour) is, however, inefficient because labour is imperfectly substitutable. Thus, the greater degree of labour churning associated with higher rates of trend inflation leads to lower average labour productivity and lower steady-state output.[12]

5.4 Sensitivity Analysis

Tables 1 and 2 present results from a set of parameter perturbation experiments. These experiments repeat the analysis presented above in Figures 1 and 2 for each new set of parameter values. In Table 1 we present the economic costs of 2 percent annual inflation and in Table 2 the same costs under 4 percent annual inflation (Table 2). The first column in each table describes the parameter that is modified and its new value. The second column displays the optimal inflation rate according to the specification analyzed and serves as the point of comparison to compute the costs of inflation. Finally, the values in the third and fourth columns represent losses in output (column 3) and welfare (column 4), relative to the optimal inflation case. In order to facilitate comparison of the results, the first row reproduces the output and welfare losses arising from our benchmark calibration.

The influence of technology growth

The second and third rows of the tables consider different rates of steady-state productivity growth. It is readily apparent that real per capita growth is an important element in understanding the costs of nominal contracting. In particular, the tables reveal that in an environment with higher rates of growth, the output and welfare costs of trend inflation increase considerably. Indeed, with steady-state inflation at 2 percent, an increase in real per capita growth, from 2 percent (the benchmark) to 3 percent, worsens output and welfare losses of inflation by almost 60 percent, from 0.8 percent to around 1.2 percent.

In the no-growth version of our model, by contrast, output and welfare costs of trend inflation are reduced by a factor of around four, to 0.2 percent. The intuition underlying this interaction between technological and nominal (price) growth is very similar that described above for nominal contracts. Recall that households do not want their contract wage to become "too low" relative to

[12] We note that under full indexation, output is independent of trend inflation. Full indexation eliminates the markup distortions leading price and wage setters to choose an optimal markup instead of a distortionary markup beyond the optimum. Under the no-growth case, wages and prices would need to be fully indexed to lagged or steady-state inflation, or some combination of the two to offset the markup distortions. Under the postive growth case, prices and wages would need to be fully indexed to inflation and productivity growth to induce output growth to be independent of trend inflation.

others in the economy because it leads to rapidly decreasing utility. In a no-growth environment, inflation is the only potential source of this costly wage dispersion so trend inflation at 2 percent moves the economy only modestly away from the no-dispersion optimum resulting in relatively small output and welfare losses. When growth is present, however, wage setters consider not only the effect of trend inflation but also that of technology growth on their relative wage over the duration of their contract. In such an environment, trend inflation and productivity growth work in the same direction to push utility further to the left where utility is declining rapidly, thus exacerbating the distortion arising from only inflation. As a result, wage markups react more strongly to trend inflation leading to a greater degree of monopoly distortion than is present in the no-growth model. Productivity growth has virtually no effect on the goods market since prices, unlike wages, do not rise with productivity growth. Finally, note from Table 2 that the sensitivity of our results to the inclusion of growth is not as acute when trend inflation increases to 4 percent as the effects arising from higher inflation start to dominate the effects from productivity growth.

The influence of utility parameters

The following two rows of Table 1 and Table 2 explore the importance of curvature in the disutility of labour. First, the tables show that a rise in the preference for smooth labour – a rise of ν from its benchmark value of 1 to 2 – increases significantly the output and welfare costs of trend inflation. The output costs of 2 percent inflation almost double, from 0.8 percent of GDP in the benchmark to 1.5 percent. Such a strong effects is to be expected from our analysis of Figure 5. Increasing the value of ν exacerbates the asymmetry of (utility) profits around the optimum, leading households to increase their markups when they reoptimize. Correspondingly, completely eliminating the desire for smoothness of labour (setting ν to 0) leads to a sharp decline in the costs of trend inflation. Table 1 shows, for example, that in this extreme case trend inflation at an annual rate of 2 percent carries output costs of less than 0.1 percent. Overall, these two rows of Table 1 and Table 2 underline the key importance of adequately modelling the interaction between nominal contracting and the labour market to measure the costs of trend inflation.

The influence of labour market parameters

Rows 6 to 9 of the tables report further evidence supporting the importance of nominal wage rigidity for our results. Row 6 reports the consequences of decreasing the elasticity of substitution between differentiated labour varieties, from $\theta = 11$ (benchmark) to $\theta = 5$. This is equivalent to increasing the markup from 10 to 25 percent in a zero-inflation, zero-growth economy. In this new environment, the monopoly power of households is strong and they maintain already higher markups. Inflation brings an additional distortion by pushing the markup even further away, but this effect is now subdued, as the original markup was much higher than in the benchmark case. As a result, the output costs of 2 percent inflation are noticeably smaller, at around 0.2 percent.

Rows 7 and 8 report experiments where the length of wage contracts is modified, first increasing it from 4 to 6 quarters (row 7) and then reducing it from 4 quarters to 1 (row 8). These results demonstrate that wage contracts is one important key to our results (when $v > 0$), as welfare and output losses increase substantially in the longer wage contract case and fall greatly in the no-wage contract calibration. Finally, the duration of price and wage contract are set to an equal length of four quarters and the results are reported in row 9. They indicate the unimportance of price contracts for the current model as the relative welfare and output losses are virtually identical to the benchmark where the price contracts were set to two quarters.

6 Conclusion

In this paper, we have studied the welfare costs of trend inflation in the presence of nominal price and wage contracts and real per capita output growth. To this end, we develop a general-equilibrium environment based on Erceg et al. (2000), modified to account for the possible effects of non-zero trend inflation and real per capita output growth on economic welfare. In this sense, the same labour market mechanism that gives rise to large dynamic welfare costs reported in Erceg et al. is at work in the present analysis. Our calculations of the welfare costs of inflation suggest that in such an environment with productivity growth, steady-state inflation at relatively modest annual rates of 2 percent or 4 percent have significant effects on the economy. We find that the quantitative importance of wage rigidity outweighs by a wide margin that of price rigidity in the computations of welfare costs. This important result does not arise from price dispersion *per se* but from effect of trend inflation on the optimal markup of monopolistically competitive labour suppliers. In particular, the inherent asymmetry in the utility function around the optimum – utility declines faster with a wage below the optimum than with a markup above the optimum – leads households to choose a higher average markup of wage over the marginal rate of substitution. This higher markup is equivalent to increasing the monopoly distortion that is already present in the model owing to imperfect competition. As well, we find that real output growth imparts an important effect on our welfare calculations. Indeed, a two percent rate of productivity growth leads, in our model, to welfare costs of inflation that are four times as important as those in the zero growth case.

Our analysis suggests at least two issues that merit further investigation. First, our results are based on the inherent features of a model that abstracts from strategic complementarity in price setting; by relaxing this assumption, subsequent research can study the implications of factor specificity for the steady-state welfare effects of trend inflation.[13] Second, the welfare costs of inflation are likely to be sensitive to assumptions regarding the structure of the labour market. In this paper, we followed Christiano et al. (2005) and Erceg et

[13] In our benchmark model (with $v = 1$) there is an element of strategic complementarity in wage setting (see Huang and Liu (2002)).

al. (2000), among others, and model the labour market as allocative. Allowing for richer non-allocative markets may change the welfare costs of trend inflation arising from labour market frictions.

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Figure 1: Output Costs of Trend Inflation

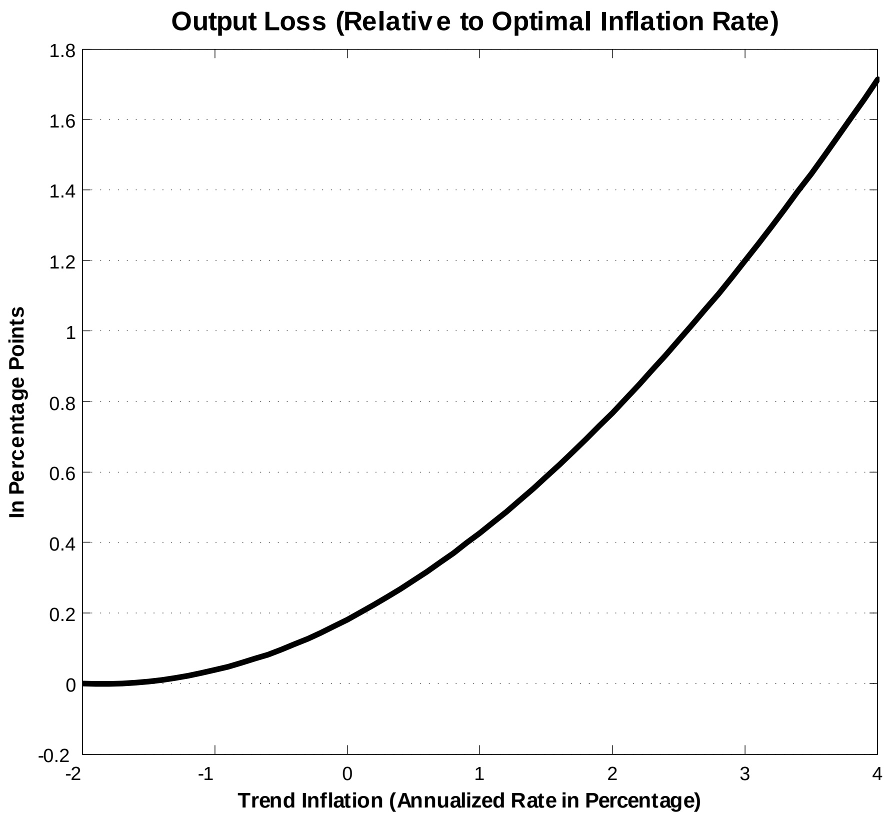

Figure 2: Welfare Cost of Trend Inflation

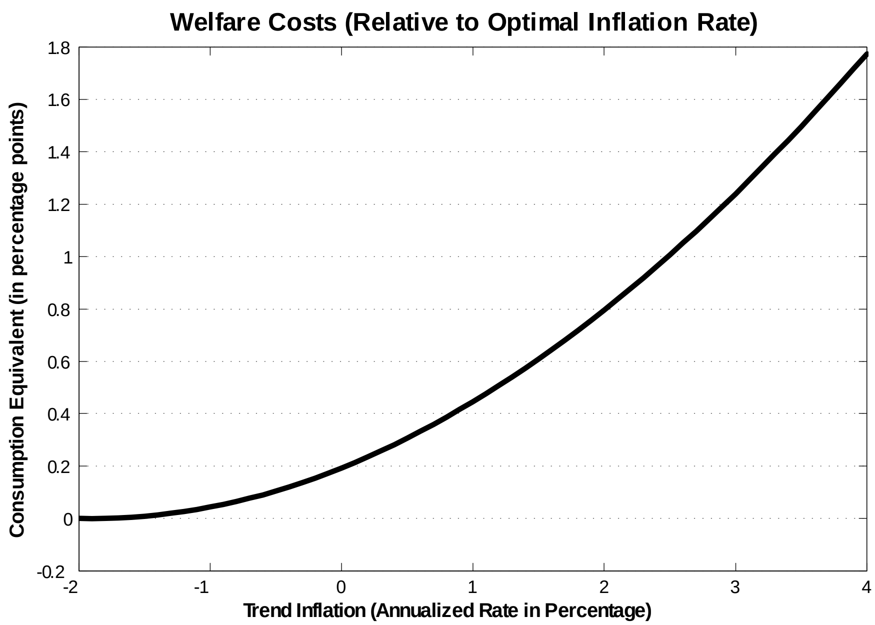

Figure 3: Markup and Price Dispersion in Market for Intermediate Goods

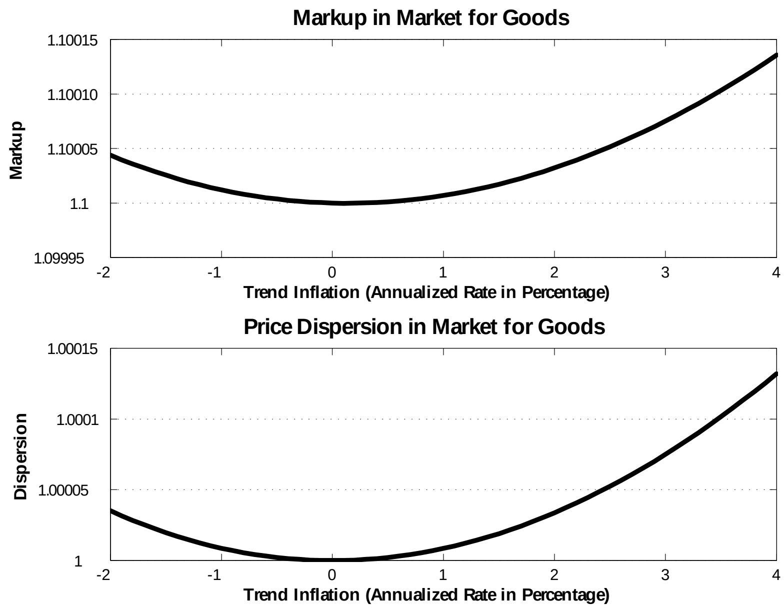

Figure 4: Markup and Wage Dispersion in Market for Labour

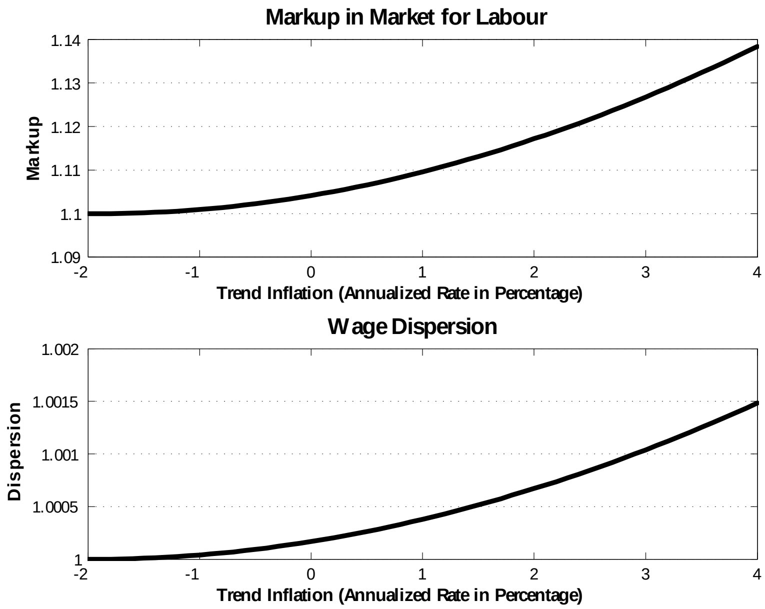

Figure 5: Asymmetry of the Monopolist's Profits around Optimum

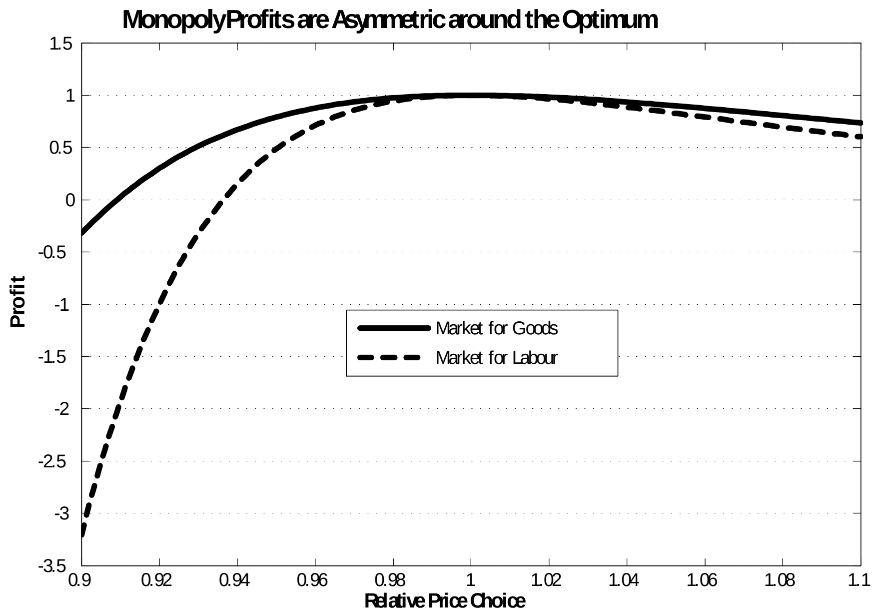

Table 1: Output and Welfare Costs From 2 Percent Annual Inflation
(Relative to Optimal Inflation Rate)

Model Parameterization	Optimal Inflation	Output (in %)	Welfare (in %)
1. Benchmark	-1.90	0.77	0.80
2. No Growth ($g = 0$)	0.039	0.17	0.20
3. Higher Trend Growth ($g = 3.0$)	-2.82	1.21	1.23
4. High curvature in Labour Disutility ($\nu = 2$)	-1.94	1.47	1.52
5. No curvature in Labour Disutility ($\nu = 0$)	-1.48	0.06	0.07
6. Lower elasticity in labour market ($\theta = 5$)	-1.71	0.17	0.18
7. Longer Wage Contracts ($I = 6$)	-1.94	1.77	1.85
8. No Wage Contracts ($I = 1$)	0.039	0.005	0.005
9. Equal Price and Wage Contracts ($J = I = 4$)	-1.71	0.77	0.80

Table 2: Output and Welfare Costs From 4 Percent Annual Inflation
(Relative to Optimal Inflation Rate)

Model Parameterization	Optimal Inflation	Output (in %)	Welfare (in %)
1. Benchmark	-1.90	1.71	1.77
2. No Growth ($g = 0$)	0.039	0.74	0.79
3. Higher Trend Growth ($g = 3.0$)	-2.82	2.33	2.39
4. High curvature in Labour Disutility ($\nu = 2$)	-1.94	3.21	3.35
5. No curvature in Labour Disutility ($\nu = 0$)	-1.48	0.16	0.16
6. Lower elasticity in labour market ($\theta = 5$)	-1.71	0.40	0.42
7. Longer Wage Contracts ($I = 6$)	-1.94	3.83	4.05
8. No Wage Contracts ($I = 1$)	0.039	0.02	0.02
9. Equal Price and Wage Contracts ($J = I = 4$)	-1.71	1.71	1.84

A Welfare Computations

Recall the expression for expected lifetime utility of the extended representative household (18):

$$U = \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \gamma \int_0^1 \frac{{L_{it}}^{1+\upsilon}}{1+\upsilon} \mathrm{d}i \right). \tag{48}$$

This expression can be rewritten as the following once it is evaluated at the steady state:

$$U = \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log c - \frac{\gamma}{1+\upsilon} \frac{1}{I} \sum\nolimits_{s=0}^{I-1} {L_s}^{1+\upsilon} \right) = (1/1-\beta) \left[\log c - \frac{\gamma}{1+\upsilon} \frac{1}{I} \sum\nolimits_{s=0}^{I-1} {L_s}^{1+\upsilon} \right]$$

where L_s represents the labour supply of the $1/I$ fraction of the household's members whose wage was modified s periods ago.

Consider now computing lifetime utility for a given benchmark level of inflation π^*, as follows:

$$U_{\pi^*} = (1/1-\beta) \left[\log c_{\pi^*} - \frac{\gamma}{1+\upsilon} \frac{1}{I} \sum\nolimits_{s=0}^{I-1} {L_{s,\pi^*}}^{1+\upsilon} \right],$$

where the subscript π^* indicates the dependence of that variable on the rate of trend inflation. Alternatively, consider lifetime utility at another rate of trend inflation π, in which, consumption is increased at every period by the factor x:

$$U_{\pi} = (1/1-\beta) \left[\log(c_{\pi}(1+x)) - \frac{\gamma}{1+\upsilon} \frac{1}{I} \sum\nolimits_{s=0}^{I-1} {L_{s,\pi}}^{1+\upsilon} \right].$$

The consumption equivalent measure of welfare changes between U_{π^*} and U_{π} is the value of x which makes the two welfare measure equal. In other words, x represents the percentage increase in consumption which would make households indifferent between living in the (steady-state) economy with inflation π and the economy with the benchmark inflation rate π^*. Considering the two expressions, we can solve for x as follows:

$$x = 100 \left\{ \exp \left[\log(\frac{c_{\pi_0}}{c_{\pi}}) - \frac{\gamma}{1+\upsilon} \left(\widehat{L_{\pi_0}} - \widehat{L_{\pi^*}} \right) \right] - 1 \right\},$$

where we have

$$\widehat{L_{\pi}} \equiv \frac{1}{I} \sum\nolimits_{s=0}^{I-1} {L_{s,\pi}}^{1+\upsilon}.$$

Using the steady-state values for expressions (16), (29), and (31) in the text, this reduces to:

$$\widehat{L_{\pi}} = \left({w^*}^{-\theta} L \right)^{1+\nu} \frac{1}{I} \sum\nolimits_{s=0}^{I-1} (\pi g)^{s\theta(1+\upsilon)}.$$

We compute the value of x for all rates of trend inflation from the optimal rate π^* to $\pi = 4$ percent.