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Abstract

We assess the implications of price indexation for estimated frequency of price adjustment in sticky price models of business cycles. These models predominantly assume that non-reoptimized prices are indexed to lagged or average inflation. The assumption of price indexation adds tractability although it is not likely reflective of the price practices of firms at the micro level. Under indexation firms have less incentive to adjust their prices, which implies downward bias in the estimated frequency of price changes. To evaluate the bias, we generate data with Calvo-type models without indexation. The artificial data are then used to estimate the frequency of price changes with indexation. Considering different assumptions about the degree of price rigidity and the level of trend inflation in the data-generating model, we find that the estimated indexation bias can be substantial, ranging up to 12 quarters in some cases.

JEL classification: E31, E37

Bank classification: Inflation and prices; Economic models; Econometric and statistical methods

Résumé

Les auteurs évaluent l'incidence de l'indexation sur la fréquence estimée de révision des prix dans les modèles du cycle économique à prix rigides. La majorité de ces modèles partent de l'hypothèse que les prix sont indexés sur l'inflation moyenne ou passée en dehors des périodes de réoptimisation. Cette hypothèse est adoptée pour accroître la maniabilité du modèle, mais elle ne reflète vraisemblablement pas les pratiques des entreprises au niveau microéconomique. En régime d'indexation, celles-ci sont moins incitées à ajuster leurs prix, ce qui donne à penser que la fréquence de révision des prix est sous-estimée. Pour évaluer la taille de ce biais, les auteurs génèrent des données à l'aide de modèles à la Calvo sans indexation, dans lesquels ils formulent diverses hypothèses concernant le degré de rigidité des prix et le taux d'inflation tendanciel. Ils estiment ensuite la fréquence de révision des prix en régime d'indexation au moyen des données ainsi créées. Les auteurs constatent que le biais dû à l'indexation peut être considérable et atteint jusqu'à douze trimestres dans certains cas.

Classification JEL : E31, E37

Classification de la Banque : Inflation et prix; Modèles économiques; Méthodes économétriques et statistiques

1. Introduction

One of the main topics in economics is the theory describing the output-inflation trade-off. For at least over the last two decades, the workhorse framework to analyze this trade-off has been the Calvo (1983) model, more recently refined by Yun (1996). In the model, price setters reoptimize their price with some probability that is assumed constant across time and population. Together with perfect staggering, the fraction of price reoptimizations is the same in every period. Since not all prices are reset each period, unanticipated changes in money growth create movements in real demand, and hence, in aggregate prices and quantities.

The attractive feature of the Calvo-Yun model is that it allows for the above relationship to be presented in a very simple form, called the "New Keynesian Phillips Curve", or NKPC. Explicitly stated, under certain assumptions, log inflation (the first-difference of log inflation) is a linear function of the log real marginal cost and log expected future inflation (the first-difference of log expected future inflation). The literature has gone a considerable length in estimating the NKPC inferring the values of its structural parameters and validating the underlying model. The most attention has been given to the estimates of the frequency of price reoptimizations. The frequencies of price adjustment estimated from the U.S. macro data imply a wide range of price durations. Gali and Gertler (1999) find that the average time between price changes is between 5.8 and 8.6 quarters. Smets and Wouters (2005) estimate a large DSGE model obtaining the duration of around 2 years. Sbordone (2002) and Gali et al. (2001) estimate shorter price spells, between $2\frac{1}{2}$ and $3\frac{1}{2}$ quarters.

To derive the NKPC, most studies above assume some form of price indexation.¹ Yet there is little evidence at the micro level of any form of indexation or deterministic durations between individual price adjustments in the empirical literature. Bills and Klenow (2004) and Klenow and Kryvtsov (2005) provide detailed evidence on the frequency of price adjustments in the U.S.. They examine BLS disaggregated consumer price data and document that the median duration between price adjustments is around 1.8 quarters. Thus, 100% price adjustment implied by price indexation at quarterly frequency is not present in these data. Klenow and Kryvtsov (2005) also report aggregate hazard rates in their data. Indexation or deterministic durations between price changes would reveal spikes at certain frequencies in the hazard rate function (e.g. a spike after one quarter for quarterly indexation). The

¹Alternatively, one can assume zero trend inflation. This assumption yields the same forward-looking NKPC that is obtained under positive inflation trend and indexation to average inflation rate.

Ascari (2004) argues that assuming zero instead of an empirically-relevant positive trend inflation has important implications for both long and short run dynamics in the Calvo model.

empirical hazard rates reveal no such spikes, except for a small increase after one year.²

Some forms of price indexation in the literature are motivated not by their empirical plausibility, but rather by methodological convenience. For example, it is known that the NKPC with a lagged inflation term matches the aggregate data much better than the standard forward-looking NKPC. To incorporate the lagged inflation term into the NKPC, Gali and Gertler (1999) introduce a subset of firms that use a backward looking rule of thumb to set prices. In contrast, Eichenbaum and Fisher (2004) assume "dynamic" indexation where prices are automatically adjusted every quarter to last period's aggregate inflation level. Sahuc (2006) studies the interaction between partial (dynamic) indexation and trend inflation. He shows that a "hybrid" backward-looking NKPC produces more precise estimates than the forward-looking NKPC when the trend inflation is positive.

To the extent that the indexation assumption is counterfactual, the NKPC econometric model used to estimate the degree of price rigidity is misspecified, and the estimated frequency of price reoptimization is therefore biased. In this paper we evaluate the incidence and the magnitude of this indexation bias. To this end we generate data with Calvo-type models with no indexation. The model data are then used to estimate NKPC equations derived under different forms of indexation. The difference between the frequency of price reoptimization assumed in the data generating model and corresponding estimated value for this parameter is what we refer to as the indexation bias.

Various degrees of price rigidity and differing values of trend inflation are considered in the data generating models. We find that indexation bias is present for almost all of these cases. When this bias is present, the estimated frequency of price adjustments obtained from the econometric model is always lower than the corresponding value assumed in the data generating model, implying counterfactually longer price durations. It is higher with larger trend inflation and with longer price durations in the data generating model. Indeed, the size of the bias is 12 quarters in one such case.

In sum, our results reconcile, at least partially, the difference between the low estimated frequencies of price adjustment in the NKPC literature and the high observed frequencies of price adjustment at the disaggregate level (e.g. Bils and Klenow (2004)). Thus, some of the discrepancy is due to the bias introduced into the NKPC models through the counterfactual

²Klenow and Kryvtsov's Figure 3 suggests that the fraction of prices that change every year is less than 2%. Dhyne et al. (2005) summarize a battery of disaggregate price data studies for Europe and also find no evidence of price indexation.

price indexation assumption.

The paper proceeds as follows. Section 2 introduces the Calvo model with and without indexation. Section 3 discusses methodology. Section 4 provides the main results and their discussion. In Section 5 we consider two extensions of the data generating model: the model with truncation of the probability of price adjustment and the model with fixed capital. Finally, Section 6 concludes.

2. Model

In this section, we lay out the workhorse sticky price model of monetary business cycles. The time is discrete and indexed by $t = 0, 1, \dots$. The uncertainty in period t is captured by a random event s_t . The history of events through period t is given by $s^t \equiv (s_0, s_1, \dots, s_t)$. Let ϑ be a measure defined on the appropriate sigma-algebra.

The economy consists of three types of agents: households, producers, and the government.

2.1 Households

There is a continuum of identical households. Each household chooses consumption bundle $\{c(i, s^t)\}$, $i \in [0, 1]$, labor $l(s^t)$, and real balances $M(s^t)/P(s^t)$ to maximize:

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \vartheta(s^t) U(c(s^t), l(s^t), M(s^t)/P(s^t)),$$

subject to the sequence of budget constraints

$$\begin{aligned} P(s^t)c(s^t) + M(s^t) + \sum_{s^{t+1}} Q(s^{t+1}|s^t)B(s^{t+1}) \\ \leq P(s^t)w(s^t)l(s^t) + M(s^{t-1}) + B(s^t) + \Pi(s^t) + T(s^t) \end{aligned}$$

definition of the aggregate consumption

$$c(s^t) = \left(\int_0^1 c(i, s^t)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}, \quad \theta > 1$$

borrowing constraints $B(s^{t+1}) \geq -P(s^t)\bar{b}$, given $M(s^{-1})$ and $B(s^0)$.

In the above, M and B are consumers' holdings of money and contingent claims, respectively, Q is the price of the claims, w is the real wage, Π are profits, T are transfers from the government, and P is the aggregate price index.

The first-order conditions are:

$$-\frac{U_l(s^t)}{U_c(s^t)} = w(s^t) \quad (1)$$

$$\frac{U_m(s^t)}{P(s^t)} - \frac{U_c(s^t)}{P(s^t)} + \beta \sum_{s^{t+1}} \vartheta(s^{t+1}|s^t) \frac{U_c(s^{t+1})}{P(s^{t+1})} = 0 \quad (2)$$

$$Q(s^t|s^{t-1}) = \beta \vartheta(s^t|s^{t-1}) \frac{U_c(s^t)}{U_c(s^{t-1})} \frac{P(s^{t-1})}{P(s^t)} \quad (3)$$

where $U(s^t)$ is notation for $U(c(s^t), l(s^t), M(s^t)/P(s^t))$. Expenditure minimization yields consumers' demand for goods

$$c(i, s^t) = \left[\frac{P(s^t)}{P(i, s^t)} \right]^\theta c(s^t) \quad (4)$$

and the aggregate price index

$$P(s^t) = \left(\int_0^1 P(i, s^t)^{1-\theta} di \right)^{\frac{1}{1-\theta}} \quad (5)$$

2.2 Producers

There is measure one of monopolistically competitive producers. Each producer faces a signal allowing her to choose price optimally. The signal process consists of i.i.d. draws from the Poisson distribution with arrival rate $1 - \lambda$. Hence λ can be interpreted as the conditional probability of not reoptimizing the price inherited from the last period. Any firm surely reoptimizes its price T periods after the last reoptimization, where $T \leq \infty$. The price setting scheme is going to allow for indexation of prices that are not reoptimized.

The problem solved by a producer is to choose sequences of prices $P(i)$ and labor inputs $l(i)$ to maximize the expected stream of profits

$$\sum_{\tau=0}^{T-1} \sum_{s^\tau} \tilde{Q}(s^\tau) \lambda^\tau [P(i, s^\tau) y(i, s^\tau) - P(s^\tau) w(s^\tau) l(i, s^\tau)]$$

subject to good demand (4), the technology constraint

$$y(i, s^t) = l(i, s^t)^\alpha$$

and constraints on prices, where $\tilde{Q}(s^\tau) = \prod_{t=1}^\tau Q(s^t|s^{t-1})$. For the benchmark case we will assume $T = \infty$ and $\alpha = 1$ (no capital) as in Gali et al. (1999). Two extensions in Section 5 consider (a) an economy with $T < \infty$, i.e. truncated probability of price adjustment; and (b) an economy with $\alpha < 1$ and fixed capital as in Sbordone (2002).

The Lagrangian for this problem in the benchmark case is

$$\begin{aligned} \mathcal{L} = & \dots + \tilde{Q}(s^t)\lambda^t \{ [P(i, s^t)^{1-\theta} \Lambda(s^t) - P(s^t)w(s^t)l(i, s^t) \\ & + \zeta(s^t) \{ l(i, s^t) - \Lambda(s^t)P(i, s^t)^{-\theta} \}] \\ & + \sum_{s^{t+1}} Q(s^{t+1}|s^t)\lambda \left[\left(\left[\pi^{1-\phi} \pi(s^t)^\phi \right]^\delta P(i, s^t) \right)^{1-\theta} \Lambda(s^{t+1}) - P(s^{t+1})w(s^{t+1})l(i, s^{t+1}) \right. \\ & \left. + \zeta(s^{t+1}) \left\{ l(i, s^{t+1}) - \Lambda(s^{t+1}) \left(\left[\pi^{1-\phi} \pi(s^t)^\phi \right]^\delta P(i, s^t) \right)^{-\theta} \right\} \right] + \dots \} \end{aligned}$$

where

$$\Lambda(s^t) = P(s^t)^\theta c(s^t), \tag{6}$$

and $\pi(s^t) = P(s^t)/P(s^{t-1})$ is the aggregate (gross) inflation rate in state s^t .

For the firm i that is not allowed to reoptimize its price in period $t + 1$, the previous period's price $P(i, s^t)$ is indexed by the term $\left[\pi^{1-\phi} \pi(s^t)^\phi \right]^\delta$, where π denotes the average (steady state) inflation rate. Indexation is therefore *static* (*dynamic*) if $\delta = 1$ and $\phi = 0$ ($\phi = 1$), and there is no indexation if $\delta = 0$ ³.

First-order conditions yield equations for firm i 's real marginal cost

$$mc(i, s^t) = w(s^t) \tag{7}$$

³Static indexation or zero trend inflation are assumed in Gali and Gertler (1999), Sbordone (2002); both static and dynamic indexation is used by Eichenbaum and Fisher (2004), Christiano et al. (2005).

and for the optimal price

$$P(i, s^t) = \frac{\theta}{\theta - 1} \frac{\sum_{\tau=0}^{T-1} \sum_{s^t} Q(s^{t+\tau}|s^t) \lambda^\tau \left[\pi^{(1-\phi)\tau} \prod_{i=1}^{\tau} \pi(s^{t-1+i})^\phi \right]^{-\delta\theta} P(s^\tau) mc(i, s^\tau) \Lambda(s^\tau)}{\sum_{\tau=0}^{T-1} \sum_{s^t} Q(s^\tau|s^t) \lambda^\tau \left[\pi^{(1-\phi)\tau} \prod_{i=1}^{\tau} \pi(s^{t-1+i})^\phi \right]^{\delta(1-\theta)} \Lambda(s^\tau)}. \quad (8)$$

Note that the marginal cost is the same for all producers. This is no longer true in the extension with fixed capital as in the Sbordone (2002) model. See the Appendix for details.

2.3 The Government

Define the money supply growth rate $\mu(s^t)$ as

$$M(s^t) = \mu(s^t) M(s^{t-1}).$$

Monetary policy in our model is a stochastic process $\{\mu(s^t)\}$ given by

$$\log \mu(s^t) = \rho_\mu \log \mu(s^{t-1}) + (1 - \rho_\mu) \log \mu + \epsilon_{\mu,t}, \quad (9)$$

where constant μ is a steady state money supply growth rate and $\epsilon_{\mu,t}$ are i.i.d. errors, drawn from $N(0, \sigma_\mu)$.

The government's budget constraint is

$$T(s^t) = M(s^t) - M(s^{t-1}),$$

where T are lump-sum transfers to consumers.

2.4 Market clearing conditions

Since consumers are identical, there is no trade in the market for state-contingent claims

$$B(s^t) = 0.$$

Goods market clearing conditions are

$$c(i, s^t) = y(i, s^t).$$

The labor market clearing condition is

$$l(s^t) = \int_0^1 l(i, s^t) di.$$

2.5 Aggregation

Let ν_0 denote the fraction of firms reoptimizing their price each period and let $\{P(0, s^{t-\tau})\}_\tau$ be the sequence of optimal prices before period t :

$$\nu_0 = \left(\sum_{\tau=0}^{T-1} \lambda^\tau \right)^{-1} = \frac{1 - \lambda}{1 - \lambda^T}$$

so that $\nu_0 = 1 - \lambda$ if $T = \infty$. Then the aggregate price index (5) can be written as

$$P(s^t)^{1-\theta} = \nu_0 \sum_{\tau=0}^{T-1} \lambda^\tau \left(\left[\pi^{(1-\phi)\tau} \prod_{i=1}^{\tau} \pi(s^{t-1+i})^\phi \right]^\delta P(0, s^{t-\tau}) \right)^{1-\theta}. \quad (10)$$

Following Yun (1996) we define the shadow price index as P^* so that

$$y(s^t) = l(s^t) \quad (11)$$

$$c(s^t) = \left(\frac{P^*(s^t)}{P(s^t)} \right)^{-\theta} y(s^t) \quad (12)$$

$$P^*(s^t)^{-\theta} = \nu_0 \sum_{\tau=0}^{T-1} \lambda^\tau \left(\left[\pi^{(1-\phi)\tau} \prod_{i=1}^{\tau} \pi(s^{t-1+i})^\phi \right]^\delta P(0, s^{t-\tau}) \right)^{-\theta} \quad (13)$$

2.6 Computation of an Equilibrium

Aggregate equilibrium is defined as sequences of prices $\{P(s^t)\}$, $\{P^*(s^t)\}$, $\{P(0, s^t)\}$, $\{w(s^t)\}$ and allocations $\{c(s^t)\}$, $\{y(s^t)\}$, $\{mc(s^t)\}$ that, for given initial money and debt holdings, satisfy the system of equilibrium conditions (1)-(3), (6)-(13).

To solve for equilibrium, the system (1)-(3), (6)-(13) is rendered stationary and log-

linearized around a deterministic steady state⁴. Appendices A and B show equilibrium equations for the steady state and the log-linearized systems, respectively. The solution is found by applying the method outlined in Blanchard and Kahn (1980).

2.7 New Keynesian Phillips Curve

Under indexation ($\delta = 1$), the pricing block is considerably simplified. In particular, we obtain $\hat{p}_t = \hat{p}_t^*$ and the pricing equations collapse to⁵

$$\hat{\pi}_t - \phi \hat{\pi}_{t-1} = \frac{(1 - \beta\lambda)(1 - \lambda)}{\lambda} \widehat{mc}_t + \beta E_t (\hat{\pi}_{t+1} - \phi \hat{\pi}_t). \quad (14)$$

Equation (14) is the standard New Keynesian Phillips Curve equation (see, for example, Galí and Gertler (1999)). Under conventional assumptions, the marginal cost deviations are proportional to deviations in aggregate output ("output gap"). Hence equation (14) can be used to estimate the inflation-output trade-off.

Without indexation ($\delta = 0$), the aggregate inflation is determined by

$$\begin{aligned} \hat{\pi}_t = & \frac{(1 - \beta\lambda\pi^{\theta-1})(1 - \lambda\pi^{\theta-1})}{\lambda\pi^{\theta-1}} \widehat{mc}_t + \beta \hat{\pi}_{t+1} \\ & + (\pi - 1) \frac{(1 - \lambda\pi^{\theta-1})}{\lambda\pi^\theta} \left(\hat{v}_t - [\hat{U}_{ct} + \widehat{mc}_t + \hat{\Lambda}_t] \right), \end{aligned} \quad (15)$$

where

$$\hat{v}_t = (1 - \beta\lambda\pi^\theta) [\hat{U}_{ct} + \widehat{mc}_t + \hat{\Lambda}_t] + (\beta\lambda\pi^\theta) E_t [\hat{v}_{t+1} + \theta \hat{\mu}_t].$$

3. Methodology and Parametrization

To find out whether models assuming a particular price indexation mechanism produce biased estimates of the frequency of price adjustment, we take the following approach:

⁴To make equations stationary, we normalize prices by the past quantity of money:

$$\begin{aligned} p(s^t) &= P(s^t)/M(s^{t-1}) \\ p(0, s^{t-\tau}) &= P(0, s^t)/M(s^{t-1-\tau}) \\ p^*(s^t) &= P^*(s^t)/M(s^{t-1}) \end{aligned}$$

⁵Hat-ed variables denote log-deviations from steady state level.

Consider a Calvo-Yun model with no indexation (see Section 2), with frequency of price adjustment $1 - \lambda$ and average inflation π . The parameters in the model are calibrated to US quarterly data (see Table 1). Preference specification and parameters values are chosen as in Chari et al. (2000), the price elasticity of goods' demand is set to 10, in line with evidence in Basu and Fernald (1997), and the serial correlation parameter of the money growth process in the model is set equal to the estimated value for the first-order serial correlation of M1 in the U.S. over the period 1959 to 2005. Finally, the standard deviation of innovations to the money growth process is chosen such that the standard deviation of inflation in the benchmark model matches 0.7% of the standard deviation of the U.S. CPI inflation (less food and energy) over the 1959-2005 period⁶.

Drawing from the innovation process for the money growth, we generate data from the above model. We refer to this model as the data-generating process or the data-generating model, and draw 4000 times, indexing the simulated datasets by $i = 1, 2, \dots, 4000$. Then, with each dataset, we estimate our econometric model (see below) obtaining 4000 parameter estimates for the λ parameter.

From (14) it follows that the econometric model with indexation weight ϕ , under rational expectations, can be written as:

$$\hat{\pi}_t - \phi \hat{\pi}_{t-1} = \beta(\hat{\pi}_{t+1} - \phi \hat{\pi}_t) + \frac{(1 - \beta\lambda)(1 - \lambda)}{\lambda} m\hat{c}_t + \epsilon_{t+1}^\phi. \quad (16)$$

In this context ϵ_t^ϕ should be orthogonal to information at time $t - 1$ and earlier, so that

$$E_t \left[\epsilon_t^\phi \cdot z_{t-1} \right] = 0,$$

where z_{t-1} is a vector of variables the values of which are known at time $t - 1$ and earlier.

From the obtained distribution of estimated frequencies of price changes $\tilde{\lambda}_i$, we calculate the mean, λ^* , the standard deviation, s.d., and the term $\Delta\lambda \equiv \lambda^* - \lambda$, which we interpret as the "indexation bias"⁷. The corresponding average duration of price stickiness bias, expressed in quarters, is also calculated: $\Delta D \equiv \frac{1}{1-\lambda^*} - \frac{1}{1-\lambda}$. Both definitions imply that positive bias is related to longer price durations.

As is usually done in the literature for these types of models, we use generalized method

⁶The benchmark model is the Calvo model without truncation, with frequency of price adjustment of twice a year and average inflation of 1% per year.

⁷Defining λ^* as the median instead of the mean of the $\tilde{\lambda}_i$ -distribution does not change the results.

of moments (GMM) to estimate the econometric models. This methodology is appropriate given that the econometric model is non-linear in its parameters, and that the right-hand side variables are endogenous. We use the instrument set $\{\hat{\mu}_{t-1}, \hat{\mu}_{t-2}\}$ for the estimation, where the instruments are strongly exogenous and informative by construction⁸.

4. Results

In Table 2 we report the results for 18 different Calvo economies that we use as data generating models. The economies differ by the assumed steady state value for inflation, and by the assumed frequency of price adjustment. The choices for steady state inflation are $\pi = 1\%$, 5% and 10% per year. As for the frequency of price adjustment, we consider the values $\lambda = 0.5$ and $\lambda = 0.75$. Three econometric models, that differ according to the assumed indexation type, are also considered. The indexation differences are captured by the value of the indexation weight in the econometric model, and the three choices are: $\phi = 0$, 0.5 and 1 . For each economy we report the mean of the 4000-point distribution of the estimated frequency of price changes, λ^* , the standard deviation of the distribution, s.d., as well as the indexation biases, $\Delta\lambda$ (in frequency) and ΔD (in quarters).

Several main results stand out. First, in all cases but one the size of the bias is larger than the standard deviation of the distribution, which we interpret as the bias being economically important. The exception is the case with low inflation ($\pi = 1\%$) and estimation under static indexation ($\phi = 0$). Second, when the bias is important it is always positive, implying that the estimated expected price duration is longer than the true one. Third, the indexation bias increases with trend inflation. For example, in the case of $\lambda = 0.75$ and static indexation, the bias increases from 0.3 quarters for low inflation to 1.9 quarters for medium inflation, and to 8.7 quarters for high inflation. The bias expressed in quarters also increases with the degree of price rigidity. For medium inflation and mixed indexation ($\phi = 0.5$), the indexation bias increases from 3.2 quarters, when the frequency of adjustment in the DGP model corresponds to 2 quarters, to 5.4 quarters when the average duration between price adjustments is assumed to be 4 quarters. Firms that are subject to indexation do not need to adjust their prices as often to generate the same variance of inflation as firms whose prices are not indexed. Hence, for economies in which firm's price on average is farther away from the desired price (e.g. economies with higher steady state inflation or longer duration between

⁸We know, from the properties of the GMM estimator, that the estimator will be asymptotically consistent if the instruments are strongly exogenous and informative. This is the case with our instrument set so that there is no need to worry about the weak instrument problem in our approach. Moreover, using other instruments such as $\{\hat{\pi}_{t-1}, \hat{\pi}_{t-2}\}$ or $\{\widehat{mc}_{t-1}, \widehat{mc}_{t-2}\}$ gives qualitatively similar results.

price changes) the implied indexation bias is larger.

Fourth, the size of the bias depends on the indexation weight ϕ , which in turn determines how misspecified the econometric model is with respect to the DGP Calvo model. In the benchmark case, the econometric model with static indexation yields smaller indexation bias than the model with mixed indexation. Furthermore, the econometric model with dynamic indexation ($\phi = 1$) does not always converge to parameter values within the $[0, 1]$ range.

Finally, the indexation bias can be large in comparison to the duration between price adjustments. For example, for the case with $\lambda = 0.75$, medium inflation and mixed indexation, the indexation bias is 5.4 quarters, which is larger than the average price duration of 4 quarters in the DGP model. The bias becomes as high as 12.6 quarters when the average inflation is high.

To see the intuition behind the results it is helpful to rewrite the NKPC equations with and without indexation, equations (14), and (15), respectively, after substituting forward for future inflation under the expectation operator on the right-hand side. With indexation, the NKPC equation takes the form

$$\hat{\pi}_t = \phi \hat{\pi}_{t-1} + \frac{(1 - \beta\lambda)(1 - \lambda)}{\lambda} \sum_{s=0}^{\infty} \beta^s \widehat{m}c_{t+s} \quad (17)$$

whereas, without indexation, aggregate inflation is determined by

$$\begin{aligned} \hat{\pi}_t = & \frac{(1 - \beta\lambda\pi^{\theta-1})(1 - \lambda\pi^{(\theta-1)})}{\lambda\pi^{(\theta-1)}} \sum_{s=0}^{\infty} \beta^s E_t \widehat{m}c_{t+s} \\ & + (\pi - 1) \frac{(1 - \lambda\pi^{(\theta-1)})}{\lambda\pi^\theta} \sum_{s=0}^{\infty} \beta^s E_t \left(\hat{v}_{t+s} - \left[\hat{U}_{ct+s} + \widehat{m}c_{t+s} + \hat{\Lambda}_{t+s} \right] \right) \end{aligned} \quad (18)$$

The second term in (18) is proportional to $\pi - 1$, which makes it smaller than the first term by at least one order of magnitude. Ignoring that term for now, we can see that *given* the time series for $\hat{\pi}_t$ and $\widehat{m}c_t$, the value of the frequency of adjustment λ based on (17) has to be larger than its value in the DGP model (18) whenever $\pi > 1$ or $\phi > 0$. This is due to the fact that the constant in front of the summation sign, $\frac{(1-\beta\lambda)(1-\lambda)}{\lambda}$, is a decreasing function of λ . Moreover, this compensating bias is larger the larger is average inflation π , or the larger is the elasticity of goods substitution θ .

5. Extensions

In this Section we consider two extensions to the standard Calvo model that we used as a DGP model. Both extensions have been used in the literature to increase inflation persistence in the model to match the inflation persistence in the data. Serial correlation of inflation in the benchmark Calvo model with 5% annual average inflation is 0.49 for $\lambda = 0.5$, and it is 0.78 for $\lambda = 0.75$. Even though both values are within the empirically plausible range, we will consider the two extensions in which inflation persistence is higher. In the first extension, the probability of price adjustment in the model is truncated, i.e. any firm adjusts its price surely after a finite number of quarters. This assumption prevents a firm from the possibility of losing profits due to the inability of changing its price for long periods of time. In the second extension, there are decreasing returns to labor in the aggregate technology, and capital is fixed. Sbordone (2002) demonstrated that the NKPC equation implied by this model is able to generate more persistent inflation fluctuations.

5.1 Truncated Calvo model

In this extension of the Calvo model the maximal age of the price set by the monopolistic firm is finite: $T = T_R < \infty$. The period of truncation T_R is chosen such that in the steady state a firm that has not adjusted its price for $T_R + 1$ periods, earns positive period-by-period profits for the first T_R periods but faces negative profits in period $T_R + 1$:

$$\Pi_{T_R} > 0, \quad \Pi_{T_R+1} < 0$$

where Π_s is the steady state profits of the firm that has not adjusted its price for s periods, with $s = 0, 1, \dots$. A similar truncation rule is used in the models with endogenous frequency of price adjustment, such as Dotsey et al. (1999) and Bakhshi et al. (2006). Table 3 provides truncation periods for truncated Calvo models for different steady state inflation rates and degrees of price rigidity.

Serial correlation of inflation in the truncated Calvo model with 5% annual inflation rate is 0.65 for $\lambda = 0.5$, and it is 0.93 for $\lambda = 0.75$. Table 4 documents our results for the estimation bias when the DGP is the truncated Calvo model.

Our main results do not change as to the bias being positive and large, the latter ranging from 0.7 to 10.9 quarters. However, contrary to the benchmark model, the econometric model with mixed and dynamic indexation on occasion predicts smaller indexation bias than

the NKPC with static indexation. For example, for average inflation of 5% per year and a frequency $\lambda = 0.75$, the bias decreases from 6.9 quarters for $\phi = 0$ to 5.7 and 5.1 for mixed and dynamic cases, respectively. Moreover, four out of six economies estimated under dynamic indexation have good convergence (versus none for the benchmark Calvo economy), whereas the high inflation economy under static indexation does not converge (all converged in the benchmark case). Hence inflation dynamics in the truncated Calvo DGP economy is better captured by econometric models with indexation weight on lagged inflation. Nonetheless, even in these cases, the indexation bias is still present and large.

5.2 Fixed capital model

In this model labor is subject to decreasing returns to scale in the aggregate technology and physical capital is fixed. Sbordone (2002) showed that this model does a good job of tracking aggregate U.S. data. Serial correlation of inflation in the Calvo model with fixed capital and 5% annual inflation rate is 0.67 for $\lambda = 0.5$, and it is 0.84 for $\lambda = 0.75$.

Our estimation results in Table 5 show that when the DGP model is the Calvo model with fixed capital, the indexation biases are smaller than in the benchmark case. Indeed, they range from 0 to 1.4 quarters for economies with half-year price durations; and from 0.1 to 2.2 quarters for economies with 1-year price durations. As in the benchmark case, the econometric model with static indexation does better than the one with mixed or dynamic indexations.

Finally, for the case of $\lambda = 0.75$ and 10% annual average inflation, equilibrium does not exist. The constant multiplier in the infinite summations in the pricing equation similar to (8) becomes $\beta\lambda\pi^{\frac{\theta}{1-\alpha}}$, and for our parameter values it is larger than 1, so that the summations are not summable. Ascari (2004) showed that equilibrium in the Calvo model ceases to exist for trend inflation above a certain threshold. When strategic complementarities in pricing decisions are present, Bakhshi et al. (2005) show that the threshold trend inflation can be as low as 5.5%.

5.3 Controlling for the small sample bias

The GMM estimator may be biased if the sample size is not very large. To assess whether our 200-observation samples imply small sample bias, we conduct the experiment where we estimate adjustment frequencies using an econometric model that actually corresponds to the underlying DGP. In one case, we consider data from a DGP where the Calvo model

assumes static indexation, and we obtain estimates based on an econometric model with static indexation. In a second case, we generate data from a Calvo model assuming dynamic indexation, and compare its λ parameter to its estimated counterpart in an econometric model with dynamic indexation. The results, reported in Tables 6 and 7, show that small sample biases for these cases are negligibly small, usually well within one standard deviation of the distribution of the estimated frequencies of price adjustment. On this basis, we conclude that small sample biases do not present an important source of concern for our comparison exercises.

6. Conclusion

The standard NKPC approach to assessing the inflation-output trade-off is often derived under the assumption of price indexation. To the extent that this assumption is not plausible whether empirically or from a theoretical point of view, the NKPC-type models will be misspecified. We show that this misspecification leads to biased estimates of the degree of price rigidity - a key factor in the sticky price models of the monetary transmission mechanism. In this respect, we find that estimated price durations are biased towards longer durations. Hence, once the bias is taken into account, the implied degree of price rigidity is smaller and closer to that found in the micro data.

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Table 1: Benchmark model parameter values

Parameter	Value
β , discount factor	$0.97^{1/4}$
θ , price elasticity of goods demand	10
η , interest elasticity of money demand	0.39
ω , consumption share in utility	0.94
ψ , leisure share in utility	1.5
σ , risk aversion	1
μ , average annual money growth	1%, 5%, 10%
ρ_μ , serial correlation of money growth	0.52
σ_μ , st. dev. of innovations to money growth	0.55%

Note: Preference specification and parameter values are as in Chari et al. (2000):

$$U(C, M, L) = \left\{ \left[\omega C^{1-1/\eta} + (1 - \omega) M^{1-1/\eta} \right]^{\frac{\eta}{\eta-1}} (1 - L)^\psi \right\}^{1-\sigma} / (1 - \sigma)$$

where C is consumption, M are real money balances, and L is hours worked. Goods demand elasticity is in line with evidence in Basu and Fernald (1997). Serial correlation of the money growth process in the model is set equal to the estimated first-order serial correlation of M1 in the U.S. from 1959 to 2005. Standard deviation of innovations to the money growth process is chosen so that the standard deviation of inflation in the benchmark model matches 0.7% of standard deviation of the U.S. CPI inflation (less food and energy) for 1959-2005.

Table 2: Indexation bias, non-truncated Calvo model

	<u>$\lambda = 0.5, D = 2$ quarters</u>											
	$\pi = 1\%$				$\pi = 5\%$				$\pi = 10\%$			
	λ^*	s.d.	$\Delta\lambda$	ΔD	λ^*	s.d.	$\Delta\lambda$	ΔD	λ^*	s.d.	$\Delta\lambda$	ΔD
$\phi = 0.0$	0.51	0.04	0.01	0.1	0.56	0.04	0.06	0.3	0.61	0.03	0.11	0.6
$\phi = 0.5$	0.80 ⁺	0.08	0.30	3.1	0.81 ⁺	0.07	0.31	3.2	0.81 ⁺	0.06	0.31	3.4
$\phi = 1.0$	—	—	—	—	—	—	—	—	—	—	—	—
	<u>$\lambda = 0.75, D = 4$ quarters</u>											
	$\pi = 1\%$				$\pi = 5\%$				$\pi = 10\%$			
	λ^*	s.d.	$\Delta\lambda$	ΔD	λ^*	s.d.	$\Delta\lambda$	ΔD	λ^*	s.d.	$\Delta\lambda$	ΔD
$\phi = 0.0$	0.77	0.03	0.02	0.3	0.83	0.02	0.08	1.9	0.92	0.02	0.17	8.7
$\phi = 0.5$	0.86	0.03	0.11	3.3	0.89	0.02	0.14	5.4	0.94 ⁺	0.02	0.19	12.6
$\phi = 1.0$	—	—	—	—	—	—	—	—	—	—	—	—

Note: The number of simulations for each Calvo economy is 4000, and the length of every generated series (per simulation) is 200 quarters. For each simulation, we estimate the frequency of price adjustment λ using GMM and each of three econometric models (16); the latter differ according to indexation weight, $\phi = 0, 0.5$ and 1. Thus, there are 4000 estimates $\tilde{\lambda}_i, i = 1, \dots, 4000$. The DGP economies differ by the assumed steady state inflation value ($\pi = 1\%, 5\%$ or 10% per year), and by the assumed frequency of price adjustment ($\lambda = 0.5$ or $\lambda = 0.75$). For each economy we report the mean of the estimated frequencies of price changes, λ^* , their standard deviation, s.d., as well as the indexation biases, $\Delta\lambda$ (in frequency) and ΔD (in quarters). — implies convergence occurred less than 1000 times, + implies convergence less than 3000 times but greater than 1000 times.

Table 3: Truncation periods in truncated Calvo model (in quarters)

	$\pi = 1\%$	$\pi = 5\%$	$\pi = 10\%$
$\lambda = 0.5$	45	11	7
$\lambda = 0.75$	47	14	10

Note: In the truncated Calvo model the maximal age of the price set by the monopolistic firm is finite: $T = T_R < \infty$. The period of truncation T_R is chosen such that, in the steady state, a firm that has not adjusted its price for T_R+1 periods earns positive period-by-period profits for the first T_R periods but faces negative profits in period T_R+1 .

Table 4: Indexation bias, truncated Calvo model

	<u>$\lambda = 0.5, D = 2$ quarters</u>											
	$\pi = 1\%$				$\pi = 5\%$				$\pi = 10\%$			
	λ^*	s.d.	$\Delta\lambda$	ΔD	λ^*	s.d.	$\Delta\lambda$	ΔD	λ^*	s.d.	$\Delta\lambda$	ΔD
$\phi = 0.0$	0.63	0.03	0.13	0.7	0.68	0.03	0.18	1.1	0.73	0.2	0.23	1.7
$\phi = 0.5$	0.81	0.06	0.31	3.3	0.81	0.04	0.31	3.2	0.81	0.03	0.31	3.2
$\phi = 1.0$	—	—	—	—	—	—	—	—	0.91	0.03	0.41	8.8
	<u>$\lambda = 0.75, D = 4$ quarters</u>											
	$\pi = 1\%$				$\pi = 5\%$				$\pi = 10\%$			
	λ^*	s.d.	$\Delta\lambda$	ΔD	λ^*	s.d.	$\Delta\lambda$	ΔD	λ^*	s.d.	$\Delta\lambda$	ΔD
$\phi = 0.0$	0.85	0.02	0.10	2.5	0.91	0.02	0.16	6.9	—	—	—	—
$\phi = 0.5$	0.86	0.02	0.11	3.2	0.90	0.01	0.15	5.7	0.93	0.01	0.18	10.9
$\phi = 1.0$	0.88	0.02	0.13	4.6	0.89	0.01	0.14	5.1	0.91	0.01	0.16	6.9

Note: The number of simulations for each Calvo economy is 4000, and the length of every generated series (per simulation) is 200 quarters. For each simulation, we estimate the frequency of price adjustment λ using GMM and each of three econometric models (16); the latter differ according to indexation weight, $\phi = 0, 0.5$ and 1. Thus, there are 4000 estimates $\tilde{\lambda}_i, i = 1, \dots, 4000$. The DGP economies differ by the assumed steady state inflation value ($\pi = 1\%, 5\%$ or 10% per year), and by the assumed frequency of price adjustment ($\lambda = 0.5$ or $\lambda = 0.75$). For each economy we report the mean of the estimated frequencies of price changes, λ^* , their standard deviation, s.d., as well as the indexation biases, $\Delta\lambda$ (in frequency) and ΔD (in quarters). — implies convergence occurred less than 1000 times, + implies convergence less than 3000 times but greater than 1000 times.

Table 5: Indexation bias, Calvo model with fixed capital

	<u>$\lambda = 0.5, D = 2$ quarters</u>											
	$\pi = 1\%$				$\pi = 5\%$				$\pi = 10\%$			
	λ^*	s.d.	$\Delta\lambda$	ΔD	λ^*	s.d.	$\Delta\lambda$	ΔD	λ^*	s.d.	$\Delta\lambda$	ΔD
$\phi = 0.0$	0.50	0.04	0.00	0.0	0.52	0.04	0.02	0.1	0.52	0.04	0.02	0.1
$\phi = 0.5$	0.70	0.07	0.20	1.4	0.71	0.07	0.21	1.4	0.71	0.07	0.21	1.4
$\phi = 1.0$	—	—	—	—	—	—	—	—	—	—	—	—
	<u>$\lambda = 0.75, D = 4$ quarters</u>											
	$\pi = 1\%$				$\pi = 5\%$				$\pi = 10\%$			
	λ^*	s.d.	$\Delta\lambda$	ΔD	λ^*	s.d.	$\Delta\lambda$	ΔD	λ^*	s.d.	$\Delta\lambda$	ΔD
$\phi = 0.0$	0.75	0.03	0.00	0.1	0.76	0.03	0.01	0.2	X	X	X	X
$\phi = 0.5$	0.84	0.04	0.09	2.1	0.84	0.04	0.09	2.2	X	X	X	X
$\phi = 1.0$	—	—	—	—	—	—	—	—	X	X	X	X

Note: The number of simulations for each Calvo economy is 4000, and the length of every generated series (per simulation) is 200 quarters. For each simulation, we estimate the frequency of price adjustment λ using GMM and each of three econometric models (16); the latter differ according to indexation weight, $\phi = 0, 0.5$ and 1 . Thus, there are 4000 estimates $\tilde{\lambda}_i, i = 1, \dots, 4000$. The DGP economies differ by the assumed steady state inflation value ($\pi = 1\%, 5\%$ or 10% per year), and by the assumed frequency of price adjustment ($\lambda = 0.5$ or $\lambda = 0.75$). For each economy we report the mean of the estimated frequencies of price changes, λ^* , their standard deviation, s.d., as well as the indexation biases, $\Delta\lambda$ (in frequency) and ΔD (in quarters). — implies convergence occurred less than 1000 times, + implies convergence less than 3000 times but greater than 1000 times, X means that equilibrium does not exist.

Table 6: Indexation bias: Calvo model with static indexation

	<u>$\lambda = 0.5, D = 2$ quarters</u>											
	$\pi = 1\%$				$\pi = 5\%$				$\pi = 10\%$			
	λ^*	s.d.	$\Delta\lambda$	ΔD	λ^*	s.d.	$\Delta\lambda$	ΔD	λ^*	s.d.	$\Delta\lambda$	ΔD
$\phi = 0.0$	0.50	0.04	0.00	0.0	0.50	0.04	0.00	0.0	0.50	0.04	0.00	0.0
	<u>$\lambda = 0.75, D = 4$ quarters</u>											
	$\pi = 1\%$				$\pi = 5\%$				$\pi = 10\%$			
	λ^*	s.d.	$\Delta\lambda$	ΔD	λ^*	s.d.	$\Delta\lambda$	ΔD	λ^*	s.d.	$\Delta\lambda$	ΔD
$\phi = 0.0$	0.75	0.03	0.00	-0.0	0.75	0.03	0.00	-0.0	0.75	0.03	0.00	-0.0

Note: The number of simulations of each Calvo economy is 4000, the length of each simulation is 200 quarters. For each simulation, we estimate the frequency of price adjustment λ using an econometric model (16) and GMM to obtain 4000 estimates $\tilde{\lambda}_i, i = 1, \dots, 4000$. The economies differ by their steady state inflation ($\pi = 1\%$, 5% or 10%) per year, and by the assumed frequency of price adjustment ($\lambda = 0.5$ and $\lambda = 0.75$). The indexation weight in the econometric model is $\phi = 0$ (static indexation). For each economy we report the mean estimated frequency of price changes, λ^* , the standard deviation of the distribution of estimated frequencies, s.d., and the indexation biases, $\Delta\lambda$ (in frequency) and ΔD (in quarters). — implies convergence occurred less than 1000 times, + implies convergence less than 3000 times but greater than 1000 times.

Table 7: Indexation bias: Calvo model with dynamic indexation

	<u>$\lambda = 0.5, D = 2$ quarters</u>											
	$\pi = 1\%$				$\pi = 5\%$				$\pi = 10\%$			
	λ^*	s.d.	$\Delta\lambda$	ΔD	λ^*	s.d.	$\Delta\lambda$	ΔD	λ^*	s.d.	$\Delta\lambda$	ΔD
$\phi = 0.0$	0.51	0.04	0.01	0.1	0.51	0.04	0.01	0.1	0.51	0.04	0.01	0.1
	<u>$\lambda = 0.75, D = 4$ quarters</u>											
	$\pi = 1\%$				$\pi = 5\%$				$\pi = 10\%$			
	λ^*	s.d.	$\Delta\lambda$	ΔD	λ^*	s.d.	$\Delta\lambda$	ΔD	λ^*	s.d.	$\Delta\lambda$	ΔD
$\phi = 0.0$	0.75	0.02	0.00	0.0	0.75	0.02	0.00	0.0	0.75	0.02	0.00	0.0

Note: The number of simulations of each Calvo economy is 4000, the length of each simulation is 200 quarters. For each simulation, we estimate the frequency of price adjustment λ using an econometric model (16) and GMM to obtain 4000 estimates $\tilde{\lambda}_i, i = 1, \dots, 4000$. The economies differ by their steady state inflation ($\pi = 1\%$, 5% or 10%) per year, and by the assumed frequency of price adjustment ($\lambda = 0.5$ and $\lambda = 0.75$). The indexation weight in the econometric model is $\phi = 1$ (dynamic indexation). For each economy we report the mean estimated frequency of price changes, λ^* , the standard deviation of the distribution of estimated frequencies, s.d., and the indexation biases, $\Delta\lambda$ (in frequency) and ΔD (in quarters). — implies convergence occurred less than 1000 times, + implies convergence less than 3000 times but greater than 1000 times.

Appendix A: Steady state equilibrium equations

Steady state is characterized by 8 equations for 8 unknowns: $p, p^*, p(0), c, l, M/P, w, mc$.

$$\begin{aligned}
 p &= \left(\frac{1 - \lambda}{1 - \lambda^T} \cdot \frac{1 - (\lambda\pi^{(1-\delta)(\theta-1)})^T}{1 - \lambda\pi^{(1-\delta)(\theta-1)}} \right)^{\frac{1}{1-\theta}} p(0) \\
 p^* &= \left(\frac{1 - \lambda}{1 - \lambda^T} \cdot \frac{1 - (\lambda\pi^{(1-\delta)\theta})^T}{1 - \lambda\pi^{(1-\delta)\theta}} \right)^{\frac{1}{-\theta}} p(0) \\
 p(0) &= \frac{\theta}{\theta - 1} \frac{1 - (\beta\lambda\pi^{\theta(1-\delta)})^T}{1 - (\beta\lambda\pi^{(\theta-1)(1-\delta)})^T} \frac{1 - \beta\lambda\pi^{(\theta-1)(1-\delta)}}{1 - \beta\lambda\pi^{\theta(1-\delta)}} mc \cdot p \\
 c &= \left(\frac{p^*}{p} \right)^{-\theta} l \\
 p &= \mu P/M \\
 w &= -U_l/U_c \\
 mc &= w \\
 U_m &= U_c(1 - \beta/\mu)
 \end{aligned}$$

Appendix B: Log-linearized equilibrium equations

First-order conditions for consumer's problem are linearized as follows

$$\begin{aligned}\hat{w}_t &= \left(\frac{U_{cl}}{U_l} - \frac{U_{cc}}{U_c} \right) c \hat{c}_t + \left(\frac{U_{ll}}{U_l} - \frac{U_{cl}}{U_c} \right) l \hat{l}_t + \left(\frac{U_{lm}}{U_l} - \frac{U_{cm}}{U_c} \right) M/P(\hat{\mu}_t - \hat{p}_t) \\ &\frac{U_m}{U_c} \left\{ \left(\frac{U_{cc}}{U_c} - \frac{U_{cm}}{U_m} \right) c \hat{c}_t + \left(\frac{U_{cl}}{U_c} - \frac{U_{lm}}{U_m} \right) l \hat{l}_t + \left(\frac{U_{cm}}{U_c} - \frac{U_{mm}}{U_m} \right) M/P(\hat{\mu}_t - \hat{p}_t) \right\} \\ &= \beta E_t \left(\frac{U_{cc}c}{U_c} (\hat{c}_{t+1} - \hat{c}_t) + \frac{U_{cl}l}{U_c} (\hat{l}_{t+1} - \hat{l}_t) \right. \\ &\quad \left. + \frac{U_{cm}M/P}{U_c} (\hat{\mu}_{t+1} - \hat{p}_{t+1} - \hat{\mu}_t + \hat{p}_t) + \hat{p}_t - \hat{p}_{t+1} - \hat{\mu}_t \right)\end{aligned}$$

Inflation is

$$\hat{\pi}_t = \hat{p}_t - \hat{p}_{t-1} + \hat{\mu}_t$$

Equations for producers are

$$\hat{m}c_t = \hat{w}_t$$

Market clearing conditions are

$$\hat{c}_t = \hat{l}_t - \theta (\hat{p}_t^* - \hat{p}_t)$$

The linearized pricing equation () is

$$\begin{aligned}\hat{p}_t(0) &= \left(\frac{1 - (\beta\lambda\pi^{\theta(1-\delta)})^T}{1 - \beta\lambda\pi^{\theta(1-\delta)}} \right)^{-1} \left\{ \hat{U}_{ct} + \hat{m}c_t + \hat{\Lambda}_t \right. \\ &\quad \left. + E_t \sum_{\tau=1}^{T-1} (\beta\lambda\pi^{(1-\delta)\theta})^\tau \left[\hat{U}_{ct+\tau} + \hat{m}c_{t+\tau} + \hat{\Lambda}_{t+\tau} + \theta ((\hat{\mu}_t - \delta\phi\hat{\pi}_t) + \dots + (\hat{\mu}_{t+\tau-1} - \delta\phi\hat{\pi}_{t+\tau-1})) \right] \right\} \\ &- \left(\frac{1 - (\beta\lambda\pi^{(\theta-1)(1-\delta)})^T}{1 - \beta\lambda\pi^{(\theta-1)(1-\delta)}} \right)^{-1} \left\{ \hat{U}_{ct} - \hat{p}_t + \hat{\Lambda}_t \right. \\ &\quad \left. + E_t \sum_{\tau=1}^{T-1} (\beta\lambda\pi^{(1-\delta)(\theta-1)})^\tau \left[\hat{U}_{ct+\tau} - \hat{p}_{t+\tau} + \hat{\Lambda}_{t+\tau} + (\theta-1) ((\hat{\mu}_t - \delta\phi\hat{\pi}_t) + \dots + (\hat{\mu}_{t+\tau-1} - \delta\phi\hat{\pi}_{t+\tau-1})) \right] \right\}\end{aligned}$$

where

$$\hat{\Lambda}_t = \theta \hat{p}_t + \hat{c}_t$$

Price indexes are

$$\hat{p}_t = \frac{1 - \lambda\pi^{(1-\delta)(\theta-1)}}{1 - (\lambda\pi^{(1-\delta)(\theta-1)})^T} \times \left[\hat{p}_t(0) + \sum_{\tau=1}^{T-1} (\lambda\pi^{(1-\delta)(\theta-1)})^\tau [\hat{p}_{t-\tau}(0) - ((\hat{\mu}_{t-1} - \delta\phi\hat{\pi}_{t-1}) + \dots + (\hat{\mu}_{t-\tau} - \delta\phi\hat{\pi}_{t-\tau}))] \right]$$

$$\hat{p}_t^* = \frac{1 - \lambda\pi^{(1-\delta)\theta}}{1 - (\lambda\pi^{(1-\delta)\theta})^T} \times \left[\hat{p}_t(0) + \sum_{\tau=1}^{T-1} (\lambda\pi^{(1-\delta)\theta})^\tau [\hat{p}_{t-\tau}(0) - ((\hat{\mu}_{t-1} - \delta\phi\hat{\pi}_{t-1}) + \dots + (\hat{\mu}_{t-\tau} - \delta\phi\hat{\pi}_{t-\tau}))] \right]$$