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Abstract

This paper asks: What is the effect of government policy on output and inequality in an environment with education and labor-supply decisions? The answer is given in a general equilibrium model, consistent with the post 1960s facts on male wage inequality and labor supply in the U.S. In the model, education and labor-supply decisions depend on progressive income taxation, the education system, the social security system, and technology-driven wage differentials. Government policies affect output and inequality through two channels. First, a policy change leads to an asymmetric adjustment of working hours and savings of schooled and unschooled individuals. Second, there is a redistribution of the workforce between schooled and unschooled workers. Using a battery of proposed government policies, we demonstrate that skill redistribution dampens the response of wage inequality to a policy change and amplifies the response of output by an additional 1 to 2 percent.

JEL classification: H52, J31, J38
Bank classification: Labour markets; Potential output; Productivity

Résumé

Les auteurs cherchent à cerner l'effet des politiques des pouvoirs publics sur la production et l'inégalité dans une économie où les agents ont à choisir entre les études et le travail. Pour y parvenir, ils recourent à un modèle d'équilibre général dont les résultats cadrent avec les faits qui caractérisent à partir de 1961 l'inégalité des salaires et l'offre de travail dans la population masculine aux États-Unis. Dans ce modèle, le choix entre les études et le travail est fonction de plusieurs facteurs : la progressivité de l'impôt sur le revenu, le système éducatif, le régime de sécurité sociale et les écarts salariaux attribuables à l'évolution de la technologie. Par ses politiques, l'État influe sur la production et l'inégalité de deux manières. Toute modification apportée à l'une de ses politiques induit d'abord un ajustement asymétrique du nombre d'heures travaillées et de l'épargne parmi les hommes scolarisés et non scolarisés. Elle entraîne ensuite une redistribution de la main-d'œuvre entre ces deux groupes de travailleurs. À partir de la simulation de divers scénarios, les auteurs montrent que la hausse de la proportion des travailleurs scolarisés atténue l'incidence d'un changement de politique sur les écarts salariaux mais amplifie la réaction de la production d'un ou de deux points de pourcentage.

Classification JEL : H52, J31, J38
Classification de la Banque : Marchés du travail; Production potentielle; Productivité

1. Introduction

This paper studies the effects of policy change on education choice, labor supply and wage inequality. The paper consists of two parts. First, we build a theoretical framework that is applicable for policy analysis. This is done by developing a general equilibrium model consistent with the recent U.S. history of male labor hours and earnings. A main feature of the model is its explicit treatment of individual choices to educate and work. In the second part, we use our framework to study the effects of policy change on labor supply and the education wage premium. The main question guiding our analysis is: What is the effect of government policy, such as progressive income taxation, the education system, and the social security system, on wage and income inequality in an environment with education and labor-supply choice?

There are two distinct trends in the U.S. post-war aggregate data on labor supply. Trend 1, the education wage premium for men, measured as the ratio of median wage per hour of college graduates to that for men with less than 4 years of college, increased from 1.43 in 1961 to 1.80 in 2002, a 26% change. Trend 2, the large increase in the relative wages across these education groups was accompanied by the comparable increase in the relative total working hours. The ratio of schooled to unschooled total hours worked tripled from 0.16 to 0.47.[1]

Remarkably, the main focus in the literature has been on the demand for working hours, whereas the study of the supply across skill groups in the aggregate setting has received far much less attention.[2] In the U.S. data, most of the increase in the relative hours comes from the increase in the number of individuals completing college. The fraction of college graduates in the employed population more than doubled, increasing from 12.3% to 29.3% from 1961 to 2002. In contrast, the mean workweek length for both skill groups changed only a little in that period, decreasing from 44.5 to 43.2 hours per week for schooled, and from 39.8 to 38.3 hours per week for unschooled workers. Therefore, individual choices of attending school and working should be an inherent ingredient of any aggregate model of labor supply and inequality.

In our model, each individual chooses the number of hours to work per week as well as whether to obtain a college degree given his realized utility cost of attending college. After the cost realization, an individual in the model decides to go to college, if the cost is

[1] See Eckstein and Nagypal (2004) and the survey by Katz and Autor (1999) for a detailed analysis of macroeconomic trends in comensation and labor supply.

[2] A few examples include Heckman et al. (1998a), Guvenen and Kuruscu (2006), He (2006), Arpad (2004).

smaller than the gain in terms of lifetime utility. The cost of going to college is assumed to be decreasing in cognitive ability. Ceteris paribus, an individual with higher cognitive ability faces a higher probability of finishing college. Based on this feature, we determine a parametric specification of the schooling cost to match the incidence of college completion among young men in the National Longitudinal Survey of Youth 1979.

In the model, the education system is broadly defined as the part of cognitive skills acquired prior to the college decision, the time spent at college, and the amount of tuition paid for college education. Individual supplies of hours and savings are affected by government policy in the form of progressive income taxation, the retirement system, and the education system, as well as by wage differentials driven by technology. We employ Statistics of Income census data on individual tax returns and social security payments to obtain an empirically plausible progressive-income tax function as well as retirement-benefit payments to retirees in each education group.

Another key element of the model is the aggregate technology suggested in Krusell et al. (2000), or KORV in short.[3] KORV's aggregate production function has technological progress embodied in capital equipment. Capital equipment, in turn, is complementary with skilled hours. Thus, a secular increase in the demand for equipment due to embodied technological progress boosts the demand for skilled labor. KORV show that their aggregate technology predicts the historical change in the education premium *given* the observed time series of hours worked by schooled and unschooled individuals, and stocks of capital structures and (quality adjusted) capital equipment. The general equilibrium framework developed in this paper adopts an empirically plausible KORV technology to drive up the demand for schooled working hours. Hence the key to the empirical success of our framework is to account for the behaviour of the supply of aggregate hours and savings *given* relative wages and returns on capital. We demonstrate that the calibrated model generates empirically plausible changes in the supply of total hours and savings for each skill group in the U.S. from 1961 to 2002 given the historical changes in neutral and skill-biased technological progress. This step completes the development and testing of our framework.

In the second and main part of the paper, we employ the framework to analyze the effects of public-policy changes on working hours, the number of individuals finishing college, and

[3]KORV's technology is a particular version of the popular theory that the positive secular comovement in hours and compensation across groups with different schooling is due to skill-biased technological change (SBTC). According to this hypothesis, technological change favours the demand for skilled hours over unskilled hours, inducing the increase in both relative price and relative quantity of skilled labor. For details see Katz and Murphy (1992) and references therein.

the education premium. In particular, we emphasize two effects of government policies on wage inequality and output.

First, a policy change has a differential effect on working hours and savings of schooled and unschooled workers. This is typically due to a redistributive effect on income (for example, after a reform of income taxation or the pension-benefit structure) or a change in the education system (for example, after an increase in tuition subsidies). It leads to a readjustment of the individual's labor, consumption and savings decisions. A result of the changed savings behavior is an adjustment of the amount of skill-intensive capital stock that influences the demand for schooled hours and through those wage inequality. Second, the resulting difference in the lifetime streams of consumption and leisure imply different incentives of attending college. The ensuing change in college enrollment will lead in the long run to an adjustment in the fraction of schooled workers in the employed population. For example, a higher relative supply of schooled hours puts a downward pressure on the wage premium while boosting the level of output.

In order to quantify the effects of the change in government policy on labor supply and wage inequality, we conduct a set of policy experiments mimicking some proposed policy reforms. Our policy experiments can be categorized into three groups. First, we study the income tax reforms that lower the level and progressivity of the income tax. Second, we look at changes in the retirement system: (i) reforms that improve the viability of the pension fund (increase in social security contributions and the increase in mandatory retirement age); and (ii) reforms that affect the redistributive features of the pension system (a move to proportional pension benefits or a switch from a pay-as-you-go-system to a fully-funded pension system). Finally, we investigate policies that improve the efficiency of the schooling system.

Two regularities stand out in these policy experiments. First, the long-run reallocation of the workforce between unschooled and schooled jobs can be quite substantial, reaching on several occasions a million or more workers within a 70-million male workforce. And second, in the long run, the skill reallocation in most experiments reverses the initial effect on wage inequality and leads to an additional 1 to 2% change in the level of output.

The rest of the paper preceeds as follows. Section 2 describes the main data facts. Section 3 presents the life-cycle model. In Section 4 the model is calibrated to the U.S. data in 1961 and 2002. We demonstrate the model's success in matching key macroeconomic facts and evaluate the importance of labor-supply and education choices for wage and working-time inequality. In Section 5 we subject the developed model to a sequence of policy experiments to study the effect of government policy on schooling and inequality. Section 6 concludes.

2. Data

In this section, we present data on working time, earnings per hour and productivity in the U.S. from 1961 to 2002. We focus on long-run trends and mostly consider changes between 1961 and 2002.

2.1 Main facts for education premium and hours from 1961 to 2002

The data on working hours and compensation are based on the March supplement of the Current Population Statistics (CPS) as provided by the IPUMS project.[4] Following a convention in the literature, we restrict our sample to males age 16 to 64. Before age 16, 85% of men go to school and after 64 (the normal retirement age) the fraction of males working decreases by about 80%.

The main reason for restricting our attention to males instead of the whole population is that earnings per hour of women are catching up with those of males. This is especially relevant for the less schooled in the 1980s.[5] If we included women, this catching up would add, we believe, a completely separate set of problems that is unrelated to the fundamentals that drive the education-premium change.

The population in our sample is divided into two education groups: college graduates (or "schooled") and less-than college graduates ("unschooled"). This is a convenient division and has been used before, for example by KORV. We define a college graduate as a person who has completed at least 4 years of college. In contrast a less-than college graduate is a person in the population with 0 years of schooling up to 3 years of college completed. A finer division of the group of less than college graduates does not alter our main facts. For example, the group of persons with 1 to 3 years of college is much closer in terms of their earnings per hour to the group of high-school graduates, than to the group of college graduates.[6]

[4] Appendices A and B provide detailed descriptions and transformations of the data.

[5] For a treatment of the gender gap we refer to Jones et al. (2003). Jones et al. point out that most of the increase in market hours is due to married women entering the labor market and the decrease of the gender-wage gap.

[6] Eckstein and Nagypal (2004) point out that there is a considerable difference between undergraduate and graduate degree holders in terms of their earnings per hour. Data limitations do not allow us to pursue subdivision of the group of college graduates.

Given these groups, we are interested in documenting the evolution of average hours worked per employed person, the number of workers, and earnings per hour for each education group over the last four decades.

Consider the following decomposition of average hours worked:

$$\begin{aligned} \frac{L}{N} &= \frac{L}{E} \times \frac{E}{N} \\ &= \left[\frac{E^S}{E} \frac{L^S}{E^S} + \frac{E^U}{E} \frac{L^U}{E^U} \right] \times \frac{E}{N} , \end{aligned} \tag{1}$$

where L^i, E^i, N stand, respectively, for the total hours worked by group i, the number of employed[7] in group i, and the working age population. The following relationships hold: $L^S + L^U = L$ and $E^S + E^U = E$. We define as the workweek length of group $i : l^i \equiv L^i/E^i$. We refer to $e \equiv E^S/E$ as the fraction of schooled workers in the employed population. The earnings per hour for each education group are measured by the (hours-weighted) median earnings per hour of employed persons in the respective group. Using the median (as opposed to the mean) helps us to circumvent the change in the CPS' top coding procedure in the 1990s. For convenience, we refer to the median earnings per hour as wage. Finally, we define the education premium as the wage of a college graduate relative to that of a less than college graduate.

From 1961 to 2002, the average hours worked by males decreased by about 9% (see Table 1). This is mostly due to a decrease in the fraction of employed in the male population, -6.5 percentage points. Even though the employment to population change for males is interesting, it is dwarfed in size and importance by the changes that happened within the square brackets in equation (1). Taking this observation into account, we focus on the compositional changes of hours worked by employed persons. In particular, we highlight three main observations for employed-male population for the period of 1961 to 2002:

(1) wages of schooled workers increased dramatically, by 26.1%, relative to those of unschooled workers,

(2) the fraction of employed schooled persons among all employed persons increased considerably, by 17 percentage points,

(3) the workweek length of both schooled and unschooled employed persons decreased,

[7] We consider a person to be employed, if that person works more than zero hours per year.

but only slightly: -1.4 hours per week (-2.9%) for schooled workers, and 1.5 hours per week (-3.8%) for unschooled workers.[8]

Figure 1 contrasts changes in wages of college graduates and less-than college graduates from 1961 to 2002. While the wage of schooled workers increased by three quarters from \$13.3 to \$23.0, the wage of unschooled increased by only a quarter from \$9.3 to \$12.8. [9] Accordingly, the education premium increased from 1.43 in 1961 to 1.80 in 2002 (see Figure 2). The 26.1% increase in relative wages is sizeable, suggesting that a large part of the 47.7% overall wage growth for males over this period was unevenly distributed.

The observed change in wages was accompanied by a large shift in working hours from unschooled to schooled workers. The fraction of total hours supplied by the college graduates has more than doubled over the last 40 years, reaching now one third of all hours worked in the market place (see Figure 3). Almost all of the reallocation of total hours is due to a change in the fraction of college graduates in the population and especially in the employed population. From 1961 to 2002 the fraction of schooled in the employed population more than doubled, increasing from 12% to 29%. In contrast, the workweek in each education group stayed virtually unchanged. Figure 4 plots the workweek for the two groups from 1961 to 2002. Schooled individuals work more on average with 44.5 hours per week in 1961 compared to 39.8 hours per week for unschooled workers.[10] Both time series observe a minor fall by 2002 down to 43.2 and 38.3 hours per week, respectively, for schooled and unschooled employed persons.

In Section 3, we present a model which can be used to account for the three highlighted facts. In particular, the model has schooling and working-time choices. To generate the transition from a low to a high fraction of schooled workers, we appeal to an idea developed by Katz and Murphy (1992) and later extended by KORV. The partial equilibrium approach undertaken by KORV, attributes the skill reallocation to skill-biased technological change which, in turn, is driven by capital-skill complementarity and capital-embodied technological change. In the next subsection, we describe the measurement of capital-embodied technological progress in the data.

[8] The workweek length is measured as the average annual hours worked per employed person at the weekly rate. We take the actual hours worked per week and the actual weeks worked per year into account in our calcuation of annual hours.

[9] Unless otherwise noted, all quantities are in constant (year 2000) US dollars.

[10] We assume 100 hours available for work in a week.

2.2 Capital-embodied technological progress

The stock of capital equipment is reported in National Income and Product Accounts (NIPA) tables and, for most of our sample period, the Bureau of Economic Analysis did not adjust the stock of capital equipment to reflect quality changes. Gordon (1990) conducts a hedonic regression analysis to document the quality component of the growth of equipment. He finds that the quality of capital equipment increased by a factor of 4 from 1961 to 1982. KORV use Gordon's quality time series and extrapolate to cover their sample period. We use KORV's data and the methodology suggested in Greenwood et al. (1997) to extend Gordon's time series through 2002 (see Appendix B.2). Equations (2) and (3) capture the measurement and quality adjustment of capital stocks:

$$\overline{K}^{st} = \frac{\text{Structures Stock}}{\text{Consumption Deflator}}, \tag{2}$$

$$\overline{K}^{eq} = \underbrace{\frac{\text{Equipment Stock}}{\text{Qual. adj. Equipment Deflator}}}_{K^{eq}} \cdot \underbrace{\frac{\text{Qual. adj. Equipment Deflator}}{\text{Consumption Deflator}}}_{q}. \tag{3}$$

Equation (3) captures the quality adjustment of the stock of equipment. $\overline{K}^{st}$ and $\overline{K}^{eq}$ are, respectively, measured real stocks of capital structures and equipment (in consumption units), K^{eq} denotes the quality-adjusted stock and q is the quality-adjustment factor.

Figure 5 presents the two stocks from 1961 to 2002. Even though the stock of capital structures grew by 136%, it is dwarfed by the 814% growth in the quality adjusted stock of capital equipment[11]. The quality-adjustment factor q, which is equal to the relative price of the quality-adjusted stock of equipment in terms of the consumption good, decreased by a factor of four from 0.93 in 1961 to 0.26 in 2002. The decrease in the relative price of equipment is used in the model in Section 3 as a stand-in for (the reciprocal of) capital-embodied technological progress. It is a key force behind the increase in the demand for schooled labor in the model.

[11] There are mainly two alternative measures of capital-embodied technological progress covering about the same period that we consider: Cummins and Violante (2002) and Polgreen and Silos (2005). The later paper provides two main series for capital-embodied technological progress, one that predicts strong growth during the 1990s (similar to Cummins and Violante) and one with very week growth in quality-adjusted capital equipment. We are closer to the more conservative estimate in capital-embodied technological growth for the 1990s but differ only slightly from the other papers' measures for the pre 1990 period.

3. Model

In this Section we present a general-equilibrium life-cycle model with education choice, labor supply, and skill-biased technological change.

There is a continuum of ex-ante identical individuals. The diagram in Figure 6 summarizes the life cycle of individuals in the economy. An individual is born at age 18.[12] At birth he faces a cost of education and decides whether or not to educate. If the individual decides to educate, he spends 4 years at college. The individual starts working at 22, or 18 if he did not go to college. All workers retire after turning 65 and die after 71.[13]

In this paper we focus on the economy in steady state. This implies that all aggregate prices and quantities are constant. The steady state assumption is reasonable given our focus on the secular change in the U.S. economy between two distant points of time - the years 1961 and 2002. Furthermore, we find that the main results are qualitatively robust even on the transition path. We start by considering an individual's choice of working time, savings, and consumption conditional on the education choice. Afterwards, we present the education choice problem.

3.1 Working time choice

An individual's age is represented by an index $a = 1, ..., I$. The problem of an unschooled individual is to choose lifetime sequences of consumption c_a^U, working hours l_a^U, and asset holdings k_{a+1}^U to solve :

$$V^U\left(k^U\right) = \max_{\left\{c_a^U, l_a^U, k_{a+1}^U\right\}} \sum_{a=1}^{I} \beta^{a-1}\left[\ln\left(c_a^U\right) + \phi \ln\left(1 - l_a^U\right)\right] \tag{4}$$

subject to

$$(1+\tau_c)\, c_a^U + k_{a+1}^U \leq w^U\ l_a^U + rk_a^U - T\left(w^U\ l_a^U + rk_a^U\right) + TR_a^U + \Pi_a^U\ , \tag{5}$$

$$l_a^U = 0,\ \text{for}\ \forall a > I_r\ , \tag{6}$$

[12] We choose the age 18 as the beginning in the model since it is the average time for high-school graduation and for most young males to begin college. This is in contrast to 16 years in the sample we considered in the data. The difference in the data facts due to this age choice is minor and compensated by the comparability of our numbers with standard publications, for example by the Bureau of Labor Statistics.

[13] In the data reported in Arias (2004), the life expectancy at age 15 increased from 69 in 1961 to 75 in 2002. In the model, we choose constant life length of 71 to match relatively constant ratio of males over 64 to males over 16 (model: 13.0%, Census data: 12.8% in 1961, 13.6% in 2002).

$$k_1^U = k^U \ . \tag{7}$$

Here (5) is the budget constraint of the unschooled at age a. He consumes, saves, receives wage income at wage rate w^U, asset returns at rate r, transfers from the government TR_a^U and divident payments Π_a^U. He faces tax payments as a (smooth) function $T(\cdot)$ of her wage and capital income, and pays a proportional consumption tax with the rate τ_c.[14] Equation (6) represents a no-working constraint for retirees, I_r is the retirement age, and (7) is the constraint on the initial asset holdings.

The problem of a schooled individual is similar to that of an unschooled worker:

$$V^S\left(k^S\right) = \max_{\left\{c_a^S, l_a^S, k_{a+1}^S\right\}} \sum_{a=1}^{I} \beta^{a-1} \left[\ln\left(c_a^S\right) + \phi \ln\left(1 - 0.33 \cdot \mathcal{I}(a \leq I_e) - l_a^S\right)\right] \tag{8}$$

subject to

$$(1+\tau_c)\, c_a^S + k_{a+1}^S \leq w^S\ l_a^S + r k_a^S - B_a - T\left(w^S\ l_a^S + r k_a^S\right) + TR_a^S + \Pi_a^S \ , \tag{9}$$

$$l_a = 0, \text{ for } \forall a > I_r \ , \tag{10}$$

$$l_a = 0, \text{ for } \forall a \leq I_e \ , \tag{11}$$

$$B_a = B > 0, \text{ for } \forall a \leq I_e \ , \tag{12}$$

$$k_1^S = k^S \ . \tag{13}$$

In addition to the budget (9), the retirement time (10) and initial asset holdings (13), individuals who go to college for I_e years cannot work (11), and pay tuition cost B (12) while studying. To reflect the time spent on studying in college, the time endowment of students is reduced by an equivalent of a full-time workweek length, where $\mathcal{I}(a \leq I_e)$ equals to 1 for $a \leq I_e$ and zero otherwise. Note that all individuals face the same income tax function and consumption tax rate.

To summarize, the factors that contribute to the difference in lifetime utility across education groups are: initial asset holdings, wages, income tax rates (if the tax function is progressive), government transfers, tuition cost, and education time. We next proceed to the problem of education choice faced by a newborn individual.

[14] We assume that capital income is taxed only as a part of personal income. In the U.S. the revenue from taxes on capital as part of the personal income taxes is considerably larger than the corporate income tax revenue. See, for example, Castaneda et al. (1998) or Altig and Carlstrom (1999) for the same approach.

3.2 Education choice

Newborn individuals face a cost of education d that has the following specification:

$$d(Q,z) = d_{\max} - \delta_d (\iota(Q) - z) \ , \tag{14}$$

where Q and z are independent random draws from $N(0,1)$, $d_{\max} \in \mathbb{R}_{++}$, $\delta_d \in \mathbb{R}_{++}$, and the function $\iota : \mathbb{R} \to \mathbb{R}$ is continuous and increasing. We interpret Q as an individual's cognitive ability, and z is the residual ("psychic") cost of education. As we show below, specification (14) implies that cognitive ability is crucial for determining the size of the response of the enrollment rate to a change in government policy. In contrast, the psychic cost of education is irrelevant for the aggregate economy.

The education-choice problem of a newborn individual is:

$$V(k) = \max_{\text{study / work}} \left\{ \begin{array}{c} V^S(k) - d(Q,z) \ \text{ if study at 18} \\ V^U(k) \ \text{ if work at 18} \end{array} \right\} . \tag{15}$$

Given continuity of $d(\cdot,\cdot)$ with respect to its second argument, for any realization of cognitive ability Q, there exists a draw z^* such that the individual is indifferent between going to college and working. Hence the education choice for an individual with realized ability Q and cost draw z is to educate if $d(Q,z) < d(Q,z^*)$, and work otherwise. We apply this decision rule to derive the probability of going to college for an individual with cognitive ability Q:

$$\Pr(d(Q,z) \leq \Delta V(k)) = F\left(\frac{\Delta V(k) - d_{\max}}{\delta_d} + \iota(Q)\right), \tag{16}$$

where F is the c.d.f. of $N(0,1)$, and $\Delta V(k) \equiv V^S(k) - V^U(k)$ is the difference in lifetime utility across two education groups. Notice that the probability of college completion is increasing in cognitive ability, Q, and does not depend on the psychic cost, z. Hence, specification (14) with two random draws allows us to capture two aspects from the data on test performance and educational attainment: (1) individuals with higher test performance have (on average) a higher educational attainment; and (2) for any low (high) test performance, there are individuals who have (have not) completed at least 4 years of college (see Section 4).

Equation (16) implies that the fraction of households that decide to educate in each period, the enrollment rate, is given by:

$$\varepsilon_a = \int_{-\infty}^{\infty} F\left(\frac{\Delta V(k) - d_{\max}}{\delta_d} + \iota(Q)\right) dF(Q). \tag{17}$$

Equation (17) relates the enrollment rate to a change in government policy. A policy change that has an asymmetric effect on lifetime streams of consumption and leasure of schooled and unschooled individuals, alters the lifetime utility gap, $\Delta V(k)$. For an individual with given cognitive ability Q, a higher (lower) gap in the lifetime stream of utility will increase (decrease) that individual's probability of completing college. On the aggregate, the size of the response of the enrollment rate to a policy change depends on the change in the lifetime utility gap and the distribution of education cost over the population, captured by the term $\frac{\iota'(Q)F''(Q)}{F'(Q)}$.[15] The advantage of our approach is that we can infer this term directly from the data by estimating function $\iota(\cdot)$ using the data on test performance and educational attainment (see Section 4).

Finally, cohorts in the model have equal size, $1/I$, so the fraction of people ever enrolled in college is

$$\varepsilon = \varepsilon_a I \ . \tag{18}$$

Given an individuals' life cycle (see Figure 6), the fraction of schooled in the employed population, e, is

$$e = \frac{\varepsilon(I_r - I_e)}{I_r - \varepsilon I_e} \ . \tag{19}$$

3.3 Bank's problem

There is a competitive industry of banks that pool individuals' savings in the form of asset k and invest them in two types of physical capital: structures and equipment. Structures are a generic form of physical capital, whereas equipment undergoes embodied technological progress - a key source of rising demand for schooled labor in the model. The problem of the

[15] To see this, note that the derivative of the enrollment rate with respect to the lifetime utility gap is:

$$\frac{d\varepsilon_a}{d(\Delta V)} = \int_{-\infty}^{\infty} \omega(Q) F\left(\frac{\Delta V - d_{\max}}{\delta_d} + \iota(Q)\right) dF(Q),$$

where

$$\omega(Q) = \frac{F'(Q)}{\delta_d \iota'(Q) F''(Q)}.$$

representative bank is

$$\max_{K^{st},K^{eq},\{k_a^S,k_a^U\}} \quad \left(r^{st}-\delta_{st}\right)K^{st}+\left(r^{eq}-\delta_{eq}q\right)K^{eq}-r\sum_{a=1}^{I}\left(\varepsilon_a k_a^S+\left(\frac{1}{I}-\varepsilon_a\right)k_a^U\right)$$

subject to

$$K^{st}+qK^{eq}\leq\sum_{a=1}^{I}\left(\varepsilon_a k_a^S+\left(\frac{1}{I}-\varepsilon_a\right)k_a^U\right) \quad .$$

Here K^{st}, K^{eq} denote aggregate stocks of structures and equipment, r^{st}, r^{eq} are their respective rental rates; δ_{st}, δ_{eq} are their respective depreciation rates; and q is the price of equipment in terms of the consumption good (with the price of structures in terms of the consumption good normalized to one).

3.4 Final good producing technology

We adopt the final goods producing technology from KORV. This technology admits both the secular rise in relative wages and the rise in schooled hours (see the discussion in footnote 16). The production function $F(\cdot)$ has four factors: aggregate stocks of capital equipment K^{eq} and structures K^{st}, total hours of schooled workers L^S and total hours of unschooled workers L^U :

$$\begin{aligned}F\left(K^{eq},K^{st},L^S,L^U\right)&=A\left(K^{st}\right)^{\alpha}\times\\&\left[\theta_u\left(A_S L^U\right)^{\eta}+(1-\theta_u)\left(\theta_k\left(K^{eq}\right)^{\kappa}+(1-\theta_k)\left(A_S L^S\right)^{\kappa}\right)^{\eta/\kappa}\right]^{(1-\alpha)/\eta} \quad .\end{aligned} \tag{20}$$

We constrain the parameters of the technology to guarantee two input relationships. Capital equipment and labor services of schooled are complementary, which requires $\kappa<0$. Total hours of unschooled, on the other hand, are substitutes with the capital-skill aggregate, implying $\eta>0$. Technological progress is embodied in capital equipment and is the source of increasing demand for hours of college graduates in the model. The remaining parameters include: share parameters α, θ_u, $\theta_k\in[0,1]$, total factor productivity $A\in\mathbb{R}_{++}$, and the labor-efficiency parameter $A_S\in\mathbb{R}_{++}$.

Under competitive factor markets, rental prices of structures, r^{st}, equipment, r^{eq}, and wages of schooled, w^S, and unschooled, w^S, hours are equal to their respective marginal products:

$$\frac{r^{st}K^{st}}{Y} = \alpha \ , \tag{21}$$

$$\frac{r^{eq}K^{eq}}{Y} = \frac{(1-\alpha)\,\Theta\left(\frac{\theta_K}{1-\theta_K}\left(\frac{K^{eq}}{A_S L^S}\right)^{\kappa}+1\right)^{\frac{\eta-\kappa}{\kappa}}}{\left(\frac{L^S}{L^U}\right)^{\eta}+\Theta\left(\frac{\theta_K}{1-\theta_K}\left(\frac{K^{eq}}{A_S L^S}\right)^{\kappa}+1\right)^{\eta/\kappa}} \cdot \frac{\theta_K}{1-\theta_K}\left(\frac{K^{eq}}{A_S L^S}\right)^{\kappa} \ , \tag{22}$$

$$\frac{w^S L^S}{Y} = \frac{(1-\alpha)\,\Theta\left(\frac{\theta_K}{1-\theta_K}\left(\frac{K^{eq}}{A_S L^S}\right)^{\kappa}+1\right)^{\frac{\eta-\kappa}{\kappa}}}{\left(\frac{L^S}{L^U}\right)^{\eta}+\Theta\left(\frac{\theta_K}{1-\theta_K}\left(\frac{K^{eq}}{A_S L^S}\right)^{\kappa}+1\right)^{\eta/\kappa}} \ , \tag{23}$$

$$\frac{w^U L^U}{Y} = \frac{(1-\alpha)\left(\frac{L^S}{L^U}\right)^{-\eta}}{\left(\frac{L^S}{L^U}\right)^{\eta}+\Theta\left(\frac{\theta_K}{1-\theta_K}\left(\frac{K^{eq}}{A_S L^S}\right)^{\kappa}+1\right)^{\eta/\kappa}} \ , \tag{24}$$

where $\Theta \equiv \frac{(1-\theta_u)(1-\theta_K)^{\frac{\eta}{\kappa}}}{\theta_u}$ and Y denotes aggregate output,

$$Y = F\left(K^{eq}, K^s, L^S, L^U\right) \ . \tag{25}$$

Equations (23) and (24) imply the following expression for the education premium:

$$\frac{w^S}{w^U} = \Theta\left(\frac{L^S}{L^U}\right)^{\eta-1}\left(\frac{\theta_K}{1-\theta_K}\left(\frac{K^{eq}}{A_S L^S}\right)^{\kappa}+1\right)^{\frac{\eta-\kappa}{\kappa}} \ . \tag{26}$$

According to (26), the education premium is ceteris paribus negatively related to relative total hours (note that $\eta - 1 < 0$). Hence, for our model to be consistent with the positive secular movement of relative wages and hours (Fact 2 in Section 2), capital equipment K^{eq} must grow fast enough relative to schooled hours (in efficiency units), $A_S L^S$. In our data, the ratio of quality-adjusted capital equipment stock to total schooled hours indeed increases by a factor of four from 1961 to 2002. Therefore, the KORV technology, as opposed to other standard aggregate technologies, is consistent with the secular increases in relative wages and hours.[16]

[16] The education premium derived from a Cobb-Douglas technology is

$$\frac{w^S}{w^U} = \text{const} \cdot \left(\frac{L^S}{L^U}\right)^{-1}$$

so that the education premium and relative total hours are negatively related.

If the CES technology is symmetric (see Katz and Murphy (1992)), relative wages and hours are (as in the

3.5 Government

The government in the model collects tax revenues, pays for government expenditures G and makes transfers to individuals. The government budget is balanced in each period:

$$
\begin{aligned}
& \tau_c \sum_{a=1}^{I} \left(\varepsilon_a c_a^S + \left(\frac{1}{I} - \varepsilon_a \right) c_a^U \right) + \sum_{a=1}^{I} \varepsilon_a \tau_a^S y_a^S + \sum_{a=1}^{I} \left(\frac{1}{I} - \varepsilon_a \right) \tau_a^U y_a^U \\
= \; & G + \sum_{a=1}^{I} \varepsilon_a TR_a^S + \sum_{a=1}^{I} \left(\frac{1}{I} - \varepsilon_a \right) TR_a^U \;, \qquad (27)
\end{aligned}
$$

where $y_a^S \left(y_a^U \right)$ is taxable income of an age a schooled (unschooled) individual; $\tau_a^S \left(\tau_a^U \right)$ is the corresponding average income tax rate, defined as $\tau_a^S = T\left(y_a^S \right) / y_a^S$ and $\tau_a^U = T\left(y_a^U \right) / y_a^U$. The part of the income tax revenue due to retirement contributions is distributed among the retirees. In the benchmark version of the model, per-capita transfers are equalized, implying a redistribution of retirement payments in favour of unschooled retirees. This captures the redistributive feature of the pay-as-you-go pension system currently in place for example in the United States. We consider other specifications of the retirement system in Section 5.

3.6 Market clearing and equilibrium

There are 4 markets in the model: capital-rental markets, and markets for labor services of schooled and unschooled individuals, and the final-goods market. The first three markets' clearing conditions are:

$$
\begin{aligned}
\sum_{a=1}^{I} \varepsilon_a k_a^S + \sum_{a=1}^{I} \left(\frac{1}{I} - \varepsilon_a \right) k_a^U &= K^{st} + qK^{eq} \;, \qquad (28) \\
\sum_{a=1}^{I} \varepsilon_a l_a^S &= L^S \;, \qquad (29) \\
\sum_{a=1}^{I} \left(\frac{1}{I} - \varepsilon_a \right) l_a^U &= L^U \;, \qquad (30)
\end{aligned}
$$

with the final-goods market clearing by Walras law.

Cobb-Douglas case) negatively related, unless labor efficiency A_S grows fast enough:

$$
\frac{w^S}{w^U} = \text{const} \cdot A_S^{\eta} \left(\frac{L^S}{L^U} \right)^{\eta - 1}
$$

KORV argue that the growth in labor efficiency required to account for facts 1-3 is implausibly large: 11% per year, or a factor of 25 over 30 years.

We define the **steady state competitive equiliubrium** as an allocation $\left\{c_a^X, l_a^X, k_{a+1}^X\right\}_{a=1,X=\{\mathrm{S,U}\}}^I$, $\left\{Y, K^{st}, K^{eq}, \varepsilon, e, L^S, L^U\right\}$ and prices r,r^{st},r^{eq},w^S,w^U such that, given prices, as well as the relative price of equipment q, productivity A, tuition cost N, income tax function $T(\cdot)$, government purchases G, and initial asset holdings k^X, the allocation solves the problems of unschooled individuals (8)-(7) and schooled individuals (8)-(12); education-choice equations (17)-(19) are satisfied; input prices are equal to marginal products (21)-(24); and all markets clear (28)-(30).

4. Model and the U.S. data

This section consists of two parts. We first calibrate the model to key characteristics in the U.S. data from 1961 to 2002. Then we investigate the capability of the benchmark model to match facts 1 through 3 outlined in Section 2: increase in the education premium, rise in the fraction of college graduates, and slight decrease in the workweek length. We emphasize the role of education choice and labor supply together with productivity (embodied and neutral) for matching these facts.

4.1 Calibration

Our calibration proceeds in three steps. We first calibrate the parameters of the aggregate technology following KORV. Then, we estimate the income tax functions. And finally, we calibrate preference parameters and parameterize and estimate the cost of education.

4.1.1 Technology

As noted in the previous Section, KORV's aggregate technology in combination with competitive factor markets implies that the education premium is a (nonlinear) function of two ratios: total hours of schooled workers per total hours of unschooled workers, $\frac{L^S}{L^U}$, and equipment per total schooled hours (in efficiency units), $\frac{K^{eq}}{A_S L^S}$. The education premium function contains six parameters: factor demand elasticities η, κ, share parameters α, θ_u, θ_k, and the labor-efficiency parameter, A_S. We choose parameter values to match six moments in the U.S. data: the 1961 level and 1961 to 2002 growth of the ratio of total wage income of schooled and unschooled workers, the 1961 level and 1961 to 2002 growth of the labor-income share, the ratio of stocks of capital structures and equipment in 1961; and real GDP in 1961.[17]

[17] A convenient feature of the calibration procedure is that input elasticities η and κ are invariant with respect to units of (quality adjusted) stock of capital equipment. To see this, let ζ scale the capital equipment stock. The following three equations relate θ_k, θ_u, A to their new values θ_k', θ_u', A':

Table 3 contains the calibrated parameter values. We borrow values for capital depreciation rates δ_{st}, δ_{eq} directly from KORV, 0.05 and 0.125, respectively. The initial asset holdings of individuals are zero.

After calibrating the parameter values of the aggregate technology, we use NIPA data for output and capital inputs as well as CPS data for hours worked by education group to infer the change in the neutral productivity A from 1961 to 2002. We find that neutral productivity increases by 50%, or about 1% per year.

The calibration procedure identifies the aggregate technology that is consistent with secular movements of relative prices and aggregate quantities of labor and capital in the U.S. from 1961 to 2002. By construction, the ability of our general equilibrium model to match the observed trend in the education premium depends on its ability to match the historic levels of aggregate supplies of total hours and savings given wages and rental rates, changes in exogenous factors (such as productivity, taxes, etc.) and the specification of aggregate technology. At the end of this Section, we show that the complete model, when calibrated, fits the data well. We then use the model in Section 5 to investigate (a) the importance of education choice and labor supply for wage inequality; and (b) the effects of alternative government policies on schooling, supply of hours and welfare.

4.1.2 Income tax function and retirement benefits

Let y denote an individual's taxable income. The effective average income tax rate $T(y)/y$, denoted by $\tau(y)$, has three components: federal $\tau_{fed}(y)$, state $\tau_{st}(y)$ and social security tax rates $\tau_{ss}(y)$. Given data availability, we first estimate each component separately and then combine them to obtain the estimated income tax rate:

$$\tau(y) = \underbrace{\tau_{fed}(y)}_{\text{federal}} + \underbrace{\tau_{st}(y)}_{\text{state}} + \underbrace{\tau_{ss}(y)}_{\text{social security}} . \quad (31)$$

$$\begin{aligned} \frac{\theta_K'}{1-\theta_K'}(\zeta/A')^{\kappa} &= \frac{\theta_K}{1-\theta_K}(1/A)^{\kappa} , \\ \frac{(1-\theta_u')(1-\theta_K')^{\frac{\eta}{\kappa}}}{\theta_u'} &= \frac{(1-\theta_u)(1-\theta_K)^{\frac{\eta}{\kappa}}}{\theta_u} , \\ \theta_u'/\theta_u &= (A'/A)^{-\eta} . \end{aligned}$$

We thank Rui Castro for bringing this point to our attention. Derivations are available upon request.

The federal tax function is approximated by a polynomial of degree five. The polynomial form is easy to estimate and it fits fairly well to both macro and micro data on income tax rates [18]:

$$\tau_{fed}(y) = a_0 + a_1 y + a_2 y^2 + a_3 y^3 + a_4 y^4 + a_5 y^5 \ .$$

The parameters are determined using micro data from the Statistics of Income (SOI) as provided by the Internal Revenue Services. To estimate parameters a_0 through a_5 we use Internal Revenue Service data on individual federal income tax returns in 1961 and 2002. We regress the average tax rate for each person on her respective income.[19] Table 4 provides the estimated parameter values. To obtain the state tax rates, we use the State Tax Handbooks for 1964 and 2002 to find the states' statutory tax schedules. From those we take the highest tax rates for each state and determine a population weighted average, $\overline{tx}_{st}$. This is then used to determine a state tax function of the following form:

$$\tau_{st}(y) = \begin{cases} \frac{y}{20{,}000}\overline{tx}_{st} \\ \overline{tx}_{st} \end{cases} \text{for } y \begin{Bmatrix} \leq \\ > \end{Bmatrix} 20,000 \ .^{20}$$

For the social security tax, we make use of data provided by the U.S. Social Security Agency on social security tax rates and the social security tax base to determine the proportional rates. There are some social security taxes that appy to all levels of income above a certain minimum, $\underline{y}$. The respective tax rate is denoted by $\overline{tx}_{\text{other SS}}$. This is not the case for social security taxes that are counted towards the retirement fund. There exists an upper bound on income, $\bar{y}_{\text{Retirement}}$, up to which the statutory retirement tax rate, $\overline{tx}_{\text{Retirement}}$, is applicable. All income above this bound is not taxed by the retirement part of social security. This leads to a lower average social security tax rate for persons with income above $\bar{y}_{\text{Retirement}}$. These facts, taken together, lead to the following formula for the social security tax rate:

$$\tau_{ss}(y) = \begin{cases} 0.7\left(\overline{tx}_{\text{Retirement}} + \overline{tx}_{\text{other SS}}\right) \\ 0.7\left(\overline{tx}_{\text{Retirement}} \times \frac{\bar{y}_{\text{Retirement}}}{y} + \overline{tx}_{\text{other SS}}\right) \end{cases} \text{for } \underline{y} \leq y \begin{Bmatrix} \leq \\ > \end{Bmatrix} \bar{y}_{\text{Retirement}} \ .$$

Note that the factor of 0.7 is required since social security taxes only apply to wage income, which is around 70% of the total income of an individual.

[18] See, for example, Gouveia and Strauss (1994).

[19] We thank the IRS and especially Michael Weber for providing us with the results of the regression analysis. Throughout the determination of the tax function we work with real (year 2000) U.S. dollars.

[20] The number 20,000 refers to year 2000 U.S. dollars and captures the approximate level of income after which the state taxes are no longer progressive.

Finally, we smooth out the resulting function $\tau(y)$ by a polynomial of degree five in order to ensure countinuous differentiability of the tax function. It turns out that the difference between the smooth approximation and the original is quite small. [21]

Figure 8 plots the estimated effective average income tax rates as a function of disposable income for 1961 and 2002. Two features stand out. First, the level of the income tax function for 2002 is virtually the same as in 1961. To illustrate this we consider college-graduates' income tax. The median income of college graduate in 2002 was around 60 thousand US$ 2000. So, the average tax rate for a median-income college graduate was 20.7%, whereas in 1961 it would have been 17.1%. Second, the income tax schedule is fairly progressive. In 2002 the average tax rate of the median income unschooled individual was 17.5%, which is lower than the average income tax rate of a median income schooled person, 20.7%. When we consider tax policies in Section 5, we find that the progressivity of income taxation is an important factor for an individual's decision to go to college, and it has a large impact on the education premium.

In the model, tax revenues are used to pay for government expenditures and lump-sum transfers to workers and retirees. In the NIPA, the government expenditures (plus net exports) to GDP ratio decreased from 25% in 1961 to 16% in 2002. In the benchmark model, the revenue from the social security tax is distributed equally among the retired population in a lump-sum fashion. This way of redistribution captures the pay-as-you-go pension benefit system in the U.S. well. It favours income-poor unschooled relative to income-rich schooled retired individuals. The residual budget balance is distributed equally among all workers.

4.1.3 Preferences and education cost

The discount factor β is 0.96 so that the interest rate is 4%. We determine the share of leisure in the utility function, ϕ, by matching the workweek of college graduates in 1961 at 44.5 hours per week. The parameter value, 1.78, is consistent with values found in the empirical literature.[22]

[21] The procedure described in this subsection is based on the analysis of Current Population Statistics data on taxes for the year 2002. As an alternative, we estimated a fifth order polynomial for the combined federal, state, and social security tax rates directly from the CPS. The difference for the estimated income tax function is small. We would have prefered to use the CPS-micro approach both for the year 1961 and the year 2002, but the CPS does not provide tax data for the 1960s. To preserve consistency, we decided to use the approach outlined in the text.

[22] We rescale dollars in the model to 2002 dollars in the data. The scale parameter is picked to match the average tax rate faced by the median income individual in the data in 1961 at 15.5%.

To calibrate the utility cost of education, we employ data on the cognitive ability test performance of individuals in The National Longitudinal Survey of Youth 1979 (NLSY79). It is a nationally representative panel of more than 12,000 men and women who are between the age of 14 and 22 when they are first interviewed. In 1980, 94% of the NLSY79 respondents took the Armed Services Vocational Aptitude Battery (ASVAB) - a series of tests measuring knowledge and skill in 10 areas, ranging from general science and knowledge of mathematics to word knowledge and paragraph comprehension (see Appendix C for details). A composite Armed Forces Qualifications Test score (AFQT) was constructed for each youth based on selected sections of the ASVAB. Taking the AFQT score as a good indicator of cognitive ability, we use the score distribution of respondents in the NLSY79 to estimate the $\iota(\cdot)$ function that relates an individual's cognitive ability in the model to their utlity cost of education (see equation (14)).[23]

For our purposes, we restrict the sample to white males who took the ASVAB tests and who were not in college in 1979. We then define standardized test scores by taking out age-specific weighted means from the raw AFQT scores and normalize by the (weighted) standard deviation of scores across the entire sample:

$$Q_i = \frac{Q_i^{AFQT}}{std\left(Q_i^{AFQT}\right)} \ .$$

Here Q_i^{AFQT} is the raw (de-meaned by age) AFQT score and Q_i is the standardized AFQT score for individual i.

For each individual in this sample we document his highest completed grade as of May 1, 1988. Let HGR_i be equal to 1 if individual i's highest completed grade was 4 or more years in college, and 0 otherwise. We estimate the parameters of the following weighted probit equation:

$$v_i = h_0 + h_1 Q_i + h_2 Q_i^2 + h_3 Q_i^3 \ . \tag{32}$$

where the probit is defined as $v_i = F^{-1}\left(\Pr(HGR_i) = 1\right)$, and F is the c.d.f. of a standard normal random variable. Table 5 contains regression results. Given the normality assumptions on cognitive ability in the model, the right hand side of (32) is a third-order polynomial approximation of function $\iota(\cdot)$. For the estimated parameter values h_i, Figure 7 plots the probability of going to college conditional on the test score. According to our estimates, an individual with the median score (65 AFQT points, or 0.4 standardized AFQT points)

[23] Due to limited availability of the data, we assume in our analysis that the distribution of cognitive ability is invariant between 1961 and 2002.

completes college with probability 17%, whereas this probability for an individual in the 75th percentile (86 AFQT points, or 1.4 standardized AFQT points) finishes college almost half the time, 46%. According to equation (17), the area below the probability line (weighted by the standard normal distribution) gives the enrollment rate. The shape of the probability line (reflecting the shape of function $\iota(\cdot)$) determines the magnitude of adjustment of the enrollment rate in response to a change in government policy.

It only remains to calibrate two parameters of the cost function (14), $d_{\max}$ and δ_d. We use equations (17) and (18) relating education cost and the utility difference between two education groups to the fraction of college graduates. To infer the utility difference consistent with the observed macro data, we calculate the steady state equilibria such that the model matches the fraction of schooled workers and quality adjusted capital equipment stock in the data in 1961 and 2002.

This procedure gives us two pairs of enrollment rates and utility-difference values, which together with the function $\iota(\cdot)$ are plugged into (17) and (18) to infer the values for $d_{\max}$ and δ_d (see Table 3).[24]

Finally, we take pure monetary tuition cost from the data. These cost include tuition and fees of full time undergraduate students and are taken from the College Board's publication "Trends in College Pricing 2005". The numbers supplied are split into public and private college cost. To get an average rate, we weighted the average of the private and public universities student tuition and fees series by their respective fractions of students, $(0.2 \text{ tuition}_{private} + 0.8 \text{ tuition}_{public})$. We find that the ratio of average annual college tuition to median annual wage income of an unschooled worker is 0.16 for 1961 and increased to 0.37 in 2002.

4.2 How well does the model match the data?

We first ask how well does the model capture the observed changes in the education premium, fraction of college graduates and average working time, given the estimated education cost function (14) and given the change in exogenous factors, reported in Table 6. The driving force of wage inequality is skill-biased technological change reflected in the dramatic fourfold fall in the price of capital equipment from 0.96 in 1961 to 0.26 in 2002. Neutral productivity increased by a solid 50%. A contribution of this paper is to study factors affecting the *supply* of schooled working time. These factors in the model include progressive income taxes (see

[24] We assume that the intercept in (32) is included in the calibrated value of parameter $d_{\max}$.

Figure 8), consumption taxes (-1.7 percentage points change), government purchases (plus net exports) to GDP (-37% change), and tuition relative to labor income of unschooled (131% increase).

Table 7 compares simulation results for the benchmark model and the data in 1961 (rows 1 and 2) and 2002 (rows 3 and 4).

By virtue of our calibration exercise, the success of the 1961 model simulation to match the data hinges on the model's ability to account for the average time of unschooled workers. The model is relatively successful in predicting 38.5 hours per workweek for an unschooled individual, which is close to 39.8 hours per week in the data. As a result, the predicted education premium of 1.42 almost matches the number in the data, 1.43.

The model for 2002 is quite close to the data, too. It predicts an increase in the education premium up to 1.75 (compared to 1.80 in the data), accounting for 89% of the observed change. It does well in predicting the large increase in the fraction of schooled working hours and a modest decrease in the average hours worked. The fraction of schooled workers in the 2002 model increases to 26.7% (compared to 29.3% in the data), whereas working time of schooled (unschooled) workers decreases somewhat to 40.7 and 32.6 respectively in the model (43.2 and 38.3 in the data).[25] We summarize that the calibrated model is successful in generating the main labor facts 1 through 3 presented in Section 2.

To understand the importance of education choice and productivity for wage and working time inequality in the model, we conduct a series of counterfactual experiments. In each experiment, we simulate a steady state equilibrium for the year 2002 keeping one of the exogenous factors at its 1961 level. We compare outcomes for this counterfactual model to those of the 2002 benchmark model. If the difference between the outcomes of the two experiments is large, then the factor that was "shut down" in the counterfactual simulation is interpreted to be important for wage inequality and education choice. Table 8 provides the results of simulations in which the productivity change or the education decisions are shut down.

When productivity (embodied and neutral) are at the 1961 level, any change in hours and in the number of workers is driven by the remaining factors from Table 6: income

[25] The larger decrease in average hours worked is due to our choice to not increase the life length in the model to keep the fraction of retirees in the population constant. The somewhat lower level of average hours in the model relative to the data has little impact on the results of policy experiments conducted in Section 5.

and consumption taxes, government expenditures and tuition costs. Tuition affects only a small fraction of population, and the rest of the changing factors experienced relatively small changes from 1961 to 2002. Hence, in this experiment all the moments are close to the 1961 level: the fraction of schooled workers is 10.7%, average hours are 42.8 and 35.3 for schooled and unschooled workers, the capital-output ratio is 2.52, output level (relative to output level in the benchmark 1961 model) is 0.91 and, finally, the education premium is 1.48. Hence, in the model without productivity changes, there is little to no changes relative to the 1961 benchmark model. One interesting finding is that neutral (in addition to capital-embodied) technological progress has a large impact on the schooling rate and the stock of equipment. Subsequently, it also accounts for some of the change in education premium.

In another experiment, we consider a world where the supply of schooled individuals is completely inelastic and thus the schooling rate cannot change from its 1961 level. This might be seen as a country that has problems to increase its university capacities, has restrictions on university access, or the schooling system is incabable of providing more than a bare minimum of the population with the opportunity to attend college. In this case, the increase in demand for capital equipment and schooled labor is not accompanied by an increase in the supply of schooled workers. As a result, the education premium skyrockets to 2.38. This increase is 36% higher than the 2002 benchmark model and 68% higher than the 1961 level. The workweek lengths of schooled and unschooled workers only see an approximately 1 hour increase. Scarce supply of schooled hours decreases the demand for capital equipment due to capital-skill complementarity in the production technology. The stock of equipment is half of that in the benchmark 2002 model. To summarize, education choice, in addition to technological change, is critical in accounting for the change in the education premium and schooled hours.

5. The effects of government policy on schooling and inequality

In this Section, we study the importance of public policy (income taxes, social security taxes, retirement benefits, tuition subsidies, etc.) in shaping secular changes in the education decisions, labor supply, the accumulation of physical capital, and the education premium.

After having documented the overall success of the model in generating the main macro facts, we turn to the analysis of the effects of alternative social policies on education choice, the education premium and the output level. We group our policy experiments in three categories. First, we study income tax reforms that lower the level and progressivity of the income tax. Second, we look at changes in the retirement system: (i) reforms that improve the viability of the pension fund (increase in social security contributions and the increase in mandatory retirement age); and (ii) reforms that affect the redistributive features of the pension system (a move to proportional pension benefits or a switch from a pay-as-you-go system to a fully-funded pension system). Finally, we derive consequences of policies that improve the efficiency of the schooling system, such as those leading to higher test scores.

Our focus will be on two effects of policy change that distinguish our analysis from previous studies. Both of these effects turn out to be important in shaping responses of output and inequality to government policy reforms. The two effects differ by the timing of their impact on the fraction of schooled workers in the economy. The fraction of schooled workers is a moving average of school enrollment rates over four or five decades. So we will think of the medium-run effect as the one occuring within one or two decades after a policy change, so that the response of the enrollment rate is not yet fully transpired into the fraction of schooled workers in the economy. In the medium run, there is a dispropotionate response of working hours and savings of schooled individuals and those of unschooled individuals. This difference is due to (i) redistribution of income under progressive income taxation and the incumbent pension system, and (ii) the change in demand for schooled hours due to the change in capital accumulation (stemming from capital-skill complementarity). In the long run, the fraction of educated persons adjusts to the policy change. Our policy experiments will demonstrate that the long-run reallocation of total hours from unschooled to schooled workers often has a dampening effect on wage inequality and an amplifying effect on the level of output.

5.1 Income tax reforms

Our first set of experiments centers around tax reforms.

In the first experiment, the progressive income tax function is replaced with a (revenue-neutral) proportional income tax rate. This reform is akin to the "fundamental" flat-tax reform proposed by Hall and Rabushka (1995), in which the marginal tax rate on income above a certain threshold is flat. Life-cycle models of wage and income inequality typically predict positive effects of such a reform on the level of output and capital accumulation, a small effect on average working hours, as well as ambiguous effects on wage and income inequality.[26]

[26] Positive effects on output and savings range from small (~1%) in Heckman et al. (1998b) to large (~10%)

In the experiment, the level of the flat-tax rate is equal to the tax rate faced by the mean income individual in the 2002 benchmark model, so that the tax reform is revenue neutral (see Table 9). The marginal tax rates are lower for both income groups, and more so for schooled workers. The reduction of marginal tax rates has positive effects on the workweek and savings. The average working time of unschooled workers goes up by 1.6 hours per week. Schooled workers also work more but only by an hour per week. The smaller effect on schooled workers, whose time is compensated at a higher wage rate, is due to the income effect. Furthermore, lower marginal tax rates on the capital part of individual's income encourages savings: the capital-output ratio increases from 2.50 to 2.55. Higher level of capital equipment, in addition to increasing output by 4%, boosts the demand for schooled hours. The wage rate of schooled workers goes up by almost a dollar, whereas wages of unschooled workers experience almost no change. This drives up the wage premium from 1.75 to 1.77 in the medium run.[27] Income inequality also rises, measured by the Gini coefficient applied to taxable income, from 0.249 to 0.263.

The positive income effect of the tax cut and higher wages for schooled hours increase the benefits of schooling. They induce a larger fraction of the population to enroll in school. In the long run, the fraction of schooled workers goes up by 2 percentage points, whereas the workweek length stays virtually unchanged. A larger supply of total hours of schooled workers and a smaller supply of unschooled hours, in turn, reduce the compensation of schooled workers relative to unschooled workers, bringing the education premium from 1.77 down to 1.71. Income inequality also falls from 0.263 in the medium run to 0.256. Hence, the wage inequality after the reform is lower than before the reform with schooled wages unchanged and unschooled wages higher by about half a dollar than before the reform. The long-un income inequality remains higher than before the reform, reflecting a faster accrual of the capital income. Finally, the increase in the fraction of more productive (schooled) workers boosts output by an additional 2%. To sum up, in the long run after the flat-tax reform, changes in the skill composition of the workforce decrease wage inequality, mitigate the increase in income inequality and lead to a positive effect on output.

In the second experiment, tax rates are lowered proportionally by 19% (that is, average

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in Ventura (1999) and Altig et al. (2001). The flat income tax reform decreases the education premium in Heckman et al. (1998b), and Gini earnings inequality in Castaneda et al. (1998). The reform increases the Gini income inequality in Heckman et al. (1998b), Altig and Carlstrom (1999). In Ventura (1999) the distributions of earnings and income become more concentrated after the reform. In Diaz-Gimenez and Pijoan-Mas (2006) the effects on earnings and income inequality are small.

[27] Note that the level of the stock of capital structures is irrelevant for education premium in the model, see equation (26).

and marginal tax rates for all incomes are multiplied by 0.81).[28] This reform is comparable to the flat-tax reform in the sense that after the reform the mean income individual faces the same marginal income tax rate of 19.5%. The overall results turn out to be very close to those of the flat-tax reform (see Table 9), lower level of marginal tax rates encourages individual's labor supply and savings, leading to the increase in output, capital-output ratio and, via capital-skill complementarity, to higher wage inequality in the medium run. The incentives to attend school increase, but not as much as in case of flat taxes, since the income tax after the cut is still progressive. In the long run, the fraction of schooled workers goes up by 1.6 percentage points, bringing the wage premium down to 1.72 and raising output by an extra 1%. Income inequality is close to its level prior the reform, 0.246.

We conclude that a proportional 19% tax cut yields implications for inequality and potential output that are quantitatively very similar to those due to a switch from progressive to flat income taxes.

5.2 Change in retirement system

Many European countries as well as the United States, Canada, and Japan experience population aging. For countries with a pay-as-you-go (PAYG) retirement system, like the United States, this can lead to severe problems of retirement funding in the near future. Various ways out of the impending retirement fund imbalance have been suggested and in some cases tried: increasing the full-benefit retirement age (this has been done in the United States, Germany, and Italy among others), increasing contributions to the pension fund or cutting retirement benefits. More drastic reforms of the pension system aim at reducing or eliminating the redistribution of income from income-rich to income-poor retirees. Examples of such reforms include decreasing the redistiribution of retirement benefits from higher- to lower-income retirees, or switching from a PAYG to a fully-funded retirement system (as suggested, for example, in Prescott (2004)).

A main source of population aging in most economies is a high proportion of individuals born after the World War II ("baby-boomers"), who are on the verge of retirement. According to U.S. interim projections released by the U.S. Census Bureau, the ratio of male population above age 64 to male population above age 16 will increase from 13.6% in 2000 to 22.4% in 2030.[29] To capture population aging in the model, we increase the life length from 71 to 78,

[28] A similar in magnitude tax cut was instituted in the U.S. by the Tax Reform Act of 1986. See Altig and Carlstrom (1999) for the analysis in a general equiliubrium life-cycle framework. In our model, the tax cut is financed by higher lump-sum taxes on workers.

[29] U.S. Census Bureau, "State Interim Population Projections by Age and Sex: 2004 - 2030",

which increases the fraction of retirees to working age population from 13.0% to 23.0%.

The size of the retirement benefits per retiree in the 2002 steady state (expressed as a fraction of the average lifetime wage income) equals 0.21 for schooled and 0.45 for unschooled retired individuals. In the 2030 steady state, the fraction of retirees in the population is higher and, because of that, the benefits per retired worker are lower at 0.10 and 0.21 respectively (see Table 10). We consider two reforms that are intended to offset the decrease in the size of individual retirement benefits due to population aging. Specifically, the revenues of the pension fund are increased by raising the social security tax rate or by increasing the mandatory retirement age. We compare the effect of these two reforms to the 2030 steady state.

First, we increase contributions to the pension fund by 7 percentage points. This reform increases the average tax rates by 7 percentage points without affecting the marginal tax rates. The resulting income effect discourages savings and labor effort as the capital-output ratio and average hours go down moderately. Wage and income dispersion change little, whereas potential output falls by 3%. This reform also decreases incentives to educate by increasing the share of retirement benefits in wage income of unschooled workers faster than the share for schooled workers (0.21 to 0.45 for unschooled, 0.10 to 0.22 for schooled workers). In the long run, the fraction of schooled employed falls from 29.7% to 28.7%.

In the alternative experiment, we increase the mandatory retirement age from 64 to 70. This brings the fraction of retired workers in the population back to its 2002 level of 13%. Hence, as in the previous case, benefits per a retired person increase to the same level as in the 2002 steady state with life expectancy at age 71 and retirement after 64 years. A longer working age and a shorter life after retirement reduce incentives to save and work. The capital per output goes down from 2.64 to 2.57, the workweek length declines by about 3 hours for both groups, and output declines by 9% in the medium run. The resulting capital equipment stock, average hours worked and output are at about the same levels as in the benchmark 2002 steady state. The fraction of schooled employed falls to 28.9%, similar to the first experiment. The overall effect on wage inequality is close to zero. The income distribution is more concentrated since the growth of individual income over the lifetime is smaller.

The next two experiments are aimed at eliminating the redistribution from schooled (income-rich) retirees to unschooled (income-poor) retired individuals that is present in the

<http://www.census.gov/population/www/projections/projectionsagesex.html>.

current U.S. retirement system. In the first 2002 experiment, pension benefits, as a fraction of wage income, are equalized for all retirees, keeping total retirement payments the same (see Table 11). After the reform the retirement benefits to wage income ratio increases from 0.21 to 0.34 for schooled retirees, and decreases from 0.45 to 0.34 for unschooled retirees.[30] The reform has a very small effect on working time and savings, since the redistribution of lump-sum benefits generates a negligible income effect. For example, the ratio of lifetime retirement payments to lifetime wage income increases by only 2.2 percentage points for schooled individuals and decreases by 1.6 percentage points for unschooled individuals. The reform only marginally improves the incentives to attend college as the fraction of schooled workers goes up to 26.9% in the long run. The effects on output and inequality are also small.

Finally, in our last experiment, the PAYG retirement system is substituted with a fully-funded system, in which each retired individual saves for his own retirement. The reform affects the economy via two distinct channels. First, it stops the redistribution of the retirement income from schooled to unschooled workers. We know from the previous experiment that this channel is weak. Second, a switch to the fully-funded retirement system implies a positive income effect on average hours worked and savings due to the elimination of the social security part of the income tax. Schooled and unschooled workers under the fully-funded system work by 3/4 and 1 hour more per week than under the PAYG system, and the capital-output ratio boosts to 2.59. As shown in Table 11, the model implies a strong long-run increase in the fraction of schooled workers to 27.7, and the reversal of the medium-run increase of the education premium, from 1.77 to 1.74. The reallocation of skill offsets the initial effect on income inequality due to higher savings rate and higher labor supply leading to no change in the dispersion of income, with the Gini coefficient hovering at 0.244 in the long run. Most importantly, output level rises by 4% in the medium run, and by 5% in the long run.

To summarize, the reforms of the retirement system aiming at maintaining the viability of the pension fund, discourage incentives to work, save and educate, leading to output losses. They typically do not affect wage inequality and slightly decrease income inequality. Quite opposite results are found when eliminating the PAYG retirement system altogether and letting agents save for their own retirement. This reform influences the economy not through

[30] The benefit-earnings ratios estimated by the social security agency at <http://www.socialsecurity.gov/cgi-bin/benefit6.cgi> for persons with pre-retirement income of 20,000 and 40,000 U.S. dollars in 2006 (roughly corresponding to earnings of unschooled and schooled individuals in 2002) are 0.42 and 0.31. The redistribution of the respective benefits in the model, 0.45 and 0.21, is higher than in the data, implying a conservative approach in our redistribution experiments.

abolition of the redistribution of pension benefits, but through lower average tax rates. Long after the reform, wage inequality decreases and income dispersion stays unchanged.

5.3 Efficiency of the schooling system

In the final part of our policy section, we consider policies that directly affect the efficiency of the education system. Such policies engage financial and non-financial factors to improve the incidence of college completion, e.g. tuition subsidy, shortening schooling time, higher grading standards, administration of graduation exams, rasing the amount of homework.

In the first experiment, college tuition is fully subsidized. Many European countries had full subsidization of higher education for many decades with the goal of providing equal access to schooling and reducing wage inequality. In the model, this reform increases disposable income of schooled individuals by removing the necessity to pay tuition costs. Higher income implies a shorter workweek by 1 hour (see Table 12). In the medium run, the lower supply of total hours of schooled workers leads to an increase in their wages relative to the wages of the unschooled workers, and also to a slight increase in hours worked of unschooled workers. Thus in the medium run, subsidizing tuition decreases inequality in hours worked and increases wage inequality, whereas the effects on aggregate output and savings are very small. In the long run, the fraction of schooled workers goes up to 39.7%, a swing of 2 million male workers, taking today's working population as the benchmark. A surge in the supply of schooled workers overturns the initial increase in wage and income inequality as the education premium and the Gini income coefficient fall relative to the baseline case, 1.68 and 0.242 respectively. The larger number of more productive workers increases output level by 3% in the long run.[31]

In the second experiment, we consider the consequences of a policy that leads to a decrease in college study time from 4 to 3 years. It is assumed that college graduates are able to complete the four-year college program in three years. Hence, by decreasing the number of years of forgone labor earnings, such a policy implies smaller costs of attending college. In this sense, it is similar to the experiment with a tuition subsidy. Indeed, as Table 12 shows, the results between the two policy experiments are quantitatively very similar: in the medium

[31] In the model, the effect of the change in tuition cost on the enrollment rate is small. This implication of the model is consistent with the general equilibrium analysis in Heckman et al. (1998a). They find that the increase in tuition cost by about a third decreases the average probability of going to college by 0.08. In our model the corresponding decrease in probability is even smaller, 0.02. We conjecture that the difference in the results is due to the redistribution of hours worked across workers in our model, whereas in Heckman et al. (1998a) hours are supplied inelastically.

run, working hours are redistributed from schooled to unschooled workers; in the long run, the redistribution of total hours is reversed as the fraction of college graduates rises. The economy achieves lower wage and income inequality and higher long-run output.

In our last experiment, we consider policies that improve the efficiency of the schooling system and lead to a higher performance on the cognitive ability test. We focus on policies that typically do not require an increase in public spending. Examples of such policies include designing a system of graduation tests, advancing homework and grading standards, and facilitating more competition among schools.[32] In the experiment, we assume that realized de-meaned AFQT scores in the model are 1 point (0.04 of standardized AFQT score) higher than in the benchmark 2002 model. As a result, more individuals qualify to go (and finish) college, so that the enrollment rate increases. Hence, this effect represents a pure shift in the supply of schooled workers. This change affects the economy only in the long run, when the fraction of schooled workers increases from 26.7% to 27.3%. As Table 12 demonstrates, even such a small increase in test performance has non-trivial effects on output and inequality. The wage premium goes down from 1.75 to 1.73 and output increases by 1%.

In sum, a more effective school system (on the pre-college or college level) leads to an increase in the supply of schooled hours resulting in an unambiguous decrease in wage and income inequality and an increase in long-run output.

6. Conclusion

In recent years macroeconomic theory has had great success in matching the historical secular increases in relative wages and hours of schooled and unshooled male workers. This paper undertakes the next natural step and asks: What are the effects of various government policies on the economy-wide differences in wages, income, working hours and savings? In answering this question we build a framework siutable for policy analysis, which we then use to conduct some proposed policy experiments. One of the main roles in driving the demand in the labor market in the data has been attributed to skill-biased technological change. This paper fills the gap in that literature by building an empirically sound general equilibrium model of aggregate supply in the labor and investment markets.

Our battery of policy experiments highlights the importance of modeling education choice

[32] Betts and Ferrall (1997) argue that, given little evidence on the benefits from increasing school spending, the more promising type of reforms for public schools in the U.S. and Canada may be changing the incentive structure for public schools.

and labor supply for gauging the effects of policy change on wage inequality and output. Table 13 summarizes medium and long-run responses in the number of college graduates, wage inequality and output level to policy reforms in our model. Even though the reforms are quite different, two regularities stand out. First, the long-run reallocation of the workforce between unschooled and schooled jobs can be quite substantial, reaching on several occasions a million or more workers. The response is caused by the differential effect of policy change on the lifetime income, consumption and leisure streams of schooled and unschooled individuals. Typically a policy change operates through the redistribution of income or through its effect on the accumulation of the skill-intensive capital stock.

The second common feature across the reforms is the effect of the workforce reallocation on wage inequality and the output level. Here we distinguish between medium and long-run effects of the policy change, depending on whether the change in the overall fraction of schooled workers in the employed population adjusts. As Table 13 demonstrates, a larger supply of schooled worker-hours and a smaller supply of unschooled worker-hours reverses (most of the time) the medium-run effect on wage inequality. The output level receives an additional change of 1 to 2% due to the different number of workers in more and less productive jobs.

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Table 1: Working time decomposition for 1961 and 2002

	1961	2002	change[1]
Working hours per capita	33.0	29.9	-3.1
Fraction of employed in population, %	81.7	75.3	-6.4
Fraction of schooled in employed population, %	12.3	29.3	17.0
Workweek length, schooled[2]	44.5	43.2	-1.3
Workweek length, unschooled[2]	39.8	38.3	-1.5
Education premium	1.43	1.80	26.1

Note: [1]change is given as a difference from 1961 to 2002, except for education premium (in %). [2]Workweek length is calculated assuming 100 hours a week available for work.

Table 2: Changes in labor supply and earnings inequality for 1961, 1980, and 2002 by gender and education group.

	Males		Females	
	1961-1980	1980-2002	1961-1980	1980-2002
Working hours per capita[1]	-3.4	0.3	4.5	5.2
Fraction of schooled in employed population[2]	9.4	7.6	6.8	12.7
Wage, schooled[3]	32.4	30.3	20.0	41.5
Wage, unschooled[3]	37.0	0.10	38.9	23.1
Education premium[3]	-3.3	30.1	-13.6	15.0

Note: [1]in hours per week, [2]in percentage points, [3]in percent.

Table 3: Calibrated parameters

Parameter	Value	Source or Moment
Elasticity of substitution between unschooled hours and equipment, η	0.693	Wage-bill ratio growth, 1961-2002
Elasticity of substitution between schooled hours and equipment, κ	-0.536	Labor share growth, 1961-2002
Share of unschooled hours, θ_u	0.467	Wage-bill ratio, 1961
Share of schooled hours, θ_k	0.479	Labor share, 1961
Share of structures, α	0.230	NIPA Structures-to-Equipment, 1961
Labour-efficiency, A_S	118	Real GDP, 1961
Depreciation rate of structures, δ_s	0.05	Krusell et al. (2000)
Depreciation rate of equipment, δ_{eq}	0.125	
Life length, I [1]	54	Life expectancy at age 15 in 1980, Arias (2004)
Mandatory retirement age, I_r [1]	47	
Time at school, I_e	4	
Discount factor, β	0.96	Real interest rate is 4%
Elasticity of labor supply, ϕ	1.60	Workweek length of schooled, 1961
Education cost, $\{d_{\max},\ \delta_d\}$	$\{25.09\ ,\ 5.00\}$	Model fits quality-adjusted equipment stock, given fraction of schooled in employed population, in 1961 and 2002

Note: [1] A newborn individual in the model corresponds to a 18 year old individual in the data.

Table 4: Tax function: parameter estimates

	Parameter	1961	2002
Federal	a_0	0.057	0.007
Income	$a_1, \times 10^{-5}$	0.231	0.306
Taxes	$a_2, \times 10^{-10}$	-0.257	-0.414
	$a_3, \times 10^{-15}$	0.246	0.348
	$a_4, \times 10^{-20}$	-0.107	-0.134
	$a_5, \times 10^{-25}$	0.016	0.019
	R^2	0.33	0.52
State taxes and	$\overline{tx}_{st}$	0.04	0.05
Social Security taxes	$\overline{tx}_{ss}$	0.06	0.12

Note: The federal tax function is estimated by regressing the average federal income tax rate on the polynomial of degree five in the disposable income:

$$\tau_{fed}(y) = a_0 + a_1 y + a_2 y^2 + a_3 y^3 + a_4 y^4 + a_5 y^5$$

The state tax function is of the following form:

$$\tau_{st}(y) = \begin{cases} \frac{y}{20{,}000}\overline{tx}_{st} \\ \overline{tx}_{st} \end{cases} \quad \text{for } y \begin{Bmatrix} \leq \\ > \end{Bmatrix} 20,000 .$$

The social security tax function follows:

$$\tau_{ss}(y) = \begin{cases} 0.7\left(\overline{tx}_{\text{Retirement}} + \overline{tx}_{\text{other SS}}\right) \\ 0.7\left(\overline{tx}_{\text{Retirement}} \times \frac{\bar{y}_{\text{Retirement}}}{y} + \overline{tx}_{\text{other SS}}\right) \end{cases} \quad \text{for } \underline{y} \leq y \begin{Bmatrix} \leq \\ > \end{Bmatrix} \bar{y}_{\text{Retirement}} .$$

Table 5: Estimating the effect of ability on college completion

Parameter	Value	St. dev.
h_0	-1.22	0.07
h_1	0.62	0.11
h_2	0.02	0.08
h_3	0.08	0.07

Note: We restrict the NLSY79 sample to white males who took the ASVAB tests and who were not in college in 1979. For each individual in this sample we document her highest grade completed as of May 1, 1988. Let HGR_i be equal to 1 if individual i's highest grade completed was 4 or more years in college, and 0 otherwise. We estimate the parameters of the following weighted probit equation:

$$HGR_i = h_0 + h_1 Q_i + h_2 Q_i^2 + h_3 Q_i^3$$

where the probit is defined as $v_i = F^{-1}\left(\Pr\left(HGR_i\right) = 1\right)$, F is the c.d.f. of a standard normal random variable, and Q_i is the standardized AFQT score for individual i:

$$Q_i = \frac{Q_i^{AFQT}}{std\left(Q_i^{AFQT}\right)}$$

and Q_i^{AFQT} is the raw (de-meaned by age) AFQT score.

Table 6: Exogenous factors in the model

	1961	2002	change
Capital-embodied progress, q	0.96	0.26	-73%
TFP, A	1	1.50	50%
Average effective tax rate, $\tau(\cdot)$	see Figure 8		
Consumption taxes, τ_c	6.5%	4.8%	-1.7%pt
Government purchases, G/Y	0.25	0.16	-37%
Tuition cost, $B/\left(w^U l_U\right)$	0.16	0.37	131%

Source data: National Income and Product Accounts, the College Board's publication "Trends in College Pricing 2005".

Table 7: Benchmark model vs data in 1961 and 2002

	1961		2002	
	Data	Model	Data	Model
Workweek length, schooled[1]	44.5	(*)44.5	43.2	40.7
Workweek length, unschooled[1]	39.8	38.5	38.3	32.6
Education premium	1.43	1.42	1.80	1.75
Capital-output ratio	2.30	2.56	2.43	2.50
Fraction of schooled workers in employed population, %	12.3	(*)12.3	29.3	26.7
Output[2]	1.00	1.01	2.26	1.86

Note: * denotes moments matched directly by calibration. [1]Workweek length is calculated assuming 100 hours in a week available for work. [2]Output is normalized to 1.00 for 1961.

Table 8: Effect of technological change and education choice in the model

	Model	Model	Experiments with 2002 Model	
	1961	2002	A. No technological progress	B. No change in schooled fraction
Workweek length, schooled[1]	44.5	40.7	42.8	41.8
Workweek length, unschooled[1]	38.5	32.6	35.3	33.8
Wages, schooled	15.5	33.5	15.9	42.2
Wages, unschooled	10.9	19.2	10.8	17.1
Education premium	1.42	1.75	1.48	2.47
Gini coefficient	0.204	0.249	0.216	0.314
Capital-output ratio	2.56	2.50	2.52	2.38
Fraction of schooled workers in employed population, %	12.3	26.7	10.7	12.3
Output[2]	1.00	1.85	0.91	1.55

Note: In experiment A, we simulate a steady state equilibrium for the year 2002 keeping the level of embodied and neutral technological change at 1961 level. In experiment B, the fraction of schooled in employed population is kept at its 1961 level.
[1]Workweek length is calculated assuming 100 hours in a week available for work. [2]Output is normalized to 1.00 in 1961 model.

Table 9: Income tax reforms

	Model	Experiments with 2002 Model			
	2002	A. Switch to proportional tax		B. Income tax cut by 19%	
		Medium run	Long run	Medium run	Long run
Mean marginal tax rate, schooled[1]	24.6	19.5	19.5	20.0	19.9
Mean marginal tax rate, unschooled[1]	23.9	19.5	19.5	19.4	19.4
Workweek length, schooled[2]	40.7	41.5	41.4	41.7	41.6
Workweek length, unschooled[2]	32.6	34.2	34.1	34.2	34.1
Wages, schooled	33.5	34.2	33.5	34.3	33.7
Wages, unschooled	19.2	19.3	19.6	19.3	19.6
Education premium	1.75	1.77	1.71	1.77	1.72
Gini taxable income	0.249	0.263	0.256	0.251	0.246
Capital-output ratio	2.50	2.55	2.57	2.57	2.58
Fraction of schooled workers in employed population, %	26.7	26.7	28.7	26.7	28.3
Output[3]	1.00	1.04	1.06	1.05	1.06

Note: In experiment A, the progressive income tax function is replaced with a (revenue neutral) proportional income tax rate. In experiment B, average and marginal tax rates are multiplied by 0.808. The fraction of schooled workers in the employed population is fixed in the medium run, and is endogenous in the long run.
[1]The marginal tax rate of the mean income individual in the group. [2]Workweek length is calculated assuming 100 hours in a week available for work. [3]Output in the 2002 model is normalized to 1.00.

Table 10: Inreasing viability of the pension fund

	Model 2002	Model 2030	Experiments with 2030 Model: A. Increase in SS tax by 7 percentage points, Medium run	A. Long run	B. Increase in retirement age from 64 to 70, Medium run	B. Long run
Ret. benefits to wage income[1],						
schooled	0.21	0.10	0.22	0.22	0.22	0.21
unschooled	0.45	0.21	0.45	0.45	0.45	0.45
Workweek length, schooled[2]	40.7	43.4	42.9	43.0	40.3	40.4
Workweek length, unschooled[2]	32.6	35.9	35.1	35.2	32.6	32.6
Wages, schooled	33.5	33.7	32.8	33.2	33.0	33.2
Wages, unschooled	19.2	19.9	19.6	19.5	19.8	19.6
Education premium	1.75	1.70	1.67	1.70	1.67	1.69
Gini coefficient	0.249	0.282	0.271	0.275	0.233	0.236
Capital-output ratio	2.50	2.64	2.54	2.53	2.57	2.56
Fraction of schooled workers in employed population, %	26.7	29.7	29.7	28.7	29.7	28.9
Output[3]	0.87	1.00	0.97	0.96	0.91	0.90

Note: The 2030 steady state corresponds to the steady state in the benchmark 2002 model with life length increased from 71 to 78 to capture population aging. In experiment A, the social security tax rate in 2030 model is increased by 7%. In experiment B, the retirement age in 2030 model is increased from 64 to 70 years of age. In both experiments, the ratio of individual retirement benefits to wage income is the same as in the 2002 model without population aging (0.21 - schooled, 0.45 - unschooled).

[1]The ratio of the mean annual retirement benefits to the mean annual wage income for a retiree in the group. [2]Workweek length is calculated assuming 100 hours in a week available for work. [3]Output in the 2030 model is normalized to 1.00.

Table 11: Eliminating redistribution of retirement benefits

	Model	Experiments with 2002 Model			
	2002	A. Equal retirement payments		B. Switch to fully-funded SS system	
		Medium run	Long run	Medium run	Long run
Ret. benefits to wage income[1],					
schooled	0.21	0.34	0.34	0	0
unschooled	0.45	0.34	0.34	0	0
Workweek length, schooled[2]	40.7	40.6	40.6	41.4	41.4
Workweek length, unschooled[2]	32.6	32.6	32.6	33.7	33.6
Wages, schooled	33.5	33.6	33.5	34.4	34.1
Wages, unschooled	19.2	19.2	19.2	19.4	19.6
Education premium	1.75	1.75	1.74	1.77	1.74
Gini coefficient	0.249	0.247	0.246	0.248	0.244
Capital-output ratio	2.50	2.51	2.51	2.59	2.60
Fraction of schooled workers in employed population, %	26.7	26.7	26.9	26.7	27.7
Output[3]	1.00	1.00	1.00	1.04	1.05

Note: In experiment A, retirement benefits (as a share of wage income) are the same for all retirees and equal to those for mean wage income individual in 2002 model. In experiment B, the social security tax rate and retirement benefits are zero. The fraction of schooled workers in the employed population is fixed in the medium run, and is endogenous in the long run.
[1]The ratio of the mean annual retirement benefits to the mean annual wage income for a retiree in the group. [2]Workweek length is calculated assuming 100 hours in a week available for work. [3]Output in the 2002 model is normalized to 1.00.

Table 12: Adjustments of the schooling system

	Model	Experiments with 2002 Model				
	2002	A. Full tuition subsidy		B. Decrease in schooling time from 4 to 3 years		C. Improvement in ability by +1 AFQT point
		Med. run	Long run	Med. run	Long run	Long run
Workweek length, schooled[1]	40.7	39.8	39.6	39.4	39.2	40.7
Workweek length, unschooled[1]	32.6	33.2	33.0	33.2	33.0	32.6
Wages, schooled	33.5	33.9	32.9	34.1	32.9	33.3
Wages, unschooled	19.2	19.1	19.6	19.1	19.6	19.3
Education premium	1.75	1.77	1.68	1.78	1.68	1.73
Gini coefficient	0.249	0.252	0.242	0.245	0.234	0.247
Capital-output ratio	2.50	2.51	2.53	2.51	2.54	2.51
Fraction of schooled workers in employed population, %	26.7	26.7	29.7	26.7	30.2	27.3
Output[2]	1.00	1.00	1.03	0.99	1.03	1.01

Note: In experiment A, college tuition is fully subsidized. In experiment B, the number of years it takes to graduate from college is decreased from 4 to 3 years. In experiment C, the realized raw AFQT scores in the model are 1 point (or 0.04 of standardized AFQT score) higher than in the benchmark 2002 model. The fraction of schooled workers in the employed population is fixed in the medium run, and is endogenous in the long run.

[1]Workweek length is calculated assuming 100 hours in a week available for work. [2]Output in the 2002 model is normalized to 1.00.

Table 13: The effect government policies on education choice, wage inequality and output

	Fraction of schooled workers in employed population[1]	Education premium[2]	Output[2]
Income tax reforms			
Switch to proportional income tax	+2.1 (0.0)	–2.1 (+1.6)	+6.3 (+4.3)
Income tax cut by 19%	+1.6 (0.0)	–1.3 (+1.6)	+6.4 (+4.8)
Change in retirement system			
Increase in SS tax by 7 percentage points	–1.1 (0.0)	+0.4 (–1.4)	–4.4 (–3.4)
Increase in retirement age from 64 to 70	–0.8 (0.0)	–0.2 (–1.6)	–10.0 (–9.4)
Equal retirment payments	+0.2 (0.0)	–0.3 (+0.1)	+0.2 (–0.0)
Switch to fully-funded SS system	+1.1 (0.0)	–0.4 (+1.5)	+5.2 (+4.2)
Efficiency of the schooling system			
Full tuition subsidy	+3.1 (0.0)	–3.7 (+1.5)	+2.8 (+0.0)
Decrease in schooling time from 4 to 3 years	+3.5 (0.0)	–4.1 (+2.0)	+2.6 (–0.5)
Improvement in ability by +1 AFQT point	+0.7 (0.0)	–1.2 (0.0)	+0.6 (0.0)

Note: The table compares the effect of government policies on the fraction of schooled workers in the employed population, education premium, and output in the long run and (in parentheses) in the medium run. The fraction of schooled workers in the employed population is fixed in the medium run, and is endogenous in the long run.
[1]percentage points change, [2]percent change.

Figure 1: Earnings per hour of schooled and unschooled, U.S. 1961 to 2002.

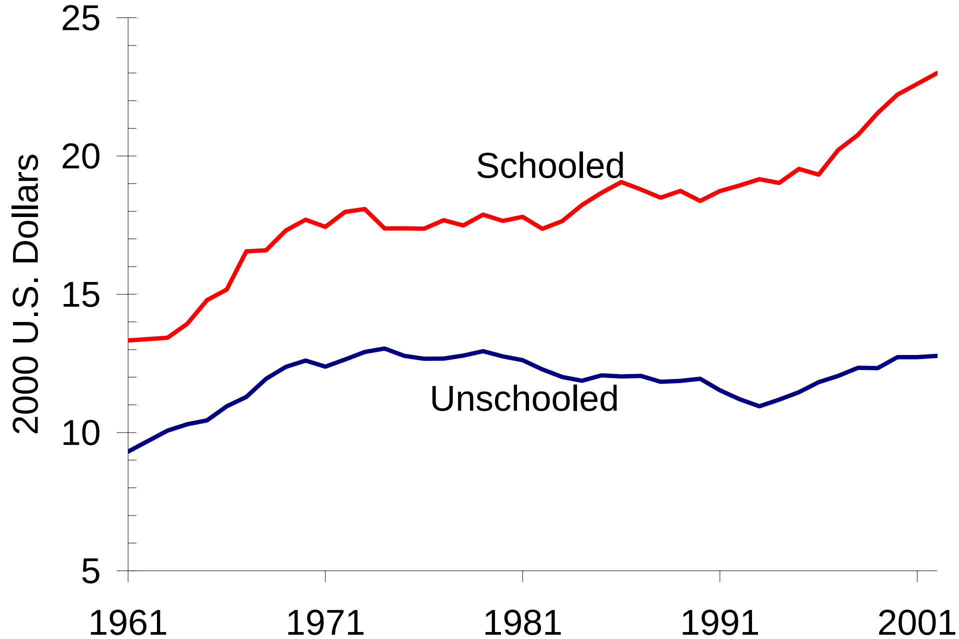


Figure 2: Education premium, U.S. 1961 to 2002.

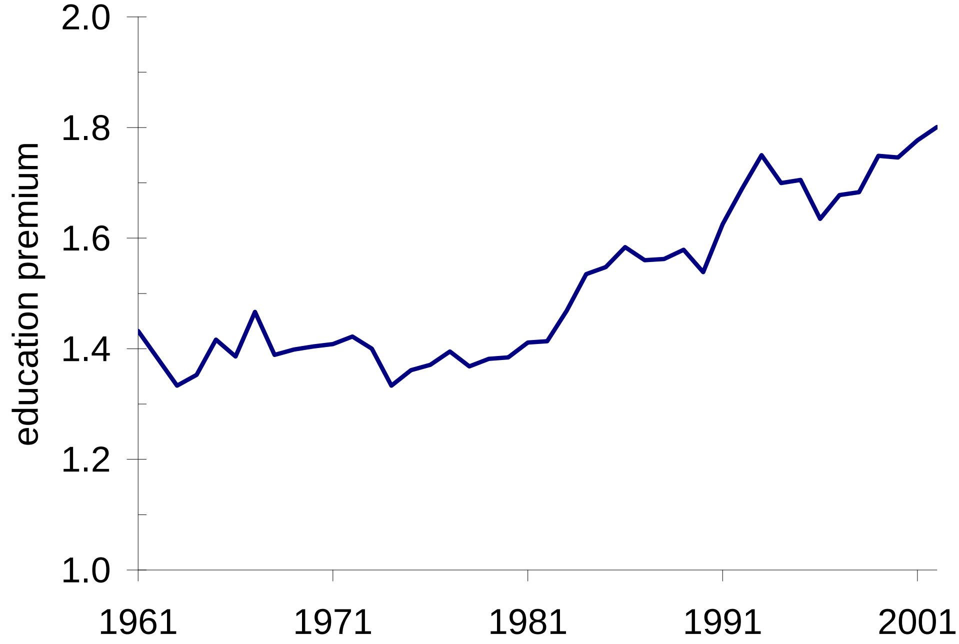


Figure 3: Change in working time contribution of schooled, U.S. 1961 to 2002.

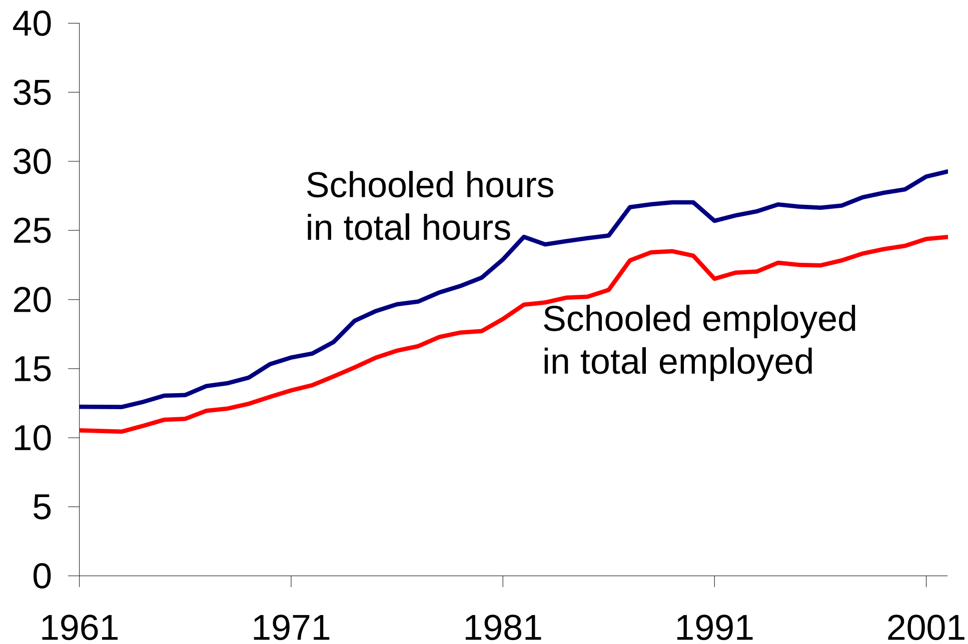


Figure 4: Workweek by group, U.S. 1961 to 2002.

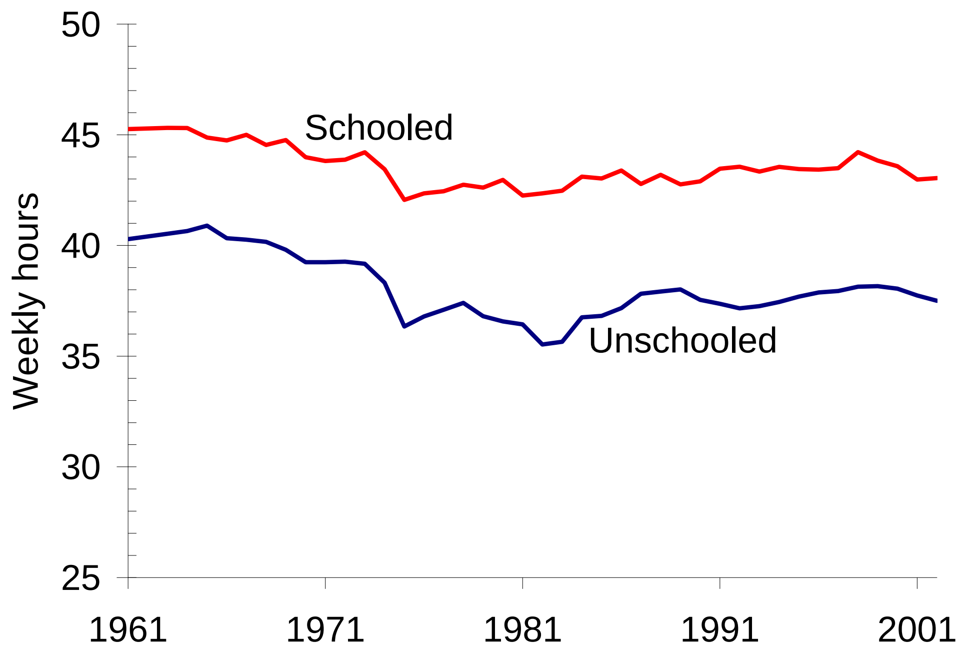


Figure 5: Measures of capital stocks and prices, U.S. 1961 to 2002.

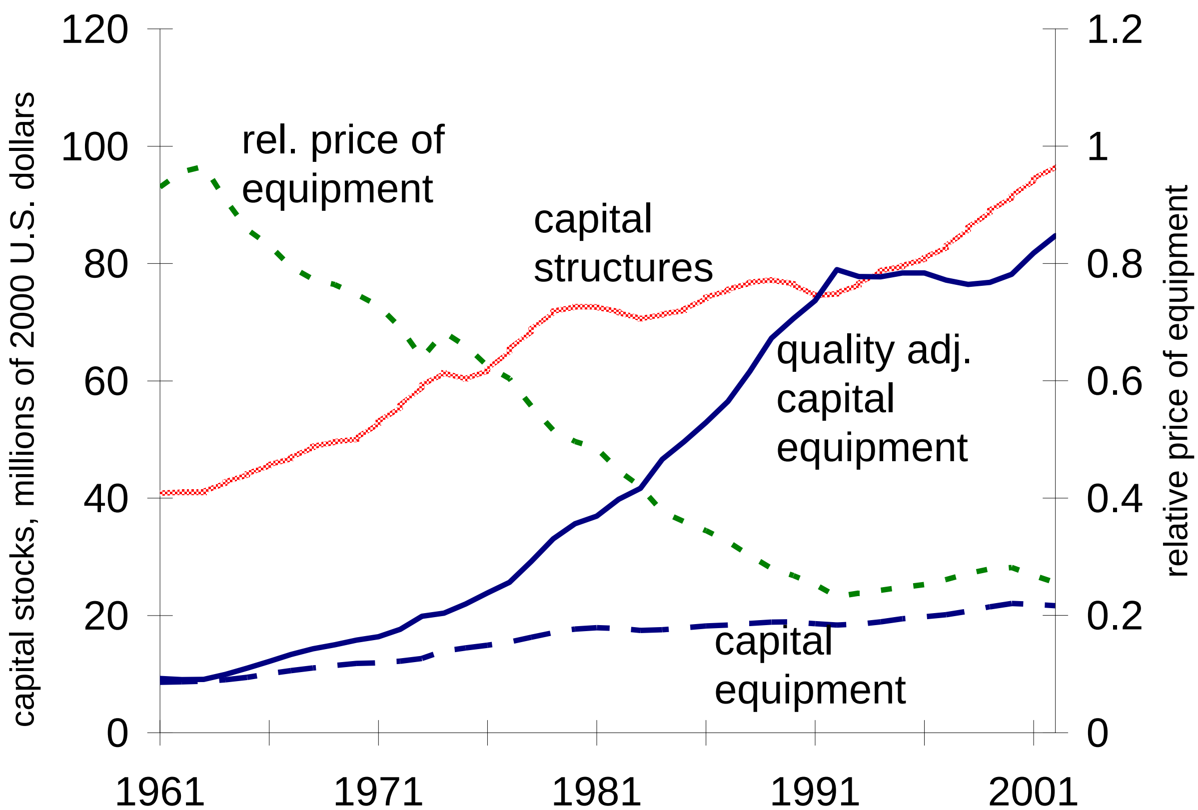


Figure 6: Life cycle of an individual in the model.

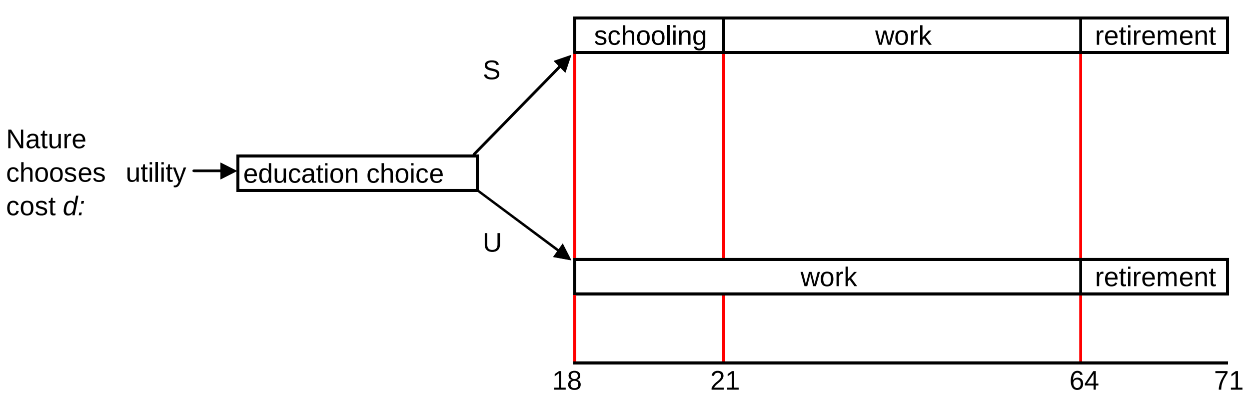


Figure 7: Probability of completing college, US 1979.

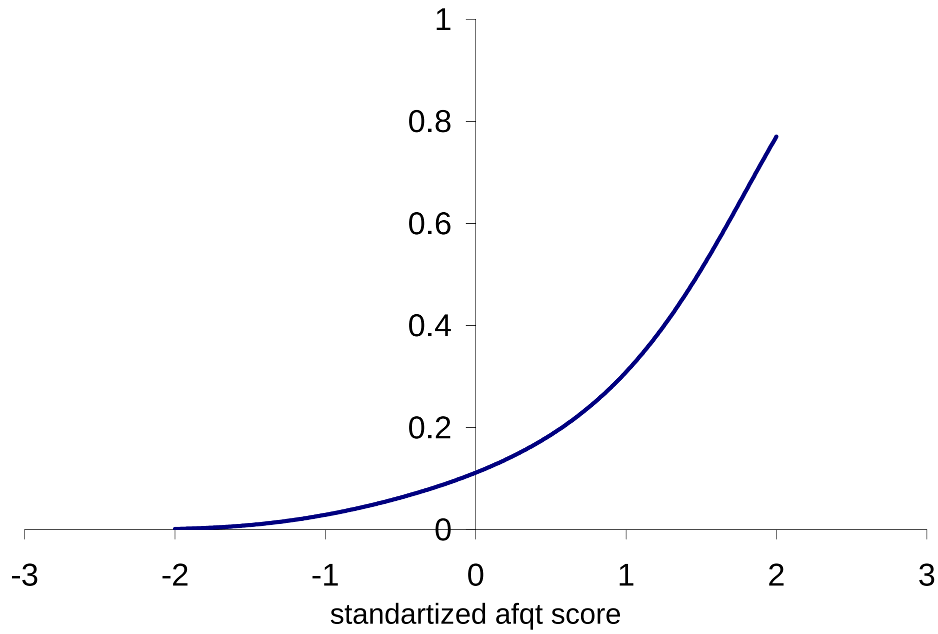
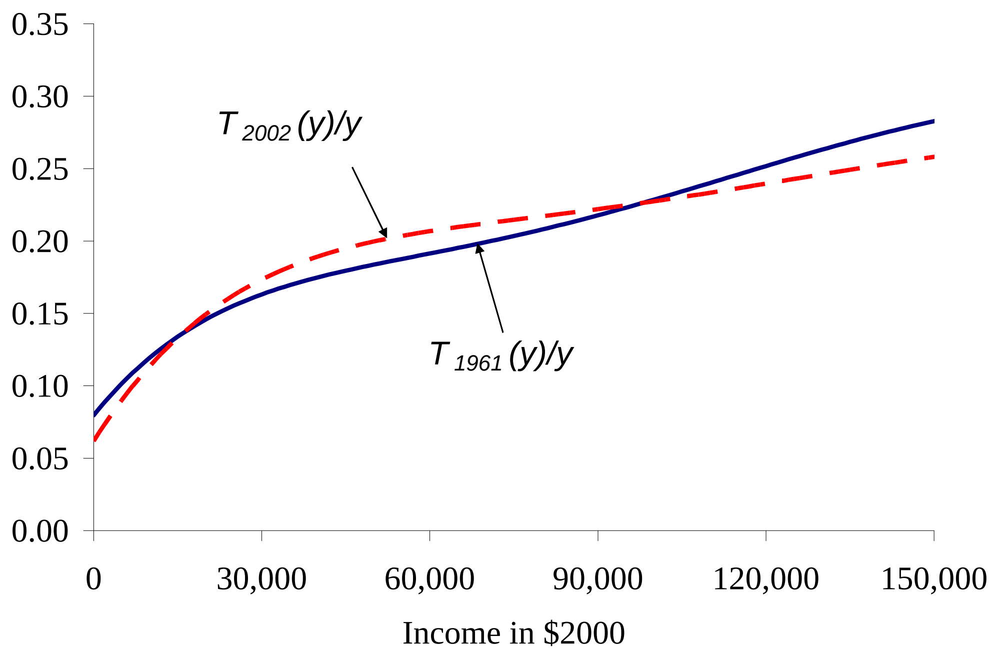


Figure 8: Average effective tax rate functions for the U.S. in 1961 and in 2002.



Appendix A: Data sources

All the labor data, including wage data, are taken from the March Supplement of the Current Population Survey (CPS). We also used CPS data for federal, state, and social security taxes and income for the year 2003 to determine coefficients of the tax function.

We downloaded the following variables from the IPUMS-CPS home at www.ipums.org/cps for the 1962 to 2003 surveys (Miriam King, Steven Ruggles, Trent Alexander, Donna Leicach, and Matthew Sobek. Integrated Public Use Microdata Series, Current Population Survey: Version 2.0. [Machine-readable database]. Minneapolis, MN: Minnesota Population Center [producer and distributor], 2004).

Table A1: CPS variables used.

Variable	Content	Restrictions used
	LABOR and INCOME data	
perwt	frequency weight	
year	year of survey	
age	age	16-64
sex	gender	male
educrec	educational attainment record	grades >=1
empstat	employment status	employed / employed at work
classwkr	class of worker	control for self employment
wkswork1	weeks worked last year	1976 to 2003
wkswork2	weeks worked last year (intervalled)	1962 to 1975
hrswork	hours worked last week	
inctot	total income last year	
incwage	total wage income last year	
	TAX data	
adjginc	adjusted gross income	0<adjginc<99980[1]
fedtax	federal income tax liability	fedtax>0
fica	social security retirement payroll deduction	fica>0
statetax	state income tax liability	statetax>0

To determine the relevant capital stock, output and income data we used the following tables from the National Income and Product accounts as supplied by the Bureau of Economic Analysis (from http://www.bea.gov/ downloaded between January and March 2006). We downloaded the tables for the year 1960 to 2002 at the annual rate.

Aside from this we have to adjust our wage income data for wage supplements. To get the right adjustment factor we make use of both the income side of the national accounts and a detailed listing of the wage supplements as provided by the BEA. The income side is also used to determine the capital income share in GDP.

Table A2: NIPA tables used.

Table	Content	
	National GDP accounts	
1.1.5.	Gross Domestic Product	Nominal
1.10	Gross Domestic Income	Nominal
1.1.4.	Price Indices for GDP	Base 2000
2.7.	Investment in Private Fixed Assets, Equipment and Software, and Structures by Type	Historical-Cost
2.8.	Investment in Private Fixed Assets, Equipment and Software, and Structures by Type	Chain-Type Quantity Indices
	Fixed assets accounts	
2.1.	Private Fixed Assets, Equipment and Software, and Structures by Type	Current-Cost Net Stock
2.2.	Price indices for Net Stock of Private Fixed Assets, Equipment and Software, and Structures by Type	Chain-Type Quantity Indices
	Wage supplements	
7.8	Supplements to wages and salaries by type	Nominal

For capital equipment from 1963 to 1992 we make use of the time series provided by KORV. We use a suggestion by Robert Gordon to extend the series from 1992 to 2000.

Concerning the tax data:

Federal taxes: *Statistics of Income, 1961 (2002)*; *Individual Income Tax Returns for 1961 (2002)*, Internal Revenue Service, U.S. Treasury Department Publication.

State taxes: *State Tax Handbook 1964 (2002)*, CCH Editorial Staff Publication, Commerce Clearing House Inc., New York, Chicago, Washington, Boston, Philadelphia, Los Angeles, San Francisco.

Consumption taxes and social security taxes for 1961 and 2002: We used the SourceOECD, OECD database to get the expenditure and income side of the National Accounts as well as the consumption taxes and social security contributions from the Revenue Statistics. The data were downloaded in March 2006 at <http://titania.sourceoecd.org/vl=12670625/cl=17/nw=1/rpsv/cgi-bin/jsearch_oecd_stats>.

The data on tuition and fees of full time undergraduate students was taken from the College Board's web page. In particular it was taken from their publication "Trends in College Pricing 2005". We took a weighted average of the private and public universities student tuition and fees series $(0.2 \text{ tuition}_{private} + 0.8 \text{ tuition}_{public})$. To extend the series back prior to 1976, we used the growth trend from 1976 to 1980.

Appendix B: Data transformation

B.1 Labor variables

For all the variables generated we restricted our sample to the male age 16 to 64 population. Furthermore, we defined college graduates (CG) to be all persons in the sample, that has at least 4 years of college completed. We define as less than college graduates (LC) all students that have either 1 to 12 years of highschool or 1 to 3 years of college. All the time series generated are generated for the respective groups.

For the work related variables, we also restrict our attention to the persons actually at work during the last week.

For the artificial cohort generation we make use of the age variable by looking at all the work variables for the respective groups and subdevide the education groups by age in 1 or 5 year steps depending on the context.

From the IPUMS samples, we determined the following for all years from 1961 to 2002:

Table B1: CPS data generated.

Name	Content	Variables used	Operation on sample
wap	Working age population	perwt	sum
emp	Employment	empstat, perwt	sum
$hours(ave)$	Average annual hours[1]	empstat, perwt, wksworkX, hrswork	average(wksworkX* hrswork)
$hours(med)$	Median annual hours	empstat, perwt, wksworkX, hrswork	median(wksworkX* hrswork)
$wph(ave)$	Average wage per hour[2]	empstat, perwt, incwage wksworkX, hrswork	average(incwage/ wksworkX*hrswork)
$wph(med)$	Median wage per hour	empstat, perwt, incwage wksworkX, hrswork	median(incwage/ wksworkX*hrswork)
Inc	Total income before taxes	inctot	median for each group

For the *hours* calculation from 1962 to 1975, we had to transform the intervalled weeks worked per year into representative weeks worked. We did this with the following mapping:

wks=9 if weeks worked in 1 to 13

wks=22 if weeks worked in 14 to 26

wks=34 if weeks worked in 27 to 39

wks=43 if weeks worked in 40 to 47

wks=48 if weeks worked in 48 to 49

wks=52 if weeks worked in 50 to 52.

We choose this mapping based on the post 1975 data. For the late period we have both the actual number of weeks worked and the intervalled weeks worked. To find the right weights we determine the median hours worked in each intervall.

Based on these primary results we determined the following objects for each education group:

$$\begin{aligned} emp \text{ fraction with college graduates} &= \frac{emp\,[S]}{emp\,[S+\ U]} \\ \text{total hours fraction of college graduates} &= \frac{emp\,[S] \times hours\,[i]}{emp\,[S+\ U] \times hours\,[S+\ U]} \\ \text{workweek}\,[i] &= \frac{hours\,[i]}{emp[i] \times 5200} \end{aligned}$$

where the square brackets represent the restriction to a subgroup (college graduates or less than college graduates). The adjustment factor 5200 in average hours worked is used to normalize the annual hours worked to the intervall zero one.

To make the wage data comparable, we used the consumption deflator from NIPA table 1.1.4.

B.2 Capital stocks

For the construction of the structures capital stocks we used the nominal private and residential fixed assets from table 2.1 and deflated it using the consumption deflator from table

1.1.4.

$$K_{structures} = \frac{\text{nominal structures}}{\text{consumption deflator}}$$

As a final step we transformed the variable into per working age terms by deviding the resulting capital stock through the number of persons age 16 to 64.

$$k_{structures} = \frac{K_{structures}}{wap}$$

For the construction of equipment capital stocks we are starting under the assumption that Gordon (1990) and subsequently KORV did a good job when determining the quality component of capital equipment and we could use their quality adjustment factor from 1963 to 1992. For the period before we assumed that their was little to no quality adjustment necessary. For the period after 1992 we followed Grodon's suggestion that the BEA did a good job on capturing the quality adjustment for computers and that for the other equipment the missed quality part amounted to 1.5%.[3] We made the appropriate adjustments to the deflators and spliced the resulting q series with that from KORV.

As first step, we calculated the shares in private equipment using the following groupings: Information processing equipment and software (Computers and peripheral equipment, Software, Communication equipment, Medical equipment and instruments, Nonmedical instruments, Photocopy and related equipment, Office and accounting equipment), Industrial equipment, Transportation equipment, Other equipment.

The only bigger catergory we split is Information processing equipment and software. This is necessary since the computer part includes quality adjustment in its deflator while the other sub-catergories don't.

[3] The approach has also been used in Greenwood et al. (1997).

As the next step we perfrom the following transformation:

$$\begin{aligned}
\frac{\tilde{d}_{i,t+1}}{\tilde{d}_{i,t}} &= \frac{d_{i,t+1}}{d_{i,t}} - 0.015;\ i \ = \ \text{all except computer} \\
\frac{\tilde{d}_{i,t+1}}{\tilde{d}_{i,t}} &= \frac{d_{i,t+1}}{d_{i,t}};\ i \ = \ \text{computer} \\
\frac{d_{Thoern,t+1}}{d_{Thoern,t}} &= \sum_i \log\left(\frac{\tilde{d}_{i,t+1}}{\tilde{d}_{i,t}}\right)\left(\frac{s_{i,t+1}+s_{i,t}}{2}\right) \\
q_{NIPA} &= \frac{d_{Thoern}}{d_{\text{consumption}}}
\end{aligned}$$

here s_i stands for the share in total non-residential investment, d_i stands for the NIPA investment deflator (or if stated the Thoern / consumption deflator), and $\tilde{d}_i$ stands for the adjusted deflator.

The NIPA q is different from the KORV q so that we have to make an ajustment. we splice the two series together assuming, that KORV is correct and get the adjusted $\tilde{q}_{NIPA}$. Then we determine the equipment deflator the following way: $\tilde{d}_{Thoern} = \tilde{q}_{NIPA} \times d_{\text{consumption}}$.

Given all these steps we find the capital equipment in the following way:

$$\begin{aligned}
K_{equipment} &= \frac{\text{nominal equipment}}{\tilde{d}_{Thoern}} \\
k_{equipment} &= \frac{K_{equipment}}{wap}
\end{aligned}$$

One implicit assumption we are making is that the residential equipment exhibits the same quality change as the non-residential equipment.

B.3 Taxes

The micro regression results for the income tax functions for 1961 and 2002 are provided by the Statistics of Income division of the Internal Revenue Agency. They used the respective years tax return froms to regress the effective income tax rate paid on the respective taxable income. We restricted the regression such that the income was between 3000 and 300000 year-2002 US$. The lower bound takes the tax-exempted income into account and the upper bound reflects the fact that the median person does not exceed that income in any given year of his life. Furthermore we restriced the average income tax rate to be between zero and

70%, excluding the boundary points.

The consumption taxes were taken from Mendoza, Rasin, and Tesar as provided on Mendoza's web page: <http://www.bsos.umd.edu/econ/mendoza/pdfs/newtaxdata.pdf>. We used SourceOECD data on expenditures and tax revenues to extend the series forward to 2004. The data were downloaded in March 2006 at <http://titania.sourceoecd.org/vl=12670625/cl=17/nw=1/rpsv/cgi-bin/jsearch_oecd_stats>. Finally, to get coverage of the early period for the consumption tax we had to extend the series using the trend of a consumption tax rate series estimated based on consumption expenditures from the BEA-NIPA together with Indirect Business Taxes net of Subsidies.

Appendix C: Armed Forces Qualification Test in NLSY79

ASVAB Administration: During the summer and fall of 1980, NLSY79 respondents participated in an effort of the U.S. Departments of Defense and Military Services to update the norms of the Armed Services Vocational Aptitude Battery (ASVAB)[1]. The Department of Defense and Congress, after questioning the appropriateness of using the World War II reference population as the primary basis for interpreting the enlistment test scores of contemporary recruits, decided in 1979 to conduct this new study. NLSY79 respondents were selected since they comprised a pre-existing nationally representative sample of young people born during the period 1957 through 1964. This testing, which came to be referred to as the "Profile of American Youth," was conducted by NORC representatives according to standard ASVAB procedural guidelines; respondents were paid $50 for their participation. Groups of five to ten persons were tested at more than 400 test sites, including hotels, community centers, and libraries throughout the United States and abroad. A total of 11,914 civilian and military NLSY79 respondents (or 94% of the 1979 sample) completed this test: 5,766 or 94.4% of the cross-sectional sample, 4,990 or 94.2% of the supplemental sample, and 1,158 or 90.5% of the military sample.

The ASVAB consists of a battery of 10 tests that measure knowledge and skill in the following areas: (1) general science; (2) arithmetic reasoning; (3) word knowledge; (4) paragraph comprehension; (5) numerical operations; (6) coding speed; (7) auto and shop information; (8) mathematics knowledge; (9) mechanical comprehension; and (10) electronics information. The following variables are available for each youth tested: raw scores, scale scores, standard errors, sampling weight, high school graduation status, and whether the test was completed under normal or altered testing conditions.

A composite score derived from select sections of the battery can be used to construct an approximate and unofficial Armed Forces Qualifications Test score (AFQT) for each youth. The AFQT is a general measure of trainability and a primary criterion of enlistment eligibility for the Armed Forces. Two methods of calculating AFQT scores, developed by the U.S. Department of Defense, have been used to create two percentile scores, an AFQT80 and an AFQT89, for each Profiles respondent. To construct AFQT80, the raw scores from the following four sections of the ASVAB are summed: Section 2 (arithmetic reasoning), Section

[1] This excerpt is taken from BLS' "NLYS79 User's Guide 1979-2002", Chapter 4, pages 95-96, <http://www.bls.gov/nls/79guide/2002/nls79g4a.pdf>.

3 (word knowledge), Section 4 (paragraph comprehension), and one half of the score from Section 5 (numerical operations). Beginning in January 1989, the Department of Defense began using a new calculation procedure. Creation of this revised percentile score, called AFQT89, involves (1) computing a verbal composite score by summing word knowledge and paragraph comprehension raw scores; (2) converting subtest raw scores for verbal, math knowledge, and arithmetic reasoning; (3) multiplying the verbal standard score by two; (4) summing the standard scores for verbal, math knowledge, and arithmetic reasoning; and (5) converting the summed standard score to a percentile.

Appendix D: Finding the equilibrium

The steady state equilibrium consists of 393 variables: allocations $\left\{c_a^X, l_a^X\right\}_{a=1,X=\{\mathrm{S,U}\}}^{I}$, $\left\{k_a^X\right\}_{a=1,X=\{\mathrm{S,U}\}}^{64}$, Y,K^{st},K^{eq},ε,L^S,L^U, transfers to retirees TR_{ss}, residual transfers TR, and prices r,r^{st},r^{eq},w^S,w^U that satisfy the system of 393 equations below, given relative price of equipment q, productivity A, tuition cost N, income tax function $T(\cdot)$, and government purchases G.

First introduce the following additional variables and notation ($X = \{\mathrm{S,U}\}$ whenever applicable):

- taxable income of age a individual: $y_a^X = w^X l_a^X - N^X + (r^s - \delta_s) k_a^X, \quad a = 1, ..., I$

- marginal tax rate of age a individual: $\tau_a^X = T'\left(y_a^X\right), \quad a = 1, ..., I$

- average tax rate of age a individual: $\bar{\tau}_a^X = T\left(y_a^X\right)/y_a^X, \quad a = 1, ..., I$

- Euler equations:

$$c_{a+1}^X = c_1^X \beta^a \prod_{k=1}^{a} \left[1 + \left(r^{st} - \delta_{st}\right)\left(1 - \tau_k^X\right)\right], \quad a = 2, ..., I$$

- labor market equations:

$$l_a^S = 1 - \phi \frac{(1-\tau_c)\, c_a^S}{\left(1-\tau_a^S\right) w^S}, \quad a = I_e + 1, ..., I_r$$

$$l_a^U = 1 - \phi \frac{(1-\tau_c)\, c_a^U}{\left(1-\tau_a^U\right) w^U}, \quad a = 1, ..., I_r$$

- budget constraints of age a individual:

$$\begin{aligned}
k_2^X &= -(1-\tau_c)\, c_1^X + \left(1-\bar{\tau}_1^X\right)\left(w^X l_1^X - N^X\right) + TR \\
k_{A+1}^X &= \left\{ \begin{array}{c} -(1-\tau_c)\, c_1^X + \left(1-\bar{\tau}_1^X\right)\left(w^X l_1^X - N^X\right) + TR - \\ \sum_{a=2}^{A} \left[1 + (r^{st} - \delta_{st})\left(1-\bar{\tau}_a^X\right)\right]^{-a+1} \left((1-\tau_c)\, c_a^X - \left(1-\bar{\tau}_a^X\right)\left(w^X l_a^X - N^X\right) - TR\right) \end{array} \right\} \\
\times & \quad \left[1 + \left(r^{st} - \delta_{st}\right)\left(1-\bar{\tau}_A^X\right)\right]^{A-1}, \quad A = 2, ..., I
\end{aligned}$$

- arbitrage conditions:

$$\begin{aligned} r^{st}-\delta_{st} &= \frac{r^{eq}}{q}-\delta_{eq} \\ r &= r^{st}-\delta_{st} \end{aligned}$$

- initial and terminal conditions on individual a's asset holdings:

$$\begin{aligned} k_1^X &= 0 \\ k_{64}^X &= 0 \end{aligned}$$

- working time constraints for individuals at college:

$$l_a^S = 0, \quad a = 1, ..., I_e$$

- working time constraints for individuals at college:

$$l_a^X = 0, \quad a = I_r + 1, ..., I$$

- education choice equation:

$$\frac{\varepsilon}{I} = \int_{-\infty}^{\infty} F\left(\frac{\Delta V(0) - d_{\max}}{\delta_d} + \iota(Q)\right) dF(Q)$$

where $\Delta V(0) = \sum_{a=1}^{I} \beta^{a-1} \left[\ln \frac{c_a^S}{c_a^U} + \phi \ln \frac{1-0.33 \cdot \mathcal{I}(a \leq I_e) - l_a^S}{1-l_a^U}\right]$

- aggregate output:

$$Y = A\left(K^{st}\right)^{\alpha}\left\{\theta_u\left(A_S L^U\right)^{\eta} + (1-\theta_u)\left(\theta_K\left(K^{eq}\right)^{\kappa} + (1-\theta_K)\left(A_S L^S\right)^{\kappa}\right)^{\eta/\kappa}\right\}^{(1-\alpha)/\eta}$$

- factor shares in aggregate production:

$$\frac{r^{st}K^{st}}{Y} = \alpha$$

$$\frac{r^{eq}K^{eq}}{Y} = \frac{(1-\alpha)\,\Theta\left(\frac{\theta_K}{1-\theta_K}\left(\frac{K^{eq}}{A_S L^S}\right)^{\kappa}+1\right)^{\frac{\eta-\kappa}{\kappa}}}{\left(\frac{L^S}{L^U}\right)^{\eta}+\Theta\left(\frac{\theta_K}{1-\theta_K}\left(\frac{K^{eq}}{A_S L^S}\right)^{\kappa}+1\right)^{\eta/\kappa}}\cdot\frac{\theta_K}{1-\theta_K}\left(\frac{K^{eq}}{A_S L^S}\right)^{\kappa}$$

$$\frac{w^S L^S}{Y} = \frac{(1-\alpha)\,\Theta\left(\frac{\theta_K}{1-\theta_K}\left(\frac{K^{eq}}{A_S L^S}\right)^{\kappa}+1\right)^{\frac{\eta-\kappa}{\kappa}}}{\left(\frac{L^S}{L^U}\right)^{\eta}+\Theta\left(\frac{\theta_K}{1-\theta_K}\left(\frac{K^{eq}}{A_S L^S}\right)^{\kappa}+1\right)^{\eta/\kappa}}$$

$$\frac{w^U L^U}{Y} = \frac{(1-\alpha)\left(\frac{L^S}{L^U}\right)^{-\eta}}{\left(\frac{L^S}{L^U}\right)^{\eta}+\Theta\left(\frac{\theta_K}{1-\theta_K}\left(\frac{K^{eq}}{A_S L^S}\right)^{\kappa}+1\right)^{\eta/\kappa}}$$

where $\Theta \equiv \frac{(1-\theta_u)(1-\theta_K)^{\frac{\eta}{\kappa}}}{\theta_u}$;

- government's budget constraint:

$$\begin{aligned} & \tau_c \frac{1}{I}\sum_{a=1}^{I}\left(\varepsilon c_a^S+(1-\varepsilon)\,c_a^U\right)+\frac{1}{I}\sum_{a=1}^{I}\varepsilon\bar{\tau}_a^S y_a^S+\frac{1}{I}\sum_{a=1}^{I}(1-\varepsilon)\,\bar{\tau}_a^U y_a^U \\ = \quad & G+TR_w+\frac{I-I_r}{I}TR_{ss} \end{aligned}$$

- social security distribution constraint:

$$\frac{1}{I}\sum_{a=1}^{I}\varepsilon\tau_{ss}y_a^S+\frac{1}{I}\sum_{a=1}^{I}(1-\varepsilon)\,\tau_{ss}y_a^U = \frac{I-I_r}{I}TR_{ss}$$

- market clearing conditions:

$$\begin{aligned} \frac{1}{I}\sum_{a=1}^{I}\varepsilon k_a^S+\frac{1}{I}\sum_{a=1}^{I}(1-\varepsilon)\,k_a^U &= K^{st}+qK^{eq} \\ \frac{1}{I}\sum_{a=1}^{I}\varepsilon l_a^S &= L^S \\ \frac{1}{I}\sum_{a=1}^{I}(1-\varepsilon)\,l_a^U &= L^U \end{aligned}$$