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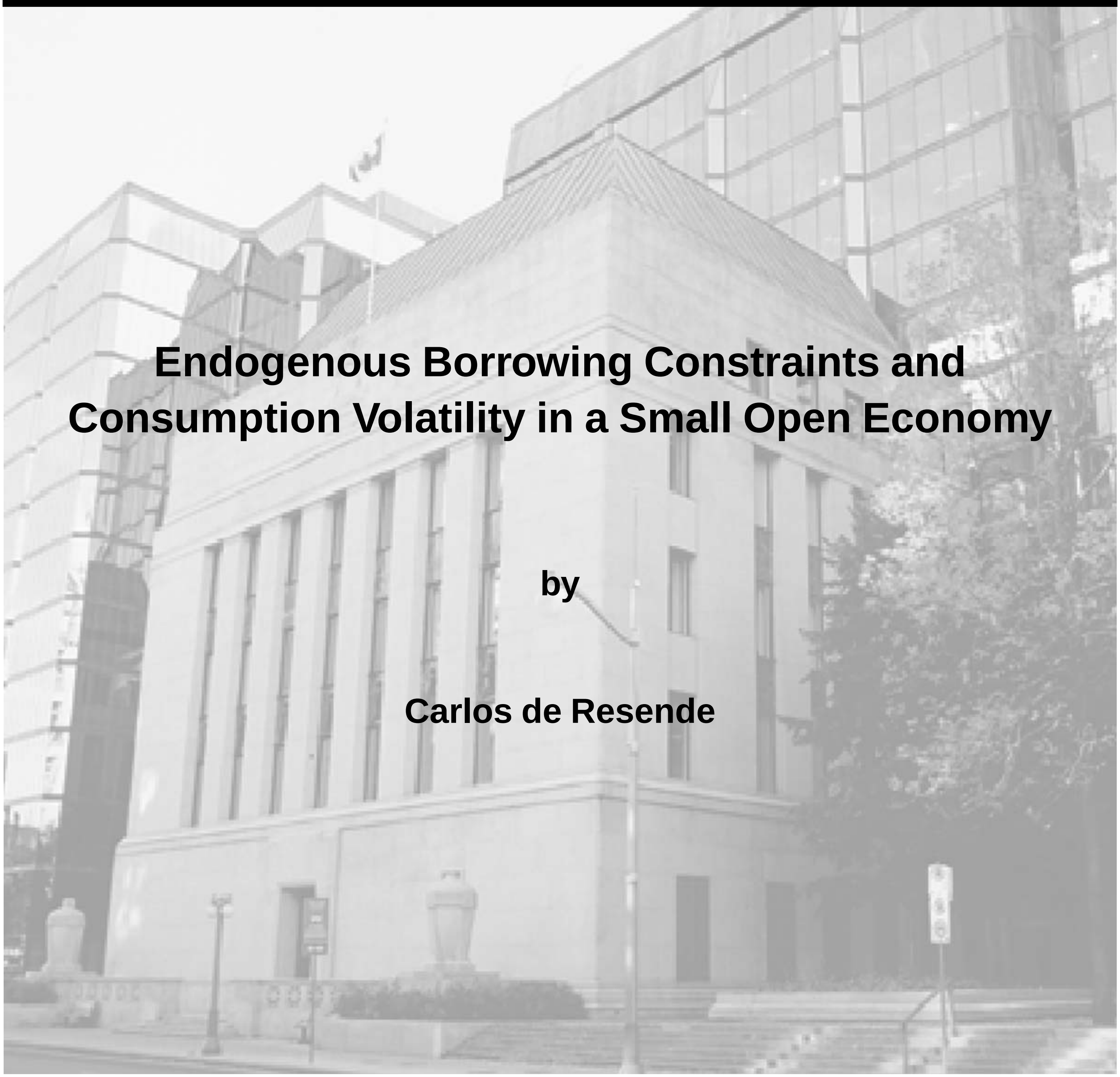

Endogenous Borrowing Constraints and Consumption Volatility in a Small Open Economy

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Abstract

Consumption volatility relative to output volatility is consistently higher in emerging economies than in developed economies. One natural explanation is that emerging economies are more likely to face borrowing constraints and, as a consequence, find it more difficult to use international capital markets to smooth consumption. The author investigates how much this mechanism alone can account for the relative consumption volatility differential between emerging and developed economies. His theoretical approach relies on a standard dynamic general-equilibrium model of a small open endowment economy that is subject to an endogenous borrowing constraint. The borrowing constraint makes the small economy exactly indifferent between two options: (i) repaying its external debt, or (ii) defaulting and having to live in financial autarky in the future. The model for the constrained economy is calibrated to match Brazilian data during the period 1980–2001. The author's findings suggest that the model is capable of accounting for more than half of the observed relative consumption volatility differential.

JEL classification: F32, F34, F41
Bank classification: International topics

Résumé

La volatilité de la consommation, par rapport à celle de la production, est systématiquement plus élevée dans les économies émergentes que dans les économies développées. Une explication naturelle de ce phénomène est que les premières sont plus susceptibles de se trouver aux prises avec des contraintes d'endettement, et qu'elles ont, par conséquent, plus de difficultés à recourir aux marchés internationaux de capitaux pour lisser la consommation. L'auteur tente de déterminer dans quelle mesure ce mécanisme peut, à lui seul, expliquer les écarts de volatilité de la consommation observés entre les économies émergentes et développées. Son approche théorique est fondée sur un modèle dynamique d'équilibre général standard d'une petite économie ouverte assujettie à une contrainte d'endettement endogène. La contrainte d'endettement rend parfaitement indifférent le choix de l'une ou de l'autre des deux options suivantes pour la petite économie : i) rembourser la dette extérieure; ou ii) faire défaut et se trouver dans l'obligation de fonctionner en autarcie financière dans l'avenir. Le modèle avec contrainte est calibré en fonction des données de l'économie brésilienne pour la période de 1980 à 2001. Les résultats obtenus par l'auteur donnent à penser que le modèle est en mesure de rendre compte de plus de la moitié de la différence observée au titre de la volatilité de la consommation.

Classification JEL : F32, F34, F41
Classification de la Banque : Questions internationales

1. Introduction

The purpose of this paper is to study the differences in consumption volatility observed in the data from emerging and developed small open economies. As a general rule, empirical evidence from business cycle statistics across countries suggests that economic activity is more volatile in emerging economies than in developed ones. In particular, the data show that output volatility is higher in the former than in the latter. Considering that output volatility may be interpreted as the underlying volatility of the economy, it is not a surprise that most macroeconomic variables, including private consumption, also tend to be more volatile in emerging economies. However, and more importantly for the purposes of this paper, standard business cycle statistics show that, even if one controls for the output volatility, the (relative) volatility of consumption is still higher in emerging economies than in small open developed economies.

Section 2 of this paper presents empirical evidence of consumption and output volatilities for two groups of small open economies. For a sample of 24 emerging economies, and 17 small open developed economies, the volatility of consumption relative to output volatility is, on average, 30 per cent higher in the emerging economies' subsample. These findings are robust to the sample period as well as to the data frequency, and confirm the results implied by studies containing business cycle statistics for developed economies (Cooley and Prescott 1995 for the United States; Mendoza 1991 for Canada; and Correia, Neves, and Rebelo 1995 for Portugal) and emerging economies (Mendoza 2001 for Mexico; Neumeyer and Perri 2004, and Aguiar and Gopinath 2004 for Argentina).

It has been shown (Neumeyer and Perri 2004; Calvo, Leiderman, and Reinhart 1993) that the excess volatility of business cycles in emerging economies may have a lot to do with a possible dominant role played by external shocks that affect these economies.[1] However, in the context of a small open economy model, one natural theoretical explanation for the differences in volatility is that, perhaps, the two groups of countries, emerging and developed economies, are subject to different external constraints in terms of their ability to borrow in the international capital markets. The obvious intuition on the relationship between borrowing constraints, including the type of constraint discussed here, and the volatility of consumption, is that they may limit consumption smoothing by risk-averse agents and produce a more volatile consumption path.

If, in fact, emerging markets are different from developed economies in that they have a lower ability to use international credit markets to smooth consumption, then the data should reveal noticeable differences in consumption volatility in those two groups of countries, as seems to be the case.[2] This empirical evidence has one important implication for the use of theoretical models

[1] Neumeyer and Perri (2004), using Argentina as a benchmark, stress the important role that shocks to the idiosyncratic interest rate (international interest rate plus a country risk factor) may play on the business cycle volatility in emerging economies. Calvo, Leiderman, and Reinhart (1993), on the other hand, suggest that external factors, such as macroeconomic variables in the United States, and capital flows in particular, may be very important to account for macroeconomic developments in Latin America.

[2] The proposition that access to international capital and credit markets is more restricted for emerging economies

applied to the study of emerging economies. If one wants to explain the high volatility observed in their business cycles, particularly in consumption, then this external borrowing constraint has to be taken into consideration and the typical assumption of unlimited access to perfect world capital markets, which is implausible in this context, must be abandoned. That is precisely the spirit of the theoretical model discussed here.

The paper is concerned with answering the following question: how much of the observed differences in relative consumption volatility in the data from small open emerging and developed economies can be accounted for by a borrowing constraint alone?

More specifically, in order to account for the facts, the paper proposes a dynamic general-equilibrium model featuring two goods (tradable and non-tradable goods) in an endowment economy that is subjected to two kinds of imperfections in international capital markets: (i) the lack of any contingent assets (incomplete markets), and (ii) a financial friction that may restrict international borrowing. The financial friction considered here is an endogenous borrowing constraint in the tradition of Eaton and Gersovitz (1981) (see also Kletzer 1984), which has been recently discussed in the international macroeconomics literature (Arellano 2004; Aguiar and Gopinath 2004). In their paper, Eaton and Gersovitz are motivated by the apparent paradox of why sovereign governments ever choose to repay their debt even when there is no credible enforcement mechanism in the international markets. Although there is some controversy (Bulow and Rogoff 1989), their answer to the "paradox" is that the threat of financial autarky induces sovereign governments to make repayments on their foreign debt in order to preserve a "reputation collateral" needed for future borrowing (see also Cole and Kehoe 1995, 1998; Cole, Dow, and English 1995; Grossman and Han 1999). Borrower countries know that if they default, lenders will be less willing to lend to them in the future. The potential exclusion from future borrowing is a cost to a small open economy populated by risk-averse agents because, in financial autarky, their ability to smooth consumption over time and over different states of nature is compromised. Default occurs whenever the present value of the (instantaneous) benefits of not paying the due services of the external debt outweighs the (intertemporal) losses in utility that will take place during an autarky state. International lenders, aware of the potential for debt repudiation, will set in motion a defensive rule to receive

in comparison to, say, OECD countries does not seem very difficult to accept. Although there is no direct evidence of that, one could mention the lower credit ratings and the higher interest rates paid by emerging economies on their sovereign debt as indirect evidence that they are more likely to be credit constrained than developed economies. Events such as the Asian crisis during the late nineties, the frequent balance-of-payments crises experienced by emerging economies that usually trigger bailouts from the IMF, and their not uncommon decisions to default on their external debt (the most recent being Argentina's default in 2002), in a sense, could also be thought of as indirect evidence that emerging economies are different in their access to international capital markets. Not surprisingly, those events gave enough motivation for a growing literature that deals with the specificities of emerging markets in explaining, among other things, how changes in their access to international credit may affect the domestic economies in various dimensions. This literature includes papers on currency crises (Eichengreen, Rose, and Wyplosz 1995; Kaminsky, Lizondo, and Reinhart 1997; Frenkel and Rose 1996), balance-of-payments crises (Kaminsky and Reinhart 1999; Calvo and Vegh 1999; Edwards 2001), and "sudden stops" (Calvo 1998a,b; Calvo and Reinhart 1999).

back the full amount of any conceded loans, including interests at the international interest rate, in all states of nature, and will never lend funds in excess of the level of credit that leaves the borrower country exactly indifferent between defaulting and fully repaying its debt.

Although some aspects of the more volatile economic fluctuations verified in emerging economies have already been studied in the literature on emerging markets' crises, a systematic attempt to explain the differences in relative consumption volatility observed in the data from emerging and developed small open economies, using a non-ad hoc, endogenous borrowing constraint, has not yet been done. Using data for 1994Q1−2000Q2 from some emerging and developed countries, Neumeyer and Perri (2004) present a broader set of facts about business cycle volatility, including information on relative consumption volatility. They find the average relative consumption volatility for their sample of emerging economies to be 78.2 per cent higher than that of Canada, which is in line with the evidence presented in section 2 of this paper. However, their explanation for the facts relies on an exogenous stochastic process for the idiosyncratic international interest rate faced by the small economy. The exogenous positive shocks on the interest rate could be interpreted as a more stringent borrowing constraint that imposes additional costs to smoothing consumption through borrowing in the international capital markets, but the mechanism does not result from optimizing behaviour on the part of lenders or borrowers.

Mendoza (2001) uses an ad hoc borrowing constraint to explain "sudden stops" in capital flows to emerging economies. The constraint takes the form of collateral, whereby the country must commit a constant (exogenous) proportion of its output before contracting any external credits. Although his model is successful in explaining the abrupt swings in capital inflows to the small emerging economy, it generates an insignificant difference in the relative volatility of consumption between the economies with and without the financial constraint.

Borrowing constraints are a way to ration out the amount of credit available to a particular economy through restriction in quantities. One could also think that, in reality, not only the quantity of credit is to be directly rationed, but the prices (i.e., the idiosyncratic interest rate that the country pays on its debt) must impose additional restrictions on the equilibrium amount of debt. One approach that allows for the interest rate on the external debt to be endogenously determined, along with the level of debt, in a model with the same kind of borrowing constraint used in this paper, is pursued by Arellano (2004) and by Aguiar and Gopinath (2004). They use the same insights that motivated this paper's endogenous borrowing constraint (in their case, to generate a positively sloped "supply of debt"), in a model that allows for default to occur in equilibrium. However, these papers do not discuss how the same model would behave without the financial constraint, nor do they try to explain the potential differences in the relative consumption volatility in constrained and unconstrained economies.

Economists have been trying to understand why emerging economies are so vulnerable to all sorts of crises, from balance-of-payments' crises and sudden stops to banking crises and currency crashes. Although the profession's explanations about the underlying mechanisms of these events have improved over the past two decades, no definitive answer has yet been presented. It is likely that the road map to a more complete understanding of these phenomena includes a clear identification of the particularities, if any, that emerging economies have in comparison with the developed world. In this sense, because it explicitly proposes an explanation to an important aspect of the differences between emerging and developed economies, the paper makes a clear contribution to the literature on emerging economies.

The rest of this paper is organized as follows. Section 2 discusses evidence of the differences in output and consumption volatility in small open economies, divided into "emerging" and "developed" groups. Section 3 presents the theoretical model featuring the endogenous borrowing constraint. Section 4 discusses the numerical solution of the model, its calibration, and some simulation results. Section 5 concludes.

2. Consumption Volatility across Countries

Table 1 displays evidence of the higher ratio of consumption volatility to output volatility, at business cycle frequencies, in emerging economies vis-à-vis small open developed countries. The table is constructed from quarterly data on real output and real private consumption (as deflated by the consumer price index),[3] for 24 emerging economies and 17 small open developed economies. The sample of countries is selected according to data availability for a relatively long period (ending in 2001Q4). All data, computed in per capita values at constant 1995 prices, come from the International Monetary Fund's *International Financial Statistics* (IMF/IFS) dataset, with the exception of Brazilian and Argentinian data, which come from national sources.[4] The series were transformed previously to the computation of their second-moment statistics, as follows. First, all the variables were expressed in logarithms. Second, a seasonal adjustment of the log variables was implemented using the multiplicative ratio-to-moving-average method. Finally, a smooth trend was subtracted using the Hodrick-Prescott filter with a smoothing parameter of 1600 for quarterly data.

[3] Typically, in the real business cycle literature, statistics on consumption exclude the consumption of durable goods (since it behaves closely to investment, being more volatile). We could not yet find the required information to do the same here. Probably, for the same reason, Neumeyer and Perri's (2004) similar empirical exercise considered only total consumption. A potential problem of this procedure would arise if, for instance, durable consumption accounts for a higher proportion of the total consumption in emerging economies than in developed countries.

[4] Argentinian data come from the *Dirección Nacional de Cuentas Nacionales* (DNCN) and Brazilian data are collected from the *Instituto de Pesquisa Economica Aplicada* (IPEA) at <http://www.ipeadata.gov.br> and from the Central Bank of Brazil. Both datasets are consistent with IMF/IFS's data, when they happen to overlap.

Table 1

Output and Consumption Volatility: Cross-Country Differences

Emerging Economies					Small Open Developed Economies				
Country	σ_c (%)	σ_y (%)	σ_c / σ_y	Data (Start)	Country	σ_c (%)	σ_y (%)	σ_c / σ_y	Data (Start)
Bulgaria	15.17	12.42	1.221	1994Q1	Austria	2.51	2.05	1.219	1980Q1
Lithuania	6.97	4.20	1.657	1993Q1	Norway	2.38	1.75	1.366	1980Q1
Latvia	6.69	5.09	1.316	1992Q1	UK	1.90	1.56	1.219	1980Q1
Ecuador	6.16	2.45	2.503	1991Q1	Italy	1.50	1.02	1.470	1980Q1
Argentina	5.61	5.34	1.051	1980Q1	Netherlands	1.45	1.14	1.269	1980Q1
Malaysia	5.34	3.52	1.528	1991Q1	Spain	1.42	1.13	1.257	1980Q1
Indonesia	5.22	4.20	1.242	1990Q1	Finland	2.22	2.47	0.898	1980Q1
Turkey	5.14	4.41	1.166	1987Q1	Ireland	1.99	2.01	0.993	1997Q1
Thailand	4.96	4.63	1.071	1993Q1	Sweden	1.86	1.88	0.990	1980Q1
Mexico	4.92	2.66	1.847	1980Q1	Denmark	1.62	1.63	0.992	1987Q1
Brazil	3.86	2.95	1.308	1980Q1	Canada	1.46	2.17	0.671	1980Q1
Croatia	3.52	2.58	1.782	1997Q1	New Zealand	1.38	1.60	0.864	1987Q1
Estonia	3.13	2.74	1.143	1993Q1	Portugal	1.32	1.70	0.780	1988Q1
Slovak Rep.	3.00	1.45	2.061	1993Q1	Australia	1.03	1.46	0.705	1980Q1
Malta	2.81	2.40	1.170	1992Q1	Switzerland	0.99	1.37	0.722	1980Q1
Czech Rep.	2.68	2.45	1.097	1994Q1	France	0.98	1.23	0.905	1980Q1
Hungary	2.64	2.23	1.186	1995Q1	Belgium	0.97	1.37	0.705	1980Q1
South Africa	2.38	1.82	1.310	1980Q1					
Colombia	2.34	2.03	1.153	1994Q1					
Slovenia	2.12	1.11	1.908	1993Q1					
Poland	1.45	1.21	1.203	1995Q1					
Peru	6.33	6.38	0.993	1980Q1					
South Korea	4.59	4.93	0.931	1980Q1					
Phillipines	3.47	3.91	0.888	1981Q1					
Emerging	**4.60**	**3.63**	**1.27**		**Developed**	**1.59**	**1.62**	**0.98**	

From Table 1 it seems clear that:

(i) The volatility of the gross domestic product (GDP), denoted as σ_y in Table 1, is more than twice as high in emerging economies compared with the developed economies. The averages are 3.6 per cent and 1.6 per cent, respectively.

(ii) The consumption volatility (σ_c) is also higher in emerging economies. On average, σ_c is almost three times as high in emerging economies. Given the results for the output volatility, this is not a surprise, since σ_y may be interpreted as the underlying volatility of the economy, affecting the volatility of all other variables.

(iii) The relative volatility of consumption tends to be higher than 1 in emerging economies (the only three exceptions are Peru, South Korea, and the Philippines) and lower than 1 in developed economies (six exceptions in the sample). The ratio between the average σ_c and the average σ_y is 30 per cent higher in emerging economies in comparison with developed economies (1.27 against 0.98).

Table 2 displays the results of four tests of equality of means for $X = \sigma_c$, σ_y, and σ_c/σ_y, between the two groups of countries. Columns 2 and 3 refer to the test of the null hypothesis $H0 : mean\,(X_{emerging}) = mean\,(X_{developed})$, against the alternative $H1(a) : mean\,(X_{emerging}) \neq mean\,(X_{developed})$, based on the ANOVA F-statistic.[5] Columns 4 and 5 also refer to the test of $H0$ against $H1(a)$, but using a simple t-statistic. Columns 6 and 7 consider t-tests of $H0$ against the alternative hypotheses $H1(b) : mean\,(X_{emerging}) > mean\,(X_{developed})$ and $H1(c) : mean\,(X_{emerging}) < mean\,(X_{developed})$, respectively.

First, consider the test of $H0$ against $H1(a)$. Note that the null hypothesis of equal means can be strongly rejected both according to the ANOVA F-test and the two-tailed t-test for all three variables. Second, regarding the one-tailed t-test of $H0$ against $H1(b)$, the null is also rejected for all variables at standard significance levels. Finally, the null cannot be rejected in the one-tailed t-test of $H0$ against $H1(c)$. The results suggest that the lower absolute and relative volatilities in emerging economies, as shown in Table 1, are statistically significant.

Table 2

Test of Equality of Means

X	Anova F-test	p-value H1(a)	t-test	p-value H1(a)	p-value H1(b)	p-value H1(c)
σ_c	19.8493	0.0001	2.0862	0.0435	0.0218	0.9782
σ_y	12.1626	0.0012	1.9177	0.0625	0.0312	0.9688
σ_c / σ_y	10.8192	0.0021	9.4635	0.0000	0.0000	1.0000

H0 : mean($X_{emerging}$) = mean($X_{developed}$)
H1(a): mean($X_{emerging}$) ≠ mean($X_{developed}$)
H1(b): mean($X_{emerging}$) > mean($X_{developed}$)
H1(c): mean($X_{emerging}$) < mean($X_{developed}$)

The results shown above are also consistent with those obtained by Neumeyer and Perri (2004). They use basically the same sample period in a comparison between Argentinian and Canadian business cycles' statistics[6] and find similar qualitative results as those in Table 1. They also compare Canada with five emerging countries (Argentina, Brazil, Mexico, South Korea, and the Philippines) for the period 1994Q1−2000Q2 and, again, their results are in the same direction.

Table 3 displays the volatilities of output and consumption, as well as their ratio, reported in Neumeyer and Perri (2004) and in other selected studies. Note that the reported relative volatility of consumption confirms the higher volatility in small open emerging economies. The information in

[5] This test is based on a single-factor, between-subjects analysis of variance (ANOVA). The basic idea is that if the subgroups have the same mean, then the variability between the sample means (between groups) should be the same as the variability within any subgroup (within group).

[6] Although we both use basically the same data, Neumeyer and Perry adjust the series of total consumption to include government consumption, changes in inventories, and a statistical discrepancy, in order to be consistent with the only available quarterly data for Argentina previous to 1993. Here, I use the information on annual series for Argentina to exclude these items from the total consumption previous to 1993, by assuming that the same proportions observed in annual data are verified in all quarters of a given year.

Tables 1, 2, and 3 seems to indicate that the basic result−a higher relative consumption volatility in emerging economies in comparison with developed economies−is robust to the sample of countries, frequency of the data, and sample period.

Table 3

Examples of Output and Consumption Volatility Statistics in the Literature				
United States	σ_y (%)	σ_c (%)	σ_c/σ_y	Data
Cooley and Prescott (1995)	1.72	1.27	0.74	1954Q1-1991Q1
Small Open Developed Economies	σ_y (%)	σ_c (%)	σ_c/σ_y	Data
Canada: Mendoza (1991)	2.81	2.46	0.88	1945-1985
Portugal: Correia, Neves, and Rebelo (1995)	3.78	3.17	0.84	1958-1991
Canada: Neumeyer and Perri (2004)	1.17	1.02	0.87	1994Q1-2002Q2
Emerging Economies	σ_y (%)	σ_c (%)	σ_c/σ_y	Data
Mexico: Mendoza (2001)	2.73	3.35	1.23	1980Q1-1997Q4
Average of 5 EE: Neumeyer and Perri (2004)	2.94	4.62	1.57	1994Q1-2002Q2
Argentina: Aguiar and Gopinath (2004)	4.08	4.86	1.19	1983Q1-2000Q2

The next section discusses a possible theoretical explanation for this empirical evidence.

3. The Model

In this section, a dynamic general-equilibrium model of a small open economy is presented. The model departs from traditional small open economy models with perfect capital mobility in that it allows for the possibility that the economy can choose optimally between defaulting or repaying its external debt. This feature introduces an endogenous borrowing constraint in the tradition of Eaton and Gersovitz (1981) and Kletzer (1984).

Consider a small open economy, where a central planner seeks to maximize the lifetime utility of a representative agent. The agent enjoys utility from a consumption index, c_t, which is a composite of the consumption of tradable $\left(c_t^T\right)$ and non-tradable goods $\left(c_t^N\right)$. There is no production and the agent receives an endowment of non-tradable goods $\left(Y^N\right)$, assumed constant for simplicity, and an endowment of tradable goods, $Y_t^T = Y^T + z_t$, which randomly fluctuates around the average level, Y^T, according to a stochastic process for the production shock, z_t.

International asset/capital markets are incomplete and no contingent contracts are signed.[7] At the beginning of every period t, the economy inherits a one-period external debt, d_{t-1}, expressed in units of the tradable good, contracted at $t-1$ at the exogenous foreign interest rate, r, and realizes the levels of the endowments. Denote $S(d_{t-1}, z_t) = \{d_{t-1}, z_t\}$ to be the current state of the economy, at time t. Once $S(d_{t-1}, z_t)$ is known, the central planner decides whether the outstanding debt, including interest services, $(1+r)\, d_{t-1}$, is going to be paid or defaulted. The central planner's decision about the full repayment of the external debt is based on the relative incentives to do so, as follows. The cost of defaulting at time t is to stay out of the international capital markets from t onwards, renouncing the possibility of using international borrowing to smooth consumption.[8] Implicitly, we are assuming that default against one lender is taken as a signal by all other international lenders and that they will not only exclude the defaulting country from borrowing again, but will seize its assets if the country eventually tries to invest any assets in another international financial institution. Given the current state, let V_t^D and V_t^R be the indirect utility of defaulting at t (and having to consume the endowments Y^N and Y_t^T from this time onwards), or of fully repaying the external debt and continuing to be able to borrow abroad. Default at time t is chosen by the country whenever $V_t^D > V_t^R$.

The international capital market consists of lenders who want to receive back the full amount of their loans in all possible states of nature. The directive proposed here is to find a borrowing constraint that, at each date and state, will induce the country to participate in the asset market, instead of defaulting. One could think of the international lenders as a representative international investor, or an outside foreign agency, that has full information about the domestic economy (for instance, its current state and the specification of the borrower/consumer's preferences) and the borrower's optimization problem. The only role played by the foreign agents is to set up and enforce the credit limits. Should the sovereign country default on its external debt, the "agency," or the pool of investors, would exclude it from intertemporal asset trading forever and, as a result, the country would be deprived of the risk-sharing opportunities in the future. Aware of potential debt repudiation, in order to prevent default, the foreign agents will impose a borrowing constraint to the small economy, by not lending any amount of funds that makes the planner choose default over repayment. That is, the external investors will set the credit limit such that the borrower's expected lifetime utility from participating in the asset market is at least as high as that of staying

[7] Kehoe and Levine (1993) discuss endogenous borrowing constraints with complete markets. The assumption of incomplete markets seems to better fit the evidence that countries tend to default during recessions. With the insurance given by contingent assets, agents tend to leave the credit contract (that is, to default) during "good times," when they have to make payments, as opposed to the "bad times," when they receive the insurance.

[8] The assumption that countries that default will stay out of the international capital markets forever is clearly at odds with the evidence that shows many of defaulting countries are able to borrow again after some renegotiation of their debts. In terms of the model presented in this paper, this assumption means, perhaps, a higher penalty for defaulting countries than what actually occurs. A standard and simple way of dealing with this issue (Arellano 2004) is to introduce an exogenous probability of leaving the default state at each period.

in financial autarky, where the country consumes its exogenous endowment output.

If $\overline{d}$ is the maximal amount of funds that the domestic economy can borrow without triggering the strategy of optimal default (that is, $\overline{d}$ is such that $V_t^D \leq V_t^R$), at every period t, then the domestic economy is constrained to borrow $d_t \leq \overline{d}$. In order to assure repayment in all states of nature, Zhang's (1997) approach is adopted by considering the worst-case scenario for the foreign lenders to define the critical level of borrowing that triggers default, given the state $S\left(d_{t-1}, z_t\right)$.

We assume that the lifetime utility of the representative agent is given by:

$$V_0 = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t\right), \tag{1}$$

where $u\left(\cdot\right)$ is concave, strictly increasing, and twice continuously differentiable; $\beta \in (0,1)$ is the subjective discount factor and c_t is a consumption index, assumed to be a constant elasticity of substitution (CES) aggregator of the consumption of tradables and non-tradables, with elasticity of substitution between c_t^T and c_t^N given by $1/\left(1+\mu\right) > 0$, and the weight of tradables in the index equal to $\omega \in [0,1]$:

$$c_t = \left[\omega\left(c_t^T\right)^{-\mu} + \left(1-\omega\right)\left(c_t^N\right)^{-\mu}\right]^{-\frac{1}{\mu}}. \tag{2}$$

The economy is subject to two resource constraints, one for each type of good. For the non-tradable good, the constraint means that the economy has to consume the endowment:

$$c_t^N = Y^N. \tag{3}$$

In contrast to Bulow and Rogoff (1989), this paper accepts the notion that default on the external debt precludes a sovereign government not only of borrowing internationally, but also excludes the country from investing its accumulated assets in the international market in the form of bank accounts, treasury bills, stocks, and other state-contingent assets, without the risk of having those assets seized by international financial institutions or governments. This assumption assures a support for a positive external debt in equilibrium.[9] However, as shown by other empirical studies that use the same type of borrowing constraint considered here (Arellano 2004; Aguiar and Gopinath 2004), for reasonable values of the structural parameters on a dynamic general-equilibrium model applied to a small open economy, the threat of autarky, although capable of

[9] Bulow and Rogoff (1989, 43) have shown that "under fairly general conditions, lending to small countries must be supported by the direct sanctions available to creditors, and cannot be supported by a country's reputation for repayment"; i.e., the penalty of no further borrowing would not deter repudiation and, consequently, a sovereign could not issue any uncollateralized debt. Bulow and Rogoff's result depends crucially on the controversial assumption that repudiation of debt does not mean that the defaulting country is to be cut off from international capital markets entirely and may keep on participating as a creditor without fearing that its assets would be seized by foreign financial institutions or governments. However, as Cole and Kehoe (1995, 1998) point out, that result has the counterfactual implication that the only explanation of why countries do not default is that there are large direct sanctions for doing so. English (1996) shows historic evidence suggesting that direct sanctions cannot explain why sovereign governments repay their debts.

producing a positive amount of debt in equilibrium, cannot generate the levels of debt-to-output ratio observed in actual indebted economies. For this reason, the model imposes an extra penalty to the defaulting country, which could be motivated by "the common view that after default there is a disruption in the countries' ability to engage international trade, and this reduces the value of output" (Cole and Kehoe 1998). We assume that, in the case of default, there is an output loss factor, $(1-\lambda)$, for $\lambda \in [0,1]$, that corresponds to the negative effects that the default state causes in the country's international trade.[10] Thus, in case of default, the resource constraint for the non-tradable good is:

$$c_t^N = \lambda Y^N. \tag{4}$$

For the tradable good, the resource constraint, in case of full repayment, means that the economy keeps the ability to borrow from international lenders, and it is given by:

$$c_t^T = Y^T + z_t + d_t - (1+r)\, d_{t-1}. \tag{5}$$

In case of default, the economy does not have to pay $(1+r)\, d_{t-1}$, but cannot contract d_t and must operate in financial autarky from t onwards. The resource constraint then implies that the consumption of tradables is to be restricted to the stochastic tradable output minus the default-state output loss:

$$c_t^T = \lambda \left(Y^T + z_t\right). \tag{6}$$

The process for the shock z_t is assumed to follow a first-order Markov chain with transition probabilities given by $f\left(z_t | z_{t-1}\right)$ and compact support. The finite support for z_t allows the use of Zhang's (1997) approach, as mentioned above:

$$z_t \in \Omega_Z = \left[z_{\min}, z_{\max}\right]. \tag{7}$$

The central planner's problem is to maximize the objective function given by equation (1) subject to (2)-(7), a standard no-Ponzi-game condition, and to the following borrowing constraint:

$$d_t \leq \overline{d},$$

where:

$$\overline{d} = \min_{\Omega_Z} \left\{\overline{d}_t\left(z_t\right) : V_t^R\left(\overline{d}_t\left(z_t\right), z_t\right) = V_t^D\left(z_t\right)\right\}.$$

The constraint described above represents a way of capturing the widespread notion that borrowers face credit limits in reality and, as such, its use in economic models can mimic important features of the real world. Borrowing constraints are typically needed to prevent default and Ponzi schemes (a "natural" borrowing constraint), and to ensure the existence of equilibrium for

[10] Chuhan and Sturzenegger (2003) find that the per cent contraction in output in Latin America, following the default episodes in the 1990s, was 2 per cent.

incomplete-markets economies. However, the borrowing constraints used in the literature are often specified arbitrarily outside economic models. The borrowing constraints used in most studies take the form of a lower bound on an investor's bond holdings, which is a certain percentage of total income that is independent of the investor's individual characteristics and income streams that in reality are important factors in determining the borrowing limit.[11]

Notice that the borrowing constraint defined above depends not only on the country's representative agent's characteristics, such as time preference rate, risk aversion, and elasticity of substitution between the consumption of tradable and non-tradable goods, but also on the representative agent's exogenous endowment income stream, here completely determined by the shock z_t. Because the constraint can be interpreted as the borrowing limit such that an investor will not default and live in autarky, Zhang (1997) refers to it as the "no default borrowing constraint." In terms of this paper, it is assumed that emerging economies (given their history and, likely, their experienced default episodes) face this type of borrowing constraint while developed economies do not. Although it is not a feature of the model, one could think of "reputation" as an additional state variable and consider that, at this particular point in time, developed economies have a higher "stock of reputation" than emerging economies−higher enough to signal a very low propensity to default.

One can explore the recursive form of the problem. In terms of notation, henceforth the time subscript t is dropped from the (indirect) utility functions V^D, V^R, and V, which are going to represent time-invariant value functions. Considering the CES consumption index in (2) and using the resource constraints for the tradable and non-tradable goods, one can denote the instantaneous utility function, $u(c_t) = u\left(c_t^T, c_t^N\right)$, by:

$$u\left(c_t^T, c_t^N\right) = u\left(\lambda\left(Y^T + z_t\right)\ ; \lambda Y^N\right),$$

in case of default, and

$$u\left(c_t^T, c_t^N\right) = u\left(Y^T + z_t + d_t - (1+r)\, d_{t-1}\ ; Y^N\right),$$

in case of full repayment.

Let z_t and d_{t-1} be in Ω_Z and $D = \{d : d_{\min} \leq d \leq d_{\max}\}$, respectively. Conditional on the state variables in $S(d_{t-1}, z_t)$, and given the Markov process governing the shock, the central planner's problem can be expressed in recursive form as:

$$V^D(z_t) = u\left(\lambda\left(Y^T + z_t\right)\ ; \lambda Y^N\right) + \beta \mathrm{E}_z V^D(z_{t+1})$$

[11] Examples of models with ad hoc borrowing constraints include Aiyagari and Gertler (1991), Telmer (1993), and Lucas (1994), in the context of using incomplete markets with borrowing constraints in order to resolve the "equity premium puzzle." In the international macroeconomics literature, examples of the use of ad hoc borrowing constraints include Mendoza (2001) and other papers in the "sudden stop" literature, as mentioned in footnote 2.

in case of default, and as the solution to the following Bellman equation:

$$
\begin{aligned}
&V^R(d_{t-1}, z_t) = \max_{(d_t)} \left\{ u\left(Y^T + z_t + d_t - (1+r)\, d_{t-1}\ ; Y^N\right) + \beta \mathrm{E}_z V(d_t, z_{t+1}) \right\} \\
&st : d_t \leq \overline{d} = \min_{\Omega_Z} \left\{ \overline{d}(z_t) : V^R\left(\overline{d}(z_t), z_t\right) = V^D(z_t) \right\} \\
&\text{with } V(d_{t-1}, z_t) = \max \left\{ V^R(d_{t-1}, z_t), V^D(z_t) \right\}
\end{aligned}
$$

in case of full repayment.

The solution of the model consists of three objects: (i) a state-contingent optimal decision rule for the level of next-period debt[12] that depends on the current realization of the states, $d(d_{t-1}, z_t)$; (ii) a set of value functions $V^D(z_t)$, $V^R(d_{t-1}, z_t)$, and $V(d_{t-1}, z_t)$; and (iii) the level of the borrowing constraint, $\overline{d}$. Given the solution, the underlying probability distribution function of the production shock, jointly with the decision rule, determines the transition and limiting distributions of all endogenous variables in the model.

In the empirical application of the model, discussed in the next section, a constant relative risk-aversion (CRRA) specification for the instantaneous utility function:

$$
\begin{aligned}
u(c_t) &= \frac{c_t^{1-\gamma} - 1}{1-\gamma}\ , \text{ if } \gamma \neq 1 \\
&= \log(c_t)\ , \text{ if } \gamma = 1
\end{aligned}
$$

is used, where $\gamma > 0$ is the (reciprocal) of the intertemporal elasticity of substitution on the consumption index (or the risk-aversion parameter).

The model also provides implications for the real exchange rate, as measured by the relative price of non-tradable with respect to tradable goods. In the model, the sectorial (shadow) prices are represented by the Lagrange multipliers on the respective resource constraints. At the optimum, there is an implied equation that links the real exchange rate to the $\left(c^T/c^N\right)$ ratio:

$$
p_t \equiv \frac{P_t^N}{P_t^T} = \frac{(1-\omega)}{\omega} \left(\frac{c_t^T}{c^N}\right)^{(1+\mu)}, \tag{8}
$$

where P_t^N and P_t^T are the Lagrange multipliers associated with the non-tradable and tradable resource constraints, respectively.

[12] The decision rule for the dynamic path of d_t implies another, $c^T(d_{t-1}, z_t)$, for the consumption of tradable goods.

4. Numerical Solution, Calibration, and Simulation Results

Because the model developed in this paper does not have an analytical solution, we explore the recursive formulation of the central planner's problem to solve it numerically. We use the value function iteration method with discretization of the state-space $[D \times \Omega_Z]$, for which, given the finite support Ω_Z for the shock, the limits $d_{\min}$ and $d_{\max}$ of the set $D = \{d : d_{\min} \leq d \leq d_{\max}\}$ are appropriately chosen to include the ergodic space.

The algorithm used in the numeric solution is the following. For each iteration j of the algorithm, given an initial guess for the borrowing constraint, $\overline{d}^{(j)}$, the model is solved and the value functions $V^{D(j)}(z_t)$ and $V^{R(j)}(d_{t-1}, z_t)$ are computed. During this step, every point in the decision rule $d^{(j)}(d_{t-1}, z_t)$ such that $d^{(j)} > \overline{d}^{(j)}$ is replaced by the critical level $\overline{d}^{(j)}$. After computing $V^{D(j)}$ and $V^{R(j)}$, an update of the borrowing constraint is obtained using

$$\overline{d}^{(j+1)} = \min_{\Omega_Z} \left\{ \overline{d}(z_t) : V^{R(j)}\left(\overline{d}(z_t), z_t\right) = V^{D(j)}(z_t) \right\}.$$

The procedure is implemented until convergence with $\overline{d}^{(j+1)} \approx \overline{d}^{(j)}$.

The artificial economy is calibrated to match some aspects of the Brazilian economy during the period 1980Q1−2001Q4, when the net external debt (total debt minus international reserves) averaged $\theta_d = 28.34$ per cent and reached a peak of 47.02 per cent of the GDP,[13] which is roughly equivalent to two standard deviations from the mean. It is assumed that Brazil is an economy subject to a borrowing constraint like the one discussed in the previous section, and, as such, it could be used as a benchmark for the simulation exercise.

In order to calibrate the exogenous sectorial outputs, the procedure used here considers the tradable output share in total GDP observed in Brazil, $\theta_T = 29.05$ per cent, and normalizes the (deterministic) steady-state values of the tradable output and the relative price of non-tradables in terms of tradables to be $Y^T_{ss} = 100$ and $p_{ss} = 1$, respectively. If one sets the average tradable output to be $Y^T = Y^T_{ss}$, these figures imply: (i) that the value of the non-tradable output is $Y^N = 244.21$ and, given a debt-to-output ratio equal to the average value θ_d, (ii) that the level of debt (in units of tradable goods) at the steady state is $d_{ss} = 97.56$. In order to capture the potential movements of the simulated series of external debt, an evenly spaced $d-$grid of 800 points is constructed from the interval $[-100, 700]$, with negative values being assets instead of liabilities. Roughly, considering the total output at the steady state $\left(Y^T + p_{ss}Y^N = 344.23\right)$ as reference, the grid implies debt-to-output ratios in the range $[-0.29, 2.03]$.

For the discretization of the $z-$grid, the Markov chain is set to mimic a first-order autoregressive process of the type $z_t = \rho z_{t-1} + \varepsilon_t$, with $\varepsilon_t \backsim N(0, \sigma_\varepsilon)$, using Tauchen's (1986) procedure. The

[13] Actually, these figures refer to the period 1982Q4−2001Q4, since quarterly data on Brazilian external debt are not available for the whole period of reference.

$z-$grid has five points, evenly spaced in the interval $[-17.11, 17.11]$ with an underlying matrix of transition probabilities given by:

$$\Pi = \begin{bmatrix} 0.3423 & 0.5984 & 0.0591 & 0.0002 & 0.0000 \\ 0.0467 & 0.5669 & 0.3744 & 0.0120 & 0.0000 \\ 0.0016 & 0.1611 & 0.6746 & 0.1611 & 0.0016 \\ 0.0000 & 0.0120 & 0.3744 & 0.5669 & 0.0467 \\ 0.0000 & 0.0002 & 0.0591 & 0.5984 & 0.3423 \end{bmatrix}.$$

Table 4 displays the values of the structural parameters used in the calibration exercise. The value for the reciprocal of the intertemporal elasticity substitution (or, equivalently, for the CRRA case, the risk-aversion parameter) is set to $\gamma = 1.5$, which is standard.[14] The exogenous interest rate is taken from what the Brazilian government pays in the international capital markets for its sovereign debt, as represented by the Federative Republic of Brazil's C bonds. Here, the idiosyncratic interest rate, r, is considered to be the quarterly equivalent of the average real annual rate on the U.S government bonds (4 per cent per year, using the inflation rate on the consumer price index) plus the average spread paid on the C bonds (803.4 basis points).[15] Following the traditional hypothesis used in the small open economy literature, in order to avoid a unit root in the current account, the subjective discount factor has to satisfy $\beta (1 + r) = 1$ and, thus, was set to $\beta = 0.9713$. It is worth mentioning that this value of β is consistent with estimations by Issler and Piqueira (2000) for the Brazilian economy.

The autocorrelation and volatility of the stochastic process of the z production shock is obtained from an ordinary least squares (OLS) estimation of the Hodrick-Prescott(HP)-detrended output of tradables against its one-period lagged value. Assuming that the output of tradables $\left(Y_t^T\right)$ has a trend component $\left(HPY_t^T\right)$ and a business cycle component with zero average (the production shock z), the following regression:

$$\left(Y_t^T - HPY_t^T\right) = k + \rho \left(Y_{t-1}^T - HPY_{t-1}^T\right) + \varepsilon_t$$

is estimated, resulting in $\rho = 0.65$ and $\sigma_\varepsilon = 4.35$.[16]

The output loss in default states, $(1 - \lambda)$, is calibrated to approximate the average level of debt-to-output ratio to the actual value ($\theta_d = 28.34$ per cent). Notice that the calibrated value $\lambda = 0.975$, which implies output losses of 2.5 per cent during default states, is not very different from the empirical findings by Chuhan and Sturzenegger (2003), mentioned in footnote 10.

[14] For instance, the value used here is the mid-range value of two very common alternatives, $\gamma = 1.001$ or $\gamma = 2$, used by Greenwood, Hercovitz, and Huffman (1988) and Mendoza (1991), for example. Issler and Piqueira (2000) estimate $\gamma = 1.7$, using Brazilian data and the same type of utility function used in this paper. The results of the simulation of the model are virtually the same if one uses this value instead of $\gamma = 1.5$.

[15] For the average foreign real interest rate, the 10-year-maturity U.S. government bond is used, whose maturity is comparable with that of the C bonds. The average spread for the C bonds refers to the period 1995Q1$-$2001Q4, since data are not available before that.

[16] The estimated parameters (p-values in parentheses) are $\widehat{k} = 0.1240$ (0.846), $\widehat{\rho} = 0.6468$ (0.000), and $\widehat{\sigma}_\varepsilon = 4.3499$.

The less-straightforward parameters to calibrate are the weight of tradables in the CES consumption aggregator (ω) and the parameter governing the elasticity of substitution between the consumption of tradables and non-tradables (μ). Given equation (8) and the calibration procedure based on the deterministic steady state−at which the external debt-to-output ratio is constant at the average level, θ_d, the share of tradable output in total output is θ_T, and the real exchange rate is at the normalized level, $p_{ss} = 1-$ the following system of "steady-state" equations must be satisfied:[17]

$$\begin{aligned}
\theta_T &= \frac{Y_{ss}^T}{Y_{ss}^T + p_{ss} Y^N} \\
\theta_d &= \frac{d_{ss}}{Y_{ss}^T + p_{ss} Y^N} \\
c_{ss}^T &= Y_{ss}^T - r d_{ss} \\
p_{ss} &= \frac{(1-\omega)}{\omega} \left(\frac{c_{ss}^T}{c_{ss}^N} \right)^{(1+\mu)} = 1 \\
c_{ss}^N &= Y^N.
\end{aligned}$$

Given the above system above, the parameter ω can be expressed as a function of μ, as follows:

$$\omega = \left\{ \left[\frac{\left(\frac{1}{\theta_T} - 1 \right)}{\left(1 - \frac{r\theta_d}{\theta_T} \right)} \right]^{(1+\mu)} + 1 \right\}^{-1}.$$

It should be noticed that, in principle, both parameters are important to the volatility of the real exchange rate. However, since the business cycle statistics are usually computed on the log variables, only μ will have an impact on the volatility of (the log of) p. For instance, by taking the logarithm on both sides of equation (8), it is easy to see that $VAR(\log p_t) = (1+\mu)^2 VAR(\log c_t^T)$, implying that the ratio between the volatilities of (the logs of) p_t and c_t^T, as measured by their standard deviations, must be constant and equal to $(1+\mu)$. Because of its effect on the volatility of p, the parameter μ has an influence in the volatilities of both total output, $Y_t^T + p_t Y^N$, and total consumption, $C_t = c_t^T + p_t c^N$. Among the different possible combinations of values for the two parameters that satisfy the above system of stationary equations, $\omega = 0.0659$ and $\mu = 1.875$ (which implies an elasticity of substitution between c^T and c^N equal to 0.35) are chosen in order to match the total output volatility, $\sigma_y = 2.95$ per cent, observed in the data (see Table 1).

[17] Technically, because of the non-linear nature of the model, which in principle should induce agents to react asymmetrically to positive and negative shocks, a "deterministic steady state" may not be relevant to reflect the long-run "average" state of the system. Ideally, in this case, a more precise method of calibration should be carried out through the solution of the whole model for a given set of parameters (all of them), and successive improvements should be made until the target average values are obtained. However, this non-linearity does not seem to be important here and the calibration procedure used, based on a deterministic steady state, is able to generate the target averages quite accurately.

Table 4

Summary of the Calibration Procedure		
Parameter	Values	Target
1. Risk-aversion	$\gamma = 1.5000$	Standard
2. Idiosyncratic interest rate	$r = 0.0295$	C bond spread over U.S. bonds
3. Subjective discount factor	$\beta = 0.9713$	$\beta(1+r) = 1$
4. Average tradable output	$Y^T = 100.00$	normalization
5. Constant non-tradable output	$Y^N = 244.23$	$\frac{Y^T}{Y^T + p_{ss}Y^N} = \theta_T = 29.05\%$
6. Elasticity of substitution between c^T and c^N	$\mu = 1.8750$	$\sigma_y = 2.95\%$
7. Weight of tradables in CES c aggregator	$\omega = 0.0659$	$p_{ss} = \frac{(1-\omega)}{\omega}\left(\frac{Y^T - rd_{ss}}{Y^N}\right)^{(1+\mu)} = 1$
8. Autocorrelation for z	$\rho = 0.6468$	OLS estimation
9. Std. dev. of the production shock z	$\sigma_\varepsilon = 4.3499$	OLS estimation
10. Output loss in state of default	$\lambda = 0.9750$	$avg\left(\frac{d_t}{Y_t^T + p_t Y^N}\right) \cong \theta_d = 28.34\%$

Table 5 shows the average results of 500 simulations of a time series of size 88, which is the number of quarterly observations for the 1980Q1−2001Q4 period. The simulated series are transformed according to the same procedure used in the actual data, as discussed in the previous section. In terms of the model, σ_c represents the volatility of (the log of) total consumption (in units of tradable goods) as given by $C_t = c_t^T + p_t c^N$. Notice that the comparison between the models for the constrained and unconstrained (perfect capital mobility) economies shows that the type of borrowing constraint used here has the effect of increasing the relative consumption volatility from 0.554 to 0.644, a 16 per cent increase. Considering that the average figure implied by the data from Table 1 is 30 per cent, one could conclude that the borrowing constraint used here is capable of accounting for 55 per cent of the difference in relative consumption volatility between emerging and developed economies.[18]

Table 5

Brazil - Output and Consumption Volatility Statistics			
	σ_y (%)	σ_c (%)	σ_c/σ_y
Brazil (1980Q1−2001Q4)	2.95	3.90	1.308
Model (constrained)	2.95	1.90	0.644
Model (unconstrained)	2.60	1.44	0.554

Although the model manages to increase the relative consumption volatility, it is not able to reproduce both the actual absolute and relative levels of consumption volatility, and cannot

[18] The constrained economy is calibrated for Brazil, rather than for an "average" of emerging economies. However, the observed values of σ_c/σ_y in Brazil and in the average of emerging economies are 1.30 and 1.27, respectively (see Table 1). At least in terms of the relative volatility of consumption, Brazil can be considered a typical representative of the group of emerging countries. In addition, as will become clear in the next subsection, the results are quite robust to a sensitivity analysis that tests different calibrations.

account for the fact that consumption is consistently more volatile than output. Neumeyer and Perri (2004) attribute this excess volatility of consumption to the dominant role played by interest rate shocks in these economies. In an economy that faces both income and interest rate volatility, consumption will be smoother than income if the transitory production shocks are dominant, and the opposite happens if, instead, the interest rate shocks are dominant. In this model, the absence of shocks that affect consumption independently of output, such as interest rate shocks, makes it impossible for consumption to fluctuate more than output. For instance, interest rate shocks affect the intertemporal decisions of consumption/savings and act on the consumption growth rate, but have only second-order effects on the production side (in a production economy, *ceteris paribus*, the main effect would be inducing a substitution of capital by labour). Aguiar and Gopinath (2004) explain the fact that $\sigma_c/\sigma_y > 1$ in emerging economies by adding permanent shocks to the growth rate of productivity. Since the model is not capable of accounting for the absolute volatility of consumption observed in the data from emerging economies, other sources of consumption volatility that should play a major role in emerging economies, while not playing much of a role in developed economies, are clearly missing here.[19]

The results of one particular simulation are shown in Figures 1 and 2, for the unfiltered and HP-filtered simulated series. Notice that the model is capable of generating a pro-cyclical behaviour for the consumption series (both tradable consumption and total consumption) as well as for the real exchange rate, as observed in the actual data from emerging economies (Arellano 2004). Also notice that the debt series in the constrained economy follows a similar path as in the unconstrained one, but at a lower level. This feature implies that the borrowing constraint affects the behaviour of the economy even when it is not binding. In terms of the supply of credits, the simple possibility of default means less credit to the small economy at all times. From the demand side, agents that consider the possibility of being credit constrained in the future will save more now (hence, less debt). The borrowing constraint will bind only when the cost of a bad production shock, in terms of reducing consumption today, is high enough to induce the agents to borrow until their limit.

Finally, it should be mentioned that the simulated average of the debt-to-output ratio for the sample is 28.35 per cent in the constrained economy, virtually identical to the actual average observed in Brazilian data. In addition, the level of the debt limit is such that it corresponds to 80.7 per cent of the simulated average GDP. Notice that this level is well above the maximal level for the debt-to-output ratio observed in Brazil, in the period 1980Q1−2001Q4 (47.02 per cent).

[19] These factors tend to be exogenously given. In order to properly assess the effect of the constraint alone, one would have to control for them anyway. The risk of not considering them is to miss some interactive effect between the exogenous factors and the endogenous borrowing constraint.

Figure 1: Simulated Series

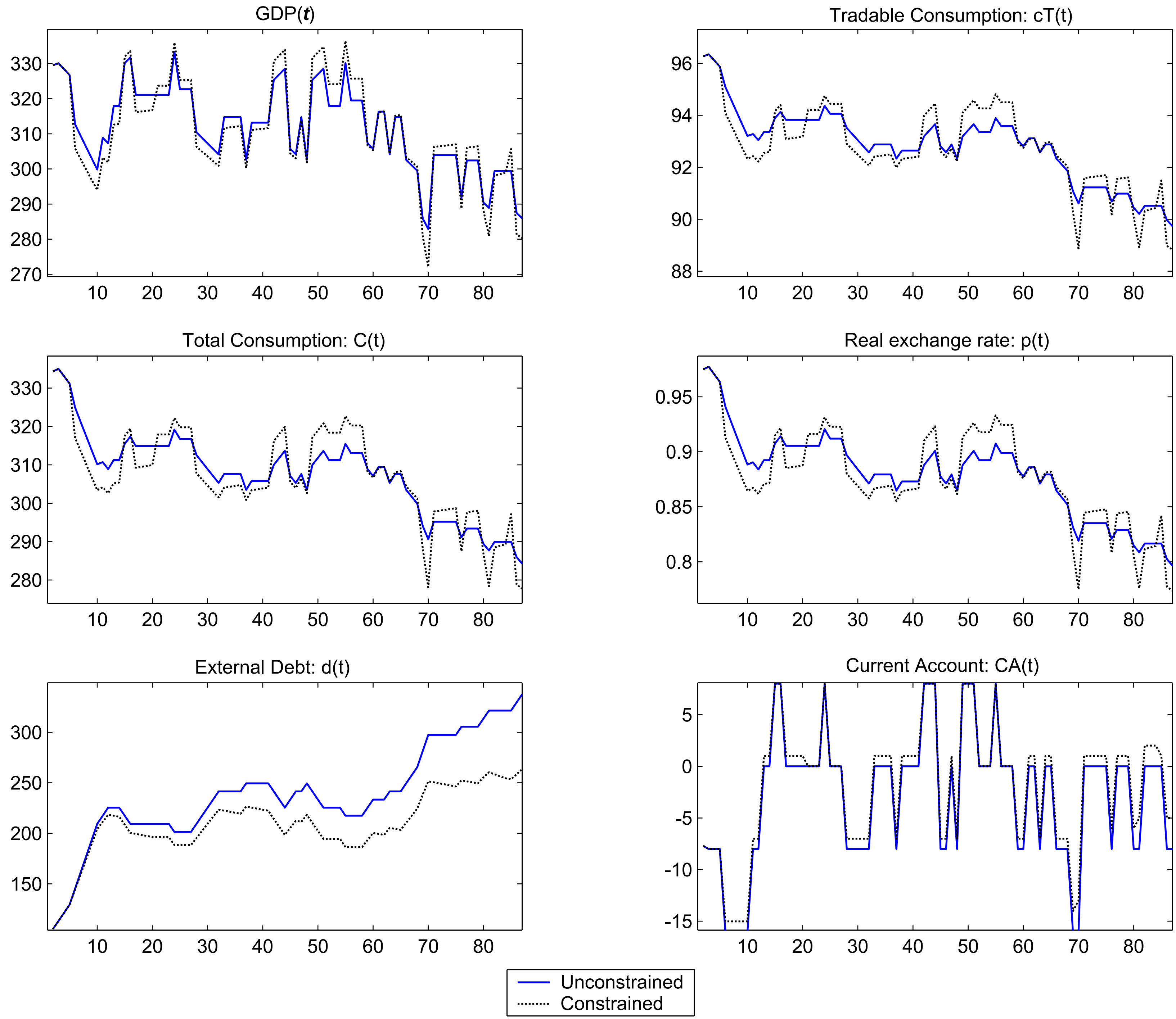
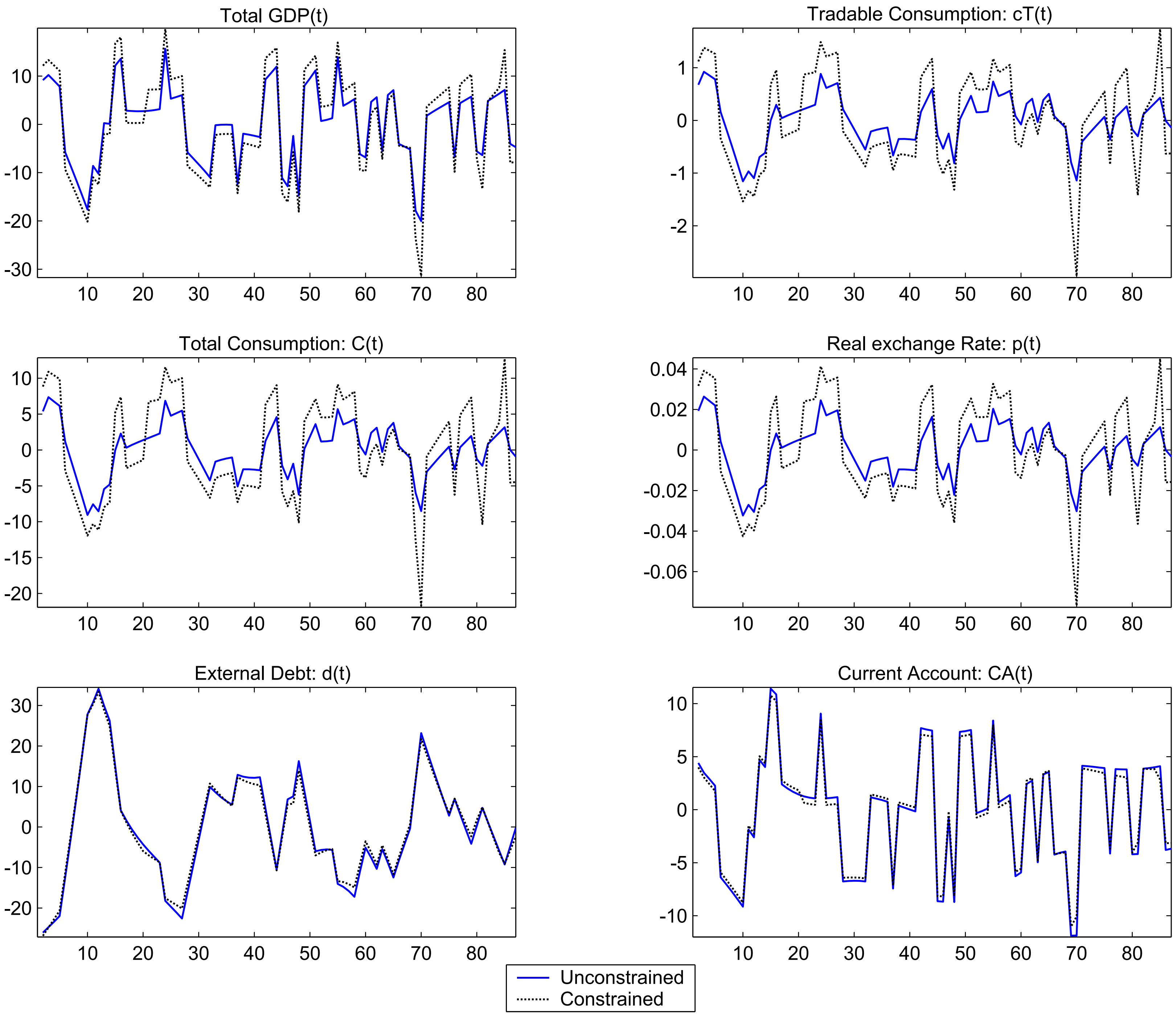


Figure 2: Simulated HP-Filtered Series (Cycle Component)



4.1 Sensitivity analysis

Information displayed in Tables 6, 7a, 8a, and 9a shows how the model for a constrained economy behaves under different values of the structural parameters. The columns in the tables, from left to right, provide information on the value of the relevant parameter (column 1), on the volatilities of output and consumption (columns 2 and 3, respectively), their ratio (column 4), the average level of debt as a percentage of the GDP (column 5), and the credit limit (column 6), both in level and as a percentage of the GDP (within parentheses). The tables also show the frequency at which the constraint binds (column 7) and a measure of the explaining power of the model (column 8). This measure of "success" is given by the proportion of the observed percentage difference in σ_c/σ_y from the data of emerging and developed economies (that is, the 30 per cent gap between $\sigma_c/\sigma_y = 1.27$ in emerging countries, and $\sigma_c/\sigma_y = 0.98$ in developed economies) that is accounted for by the percentage difference in the relative consumption volatility obtained from the simulated model for the constrained and unconstrained economies. Tables 7b, 8b, and 9b, in the appendix, show the results for the unconstrained economy.

Table 6

Constrained Model: Sensitivity to Changes in λ

λ	σ_y (%)	σ_c (%)	σ_c/σ_y	*avg d* (% *GDP*)	$\overline{d}$ (% *GDP*)	% bind	"success" (%)
0.9725	2.88	1.80	0.625	30.18	300.5 (88.08)	0.22	43.5
*0.9750	2.95	1.90	0.644	28.35	279.5 (80.67)	0.31	55.2
0.9775	3.03	1.99	0.657	26.14	255.4 (73.51)	0.40	62.9
0.9800	3.11	2.10	0.675	23.10	229.4 (65.73)	0.51	74.2
0.9825	3.19	2.20	0.690	19.41	203.4 (57.92)	0.61	83.0
0.9850	3.29	2.32	0.705	15.43	177.4 (50.13)	0.71	92.5
0.9900	3.54	2.64	0.746	5.78	123.3 (34.15)	0.88	117.3
1.0000	4.83	4.12	0.853	-12.53	9.14 (2.42)	1.85	182.9

Note: Column 6 shows the borrowing constraint both in level and, inside the parentheses, as a percentage of the GDP.
(*): this row shows the baseline case.

Table 6 shows how the model for the constrained economy behaves under different values of the parameter λ, which represent the indirect costs of default. The economic principle at work is based on changes in the cost/benefit of defaulting. Notice that the credit limit $\overline{d}$ falls (rises) with increases (decreases) in the value of λ. In order to understand why this happens, one should recall that a higher (lower) value of λ means that the output losses during default states are less (more) important, which reduces (increases) the penalty for staying out of international capital markets. Thus, the higher the parameter λ is, the more likely are the domestic agents to default (because it costs less), *ceteris paribus,* and the more likely it is to trigger a defensive response from the external creditors, who will have to reduce their maximal level of conceded credits to avoid default. On the other hand, as λ decreases, it becomes more costly for the country to default and foreign investors can relax the borrowing constraint without fearing default.

Notice that as λ increases, and the constraint becomes more stringent, both output and consumption become more volatile, although the effect is more important on consumption, since the ratio σ_c/σ_y consistently increases. The intuition behind this result is that a lower credit limit imposes additional difficulties to risk sharing and consumption smoothing, causing the consumption of tradables to be more volatile. A more volatile c_t^T, in turn, reflects on a more volatile real exchange rate through equation (8).[20] Since total consumption is defined as $C_t = c_t^T + p_t c^N$, the more volatile consumption of tradables increases total consumption volatility directly and indirectly, through its effect on p_t (the effects cannot cancel each other, since c_t^T and p_t are positively correlated). The same is not true for total output $Y_t = Y_t^T + p_t Y^N$, which only suffers the effect of the more volatile real exchange rate.

Table 6 also shows that a higher λ induces a lower average level of debt-to-output ratio (which eventually becomes negative for the extreme value $\lambda = 1.0$) and, at the same time, increases the frequency at which the borrowing constraint binds, suggesting that the effect of an increasing λ is more important on reducing the credit limit $\overline{d}$ than on decreasing the domestic agents' borrowing motivation. One should expect that, as $\overline{d}$ is reduced, with incomplete markets, risk-averse agents would save more (hold less debt), because the risk of being credit constrained in the future is higher the lower the credit limit is.

Finally, notice that the explanatory power of the model would be improved if a higher value of λ were used, although the target values for the output volatility and debt-to-output ratio would be missed.

Table 7a

Constrained Model: Sensitivity to Changes in γ

γ	σ_y (%)	σ_c (%)	σ_c/σ_y	*avg d (% GDP)*	$\overline{d}$ (% GDP)	% bind	"success" (%)
0.50	2.95	1.90	0.644	28.58	279.5 (80.70)	0.33	53.7
1.00	2.95	1.90	0.644	28.44	279.5 (80.68)	0.32	53.7
*1.50	2.95	1.90	0.644	28.35	279.5 (80.67)	0.31	55.2
2.00	2.95	1.90	0.644	28.26	279.5 (80.66)	0.30	52.5
2.50	2.96	1.90	0.642	28.17	279.5 (80.65)	0.29	51.1
3.00	2.96	1.90	0.642	28.11	279.5 (80.64)	0.29	51.1
4.00	2.96	1.90	0.642	27.87	279.5 (80.61)	0.26	51.1

Note: Column 6 shows the borrowing constraint both in level and, inside the parentheses, as a percentage of the GDP.
(*): this row shows the baseline case.

[20] Throughout the values of λ in Table 6, the volatilities of c^T and p rise monotonically from 0.7 per cent to 1.7 per cent and from 2.2 per cent to 5 per cent, respectively. This information is not displayed in the tables.

Table 7a shows that the results obtained for σ_y and σ_c in the benchmark (constrained economy) are relatively robust to changes in the coefficient of risk aversion, γ. In terms of the volatilities, observe that the results barely change (for σ_y) or are completely unchanged (σ_c) from the baseline case. The absolute value of the constraint, $\overline{d}$, is also the same. In addition, in terms of the "success" of the model in matching the data, no gain is possible by choosing alternative values for γ. There are a few changes, though. For instance, notice that as γ increases and agents become more risk-averse, given that markets are incomplete, they tend to save more or, equivalently, hold lower amounts of debt, since they become too scared of being credit constrained in the future. That explains why the average level of debt held by domestic agents falls with γ and, given that $\overline{d}$ remains unchanged, explains the reduction in the frequency at which the constraint is binding.

On the other hand, one should also expect that more risk-averse agents would be less inclined to default, *ceteris paribus*, since they tend to care more about risk sharing, and the cost of defaulting and being deprived of risk sharing in the future becomes higher. In that case, agents do not want to default unless they hold a large amount of debt and/or are hit by a bad enough production shock. Since the cost of default increases for the country, the external investors may relax the credit limit and still receive back the conceded loans. Conversely, if agents have low risk aversion, then they do not care very much about risk sharing in the future, which means that not paying back the debt becomes relatively attractive, forcing the external investors to make the borrowing constraint more stringent to avoid default. However, for the range of values of γ considered in Table 7a, this effect is not quantitatively important and the level of $\overline{d}$ turns out to be constant. In terms of $\overline{d}$ as a percentage of the average GDP, the observed reduction is explained as follows. A lower level of (average) debt induces a higher level of average consumption of tradables, which can be fairly approximated by $avg\left(c^T\right) \approx Y^T - r.avg\left(d\right)$, provided that $avg\left(d\right) \approx \theta_d\left[Y^T + avg\left(p\right)Y^N\right]$ and (μ, ω) satisfy $avg\left(p\right) \approx 1$, as is the case. A higher average level of c^T combined with an inelastic (here, constant) level of c^N, in turn, means a higher average relative price of non-tradable goods, p (see equation (8)). The consequence of this appreciation of the real exchange rate is a higher level of total GDP in units of tradable goods, which explains why the constant level of $\overline{d}$ falls as a percentage of the average GDP as γ increases. The fact that the borrowing constraint is not very sensitive to changes in γ while the average level of debt decreases explains why the borrowing constraint binds less frequently as γ rises.

Table 8a displays the sensitivity analysis to changes in the weight of the tradable good in the CES consumption aggregator, ω. One could think of two opposite effects of ω in terms of the incitation to default. Since a higher ω increases the marginal utility of the consumption of tradable goods at all times, first, there would be higher instantaneous gain from default because, in that event, the country would be able to consume more of a good (tradables) that has a higher weight on the consumption index. On the other hand, intertemporally, there would be a higher cost of default by the same motive (one could also think that a higher ω makes the agent care more about risk sharing, since the "insurable" part of the agent's consumption becomes more important for their utility). Again, higher benefits of default induce external agents to reduce the level of maximal credit available to the country and higher costs of default make the constraint less stringent. Thus, the first effect would reduce the level of $\overline{d}$, while the second effect would increase it. Notice that, since the level of $\overline{d}$ falls (although it increases as a percentage of the GDP because of a real depreciation that more than proportionally reduces the level of the average GDP in units of tradable goods) as ω increases, the quantitative relevance of the instantaneous benefits seems to dominate the intertemporal costs of default.

Table 8a

Constrained Model: Sensitivity to Changes in ω

ω	σ_y (%)	σ_c (%)	σ_c/σ_y	*avg d (% GDP)*	$\overline{d}$ (% GDP)	% bind	"success" (%)
0.0100	1.88	1.71	0.910	6.86	658.0 (36.38)	0.00	0.00
*0.0659	2.95	1.90	0.644	28.35	279.5 (80.67)	0.31	55.2
0.1000	3.42	1.98	0.579	28.63	215.4 (83.22)	0.56	99.2
0.2500	4.35	1.81	0.416	17.35	133.3 (86.04)	0.90	216.7
0.5000	4.97	1.53	0.308	6.27	104.3 (87.88)	0.98	297.4
0.7500	5.27	1.35	0.256	0.76	94.3 (88.69)	1.02	327.3
0.9900	5.44	1.24	0.228	-2.75	89.2 (89.05)	1.03	349.8

Note: Column 6 shows the borrowing constraint both in level and, inside the parentheses, as a percentage of the GDP.
(*): this row shows the baseline case.

The effects of the constraint are very clear if one compares the sensitivity of the model to changes in ω in the constrained (Table 8a) and unconstrained (Table 8b, in the appendix) economies. Notice that, at the very low value $\omega = 0.01$, the two economies are virtually identical, since tradable consumption has a very small impact on the consumption index and the borrowing constraint is set at a very high level, as discussed in the previous paragraph. The level of $\overline{d}$ is high enough to imply a very low frequency at which the constraint is binding, which makes the two models very close in behaviour. Numerically, in the simulations, this frequency is zero, for two decimal places, although it is likely that a high-enough number of simulations would show some cases in which the constraint binds, since, theoretically, the two models are still different.

However, as ω rises, interesting differences show up regarding the constrained and the unconstrained economies. First, notice that the volatility of output departs from 1.88 per cent and rises in both economies, but it increases more rapidly in the constrained case. The intuition of this result is the following: since $Y_t = Y_t^T + p_t Y^N$, the volatility of output depends on the (exogenous) volatility of Y_t^T, as well as on the (endogenous) volatility of p_t and the (also endogenous) covariance between the two, $cov\left(Y_t^T, p_t\right) > 0$. In the unconstrained economy, the volatility of p_t (not shown in the tables) is almost insensitive to changes in ω (it goes from 1.76 per cent to 1.75 per cent as ω changes from 0.01 to 0.99), and the volatility of Y_t^T is exogenously given. Thus, the only way that Y_t can become more volatile is through increases in $cov\left(Y_t^T, p_t\right)$, possibly due to the fact that the proportion of tradables in total consumption and total GDP increases with ω. In the constrained economy, on the other hand, on top of the effect described above, the volatility of p_t rises (from 1.77 per cent to 3.56 per cent as ω goes from 0.01 to 0.99), rather than stay constant, which explains the sharper increase in σ_y verified in Table 8a in comparison with Table 8b.

The rising volatility of p_t in the constrained economy in response to changes in ω, while constant in the unconstrained economy, is certainly an effect of the borrowing constraint that becomes even more stringent with increases in ω, and it makes tradable consumption smoothing more difficult. Not surprisingly, the same happens with the volatility of c_t^T (constant at 0.61 per cent in the unconstrained economy and rising from 0.61 per cent to 1.24 per cent in the constrained economy, as ω changes in Table 8a). Recall that, since the same standard procedure for business cycle statistics is being used here (in particular, the variables are treated in logarithmic scale), the ratio between the volatilities of p_t and c_t^T has to be equal to $(1 + \mu) = 2.875$.[21]

A second difference observed in Tables 8a and 8b, for the constrained and unconstrained economies, is that the volatilities of total consumption are identical in both economies for $\omega = 0.01$, but, similar to what happens with σ_y, they become different as ω rises. In the unconstrained economy, σ_c falls monotonically with increases in ω, while in the constrained economy there is an initial phase in which σ_c rises. In the case of an unconstrained economy, the monotonic fall in σ_c is purely mechanical, a consequence of the reduction of the term $(1 - \omega)/\omega$. Note that, since $C_t = c_t^T + p_t c^N$ and p_t is given by equation (8), one can write:

$$C_t = c_t^T + \left[\frac{(1-\omega)}{\omega}\left(c^N\right)^{-\mu}\right]\left(c_t^T\right)^{(1+\mu)},$$

and, as ω goes from 0 to 1, the term $(1 - \omega)/\omega$ goes from infinity to zero and the volatility of total consumption converges (falls) to the volatility of tradable consumption, which does not change with ω, as discussed above. That is also the reason for the more depreciated real exchange rate (lower p_t) that follows from the increase in ω (see equation (8)). The same effects occur in the constrained case, with the important difference that, because the constraint becomes more stringent with a rising value of ω, tradable consumption volatility increases sharply. The net effect on σ_c depends on the relative importance of these direct and indirect effects (through increases in tradable consumption volatility) induced by a rise in ω. The direct effect makes σ_c fall, while the indirect

[21] For instance, up to a rounding error effect (the values are presented with only two decimal places):

$$\frac{1.76\%}{0.61\%} \simeq \frac{1.75\%}{0.61\%} \simeq \frac{1.77\%}{0.61\%} \simeq \frac{3.56\%}{1.24\%} \simeq 2.875.$$

effect acts in the opposite direction. It seems that the indirect effect dominates for small values of ω (up to 0.1 in Table 8a) and, as $(1-\omega)/\omega$ converges to zero, for higher values of ω, the direct effect becomes more important and forces σ_c down.

In terms of the effects of different values of ω, a final difference between the constrained and unconstrained economies is the behaviour of the average level of debt-to-output ratio. Since a higher ω makes tradable consumption more important for the CES consumption aggregator index and for utility, it makes the representative agent attach more importance to risk sharing at all times. If markets were complete, this would probably not affect the agent's total savings, since there would be complete risk sharing and a reallocation of contingent assets would occur without important effects on total savings. However, with no contingent assets, agents more concerned with risk sharing will tend to save more for self-insurance. In fact, in both the constrained and unconstrained economies, the average *level* (not shown in Table 8a) of debt falls.

In the unconstrained economy, where there is no risk of a shortage of credits, the average level of debt falls by 11 per cent (124 to 110.6), but in the constrained economy, where the risk of becoming credit constrained is real, and increasing with ω, the average level of debt falls from the same 124 as in the unconstrained economy to -2.75, and the agent becomes a net creditor.

In terms of the debt-to-output ratio, in the constrained economy, the fall in the level of debt is less than proportional to the fall in the value of the GDP for lower values of ω, and the debt-to-output ratio actually rises. But for $\omega \geq 0.1$, the higher motivation for savings dominates the real depreciation, debt falls quicker than GDP, and the opposite occurs. In the unconstrained economy, since there is no risk of being credit constrained, the fall in debt is smoother and the effects of the real depreciation on total GDP always dominate, which makes the debt-to-output ratio grow monotonically with ω.

Table 9a displays the sensitivity of the model to changes in the elasticity of substitution between c^T and c^N. The most obvious effect of an increase in μ,[22] which means that c^T and c^N tend to work more as complements than as substitutes, is a rise in the volatility of p_t for a given volatility of tradable consumption, according to equation (8). For a given volatility of tradable consumption, a lower elasticity of substitution between c^T and c^N implies a lower percentage variation in c^N/c^T for a given percentage change in p, or, alternatively, that a higher proportional change in p is required for a given change in the consumption of tradable, relative to the consumption of non-tradable, goods. Notice that, as μ rises, both σ_y and σ_c increase as a consequence of the higher volatility of

[22] The results for $\mu = 1.0$ and $\mu = -0.25$ are particularly important, because they represent a possible alternative for the calibration procedure, if one wants to consider values of μ close to those implied by the estimates of the elasticity of substitution between c^T and c^N used in Arellano (2004) and Mendoza and Uribe (1999). Arellano relies on estimation of the elasticity of substitution between tradable and non-tradable consumption for Argentina by Gonzales-Rosada and Neumeyer (2003), who find it to be 0.48, implying $\mu = 1.0833$. Mendoza and Uribe (1999) use $\mu = -0.218$, the same value as used in Mendoza (1995), which implies an elasticity of substitution of 1.28. Needless to say, the existence of empirical studies that provide estimations of μ that are lower than the value used in the baseline case is an important caveat for the results of this paper.

the real exchange rate. At first, for lower values of μ, the effect on σ_c is stronger than that on σ_y, and σ_c/σ_y rises, but the inverse occurs after $\mu \geq 0.01$.[23]

Table 9a

Constrained Model: Sensitivity to Changes in μ

μ	σ_y (%)	σ_c (%)	σ_c/σ_y	*avg d* (% *GDP*)	$\bar{d}$ (% *GDP*)	% bind	"success" (%)
-0.750	0.31	0.17	0.548	3.89	641.9 (22.66)	0.00	0.0
-0.250	0.67	0.47	0.701	6.17	641.9 (35.23)	0.00	0.0
0.010	0.86	0.62	0.721	7.81	641.9 (43.93)	0.00	0.0
0.250	1.06	0.75	0.708	9.67	651.9 (54.74)	0.00	0.0
0.500	1.26	0.88	0.698	12.02	670.9 (69.42)	0.00	0.0
1.000	1.73	1.16	0.671	18.18	493.7 (76.35)	0.00	5.0
1.500	2.37	1.53	0.646	25.02	351.6 (78.89)	0.09	23.0
*1.875	2.95	1.90	0.644	28.35	279.5 (80.67)	0.31	55.2
2.000	3.14	2.00	0.637	28.87	260.5 (81.26)	0.37	61.9
3.000	4.39	2.47	0.563	24.35	163.3 (84.91)	0.78	140.9
4.000	5.12	2.37	0.463	14.78	121.3 (87.33)	0.95	222.1
5.000	5.42	2.02	0.373	6.82	102.3 (89.72)	1.02	283.9

Note: Column 6 shows the borrowing constraint both in level and, inside the parentheses, as a percentage of the GDP.

(*): this row shows the baseline case.

As in the case of changes in ω, there are two effects caused by variations in μ, one instantaneous and the other intertemporal. The relative importance of how the changing μ will affect the two effects will ultimately determine what happens with the level of the borrowing constraint. For instance, if the two goods are substitutes (low μ), then risk sharing is relatively less important at all times because, when facing a bad tradable output shock, agents can always substitute away their tradable consumption for non-tradable consumption. Thus, the instantaneous gain in terms of a higher tradable consumption in case of default is reduced with reductions in μ. However, since this substitution is also possible in the future, the intertemporal cost of default is also reduced. The opposite occurs when μ rises: the instantaneous benefits are higher and, also, the intertemporal costs of default are higher, since substitutability between the two goods becomes weak and a bad tradable output shock hurts more at all times. Notice, in Table 8a, that the intertemporal effect

[23] Notice that, since:

$$C_t = c_t^T + \left[\frac{(1-\omega)}{\omega}\left(c^N\right)^{-\mu}\right]\left(c_t^T\right)^{(1+\mu)},$$

and

$$Y_t = Y_t^T + \left[\frac{(1-\omega)}{\omega}\left(c^N\right)^{-\mu}\right]\left(c_t^T\right)^{(1+\mu)},$$

the absolute effects of μ are the same in both σ_c and σ_y, given the volatilities of c_t^T and Y_t^T. However, the percentage increase depends on the relative share of the volatilities of c_t^T and Y_t^T, respectively, on σ_c and σ_y.

dominates for lower values ($\mu \leq 0.5$) and, as μ increases, the borrowing constraint, $\overline{d}$, becomes less stringent. For $\mu \geq 0.5$, on the other hand, the benefits of default increase faster than the costs, and external investors have to reduce the credit limit to avoid default.

The borrowing constraint as a percentage of the average GDP is monotonically increasing with μ, even when the borrowing constraint becomes more stringent. Again, the reason for this is a sharp real depreciation that follows the increase in μ, which causes the GDP in units of tradable goods to fall more than proportionally to the fall in $\overline{d}$. This real depreciation is a consequence of the fact that non-tradable consumption is constant in equilibrium and the two goods tend to become complements, as μ increases. With low values of μ and higher substitution between the two goods, given that non-tradable output and consumption are constant, the relative scarcity of tradable goods is reduced, which requires a lower price of tradables relative to non-tradables (that is, p has to rise); the opposite (i.e., real depreciation; a fall in p) happens for high values of μ.

As the value of μ rises, the *level* of the average debt increases initially and falls afterwards (this information is not displayed in Table 9a). For $\mu \leq 1.0$, the debt level rises by 6.2 per cent (from 110.7 to 117.6) as μ goes from -0.75 to 1.0. For values of μ that are higher than 1.0, the level of debt falls by 93.3 per cent (from 117.6 to 7.9) as μ goes from 1.0 to 5.0. This result is a consequence of the effect that μ has on the borrowing constraint, $\overline{d}$. While μ is still low, and the borrowing constraint becomes less stringent as μ rises, agents that are risk-averse and fear being credit constrained will save less, because $\overline{d}$ is too high. Actually, this explains why the constraint does not bind at low values of μ and, also, why the constrained and unconstrained economies are virtually the same for values of μ that are lower than 1.0 (the constraint is so loose that, numerically, the two economies behave almost the same). However, as μ increases and the constraint becomes more stringent, the risk of being credit constrained increases and agents will tend to start saving more, reducing their debt.

In terms of the debt-to-output ratio, the initial increase is due both to the rise in the average level of debt and to the reduction in the value of total GDP in units of tradables that follows the real depreciation. The fall observed for $\mu \geq 2.0$ is explained by the fact that the level of debt decreases more than proportionally to the fall in the value of GDP.

5. Conclusion

This paper presented empirical evidence of higher relative consumption volatility (to output volatility) experienced by emerging economies compared with developed small open economies. The data indicate that emerging economies have 30 per cent more relative consumption volatility than small open developed economies, and this difference is statistically significant. Using a dynamic-general equilibrium model of an endowment, two-goods, small open economy subject to an endogenous borrowing constraint, the paper suggests that the constraint alone, although having limited explanatory power on the relative consumption volatility differential, is able to increase the relative consumption volatility by 16.3 per cent, which corresponds to more than 55 per cent of the gap observed in the data from emerging (likely to be constrained) and small developed open economies.

The model does relatively well, quantitatively, in explaining the empirical evidence discussed here and, qualitatively, in a number of dimensions such as the pro-cyclical movements of consumption and the real exchange rate, as mentioned in the previous section. However, the model does not perform well in other aspects. For example, it is not able to reproduce actual levels of absolute output and consumption volatilities, nor is it capable of explaining the fact that consumption is consistently more volatile than output in emerging economies. Also, since there is no investment or production in the model, any positive production shock translates into an amelioration of the current account, since only the consumption-smoothing mechanism is at work and the investment motive does not exist. Future extensions of this paper intend to address those matters.

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Appendix

Tables 7b, 8b, and 9b display information about the sensitivity analysis of the model for the unconstrained economy.

Table 7b

Unconstrained Model: Sensitivity to Changes in γ

γ	σ_y (%)	σ_c (%)	σ_c/σ_y	$avg\ d$ (% GDP)
0.50	2.59	1.44	0.556	34.95
1.00	2.59	1.44	0.556	34.93
*1.50	2.60	1.44	0.554	34.91
2.00	2.60	1.45	0.558	34.89
2.50	2.60	1.45	0.558	34.88
3.00	2.60	1.45	0.558	34.87
4.00	2.60	1.45	0.558	34.83

Table 8b

Unconstrained Model: Sensitivity to Changes in ω

ω	σ_y (%)	σ_c (%)	σ_c/σ_y	$avg\ d$ (% GDP)
0.0100	1.88	1.71	0.910	6.86
*0.0659	2.60	1.44	0.554	34.91
0.1000	2.97	1.33	0.448	46.51
0.2500	4.02	1.02	0.254	75.75
0.5000	4.82	0.79	0.164	95.86
0.7500	5.22	0.68	0.130	105.22
0.9900	5.44	0.61	0.112	110.41

Table 9b

Unconstrained Model: Sensitivity to Changes in μ

μ	σ_y (%)	σ_c (%)	σ_c/σ_y	$avg\ d$ (% GDP)
-0.750	0.31	0.17	0.548	3.89
-0.250	0.66	0.47	0.712	6.17
0.010	0.86	0.62	0.721	7.81
0.250	1.06	0.75	0.708	9.67
0.500	1.26	0.88	0.698	12.02
1.000	1.71	1.13	0.661	18.27
1.500	2.20	1.33	0.605	26.86
*1.875	2.60	1.44	0.554	34.91
2.000	2.73	1.47	0.539	37.86
3.000	3.80	1.51	0.397	63.92
4.000	4.65	1.30	0.280	86.65
5.000	5.13	1.04	0.203	100.37

(*): this row shows the baseline case.

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